EE4620 - Spectral Domain Methods in Electromagnetics

Topic # 4

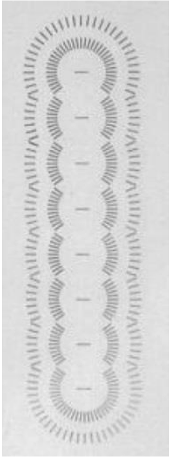
Surface Waves

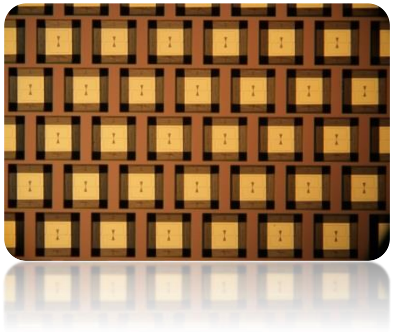
Nuria Llombart

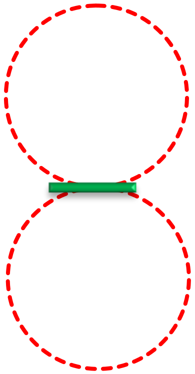
Integrated Antennas

Radar, Space, Sensing applications…
The objective is to radiate the power into a certain direction

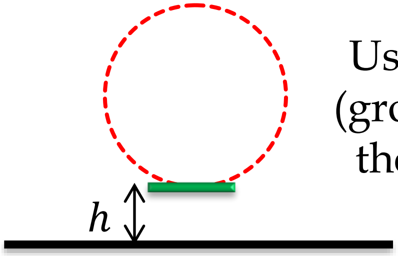
Dielectric allows using integrated technology (PCB, lithography)







A dipole in free space radiates the same power in the upper and lower regions

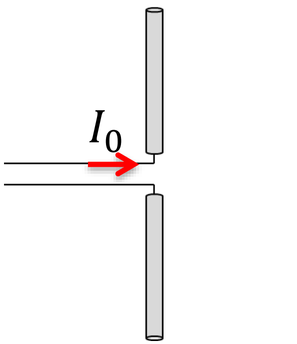


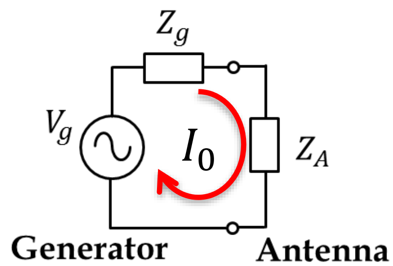
Using a back reflector (ground plane) reduces the radiation into the lower region

$$h_{opt} = \lambda/4$$

Power Budget

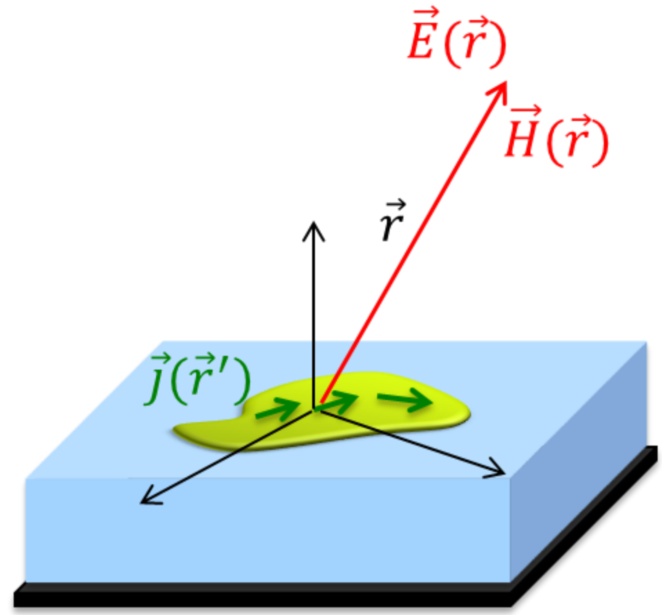
For antennas in free space
with no ohmic losses





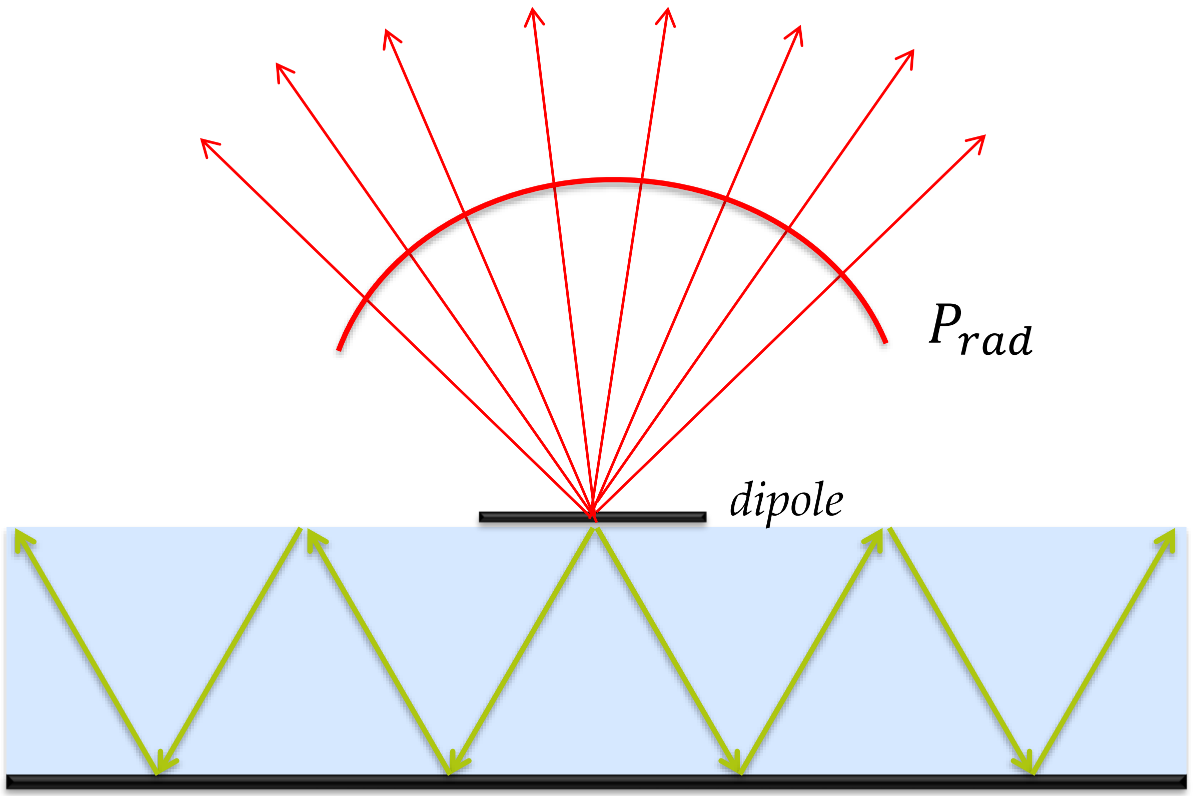
$$\boldsymbol{j}_{eq} = I_0 \boldsymbol{f}(x, y)$$

$$P_{rad} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} U(\theta, \phi) \sin\theta \, d\theta d\phi = P_{Z_A} = \frac{1}{2} |I_0|^2 Re\{Z_A\}$$



What happens in antennas radiating into infinitely extended dielectrics?

Surface Waves



There is a part of the power delivered to the antenna that is radiated inside the dielectric!

P_{sw}

Today, we will focus on how to characterize the power launched into SWs!

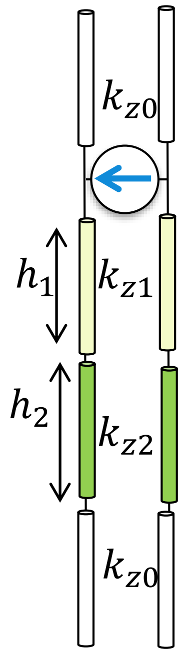
$$P_{Z_A} = \frac{1}{2}|I_0|^2 Re\{Z_A\} = P_{rad} + P_{sw}$$

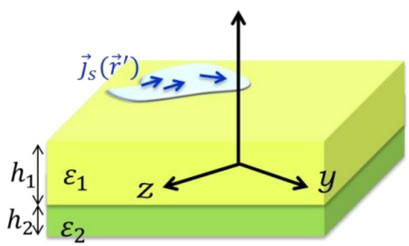
Dyadic Green's Function for Stratified Media

$$\boldsymbol{f}(\vec{r}) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\tilde{\boldsymbol{G}}^{fc}(k_x,k_y,z,z')\boldsymbol{C}_s(k_x,k_y)e^{-jk_x x}e^{-jk_y y}\,dk_x dk_y$$

$$\tilde{\boldsymbol{G}}^{ej} = \begin{bmatrix} -\dfrac{v_{TM}k_x^2 + v_{TE}k_y^2}{k_\rho^2} & \dfrac{(v_{TE} - v_{TM})k_x k_y}{k_\rho^2} \\ \dfrac{(v_{TE} - v_{TM})k_x k_y}{k_\rho^2} & -\dfrac{v_{TE}k_x^2 + v_{TM}k_y^2}{k_\rho^2} \\ \varsigma\dfrac{k_x}{k} i_{TM} & \varsigma\dfrac{k_y}{k} i_{TM} \end{bmatrix}$$

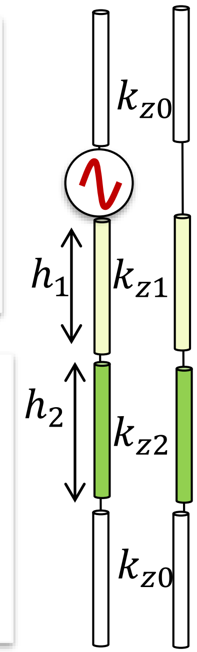
$$\tilde{\boldsymbol{G}}^{hj} = \begin{bmatrix} \dfrac{(i_{TM} - i_{TE})k_x k_y}{k_\rho^2} & \dfrac{i_{TE}k_x^2 + i_{TM}k_y^2}{k_\rho^2} \\ \dfrac{i_{TE}k_y^2 + i_{TM}k_x^2}{k_\rho^2} & \dfrac{(i_{TE} - i_{TM})k_x k_y}{k_\rho^2} \\ \dfrac{k_y}{\varsigma k} v_{TE} & \dfrac{k_x}{\varsigma k} v_{TE} \end{bmatrix}$$

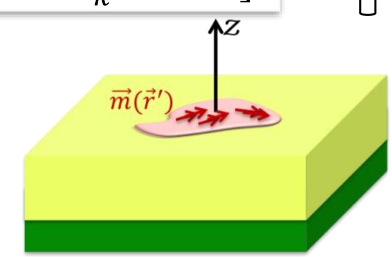




$$\tilde{\boldsymbol{G}}^{hm} = \begin{bmatrix} -\dfrac{i_{TE}k_x^2 + i_{TM}k_y^2}{k_\rho^2} & \dfrac{(i_{TM} - i_{TE})k_x k_y}{k_\rho^2} \\ \dfrac{(i_{TM} - i_{TE})k_x k_y}{k_\rho^2} & -\dfrac{i_{TM}k_x^2 + i_{TE}k_y^2}{k_\rho^2} \\ \dfrac{k_x}{\varsigma k} v_{TE} & \dfrac{k_y}{\varsigma k} v_{TE} \end{bmatrix}$$

$$\tilde{\boldsymbol{G}}^{em} = \begin{bmatrix} \dfrac{(v_{TM} - v_{TE})k_x k_y}{k_\rho^2} & -\dfrac{v_{TE}k_y^2 + v_{TM}k_x^2}{k_\rho^2} \\ \dfrac{v_{TE}k_x^2 + v_{TM}k_y^2}{k_\rho^2} & \dfrac{(v_{TE} - v_{TM})k_x k_y}{k_\rho^2} \\ -\varsigma\dfrac{k_y}{k} i_{TM} & \varsigma\dfrac{k_x}{k} i_{TM} \end{bmatrix}$$



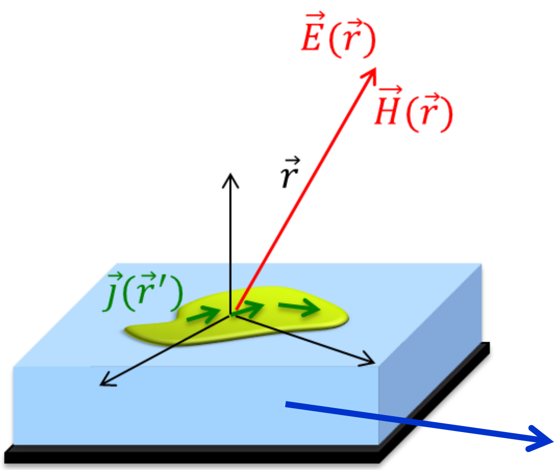


$$k_{zi} = \sqrt{k_i^2 - k_x^2 - k_y^2}$$

$$Z_{TMi} = \zeta_i k_{zi}/k_i \qquad Z_{TEi} = \zeta_i k_i/k_{zi}$$

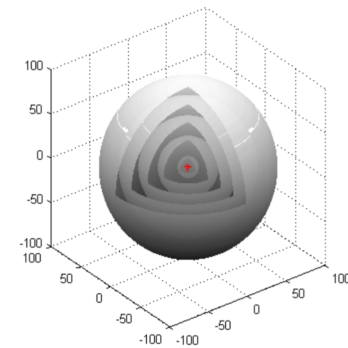
Fields Radiated by Printed Antennas

$$\boldsymbol{f}(\vec{r}) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\widetilde{\boldsymbol{G}}^{fc}(k_x,k_y,z,z')\boldsymbol{C}_s(k_x,k_y)e^{-jk_x x}e^{-jk_y y}dk_x dk_y$$



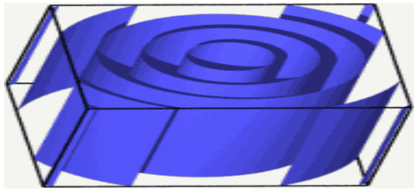
Far field: Spherical wave is emerging when the observation point is in the infinite medium

$$\boldsymbol{\frac{e^{-jk_0 r}}{r}}$$



Surface wave: Cylindrical wave is emerging when the observation point is inside a dielectric substrate with finite thickness

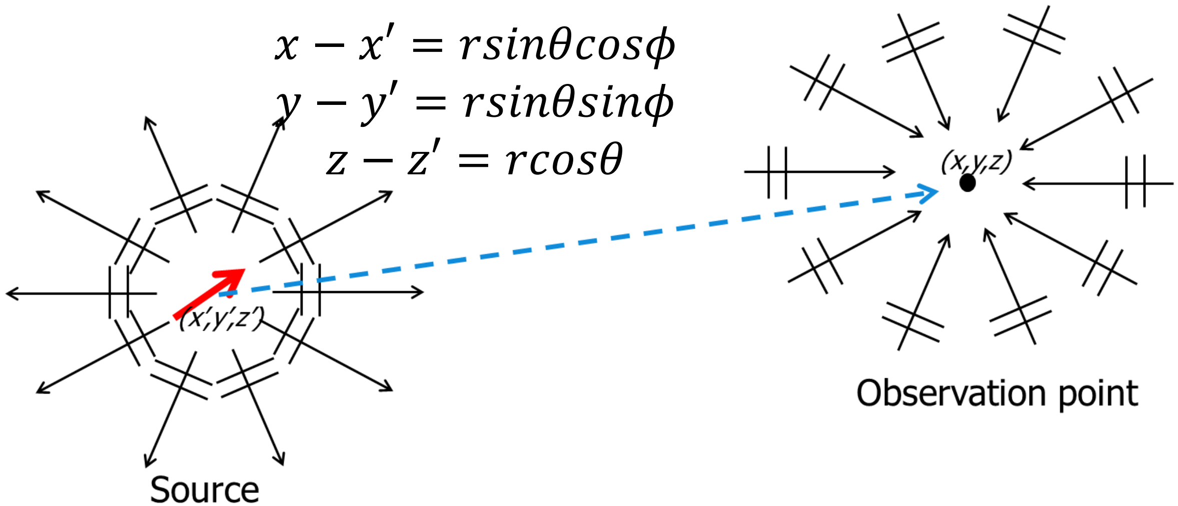
$$\boldsymbol{\frac{e^{-jk_{sw}\rho}}{\sqrt{\rho}}}$$



Dominant PW in the Far Field

$$\vec{f}^{far}(\vec{\boldsymbol{r}}) = jk_{zs}\widetilde{\boldsymbol{G}}^{fc}(k_{xs}, k_{ys}, z, z')\vec{\boldsymbol{C}}(k_{xs}, k_{ys})e^{jk_{zs}|z-z'|}\frac{e^{-jkr}}{2\pi r}$$

$$x - x' = r sin\theta cos\phi$$
$$y - y' = r sin\theta sin\phi$$
$$z - z' = r cos\theta$$



The plane wave with the dominant role is the one given by the direct ray

$$k_{xs} = k_0 sin\theta cos\phi$$
$$k_{ys} = k_0 sin\theta sin\phi$$
$$k_{zs} = k_0 cos\theta$$

It corresponds to a stationary phase point

The far field of any source is proportional to $\frac{e^{-jkr}}{r}$ *Spherical Wave*

The region of the spectrum that impacts the far field is $\boldsymbol{k_\rho < k_0}$

Can we simplify the integration?

$$\vec{f}(\vec{r}) = \frac{1}{4\pi^2}\int_0^{2\pi}\int_{-\infty}^{\infty} \tilde{G}^{fc}(k_\rho, \alpha, z, z')\,\vec{C}(k_\rho, \alpha)\, e^{-j\vec{k}_\rho\cdot\vec{\rho}}\; k_\rho dk_\rho d\alpha$$

If we want to calculate the field at a *relatively large distance* from the source:

- Far field
- Surface wave contribution

Apart from space waves, the integral is dominated by the singularities (critical spectral points) of the SGF

The current is slow varying function at a critical points ($k_{\rho C},$), and saddle points therefore it can be extracted from the integral

$$\vec{f}_c(\vec{r}) \approx \frac{1}{4\pi^2}\int_0^{2\pi}\int_{-\infty}^{\infty} \tilde{G}^{fc}(k_\rho, \alpha, z, z')\, e^{-j\vec{k}_\rho\cdot\vec{\rho}}\; k_\rho dk_\rho d\alpha\, \vec{C}(k_{\rho c}, \alpha_{cs})$$

critical point In kρ

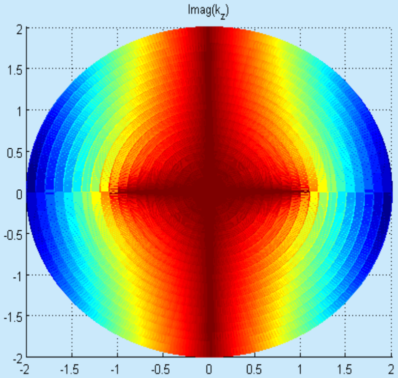
Saddle point in α

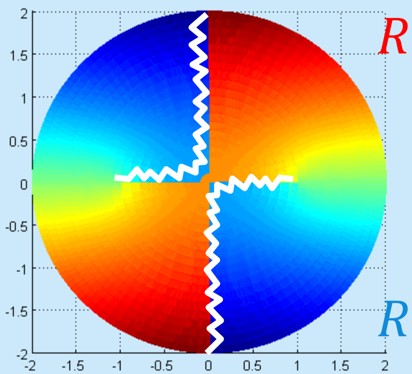
Preferred Branch Convention

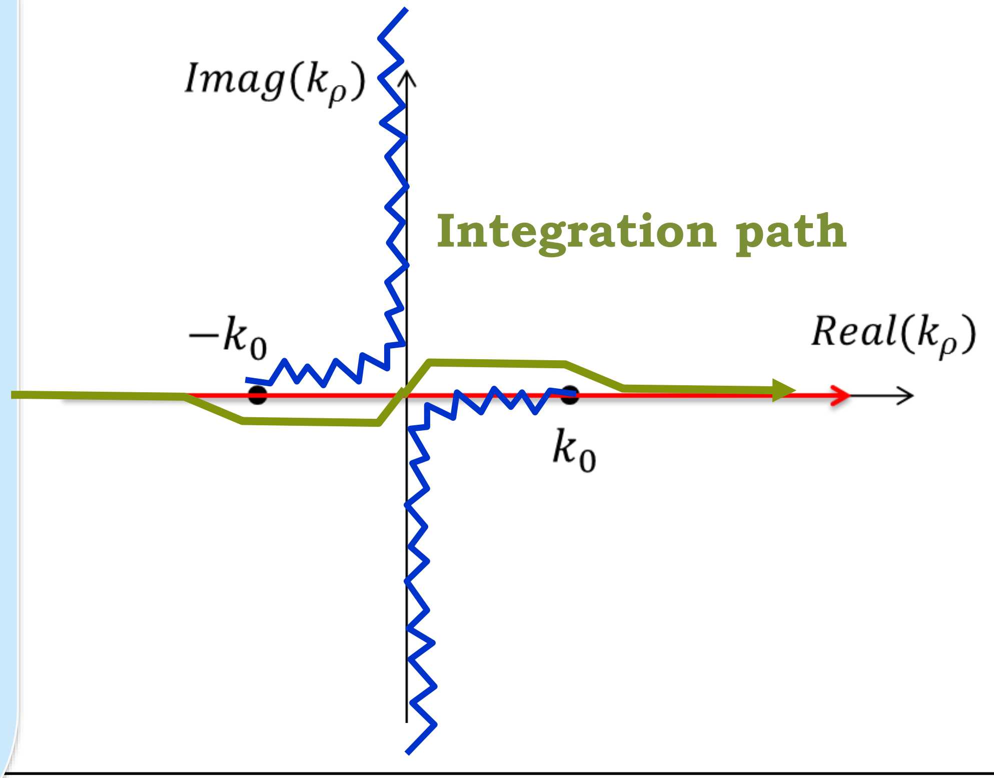
Selection of square root multivalues on the Riemann sheets to obtain a **unique specification of the integrand in the complex plane**

$$k_z = -j\sqrt{-\left(k^2 - k_\rho^2\right)}$$

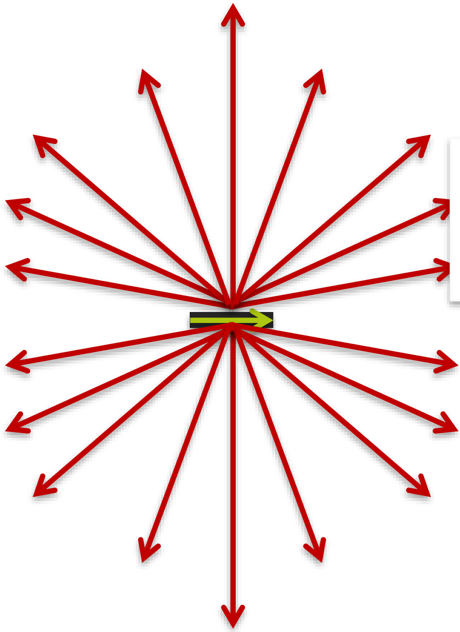
Top Riemann Sheet:



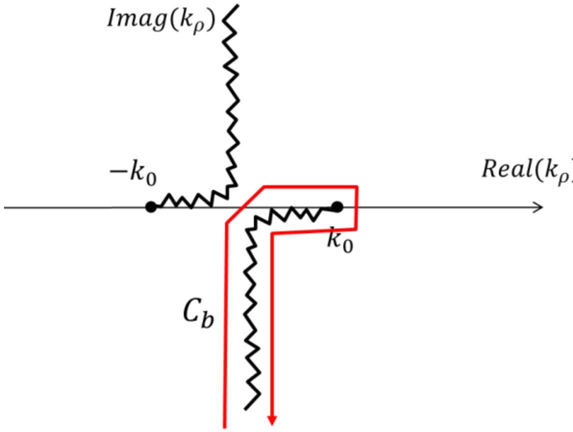




Space Wave



$$I_b = \int_{C_b} \tilde{\boldsymbol{G}}^{fc}(k_\rho, z, z')e^{-j\vec{\boldsymbol{k}}_\rho \cdot \vec{\boldsymbol{\rho}}} \; k_\rho dk_\rho$$



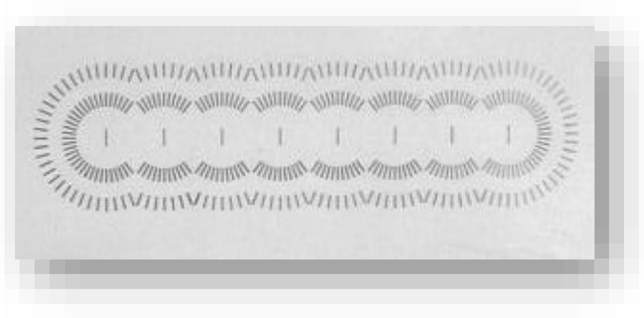
The field associated has a decay of **1/r**

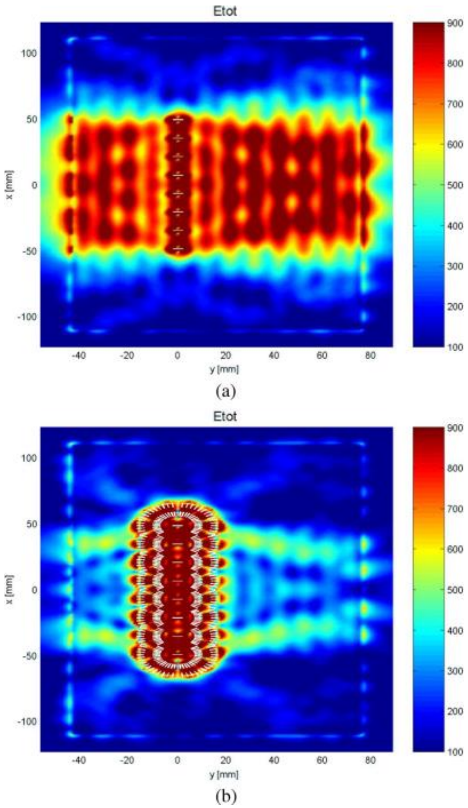
The far field is a space wave evaluated at large observation distances

$$\vec{f}^{far}(\vec{\boldsymbol{r}}) = jk_{zs}\tilde{\boldsymbol{G}}^{fc}(k_{xs}, k_{ys}, z, z')\vec{\boldsymbol{C}}(k_{xs}, k_{ys})e^{jk_{zs}|z-z'|}\frac{e^{-jkr}}{2\pi r}$$

$$\boldsymbol{k_\rho < k_0}$$

Fields radiated into the dielectric (surface wave)





General Stratification

$$\int_{-\infty}^{\infty} \tilde{\boldsymbol{G}}^{fc}(k_\rho,\alpha,z,z')e^{-j\vec{\boldsymbol{k}}_\rho\cdot\vec{\boldsymbol{\rho}}}\ k_\rho dk_\rho$$

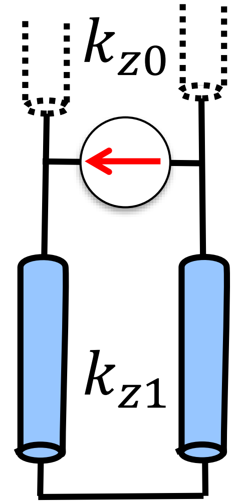
For n-dielectric layers, we have n different square roots

$$k_{zi} = \pm\sqrt{k_i^2 - k_\rho^2}$$

The dependency of $\tilde{G}^{fc}(k_\rho,\alpha,z,z')$ on k_{zi} is always even except for infinite mediums

There are not two different values associated to the $\pm$

$$Z_d = jZ_s\tan(k_{zs}h)$$



Branch cuts are only present in infinite open mediums. They give rise to the space wave field.

Pole singularities in k_ρ arise in dielectric stratifications.

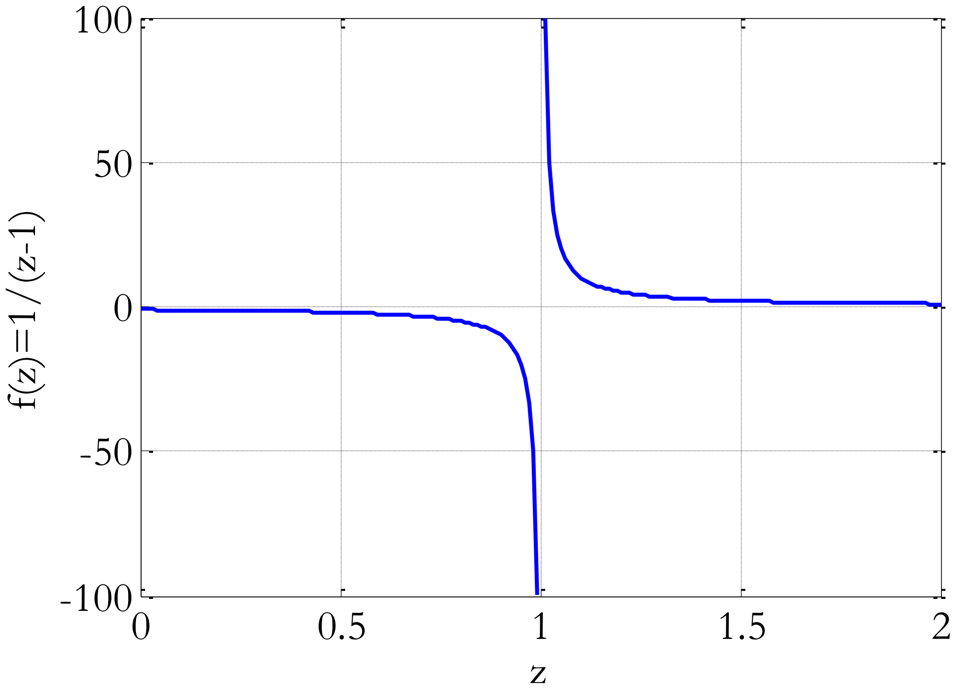
Pole Singularities

$$f(z) = \frac{g(z)}{(z-a)^n}$$

a is a pole of $f(z)$of order n provided that $g(a) \neq 0$

$$f(z) = \frac{1}{h(z)}$$

The zeros of $h(z)$ are the poles of $f(z)$

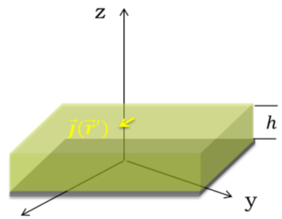


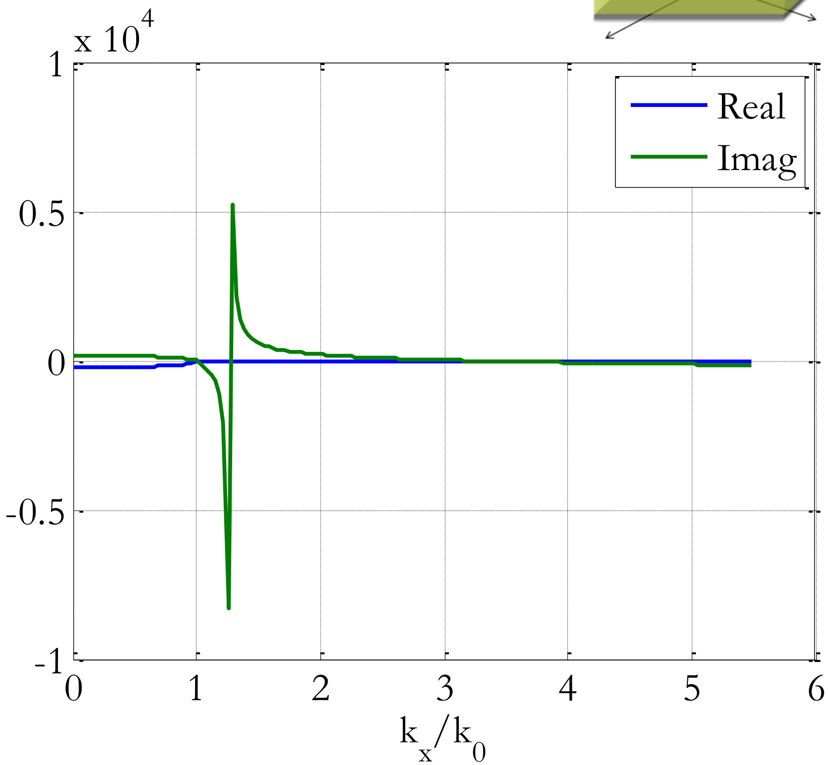
$$\boxed{h(z) = 0}$$

Dispersion equation to find the pole singularities

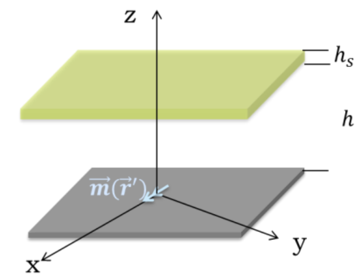
Examples

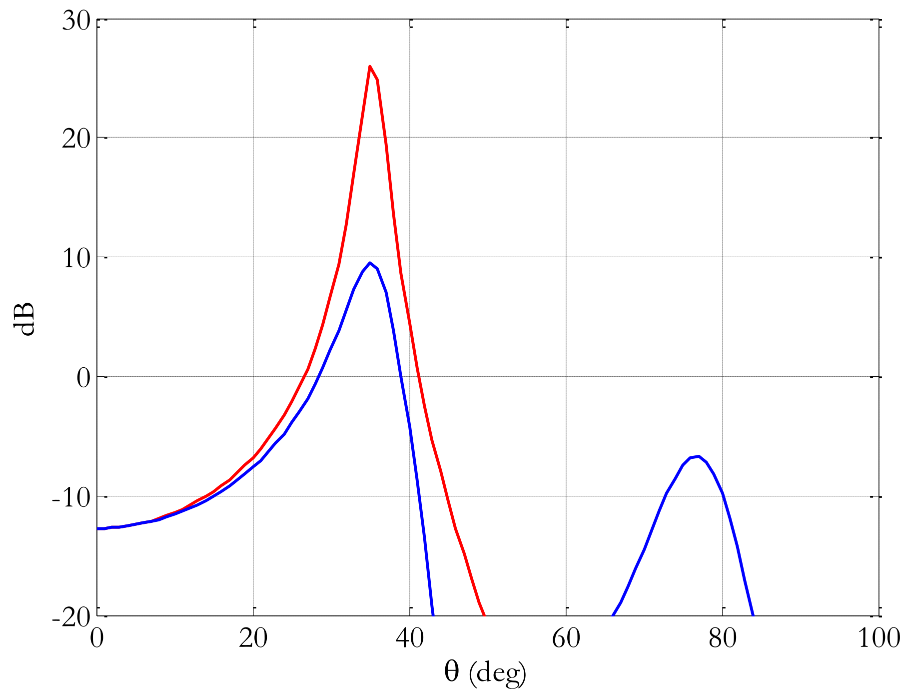
Surface Waves





Leaky Waves





Pole Singularities in the Trx Line Representation

$$\int_{-\infty}^{\infty} \tilde{\boldsymbol{G}}^{fc}(k_\rho, \alpha, z, z') e^{-j\vec{\boldsymbol{k}}_\rho \cdot \vec{\boldsymbol{\rho}}} \; k_\rho dk_\rho$$

$$\tilde{\boldsymbol{G}}^{ej} = \begin{bmatrix} -\frac{v_{TM}k_x^2 + v_{TE}k_y^2}{k_\rho^2} & \frac{(v_{TE} - v_{TM})k_x k_y}{k_\rho^2} \\ \frac{(v_{TE} - v_{TM})k_x k_y}{k_\rho^2} & -\frac{v_{TE}k_x^2 + v_{TM}k_y^2}{k_\rho^2} \\ \varsigma \frac{k_x}{k} i_{TM} & \varsigma \frac{k_y}{k} i_{TM} \end{bmatrix}$$

- The pole singularities come from the transmission line solution:

$$v_{TM/TE}(k_\rho, z, z') \;\&\; i_{TM/TE}(k_\rho, z, z')$$

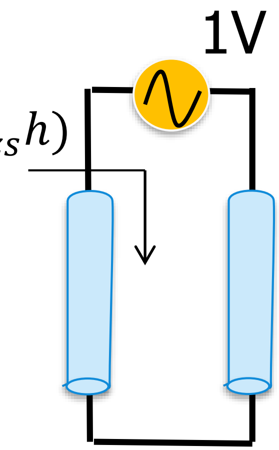
- They arise from the zeros of the denominator

$$v_{TM/TE}(k_\rho, z, z') = \frac{N_{TM/TE}(k_\rho, z, z')}{D_{TM/TE}(k_\rho)} \qquad i_{TM/TE}(k_\rho, z, z') = \frac{F_{TM/TE}(k_\rho, z, z')}{D_{TM/TE}(k_\rho)}$$

- Two type of poles can be found: TE and TM associated to the two types of transmission lines
- Different type of sources (J/M) on the same stratification will be characterized by the same surface waves (pole singularities)

Example: Parallel Plate Waveguide

$$Z_{in} = jZ_s \tan(k_{zs}h)$$



$$\tilde{\boldsymbol{G}}^{hm} = \begin{bmatrix} -\dfrac{i_{TE}k_x^2 + i_{TM}k_y^2}{k_\rho^2} & \dfrac{(i_{TM} - i_{TE})k_x k_y}{k_\rho^2} \\ \dfrac{(i_{TM} - i_{TE})k_x k_y}{k_\rho^2} & -\dfrac{i_{TM}k_x^2 + i_{TE}k_y^2}{k_\rho^2} \\ \dfrac{k_x}{\varsigma k} v_{TE} & \dfrac{k_y}{\varsigma k} v_{TE} \end{bmatrix}$$

$$i(z = z_s) = Y_{in} = \frac{1}{jZ_s \tan(k_{zs}h)}$$

The poles in the transversal complex plane-k_ρ can be found solving the following ***dispersion equation***

$$D(k_\rho) = jZ_s \tan(k_{zs}h) = 0$$

Example: Parallel Plate Waveguide

$Z_{TMi} = \zeta_i k_{zi}/k_i$

$D(k_\rho) = jZ_s \tan(k_{zs}h) = 0$

$Z_{TEi} = \zeta_i k_i/k_{zi}$

for TM $\quad k_{zs} \tan(k_{zs}h) = 0$

$k_{zs} = 0 \qquad \tan(k_{zs}h) = 0$

$k_{\rho_0} = k \qquad k_{zs}h = n\pi$

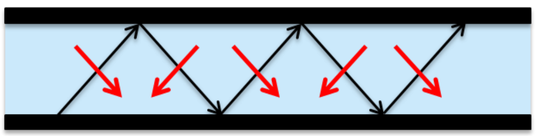
$$k_{\rho_n} = \sqrt{k^2 - \left(\frac{n\pi}{h}\right)^2}$$

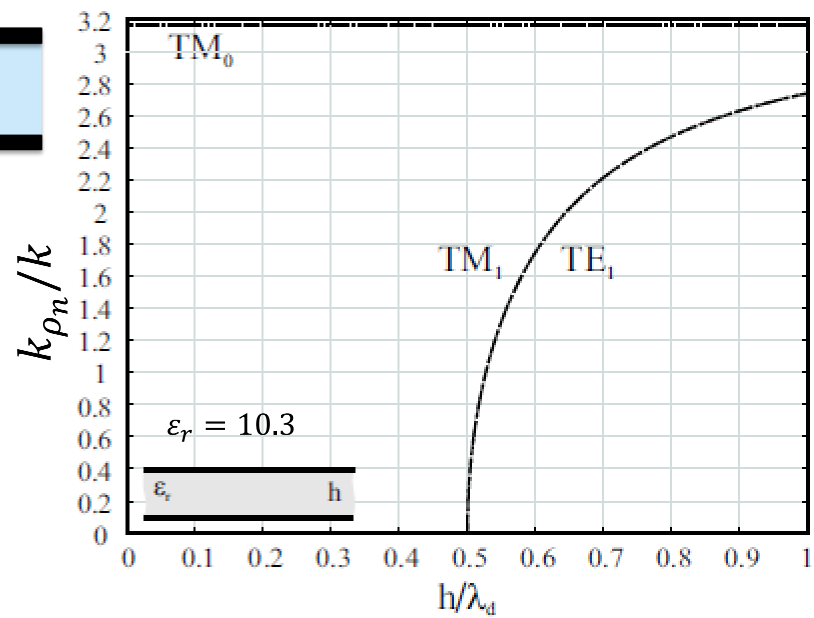
for TE $\quad \tan(k_{zs}h) = 0$

$k_{zs}h = n\pi$

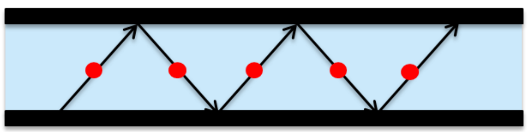
$$k_{\rho_n} = \sqrt{k^2 - \left(\frac{n\pi}{h}\right)^2}$$

TM Wave

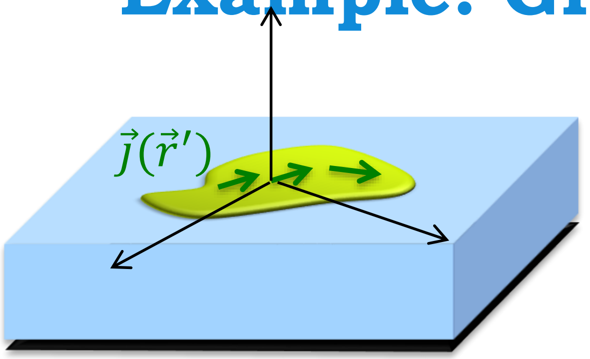




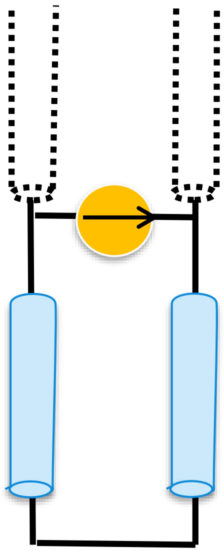
TE Wave



Example: Grounded Dielectric Substrate



$$Z_{in} = \frac{Z_u Z_d}{Z_u + Z_d} \quad \begin{cases} Z_u = Z_0 \\ Z_d = jZ_s \tan(k_{zs} h) \end{cases}$$



$$v(z = z_s) = Z_{in}$$

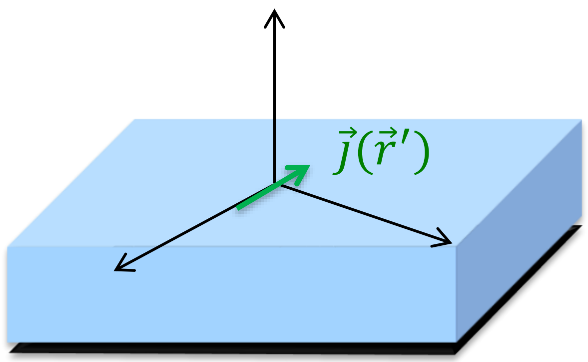
All the voltage and current solutions at any z-quote are expressed as a function of this voltage

The poles in the complex plane can be found solving the following **dispersion equation**

$$\boxed{D(k_\rho) = Z_u + Z_d = 0}$$

for TE and TM

Example: Grounded Dielectric Substrate



All the voltage and current solutions at any z-quote are expressed as a function of the same denominator, $D(k_\rho)$

Voltage in the slab:

$$V_s = \frac{Z_u Z_d}{Z_u + Z_d} \frac{sin(k_{zs}z)}{sin(k_{zs}h)} = \frac{Z_u Z_d}{D(k_\rho)} \frac{sin(k_{zs}z)}{sin(k_{zs}h)}$$

Current in the slab:

$$I_s = \frac{1}{Z_s} \frac{Z_u Z_d}{D(k_\rho)} \frac{j\cos(k_{zs}z)}{sin(k_{zs}h)}$$

Voltage in the air:

$$V_0 = \frac{Z_u Z_d}{D(k_\rho)} e^{jk_{z0}h} e^{-jk_{z0}z}$$

Current in the air:

$$I_0 = \frac{1}{Z_0} \frac{Z_u Z_d}{D(k_\rho)} e^{jk_{z0}h} e^{-jk_{z0}z}$$

Solving Dispersion Equations

If we know a good guess of the solution, k_ρ^g

1) One can expand the denominator around this point using the Taylor's series:

$$D(k_\rho) \approx D(k_\rho^g) + D'(k_\rho^g)(k_\rho - k_\rho^g)$$

2) Evaluating this expansion in the actual zero, $k_{\rho 0}$:

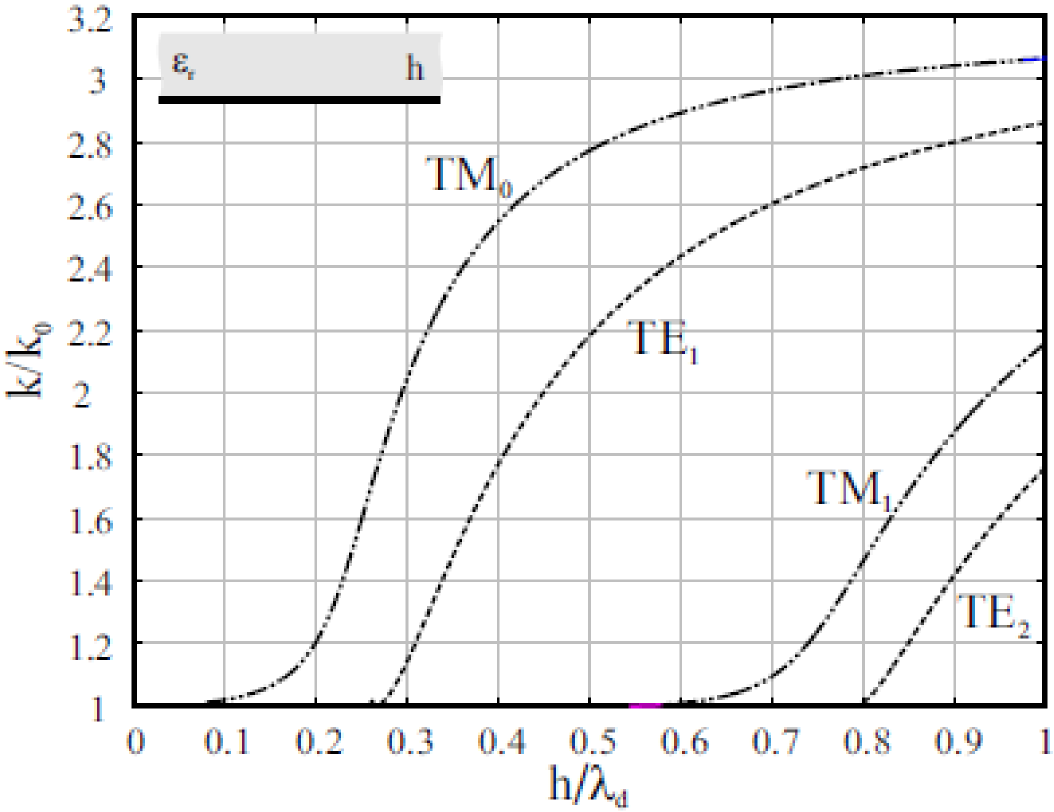
$$D(k_{\rho 0}) \approx D(k_\rho^g) + D'(k_\rho^g)(k_{\rho 0} - k_\rho^g) = 0$$

The derivative can be done numerically

$$k_{\rho 0} = k_\rho^g - \frac{D(k_\rho^g)}{D'(k_\rho^g)}$$

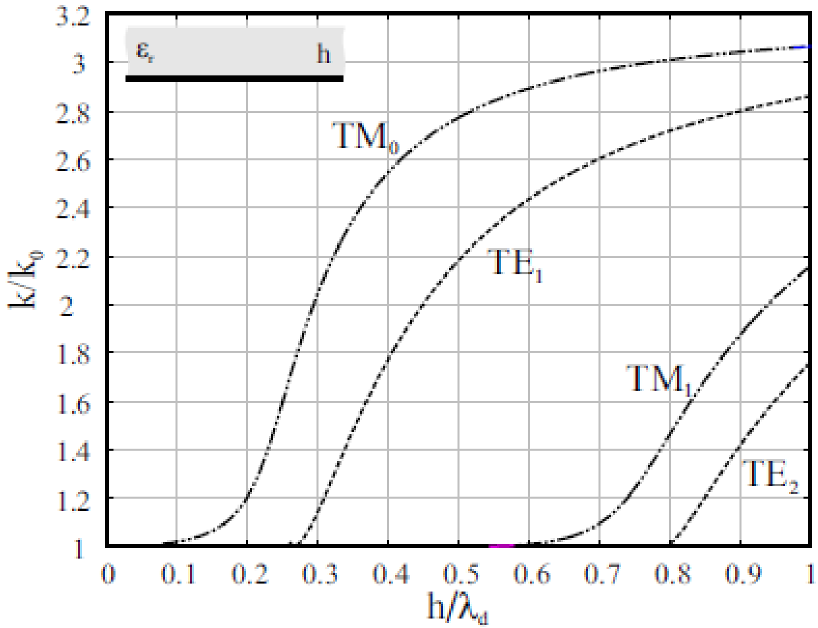
$$D'(k_\rho^g) \approx \frac{D(k_\rho^g + \Delta k/2) - D(k_\rho^g - \Delta k/2)}{\Delta k} \qquad \Delta k = k_0/500$$

SWs in a Grounded Slab



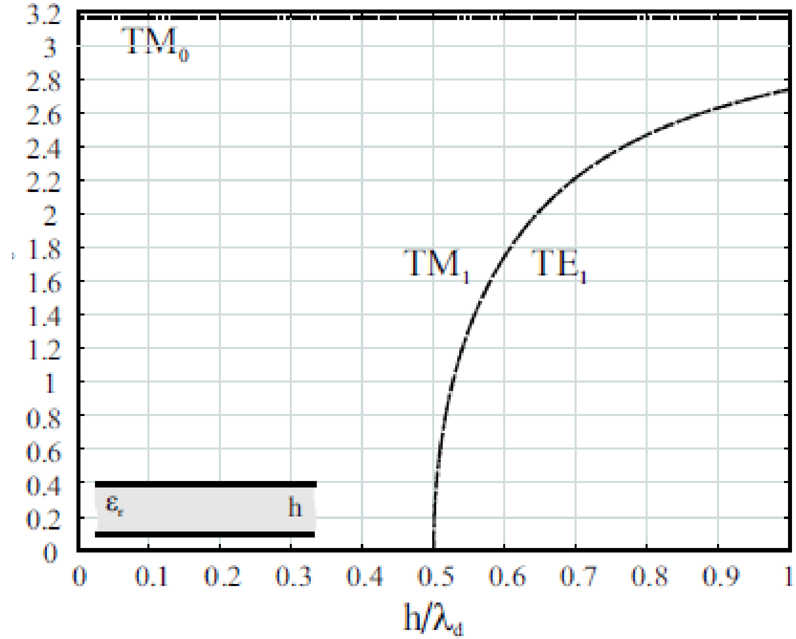
Grounded Slab vs PPW

Grounded Slab



Not a TEM (propagation constant goes from *k0* to *kd*)
TE1 starts propagation at $\lambda_d/4$

PPW



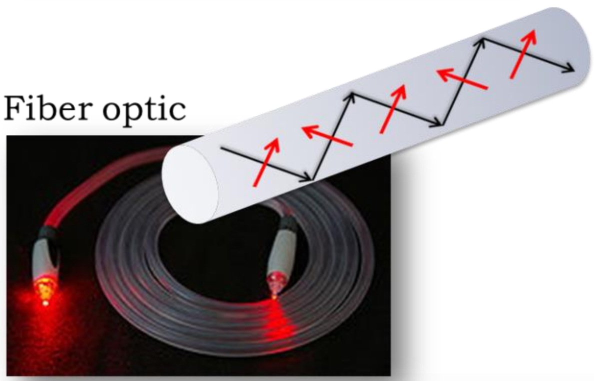
Fundamental mode, no cut-off
TEM mode with kd propagation constant
TE1/TM1 starts propagation at $\lambda_d/2$

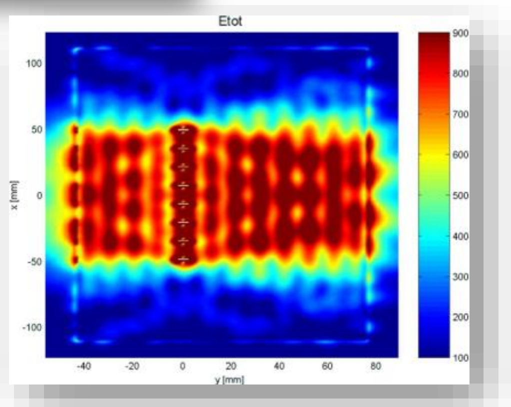
Surface Waves

Real transverse propagation constant

$$k_{\rho sw} \equiv \beta_{sw} > k_0$$

Fiber optic





Waves that propagate in the transverse direction without attenuation: $e^{-jk_{\rho sw}\rho}$

They correspond to *guided waves* inside the dielectric substrates

- They can be used as a dielectric waveguides
- They constitute a loss of power in antennas

Surface Waves

$$k_\rho^{sw} = \beta^{sw} = k_0\sqrt{\epsilon_r^{sw}}$$

$$k_0 < k_\rho^{sw} < k_d$$

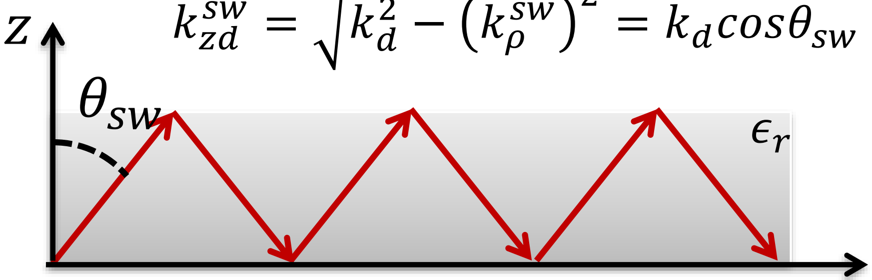
They are also referred as **slow waves**

$$k_\rho^{sw} = \frac{2\pi f}{v_{sw}} > k_0 \qquad v_{sw} < v_0$$

It can be seen as a couple of **homogenous waves propagating inside the dielectric** with a direction characterized by a real angle $\pm\theta_{sw}$:

$$\beta^{sw} = k_d sin\theta_{sw} < k_d \qquad sin\theta_{sw} = \frac{\sqrt{\epsilon_r^{sw}}}{\sqrt{\epsilon_r}}$$

$$k_{zd}^{sw} = \sqrt{k_d^2 - \left(k_\rho^{sw}\right)^2} = k_d cos\theta_{sw}$$



There is no attenuation, therefore the energy carried by the surface wave will reach *infinity in* ρ

It is an **inhomogenous wave in** the air region:

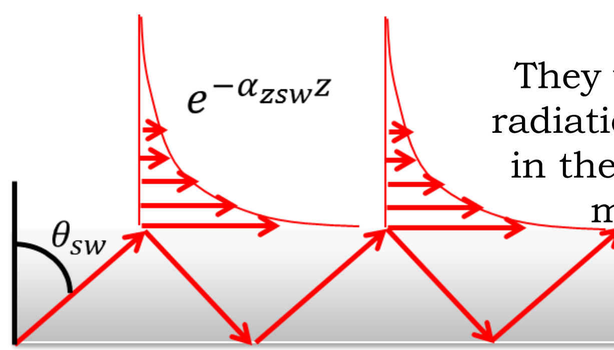
$$k_{z0sw} = -j\sqrt{-\left(k_0^2 - \left(k_\rho^{sw}\right)^2\right)} = -j\alpha_{zsw}$$

The sw angle is above the critical angle

$$\sqrt{\epsilon_r} sin\theta_{sw} = sin\theta_0$$

$$\sqrt{\epsilon_{rsw}} = sin\theta_0$$

Therefore θ_0 *is imaginary*



They verified the radiation condition in the infinite air medium

Asymptotic Evaluation of the SW Fields

How does the EM field of the SW depends on the observation point and on the excitation current shape?

$$\boldsymbol{f}(\vec{r}) = \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{\infty} \widetilde{\boldsymbol{G}}^{fc}(k_\rho,\alpha,z,z')\boldsymbol{C}_s(k_\rho,\alpha)e^{-jk_\rho\rho\cos(\alpha-\phi)}\, k_\rho dk_\rho d\alpha$$

$k_x = k_\rho cos\alpha \quad k_y = k_\rho sin\alpha$

$x = \rho cos\phi \qquad y = sin\phi$

The function $\boldsymbol{C}_s(k_\rho,\alpha)$ does not contain any singularity, and we assume that that this function is slowly varying function of $k_\rho$and α **for large observation points**.

$$\vec{\boldsymbol{f}}_i^{sw}(\vec{r}) \approx \frac{1}{4\pi^2}\int_0^{\pi}\int_{-\infty}^{\infty} \widetilde{\boldsymbol{G}}^{fc}(k_\rho,\alpha,z,z')e^{-jk_\rho\rho\cos(\alpha-\phi)}\, k_\rho dk_\rho d\alpha \boldsymbol{C}_s(k_{\rho swi},\alpha_s)$$

Note $\boldsymbol{C}_s(k_\rho,\alpha)$: k_ρ in surface wave pole, α in saddle point

Cauchy's theorem

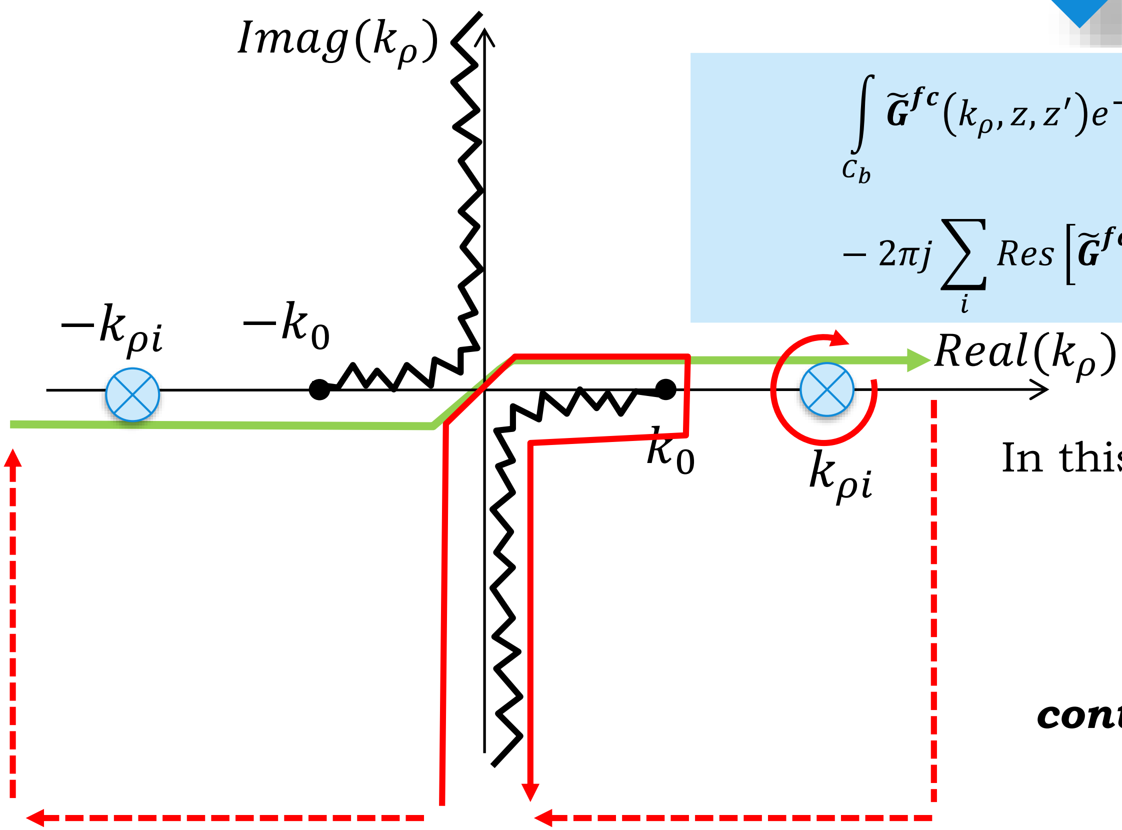
$$\int_{-\infty}^{\infty} \tilde{\boldsymbol{G}}^{fc}(k_\rho, \alpha, z, z') e^{-j\vec{k}_\rho \cdot \vec{\rho}} \; k_\rho dk_\rho$$

$$\int_R f(k_\rho) \mathrm{d}k_\rho = \int_{C_b} f(k_\rho) \mathrm{d}k_\rho - 2\pi j \sum_i Res[k_{\rho i}]$$

The field is divided into a space wave and a ***surface wave contribution***

$$\int_{C_b} \tilde{\boldsymbol{G}}^{fc}(k_\rho, z, z') e^{-j\vec{k}_\rho \cdot \vec{\rho}} \; k_\rho dk_\rho$$

$$- 2\pi j \sum_i Res\left[\tilde{\boldsymbol{G}}^{fc}(k_\rho, z, z') e^{-j\vec{k}_\rho \cdot \vec{\rho}} \; k_\rho\right]\Big|_{k_\rho = k_{\rho i}}$$



In this deformation path, only poles in the top Riemann sheet are captured

Poles gives rise to discrete contributions to the field in the form of waves

SW Fields

Evaluating current in the spectral critical point

$$\vec{\boldsymbol{f}}_c(\vec{r}) \approx \frac{1}{4\pi^2}\int_0^{\pi}\int_{-\infty}^{\infty} \widetilde{\boldsymbol{G}}^{fc}(k_\rho,\alpha,z,z')\, e^{-j\vec{\boldsymbol{k}}_\rho\cdot\vec{\boldsymbol{\rho}}}\; k_\rho dk_\rho d\alpha\, \vec{\boldsymbol{C}}(k_{\rho c},\alpha_{cs})$$

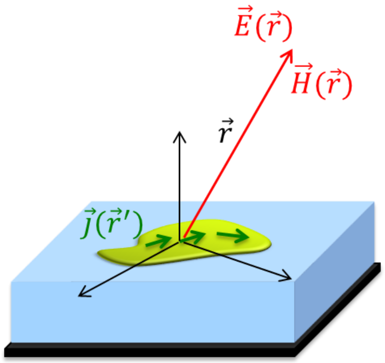
α_{cs}=*Saddle point in* α

Residue theorem

$$-2\pi j\sum_i Res\left[\widetilde{\boldsymbol{G}}^{fc}(k_\rho,z,z')e^{-j\vec{\boldsymbol{k}}_\rho\cdot\vec{\boldsymbol{\rho}}}\;k_\rho\right]\Big|_{k_\rho=k_{\rho i}}$$

$$\vec{\boldsymbol{f}}_i^{sw}(\vec{r}) \approx \frac{1}{4\pi^2}\int_0^{\pi}\left(-2\pi j\, Res\left[\widetilde{\boldsymbol{G}}^{fc}(k_\rho,z,z')\right]\Big|_{k_\rho=k_{\rho i}} k_{\rho i}e^{-j\vec{\boldsymbol{k}}_{\rho i}\cdot\vec{\boldsymbol{\rho}}}\right)d\alpha\vec{\boldsymbol{C}}(k_{\rho_i},\alpha_{cs})$$

$k_{\rho_i} = surface\ wave\ propagation\ constant$



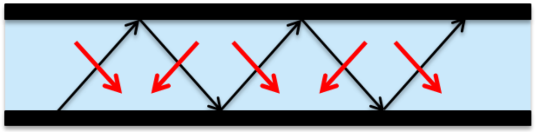
SW Field Contribution

Poles gives rise to discrete contributions to the field in the form of waves

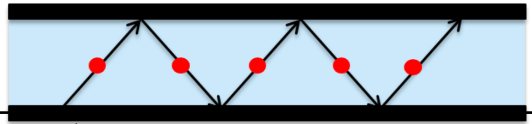
$$\vec{\boldsymbol{f}}_i^{sw}(\vec{r}) \approx \frac{1}{4\pi^2}\int_0^{\pi}\left(-2\pi j\, Res\left[\tilde{\boldsymbol{G}}^{fc}(k_\rho, z, z')\right]\Big|_{k_\rho = k_{\rho i}} k_{\rho i} e^{-j\vec{\boldsymbol{k}}_{\rho i}\cdot\vec{\boldsymbol{\rho}}}\right) d\alpha \vec{\boldsymbol{C}}(k_{\rho_i}, \alpha_s)$$

There is two kind of surface waves depending on the field polarization, related to the two transmission line field representation

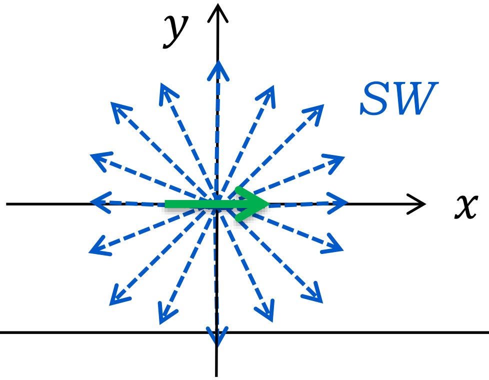
TM Wave



TE Wave



The surface waves propagate in radial direction ($\vec{\boldsymbol{\rho}}$) with a propagation constant given by the pole spectral location, $k_{\rho i}$

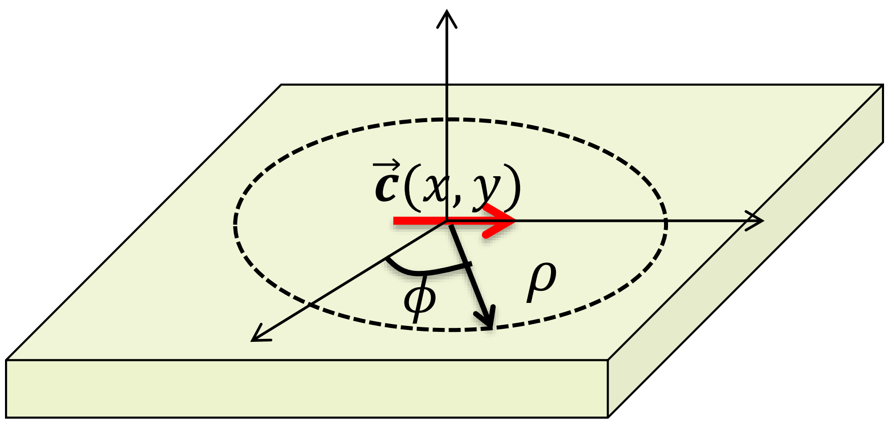


The amount of energy launched into the surface wave and its spatial distribution will also depend on the shape current via its FT

α-Integral

Since SWs do not attenuate in the dielectric (real propagation constant), for large distances, we can approximate the α-integrands similar to what we did for the far field, via a **saddle points**

$$\vec{\boldsymbol{f}}_i^{sw}(\vec{r}) \approx \frac{1}{4\pi^2}\int_0^{\pi}\left(-2\pi j\, Res\left[\widetilde{\boldsymbol{G}}^{fc}(k_\rho, z, z')\right]\Big|_{k_\rho=k_{\rho i}} k_{\rho i} e^{-j\vec{\boldsymbol{k}}_{\rho i}\cdot\vec{\boldsymbol{\rho}}}\right) d\alpha \vec{\boldsymbol{C}}(k_{\rho_i}, \alpha_s)$$



The Integral in α will be evaluated asymptotically

$$\vec{\boldsymbol{f}}_i^{sw}(\vec{r}) \approx \frac{-jk_{\rho i}}{2\pi} Res\left[\widetilde{\boldsymbol{G}}^{fc}(k_\rho, z, z')\right]\Big|_{k_\rho=k_{\rho i}} \vec{\boldsymbol{C}}(k_{\rho_i}, \alpha_s) \int_0^{\pi} e^{-j\vec{\boldsymbol{k}}_{\rho i}\cdot\vec{\boldsymbol{\rho}}}\, d\alpha$$

Pole-Residue

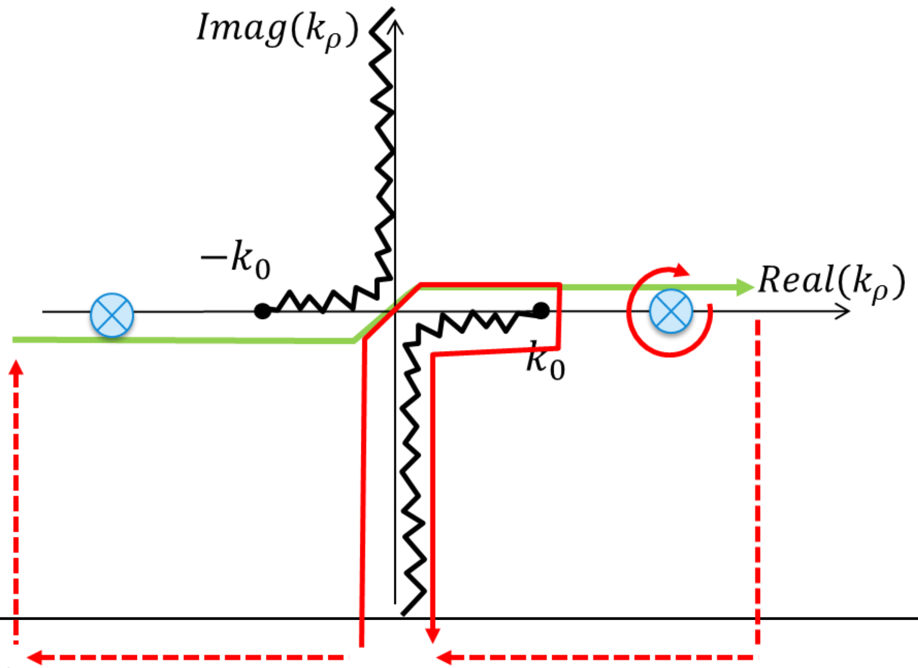
$$\int_0^\pi e^{-j\vec{\boldsymbol{k}}_{\rho i}\cdot\vec{\boldsymbol{\rho}}}\, d\alpha$$

$$e^{-j\vec{k}_{\rho i}\cdot\vec{\rho}} = e^{-jk_{\rho i}\rho\cos(\alpha-\phi)}$$

Let us consider:

$\vec{k}_{\rho i}\cdot\vec{\rho} > 0 \quad \Longleftrightarrow \quad k_{\rho i}\rho\cos(\alpha-\phi) > 0 \quad \cos(\alpha-\phi) > 0$

the poles with positive ksw are captured



We are now left with this integral

$$\int_0^\pi e^{-jk_{\rho i}\rho\cos(\alpha-\phi)}\, d\alpha$$

Stationary phase method

$$I = \int_{-\infty}^{\infty} f(x) e^{-j\Omega q(x)} dx \;\Rightarrow\; I \approx f(x_s) e^{-j\Omega q(x_s)} \sqrt{\frac{\pi}{\frac{1}{2}\Omega |q''(x_s)|}} e^{\mp j\pi/4}$$

Where $q(x)$ is a function with a minimum/ maximum in x_s

$$q'(x_s) = 0$$

$$\Omega > 0 \qquad \mp q''(x_s) \;{}^{>}/_{<}\; 0$$

$$I = \int_{0}^{\pi} e^{-jk_{\rho i}\rho \cos(\alpha - \phi)} d\alpha \;\Rightarrow\; \boxed{I = \int_{-\infty}^{\infty} f(\alpha) e^{-j\Omega q(\alpha)} d\alpha \qquad k_{\rho i}\rho = \Omega \quad q(\alpha) = cos(\alpha - \phi) \quad f(\alpha) = 1}$$

Saddle point $\quad q'(\alpha) = -sin(\alpha - \phi) \;\Rightarrow\; \alpha = \phi$

Stationary phase method

$$I = \int_0^{\pi} e^{-jk_{\rho i}\rho\cos(\alpha-\phi)}\, d\alpha$$

$$I = \int_{-\infty}^{\infty} f(\alpha) e^{-j\Omega q(\alpha)} d\alpha$$

$$k_{\rho i}\rho = \Omega$$

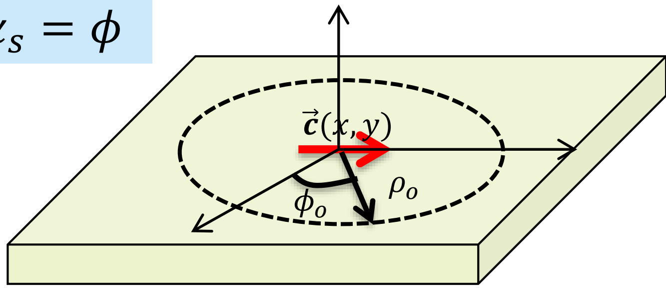
$$q(\alpha) = cos(\alpha - \phi)$$

$$f(\alpha) = 1$$

$$I \approx f(\alpha_s) e^{-j\Omega q(\alpha_s)} \sqrt{\frac{\pi}{\frac{1}{2}\Omega |q''(\alpha_s)|}}\, e^{j\pi/4}$$

$$q''(\alpha_s) = -cos(\alpha_s - \phi) = -1$$

$$\alpha_s = \phi$$



$$I \approx \frac{e^{-jk_{\rho i}\rho}}{\sqrt{\rho}} \left[\sqrt{\frac{\pi}{\frac{1}{2} k_{\rho i}}}\, e^{j\pi/4} \right]$$

SW Field Contribution

$$\vec{f}_i^{sw}(\vec{r}) \approx \frac{-jk_{\rho i}}{2\pi} Res\left[\tilde{\boldsymbol{G}}^{fc}(k_\rho, z, z')\right]\Big|_{k_\rho = k_{\rho i}} \vec{\boldsymbol{C}}(k_{\rho_i}, \alpha_s) \int_0^\pi e^{-j\vec{\boldsymbol{k}}_{\rho i}\cdot\vec{\boldsymbol{\rho}}}\, d\alpha$$

$$\alpha_s = \phi$$

$$\int_0^\pi e^{-j\vec{\boldsymbol{k}}_{\rho i}\cdot\vec{\boldsymbol{\rho}}}\, d\alpha = \frac{e^{-jk_{\rho i}\rho}}{\sqrt{\rho}}\left[\sqrt{\frac{\pi}{\frac{1}{2}k_{\rho i}}}\, e^{j\pi/4}\right]$$

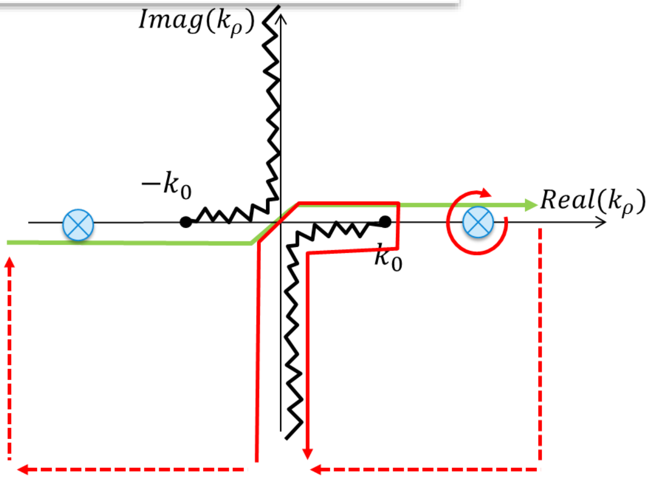
$$\vec{f}_i^{sw}(\vec{r}) \approx \frac{-jk_{\rho i}}{2\pi} Res\left[\tilde{\boldsymbol{G}}^{fc}(k_\rho, z, z')\right]\Big|_{k_\rho = k_{\rho i}} \vec{\boldsymbol{C}}(k_{\rho i}, \phi) \frac{e^{-jk_{\rho i}\rho}}{\sqrt{\rho}}\left[\sqrt{\frac{\pi}{\frac{1}{2}k_{\rho i}}}\, e^{j\pi/4}\right]$$

$$\vec{f}_i^{sw}(\vec{r}) \approx -je^{j\pi/4}\sqrt{\frac{k_{\rho i}}{2\pi}}\, Res\left[\tilde{\boldsymbol{G}}^{fc}(k_\rho, z, z')\right]\Big|_{k_\rho = k_{\rho i}} \vec{\boldsymbol{C}}(k_{\rho i}, \phi) \frac{e^{-jk_{\rho i}\rho}}{\sqrt{\rho}}$$

SW Field Contribution

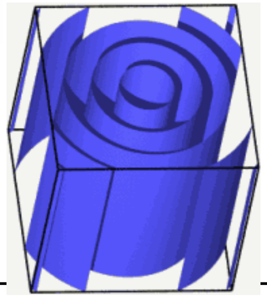
$$\boldsymbol{f}(\vec{r}) = \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{\infty} \widetilde{\boldsymbol{G}}^{fc}(k_\rho,\alpha,z,z')\boldsymbol{C}_s(k_\rho,\alpha)e^{-jk_\rho\rho\cos(\alpha-\phi)}\, k_\rho dk_\rho d\alpha$$

Pole singularities:
Discrete field contributions in the form of waves



$$\vec{\boldsymbol{f}}_i^{sw}(\vec{r}) \approx -je^{j\pi/4}\sqrt{\frac{k_{\rho i}}{2\pi}}\,Res\left[\widetilde{\boldsymbol{G}}^{fc}(k_\rho,z,z')\right]\Big|_{k_\rho=k_{\rho i}}\vec{\boldsymbol{C}}(k_{\rho i},\phi)\frac{e^{-jk_{\rho i}\rho}}{\sqrt{\rho}}$$

$$\boldsymbol{\frac{e^{-jk_{sw}\rho}}{\sqrt{\rho}}}$$



Residue

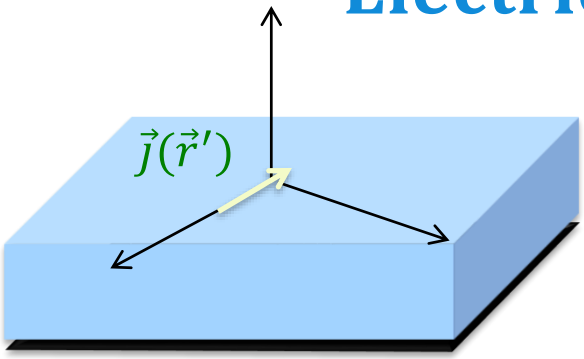
$$f(z) = \frac{g(z)}{z-a} \quad \Rightarrow \quad Res(f(z)) = g(a)$$

$$f(z) = \frac{g(z)}{h(z)} \quad \Rightarrow \quad Res(f(z)) = \frac{g(a)}{h'(a)}$$

$$v_{TM}(k_\rho, z, z') = \frac{N(k_\rho)}{D(k_\rho)} \quad \Rightarrow \quad Res\left[v_{TM}(k_\rho, z, z')\right]\Big|_{k_{\rho i}} = \frac{N(k_{\rho i})}{D'(k_{\rho i})}$$

$$D'(k_\rho^g) \approx \frac{D\left(k_\rho^g + \Delta k/2\right) - D\left(k_\rho^g - \Delta k/2\right)}{\Delta k}$$

Electric current oriented along x



$$\vec{f}_i^{sw}(\vec{r}) \approx Res\left[\tilde{\boldsymbol{G}}^{fj}(k_\rho, z, z')\right]\Big|_{k_\rho = k_{\rho i}} \vec{\boldsymbol{J}}(k_{\rho i}, \phi)\frac{Ce^{-jk_{\rho i}\rho}}{\sqrt{\rho}}$$

$$C = j\sqrt{\frac{k_{\rho i}}{2\pi}}e^{j\frac{\pi}{4}}$$

$$\tilde{\boldsymbol{G}}^{ej} = \begin{bmatrix} -\frac{v_{TM}k_x^2 + v_{TE}k_y^2}{k_\rho^2} & \frac{(v_{TE} - v_{TM})k_xk_y}{k_\rho^2} \\ \frac{(v_{TE} - v_{TM})k_xk_y}{k_\rho^2} & -\frac{v_{TE}k_x^2 + v_{TM}k_y^2}{k_\rho^2} \\ \varsigma\frac{k_x}{k}i_{TM} & \varsigma\frac{k_y}{k}i_{TM} \end{bmatrix}$$

$$k_x = k_\rho \cos\phi$$
$$k_y = k_\rho \sin\phi$$

TM Wave

$$E_x(\rho, z) \approx Res[v_{TM}(k_\rho, z, z')]\Big|_{k_{\rho i}} J_x(k_{\rho i}, \phi)\frac{Ce^{-jk_{\rho i}\rho}}{\sqrt{\rho}}\cos^2\phi$$

$$E_y(\rho, z) \approx Res[v_{TM}(k_\rho, z, z')]\Big|_{k_{\rho i}} J_x(k_{\rho i}, \phi)\frac{Ce^{-jk_{\rho i}\rho}}{\sqrt{\rho}}cos\phi sin\phi$$

$$E_z(\rho, z) \approx -\frac{\varsigma k_{\rho i}}{k}Res[i_{TM}(k_\rho, z, z')]\Big|_{k_{\rho i}} J_x(k_{\rho i}, \phi)\frac{Ce^{-jk_{\rho i}\rho}}{\sqrt{\rho}}\cos\phi$$

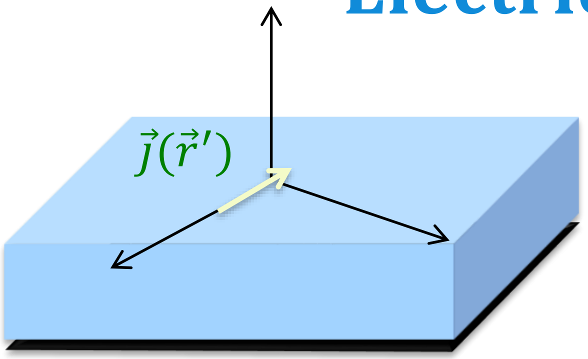
TE Wave

$$E_x(\rho, z) \approx Res[v_{TE}(k_\rho, z, z')]\Big|_{k_{\rho i}} J_x(k_{\rho i}, \phi)\frac{Ce^{-jk_{\rho i}\rho}}{\sqrt{\rho}}\sin^2\phi$$

$$E_y(\rho, z) \approx -Res[v_{TE}(k_\rho, z, z')]\Big|_{k_{\rho i}} J_x(k_{\rho i}, \phi)\frac{Ce^{-jk_{\rho i}\rho}}{\sqrt{\rho}}cos\phi sin\phi$$

$$E_z(\rho, z) \approx 0$$

Electric current oriented along x



$$f_\rho = f_x cos\phi + f_y sin\phi$$

$$f_\phi = -f_x sin\phi + f_y cos\phi$$

Cylindrical field components

TM Wave

$$E_\rho(\rho,z) \approx Res[v_{TM}(k_\rho,z,z')]\Big|_{k_{\rho i}} J_x(k_{\rho i},\phi)\frac{C\cos\phi\, e^{-jk_{\rho i}\rho}}{\sqrt{\rho}}$$

$$E_z(\rho,z) \approx -\frac{\varsigma k_{\rho i}}{k} Res[i_{TM}(k_\rho,z,z')]\Big|_{k_{\rho i}} J_x(k_{\rho i},\phi)\frac{C\cos\phi\, e^{-jk_{\rho i}\rho}}{\sqrt{\rho}}$$

$$H_\phi(\rho,z) \approx Res[i_{TM}(k_\rho,z,z')]\Big|_{k_{\rho i}} J_x(k_{\rho i},\phi)\frac{C\cos\phi\, e^{-jk_{\rho i}\rho}}{\sqrt{\rho}}$$

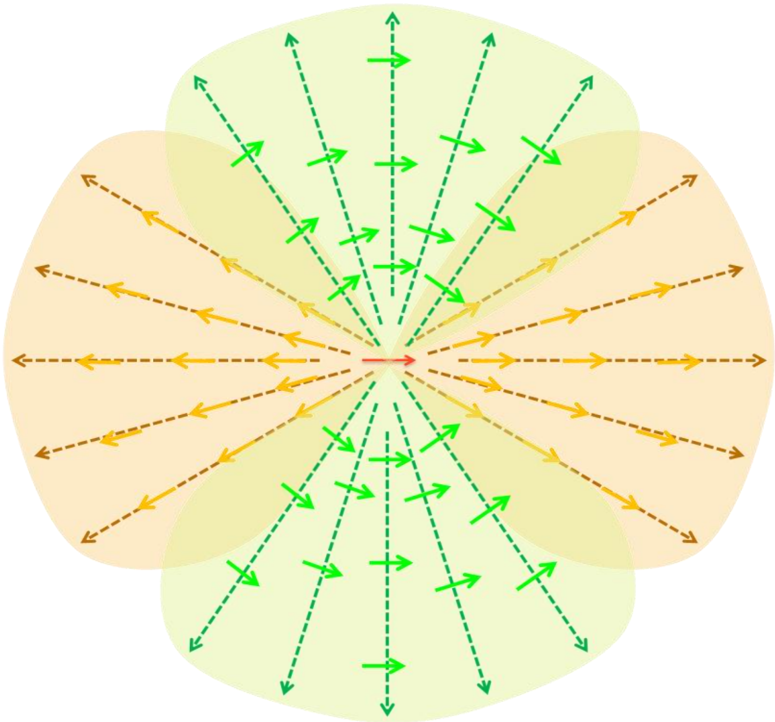
TE Wave

$$E_\phi(\rho,z) \approx -Res[v_{TE}(k_\rho,z,z')]\Big|_{k_{\rho i}} J_x(k_{\rho i},\phi)\frac{C\sin\phi\, e^{-jk_{\rho i}\rho}}{\sqrt{\rho}}$$

$$H_\rho(\rho,z) \approx Res[i_{TE}(k_\rho,z,z')]\Big|_{k_{\rho i}} J_x(k_{\rho i},\phi)\frac{C\sin\phi\, e^{-jk_{\rho i}\rho}}{\sqrt{\rho}}$$

$$H_z(\rho,z) \approx -\frac{k_{\rho i}}{k\varsigma} Res[v_{TE}(k_\rho,z,z')]\Big|_{k_{\rho i}} J_x(k_{\rho i},\phi)\frac{C\sin\phi\, e^{-jk_{\rho i}\rho}}{\sqrt{\rho}}$$

Cylindrical TE/TM Surface Waves

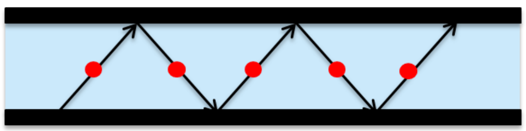


TE Wave

Electric field oriented in the azimuthal direction
Maximum in the Hplane of the antenna

$$E_\phi(\rho, z) \approx -Res\left[v_{TE}(k_\rho, z, z')\right]\Big|_{k_{\rho i}} j_x(k_{\rho i}, \phi)\frac{C\sin\phi\, e^{-jk_{\rho i}\rho}}{\sqrt{\rho}}$$

TE Wave



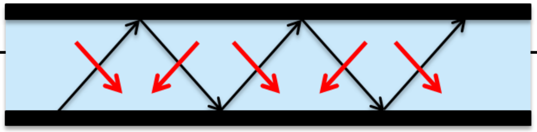
TM Wave

Electric field oriented in the radial and z direction
Maximum in the Eplane of the antenna

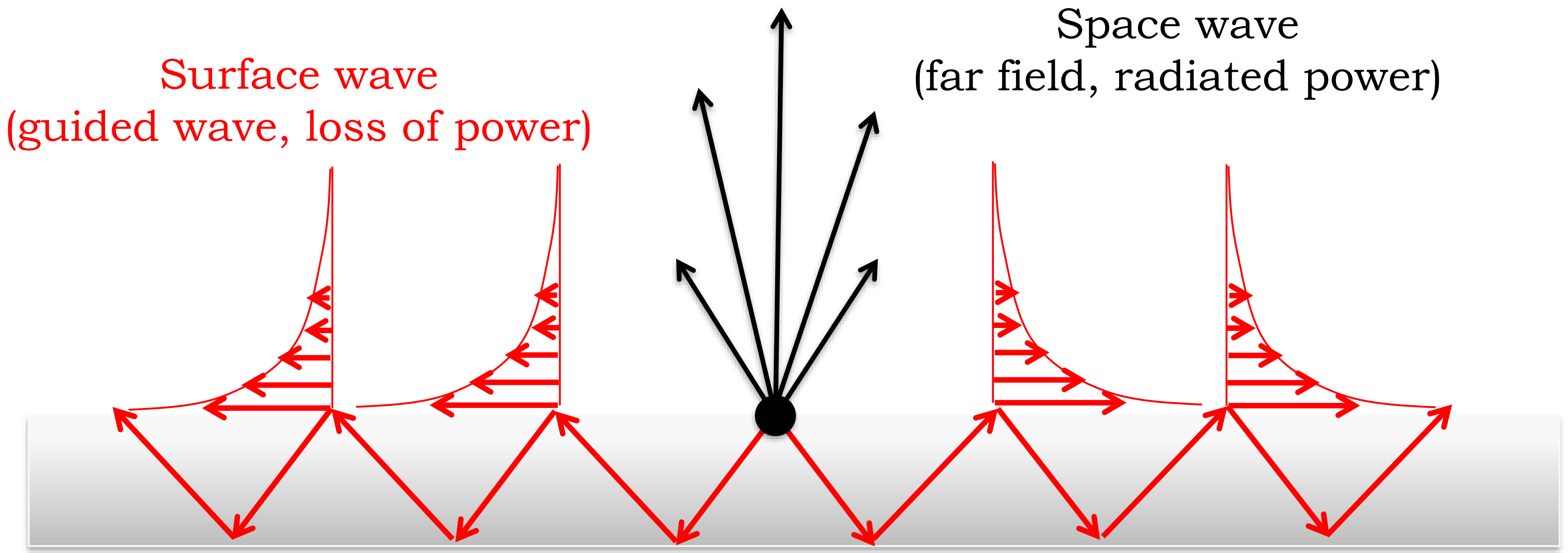
$$E_\rho(\rho, z) \approx Res\left[v_{TM}(k_\rho, z, z')\right]\Big|_{k_{\rho i}} j_x(k_{\rho i}, \phi)\frac{C\cos\phi\, e^{-jk_{\rho i}\rho}}{\sqrt{\rho}}$$

$$E_z(\rho, z) \approx -\frac{\varsigma k_{\rho i}}{k} Res\left[i_{TM}(k_\rho, z, z')\right]\Big|_{k_{\rho i}} j_x(k_{\rho i}, \phi)\frac{C\cos\phi\, e^{-jk_{\rho i}\rho}}{\sqrt{\rho}}$$

TM Wave



SW Characterization

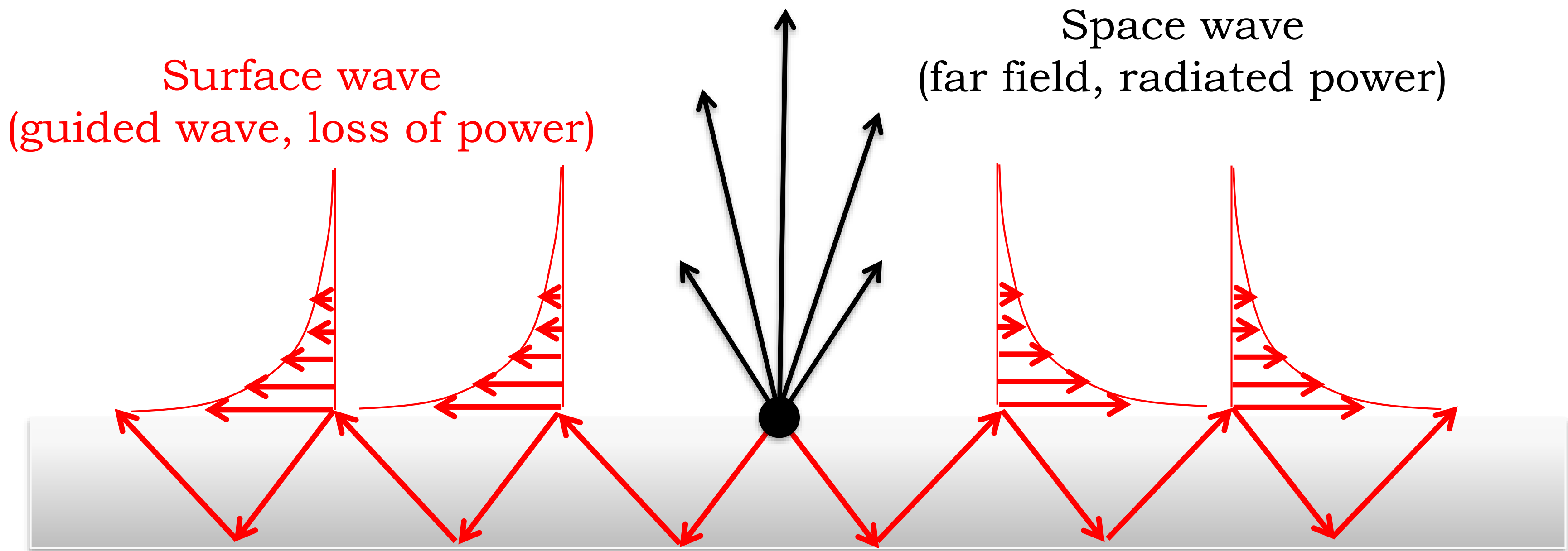


1) Find the SW propagation constant (**dispersion equation**)

2) Calculate the EM fields associated to a surface wave (**Residue**)

3) Power launched into the SW (**Poynting vector**)

Antenna SW Efficiency



We are going to characterize the lose of efficiency into SW:

1. Find the SW propagation constant (dispersion equation)
2. Calculate the SW fields (Residue)
3. Power into SW vs radiated power

$$\eta_{sw} = \frac{P_{rad}}{P_{rad} + P_{sw}}$$

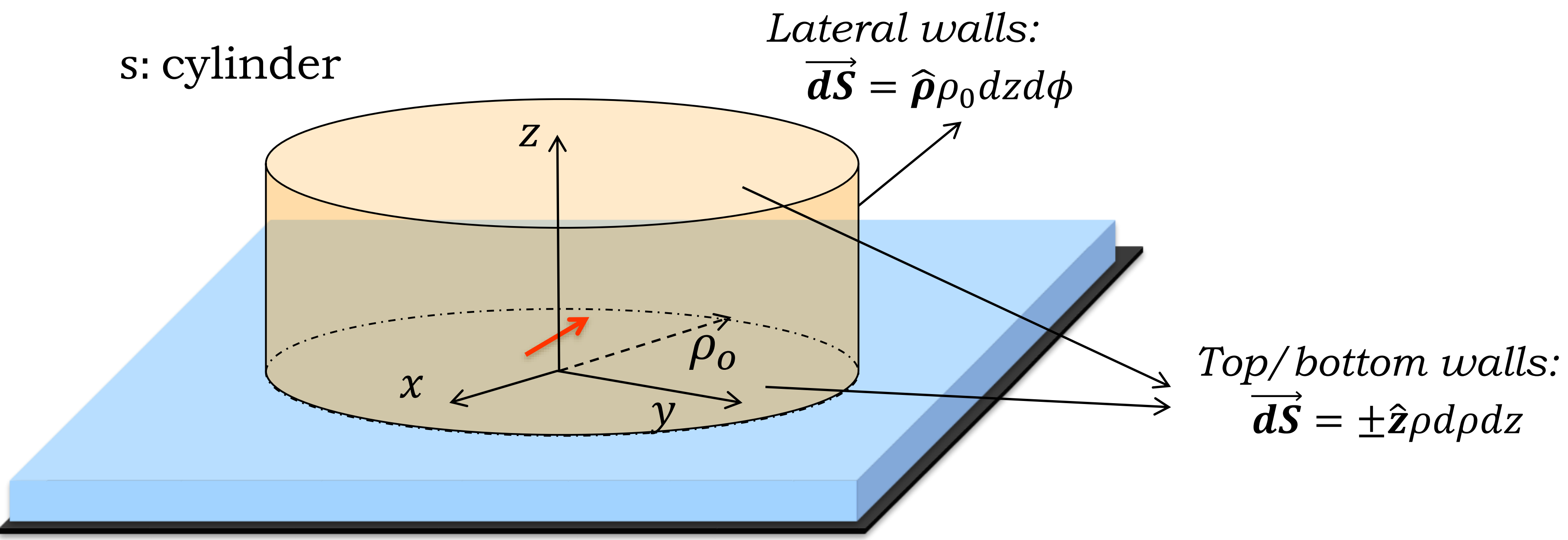
SW Power

We need to characterize the power launched into surface waves

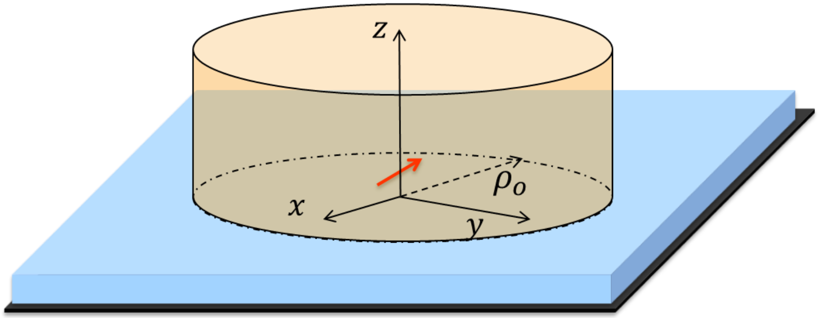
By integrating the Poynting vector around a closed surface surrounding the antenna

$$P_{sw} = \frac{1}{2}\sum_i \iint_S Re\left[\vec{\boldsymbol{E}}_{\boldsymbol{sw_i}} \times \vec{\boldsymbol{H}}^*_{\boldsymbol{sw_i}}\right] \cdot \boldsymbol{d}\vec{\boldsymbol{s}}$$

s: cylinder



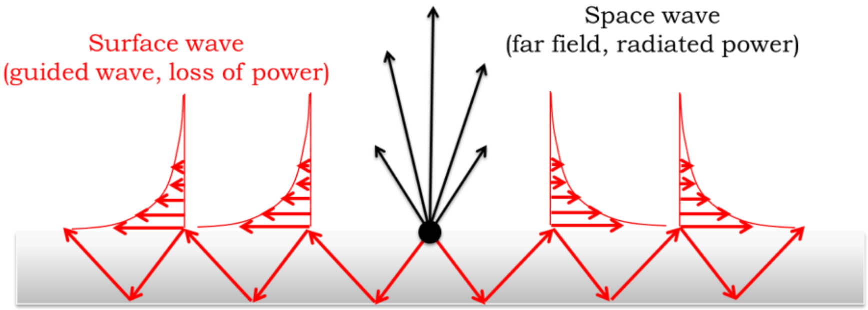
SW Poynting's Vector



$$\vec{S} = \frac{1}{2}\left(E_\rho H_\phi^* - E_\phi H_\rho^*\right)\hat{\mathbf{z}} + \frac{1}{2}\left(E_\phi H_z^* - E_z H_\phi^*\right)\hat{\boldsymbol{\rho}}$$

$$\downarrow$$

$\vec{S}\cdot\hat{\boldsymbol{n}} = 0$ *at top/bottom walls:*



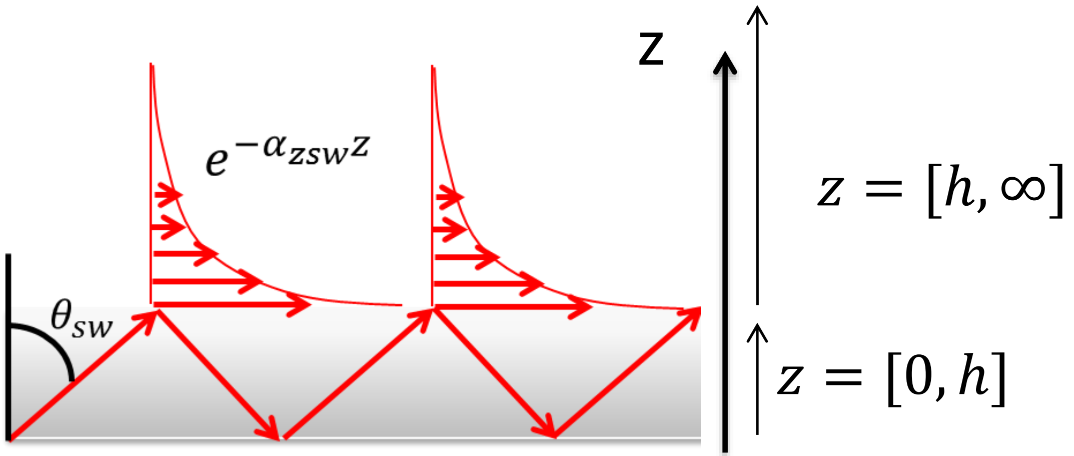
$$P_{sw}^{TM} = \frac{1}{2}\int_0^{2\pi}\int_0^{\infty} Re\left[-E_z H_\phi^*\right]\rho_s dz d\phi$$

$$P_{sw}^{TE} = \frac{1}{2}\int_0^{2\pi}\int_0^{\infty} Re\left[E_\phi H_z^*\right]\rho_s dz d\phi$$

SW Power

The integral along z is performed for every dielectric layer individually since the fields solutions of the trx line changes

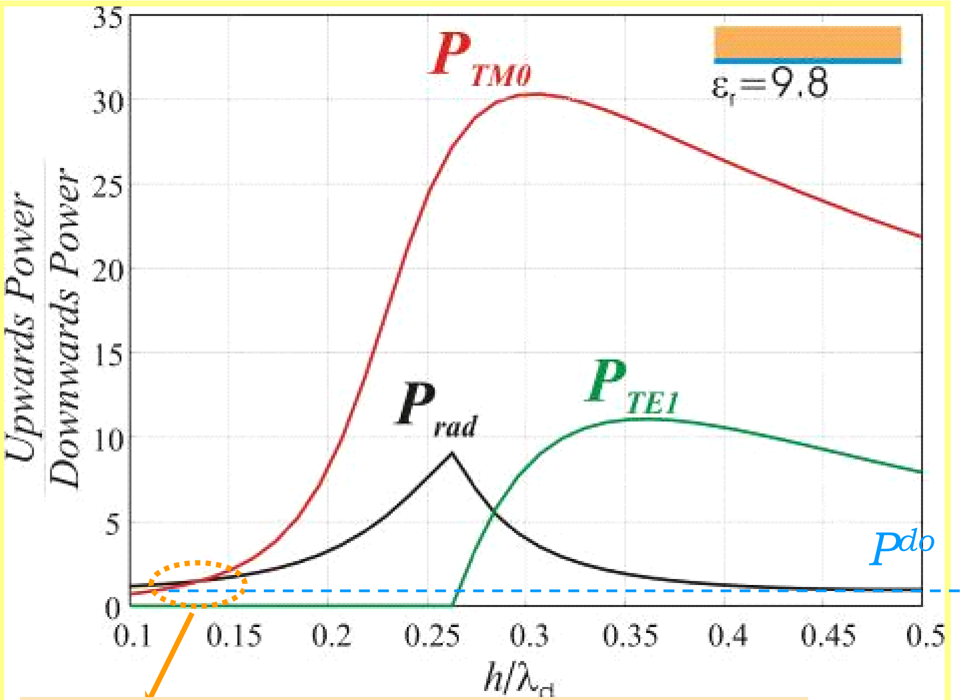
$$P_{sw}^{TM} = \frac{1}{2}\int_0^{2\pi}\int_0^{\infty} Re\left[-E_z H_\phi^*\right]\rho_s dz d\phi$$

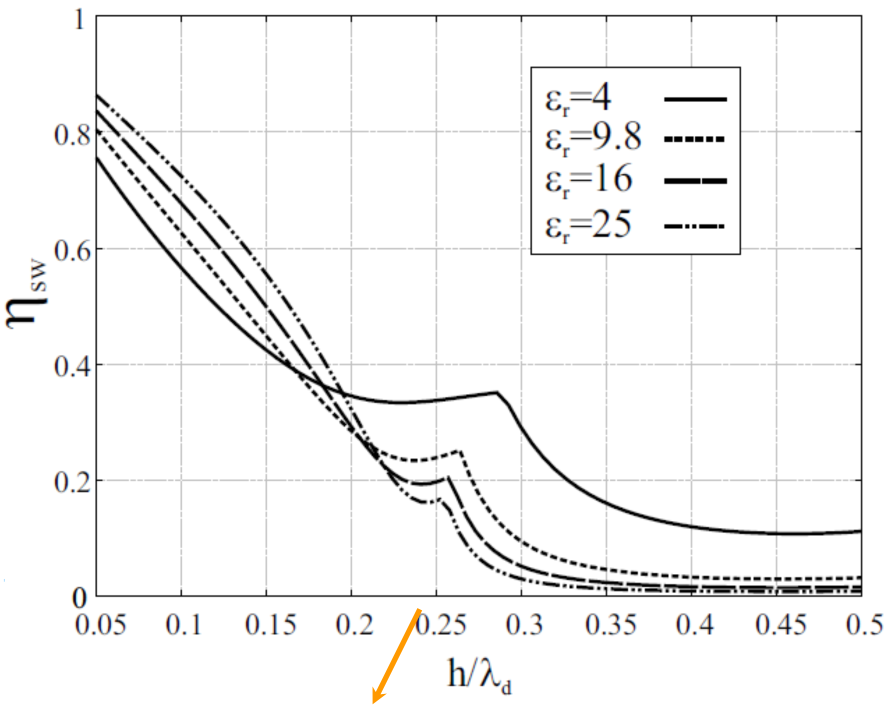


SWs in Integrated Antennas

$$P_{Z_A} = \frac{1}{2}|I_0|^2 Re\{Z_A\} = P_{rad} + P_{sw}$$

Radiation from elementary slot

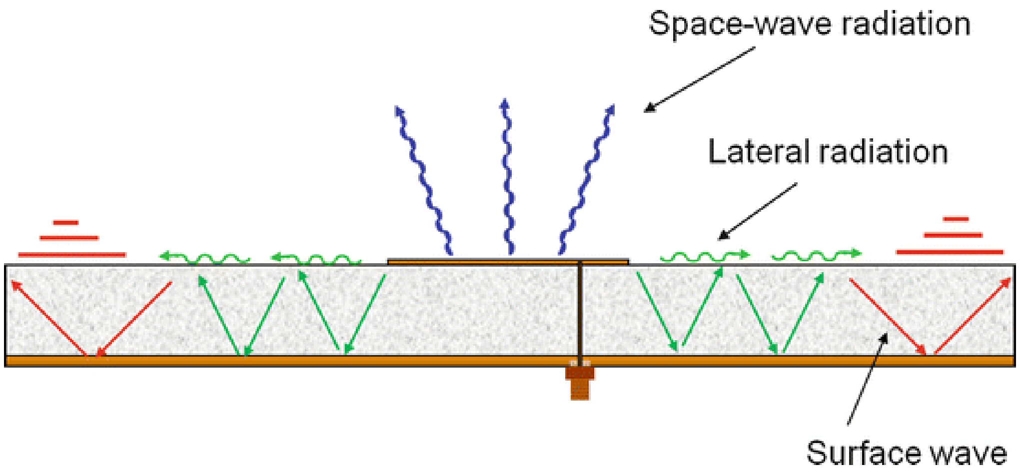




Standard antenna
Designs, poor front to back or very small bandwidths (<10%)

Optimum front to back
Poor surface efficiency

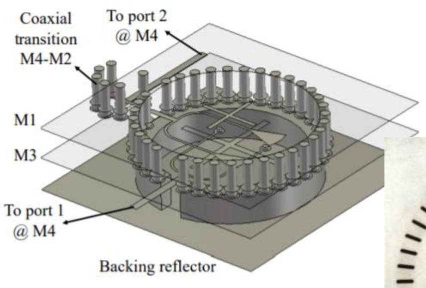
SWs in Integrated Antennas

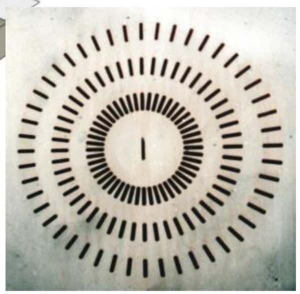


Edge Radiation

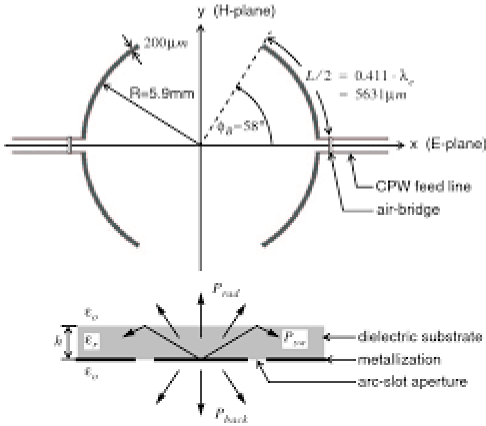
Ways to suppress surface waves

1) Cavity via vias/EBGs

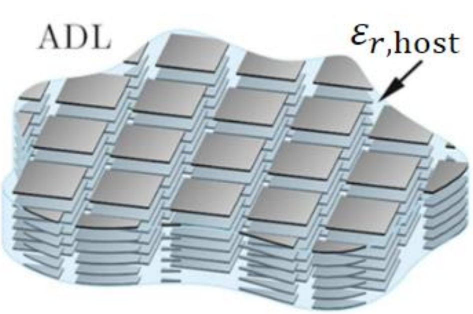




2) Double arc slot



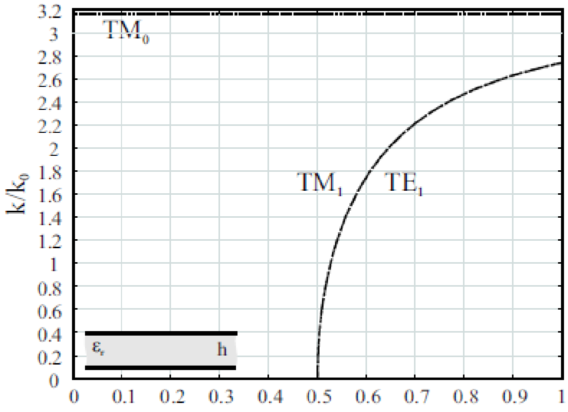
3) Artificial dielectrics

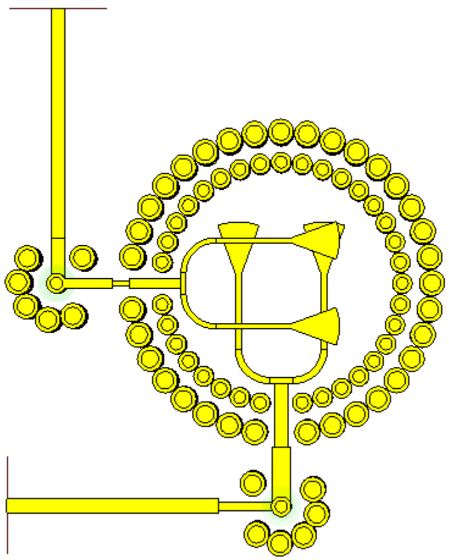


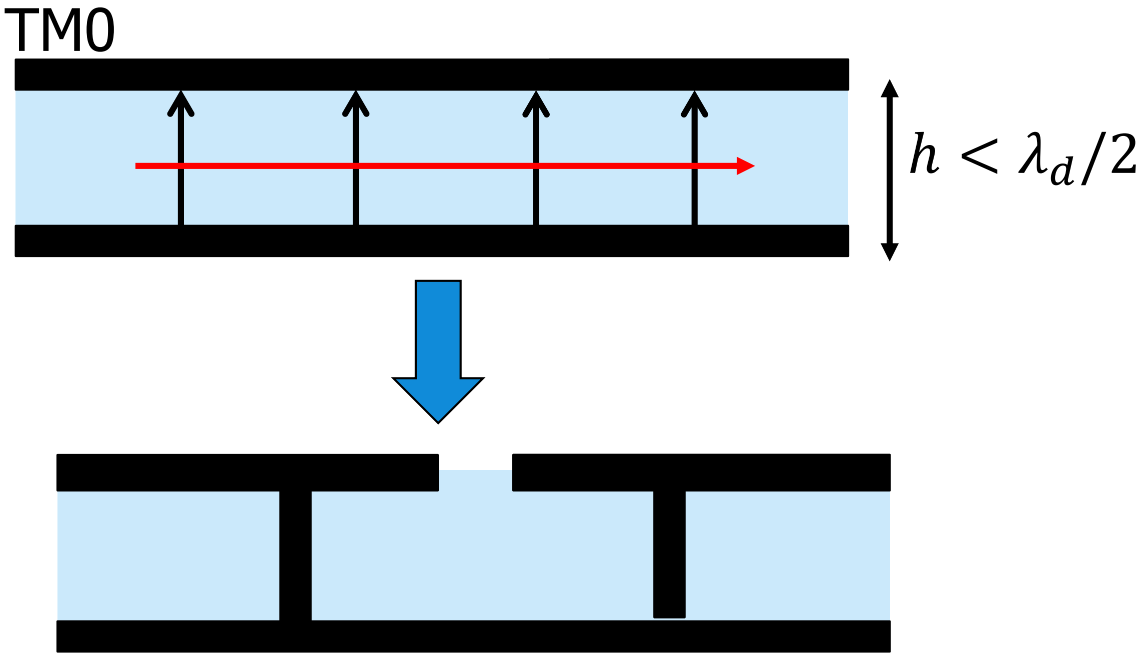
Controlling SW in thick dielectrics

1) Vertical vias

Suppression of the TM0 mode in a PPW







Vertical pins will bock the TM surface

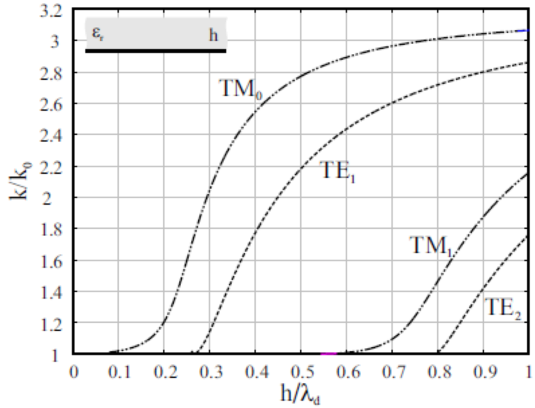
Power reflected into the antenna

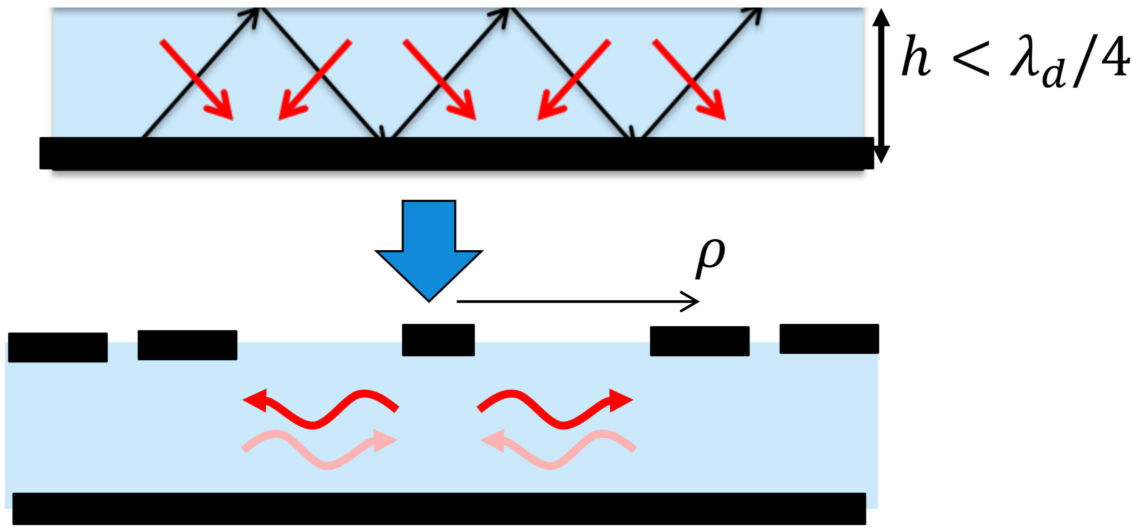
Resonant cavity (bandwidths of about 20%)

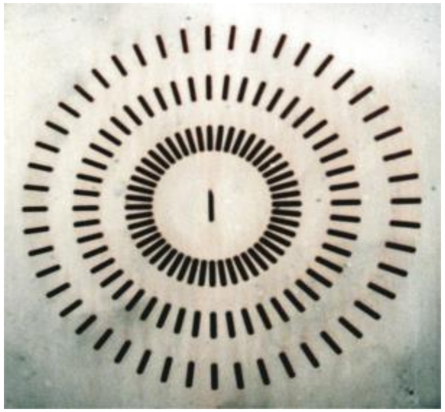
Controlling SW in thick dielectrics

2) Periodic planar structures (EBG)

Suppression of the TM0 mode in a grounded slab





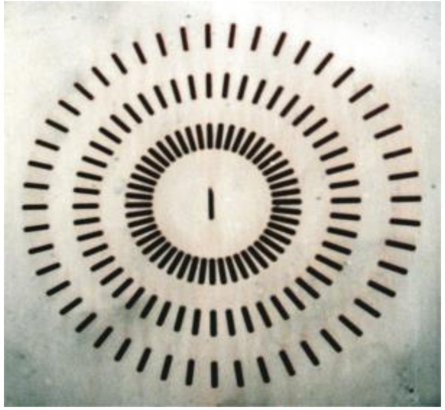


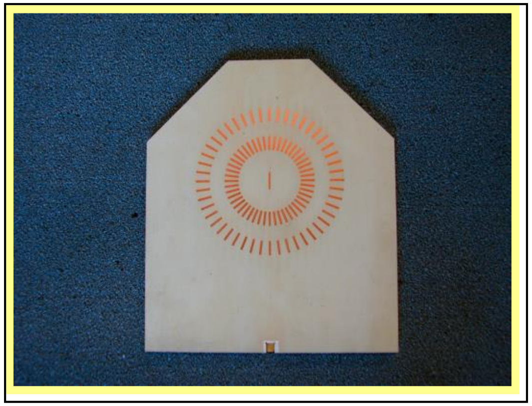
Resonant cavity (bandwidths of about 20%)

Optimum radius is $\lambda_{sw}/2$

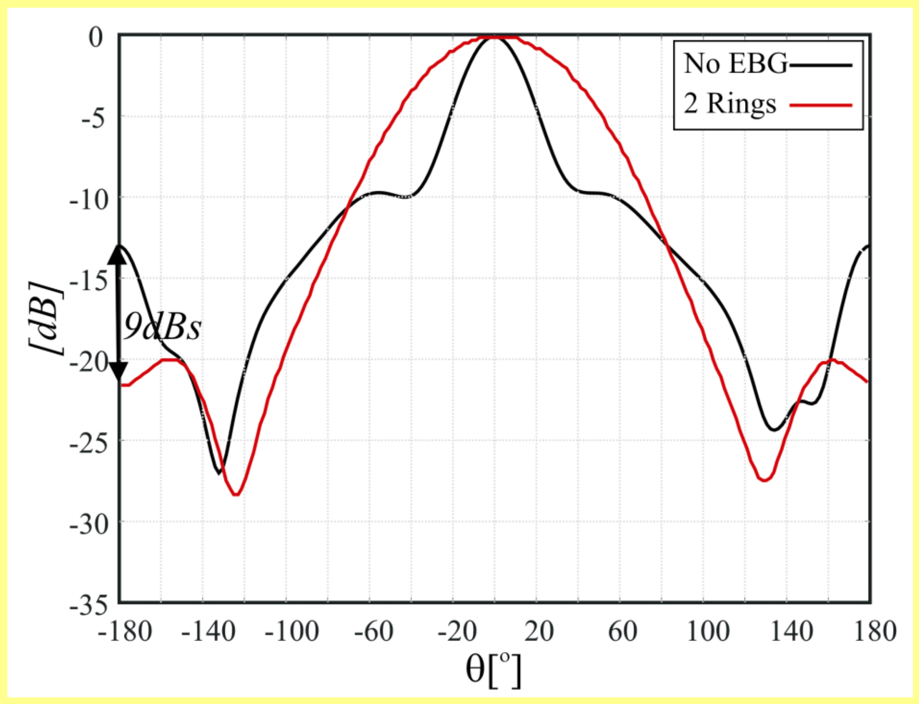
In this situation the wave reflected from the EBG cancels out the outgoing waves emanated by the source so all the power delivered to the slot is then radiated.

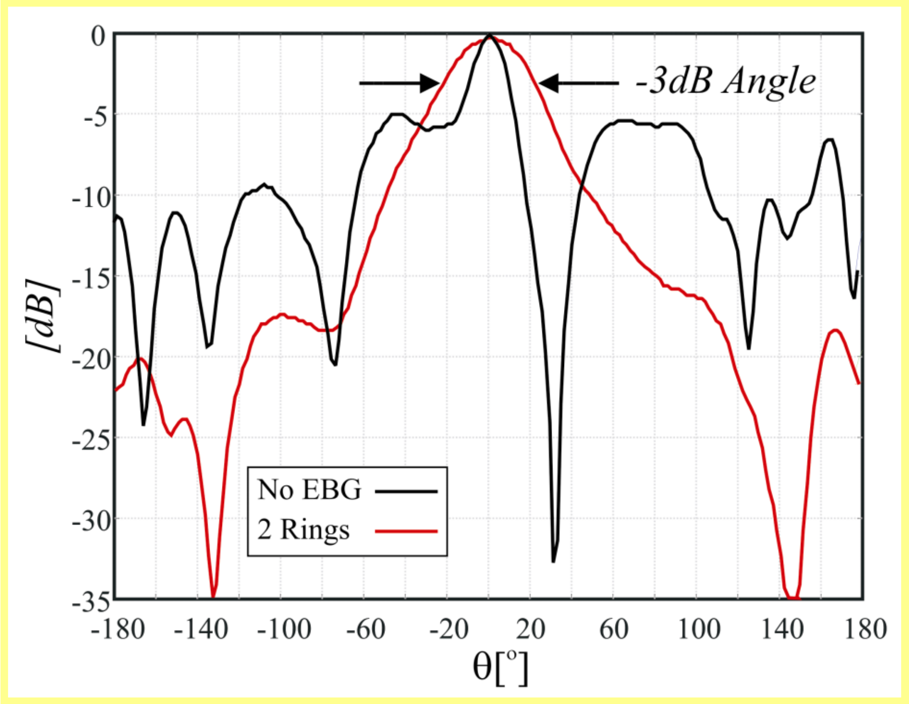
Example





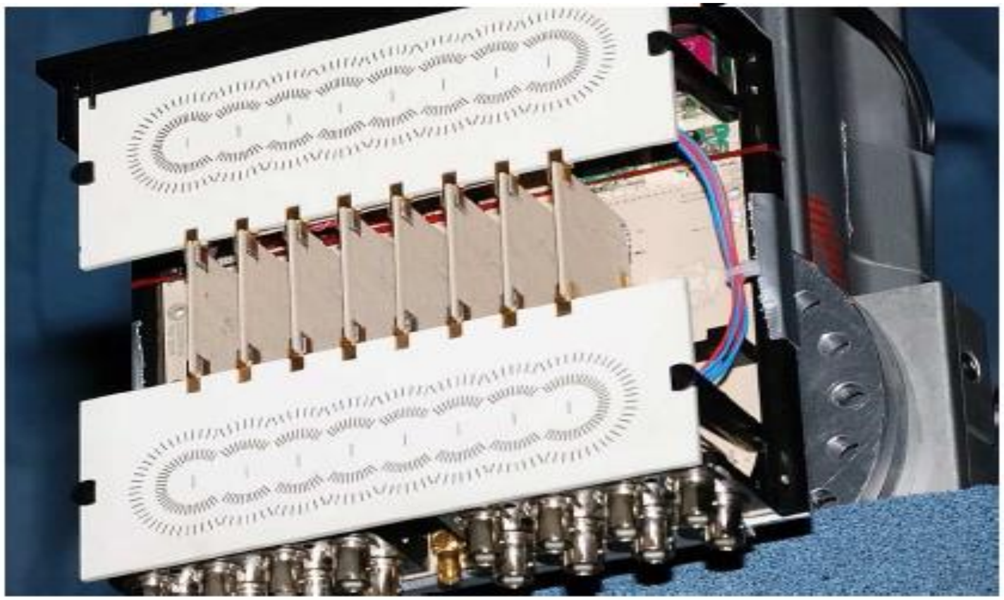
The on-blocked SW power arrives to the substrate edges and gets diffracted



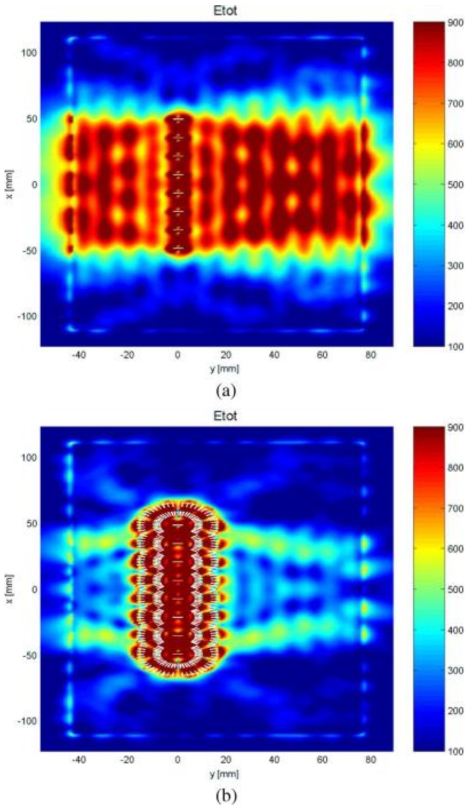


Example

1D Phased Array, TNO



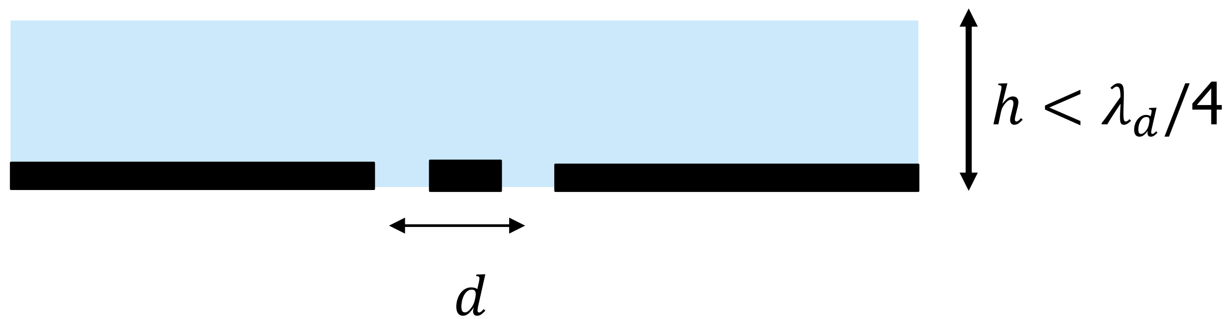
IEEE Best Antenna Design Award



Controlling SW in thick dielectrics

3) Double slot

Suppression of the TM0 mode in a grounded slab



Fourier transform of the current

$$e^{-jk_x d/2} + e^{jk_x d/2} = 2\cos\left(\frac{k_x d}{2}\right)$$

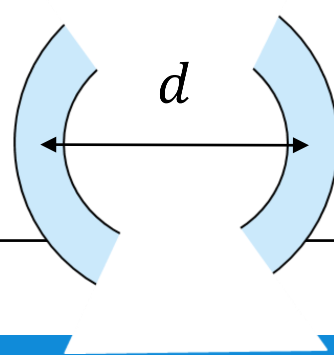
Cancelling the sw

$$\cos\left(\frac{k_{xsw} d}{2}\right) = 0$$

$$\frac{k_{xsw} d}{2} = \frac{\pi}{2}$$

$$\boxed{d = \frac{\lambda_{sw}}{2}}$$

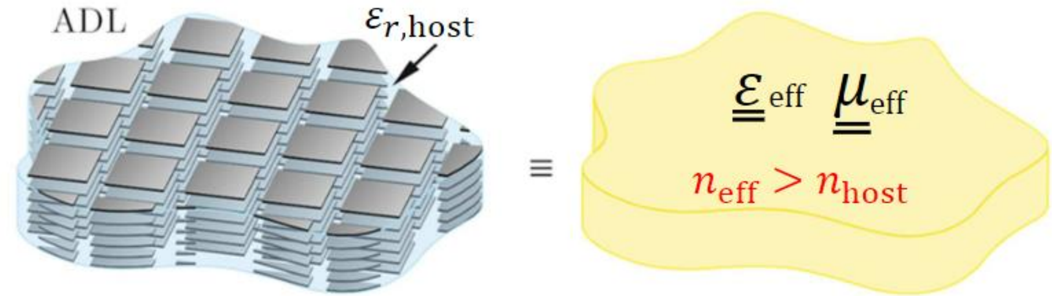
Cancelation in all planes:



Only possible over narrow band

Controlling SW in *thick* dielectrics

4) Artificial dielectrics

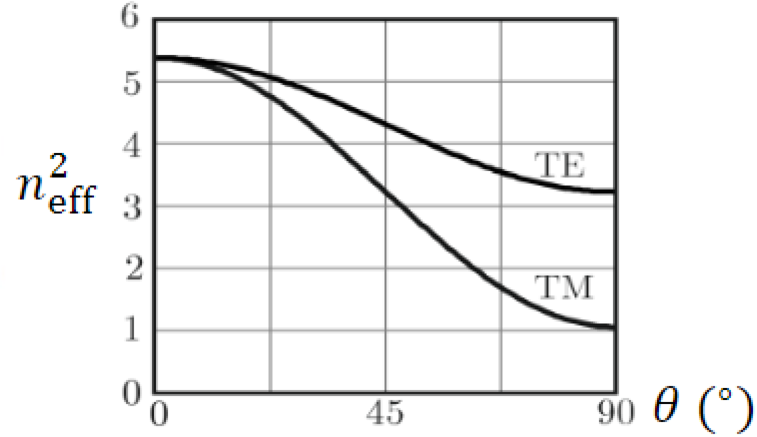


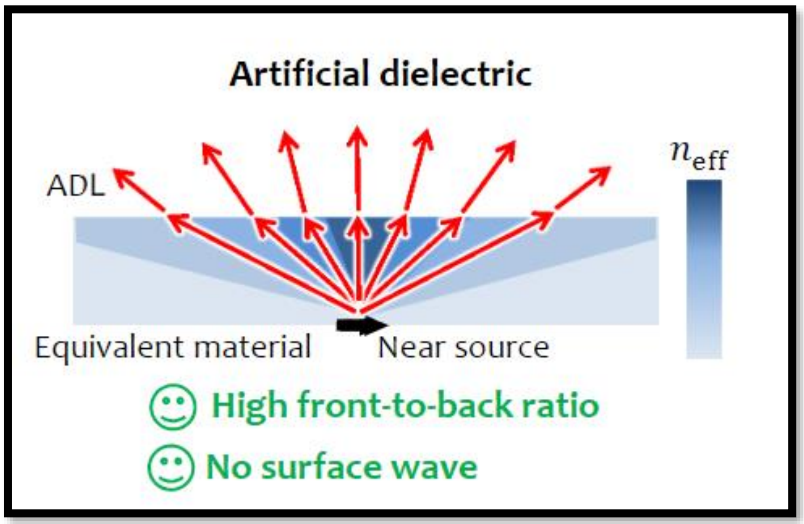
Sub-wavelength periodic patches

$n_{\mathrm{eff}}(\hat{\mathbf{p}}, \theta)$

$\hat{\mathbf{p}}$: polarization of the incident plane wave (TE/TM)
θ: angle of incidence

n_{eff} decreases with angle





Reduce the amount of energy radiated into surface waves by using a thin and low permitivity dielectric
Enhance the front-to-back in the broadside direction with the ADL

Wideband option (>20%)

Important Points

- The spectral Green´s function has two **main singularities**: branch points and poles
- **Real pole singularities** give rise to surface waves.
- **Surface waves** are slow waves that remain close to surface.
- At large distance form the antenna, the surface waves are **cylindrical waves** travelling inside the dielectric
- They constitute a **loss of efficiency (or pattern quality)** in printed antennas since the power carried by the surface wave is uncontrolled (reaches edges of the substrate, introduce coupling between antennas, etc)

Related IEEE Papers on Antenna Design

- Meide Qiu and G. V. Eleftheriades, "Highly efficient unidirectional twin arc-slot antennas on electrically thin substrates," in IEEE Transactions on Antennas and Propagation, vol. 52, no. 1, pp. 53-58, Jan. 2004, doi: 10.1109/TAP.2003.822412.

- M. A. Hickey, Meide Qiu and G. V. Eleftheriades, "A reduced surface-wave twin arc-slot antenna for millimeter-wave applications," in IEEE Microwave and Wireless Components Letters, vol. 11, no. 11, pp. 459-461, Nov. 2001

- N. Llombart, A. Neto, G. Gerini, P. De Maagt, “Planar Circularly Symmetric EBG Structures for Reducing Surface Waves in Printed Antennas” IEEE TAP, Vol.53, no.10, pp.3210-3218, Oct. 2005.

- A. Neto, N. Llombart, G. Gerini, P. De Maagt, “On the Optimal Radiation Bandwidth of Printed Slot Antennas Surrounded by EBGs”, IEEE TAP, vol. 54, no. 4, pp.1074-1083, Apr. 2006

- N. Llombart, A. Neto, G. Gerini, P. De Maagt, “1D Scanning Arrays on Dense Dielectrics Using PCS-EBG Technology”, IEEE TAP, vol. 55, no. 1, pp.26-35, Jan. 2007

- H. Zhang, S. Bosma, A. Neto and N. Llombart, "A Dual-Polarized 27-dBi Scanning Lens Phased Array Antenna for 5G Point-to-Point Communications," in IEEE Transactions on Antennas and Propagation, doi: 10.1109/TAP.2021.3069494.