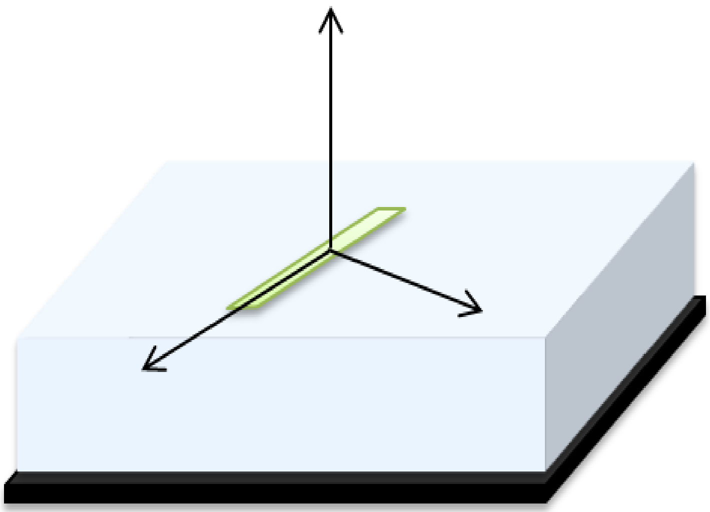
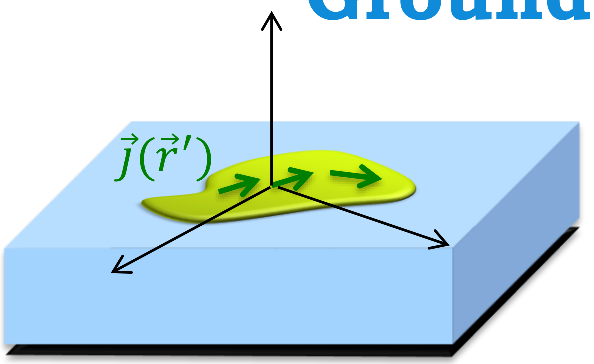
Question 1

Write a Matlab routine that finds the propagation constant of the first two surface waves (TM0 and TE1) supported by a grounded slab.

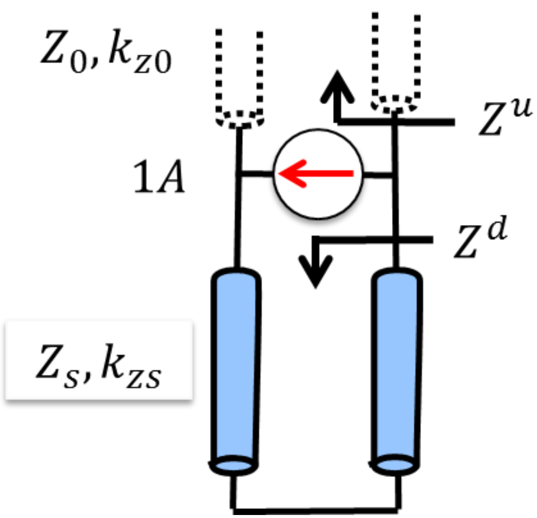
To check the program, provide a plot showing the propagation constant, normalized to k_0, of the TM0 and TE1 surface waves for a frequency range from 1 to 20 GHz. Consider $h = 2$ mm, $\varepsilon_r = 10$ and $\varepsilon_r = 5$.



Grounded Dielectric Substrate



$$Z_{in} = \frac{Z_u Z_d}{Z_u + Z_d} \quad \begin{cases} Z_u = Z_0 \\ Z_d = jZ_s \tan(k_{zs} h) \end{cases}$$



$$v(z = z_s) = Z_{in}$$

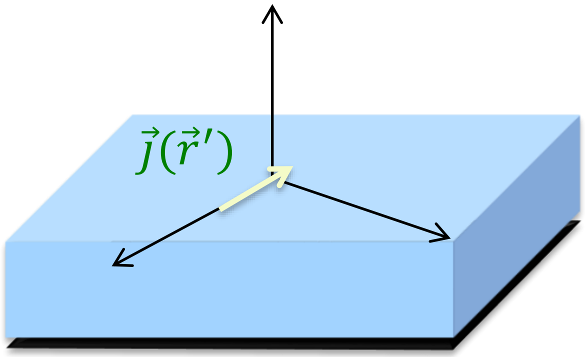
All the voltage and current solutions at any z-quote are expressed as a function of this voltage

The poles in the transversal complex plane can be found solving the following **dispersion equation**

$$D(k_\rho) = Z_u + Z_d = 0$$

for TE and TM

Grounded Dielectric Substrate



All the voltage and current solutions at any z-quote are expressed as a function of the same denominator

Voltage in the slab:

$$V_s = \frac{Z_u Z_d}{Z_u + Z_d} \frac{sin(k_{zs}z)}{sin(k_{zs}h)} = \frac{Z_u Z_d}{D(k_\rho)} \frac{sin(k_{zs}z)}{sin(k_{zs}h)}$$

Current in the slab:

$$I_s = \frac{1}{Z_s} \frac{Z_u Z_d}{D(k_\rho)} \frac{j\cos(k_{zs}z)}{sin(k_{zs}h)}$$

Voltage in the air:

$$V_0 = \frac{Z_u Z_d}{D(k_\rho)} e^{jk_{z0}h} e^{-jk_{z0}z}$$

Current in the air:

$$I_0 = \frac{1}{Z_0} \frac{Z_u Z_d}{D(k_\rho)} e^{jk_{z0}h} e^{-jk_{z0}z}$$

Solving Dispersion Equations

If we know a good guess of the solution, k_ρ^g

1) One can expand the denominator around this point using the Taylor's series:

$$D(k_\rho) \approx D(k_\rho^g) + D'(k_\rho^g)(k_\rho - k_\rho^g)$$

2) Evaluating this expansion in the actual zero, $k_{\rho 0}$:

$$D(k_{\rho 0}) \approx D(k_\rho^g) + D'(k_\rho^g)(k_{\rho 0} - k_\rho^g) = 0$$

The derivative can be done numerically

$$k_{\rho 0} = k_\rho^g - \frac{D(k_\rho^g)}{D'(k_\rho^g)}$$

$$D'(k_\rho^g) \approx \frac{D(k_\rho^g + \Delta k/2) - D(k_\rho^g - \Delta k/2)}{\Delta k} \qquad \Delta k = k_0/500$$

- Write a separate routine to calculate the dispersion equation:

$$[D] = Den\_GroundSlab(k0, er, h, kro, 'TE/TM')$$

$$D(k_\rho) = Z_u + Z_d$$

- Write a separate routine to calculate the propagation constant:

$$[krho] = findprop(k0, er, h, \boldsymbol{k_\rho^g}, 'TE/TM')$$

$$k_\rho = k_\rho^g - \frac{D(k_\rho^g)}{D'(k_\rho^g)}$$

$$D'(k_\rho^g) \approx \frac{D(k_\rho^g + \Delta k/2) - D(k_\rho^g - \Delta k/2)}{\Delta k}$$

$$\Delta k = k_0/500$$

Iterative Frequency Loop for solving the dispersion equation

1) Plot the Denominator vs $k_\rho\ /k_0$ at the highest frequency to find a good initial guess: k_ρ^g

2) Use this guess to find the propagation constant at the highest frequency

$$k_\rho\ (f_i) = k_\rho^g - \frac{D\left(k_\rho^g\right)}{D'\left(k_\rho^g\right)}$$

3) Do a loop from the highest to the lowest frequency where you use the *normalized* propagation constant, $k_\rho^{gn} = k_\rho\ (f_i)/k_0(f_i)$, of the previous step as a guess to the next frequency k_ρ^0:

$$k_\rho\ (f_i + 1) = k_\rho^g - \frac{D\left(k_\rho^g\right)}{D'\left(k_\rho^g\right)}$$

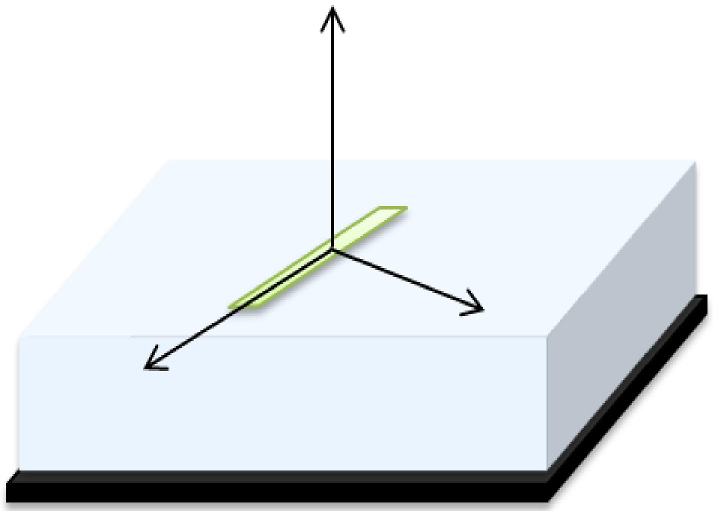
with $k_\rho^g = k_\rho^{gn} k_0(f_i + 1)$

Question 2

Write a Matlab routine to calculate the electric field contribution, in cylindrical coordinates, associated to the TM0 surface wave excited by a half-wavelength dipole on top of a grounded slab.

Provide the following plots ($h = 2$ mm, $\varepsilon_r = 10$, $f = 10$ GHz):

- Real and imaginary part variation of the electric field as a function of the radial distance.
- Amplitude variation of the electric field as a function of z.
- Amplitude variation of the electric field as a function of ϕ.



TM surface wave

Expressions for an *electric current oriented along x*

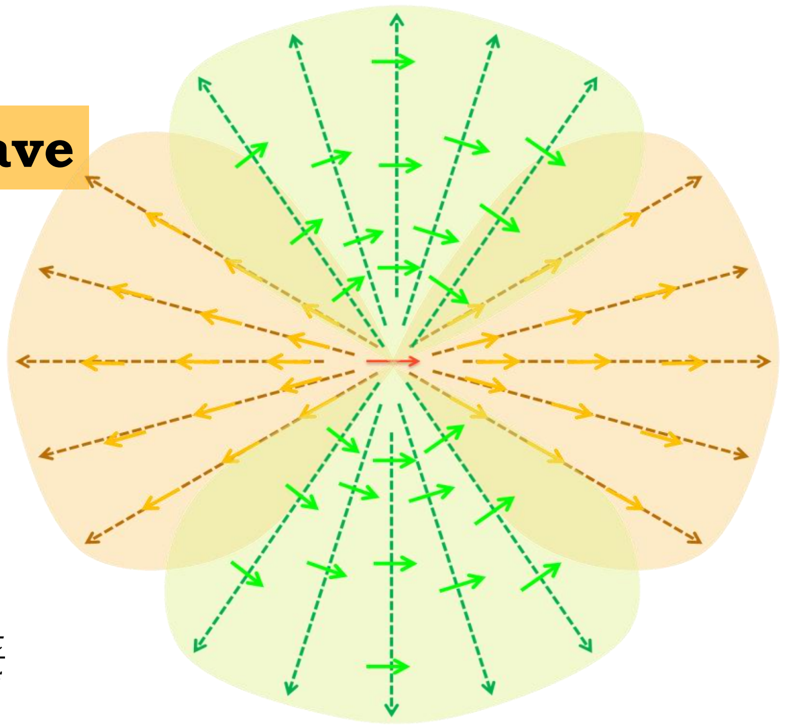
$$E_\rho(\rho,z) \approx Res\left[v_{TM}(k_\rho,z,z')\right]\Big|_{k_{\rho i}} J_x(k_{\rho i},\phi)\,\frac{C\cos\phi\, e^{-jk_{\rho i}\rho}}{\sqrt{\rho}}$$

$$E_z(\rho,z) \approx -\frac{\varsigma k_{\rho i}}{k} Res\left[i_{TM}(k_\rho,z,z')\right]\Big|_{k_{\rho i}} J_x(k_{\rho i},\phi)\,\frac{C\cos\phi\, e^{-jk_{\rho i}\rho}}{\sqrt{\rho}}$$

$$H_\phi(\rho,z) \approx Res\left[i_{TM}(k_\rho,z,z')\right]\Big|_{k_{\rho i}} J_x(k_{\rho i},\phi)\,\frac{C\cos\phi\, e^{-jk_{\rho i}\rho}}{\sqrt{\rho}}$$

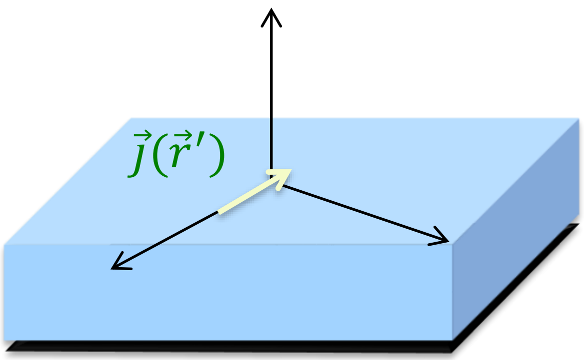
$$C = j\sqrt{\frac{k_{\rho i}}{2\pi}}\,e^{j\frac{\pi}{4}}$$

TM Wave



$k_{\rho i}$: ***pole location in the spectrum***

Residues



$$E_\rho(\rho,z) \approx Res\left[v_{TM}(k_\rho,z,z')\right]\Big|_{k_{\rho i}} \boldsymbol{J}_x(k_{\rho i},\phi)\frac{C\cos\phi\, e^{-jk_{\rho i}\rho}}{\sqrt{\rho}}$$

$$E_z(\rho,z) \approx -\frac{\varsigma k_{\rho i}}{k} Res\left[i_{TM}(k_\rho,z,z')\right]\Big|_{k_{\rho i}} \boldsymbol{J}_x(k_{\rho i},\phi)\frac{C\cos\phi\, e^{-jk_{\rho i}\rho}}{\sqrt{\rho}}$$

$$H_\phi(\rho,z) \approx Res\left[i_{TM}(k_\rho,z,z')\right]\Big|_{k_{\rho i}} \boldsymbol{J}_x(k_{\rho i},\phi)\frac{C\cos\phi\, e^{-jk_{\rho i}\rho}}{\sqrt{\rho}}$$

Fields inside the slab:

$$Res\left[v_{TE/TM}(k_\rho,z,z')\right]\Big|_{k_{\rho i}} = \frac{Z_u Z_d}{D'(k_{\rho i})}\frac{sin(k_{zs}z)}{sin(k_{zs}h)}$$

$$Res\left[i_{TE/TM}(k_\rho,z,z')\right]\Big|_{k_{\rho i}} = \frac{Z_u Z_d}{D'(k_{\rho i})}\frac{1}{Z_s}\frac{j\cos(k_{zs}z)}{sin(k_{zs}h)}$$

Fields inside the air:

$$Res\left[v_{TE/TM}(k_\rho,z,z')\right]\Big|_{k_{\rho i}} = \frac{Z_u Z_d}{D'(k_{\rho i})} e^{jk_{z0}h} e^{-jk_{z0}z}$$

$$Res\left[i_{TE/TM}(k_\rho,z,z')\right]\Big|_{k_{\rho i}} = \frac{Z_u Z_d}{D'(k_{\rho i})}\frac{1}{Z_0} e^{jk_{z0}h} e^{-jk_{z0}z}$$

$$D'(k_\rho^g) \approx \frac{D\left(k_\rho^g + \Delta k/2\right) - D\left(k_\rho^g - \Delta k/2\right)}{\Delta k}$$

- Write a separate routine to calculate the residues:

$$[VtmR, ItmR] = Residue\_GroundSlab(k0, er, h, ksw, z)$$

Fields inside the slab:

$$Res[v_{TE/TM}(k_\rho, z, z')]\Big|_{k_{\rho i}} = \frac{Z_u Z_d}{D'(k_{\rho i})} \frac{sin(k_{zs} z)}{sin(k_{zs} h)}$$

$$Res[i_{TE/TM}(k_\rho, z, z')]\Big|_{k_{\rho i}} = \frac{Z_u Z_d}{D'(k_{\rho i})} \frac{1}{Z_s} \frac{j\cos(k_{zs} z)}{sin(k_{zs} h)}$$

Fields inside the air:

$$Res[v_{TE/TM}(k_\rho, z, z')]\Big|_{k_{\rho i}} = \frac{Z_u Z_d}{D'(k_{\rho i})} e^{jk_{z0}h} e^{-jk_{z0}z}$$

$$Res[i_{TE/TM}(k_\rho, z, z')]\Big|_{k_{\rho i}} = \frac{Z_u Z_d}{D'(k_{\rho i})} \frac{1}{Z_0} e^{jk_{z0}h} e^{-jk_{z0}z}$$

$$k_{\rho i} = ksw$$

- Write a separate routine to calculate the field components:

$$[Erho, Ez, Hphi] = TMSwFields(k0, ksw, er, VtmR, ItmR, \rho, \phi, z)$$

$$E_\rho(\rho, z) \approx Res[v_{TM}(k_\rho, z, z')]\Big|_{k_{\rho i}} J_x(k_{\rho i}, \phi) \frac{C \cos\phi \, e^{-jk_{\rho i}\rho}}{\sqrt{\rho}}$$

$$E_z(\rho, z) \approx -\frac{\varsigma k_{\rho i}}{k} Res[i_{TM}(k_\rho, z, z')]\Big|_{k_{\rho i}} J_x(k_{\rho i}, \phi) \frac{C \cos\phi \, e^{-jk_{\rho i}\rho}}{\sqrt{\rho}}$$

$$H_\phi(\rho, z) \approx Res[i_{TM}(k_\rho, z, z')]\Big|_{k_{\rho i}} J_x(k_{\rho i}, \phi) \frac{C \cos\phi \, e^{-jk_{\rho i}\rho}}{\sqrt{\rho}}$$

- FT of the current distribution (PWS):

$$Jx = FTcurrent(k0, kx, ky, l, w)$$

$$k_x = k_{sw} cos\phi$$
$$k_y = k_{sw} sin\phi$$

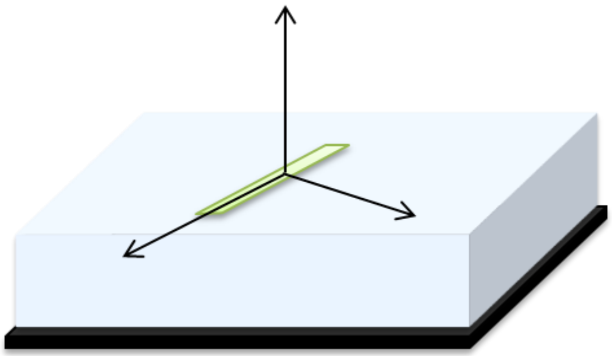
$$C = j\sqrt{\frac{k_{\rho i}}{2\pi}} e^{j\frac{\pi}{4}}$$

Question 3

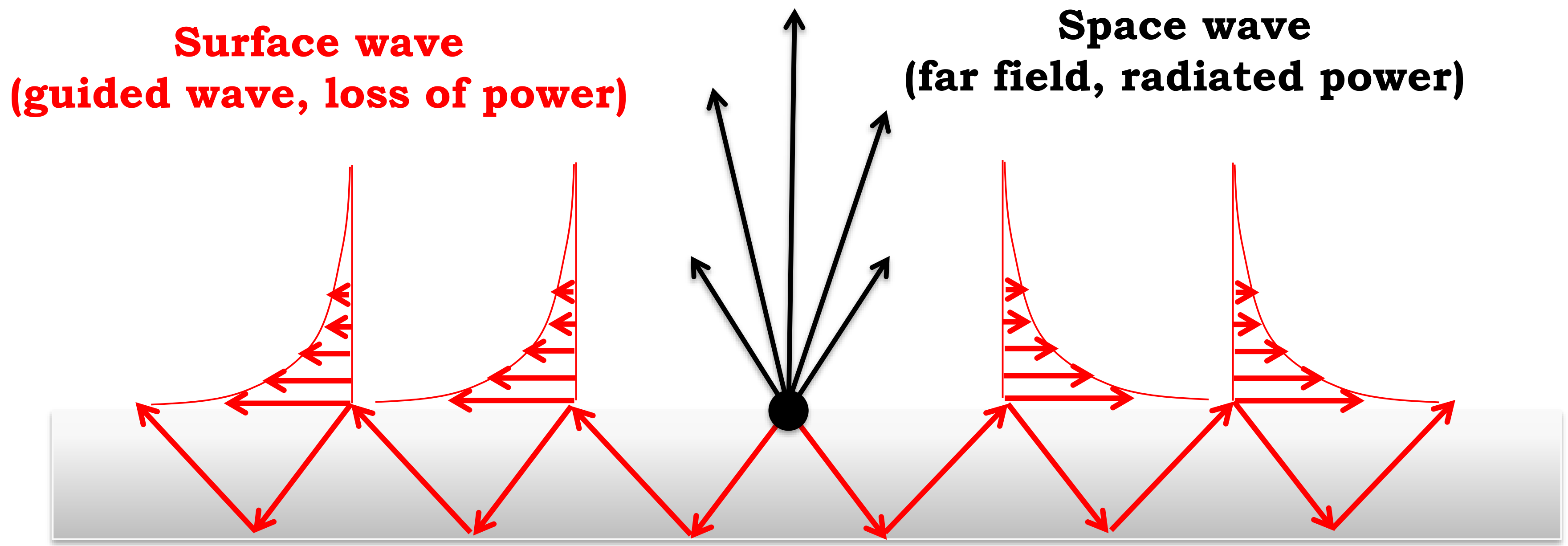
Write a Matlab routine to calculate the power radiated into the TM0 surface wave of a dipole antenna printed on top of a grounded slab.

Provide the following plots ($h = 2$ mm, $\varepsilon_r = 10$, $f = 1 - 15$ GHz):

- The far field power and surface wave power radiated by an elementary dipole ($J = 1$) normalized to the power radiated by the same dipole in free space.
- The TM0 surface wave efficiency for the elementary dipole and for a dipole with $l = 5.3$ mm and $w = 0.5$ mm (assume a PWS distribution).
- The TM0 surface efficiency for a *uniform* current distribution in both x and y directions with a dimension of $l = w = 25$ mm.



Antenna SW Efficiency



We are going to characterize the loss of efficiency into SW:

1. Find the SW propagation constant (dispersion equation)
2. Calculate the SW fields (Residue)
3. Power into SW vs radiated power

$$\eta_{sw} = \frac{P_{rad}}{P_{rad} + P_{sw}}$$

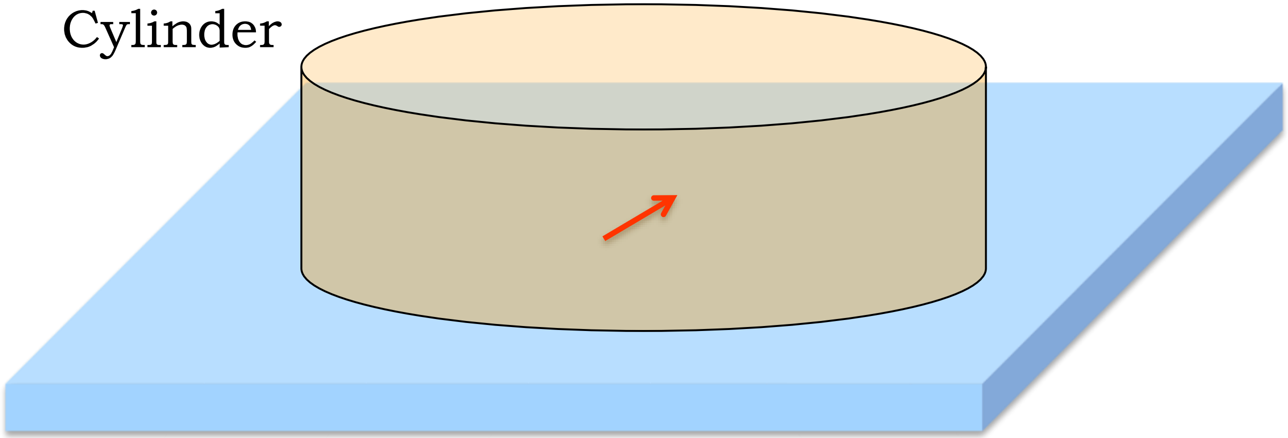
SW Power

We need to characterize the power launched into surface waves

By integrating the Poynting vector around a closed surface surrounding the antenna

$$P_{sw} = \frac{1}{2}\sum_{i}\iint_{S} Re\left[\vec{\boldsymbol{E}}_{\boldsymbol{sw}_i} \times \vec{\boldsymbol{H}}^*_{\boldsymbol{sw}_i}\right]\boldsymbol{d}\vec{\boldsymbol{s}}$$

Cylinder



Since the fields at the top and bottom of the cylinder are zero

$$P_{sw}^{TM} = \frac{1}{2}\int_0^{2\pi}\int_0^{\infty} Re\left[-E_z H_\phi^*\right]\rho_s dz d\phi$$

TM SW Power

$$P_{sw}^{TM} = \frac{1}{2}\int_0^{2\pi}\int_0^{\infty} Re\left[-E_z H_\phi^*\right]\rho_s dz d\phi$$

$$E_z(\rho,z) \approx -\frac{\zeta k_{\rho i}}{k} Res\left[i_{TM}(k_\rho,z,z')\right]\Big|_{k_{\rho i}} C_x(k_{\rho i},\phi)\frac{C\cos\phi\, e^{-jk_{\rho i}\rho}}{\sqrt{\rho}}$$

$$H_\phi(\rho,z) \approx Res\left[i_{TM}(k_\rho,z,z')\right]\Big|_{k_{\rho i}} C_x(k_{\rho i},\phi)\frac{C\cos\phi\, e^{-jk_{\rho i}\rho}}{\sqrt{\rho}}$$

$$C = j\sqrt{\frac{k_{\rho i}}{2\pi}}e^{j\frac{\pi}{4}}$$

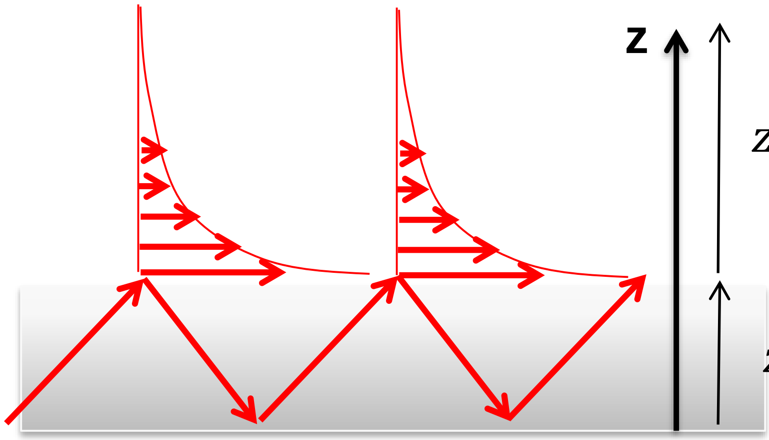
$$P_{sw}^{TM} = \frac{1}{2}\frac{k_{\rho i}^2}{2\pi}\underbrace{\int_0^{\infty}\frac{\zeta}{k}\left|Res\left[i_{TM}(k_\rho,z,z')\right]\Big|_{k_{\rho i}}\right|^2 dz}\ \underbrace{\int_0^{2\pi}\left|C_x(k_{\rho i},\phi)\right|^2\cos^2\phi\, d\phi}$$

The integral in z and phi are not coupled
They can be performed independently for faster computation

$$P_{sw}^{TM} = \frac{1}{2}\frac{k_{\rho i}^2}{2\pi}I_z I_\phi$$

Z-integration

$$I_z = \int_0^\infty \frac{\zeta}{k} \left| Res[i_{TM}(k_\rho, z, z')] \Big|_{k_{\rho i}} \right|^2 dz = \frac{\zeta_0}{k_0} \int_0^h \frac{1}{\epsilon_r} \left| Res[i_{TM}(k_\rho, z, z')] \Big|_{k_{\rho i}} \right|^2 dz + \frac{\zeta_0}{k_0} \int_h^\infty \left| Res[i_{TM}(k_\rho, z, z')] \Big|_{k_{\rho i}} \right|^2 dz$$



$z = [h, \infty]$

$z = [0, h]$

The z-integration domain is divided in two regions associated to the different dielectrics

$$Res[i_{TM}(k_\rho, z, z')] = \begin{cases} \dfrac{Z_u Z_d}{D'(k_{\rho i})} \dfrac{1}{Z_s} \dfrac{j\cos(k_{zs} z)}{\sin(k_{zs} h)} \\ \dfrac{Z_u Z_d}{D'(k_{\rho i})} \dfrac{1}{Z_0} e^{jk_{z0} h} e^{-jk_{z0} z} \end{cases}$$

$$k_{z0} = \sqrt{k_0^2 - k_{\rho i}^2} = -j\sqrt{k_{\rho i}^2 - k_0^2} = -j\alpha_{z0}$$

It is possible doing the z-integration in analytical form

$$\int_0^h \cos^2(k_{zs} z)\, dz = \frac{h}{2}(1 + sinc(2k_{zs} h))$$

$$\int_h^\infty e^{-2\alpha_{z0}(z-h)} dz = \frac{1}{2\sqrt{k_{\rho i}^2 - k_0^2}}$$

phi-integration: numerical

$$I_\phi = \int_0^{2\pi} \left|\boldsymbol{J}_x(k_{\rho i}, \phi)\right|^2 \cos^2\phi \, d\phi$$

The integral in phi can be done numerically for any current distribution

However if we consider a short dipole $(J_x(k_{\rho i}, \phi) = I_0 l)$

$$I_\phi = \pi$$

- Write a separate routine to calculate the power radiated into the surface wave for an elementary dipole:

$$[Psw] = PswTMelem(k0, er, h, ksw)$$

$$P_{sw}^{TM}\Big|_{\delta} = \frac{1}{2}\frac{k_{\rho i}^2}{2\pi} I_{\boldsymbol{z}} I_{\phi}^{\delta}$$

$$I_{\phi}^{\delta} = \pi$$

$$I_z = \int_0^{\infty} \frac{\zeta}{k} \left| Res\left[i_{TM}(k_\rho, z, z')\right]\Big|_{k_{\rho i}} \right|^2 dz$$

Numeric or analytic

- Write a separate routine to calculate the FT of a uniform current distribution

$$\tilde{J}_x(k_x, k_y) = Lsinc(\frac{k_x L}{2})sinc(\frac{k_y w}{2})$$

- Write a separate routine to calculate the phi integral for the specific current distribution:

$$[I_\phi] = PhiInt\_PWS(k0, ksw, l, w)$$

$$[I_\phi] = PhiInt\_Uniform(k0, ksw, l, w)$$

$$I_\phi = \int_0^{2\pi} \left|\boldsymbol{J}_x(k_{\rho i}, \phi)\right|^2 \cos^2\phi \, d\phi$$

The power can be evaluated using the power of the elementary current ($P_{sw}^{TM}|_{\delta}$) via

$$P_{sw}^{TM} = P_{sw}^{TM}\Big|_{\delta} \frac{I_\phi}{I_\phi^\delta}$$

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