Delft University of Technology

Spectral Domains In Electromagnetics

EE4620

Assignment 6: Connected Array Derivations Part 2

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Equivalent Circuit Derivations

The connected array impedance for doubly connected slots can be given by equation 1.

$$Z_{a,slot} = \frac{-1}{4 \cdot 2\pi} \sum_{-\infty}^{\infty} \frac{\operatorname{sinc}^2\left(\frac{k_{xm}\delta_s}{2}\right)}{D(k_{xm})} \tag{1}$$

Where it can be decomposed into the equation 2.

$$Z_{slot}^{a} = Z_{mx=0} + Z_{mx\neq 0} \tag{2}$$

And after further decomposition equation3 is obtained.

$$Y_{\infty} = -n^2 G_{xx}^{hm}\left(k_{x0}, k_{y0}\right) \tag{3}$$

Where n^2 is given by equation 4 and $-G_{xx}^{hm}$ is given by equation 5

$$n^2 = \frac{4 d_x J_0\left(\frac{k_{y0} w_s}{2}\right)}{d_y \operatorname{sinc}^2\left(\frac{k_{x0}\delta_s}{2}\right)} \tag{4}$$

$$-G_{xx}^{hm} = i_{te}\cos^2(\phi) - i_{tm}\sin^2(\phi) \tag{5}$$

And Fundamental mode of admittance is given as the in the equation 6

$$Y_{00} = n^2\left(i_{te}\cos^2(\phi) - i_{tm}\sin^2(\phi)\right) \tag{6}$$

The final equivalent circuit derived is shown on figure 1.

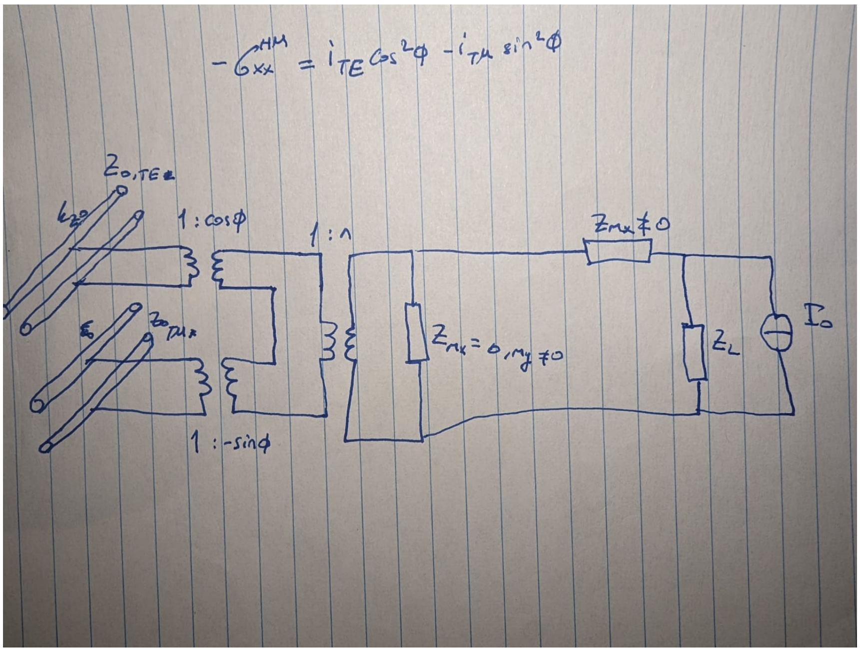

Figure 1: The final equivalent circuit obtained after the decompositions.

1 Appendix

Handwritten Solutions are given in this section.

Equivalent Circuits on Connected Arrays

$$\boxed{Z_{in,slot} = \frac{1}{4d_x} \sum_{m_x=-\infty}^{\infty} \frac{-\operatorname{sinc}^2\left(\frac{k_{xm}\delta_s}{2}\right)}{D_{\infty}(k_{xm})}}$$

$$\boxed{D_{\infty}(k_{xm}) = \frac{1}{d_y} \sum_{m_y=-\infty}^{\infty} G_{xx}^{hm}(k_{xm}, k_{ym})\, J_0\left(\frac{k_{ym} w_s}{2}\right)}$$

$$Z_{slot}^{a} = Z_{m_x=0} + Z_{m_x\neq 0} = -\frac{1}{d_x} \frac{\operatorname{sinc}^2\left(\frac{k_{x0}\delta_s}{2}\right)}{4 D_{\infty}(k_{x0})}$$

$$-\frac{1}{4d_x} \sum_{m_x\neq 0} \frac{\operatorname{sinc}^2\left(\frac{k_{xm}\delta_s}{2}\right)}{D_{\infty}(k_{xm})}$$

we can decompose sum in infinity

$$Y_{m_x=0} = -\frac{4d_x}{\operatorname{sinc}^2\left(\frac{k_{x0}\delta_s}{2}\right)} D_{\infty}(k_{x0}) = Z_{00} + Z_{m_x=0, m_y\neq 0}$$

$$= -\frac{4 d_x J_0\left(\frac{k_{y0} w_s}{2}\right) G_{xx}^{hm}(k_{x0}, k_{y0})}{d_y \operatorname{sinc}^2\left(\frac{k_{x0}\delta_s}{2}\right)} - \frac{4d_x}{d_y \operatorname{sinc}^2\left(\frac{k_{x0}\delta_s}{2}\right)} \sum_{m_y\neq 0} G_{xx}^{hm}(k_x, k_{ym})\, J_0\left(\frac{k_{ym} w_s}{2}\right)$$

$$Y_{00} = -n^2\, G_{xx}^{hm}(k_{x0}, k_{y0}), \text{ where } n^2 \text{ is:}$$

$$n^2 = \frac{4d_x J_0\left(\frac{k_{y0} w_s}{2}\right)}{d_y \operatorname{sinc}^2\left(\frac{k_{x0}\delta_s}{2}\right)}$$

$$-G_{xx}^{hm}(k_{x0}, k_{y0}) = \frac{i_{TE} k_x^2 - i_{TM} k_y^2}{k_\rho^2} = i_{TE}\cos^2\phi - i_{TM}\sin^2\phi$$

$$Y_{00} = n^2 \left(i_{TE}\cos^2\phi - i_{TM}\sin^2\phi\right)$$

$$-G_{xx}^{HM} = i_{TE}\cos^2\phi - i_{TM}\sin^2\phi$$