

# The Art of Electronics Solutions

May 31, 2023

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**Exercise.** This is the solution to exercise 1.1 in the book.

**Solution.** Series

$$R = R_1 + R_2 = 15 \text{ k}\Omega$$

Parallel

$$R = \frac{R_1 R_2}{R_1 + R_2} = 3.33 \text{ k}\Omega$$

---

**Exercise.** This is the solution to exercise 1.2 in the book.

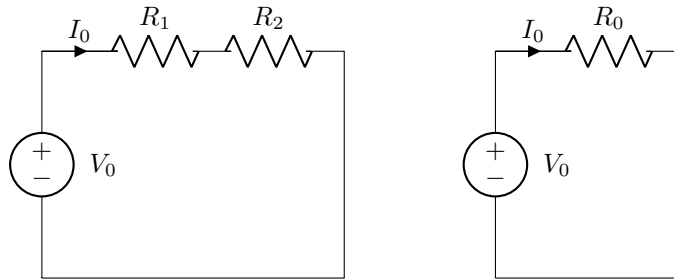
**Solution.**

$$P = UI = U \frac{U}{R} = \frac{144 \text{ V}^2}{1 \text{ }\Omega} = 144 \text{ W}$$

---

**Exercise.** This is the solution to exercise 1.3 in the book.

**Solution.** For series resistors we have the equivalent circuits:



We have that:

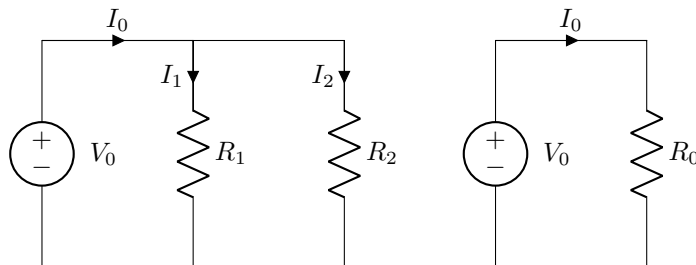
$$\begin{aligned} V_1 &= I_0 R_1 \\ V_2 &= I_0 R_2 \\ V_0 &= V_1 + V_2 \end{aligned}$$

and

$$V_0 = I_0 R_0$$

Therefore:

$$R_0 = R_1 + R_2$$



$$\begin{aligned} V_0 &= I_1 R_1 \\ V_0 &= I_2 R_2 \\ I_0 &= I_1 + I_2 \end{aligned}$$

and

$$V_0 = I_0 R_0$$

hence:

$$\frac{V_0}{R_0} = \frac{V_0}{R_1} + \frac{V_0}{R_2}$$

or

$$\frac{1}{R_0} = \frac{1}{R_1} + \frac{1}{R_2}$$

These can be generalized to more than two resistors by taking two at a time and reducing to one.

---

**Exercise.** This is the solution to exercise 1.4 in the book.

**Solution.** The expression can be re-written as:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

Using the result in [exercise 1.3](#) we can replace  $R_1$  and  $R_2$  with  $R_{12}$  to get:

$$\frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2}$$

We can then replace  $R_{12}$  and  $R_3$  with  $R_{123}$ . Thus:

$$\frac{1}{R_{123}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Extending this up to more than three resistors we thus get:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

---

**Exercise.** This is the solution to exercise 1.5 in the book.

**Solution.** When the resistor shorts the voltage source the current is:

$$I = \frac{V}{R} = 15 \text{ mA}$$

The power dissipated in the resistor is:

$$P = VI = 15 \times 15 \cdot 10^{-3} \text{ W} = 0.225 \text{ W}$$

Hence, power rating will not be exceeded.

---

**Exercise.** This is the solution to exercise 1.6 in the book.

**Solution.** (a) In order to deliver  $10^{10}$  W of power at 115 V the DC current through the cable must be:

$$I = \frac{P}{U} = \frac{10^{10}}{115} = 86.956 \cdot 10^6 \text{ A}$$

The  $I^2R$  losses are then:

$$P_{loss} = (86.956 \cdot 10^6 \text{ A})^2 \cdot 5 \cdot 10^{-8} \frac{\Omega}{\text{ft}} = 378.071 \cdot 10^6 \frac{\text{W}}{\text{ft}}$$

(b) The length is:

$$L = \frac{10^{10}}{378.071 \cdot 10^6} = 26.45 \text{ ft}$$

This is obviously not practical.

(c) From Stefan-Boltzmann law:

$$P = \sigma AT^4$$

where  $A$  is the area of the radiating surface. In this case  $A = \pi DL$  with  $D = 1$  ft and  $L$  as determined above. We get:

$$A = 74.785 \cdot 10^3 \text{ cm}^2$$

Using the provided value for  $\sigma$  and the  $10^{10}$  W of power we get:

$$T^4 = 2.23 \cdot 10^{16} \text{ K}^4$$

or

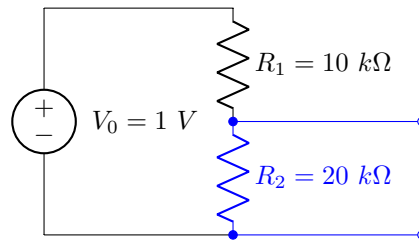
$$T = 12220 \text{ K}$$

which is also not practical.

The solution to this is to transmit power to the city at high voltage. In that case the current is much reduced and  $I^2R$  losses are also much smaller.

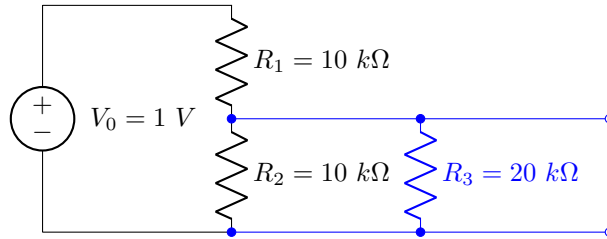
**Exercise.** This is the solution to exercise 1.7 in the book.

**Solution.** Remember that the meter will determine the voltage using the current thru the  $20\text{k}\Omega$  resistor internal to the meter. Hence, when measuring the voltage the following equivalent circuit is exercised, with the part in blue corresponding to the internals of the meter:



We then have:

$$V_{out} = \frac{R_2}{R_1 + R_2} V_0 = \frac{20 \text{ k}\Omega}{30 \text{ k}\Omega} \cdot 1 \text{ V} = 0.67 \text{ V}$$



In this case:

$$R_{23} = \frac{R_2 R_3}{R_2 + R_3} = 6.67 \text{ k}\Omega$$

and

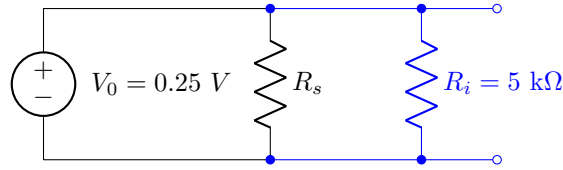
$$V_{out} = \frac{R_{23}}{R_1 + R_{23}} V_0 = \frac{6.67 \text{ k}\Omega}{16.67 \text{ k}\Omega} \cdot 1 \text{ V} = 0.40 \text{ V}$$

**Exercise.** This is the solution to exercise 1.8 in the book.

**Solution.** Without the shunt resistance the voltage drop across the meter is:

$$V_m = I_m \cdot R_m = 50 \mu\text{A} \cdot 5 \text{ k}\Omega = 0.25 \text{ V}$$

To change the scale of the movement from  $(0,50) \mu\text{A}$  to  $(0,1) \text{ A}$ , a shunt resistor must be installed between the meter terminals:



The current through the circuit must be 1 A, therefore the equivalent resistance must be:

$$R_e = \frac{V_0}{I_0} = \frac{0.25 \text{ V}}{1 \text{ A}} = 0.25 \Omega$$

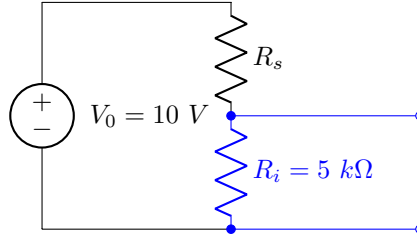
The equivalent resistance is:

$$\frac{1}{R_e} = \frac{1}{R_s} + \frac{1}{R_i}$$

$$\frac{1}{R_s} = \frac{1}{R_e} - \frac{1}{R_i}$$

Therefore the shunt resistance is:

$$R_s = \frac{R_i R_e}{R_i - R_e} = \frac{5000 \cdot 0.25}{5000 - 0.25} = 0.25 \Omega$$



The current thru the resistors must be the full scale movement current  $I_0 = 50 \mu\text{A}$  when the voltage is  $V_0 = 10 \text{ V}$ . Therefore:

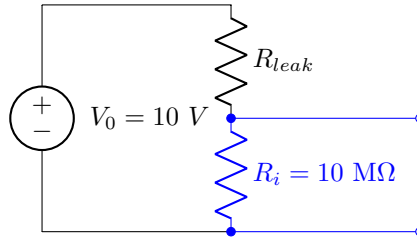
$$R_e = R_i + R_s = \frac{V_0}{I_0} = 200 \text{ k}\Omega$$

Therefore the series resistance is:

$$R_i = R_e - R_s = 195 \text{ k}\Omega$$

**Exercise.** This is the solution to exercise 1.9 in the book.

**Solution.** The measurement can be achieved with the following circuit:



When connecting the 10 V source to the leakage sink the current is:

$$I_{leak} = \frac{10 \text{ V}}{(R_{leak} + R_i) \Omega} = \frac{10}{R_{leak} + 1 \cdot 10^7} \text{ A}$$

The voltage measurement on the meter's 2V scale is in this case:

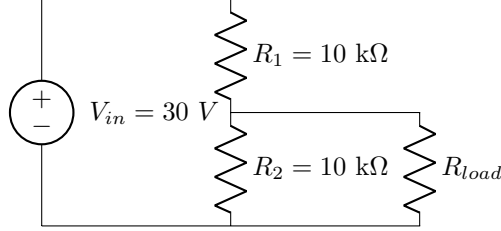
$$V_{leak} = I_{leak} \cdot R_i = \frac{1 \cdot 10^8}{R_{leak} + 1 \cdot 10^7} \text{ V}$$

For example if  $R_{leak} = 1000 \text{ M}\Omega$  then:

$$V_{leak} = \frac{1 \cdot 10^8}{1 \cdot 10^9 + 1 \cdot 10^7} \text{ V} = 0.099 \text{ V}$$

**Exercise.** This is the solution to exercise 1.10 in the book.

**Solution.**



(a) The open circuit voltage is:

$$V_{open} = \frac{R_2}{R_1 + R_2} V_{in} = 0.5 \cdot 30 \text{ V} = 15 \text{ V}$$

(b) We have the equivalent resistor of  $R_2$  and  $R_{load}$  as:

$$R_{out} = \frac{R_2 R_{load}}{R_2 + R_{load}} = \frac{100}{20} \text{ k}\Omega = 5 \text{ k}\Omega$$

$$V_{out} = \frac{R_{out}}{R_1 + R_{out}} V_{in} = \frac{5}{10 + 5} \cdot 30 \text{ V} = 10 \text{ V}$$

(c) The open circuit/Thévenin voltage is:

$$V_{Th} = \frac{R_2}{R_1 + R_2} V_{in}$$

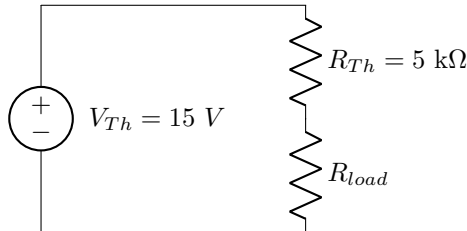
The short-circuit current is:

$$I_{sc} = \frac{V_{in}}{R_1}$$

and the Thévenin resistance is:

$$R_{Th} = \frac{V_{Th}}{I_{sc}} = V_{Th} \frac{R_1}{V_{in}} = \frac{R_1 R_2}{R_1 + R_2} = 5 \text{ k}\Omega$$

Hence the equivalent circuit is:



(d) Using the Thévenin equivalent circuit from (c):

$$V_{out} = \frac{R_{load}}{R_{Th} + R_{load}} V_{Th} = \frac{10}{5 + 10} \cdot 15 \text{ V} = 10 \text{ V}$$

We obtain the same value as in (b).



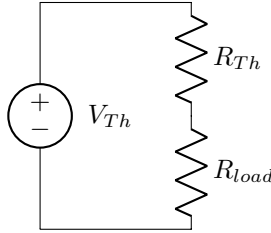
(e)

$$\begin{aligned}P_{load} &= \frac{V_{out}^2}{R_{load}} = \frac{100 \text{ V}^2}{10000 \text{ } \Omega} = 0.01 \text{ W} \\P_1 &= \frac{(V_{in} - V_{out})^2}{R_1} = \frac{400 \text{ V}^2}{10000 \text{ } \Omega} = 0.04 \text{ W} \\P_2 &= \frac{V_{out}^2}{R_2} = \frac{100 \text{ V}^2}{10000 \text{ } \Omega} = 0.01 \text{ W}\end{aligned}$$

---

**Exercise.** This is the solution to exercise 1.11 in the book.

**Solution.** Remember that the Thévenin equivalent circuit with a load is:



The load voltage is:

$$V_{load} = \frac{R_{load}}{R_{Th} + R_{load}} V_{Th}$$

and the power dissipated by load is:

$$P_{load} = \frac{V_{load}^2}{R_{load}} = \frac{R_{load} V_{Th}^2}{(R_{Th} + R_{load})^2}$$

To maximize power dissipated in the load with respect to  $R_{load}$  (and for a given  $R_{Th}$ ):

$$\frac{dP_{load}}{dR_{load}} = 0 = \frac{V_{Th}^2}{(R_{Th} + R_{load})^2} - 2 \frac{R_{load} V_{Th}^2}{(R_{Th} + R_{load})^3}$$

or

$$R_{Th} + R_{load} - 2R_{load} = 0$$

or

$$R_{load} = R_{Th}$$

---

**Exercise.** This is the solution to exercise 1.12 in the book.

**Solution.** The power ratio is:

$$\frac{P_1}{P_2} = 10^{\frac{\text{dB}}{10}}$$

and the amplitude (voltage) ratio is:

$$\frac{A_1}{A_2} = 10^{\frac{\text{dB}}{20}}$$

We get:

(a)

$$\frac{P_1}{P_2} = 2 \quad \frac{A_1}{A_2} = 1.41$$

(b)

$$\frac{P_1}{P_2} = 3.98 \quad \frac{A_1}{A_2} = 2$$

(c)

$$\frac{P_1}{P_2} = 10 \quad \frac{A_1}{A_2} = 3.16$$

(d)

$$\frac{P_1}{P_2} = 100 \quad \frac{A_1}{A_2} = 10$$

---

**Exercise.** This is the solution to exercise 1.13 in the book.

**Solution.** We can use the fact that 3 dB difference corresponds to a doubling (or halving) of power while a 10 dB difference corresponds to increasing/decreasing the power  $10\times$ . So we first find the values for 7 dB, 4 dB and 1 dB. Then we estimate the values for 11 dB and then 8 dB, 5 dB and 2 dB.

dB	ratio ( $P/P_0$ )
0	1
1	1.25
2	$\frac{\pi}{2}$
3	2
4	2.5
5	$\pi$
6	4
7	5
8	6.25
9	8
10	10
11	12.5

---

**Exercise.** This is the solution to exercise 1.14 in the book.

**Solution.** We have that:

$$dU = VIdt$$

but

$$I = C \frac{dV}{dt}$$

therefore:

$$dU = CVdV$$

or

$$\int_0^{U_c} = C \int_0^{V_f} VdV$$

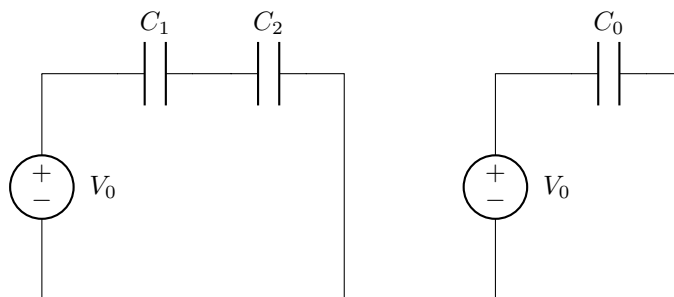
or

$$U_c = C \frac{V_f^2}{2}$$

---

**Exercise.** This is the solution to exercise 1.15 in the book.

**Solution.**



We have that:

$$\begin{aligned} V_0 &= V_1 + V_2 \\ Q_0 &= C_0 V_0 \\ Q_1 &= C_1 V_1 \\ Q_2 &= C_2 V_2 \end{aligned}$$

Hence:

$$\frac{Q_0}{C_0} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

Since charge neutrality between  $C_1$  and  $C_2$  must be maintained then  $Q_1 = Q_2$ . Therefore, the series capacitors have  $Q_{12}$  charge on their terminals exposed to  $V_0$ . Hence the most obvious choice of an equivalent capacitance is to set  $Q_{12} = Q_0$  and thus:

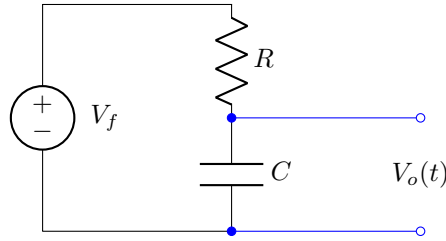
$$\frac{Q_0}{C_0} = Q_{12} \left( \frac{1}{C_1} + \frac{1}{C_2} \right) = Q_0 \left( \frac{1}{C_1} + \frac{1}{C_2} \right)$$

and thus:

$$\frac{1}{C_0} = \frac{1}{C_1} + \frac{1}{C_2}$$

**Exercise.** This is the solution to exercise 1.16 in the book.

**Solution.**



The current through the resistor and capacitor while the capacitor is charging is:

$$I = C \frac{dV_o(t)}{dt} = \frac{V_f - V_o(t)}{R}$$

or

$$C \frac{dV_o(t)}{dt} + \frac{V_o(t)}{R} = \frac{V_f}{R}$$

or

$$\frac{dV_o(t)}{dt} + \frac{V_o(t)}{RC} = \frac{V_f}{RC}$$

The solution to the homogenous equation is:

$$V_{oh}(t) = Ae^{-\frac{t}{RC}}$$

A particular solution to the non-homogenous equation is:

$$V_{op} = V_f$$

hence:

$$V_o(t) = V_f + Ae^{-\frac{t}{RC}}$$

Using the initial condition:

$$V_o(0) = 0$$

we get:

$$V_o(t) = V_f(1 - e^{-\frac{t}{RC}})$$

or

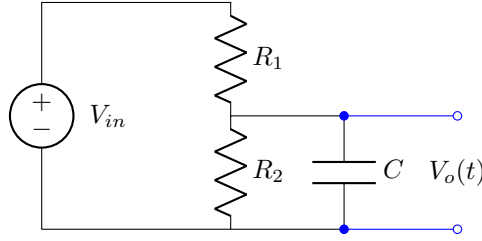
$$t = RC \ln \frac{V_f}{V_f - V_o(t)}$$

or

$$\begin{aligned}\Delta t_{12} &= RC \ln \frac{V_f - V_o(t_2)}{V_f - V_o(t_1)} \\ \Delta t_{10\% \rightarrow 90\%} &= RC \ln \frac{V_f - 0.1V_f}{V_f - 0.9V_f} \\ \Delta t_{10\% \rightarrow 90\%} &= 2.2RC\end{aligned}$$

**Exercise.** This is the solution to exercise 1.17 in the book.

**Solution.**



We have that:

$$\begin{aligned}I_2(t) &= \frac{V_o(t)}{R_2} \\ I_1(t) &= \frac{V_{in} - V_o(t)}{R_1} \\ I_C(t) &= I_1(t) - I_2(t) = C \frac{dV_o(t)}{dt} \\ \frac{V_{in} - V_o(t)}{R_1} - \frac{V_o(t)}{R_2} &= C \frac{dV_o(t)}{dt} \\ C \frac{dV_o(t)}{dt} + \frac{R_1 + R_2}{R_1 R_2} V_o(t) &= \frac{V_{in}}{R_1}\end{aligned}$$

The homogenous solution is:

$$V_{oh}(t) = Ae^{-\frac{t(R_1 + R_2)}{R_1 R_2 C}}$$

and a particular solution is:

$$V_{op} = V_{in} \frac{R_2}{R_1 + R_2}$$

hence the general solution is:

$$V_o(t) = Ae^{-\frac{t(R_1 + R_2)}{R_1 R_2 C}} + V_{in} \frac{R_2}{R_1 + R_2}$$

When  $t = 0$   $V_o(t) = 0$  and then:

$$\begin{aligned}A &= -V_{in} \frac{R_2}{R_1 + R_2} \\ V_o(t) &= V_{in} \frac{R_2}{R_1 + R_2} \left[ 1 - e^{-\frac{t(R_1 + R_2)}{R_1 R_2 C}} \right]\end{aligned}$$

Plotting can be done with the following script:

```

import matplotlib.pyplot as plt
import numpy as np

plt.ion()
plt.close('all')

R1=10000
R2=10000
C=1.0E-7

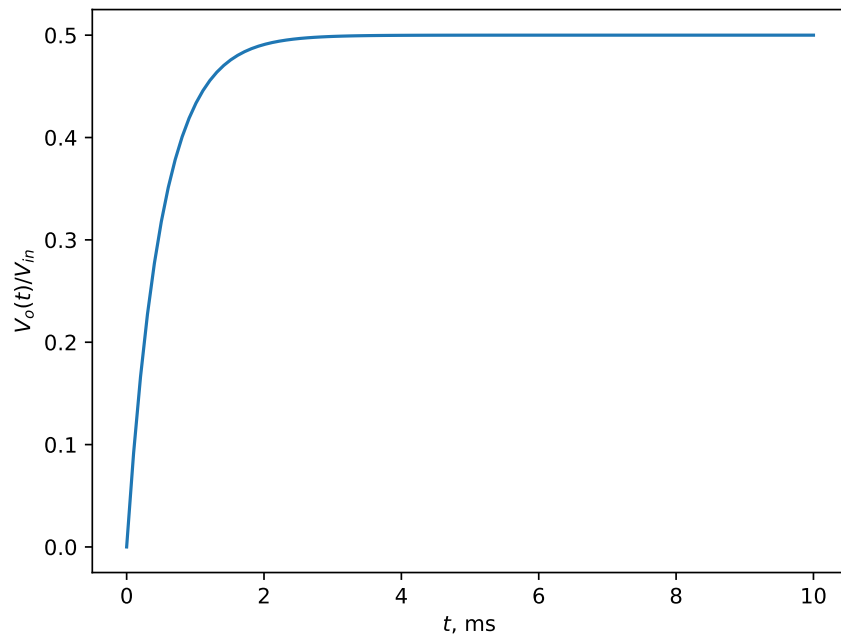
def calc_vr(tin):
    retval = R2/(R1+R2)
    retval*=(1-np.exp(-tin*(R1+R2)/R1/R2/C))
    return retval

t=np.linspace(0,10,100)
vr=np.empty(shape=[100,1])
for i in range(len(t)):
    vr[i] = calc_vr(t[i]/1000.0)

fig,ax=plt.subplots(1,1)
ax.set_xlabel('$t$, ms')
ax.set_ylabel('$V_o(t)/V_{in}$')
ax.plot(t, vr)
#plt.savefig('c01ex17i02.pdf',format='pdf',bbox_inches='tight')

```

to produce the result:




---

**Exercise.** This is the solution to exercise 1.18 in the book.

**Solution.** We have that:

$$I(t) = C \frac{dV(t)}{dt}$$

Since the charging current is constant:

$$dV(t) = \frac{I}{C} dt$$

or

$$V_f(t) = \frac{I}{C} t$$

or:

$$t = \frac{1 \cdot 10^{-6} \text{ F} \cdot 10 \text{ V}}{1 \cdot 10^{-3} \text{ A}} = 0.01 \text{ s}$$

**Exercise.** This is the solution to exercise 1.19 in the book.

**Solution.** Self-inductance of a coil is related to the magnetic flux through one turn of the coil, the number of turns in the coil  $n$  and the current through the coil  $I$  as:

$$L = \frac{n\Phi_m}{I}$$

The magnetic flux  $\Phi_m$  through one turn of the coil is:

$$\Phi_m = B \cdot S$$

where  $B$  is the magnetic flux density and  $S$  is the area of each turn of the coil. The magnetic flux density thru the coil is:

$$B = \mu H = n\mu \frac{I}{l}$$

where  $\mu$  is the magnetic constant and  $H$  is the magnetic field strength produced by the coil with  $n$  turns and length  $l$  and  $I$  is the current through the coil.

Therefore:

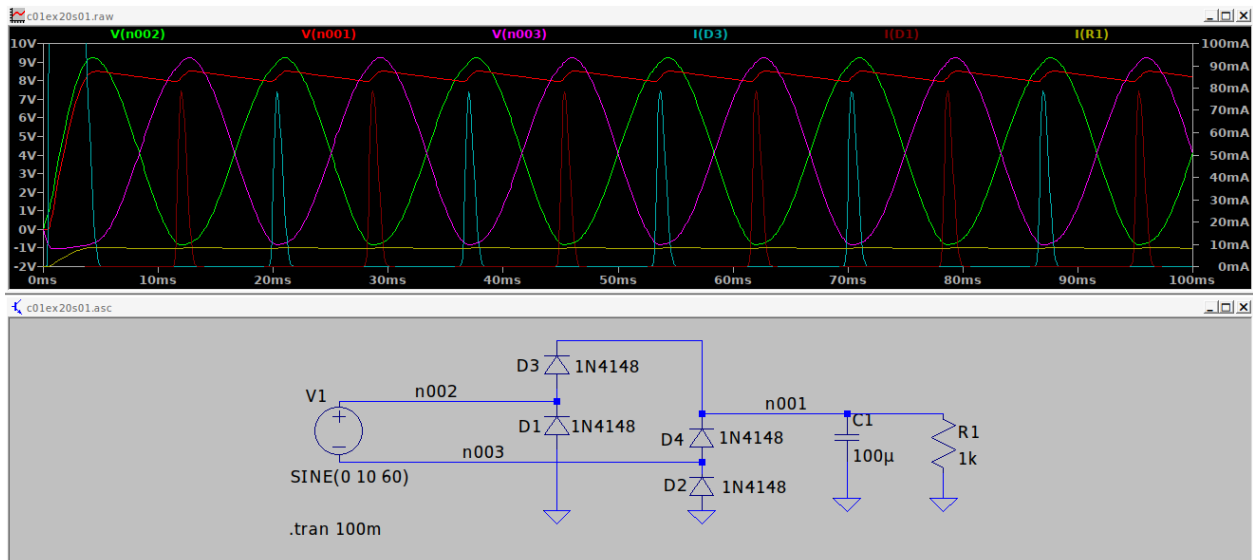
$$L = n^2 \mu \frac{S}{l}$$

Notice that  $l$  is the total length of the coil, which does not depend on the number of turns (more turns can be accommodated if the surface of each turn is reduced).

**Exercise.** This is the solution to exercise 1.20 in the book.

**Solution.** Let's try with a sine wave with amplitude of 10V and 60 Hz frequency.

Please refer to the SPICE simulation below:



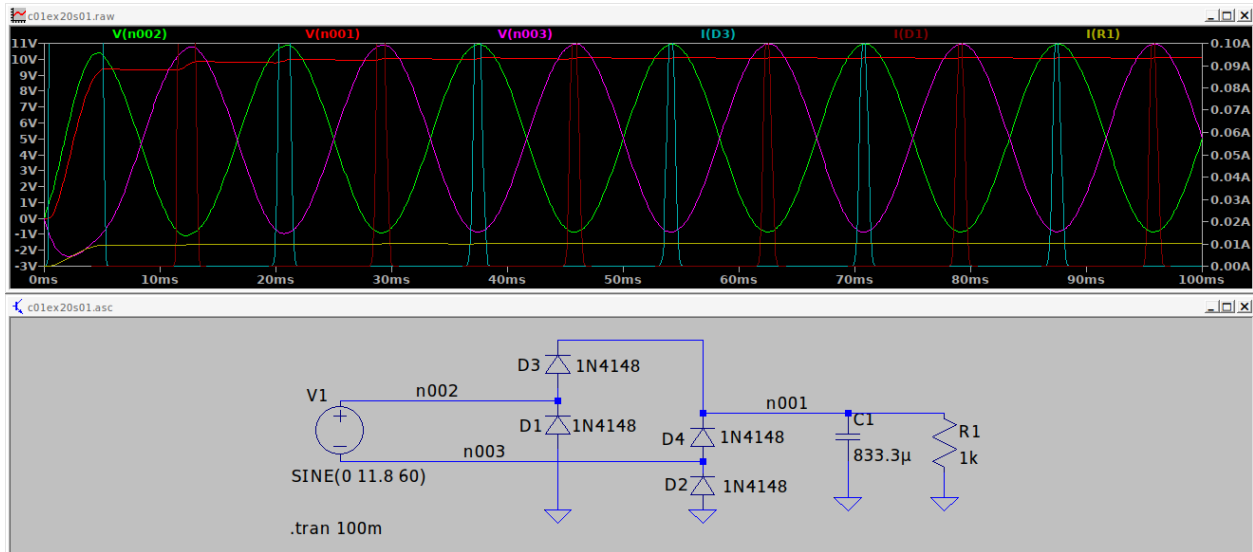
Notice that following the initial transient that charges up the capacitor  $V_{dc}(t)$  oscillates between 8.52V and 7.91V. Hence the desired DC voltage has too much ripple and averages around 8.22V. Additionally notice that the peak AC voltage is 9.23 volts. The difference in peak voltage between AC and DC is due to the voltage drop across the forward biased diodes which is 0.7V in this case. Notice that the current thru R1 is indeed approximately 10mA. The DC ripple is expected base on the capacitor we chose:

$$\Delta V = \frac{I_{load}}{2fC} = \frac{8.22 \cdot 10^{-3}}{120 \cdot 100 \cdot 10^{-6}} = 0.68V$$

where we have used the load current of 8.22mA given the average  $V_{dc}=8.22V$ . Hence we need the source voltage to be a bit larger to bring the DC voltage to 10V. Additionally the capacitor should be:

$$C = \frac{I_{load}}{2f\Delta V} = \frac{10 \cdot 10^{-3}}{120 \cdot 0.1} = 833\mu F$$

to reduce the ripple in the DC voltage. Hence we increase the amplitude of the AC voltage to 11.8V and change the capacitor accordingly:



In this case the DC voltage ranges from 9.91V to 9.99V hence the ripple is reduced to spec and  $V_{dc} \approx 10V$ . And the load current is 10mA.

Here are the LTSpice [schematic](#) and [plotting](#) files.

**Exercise.** This is the solution to exercise 1.21 in the book.

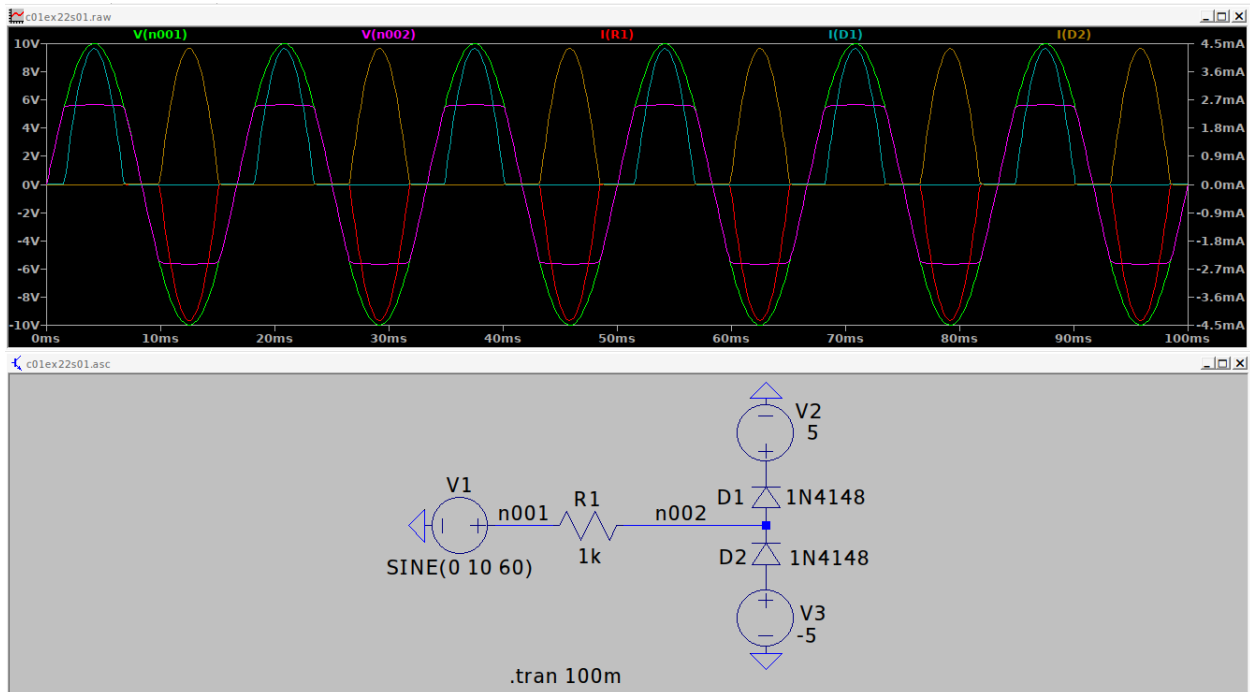
**Solution.** We use the hint provided to calculate the average  $I^2$  current for the given waveform. If  $\tau = \frac{t}{T}$  where  $T$  is the period of one cycle. Therefore:

$$\langle I^2 \rangle = \int_0^1 I^2(t) d\tau = \int_{1/2}^1 4 d\tau = 2 \text{ A}^2$$

Therefore the fuse must be minimally rated for  $I_{fuse} = \sqrt{\langle I^2 \rangle} = 1.41 \text{ A}$ .

**Exercise.** This is the solution to exercise 1.22 in the book.

**Solution.** An implementation is provided below:



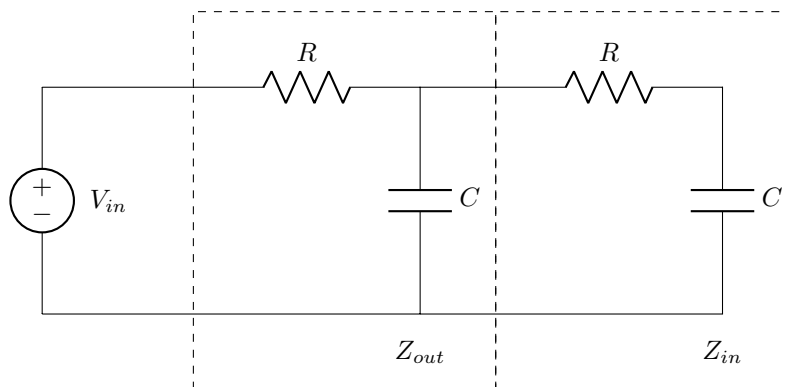
Notice that as the voltage in front of  $R1$  ( $V(n001)$ ) becomes greater than 5.6 V diode  $D1$  becomes forward biased and current starts to flow through it from source  $V_1$ . There is no current flowing thru  $D2$ . Similarly, as the voltage decreases below  $-5.6$  V then diode  $D2$  becomes forward biased and the current flows back into source  $V_1$ . There is no current flowing thru  $D1$  in this case.

Hence  $V(n002)$  is clamped between  $-5.6$  V and  $5.6$  V.

Here are the LTSpice [schematic](#) and [plotting](#) files.

**Exercise.** This is the solution to exercise 1.23 in the book.

**Solution.** Let's consider the following circuit. It encompasses an RC low-pass filter as source output impedance and another RC low-pass filter as load input impedance.



We have that:

$$Z_{out} = \frac{RX_C}{R + X_C} = \frac{-i/\omega/C \cdot R}{R - i/\omega/C} = \frac{R}{1 + i\omega RC}$$

$$|Z_{out}| = \frac{R}{\sqrt{1 + \omega^2 R^2 C^2}}$$

Therefore the maximum output impedance is for  $\omega = 0$ :

$$|Z_{out}| = R$$



We also have that:

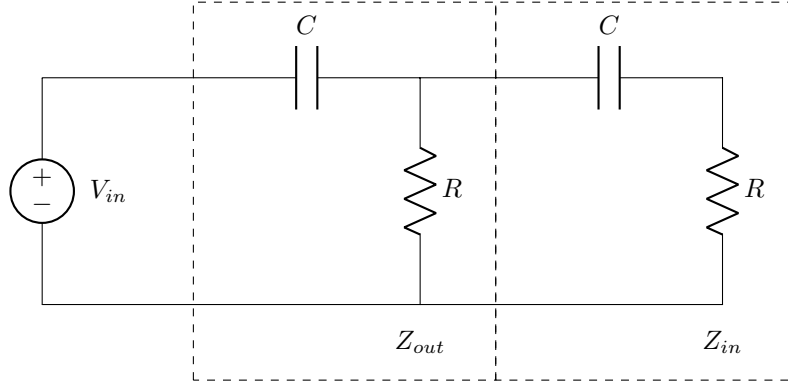
$$Z_{in} = R + X_C = R - \frac{j}{\omega C}$$

$$|Z_{in}| = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

The minimum input impedance is for  $\omega \rightarrow \infty$ :

$$|Z_{in}| = R$$

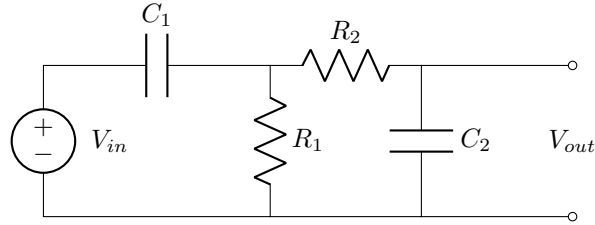
Now consider the case of a high-pass filter:



Notice that impedances are calculated in exactly the same way. Hence, indeed, the worst-case impedance is indeed just  $R$ .

**Exercise.** This is the solution to exercise 1.24 in the book.

**Solution.** Consider the circuit as follows:



Since the source impedance is  $100 \Omega$  then we must choose the minimum input impedance of the high pass RC filter to be  $1 \text{ k}\Omega$ . Hence we choose  $R_1 = 1 \text{ k}\Omega$  and given the breakpoint of  $100 \text{ Hz}$ :

$$\omega = 2\pi f = \frac{1}{RC}$$

hence:

$$C_1 = \frac{1}{2\pi f R_1}$$

or:

$$C_1 = 1.59 \mu\text{F}$$

The worst-case output impedance of the second low-pass filter must be  $10 \text{ k}\Omega$  hence we choose  $R_2 = 10 \text{ k}\Omega$ . Using:

$$C_2 = \frac{1}{2\pi f R_2}$$

with  $f = 10 \text{ kHz}$  we get:

$$C_2 = 1.59 \text{ nF}$$

Since the output impedance of the low-pass filter is  $R_2 = 10 \text{ k}\Omega$  then the minimum input impedance of the load must be:

$$|Z_{load}| = 10R_2 = 100 \text{ k}\Omega$$

**Exercise.** This is the solution to exercise 1.25 in the book.

**Solution.** When the capacitors are in series then impedances add:

$$Z = Z_1 + Z_2$$

But:

$$Z_1 = -\frac{i}{\omega C_1} \quad Z_2 = -\frac{i}{\omega C_2}$$

and

$$Z = -\frac{i}{\omega C}$$

where  $C$  is the capacitance of the capacitor equivalent to the two capacitors in series. Hence:

$$-\frac{i}{\omega C} = -\frac{i}{\omega C_1} - \frac{i}{\omega C_2}$$

or

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

or

$$C = \frac{C_1 C_2}{C_1 + C_2}$$

which is exactly equation 1.18 in the text. When the capacitors are in parallel the impedances are related as:

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

or

$$i\omega C = i\omega C_1 + i\omega C_2$$

or

$$C = C_1 + C_2$$

which is exactly equation 1.17 in the text.

**Exercise.** This is the solution to exercise 1.26 in the book.

**Solution.** Following the hint and using:

$$\begin{aligned} \mathbf{A} &= Ae^{\theta_A} \\ \mathbf{B} &= Be^{\theta_B} \\ \mathbf{C} &= Ce^{\theta_C} \end{aligned}$$

we get:

$$A = BCe^{\theta_B + \theta_C - \theta_A}$$

Since  $A, B, C$  are real numbers we must have that  $e^{\theta_B + \theta_C - \theta_A}$  is also a real number. Hence  $e^{\theta_B + \theta_C - \theta_A} = 1$  and:

$$A = BC$$

**Exercise.** This is the solution to exercise 1.27 in the book.

**Solution.** We can compute the power over one period as:

$$P = \frac{1}{T} \int_0^T V(t)I(t)dt$$

Since we can have:

$$V(t) = V_0 \sin 2\pi ft$$

and

$$I(t) = I_0 \cos 2\pi ft$$

such that voltage and current are  $90^\circ$  out of phase then:

$$P = \frac{V_0 I_0}{T} \int_0^T \sin 2\pi ft \cos 2\pi ft dt = \frac{V_0 I_0}{2T} \int_0^T \sin 4\pi ft dt$$

or

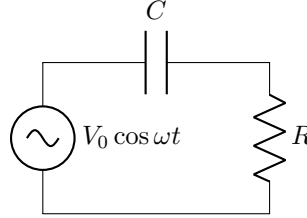
$$P = \frac{V_0 I_0}{8\pi fT} \cos 4\pi ft \Big|_0^T = \frac{V_0 I_0}{8\pi fT} (1 - \cos 4\pi fT)$$

But  $fT = 1$  hence:

$$P = \frac{V_0 I_0}{8\pi} (1 - \cos 4\pi) = 0$$

**Exercise.** This is the solution to exercise 1.28 in the book.

**Solution.**



Consider that  $I(t)$  is the current through the circuit. We have then:

$$I(t) = C \frac{dV_C(t)}{dt} \quad V_C(t) = V(t) - RI(t)$$

or

$$I(t) = C \frac{dV(t)}{dt} - RC \frac{dI(t)}{dt}$$

Using the expression:

$$V(t) = V_0 \cos \omega t$$

$$\frac{dI(t)}{dt} + \frac{1}{RC} I(t) = -\frac{\omega V_0 \sin \omega t}{R}$$

The differential equation can be solved and using  $I(0) = 0$  we get:

$$I(t) = \frac{V_0 \omega C (\omega RC \cos \omega t - \sin \omega t)}{1 + \omega^2 R^2 C^2} - \frac{V_0 \omega^2 R C^2}{1 + \omega^2 R^2 C^2} e^{-\frac{t}{RC}}$$

Since we are interested in the steady state current when the RC decays:

$$I(t) = \frac{V_0 \omega C (\omega RC \cos \omega t - \sin \omega t)}{1 + \omega^2 R^2 C^2}$$

The power dissipated in the resistor is:

$$P_R(t) = I(t)^2 R = \frac{V_0^2 \omega^2 R C^2 (\omega RC \cos \omega t - \sin \omega t)^2}{(1 + \omega^2 R^2 C^2)^2}$$

The average power dissipated over one period is:

$$\bar{P}_R = \frac{1}{T} \int_0^T P_R(t) dt = \frac{V_0^2 \omega^2 R C^2}{T(1 + \omega^2 R^2 C^2)^2} \int_0^T (\omega RC \cos \omega t - \sin \omega t)^2 dt$$

or

$$\bar{P}_R = \frac{V_0^2 \omega^2 R C^2}{(1 + \omega^2 R^2 C^2)^2} \int_0^1 (\omega R C \cos 2\pi\tau - \sin 2\pi\tau)^2 d\tau$$

The integral can be evaluated with Mathematica for example to yield:

$$\bar{P}_R = \frac{V_0^2 \omega^2 R C^2}{(1 + \omega^2 R^2 C^2)^2} \frac{1 + \omega^2 R^2 C^2}{2}$$

or

$$\bar{P}_R = \frac{V_0^2 \omega^2 R C^2}{2(1 + \omega^2 R^2 C^2)}$$

The instantaneous power produced by the source (after the decay of the RC transient) is:

$$P_S(t) = V(t)I(t) = \frac{V_0^2 \omega C}{1 + \omega^2 R^2 C^2} (\omega R C \cos^2 \omega t - \sin \omega t \cos \omega t)$$

The average power produced over one period is:

$$\bar{P}_S = \frac{1}{T} \int_0^T P_S(t) dt = \frac{V_0^2 \omega C}{T(1 + \omega^2 R^2 C^2)} \int_0^T (\omega R C \cos^2 \omega t - \sin \omega t \cos \omega t) dt$$

or

$$\bar{P}_S = \frac{V_0^2 \omega C}{(1 + \omega^2 R^2 C^2)} \int_0^1 (\omega R C \cos^2 2\pi\tau - \sin 2\pi\tau \cos 2\pi\tau) d\tau$$

The integral can be evaluated with Mathematica for example to yield:

$$\bar{P}_S = \frac{V_0^2 \omega C}{(1 + \omega^2 R^2 C^2)} \frac{\omega R C}{2}$$

or

$$\bar{P}_S = \frac{V_0^2 \omega^2 R C^2}{2(1 + \omega^2 R^2 C^2)}$$

Therefore,

$$\bar{P}_S = \bar{P}_R$$

that is all average power produced by the source is dissipated by the resistor.

For  $V_0 = 115\sqrt{2}$  V,  $\omega = 120\pi$  s<sup>-1</sup>,  $C = 1 \cdot 10^{-6}$  F, and  $R = 1000$   $\Omega$ .

$$P = \frac{115^2 \cdot (120\pi)^2 \cdot 10^{-12} \cdot 1000}{1 + (120\pi)^2 \cdot 10^6 \cdot 10^{-12}} \text{ W} = 1.646 \text{ W}$$

**Exercise.** This is the solution to exercise 1.29 in the book.

**Solution.** For an RLC circuit in series the impedance is:

$$Z = R + X_L + X_C$$

$$Z = R + i\omega L - \frac{i}{\omega C} = R + i\omega L - \frac{i\omega^2 L}{\omega} = R$$

hence the impedance of the circuit is real and the reactance component is zero. Thus the power factor is 1 since impedance is purely resistive.

For a parallel RLC circuit:

$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{X_L} + \frac{1}{X_C}$$

$$Z = \frac{1}{\frac{1}{R} - \frac{i}{\omega L} + i\omega C} = \frac{1}{\frac{1}{R} - \frac{i}{\omega L} + \frac{i\omega}{\omega^2 L}} = R$$

hence the impedance of the circuit is real and the reactance component is zero. Thus the power factor is 1 since impedance is purely resistive.

---

**Exercise.** This is the solution to exercise 1.30 in the book.

**Solution.** We have that:

$$\frac{V_{out}}{V_{in}} = \frac{X_C}{R + X_C} = \frac{-\frac{i}{\omega C}}{R - \frac{i}{\omega C}} = \frac{1}{1 + i\omega RC}$$

or

$$V_{out} = \frac{1 - i\omega RC}{1 + \omega^2 R^2 C^2} V_{in}$$

Hence:

$$|V_{out}|^2 = \frac{1 + \omega^2 R^2 C^2}{(1 + \omega^2 R^2 C^2)^2} |V_{in}|^2$$

or

$$|V_{out}| = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}} |V_{in}|$$

---

**Exercise.** This is the solution to exercise 1.31 in the book.

**Solution.** For lowpass filter we have that:

$$\mathbf{V}_{out} = \mathbf{V}_{in} \frac{-\frac{i}{\omega C}}{R - \frac{i}{\omega C}}$$

or

$$\mathbf{V}_{out} = \mathbf{V}_{in} \frac{1}{1 + i\omega RC}$$

or

$$\mathbf{V}_{out} = \mathbf{V}_{in} \frac{1 - i\omega RC}{1 + \omega^2 R^2 C^2}$$

Therefore the phase angle is:

$$\phi = -\arctan \frac{\frac{\omega RC}{1 + \omega^2 R^2 C^2}}{\frac{1}{1 + \omega^2 R^2 C^2}} = -\arctan \omega RC$$

or

$$\phi = -\arctan(2\pi f RC) = -\arctan \frac{f}{f_{3dB}}$$

We therefore get:

$$\phi_{0.1\times} = -5.71^\circ \quad \phi_{10\times} = -84.29^\circ$$

Therefore  $\phi_{0.1\times}$  is about  $6^\circ$  from the value  $\phi_{0\times} = 0^\circ$  and  $\phi_{10\times}$  is about  $6^\circ$  from the asymptotic value  $\phi_{\infty\times} = -90^\circ$ .

---

**Exercise.** This is the solution to exercise 1.32 in the book.

**Solution.** We have that:

$$\begin{aligned} \mathbf{V}_{out} &= \mathbf{I}R \\ \mathbf{V}_{in} &= \mathbf{I} \left( R - \frac{i}{\omega C} \right) \end{aligned}$$

or

$$\mathbf{V}_{out} = \mathbf{V}_{in} \frac{R}{R - \frac{i}{\omega C}}$$

Therefore the magnitude is:

$$|\mathbf{V}_{out}| = |\mathbf{V}_{in}| \frac{R}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

---

**Exercise.** This is the solution to exercise 1.33 in the book.

**Solution.**

$$\begin{aligned} \mathbf{V}_{out} &= -\mathbf{I} \frac{i}{\omega C} \\ \mathbf{V}_{in} &= \mathbf{I} \left( R - \frac{i}{\omega C} \right) \end{aligned}$$

or

$$\mathbf{V}_{out} = \mathbf{V}_{in} \frac{-\frac{i}{\omega C}}{R - \frac{i}{\omega C}}$$

or

$$\mathbf{V}_{out} = \mathbf{V}_{in} \frac{1}{1 + i\omega RC}$$

Therefore the magnitude is:

$$|\mathbf{V}_{out}| = |\mathbf{V}_{in}| \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}}$$

For 6 dB the ratio must be:

$$\frac{|\mathbf{V}_{out}|}{|\mathbf{V}_{in}|} = \frac{1}{2} = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}}$$

$$\omega^2 R^2 C^2 = 3$$

or

$$\omega = \frac{\sqrt{3}}{RC}$$

or

$$f_{6dB} = \frac{\sqrt{3}}{2\pi RC} = \sqrt{3} f_{3dB}$$

Since for lowpass RC filter we determined in [exercise 1.31](#):

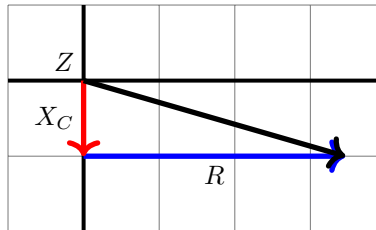
$$\phi_{6dB} = -\arctan \frac{f_{6dB}}{f_{3dB}} = -\arctan \sqrt{3}$$

Thus the phase shift is:

$$\phi_{6dB} = -60^\circ$$

**Exercise.** This is the solution to exercise 1.34 in the book.

**Solution.**



The phase shift between  $V_{out}$  and  $V_{in}$  is the angle between phasor  $X_C$  and the resultant phasor  $X_C + R$ . The filter response  $V_{out}/V_{in}$  is the ratio of the two phasors:

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{X_C}{X_C + R} = \frac{-\frac{i}{\omega C}}{R - \frac{i}{\omega C}}$$

or taking the magnitude:

$$|\mathbf{V}_{\text{out}}| = |\mathbf{V}_{\text{in}}| \frac{\frac{1}{\omega C}}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

or

$$|\mathbf{V}_{\text{out}}| = |\mathbf{V}_{\text{in}}| \frac{1}{\sqrt{1 + \omega^2 C^2 R^2}}$$

which is exactly the expression of equation 1.36 in the text.

**Exercise.** This is the solution to exercise 1.35 in the book.

**Solution.** We have that:

$$\frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{X_C + X_L}{R + X_C + X_L} = \frac{i\omega L - \frac{i}{\omega C}}{R + i(\omega L - \frac{1}{\omega C})}$$

Hence, taking the magnitude:

$$|\mathbf{V}_{\text{out}}| = |\mathbf{V}_{\text{in}}| \frac{\omega L - \frac{1}{\omega C}}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

or

$$|\mathbf{V}_{\text{out}}| = |\mathbf{V}_{\text{in}}| \frac{\omega^2 LC - 1}{\sqrt{\omega^2 R^2 C^2 + (\omega^2 LC - 1)^2}}$$

Therefore when  $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$  then:

$$\frac{|\mathbf{V}_{\text{out}}|}{|\mathbf{V}_{\text{in}}|} = 0$$

When  $\omega^2 LC \ll 1$  then:

$$\frac{|\mathbf{V}_{\text{out}}|}{|\mathbf{V}_{\text{in}}|} = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}}$$

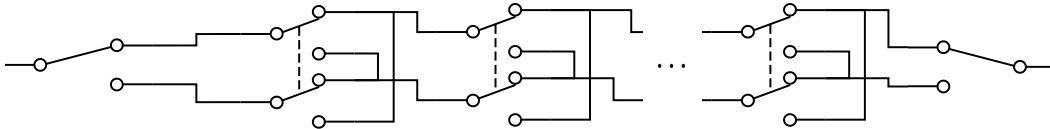
which is the response of an RC circuit. When  $\omega^2 LC \gg 1$  then:

$$\frac{|\mathbf{V}_{\text{out}}|}{|\mathbf{V}_{\text{in}}|} = \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}}$$

which is the response of the RL circuit.

**Exercise.** This is the solution to exercise 1.36 in the book.

**Solution.** We can replace the assembly of two SPST switches with the assembly of 2 SPST and  $N - 2$  DPDT switches:



**Exercise.** This is the solution to exercise 1.37 in the book.

**Solution.** First we place a short across the load (opposite to calculating the Thévenin equivalent voltage where load is open). The Norton current is:

$$I_N = \frac{10 \text{ V}}{10 \text{ k}\Omega} = 1 \text{ mA}$$

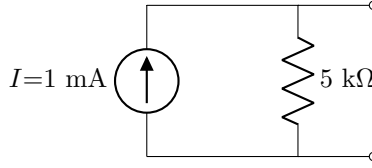
Second find the equivalent Norton resistance by replacing the voltage source with a short. The resistance is:

$$\frac{1}{R_N} = \frac{1}{10 \text{ k}\Omega} + \frac{1}{10 \text{ k}\Omega} = \frac{1}{5 \text{ k}\Omega}$$

therefore:

$$R_N = 5 \text{ k}\Omega$$

Hence the Norton equivalent is:



When the load is  $5 \text{ k}\Omega$  the equivalent resistance is:

$$\frac{1}{R_{eq}} = \frac{1}{5 \text{ k}\Omega} + \frac{1}{5 \text{ k}\Omega}$$

or

$$R_{eq} = 2.5 \text{ k}\Omega$$

and the voltage is  $V_{out} = IR_{eq} = 2.5 \text{ V}$ .

If the load of  $R_{load} = 5 \text{ k}\Omega$  is used in the original circuit the voltage across the load is:

$$V_{out} = V_{in} \frac{R_{eq}}{10 \text{ k}\Omega + R_{eq}}$$

where

$$\frac{1}{R_{eq}} = \frac{1}{10 \text{ k}\Omega} + \frac{1}{5 \text{ k}\Omega}$$

or

$$R_{eq} = \frac{10}{3} \text{ k}\Omega$$

Hence:

$$V_{out} = 10 \text{ V} \cdot \frac{\frac{10}{3}}{10 + \frac{10}{3}} = 2.5 \text{ V}$$

Hence the same output value is obtained as that found above from the Norton equivalent circuit.

**Exercise.** This is the solution to exercise 1.38 in the book.

**Solution.** For [exercise 1.37](#) the Thévenin equivalent is:

$$V_{Th} = V_{in} \frac{R_1}{R_1 + R_2} = 10 \text{ V} \frac{10 \text{ k}\Omega}{10 \text{ k}\Omega + 10 \text{ k}\Omega} = 5 \text{ V}$$

and

$$R_{Th} = \frac{R_1 R_2}{R_1 + R_2} = \frac{10 \text{ k}\Omega \cdot 10 \text{ k}\Omega}{10 \text{ k}\Omega + 10 \text{ k}\Omega} = 5 \text{ k}\Omega$$

For this exercise the open circuit voltage leads to:

$$V_{Th} = IR_2 = 0.5 \text{ mA} \cdot 10 \text{ k}\Omega = 5 \text{ V}$$

The short-circuit resistance is:

$$R_{Th} = \frac{V_{Th}}{I} = \frac{5}{0.5} = 10 \text{ k}\Omega$$

Hence the two circuits are not equivalent, meaning that even though the open circuit  $V_+ = 10 \text{ V}$  we cannot replace the current source with a voltage source of  $10 \text{ V}$ .

**Exercise.** This is the solution to exercise 1.39 in the book.

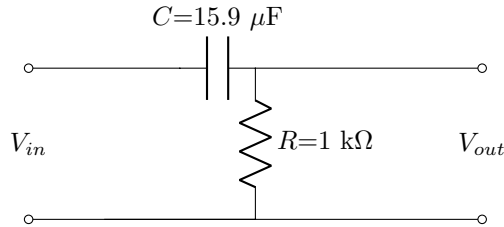


**Solution.** This is a high-pass RC filter. If  $f_{3\text{ dB}} = 10\text{ Hz}$  then:

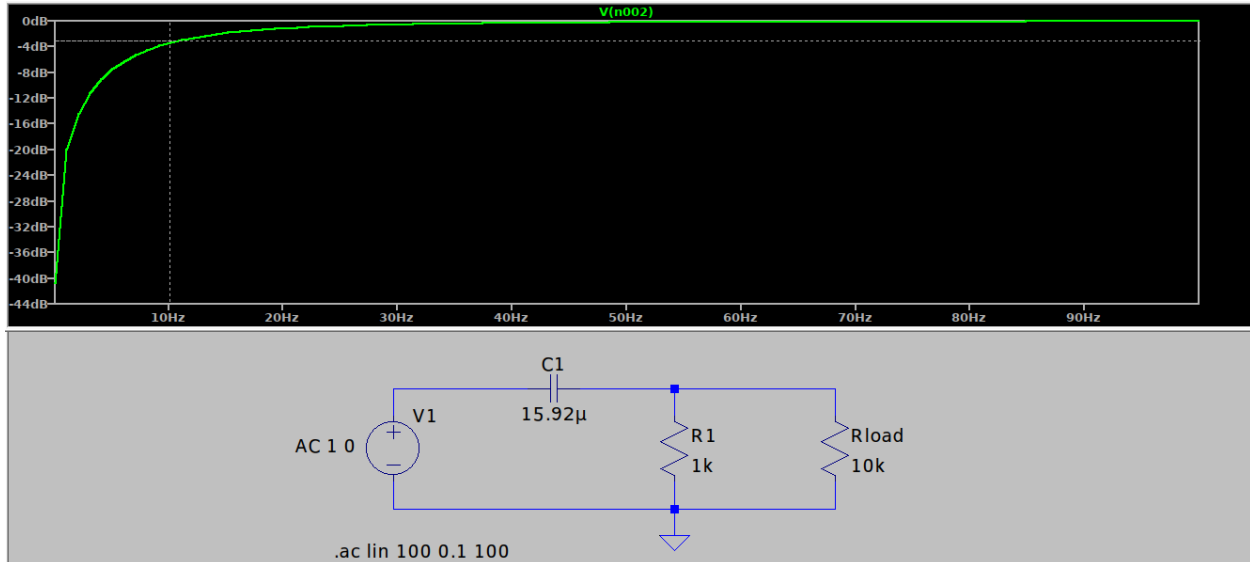
$$RC = \frac{1}{2\pi f_{3\text{ dB}}} = 0.015915\text{ s}$$

Since the minimum load impedance is  $10\text{ k}\Omega$  then the output impedance of the filter must be at most  $1\text{ k}\Omega$  (following the 1:10 source to load impedance ratio). We thus choose  $R = 1\text{ k}\Omega$  and:

$$C = 15.92\text{ }\mu\text{F}$$



We can simulate the circuit with the schematic from [here](#) and the plot [here](#):




---

**Exercise.** This is the solution to exercise 1.40 in the book.

**Solution.** The scratch filter is a low-pass filter. Again for  $f_{3\text{ dB}} = 10\text{ kHz}$  we get:

$$RC = \frac{1}{2\pi f_{3\text{ dB}}} = 15.92\text{ }\mu\text{s}$$

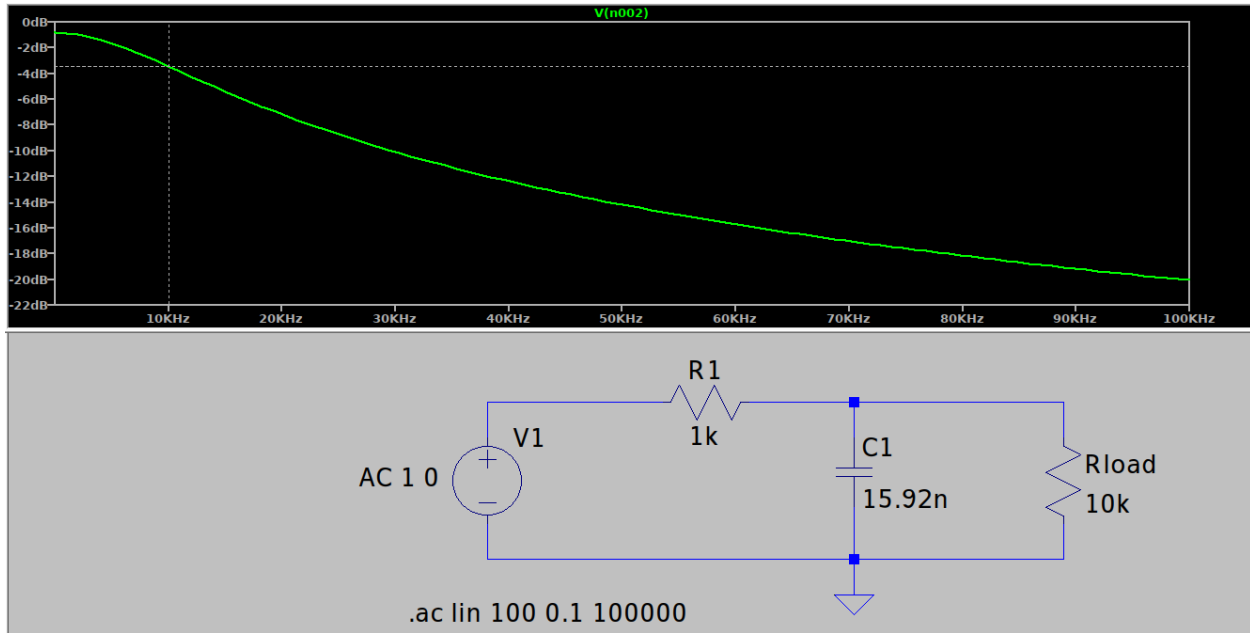
Using a  $10\text{ k}\Omega$  minimum load impedance and using the 1:10 rule for source to load impedance we set the capacitor impedance at  $10\text{ kHz}$  to  $1\text{ k}\Omega$  and hence:

$$C = \frac{1}{2\pi \cdot 10\text{ kHz} \cdot 1\text{ k}\Omega} = 15.92\text{ nF}$$

Resistor value is then:

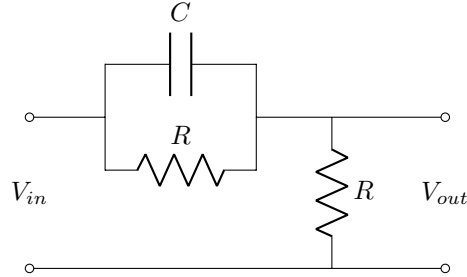
$$R = 1\text{ k}\Omega$$

We can simulate the circuit with the schematic from [here](#) and the plot [here](#):



**Exercise.** This is the solution to exercise 1.41 in the book.

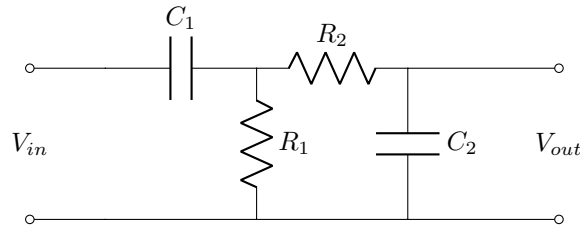
**Solution.** At low frequencies the circuit behaves like a voltage divider with  $R_1 = R_2$  such that  $V_{out} = \frac{1}{2}V_{in}$ . At high frequencies the circuit behaves as if the voltage source is perfect with zero source resistance. Hence a possible circuit is:



The 3 dB frequency  $\omega_{3 \text{ dB}}$  is for  $\omega_{3 \text{ dB}} = \frac{1}{RC} = \omega_0$ . Hence once a value for  $R$  was chosen, the capacitor value is  $C = \frac{1}{\omega_0 R}$ .

**Exercise.** This is the solution to exercise 1.42 in the book.

**Solution.** According to the numbering the high-pass filter is the first stage and the low-pass filter is the second stage:



The requirement for a 1:10 ratio of source to load impedance implies that  $R_2 = 10R_1$ . We have that:

$$\omega_1 = \frac{1}{R_1 C_1} \quad \omega_2 = \frac{1}{R_2 C_2}$$

$R_2$  value is selected based on the impedance of the load downstream of the bandpass filter. Then  $C_2$  is selected as:

$$C_2 = \frac{1}{\omega_2 R_2}$$

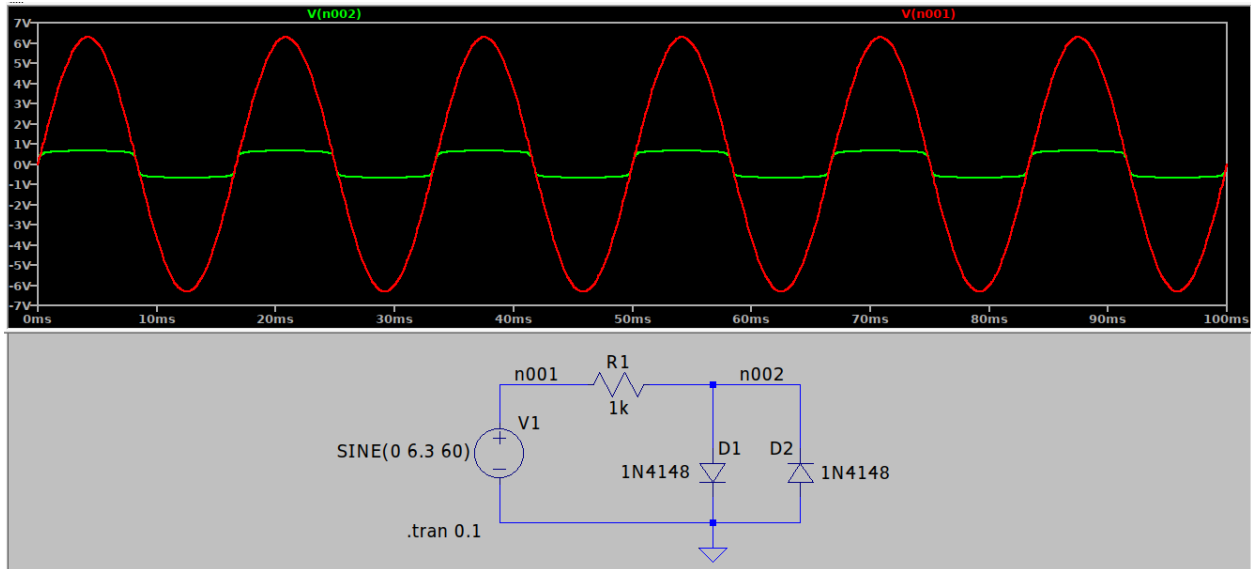
and then we select  $C_1$  as:

$$C_1 = \frac{1}{\omega_1 R_1} = \frac{10}{\omega_1 R_2}$$

**Exercise.** This is the solution to exercise 1.43 in the book.

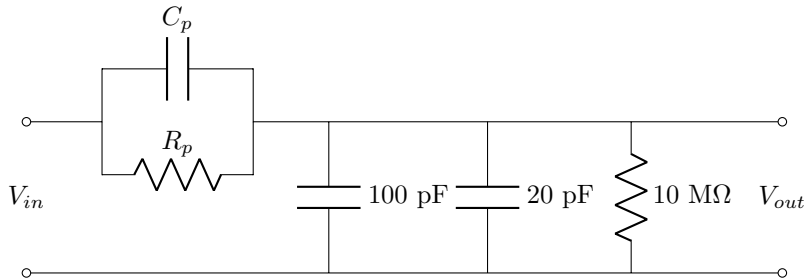
**Solution.** The circuit is just a voltage clamping circuit. It clamps the output  $V_{out}$  between -0.6V and +0.6V.

We can simulate the circuit with the schematic from [here](#) and the plot [here](#):



**Exercise.** This is the solution to exercise 1.44 in the book.

**Solution.** The circuit is:



The parallel capacitors are equivalent to another capacitor with  $C_s = 120$  pF. Hence the impedance at the scope input is:

$$Z_s = R_s - \frac{i}{\omega C_s} = 1 \text{ M}\Omega - \frac{i}{\omega 120 \text{ pF}}$$

The probe parallel RC results in impedance:

$$Z_p = R_p - \frac{i}{\omega C_p}$$

Therefore the input-output relationship can be written:

$$\frac{V_{out}}{V_{in}} = \frac{Z_s}{Z_s + Z_p} = \frac{1}{10}$$

for 20 dB attenuation. Thus:

$$Z_p = 9Z_s$$

and therefore:

$$\begin{aligned} R_p &= 9R_s \\ C_p &= \frac{C_s}{9} \end{aligned}$$

or

$$R_p = 9 \text{ M}\Omega \quad C_p = 13.33 \text{ pF}$$

**Exercise.** This is the solution to exercise 2.1 in the book.

**Solution.** If  $V_{led}$  is the voltage drop across the LED then the LED current must be (with the given collector resistor of  $330 \Omega$ ):

$$I_{led} = 10 - 3V \text{ mA}$$

The current should be much less than 10 mA so that the LED is not too bright but much larger than 1-2 mA so that the LED is not too faint. For this constraint to hold then the LED must be a red, amber or green (GaP) led. If  $V=1.7 \text{ V}$  then  $I_{led} = I_C = 5 \text{ mA}$ .

Assuming a B to E voltage drop of a typical diode of  $0.6 \text{ V}$  then  $V_B = 0.6 \text{ V}$  and thus:

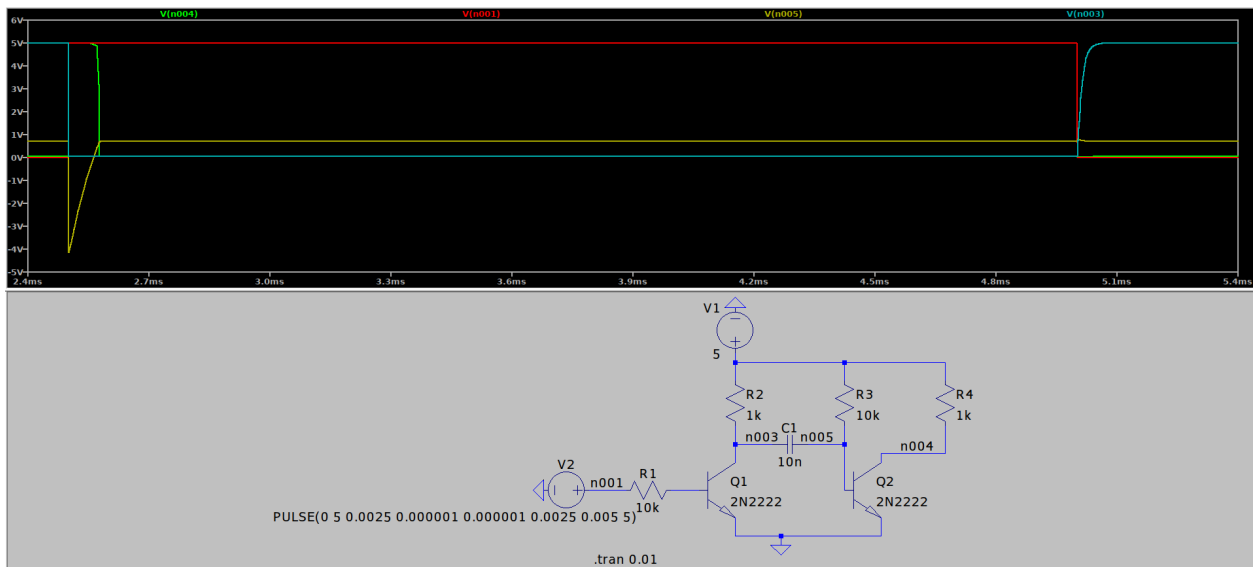
$$I_B = \frac{3.3 - 0.6}{10000} = 0.27 \text{ mA}$$

Hence:

$$\beta = \frac{I_C}{I_B} = \frac{5}{0.27} = 18.5$$

**Exercise.** This is the solution to exercise 2.2 in the book.

**Solution.** Spice simulation of the circuit shows the following behavior ([schematic](#) and [plotting](#)):



Before Q1 is saturated,  $V_{B,Q2} = 0.6\text{V}$  and  $I_{B,Q2} = 0.5 \text{ mA}$  Q2 is saturated and  $V_{C,Q2} = 0\text{V}$ .  $V_{C,Q1} = 5\text{V}$  and hence there's a  $4.4\text{V}$  drop across the capacitor  $\Delta V_{n003-n005}$  and the capacitor is charged.

When V2 is turned on this immediately saturates Q1 as  $I_{B,Q1} = \frac{5-0.6}{10k} = 0.44$  mA. and  $I_{C,Q1} = \frac{5}{1k} = 5$  mA and most likely  $\beta > 12$  and thus its collector drops from 5V to 0V. The drop in potential at the positive plate of C1 from 5V to 0V results in the potential at the negative plate of C1 to drop from 0.6V to -4.4V. This implies that the base-emitter diode of Q2 becomes unbiased and current starts being diverted from flowing to base of Q2 to flowing into capacitor to raise  $V_{B,Q2}$ .

The current charging the capacitor is:

$$I(t) = C \frac{dV(t)}{dt}$$

and

$$V_{CC} - I(t)R_3 = V(t)$$

This holds while  $V(t) < 0.6$  V and the base-emitter diode of Q2 is unbiased, hence no current flows into the base of Q2. Therefore:

$$\frac{dV(t)}{dt} + \frac{V(t)}{R_3 C} = \frac{V_{CC}}{R_3 C}$$

The solution is the sum of the homogenous solution and a particular solution. We get:

$$V(t) = V_{CC} + V_0 e^{-\frac{t}{R_3 C}}$$

Since  $V(0) = -4.4$ V and  $V_{CC} = 5$ V then  $V_0 = -9.4$ V. Therefore the pulse resets when  $V(t) = 0.6$ V and Q2 turns on again. Therefore:

$$0.6 = 5 - 9.4 e^{-\frac{t_{pulse}}{R_3 C}}$$

or

$$\frac{t_{pulse}}{R_3 C} = 0.759$$

or

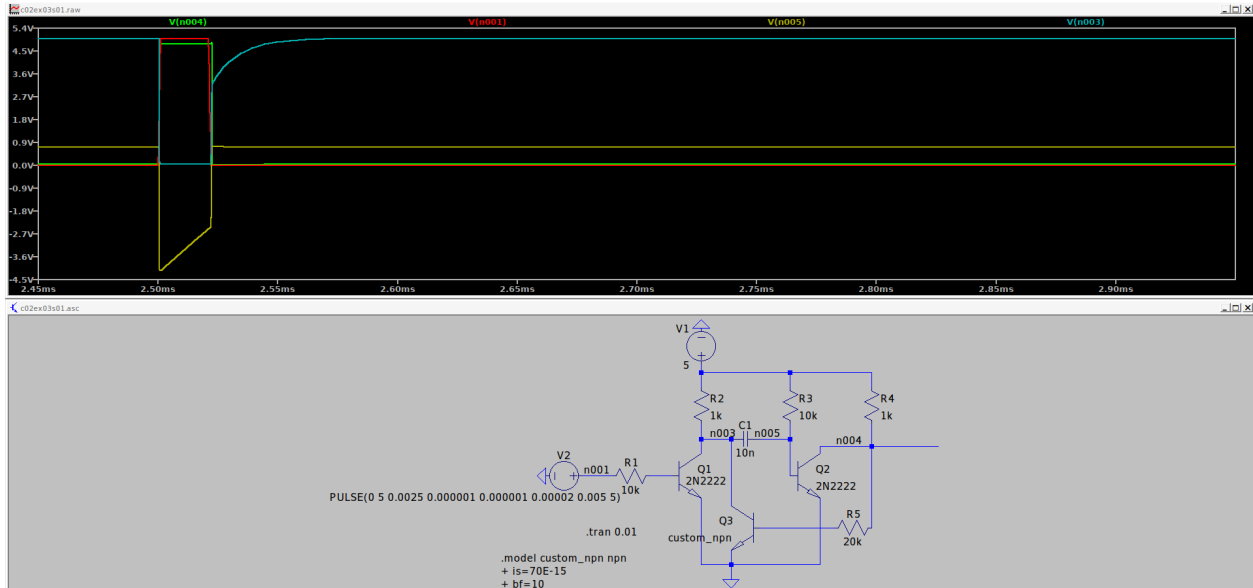
$$t_{pulse} = 0.759 R_3 C$$

For the values  $C = 10$ nF and  $R_3 = 10$ k $\Omega$ :

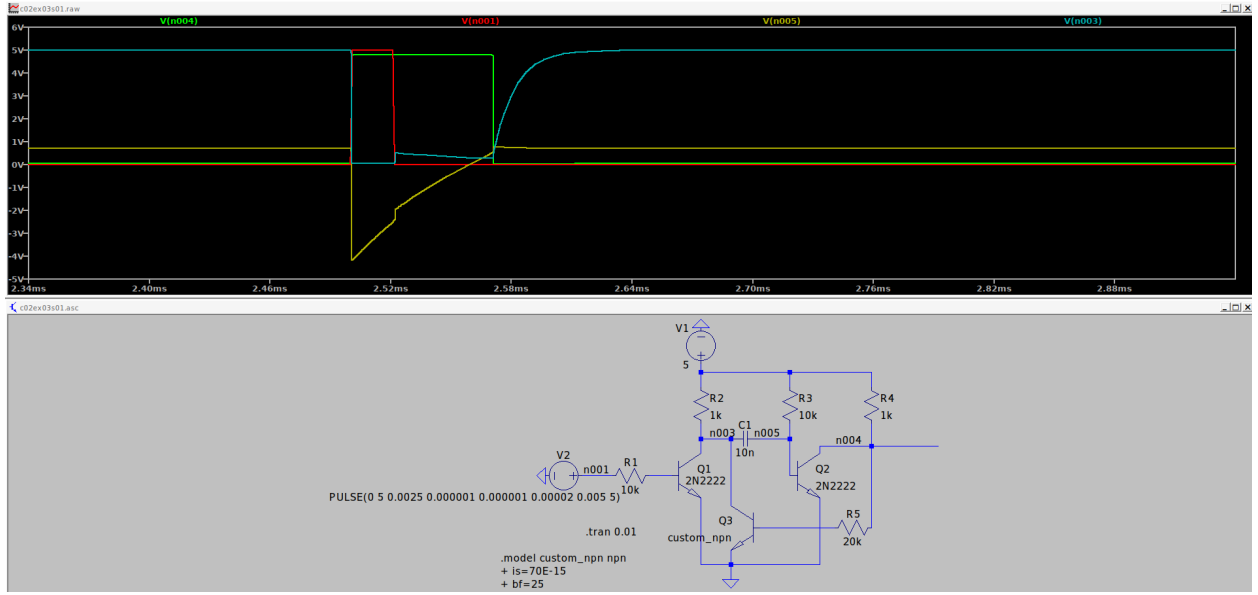
$$t_{pulse} = 0.759 \cdot 10^{-4} \text{ s} = 76 \mu\text{s}$$

**Exercise.** This is the solution to exercise 2.3 in the book.

**Solution.** Spice simulation of the circuit shows the following behavior ([schematic](#) and [plotting](#)):



when transistor  $\beta$  value is below the minimum required ( $\beta = 10$ ) and when it is above that minimum ( $\beta = 25$ ):



Notice that in the first case the output pulse width follows the input pulse width (in this case some  $20 \mu\text{s}$ ) rather than obeying the duration determined by the  $R_3C$  time constant of  $76 \mu\text{s}$ . In the second case the output pulse width is as expected.

Now let's calculate the voltage of the output pulse and the minimum  $\beta$  for Q3.

During the output pulse  $V_{B,Q3} = 0.6\text{V}$  and Q2 is turned off. Therefore the current through R4 and R5 is:

$$I_{R4,R5} = I_{B,Q3} = \frac{V_{CC}}{R4 + R5} = \frac{5}{21000} = 0.238 \text{ mA}$$

Hence

$$V_{out} = V_{CC} - I_{R4,R5}R4 = 4.76 \text{ V}$$

While the input pulse is ongoing Q1 is saturated and it is irrelevant if Q3 is saturated yet or not. However, Q3 must reach saturation by the time the input pulse is reset. Then Q3 must be able to handle:

$$I_{C,Q3} = \frac{V_{CC}}{R2} = \frac{5}{1000} = 5 \text{ mA}$$

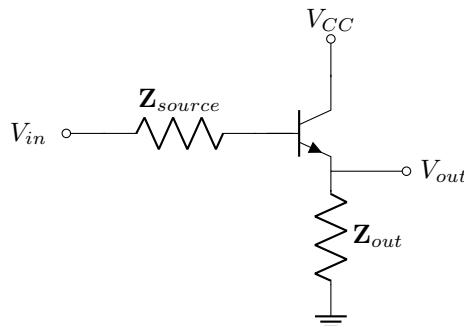
Therefore, minimally:

$$\beta_{Q3} = \frac{I_{C,Q3}}{I_{B,Q3}} = 21$$

---

**Exercise.** This is the solution to exercise 2.4 in the book.

**Solution.** Consider the generic circuit:



We follow the provided hint to compute the change in emitter voltage:

$$\Delta I_E = \frac{\Delta V_{out}}{Z_{out}}$$

We also have:

$$I_B = \frac{V_{in} - V_B}{Z_{source}}$$

hence:

$$\Delta I_B = \frac{\Delta V_B}{Z_{source}} = \frac{\Delta V_{out}}{Z_{source}}$$

Also:

$$\Delta I_C = \beta \Delta I_B = \beta \frac{\Delta V_B}{Z_{source}}$$

But:

$$\Delta I_E = \Delta I_B + \Delta I_C$$

and thus:

$$\frac{\Delta V_{out}}{Z_{out}} = \frac{\Delta V_{out}}{Z_{source}} + \beta \frac{\Delta V_{out}}{Z_{source}}$$

or

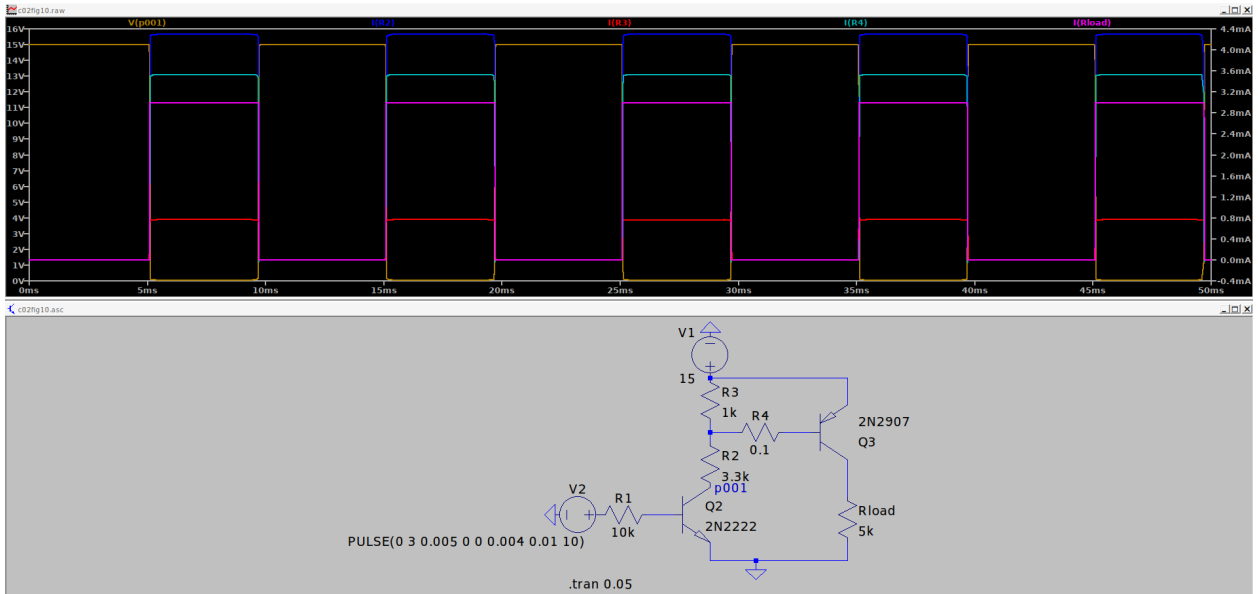
$$\frac{1}{Z_{out}} = \frac{\beta + 1}{Z_{source}}$$

or

$$Z_{out} = \frac{Z_{source}}{\beta + 1}$$

**Notes.** Extra derivations for chapter 2.

There is a discussion around figure 2.10 in the text that is investigated below.



Notice that when the V2 pin is on at 3V Q2 becomes saturated and hence the voltage at collector  $V_{C,Q2} = 0$ . This turns on Q3, whose base is now forward biased to about 14.4V. Therefore the current through R3 is about 0.6mA and the current thru R2 is  $14.4/3.3 = 4.4\text{mA}$ . The base current  $I_{B,Q3} = 3.7\text{mA}$ . The collector at Q3 sits at  $V_{C,Q3} = 15\text{V}$  and hence the current thru the  $5\text{k}\Omega$  load is 3mA. Indeed, as the text suggests the voltage divider would sit at 11.5V were it not for the forward biased BE diode of Q3 as  $11.5 = 15 \frac{3.3}{3.3+1}$ .