EE4620 - Spectral Methods in Electromagnetics: Spectral Green's Function for Stratified Media MATLAB Instruction 1

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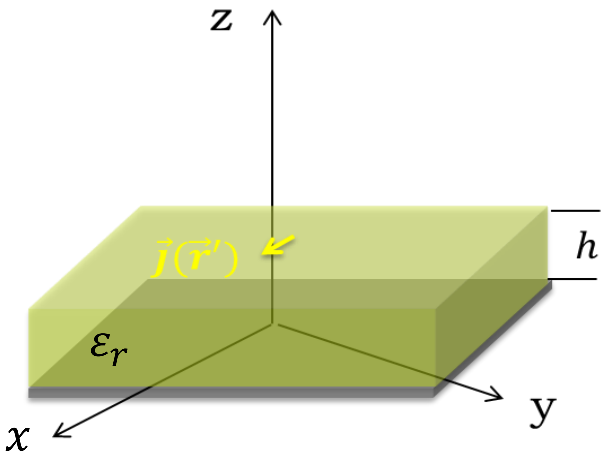
Question 1

Write a MATLAB routine to calculate the spectral Green's function for the electric field given an elementary electric source placed at the top of a grounded slab of thickness h and dielectric constant ε_r as shown in the figure.
Consider $h = 4.5\text{mm}$, $\varepsilon_r = 6$ and the source oriented along x.

Make a plot of the amplitude variation of the x-component of spectral field at $z = h^+$ as a function of k_x from 0 to $3k_0$ with $k_y = 0$ at 10GHz and 20GHz.



Spectral Green's function of stratified media

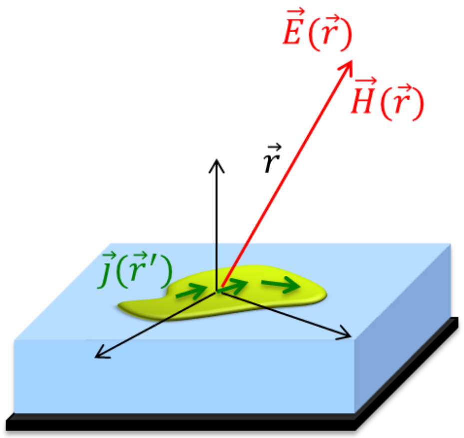
$$\vec{e}(\vec{r}) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} \tilde{\tilde{G}}^{ej}(k_x, k_y, z, z') e^{-jk_x x} e^{-jk_y y} dk_x dk_y$$

$$\tilde{\boldsymbol{G}}^{ej} = \begin{bmatrix} -\frac{v_{TM}k_x^2 + v_{TE}k_y^2}{k_\rho^2} & \frac{(v_{TE} - v_{TM})k_x k_y}{k_\rho^2} \\ \frac{(v_{TE} - v_{TM})k_x k_y}{k_\rho^2} & -\frac{v_{TE}k_x^2 + v_{TM}k_y^2}{k_\rho^2} \\ \zeta_i \frac{k_x}{k_i} i_{TM} & \zeta_i \frac{k_y}{k_i} i_{TM} \end{bmatrix}$$

z: observation point in z (voltage/current output of the transmission line)
z': source location (generator in the transmission line)

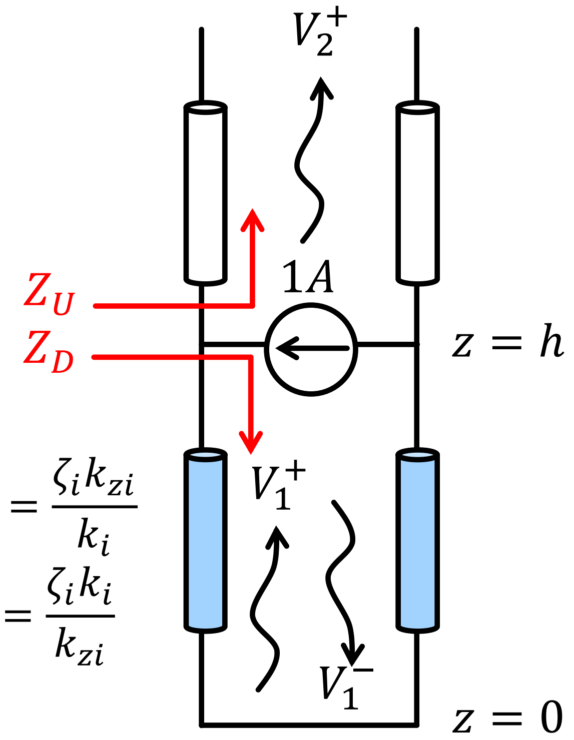
Implementation of the square root: $k_{z0} = -j\sqrt{-(k_0^2 - k_\rho^2)}$

Transmission line Solution

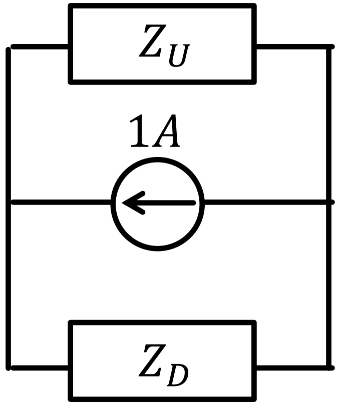


$$Z_i^{TM} = \frac{\zeta_i k_{zi}}{k_i}$$

$$Z_i^{TE} = \frac{\zeta_i k_i}{k_{zi}}$$



Boundary 1:



$$\boldsymbol{V(z = h) = Z_U || Z_D \times 1A}$$

Boundary 2:

$$\boldsymbol{V(z = 0) = 0}$$

Voltage and current in air ($z > h$)

$$V_2(z) = V_2^+ \, e^{-jk_{z0}z}$$

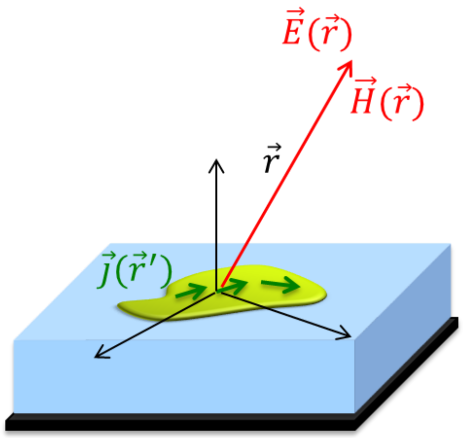
$$I_2(z) = \frac{V_2^+}{Z_0} \, e^{-jk_{z0}z}$$

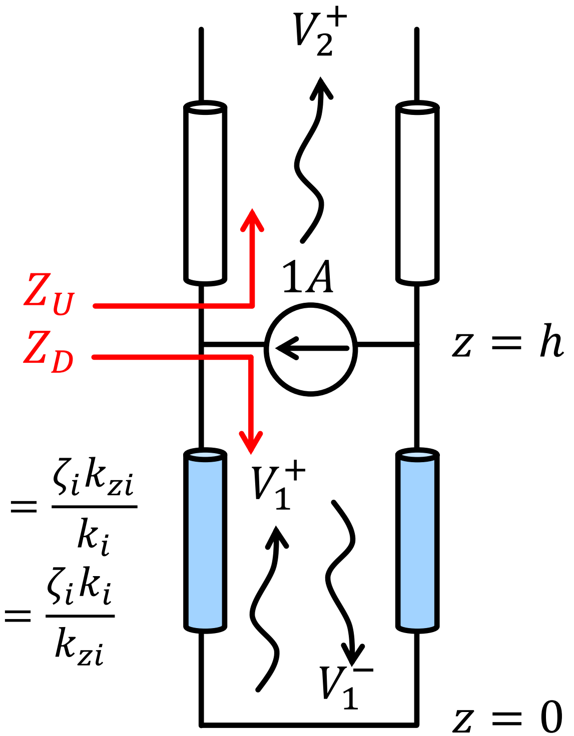
Voltage and current in substrate ($0 < z < h$)

$$V_1(z) = V_1^+ \, e^{-jk_{zs}z} + V_1^- \, e^{+jk_{zs}z}$$

$$I_1(z) = \frac{V_1^+}{Z_s} \, e^{-jk_{zs}z} - \frac{V_1^-}{Z_s} \, e^{+jk_{zs}z}$$

Transmission line Solution

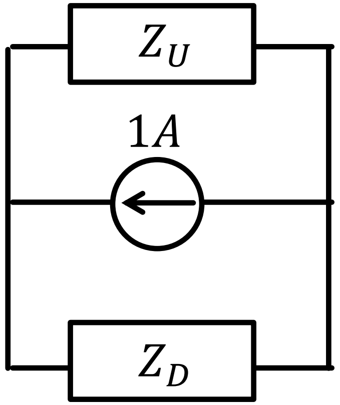




$$Z_i^{TM} = \frac{\zeta_i k_{zi}}{k_i}$$

$$Z_i^{TE} = \frac{\zeta_i k_i}{k_{zi}}$$

Boundary 1:



$$\boldsymbol{V(z=h) = Z_U || Z_D \times 1A}$$

$$V_2(z) = V_2^+ \, e^{-jk_{z0}z} \xrightarrow{\text{Boundary 1}} V_2^+ \, e^{-jk_{z0}h} = Z_U||Z_D \longrightarrow V_2^+ = Z_U||Z_D \; e^{+jk_{z0}h}$$

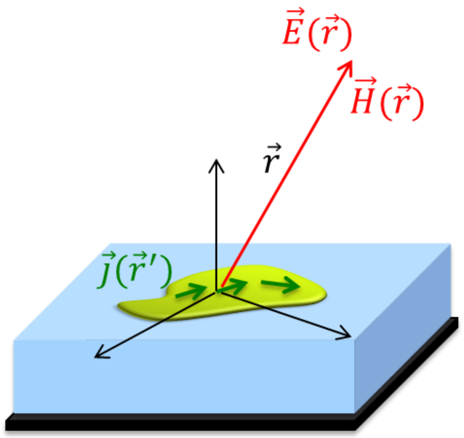
$$V_2(z=h) = \boldsymbol{Z_U||Z_D}$$

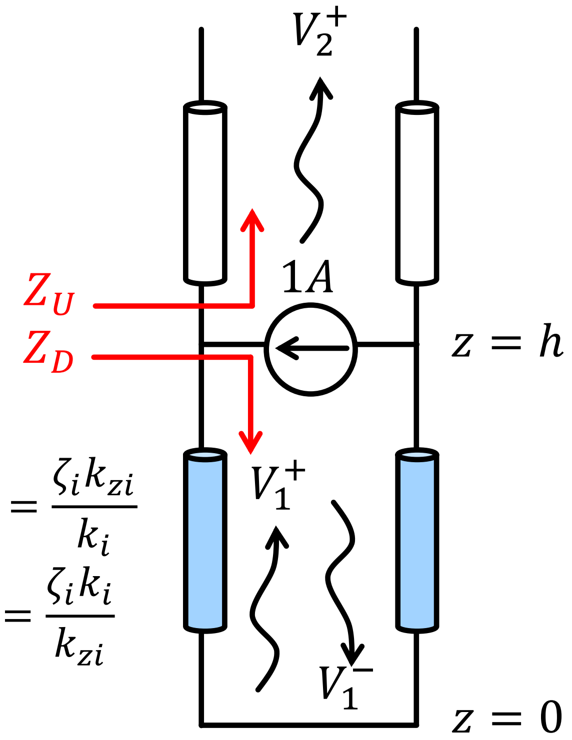
Voltage and current in air ($z > h$)

$$V_2(z) = Z_U||Z_D \; e^{+jk_{z0}h} e^{-jk_{z0}z}$$

$$I_2(z) = \frac{Z_U||Z_D}{Z_0} e^{+jk_{z0}h} e^{-jk_{z0}z}$$

Transmission line Solution

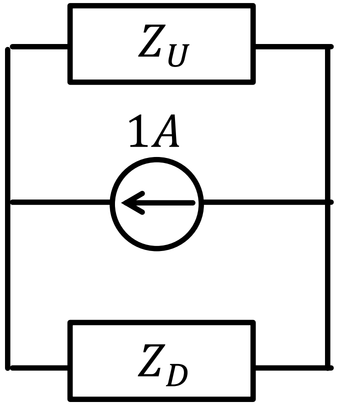




$$Z_i^{TM} = \frac{\zeta_i k_{zi}}{k_i}$$

$$Z_i^{TE} = \frac{\zeta_i k_i}{k_{zi}}$$

Boundary 1:



$$\boldsymbol{V(z = h) = Z_U || Z_D \times 1A}$$

Boundary 2:

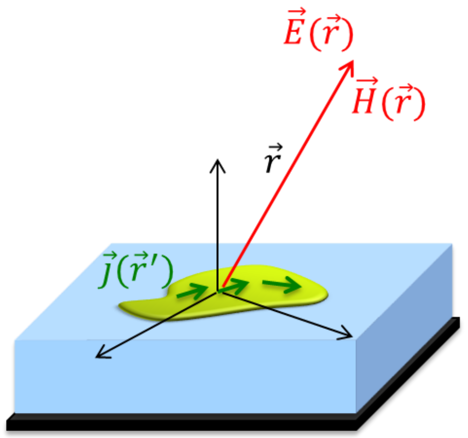
$$\boldsymbol{V(z = 0) = 0}$$

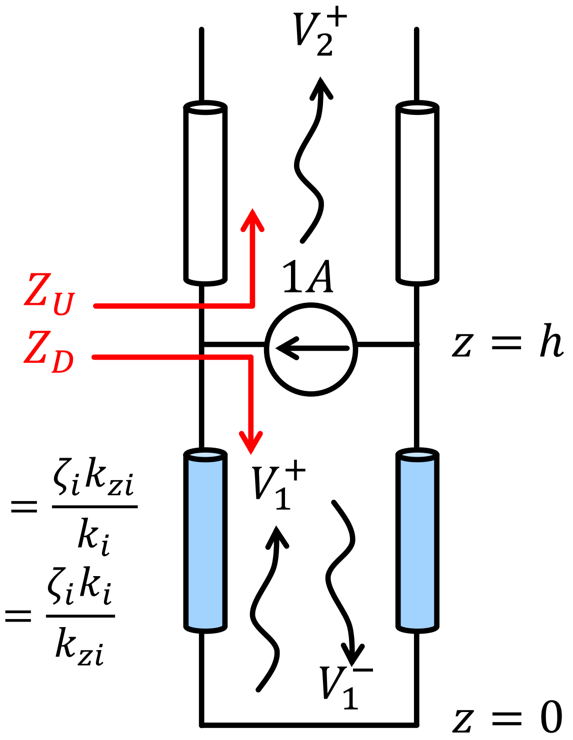
$$V_1(z) = V_1^+ \, e^{-jk_{zs}z} + V_1^- \, e^{+jk_{zs}z} \xrightarrow{\textbf{Boundary 2}} V_1(z=0) = V_1^+ + V_1^- = 0 \longrightarrow V_1^+ = -V_1^-$$

$$V_1(z = h) = V_1^+ \, e^{-jk_{zs}h} - V_1^+ \, e^{+jk_{zs}h} = V_1^+(-2j)\sin(k_{zs}h) \xrightarrow{\textbf{Boundary 1}}$$

$$V_1^+(-2j)\sin(k_{zs}h) = Z_U || Z_D \longrightarrow V_1^+ = \frac{Z_U || Z_D}{(-2j)} \frac{1}{\sin(k_{zs}h)}$$

Transmission line Solution

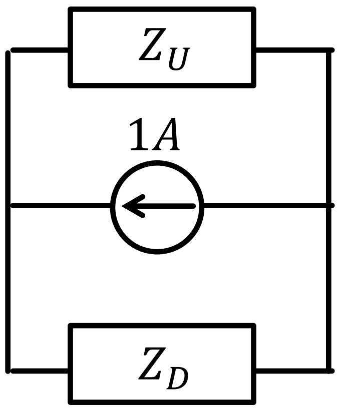




$$Z_i^{TM} = \frac{\zeta_i k_{zi}}{k_i}$$

$$Z_i^{TE} = \frac{\zeta_i k_i}{k_{zi}}$$

Boundary 1:



$$\boldsymbol{V(z = h) = Z_U || Z_D \times 1A}$$

Boundary 2:

$$\boldsymbol{V(z = 0) = 0}$$

Voltage and current in substrate $(0 < z < h)$

$$V_1(z) = V_1^+ \, e^{-jk_{zs}z} + V_1^- \, e^{+jk_{zs}z} = \frac{Z_U || Z_D}{(-2j)} \frac{1}{\sin(k_{zs}h)} \left(e^{-jk_{zs}z} - e^{+jk_{zs}z}\right) = Z_U || Z_D \, \frac{\sin(k_{zs}z)}{\sin(k_{zs}h)}$$

$$I_1(z) = \frac{V_1^+}{Z_s} \, e^{-jk_{zs}z} - \frac{V_1^-}{Z_s} \, e^{+jk_{zs}z} = \frac{Z_U || Z_D}{Z_s(-2j)} \frac{1}{\sin(k_{zs}h)} \left(e^{-jk_{zs}z} + e^{+jk_{zs}z}\right) = \frac{Z_U || Z_D}{Z_s} \frac{j\cos(k_{zs}z)}{\sin(k_{zs}h)}$$

Transmission line Solution: Summary

Voltage solution in the slab:

$$V_s(z) = \frac{Z_u Z_d}{Z_u + Z_d} \frac{sin(k_{zs}z)}{sin(k_{zs}h)}$$

Current solution in the slab:

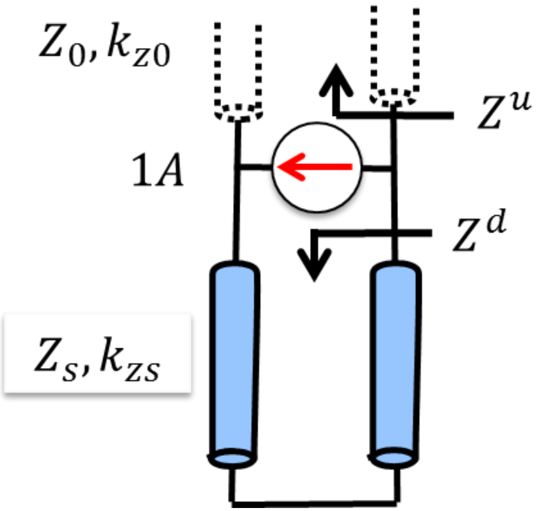
$$I_s(z) = \frac{V_s^+}{Z_s}\left(e^{-jk_{zs}z} + e^{jk_{zs}z}\right) = \frac{1}{Z_s} \frac{Z_u Z_d}{Z_u + Z_d} \frac{j\cos(k_{zs}z)}{sin(k_{zs}h)}$$

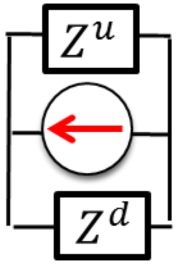
Voltage solution in the air:

$$V_0(z) = \frac{Z_u Z_d}{Z_u + Z_d} e^{jk_{z0}h} e^{-jk_{z0}z}$$

Current solution in the air:

$$I_0(z) = \frac{1}{Z_0} \frac{Z_u Z_d}{Z_u + Z_d} e^{jk_{z0}h} e^{-jk_{z0}z}$$





$Z_u = Z_0$

$Z_d = jZ_s \tan(k_{zs}h)$

Routines

- Solution of the equivalent transmission line:

$$[v_{TM}, v_{TE}, i_{TM}, i_{TE}] = trxline\_GroundSlab(k0, zeta0, er, h, kro, z)$$

- Dyadic SGF :

$$[Gxx, Gyx, Gzx] = SpectralGFej(k0, er, kx, ky, v_{TM}, v_{TE}, i_{TM}, i_{TE}, zeta0)$$

How to code spectral GF in MATLAB

$$\vec{e}(\vec{r}) = \frac{1}{(2\pi)^2}\iint_{-\infty}^{\infty} \tilde{\tilde{G}}^{ej}(k_x, k_y, z, z')e^{-jk_x x}e^{-jk_y y}dk_x dk_y$$

- Defining the independent variables: k_x and k_y
  - Radial propagation vector is related to these two:

$$k_\rho = \sqrt{k_x^2 + k_y^2}$$

- Calculating the perpendicular propagation vector:

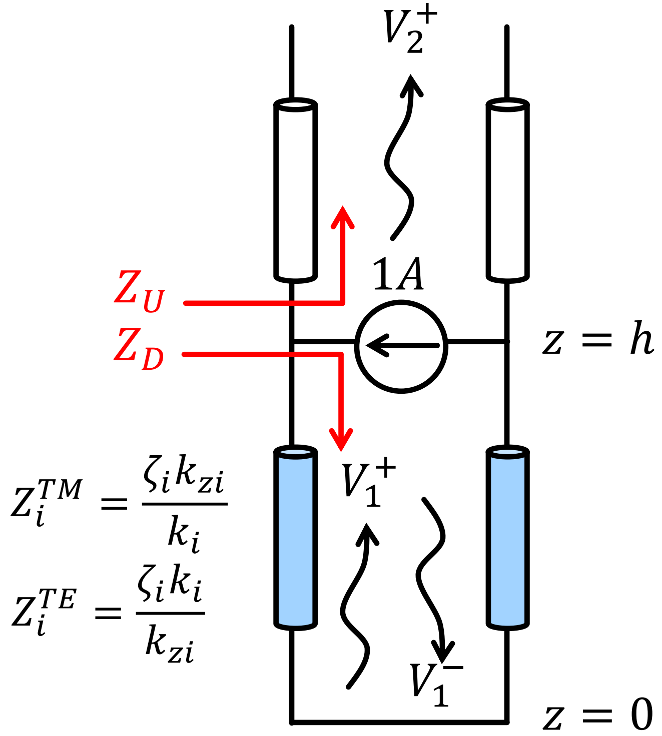
$$k_{z0} = -j\sqrt{-(k_0^2 - k_\rho^2)}$$

$$k_{zs} = -j\sqrt{-(k_s^2 - k_\rho^2)}$$

- Defining the TE and TM characteristic transmission line impedances

$$Z_i^{TM} = \frac{\zeta_i k_{zi}}{k_i} \qquad Z_i^{TE} = \frac{\zeta_i k_i}{k_{zi}}$$

- Code the solution of TL line for TE and TM transmission lines

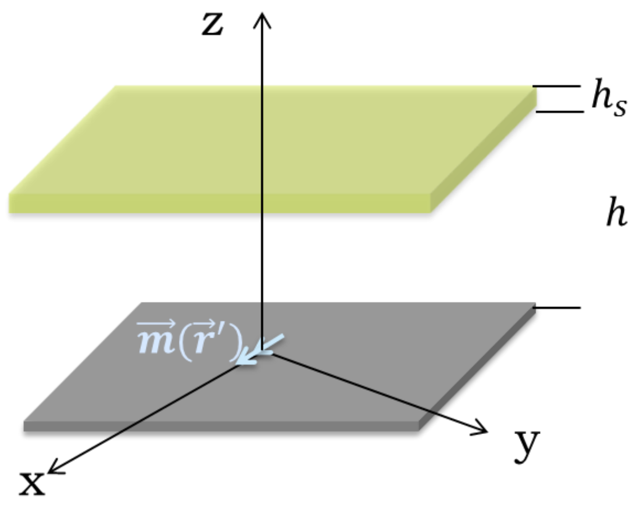


$$\tilde{G}^{ej} = \begin{bmatrix} -\frac{v_{TM}k_x^2 + v_{TE}k_y^2}{k_\rho^2} & \frac{(v_{TE} - v_{TM})k_x k_y}{k_\rho^2} \\ \frac{(v_{TE} - v_{TM})k_x k_y}{k_\rho^2} & -\frac{v_{TE}k_x^2 + v_{TM}k_y^2}{k_\rho^2} \\ \zeta_i \frac{k_x}{k_i} i_{TM} & \zeta_i \frac{k_y}{k_i} i_{TM} \end{bmatrix}$$

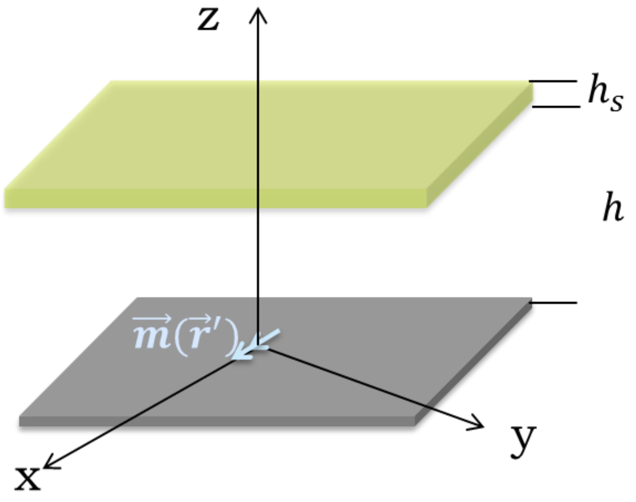
Question 2

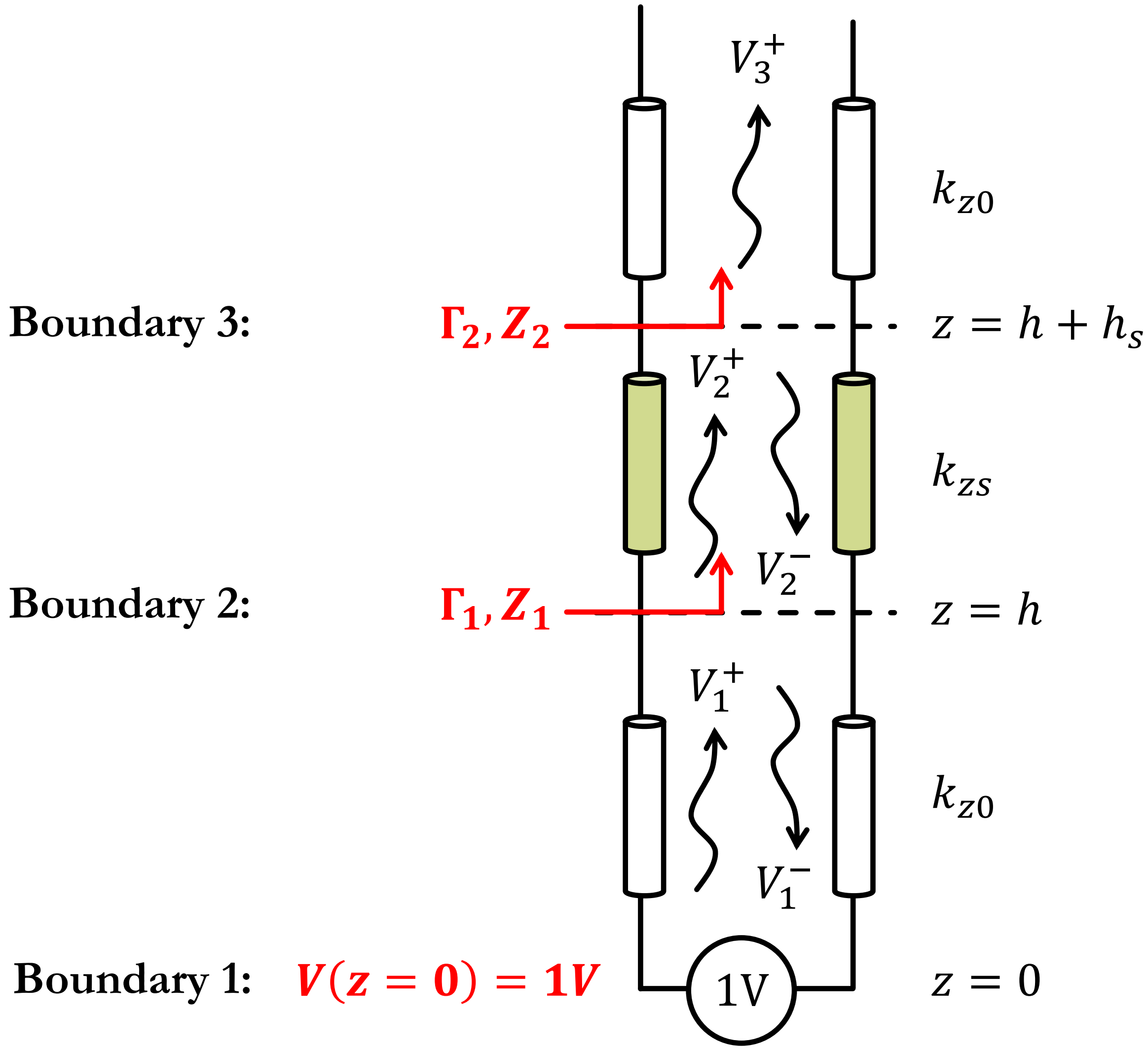
Write a MATLAB routine to calculate the spectral Green's function for the electric field given by an elementary x-oriented magnetic source radiating at $z = 0$ with the presence of a ground plane and a dielectric layer of thickness h_s located at a distance of h from the ground plane, as shown in the figure.
Consider $h = 15.6$mm, $h_s = 2.6$mm, $\varepsilon_r = 10$.

Make a plot of the amplitude variation of the y-component of spectral field at $z = h + h_s^+$ as a function of k_y from 0 to k_0 with $k_x = 0$ for the following frequencies: 8GHz, 8.5GHz, 9GHz, 9.5GHz and 10GHz



Transmission Line Solution



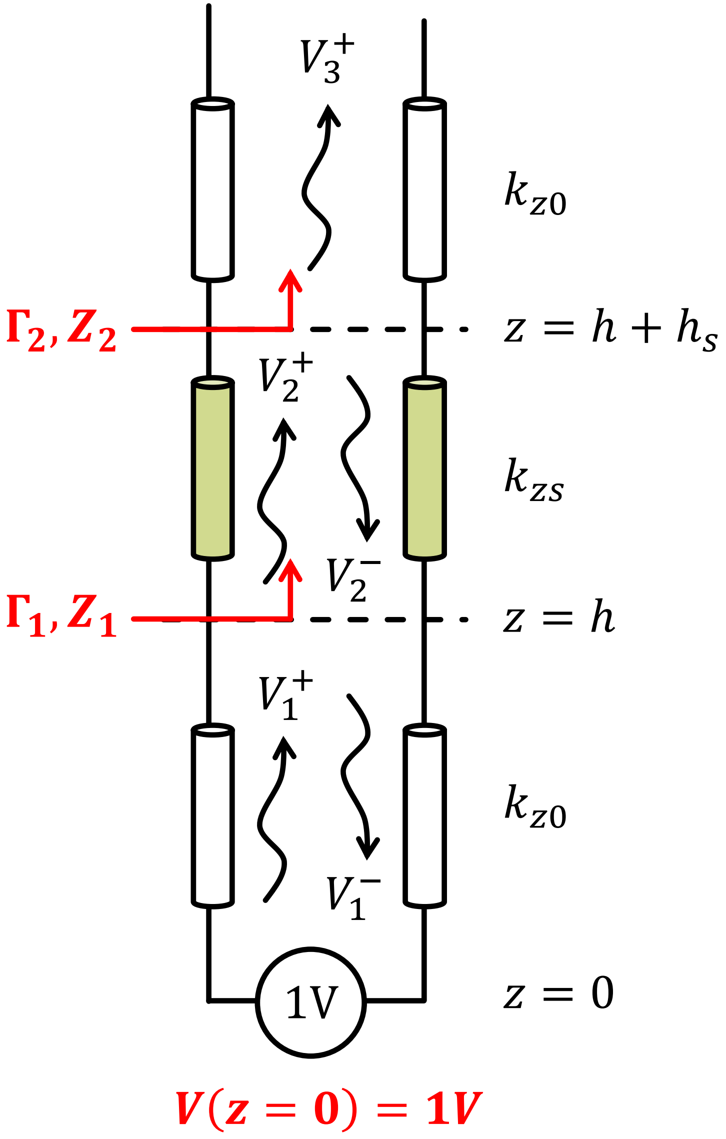


$$Z_i^{TM} = \frac{\zeta_i k_{zi}}{k_i}$$

$$Z_i^{TE} = \frac{\zeta_i k_i}{k_{zi}}$$

3 boundary to use and 3 parameters to calculate:
V_1^+, V_2^+, V_3^+

Transmission Line Solution



Starting from bottom and going up:

In region $0 \le z < h$:

$$V_1(z) = V_1^+ e^{-jk_{z0}z} + V_1^- e^{+jk_{z0}z}$$
$$I_1(z) = \frac{V_1^+}{Z_0} e^{-jk_{z0}z} - \frac{V_1^-}{Z_0} e^{+jk_{z0}z}$$

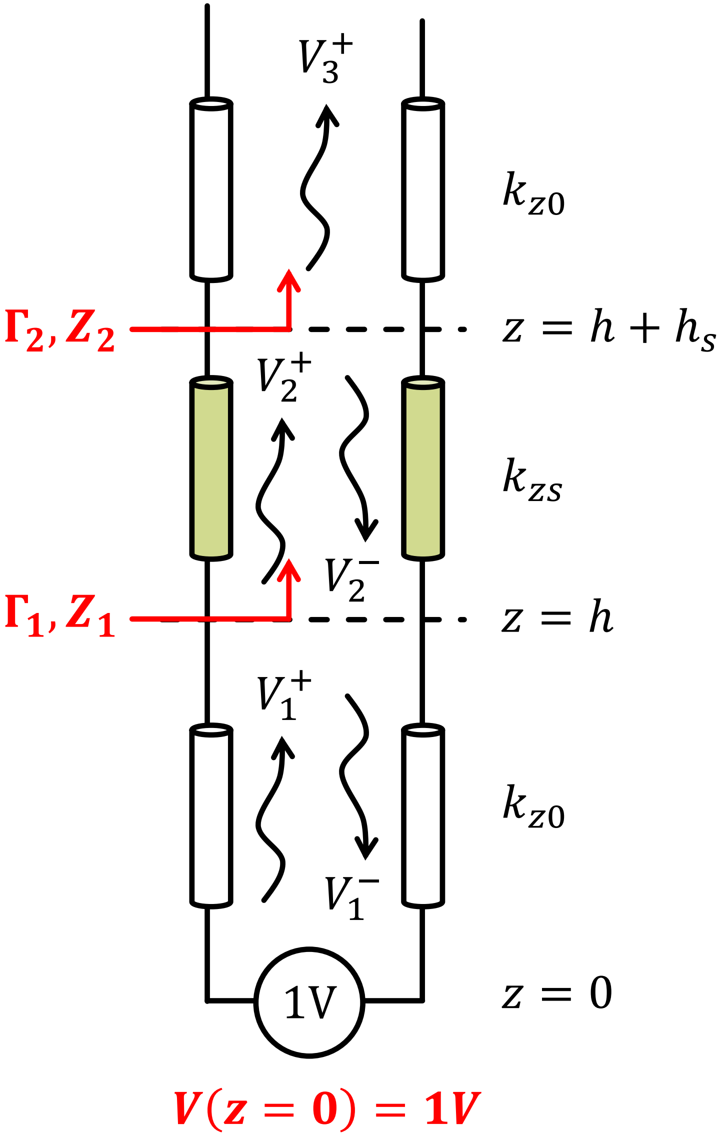
$$V_1(z=0) = 1 \rightarrow V_1^- = 1 - V_1^+$$

$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0} = \frac{V_1^- e^{+jk_{z0}h}}{V_1^+ e^{-jk_{z0}h}} \rightarrow$$

$$\frac{(1 - V_1^+)}{V_1^+} e^{+2jk_{z0}h} = \Gamma_1 \rightarrow$$

$$\boxed{V_1^+ = \frac{e^{+2jk_{z0}h}}{\Gamma_1 + e^{+2jk_{z0}h}}}$$

Transmission Line Solution



In region $0 \le z < h$:

$$V_1(z) = V_1^+ e^{-jk_{z0}z} + V_1^- e^{+jk_{z0}z}$$

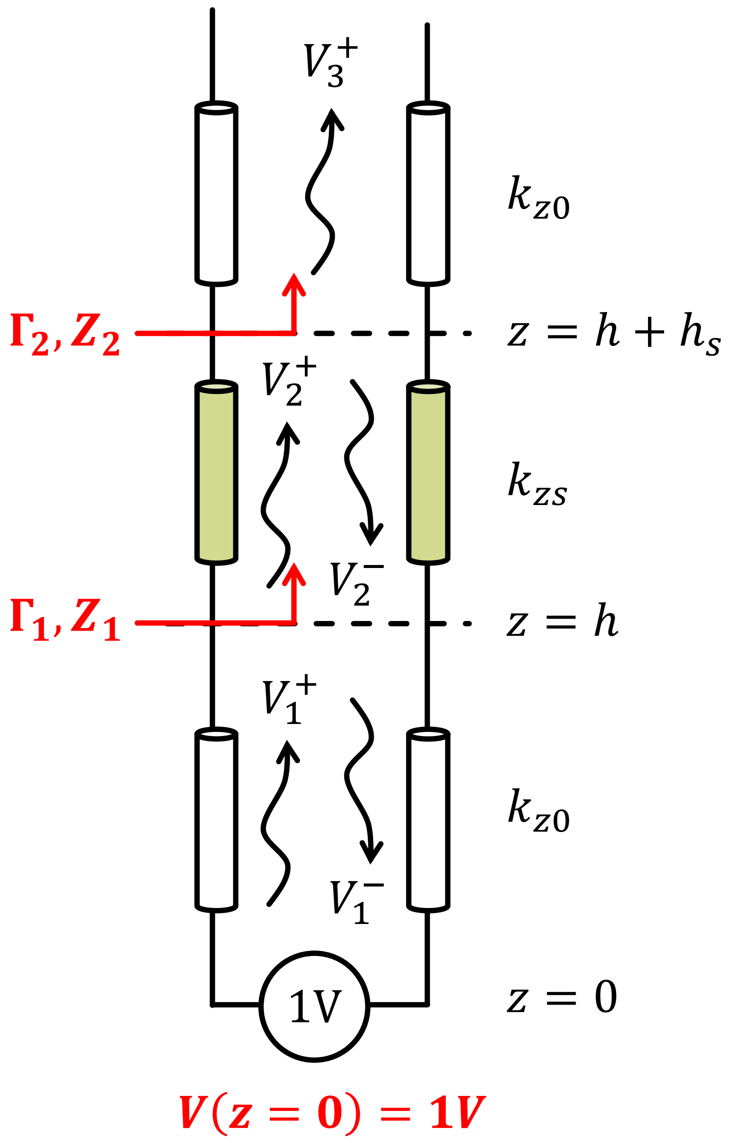
$$I_1(z) = \frac{V_1^+}{Z_0} e^{-jk_{z0}z} - \frac{V_1^-}{Z_0} e^{+jk_{z0}z}$$

$$\boxed{V_1^+ = \frac{e^{+2jk_{z0}h}}{\Gamma_1 + e^{+2jk_{z0}h}}}$$

$$V_1(z) = \frac{e^{+2jk_{z0}h}}{\Gamma_1 + e^{+2jk_{z0}h}} e^{-jk_{z0}z}\left[1 + \Gamma_1 e^{-2jk_{z0}h} e^{+2jk_{z0}z}\right]$$

$$I_1(z) = \frac{e^{+2jk_{z0}h}}{Z_0(\Gamma_1 + e^{+2jk_{z0}h})} e^{-jk_{z0}z}\left[1 - \Gamma_1 e^{-2jk_{z0}h} e^{+2jk_{z0}z}\right]$$

Transmission Line Solution



In region $h \le z < h + h_s$:

$$V_2(z) = V_2^+ e^{-jk_{zs}z} + V_2^- e^{+jk_{zs}z}$$

$$I_2(z) = \frac{V_2^+}{Z_s} e^{-jk_{zs}z} - \frac{V_2^-}{Z_s} e^{+jk_{zs}z}$$

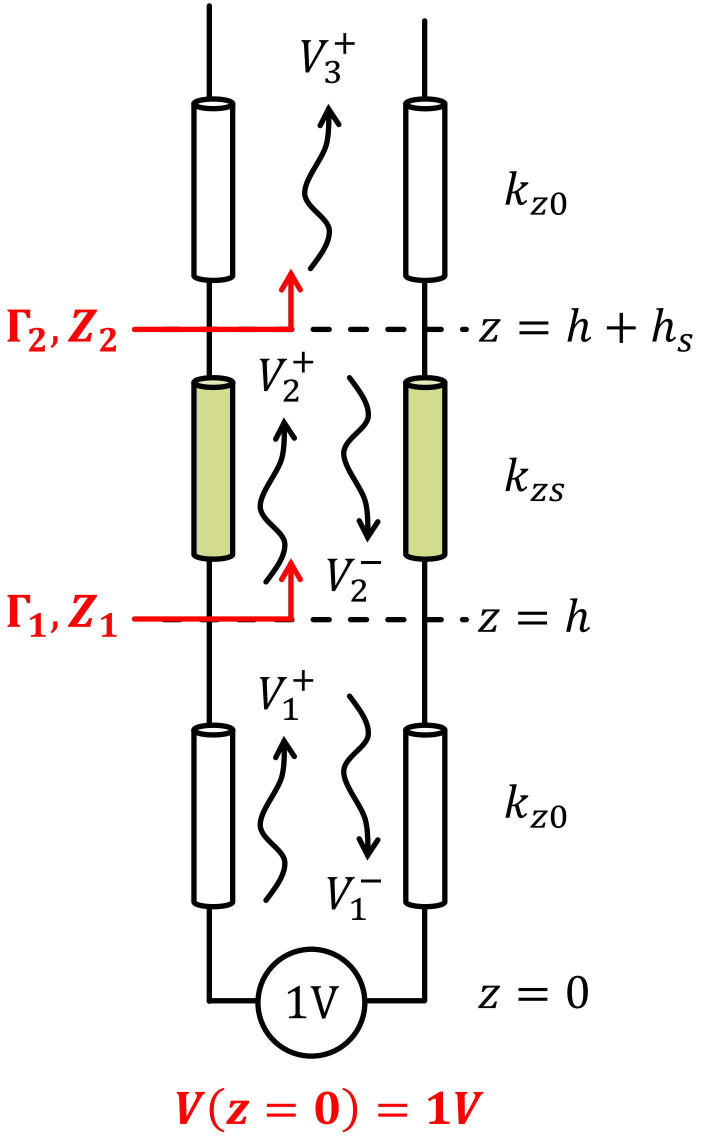
$$\Gamma_2 = \frac{Z_2 - Z_s}{Z_2 + Z_s} = \frac{V_2^- e^{+jk_{zs}(h+h_s)}}{V_2^+ e^{-jk_{zs}(h+h_s)}} \rightarrow$$

$$V_2^- = \Gamma_2 V_2^+ e^{-2jk_{zs}(h+h_s)}$$

$$\boldsymbol{V_2(z=h) = V_1(z=h)}$$

$$\boxed{V_2^+ = \frac{e^{jk_{z0}h} e^{+jk_{zs}h}(1+\Gamma_1)}{(\Gamma_1 + e^{2jk_{z0}h})(1 + \Gamma_2 e^{-2jk_{zs}h_s})}}$$

Transmission Line Solution



In region $h \leq z < h + h_s$:

$$V_2(z) = V_2^+ e^{-jk_{zs}z} + V_2^- e^{+jk_{zs}z}$$

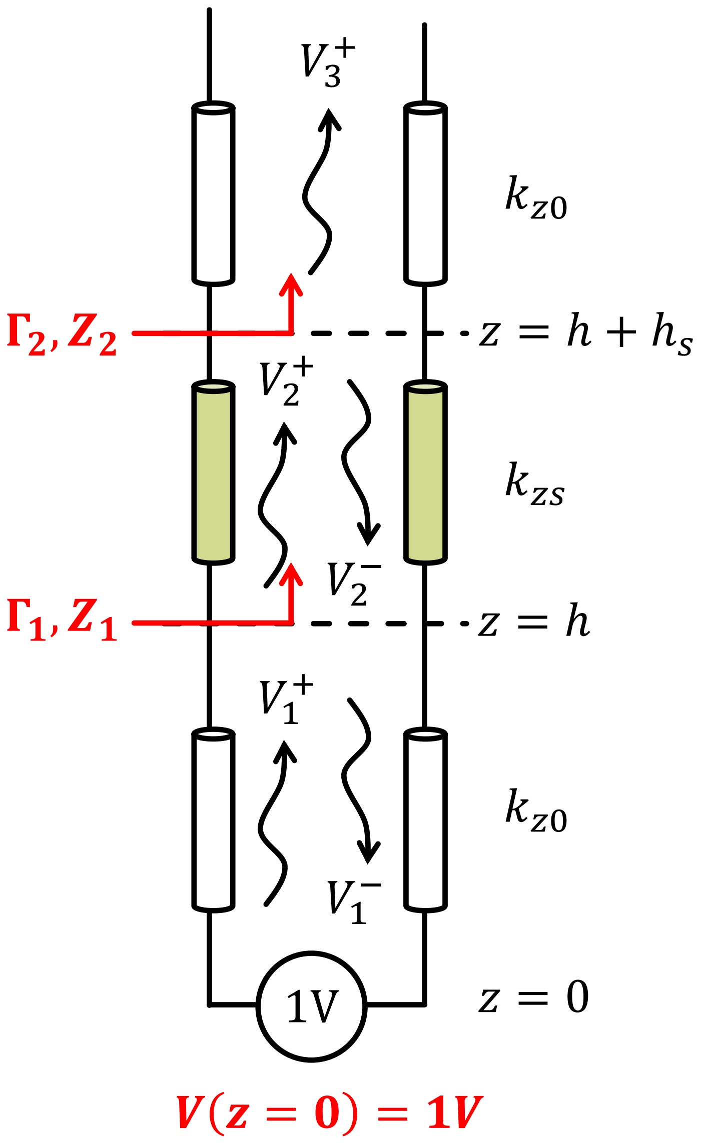
$$I_2(z) = \frac{V_2^+}{Z_s} e^{-jk_{zs}z} - \frac{V_2^-}{Z_s} e^{+jk_{zs}z}$$

$$\boxed{V_2^+ = \frac{e^{jk_{z0}h} e^{+jk_{zs}h}(1+\Gamma_1)}{(\Gamma_1 + e^{2jk_{z0}h})(1 + \Gamma_2 e^{-2jk_{zs}h_s})}}$$

$$V_2(z) = \frac{e^{jk_{z0}h} e^{+jk_{zs}h}(1+\Gamma_1)}{(\Gamma_1 + e^{2jk_{z0}h})(1 + \Gamma_2 e^{-2jk_{zs}h_s})}\left[e^{-jk_{zs}z} + e^{+jk_{zs}z}\Gamma_2 e^{-2jk_{zs}(h+h_s)}\right]$$

$$I_2(z) = \frac{e^{jk_{z0}h} e^{+jk_{zs}h}(1+\Gamma_1)}{Z_s(\Gamma_1 + e^{2jk_{z0}h})(1 + \Gamma_2 e^{-2jk_{zs}h_s})}\left[e^{-jk_{zs}z} - e^{+jk_{zs}z}\Gamma_2 e^{-2jk_{zs}(h+h_s)}\right]$$

Transmission Line Solution



In region $z \geq h + h_s$:

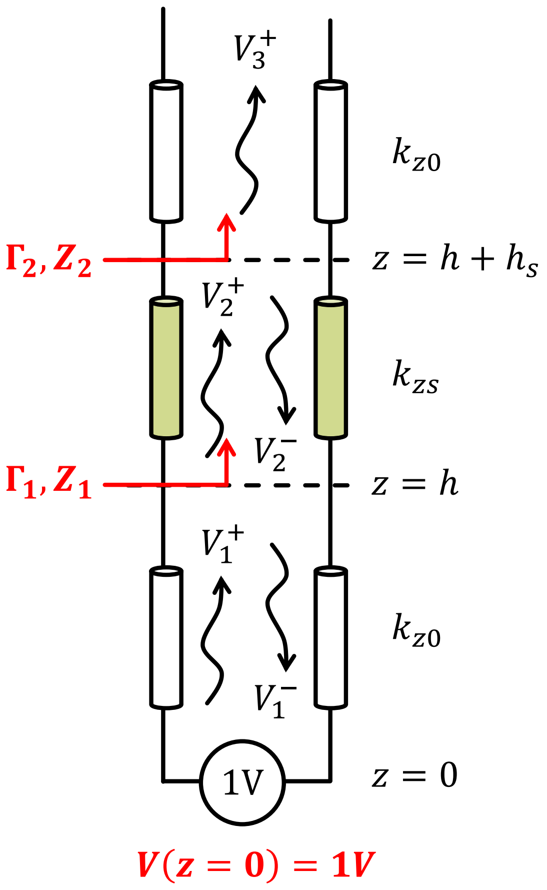
$$V_3(z) = V_3^+ e^{-jk_{z0}z}$$

$$I_3(z) = \frac{V_3^+}{Z_0} e^{-jk_{z0}z}$$

$$\boldsymbol{V_3(z = h + h_s) = V_2(z = h + h_s) \rightarrow}$$

$$\boxed{V_3^+ = \frac{(1+\Gamma_1)(1+\Gamma_2)\, e^{+jk_{z0}h} e^{-jk_{zs}h_s} e^{+jk_{z0}(h+h_s)}}{(\Gamma_1 + e^{2jk_{z0}h})(1 + \Gamma_2 e^{-2jk_{zs}h_s})}}$$

Transmission Line Solution: Summary



$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

$$\Gamma_2 = \frac{Z_2 - Z_s}{Z_2 + Z_s}$$

In region $0 \le z < h$:

$$V_1(z) = \frac{e^{+2jk_{z0}h}}{\Gamma_1 + e^{+2jk_{z0}h}} e^{-jk_{z0}z}\left[1 + \Gamma_1 e^{-2jk_{z0}h} e^{+2jk_{z0}z}\right]$$

$$I_1(z) = \frac{e^{+2jk_{z0}h}}{Z_0\left(\Gamma_1 + e^{+2jk_{z0}h}\right)} e^{-jk_{z0}z}\left[1 - \Gamma_1 e^{-2jk_{z0}h} e^{+2jk_{z0}z}\right]$$

In region $h \le z < h + h_s$:

$$V_2(z) = \frac{e^{jk_{z0}h} e^{+jk_{zs}h}(1 + \Gamma_1)}{\left(\Gamma_1 + e^{2jk_{z0}h}\right)\left(1 + \Gamma_2 e^{-2jk_{zs}h_s}\right)}\left[e^{-jk_{zs}z} + e^{+jk_{zs}z}\Gamma_2 e^{-2jk_{zs}(h+h_s)}\right]$$

$$I_2(z) = \frac{e^{jk_{z0}h} e^{+jk_{zs}h}(1 + \Gamma_1)}{Z_s\left(\Gamma_1 + e^{2jk_{z0}h}\right)\left(1 + \Gamma_2 e^{-2jk_{zs}h_s}\right)}\left[e^{-jk_{zs}z} - e^{+jk_{zs}z}\Gamma_2 e^{-2jk_{zs}(h+h_s)}\right]$$

In region $z \ge h + h_s$:

$$V_3(z) = V_3^+ e^{-jk_{z0}z} \qquad I_3(z) = \frac{V_3^+}{Z_0} e^{-jk_{z0}z}$$

$$V_3^+ = \frac{(1 + \Gamma_1)(1 + \Gamma_2)\, e^{+jk_{z0}h} e^{-jk_{zs}h_s} e^{+jk_{z0}(h+h_s)}}{\left(\Gamma_1 + e^{2jk_{z0}h}\right)\left(1 + \Gamma_2 e^{-2jk_{zs}h_s}\right)}$$

Routines

- Solution of the equivalent transmission line:

$$[v_{TM}, v_{TE}, i_{TM}, i_{TE}] = trxline\_Superstrate(k0, zeta0, er, h, hs, kro, z)$$

- Dyadic SGF :

$$[Gxx, Gyx, Gzx] = SpectralGFem(k0, er, kx, ky, v_{TM}, v_{TE}, i_{TM}, i_{TE}, zeta0)$$

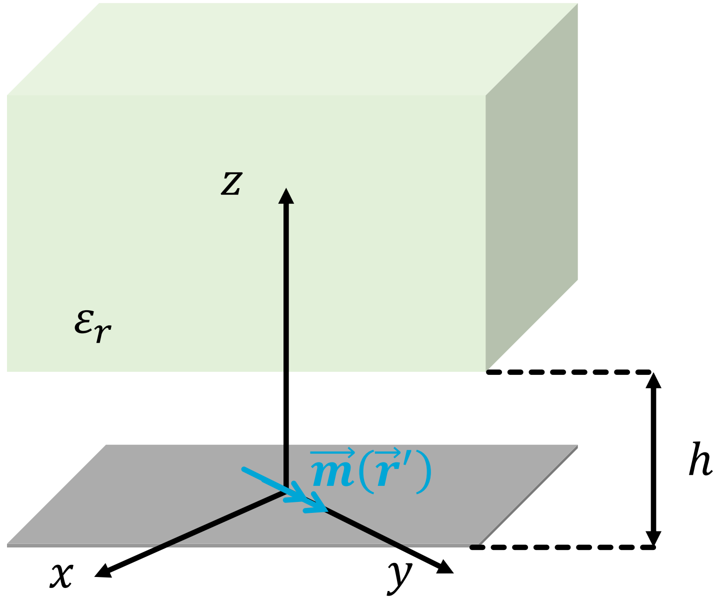
$$\underline{\underline{\mathbf{G}}}^{em}(k_x, k_y, z, z') =$$

$$\begin{bmatrix} \frac{k_x k_y}{k_\rho^2}\left({}^mV_{TM}(k_\rho, z, z') - {}^mV_{TE}(k_\rho, z, z')\right) & \frac{-1}{k_\rho^2}\left({}^mV_{TM}(k_\rho, z, z')k_x^2 + {}^mV_{TE}(k_\rho, z, z')k_y^2\right) \\ \frac{1}{k_\rho^2}\left({}^mV_{TM}(k_\rho, z, z')k_y^2 + {}^mV_{TE}(k_\rho, z, z')k_x^2\right) & \frac{k_x k_y}{k_\rho^2}\left({}^mV_{TE}(k_\rho, z, z') - {}^mV_{TM}(k_\rho, z, z')\right) \\ -\frac{k_y}{k_{zi}} Z_{TMi}\ {}^mI_{TM}(k_\rho, z, z') & \frac{k_x}{k_{zi}} Z_{TMi}\ {}^mI_{TM}(k_\rho, z, z') \end{bmatrix}$$

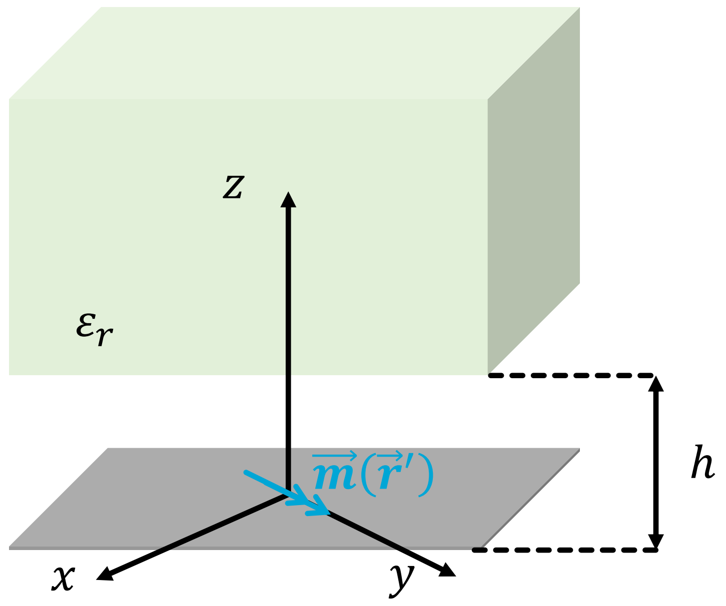
Question 3

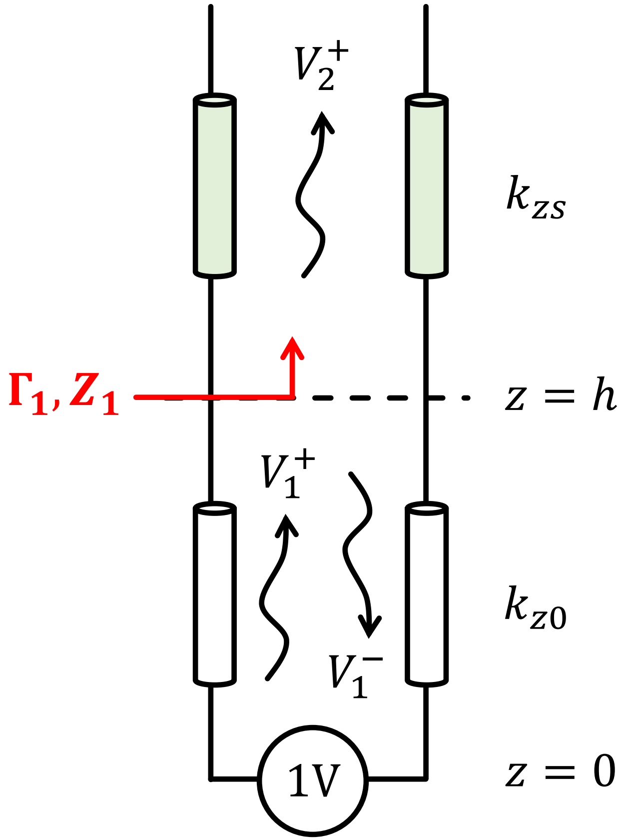
Write a MATLAB routine to calculate the spectral Green's function for the electric field given by an elementary y-oriented magnetic source at $z = 0$ radiating into an infinite medium with a permittivity of ε_r in the presence of a ground plane and an air layer of thickness h, as shown in the figure.
Consider $h = 5$mm and a frequency of 30GHz.

Make a plot of the amplitude variation of the x-component of spectral field at $z = h^+$ as a function of k_x from 0 to $2k_0$ with $k_y = 0$ for the following values of the permittivity $\varepsilon_r = 2.5, 6$ and 12.



Transmission Line Solution





Voltage and current in air ($0 \leq z < h$)

$$V_1(z) = V_1^+ \, e^{-jk_{z0}z} + V_1^- \, e^{+jk_{z0}z}$$

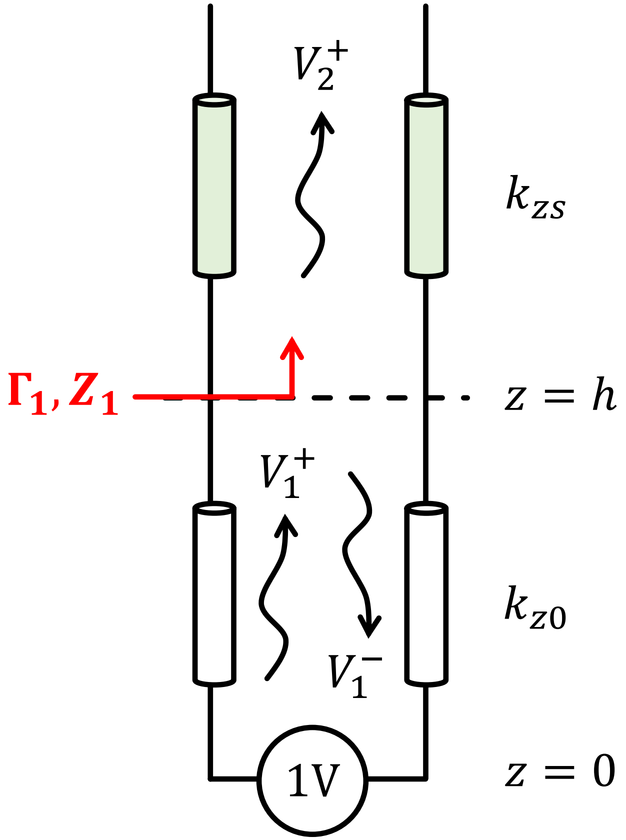
$$I_1(z) = \frac{V_1^+}{Z_0} \, e^{-jk_{z0}z} - \frac{V_1^-}{Z_0} \, e^{+jk_{z0}z}$$

Voltage and current in superstrate ($z \geq h$)

$$V_2(z) = V_2^+ \, e^{-jk_{zs}z}$$

$$I_2(z) = \frac{V_2^+}{Z_s} \, e^{-jk_{zs}z}$$

Transmission Line Solution



In air region $(0 \leq z < h)$

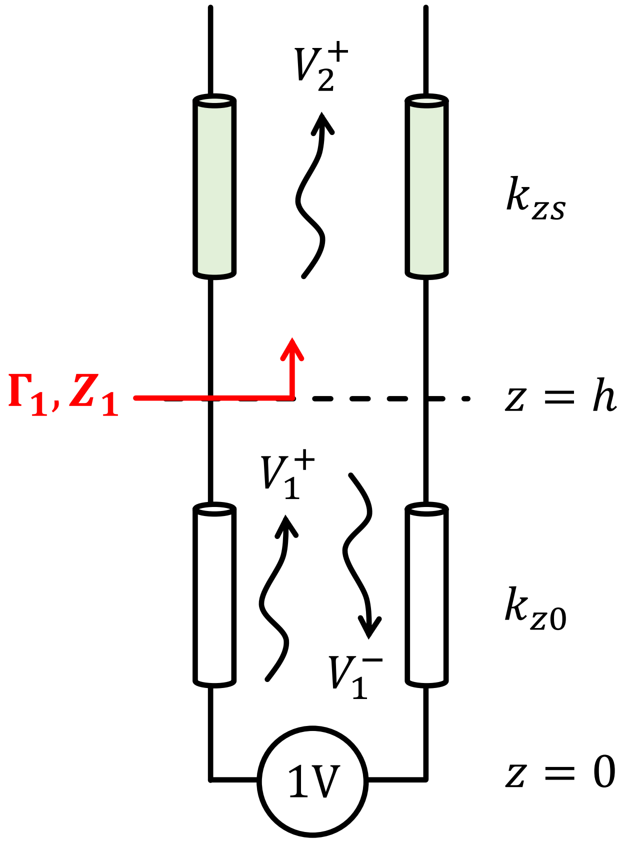
$$\left.\begin{aligned} &V_1(z=0) = 1V \\ &\Gamma_1 = \frac{Z_s - Z_0}{Z_s + Z_0} = \frac{V_1^- e^{+jk_{z0}h}}{V_1^+ e^{-jk_{z0}h}} \end{aligned}\right\} \rightarrow$$

$$\boxed{V_1^+ = \frac{1}{1 + \Gamma_1 e^{-2jk_{z0}h}}}$$

$$V_1(z) = \frac{e^{-jk_{z0}z}}{1 + \Gamma_1 e^{-2jk_{z0}h}} \left[1 + \Gamma_1 e^{-2jk_{z0}h} e^{+2jk_{z0}z}\right]$$

$$I_1(z) = \frac{e^{-jk_{z0}z}}{Z_0(1 + \Gamma_1 e^{-2jk_{z0}h})} \left[1 - \Gamma_1 e^{-2jk_{z0}h} e^{+2jk_{z0}z}\right]$$

Transmission Line Solution



In superstrate region ($z \geq h$)

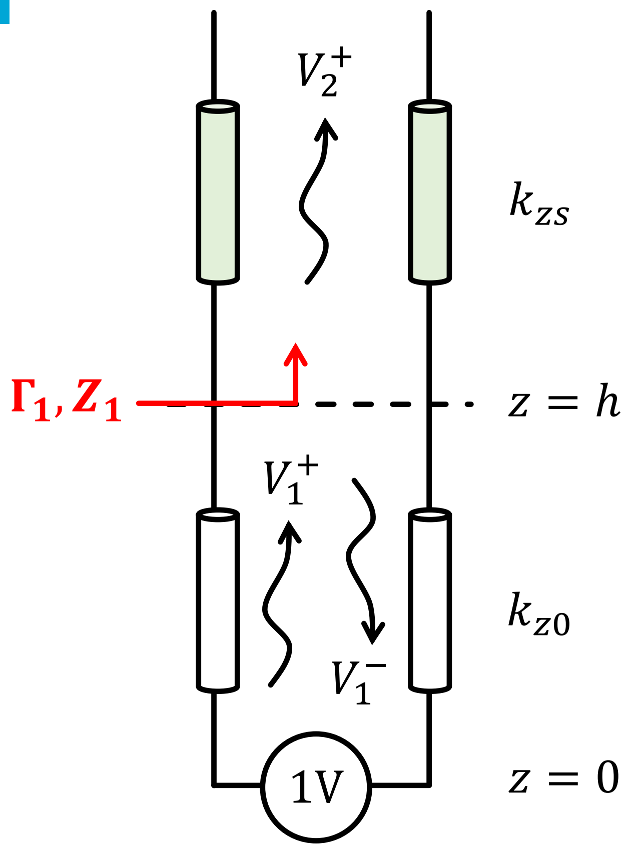
$$V_1(z = h) = V_2(z = h) \quad \rightarrow$$

$$\boxed{V_2^+ = \frac{e^{-jk_{z0}h}e^{+jk_{zs}h}}{1 + \Gamma_1 e^{-2jk_{z0}h}}[1 + \Gamma_1]}$$

$$V_2(z) = \frac{e^{-jk_{z0}h}e^{+jk_{zs}h}}{1 + \Gamma_1 e^{-2jk_{z0}h}}[1 + \Gamma_1]e^{-jk_{zs}z}$$

$$I_2(z) = \frac{e^{-jk_{z0}h}e^{+jk_{zs}h}}{Z_s(1 + \Gamma_1 e^{-2jk_{z0}h})}[1 + \Gamma_1]e^{-jk_{zs}z}$$

Transmission Line Solution: Summary



$$\Gamma_1 = \frac{Z_s - Z_0}{Z_s + Z_0}$$

In air region ($0 \leq z < h$)

$$V_1(z) = \frac{e^{-jk_{z0}z}}{1 + \Gamma_1 e^{-2jk_{z0}h}}\left[1 + \Gamma_1 e^{-2jk_{z0}h} e^{+2jk_{z0}z}\right]$$

$$I_1(z) = \frac{e^{-jk_{z0}z}}{Z_0(1 + \Gamma_1 e^{-2jk_{z0}h})}\left[1 - \Gamma_1 e^{-2jk_{z0}h} e^{+2jk_{z0}z}\right]$$

In superstrate region ($z \geq h$)

$$V_2(z) = \frac{e^{-jk_{z0}h} e^{+jk_{zs}h}}{1 + \Gamma_1 e^{-2jk_{z0}h}}[1 + \Gamma_1] e^{-jk_{zs}z}$$

$$I_2(z) = \frac{e^{-jk_{z0}h} e^{+jk_{zs}h}}{Z_s(1 + \Gamma_1 e^{-2jk_{z0}h})}[1 + \Gamma_1] e^{-jk_{zs}z}$$

Routines

- Solution of the equivalent transmission line:

$$[v_{TM}, v_{TE}, i_{TM}, i_{TE}] = trxline\_semi\_inf\_superstrate(k0, zeta0, er, h, kro, z)$$

- Dyadic SGF :

$$[Gxy, Gyy, Gzy] = SpectralGFem(k0, er, kx, ky, v_{TM}, v_{TE}, i_{TM}, i_{TE}\ , zeta0)$$

$$\underline{\underline{\mathbf{G}}}^{em}(k_x, k_y, z, z') =$$

$$\begin{bmatrix} \frac{k_x k_y}{k_\rho^2}\left({}^mV_{TM}(k_\rho, z, z') - {}^mV_{TE}(k_\rho, z, z')\right) & \frac{-1}{k_\rho^2}\left({}^mV_{TM}(k_\rho, z, z')k_x^2 + {}^mV_{TE}(k_\rho, z, z')k_y^2\right) \\ \frac{1}{k_\rho^2}\left({}^mV_{TM}(k_\rho, z, z')k_y^2 + {}^mV_{TE}(k_\rho, z, z')k_x^2\right) & \frac{k_x k_y}{k_\rho^2}\left({}^mV_{TE}(k_\rho, z, z') - {}^mV_{TM}(k_\rho, z, z')\right) \\ -\frac{k_y}{k_{zi}} Z_{TMi}\ {}^mI_{TM}(k_\rho, z, z') & \frac{k_x}{k_{zi}} Z_{TMi}\ {}^mI_{TM}(k_\rho, z, z') \end{bmatrix}$$