EE4620 - Spectral Domain Methods in Electromagnetics

Topic # 5

Introduction to Leaky Wave Antennas

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Antennas Printed in Stratified Dielectrics

Dielectric allows using integrated technology (PCB, lithography)



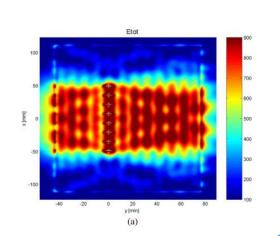
(ground plane) reduces the radiation into the lower region

 $h_{opt} = \lambda/4$

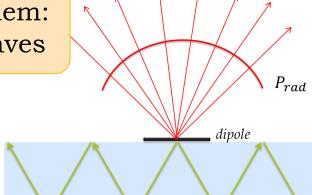
Using a back reflector

E.g. of a linear array





Main problem: surface waves

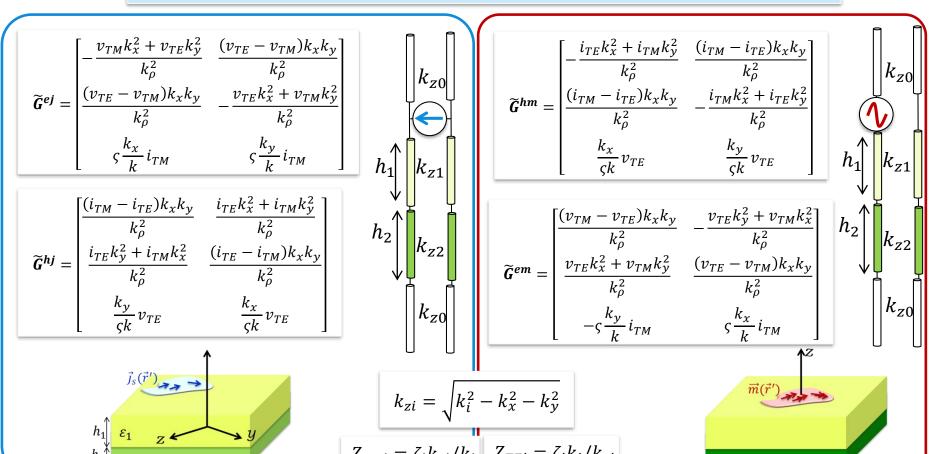


There is a part of the power delivered to the antenna that is radiated inside the dielectric!



Dyadic Green's Function for Stratified Media

$$\boldsymbol{f}(\vec{r}) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \widetilde{\boldsymbol{G}}^{fc}(k_x, k_y, z, z') \boldsymbol{C}_s(k_x, k_y) e^{-jk_x x} e^{-jk_y y} dk_x dk_y$$



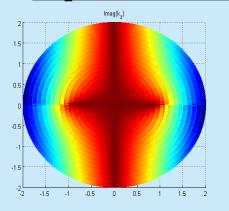


Preferred Branch Convention

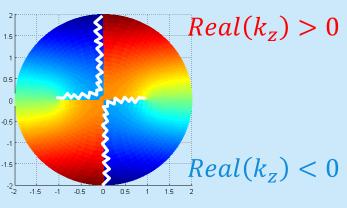
Selection of square root multivalues on the Riemann sheets to obtain a unique specification of the integrand in the complex plane

$$k_z = -j\sqrt{-\left(k^2 - k_\rho^2\right)}$$

Top Riemann Sheet:

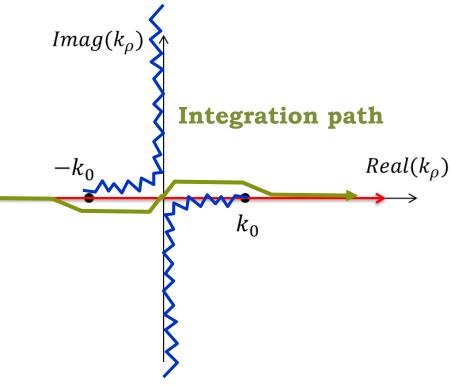


 $Imag(k_z) < 0$



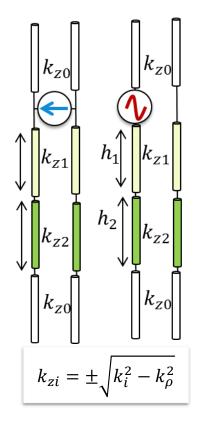
The radiation condition is verified

Branch cuts



Pole Singularities in Stratified Media

Pole singularities in k_{ρ} arise in dielectric stratifications



The solutions of the transmission lines can be expressed as:

$$v_{TM/TEi}(k_{\rho}, z) = \frac{N_{TM/TEi}^{v}(k_{\rho}, z)}{D_{TM/TE}(k_{\rho})}$$

$$i_{TM/TEi}(k_{\rho}, z) = \frac{N_{TM/TEi}^{i}(k_{\rho}, z)}{D_{TM/TE}(k_{\rho})}$$

Dispersion equation to find transversal propagation constants:

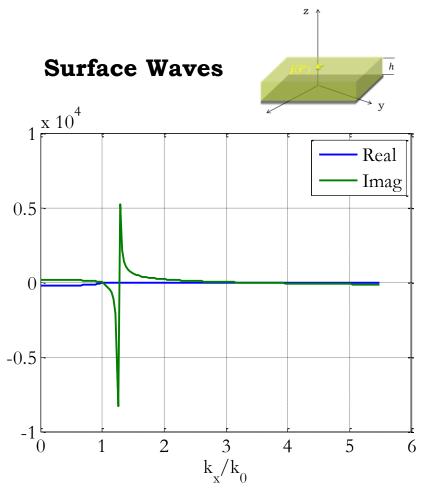
$$D_{TM/TE}(k_{\rho}) = 0 \qquad \qquad k_{\rho 0} = k_{\rho}^{g} - \frac{D(k_{\rho}^{g})}{D'(k_{\rho}^{g})}$$

 Residues to find the surface wave field distribution and associated power

$$v_{TM}(k_{\rho},z) = \frac{N(k_{\rho})}{D(k_{\rho})} \qquad \qquad \qquad \left| Res[v_{TM}(k_{\rho},z)] \right|_{k_{\rho i}} = \frac{N(k_{\rho i})}{D'(k_{\rho i})}$$



Examples of Surface wave poles



Polar singularity in the real axis with $k_{\rho} > k_0$

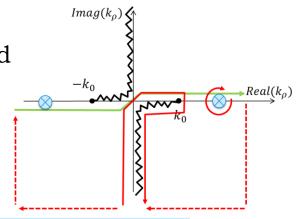


SW Field Contribution

$$\vec{f}(\vec{r}) \approx \frac{1}{4\pi^2} \int_{0}^{\pi} \int_{-\infty}^{\infty} \tilde{G}^{fc}(k_{\rho}, z, z') k_{\rho} e^{-jk_{\rho}\rho} k_{\rho} dk_{\rho} d\alpha \vec{C}_{0}(k_{\rho c}, \phi)$$



Surface wave poles are found on the real axis in the Top Riemann Sheet



$$\vec{f}_{SW}(\vec{r}) = -2\pi j \sum_{i} Res \left[\frac{e^{\frac{j\pi}{4}} \sqrt{k_{\rho}} e^{-jk_{\rho}\rho}}{2\pi \sqrt{2\pi\rho}} \widetilde{\boldsymbol{G}}^{fc}(k_{\rho}, z, z') \overrightarrow{\boldsymbol{C}}_{\mathbf{0}}(k_{\rho}, \phi) \right]_{k_{\rho} = k_{\rho i}}$$

Field contributions in the form of cylindrical propagating waves inside the stratified dielectrics



Surface Waves

$$k_{\rho}^{sw} = \beta^{sw} = k_0 \sqrt{\epsilon_r^{sw}}$$

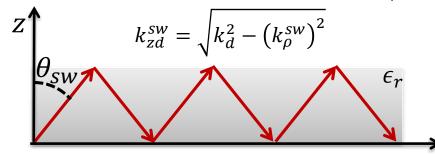
$$k_0 < k_\rho^{sw} < k_d$$

They are also referred as **slow waves**

$$k_{\rho}^{sw} = \frac{2\pi f}{v_{sw}} > k_0 \qquad v_{sw} < v_0$$

It can be seen as a couple of homogenous waves propagating inside the dielectric with a direction characterized by a real angle $\pm \theta_{sw}$:

$$\beta^{sw} = k_d sin\theta_{sw} < k_d \qquad sin\theta_{sw} = \frac{\sqrt{\epsilon_r^{sw}}}{\sqrt{\epsilon_r}}$$



There is no attenuation, therefore the energy carried by the surface wave will reach *infinity* It is an **inhomogenous wave in** the air region:

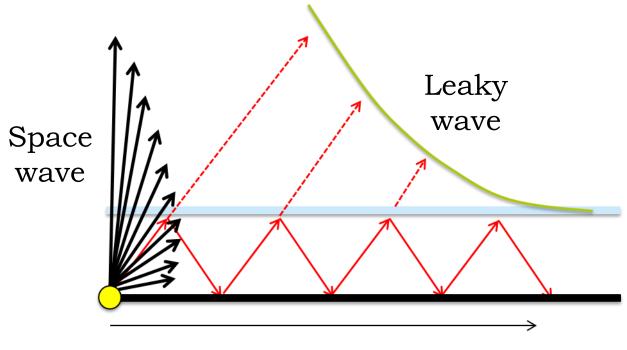
$$k_{z0sw} = -j\sqrt{-\left(k_0^2 - \left(k_\rho^{sw}\right)^2\right)} = -j\alpha_{zsw}$$
They verified the radiation condition in the infinite air medium

The sw angle is above the critical angle $\sqrt{\epsilon_r} sin\theta_{sw} = sin\theta_0$ Therefore

$$\sqrt{\epsilon_{rsw}} = sin\theta_0$$

Therefore θ_0 is imaginary

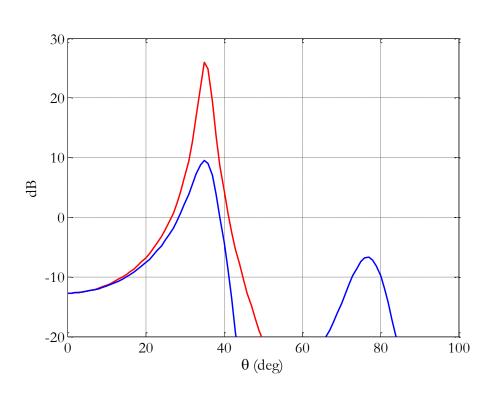
Introduction to leaky waves in stratified media

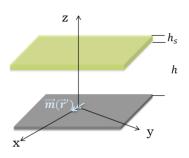


Attenuation in rho



Leaky Wave Poles





The spectrum seems to peak at a certain location in the visible spectrum, but does not go to infinity

Polar singularity in the complex plane close to the real axis with $Re[k_{\rho}] < k_0$

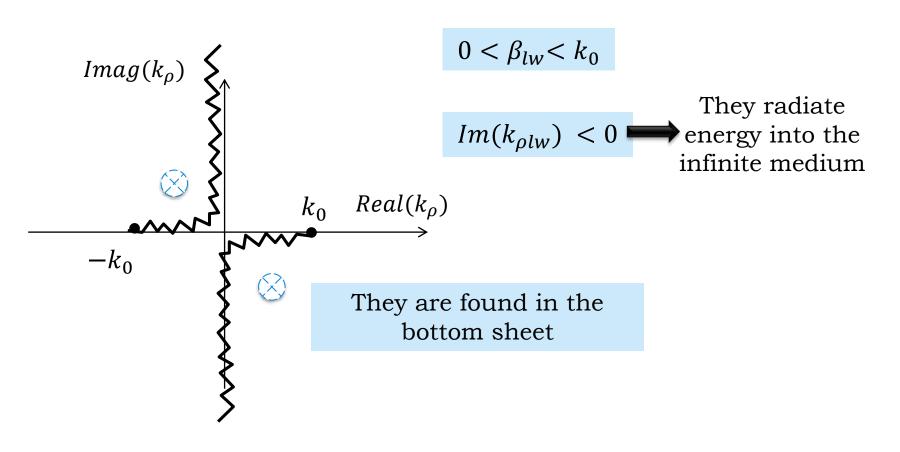


Pole Singularity of Leaky Waves

Complex pole singularities:

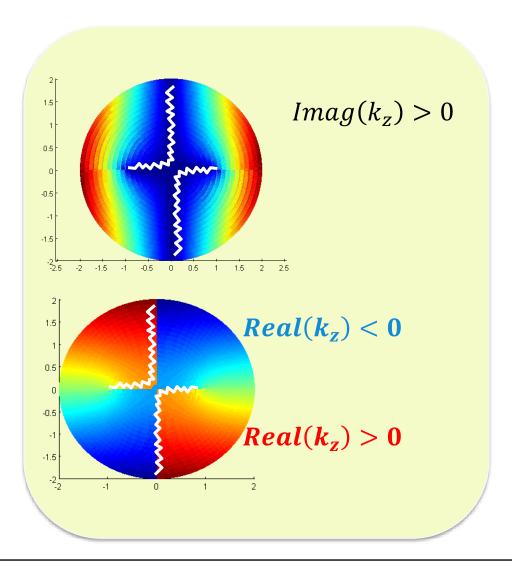
$$k_{\rho lw} = \beta_{lw} - j\alpha_{lw}$$

They are present only in certain stratifications





Bottom Riemann Sheet



$$k_z = +j\sqrt{-\left(k^2 - k_\rho^2\right)}$$

This is opposite to the standard choice



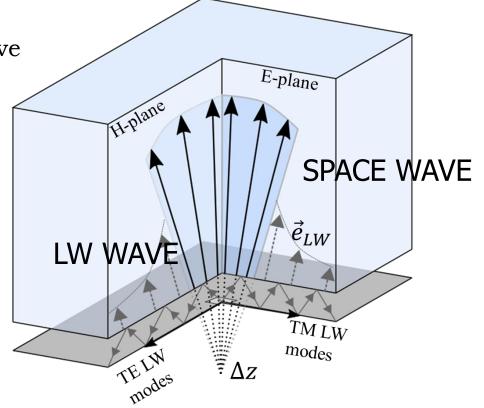
They are present close to the source

They enlarge the effective area of the source

The leaky wave pole is only captured for observation points near the source

The residue contribution does not arrive to infinity

But it is responsible to the power transferred to the far field of the antenna: modulation of the SFG in the region around θ_{lw}^0





They can be used to do pattern shaping!



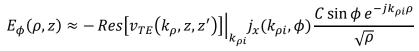
They are cylindrical waves with the same ϕ -pattern and polarization than those of surface waves

Cylindrical LWs

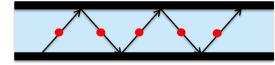
TE Wave

Electric field oriented in the azimuthal direction

Maximum in the Hplane of the antenna



TE Wave

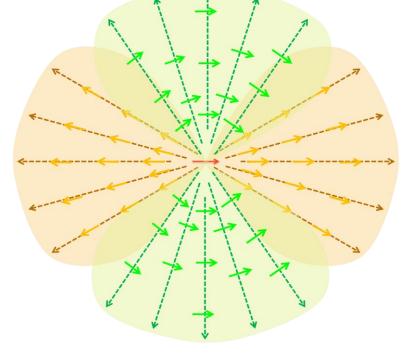


TM Wave Electric field oriented in the radial and z direction Maximum in the Eplane of the antenna

$$E_{\rho}(\rho,z) \approx Res \left[v_{TM}(k_{\rho},z,z') \right] \Big|_{k_{\rho i}} j_{x}(k_{\rho i},\phi) \frac{C\cos\phi \, e^{-jk_{\rho i}\rho}}{\sqrt{\rho}}$$

$$E_z(\rho,z) \approx -\frac{\varsigma k_{\rho i}}{k} Res \big[i_{TM} \big(k_\rho, z, z' \big) \big] \bigg|_{k_{\rho i}} j_x(k_{\rho i}, \phi) \frac{C \cos \phi \, e^{-j k_{\rho i} \rho}}{\sqrt{\rho}}$$

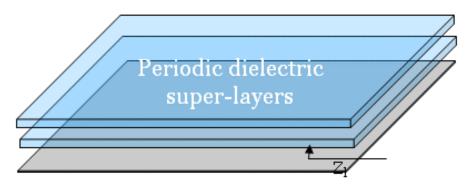
TM Wave



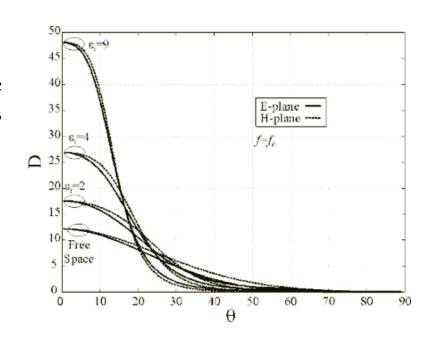




Main Purpose of discussing them is LW Antennas

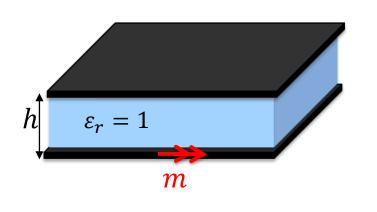


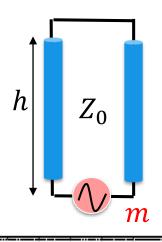
They can be used to enhance the directivity at broadside of small antennas





Parallel Plate in Air

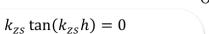




 $D(k_{\rho}) = jZ_0 \tan(k_{z0}h) = 0$

TM Wave

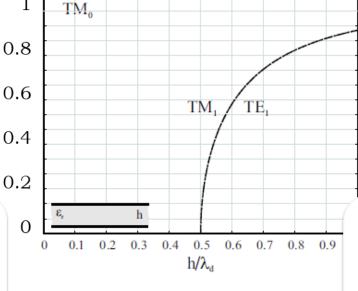




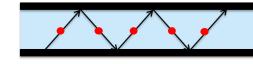
$$k_{ZS} = 0 tan(k_{ZS}h) = 0$$

$$k_{\rho_0} = k \qquad k_{ZS}h = n\pi$$

$$k_{\rho_n} = \sqrt{k^2 - \left(\frac{n\pi}{h}\right)^2}$$



TE Wave



for TE

$$\tan(k_{zs}h)=0$$

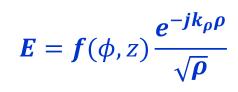
$$k_{zs}h=n\pi$$

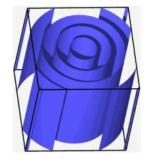
$$k_{\rho_n} = \sqrt{k^2 - \left(\frac{n\pi}{h}\right)^2}$$

ŤUDelft

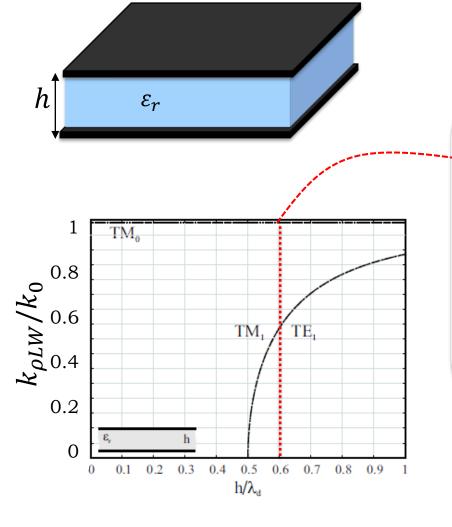
for TM

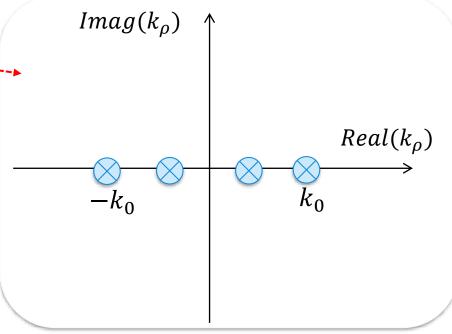
Parallel Plate in Air





 k_{ρ} -Spectrum

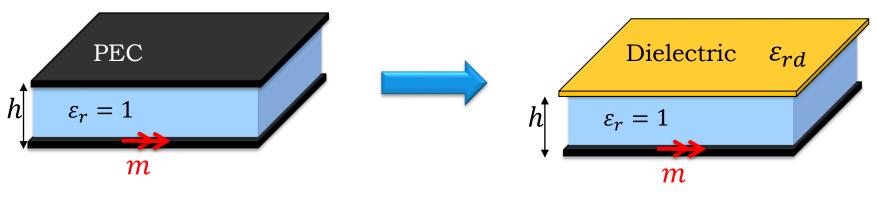


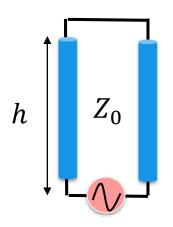


A PPW has no branch singularity, only real poles. These ones are SWs with propagation constant that can be $\leq k_0$



Open Parallel Plate Waveguide



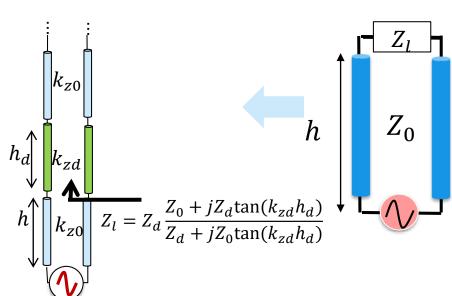


$$Z_0(k_\rho = 0) = \zeta_0$$

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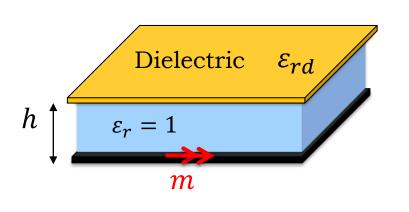
$$Z_d(k_\rho = 0) = \frac{\zeta_0}{\epsilon_r}$$

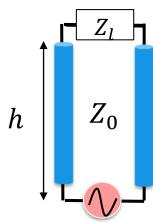
$$Z_0(k_\rho=0)=\zeta_0$$





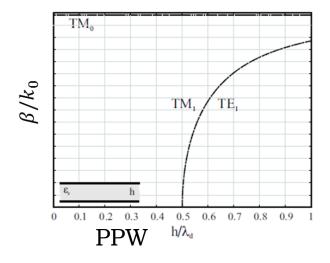
Open Parallel Plate Waveguide

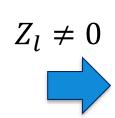


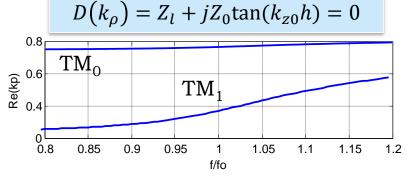


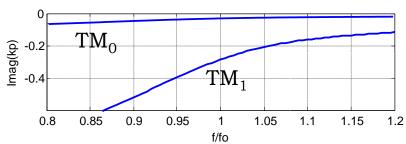
$$i(z = z_s) = Y_{in} = Y_0 \frac{Z_0 + jZ_l \tan(k_{z0}h)}{Z_l + jZ_0 \tan(k_{z0}h)}$$

$$D(k_{\rho}) = jZ_0 \tan(k_{z0}h) = 0$$



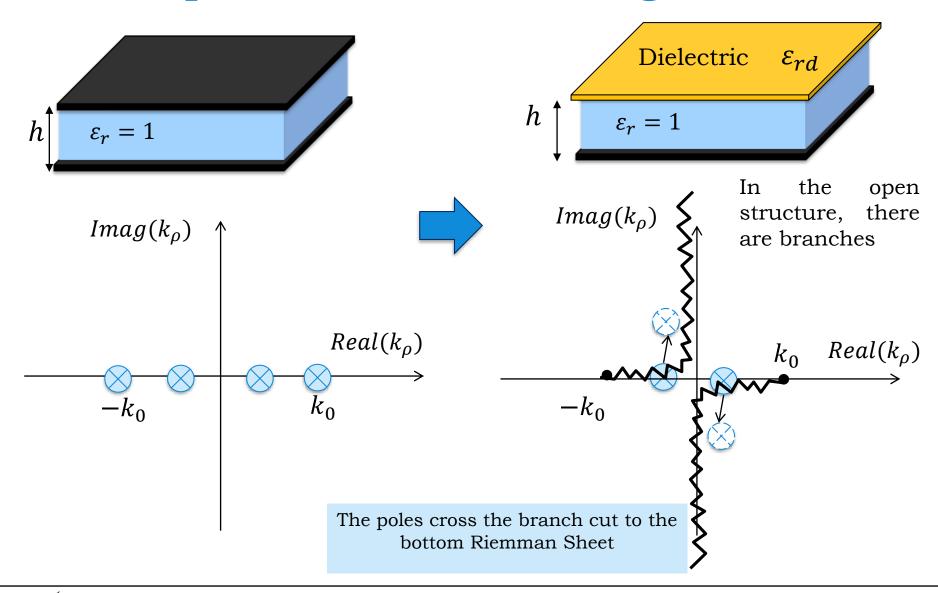






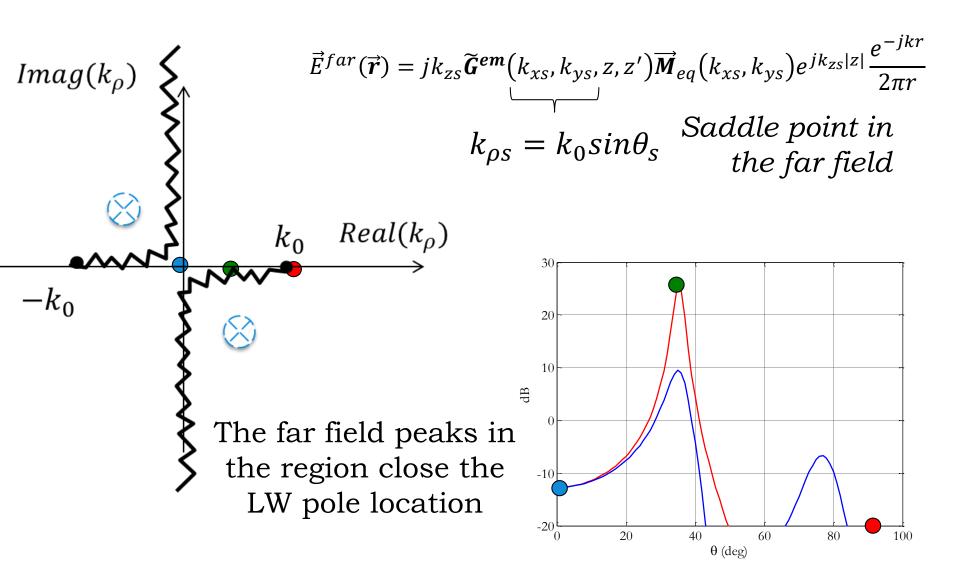


Open Parallel Plate Waveguide





Pattern shaping with leaky waves

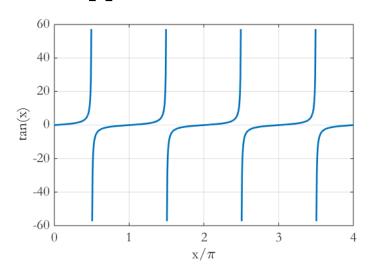




Dispersion Equation: Analytical Approximated Solution

$$D(k_{\rho}) = Z_l + jZ_0 \tan(k_{z0}h)$$

Approximation 1:



The tan function can be approximated linearly around its zeroes for arguments in the surrounding of $x = \pm n\pi$, resorting to

$$\tan(k_{z0}h)_{|x\pm n\pi|\approx 0} \approx k_{z0}h - n\pi$$

$$n=1$$

$$k_{z0} = j\frac{Z_l}{hZ_0} + \frac{\pi}{h} = \sqrt{k_0^2 - k_\rho^2}$$

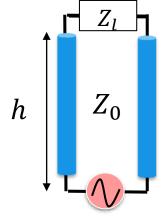
A. Neto and N. Llombart, "Wideband Localization of the Dominant Leaky Wave Poles in Dielectric Covered Antennas," in IEEE Antennas and Wireless Propagation Letters, vol. 5, pp. 549-551, 2006, doi: 10.1109/LAWP.2006.889558.



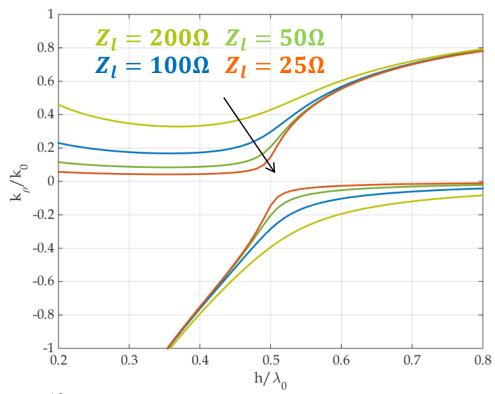
Impact of the cavity top impedance

$$k_{z0} = j\frac{Z_l}{hZ_0} + \frac{\pi}{h} = \sqrt{k_0^2 - k_\rho^2}$$

The propagation constant highly depends on Zl



$$k_{\rho} = \sqrt{k_0^2 - k_{z0}^2} = \beta - j\alpha$$



$$\beta = k_0 sin\theta_{LW}$$

$$Z_l \downarrow \theta_{lw} \downarrow$$

Resonant condition:

$$h \approx \lambda_0/2$$

Tunning the cavity top impedance

$$Z_{0}(k_{\rho} = 0) = \zeta_{0}$$

$$Z_{d}(k_{\rho} = 0) = \frac{\zeta_{0}}{\epsilon_{r}}$$

$$h_{d} \downarrow k_{zd}$$

$$k_{zd}$$

$$k_{zd}$$

$$k_{zd}$$

$$k_{zd}$$

$$k_{zd}$$

$$Z_{l} = Z_{d} \frac{Z_{0} + jZ_{d} \tan(k_{zd}h_{d})}{Z_{d} + jZ_{0} \tan(k_{zd}h_{d})}$$

For a quarter wavelength thick substrate (Z_l tends to zero for large ϵ_r):

$$h_d = \lambda_d/4$$

$$k_\rho = 0$$

$$Z_l = \zeta_0/\epsilon_r$$



Dispersion Equation: Analytical Solution

$$D(k_{\rho}) = Z_l + jZ_0 \tan(k_{z0}h)$$



$$k_{z0} = j\frac{Z_l}{hZ_0} + \frac{\pi}{h} = \sqrt{k_0^2 - k_\rho^2}$$

Approximation 2:

$$Z_l = Z_d \frac{Z_0 + jZ_d \tan(k_{zd}h_d)}{Z_d + jZ_0 \tan(k_{zd}h_d)}$$



$$Z_l \approx \zeta_0/\epsilon_r$$

For poles radiating close to broadside



Analytical Approximated Solution

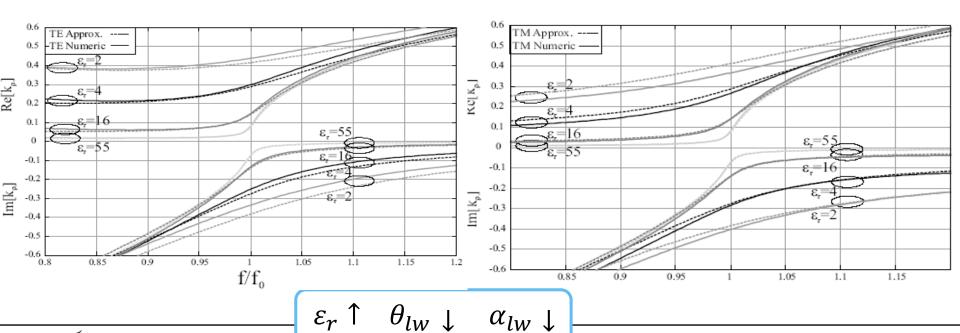
$$k_{z0} = j \frac{\zeta_0}{h Z_0 \epsilon_r} + \frac{\pi}{h} = \sqrt{k_0^2 - k_\rho^2}$$

$$Z_{TE} = \zeta_i k_i / k_{zi}$$

$$k_{z0}^{TE} = k_0 \left(\frac{1}{2\bar{h}} + j \frac{1}{4\bar{h}^2 \pi \epsilon_r} \right) \qquad \bar{h} = h/\lambda_0$$

$$Z_{TM} = \zeta_i k_{zi} / k_i$$

$$k_{z0}^{TM} = k_0 \left(\frac{1}{2\bar{h}} + j \frac{1}{\pi \epsilon_r} \right)$$



Leaky Waves

$$k_{\rho}^{lw} = \beta^{lw} - j\alpha^{lw}$$

$$\beta^{lw} < k_0$$

They are also referred as **fast waves**

$$k_{\rho}^{lw} = \frac{2\pi f}{v_{lw}} < k_0 \qquad v_{lw} > v_0$$

It can be seen as a couple of waves propagating with a direction characterized by a real angle $\pm \theta_{lw}$ and an attenuation:

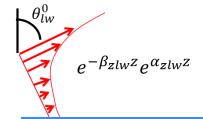
$$\beta^{lw} = k_o sin\theta_{lw} < k_0$$



There is attenuation, therefore the the leaky wave looses energy while propagates

There is also propagation in the air region that can be related to a real angle:

$$\sqrt{\epsilon_r} \sin \theta_{lw} = \sin \theta_{lw}^0$$



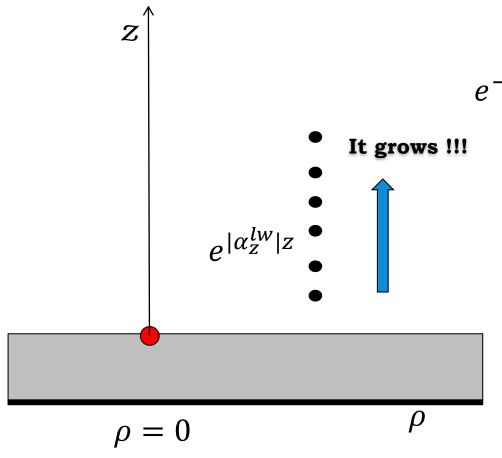
They do not verify the radiation condition in the infinite air medium



$$k_{z0}^{lw} = j\sqrt{-(k_0^2 - k_{\rho lw}^2)} = \beta_{zlw} + j\alpha_{zlw}$$

The difficulty with leaky Waves

$$\vec{f}_{Lw}(\vec{r}) = -2\pi j \sum_{i} Res \left[\frac{e^{\frac{j\pi}{4}} \sqrt{k_{\rho LW}} e^{-jk_{\rho LW}\rho}}{2\pi \sqrt{2\pi\rho}} e^{-jk_{z LW}z} \tilde{\boldsymbol{G}}^{fc} (k_{\rho LW}, z = 0, z') \vec{\boldsymbol{C}}_{\boldsymbol{0}}(k_{\rho Lw}, \phi) \right]_{k_{\rho} = k_{\rho LW}}$$



$$e^{-jk_{zLW}z} = e^{-j\beta_{zLW}z}e^{-j(j\alpha_{zLW})z}$$
$$= e^{-j\beta_{zLW}z}e^{\alpha_{zLW}z}$$

This scared people for decades
Their claimed was they did not exist



Integration path: far observation point

Complex pole singularities (Bottom sheet poles)

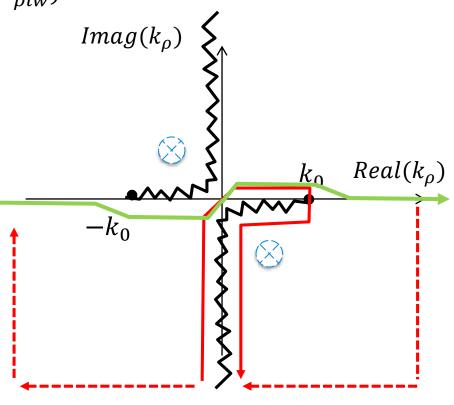
$$k_{\rho lw} = \beta_{lw} - j\alpha_{lw}$$
 $k_{zlw} = j\sqrt{-(k_0^2 - k_{\rho lw}^2)}$



Waves that propagate in the transverse direction with attenuation

However it is captured in other deformations: near-field!!!

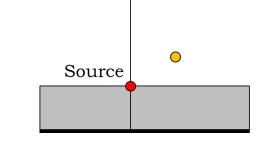
Not captured in the deformation around the branch





Near field observation points

For near field **steepest descent path** are the integration paths that guarantee most rapid convergence of the integral



Observation

point

Saddle point exist in these paths

 $Real(k_{\rho})$

$$k_{\rho 1} = k_0 sin 0 = 0$$

$$k_{\rho 2} = k_0 \sin \frac{\pi}{4} = k_0 / \sqrt{2}$$

$$k_{\rho 3} = k_0 \sin \frac{\pi}{2} = k_0$$

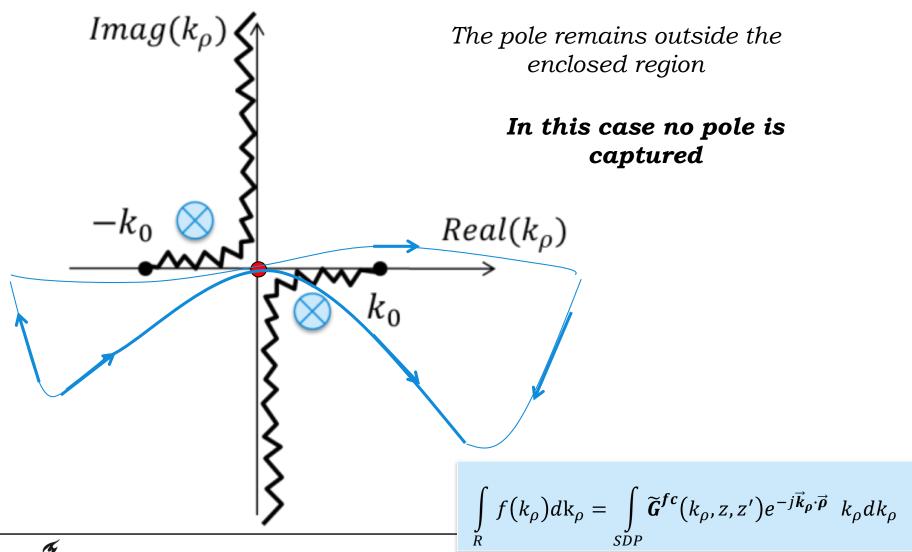
When is the LW pole captured?



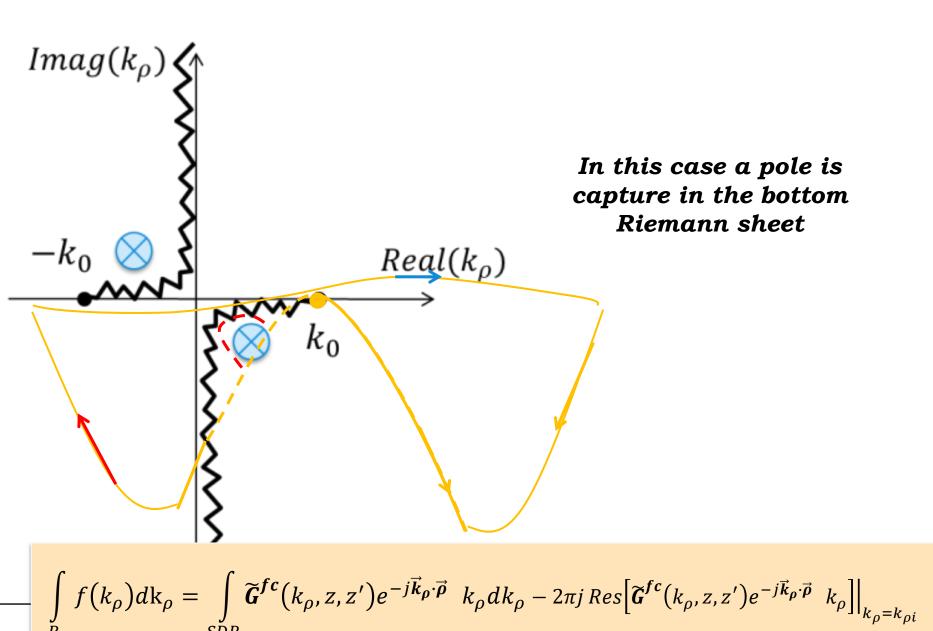
 $Imag(k_{\rho})$

Deformation on SDP

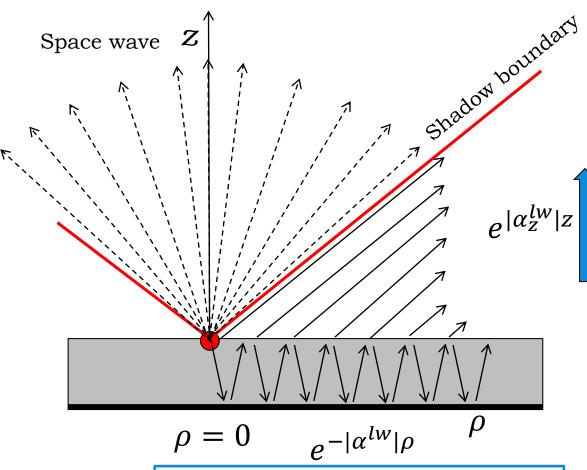
From the original path to SDP via deformation



Near field shaping with leaky waves



Existence Region



If you take z = 0, and then increase ρ from zero, the wave attenuates exponentially to zero

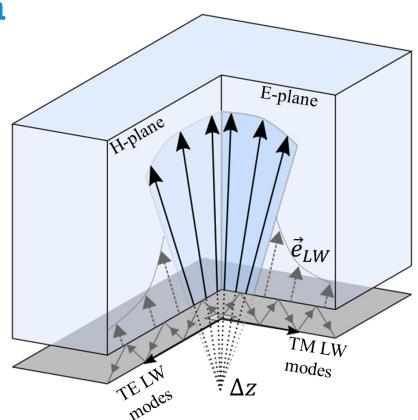
If you take any ρ , and then increase z from zero, the wave becomes larger... until it disappears at the shadow boundary

The leaky wave field contribution is in fact limited to a region of existence.



Existence Region

The leaky wave pole is only captured for observation points near the source



The residue contribution does not arrive to infinity

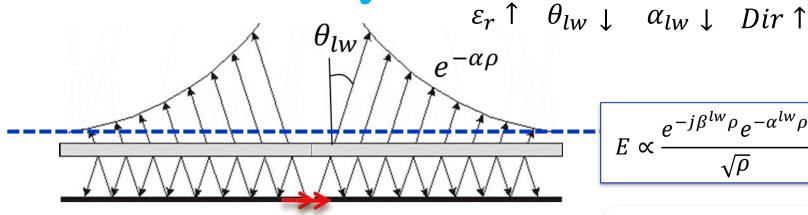
But it is responsible to the power transferred to the far field of the antenna: modulation of the SFG in the region around θ_{lw}^0

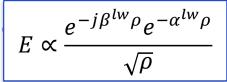


They can be used to do pattern shaping!



Enhanced Directivity





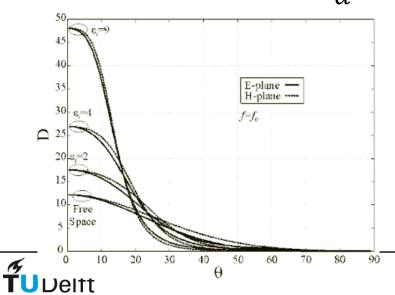
A_{source}

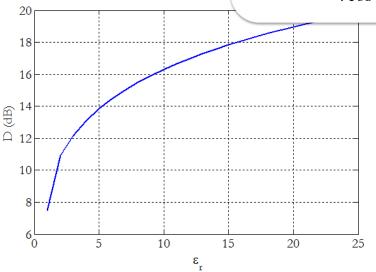
$$A_{eff} \propto \frac{1}{\alpha^2} \gg A_{source}$$

Resonant condition:

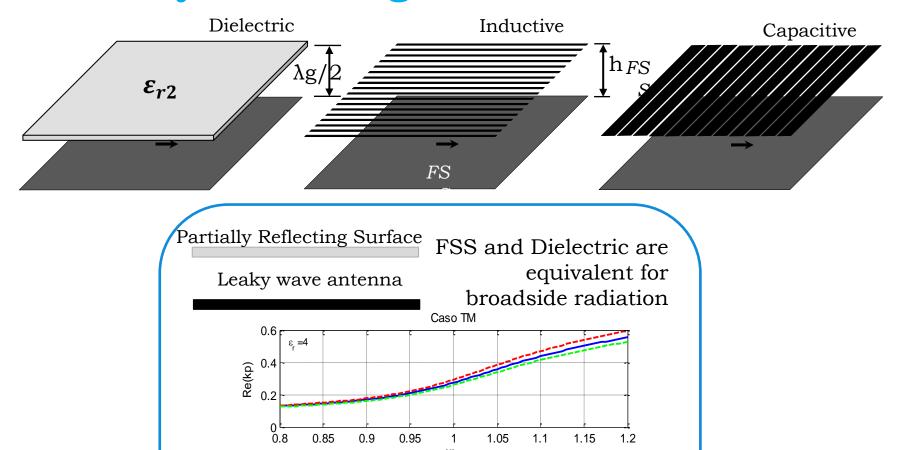
$$h = \frac{\lambda_0}{2\cos\theta_0}$$

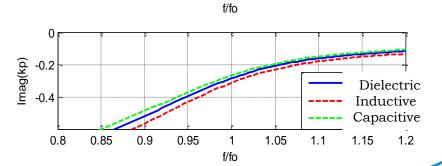
$$h_d = \frac{\lambda_{d2}}{4\cos\theta_d}$$





Partially Reflecting Surfaces







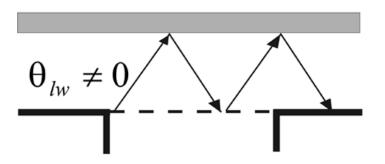
Bandwidth Issues

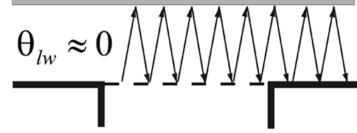
Input Admittance:

$$Y_{in} = \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} \left| M(k_x, k_y) \right|^2 G^{hm}(k_x, k_y) dk_x dk_y$$

Highly dominated by the presence of the leaky wave poles $Res\left(G^{hm}(k_x,k_y)\right) \propto \frac{1}{sin\theta_{lw}}$

$$\mathbf{BW} \iff \frac{Y^{ebg}(f_1)}{Y^{ebg}(f_2)} \approx \frac{k_{lw}(f_2)}{k_{lw}(f_1)}$$

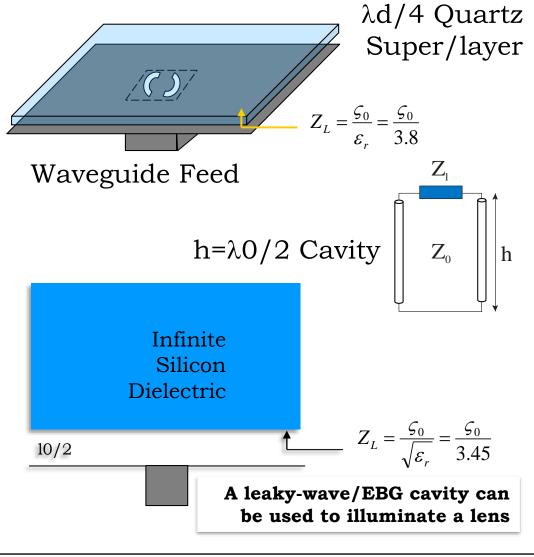


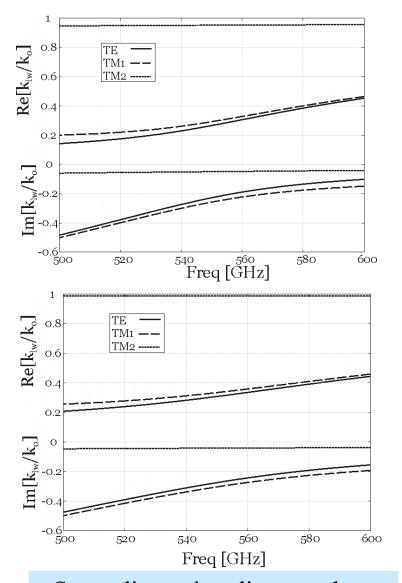


$$\varepsilon_r \uparrow \theta_{lw} \downarrow \alpha_{lw} \downarrow Dir \uparrow BW \downarrow$$



Leaky Wave Lens Feed

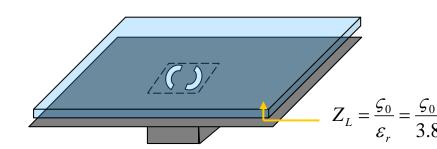


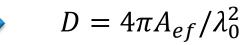


Same dispersion diagram than quarter-wavelenght superstrates



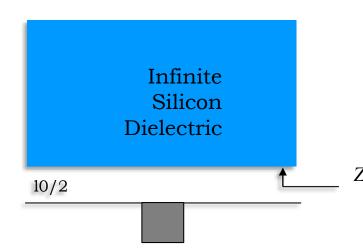
Leaky Wave Lens Feed





Same effective area given by the same leaky wave mode

 ε_r higher directivity for the same bandwidth or leaky wave frequency dispersion



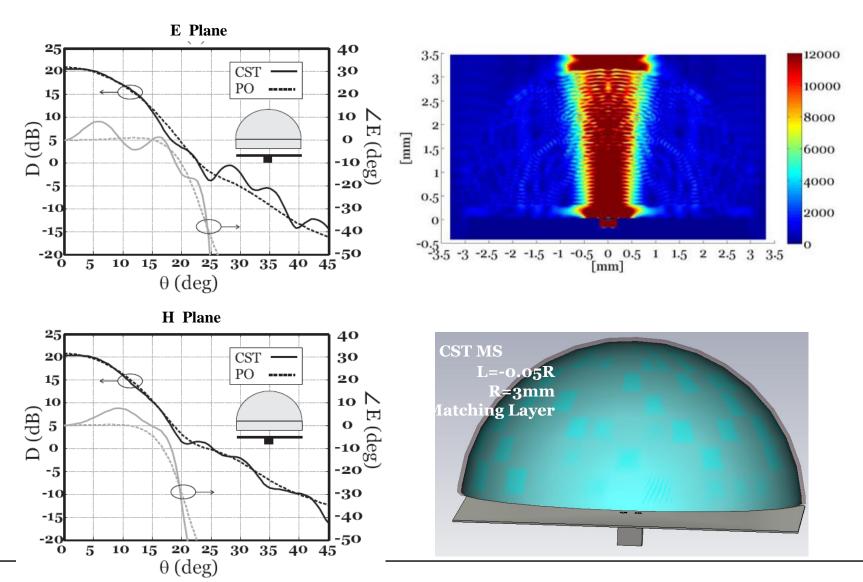


$$Z_L = \frac{\zeta_0}{\sqrt{\varepsilon_r}} = \frac{\zeta_0}{3.45}$$

$$D = 4\pi A_{ef}/\lambda_d^2 = 4\pi A_{ef} \varepsilon_r/\lambda_0^2$$

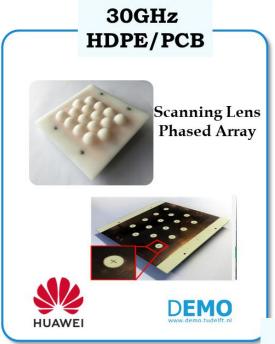


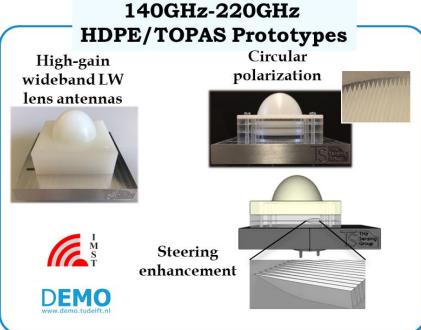
Lens Radiated Fields in standard lenses





Examples of Lens LW Prototypes





Silicon Prototypes In-package wideband LW lens antenna Scanning Lens Phased Array Fraunhofer In Fraunhofer

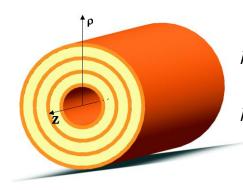




Waveguide Applications

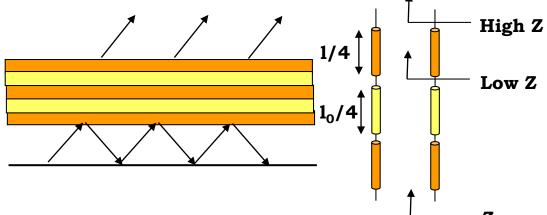
Same Resonant condition than leaky wave antenna:

Bragg Fibers

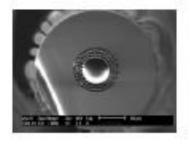


$$h_{d1} = \frac{\lambda_{d1}}{4\cos\theta_{d1}}$$

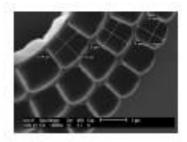
$$h_{d2} = \frac{\lambda_{d2}}{4\cos\theta_{d2}}$$



- •They are used at near infrared and optical regimes.
- •Field distribution similar than a circular waveguide





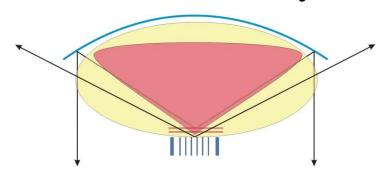


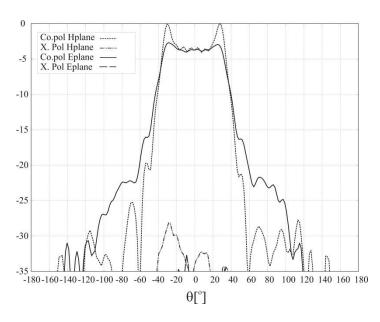
Loss in the order of dB/km

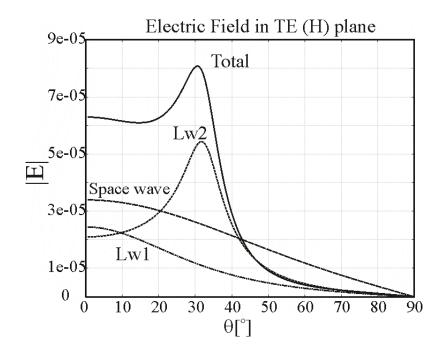


Field Shaping Application

Illumination of reflector with 90% efficiency

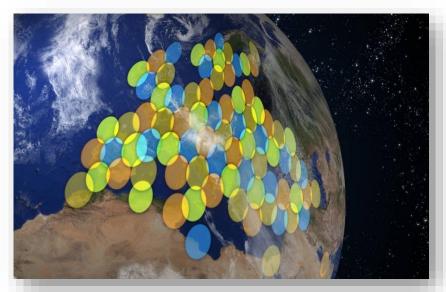






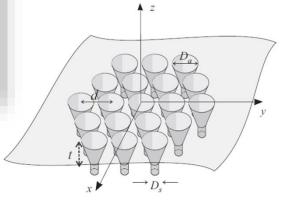


Multi Beam Imaging for Telecom





Ka Band
Telecommunication
Applications
Earth Coverage by using
Multiple Antenna Beams



<u>Technological Solutions</u>

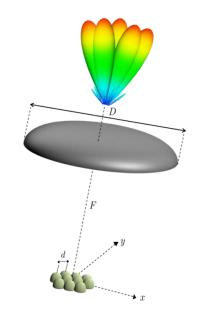
Use of reflectors for high directivity in combination with

- 1) Phased Arrays generating multiple beams
- 2) Focal Plane Arrays with Single Fed per Beam



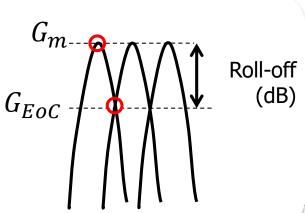
Edge of Coverage Gain

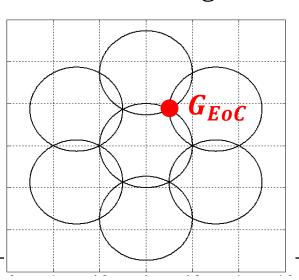
The goal is to maximize the gain at the edge of coverage and keep the inter-beam interferences low



The best packaging is achieve for an hexagonal array

Far Field Beams





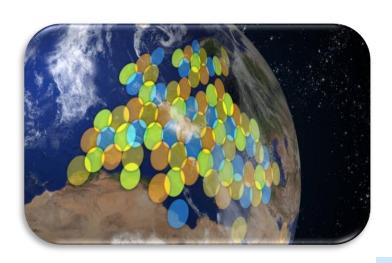


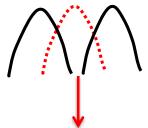
Standard Solution for ka Satellites

Use of 4 reflectors to increase the Geoc with $2\lambda_0 f_{\#}$ FPA

Diminish roll-off without increasing spill over







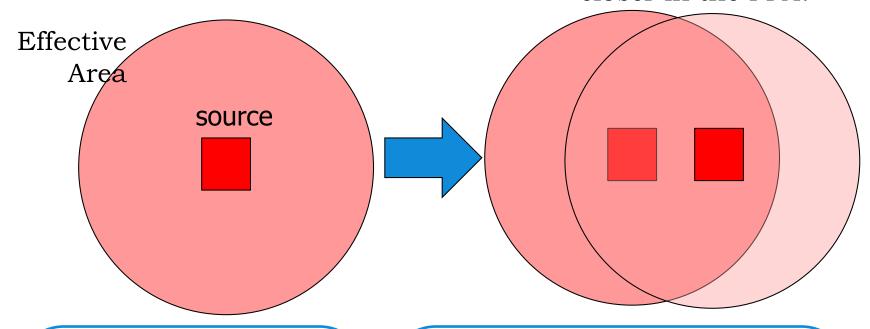
Beam generated by another reflector

 $G_{EoC} \approx D_m + SO + RO = D_m + 3.5dB$

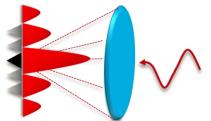


Overlapped Feeds

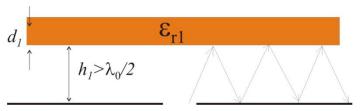
Is it possible to place them closer in the FPA?



Use of Phased arrays: Coherent beamforming of 3x3 or 5x5 sub arrays



Leaky wave/Fabry Perot Antennas



Antennas with directive patterns and 'small physical areas'

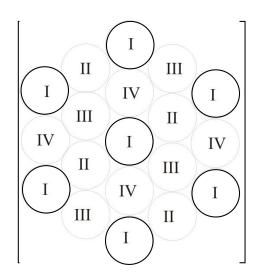


Overlapped Feeds FPA

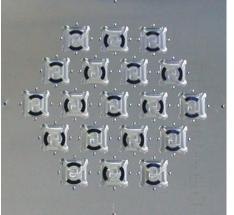
Loss due to the mutual coupling



Use of reactive loading to compensate for it: **Equivalent short circuits loads** for neighboring waveguides



1) Polarization discrimination

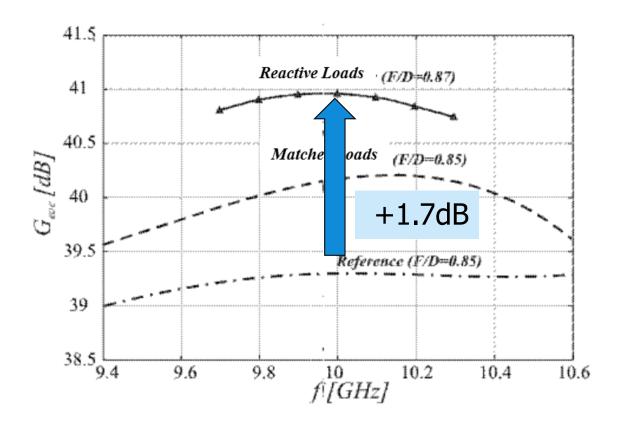


2) Frequency discrimination





Reactively Loaded case



10% Bandwidth



Important Points

Stratified GFs contain poles on bottom Riemann sheet

They correspond to leaky waves

These poles are complex and located closed to the real axes on the bottom Riemann sheet

These poles are not captured in deformations around branch, but in other deformations useful to obtain analytic evaluations of the near field

In the near field, the leaky waves are cylindrical waves, similar to surface waves but with an attenuation associated to radiation

Leaky waves can be used to shape the far field of small sources



Related IEEE Papers on LWAs

- D. R. Jackson and A. A. Oliner, "A leaky-wave analysis of the high-gain printed antenna configuration," IEEE Transactions on Antennas and Propagation, vol. 36, no. 7, pp. 905–910, Jul. 1988, doi: 10.1109/8.7194.
- G. Lovat, P. Burghignoli, and D. R. Jackson, "Fundamental properties and optimization of broadside radiation from uniform leaky-wave antennas," IEEE Transactions on Antennas and Propagation, vol. 54, no. 5, pp. 1442–1452, May 2006, doi: 10.1109/TAP.2006.874350
- R. Gardelli, M. Albani, and F. Capolino, "Array thinning by using antennasinaFabry-erot cavity for gainenhancement," IEEETrans. Antennas Propag., vol. 54, no. 7, pp. 1979–1990, Jul. 2006
- A. Neto, N. Llombart, G. Gerini, M. D. Bonnedal, and P. de Maagt, "EBG Enhanced Feeds for the Improvement of the Aperture Efficiency of Reflector Antennas," IEEE Transactions on Antennas and Propagation, vol. 55, no. 8, pp. 2185–2193, Aug. 2007,
- N. Llombart, G. Chattopadhyay, A. Skalare, and I. Mehdi, "Novel Terahertz Antenna Based on a Silicon Lens Fed by a Leaky Wave Enhanced Waveguide," IEEE Transactions on Antennas and Propagation, vol. 59, no. 6, pp. 2160–2168, Jun. 2011, doi: 10.1109/TAP.2011.2143663.
- M. Arias Campo, D. Blanco, S. Bruni, A. Neto, and N. Llombart, "On the Use of Fly's Eye Lenses with Leaky-Wave Feeds for Wideband Communications," IEEE Transactions on Antennas and Propagation, vol. 68, no. 4, pp. 2480–2493, Apr. 2020,
- M. Alonso-delPino, S. Bosma, C. Jung-Kubiak, G. Chattopadhyay, and N. Llombart, "Wideband Multi-Mode Leaky-Wave Feed for Scanning Lens Phased Array at Submillimeter Wavelengths," IEEE Transactions on Terahertz Science and Technology, 2020,

