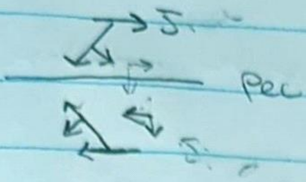

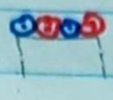

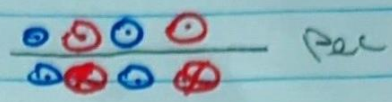

$\vec{E}_{tot} = \vec{E}_{inc} + \vec{E}_{scattered}$

$S_{eq} = 0$

$M_{eq} = -2\hat{n} \times \vec{E}_{inc}$

$M_s = m_s(z=0^+) + m_s(z=0^-)$

on the gap where j_{inc} is located:

$e_{tot} = Z_s j$ we assume $Z_s = 0$ so $e_{tot} = 0$

$e_{tot} = e_{inc} + e_{scat} = 0$

narrow slot ∴ ($w_s << \lambda$) oriented along x, we can assume $E_x \neq 0$

$e_{scat} = -e_{inc}$

the incident field is field in δ gap, which is constant over gap region.

$$-g^{hm}_{xx} * 4m_x(x,y) = j_{inc,y}(x,y)$$

$$-g^{hm}_{xx} * 4m_x(x,y) = I_0 \frac{rect_{\delta_s}(x)}{\delta_s}$$

Magnetic field Integral Equation (MFIE)

$$-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g^{hm}_{xx}(x-x', y-y')\, 4m_x(x',y')\, dx'dy' = I_0 \frac{rect(x,y)}{\delta_d}$$

transverse magnetic current (The m_x is written as a product of long term and trans term)

m_L is the longitudinal term. $m_x(x',y') = m_L(x')\, m_t(y')$

$$m_t(y) = \frac{2}{w_s \pi} \frac{1}{\sqrt{1-\left(\frac{2y}{w_s}\right)^2}}$$

$$-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g^{hm}_{xx}(x-x', y-y')\, 4m_L(x')\, m_t(y')\, dx'\, dy' = I_0 \frac{rect(x,y)}{\delta_d}$$

$$-4\int_{-\infty}^{\infty} m_L(x') \left(\int_{-\infty}^{\infty} g^{hm}_{xx}(x-x', y-y')\, m_t(y')\, dy' \right) dx' = I_0 \frac{rect_{gap}(x)}{\delta_d}$$

we can define the term inside parenthesis as:

$$d(x-x') = \int_{-\infty}^{\infty} g_{xx}^{hm}(x-x', y-y')\, m_t(y')\, dy'$$

using IFT def. we can write

$$d(x-x') = \frac{1}{2\pi}\int_{-\infty}^{\infty} D(k_x)\, e^{-jk_x(x-x')}\, dk_x$$

we substitute above formula into one of prev formulas (8b)

$$-\frac{4}{2\pi}\int_{-\infty}^{\infty} D(k_x)\underbrace{\left(\int_{-\infty}^{\infty} m_\ell(x')\, e^{jk_x x'}\, dx'\right)}_{\text{Longitudinal Current}} e^{-jk_x x'}\, dk_x = I_0 \frac{rect_{gap}(x)}{\delta_d} \quad (a)$$

so we can write Long. current as:

Magnetic Field Integral Equation (MFIE)

$$I(k_x) = \int_{-\infty}^{\infty} m_\ell(x')\, e^{jk_x x'}\, dx' \quad (b)$$

the right side of MFIE can be represented as IFT.

$$I_0 \frac{rect_{gap}(x)}{\delta_d} = I_0 \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{sinc}\left(\frac{k_x \delta_d}{2}\right) e^{-jk_x x}\, dk_x \quad (c)$$

substitute b & c into (a).

$$-\frac{2}{\pi}\int_{-\infty}^{\infty} D(k_x)\, I(k_x)\, e^{-jk_x x'} = I_0 \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{sinc}\left(\frac{k_x \delta_d}{2}\right) e^{-jk_x x}\, dk_x$$

Now we need to find an equation for $D(k_x)$

$$k_{zi} = \sqrt{k_i^2 - \underbrace{k_x^2 - k_y^2}_{-(k_x^2 + k_y^2)}} = \sqrt{k_i^2 - k_\rho^2} \qquad k_\rho^2 = k_x^2 + k_y^2$$

$$d(x-x') = \int_{-\infty}^{\infty} \left(\frac{1}{4\pi^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G_{xx}^{hm}(k_x, k_y)\, e^{-jk_x(x-x')}\, e^{jk_y y'}\, dk_x\, dk_y \right) m_t(y')\, dy'$$

Regroup terms that depend on y'

$$d(x-x') = \frac{1}{4\pi^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G_{xx}^{hm}(k_x, k_y)\, e^{-jk_x(x-x')} \left(\int_{-\infty}^{\infty} m_t(y')\, e^{jk_y y'}\, dy' \right) dk_x\, dk_y$$

the term within brackets is FT, FT of edge sing dist is known in closed form and is equal to a zeroth order Bessel func of first kind:

$$d(x-x') = \frac{1}{4\pi^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G_{xx}^{hm}(k_x, k_y)\, e^{-jk_x(x-x')}\, J_0\left(\frac{k_y w_s}{2}\right) dk_x\, dk_y$$

regroup terms that depend on k_y.

$$d(x-x') = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(\frac{1}{2\pi} \int_{-\infty}^{\infty} G_{xx}^{hm}(k_x, k_y)\, J_0\left(\frac{k_y w_s}{2}\right) dk_y \right) e^{-jk_x(x-x')}\, dk_x$$

it is found that $d(x-x')$ is the IFT of term within brackets.

$$D(k_x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G_{xx}^{hm}(k_x, k_y)\, J_0\left(\frac{k_y w_s}{2}\right) dk_y$$

G_{xx}^{hm} can be found by solving for the transmission line theorem for the required strat. media.

$$D(k_x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(\frac{-i_{TE} k_x^2 + i_{TM} k_y^2}{k_x^2 + k_y^2} \right) J_0\left(\frac{k_y w_s}{2}\right) dk_y$$

$V = I R$

for current dist we can equate integrand

$$\frac{-2}{\pi} D(k_x) V(k_x) = I_0 \frac{1}{2\pi} \operatorname{sinc}\left(\frac{k_x \delta_s}{2}\right)$$

$$\boxed{\begin{aligned} V(k_x) &= -\frac{I_0 \operatorname{sinc}\left(\frac{k_x \delta_s}{2}\right)}{4 D(k_x)} \\ v(x) &= -\frac{I_0}{4} \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\operatorname{sinc}\left(\frac{k_x \delta_d}{2}\right)}{D(k_x)} e^{-jk_x x} \, dk_x \end{aligned}}$$

$$Z_{in} = \frac{V_0}{I_0} = \frac{1}{I_0 \delta_d} \int_{-\delta_d/2}^{\delta_d/2} v(x) \, dx$$

$$Z_{in} = \frac{1}{I_0 \delta_d} \int_{-\delta_d/2}^{\delta_d/2} \left(-\frac{I_0}{4} \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\operatorname{sinc}\left(\frac{k_x \delta_d}{2}\right)}{D(k_x)} e^{-jk_x x} \, dk_x \right) dx$$

$$Z_{in} = -\frac{1}{4} \cdot \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\operatorname{sinc}\left(\frac{k_x \delta_d}{2}\right)}{D(k_x)} \underbrace{\left(\frac{1}{\delta_s} \int_{-\delta_s/2}^{\delta_s/2} e^{-jk_x x} \, dx \right)}_{\operatorname{sinc}\left(\frac{k_x \delta_d}{2}\right)} dk_x$$

$$\boxed{Z_{in} = -\frac{1}{4} \cdot \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\operatorname{sinc}^2\left(\frac{k_x \delta_s}{2}\right)}{D(k_x)} \, dk_x}$$

Part 2 Analysis of doubly connected slot arrays

$$m_t(x'+n_x d_x) = m_t(x')e^{-jk_{x0} n_x d_x}$$
$$m_t(y'+n_y d_x) = m_t(y')e^{-jk_{y0} n_y d_y}$$

$$h_{inc}(x, y=0) = \sum_{n_x=-\infty}^{\infty} \frac{I_0 \operatorname{rect}_{\delta_d}(x-n_x d_x)}{\delta_d} e^{-jk_{x0} n_x d_x}$$

MFIE of double periodic connected array of slots is given below

$$4\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g_{xx}^{hm}(x-x', y-y')m_t(x')m_t(y')dx'dy' =$$

$$-4\sum_{n_x=-\infty}^{\infty} \frac{I_0}{\delta_d}\operatorname{rect}(x-n_x d_x)e^{-jk_{x0} n_x d_x}$$

the left hand-side of the equation above can be decomposed into infinite amount of finite integration domains.

$$4\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g_{xx}^{hm}(x-x', y-y')m_t(x')m_t(y')\, dx'dy' =$$

$$4\sum_{n_x=-\infty}^{\infty}\sum_{n_y=-\infty}^{\infty}\int_{n_x d_x-d_x/2}^{n_x d_x+d_x/2}\int_{n_y d_y-d_y/2}^{n_y d_y+d_y/2} g_{xx}^{hm}(x-x', y-y')m_t(x')m_t(y')dx'dy'$$

applying changes of variables $x''=x'-n_x d_x$ & $y''=y'-n_y d_y$

$$4\sum_{n_x, n_y}\int_{-d_x/2}^{d_x/2}\int_{-d_y/2}^{d_y/2} g_{xx}^{hm}(x-x''-n_x d_x, y-y''-n_y d_y)m_t(x''+n_x d_x)m_t(y''+n_y d_y)dx''dy''$$

$$\sum_{n_x, n_y} \int_{-dx/2}^{dx/2} \int_{-dy/2}^{dy/2} g_{xx}^{hm}(x - x'' - n_x d_x, -y'' - n_y d_y) m_t(x'') e^{-jk_{x0} n_x d_x} \cdot m_t(y'') e^{-jk_{y0} n_y d_y} \, dx'' \, dy''$$

periodic connected array green's function.

$$d_\infty = \sum_{n_y=-\infty}^{\infty} \int_{-dy/2}^{dy/2} g_{xx}^{hm}(x, -y'' - n_y d_y) m_t(y'') e^{-jk_{y0} n_y d_y} \, dy''$$

$$4 \sum_{n_x=-\infty}^{\infty} \int_{-\frac{dx}{2}}^{dx/2} d_\infty(x - x'' - n_x d_x) M_L(x'') e^{-jk_{x0} n_x d_x} \, dx'' = -\sum_{n_x=-\infty}^{\infty} \frac{I_0}{\delta_d} \mathrm{rect}(x - n_x d_x) e^{-jk_{x0} n_x d_x}$$

using def of IFT egu 38

$$4 \sum_{n_x} \int_{-dx/2}^{dx/2} \left(\frac{1}{2\pi} \int_{-\infty}^{\infty} D_\infty(k_x) e^{-jk_x(x - x'' - n_x d_x)} \, dk_x \right) m_L(x'') e^{-jk_{x0} n_x d_x} \, dx''$$

$$\frac{4}{2\pi} \int_{-\infty}^{\infty} D_\infty(k_x) \int_{-dx/2}^{dx/2} m_L(x'') e^{jk_x x''} \, dx'' \underbrace{\sum_{n_x=-\infty}^{\infty} e^{j(k_x - k_{x0}) n_x d_x}}_{\text{apply Poisson Formula}} e^{-jk_x x} \, dk_x$$

$$\frac{4}{2\pi} \int_{-\infty}^{\infty} D_\infty(k_x) M_L(k_x) \frac{2\pi}{d_x} \sum_{m_x=-\infty}^{\infty} \delta(k_x - k_{xm}) e^{-jk_x x} \, dk_x$$

where $k_{xm} = k_{x0} - \frac{2\pi m_x}{d_x}$ Floquet wave numbers

$$-g_{xx}^{hm} * 4m_x = \frac{I_0 \mathrm{rect}\left(\frac{x}{\delta_s}\right)}{\delta_s}$$

finally

$$D_\infty(k_x) = \frac{1}{d_y} \sum_{n_y=-\infty}^{\infty} G_{xx}^{hm} \, \delta_0\left(\frac{k_{yn} w_s}{2}\right)$$

applying the Floquet theorem also on the right handside

$$\frac{4}{d_x} \sum_{m_x=-\infty}^{\infty} D_\infty(k_{xm}) V(k_{xm}) e^{-jk_{xm}x} = -\frac{1}{d_x} I_0 \sum_{m_x=-\infty}^{\infty} \mathrm{sinc}\left(\frac{k_{xm}\delta_d}{2}\right) e^{-jk_{xm}x}$$

$$\boxed{\begin{aligned} V(k_{xm}) &= -\frac{I_0 \,\mathrm{sinc}\left(\frac{k_{xm}\delta_d}{2}\right)}{D_\infty(k_{xm})} \\ v(x) &= \frac{1}{d_x} \sum_{m_x=-\infty}^{\infty} \frac{-I_0 \,\mathrm{sinc}\left(\frac{k_{xm}\delta_d}{2}\right)}{D_\infty(k_{xm})} e^{-jk_{xm}x} \end{aligned}}$$

$$d_\infty(x) = \sum_{n_y} \int_{-d_y/2}^{d_y/2} g_{xx}^{hm}(x, -y'' - n_y d_y) \, m_t(y'' + n_y d_y) \, dy''$$

$$d_\infty(x) = \sum_{n_y} \int_{-d_y/2}^{d_y/2} \left(\frac{1}{4\pi^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G_{xx}^{hm}(k_x, k_y) e^{-jk_x x} e^{-jk_y(y'' + n_y d_y)} dk_x \, dk_y\right) m_t(y'') e^{-jk_{y0} n_y d_y} \, dy''$$

Regroup terms in y'' and n_y

$$d_\infty(x) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G_{xx}^{hm}(k_x, k_y) e^{-jk_x x} \int_{-d_y/2}^{d_y/2} m_t(y'') e^{-jk_y y''} \sum_{n_y=-\infty}^{\infty} e^{-j(k_y - k_{y0}) n_y d_y} \, dk_x \, dk_y$$

FT of edge singular = zeroth order Bessel function J_0 + Poisson formulas

$$d_\infty(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{d_y} \sum_{n_y=-\infty}^{\infty} G_{xx}^{hm}(k_x, k_{yn}) J_0\left(\frac{k_{yn} w_s}{2}\right) e^{-jk_x x} dk_x$$

$$V_0 = I_0 \cdot Z_{in,a}$$

$$\boxed{D_\infty(k_x) = \frac{1}{d_y} \sum_{n_y=-\infty}^{\infty} G_{xx}^{hm}(k_x, k_{ym}) J_0\left(\frac{k_{ym} w_s}{2}\right)}$$

$$k_{ym} = k_{y0} = \frac{2\pi m_y}{d_y}$$

$$\frac{V_0}{d_x} \sum_{m_x=-\infty}^{\infty} V(k_{xm}) D_\infty(k_{xm}) e^{-jk_{xm} x} = -\frac{I_0}{d_x} \operatorname{sinc}\left(\frac{k_x \delta_d}{2}\right) e^{-jk_{xm} x}$$

$$\boxed{V(k_{xm}) = \frac{-I_0 \operatorname{sinc}\left(\frac{k_{xm} \delta_d}{2}\right)}{4\, D_\infty(k_{xm})}}$$

$$\boxed{v(x) = \frac{1}{4 d_x} \sum_{m_x=-\infty}^{\infty} \frac{-I_0 \operatorname{sinc}\left(\frac{k_{xm} \delta_d}{2}\right)}{D_\infty(k_{xm})} e^{-jk_{xm} x}}$$

$$Z_{in,slot} = \frac{V_0}{I_0} = \frac{1}{I_0 \delta_d} \int_{-\delta_d/2}^{\delta_d/2} v(x)\, dx$$

$$Z_{in,slot} = \frac{1}{4 I_0 \delta_d} \int_{-\delta_d/2}^{\delta_d/2} \left(\frac{1}{d_x} \sum_{m_x=-\infty}^{\infty} \frac{-I_0 \operatorname{sinc}\left(\frac{k_{xm} \delta_d}{2}\right)}{D_\infty(k_{xm})} e^{-jk_{xm} x} \right) dx$$

use $\displaystyle \frac{1}{\delta_d} \int_{-\delta_d/2}^{\delta_d/2} e^{-jk_{xm} x} = \operatorname{sinc}\left(\frac{k_{xm} \delta_s}{2}\right)$

$$\boxed{Z_{in,slot} = \frac{1}{4 d_x} \sum_{m_x=-\infty}^{\infty} \frac{-\operatorname{sinc}^2\left(\frac{k_{xm} \delta_d}{2}\right)}{D_\infty(k_{xm})}}$$