Lecture 5
Representation-based Search

Nadia Polikarpova

This week

Topics:

- Representation-based search
- Stochastic search

Paper: Rishabh Singh: BlinkFill: Semisupervised Programming By Example for Syntactic String Transformations. VLDB’16

Projects:

- Proposals due Friday
- 1 page, PDF or Google Doc
- Upload to “Proposals” inside the shared Google Folder
- Doc name **must be** TeamN, where N is your team ID

The problem statement

Behavioral constraints = examples

```
[1,4,7,2,0,6,9,2,5]  →  [1,2,4,7,0]
[0] → [0]
[5,1] → [1,5,0]
```

Search strategy?

Enumerative
Representation-based
Stochastic
Constraint-based

Structural constraints = grammar

```
L ::= sort(L)  |  L[N..N]
    |  L + L  |  [N]  |  x
N ::= find(L,N)  |  0
```

Representation-based search

Idea:

1. build a data structure that compactly represents good parts of the program space
2. extract solution from that data structure

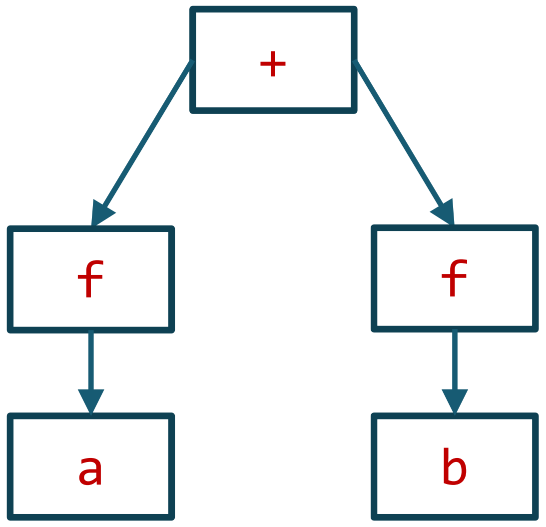
Compact term representation

Consider the space of 9 programs:

```
f(a) + f(a)    f(a) + f(b)    f(a) + f(c)
f(b) + f(a)    f(b) + f(b)    f(b) + f(c)
f(c) + f(a)    f(c) + f(b)    f(c) + f(c)
```

Can we represent this compactly?

- observation 1: same top level structure, independent subterms



Compact term representation

Consider the space of 9 programs:

```
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f(c) + f(a)   f(c) + f(b)   f(c) + f(c)
```
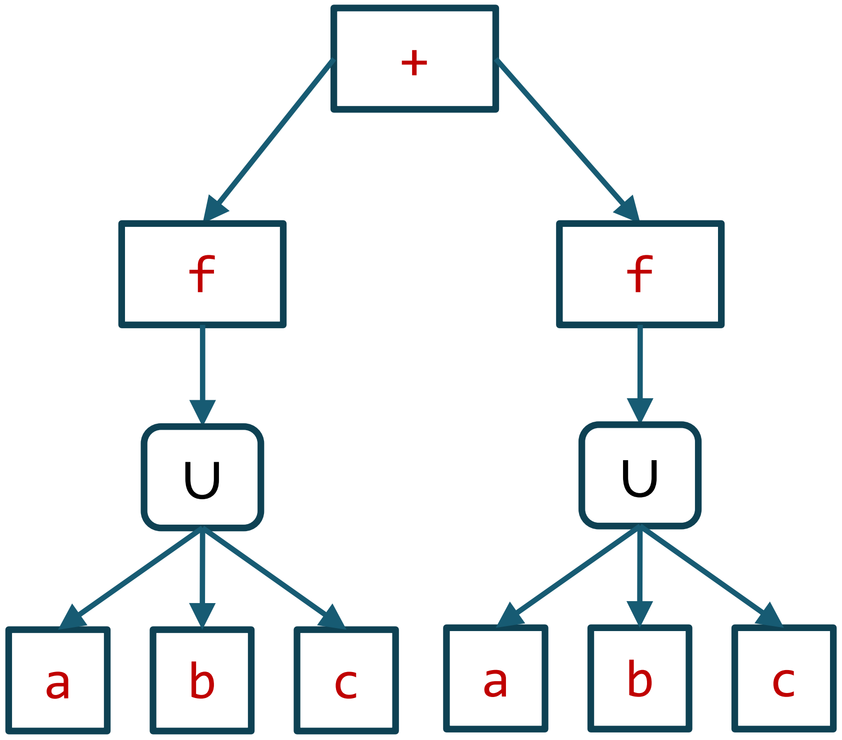

Can we represent this compactly?

- observation 1: same top level structure, independent subterms
- observation 2: shared sub-spaces



Compact term representation

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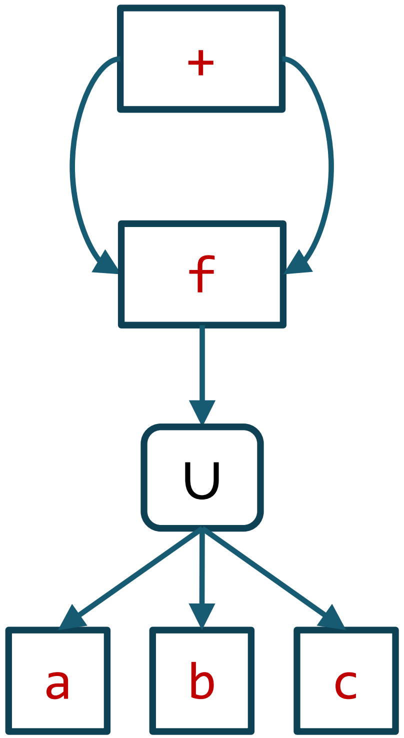
f(c) + f(a)   f(c) + f(b)   f(c) + f(c)
```

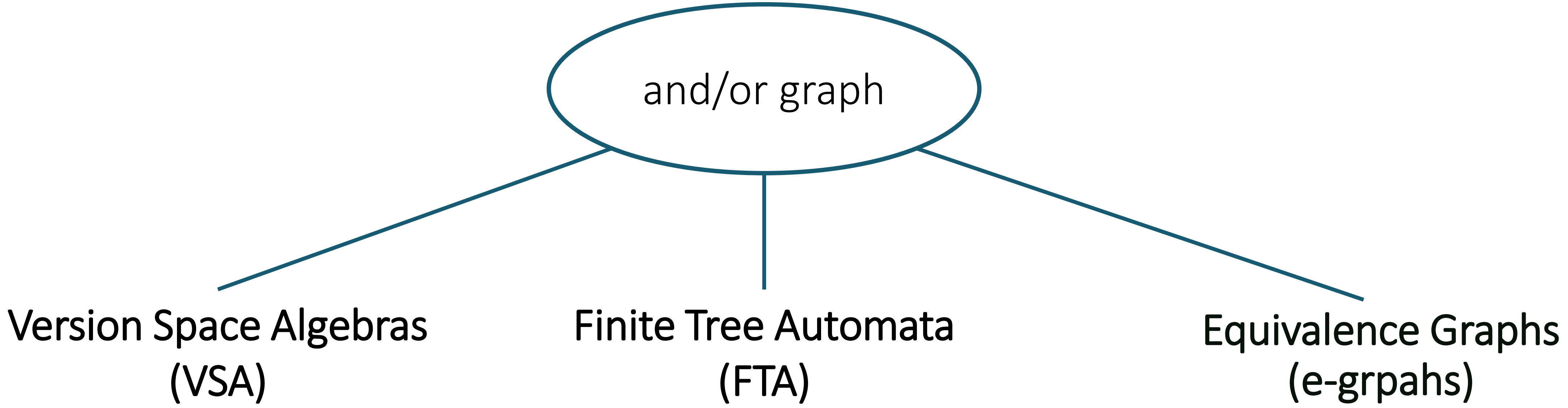
Can we represent this compactly?

- observation 1: same top level structure, independent subterms
- observation 2: shared sub-spaces

Key idea: use and-or graph!



Representation-based search



Version Space Algebra

Idea: build a graph that succinctly represents the space of *all* programs consistent with examples

- called a **version space**

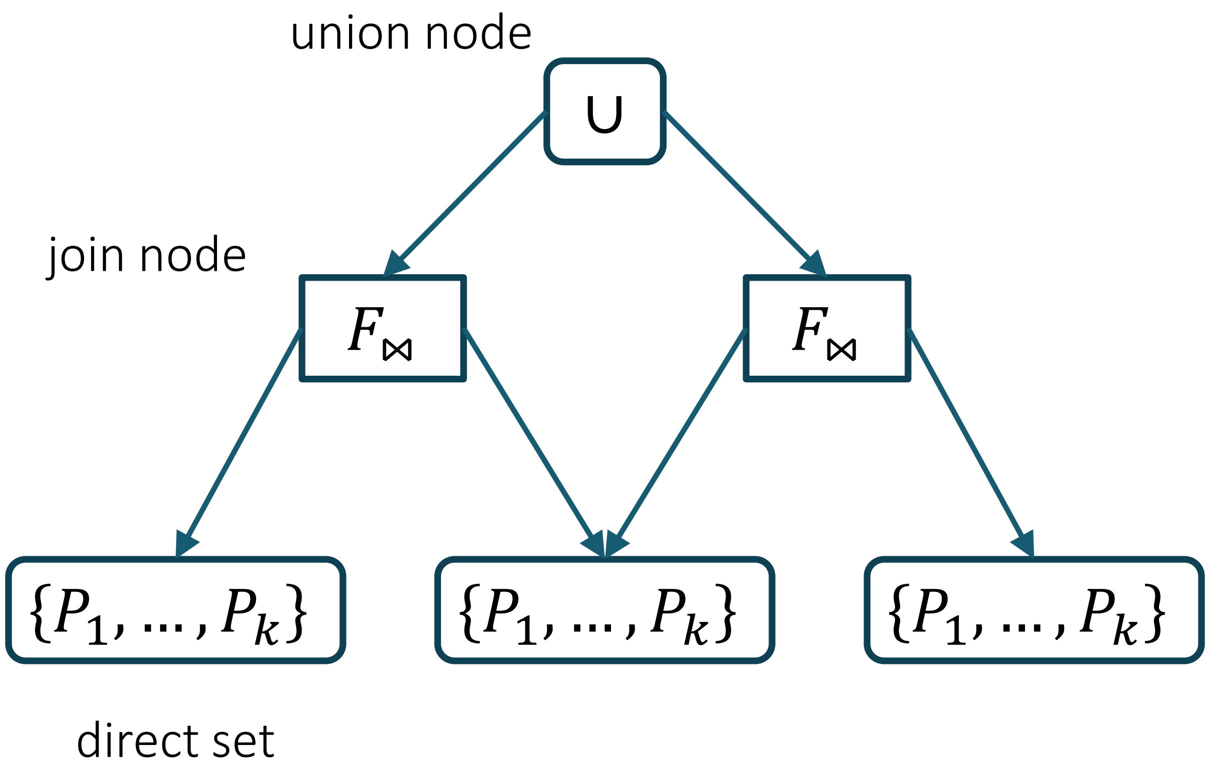
Operations on version spaces:

- learn <i, o> → VS
- $VS_1 \cap VS_2 \rightarrow VS$
- extract VS → program

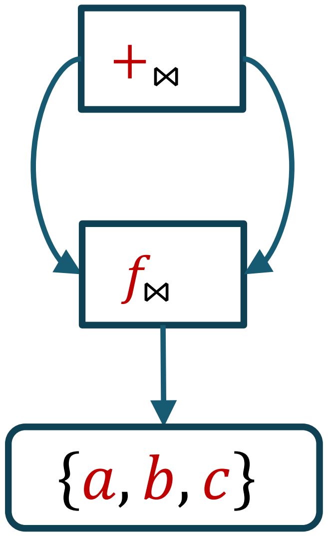
Algorithm:

1. learn a VS for each example
2. intersect them all
3. extract any (or best) program

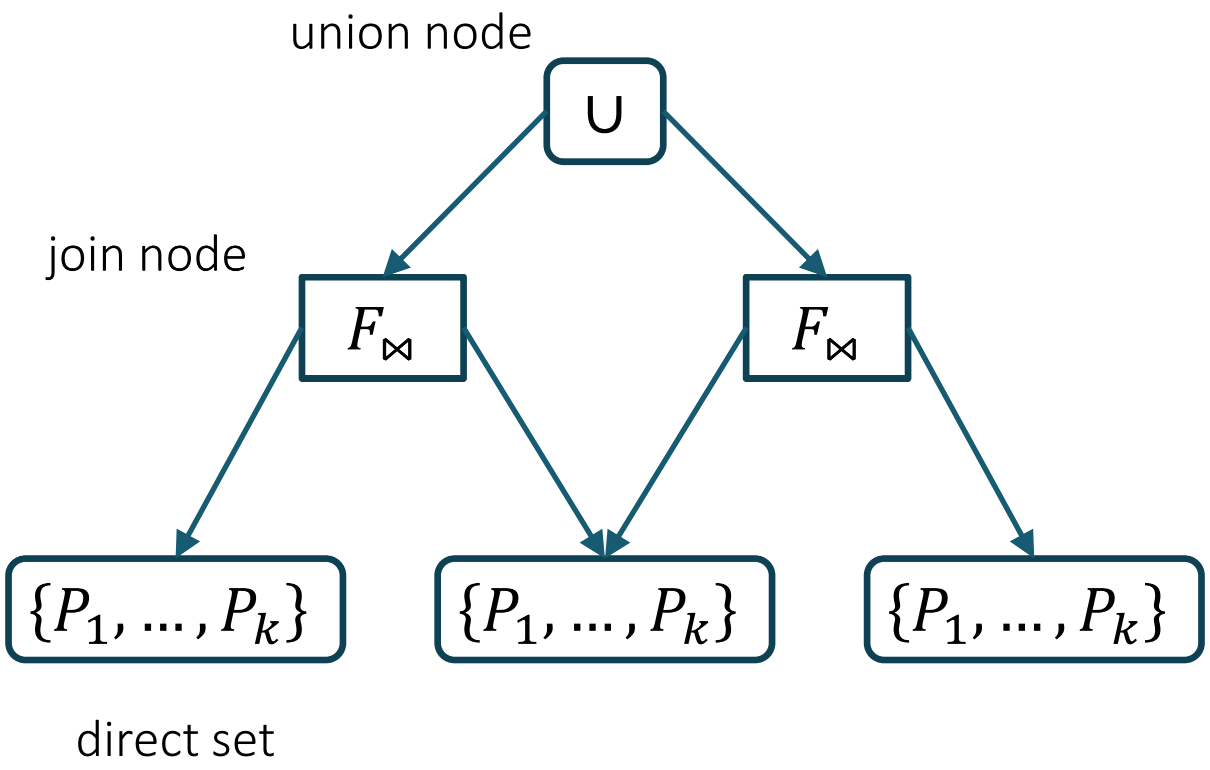
Version Space Algebra



example:



Version Space Algebra



Volume of a VSA (the number of nodes) $V(VSA)$

Size of a VSA (the number of programs) $|VSA|$

$$V(VSA) = O(\log|VSA|)$$

VSA-based search

Mitchell: *Generalization as search*. AI 1982

Lau, Domingos, Weld. *Version space algebra and its application to programming by example*. ICML 2000

Gulwani: *Automating string processing in spreadsheets using input-output examples.* POPL 2011.

- Follow-up work: BlinkFill, FlashExtract, FlashRelate, …
- generalized in the PROSE framework

FlashFill

[Gulwani '11]

Simplified grammar:

```
E ::= F | concat(F, E)              "Trace" expression
F ::= cstr(str) | sub(P, P)         Atomic expression
P ::= cpos(num) | pos(R, R)         Position expression
R ::= tokens(T1, ..., Tn)           Regular expression
T ::= C | C+                        Token expression
C ::= ws | digit | alpha | Alpha | $ | ^ | …
```

FlashFill: example

0 1 2 3 4 5 6 7 8 9 …

“Hello POPL 2023” → “POPL’2023”

“Goodbye PLDI 2021” → “PLDI’2021”

```
concat(
  sub(pos(ws, Alpha), pos(Alpha, ws)),
  concat(
    cstr(“’”),
    sub(pos(ws, digit), pos(digit, $))))
```

$$
\begin{aligned}
E &::= F \mid \mathtt{concat}(F, E) \\
F &::= \mathtt{cstr}(\mathit{str}) \mid \mathtt{sub}(P, P) \\
P &::= \mathtt{cpos}(\mathit{num}) \mid \mathtt{pos}(R, R) \\
R &::= \mathtt{tokens}(T_1, \ldots, T_n) \\
T &::= C \mid C+
\end{aligned}
$$

VSAs for Flashfill

Recall operations on version spaces:

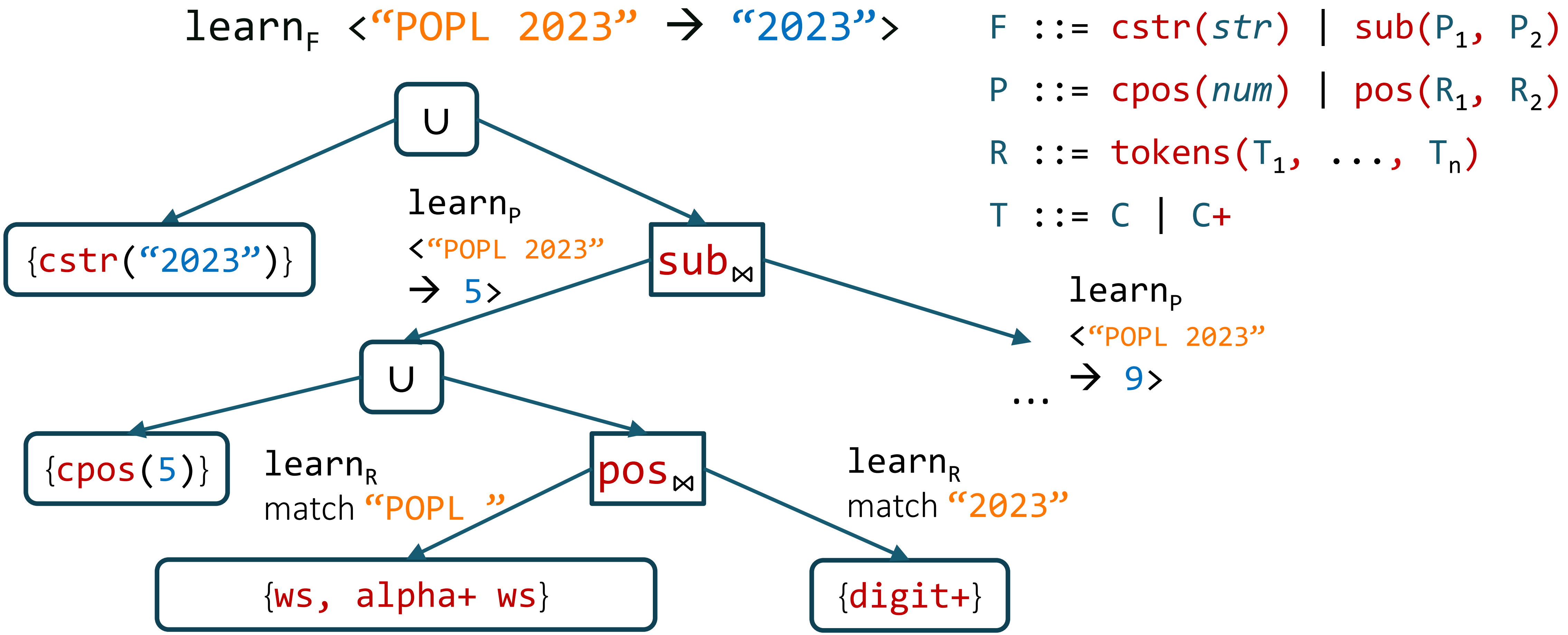
- `learn <i, o>` → `VS`
- $VS_1 \cap VS_2 \rightarrow VS$
- `extract VS` → `program`

How do we implement `learn`?

- define $\texttt{learn}_N$ `<i, o>` for every non-terminal `N`
- build VS top-down, propagating `<i, o>` the example

```
E ::= F | concat(F, E)
F ::= cstr(str) | sub(P, P)
P ::= cpos(num) | pos(R, R)
R ::= tokens(T1, ..., Tn)
T ::= C | C+
```

Learning atomic expressions

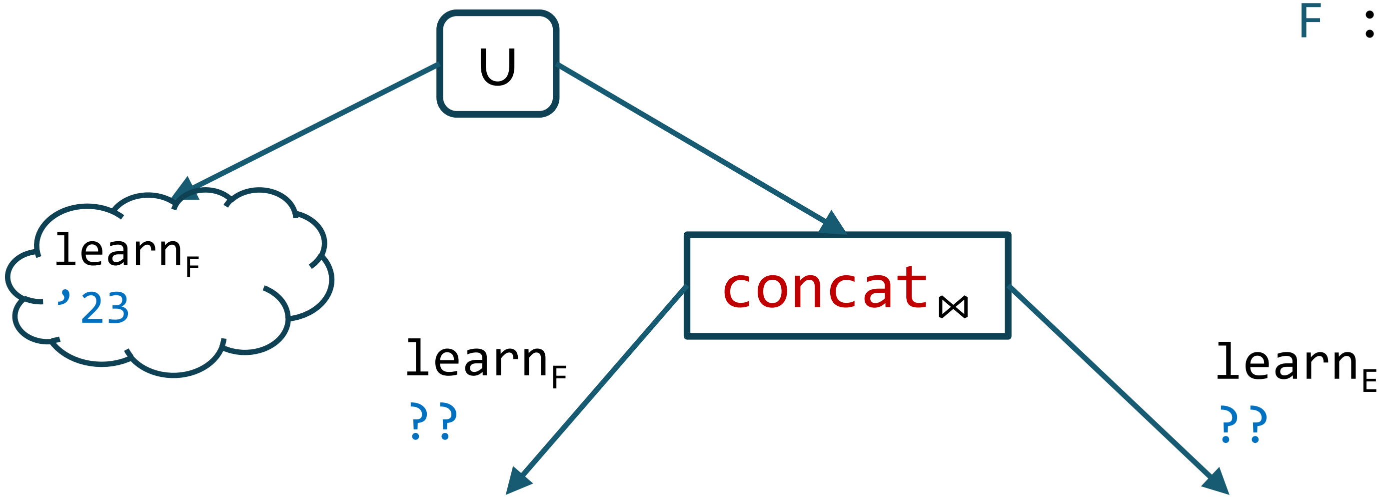


Learning trace expressions

learn$_E$ <"POPL 2023" → "'23">

E ::= F | concat(F, E)

F ::= ...



Learning trace expressions

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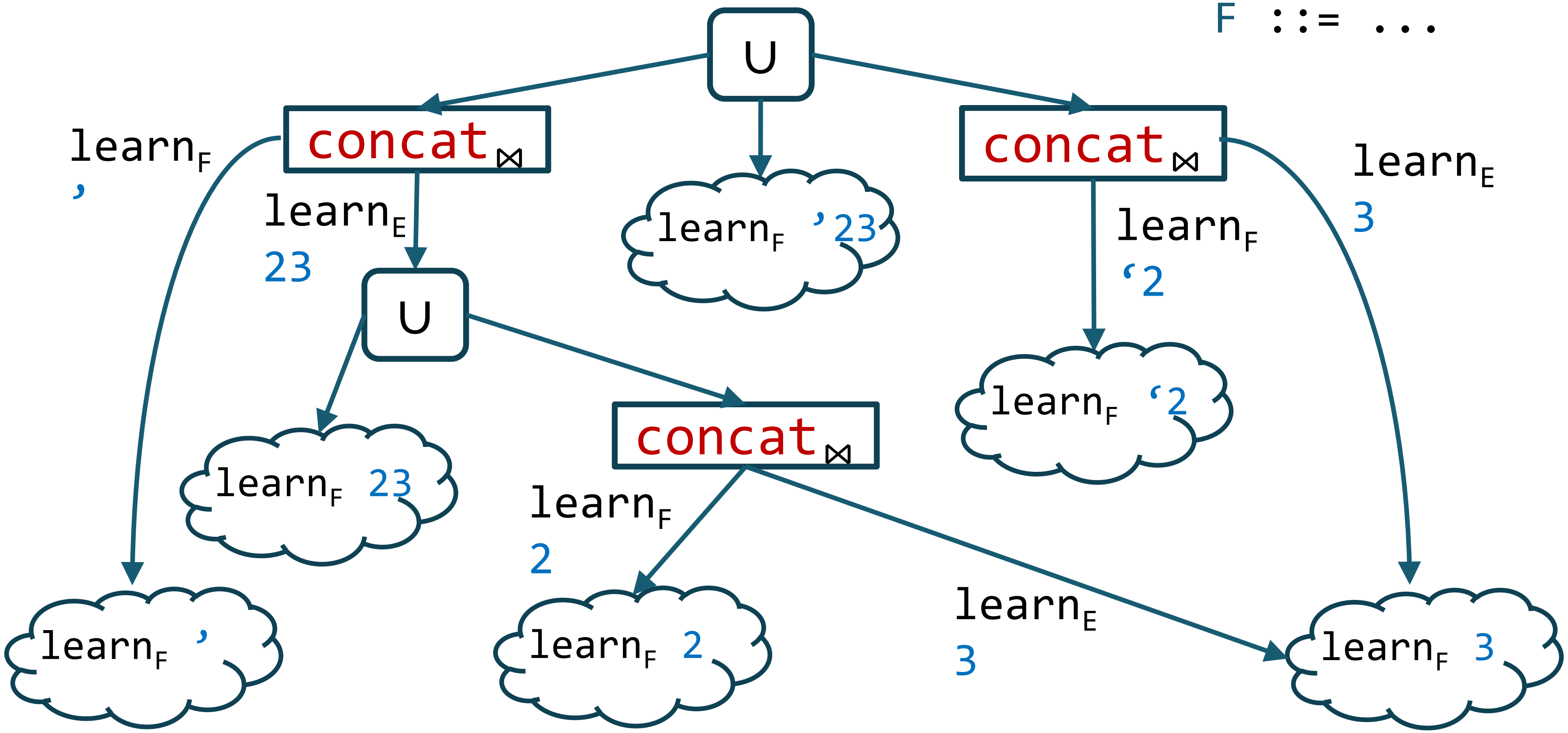


VSAs for Flashfill

Recall operations on version spaces:

- `learn <i, o> → VS`
- $VS_1 \cap VS_2 \rightarrow VS$
- `extract VS → program`

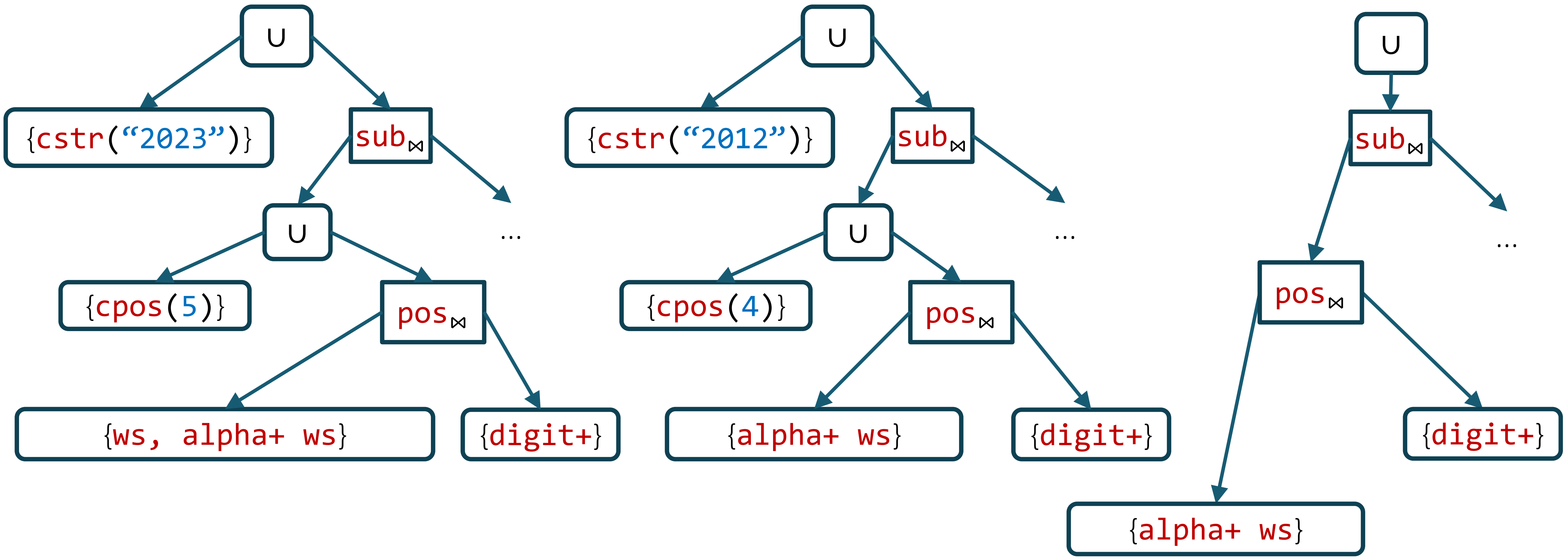
How do we implement intersection?

- top-down
- union: intersect all pairs of children
- join: intersect children pairwise

```
E ::= F | concat(F, E)
F ::= cstr(str) | sub(P, P)
P ::= cpos(num) | pos(R, R)
R ::= tokens(T1, ..., Tn)
T ::= C | C+
```

Intersection

“POPL 2023” → “2023”　　“ 3M 2012” → “2012”



VSAs for Flashfill

Recall operations on version spaces:

- `learn <i, o> → VS`
- $VS_1 \cap VS_2 \rightarrow VS$
- `extract VS → program`

How do we implement extract?

- any program: just pick one child from every union
- best program: shortest path in a DAG

```
E ::= F | concat(F, E)
F ::= cstr(str) | sub(P, P)
P ::= cpos(num) | pos(R, R)
R ::= tokens(T1, ..., Tn)
T ::= C | C+
```

Discussion

What do VSAs remind you of in the enumerative world?

- VSA learning ~ top-down search with top-down propagation

How are they different?

- Caching of sub-problems (DAG!)
- Can construct one per example and intersect
- This allows it to guess arbitrary constants!

Discussion

Why could we build a finite representation of all solutions?

- Could we do it for this language?

```
E ::= F + F
F ::= k | x
```

$k \in \mathbb{Z}$ + is integer addition

- What about this language?

```
E ::= E + 1 | x
```

DSL restrictions: efficiently invertible

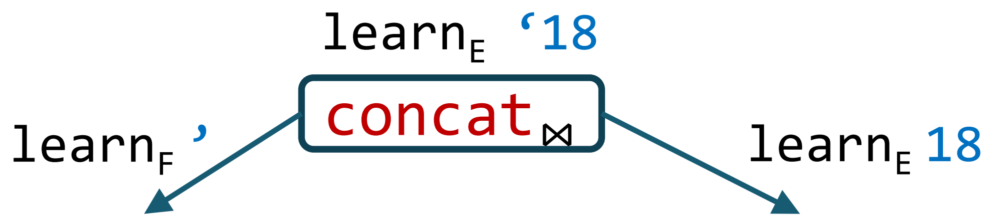
Every operator has a small, easily computable inverse

- Example when an inverse is small but hard to compute?

The space of sub-specs is finite

- either non-recursive grammar
- or finite space of values for the recursive non-terminal (e.g. bit-vectors)
- or every recursive production generates a strictly smaller spec

```
E ::= F | concat(F, E)
```



PROSE

[Polozov, Gulwani '15]

Ranking function

```
r :: Program -> Int
```

Grammar

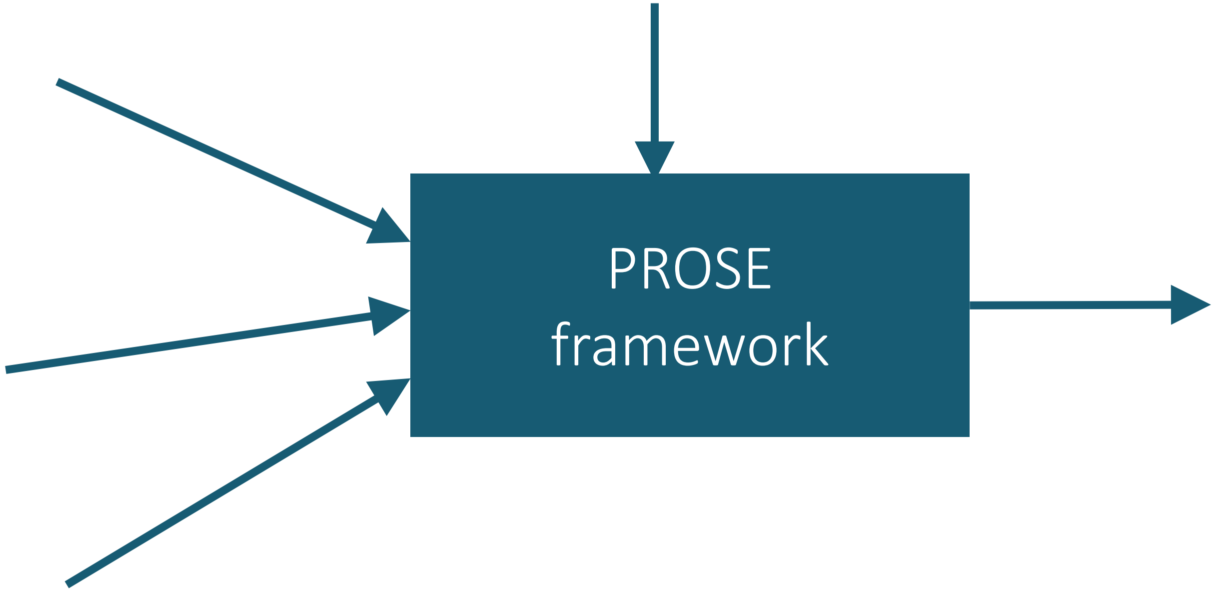
```
A ::= f(B, C) | ...
B ::= g(D, E) | ...
```

Semantics

```
f :: (B, C) -> A
f (b, c) = ...
```

Inverse semantics

```
f-1 :: A -> {(B, C)}
f-1 a = ...
```



VSA learning algorithm

https://microsoft.github.io/prose/

Representation-based search

and/or graph

Version Space Algebras
(VSA)

Finite Tree Automata
(FTA)

Equivalence Graphs
(e-grpahs)

ops: learn-1, intersect, extract
DSL: efficiently invertible
similar to: top-down prop,
but can infer constants

Finite Tree Automata

states

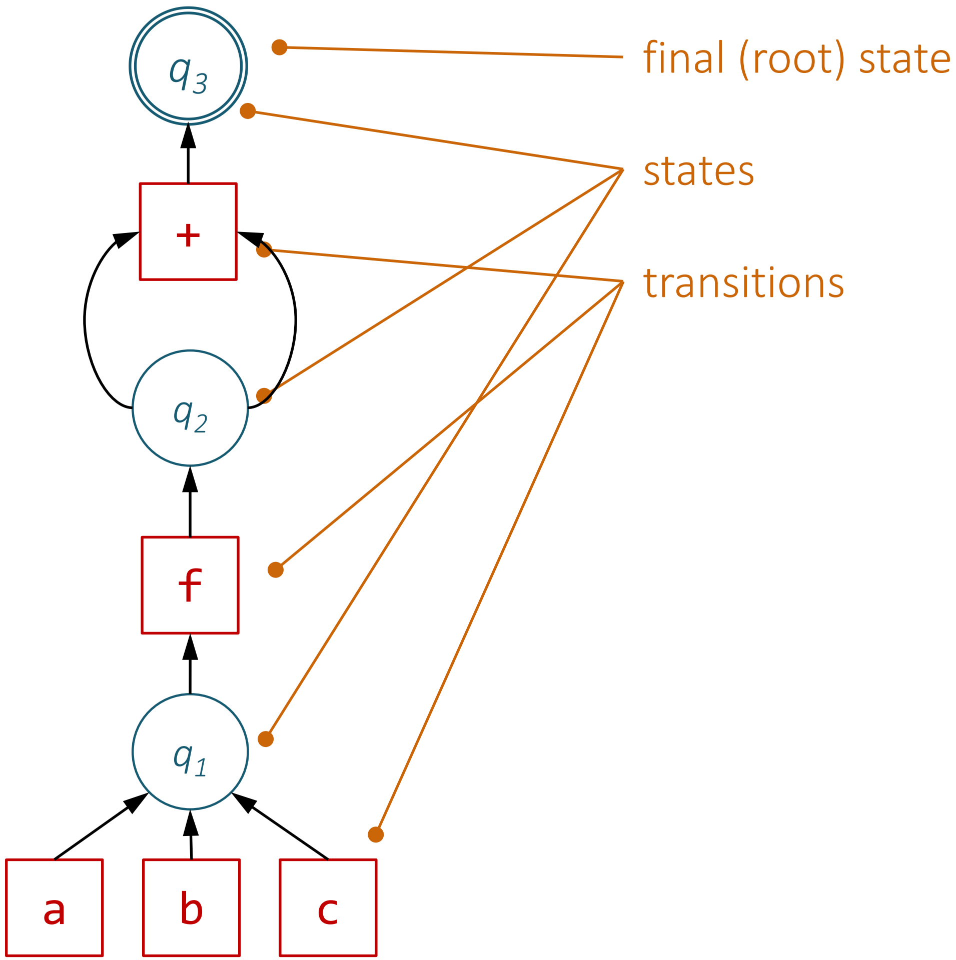
final state(s)

$$\mathcal{A} = \langle Q, F, Q_f, \Delta \rangle$$

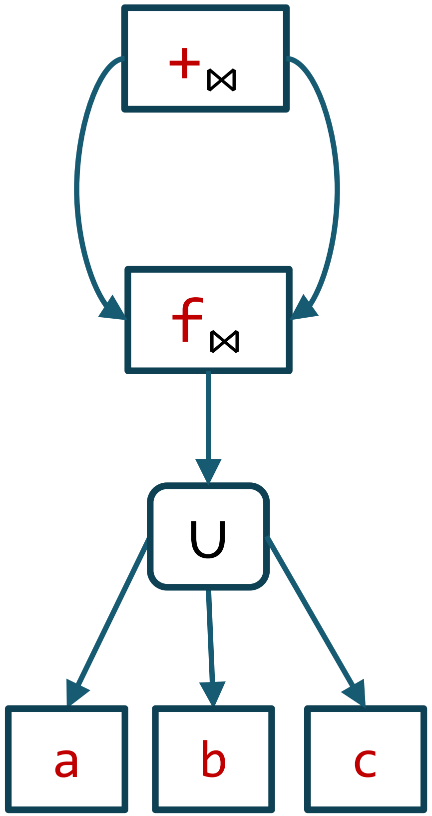
ranked alphabet

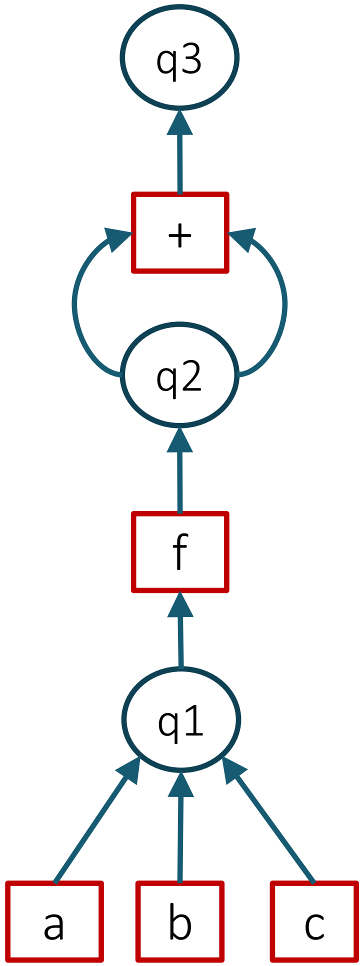
transitions

$$f(q_1, \ldots, q_n) \rightarrow q$$



VSA vs FTA





Both are and-or graphs

FTA state = VSA union node

- in VSAs singleton joins are omitted

FTA transition = VSA join node

FTA-based search

Synthesis of Data Completion Scripts using Finite Tree Automata
Xinyu Wang, Isil Dillig, Rishabh Singh, *OOPSLA’17*

Program Synthesis using Abstraction Refinement
Xinyu Wang, Isil Dillig, Rishabh Singh, *POPL’18*

Searching Entangled Program Spaces
James Koppel, Zheng Guo, Edsko de Vries, Armando Solar-Lezama, Nadia Polikarpova. *ICFP'22*

FTA-based search

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Example

Grammar

```
N ::= id(V) | N + T | N * T
T ::= 2 | 3
V ::= x
```

Spec

$1 \rightarrow 9$

PBE with Finite Tree Automata

$\langle A, \mathbb{Z} \rangle$

$A \in \{\mathsf{N}, \mathsf{T}, \mathsf{X}\}$

$\{\langle \mathsf{N}, 9 \rangle\}$

states

final states

$$\mathcal{A} = \langle Q, F, Q_f, \Delta \rangle$$

alphabet

transitions

`id`, `+`, `*`

$$f(q_1, \dots, q_n) \to q$$

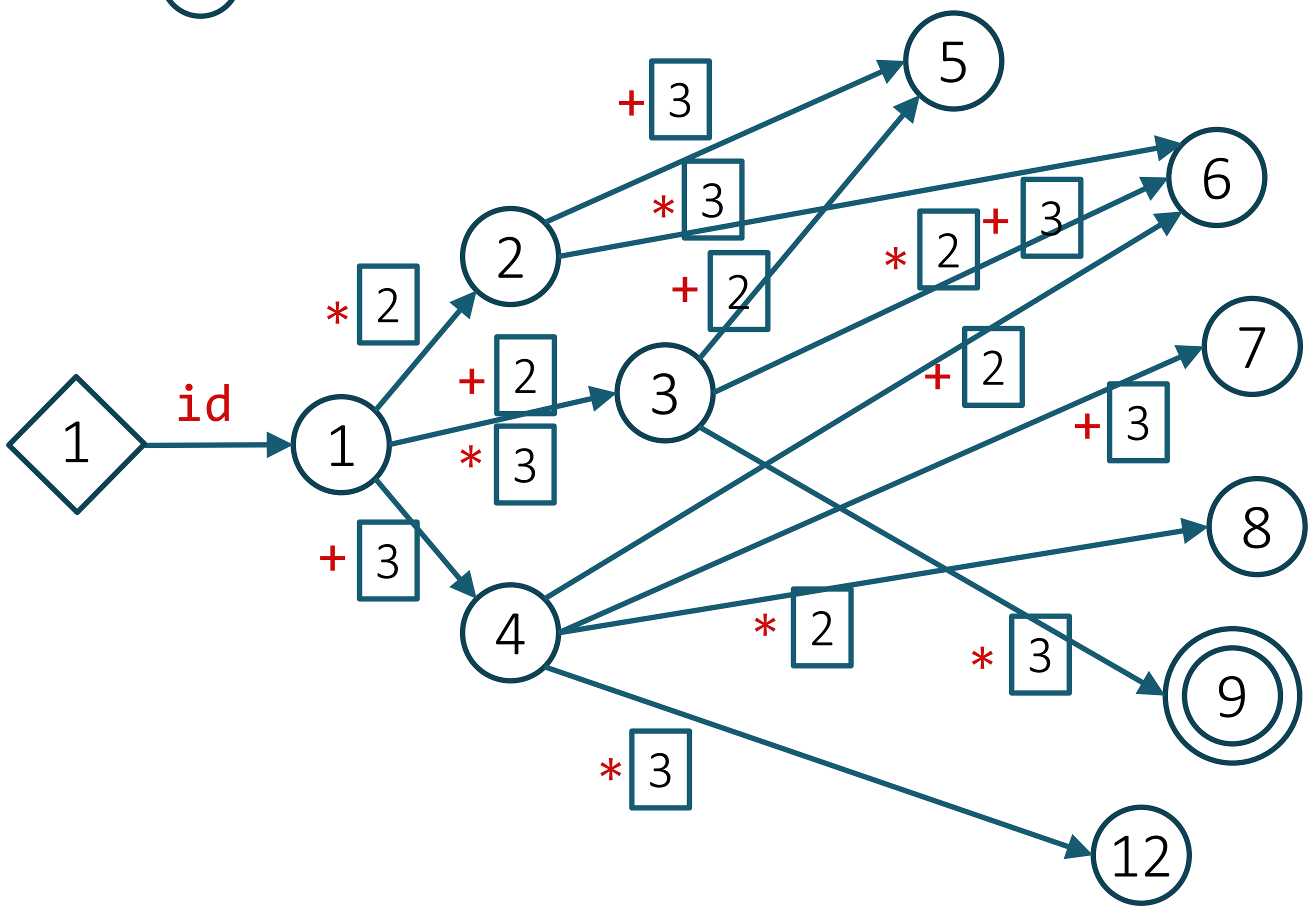
`+(<N,1>,<T,2>) → <N,3>`

`...`

PBE with Finite Tree Automata

```
N ::= id(V) | N + T | N * T
T ::= 2 | 3
V ::= x
```

1 → 9



Discussion

What do FTAs remind you of in the enumerative world?

- FTA ~ bottom-up search with OE

How are they different?

- More size-efficient: sub-terms in the bank are replicated, while in the FTA they are shared
- Hence, can store all terms, not just one representative per class
- Can construct one FTA per example and intersect
- More incremental in the CEGIS context!

FTA-based search

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Abstract FTA

Challenge: FTA still has too many states

Idea:

- instead of one state = one value
- we can do one state = set of values (= abstract value)

Abstract FTA

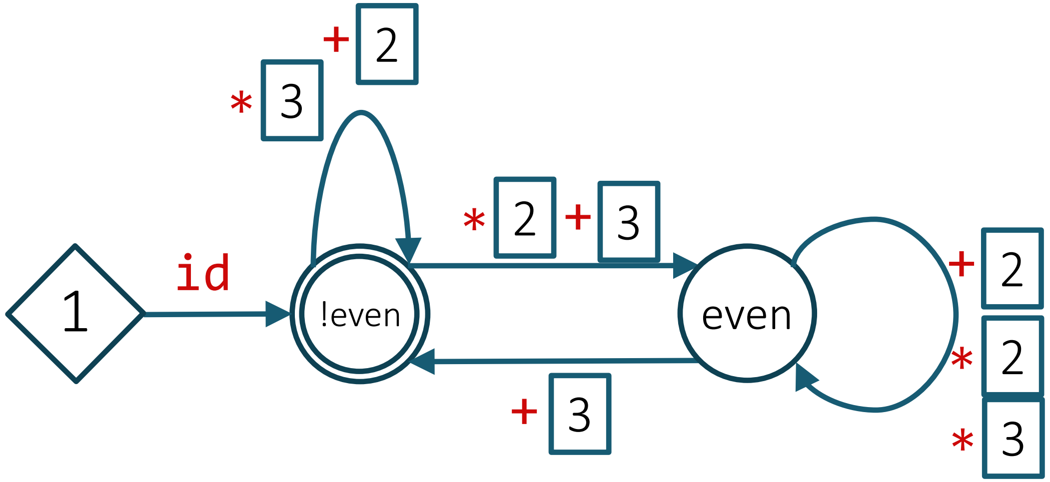
[Wang, Dillig, Singh POPL'18]

N ::= id(V) | N + T | N * T

T ::= 2 | 3

V ::= x

1 → 9



What now?

- idea 1: enumerate from reduced space
- idea 2: refine abstraction!

Abstract FTA

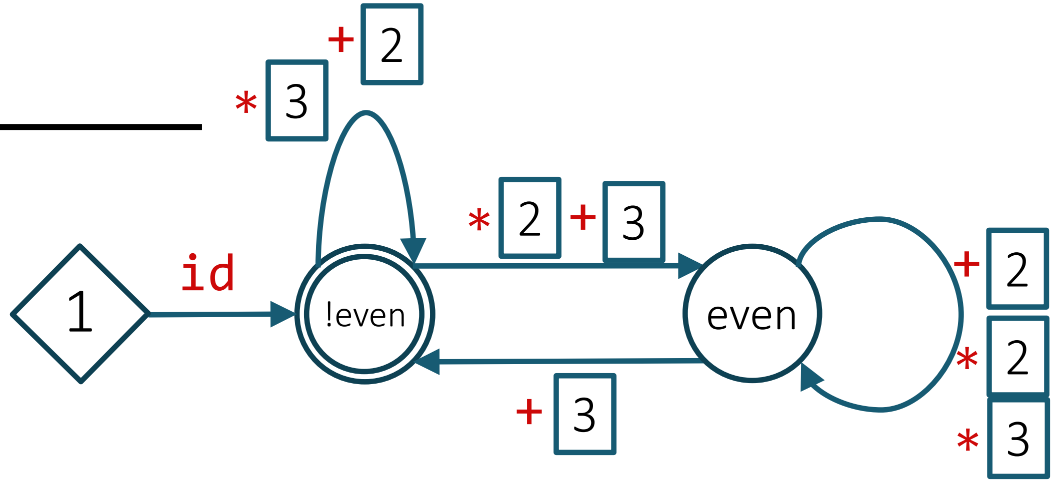
`N ::= id(V) | N + T | N * T`

`T ::= 2 | 3`

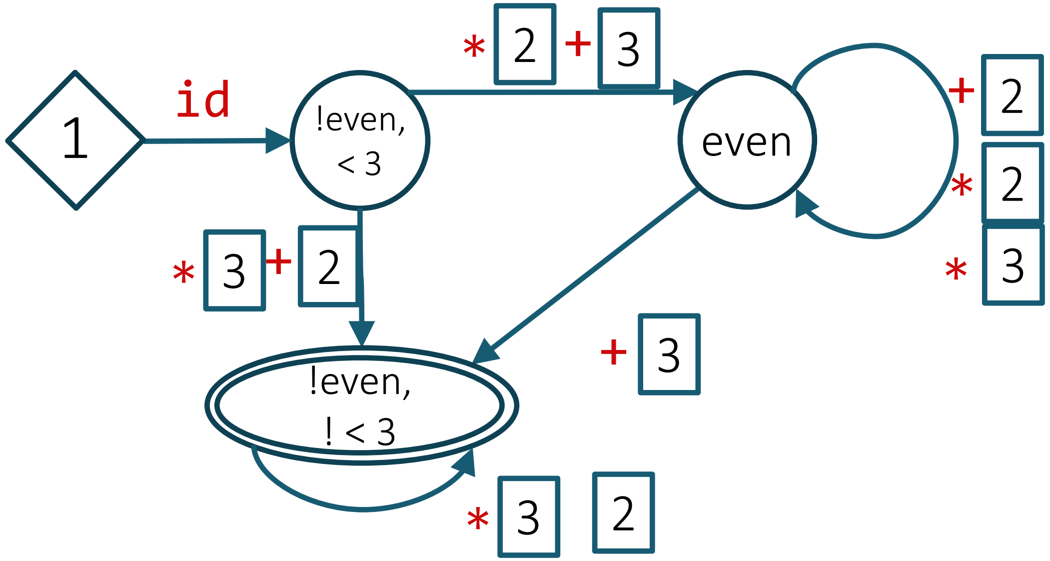
`V ::= x`

1 → 9

Predicates: {even, < 3, ...}



solution: `id(x)`



solution: `id(x)*3`

Representation-based search

and/or graph

Version Space Algebras (VSA)

Finite Tree Automata (FTA)

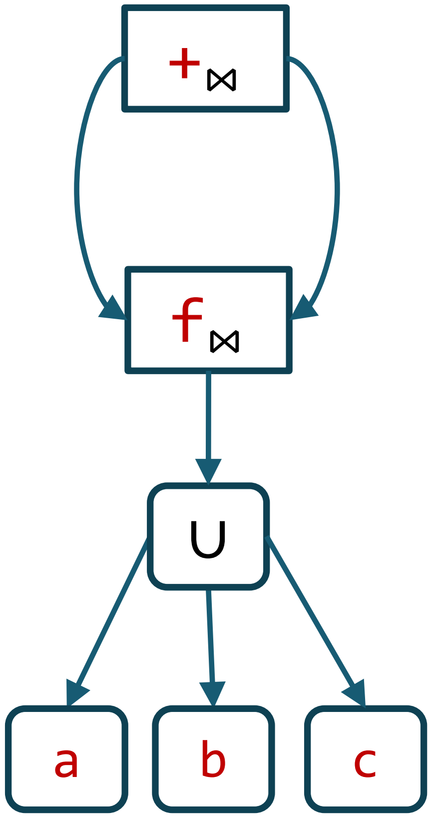
Equivalence Graphs (e-grpahs)

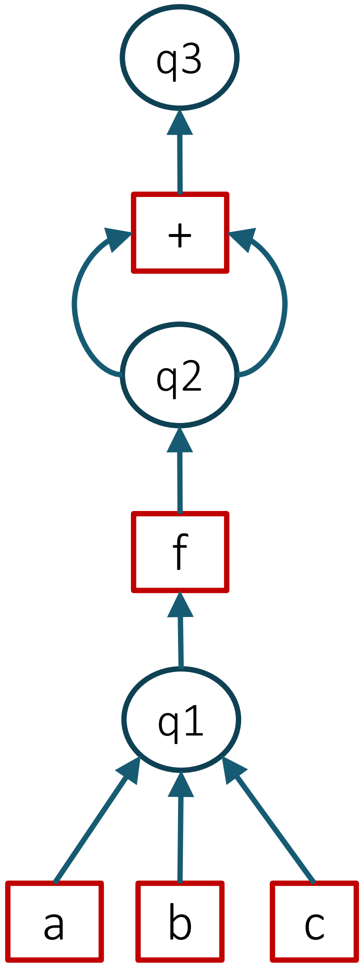
ops: learn-1, intersect, extract
DSL: efficiently invertible
similar to: top-down prop, but can infer constants

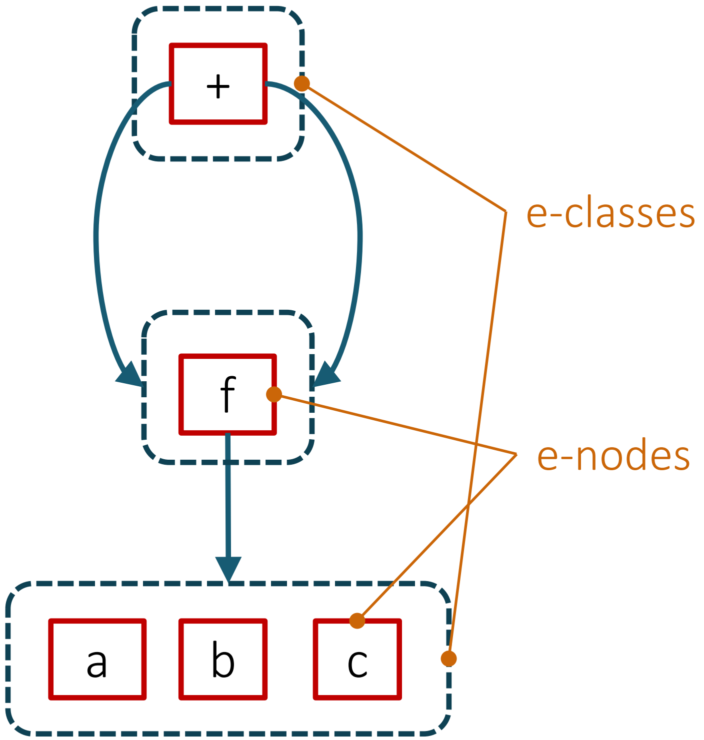
ops: learn-1, intersect, extract
DSL: efficiently enumerable
similar to: bottom-up with OE, but can store all programs (and add examples incrementally)

state: represents a set of observationally-equivalent programs

VSA vs FTA vs E-Graphs







Program search with e-grpahs

Equality saturation: a new approach to optimization
Ross Tate, Michael Stepp, Zachary Tatlock, Sorin Lerner, *POPL'09*

egg: Fast and Extensible Equality Saturation
Max Willsey, Chandrakana Nandi, Yisu Remy Wang, Oliver Flatt, Zachary Tatlock, Pavel Panchekha, *POPL'21*

Semantic Code Search via Equational Reasoning
Varot Premtoon, James Koppel, Armando Solar-Lezama. *PLDI'20*

Equality saturation

Program optimization via rewriting:

```
  (a * 2) / 2
⇒ a * (2 / 2)
⇒ a * 1
⇒ a
```

useful rules:

```
(x * y) / z = x * (y / z)
      x / x = 1
      x * 1 = x
```

not so useful:

```
x * 2 = x << 1
x * y = y * x
```

Challenge: which ones to apply and in what order?

Idea: all of them all the time!

Equality saturation

Initial term: `(a * 2) / 2`

Rewrite rules:

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(x * y) / z = x * (y / z)
          x / x = 1
          x * 1 = x
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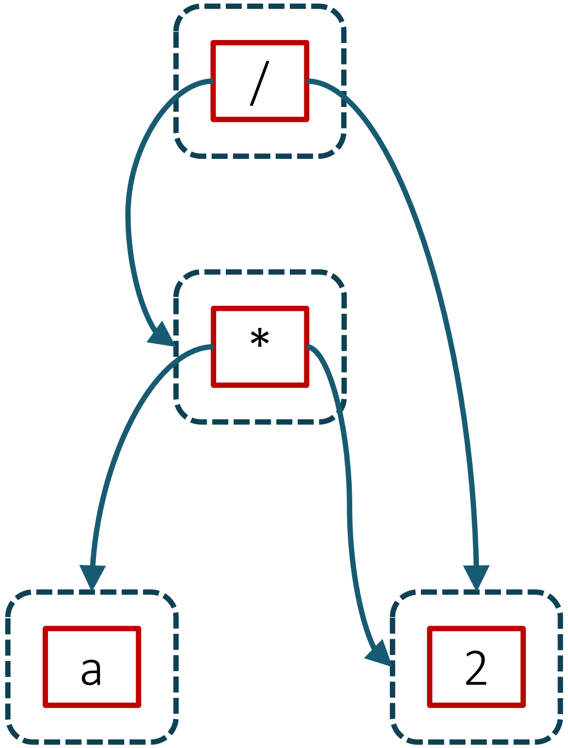

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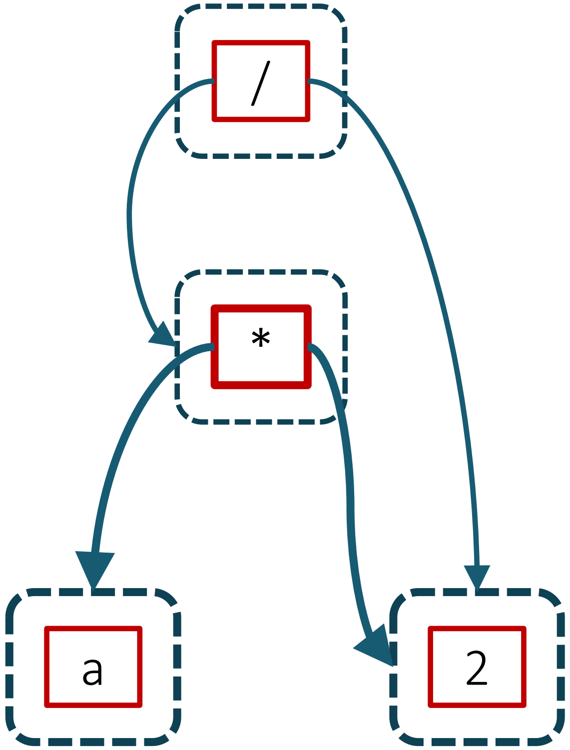

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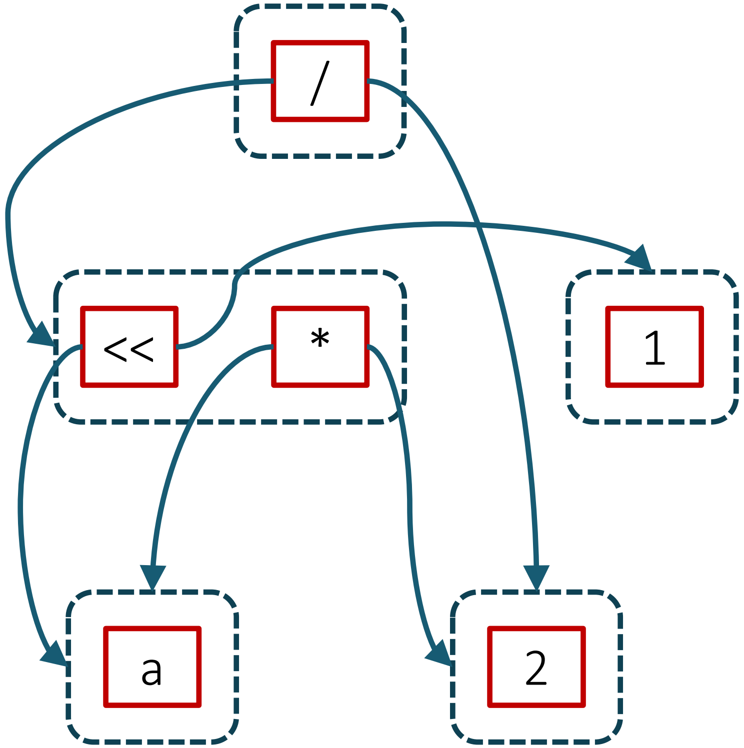

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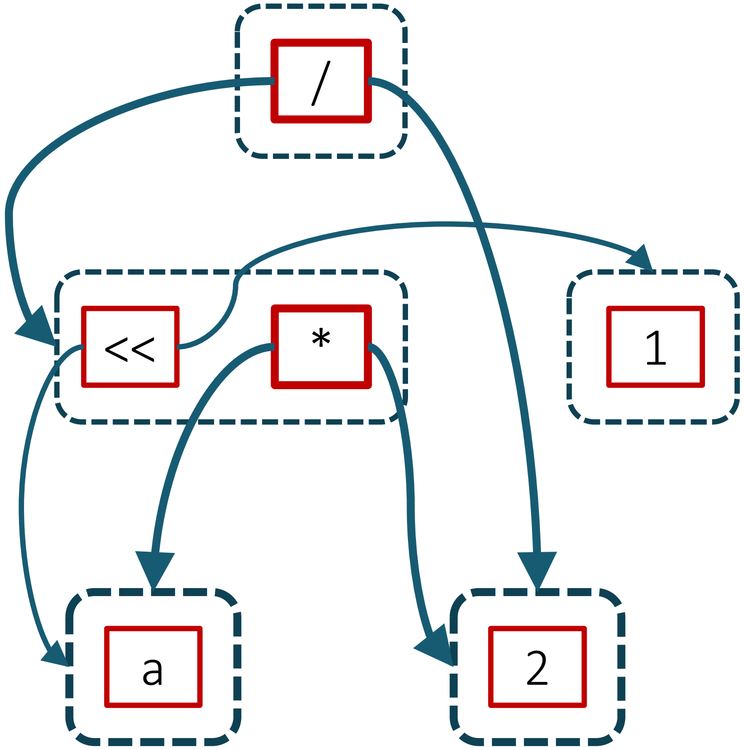

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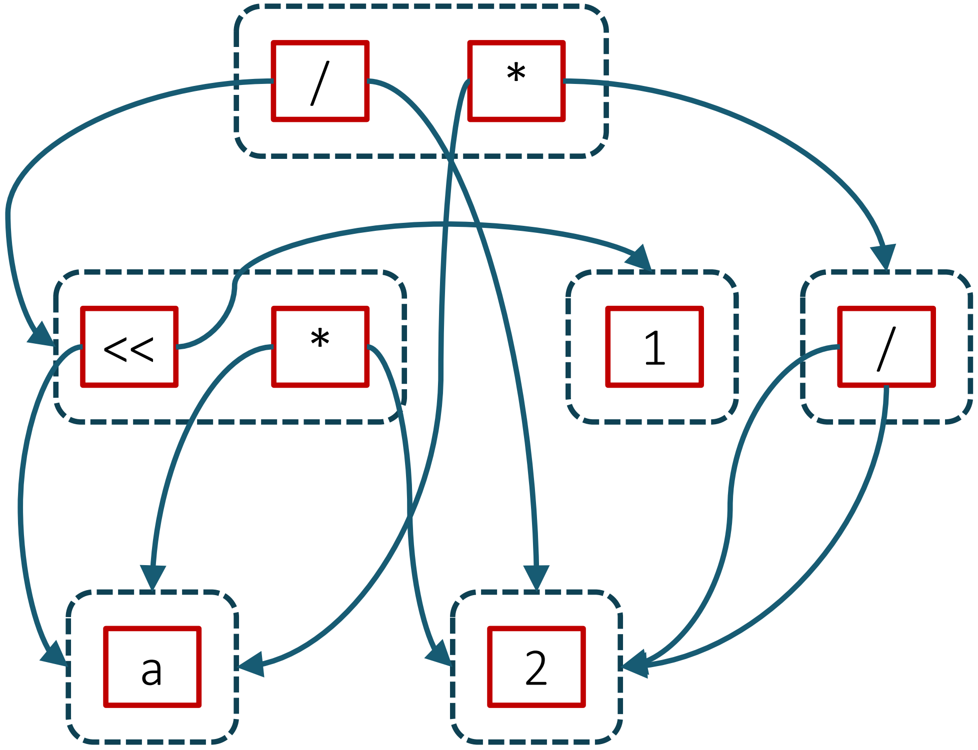

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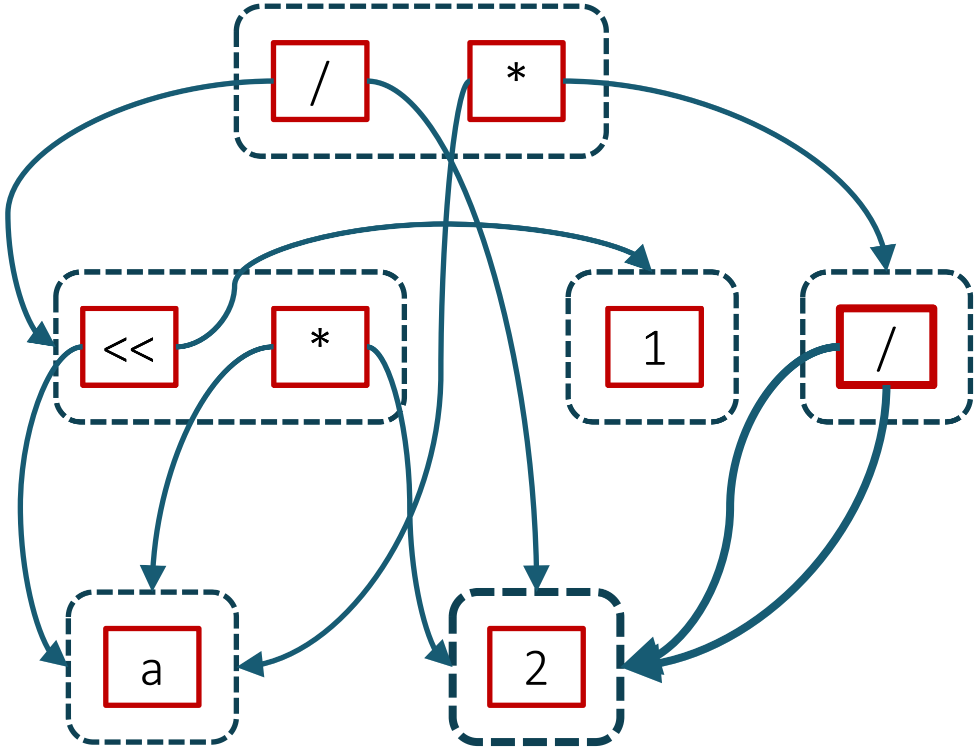

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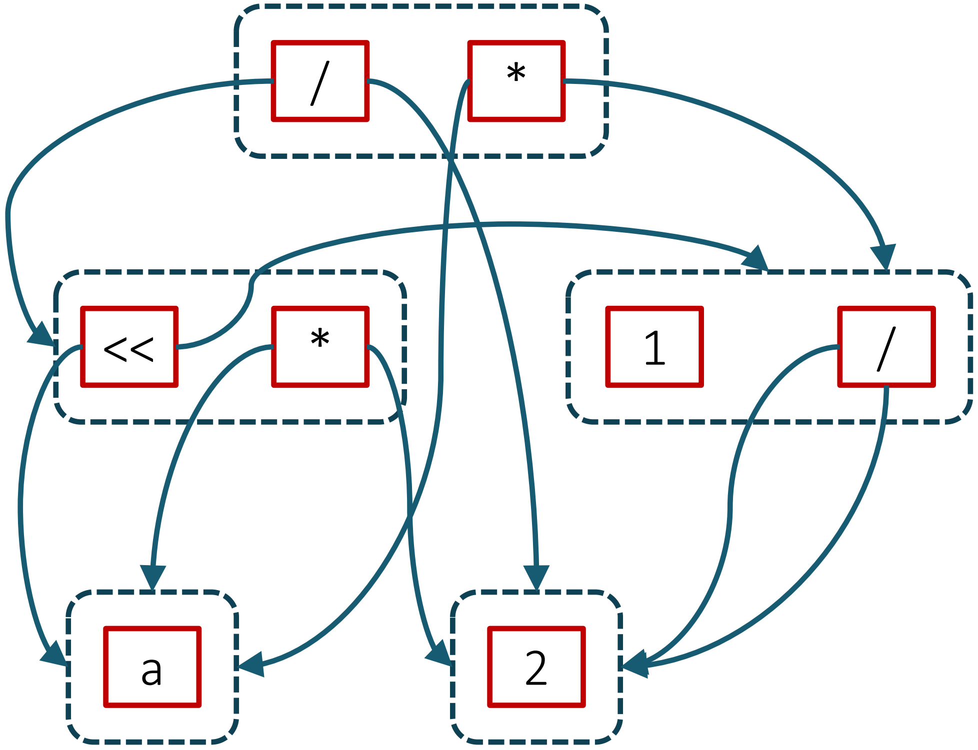

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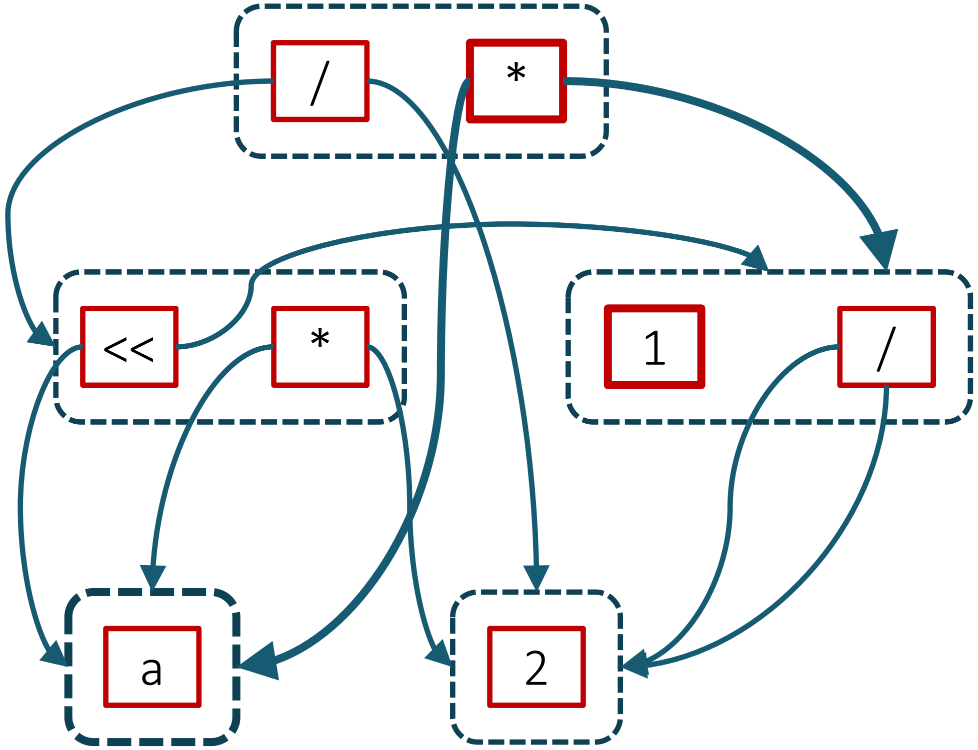

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```
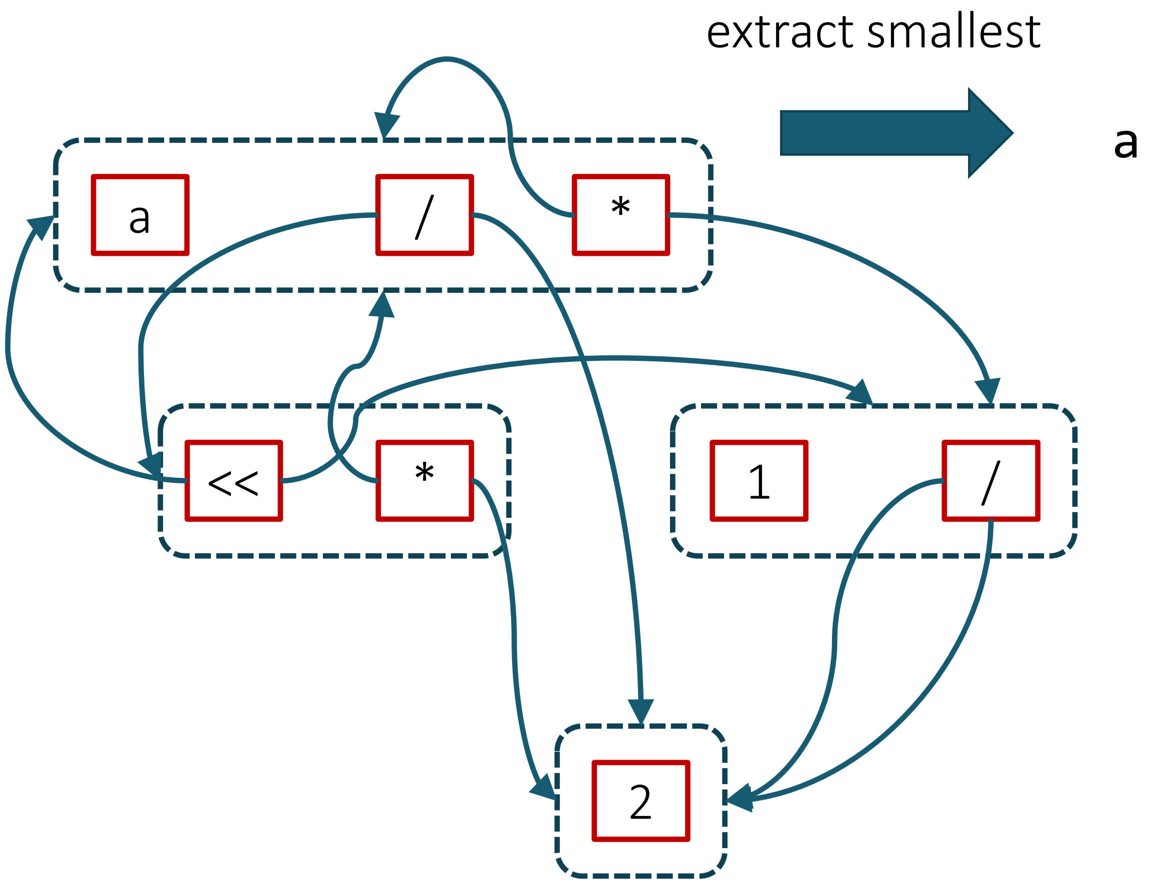



Representation-based search

and/or graph

Version Space Algebras (VSA)

ops: learn-1, intersect, extract
DSL: efficiently invertible
similar to: top-down prop, but can infer constants

Finite Tree Automata (FTA)

ops: learn-1, intersect, extract
DSL: efficiently enumerable
similar to: bottom-up with OE, but can store all programs (and add examples incrementally)

state: represents a set of observationally-equivalent programs

Equivalence Graphs (e-grpahs)

ops: rewrite, extract

similar to: term rewriting, but can store all programs

e-class: represents a set of programs equivalent up to rewrites

BlinkFill

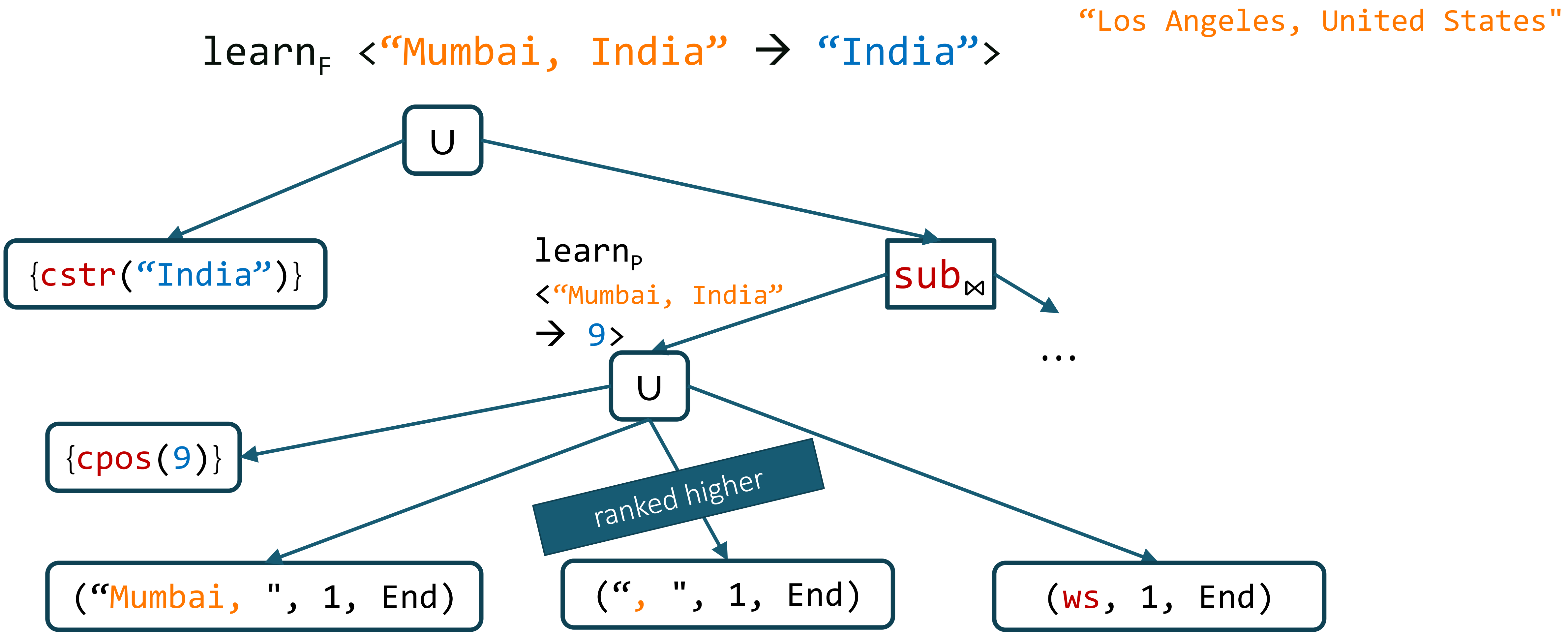
What does BlinkFill use as behavioral constraints? Structural constraints? Search strategy?

- input-output examples (+ input examples); custom string DSL; VSA

What is the main technical insight of BlinkFill wrt FlashFill?

- BlinkFill uses the available inputs (with no outputs) to infer structure (segmentation) common to all inputs
- it uses this structure to shrink the DAG and to rank substring expressions

Example



"Los Angeles, United States"
learn_F <"Mumbai, India" → "India">
∪
{cstr("India")}
sub⋈
learn_P
<"Mumbai, India"
→ 9>
...
∪
{cpos(9)}
ranked higher
("Mumbai, ", 1, End)
(", ", 1, End)
(ws, 1, End)

BlinkFill

Write a BlinkFill program that satisfies:

- `"Programming Language Design and Implementation (PLDI), 2019, Phoenix AZ" -> "PLDI 2019"`
- `"Principles of Programming Languages (POPL), 2020, New Orleans LA" -> "POPL 2020"`

- Between first parentheses and between first and last comma:

```
Concat(SubStr(v1, ("(", 1, End), (")",1, Start)),
       SubStr(v1, (",", 1, End), (",", -1, Start)))
```

BlinkFill

Could we extend the algorithm to support sequences of tokens?

- More expressive:
  - "Programming Language Design and Implementation: PLDI 2019" -> "PLDI 2019"
  - “POPL 2020 started on January 22" -> "POPL 2020“
  - `SubStr(v1, (C ws d, 1, Start), (C ws d, 1, End))`
- Each edge of the single-string IDG would have more labels
- Extra edges from 0 and to the last node
- More edges left after intersection (might be a problem, but unclear)
- Need fewer primitive tokens (no need for ProperCase)

BlinkFill

Strengths? Weaknesses?

- differences between FlashFill and BlinkFill language? which one is more expressive?