İsmail Çağdaş Yılmaz

MPC-Based Approach to Active Steering for Autonomous Vehicle Systems

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Contents of the Presentation

Introduction

- Recent trends in automotive industry point in the direction of increased content of electronics, computers, and controls.
- Passive safety is primarily focused on structural integrity of vehicle.
- Active safety on the other hand is primarily used to avoid accidents and at the same time facilitate better vehicle controllability and stability especially in emergency situations.

The progress of safety functions

1. Longitudinal dynamics part of motion
   a) on more effective braking (ABS)
   b) traction control

2. work on different vehicle stability control systems (some of them are same acronyms)
   a) Electronic Stability Program (ESP)
   b) Vehicle Stability Control (VSC)
   c) Interactive Vehicle Dynamics (IVD)
   d) Dynamic Stability Control (DSC)

Essentially, these systems use brakes on one side to stabilize the vehicle in extreme limit handling situations through controlling the yaw motion.

Effectiveness of active safety provided:

1. not only by
    a) Four wheel steering (4WS)
    b) active steering
    c) active suspensions (active differentials)
    d) ...

2. but also by additional sensor information
    a) onboard 360 degree cameras
    b) Infrared sensors (RADAR, LIDAR)
    c) GPS and compass
    d) Inertial Measurements Unit (IMU)
    e) ...

Scope of the Study

- The Proposal:
  - a double lane change scenario on a slippery road
  - Assumption: a vehicle equipped with a fully autonomous steering system
- The Control Input:
  - front steering angle, δ_f
  - forward speed of the vehicle, $v_x \Rightarrow$ constant
- The Goal:
  - follow the desired trajectory or target as close as possible
  - Fulfilling various constraints reflecting vehicle physical limits

Selection of the Controller

- Question?: What is the best and optimum way in controlling the vehicle maneuver for given obstacle avoidance situation?
- Answer: **Model Predictive Control (MPC)**
  - a nonlinear model of the plant to *predict* the future evolution of the system
  - At each time step t a performance index is optimized under operating constraints with respect to a sequence of future steering moves in order to best follow the given reference
  - The first of such optimal moves is the *control* action applied to the plant at time t.
  - At time t + 1, a new optimization is solved over a shifted prediction horizon.

Vehicle Model

- **Bicycle model** to describe the dynamics of the car
- **Assumption**: Constant normal tire load, i.e., F_{z_f}, F_{z_r} = constant

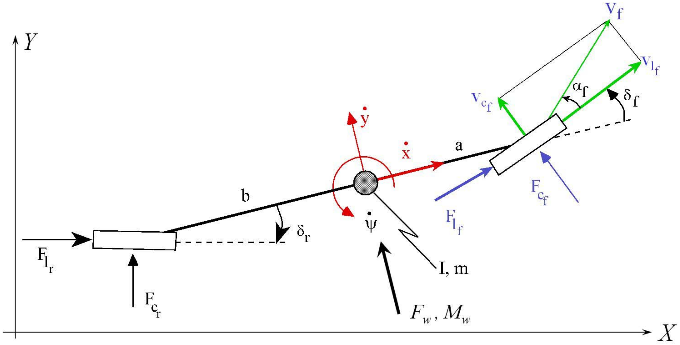

Figure 1: The simplified vehicle dynamical model. [1]

[1] F. Borrelli, P. Falcone, T. Keviczky, J. Asgari, and D. Hrovat (2005). "MPC-Based Approach to Active Steering for Autonomous Vehicle Systems". In: *International Journal of Vehicle Autonomous Systems* 3, pp. 265–291

Dynamics and Kinematics Equations

- **Nonlinear Dynamical System Equations**

$$\ddot{x} = \dot{y}\dot{\psi} + \frac{2(F_{x_f} + F_{x_r})}{m} \tag{1a}$$

$$\ddot{y} = -\dot{x}\dot{\psi} + \frac{2(F_{y_f} + F_{y_r})}{m} \tag{1b}$$

$$\ddot{\psi} = \dot{r} = \frac{2(aF_{y_f} - bF_{y_r})}{I_z} \tag{1c}$$

- **Nonlinear Kinematics Equations (Equations of Motion)**

$$\dot{Y} = \dot{x}\sin(\psi) + \dot{y}\cos(\psi) \tag{2a}$$

$$\dot{X} = \dot{x}\cos(\psi) - \dot{y}\sin(\psi) \tag{2b}$$

$$\dot{\psi} = r \tag{2c}$$

Longitudinal and lateral tire forces

Forces acting on the center of gravity:

$$F_y = F_l \sin(\delta) + F_c \cos(\delta) \tag{3a}$$

$$F_x = F_l \cos(\delta) - F_c \sin(\delta) \tag{3b}$$

Tire forces for each tire are given by

$$F_l = f_l(\alpha, s, \mu, F_z) \tag{4a}$$

$$F_c = f_c(\alpha, s, \mu, F_z) \tag{4b}$$

where α, **slip angle** of the tire, s is the **slip ratio**, $F_{z_f} = \frac{bmg}{2(a+b)}$, $F_{z_r} = \frac{amg}{2(a+b)}$ forward and rear normal tire loads.

$$s = \begin{cases} \frac{r\omega}{v} - 1 \text{ if } v > r\omega,\ v \neq 0 \text{ for braking} \\ 1 - \frac{r\omega}{v} \text{ if } v < r\omega,\ \omega \neq 0 \text{ for driving} \end{cases} \tag{5}$$

$$\alpha = \tan^{-1} \frac{v_c}{v_l} \tag{6}$$

Nonlinear State Space Equations

Using above equations, the nonlinear vehicle dynamics can be described,

$$
\dot{\boldsymbol{\xi}}(t) = \begin{pmatrix}
\dot{y} \Rightarrow & v\sin(\psi+\beta) \\
\ddot{y} \Rightarrow & -\dot{x}\dot{\psi} + \frac{2(F_{y_f}+F_{y_r})}{m} = -\dot{x}\dot{\psi} + \frac{2}{m}(F_{c,f}\cos(\delta_f) + F_{c,r}) \\
\dot{\psi} \Rightarrow & r \\
\ddot{\psi} = \dot{r} \Rightarrow & \frac{1}{I_z}(aF_{y_f}\cos(\delta) - bF_{y_r}) \\
\dot{Y} \Rightarrow & \dot{x}\sin(\psi) + \dot{y}\cos(\psi) \\
\dot{X} \Rightarrow & \dot{x}\cos(\psi) - \dot{y}\sin(\psi)
\end{pmatrix} \tag{7}
$$

where $\beta = \tan^{-1}(\frac{b}{a+b}\tan(\delta_f))$ and acceleration of the car is constant, i.e. $\dot{v} = 0$. Forces in x- and y-directions, F_x and F_y, are the function of the forward steering angle, δ_f and longitudinal and lateral tire forces acting on the center.

Nonlinear State Space Equations cont'd

Forward α_f and backward α_r slip angles calculated as:

$$\alpha_f = -\arctan\left(\frac{ra + v_y}{vx}\right) + \delta_f \tag{8a}$$

$$\alpha_r = \arctan\left(\frac{rb - v_y}{vx}\right) \tag{8b}$$

Closed form with output equations:

$$\dot{\boldsymbol{\xi}} = f_{s,\mu}(\boldsymbol{\xi}, u) \tag{9a}$$

$$\boldsymbol{\eta} = h(\boldsymbol{\xi}) \tag{9b}$$

where the state and input vectors are $\boldsymbol{\xi} = [Y \;\, X \;\, \psi \;\, \dot{y} \;\, \dot{\psi}]^T$ and $u = \delta_f$, respectively, and the output map is given as

$$\boldsymbol{\eta} = \begin{bmatrix} \psi \\ Y \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \boldsymbol{\xi} \tag{10}$$

Tire Model

- Three possible ways to represent measured tyre data are in use [2]
  - representation by tables,
  - representation by graphs,
  - representation by formula.

- The formula should, if possible, be able to describe:
  - the side force as a function of slip angle, α_f & α_r
  - the brake force as a function of longitudinal slip,
  - the self aligning torque as a function of slip angle.

[2] E. Bakker, L. Nyborg, and H. B. Pacejka (Feb. 1987). "Tyre Modelling for Use in Vehicle Dynamics Studies". In: *SAE Technical Paper*. SAE International. DOI: 10.4271/870421

Steady-State Tyre characteristics

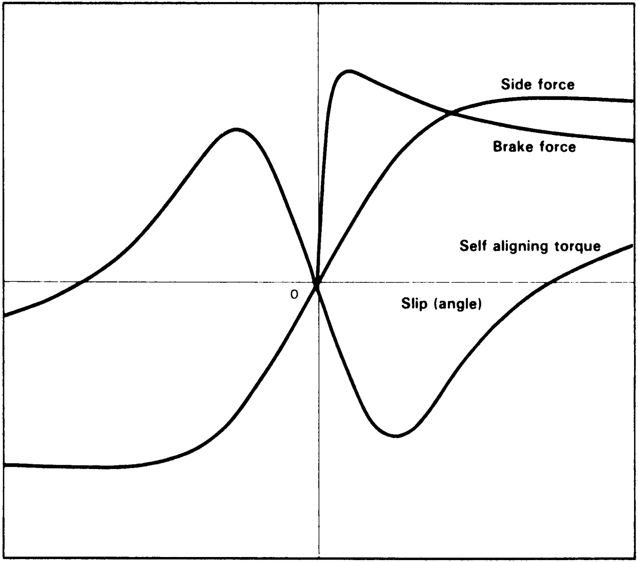

Figure 2: Steady-state tyre characteristics.

Basic form of the tyre force

$$F = D\sin(B\alpha) \tag{11}$$

with F standing for either side force, self aligning torque or break force and α denoting slip angle. D is the peak value and the product DB equals the slip stiffness at zero slip.

$$F = D\sin(C\arctan(B\alpha)) \tag{12}$$

D is still the peak value, the slip stiffness at zero slip is now equal to the BCD (from now on called the stiffness). The coefficient C governs the shape of the curve in Fig 2.

$$F = D\sin(C\arctan(B\Phi)) \tag{13a}$$

$$\Phi = (1-E)\alpha + \frac{E}{B}\arctan(B\alpha) \tag{13b}$$

The four coefficients are:

- $B \Rightarrow$ stiffness factor
- $C \Rightarrow$ shape factor
- $D \Rightarrow$ peak factor
- $E \Rightarrow$ curvature factor

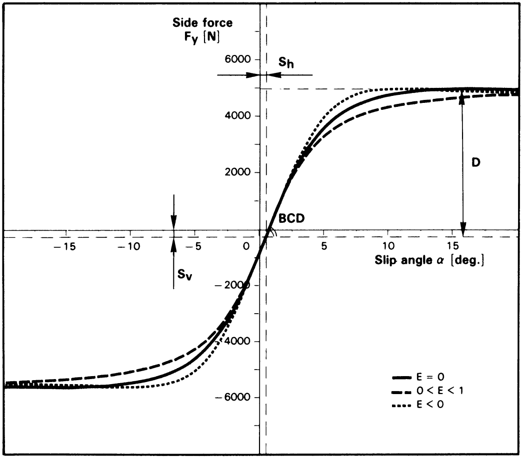

Figure 3: Coefficients appearing in tyre formula.

Side Force, F_y

$$\begin{aligned}
F_y =& D\sin(C\arctan(B\Phi)) + \cancel{\Delta S_v} && \text{(14a)}\\
\Phi =& (1-E)(\alpha + \cancel{\Delta S_h}) + (E/B)\arctan(B(\alpha + \cancel{\Delta S_h})) && \text{(14b)}\\
D =& a_1 {F_z}^2 + a_2 F_z && \text{(14c)}\\
C =& 1.30 && \text{(14d)}\\
B =& \left(\frac{a_3\sin(a_4\arctan(a_5 F_z))}{CD}\right)(1 - \cancel{a_{12}|\gamma|}) && \text{(14e)}\\
E =& a_6 {F_z}^2 + a_7 F_z + a_8 && \text{(14f)}\\
\Delta S_h =& a_9\gamma && \text{(14g)}\\
\Delta S_v =& (a_{10}{F_z}^2 + a_{11}F_z)\gamma && \text{(14h)}
\end{aligned}$$

Side Force, F_y, Graph MATLAB Simulation

Mass of the vehicle is taken as 1700 kg.

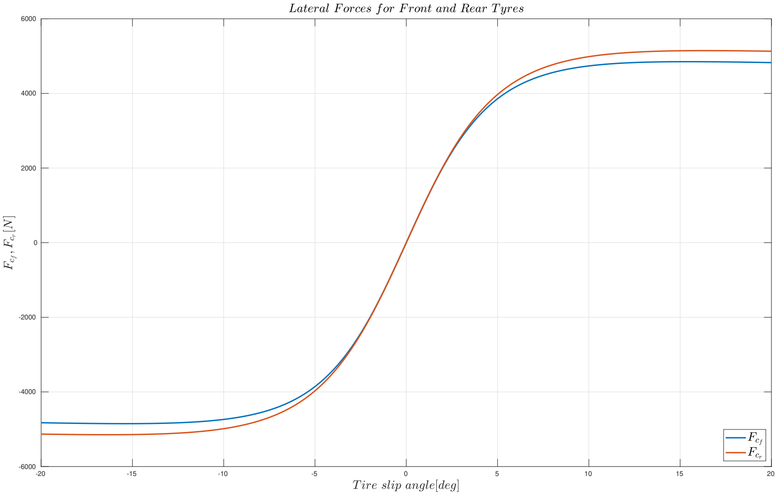

Figure 4: Lateral forces of front and rear tyres.

Problem Formulation

Discretizing the system dynamics as indicated in (9) with the Euler method,

$$\boldsymbol{\xi}(k+1) = f^{dt}_{s,\mu}(\boldsymbol{\xi}(k), u(k)) \tag{15a}$$

$$\boldsymbol{\eta}(k) = h(\boldsymbol{\xi}(k)) \tag{15b}$$

where $u(k) = u(k-1) + \Delta u(k)$ and $u(k) = \delta_f(k)$, $\Delta u(k) = \Delta\delta_f(k)$. Objective or the cost function of the system can be defined as follows:

$$\begin{aligned} J(\boldsymbol{\xi}(k), \Delta \boldsymbol{U}_t) &= \sum_{i=1}^{H_p} (\hat{\boldsymbol{\eta}}_{t+i,t} - \boldsymbol{\eta}_{ref_{t+i,t}})^T \boldsymbol{Q} \, (\hat{\boldsymbol{\eta}}_{t+i,t} - \boldsymbol{\eta}_{ref_{t+i,t}}) \\ &+ \sum_{i=0}^{H_c-1} \Delta u_{t+i,t} \, R \, \Delta u_{t+i,t} \\ &= \sum_{i=1}^{H_p} \left\| \hat{\boldsymbol{\eta}}_{t+i,t} - \boldsymbol{\eta}_{ref_{t+i,t}} \right\|^2_{\boldsymbol{Q}} + \sum_{i=0}^{H_c-1} \left\| \Delta u_{t+i,t} \right\|^2_R \end{aligned} \tag{16}$$

Nonlinear Model Predictive Control Formulation

At each time step t the following finite horizon optimal control problem is solved

$$
\begin{aligned}
\min_{\boldsymbol{\Delta U}} \quad & J(\boldsymbol{\xi}(k), \Delta \boldsymbol{U}_t) \\
\text{subject to} \quad & \boldsymbol{\xi}(k+1) = f^{dt}_{s,\mu}(\boldsymbol{\xi}(k), u(k)), \;\; k = t, \dots, t + H_p \\
& \boldsymbol{\eta}(k) = h(\boldsymbol{\xi}(k)), \;\; k = t, \dots, t + H_p \\
& \delta_{f,min} \leq u_{k,t} \leq \Delta\delta_{f,max} \;\; k = t, \dots, t + H_c - 1 \\
& \Delta\delta_{f,min} \leq \Delta u_{k,t} \leq \Delta\delta_{f,max} \;\; k = t, \dots, t + H_c - 1 \\
& u_{k,t} = u_{k,t-1} + \Delta u_{k,t}
\end{aligned}
\tag{17}
$$

Above problem is solved at time t for $\boldsymbol{\xi}_{t,t}$, sequence of the optimal control inputs, $\Delta \boldsymbol{U}^*_t = [\Delta u^*_{t,t}, \dots, \Delta u^*_{t+H_c-1,t}]^T$, within the specified control horizon. The state feedback control law is,

$$
\delta_f(t) = \delta_f(t-1) + \Delta u^*_{t,t} \tag{18}
$$

Double Lane Change Using Active Steering

- Scenario Description: The reference signals (desired tracking signals) $\boldsymbol{\eta}_{ref} = \begin{bmatrix} \psi_{ref} \\ Y_{ref} \end{bmatrix}$ are specified by the following set of equations:

$$Y_{ref} = \frac{d_{y_1}}{2}(1 + \tanh(z_1)) - \frac{d_{y_2}}{2}(1 + \tanh(z_2)) \tag{19a}$$

$$\psi_{ref} = \arctan\left(d_{y_1}\Big(\frac{1}{\cosh(z_1)}\Big)^2\Big(\frac{1.2}{d_{x_1}}\Big) - d_{y_2}\Big(\frac{1}{\cosh(z_2)}\Big)^2\Big(\frac{1.2}{d_{x_2}}\Big)\right) \tag{19b}$$

$$z_1 = \frac{shape}{d_{x_1}}(X - X_{s_1}) - \frac{shape}{2} \tag{19c}$$

$$z_2 = \frac{shape}{d_{x_2}}(X - X_{s_2}) - \frac{shape}{2} \tag{19d}$$

where $shape = 2.4$, $d_{x_1} = 25$, $d_{x_2} = 21.95$, $d_{y_1} = 4.05$, $d_{y_2} = 5.7$, $X_{s_1} = 27.19$ and $X_{s_2} = 56.46$.

Reference Yaw Angle, $\hat{\psi}$ vs X Position, obtained by MATLAB Simulation

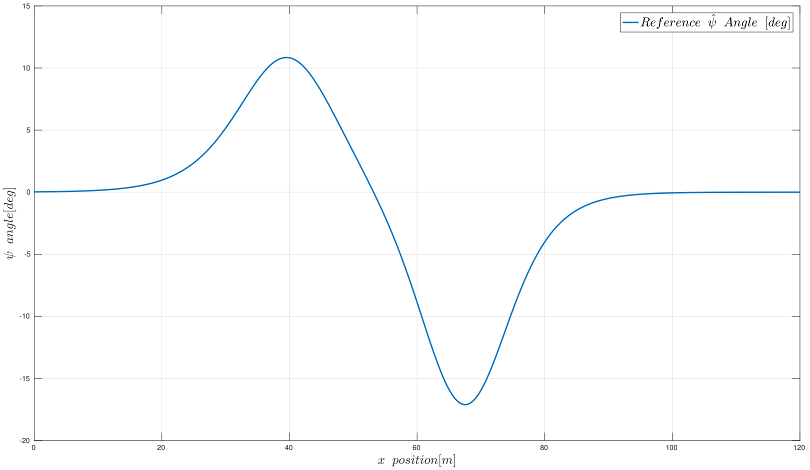

Figure 5: Reference ψ angle in degree.

Reference Yaw Angle, $\hat{Y}$ vs X Position, obtained by MATLAB Simulation

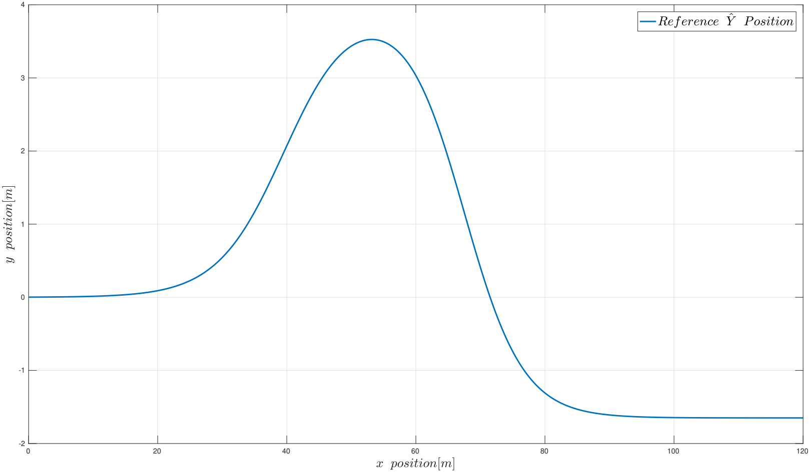

Figure 6: Reference ψ angle in degree.

MATLAB Simulations via YALMIP Toolbox

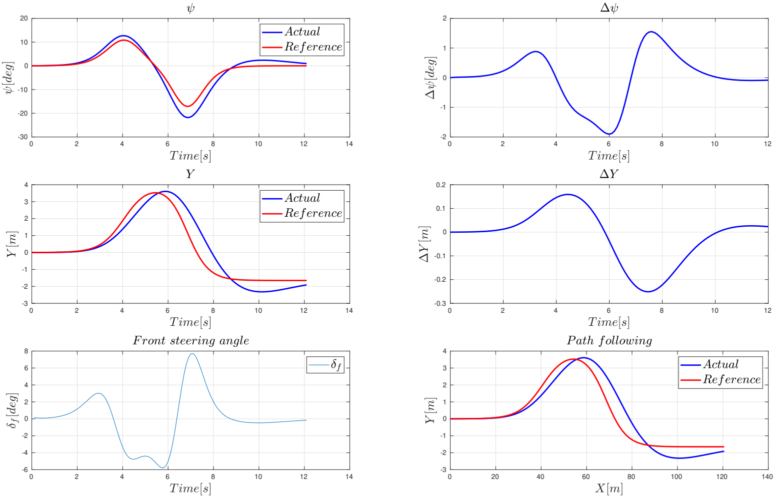

Figure 7: (MATLAB) Double lane change maneuver at 10 m/s with H_p = 7 and H_c = 7

MATLAB Simulations via YALMIP Toolbox cont'd

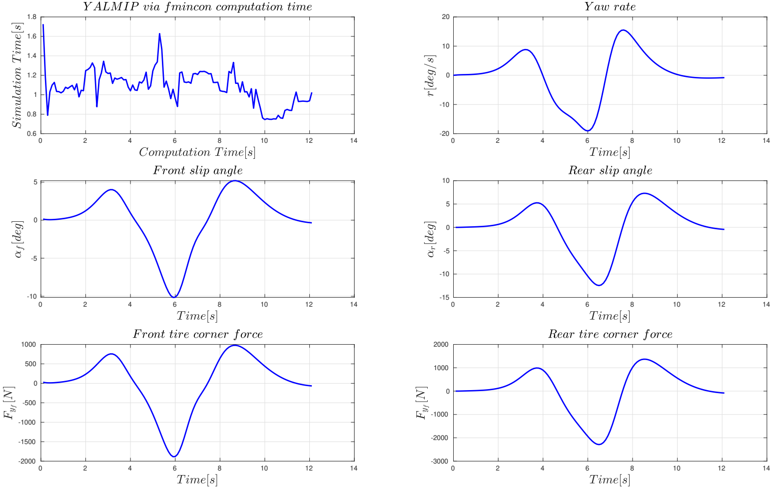

Figure 8: (MATLAB) Double lane change maneuver at 10 m/s with H_p = 7 and H_c = 7. YALMIP computation time, yaw rate, tire forces and slip angles.

C++ Simulations via (IPOPT, cppAD, matplotlibcpp)

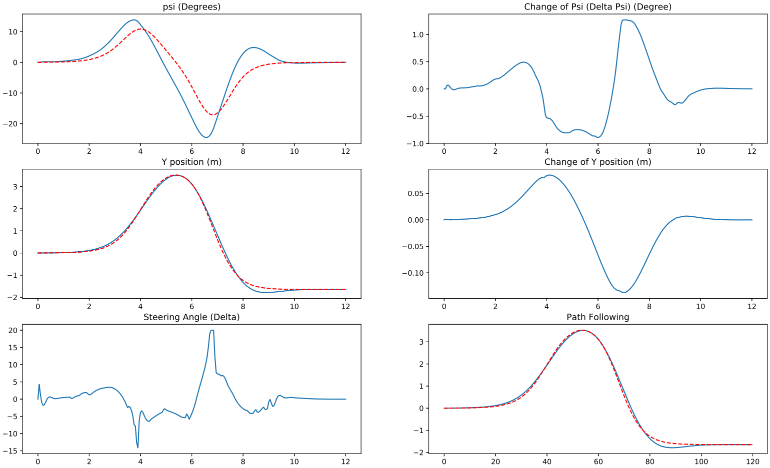

Figure 9: (c++) Double lane change maneuver at 10 m/s with $H_p = 20$ and $H_c = 20$

C++ Simulations via (IPOPT, cppAD, matplotlibcpp) cont'd

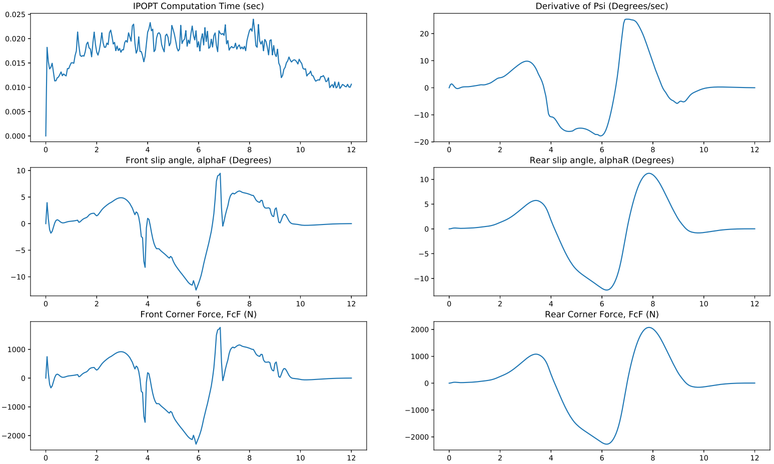

Figure 10: (c++) Double lane change maneuver at 10 m/s with $H_p = 20$ and $H_c = 20$. yaw rate, tire forces and slip angles.