Leaky-wave antennas instruction lecture

EE4620
Spectral Domain Methods in Electromagnetics
May 2022

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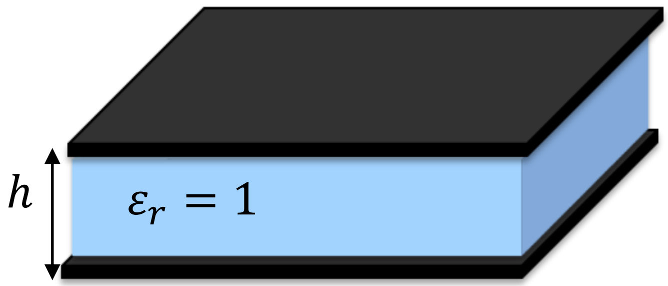
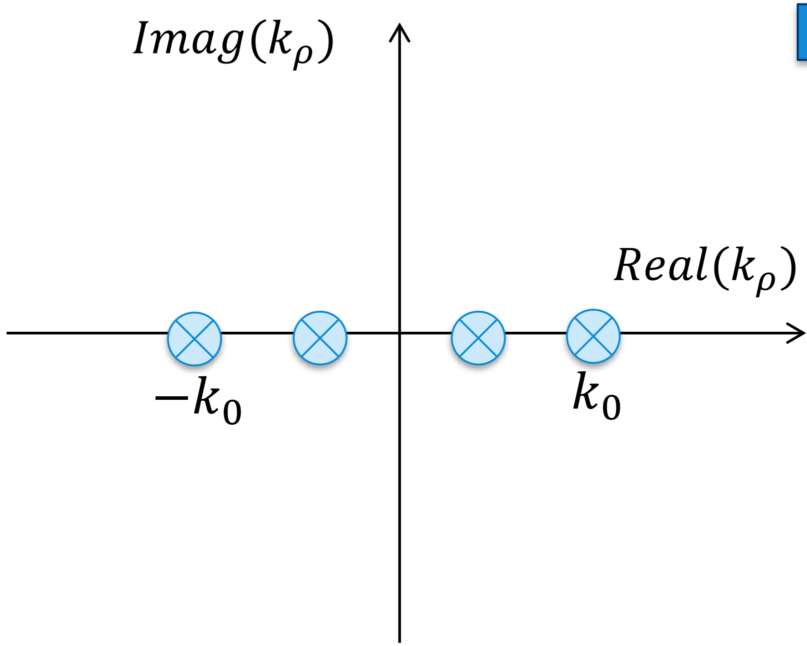
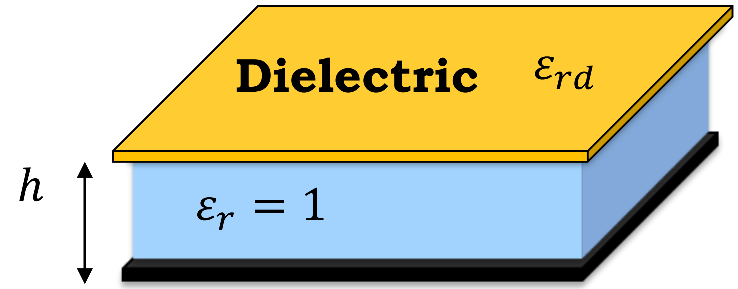
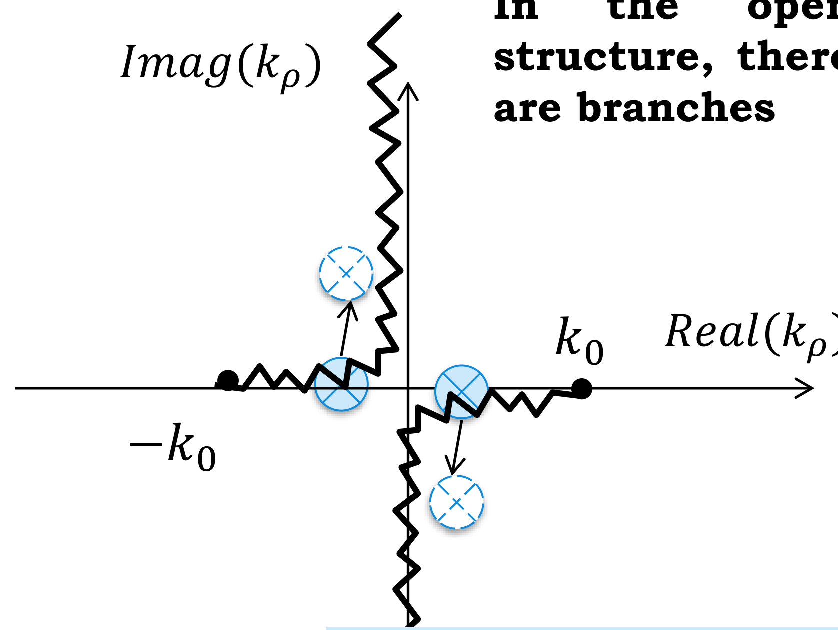
Open Parallel Plate Waveguide







In the open structure, there are branches

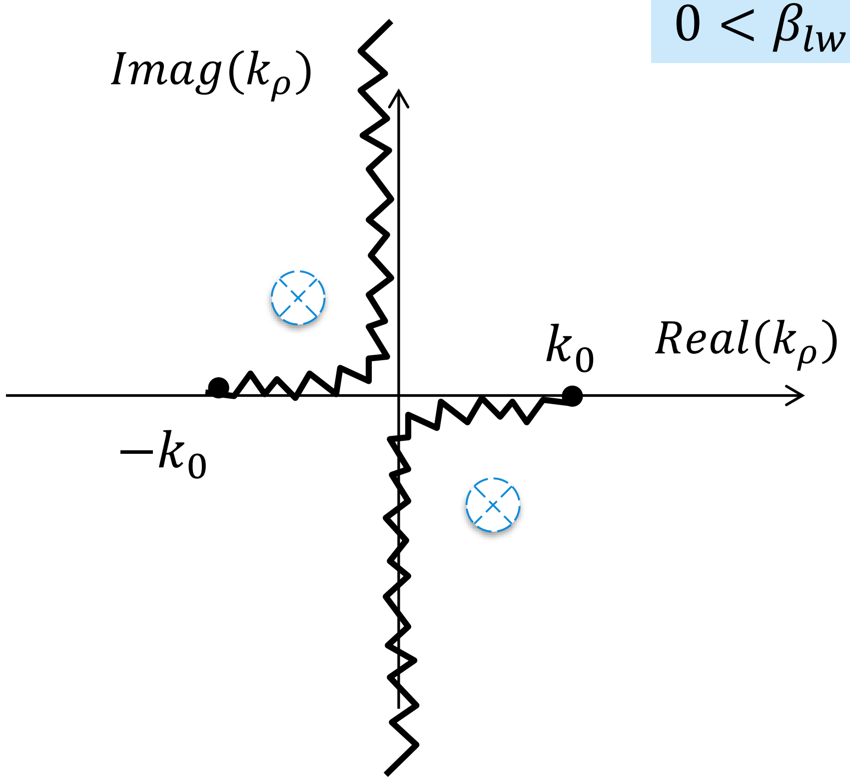


The poles cross the branch cut to the bottom Riemman Sheet

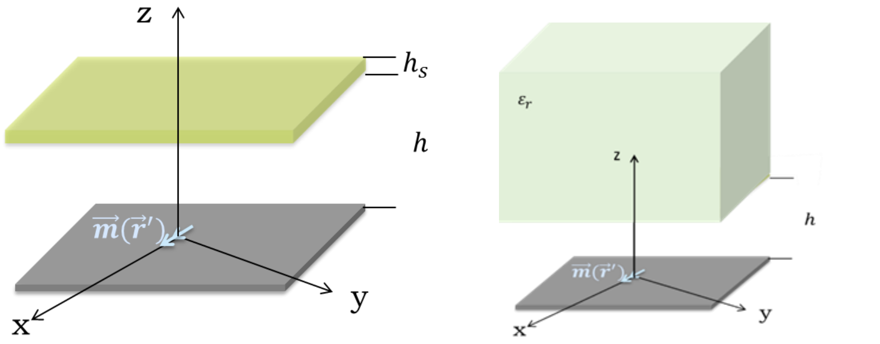
Pole Singularity of Leaky Waves

Complex pole singularities: $k_{\rho lw} = \beta_{lw} - j\alpha_{lw}$

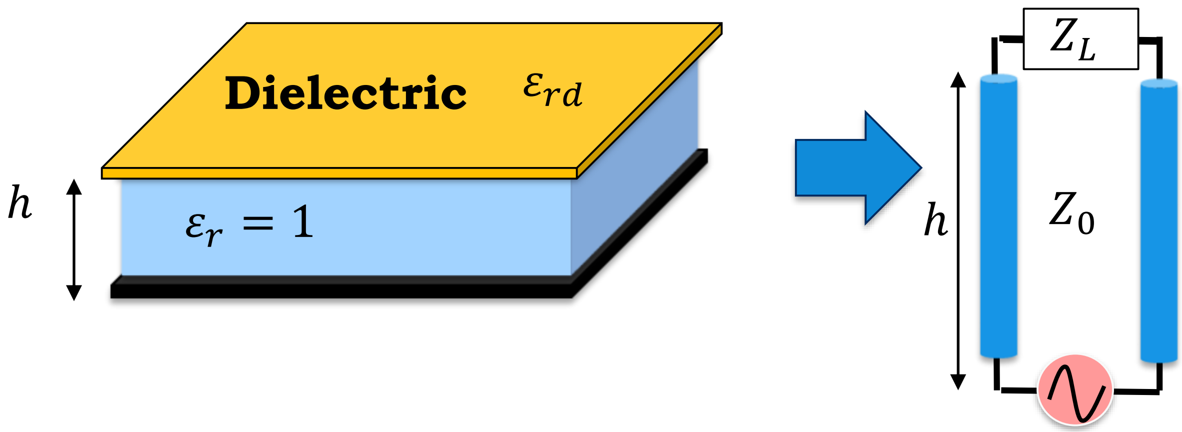
$0 < \beta_{lw} < k_0$ $Im(k_{\rho lw}) < 0$



LWs are present only in certain stratifications, for example the ones we are now familiar with:



Dispersion Equation: Finding the poles



$$Y_{in} = Y_0 \frac{Z_o + jZ_L \tan(k_{z0}h)}{Z_L + jZ_0 \tan(k_{z0}h)}$$

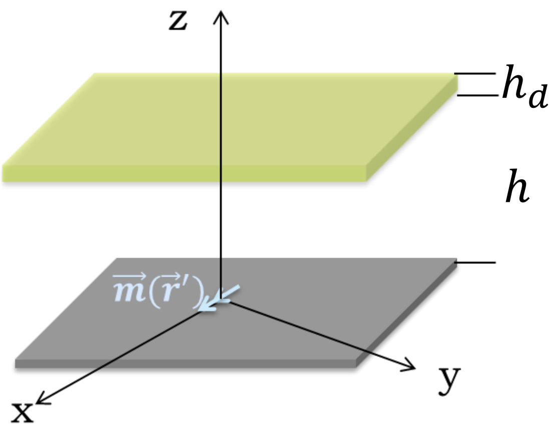
The zeros of the dispersion equation correspond to poles of Y_{in}

$$\boldsymbol{D(k_\rho) = Z_L + jZ_0 \tan(k_{z0}h)}$$

To find the poles:

$$k_{z0} = +j\sqrt{(k_0^2 - k_\rho^2)}$$

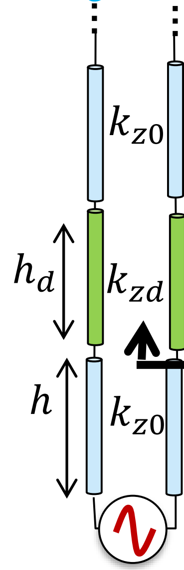
Dispersion Equation: Fabry-Perot Antenna



$Z_0(k_\rho = 0) = \zeta_0$

$Z_d(k_\rho = 0) = \dfrac{\zeta_0}{\epsilon_r}$

$Z_0(k_\rho = 0) = \zeta_0$



$h = \lambda/2$

$h_d = \lambda_d/4$

$$\boldsymbol{Z_L = Z_d \frac{Z_0 + jZ_d \tan(k_{zd} h_d)}{Z_d + jZ_0 \tan(k_{zd} h_d)}}$$

Approximations to solve dispersion equation:

$\tan(x) \approx x \pm n\pi$
when $|x \pm n\pi| \approx 0$
$\rightarrow \tan(k_{z0} h) \approx k_{z0} h - n\pi$

$k_\rho = 0 \rightarrow Z_L = \zeta_0/\varepsilon_r$

$$D(k_\rho) \approx \frac{\zeta_0}{\varepsilon_r} + jZ_0(k_{z0} h - n\pi)$$

A. Neto and N. Llombart, “Wideband localization of the dominant leaky wave poles in dielectric covered antennas,” *IEEE Antennas Wirel. Propag. Lett.*, vol. 5, no. 1, pp. 549–551, 2006.

Dispersion Equation: Fabry-Perot Antenna

$$D(k_\rho) \approx \frac{\zeta_0}{\varepsilon_r} + jZ_0(k_{z0}h - n\pi) = 0$$

gives

$$k_{z0} = \frac{j\zeta_0}{\varepsilon_r h Z_0} + \frac{n\pi}{h}$$

$$\bar{h} = h/\lambda$$
$$n = 1$$

TE1 approximate solution:

$$Z_0^{TE} = \frac{\zeta_0 k_0}{k_{z0}^{TE}}$$

$$k_{z0}^{TE} \approx \frac{k_0}{\pi\varepsilon_r(2\bar{h})^2} \frac{2\pi\bar{h}\varepsilon_r + j}{1 + \frac{1}{(2\pi\bar{h}\varepsilon_r)^2}}$$

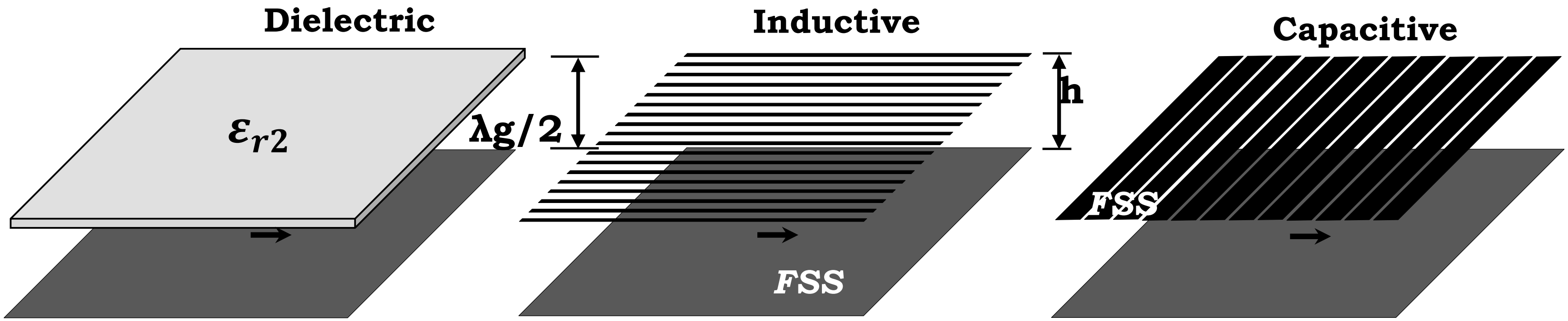
TM1 approximate solution:

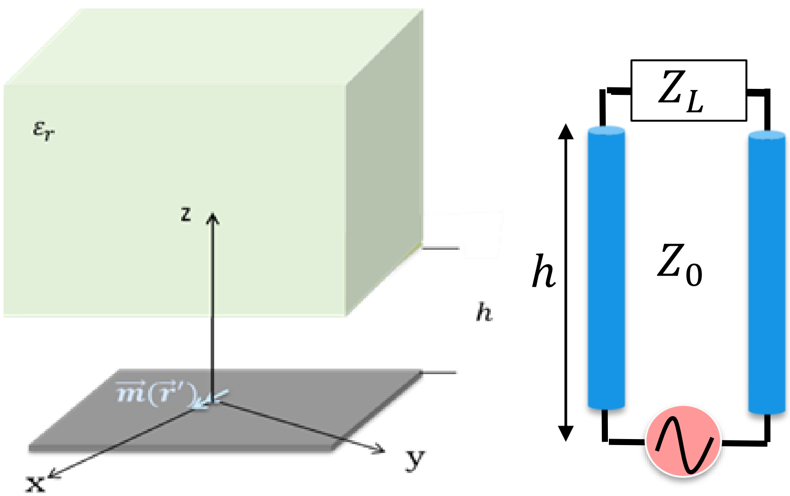
$$Z_0^{TM} = \frac{\zeta_0 k_{z0}^{TM}}{k_0}$$

$$k_{z0}^{TM} \approx \frac{k_0}{4\bar{h}}\left(1 \pm \sqrt{1 + 8j\frac{\bar{h}}{\pi\varepsilon_r}}\right)$$

A. Neto and N. Llombart, “Wideband localization of the dominant leaky wave poles in dielectric covered antennas,” *IEEE Antennas Wirel. Propag. Lett.*, vol. 5, no. 1, pp. 549–551, 2006.

What about other antennas?



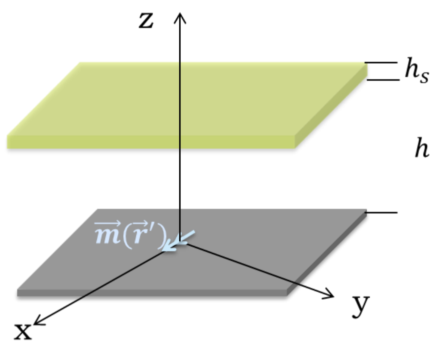


$$D(k_\rho) = Z_L + jZ_0 \tan(k_{z0}h)$$

We can apply the same principle of solving the dispersion equation by deriving the appropriate Z_L!

Question 1 (3 points)

Write a Matlab routine to calculate the leaky wave propagation constant of the TE1/TM1 modes of the geometry shown in the figure. Solve the dispersion equation with the approximate expressions as well as numerically. Consider h=15mm, h_s=2.1mm, ε_r=12 and a half-wavelength magnetic dipole with W=λ/20.



1. TE1/TM1 propagation constants for 9 GHz to 11 GHz
2. TE1/TM1 propagation constants for $\varepsilon_r = 1$ to $\varepsilon_r = 25$ at 10 GHz. Take $h_s = \lambda_0/4\sqrt{\varepsilon_r}$
3. Relate results to far-fields of Instruction 2 on far-field radiation
4. Investigate the bandwidth of the antenna for various ε_r

$$\boldsymbol{BW = 200\frac{f_H - f_L}{f_H + f_L}}\ [\%]$$

Suggestions to solve Question 1

Last week …

- Write a separate routine to calculate the propagation constant:

$$[krho] = findPropSuperStrate\left(k0, er, h, \boldsymbol{k_\rho^g}, 'TE/TM'\right)$$

$$k_\rho = k_\rho^g - \frac{D\left(k_\rho^g\right)}{D'(k_\rho^g)}$$

$$D'(k_\rho^g) \approx \frac{D\left(k_\rho^g + \Delta k/2\right) - D\left(k_\rho^g - \Delta k/2\right)}{\Delta k}$$

$$\Delta k = k_0/500$$

… what part needs to be modified?

Suggestions to solve Question 1

Solve the dispersion equation with the approximate expressions as well as numerically.

Approximations (slide 5):

$$k_{z0}^{TE1} \approx \frac{k_0}{\pi\varepsilon_r(2\bar{h})^2} \frac{2\pi\bar{h}\varepsilon_r + j}{1 + \frac{1}{(2\pi\bar{h}\varepsilon_r)^2}}$$

$$k_{z0}^{TM1} \approx \frac{k_0}{4\bar{h}}\left(1 + \sqrt{1 + 8j\frac{\bar{h}}{\pi\varepsilon_r}}\right)$$

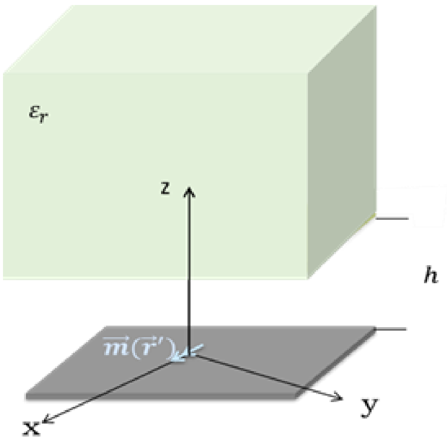
with $\bar{h} = \frac{h}{\lambda_0}$

$$k_\rho^{LW} = \sqrt{k_0^2 - (k_{z0}^{LW})^2}$$

```
function klw = approxSuperStrate(k0, er, hbar, TE_or_TM);
% your code here
switch TE_or_TM
        case 'TE'
                kz0 = …;
        case 'TM'
                kz0 = …;
end
klw = …;
end
```

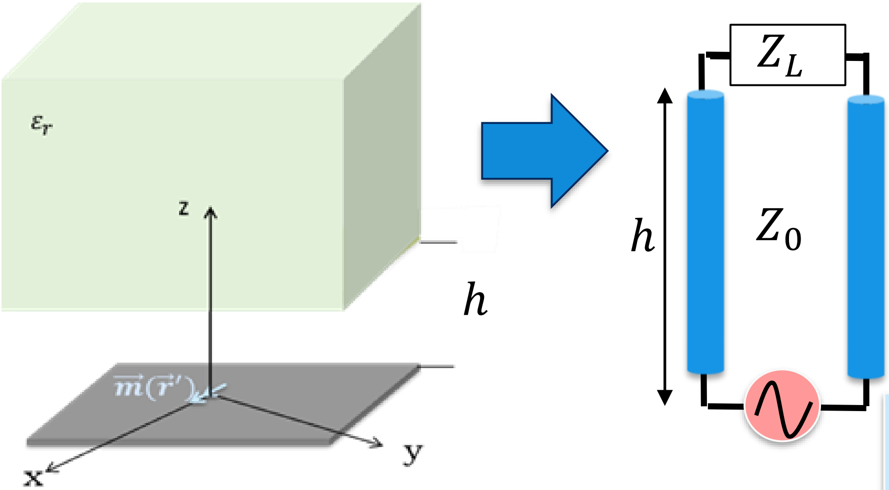
Question 2 (3 points)

Write a Matlab routine to calculate the leaky wave propagation constant of the TE1/TM1 modes of the geometry shown in the figure. Consider h=5mm, a frequency of 30GHz and a half-wavelength magnetic dipole with W=$\lambda/20$.



1. TE1/TM1 propagation constants for ε_r=1 to ε_r=25 at 30 GHz
2. Investigate the bandwidth of the antenna for various ε_r
3. Compare the results to the results of Question 1.4.

Dispersion Equation: Resonant Lens Feed



$$h = \lambda/2$$

$$\boldsymbol{D(k_\rho) = Z_L + jZ_0 \tan(k_{z0} h)}$$

Approximations to solve dispersion equation:

$$\tan(x) \approx x \pm n\pi$$
$$\text{when } |x \pm n\pi| \approx 0$$
$$\rightarrow \tan(k_{z0} h) \approx k_{z0} h - n\pi$$

$$Z_L = \zeta_0 / \sqrt{\varepsilon_r}$$

$$D(k_\rho) \approx \frac{\zeta_0}{\sqrt{\varepsilon_r}} + jZ_0 (k_{z0} h - n\pi)$$

A. Neto and N. Llombart, "Wideband Localization of the Dominant Leaky Wave Poles in Dielectric Covered Antennas," IEEE Antennas and Wireless Propagation Letters, vol. 5, pp. 549–551, 2006, doi: 10.1109/LAWP.2006.889558..

Dispersion Equation: Resonant Lens Feed

$$D(k_\rho) \approx \frac{\zeta_0}{\sqrt{\varepsilon_r}} + jZ_0(k_{z0}h - n\pi) = 0$$

gives

$$k_{z0} = \frac{j\zeta_0}{\sqrt{\varepsilon_r} h Z_0} + \frac{n\pi}{h}$$

$$\bar{h} = h/\lambda$$
$$n = 1$$

TE1 approximate solution:

$$Z_0^{TE} = \frac{\zeta_0 k_0}{k_{z0}^{TE}}$$

$$k_{z0}^{TE} \approx \frac{k_0}{\pi\sqrt{\varepsilon_r}(2\bar{h})^2} \frac{2\pi\bar{h}\sqrt{\varepsilon_r} + j}{1 + \frac{1}{(2\pi\bar{h})^2 \varepsilon_r}}$$

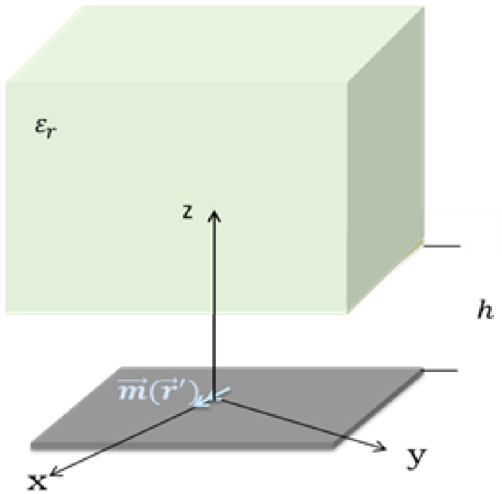
TM1 approximate solution:

$$Z_0^{TM} = \frac{\zeta_0 k_{z0}^{TM}}{k_0}$$

$$k_{z0}^{TM} \approx \frac{k_0}{4\bar{h}} \left(1 \pm \sqrt{1 + 8j \frac{\bar{h}}{\pi\sqrt{\varepsilon_r}}} \right)$$

A. Neto and N. Llombart, “Wideband Localization of the Dominant Leaky Wave Poles in Dielectric Covered Antennas,” IEEE Antennas and Wireless Propagation Letters, vol. 5, pp. 549–551, 2006, doi: 10.1109/LAWP.2006.889558..

Resonant Lens Feed: Notes



We are now radiating into a dense dielectric (silicon), not free space.

Remember:

(1) $k_{zd} = j\sqrt{-\left(k_d^2 - k_\rho^2\right)}$

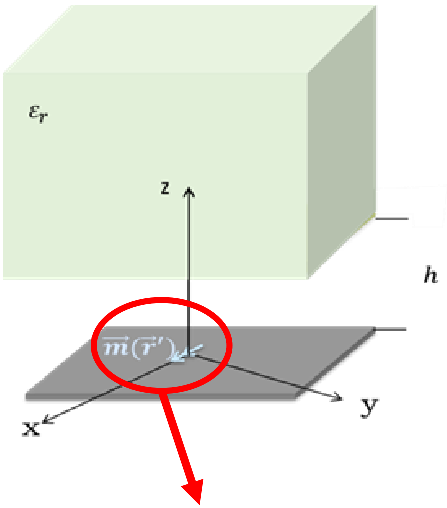
(2) We approximate $k_{z0}^{TE/TM}$

(2) Calculate P_{rad} using ζ_d

A. Neto and N. Llombart, “Wideband Localization of the Dominant Leaky Wave Poles in Dielectric Covered Antennas,” IEEE Antennas and Wireless Propagation Letters, vol. 5, pp. 549–551, 2006, doi: 10.1109/LAWP.2006.889558..

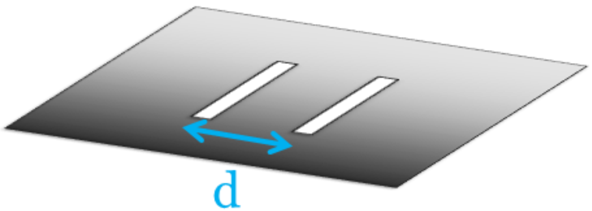
Question 3 (4 points)

We now look again at the geometry of Question 2, with ε_r=12 and h=5mm. Besides the TE1/TM1, there is also a TM0 leaky wave mode which points to large angles. This mode therefore affects the directivity of a small antenna placed in the ground plane. Write a Matlab routine to calculate the TM0 propagation constant.



1. TM0 propagation constant for BW of Question 2.2

We can use two half-wavelength slots ($W = \lambda/20$) to reduce the impact of this TM0 mode.



2. Calculate optimum distance d at 30 GHz. Compare the far-field radiation of this dual-slot antenna with a single slot antenna.
3. Evaluate the directivity of the double-slot antenna

Dispersion Equation: Resonant Lens Feed

$$D(k_\rho) \approx \frac{\zeta_0}{\sqrt{\varepsilon_r}} + jZ_0(k_{z0}h - n\pi) = 0$$

gives

$$k_{z0} = \frac{j\zeta_0}{\sqrt{\varepsilon_r} h Z_0} + \frac{n\pi}{h}$$

$\bar{h} = h/\lambda$

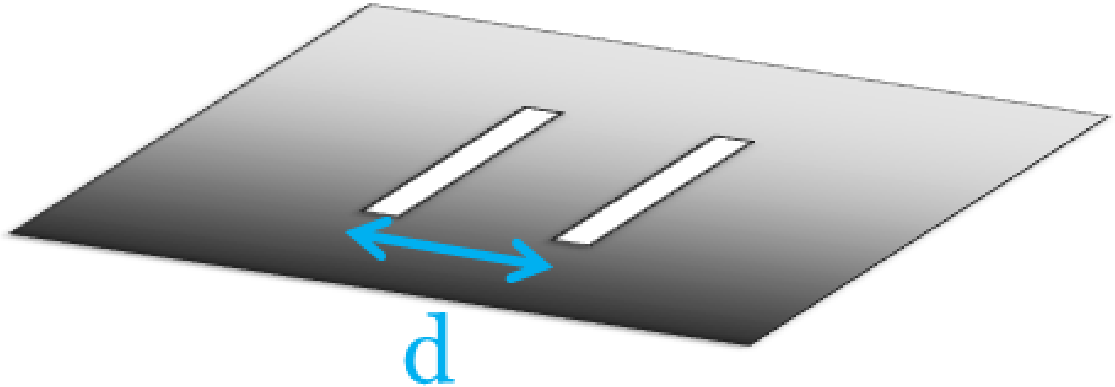
$n = 0$

TM0 approximate solution:

$$Z_0^{TM} = \frac{\zeta_0 k_{z0}^{TM}}{k_0}$$

$$(k_{z0}^{TM})^2 \approx \frac{jk_0}{\sqrt{\varepsilon_r} h}$$

Double-slot antenna distance



Fourier transform of the current

$$e^{-jk_x d/2} + e^{jk_x d/2} = 2\cos\left(\frac{k_x d}{2}\right)$$

Cancelling the LW

$$\cos\left(\frac{k_{xLW} d}{2}\right) = 0$$

$$\frac{k_{xLW} d}{2} = \frac{\pi}{2}$$

$$d = \frac{\lambda_{LW}}{2}$$

Analogous to solution for surface wave suppression of previous lecture