Lecture 8
Constraint-based search

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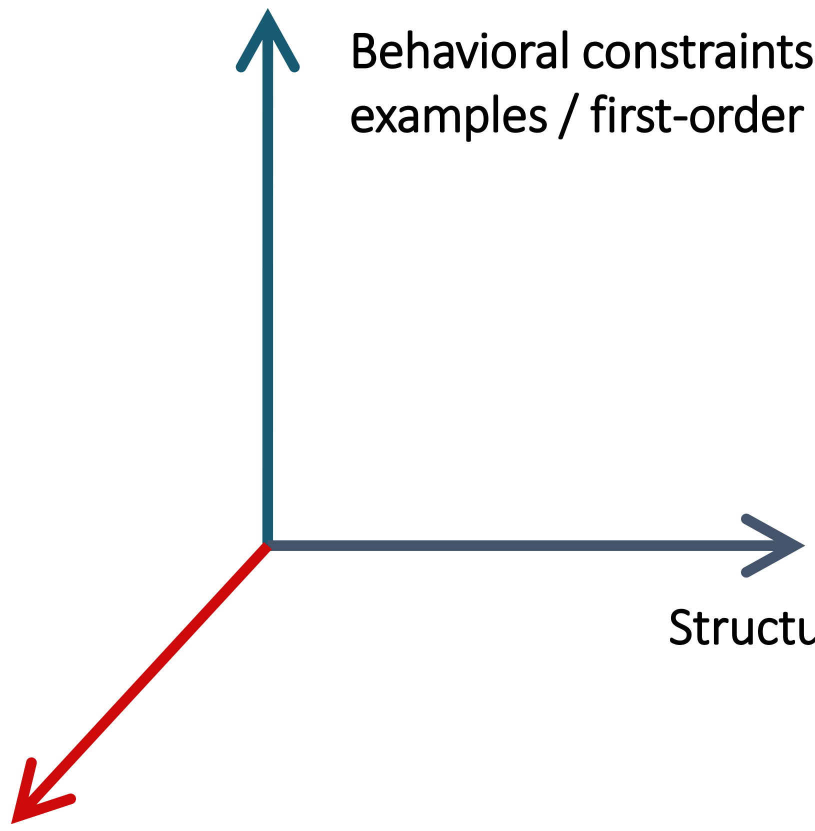
The problem statement



Behavioral constraints = examples / first-order formula

Structural constraints = we'll see...

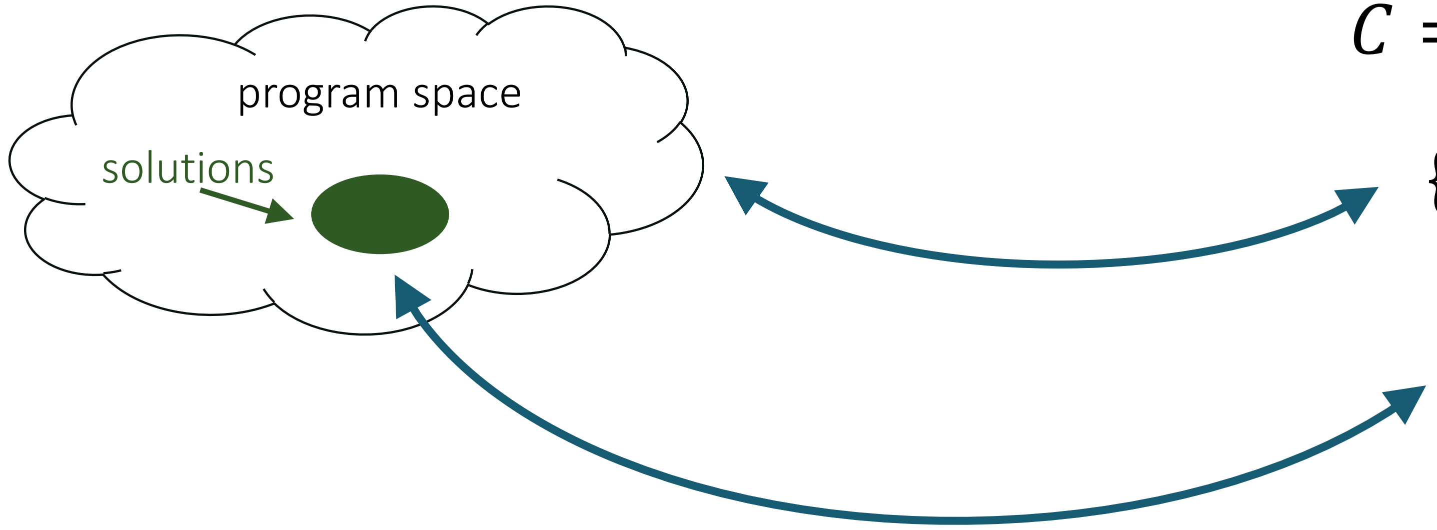
Search strategy?

Enumerative
Representation-based
Stochastic
Constraint-based

Constraint-based search

Idea: encode the synthesis problem as a SAT/SMT problem and let a solver deal with it

What is an encoding?



parameter space

$$C = \{c_1, \dots, c_n\}$$

$$\{C \mid \mathrm{wf}(C)\}$$

well-formedness constraint

$$\{C \mid \mathrm{wf}(C) \wedge \mathrm{sem}(C)\}$$

semantic correctness constraint

$$\mathrm{sem}(C) = \bigwedge_{\langle i, o\rangle \in E} \varphi(C, i, o)$$

C denotes a program that gives output *o* on input *i*

How to define an encoding

Define the parameter space $C = \{c_1, \dots, c_n\}$

- `decode : C → Prog` (might not be defined for all C)

Define a formula $\mathrm{wf}(c_1, \dots, c_n)$

- that holds iff `decode[C]` is defined

Define a formula $\varphi(c_1, \dots, c_n, i, o)$

- that holds iff `(decode[C])(i) = o`

Constraint-based search

```
constraint-based (wf, $\varphi$, E = [i → o]) {
  match SAT($\mathrm{wf}(C) \wedge \bigwedge_{\langle i,o\rangle \in E} \varphi(C, i, o)$) with
    Unsat -> return “No solution”
    Model C* -> return decode[C*]
}
```

Find a satisfying assignment for `c`$_1$`, ..., c`$_n$ (`i` and `o` are fixed)

SAT encoding: example

program space

`{x, x & 1}`

x is a two-bit word
`(x = x`$_h$`x`$_l$`)`

`E = [11 → 01]`

parameter space

$$C = \{c: \mathrm{Bool}\}$$

```
decode[0] → x
decode[1] → x & 1
```

$$\mathrm{wf}(c) \equiv \top$$

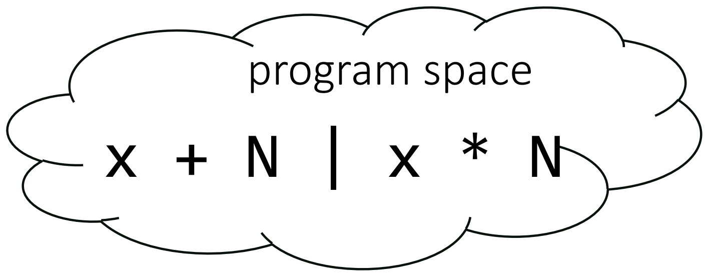
$$\varphi(c, i_h, i_l, o_h, o_l) \equiv (\neg c \;\Rightarrow\; o_h = i_h \wedge o_l = i_l)$$
$$\wedge\; (c \;\Rightarrow\; o_h = 0 \wedge o_l = i_l)$$

$$\mathrm{SAT}(\varphi(c, 1,1,0,1))$$

$$\mathrm{SAT}((\neg c \;\Rightarrow\; 0 = 1 \wedge 1 = 1) \wedge (c \;\Rightarrow 0 = 0 \wedge 1 = 1)) \xrightarrow{\text{SAT solver}} \texttt{Model \{c→1\}}$$

return `decode[1]` i.e. `x & 1`

SMT encoding: example



N is an in integer literal
x is an integer input

```
E = [2 → 9]
```

parameter space

$$C = \{c_{op}: \text{Bool}, c_N: \text{Int}\}$$

```
decode[0,N] → x + N
decode[1,N] → x * N
```

$$\text{wf}(c_{op}, c_N) \equiv \top$$

$$\varphi(c_{op}, c_N, i, o) \equiv (\neg c_{op} \Rightarrow o = i + c_N) \wedge (c_{op} \Rightarrow o = i * c_N)$$

$$\text{SAT}(\varphi(c_{op}, c_N, 2, 9))$$

$$\text{SAT}((\neg c_{op} \Rightarrow 9 = 2 + c_N) \wedge (c_{op} \Rightarrow 9 = 2 * c_N)) \xrightarrow{\text{SMT solver}} \texttt{Model } \{c_{op} \to 0,\ c_N \to 7\}$$

return decode[0,7] i.e. x + 7

What is a good encoding?

Sound

- if $\mathrm{wf}(C) \wedge \mathrm{sem}(C)$ then `decode[C]` is a solution

Complete

- if `decode[C]` is a solution then $\mathrm{wf}(C) \wedge \mathrm{sem}(C)$

Small parameter space

- avoid symmetries

Solver-friendly

- decidable logic, compact constraint

DSL limitations

Program space can be parameterized with a finite set of parameters

- Counterexample:

```
L ::= sort(L)  |  L[N..N]
    |  L + L  |  [N]  |  x
N ::= find(L,N)  |  0
```

- Workaround

```
L0 ::= x    L1 ::= sort(L0)  |  L0[N0..N0]
N0 ::= 0       |  L0 + L0  |  [N0]  |  L0
            N1 ::= find(L0,N0)  |  N0
```

Program semantics $\varphi(C, i, o)$ is expressible as a (decidable) SAT/SMT formula

- Counterexample

Brahma

Idea: encode the space of loop-free (bit-vector) programs as an SMT constraint

Brahma encoding: take 1

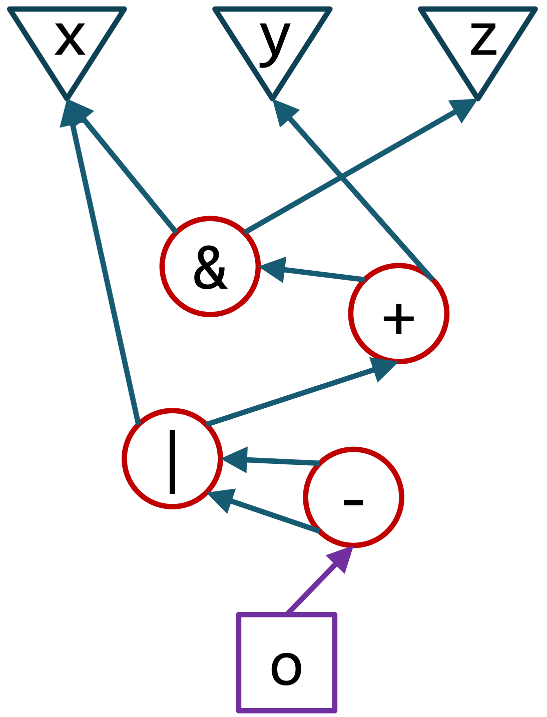
program = DAG

I

inputs

F

components

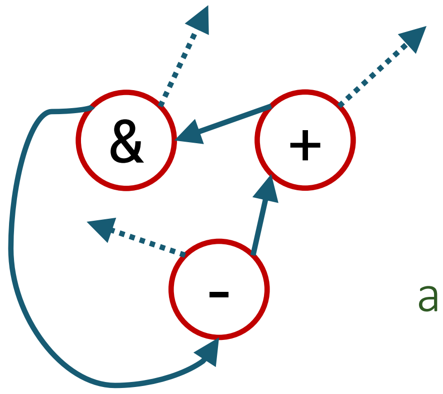


$V = I \cup F$ nodes

parameter space

$$C = \{c_o : V\} \cup \bigcup_{f \in F} \{c_1^f, c_2^f : V\}$$

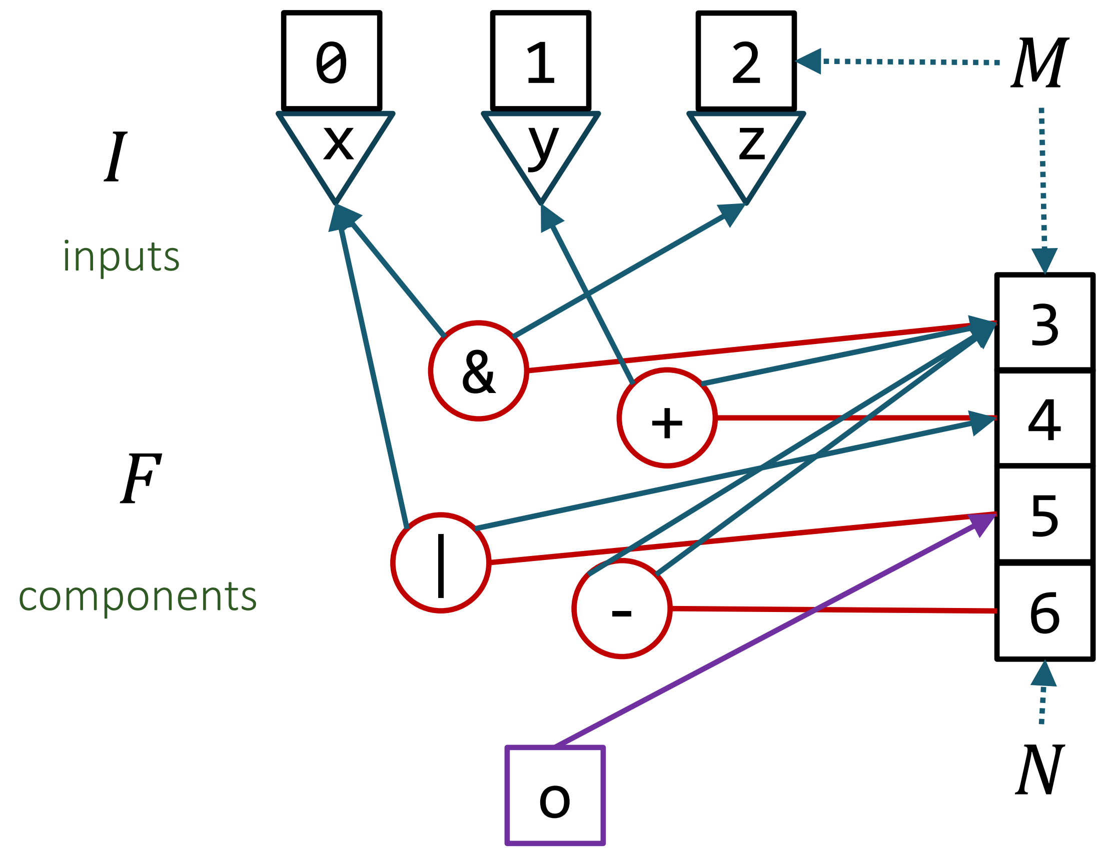
$\mathrm{wf}(C) \equiv ?$



hard to encode acyclicity!

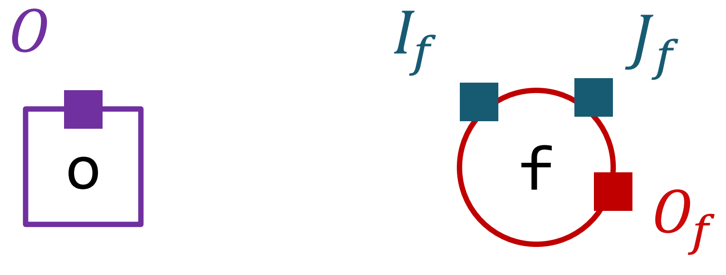
Brahma encoding: take 2

program = DAG



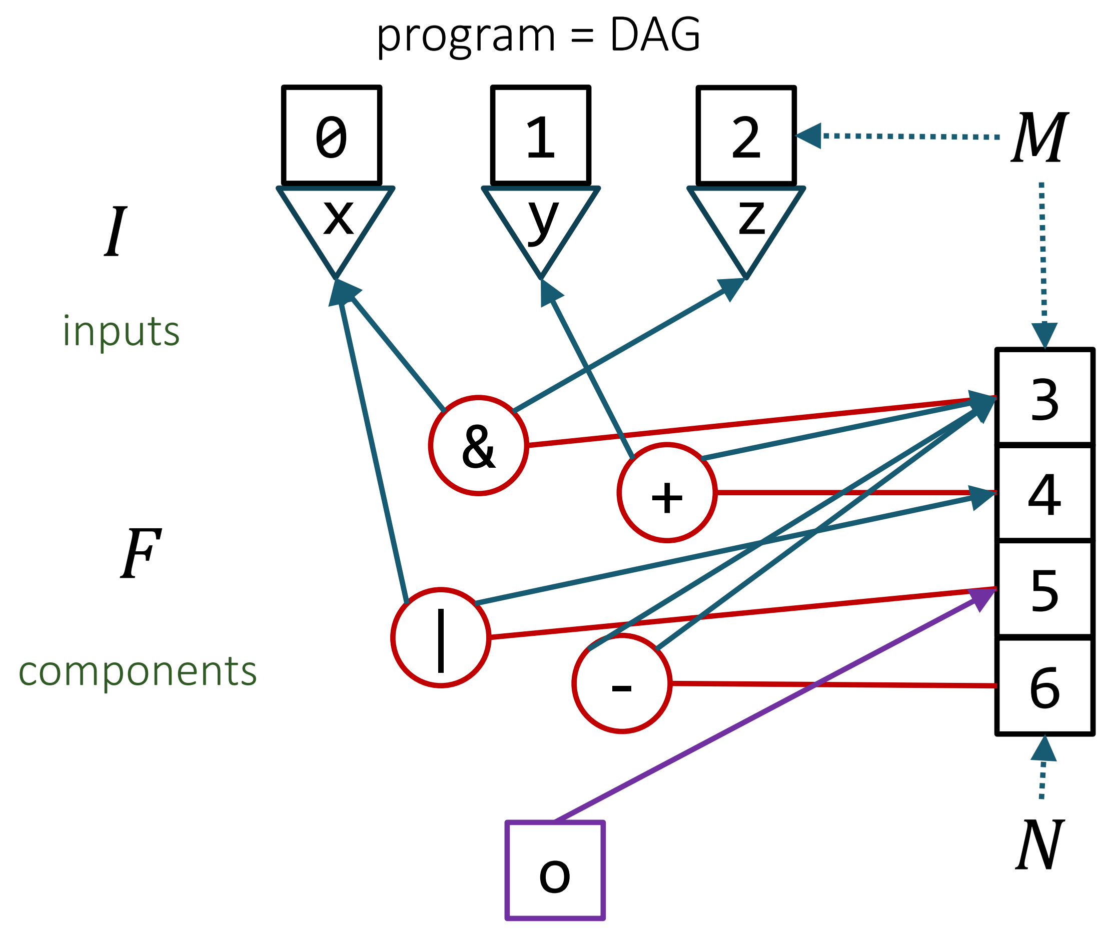
parameter space

$$C = \{c_O : \text{Int}\} \cup \bigcup_{f \in F} \{c_{O_f}, c_{I_f}, c_{J_f} : \text{Int}\}$$



$$\text{wf}(C) \equiv c_O \in M \ \wedge \bigwedge_{f \in F} c_{O_f} \in N \wedge c_{I/J_f} \in M$$

Brahma encoding: take 2



parameter space

$$C = \{c_O : \text{Int}\} \cup \bigcup_{f \in F} \{c_{O_f}, c_{I_f}, c_{J_f} : \text{Int}\}$$

$$T = \bigcup_{f \in F} \{I_f, J_f, O_f\}$$

$$\varphi(C, I, O) \equiv \exists T. \bigwedge_{f \in F} O_f = F(I_f, J_f)$$

$$\wedge \bigwedge_{x,y \in T \cup I \cup \{O\}} c_x = c_y \Rightarrow x = y$$

Brahma: contributions

SMT encoding of program space

- sound?
- complete?
- solver-friendly?

SMT solver can guess constants

- e.g. 0x55555555 in P23

Brahma: limitations

Requires component multiplicities

- If we didn't have multiplicities, where would their encoding break? How could we fix it?
- What happens if user provides too many? too few?
- What's the alternative to including dead code?

Requires *precise* SMT specs for components

- What happens if we give an over-approximate spec?

No loops, no types, no ranking

Brahma: questions

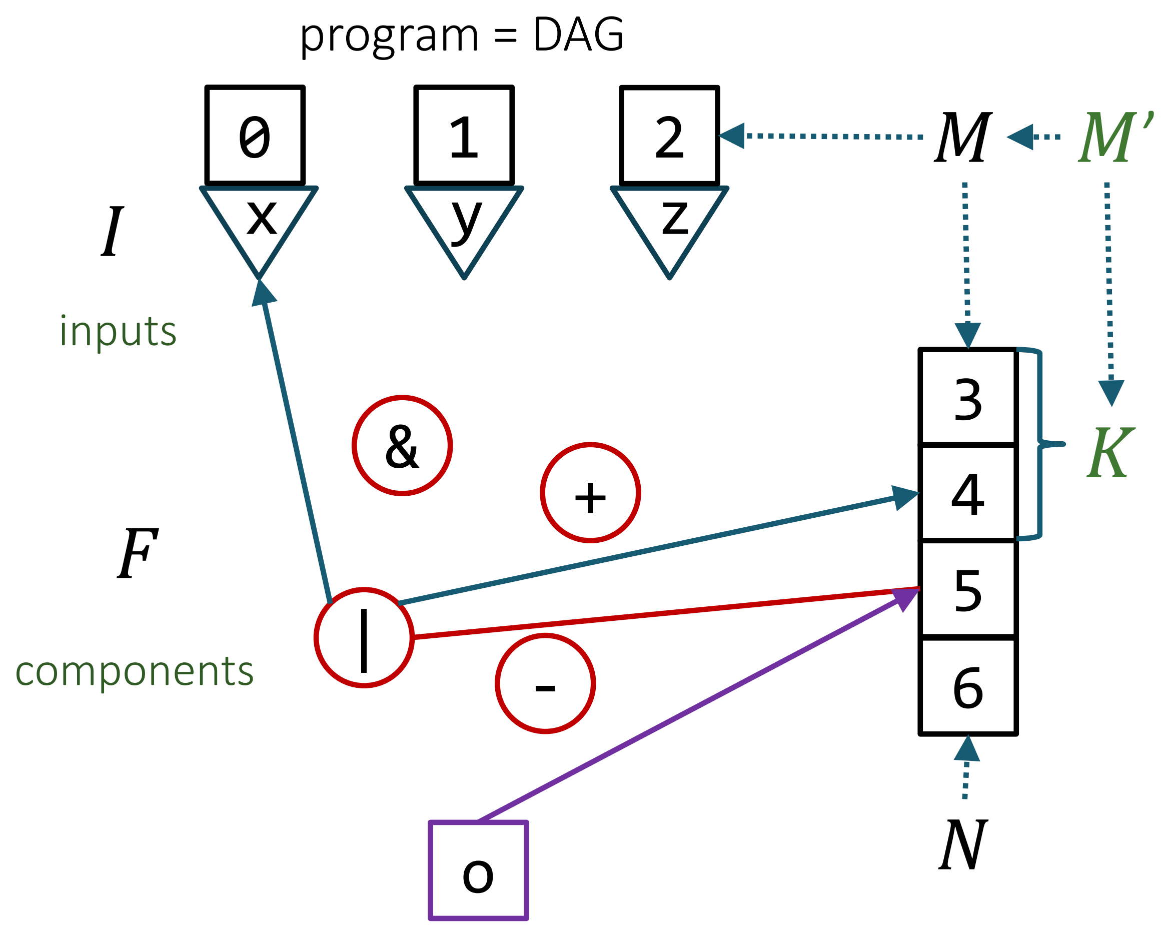
Behavioral Constraints? Structural Constraints? Search Strategy?

- First-order formula
- A multiset of components + straight-line program
- Constraint based + CEGIS

Can we represent these structural constraints as a grammar?

- Yes and no
- No because grammars cannot encode multiplicities
  - also: you can have let-bindings in SyGus but CFG cannot encode well-formedness
- Yes because the set is finite, so we can simply enumerate all possible programs
  - but this is not useful for synthesis

Limit #components to K?



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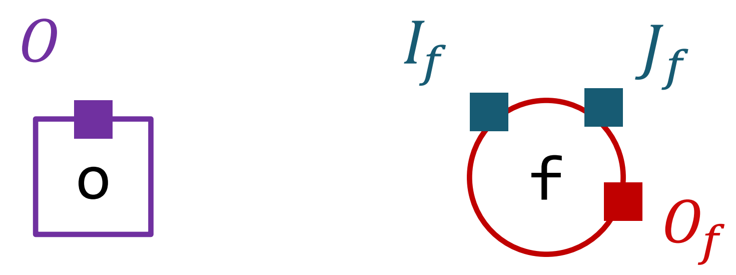
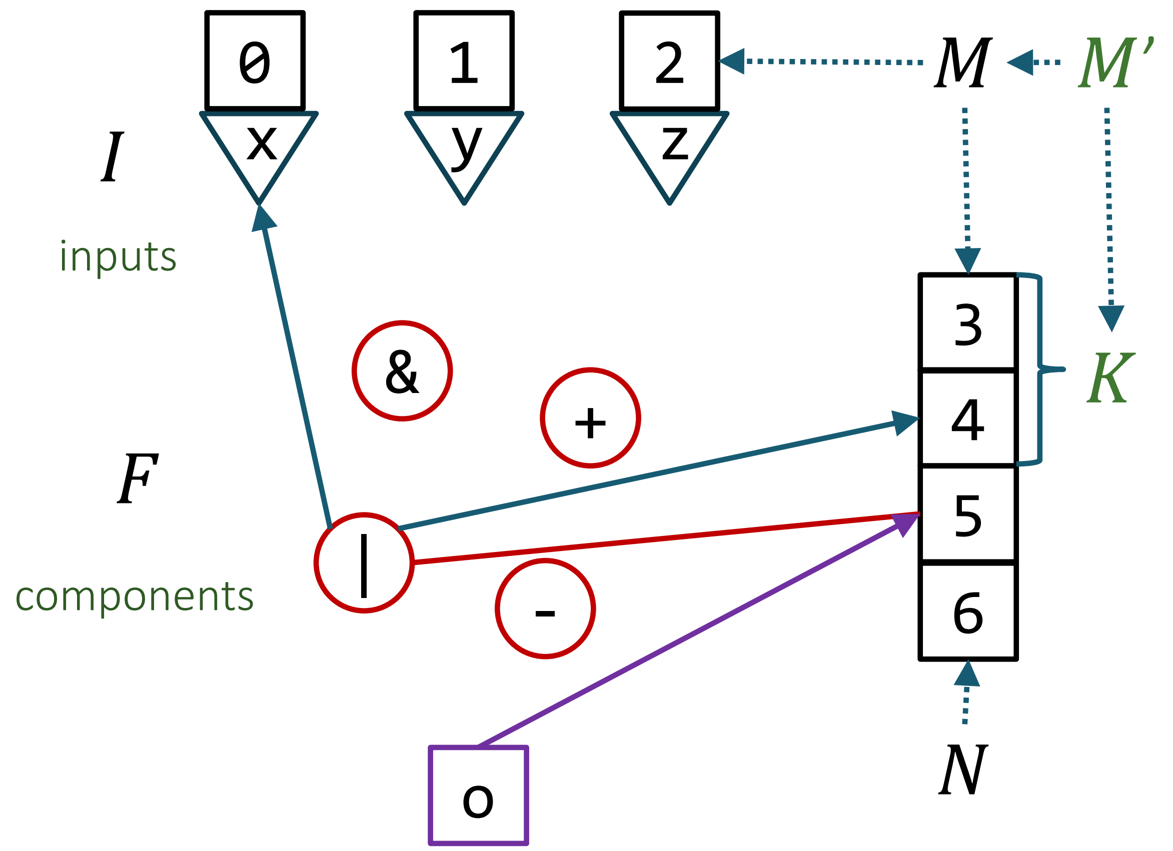


$$\mathrm{wf}(C) \equiv c_O \in \overset{M'}{\cancel{M}} \wedge \bigwedge_{f \in F} c_{O_f} \in \overset{K}{\cancel{N}} \wedge c_{I/J_f} \in \overset{M'}{\cancel{M}}$$

$$\wedge \bigwedge_{f,g \in F, f \not\equiv g} c_{O_f} \neq c_{O_g} \wedge \bigwedge_{f \in F} c_{I/J_f} < c_{O_f}$$

Limit #components to K?

program = DAG



parameter space

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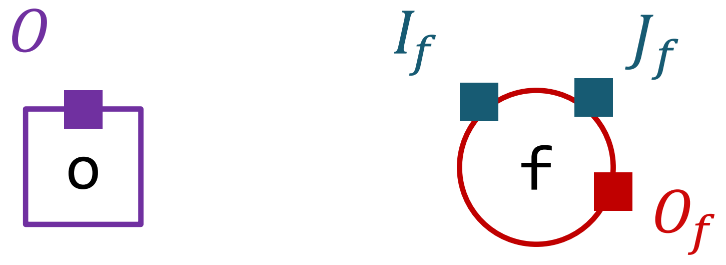
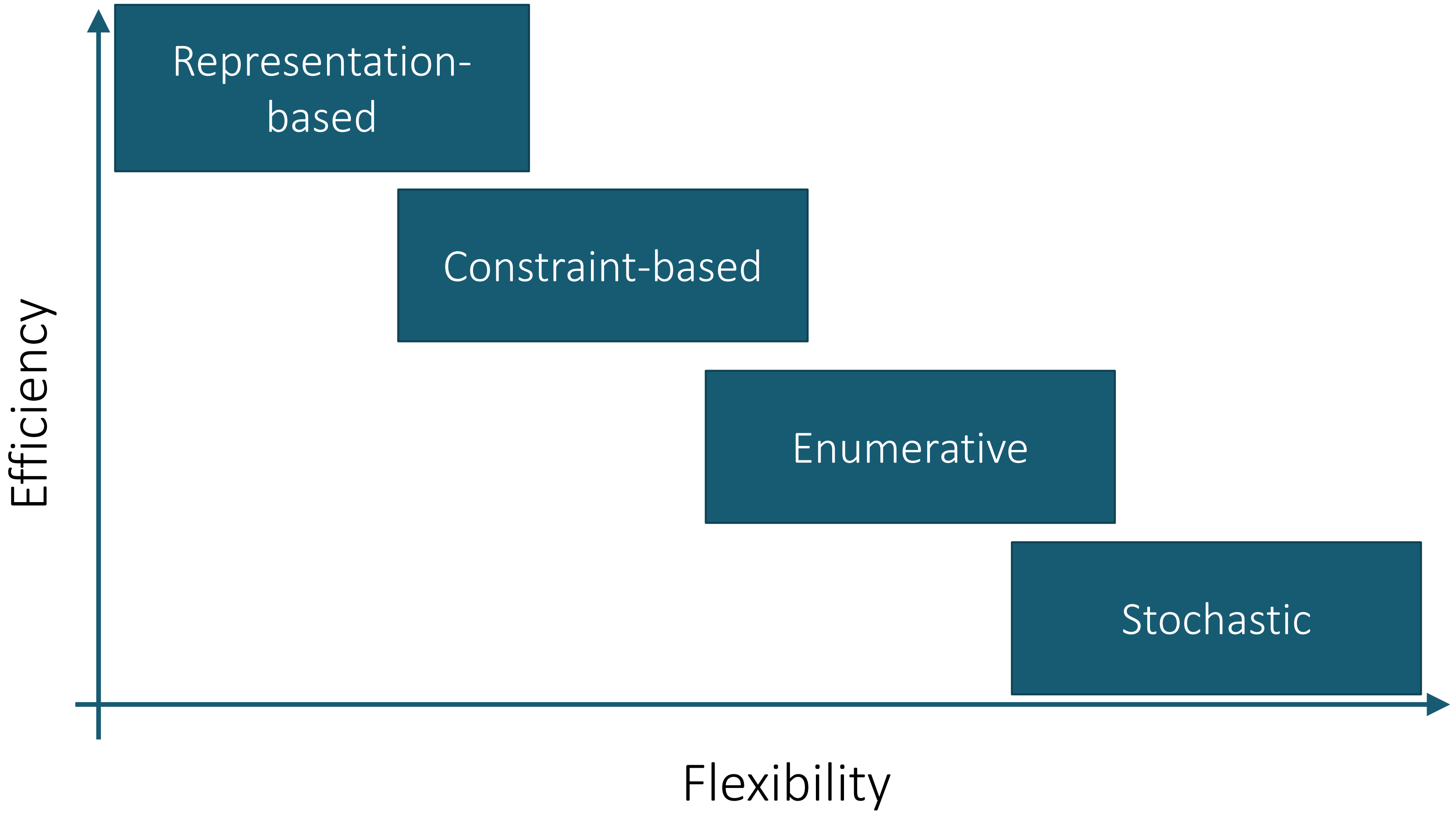


$$\text{wf}(C) \equiv c_O \in \overset{M'}{\cancel{M}} \wedge \bigwedge_{f \in F} c_{O_f} \in N \wedge c_{I/J_f} \in M$$

$$\wedge \bigwedge_{f,g \in F, f \neq g} c_{O_f} \neq c_{O_g} \wedge \bigwedge_{f \in F} c_{I/J_f} < c_{O_f}$$

Comparison of search strategies


Representation-based
Constraint-based
Enumerative
Stochastic
Efficiency
Flexibility