

# EE4620 - Spectral Domain Methods in Electromagnetics

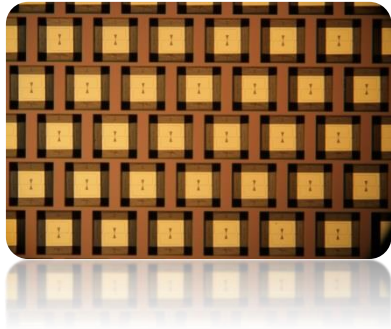
Topic # 5

## Introduction to Leaky Wave Antennas

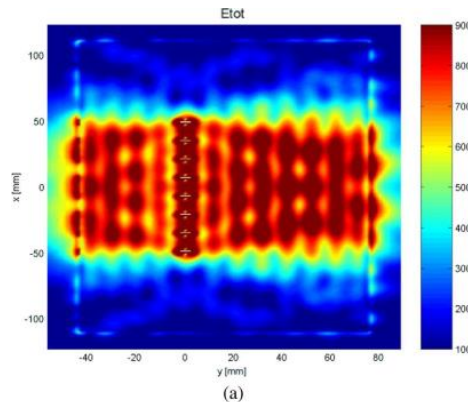
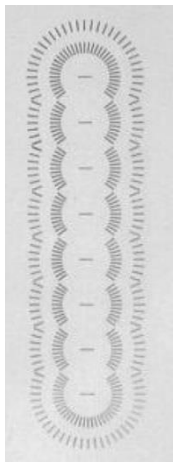
Nuria Llobart

# Antennas Printed in Stratified Dielectrics

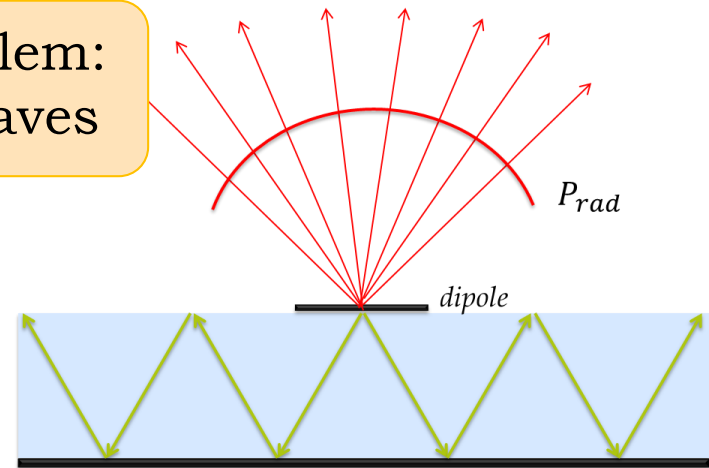
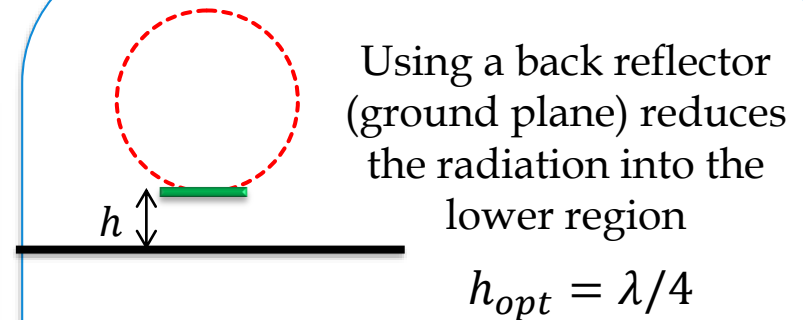
Dielectric allows using integrated technology (PCB, lithography)



E.g. of a linear array



Main problem:  
surface waves



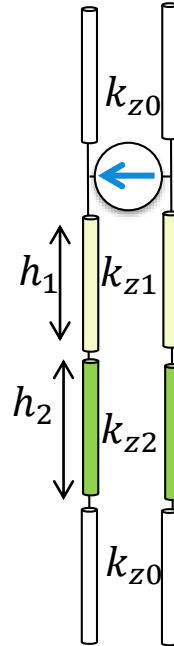
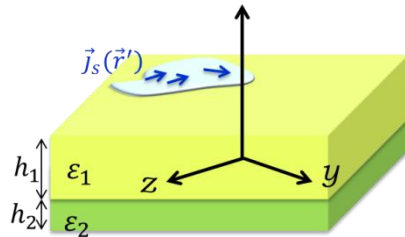
There is a part of the power delivered to the antenna that is radiated inside the dielectric!

# Dyadic Green's Function for Stratified Media

$$\mathbf{f}(\vec{r}) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{\mathbf{G}}^{fc}(k_x, k_y, z, z') \mathbf{C}_s(k_x, k_y) e^{-jk_x x} e^{-jk_y y} dk_x dk_y$$

$$\tilde{\mathbf{G}}^{ej} = \begin{bmatrix} -\frac{v_{TM}k_x^2 + v_{TE}k_y^2}{k_\rho^2} & \frac{(v_{TE} - v_{TM})k_x k_y}{k_\rho^2} \\ \frac{(v_{TE} - v_{TM})k_x k_y}{k_\rho^2} & -\frac{v_{TE}k_x^2 + v_{TM}k_y^2}{k_\rho^2} \\ \varsigma \frac{k_x}{k} i_{TM} & \varsigma \frac{k_y}{k} i_{TM} \end{bmatrix}$$

$$\tilde{\mathbf{G}}^{hj} = \begin{bmatrix} \frac{(i_{TM} - i_{TE})k_x k_y}{k_\rho^2} & \frac{i_{TE}k_x^2 + i_{TM}k_y^2}{k_\rho^2} \\ \frac{i_{TE}k_y^2 + i_{TM}k_x^2}{k_\rho^2} & \frac{(i_{TE} - i_{TM})k_x k_y}{k_\rho^2} \\ \frac{k_y}{\varsigma k} v_{TE} & \frac{k_x}{\varsigma k} v_{TE} \end{bmatrix}$$



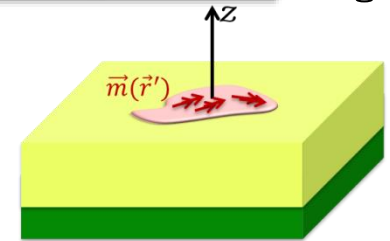
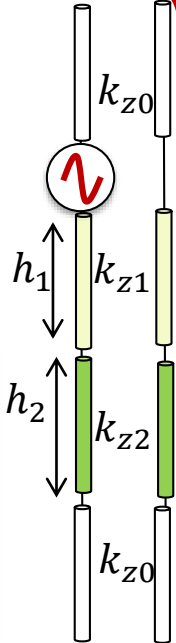
$$k_{zi} = \sqrt{k_i^2 - k_x^2 - k_y^2}$$

$$Z_{TMi} = \zeta_i k_{zi} / k_i$$

$$Z_{TEi} = \zeta_i k_i / k_{zi}$$

$$\tilde{\mathbf{G}}^{hm} = \begin{bmatrix} -\frac{i_{TE}k_x^2 + i_{TM}k_y^2}{k_\rho^2} & \frac{(i_{TM} - i_{TE})k_x k_y}{k_\rho^2} \\ \frac{(i_{TM} - i_{TE})k_x k_y}{k_\rho^2} & -\frac{i_{TM}k_x^2 + i_{TE}k_y^2}{k_\rho^2} \\ \frac{k_x}{\varsigma k} v_{TE} & \frac{k_y}{\varsigma k} v_{TE} \end{bmatrix}$$

$$\tilde{\mathbf{G}}^{em} = \begin{bmatrix} \frac{(v_{TM} - v_{TE})k_x k_y}{k_\rho^2} & -\frac{v_{TE}k_y^2 + v_{TM}k_x^2}{k_\rho^2} \\ \frac{v_{TE}k_x^2 + v_{TM}k_y^2}{k_\rho^2} & \frac{(v_{TE} - v_{TM})k_x k_y}{k_\rho^2} \\ -\varsigma \frac{k_y}{k} i_{TM} & \varsigma \frac{k_x}{k} i_{TM} \end{bmatrix}$$

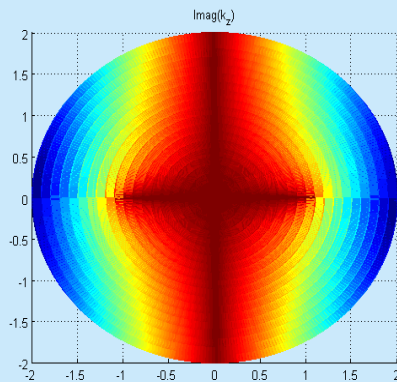


# Preferred Branch Convention

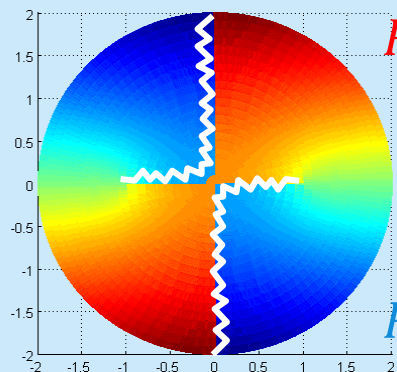
Selection of square root multivalues on the Riemann sheets to obtain a **unique specification of the integrand in the complex plane**

$$k_z = -j \sqrt{-(k^2 - k_\rho^2)}$$

## Top Riemann Sheet:



$$\text{Imag}(k_z) < 0$$

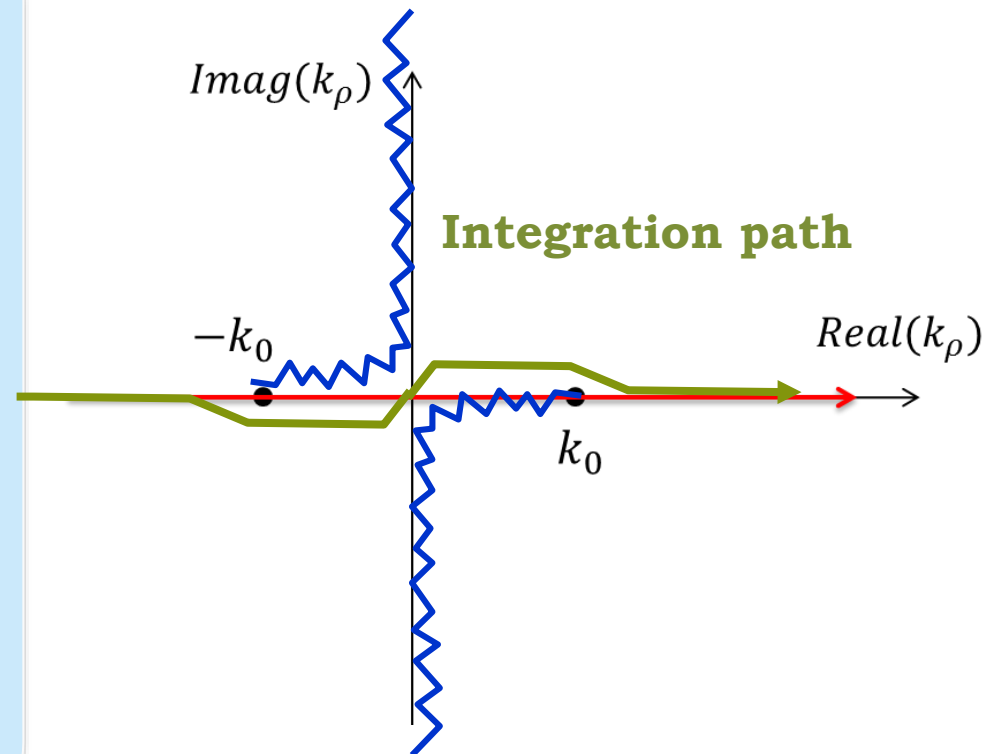


$$\text{Real}(k_z) > 0$$

$$\text{Real}(k_z) < 0$$

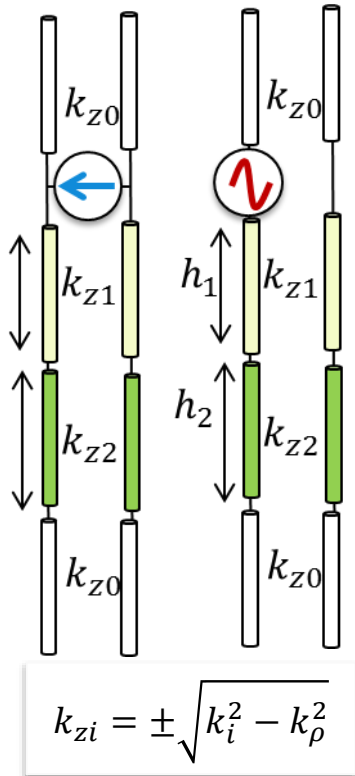
The radiation condition is verified

## Branch cuts



# Pole Singularities in Stratified Media

*Pole singularities in  $k_\rho$  arise in dielectric stratifications*



The solutions of the transmission lines can be expressed as:

$$v_{TM/TEi}(k_\rho, z) = \frac{N_{TM/TEi}^v(k_\rho, z)}{D_{TM/TE}(k_\rho)}$$

$$i_{TM/TEi}(k_\rho, z) = \frac{N_{TM/TEi}^i(k_\rho, z)}{D_{TM/TE}(k_\rho)}$$

Q: Which ones are the transversal propagation constants?  
>  $k_\rho$  probably

- Dispersion equation to find transversal propagation constants:

$$D_{TM/TE}(k_\rho) = 0$$



$$k_{\rho 0} = k_\rho^g - \frac{D(k_\rho^g)}{D'(k_\rho^g)}$$

- Residues to find the surface wave field distribution and associated power

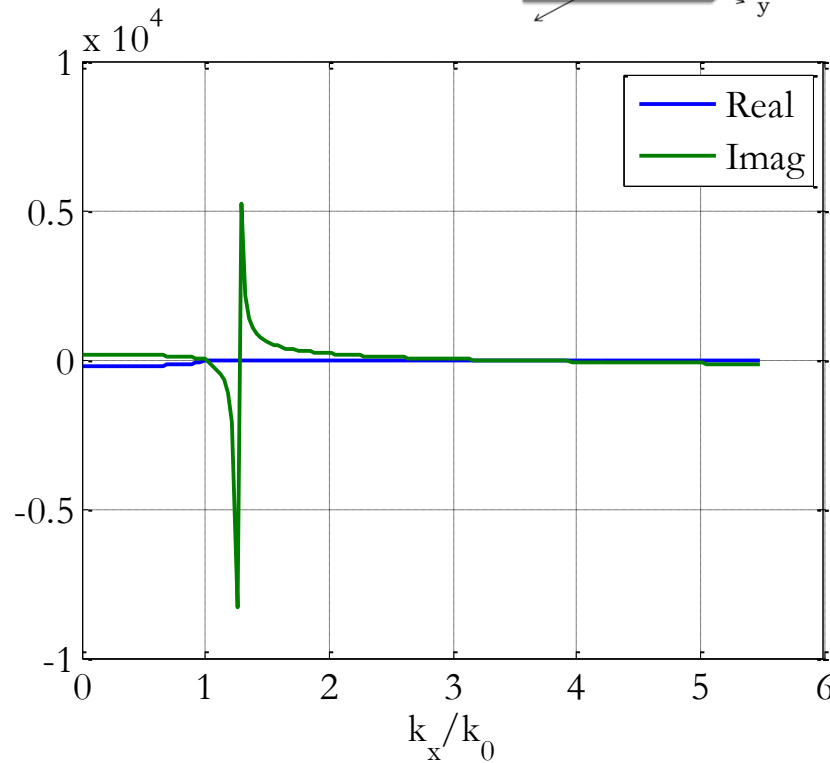
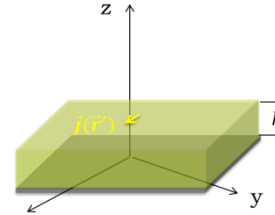
$$v_{TM}(k_\rho, z) = \frac{N(k_\rho)}{D(k_\rho)}$$



$$\text{Res}[v_{TM}(k_\rho, z)]|_{k_{\rho i}} = \frac{N(k_{\rho i})}{D'(k_{\rho i})}$$

# Examples of Surface wave poles

## Surface Waves



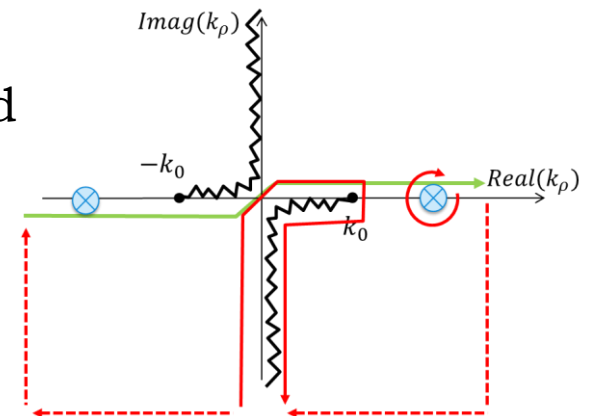
Polar singularity in the real axis  
with  $k_\rho > k_0$

# SW Field Contribution

$$\vec{f}(\vec{r}) \approx \frac{1}{4\pi^2} \int_0^\pi \int_{-\infty}^\infty \tilde{\mathbf{G}}^{fc}(k_\rho, z, z') k_\rho e^{-jk_\rho \rho} k_\rho dk_\rho d\alpha \vec{\mathbf{C}}_0(k_{\rho c}, \phi)$$



Surface wave poles are found on the real axis in the Top Riemann Sheet



$$\vec{f}_{sw}(\vec{r}) = -2\pi j \sum_i \text{Res} \left[ \frac{e^{\frac{j\pi}{4}} \sqrt{k_\rho} e^{-jk_\rho \rho}}{2\pi \sqrt{2\pi \rho}} \tilde{\mathbf{G}}^{fc}(k_\rho, z, z') \vec{\mathbf{C}}_0(k_\rho, \phi) \right] \Bigg|_{k_\rho = k_{\rho i}}$$

Field contributions in the form of cylindrical propagating waves inside the stratified dielectrics

# Surface Waves

$$k_{\rho}^{sw} = \beta^{sw} = k_0 \sqrt{\epsilon_r^{sw}}$$

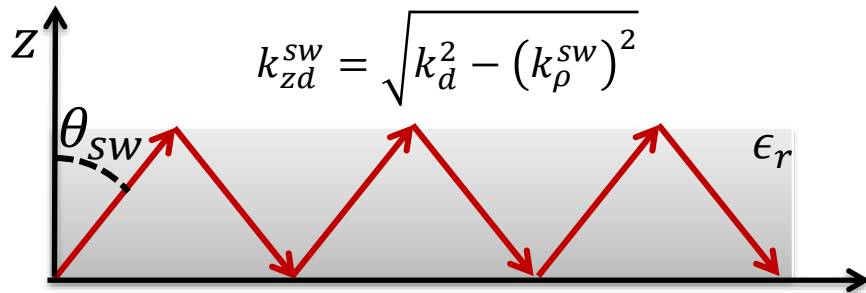
$$k_0 < k_{\rho}^{sw} < k_d$$

They are also referred as  
**slow waves**

$$k_{\rho}^{sw} = \frac{2\pi f}{v_{sw}} > k_0 \quad v_{sw} < v_0$$

It can be seen as a couple of **homogenous waves propagating inside the dielectric** with a direction characterized by a real angle  $\pm\theta_{sw}$ :

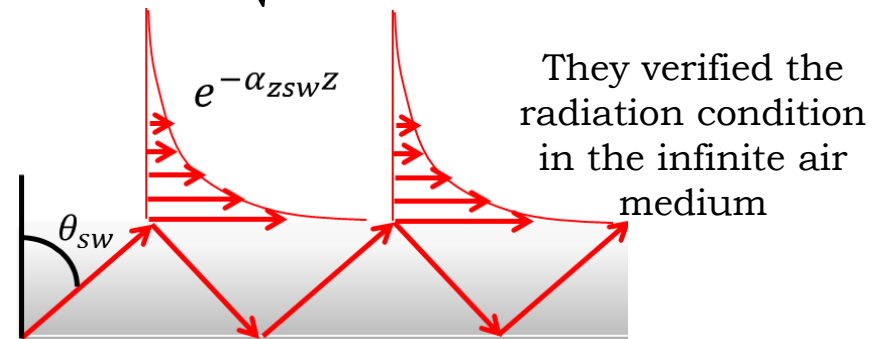
$$\beta^{sw} = k_d \sin\theta_{sw} < k_d \quad \sin\theta_{sw} = \frac{\sqrt{\epsilon_r^{sw}}}{\sqrt{\epsilon_r}}$$



There is no attenuation, therefore the energy carried by the surface wave will reach *infinity*

It is an **inhomogenous wave in the air region**:

$$k_{z0sw} = -j\sqrt{-\left(k_0^2 - (k_{\rho}^{sw})^2\right)} = -j\alpha_{zsw}$$



The sw angle is above the critical angle

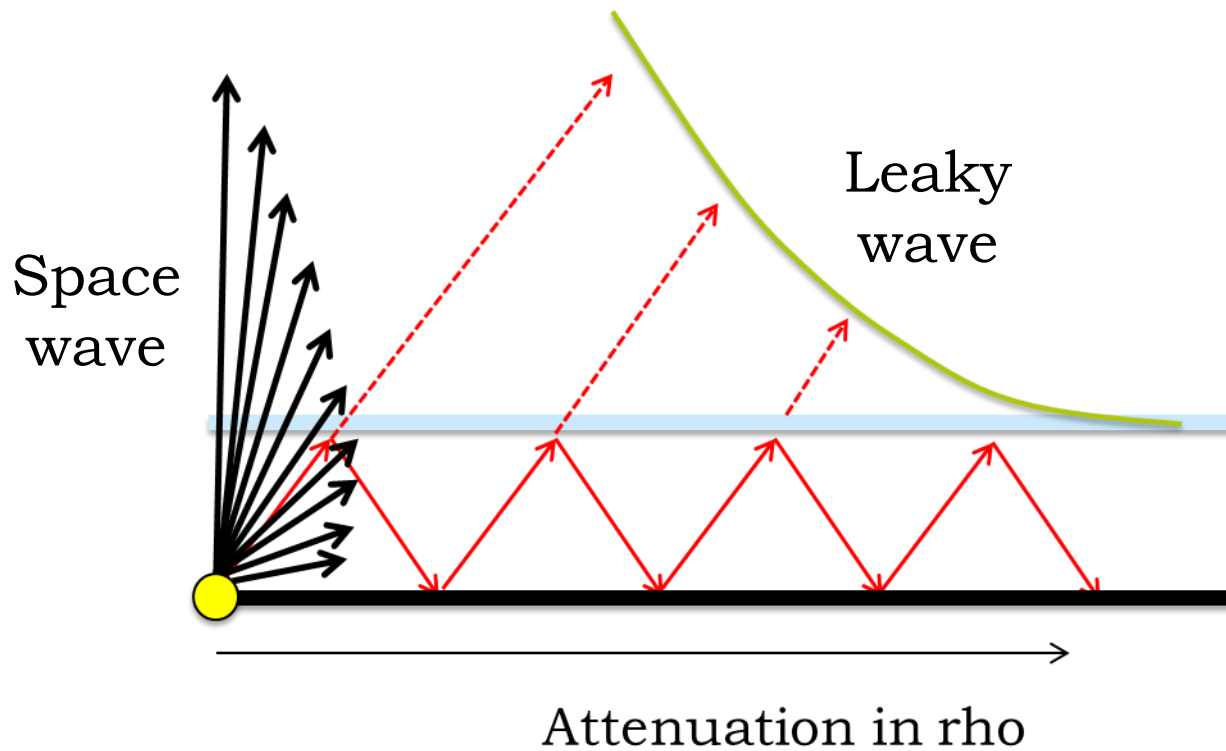
$$\sqrt{\epsilon_r} \sin\theta_{sw} = \sin\theta_0$$

$$\sqrt{\epsilon_{rsw}} = \sin\theta_0$$

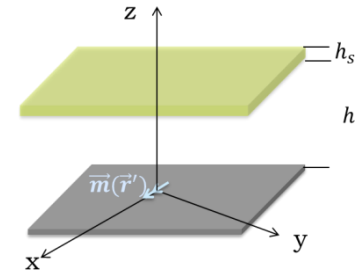
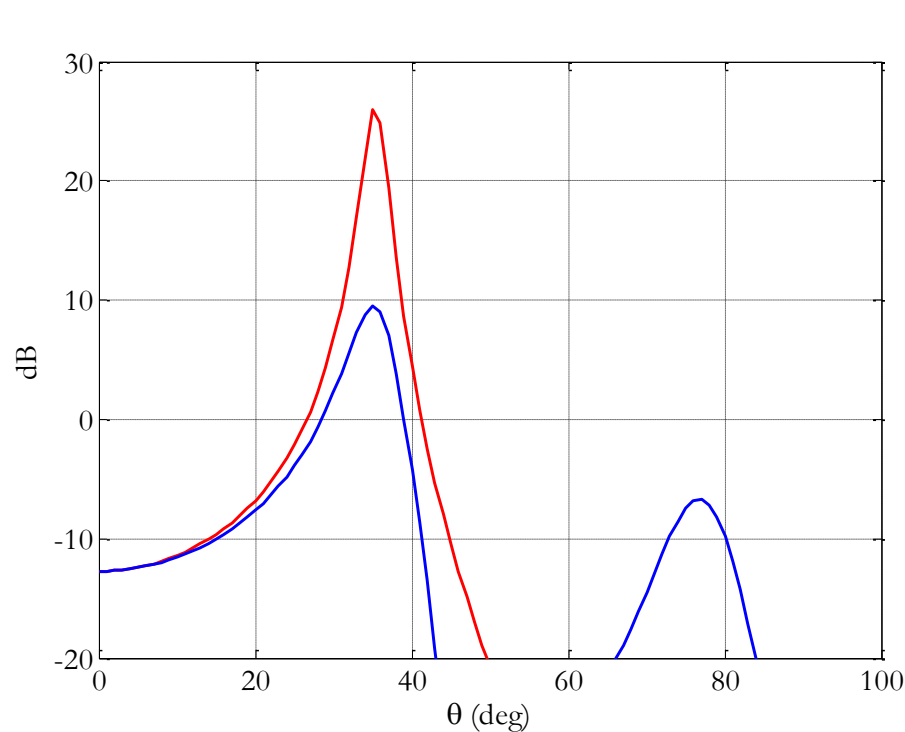
Therefore  $\theta_0$  is imaginary



# Introduction to leaky waves in stratified media



# Leaky Wave Poles



The spectrum seems to peak at a certain location in the visible spectrum, but does not go to infinity

Polar singularity in the complex plane  
close to the real axis with  $\text{Re}[k_\rho] < k_0$

# Pole Singularity of Leaky Waves

Complex pole singularities:

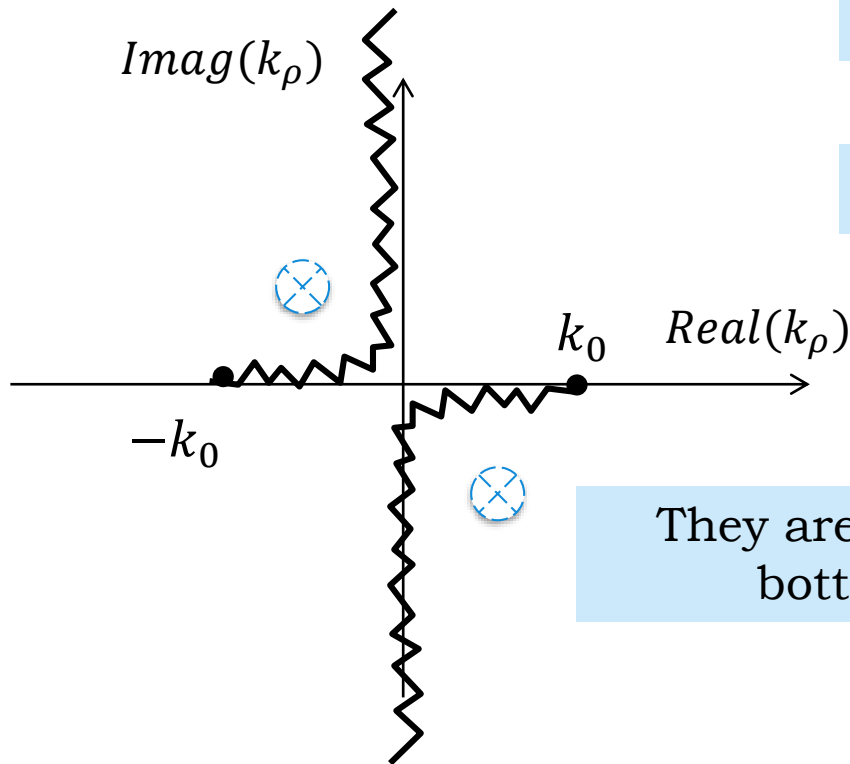
$$k_{\rho lw} = \beta_{lw} - j\alpha_{lw}$$

They are present only in certain stratifications

$$0 < \beta_{lw} < k_0$$

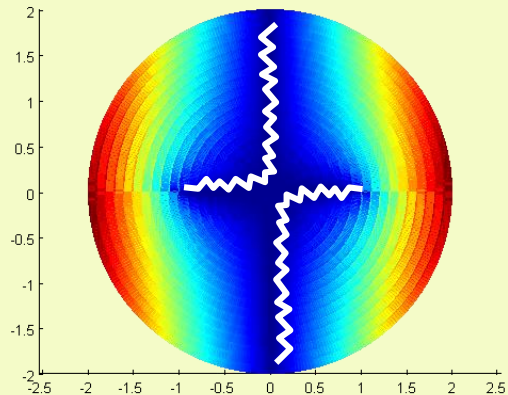
$$Im(k_{\rho lw}) < 0$$

They radiate energy into the infinite medium

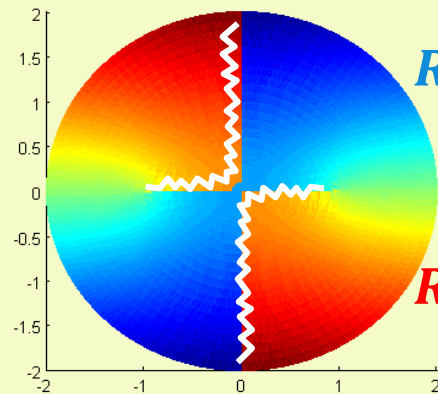


They are found in the bottom sheet

# Bottom Riemann Sheet



$Imag(k_z) > 0$



$Real(k_z) < 0$

$Real(k_z) > 0$

$$k_z = +j\sqrt{-(k^2 - k_\rho^2)}$$

This is opposite to the standard choice

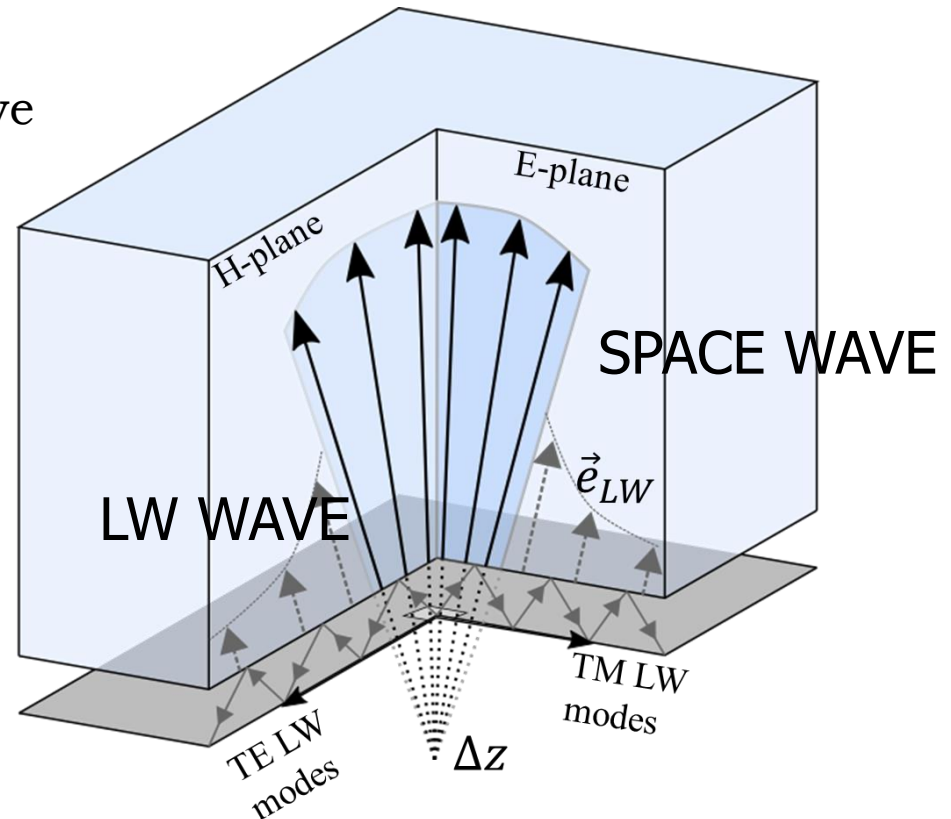
# They are present close to the source

They enlarge the effective area of the source

The leaky wave pole is only captured for observation points near the source

The residue contribution does not arrive to infinity

But it is responsible to the power transferred to the far field of the antenna: **modulation of the SFG in the region around  $\theta_{lw}^0$**



They can be used to do pattern shaping!

They are cylindrical waves with the same  $\phi$ -pattern and polarization than those of surface waves

# Cylindrical LWs

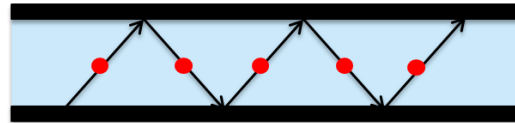
## TE Wave

Electric field oriented in the azimuthal direction

Maximum in the Hplane of the antenna

$$E_\phi(\rho, z) \approx -\text{Res}[v_{TE}(k_\rho, z, z')] \Big|_{k_{\rho i}} j_x(k_{\rho i}, \phi) \frac{C \sin \phi e^{-jk_{\rho i} \rho}}{\sqrt{\rho}}$$

TE Wave



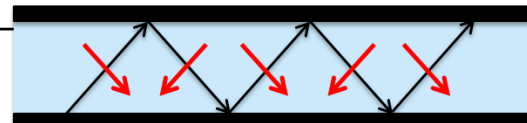
## TM Wave

Electric field oriented in the radial and z direction  
Maximum in the Eplane of the antenna

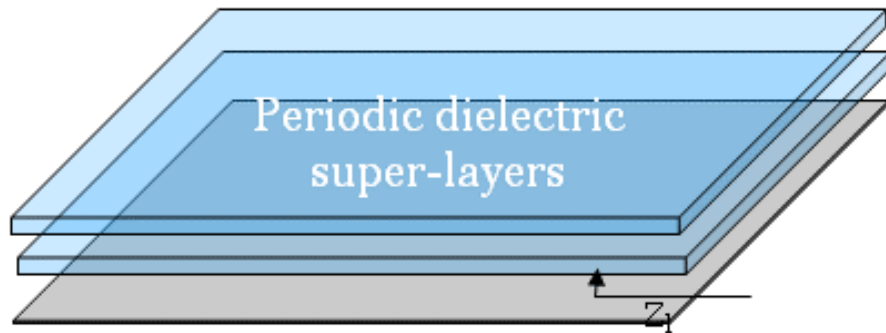
$$E_\rho(\rho, z) \approx \text{Res}[v_{TM}(k_\rho, z, z')] \Big|_{k_{\rho i}} j_x(k_{\rho i}, \phi) \frac{C \cos \phi e^{-jk_{\rho i} \rho}}{\sqrt{\rho}}$$

$$E_z(\rho, z) \approx -\frac{\zeta k_{\rho i}}{k} \text{Res}[i_{TM}(k_\rho, z, z')] \Big|_{k_{\rho i}} j_x(k_{\rho i}, \phi) \frac{C \cos \phi e^{-jk_{\rho i} \rho}}{\sqrt{\rho}}$$

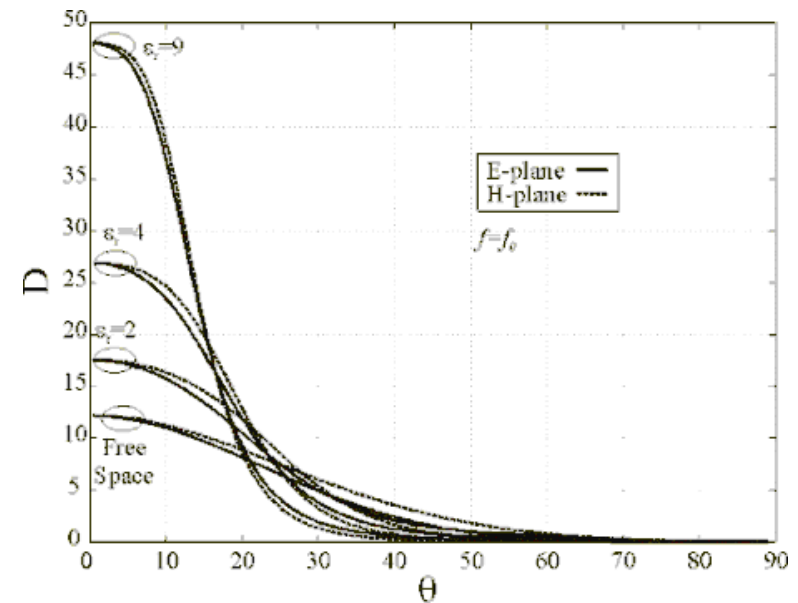
TM Wave



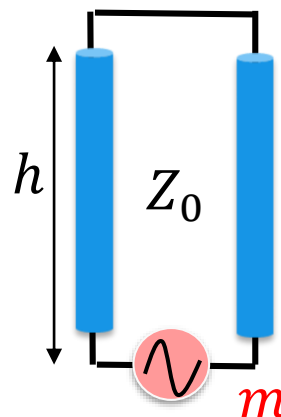
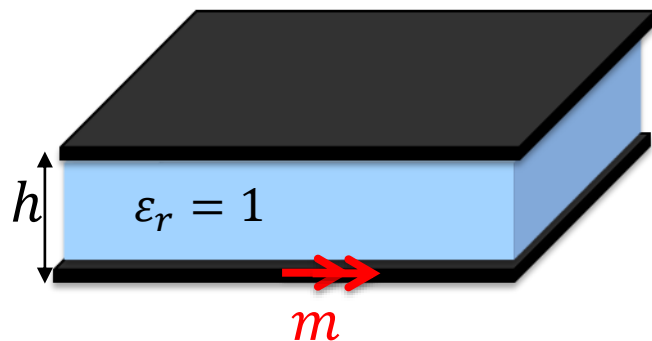
# Main Purpose of discussing them is LW Antennas



They can be used to enhance the directivity at broadside of small antennas

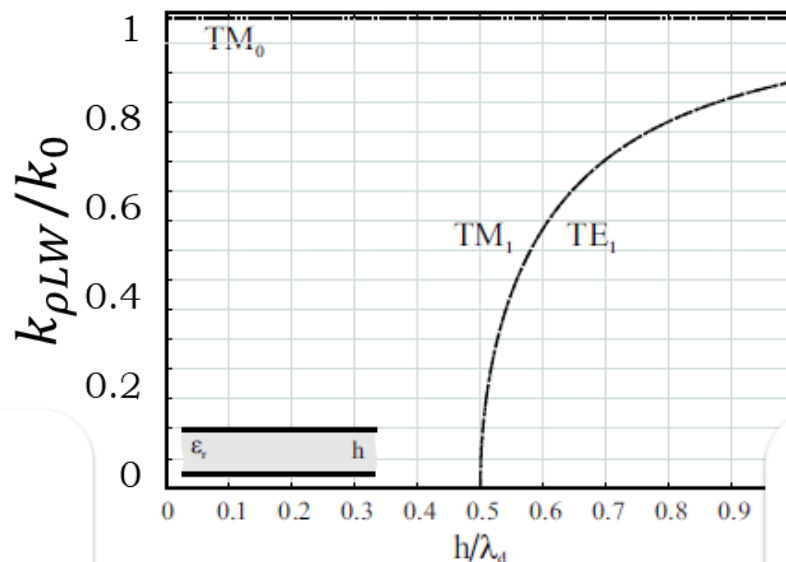
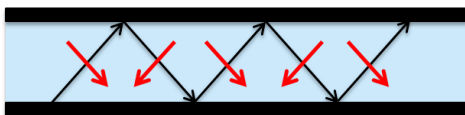


# Parallel Plate in Air

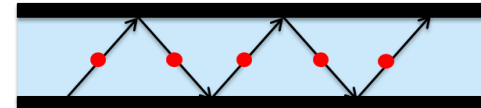


$$D(k_\rho) = jZ_0 \tan(k_{z0}h) = 0$$

TM Wave



TE Wave



for TM  $k_{zs} \tan(k_{zs}h) = 0$

$k_{zs} = 0$   $\tan(k_{zs}h) = 0$

$k_{\rho 0} = k$   $k_{zs}h = n\pi$

$$k_{\rho n} = \sqrt{k^2 - \left(\frac{n\pi}{h}\right)^2}$$

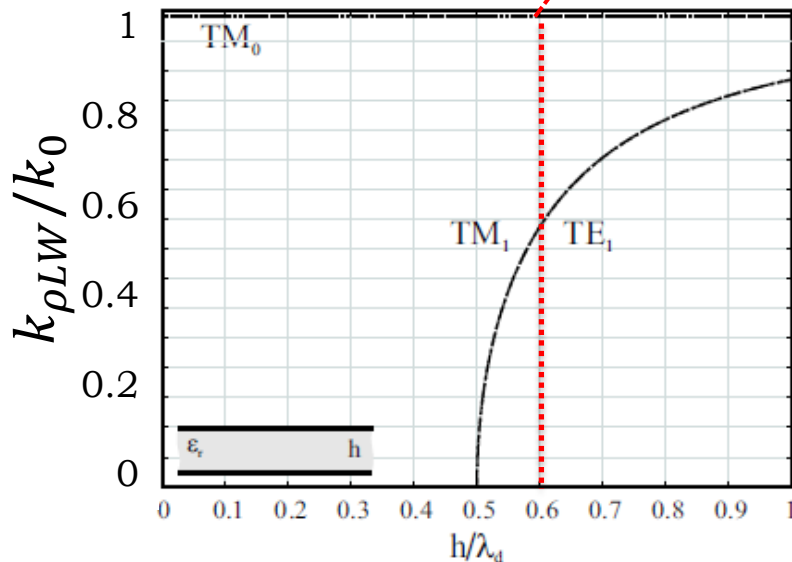
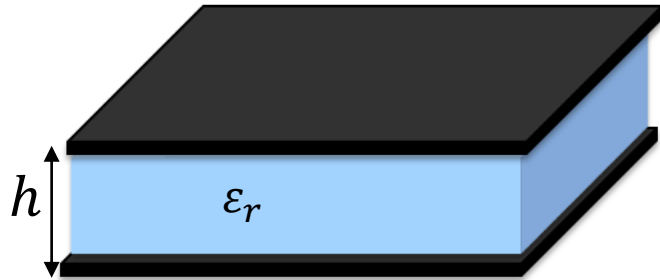
for TE  $\tan(k_{zs}h) = 0$

$k_{zs}h = n\pi$

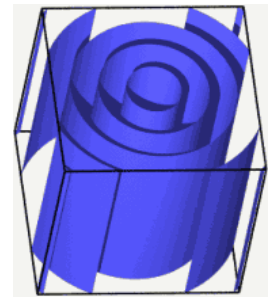
$$k_{\rho n} = \sqrt{k^2 - \left(\frac{n\pi}{h}\right)^2}$$



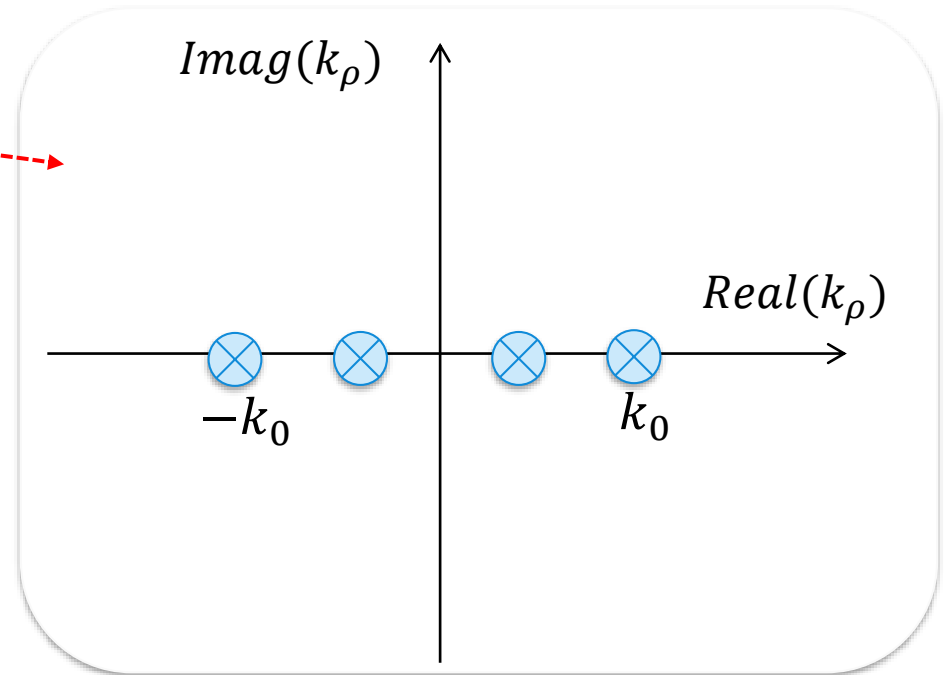
# Parallel Plate in Air



$$E = f(\phi, z) \frac{e^{-jk_\rho \rho}}{\sqrt{\rho}}$$

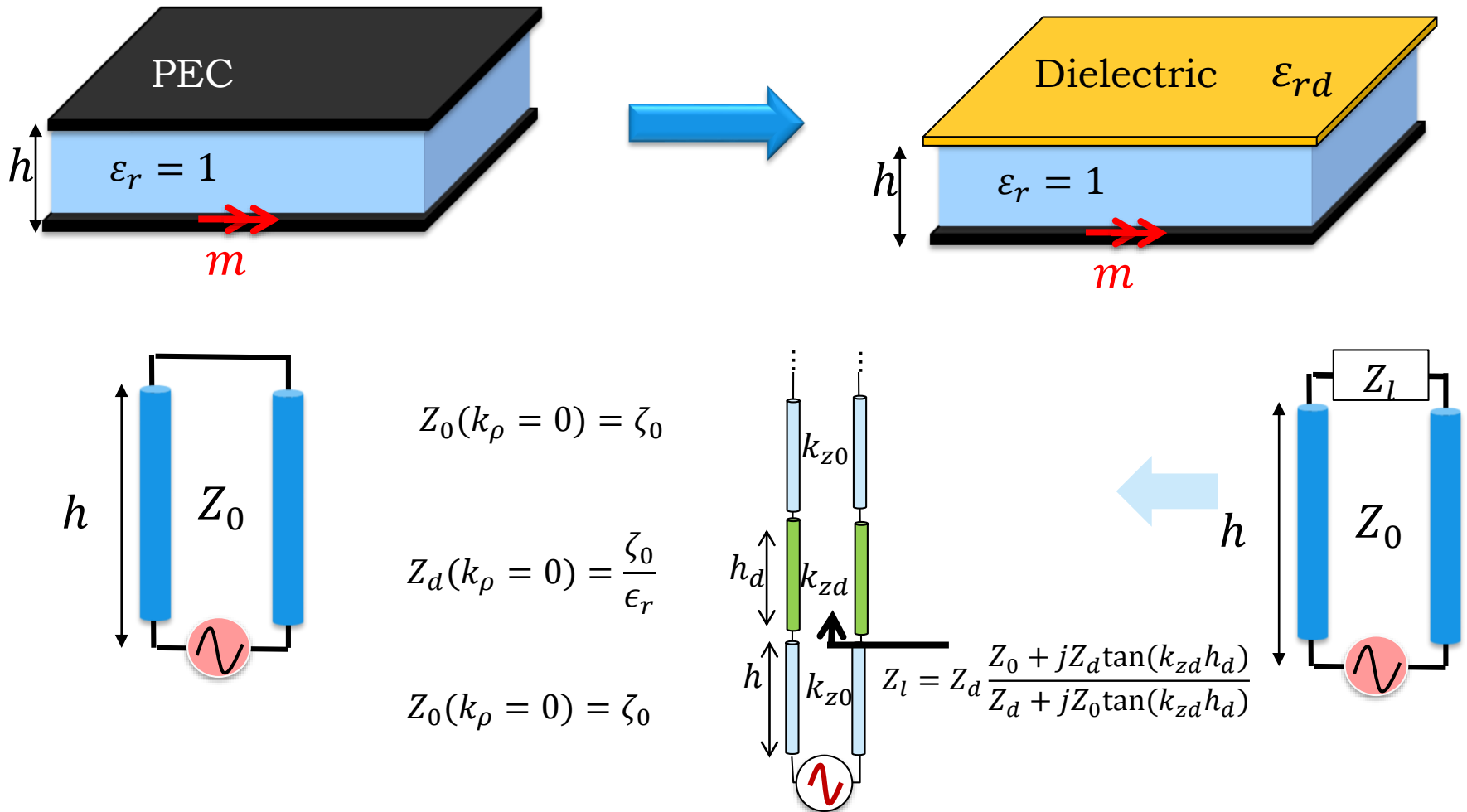


$k_\rho$ -Spectrum

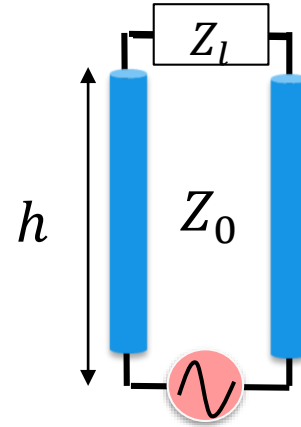
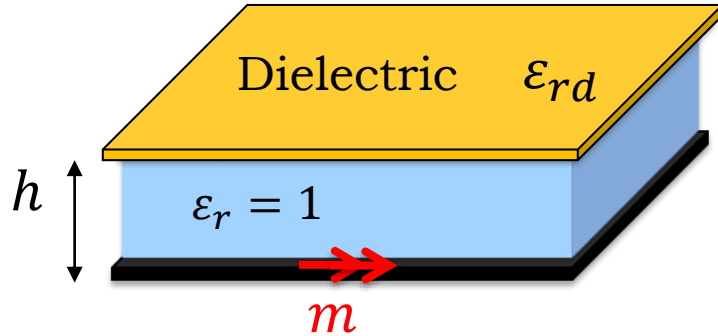


A PPW has no branch singularity, only real poles. These ones are SWs with propagation constant that can be  $\leq k_0$

# Open Parallel Plate Waveguide

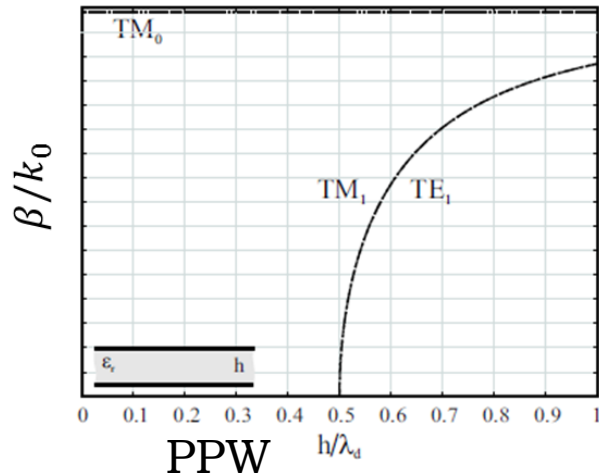


# Open Parallel Plate Waveguide



$$i(z = z_s) = Y_{in} = Y_0 \frac{Z_0 + jZ_l \tan(k_{z0}h)}{Z_l + jZ_0 \tan(k_{z0}h)}$$

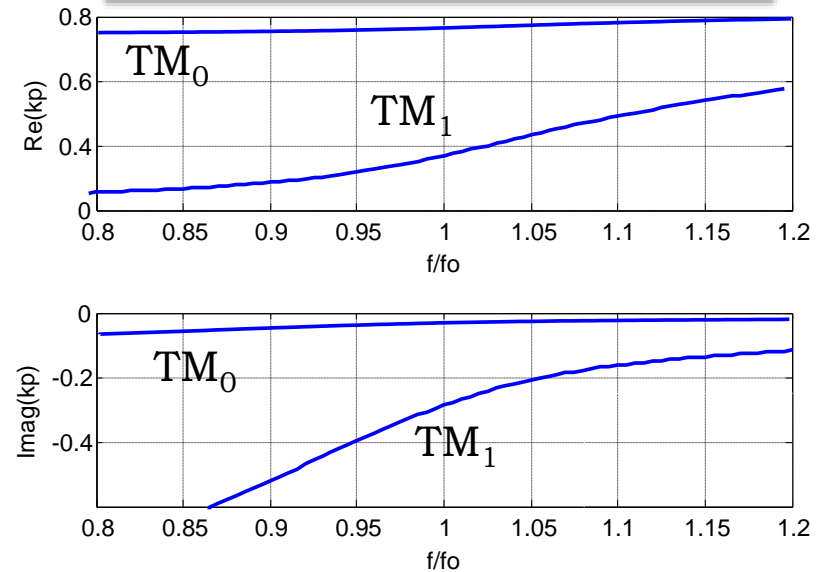
$$D(k_\rho) = jZ_0 \tan(k_{z0}h) = 0$$



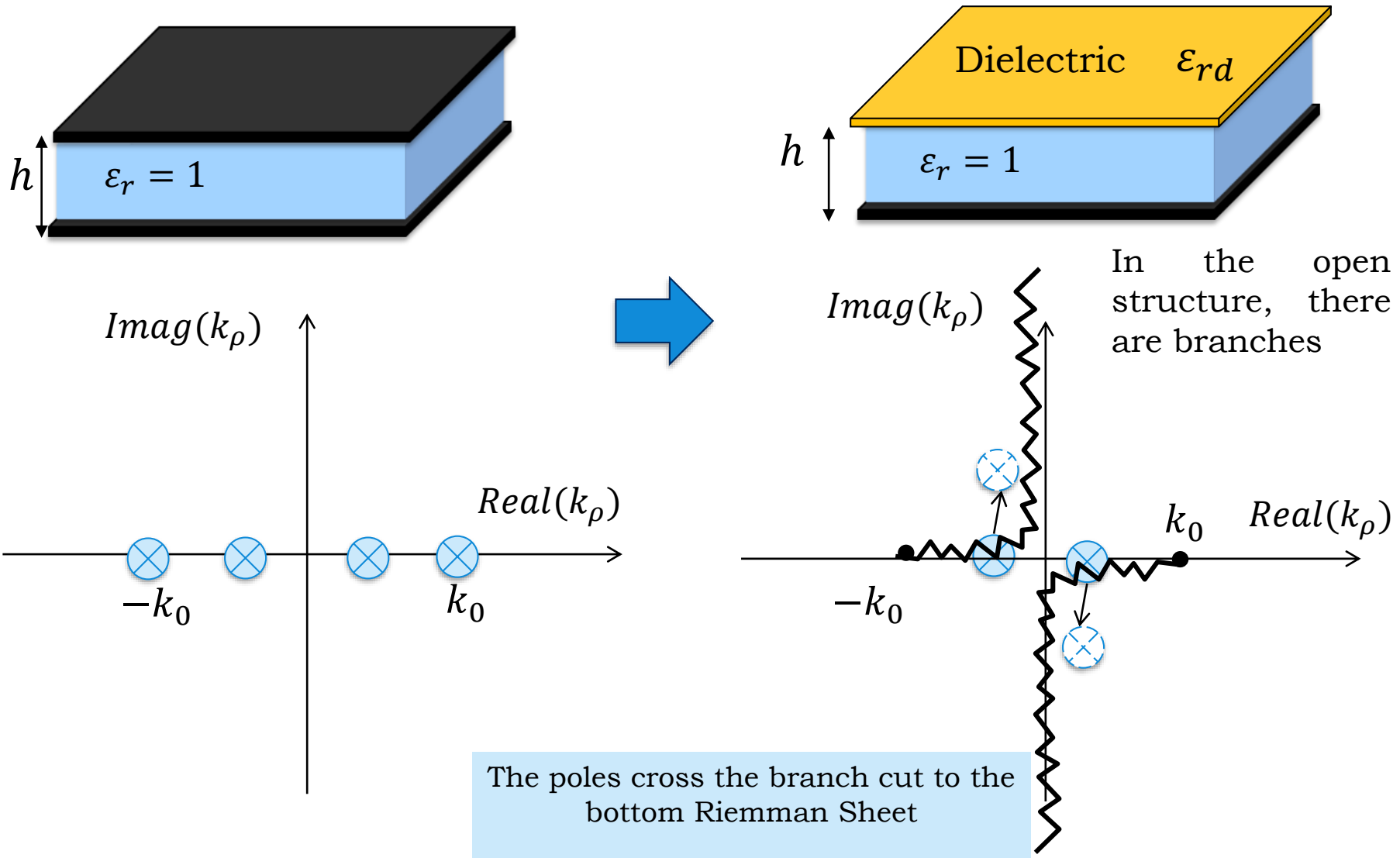
$$Z_l \neq 0$$



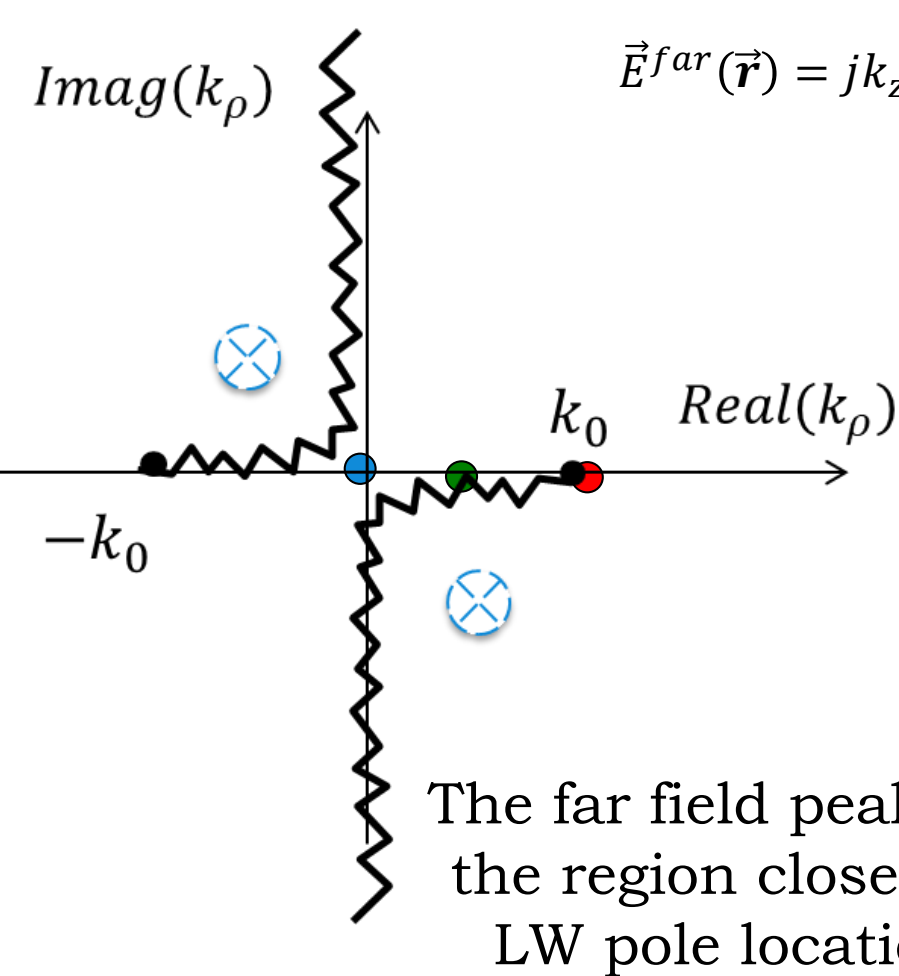
$$D(k_\rho) = Z_l + jZ_0 \tan(k_{z0}h) = 0$$



# Open Parallel Plate Waveguide



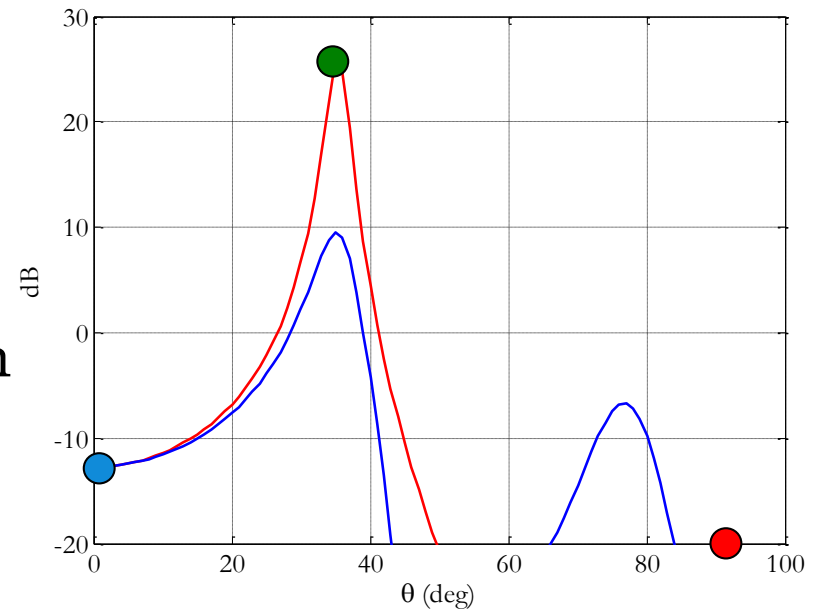
# Pattern shaping with leaky waves



The far field peaks in the region close the LW pole location

$$\vec{E}^{far}(\vec{r}) = jk_{zs} \underbrace{\tilde{\mathbf{G}}^{em}(k_{xs}, k_{ys}, z, z')} \vec{\mathbf{M}}_{eq}(k_{xs}, k_{ys}) e^{jk_{zs}|z|} \frac{e^{-jkr}}{2\pi r}$$

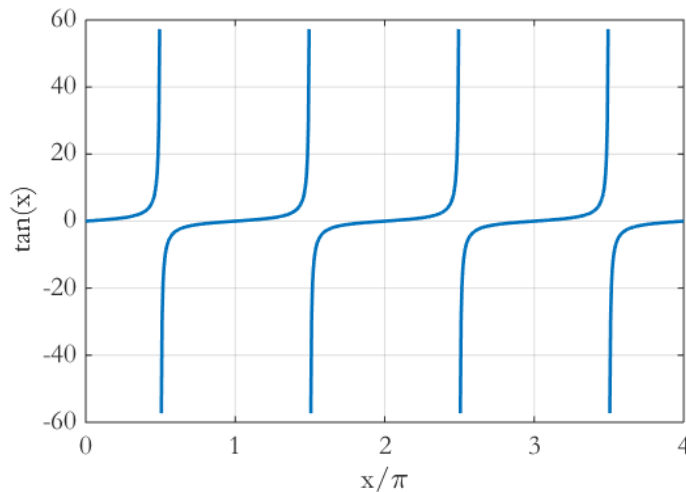
$k_{\rho s} = k_0 \sin \theta_s$  Saddle point in the far field



# Dispersion Equation: Analytical Approximated Solution

$$D(k_\rho) = Z_l + jZ_0 \tan(k_{z0}h)$$

Approximation 1:



The tan function can be approximated linearly around its zeroes for arguments in the surrounding of  $x = \pm n\pi$ , resorting to

$$\tan(k_{z0}h)|_{x \pm n\pi \approx 0} \approx k_{z0}h - n\pi$$

$n=1$

$$k_{z0} = j \frac{Z_l}{hZ_0} + \frac{\pi}{h} = \sqrt{k_0^2 - k_\rho^2}$$

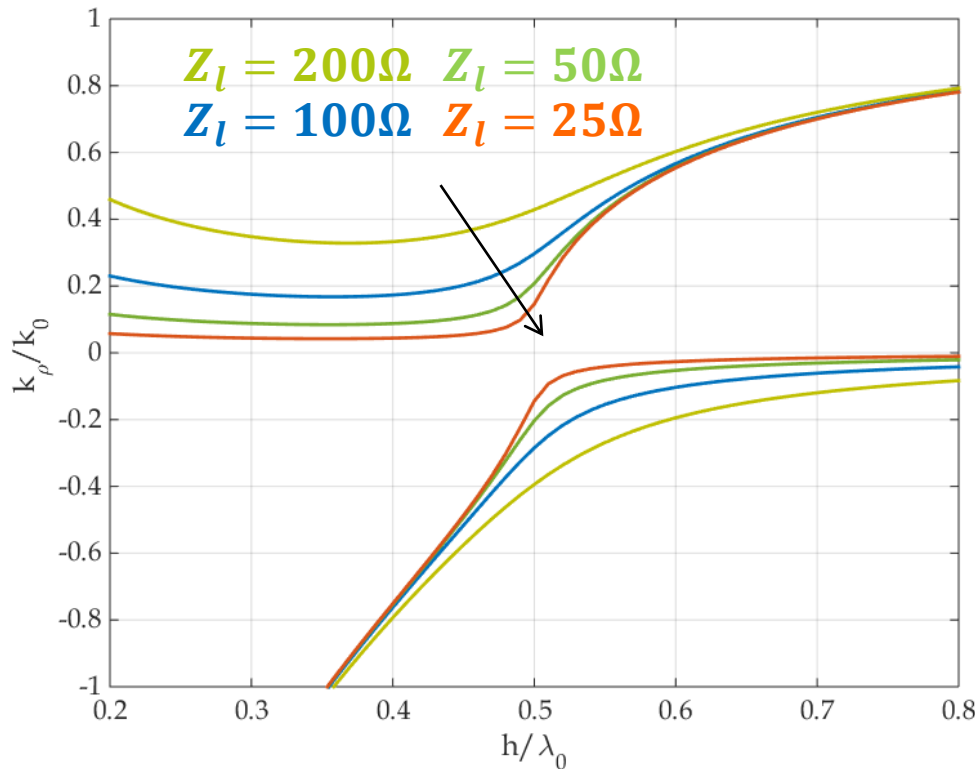
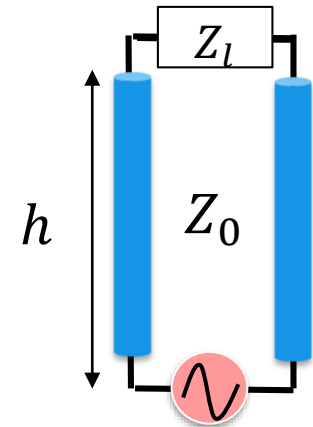
A. Neto and N. Llombart, "Wideband Localization of the Dominant Leaky Wave Poles in Dielectric Covered Antennas," in IEEE Antennas and Wireless Propagation Letters, vol. 5, pp. 549-551, 2006, doi: 10.1109/LAWP.2006.889558.

# Impact of the cavity top impedance

$$k_{z0} = j \frac{Z_l}{hZ_0} + \frac{\pi}{h} = \sqrt{k_0^2 - k_\rho^2}$$

The propagation constant highly depends on  $Z_l$

$$k_\rho = \sqrt{k_0^2 - k_{z0}^2} = \beta - j\alpha$$



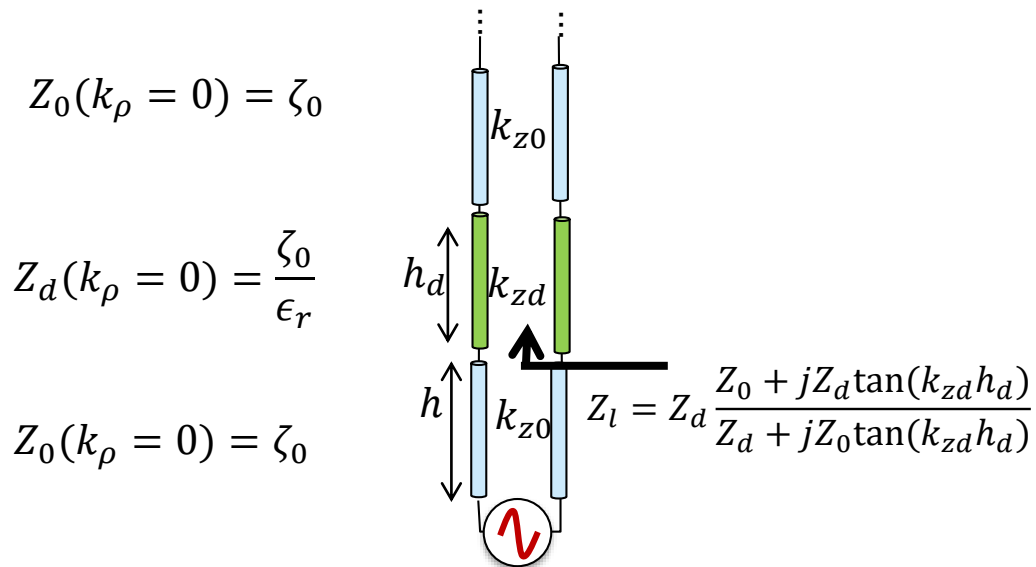
$$\beta = k_0 \sin \theta_{LW}$$

$$Z_l \downarrow \theta_{LW} \downarrow$$

Resonant condition:

$$h \approx \lambda_0/2$$

# Tunning the cavity top impedance



For a quarter wavelength thick substrate ( $Z_l$  tends to zero for large  $\epsilon_r$ ):

$$h_d = \lambda_d/4$$

$$k_\rho = 0$$



$$Z_l = \zeta_0/\epsilon_r$$



# Dispersion Equation: Analytical Solution

$$D(k_\rho) = Z_l + jZ_0 \tan(k_{z0}h)$$



$$k_{z0} = j \frac{Z_l}{hZ_0} + \frac{\pi}{h} = \sqrt{k_0^2 - k_\rho^2}$$

Approximation 2:

$$Z_l = Z_d \frac{Z_0 + jZ_d \tan(k_{zd}h_d)}{Z_d + jZ_0 \tan(k_{zd}h_d)} \quad \longrightarrow \quad Z_l \approx \zeta_0 / \epsilon_r$$

For poles radiating  
close to broadside

# Analytical Approximated Solution

$$k_{z0} = j \frac{\zeta_0}{h Z_0 \epsilon_r} + \frac{\pi}{h} = \sqrt{k_0^2 - k_\rho^2}$$

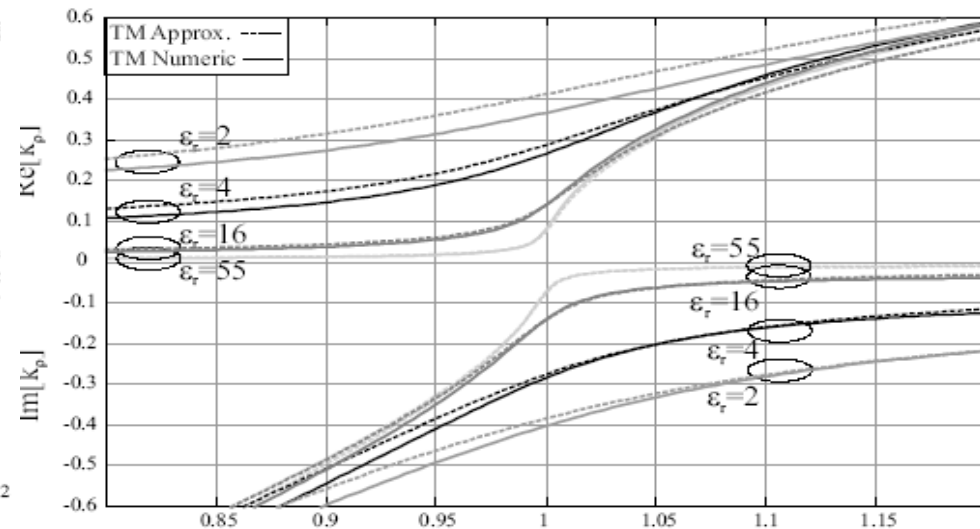
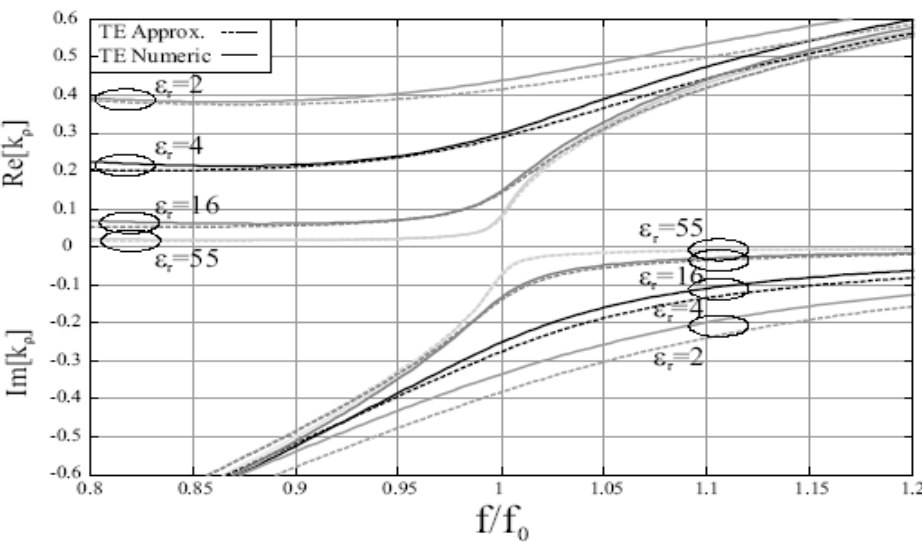
$$Z_{TE} = \zeta_i k_i / k_{zi}$$

$$Z_{TM} = \zeta_i k_{zi} / k_i$$

$$k_{z0}^{TE} = k_0 \left( \frac{1}{2\bar{h}} + j \frac{1}{4\bar{h}^2 \pi \epsilon_r} \right)$$

$$\bar{h} = h / \lambda_0$$

$$k_{z0}^{TM} = k_0 \left( \frac{1}{2\bar{h}} + j \frac{1}{\pi \epsilon_r} \right)$$



$$\epsilon_r \uparrow \quad \theta_{lw} \downarrow \quad \alpha_{lw} \downarrow$$

# Leaky Waves

$$k_{\rho}^{lw} = \beta^{lw} - j\alpha^{lw}$$

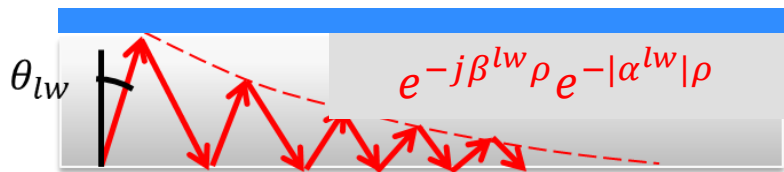
$$\beta^{lw} < k_0$$

They are also referred as  
**fast waves**

$$k_{\rho}^{lw} = \frac{2\pi f}{v_{lw}} < k_0 \quad v_{lw} > v_0$$

It can be seen as a couple of waves propagating with a direction characterized by a real angle  $\pm\theta_{lw}$  and an attenuation:

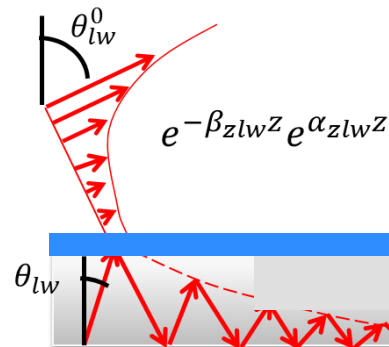
$$\beta^{lw} = k_0 \sin\theta_{lw} < k_0$$



There is attenuation, therefore the leaky wave loses energy while propagates

There is also propagation in the air region that can be related to a real angle:

$$\sqrt{\epsilon_r} \sin\theta_{lw} = \sin\theta_{lw}^0$$

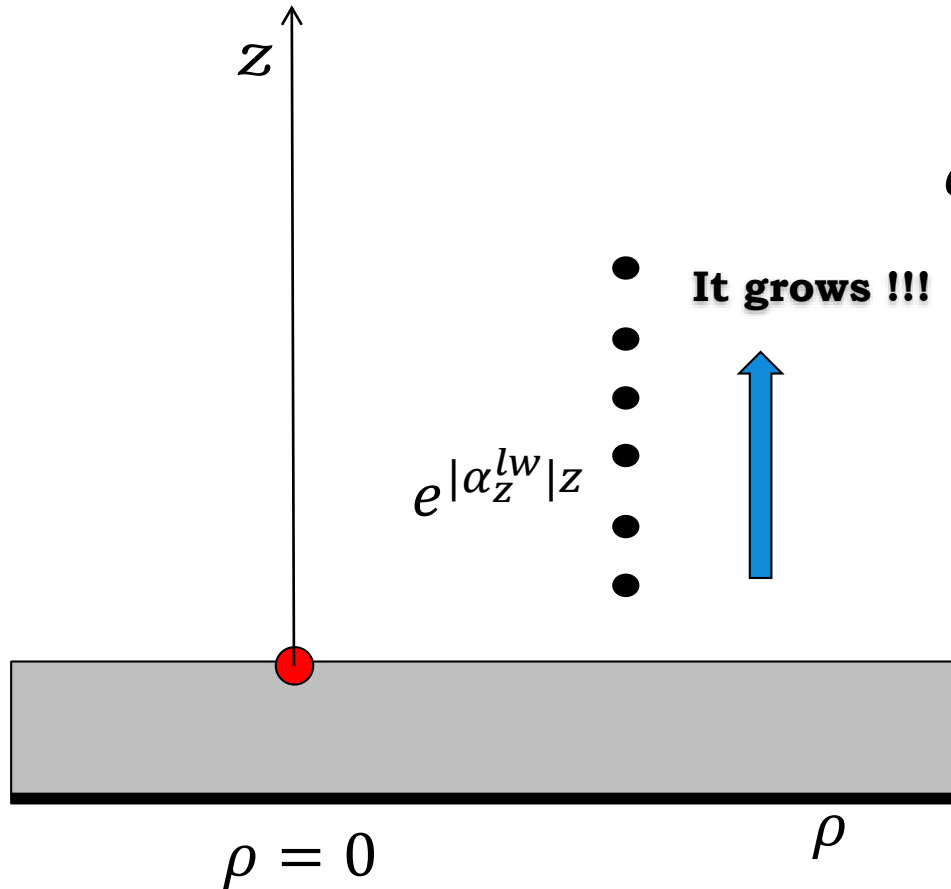


They do not verify the radiation condition in the infinite air medium

$$k_{z0}^{lw} = j\sqrt{-(k_0^2 - k_{\rho}^{lw2})} = \beta_{zlw} + j\alpha_{zlw}$$

# The difficulty with leaky Waves

$$\vec{f}_{LW}(\vec{r}) = -2\pi j \sum_i \text{Res} \left[ \frac{e^{\frac{j\pi}{4}} \sqrt{k_{\rho LW}} e^{-jk_{\rho LW} \rho}}{2\pi \sqrt{2\pi \rho}} e^{-jk_{zLW} z} \tilde{\mathbf{G}}^{fc}(k_{\rho LW}, z=0, z') \vec{\mathbf{C}}_0(k_{\rho LW}, \phi) \right] \Bigg|_{k_{\rho}=k_{\rho LW}}$$



$$e^{-jk_{zLW}z} = e^{-j\beta_{zLW}z} e^{-j(j\alpha_{zLW})z}$$

$$= e^{-j\beta_{zLW}z} e^{\alpha_{zLW}z}$$

This scared people for decades  
Their claimed was they did not exist

# Integration path: far observation point

## Complex pole singularities (Bottom sheet poles)

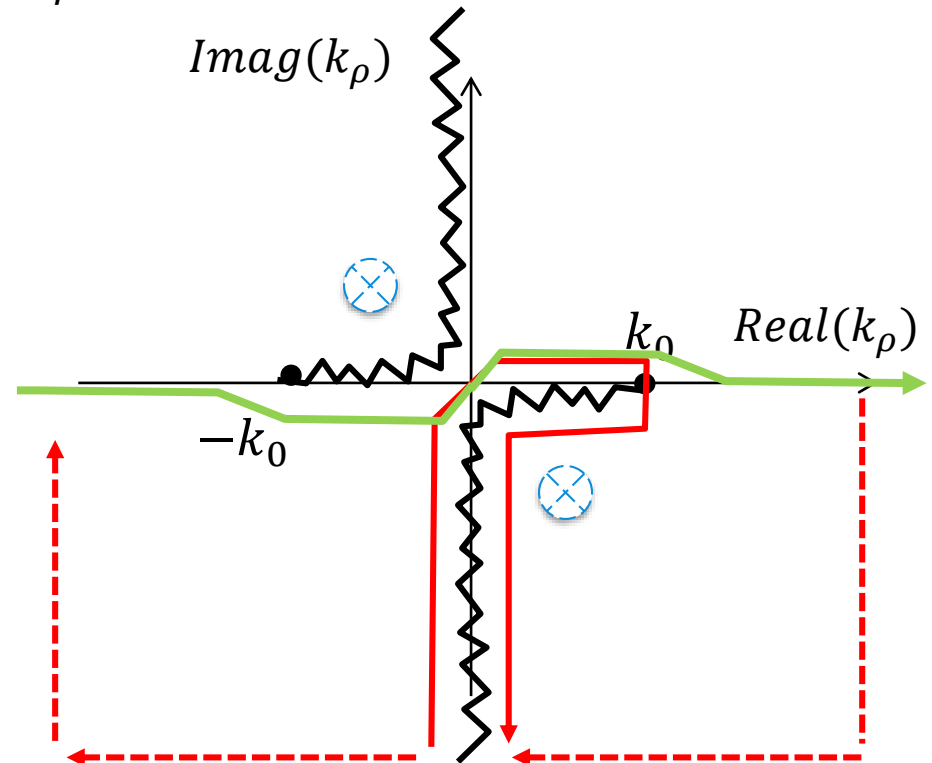
$$k_{\rho lw} = \beta_{lw} - j\alpha_{lw} \quad k_{zlw} = j\sqrt{-(k_0^2 - k_{\rho lw}^2)}$$



Waves that propagate in  
the transverse direction  
with attenuation

However it is captured  
in other deformations:  
near-field!!!

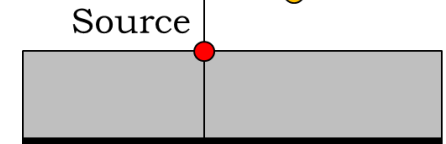
Not captured in the  
deformation around  
the branch



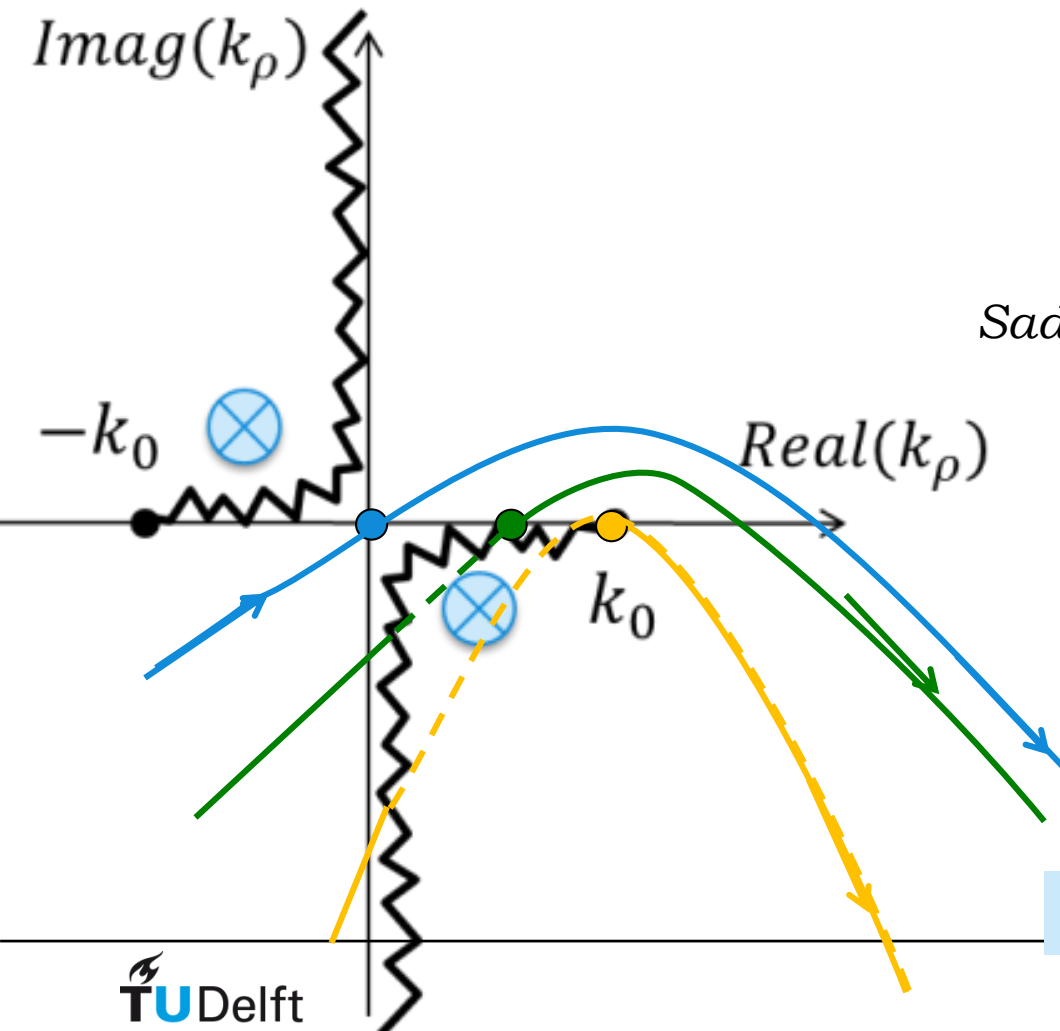
# Near field observation points

For near field **steepest descent path** are the integration paths that guarantee most rapid convergence of the integral

Observation point



Saddle point exist in these paths



$$k_{\rho 1} = k_0 \sin 0 = 0$$

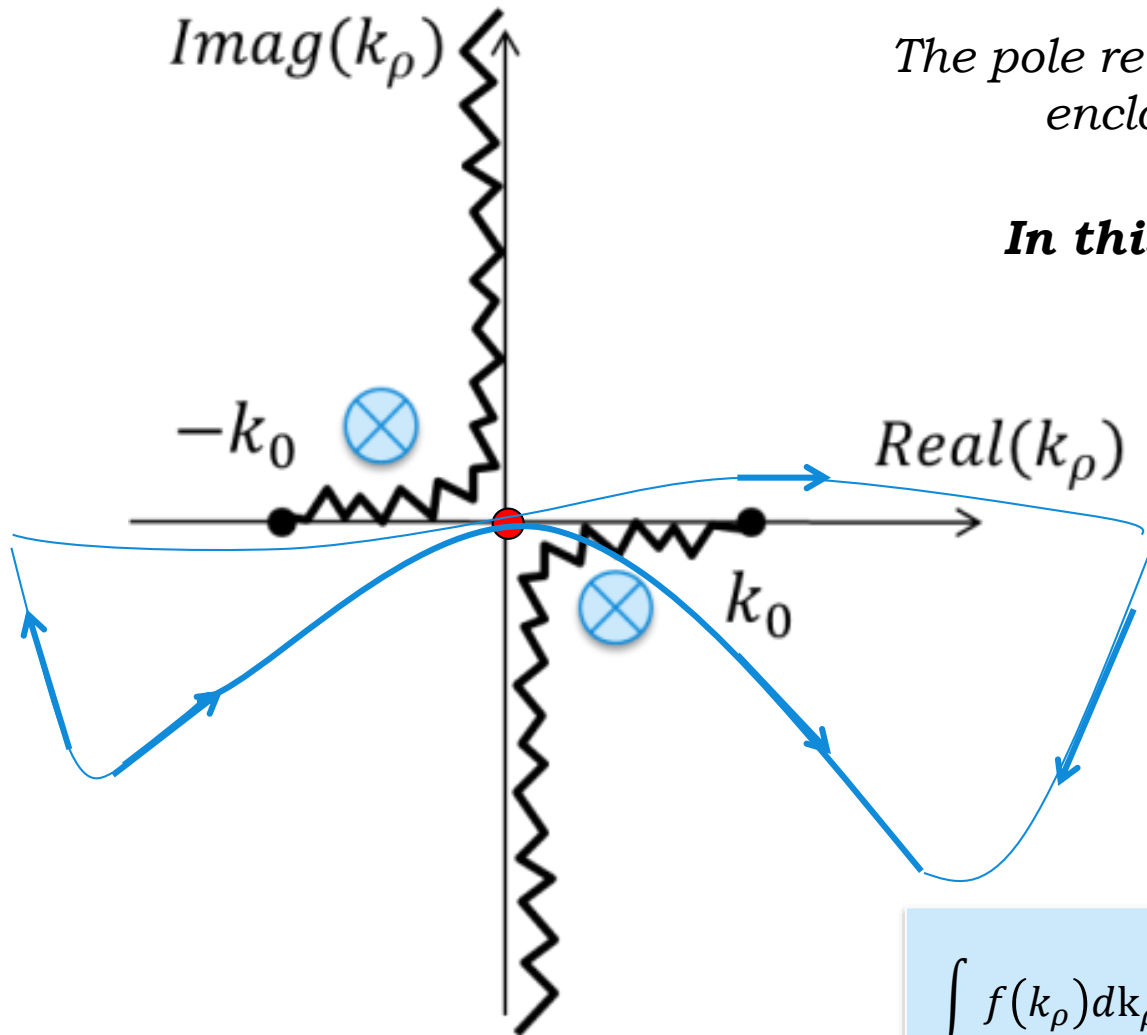
$$k_{\rho 2} = k_0 \sin \frac{\pi}{4} = k_0 / \sqrt{2}$$

$$k_{\rho 3} = k_0 \sin \frac{\pi}{2} = k_0$$

When is the LW pole captured?

# Deformation on SDP

*From the original path to SDP via deformation*

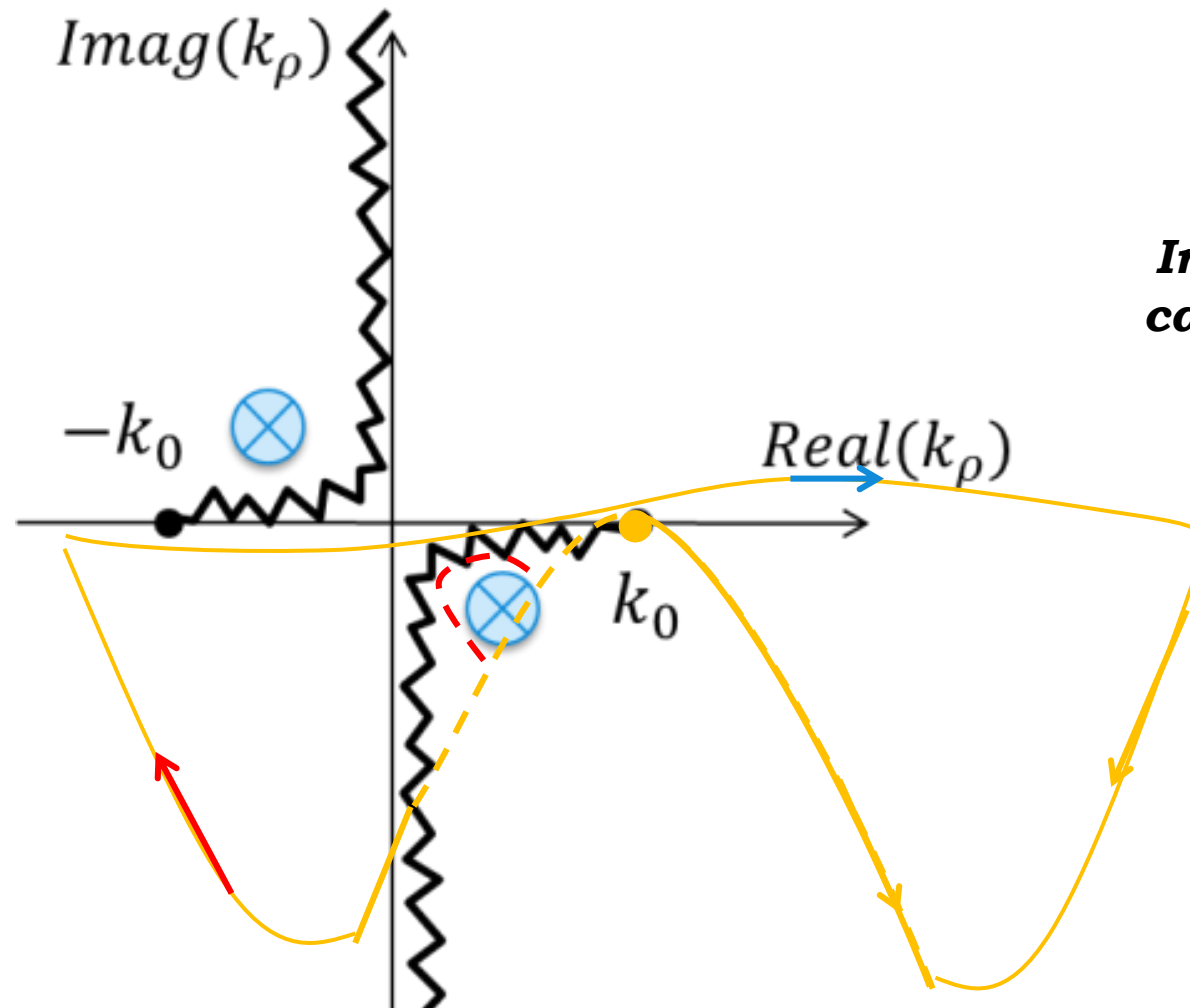


*The pole remains outside the enclosed region*

***In this case no pole is captured***

$$\int_R f(k_\rho) dk_\rho = \int_{SDP} \tilde{G}^{fc}(k_\rho, z, z') e^{-j\vec{k}_\rho \cdot \vec{p}} k_\rho dk_\rho$$

# Near field shaping with leaky waves

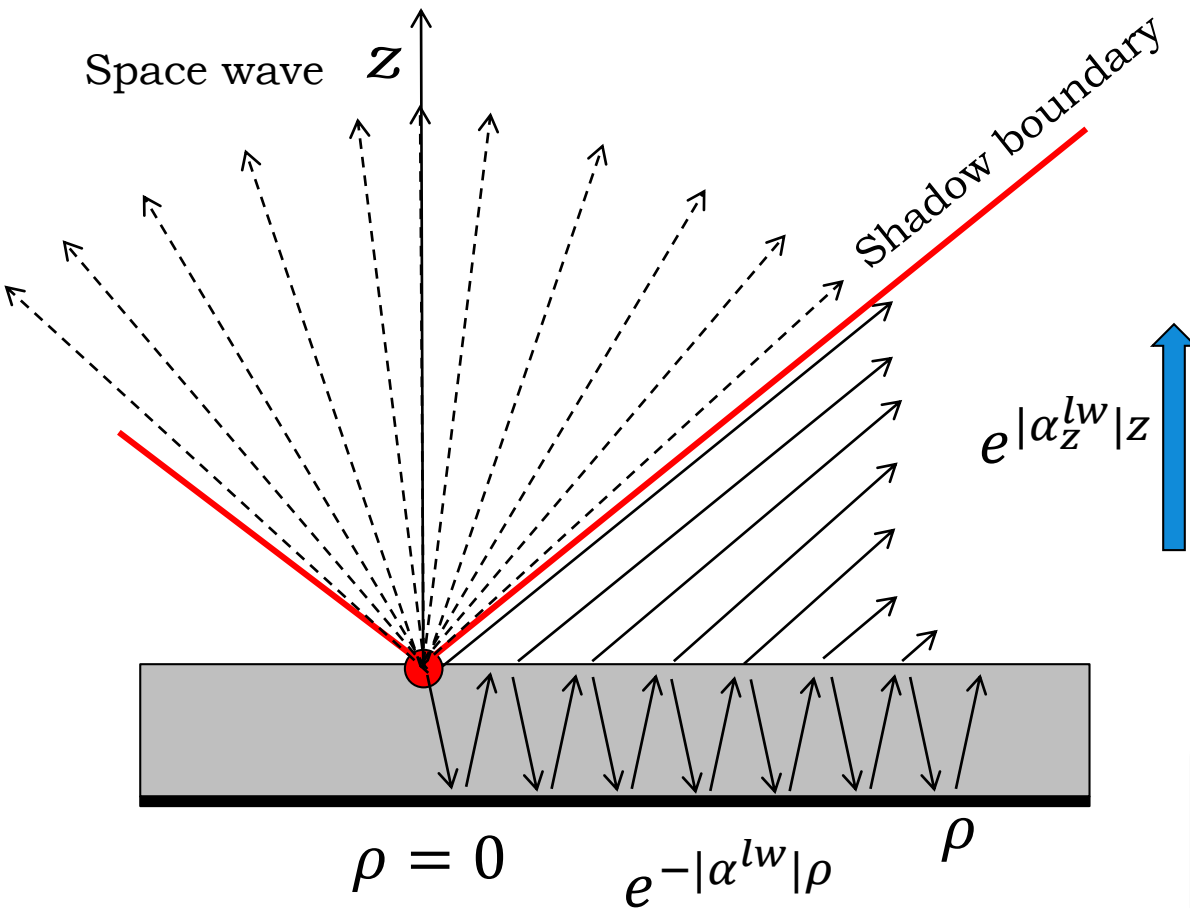


***In this case a pole is capture in the bottom Riemann sheet***

$$\int_R f(k_\rho) dk_\rho = \int_{SDP} \tilde{\mathbf{G}}^{fc}(k_\rho, z, z') e^{-j\vec{k}_\rho \cdot \vec{\rho}} k_\rho dk_\rho - 2\pi j \text{Res}[\tilde{\mathbf{G}}^{fc}(k_\rho, z, z') e^{-j\vec{k}_\rho \cdot \vec{\rho}} k_\rho] \Big|_{k_\rho = k_{\rho i}}$$



# Existence Region



If you take any  $\rho$ , and then increase  $z$  from zero, the wave becomes larger... until it disappears at the shadow boundary

The leaky wave field contribution is in fact limited to a region of existence.

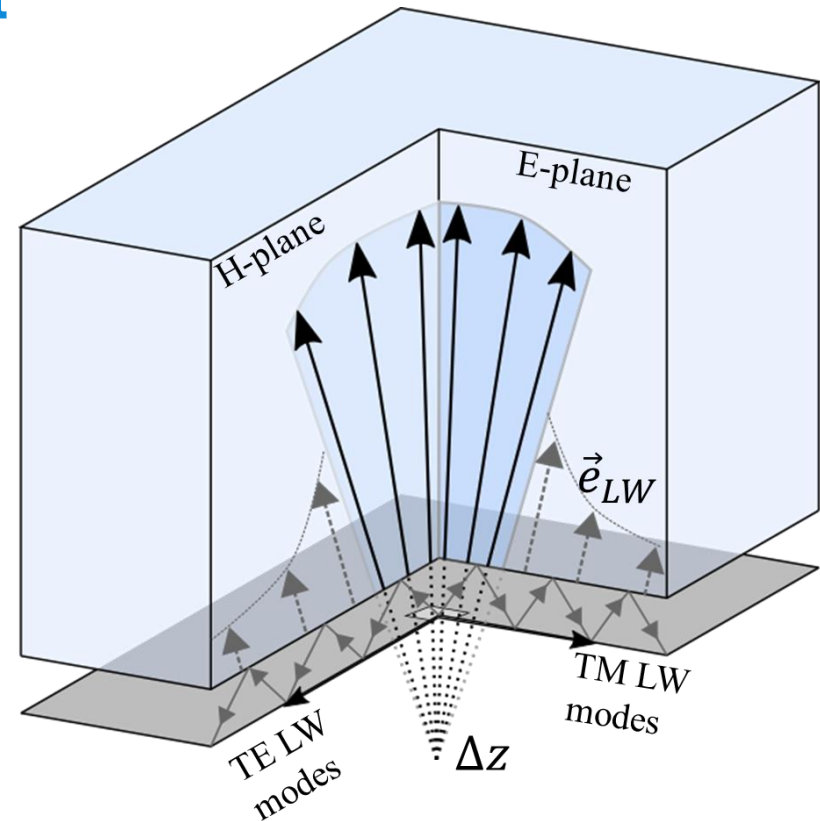
If you take  $z = 0$ , and then increase  $\rho$  from zero, the wave attenuates exponentially to zero

# Existence Region

The leaky wave pole is only captured for observation points near the source

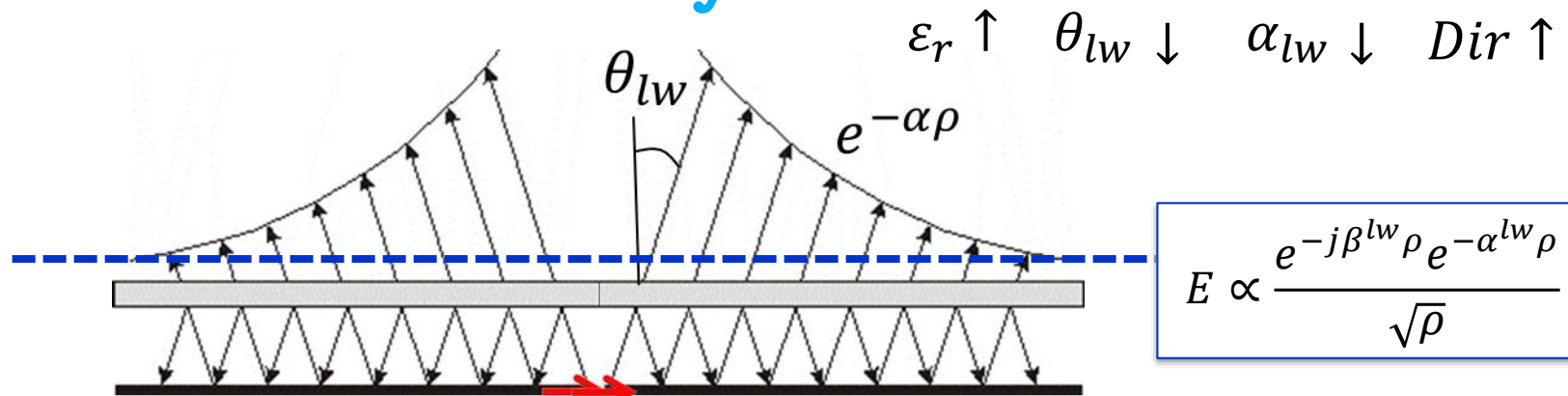
The residue contribution does not arrive to infinity

But it is responsible to the power transferred to the far field of the antenna: **modulation of the SFG in the region around  $\theta_{lw}^0$**



They can be used to do pattern shaping!

# Enhanced Directivity

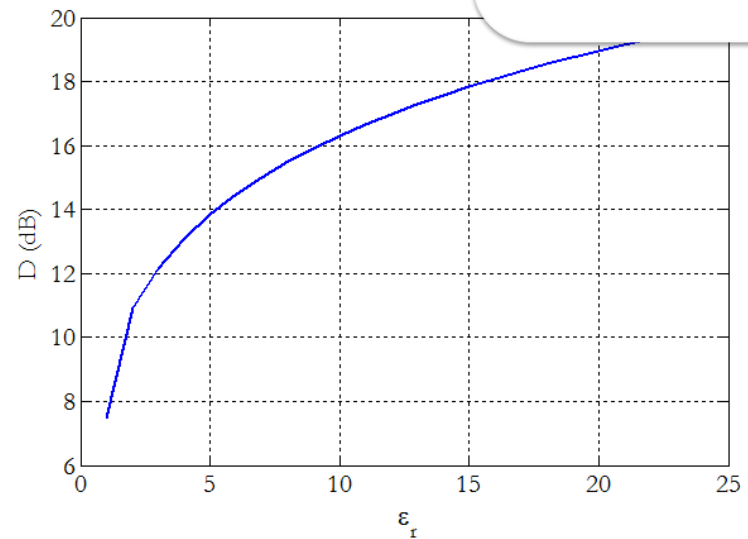
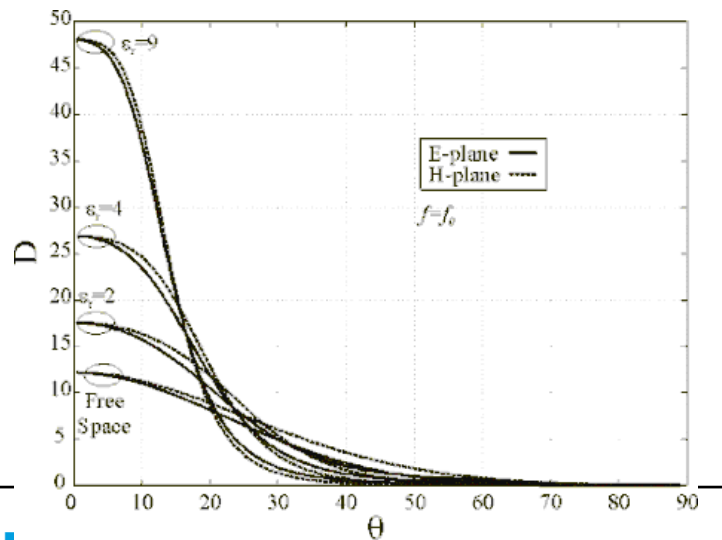


$$A_{eff} \propto \frac{1}{\alpha^2} \gg A_{source}$$

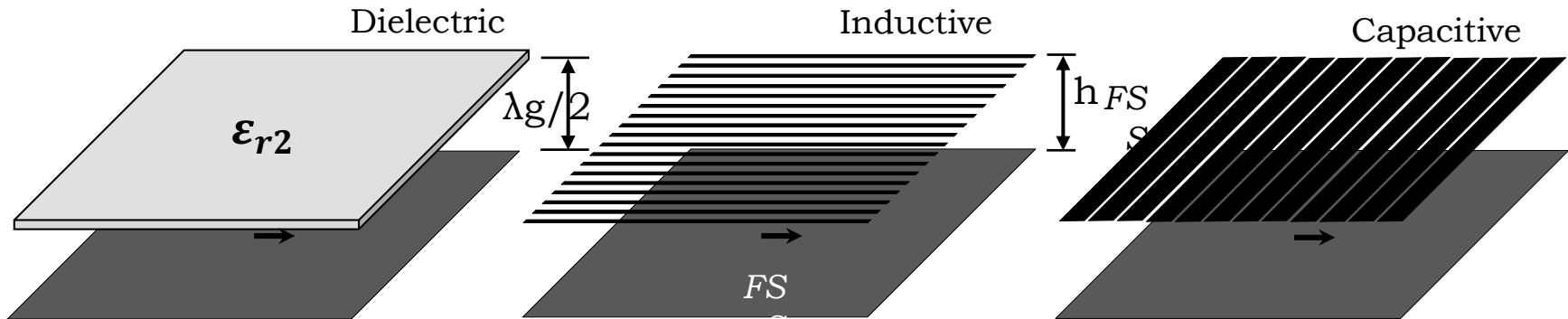
Resonant condition:

$$h = \frac{\lambda_0}{2 \cos \theta_0}$$

$$h_d = \frac{\lambda_{d2}}{4 \cos \theta_d}$$



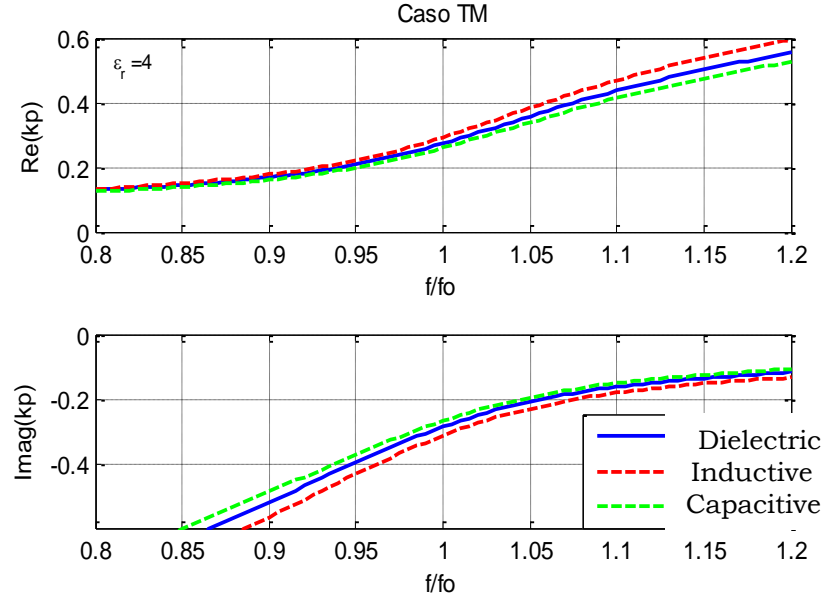
# Partially Reflecting Surfaces



Partially Reflecting Surface

Leaky wave antenna

FSS and Dielectric are equivalent for broadside radiation



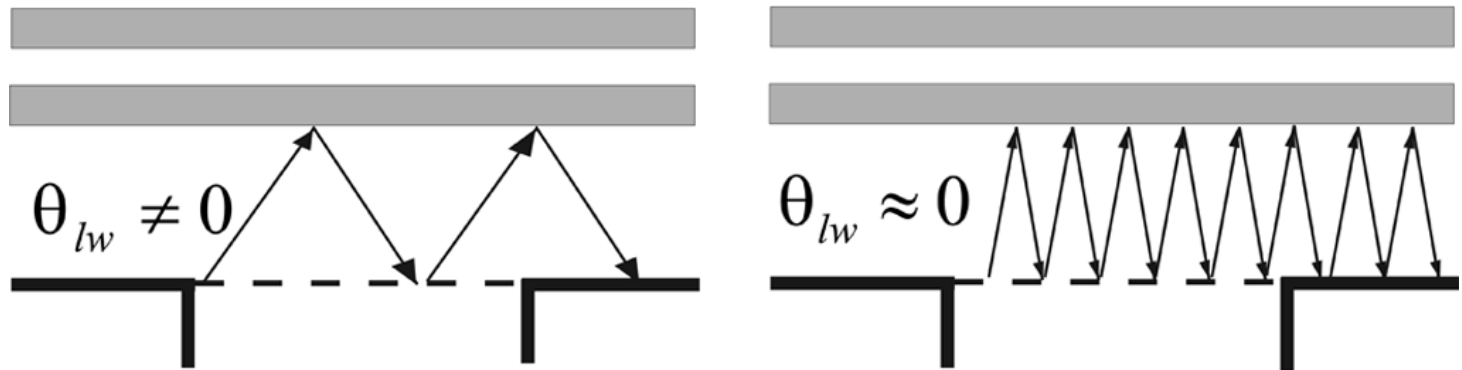
# Bandwidth Issues

Input Admittance:

$$Y_{in} = \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} |M(k_x, k_y)|^2 G^{hm}(k_x, k_y) dk_x dk_y$$

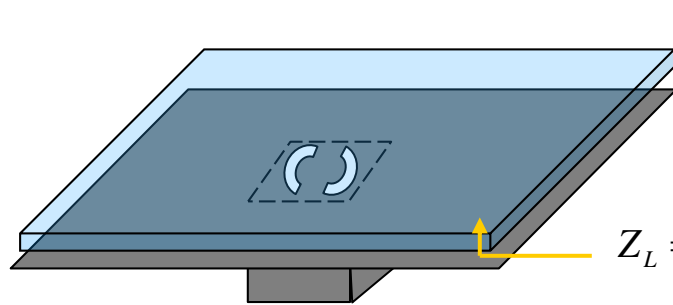
Highly dominated by the presence of the leaky wave poles  $\text{Res}(G^{hm}(k_x, k_y)) \propto \frac{1}{\sin\theta_{lw}}$

**BW**  $\longleftrightarrow$   $\frac{Y^{ebg}(f_1)}{Y^{ebg}(f_2)} \approx \frac{k_{lw}(f_2)}{k_{lw}(f_1)}$



$\epsilon_r \uparrow \quad \theta_{lw} \downarrow \quad \alpha_{lw} \downarrow \quad Dir \uparrow \quad BW \downarrow$

# Leaky Wave Lens Feed

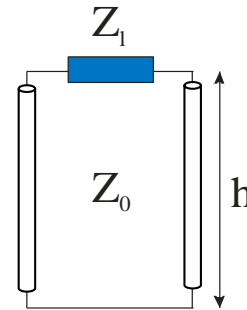


$\lambda d/4$  Quartz  
Super/layer

$$Z_L = \frac{\zeta_0}{\epsilon_r} = \frac{\zeta_0}{3.8}$$

Waveguide Feed

$h = \lambda_0/2$  Cavity

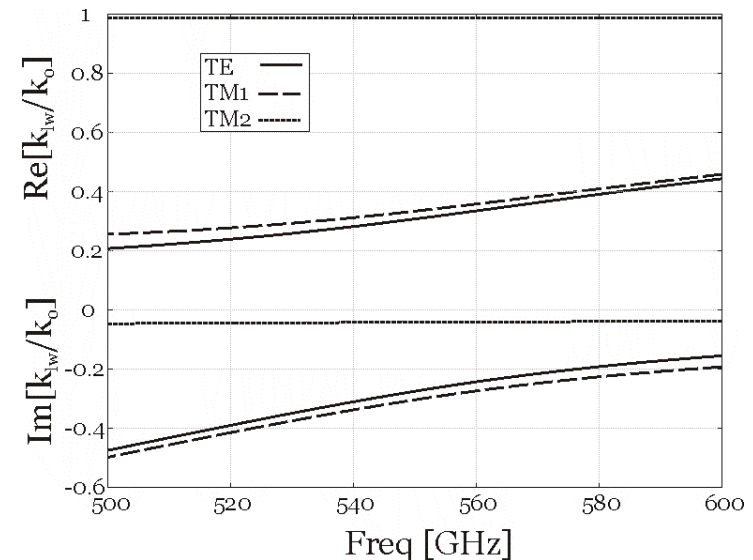
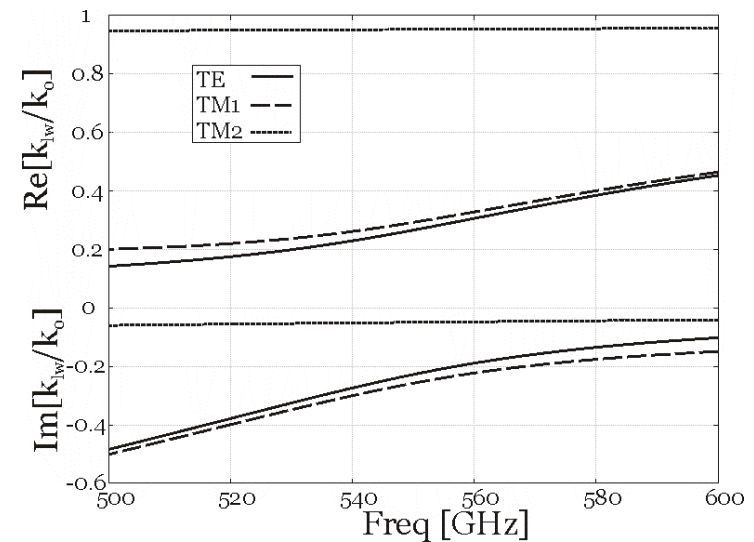


Infinite  
Silicon  
Dielectric

10/2

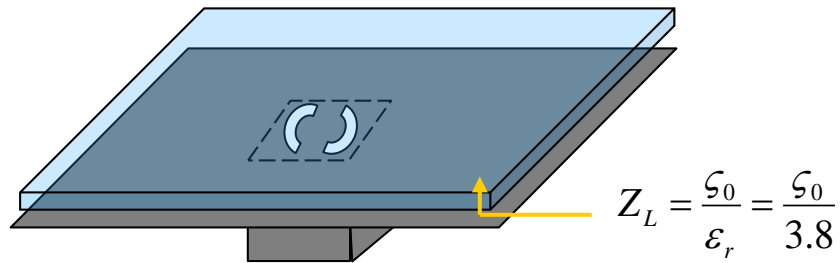
$$Z_L = \frac{\zeta_0}{\sqrt{\epsilon_r}} = \frac{\zeta_0}{3.45}$$

**A leaky-wave/EBG cavity can  
be used to illuminate a lens**



Same dispersion diagram than  
quarter-wavelength superstrates

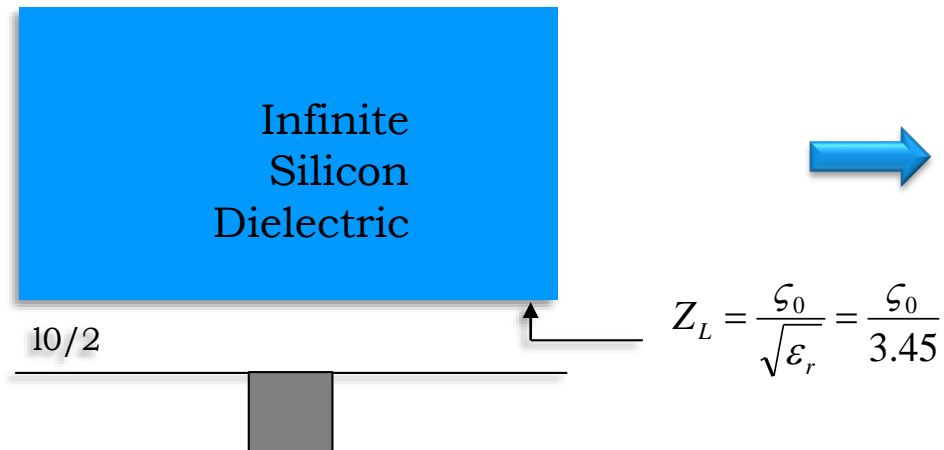
# Leaky Wave Lens Feed



$$D = 4\pi A_{ef} / \lambda_0^2$$

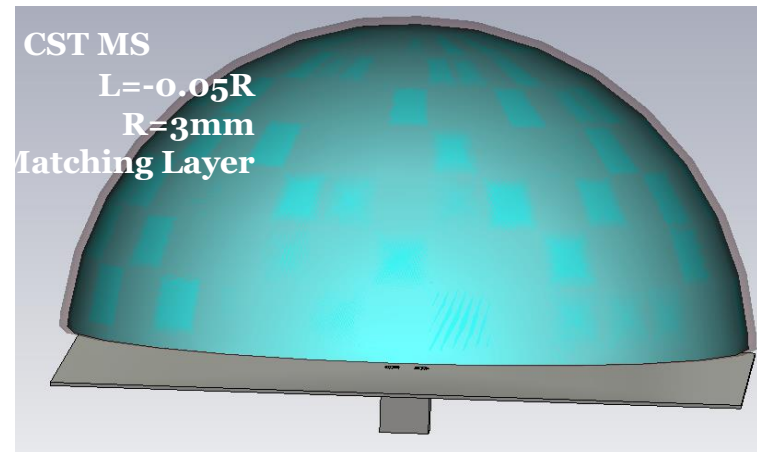
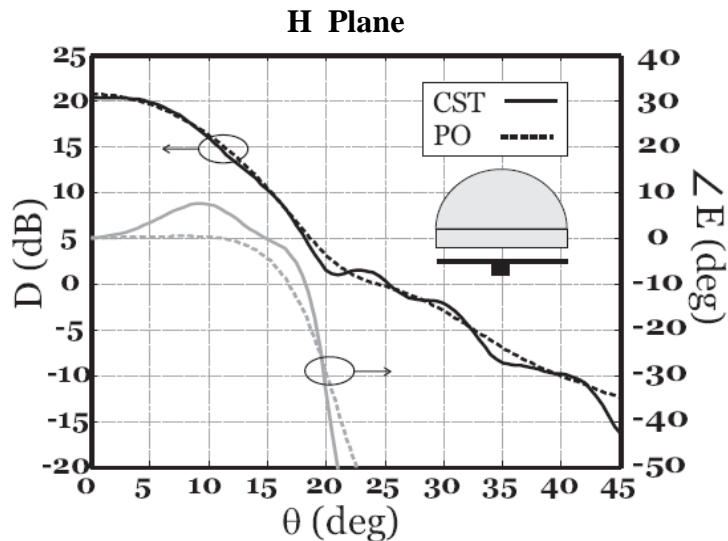
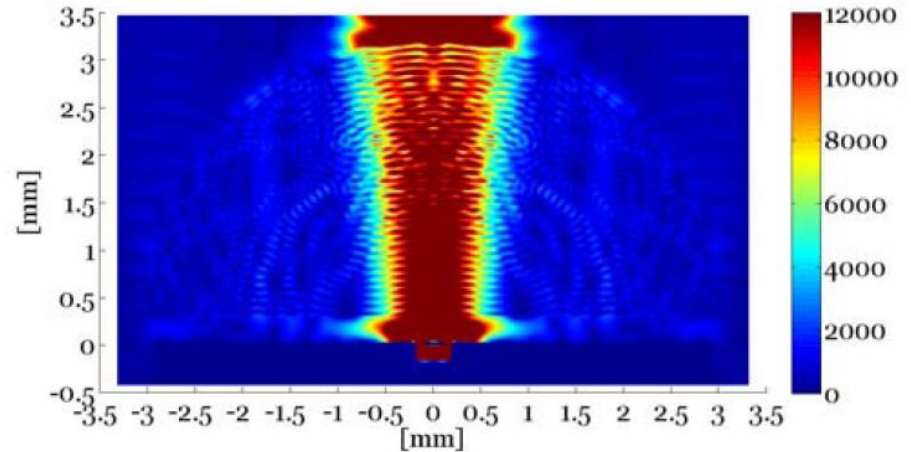
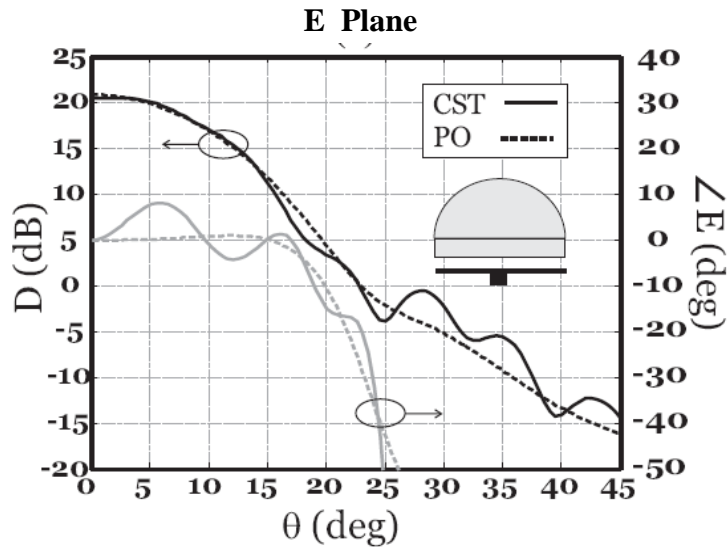
Same effective area given by the same leaky wave mode

$\epsilon_r$  higher directivity for the same bandwidth or leaky wave frequency dispersion



$$D = 4\pi A_{ef} / \lambda_d^2 = 4\pi A_{ef} \epsilon_r / \lambda_0^2$$

# Lens Radiated Fields in standard lenses



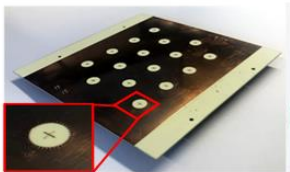


# Examples of Lens LW Prototypes

## 30GHz HDPE/PCB



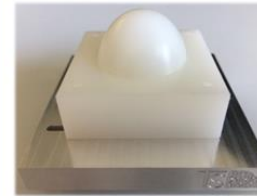
Scanning Lens  
Phased Array



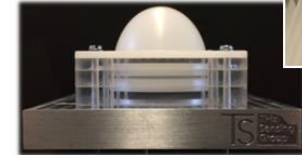
**DEMO**  
www.demo.tudelft.nl

## 140GHz-220GHz HDPE/TOPAS Prototypes

High-gain  
wideband LW  
lens antennas

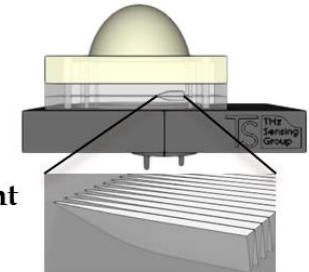


Circular  
polarization



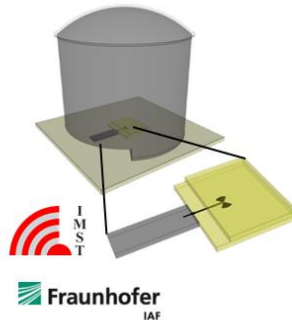
**DEMO**  
www.demo.tudelft.nl

Steering  
enhancement



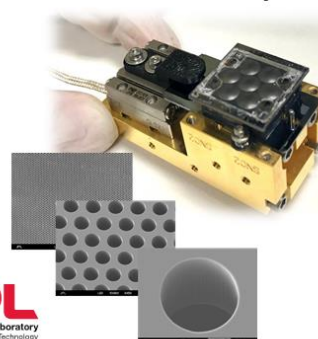
## 300&500GHz Silicon Prototypes

In-package wideband  
LW lens antenna



**Fraunhofer**  
IAF

Scanning Lens  
Phased Array

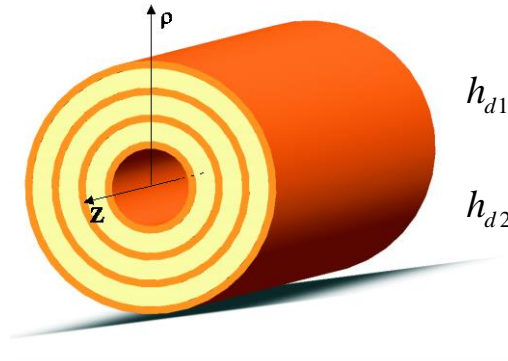


**JPL**  
Jet Propulsion Laboratory  
California Institute of Technology



# Waveguide Applications

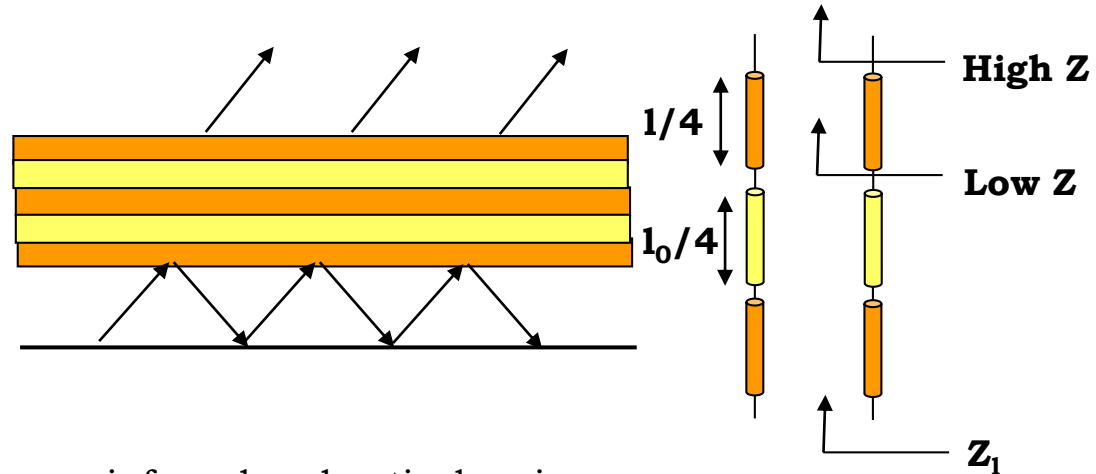
## Bragg Fibers



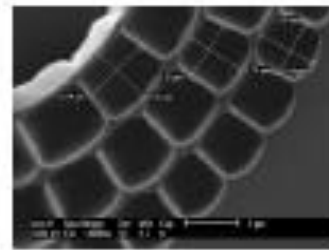
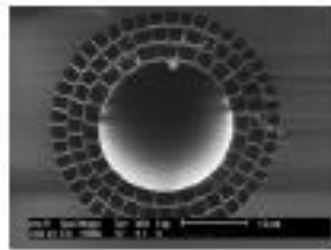
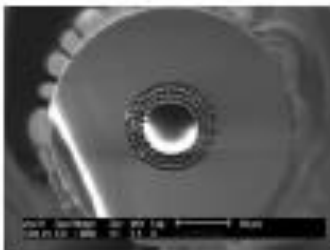
$$h_{d1} = \frac{\lambda_{d1}}{4 \cos \theta_{d1}}$$

$$h_{d2} = \frac{\lambda_{d2}}{4 \cos \theta_{d2}}$$

Same Resonant condition than  
leaky wave antenna:



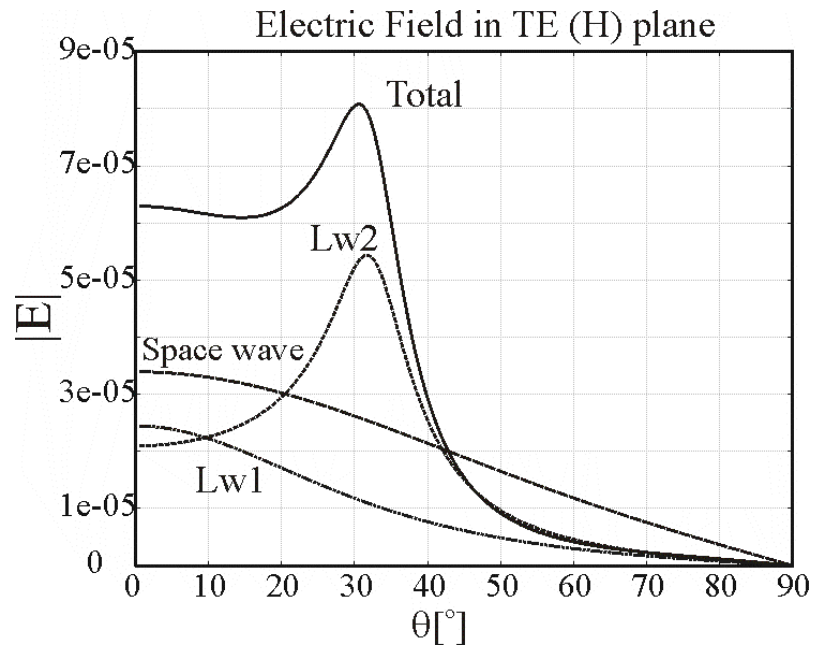
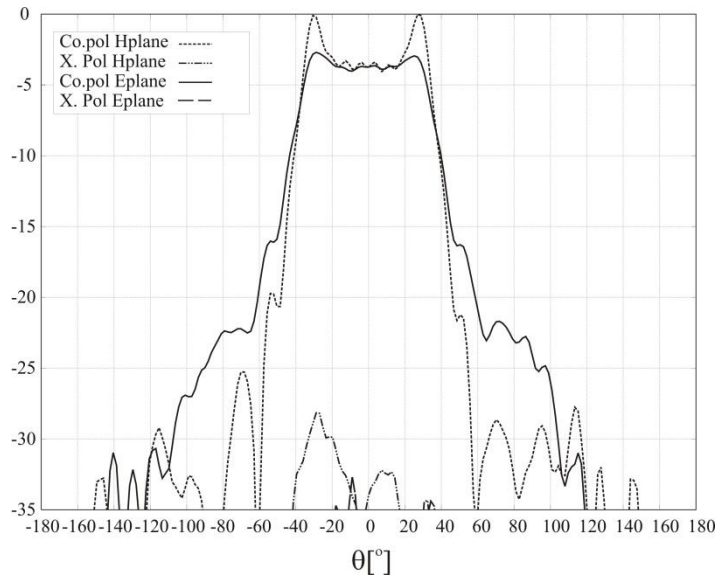
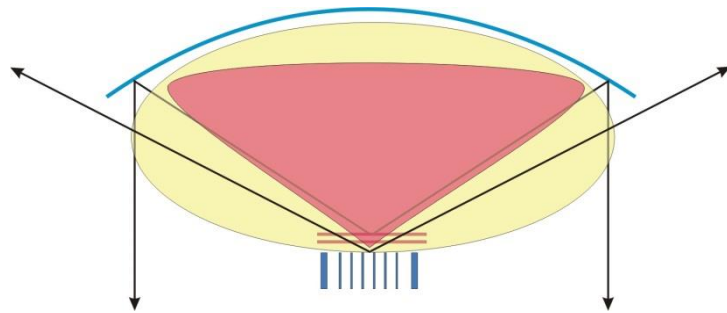
- They are used at near infrared and optical regimes.
- Field distribution similar than a circular waveguide



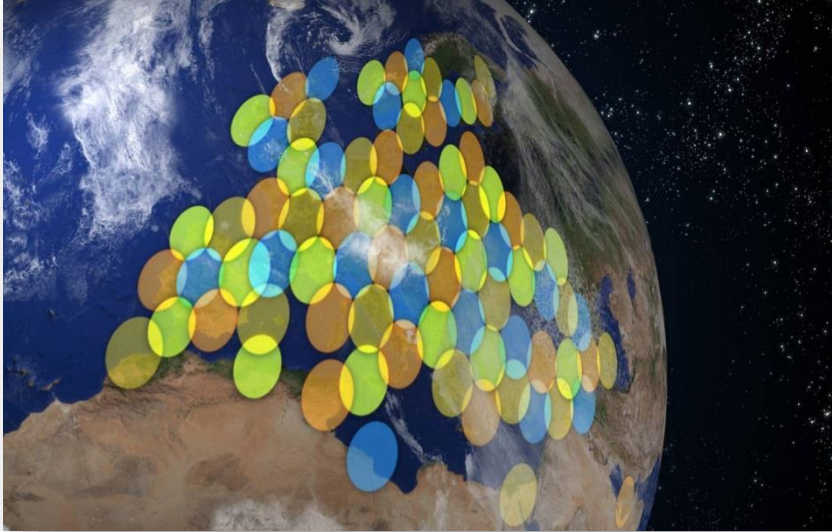
Loss in the  
order of  
dB/km

# Field Shaping Application

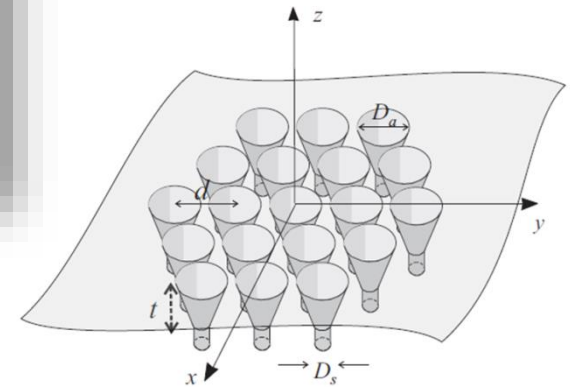
Illumination of reflector  
with 90% efficiency



# Multi Beam Imaging for Telecom



Ka Band  
Telecommunication  
Applications  
Earth Coverage by using  
Multiple Antenna Beams



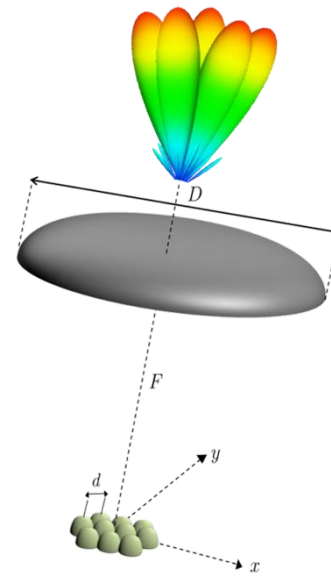
## Technological Solutions

Use of reflectors for high directivity in combination with

- 1) Phased Arrays generating multiple beams
- 2) Focal Plane Arrays with Single Fed per Beam

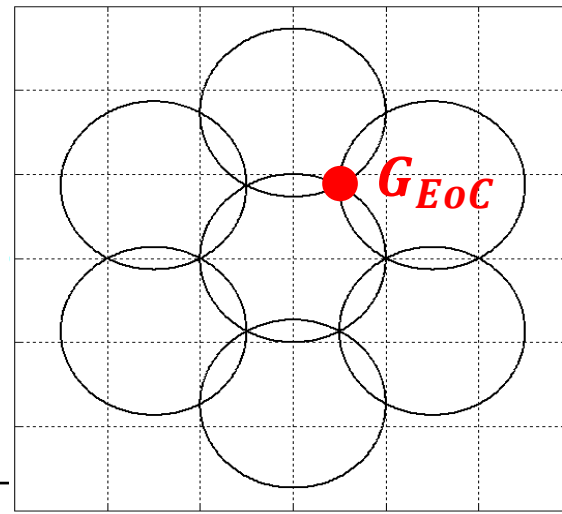
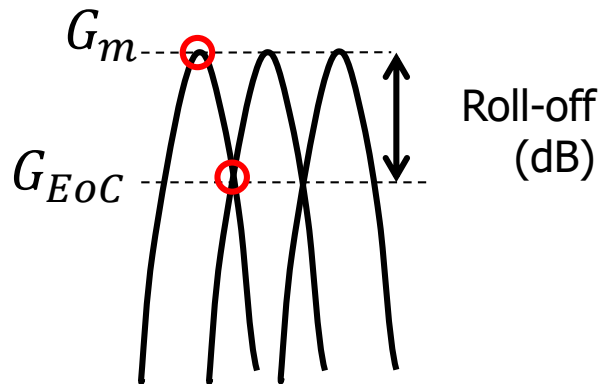
# Edge of Coverage Gain

The goal is to maximize the gain at the edge of coverage and keep the inter-beam interferences low



The best packaging is achieved for a hexagonal array

Far Field Beams

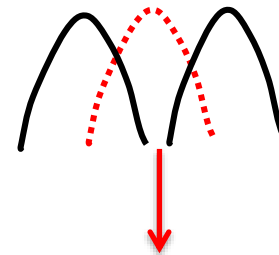
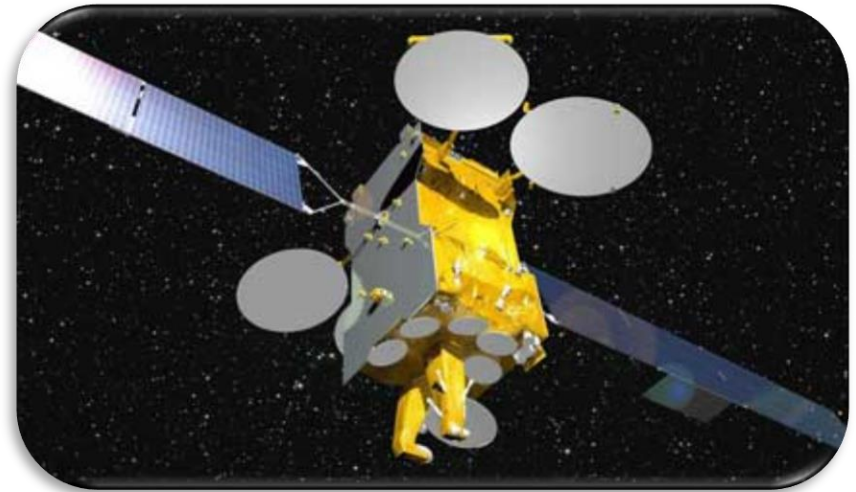
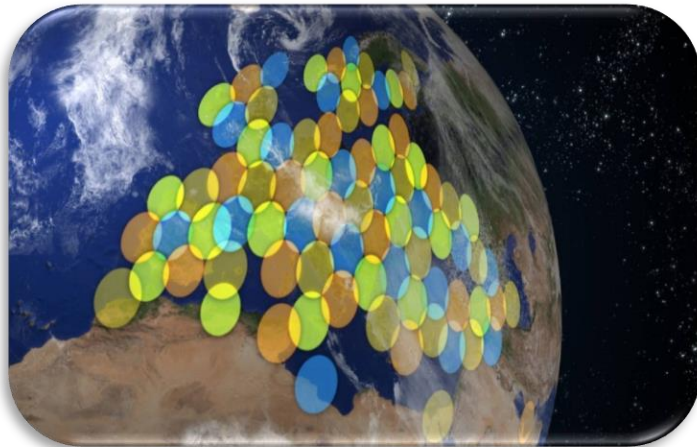




# Standard Solution for ka Satellites

Use of 4 reflectors to increase the Geoc with  $2\lambda_0 f_{\#}$  FPA

Diminish roll-off without increasing spill over

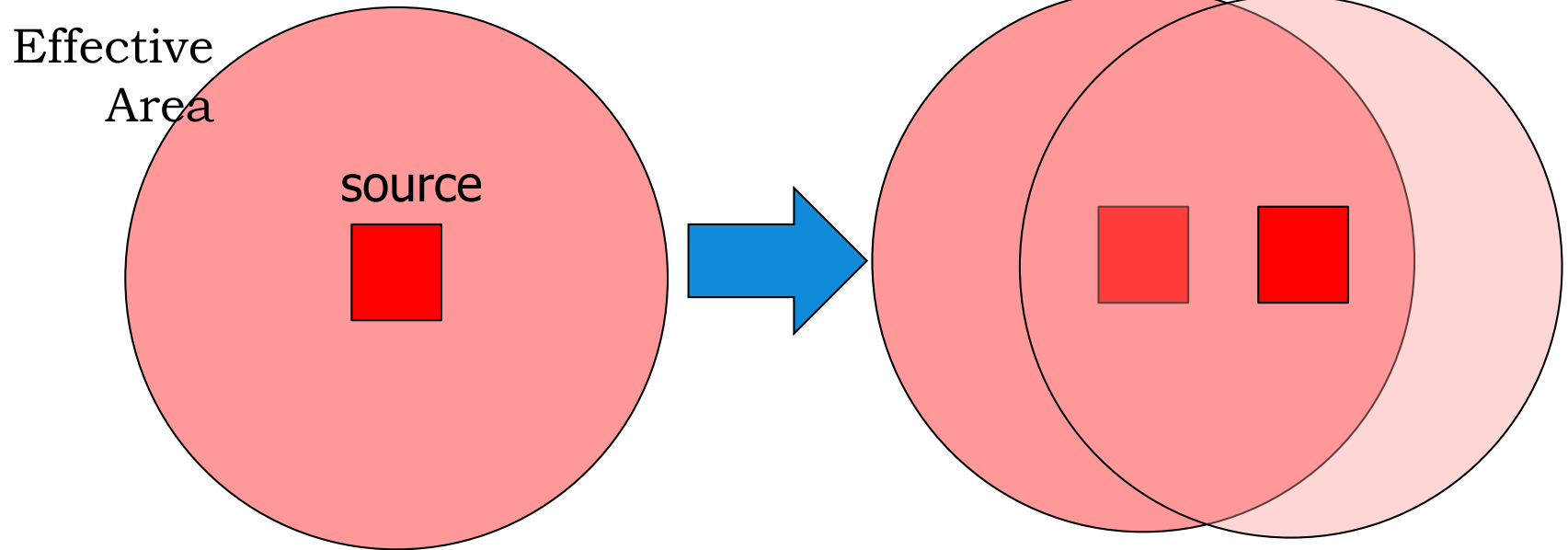


Beam generated by another reflector

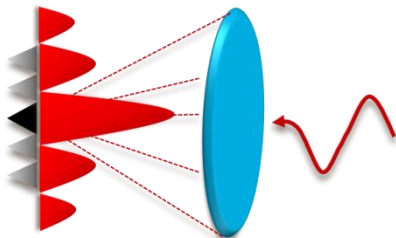
$$G_{EOC} \approx D_m + SO + RO = D_m + 3.5dB$$

# Overlapped Feeds

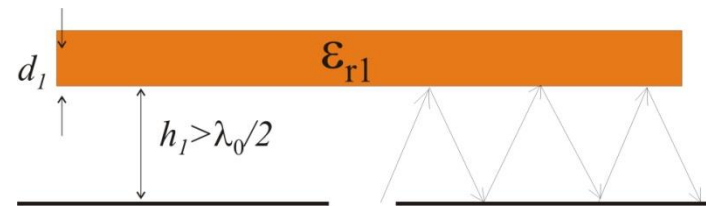
Is it possible to place them  
closer in the FPA?



Use of Phased arrays:  
Coherent beamforming of  
3x3 or 5x5 sub arrays



Leaky wave/Fabry Perot Antennas



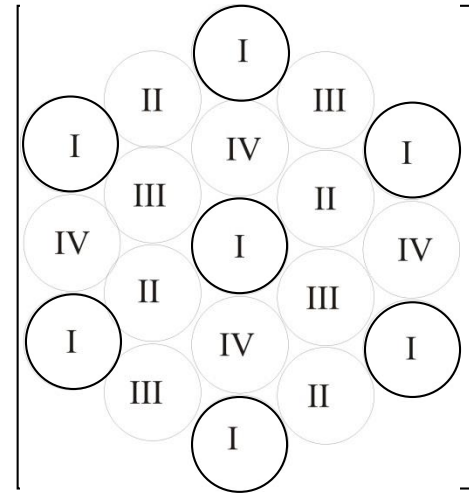
Antennas with directive patterns  
and 'small physical areas'

# Overlapped Feeds FPA

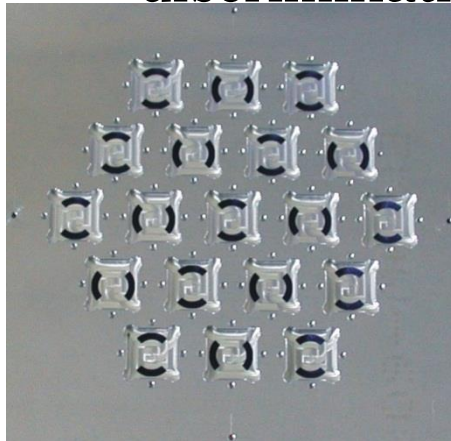
Loss due to the mutual coupling



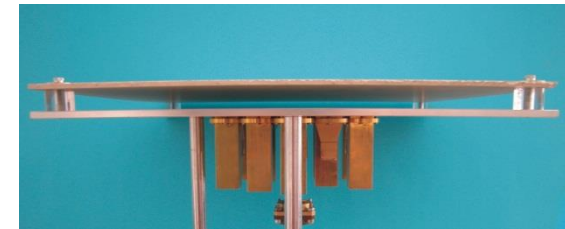
Use of reactive loading to compensate for it: **Equivalent short circuits loads** for neighboring waveguides



1) Polarization discrimination

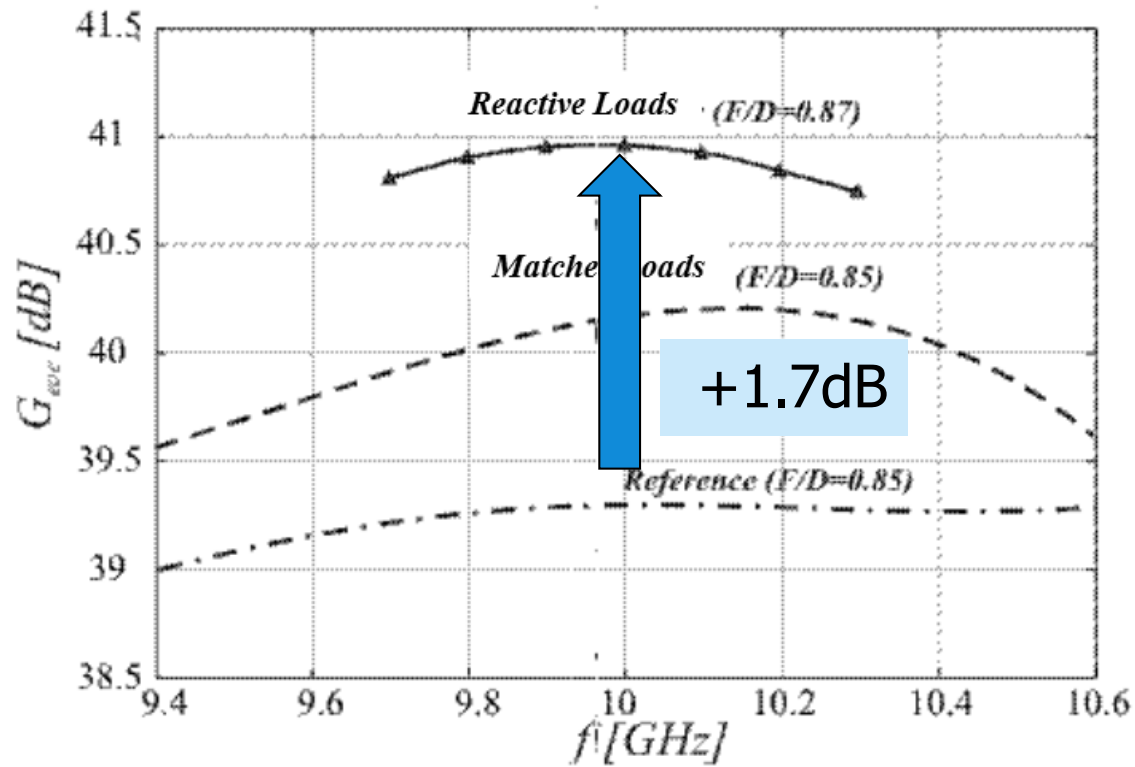


2) Frequency discrimination





# Reactively Loaded case



+1.7dB

10% Bandwidth

# Important Points

Stratified GFs contain poles on bottom Riemann sheet

They correspond to leaky waves

These poles are complex and located closed to the real axes on the bottom Riemann sheet

These poles are not captured in deformations around branch, but in other deformations useful to obtain analytic evaluations of the near field

In the near field, the leaky waves are cylindrical waves, similar to surface waves but with an attenuation associated to radiation

Leaky waves can be used to shape the far field of small sources

# Related IEEE Papers on LWAs

- D. R. Jackson and A. A. Oliner, “A leaky-wave analysis of the high-gain printed antenna configuration,” IEEE Transactions on Antennas and Propagation, vol. 36, no. 7, pp. 905–910, Jul. 1988, doi: 10.1109/8.7194.
- G. Lovat, P. Burghignoli, and D. R. Jackson, “Fundamental properties and optimization of broadside radiation from uniform leaky-wave antennas,” IEEE Transactions on Antennas and Propagation, vol. 54, no. 5, pp. 1442–1452, May 2006, doi: 10.1109/TAP.2006.874350
- R. Gardelli, M. Albani, and F. Capolino, “Array thinning by using antennas in a Fabry-Pérot cavity for gain enhancement,” IEEE Trans. Antennas Propag., vol. 54, no. 7, pp. 1979–1990, Jul. 2006
- A. Neto, N. Llombart, G. Gerini, M. D. Bonnedal, and P. de Maagt, “EBG Enhanced Feeds for the Improvement of the Aperture Efficiency of Reflector Antennas,” IEEE Transactions on Antennas and Propagation, vol. 55, no. 8, pp. 2185–2193, Aug. 2007,
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