Algorithm 1: The AEKF Algorithm

Input: $u[k] = I[k],\ k \in \{k_0, \dots N-1\}$

Output: $\widehat{SOC}[k],\ k \in \{k_0, \dots N-1\}$

Data: $y[k] = U_t^{exp}[k],\ k \in \{k_0, \dots N\}$, $R_0[k]$, $R_1[k]$, $C_1[k]$, $R_2[k]$, $C_2[k]$

Step 1: Initialization:

$$\hat{\boldsymbol{x}}[k_0-1]^+ = E\Big[\boldsymbol{x}[k_0-1]\Big],\ \hat{\boldsymbol{P}}[k_0-1]^+ = E\Bigg[\bigg(\boldsymbol{x}[k_0-1] - E\Big[\hat{\boldsymbol{x}}[k_0-1]^+\Big]\bigg)\bigg(\boldsymbol{x}[k_0-1] - E\Big[\hat{\boldsymbol{x}}[k_0-1]^+\Big]\bigg)^T\Bigg]$$

Step 2: Calculation:

for $\underline{k = k_0 \to N}$ **do**

- **State Estimation Propagation:** $\hat{\boldsymbol{x}}[k]^- = f\Big(\hat{\boldsymbol{x}}[k-1]^+, u[k]\Big)$ (look 27)
- **State Estimation Covariance:** $\boldsymbol{P}[k]^- = \boldsymbol{A_d}[k]\boldsymbol{P}[k-1]\boldsymbol{A_d}[k]^T + \boldsymbol{Q}[k-1]$
- **Error Innovation:** $e[k] = y[k] - h\Big(\hat{\boldsymbol{x}}[k]^-, u[k]\Big)$
- **Adaptive Law:** $H[k] = \dfrac{1}{M}\sum_{i=k-M+1}^{k} e[k]^2,\ R[k] = H[k] - \boldsymbol{C}[k]\boldsymbol{P}[k]^-\boldsymbol{C}[k]^T$
- **Kalman Gain Matrix:** $\boldsymbol{K}[k] = \boldsymbol{P}[k]^-\boldsymbol{C}[k]^T\Big(\boldsymbol{C}[k]\boldsymbol{P}[k]^-\boldsymbol{C}[k]^T + R[k]\Big)^{-1}$
- **State Estimate Measurement Update:** $\hat{\boldsymbol{x}}[k]^+ = \hat{\boldsymbol{x}}[k]^- + \boldsymbol{K}[k]e[k]$
- **State Covariance Measurement Update:** $\boldsymbol{Q}[k] = \boldsymbol{K}[k]H[k]\boldsymbol{K}[k]^T$,

  $$\boldsymbol{P}[k]^+ = \Big(\boldsymbol{I} - \boldsymbol{K}[k]\boldsymbol{C}[k]\Big)\boldsymbol{P}[k]^-\Big(\boldsymbol{I} - \boldsymbol{K}[k]\boldsymbol{C}[k]\Big)^T + \boldsymbol{K}[k]\boldsymbol{R}[k]\boldsymbol{K}[k]^T$$

  /* $\boldsymbol{A}_d[k] = \dfrac{\partial f(\boldsymbol{x}[k], u[k])}{\partial \boldsymbol{x}}\Big|_{x=\hat{x}[k]^-}$, $\boldsymbol{C}[k] = \dfrac{\partial h(\boldsymbol{x}[k], u[k])}{\partial \boldsymbol{x}}\Big|_{x=\hat{x}[k]^-}$ */