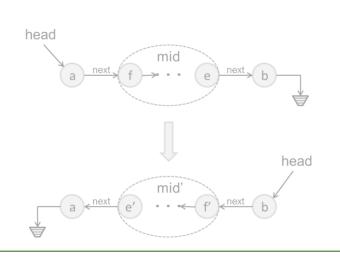
$\exists c \forall in \ Q(c, in)$

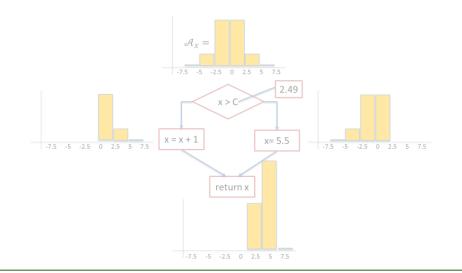
```
/* Average of x and y without using x+y (avoid overflow)*/
int avg(int x, int y) {
  int t = expr({x/2, y/2, x%2, y%2, 2 }, {PLUS, DIV});
  assert t == (x+y)/2;
  return t;
}
```

```
f_1
f_2
f_3
f_3
f_3
f_4
f_5
f_7
```

```
{
    s = n.succ;
    p = n.pred;
    p.succ = s;
    s.pred = p;
}
```

Module II: Synthesizing Complex Programs





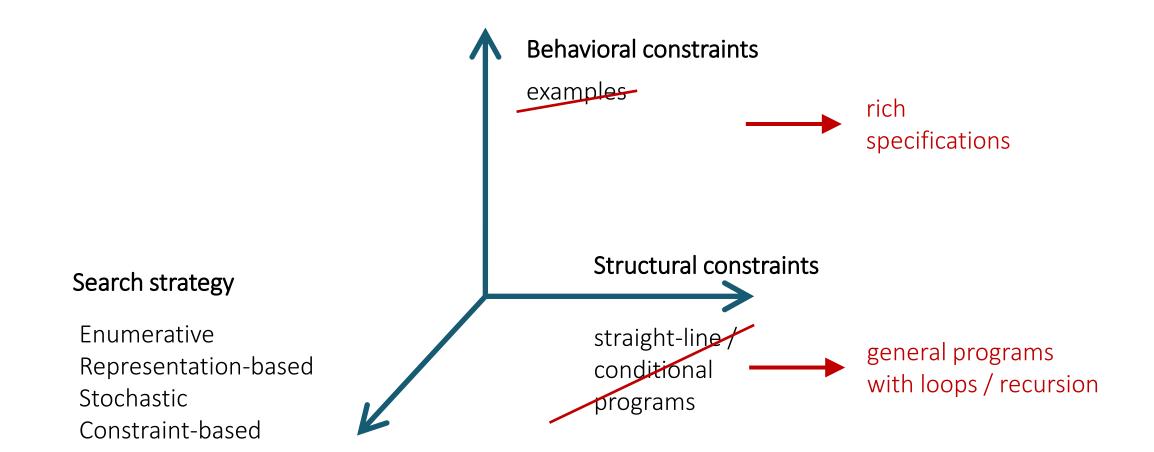


Sk[c](in)

Lecture 9 Specifications

Nadia Polikarpova

Module I vs Module II



Examples of rich specifications

Reference implementation

Assertions

Pre- and post-condition

Refinement type

Reference Implementation

Easy to compute the result, but hard to compute it efficiently or under structural constraints

```
bit[W] AES_round (bit[W] in, bit[W] rkey)
{
    ... // Transcribe NIST standard
}
bit[W] AES_round _sk (bit[W] in, bit[W] rkey) implements AES_round
{
    ... // Sketch for table lookup
}
```

Assertions

Hard to compute the result, but easy to check its desired properties

```
split_seconds (int totsec) {
  int h := ??;
  int m := ??;
  int s := ??;
  assert totsec == h*3600 + m*60 + s;
  assert 0 <= h && 0 <= m < 60 && 0 <= s < 60;
}</pre>
```

Pre-/post-conditions

Hard to compute the result but easy to express its properties in logic

```
sort (int[] in, int n) returns (int[] out)
requires n \ge 0
ensures \forall i \ j. \ 0 \le i < j < n \Rightarrow out[i] \le out[j]
\forall i. \ 0 \le i < n \Rightarrow \exists j. \ 0 \le j < n \land in[i] = out[j]
{
??
```

Refinement types

Same as pre-/post-conditions but logic goes inside the types

```
binary search tree
                                         red nodes have
data RBT a where
                                         black children
  Empty :: RBT a
  Node :: x: a ->
    black: Bool ->
                                  !black ==> isBlack
    left: { RBT {a
                     V < X
    right: { RBT \{a \mid x < v\}
                                  (!black ==> isBlack
                                                         v) &&
                 (blackHeight _v == blackHeight left)
    RBT a
                                                                        same number of
                                                                        black nodes on
insert :: x: a -> t: RBT a -> {RBT a | elems _v == elems t + [x]}
                                                                        every path to leaves
insert = ??
```

Why go beyond examples?

Might need too many

- Example: Myth needs 12 for insert_sorted, 24 for list_n_th
- Examples contain *too little* information
- Successful tools use domain-specific ranking

Output difficult to construct

- Example: AES cypher, RBT
- Examples also contain too much information (concrete outputs)

Need strong guarantees

• Example: AES cypher

Reasoning about non-functional properties

• Example: security protocols

Why is this hard?

```
gcd (int a, int b) returns (int c)
                                                                infinitely many inputs
  requires a > 0 \land b > 0
                                                               cannot validate by testing
  ensures a \% c = 0 \land b \% c = 0
             \forall d \cdot c < d \Rightarrow a \% d \neq 0 \lor b \% d \neq 0
  int x , y := a, b;
                                                            infinitely many paths!
  while (x != y) {
                                                            hard to generate constraints
     if (x > y) x := ??;
     else y := ??;
}}
```

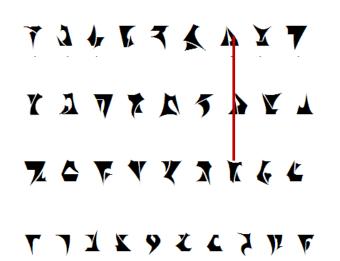
Why is this hard?

Synthesis from examples



validation was easy!

Synthesis from specifications



SEE IF YOU CAN FIND ANY KLINGON FRUIT!

validation is hard! (and search is still hard)

Module II

