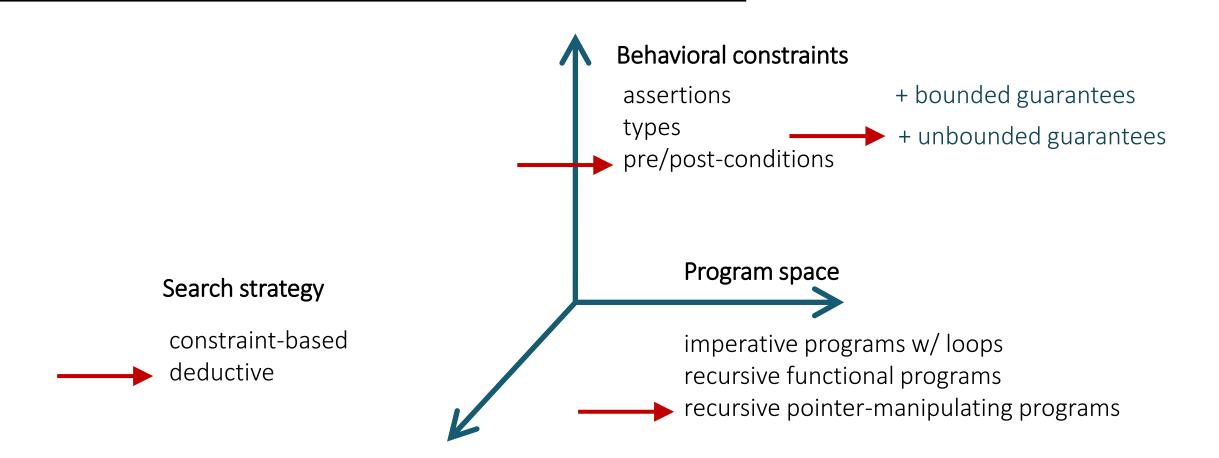
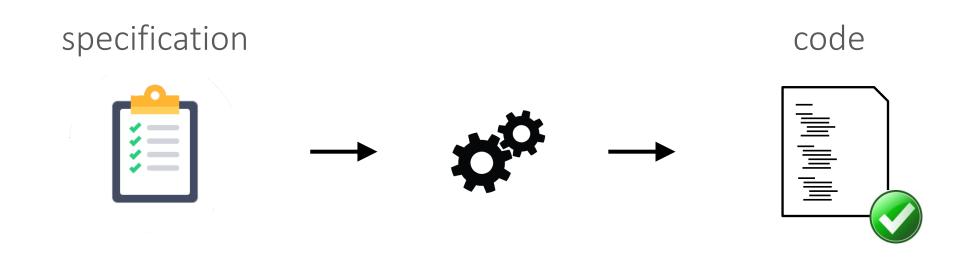
Lecture 13 Separation Logic and Deductive Synthesis

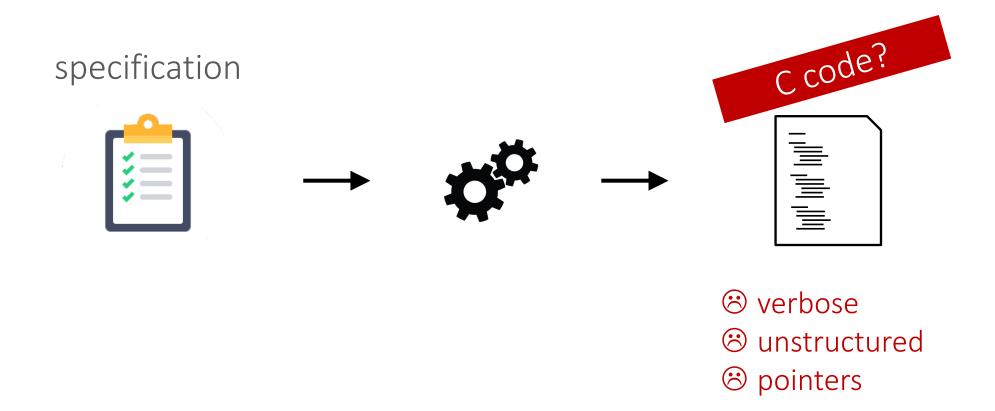
Today



Program synthesis



Program synthesis



The trouble with pointers

Can we naively apply Hoare logic to programs with pointers?

$$\{* x = 10 \land * y = 10\}$$

$$\Rightarrow$$

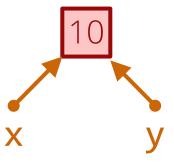
$$\{* x + 5 = 15 \land * y - 5 = 5\}$$

$$*x = *x + 5;$$

$$\{* x = 15 \land * y - 5 = 5\}$$

$$*y = *y - 5;$$

$$\{* x = 15 \land * y = 5\}$$

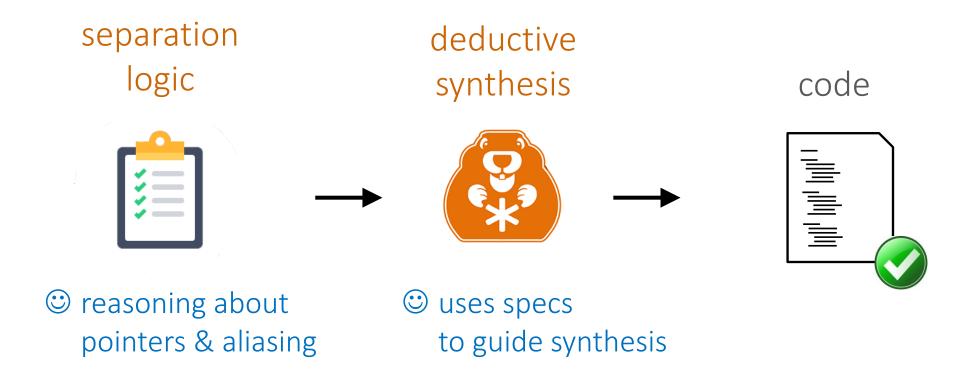


SuSLik



Synthesis Using Separation Logik

The SuSLik approach



Outline

example: swap

a taste of SuSLik

separation logic

specifying pointer-manipulating programs

deductive synthesis

from SL specifications to programs

Outline

example: swap

separation logic

deductive synthesis

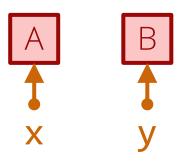
Example: swap

Swap values of two *distinct* pointers

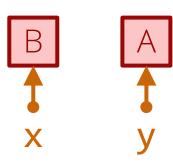
void swap(loc x, loc y)

Example: swap

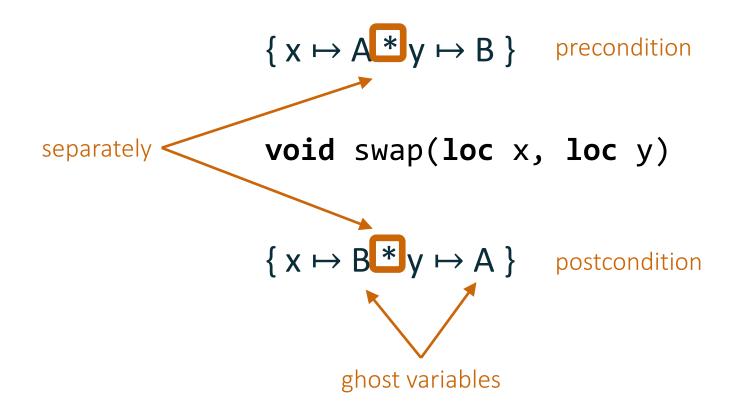
start state:



end state:



in separation logic:



Demo: swap

Swap values of two *distinct* pointers

void swap(loc x, loc y)

$$\{x \mapsto A * y \mapsto B\}$$

5

$$\{ x \mapsto B * y \mapsto A \}$$

let a1 = *x;

$$\{x \mapsto a1 * y \mapsto B\}$$

??
 $\{x \mapsto B * y \mapsto a1\}$

```
let a1 = *x;
let b1 = *y;

{x → a1 * y → b1}

??

{x → b1 * y → a1}
```

```
let a1 = *x;
let b1 = *y;
    *x = b1;

{x → b1 * y → b1}
    ??
{x → b1 * y → a1}
```

```
let a1 = *x;
     let b1 = *y;
     *x = b1;
     *y = a1;
\{x \mapsto b1 * y \mapsto a1\}
             33
                                      same
\{x \mapsto b1 * y \mapsto a1\}
```

```
let a1 = *x;
let b1 = *y;

*x = b1;

*y = a1;
```

```
void swap(loc x, loc y) {
    let a1 = *x;
    let b1 = *y;
    *x = b1;
    *y = a1;
}
```

Outline

example: swap

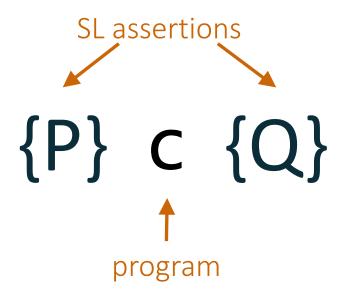
separation logic

deductive synthesis

Separation logic (SL)

Hoare logic "about the heap"

Separation logic (SL)



starting in a state that satisfies P program **c** will execute without memory errors, and upon its termination the state will satisfy Q

Outline

example: swap

separation logic

- 2.1. programs
- 2.2. assertions
- 2.3. specifying data transformations

deductive synthesis

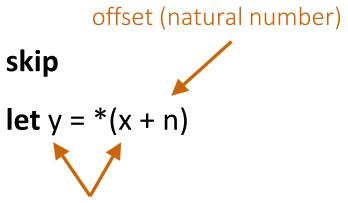
Separation logic (SL)

do nothing

skip

do nothing

read from heap



variables

do nothing

read from heap

write to heap

skip

$$let y = *(x + n)$$

*
$$(x + n) = e$$
 expression

(arithmetic, boolean)

do nothing

skip

read from heap

let y = *(x + n)

write to heap

*(x + n) = e

allocate block

let y = malloc(n)

do nothing

read from heap

write to heap

allocate block

free block

skip

let y = *(x + n)

*(x + n) = e

let y = malloc(n)

free(x)

do nothing

read from heap

write to heap

allocate block

free block

procedure call

skip

let y = *(x + n)

*(x + n) = e

let y = malloc(n)

free(x)

 $p(e_1, ..., e_n)$

read from heap

write to heap

allocate block

free block

procedure call

assignment

$$let y = *(x + n)$$

$$*(x + n) = e$$

let
$$y = malloc(n)$$

free(x)

$$p(e_1, ..., e_n)$$

only heap is mutable, not stack variables!

do nothing

read from heap

write to heap

allocate block

free block

procedure call

sequential composition

conditional

skip

let y = *(x + n)

*(x + n) = e

let y = malloc(n)

free(x)

 $p(e_1, ..., e_n)$

c₁; c₂

if (e) {c₁} else {c₂}

Outline

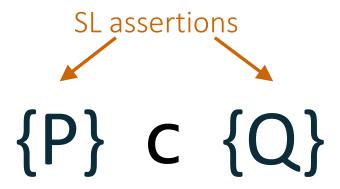
example: swap

separation logic

- 2.1. programs
- 2.2. assertions
- 2.3. specifying data transformations

deductive synthesis

Separation logic (SL)



SL assertions

empty heap { emp }

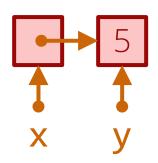
SL assertions

empty heap $\{emp\}$ singleton heap $\{y \mapsto 5\}$



SL assertions

empty heap $\{emp\}$ singleton heap $\{y \mapsto 5\}$ separating conjunction $\{x \mapsto y * y \mapsto 5\}$ heaplets

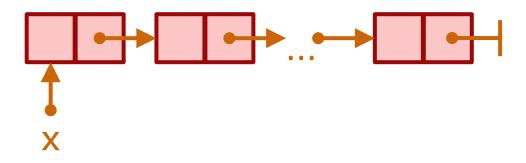


SL assertions

empty heap $\{emp\}$ singleton heap $\{y \mapsto 5\}$ separating conjunction $\{x \mapsto y * y \mapsto 5\}$ memory block $\{[x, 2] * x \mapsto 5 * (x + 1) \mapsto 10\}$

SL assertions

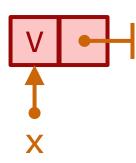
empty heap	{ emp }	
singleton heap	{ y → 5 }	
separating conjunction	$\{x \mapsto y * y \mapsto 5\}$	A
memory block	$\{ [x, 2] * x \mapsto 5 * (x + 1) \mapsto 10 \}$	X
+ pure formula	$\{A > 5; x \mapsto A\}$	



linked list $\{x = 0; emp\}$



linked list
$$\{[x, 2] * x \mapsto V * (x + 1) \mapsto 0\}$$

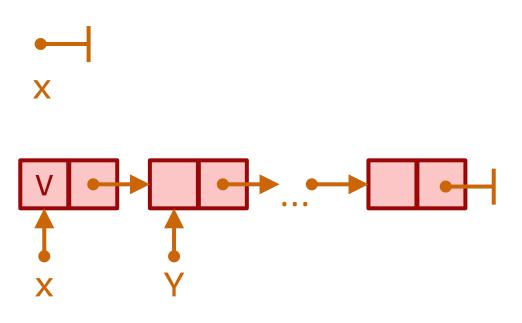


```
linked list \{[x, 2] * x \mapsto V * (x + 1) \mapsto Y *
[Y, 2] * Y \mapsto V' * (Y + 1) \mapsto 0
```

```
linked list \{[x, 2] * x \mapsto V * (x + 1) \mapsto Y *
[Y, 2] * Y \mapsto V' * (Y + 1) \mapsto Y' *
...
\{[x, 2] * x \mapsto V * (x + 1) \mapsto Y *
[Y, 2] * Y \mapsto V' * (Y + 1) \mapsto Y' *
```

inductive predicates to the rescue!

The linked list predicate



Outline

example: swap

separation logic

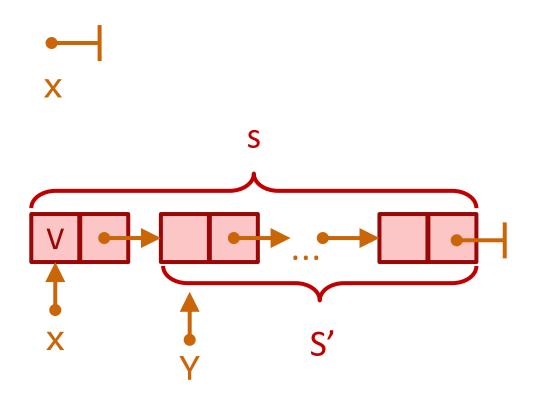
- 2.1. programs
- 2.2. assertions
- 2.3. specifying data transformations

deductive synthesis

Demo: dispose a list

```
void dispose(loc x)
{ list(x) }
{ emp }
```

Linked list with elements



Demo: copy a list

Outline

example: swap

the logic

deductive synthesis

Deductive synthesis

synthesis as proof search

this talk

example: swap

the logic

deductive synthesis

- 3.1. proof system
- 3.2. proof search

transforming entailment

a state that satisfies P can be transformed into a state that satisfies Q using a program **c**

Synthetic separation logic (SSL)

proof system for transforming entailment

{emp} ---> {emp} | ??

(Emp)

{emp} ---> {emp} | **skip**

(Frame)

```
\{P\} \rightsquigarrow \{Q\} \mid C
\{P * R\} \rightsquigarrow \{Q * R\} \mid ??
```

(Write)

```
\{x \mapsto e * P\} \rightsquigarrow \{x \mapsto e * Q\} \mid c
\{x \mapsto - * P\} \rightsquigarrow \{x \mapsto e * Q\} \mid ??
```

(Read)

```
[y/A]\{x \mapsto A * P\} \rightsquigarrow [y/A]\{Q\} \mid c
\{x \mapsto A * P\} \rightsquigarrow \{Q\} \mid ??
```

SSL: basic rules

```
(Read)
(Emp)
                                                           [y/A]{x \mapsto A * P} \rightsquigarrow [y/A]{Q} 
   {emp} --> {emp} | skip
                                                       \{x \mapsto A * P\} \rightsquigarrow \{Q\} \mid let y = *x; c
(Frame)
                                                    (Write)
                                                            \{x \mapsto e * P\} \rightsquigarrow \{x \mapsto e * Q\} \mid c
        \{P\} \rightsquigarrow \{Q\} \mid C
                                                    \{x \mapsto \_ * P\} \rightsquigarrow \{x \mapsto e * Q\} \mid *x = e; c
  \{P*R\} \rightsquigarrow \{Q*R\} \mid C
```

Example: swap

$$\{ \times \mapsto A * y \mapsto B \} \rightsquigarrow \{ \times \mapsto B * y \mapsto A \}$$
??

 $\{ \times \mapsto A * y \mapsto B \} \rightsquigarrow \{ \times \mapsto B * y \mapsto A \} \mid ??$

```
\{y \mapsto a1\} \rightsquigarrow \{y \mapsto a1\} \mid \dot{y};
                                                                                               (Write)
                               \{y \mapsto b1\} \rightsquigarrow \{y \mapsto a1\} \mid *y = a1; ??
                                                                                                        (Frame)
              \{x \mapsto b1 * y \mapsto b1\} \rightsquigarrow \{x \mapsto b1 * y \mapsto a1\} ??
                                                                                                              (Write)
    \{x \mapsto a1 * y \mapsto b1\} \implies \{x \mapsto b1 * y \mapsto a1\} \mid *x = b1; ??
                                                                                                             (Read)
\{x \mapsto a1 * y \mapsto B\} \rightsquigarrow \{x \mapsto B * y \mapsto a1\} \mid let b1 = *y; ??
                                                                                                              (Read)
  \{ \times \mapsto A * y \mapsto B \} \rightsquigarrow \{ \times \mapsto B * y \mapsto A \} \mid let a1 = *x; ??
```

```
{ emp } ----> { emp } | ??
                                                                                   <del>------</del> (Frame)
                                     \{y \mapsto a1\} \rightsquigarrow \{y \mapsto a1\} \mid \vdots
                                                                                               (Write)
                               \{y \mapsto b1\} \rightsquigarrow \{y \mapsto a1\} \mid *y = a1; ??
                                                                                                        (Frame)
              \{x \mapsto b1 * y \mapsto b1\} \rightsquigarrow \{x \mapsto b1 * y \mapsto a1\} ??
                                                                                                              (Write)
    \{x \mapsto a1 * y \mapsto b1\} \xrightarrow{w} \{x \mapsto b1 * y \mapsto a1\} \mid *x = b1; ??
                                                                                                             (Read)
\{x \mapsto a1 * y \mapsto B\} \rightsquigarrow \{x \mapsto B * y \mapsto a1\} \mid let b1 = *y; ??
                                                                                                              (Read)
  \{ \times \mapsto A * y \mapsto B \} \rightsquigarrow \{ \times \mapsto B * y \mapsto A \} \mid \text{let a1} = *x; ??
```

```
(Emp)
                                        { emp } ••• { emp }
                                                                                          (Frame)
                                    \{y \mapsto a1\} \rightsquigarrow \{y \mapsto a1\} \mid \vdots
                                                                                              (Write)
                              \{y \mapsto b1\} \rightsquigarrow \{y \mapsto a1\} \mid *y = a1; ??
                                                                                                       (Frame)
              \{x \mapsto b1 * y \mapsto b1\} \rightsquigarrow \{x \mapsto b1 * y \mapsto a1\} ??
                                                                                                            (Write)
    \{x \mapsto a1 * y \mapsto b1\} \rightsquigarrow \{x \mapsto b1 * y \mapsto a1\} \mid x = b1;??
                                                                                                           (Read)
\{x \mapsto a1 * y \mapsto B\} \rightsquigarrow \{x \mapsto B * y \mapsto a1\} \mid let b1 = *y; ??
                                                                                                            (Read)
  \{ \times \mapsto A * y \mapsto B \} \rightsquigarrow \{ \times \mapsto B * y \mapsto A \} \mid \text{let a1} = *x; ??
```

```
\{x \mapsto A * y \mapsto B \}
let a1 = *x; let b1 = *y; *x = b1; *y = a1; skip
\{x \mapsto B * y \mapsto A \}
```

Synthetic separation logic (SSL)

```
basic rules
(Emp), (Read), (Write), (Frame)
(Alloc), (Free)
pure reasoning and unification
inductive predicates and recursion
```

Synthetic separation logic (SSL)

```
basic rules
(Emp), (Read), (Write), (Frame)
(Alloc), (Free)

pure reasoning and unification
inductive predicates and recursion
```

Example: dispose a list

```
void dispose(loc x)
{ list(x) }
{ emp }
```

```
{list<sup>1</sup> (x)} void dispose(loc x) { emp }
```

```
{list<sup>0</sup>(x)}
?? (Induction)
{ emp }
```

```
{ list<sup>0</sup>(x) }
?? (Open)
{ emp }
```

```
{ list<sup>1</sup> (x) } void dispose(loc x) { emp }
```

```
predicate list (loc x) {
  | x = 0 = > \{ emp \}
  | x \neq 0 = \{ [x, 2] * x \mapsto V * (x + 1) \mapsto Y * list(Y) \}
          if (x == 0) {
          {x = 0; emp}
                  55
          { emp }
           } else {
          \{x \neq 0 ; [x, 2] * x \mapsto V * (x + 1) \mapsto Y * list^{1}(Y) \}
                 33
          { emp }
```

{ list¹ (x) } **void** dispose(**loc** x) { emp }

```
predicate list (loc x) {
  | x = 0 = > \{ emp \}
  | x \neq 0 = \{ [x, 2] * x \mapsto V * (x + 1) \mapsto Y * list(Y) \}
          if (x == 0) {
          {x = 0 ; emp}
                  3 5
                                      (Emp)
          { emp }
           } else {
          \{x \neq 0 ; [x, 2] * x \mapsto V * (x + 1) \mapsto Y * list^{1}(Y) \}
                 33
          { emp }
```

{ list¹ (x) } **void** dispose(**loc** x) { emp }

```
predicate list (loc x) {
  | x = 0 => \{emp\}
 | x \neq 0 = \{ [x, 2] * x \mapsto V * (x + 1) \mapsto Y * list(Y) \}
         if (x == 0) {
         {x = 0; emp}
                  skip
          { emp }
          } else {
          \{x \neq 0 ; [x, 2] * x \mapsto V * (x + 1) \mapsto Y * list^{1}(Y) \}
                 33
          { emp }
```

{ list¹ (x) } **void** dispose(**loc** x) { emp }

```
predicate list (loc x) {
  | x = 0 => \{emp\}
 | x \neq 0 = \{ [x, 2] * x \mapsto V * (x + 1) \mapsto Y * list(Y) \}
          if (x == 0) { skip } else {
           \{x \neq 0; [x, 2] * x \mapsto V * (x + 1) \mapsto Y * list^{1}(Y)\}
                  55
          { emp }
```

```
{ list<sup>1</sup> (x) } void dispose(loc x) { emp }
```

```
predicate list (loc x) {
                                                                                  { list<sup>1</sup> (x) } void dispose(loc x) { emp }
  | x = 0 => \{emp\}
  | x \neq 0 = \{ [x, 2] * x \mapsto V * (x + 1) \mapsto Y * list(Y) \}
          if (x == 0) { skip } else {
           \{x \neq 0; [x, 2] * x \mapsto V * (x + 1) \mapsto Y * list^{1}(Y)\}
                                                                                     (Read)
                  55
           { emp }
```

```
predicate list (loc x) {
                                                                               { list<sup>1</sup> (x) } void dispose(loc x) { emp }
  | x = 0 => \{emp\}
 | x \neq 0 = \{ [x, 2] * x \mapsto V * (x + 1) \mapsto Y * list(Y) \}
          if (x == 0) { skip } else {
                let y1 = *(x + 1);
          \{x \neq 0 ; [x, 2] * x \mapsto V * (x + 1) \mapsto y1 * list^{1}(y1) \}
                   55
          { emp }
```

```
predicate list (loc x) {
                                                                               { list<sup>1</sup> (x) } void dispose(loc x) { emp }
  | x = 0 => \{emp\}
 | x \neq 0 = \{ [x, 2] * x \mapsto V * (x + 1) \mapsto Y * list(Y) \}
          if (x == 0) { skip } else {
                 let y1 = *(x + 1);
          \{x \neq 0 ; [x, 2] * x \mapsto V * (x + 1) \mapsto y1 * list^{1}(y1) \}
                   55
                                                                                    (Free)
          { emp }
```

```
predicate list (loc x) {
  | x = 0 => \{emp\}
 | x \neq 0 = \{ [x, 2] * x \mapsto V * (x + 1) \mapsto Y * list(Y) \}
         if (x == 0) { skip } else {
               let y1 = *(x + 1);
               free x;
         \{ x \neq 0 ; list^1 (y1) \}
                 55
         { emp }
```

```
{ list<sup>1</sup> (x) } void dispose(loc x) { emp }
```

```
predicate list (loc x) {
  | x = 0 => \{emp\}
 | x \neq 0 = \{ [x, 2] * x \mapsto V * (x + 1) \mapsto Y * list(Y) \}
         if (x == 0) { skip } else {
               let y1 = *(x + 1);
               free x;
         \{ x \neq 0 ; list^1 (y1) \}
                                                       (Call)
                 55
         { emp }
```

```
{ list<sup>1</sup> (x) } void dispose(loc x) { emp }
```

```
predicate list (loc x) {
  | x = 0 => \{emp\}
 | x \neq 0 = \{ [x, 2] * x \mapsto V * (x + 1) \mapsto Y * list(Y) \}
        if (x == 0) { skip } else {
              let y1 = *(x + 1);
              free x;
              dispose(y1);
         \{x \neq 0; emp\}
         { emp }
```

```
{ list<sup>1</sup> (x) } void dispose(loc x) { emp }
```

```
predicate list (loc x) {
  | x = 0 => \{emp\}
 | x \neq 0 = \{ [x, 2] * x \mapsto V * (x + 1) \mapsto Y * list(Y) \}
        if (x == 0) { skip } else {
              let y1 = *(x + 1);
              free x;
              dispose(y1);
         \{x \neq 0; emp\}
                                                      (Emp)
         { emp }
```

```
{ list<sup>1</sup> (x) } void dispose(loc x) { emp }
```

```
predicate list (loc x) {
 | x = 0 => \{emp\}
 | x \neq 0 = \{ [x, 2] * x \mapsto V * (x + 1) \mapsto Y * list(Y) \}
        if (x == 0) { skip } else {
              let y1 = *(x + 1);
              free x;
              dispose(y1);
              skip
```

```
{ list¹ (x) } void dispose(loc x) { emp }
```

```
void dispose(loc x) {
   if (x == 0) {
    else {
      let y1 = *(x + 1);
      free x;
      dispose(y1)
   }
}
```

Synthetic separation logic (SSL)

```
basic rules
(Emp), (Read), (Write), (Frame), (Alloc), (Free)
pure reasoning and unification
inductive predicates and recursion
(Open), (Close), (Induction), (Call)
```

Outline

example: swap

the logic

deductive synthesis

- 3.1. proof system
- 3.2. proof search

SuSLik

best-first and-or search in SSL + optimizations

Optimizations

invertible rules
early failure
multi-phase search
goal memoization

Optimization: invertible rules

invertible rules do not restrict the set of derivable programs idea: invertible rules need not be backtracked

Optimization: early failure

idea: sometimes you know that a goal is unsatisfiable

(Post-Inconsistent) $\psi \neq \bot \qquad \vdash \varphi \land \psi \Rightarrow \bot \qquad \{emp\} \rightsquigarrow \{\bot; emp\} \mid c$ $\{\varphi; P\} \rightsquigarrow \{\psi; Q\} \mid c$

Optimization: multi-phase search

```
unfolding phase: eliminates inductive predicates
 (Open), (Close), (Call), (Unify), (Frame)
block phase: eliminates blocks
 (Alloc), (Free), (Unify), (Frame)
pointer phase: aligns all pointers
 (Unify), (Frame)
pure phase: deals with the content of pointers
 (Write), (Unify), (Pure)
idea: if a phase fails, don't start the next one
```

SuSLik: contributions

Synthesis with unbounded guarantees for recursive heapmanipulating programs

Does not require sketches

Optimizations from proof search

SuSLik: limitations

No auxiliary functions

Only structurally recursive programs

Requires the spec to be inductive

Does not support loops

• But we can turn recursion into loops (?)

Requires SL expertise

SuSLik: questions

Behavioral constraints? Structural constraints? Search strategy?

- SL specs
- Set of components + built-in language constraints
- Deductive search

SuSLik: questions

Why is Frame rule unsound in the presence of assignments?

$$\{ x = 0; y \mapsto x \}$$

$$x := 1$$

provable with this frame rule, but incorrect! which side condition should we add to Frame?

$$\{ x = 1; y \mapsto x \}$$

SuSLik: questions

Why is Frame rule not invertible?

What other rule can it prevent from applying?