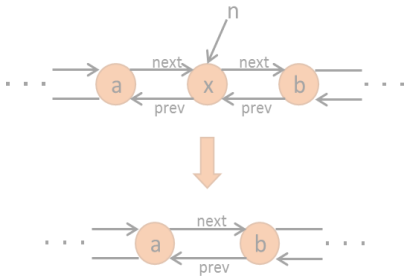
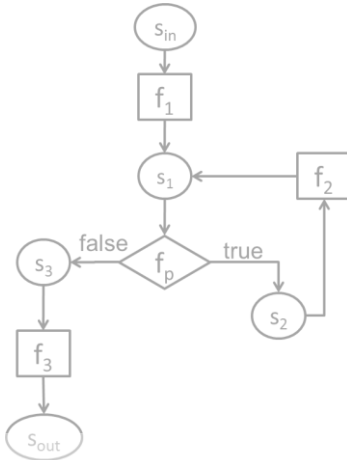


$$\exists c \forall in \ Q(c, in)$$

```

/* Average of x and y without using x+y (avoid overflow)*/
int avg(int x, int y){
    int t = expr({x/2, y/2, x%2, y%2, 2 }, {PLUS, DIV});
    assert t == (x+y)/2;
    return t;
}

```

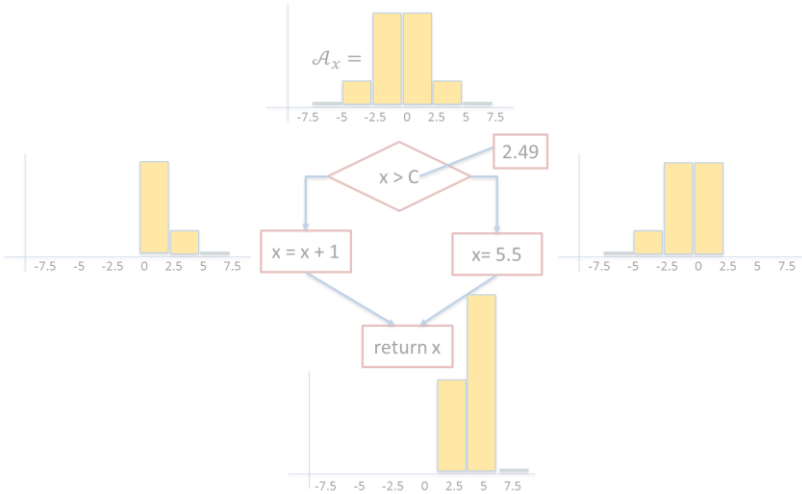
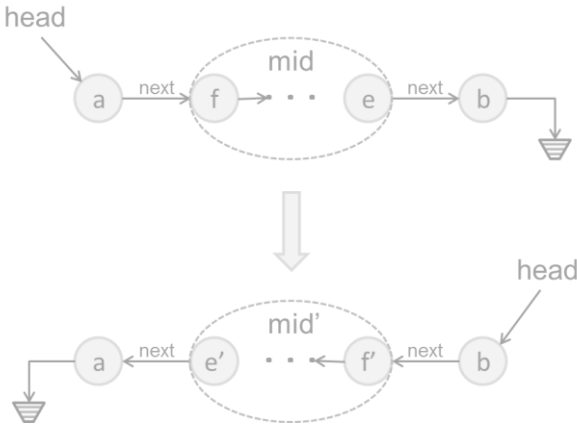


```

{
    s = n.succ;
    p = n.pred;
    p.succ = s;
    s.pred = p;
}

```

Module I: Synthesizing Simple Programs



$$\varphi(p)$$

$$Sk[c](in)$$

Lecture 2

Syntax-Guided Synthesis and Enumerative Search

Nadia Polikarpova

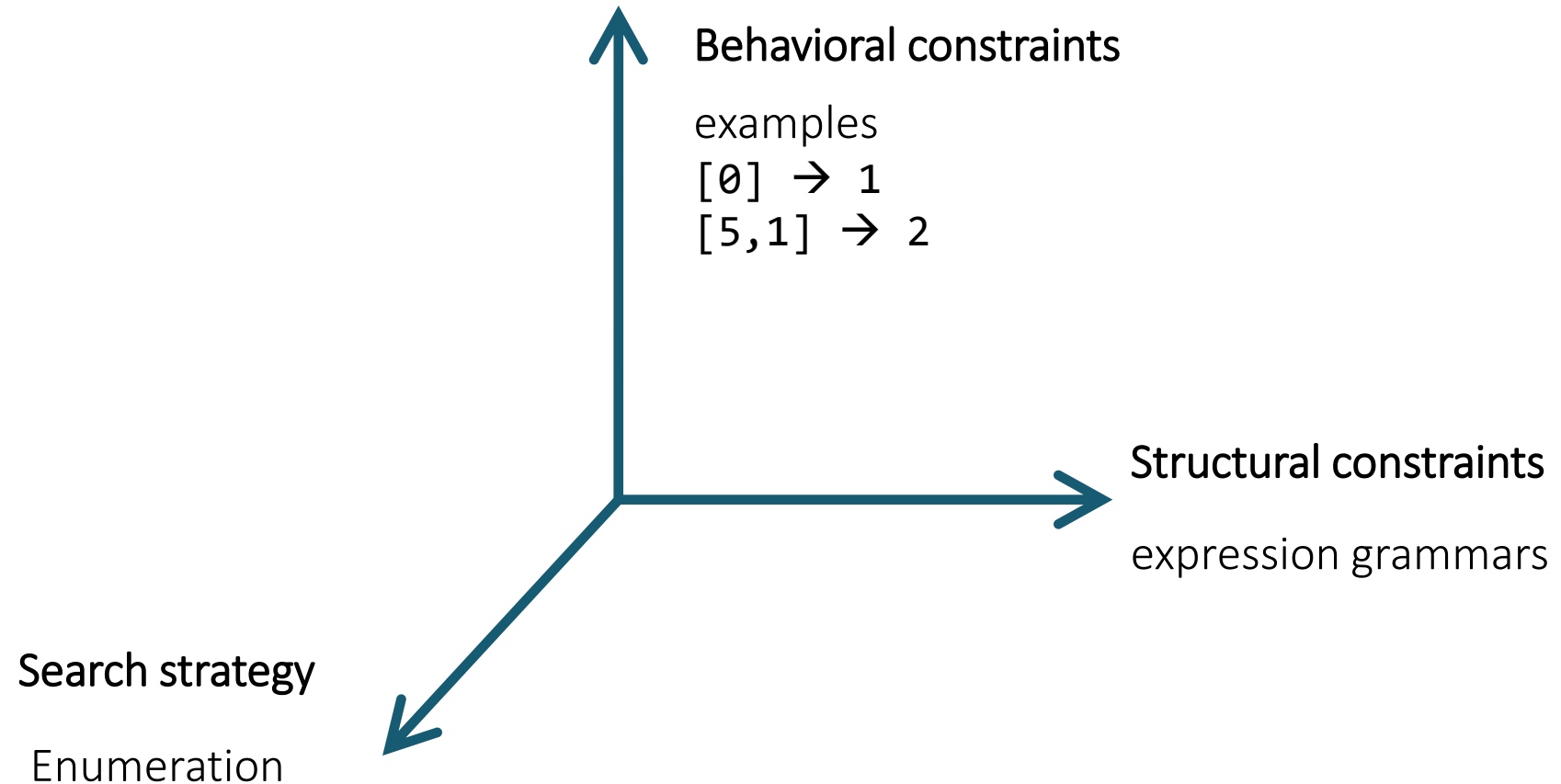
Logistics

Shared Google folder

- Does everyone have access?
- Register your team by next Friday

Other questions?

Week 1-2



Today

Synthesis from examples: motivation and history

Syntax-guided synthesis

- expression grammars as structural constraints
- the SyGuS project

Enumerative search

- enumerating all programs generated by a grammar
- bottom-up vs top-down

Synthesis from examples

Synthesis from Examples

=

Programming by Example

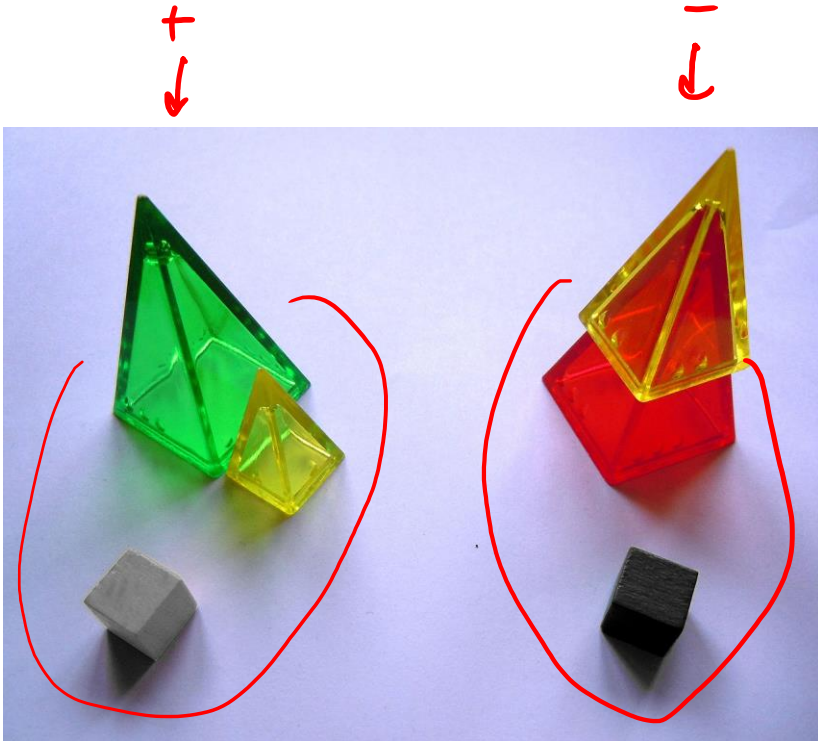
=

Inductive Synthesis

Inductive Programming

Inductive Learning

The Zendo game



This is called inductive learning!

The **teacher** makes up a secret rule

- e.g. all pieces must be grounded

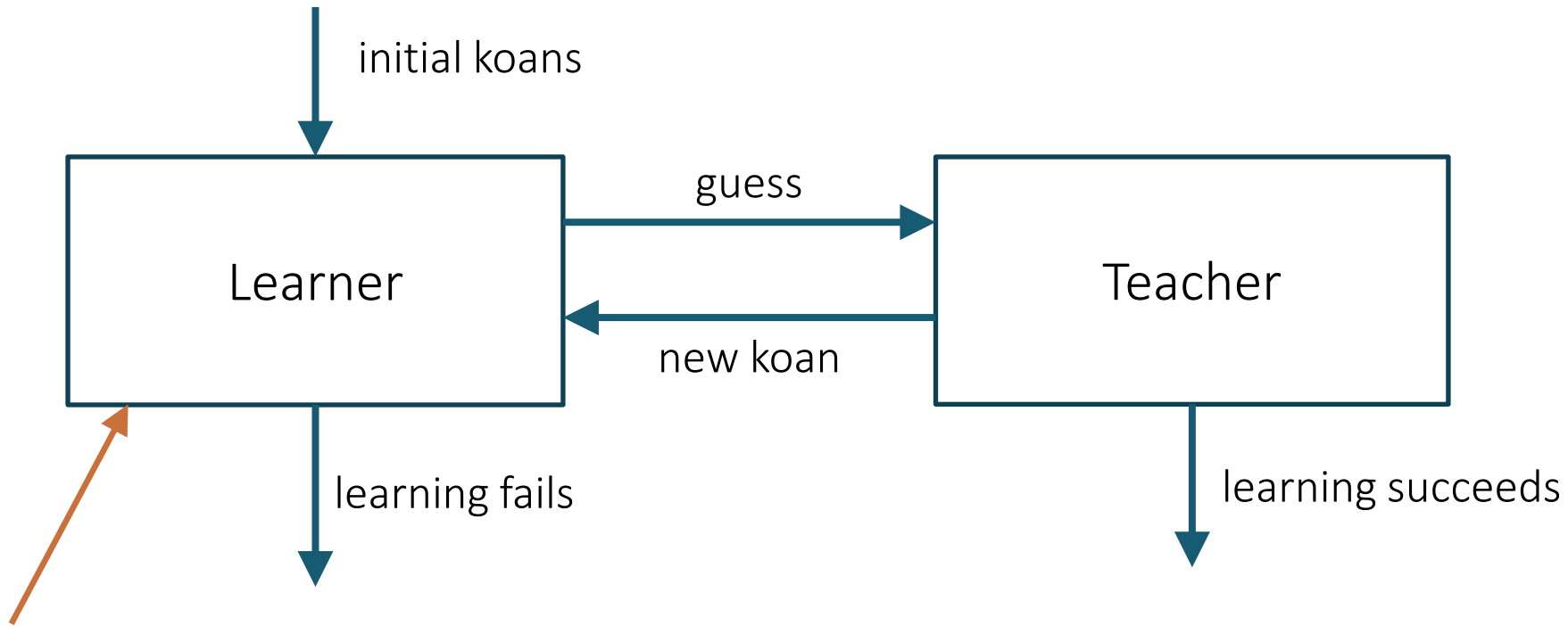
The teacher builds two **koans** (a positive and a negative)

Students take turns to build koans and ask the teacher to label them

A student can try to guess the rule

- if they are right, they win
- otherwise, the teacher builds a koan on which the two rules disagree

The Zendo game



1960s: humans are good at this...
can computers do this?

A little bit of history: inductive learning

MIT/LCS/TR-76
LEARNING STRUCTURAL DESCRIPTIONS FROM EXAMPLES
Patrick H. Winston
September 1970



Patrick
Winston

Explored the question of generalizing from a set of observations

- Similar to Zendo

Became the foundation of machine learning

A little bit of history: PBE/PBD

1980s: searching a predefined list of programs

1990s (Tessa Lau): bring inductive learning techniques into PBE

Programming by Demonstration: An Inductive Learning Formulation*

Tessa A. Lau and Daniel S. Weld
Department of Computer Science and Engineering
University of Washington
Seattle, WA 98195-2350
October 7, 1998
{tlau, weld}@cs.washington.edu

ABSTRACT

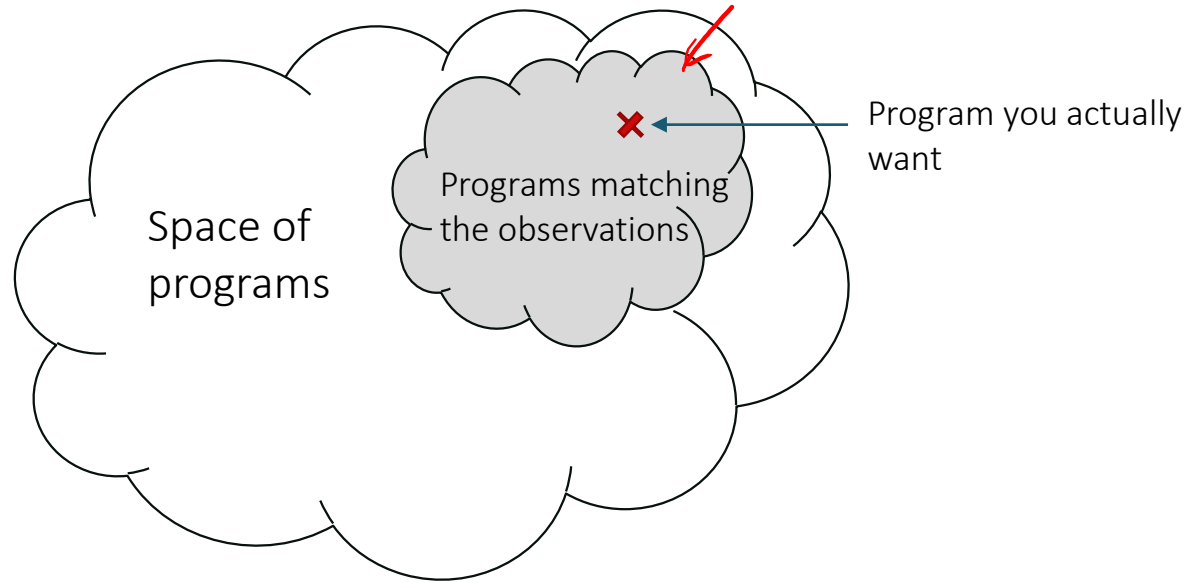
Although Programming by Demonstration (PBD) has the potential to improve the productivity of knowledge

- Applications that support macros allow users to record a fixed sequence of actions and later replay this sequence using a shortcut such as a mouse click and a



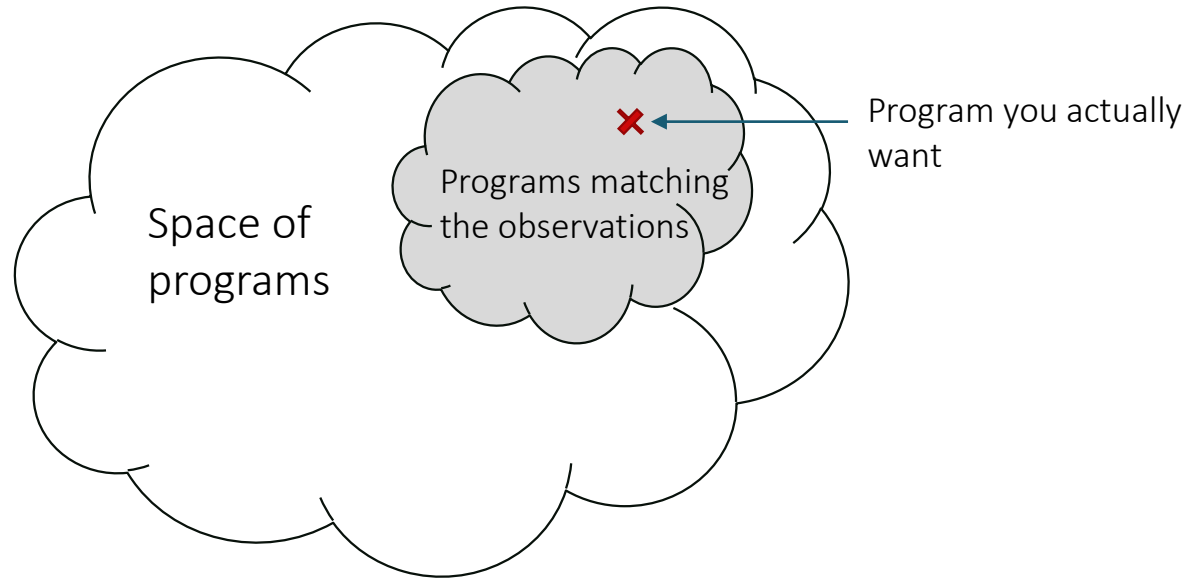
Tessa Lau

Key issues in inductive learning



- (1) How do you find a program that matches the observations?
- (2) How do you know it is the program you are looking for?

Key issues in inductive learning



Traditional ML emphasizes (2)

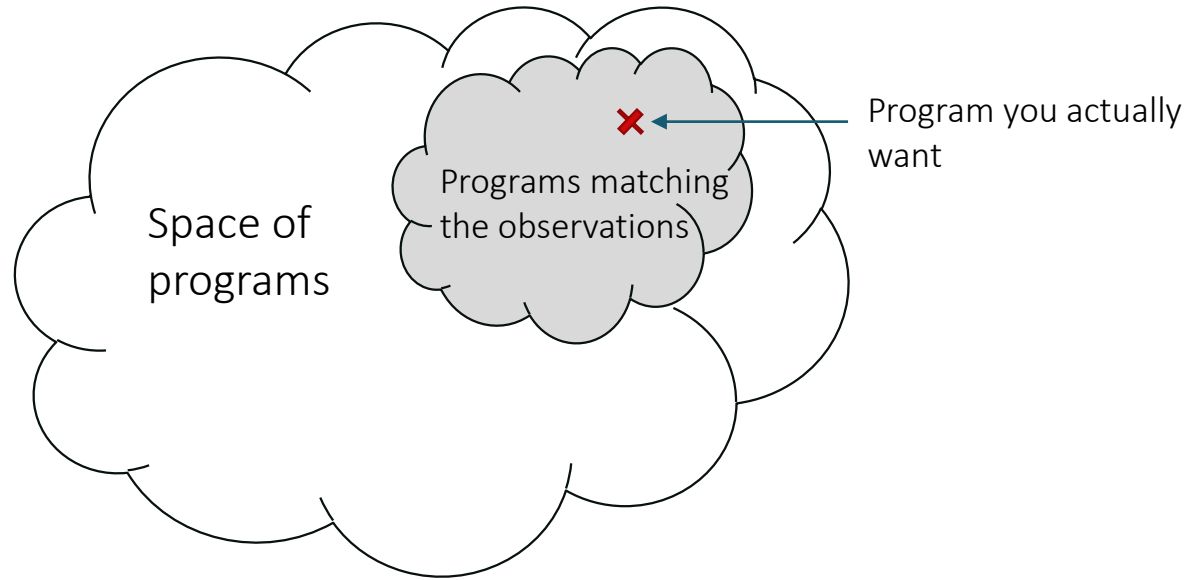
- Fix the space so that (1) is easy

So did a lot of PBD work

(1) How do you find a program that matches the observations?

(2) How do you know it is the program you are looking for?

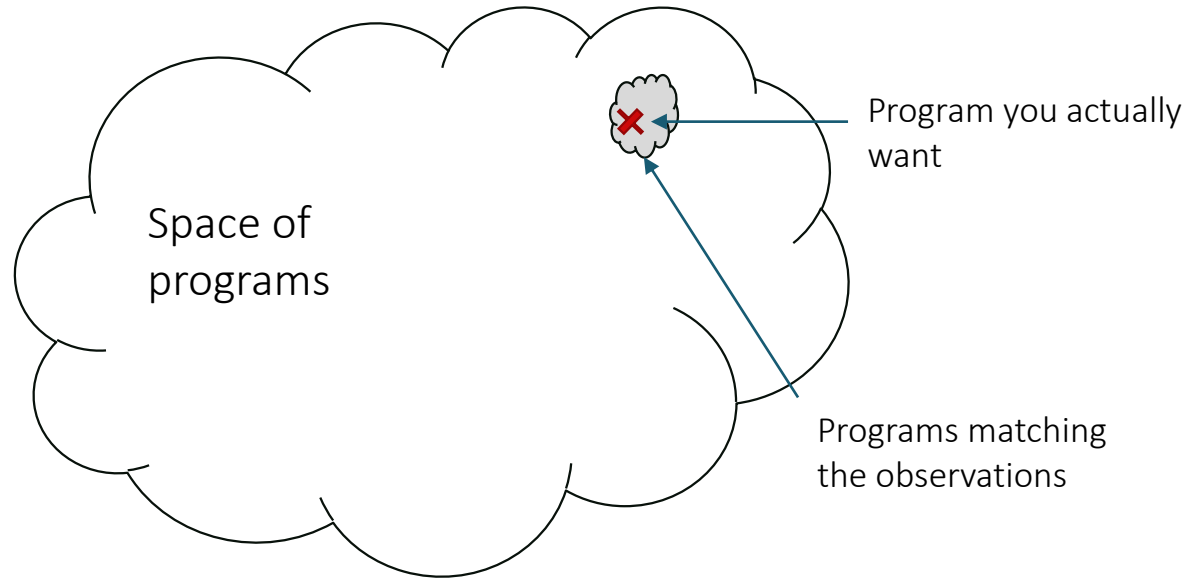
The synthesis approach



Modern emphasis

- (1) How do you find a program that matches the observations?
- (2) How do you know it is the program you are looking for?

The synthesis approach



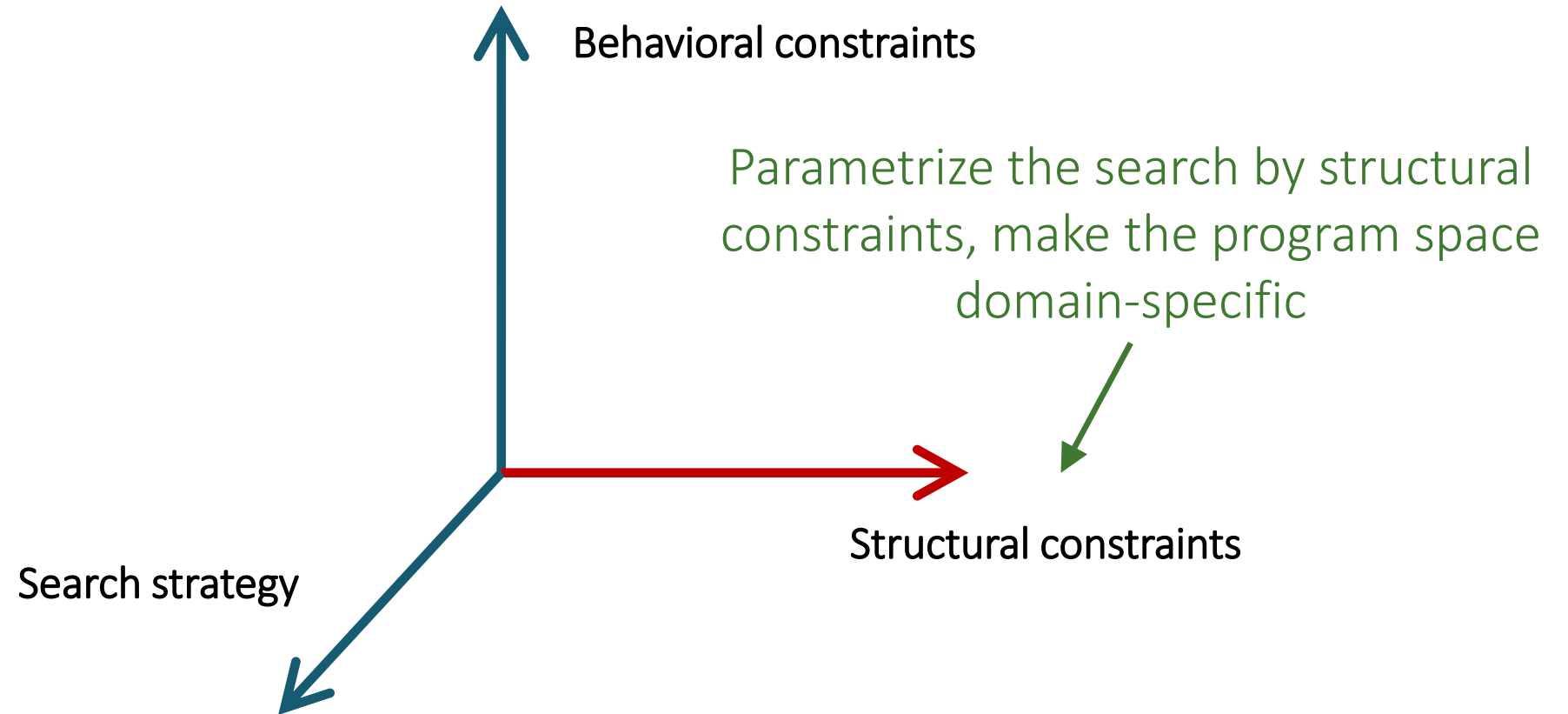
Modern emphasis

- If you can do really well with (1) you can win
- (2) is still important

(1) How do you find a program that matches the observations?

(2) How do you know it is the program you are looking for?

Key idea



Syntax-Guided Synthesis

Example

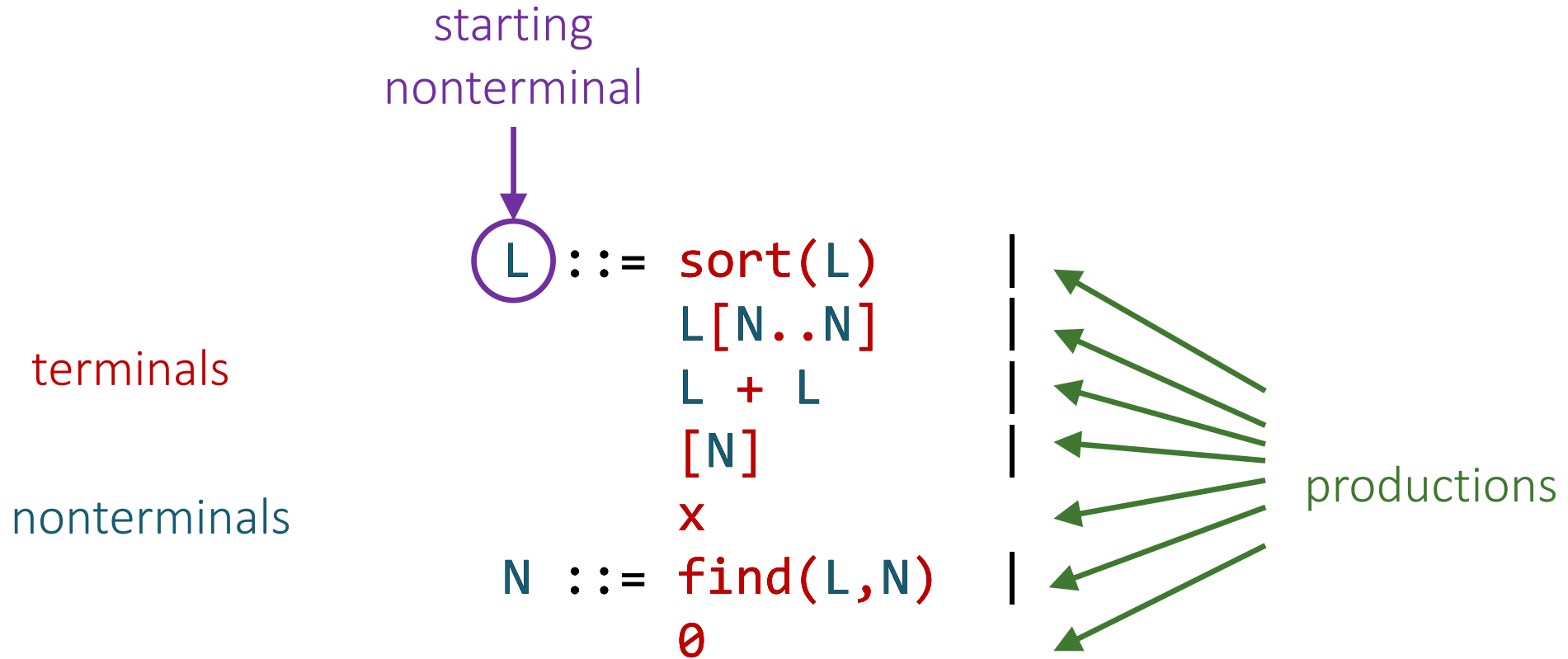
$[1, 4, 7, 2, 0, 6, 9, 2, 5, 0] \rightarrow [1, 2, 4, 7, 0]$

$f(x) := \text{sort}(x[0..\text{find}(x, 0)]) + [0]$

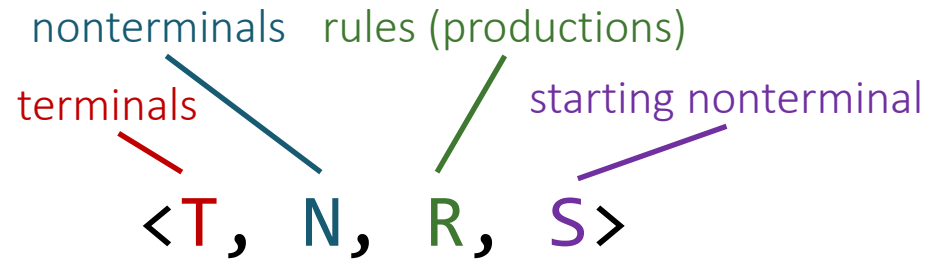
DSL

Let exp
 $\rightarrow L ::= \text{sort}(L)$
 $L[N..N]$
 $L + L$
 $[N]$
Int exp
 $\rightarrow N ::= \text{find}(L, N)$
 0

Context-free grammars (CFGs)



Context-free grammars (CFGs)



Sentential forms: $\{\alpha \in (N \cup T)^*\}$

Rewrites to: $\alpha A \beta \rightarrow \alpha \gamma \beta \equiv (A \rightarrow \gamma) \in R$

(Incomplete) terms/programs: $\{\alpha \in (N \cup T)^* | A \rightarrow^* \alpha\}$

Ground programs: $\{\alpha \in T^* | A \rightarrow^* \alpha\}$

= programs without holes, complete programs

Whole programs: $\{\alpha \in (N \cup T)^* | S \rightarrow^* \alpha\}$

= roughly, programs of the right type

$x \ N \ x$

$x \ N \ x \rightarrow x \ \text{find}(L, N) \ x$

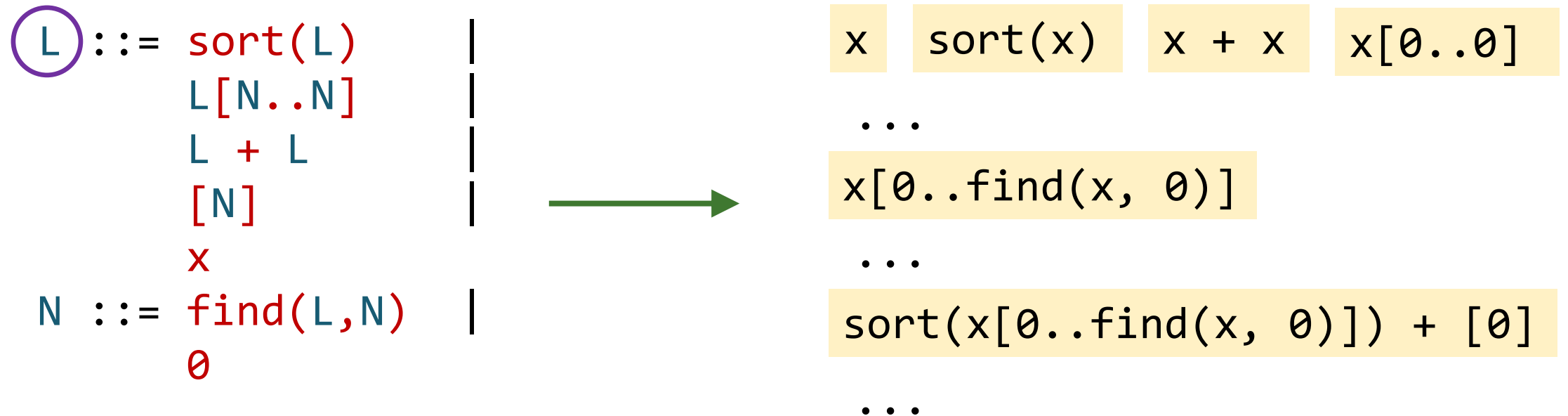
$\text{find}(L+L, N)$

$\text{find}(x+x, \theta)$

$\text{sort}(L+L)$

CFGs as structural constraints

Space of programs
=
all **ground**, **whole** programs



How big is the space?

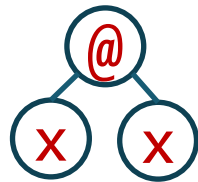
$E ::= x \mid E @ E$

depth ≤ 0



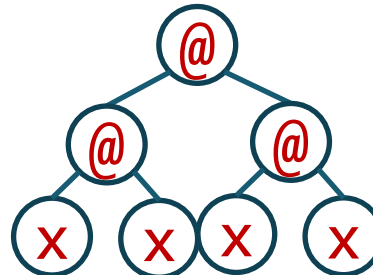
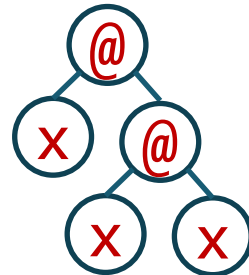
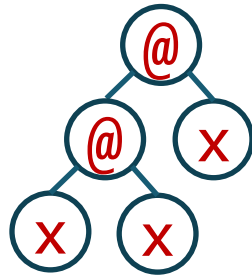
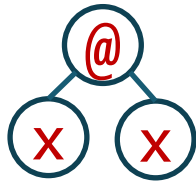
$$N(0) = 1$$

depth ≤ 1



$$N(1) = 2$$

depth ≤ 2



$$N(2) = 5$$

$$N(d) = 1 + N(d - 1)^2$$

How big is the space?

$E ::= x \mid E @ E$

$$N(d) = 1 + N(d - 1)^2$$

$$N(d) \sim c^{2^d} \quad (c > 1)$$

$$N(1) = 1$$

$$N(2) = 2$$

$$N(3) = 5$$

$$N(4) = 26$$

$$N(5) = 677$$

$$N(6) = 458330$$

$$N(7) = 210066388901$$

$$N(8) = 44127887745906175987802$$

$$N(9) = 1947270476915296449559703445493848930452791205$$

$$N(10) = 3791862310265926082868235028027893277370233152247388584761734150717768254410341175325352026$$

How big is the space?

$$E ::= E \overset{x_1}{@_1} E \mid \dots \mid E \overset{x_k}{@_m} E$$

$$N(\emptyset) = k$$

$$N(d) = k + m * N(d - 1)^2$$

$$N(1) = 3$$

$$N(2) = 30$$

$$N(3) = 2703$$

$$N(4) = 21918630$$

$$N(5) = 1441279023230703$$

$$N(6) = 6231855668414547953818685622630$$

$$N(7) = 116508075215851596766492219468227024724121520304443212304350703$$

$$k = m = 3$$

The SyGuS project

[Alur et al. 2013]

<https://sygus.org/>

Goal: Unify different syntax-guided approaches

Collection of synthesis benchmarks + yearly competition

- 6 competitions since 2013
- consider writing a SyGuS solver for your project!

Common input format + supporting tools

- parser, baseline synthesizers

SyGuS problems

SyGuS problem = $\langle \text{theory, spec, grammar} \rangle$

A “library” of types and function symbols

Example: Linear Integer Arithmetic (LIA)

True, False

0, 1, 2, ...

\wedge , \vee , \neg , $+$, \leq , ite

CFG with terminals in the theory
(+ input variables)

Example: Conditional LIA
expressions w/o sums

$E ::= x \mid \text{ite } C \ E \ E$

$C ::= E \leq E \mid C \wedge C \mid \neg C$

SyGuS problems

SyGuS problem = $\langle \text{theory, spec, grammar} \rangle$

A first-order logic formula over
the theory

Examples:

$$f(0, 1) = 1 \wedge$$

$$f(1, 0) = 1 \wedge$$

$$f(1, 1) = 1 \wedge$$

$$f(2, 0) = 2$$

SyGuS demo

SyGuS problems

SyGuS problem = $\langle \text{theory, spec, grammar} \rangle$

A first-order logic formula over
the theory

Examples:

$f(0, 1) = 1 \wedge$
 $f(1, 0) = 1 \wedge$
 $f(1, 1) = 1 \wedge$
 $f(2, 0) = 2$

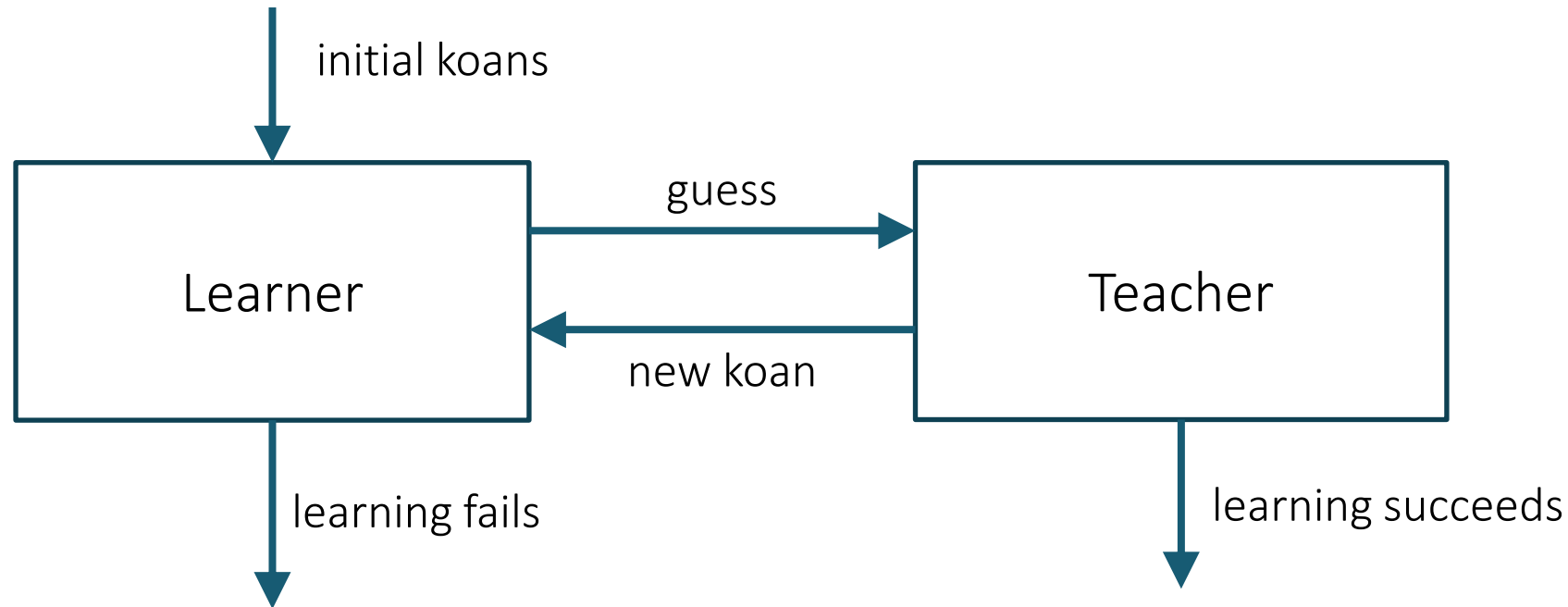
Formula with free variables:

$x \leq f(x, y) \wedge$
 $y \leq f(x, y) \wedge$
 $(f(x, y) = x \vee f(x, y) = y)$

can inductive synthesis
handle these?

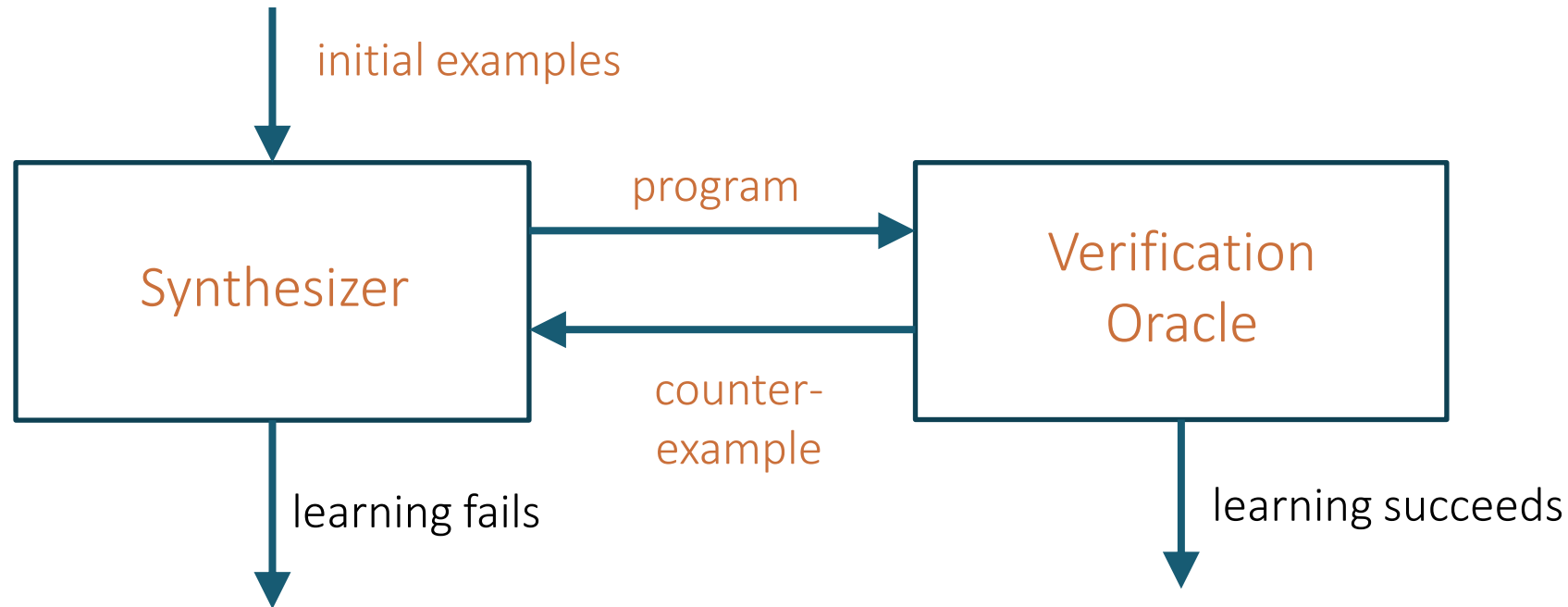
Counter-example guided inductive synthesis (CEGIS)

The Zendo of program synthesis

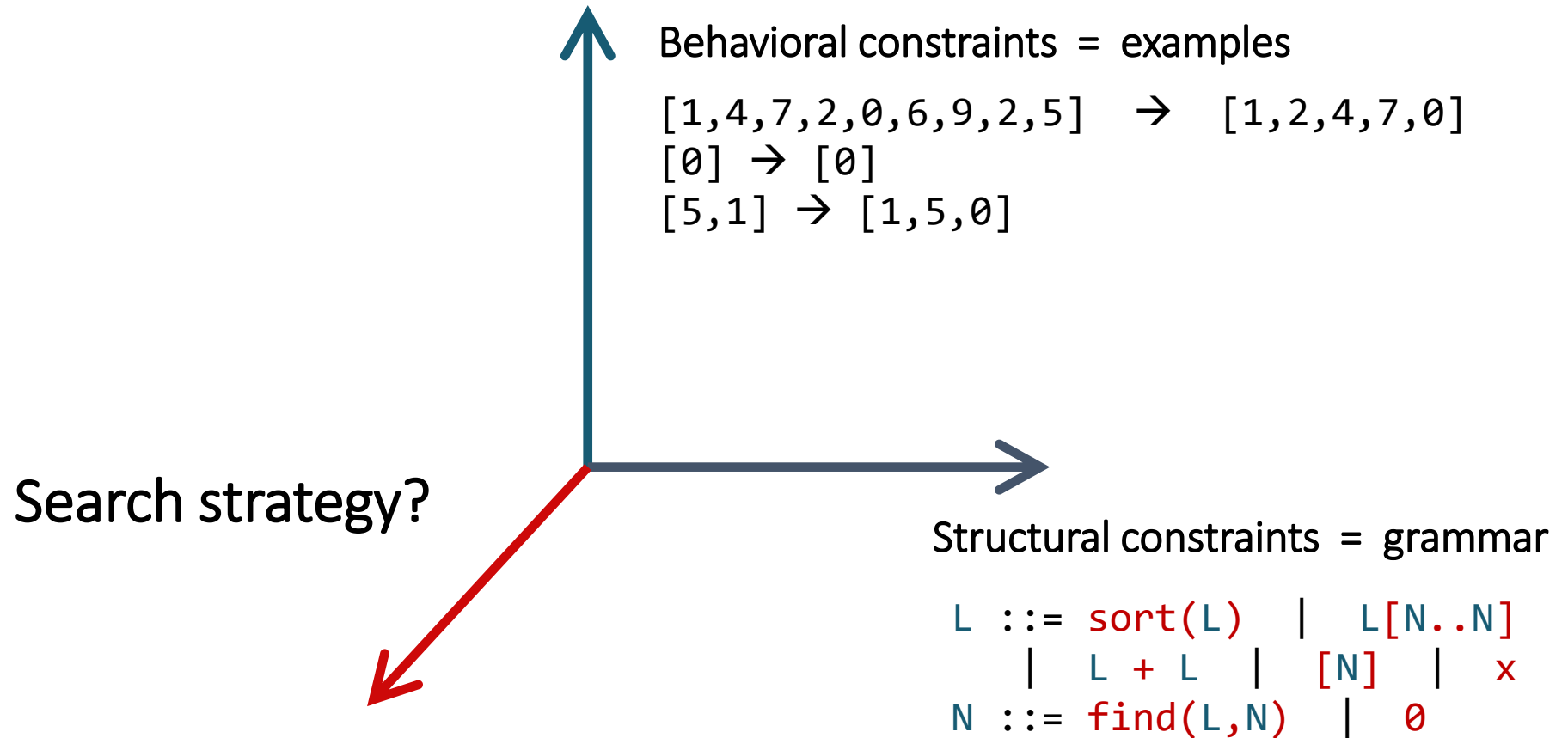


Counter-example guided inductive synthesis (CEGIS)

The Zendo of program synthesis



The problem statement



Enumerative search

Enumerative search

=

Explicit / Exhaustive Search

Idea: Sample programs from the grammar one by one and test them on the examples

Challenge: How do we systematically enumerate all programs?

bottom-up **vs** top-down

Bottom-up enumeration

Start from nullary production (variables and constant)

Combine sub-programs into larger programs using productions

```
L ::= sort(L)      |  
      L[N..N]      |  
      L + L        |  
      [N]           |  
      x
```

```
N ::= find(L,N)    |  
      0
```

`[[1,4,0,6]] → [1,4]`

Bottom-up enumeration

nonterminals rules (productions)

terminals starting nonterminal

```
bottom-up (<T, N, R, S>, [i → o]) {  
  bank := [t | A ::= t in R]  
  while (true)  
    forall (p in bank)  
      if (p([i]) = [o])  
        return p;  
    bank += grow(bank);  
}  
  
grow (bank) {  
  bank' := []  
  forall (A ::= rhs in R)  
    bank' += [rhs[B → p] | p in bank, B →* p]  
  return bank';  
}
```

Q: “Run” bottom-up on the board with:

```
L ::= sort(L) |  
      L[N..N] |  
      L + L |  
      [N] |  
      x  
N ::= find(L,N) |  
      0  
[[1,4,0,6] → [1,4]]
```

Bottom-up: example

Program bank P

iter 0:

x0

iter 1:

sort(x)

x[0..0]

x + x

[0]

find(x,0)

iter 2:

sort(sort(x))

sort(x[0..0])

sort(x + x)

sort([0])

x[0..find(x,0)]

x[find(x,0)..0]

x[find(x,0)..find(x,0)]

sort(x)[0..0]

x[0..0][0..0]

(x + x)[0..0]

[0][0..0]

x + (x + x)

x + [0]

sort(x) + x

x[0..0] + x

(x + x) + x

[0] + x

x + x[0..0]

x + sort(x)

L ::= sort(L)

L[N..N]

L + L

[N]

x

N ::= find(L,N)

0

←

←

←

←

←

←

←

[[1,4,0,6]]

→

[1,4]

...

Top-down enumeration

Start from the start non-terminal
Expand remaining non-terminals using
productions

$L ::= L[N..N] \quad |$

$N ::= \overset{x}{\text{find}}(L, N) \quad |$
 $\quad \quad \quad \theta$

$[[1, 4, \theta, 6] \rightarrow [1, 4]]$

Top-down enumeration

```
top-down(<T, N, R, S>, [i → o]) {  
  w1 := [S]  
  while (w1 != [])  
    p := w1.dequeue();  
    if (ground(p) && p([i]) = [o])  
      return p;  
    w1.enqueue(unroll(p));  
}
```

```
unroll(p) {  
  w1' := []  
  A := left-most non-term in p  
  forall (A ::= rhs in R)  
    w1' += p[A -> rhs]  
  return w1';  
}
```

Q: “Run” top-down on the board with


```
L ::= L[N..N]    |  
      x  
N ::= find(L,N)  |  
      0
```

$[[1, 4, 0, 6] \rightarrow [1, 4]]$

Top-down: example

Worklist w1


iter 0: L

iter 1:  x L[N..N]

iter 2: L[N..N]


iter 3: x[N..N] L[N..N][N..N]

iter 4: x[0..N] x[find(L,N)..N] L[N..N][N..N]


iter 5: x[0..0]  x[0.. find(L,N)] x[find(L,N)..N] ...



iter 6: x[0.. find(L,N)] x[find(L,N)..N]


iter 7: x[0.. find(x,N)] x[0.. find(L[N..N],N)]



iter 8: x[0.. find(x,0)]  x[0.. find(x,find(L,N))]



iter 9:

L ::= L[N..N] | 

 x 

N ::= find(L,N) | 

 → 

Bottom-up vs top-down

Bottom-up

Smaller to larger

- Has to explore between $3 \cdot 10^9$ and 10^{23} programs to find `sort(x[0..find(x, 0)]) + [0]` (depth 6)

Candidates are **ground** but might not be **whole**

- Can always run on inputs
- Cannot always relate to outputs

Top-down

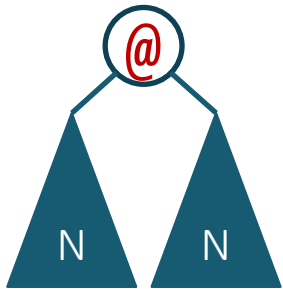
Candidates are **whole** but might not be **ground**

- Cannot always run on inputs
- Can always relate to outputs (?)

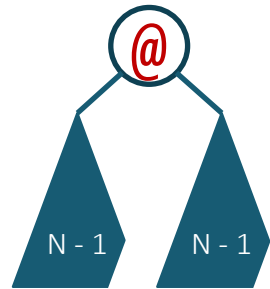
How to make it scale

Prune

Discard useless subprograms



$$m * N^2$$



$$m * (N - 1)^2$$

Prioritize

Explore more promising candidates first

$$P = \{ \begin{array}{l} [0][N..N] \\ x[N..N] \\ \dots \end{array} , \quad \leftarrow \text{dequeue this first}$$