Lecture 10
Program Sketching

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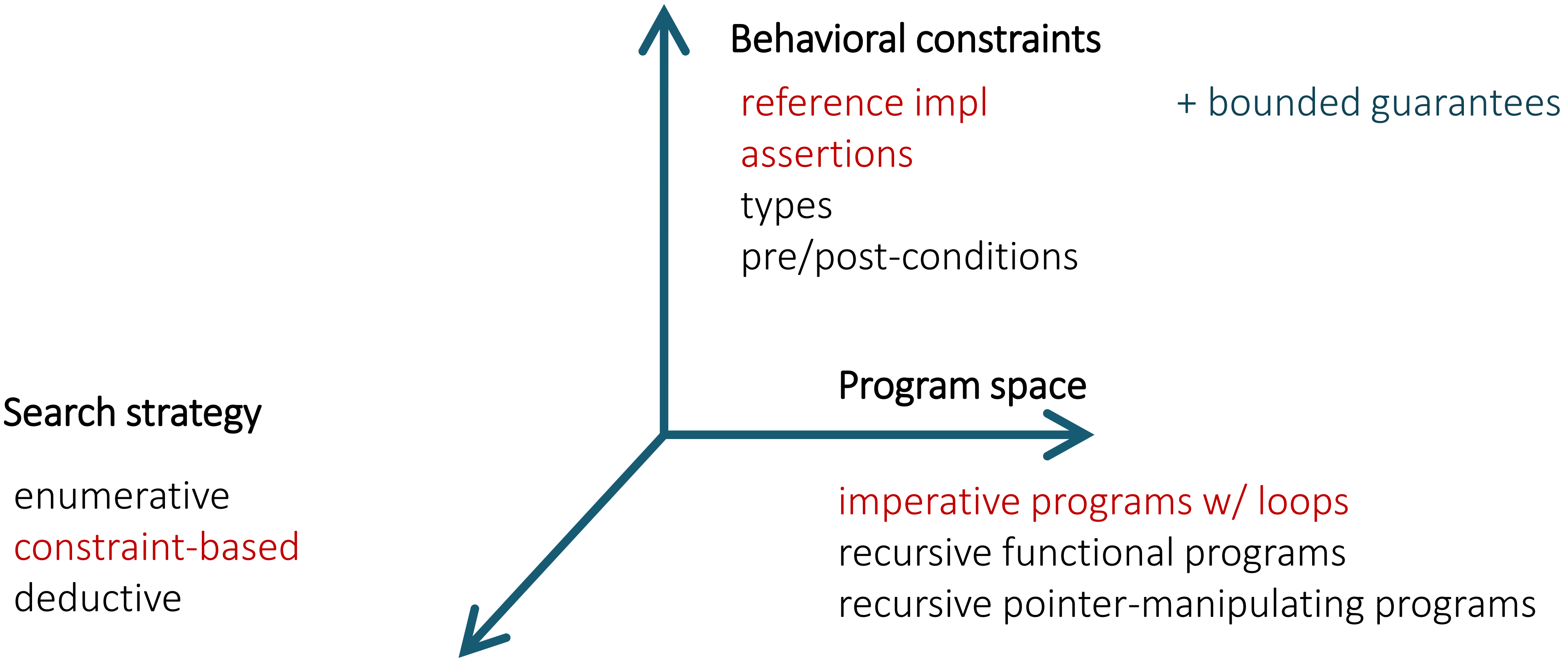
Program Sketching



Constraint-based synthesis

Behavioral constraints

Structural constraints

$$\xrightarrow{\text{encoding}} \quad \exists C\,.\,\mathrm{spec}(C)$$

CBS for complex programs

2. How to encode the behavior of complex programs?

Behavioral constraints
= assertions / reference implementation

Structural constraints

encoding

$$\exists C\,.\,\mathrm{spec}(C)$$

1. How to specify for complex programs?

3. How to solve for complex specs?

Program Sketching

2. How to encode the behavior of complex programs?

Symbolic execution

Behavioral constraints
= assertions / reference implementation

Structural constraints

encoding

$$\exists C \,.\, \mathrm{spec}(C)$$

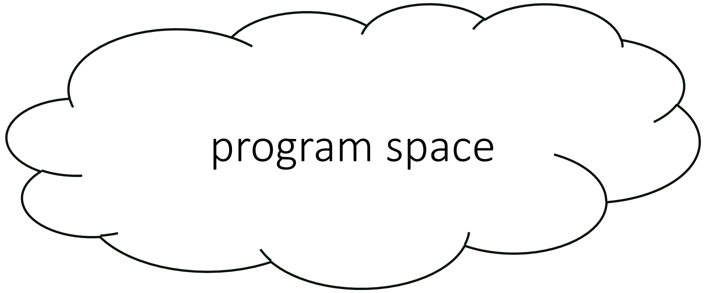
3. How to solve for complex specs?

CEGIS

1. How to specify for complex programs?

Sketches

CBS from examples



`E = [i → o]`

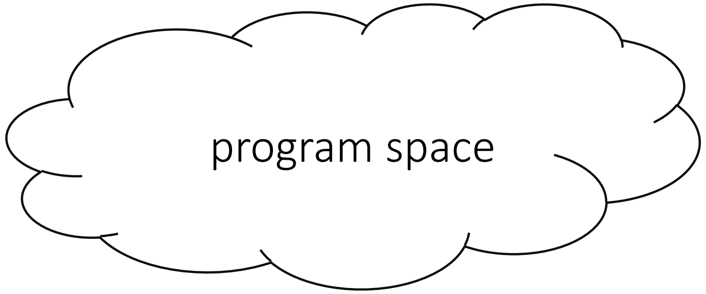
encoding

Parameters (control vector)

"*C* denotes a program that gives output *o* on input *i*"

$$\exists C \,.\, \bigwedge_{\langle i,o\rangle \in E} \varphi(C, i, o)$$

CBS from specifications



$\cancel{E = [i \to o]}$

$$\forall\, i\ o\, .\, \psi(i,o)$$

encoding

"*C* denotes a program that gives output *o* on input *i*"

$$\cancel{\exists C\, .\, \bigwedge_{\langle i,o\rangle \in E} \varphi(C,i,o)}$$

$$\boxed{\exists C\, .\, \forall i\ o}\, .\, \varphi(C,i,o) \Rightarrow \psi(i,o)$$

doubly-quantified constraint:
not solver-friendly

Example

```
harness void main(int x) {
  int y := ?? * x + ??;
  assert y - 1 == x + x;
}
```

encoding

$$\exists C\,.\,\forall i\;o\,.\,\varphi(C,i,o) \Rightarrow \psi(i,o)$$

$$\exists c_1 c_2\,.\,\forall x\;y\,.\,y = c_1 * x + c_2 \Rightarrow y - 1 = x + x$$

simplify

$$\exists c_1 c_2\,.\,\forall x\,.\,c_1 * x + c_2 - 1 = x + x$$

How do we solve this constraint?

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$$\exists c \,.\, \forall x \,.\, Q(c, x)$$

Idea 1: Bounded Observation Hypothesis

- Assume there exists a small set of inputs $\mathrm{X} = \{x_1, x_2, \ldots x_n\}$ such that whenever c satisfies

$$\bigwedge_{i \in 1..n} Q(c, x_i)$$

No quantifiers here, can give to SAT / SMT

it also satisfies

$$\forall x. Q(c, x)$$

Example

This is a linear constraint, two inputs are enough!

$$\exists c_1 c_2 \,.\, \forall x \,.\, c_1 * x + c_2 - 1 = x + x$$

$$X = \{0, 1\}$$

$$Q(c_1, c_2, 0) \equiv c_2 - 1 = 0$$

$$Q(c_1, c_2, 1) \equiv c_1 + c_2 - 1 = 2$$

$$\{c_1 \to 2, c_2 \to 1\}$$

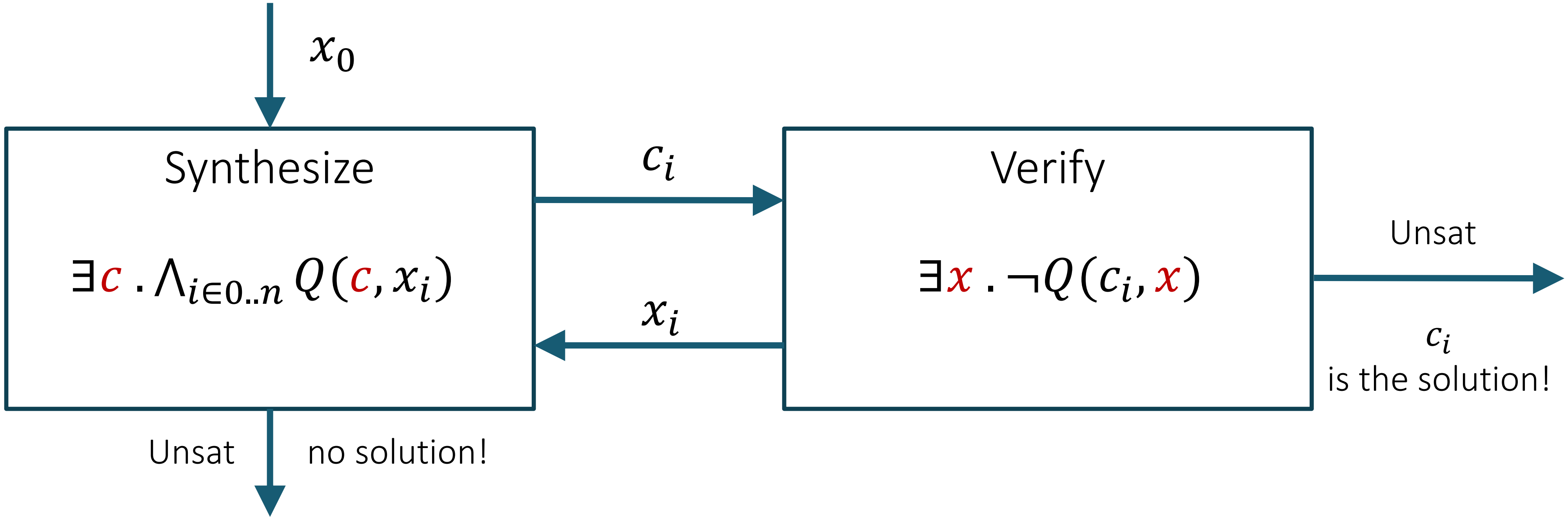
```
harness void main(int x) {
  int y := 2 * x + 1;
  assert y - 1 == x + x;
}
```

How do we find X in a general case?

CEGIS

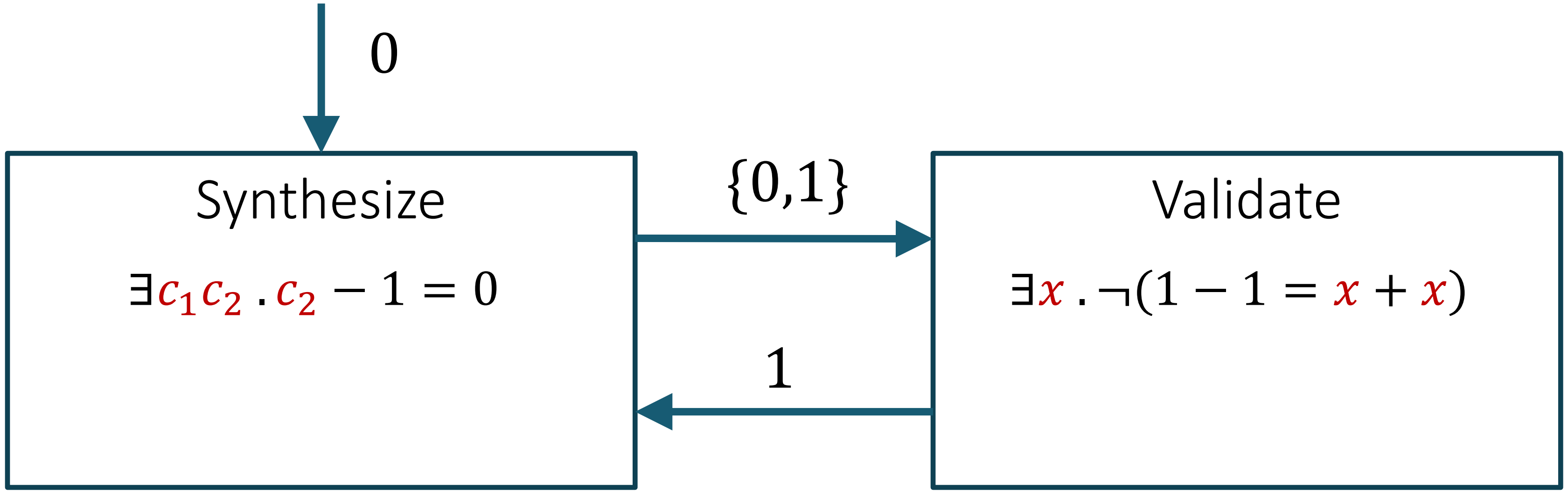
$$\exists c\,.\,\forall x\,.\,Q(c,x)$$

Idea 2: Rely on verification oracle to generate counterexamples



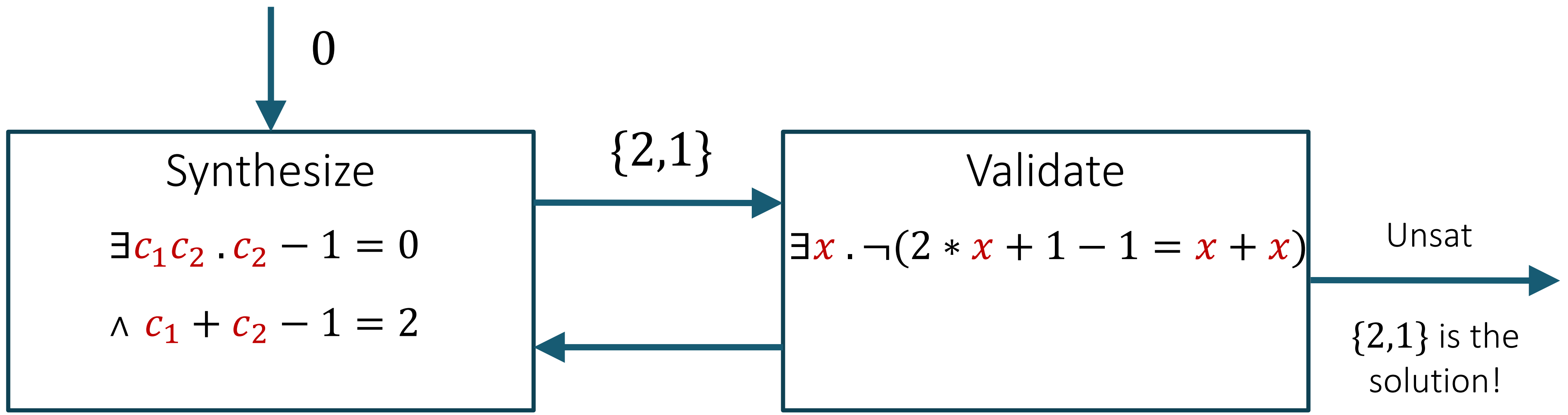
Example

$$\exists c_1 c_2 \,.\, \forall x \,.\, c_1 * x + c_2 - 1 = x + x$$



Example

$$\exists c_1 c_2 . \forall x . c_1 * x + c_2 - 1 = x + x$$



Program Sketching

2. How to encode the behavior of complex programs?

Symbolic execution

Behavioral constraints
= assertions / reference
implementation

Structural constraints

encoding

$$\exists C \,.\, \forall x \,.\, Q(C, x)$$

3. How to solve for complex specs?

CEGIS

1. How to specify for complex programs?

Sketches

Structural constraints in Sketch

Different constraints good for different problems

- CFGs
- Components
- Just figure out the constants

Idea: Allow the programmer to encode all kinds of constraints using... programs (duh!)

Language Design Strategy

Extend base language with one construct

Constant hole: ??

```
int bar (int x)
{
    int t = x * ??;
    assert t == x + x;
    return t;
}
```

```
int bar (int x)
{
    int t = x * 2;
    assert t == x + x;
    return t;
}
```

Synthesizer replaces ?? with a natural number

Constant holes → sets of expressions

Expressions with `??` == sets of expressions

- linear expressions `x*?? + y*??`
- polynomials `x*x*?? + x*?? + ??`
- sets of variables `?? ? x : y`

Example: swap without a temporary

Swap two integers without an extra temporary

```
void swap(ref int x, ref int y){
    x = ... // sum or difference of x and y
    y = ... // sum or difference of x and y
    x = ... // sum or difference of x and y
}

harness void main(int x, int y){
    int tx = x; int ty = y;
    swap(x, y);
    assert x==ty && y == tx;
}
```

Syntactic sugar

`{| RegExp |}`

RegExp supports choice ‘|’ and optional ‘?’

- can be used arbitrarily within an expression
  - to select operands `{| (x | y | z) + 1 |}`
  - to select operators `{| x (+ | -) y |}`
  - to select fields `{| n(.prev | .next)? |}`
  - to select arguments `{| foo( x | y, z) |}`

Set must respect the type system

- all expressions in the set must type-check
- all must be of the same type

Complex program spaces

Idea: To build complex program spaces from simple program spaces, borrow abstraction devices from programming languages

Function: abstracts expressions

Generator: abstracts set of expressions

- Like a function with holes...
- ...but different invocations → different code

Example: swap without a temporary

```
generator int sign() {
    if ?? {return 1;} else {return -1;}
}

void swap(ref int x, ref int y){
    x = x + sign()*y;          ➔ 1
    y = x + sign()*y;          ➔ -1
    x = x + sign()*y;          ➔ -1
}

harness void main(int x, int y){
    int tx = x; int ty = y;
    swap(x, y);
    assert x==ty && y == tx;
}
```

Recursive generators

Can generators encode a CFG?

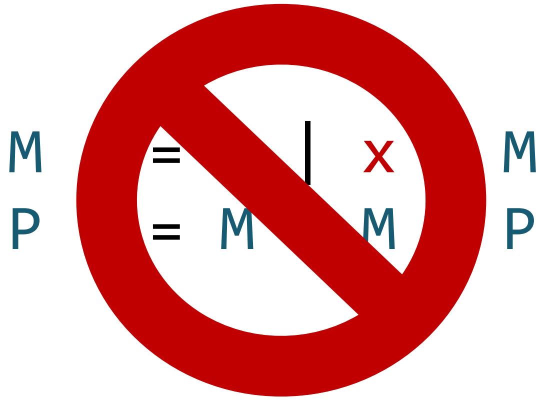
```
M ::= n | x * M
P ::= M | M + P
```

```
generator int mono(int x) {
    if (??) {return ??;}
    else {return x * mono(x);}
}

generator int poly(int x) {
    if (??) {return mono(x);}
    else {return mono(x) + poly(x);}
}
```

Recursive generators

What if monomial of every degree can occur at most once?




```
generator int mono(int x, int n) {
    if (n <= 0) {return ??;}
    else {return x * mono(x, n - 1);}
}

generator int poly(int x, int n) {
    if (n <= 0) {return mono(x,0);}
    else {return mono(x,n) + poly(x, n - 1);}
}
```

Program Sketching

2. How to encode the behavior of complex programs?

Symbolic execution

Program c has no assertion violations on input x

Behavioral constraints = assertions / reference implementation

Structural constraints

encoding

$$\exists C \,.\, \forall x \,.\, Q(C, x)$$

3. How to solve for complex specs?

CEGIS

1. How to specify for complex programs?

Sketches

Symbolic execution

Semantics of a simple imperative language

How to use it for symbolic execution?

Adding while loops

Adding holes

Semantics of a simple language

```
e   :=  n | x | e₁ + e₂
c   :=  x := e | assert e
      | c₁ ; c₂ | if e then c₁ else c₂ | while e do c
```

What does an expression mean?

- An expression reads the state and produces a value
- The state is modeled as a map σ from variables to values
- $\mathcal{A}[\![\cdot]\!] : e \to \Sigma \to \mathbb{Z}$

Ex:

- $\mathcal{A}[\![x]\!] = \lambda\sigma.\, \sigma[x]$
- $\mathcal{A}[\![n]\!] = \lambda\sigma.\, n$
- $\mathcal{A}[\![e_1 + e_2]\!] = \lambda\sigma.\, \mathcal{A}[\![e_1]\!]\sigma + \mathcal{A}[\![e_2]\!]\sigma$

Semantics of a simple language

```
e   :=  n | x | e₁ + e₂
c   :=  x := e | assert e
      | c₁ ; c₂ | if e then c₁ else c₂ | while e do c
```

What does a command mean?

- A command modifies the state
- $\mathcal{C}[\![\cdot]\!] : c \to \Sigma \to \Sigma$

Ex:

- $\mathcal{C}[\![x := e]\!] = \lambda\sigma.\sigma[x \mapsto \mathcal{A}[\![e]\!]\sigma]$
- $\mathcal{C}[\![c_1; c_2]\!] = \lambda\sigma.\mathcal{C}[\![c_2]\!](\mathcal{C}[\![c_1]\!]\sigma)$
- $\mathcal{C}[\![\text{if } e \text{ then } c_1 \text{ else } c_2]\!] = \lambda\sigma.\mathcal{A}[\![e]\!]\sigma \neq 0 \ \ ? \ \ \mathcal{C}[\![c_1]\!]\sigma \ : \ \ \mathcal{C}[\![c_2]\!]\sigma$

Semantics of assertions

```
e   :=  n | x | e₁ + e₂
c   :=  x := e | assert e
      | c₁ ; c₂ | if e then c₁ else c₂ | while e do c
```

What does a command mean?

- Commands also generate constraints on valid executions
- $\mathcal{C}[\![\cdot]\!] : c \to \langle \Sigma, \Psi \rangle \to \langle \Sigma, \Psi \rangle$

Constraints on values in initial σ

Ex:

- $\mathcal{C}[\![assert\ e]\!] = \lambda\langle \sigma, \psi \rangle . \langle \sigma, \psi \wedge \mathcal{A}[\![e]\!]\sigma \neq 0 \rangle$

Symbolic execution

Semantics of a simple imperative language

How to use it for symbolic execution?

Adding while loops

Adding holes

Concrete execution: example 1

Let's run this with `x = 2`

```
void main(int x){
  int y = 2 * x;
  assert y > x;
}
```

$\sigma = \{x \rightarrow 2\}, \qquad \psi = T$

$\sigma = \{x \rightarrow 2, y \rightarrow 4\}, \psi = T$

$\sigma = \{x \rightarrow 2, y \rightarrow 4\}, \psi = \{4 > 2\}$

Test passed

Symbolic execution: example 1

```
void main(int x){
  int y = 2 * x;
  assert y > x;
}
```

$\sigma = \{x \to X\}, \psi = \top$

$\sigma = \{x \to X, y \to 2X\}$

$\psi = \{\, 2X > X\}$

$$\mathcal{C}[\![p]\!]\langle\{\}, \top\rangle = \langle\{x \to X, y \to 2X\}, 2X > X\rangle$$

Verification constraint

$\{X \mapsto 0\}$ ✗ $\xleftarrow{\text{SMT solver}}$ $\forall X.\, 2X > X$

Symbolic execution: example 2

```
void main(int x, int u){
  int y = 0;
  if (u > 0) {
    y = 2 * x;
  } else {
    y = x + x;
  }
  assert y == 2*x;
}
```

$$\sigma = \{x \to X, u \to U\}$$

$$\sigma = \{x \to X, u \to U, y \to 0\}$$

$$\sigma = \{x \to X, u \to U, y \to 2X\}$$

$$\sigma = \{x \to X, u \to U, y \to X + X\}$$

$$\sigma = \{x \to X, u \to U, y \to U > 0\ ?\ 2X : X + X\}$$

$$\psi = \{(U > 0\ ?\ 2X : X + X) = 2X\}$$

Symbolic execution

Semantics of a simple imperative language

How to use it for symbolic execution?

Adding while loops

Adding holes

What about loops?

Semantics of a while loop

- Let $W : \Sigma \to \Sigma = \mathcal{C}[\![while\ e\ do\ c]\!]$
- W satisfies the following equation:
$$W\ \sigma \quad = \quad \mathcal{A}[\![e]\!]\sigma \neq 0\ \ ?\ \ W(\mathcal{C}[\![c]\!]\sigma)\ :\ \sigma$$
- One strategy: find a fixpoint (see later in class)
- We'll settle for a simpler strategy: unroll k times and then give up

Symbolic execution: example 3

```
void main(int x){
  int y = 0;
  int i = 0;
  while (i < 2) {
    y = y + x;
    i = i + 1;
  }
  assert y == i * x;
}
```

Step 1: unroll with depth = 2

```
if (i < 2) {
  y = y + x;
  i = i + 1;
  if (i < 2) {
    y = y + x;
    i = i + 1;
    assert !(i < 2);
  }
}
```

Symbolic execution: example 3

```
void main(int x){
  int y = 0;
  int i = 0;
  if (i < 2) {
    y = y + x;
    i = i + 1;
    if (i < 2) {
      y = y + x;
      i = i + 1;
      assert !(i < 2);
    }
  }
  assert y == i*x;
}
```

$$\sigma = \{x \to X\}$$

$$\sigma = \{x \to X, y \to 0, i \to 0\}$$

$$\sigma = \{x \to X, y \to X, i \to 1\}$$

Simplified from *0 < 2 ? (1 < 2 ? X + X : X) : 0*

$$\sigma = \{x \to X, y \to \quad \to 2\}$$

$$\psi = \{\neg(2 > 2)\}$$

$$\sigma = \{x \to X, y \to X + X, i \to 2\}$$

$$\psi = \{\neg(2 > 2) \ \wedge \ X + X = 2X\}$$

Symbolic execution

Semantics of a simple imperative language

How to use it for symbolic execution?

Adding while loops

Adding holes

Semantics of sketches

```
e   :=  n | x | e₁ + e₂ | ??ᵢ
c   :=  x := e | assert e
      | c₁ ; c₂ | if e then c₁ else c₂ | while e do c
```

What does an expression mean?

- Like before, but with a “hole environment” ϕ
- $\mathcal{A}[\![\cdot]\!] : e \to \mathbf{\Phi} \to \Sigma \to \mathbb{Z}$

Ex:

- $\mathcal{A}[\![x]\!] = \lambda\phi.\,\lambda\sigma.\,\sigma[x]$
- $\mathcal{A}[\![??_i]\!] = \lambda\phi.\,\lambda\sigma.\,\phi[i]$
- $\mathcal{A}[\![e_1 + e_2]\!] = \lambda\phi.\,\lambda\sigma.\,\mathcal{A}[\![e_1]\!]\phi\sigma + \mathcal{A}[\![e_2]\!]\phi\sigma$

Symbolic Evaluation of Commands

Commands have two roles

- Modify the symbolic state
- Generate constraints

$$\mathcal{C}[\![\cdot]\!] : c \rightarrow \Phi \rightarrow \langle \Sigma, \Psi \rangle \rightarrow (\Sigma, \Psi)$$

Symbolic Evaluation of Commands

Example: assignment and assertion

$$\mathcal{C}[\![x \coloneqq e]\!]\phi\langle\sigma, \psi\rangle = \langle\sigma[x \mapsto \mathcal{A}[\![e]\!]\phi\sigma], \psi\rangle$$

$$\mathcal{C}[\![\textbf{assert}\ e]\!]\phi\langle\sigma, \psi\rangle = \langle\sigma, \psi \wedge \mathcal{A}[\![e]\!]\phi\sigma \neq 0\rangle$$

Symbolic execution of sketches: example

```
void main(int x){
  int z = ??₁ * x;
  int y = 0;
  int i = 0;
  if (i < 2) {
    y = y + x;
    i = i + 1;
    if (i < 2) {
      y = y + x;
      i = i + 1;
      assert !(i < 2);
    }
  }
  assert y == z;
}
```

$\sigma = \{x \to X\}$ $\quad \psi = \top$

$\sigma = \{x \to X, z \to \phi_1 * X, y \to 0, i \to 0\}$

$\sigma = \{x \to X, z \to \phi_1 * X, y \to X, i \to 1\}$

$\sigma = \{x \to X, z \to \phi_1 * X, y \to X + X, i \to 2\}$

$\psi = \{\neg(2 > 2)\}$

$\psi = \{\neg(2 > 2) \ \wedge \ X + X = \phi_1 * X\}$

$$\exists \phi_1 . \forall X . X + X = \phi_1 * X$$

$$\{\phi_1 \mapsto 2\} \xleftarrow{\text{CEGIS}}$$

Controls for generators

```
harness void main(int x, int y){
→  z = mono(x)   +   mono(y);
   assert z == x + x + 3;
}

generator int mono(int x) {
    if (??1) {return ??2;}
    else {return x * mono(x);}
}
```

$$\sigma = \{z \rightarrow (\phi_1 \mathbin{?} \phi_2 : X * \phi_2) + (\phi_1 \mathbin{?} \phi_2 : Y * \phi_2)\}$$

No solution!

unroll with
depth = 1

```
if (??1) {return ??2;}
else {return x * ??2;}
```

We need to map different calls to `mono` to different controls!

Controls for generators: context

```
harness void main(int x, int y){
  z = mono¹(x,1) +  mono²(y,2);
  assert z == x + x + 3;
}

generator int mono(int x, context τ) {
    if (??τ₁) {return ??τ₂;}
    else {return x * mono³(x, τ.3);}
}
```

$$\sigma = \{z \to (\phi_1^1 \,?\, \phi_2^1 : X * \phi_2^{1.3}) + (\phi_1^2 \,?\, \phi_2^2 : X * \phi_2^{2.3})\}$$

$$\{\phi_1^1 \mapsto 0, \phi_2^{1.3} \mapsto 2, \phi_1^2 \mapsto 1, \phi_2^{1.3} \mapsto 3\}$$

Sketch: contributions

Expressing structural and behavioral constraints as programs

- the only primitive extension is an integer hole ??
- why is it important to keep extensions minimal?

Synthesis by translating to SAT

CEGIS

- became extremely popular!

Handles imperative programs with loops

- and proposes an encoding for those

Can discover constants

Sketch: limitations

Everything is bounded

- loops are unrolled
- integers are bounded
- are any of the above easily fixable?

Too much input from the programmer?

- but: as search gets better, less user input is required

CEGIS relies on the Bounded Observation Hypothesis

Sketches hard to debug

No bias, no non-functional constraints

Sketch: questions

Behavioral constraints? structural constraints? search strategy?

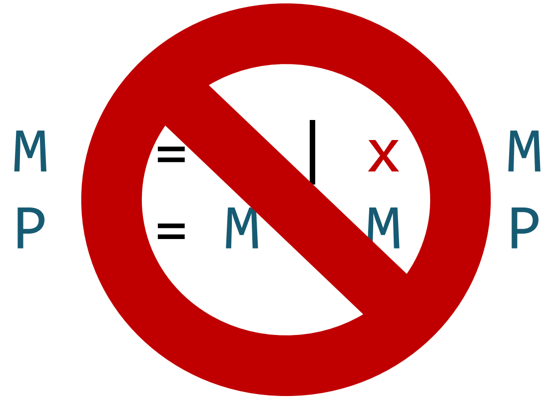
- assertions / reference implementation
- sketches
- constraint-based (CEGIS + SAT)

Sketches vs CFGs? Brahma’s components?

- A generator can encode a multiset of components (although it’s not very straightforward)
- Can a generator encode a CFG?

Recursive generators

What if monomial of every degree can occur at most once?




```
generator int mono(int x, int n) {
    if (n <= 0) {return ??;}
    else {return x * mono(x, n - 1);}
}

generator int poly(int x, int n) {
    if (n <= 0) {return mono(x,0);}
    else {return mono(x,n) + poly(x, n - 1);}
}
```

Generators are more expressive than CFGs!

- but unbounded generators cannot be encoded into constraints
- need to bound unrolling depth
- bounded generators less expressive than CFGs (but more convenient)

Semantics of abort

$$\mathcal{C}[\![\textbf{abort}]\!]\langle \sigma, \psi \rangle = \langle \sigma, \perp \rangle$$

CEGIS: the worst case

Satisfiable constraint $\exists c. \forall x. Q(c,x)$ that violates the Bounded Observation Hypothesis

$$\boxed{Q(c,x) \equiv c = x}$$

unsatisfiable, rejected in 2 iterations

$$\boxed{Q(c,x) \equiv (x \oplus c) \neq 111}$$

"every x has at least one bit in common with c"

unsatisfiable, but only rejected in 2^N iterations

$$\boxed{Q(c,x) \equiv x = (x \,\&\, c)}$$

solved in max n iterations

$$\boxed{Q(c,x) \equiv x \leq c}$$

solved in one iteration with x = 111

(but will require 2^N iterations with worst-case counterexamples)

$$\boxed{Q(c,x) \equiv x \neq c \vee c = 111}$$

violates BOH:

no small set of counter-examples exists