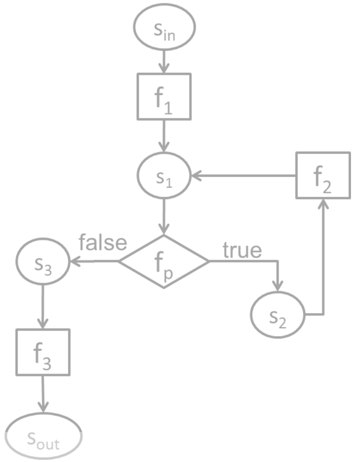
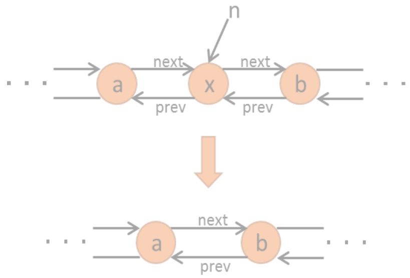
$$\exists c \forall in \; Q(c, in)$$

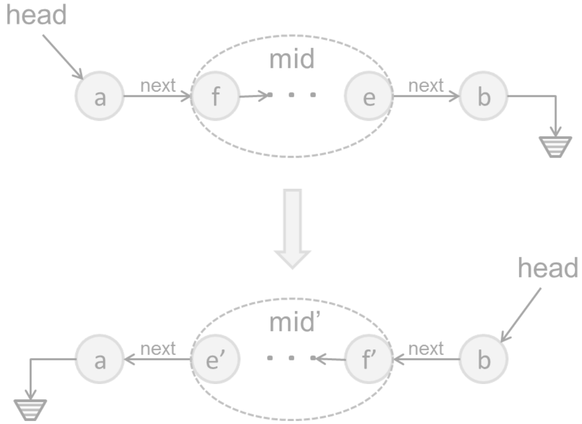
```
/* Average of x and y without using x+y (avoid overflow)*/
int avg(int x, int y){
  int t = expr({x/2, y/2, x%2, y%2, 2 }, {PLUS, DIV});
  assert t == (x+y)/2;
  return t;
}
```

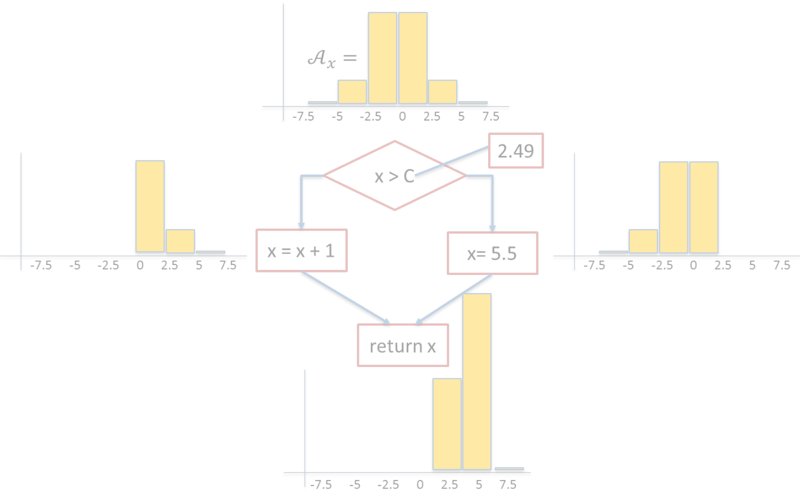





```
{
  s = n.succ;
  p = n.pred;
  p.succ = s;
  s.pred = p;
}
```

Module I: Synthesizing Simple Programs





$$\varphi(p)$$

$$Sk[c](in)$$

Lecture 2
Syntax-Guided Synthesis and Enumerative Search

Nadia Polikarpova

Logistics

Slack

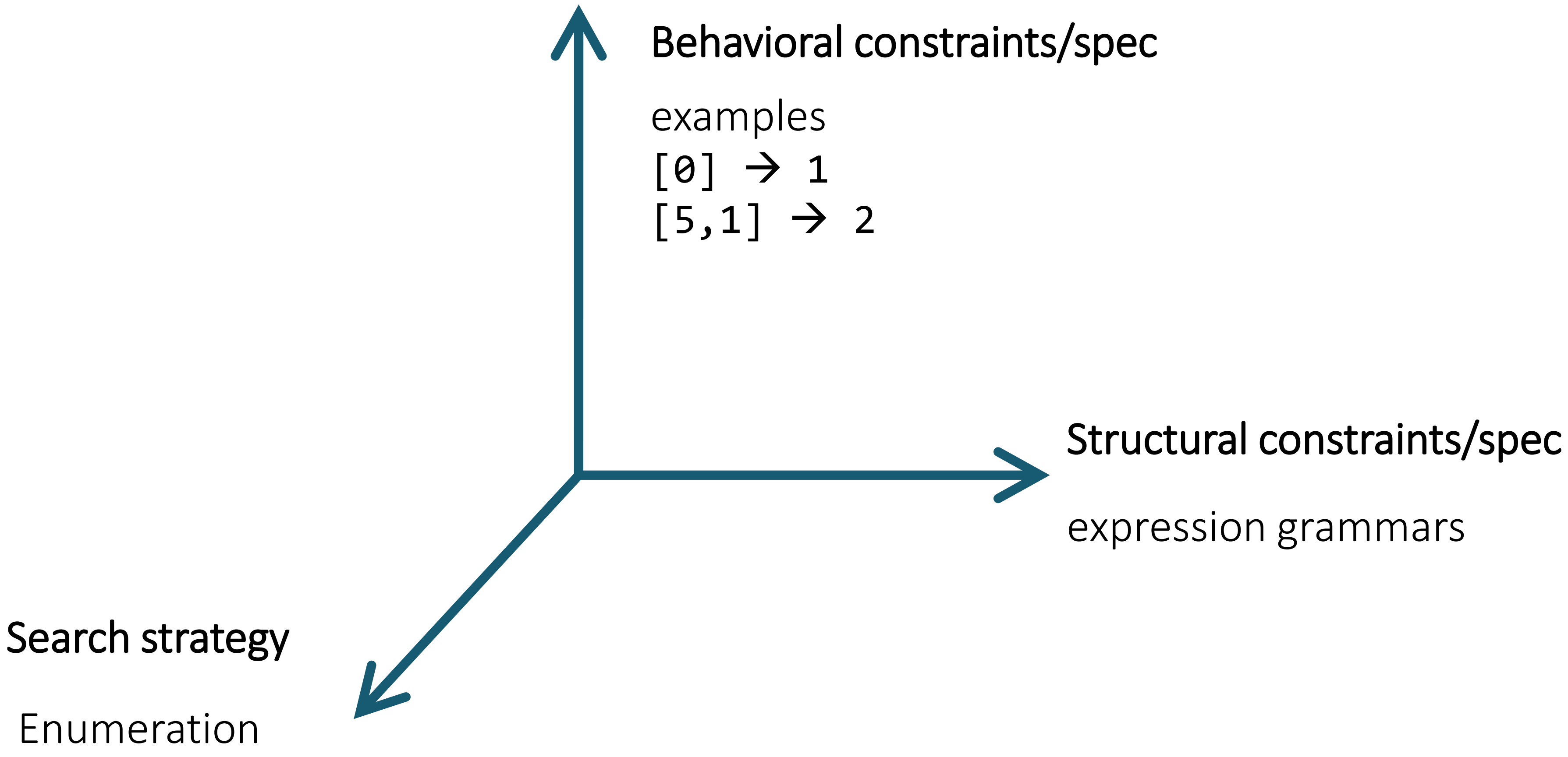
- Has everyone signed up?
- “Search for Teammates” channel

Shared Google folder

- Does everyone have access?
- Register your team by next Friday

Other questions?

Week 1-2


Behavioral constraints/spec
examples
[0] → 1
[5,1] → 2
Structural constraints/spec
expression grammars
Search strategy
Enumeration

Today

Synthesis from examples

Syntax-guided synthesis

- expression grammars as structural constraints
- the SyGuS project

Enumerative search

- enumerating all programs generated by a grammar
- bottom-up vs top-down

Synthesis from examples

Synthesis from Examples

=

Programming by Example

=

Inductive Programming

Inductive Learning

A little bit of history: inductive learning

MIT/LCS/TR-76

LEARNING STRUCTURAL DESCRIPTIONS FROM EXAMPLES

Patrick H. Winston

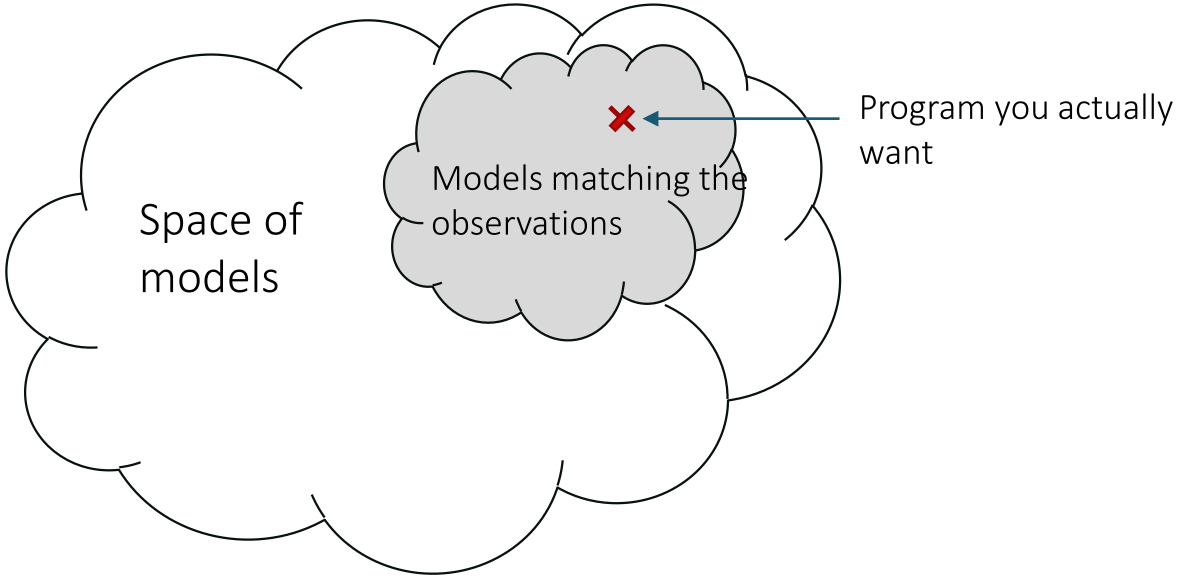
September 1970

Patrick
Winston

Explored the question of generalizing from a set of observations

Became the foundation of machine learning

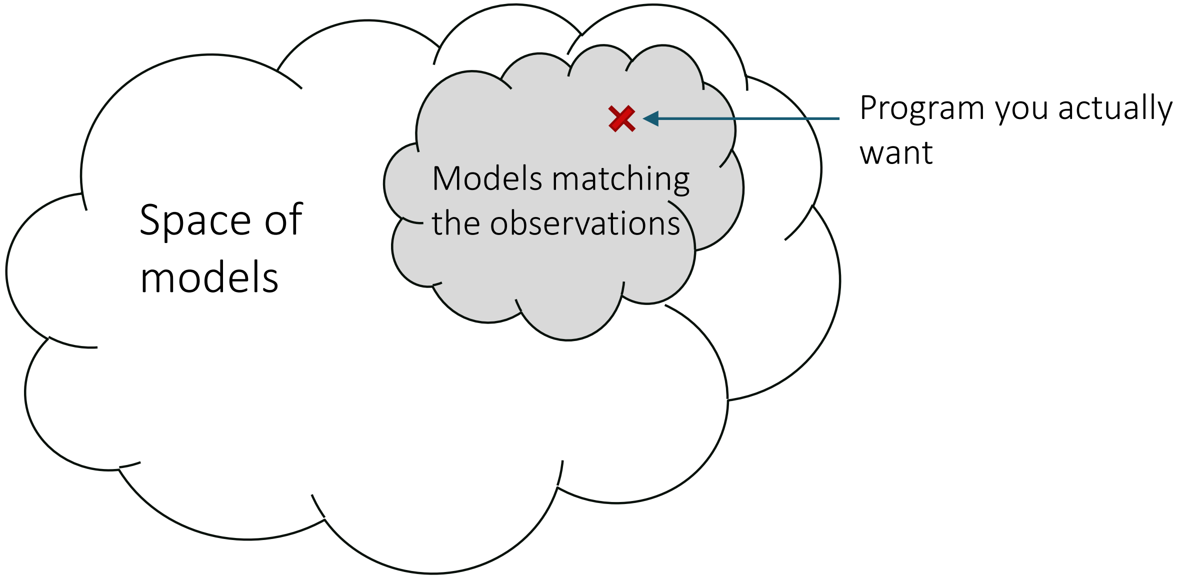
Key issues in inductive learning



(1) How do you find a model that matches the observations?

(2) How do you know it is the model you are looking for?

Key issues in inductive learning



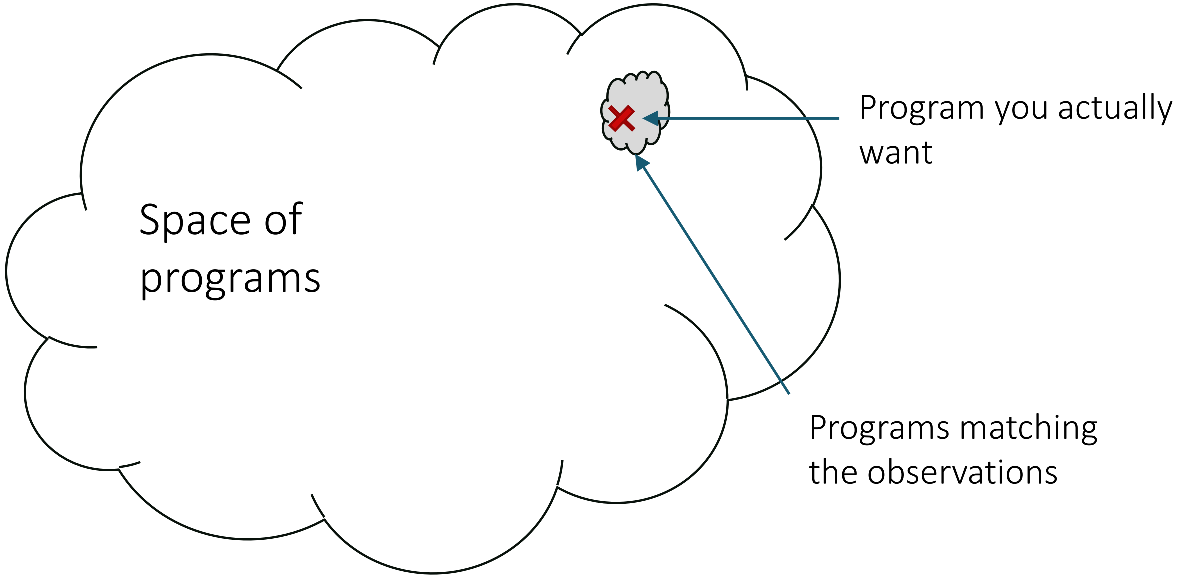
Traditional ML:

- Fix the space so that (1) is easy
- (2) becomes the main challenge

(1) How do you find a model that matches the observations?

(2) How do you know it is the model you are looking for?

The synthesis approach



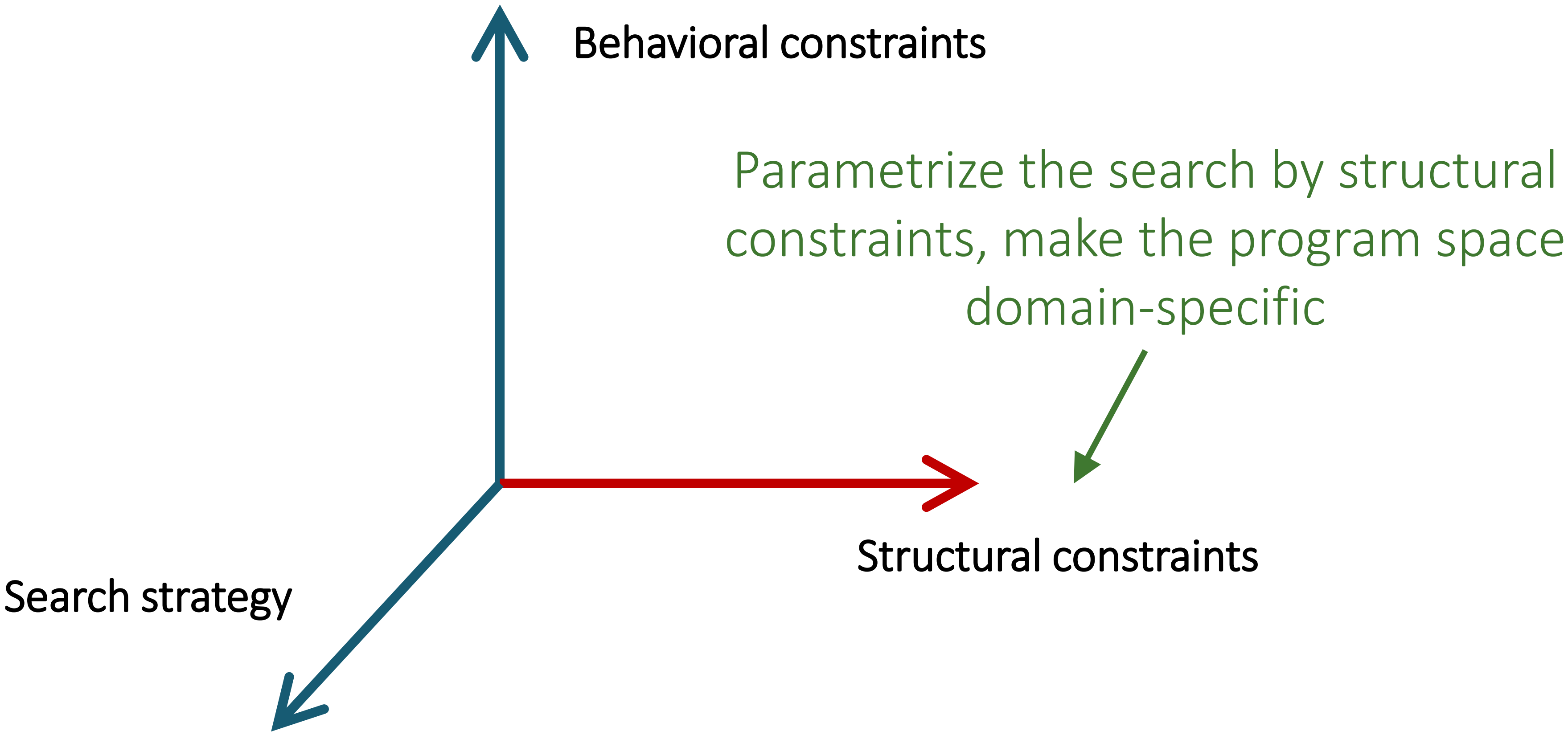
Program synthesis:

- Customize the space so that (2) becomes easier
- (1) is now the main challenge

(1) How do you find a program that matches the observations?

(2) How do you know it is the program you are looking for?

Key idea



Behavioral constraints
Parametrize the search by structural constraints, make the program space domain-specific
Structural constraints
Search strategy

Syntax-Guided Synthesis

Example

```
[1,4,7,2,0,6,9,2,5,0]  →  [1,2,4,7,0]

L ::= x              |     the input
      single(N)      |     single(1) = [1]
      sort(L)        |     sort([6,9,2,5]) = [2,5,6,9]
      slice(L,N,N)   |     slice([6,9,2,5],0,2) = [6,9]
      concat(L,L)          concat([6,9],[2,5]) = [6,9,2,5]
N ::= find(L,N)      |     find([6,9],9) = 1
      0                    0

f(x) := concat( sort(slice(x,0,find(x,0))),  single(0))
```

Regular tree grammars (RTGs)

starting nonterminal

ranked alphabet (terminals)

nonterminals

```
L ::= x             |
      single(N)     |
      sort(L)       |
      slice(L,N,N)  |
      concat(L,L)
N ::= find(L,N)     |
      0
```

productions

Regular tree grammars (RTGs)

nonterminals rules (productions)

alphabet starting nonterminal

$$\langle \Sigma,\ N,\ R,\ S \rangle$$

Trees: $\tau \in T_\Sigma(N)$ = all trees made from $N \cup \Sigma$

Rules are of the form: $A \to \sigma(A_1, \ldots, A_n)$

Derives in one step: $\mathcal{C}[A] \to \mathcal{C}[t]$ if $(A \to t) \in R$

  A is the leftmost non-terminal in $\mathcal{C}[A]$

Incomplete terms/programs: $\{\tau \in T_\Sigma(N) | A \to^* \tau\}$

Complete terms/programs: $\{t \in T_\Sigma | A \to^* t\}$

  = programs without holes

Whole programs: $\{t \in T_\Sigma | S \to^* t\}$

  = roughly, programs of the right type

```
concat(L,0)
```

$L \to concat(L, L)$

```
concat(L,L) -> concat(x,L)

find(concat(L,L),N)

find(concat(x,x),0)

sort(concat(L,L))
```

RTGs as structural constraints

Space of programs
= the *language* of an RTG $L(G)$
= all complete, whole programs

```
L ::= x              |
      single(N)      |
      sort(L)        |
      slice(L,N,N)   |
      concat(L,L)    |
N ::= find(L,N)      |
      0
```

```
x    sort(x)    concat(x, x)    slice(x,0,0)

 ...

slice(x,0,find(x,0))

 ...

concat(sort(slice(x,0,find(x,0))), single(0))

 ...
```

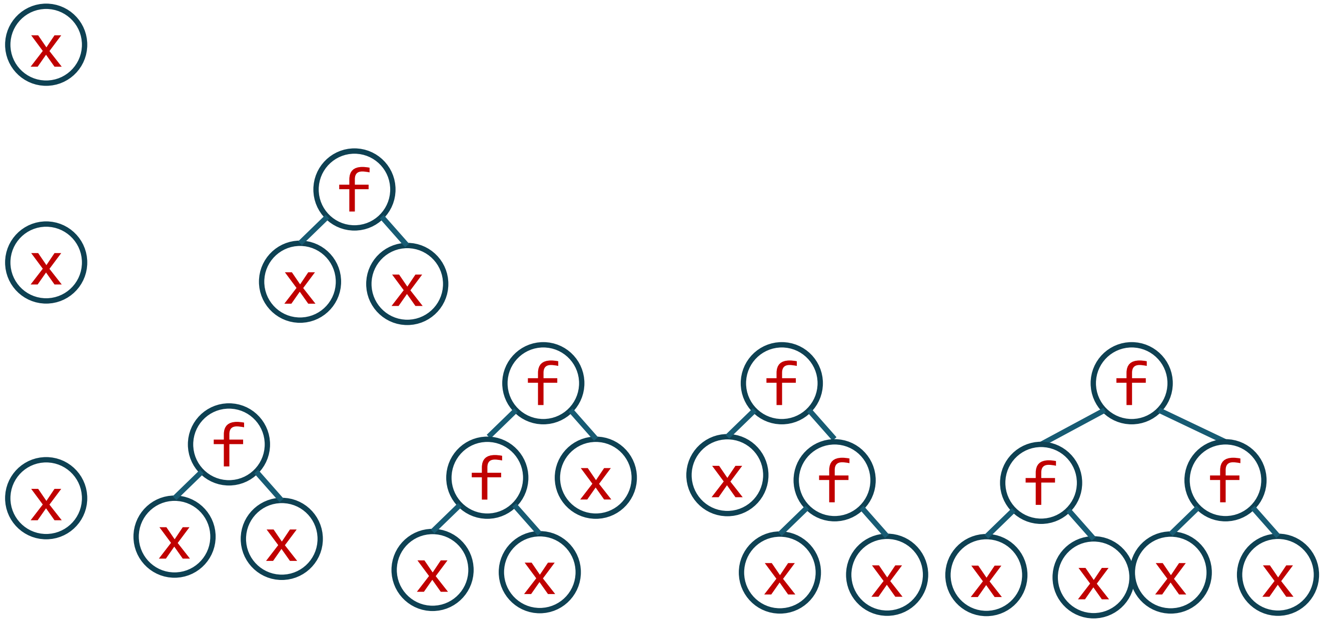
How big is the space?

E ::= x | f(E,E)

depth <= 0 N(0) = 1

depth <= 1 N(1) = 2

depth <= 2 N(2) = 5



$$N(d) = 1 + N(d - 1)^2$$

How big is the space?

```
E ::= x | f(E,E)
```

$$N(d) = 1 + N(d - 1)^2 \qquad N(d) \sim c^{2^d} \qquad (c > 1)$$

N(1) = 1
N(2) = 2
N(3) = 5
N(4) = 26
N(5) = 677
N(6) = 458330
N(7) = 210066388901
N(8) = 44127887745906175987802
N(9) = 1947270476915296449559703445493848930452791205
N(10) = 3791862310265926082868235028027893277370233152247388584761734150717768254410341175325352026

How big is the space?

$$\begin{aligned} E ::= \quad & x_1 \mid \ldots \mid x_k \mid \\ & f_1(E,E) \mid \ldots \mid f_m(E,E) \end{aligned}$$

$$N(0) = k$$

$$N(d) = k + m * N(d - 1)^2$$

N(1) = 3
N(2) = 30
N(3) = 2703
N(4) = 21918630
N(5) = 1441279023230703
N(6) = 6231855668414547953818685622630
N(7) = 116508075215851596766492219468227024724121520304443212304350703

k = m = 3

Syntactic sugar

Instead of this:

```
L ::= x              |
      single(N)      |
      sort(L)        |
      slice(L,N,N)   |
      concat(L,L)
N ::= find(L,N)      |
      0
```

We will often write this:

```
L ::= x          |
      [N]        |
      sort(L)    |
      L[N..N]    |
      L + L
N ::= find(L,N)  |
      0
```

- allow custom syntax for terminal symbols
- not an RTG strictly speaking, but you know what we mean…

Syntactic sugar

```
L ::= sort(L)     |
      L[N..N]     |
      L + L       |
      [N]         |
      x
N ::= find(L,N)   |
      0
```

```
x    sort(x)    x + x    x[0..0]
...
x[0..find(x, 0)]
...
sort(x[0..find(x, 0)]) + [0]
...
```

The SyGuS project

[Alur et al. 2013]

https://sygus.org/

Goal: Unify different syntax-guided approaches

Collection of synthesis benchmarks + yearly competition

- 6 competitions since 2013
- consider writing a SyGuS solver for your project!

Common input format + supporting tools

- parser, baseline synthesizers

SyGuS problems

SyGuS problem = < theory, spec, grammar >

A "library" of types and function symbols

Example: Linear Integer Arithmetic (LIA)

```
True, False
0,1,2,...
∧, ∨, ¬, +, ≤, ite
```

RTG with terminals in the theory (+ input variables)

Example: Conditional LIA expressions w/o sums

```
E ::= x | 0 | ite C E E
C ::= E ≤ E | C ∧ C | ¬C
```

SyGuS problems

SyGuS problem = < theory, spec, grammar >

A first-order logic formula over the theory

Examples:

```
f(0, 1) = 1 ∧
f(1, 0) = 1 ∧
f(1, 1) = 1 ∧
f(2, 0) = 2
```

SyGuS demo

SyGuS problems

SyGuS problem = < theory, spec, grammar >

A first-order logic formula over the theory

can inductive synthesis handle these?

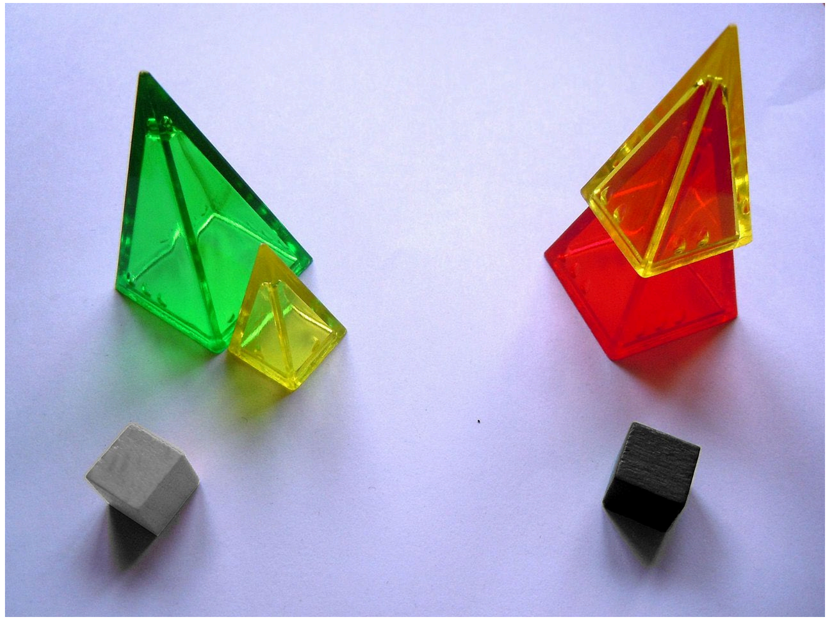
Examples:

$$
\begin{aligned}
&f(0, 1) = 1 \land \\
&f(1, 0) = 1 \land \\
&f(1, 1) = 1 \land \\
&f(2, 0) = 2
\end{aligned}
$$

Formula with free variables:

$$
\begin{aligned}
&x \leq f(x, y) \land \\
&y \leq f(x, y) \land \\
&(f(x, y) = x \lor f(x, y) = y)
\end{aligned}
$$

The Zendo game



The teacher makes up a secret rule

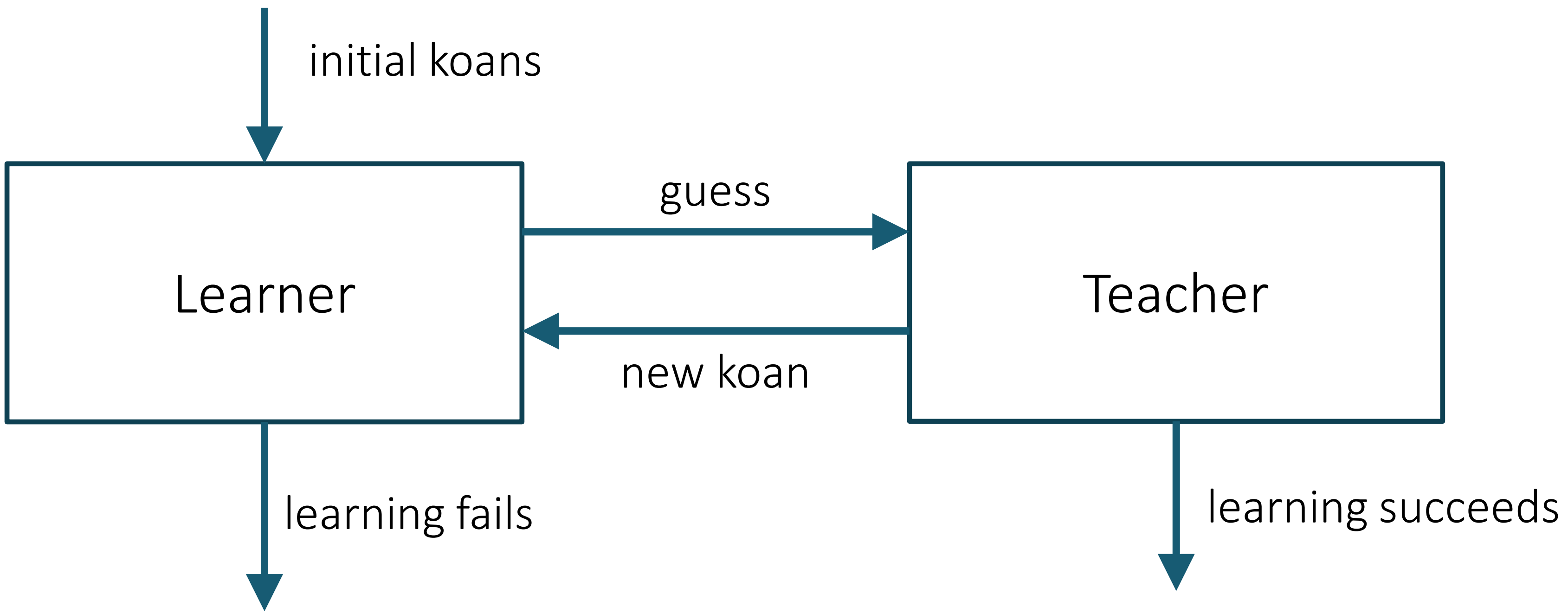
- e.g. all pieces must be grounded

The teacher builds two koans (a positive and a negative)

A student can try to guess the rule

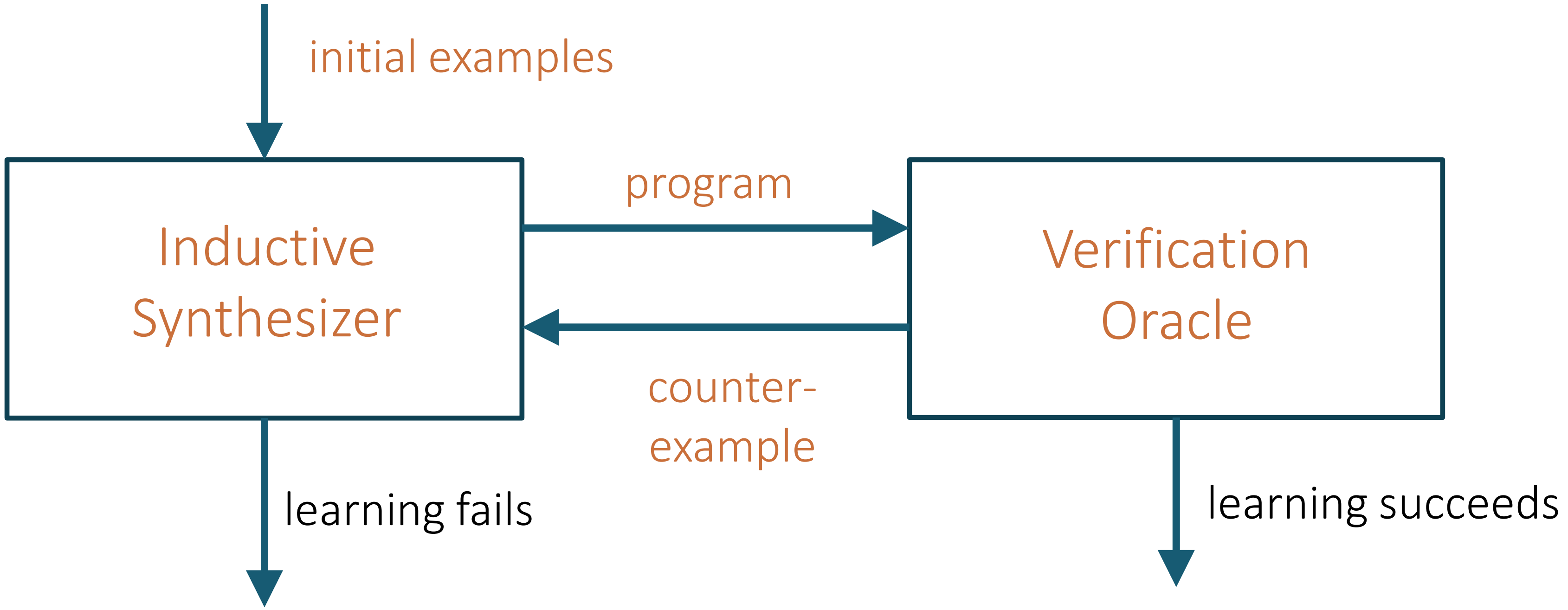
- if they are right, they win
- otherwise, the teacher builds a koan on which the two rules disagree

The Zendo game



Counter-example guided inductive synthesis (CEGIS)

The Zendo of program synthesis



The problem statement

Behavioral constraints = examples

```
[1,4,7,2,0,6,9,2,5]  →  [1,2,4,7,0]
[0] → [0]
[5,1] → [1,5,0]
```

Search strategy?

Structural constraints = grammar

```
L ::= sort(L)  |  L[N..N]
    |  L + L  |  [N]  |  x
N ::= find(L,N)  |  0
```

Enumerative search

Enumerative search

=

Explicit / Exhaustive Search

Idea: Enumerate programs from the grammar one by one and test them on the examples

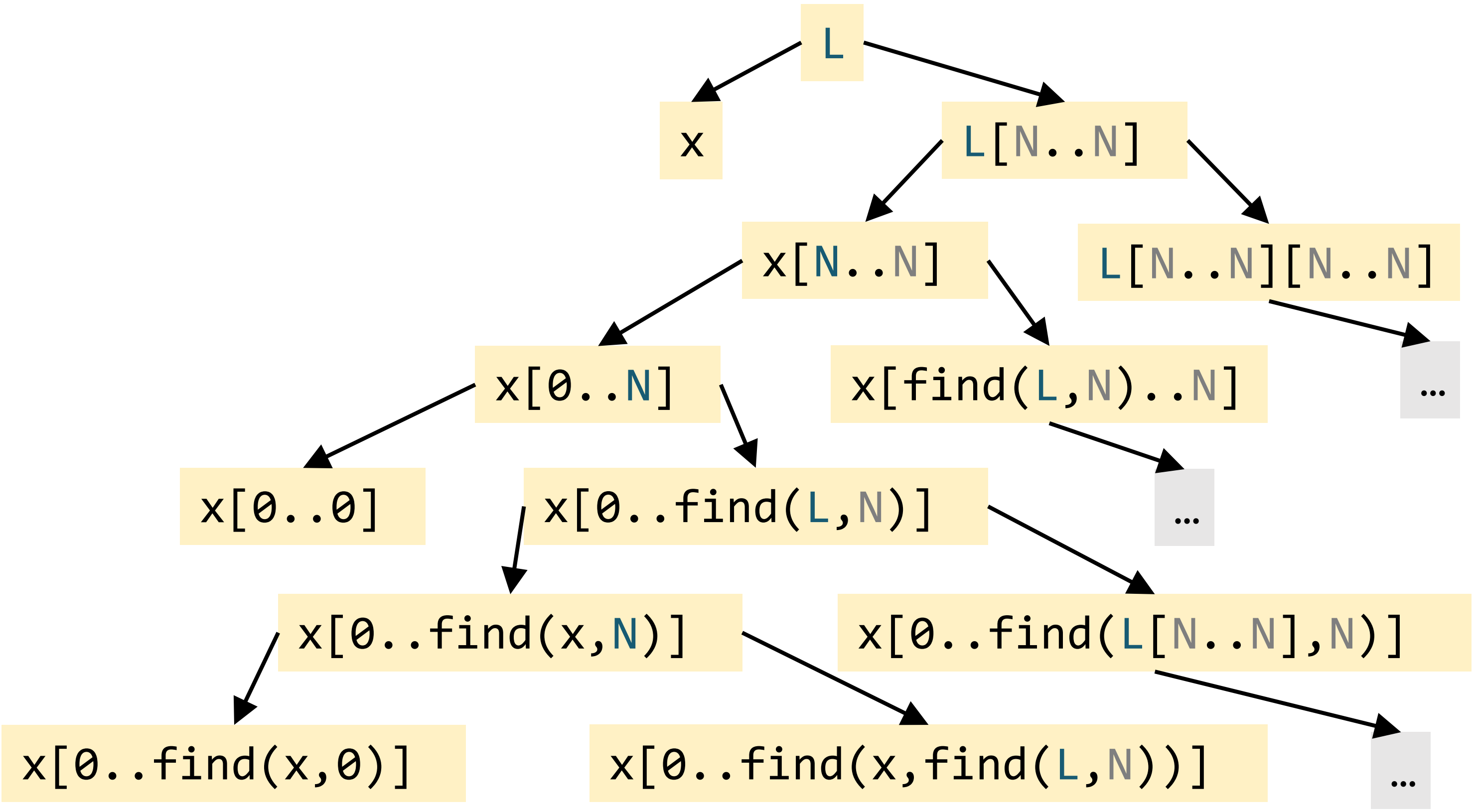
Challenge: How do we systematically enumerate all programs?

top-down vs bottom-up

Top-down enumeration: search space

Search space is a tree where

- nodes are whole incomplete programs
- edges are “derives in one step”




```
L ::= L[N..N]    |
      x
N ::= find(L,N)  |
      0
```

[[1,4,0,6] → [1,4]]

Top-down enumeration = traversing the tree

Search tree can be traversed:

- depth-first (for fixed max depth)
- breadth-first
- later in class: best-first

General algorithm:

- Maintain a worklist of incomplete programs
- Initialize with the start non-terminal
- Expand left-most non-terminal using all productions

```
L ::= L[N..N]    |
      x
N ::= find(L,N)  |
      0
```

```
[[1,4,0,6]  →  [1,4]]
```

Top-down: algorithm

nonterminals rules (productions)

alphabet starting nonterminal

```
top-down(<Σ, N, R, S>, [i → o]):
  wl := [S]
  while (wl != []):
    τ:= wl.dequeue()
    if (complete(τ) ∧ τ([i]) = [o]):
      return τ
    wl.enqueue(unroll(τ))

unroll(τ):
  wl' := []
  A := left-most non-term in τ
  forall (A → rhs) in R:
    τ' = τ[A -> rhs]
    if !exceeds_bound(τ'): wl' += τ'
  return wl'
```

depth- or breadth-first

depending on where you enqueue

```
L ::= L[N..N]     |
      x
N ::= find(L,N)   |
      0

[[1,4,0,6]  →  [1,4]]
```

can impose bounds on depth/size

Top-down: example (depth-first)

Worklist `wl`

iter 0: `L`

iter 1: `x` `L[N..N]`

iter 2: `L[N..N]`

iter 3: `x[N..N]` `L[N..N][N..N]`

iter 4: `x[0..N]` `x[find(L,N)..N]` `L[N..N][N..N]`

iter 5: `x[0..0]` `x[0.. find(L,N)]` `x[find(L,N)..N]` …

iter 6: `x[0.. find(L,N)]` `x[find(L,N)..N]` …

iter 7: `x[0.. find(x,N)]` `x[0.. find(L[N..N],N)]` …

iter 8: `x[0.. find(x,0)]` `x[0.. find(x,find(L,N))]` …

iter 9:

```
L ::= L[N..N]    |
      x
N ::= find(L,N)  |
      0
```

`[[1,4,0,6] → [1,4]]`

Bottom-up enumeration

The dynamic programming approach

Maintain a bank of complete programs

Combine programs in the bank into larger programs using productions

```
L ::= sort(L)     |
      L[N..N]     |
      L + L       |
      [N]         |
      x
N ::= find(L,N)   |
      0
```

```
[[1,4,0,6] → [1,4]]
```

Bottom-up: algorithm (take 1)

nonterminals rules (productions)

alphabet starting nonterminal

```
bottom-up (<Σ, N, R, S>, [i → o]):
  bank := {}
  for d in [0..]:
    forall (A → rhs) in R:
      forall t in new-terms(A→rhs, d, bank):
        if (A = S ∧ t([i]) = [o]):
          return t
        bank += t;

new-terms(A → σ(A1…Ak), d, bank):
 if (d = 0 ∧ k = 0) yield σ
 else forall <t1,…,tk> in bank^k:
        if Ai ->* ti: yield σ(t1,…,tk)
```

```
L ::= sort(L)    |
      L[N..N]    |
      L + L      |
      [N]        |
      x
N ::= find(L,N)  |
      0
```

[[1,4,0,6] → [1,4]]

inefficient, better index bank by non-terminal!

Bottom-up: algorithm (take 2)

```
bottom-up (<Σ, N, R, S>, [i → o]):
  bank[A] := {} forall A
  for d in [0..]:
    forall (A → rhs) in R:
      forall t in new-terms(A→rhs, d, bank):
        if (A = S ∧ t([i]) = [o]):
          return t
        bank[A] += t;

new-terms(A → σ(A₁…Aₖ), d, bank):
 if (d = 0 ∧ k = 0) yield σ
 else forall <t₁,…,tₖ> in bank[A₁] × … × bank[Aₖ]:
             yield σ(t₁,…,tₖ)
```

```
L ::= sort(L)    |
      L[N..N]    |
      L + L      |
      [N]        |
      x
N ::= find(L,N)  |
      0
```

[[1,4,0,6] → [1,4]]

inefficient, generating same terms again and again!
better index bank by depth

Bottom-up enumeration

```
bottom-up (<Σ, N, R, S>, [i → o]):
  bank[A,d] := {} forall A, d
  for d in [0..]:
    forall (A → rhs) in R:
      forall t in new-terms(A→rhs, d, bank):
        if (A = S ∧ t([i]) = [o]):
          return t
        bank[A,d] += t;

new-terms(A → σ(A₁…Aₖ), d, bank):
 if (d = 0 ∧ k = 0) yield σ
 else forall <d₁,…,dₖ> in [0..d-1]ᵏ s.t. max(d₁,…,dₖ) = d-1:
      forall <t₁,…,tₖ> in bank[A₁,d₁] × … × bank[Aₖ,dₖ]:
        yield σ(t₁,…,tₖ)
```

```
L ::= sort(L)    |
      L[N..N]    |
      L + L      |
      [N]        |
      x
N ::= find(L,N)  |
      0
```

[[1,4,0,6] → [1,4]]

Bottom-up: example

Program bank

d= 0: `x` `0`

d =1: `sort(x)` `x + x` `x[0..0]` `[0]` `find(x,0)`

d = 2: `sort(sort(x))` `sort(x[0..0])` `sort(x + x)` `sort([0])` `x + (x + x)` `x + [0]` `sort(x) + x` `x[0..0] + x` `(x + x) + x` `[0] + x` `x + x[0..0]` `x + sort(x)` `x[0..find(x,0)]`

```
L ::= sort(L)   |
      L + L     |
      L[N..N]   |
      [N]       |
      x
N ::= find(L,N) |
      0
```

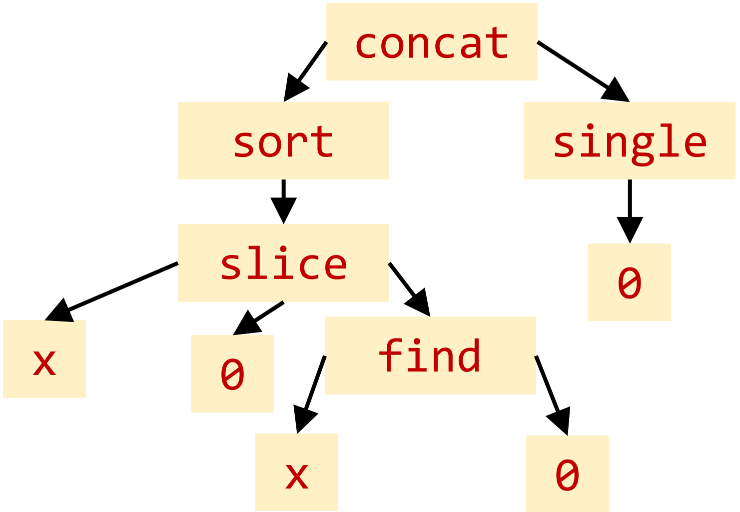
`[[1,4,0,6]` → `[1,4]]`

Bottom-up: discussion

What are some optimizations that come to mind?

Instead of by depth, we can enumerate by size

- Why would we want that?



depth = 4, size = 10

programs of size <= 10: 8667

programs of depth <= 4: >1M

- Which parts of the algo would we need to change?

Bottom-up vs top-down

Top-down | **Bottom-up**

Smaller to larger depth

- Has to explore between $3*10^9$ and 10^{23} programs to find `sort(x[0..find(x, 0)]) + [0]` (depth 6)

Top-down

Candidates are whole but might not be complete

- Cannot always run on inputs
- Can always relate to outputs (?)

Bottom-up

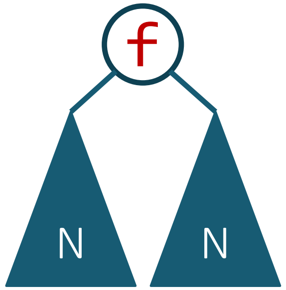
Candidates are complete but might not be whole

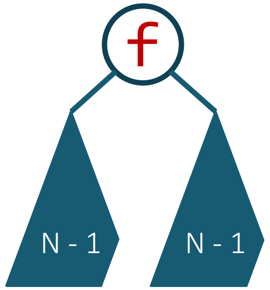
- Can always run on inputs
- Cannot always relate to outputs

How to make it scale

Prune

Discard useless subprograms





$m * N^2$ $m * (N - 1)^2$

Prioritize

Explore more promising candidates first

```
P = { [0][N..N] ,
      x[N..N]  ,
      ... }
```

dequeue this first