Parameter Identification and State Estimation of an Battery by Using Second Order Equivalent Circuit Model

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Contents of the Presentation

Introduction

For advanced battery management system (BMS) of electric vehicles (EVs)

- Accurate identification of the key state parameters
- State of Charge (SOC) estimation method

Providing an nearly accurate model and SOC framework is of a great significant in

- Real-time control of battery
- High performance operation
- Diagnosis and Prognosis of a battery behavior

Introduction

The knowledge of the internal battery parameters is mainly required for

1. Estimation of the energy losses in the battery during the operation
2. SOC estimation based on electrical models
3. Prediction of the available power of the battery

Equivalent Second-Order Circuit Model (ECM)

Second-order RC model has the moderate precision and is more suitable for the voltage estimation of battery cell.

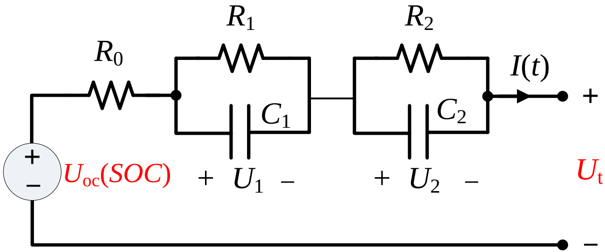


Figure 1: Schematic diagram of the second-order RC model.

where $R_0 = R_\Omega$ is ohmic resistances. R_1 and C_1 electro-chemical polarization RC parameters. R_2 and C_2 represents the concentration polarization RC parameters.

Dynamic Equations of the Second Order ECM

The electrical behavior of the second-order RC battery model:

$$\dot{U}_1(t) = -\frac{1}{R_1C_1}U_1(t) + \frac{1}{C_1}I(t) \tag{1a}$$

$$\dot{U}_2(t) = -\frac{1}{R_2C_2}U_2(t) + \frac{1}{C_2}I(t) \tag{1b}$$

$$U_t(t) = U_{oc}(SOC)(t) - U_1(t) - U_2(t) - I(t)R_0 \tag{1c}$$

Mathematical relations involving SOC in continuous time domain:

$$SOC(t) = SOC(t_0) - \frac{1}{Q}\int_{t_0}^{t} \eta I(\tau)d\tau \quad \text{or} \quad S\dot{O}C(t) = -\frac{\eta I(t)}{Q} \tag{2}$$

in discrete time

$$SOC[k+1] = SOC[k] - \left(\frac{\eta T}{Q}\right)I[k] \tag{3}$$

The LTI State Space Representation of the ECM

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}u(t) \\ h(t) = y(t) - U_{OC}(SOC(t)) = \boldsymbol{C}\boldsymbol{x}(t) + Du \end{cases} \tag{4}$$

where $\boldsymbol{A} = \begin{bmatrix} -\frac{1}{R_1C_1} & 0 & 0 \\ 0 & -\frac{1}{R_2C_2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\boldsymbol{B} = \begin{bmatrix} \frac{1}{C_1} \\ \frac{1}{C_2} \\ -\frac{\eta}{Q} \end{bmatrix}$, $\boldsymbol{C} = \begin{bmatrix} -1 & -1 & 0 \end{bmatrix}$, $D = R_0$. State vector of the state space system is denoted by $\boldsymbol{x} = [U_1 \; U_2 \; SOC]^T$ and $u(t) = I(t)$ indicates the input of the battery system and $y(t) = U_t(t) - U_{OC}(SOC)$ is the output.

SOC - OCP Relationship

The accurate relationship between SOC and OCV:

1. Great influence on the estimated SOC value
2. Necessary for reasonable parameter identification

This relation in the experimental data is interpreted as:

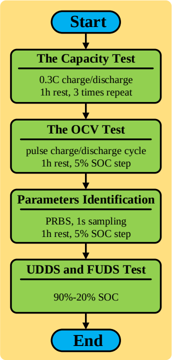


Figure 2: Battery cell test scheme. [1]

[1] F. Wen, B. Duan, C. Zhang, R. Zhu, Y. Shang, and J. Zhang (2019). "High-Accuracy Parameter Identification Method for Equivalent-Circuit Models of Lithium-Ion Batteries Based on the Stochastic Theory Response

SOC - OCP Relationship

Charging and discharging with C/3 capacity rate is taken into consideration for OCV-SOC calculation

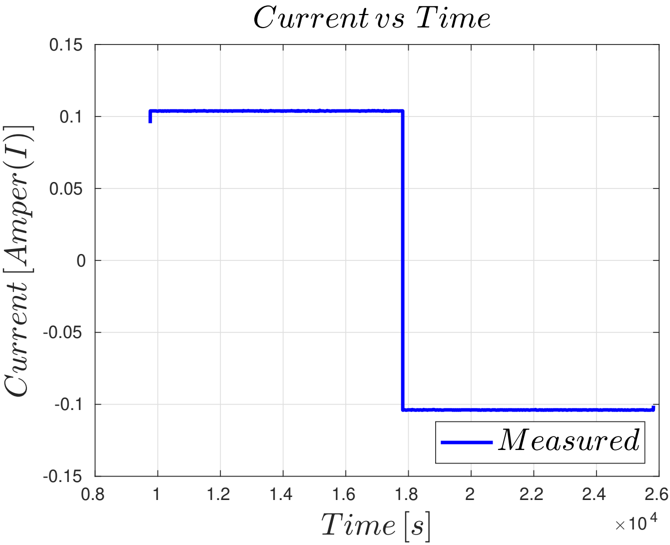


Figure 3: Current measurement to obtain OCV-SOC Relation.

SOC - OCP Relationship

Charging and discharging with C/3 capacity rate is taken into consideration to OCV-SOC relation:

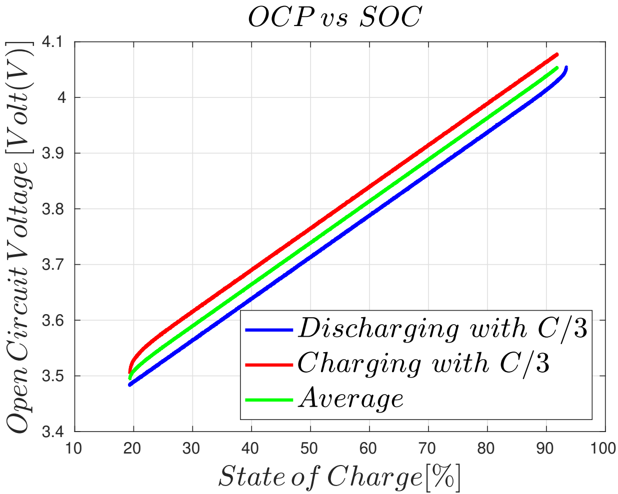


Figure 4: Experimental SOC-OCV mapping when battery is charged/discharged with $C/3$ capacity. .

SOC - OCP Models

#	Reference	RMS Error (mV)
1	[2]	0.4798
2	[3]	0.2431
3	[4]	0.3060

Table 1: Compared fitting results of OCV models.

#1: $U_{OC}(s) = a + b \times (-ln(s))^m + c \times s + d \times \exp^{n(s-1)}$
#2: $U_{OC}(s) = K_0 - \frac{K_1}{s} - K_2 s + K_3 ln(s) + K_4 ln(1-s)$
#3: $U_{OC}(s) = K_0 + K_1 \exp^{-\alpha(1-s)} - \frac{K_2}{s}$

[2] C. Zhang, J. Jiang, L. Zhang, S. Liu, L. Wang, and P. C. Loh (2016). "A Generalized SOC-OCV Model for Lithium-Ion Batteries and the SOC Estimation for LNMCO Battery". In: *Energies* 9.11. ISSN: 1996-1073

[3] G. L. Plett (2004). "Extended Kalman filtering for battery management systems of LiPB-based HEV battery packs: Part 2. Modeling and identification". In: *Journal of Power Sources* 134.2, pp. 262–276. ISSN: 0378-7753. DOI: https://doi.org/10.1016/j.jpowsour.2004.02.032

[4] D. E. Neumann and S. Lichte (2011). "A multi-dimensional battery discharge model with thermal feedback applied to a lithium-ion battery pack". In: *2011 NDIA GROUND VEHICLE SYSTEMS ENGINEERING AND TECHNOLOGY SYMPOSIUM*

Classification of the Identification Methods

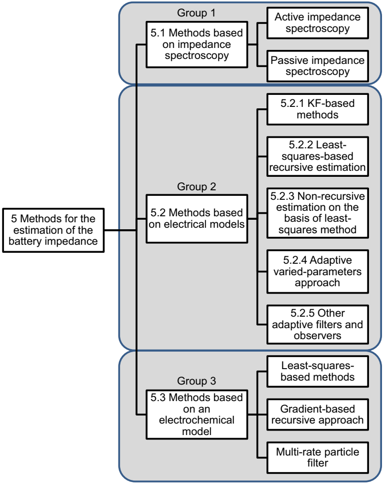


Figure 5: Classification of the methods for the estimation of the battery parameters.[5]

[5] W. Waag, C. Fleischer, and D. U. Sauer (2014). "Critical review of the methods for monitoring of lithium-ion batteries in electric and hybrid vehicles". In: *Journal of Power Sources* 258, pp. 321–339. ISSN: 0378-7753. DOI:

Preliminary Work

Before starting the parameter identification, continuous time dynamic equation in 4, i.e. $\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}u(t)$, must be discretized.

$\boldsymbol{x} = [U_1 \; U_2]^T$, and state transition matrix becomes $\boldsymbol{A} = \begin{bmatrix} -\frac{1}{R_1 C_1} & 0 \\ 0 & -\frac{1}{R_2 C_2} \end{bmatrix}$.

Input matrix denoted as $\boldsymbol{B} = \begin{bmatrix} \frac{1}{C_1} \\ \frac{1}{C_2} \end{bmatrix}$.

$$e^{-\boldsymbol{A}t}\dot{\boldsymbol{x}}(t) - e^{-\boldsymbol{A}t}\boldsymbol{A}\boldsymbol{x}(t) = \frac{d}{dt}\left(e^{-\boldsymbol{A}t}\boldsymbol{x}(t)\right) = e^{-\boldsymbol{A}t}\boldsymbol{B}u(t) \tag{5}$$

Solution is,

$$\boldsymbol{x}(t) = e^{\boldsymbol{A}t}\boldsymbol{x}(0) + e^{\boldsymbol{A}t}\int_0^t e^{-\boldsymbol{A}\tau}\boldsymbol{B}u(\tau)d\tau \tag{6a}$$

$$\boldsymbol{x}(t) = e^{\boldsymbol{A}t}\boldsymbol{x}(0) + \int_0^t e^{\boldsymbol{A}(t-\tau)}\boldsymbol{B}u(\tau)d\tau \tag{6b}$$

Preliminary Work cont'd

A continuous time signal $\{x(t)\}$ can be obtained from a discrete time (DT) signal $x[k]$:

$$x(t) = x[k], \quad kT \leq t < (k+1)T \tag{7}$$

$$\begin{aligned} \boldsymbol{x}[k+1] &= \boldsymbol{A_d}\boldsymbol{x}[k] + \boldsymbol{B_d}u[k] \\ y[k] &= \boldsymbol{C_d}\boldsymbol{x}[k] + D_d u[k] \end{aligned} \tag{8}$$

Starting from the solution of the continuous state-space equation 6

$$\boldsymbol{A_d} = e^{\boldsymbol{A}T} \quad \text{and} \quad \boldsymbol{B_d} = \left(\int_0^T e^{A\tau} d\tau \right) \boldsymbol{B} \tag{9}$$

It should be note that that this is the *exact* solution to the differential equation.

Preliminary Work cont'd

The discrete time dynamic equation can be explicity written as:

$$\boldsymbol{x}[k+1] = \underbrace{\begin{bmatrix} e^{-\frac{Tt}{\tau_1}} & 0 \\ 0 & e^{-\frac{T}{\tau_2}} \end{bmatrix}}_{\boldsymbol{A}_d} \boldsymbol{x}[k] + \underbrace{\begin{bmatrix} R_1(1 - e^{\frac{T}{\tau_1}}) \\ R_2(1 - e^{\frac{T}{\tau_2}}) \end{bmatrix}}_{\boldsymbol{B}_d} u[k] \tag{10}$$

where state vector is of the form $\boldsymbol{x}[k] = \big[U_1[k] \; U_2[k]\big]^T$, and input $u[k]$ is the measured current at each sampling time $I[k]$. And the continuous time matrices $\boldsymbol{C} = \boldsymbol{C_d}$ and $D = D_d$ remain same in discrete time.

$$\begin{aligned} U_1[k+1] &= U_1[k]e^{-\frac{T}{\tau_1}} + I[k]R_1\Big(1 - e^{\frac{T}{\tau_1}}\Big) \\ U_2[k+1] &= U_2[k]e^{-\frac{T}{\tau_2}} + I[k]R_2\Big(1 - e^{\frac{T}{\tau_2}}\Big) \\ U_t[k] &= U_{OC}\big[SOC[k]\big] - I[k]R_0 - U_1[k] - U_2[k] \end{aligned} \tag{11}$$

Parameter Identification Based On Recursive Least Square[6]

The transfer function of the second-order equivalent circuit model can be written as:

$$G(s) = \frac{U_t(s) - U_{oc}(s)}{I(s)} = -\left(R_0 + \frac{R_1}{1 + \tau_1 s} + \frac{R_2}{1 + \tau_2 s}\right) \tag{12}$$

Transformation from L (Laplace) domain to z domain based on bilinear transformation:

$$s = \frac{2(1 - z^{-1})}{T(1 + z^{-1})} \tag{13}$$

$$G(z^{-1}) = \frac{\theta_3 + \theta_4 z^{-1} + \theta_5 z^{-2}}{1 - \theta_1 z^{-1} - \theta_2 z^{-2}} \tag{14}$$

where parameters $\theta_i (i = 1, 2, ..., 5)$ being identified.

[6] F. Wen, B. Duan, C. Zhang, R. Zhu, Y. Shang, and J. Zhang (2019). "High-Accuracy Parameter Identification Method for Equivalent-Circuit Models of Lithium-Ion Batteries Based on the Stochastic Theory Response Reconstruction". In: *Electronics* 8.8. ISSN: 2079-9292. DOI: 10.3390/electronics8080834

Parameter Identification Based On RLS cont'd

Let $y[k] = U_t[k] - U_{oc}[k]$, Equation 14 can be written as follows:

$$y[k] = \theta_1 y[k-1] + \theta_2 y[k-1] + \theta_3 I[k] + \theta_4 I[k-1] + \theta_5 I[k-2] \quad (15)$$

Defining $\boldsymbol{\phi}$ and new parameter vector, $\boldsymbol{\theta}$ as,

$$\boldsymbol{\phi}[k] = \Big[y[k-1] \ y[k-2] \ I[k] \ I[k-1] \ I[k-2] \Big]^T \quad (16)$$

and

$$\boldsymbol{\theta}[k] = \big[\theta_1 \ \theta_2 \ \theta_3 \ \theta_4 \ \theta_5 \big]^T \quad (17)$$

Then Equation 15 can be written in vector form as follows:

$$y[k] = \boldsymbol{\phi}[k]^T \boldsymbol{\theta}[k] \quad (18)$$

Parameter Identification Based On RLS cont'd

Defining the estimator of $\boldsymbol{\theta}$ as $\hat{\boldsymbol{\theta}}$, Equation 18 can be expressed as:

$$y[k] = \boldsymbol{\phi}[k]^T \hat{\boldsymbol{\theta}}[k] + \varepsilon[k] \tag{19}$$

When the square sum of the output error is minimum, the parameters are optimal, and the mathematical formula can be written as

$$\min_{\hat{\boldsymbol{\theta}}} \boldsymbol{J}(\hat{\boldsymbol{\theta}}, k) = \min_{\hat{\boldsymbol{\theta}}} \sum_{k=k_0}^{N} \varepsilon[k]^2 = \min_{\hat{\boldsymbol{\theta}}} \sum_{k=k_0}^{N} \left(y[k] - \boldsymbol{\phi}[k]^T \hat{\boldsymbol{\theta}}[k] \right)^2 \tag{20}$$

$$\hat{\boldsymbol{\theta}} = (\boldsymbol{\Psi}^T \boldsymbol{\Psi})^{-1} \boldsymbol{\Psi}^T \boldsymbol{Y} \tag{21}$$

where vectors compose of past measurements $\boldsymbol{\Psi} = \left[\boldsymbol{\phi}[k_0] \ \boldsymbol{\phi}[k_0 + 1] \ \ldots \ \boldsymbol{\phi}[N] \right]^T$ and big output vector $\boldsymbol{Y} = \left[y[k_0] \ y[k_0{+}1] \ \ldots \ y[N] \right]^T$.

The Measurement of Current and Terminal Voltage For Estimation Algorithm

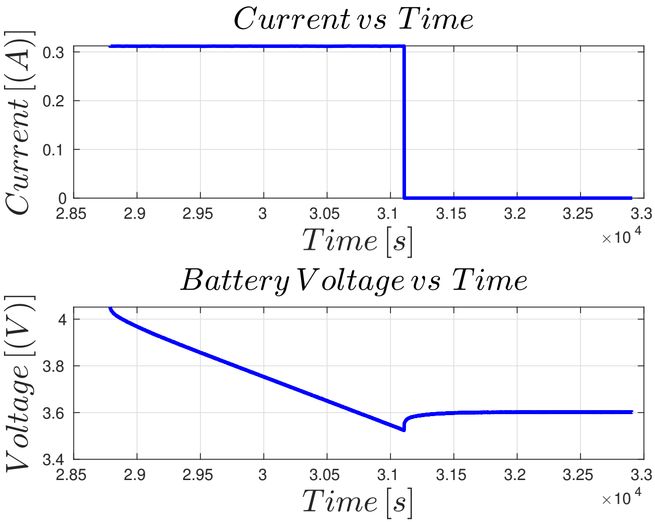


Figure 6: Measurement of current and voltage from a pulse response for second order equivalent circuit parameters identification.

Identified Parameters

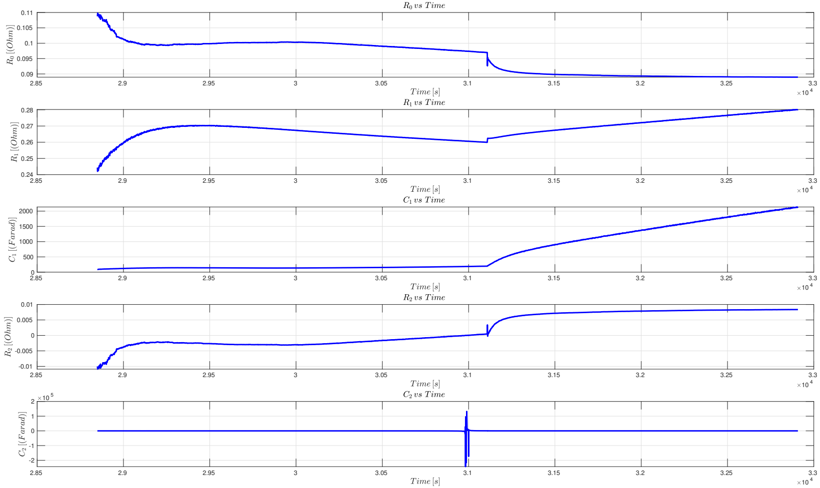


Figure 7: Parameter identification results of the RLS algorithm for parameters R_0, R_1, R_2, C_1 and C_2.

SOC Estimation

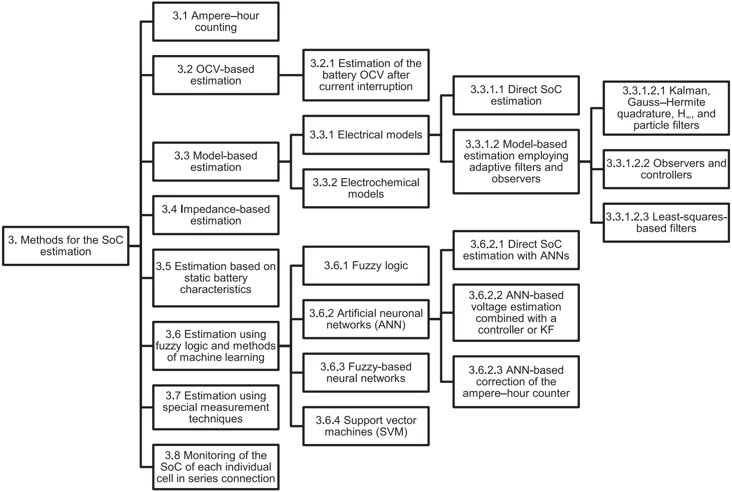


Figure 8: Classification of the methods for the SOC estimation. [7]

[7] W. Waag, C. Fleischer, and D. U. Sauer (2014). "Critical review of the methods for monitoring of lithium-ion batteries in electric and hybrid vehicles". In: *Journal of Power Sources* 258, pp. 321–339. ISSN: 0378-7753. DOI: https://doi.org/10.1016/j.jpowsour.2014.02.064

Adaptive EKF

Reform a state-space form in Equation 10:

$$
\begin{aligned}
\boldsymbol{x}[k+1] &= f(\boldsymbol{x[k]}, u[k]) = \boldsymbol{A_d}\boldsymbol{x}[k] + \boldsymbol{B_d}u[k] + \boldsymbol{w}[k] \\
y[k] &= h(\boldsymbol{x[k]}, u[k]) + v[k]
\end{aligned}
\tag{22}
$$

Explicity,

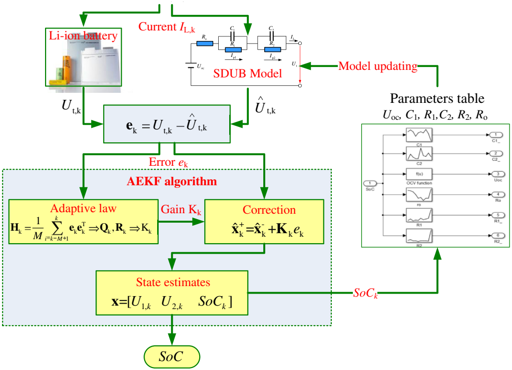
$$
\boldsymbol{x}[k+1] = \underbrace{\begin{bmatrix} e^{-\frac{Tt}{\tau_1}} & 0 & 0 \\ 0 & e^{-\frac{T}{\tau_2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\boldsymbol{A}_d} \boldsymbol{x}[k] + \underbrace{\begin{bmatrix} R_1(1 - e^{\frac{T}{\tau_1}}) \\ R_2(1 - e^{\frac{T}{\tau_2}}) \\ -\frac{\eta T}{Q} \end{bmatrix}}_{\boldsymbol{B}_d} u[k] + \boldsymbol{w}[k] \tag{23}
$$

The state vector with SOC is denoted by $\boldsymbol{x}[k] = \begin{bmatrix} U_1[k] \; U_2[k] \; SOC[k] \end{bmatrix}^T$.

$\boldsymbol{w}[k] \sim N(0, \boldsymbol{Q})$ is the unmeasured process noise, $v[k] \sim N(0, \boldsymbol{R})$ is the measurement noise.

Adaptive EKF

The matrix C is the derivative of the output equation (22) w.r.t. state vector before estimation time, $C[k] = \left.\frac{\partial h(\boldsymbol{x}[k], u[k])}{\partial \boldsymbol{x}}\right|_{x=\hat{x}[k]^-} = \left[-1 \;\; -1 \;\; \left.\frac{dU_{oc}(SOC)}{dSOC}\right|_{\hat{SOC}[k]^-} \right]$ where $\frac{dU_{oc}(SOC)}{dSOC} = K_1 SOC^{-2} - K_2 + \frac{K_3}{SOC} - \frac{K_4}{1-SOC}$.



Adaptive EKF Algorithm

Algorithm 1: The AEKF Algorithm

Input: $u[k] = I[k],\ k \in \{k_0, \dots N-1\}$

Output: $\widehat{SOC}[k],\ k \in \{k_0, \dots N-1\}$

Data: $y[k] = U_t^{exp}[k],\ k \in \{k_0, \dots N\}$, $R_0[k]$, $R_1[k]$, $C_1[k]$, $R_2[k]$, $C_2[k]$

Step 1: Initialization:

$\hat{\boldsymbol{x}}[k_0-1]^+ = E\big[\boldsymbol{x}[k_0-1]\big]$, $\hat{\boldsymbol{P}}[k_0-1]^+ =$

$$E\left[\left(\boldsymbol{x}[k_0-1] - E\big[\hat{\boldsymbol{x}}[k_0-1]^+\big]\right)\left(\boldsymbol{x}[k_0-1] - E\big[\hat{\boldsymbol{x}}[k_0-1]^+\big]\right)^T\right]$$

Step 2: Calculation:

for $k = k_0 \to N$ **do**

State Estimation Propagation: $\hat{\boldsymbol{x}}[k]^- = f\left(\hat{\boldsymbol{x}}[k-1]^+, u[k]\right)$ (look 27)

State Estimation Covariance: $\boldsymbol{P}[k]^- = \boldsymbol{A_d}[k]\boldsymbol{P}[k-1]\boldsymbol{A_d}[k]^T + \boldsymbol{Q}[k-1]$

Error Innovation: $e[k] = y[k] - h\left(\hat{\boldsymbol{x}}[k]^-, u[k]\right)$

Adaptive Law: $H[k] = \dfrac{1}{M}\sum_{i=k-M+1}^{k} e[k]^2$, $R[k] = H[k] - \boldsymbol{C}[k]\boldsymbol{P}[k]^-\boldsymbol{C}[k]^T$

Kalman Gain Matrix: $\boldsymbol{K}[k] = \boldsymbol{P}[k]^-\boldsymbol{C}[k]^T\left(\boldsymbol{C}[k]\boldsymbol{P}[k]^-\boldsymbol{C}[k]^T + R[k]\right)^{-1}$

State Estimate Measurement Update: $\hat{\boldsymbol{x}}[k]^+ = \hat{\boldsymbol{x}}[k]^- + \boldsymbol{K}[k]e[k]$

State Covariance Measurement Update: $\boldsymbol{Q}[k] = \boldsymbol{K}[k]H[k]\boldsymbol{K}[k]^T$,

$$\boldsymbol{P}[k]^+ = \left(\boldsymbol{I} - \boldsymbol{K}[k]\boldsymbol{C}[k]\right)\boldsymbol{P}[k]^-\left(\boldsymbol{I} - \boldsymbol{K}[k]\boldsymbol{C}[k]\right)^T + \boldsymbol{K}[k]\boldsymbol{R}[k]\boldsymbol{K}[k]^T$$

/* $\boldsymbol{A}_d[k] = \dfrac{\partial f(\boldsymbol{x}[k], u[k])}{\partial \boldsymbol{x}}\Big|_{x=\hat{x}[k]^-}$, $\boldsymbol{C}[k] = \dfrac{\partial h(\boldsymbol{x}[k], u[k])}{\partial \boldsymbol{x}}\Big|_{x=\hat{x}[k]^-}$ */

Estimated SOC vs Time

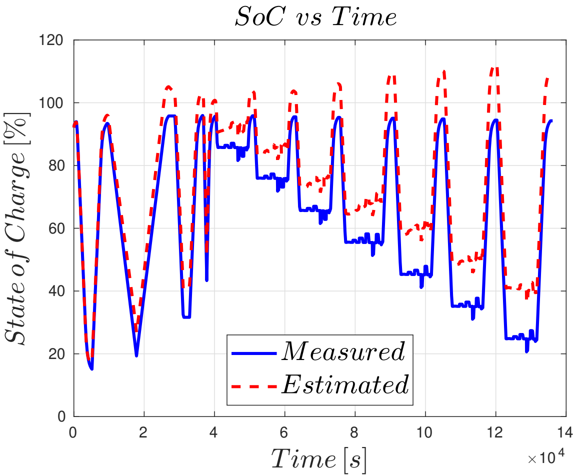


Figure 10: Validation results for SOC estimation by using AEKF.

Estimated Terminal Battery Voltage , U_t, vs Time

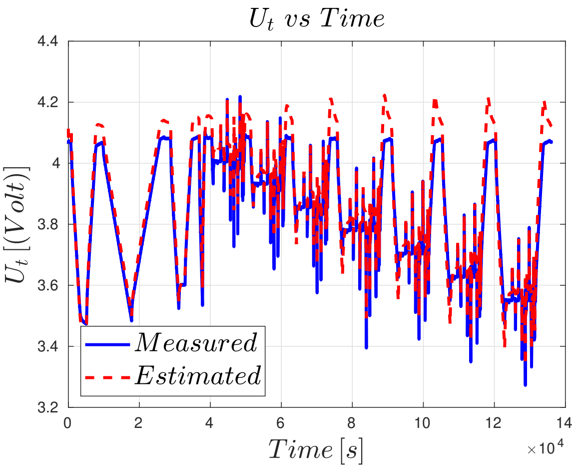


Figure 11: Validation results for terminal voltage, U_t, by using AEKF

Terminal Battery Voltage Error, e_t, vs Time

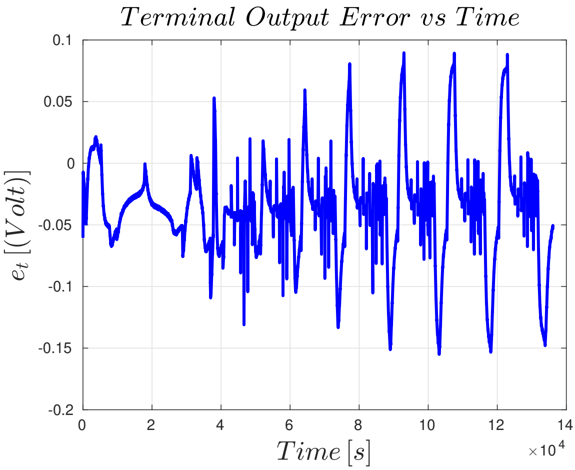


Figure 12: Error between estimated battery terminal voltage and terminal voltage, U_t, by using AEKF

State estimation for RC voltages vs Time

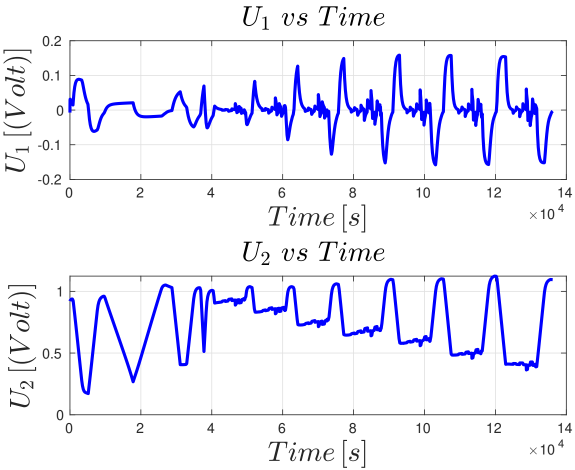


Figure 13: AEKF's state estimation for RC voltages U_1 and U_2, respectively

Estimated Parameters by using RLS

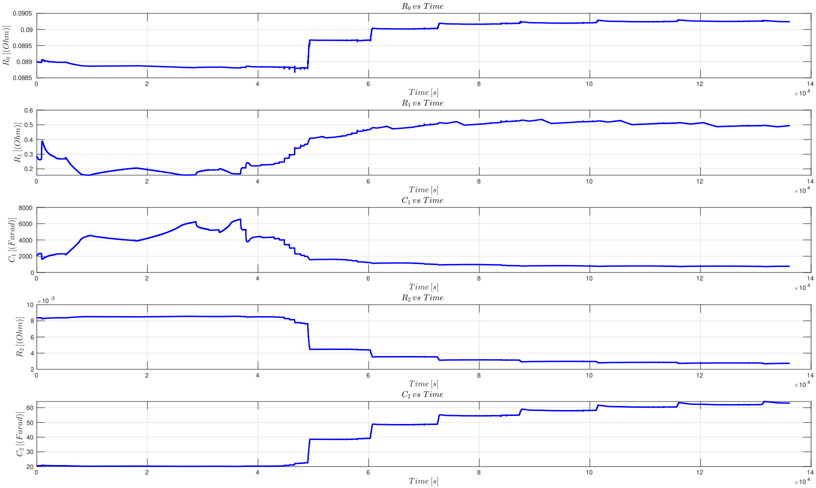


Figure 14: Estimated Parameters by using RLS through whole experiment time.