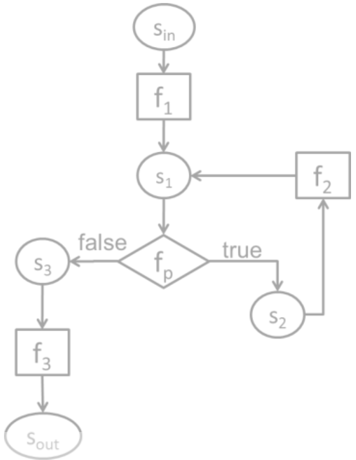
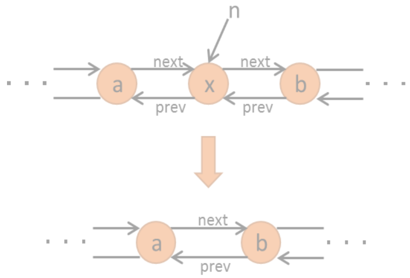
$$\exists c \forall in \; Q(c, in)$$

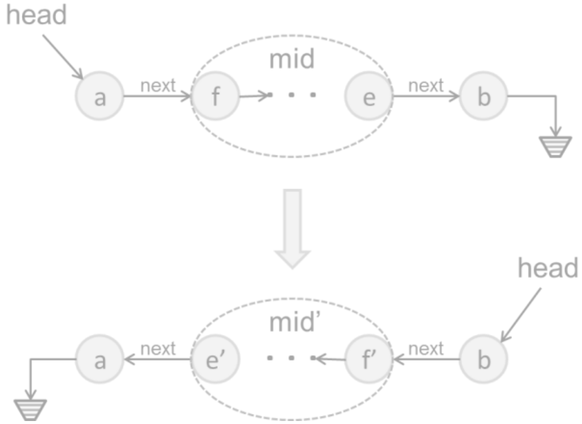
```
/* Average of x and y without using x+y (avoid overflow)*/
int avg(int x, int y){
  int t = expr({x/2, y/2, x%2, y%2, 2 }, {PLUS, DIV});
  assert t == (x+y)/2;
  return t;
}
```

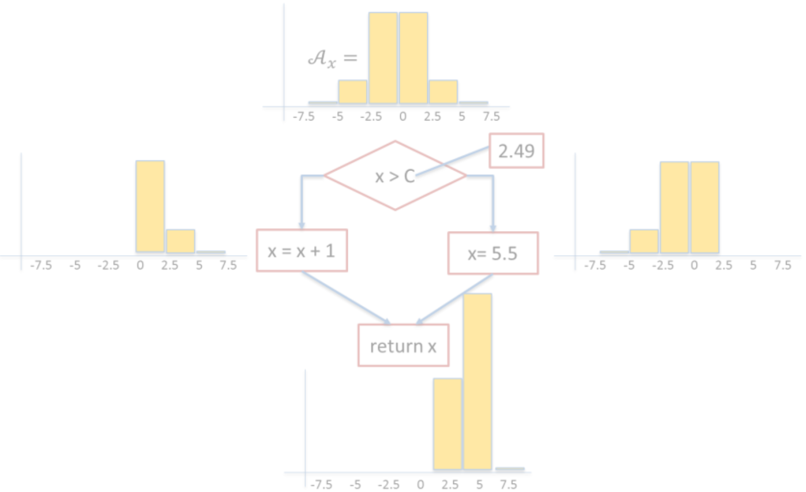





```
{
   s  = n.succ;
   p = n.pred;
   p.succ = s;
   s.pred = p;
}
```

Module II: Synthesizing Complex Programs





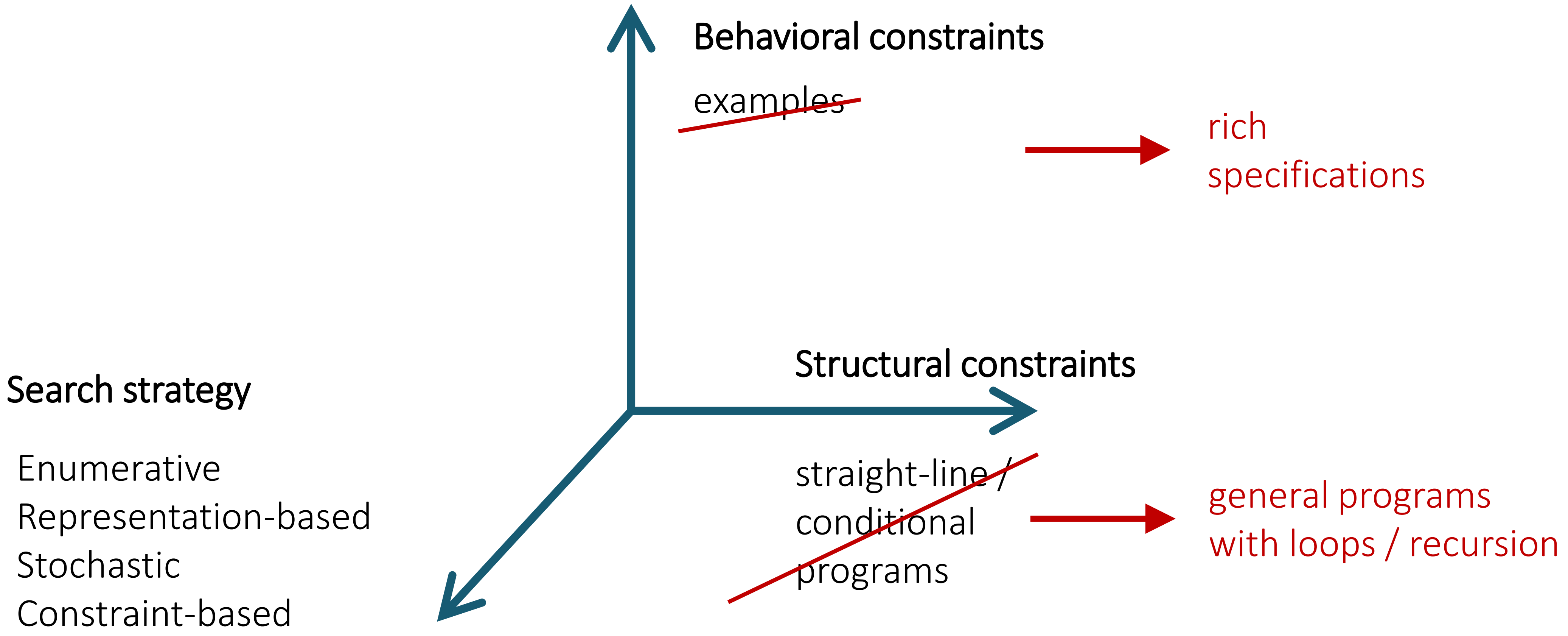
$$\varphi(p)$$

$$Sk[c](in)$$

Lecture 9
Specifications and Reduction to Inductive Synthesis

Nadia Polikarpova

Module I vs Module II



Examples of rich specifications

Reference implementation

Assertions

Pre- and post-condition

Refinement type

Reference Implementation

Easy to compute the result, but hard to compute it efficiently or under structural constraints

```
bit[W] AES_round (bit[W] in, bit[W] rkey)
{
  ... // Transcribe NIST standard
}
bit[W] AES_round _sk (bit[W] in, bit[W] rkey) implements AES_round
{
  ... // Sketch for table lookup
}
```

Assertions

Hard to compute the result, but easy to check its desired properties

```
split_seconds (int totsec) {
  int h := ??;
  int m := ??;
  int s := ??;
  assert totsec == h*3600 + m*60 + s;
  assert 0 <= h && 0 <= m < 60 && 0 <= s < 60;
}
```

Pre-/post-conditions

Hard to compute the result but easy to express its properties in logic

```
sort (int[] in, int n) returns (int[] out)
  requires  n ≥ 0
  ensures   ∀i j. 0 ≤ i < j < n ⇒ out[i] ≤ out[j]
            ∀i . 0 ≤ i < n ⇒ ∃j. 0 ≤ j < n ∧ in[i] = out[j]
{
  ??
}
```

Refinement types

Same as pre-/post-conditions but logic goes inside the types

binary search tree

red nodes have
black children

```
data RBT a where
  Empty :: RBT a
  Node  :: x: a ->
    black: Bool ->
    left:  { RBT {a | _v < x} | !black ==> isBlack _v } ->
    right: { RBT {a | x < _v} | (!black ==> isBlack _v) &&
             (blackHeight _v == blackHeight left)} ->
    RBT a

insert :: x: a -> t: RBT a -> {RBT a | elems _v == elems t + [x]}
insert = ??
```

same number of
black nodes on
every path to leaves

Why go beyond examples?

Might need too many
- **Example:** Myth needs 12 for `insert_sorted`, 24 for `list_n_th`
- Examples contain *too little* information
- Successful tools use domain-specific ranking

Output difficult to construct
- **Example:** AES cypher, RBT
- Examples also contain *too much* information (concrete outputs)

Need strong guarantees
- **Example:** AES cypher

Reasoning about non-functional properties
- **Example:** security protocols

Why is this hard?

```
gcd (int a, int b) returns (int c)
  requires a > 0 ∧ b > 0
  ensures  a % c = 0 ∧ b % c = 0
           ∀d . c < d  ⇒  a % d ≠ 0 ∨ b % d ≠ 0
{
  int x , y := a, b;
  while (x != y) {
    if (x > y) x := ??;
    else y := ??;
}}
```

infinitely many inputs

cannot validate by testing

infinitely many paths!

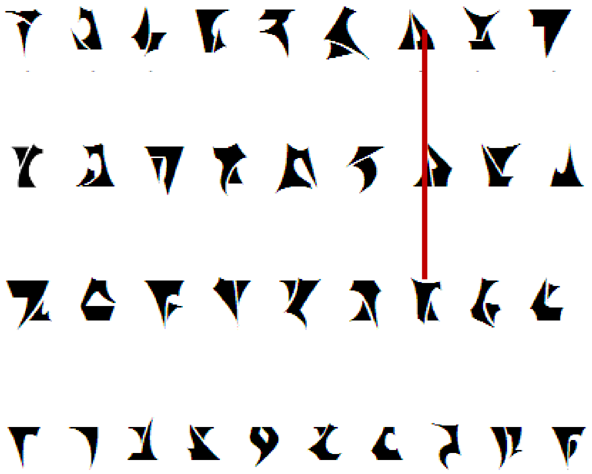
hard to generate constraints

Why is this hard?

Synthesis from examples

validation was easy!

Synthesis from specifications



SEE IF YOU CAN FIND ANY KLINGON FRUIT!

validation is hard!
(and search is still hard)

Module II

this week

Behavioral constraints

assertions → + bounded guarantees

types

pre/post-conditions

+ unbounded guarantees

Program space

imperative programs w/ loops

recursive functional programs

recursive pointer-manipulating programs

Search strategy

enumerative

constraint-based

deductive

Constraint-based synthesis from specifications

Why is this hard?

```
gcd (int a, int b) returns (int c)
  requires a > 0 ∧ b > 0
  ensures  a % c = 0 ∧ b % c = 0
           ∀d . c < d ⇒ a % d ≠ 0 ∨ b % d ≠ 0
{
  int x , y := a, b;
  while (x != y) {
    if (x > y) x := ??;
    else y := ??;
}}
```

1: how to solve constraints about infinitely many inputs?

2: how to encode semantics of a complex program as a constraint?

CBS from examples

program space

E = [i → o]

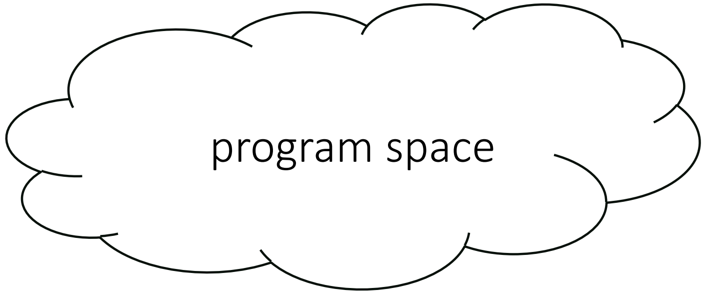
encoding

Parameters (control vector)

"*C* denotes a program that gives output *o* on input *i*"

$$\exists C\ .\ \bigwedge_{\langle i,o\rangle \in E} \varphi(C, i, o)$$

CBS from specifications



~~E = [i → o]~~

$$\forall\, i\ o\,.\,\psi(i,o)$$

$$\xrightarrow{\text{encoding}}$$

"*C* denotes a program that gives output *o* on input *i*"

$$\cancel{\exists C\,.\bigwedge_{\langle i,o\rangle\in E}\varphi(C,i,o)}$$

$$\boxed{\exists C\,.\,\forall i\ o}\,.\,\boxed{\varphi(C,i,o)} \Rightarrow \psi(i,o)$$

doubly-quantified constraint: not solver-friendly

hard to define for programs with loops / recursion

Example

```
harness void main(int x) {
  int y := ?? * x + ??;
  assert y - 1 == x + x;
}
```

$\xrightarrow{\text{encoding}}$

$$\exists C\,.\,\forall i\ o\,.\,\varphi(C, i, o) \Rightarrow \psi(i, o)$$

$$\exists c_1 c_2\,.\,\forall x\ y\,.\,y = c_1 * x + c_2 \\ \Rightarrow y - 1 = x + x$$

simplify $\downarrow$

$$\exists c_1 c_2\,.\,\forall x\,.\,c_1 * x + c_2 - 1 = x + x$$

How do we solve this constraint?

CEGIS

$$\exists c\,.\,\forall x\,.\,Q(c,x)$$

Idea 1: Bounded Observation Hypothesis

- Assume there exists a small set of inputs $X = \{x_1, x_2, \ldots x_n\}$ such that whenever c satisfies

$$\bigwedge_{i \in 1..n} Q(c, x_i)$$

No quantifiers here, can give to SAT / SMT

it also satisfies

$$\forall x.\, Q(c, x)$$

Example

This is a linear constraint, two inputs are enough!

$$\exists c_1 c_2 . \forall x . c_1 * x + c_2 - 1 = x + x$$

$X = \{0, 1\}$

$$Q(c_1, c_2, 0) \equiv c_2 - 1 = 0$$
$$Q(c_1, c_2, 1) \equiv c_1 + c_2 - 1 = 2$$

$$\{c_1 \to 2, c_2 \to 1\}$$

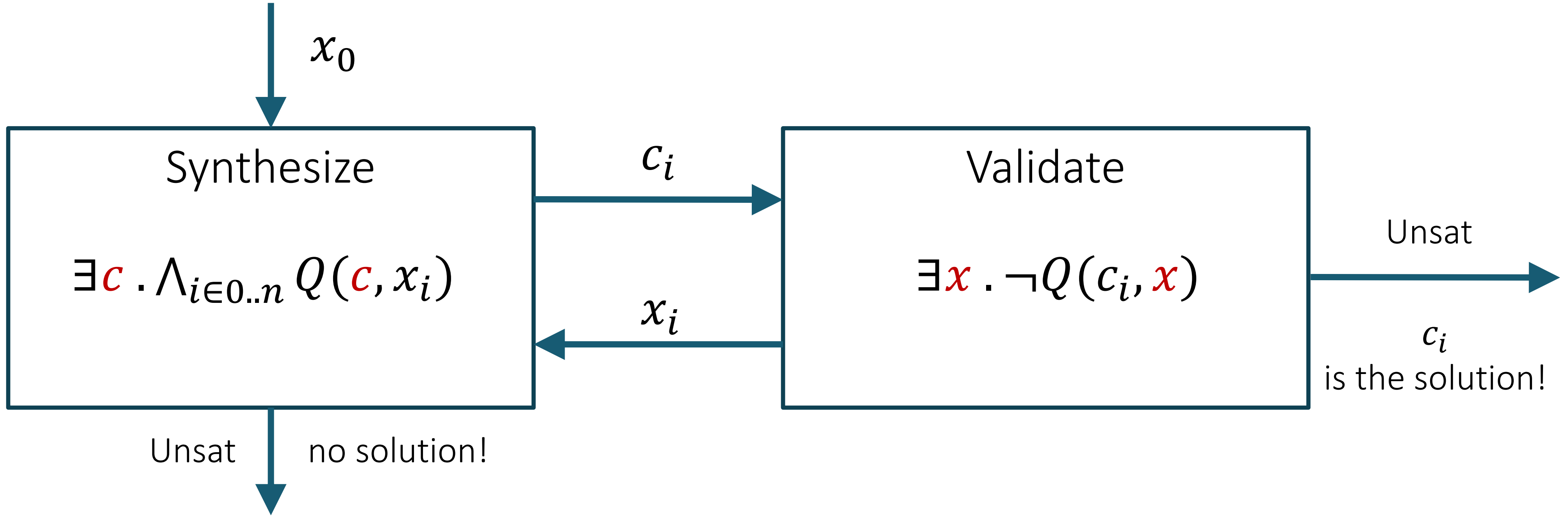
```
harness void main(int x) {
  int y := 2 * x + 1;
  assert y - 1 == x + x;
}
```

How do we find X in a general case?

CEGIS

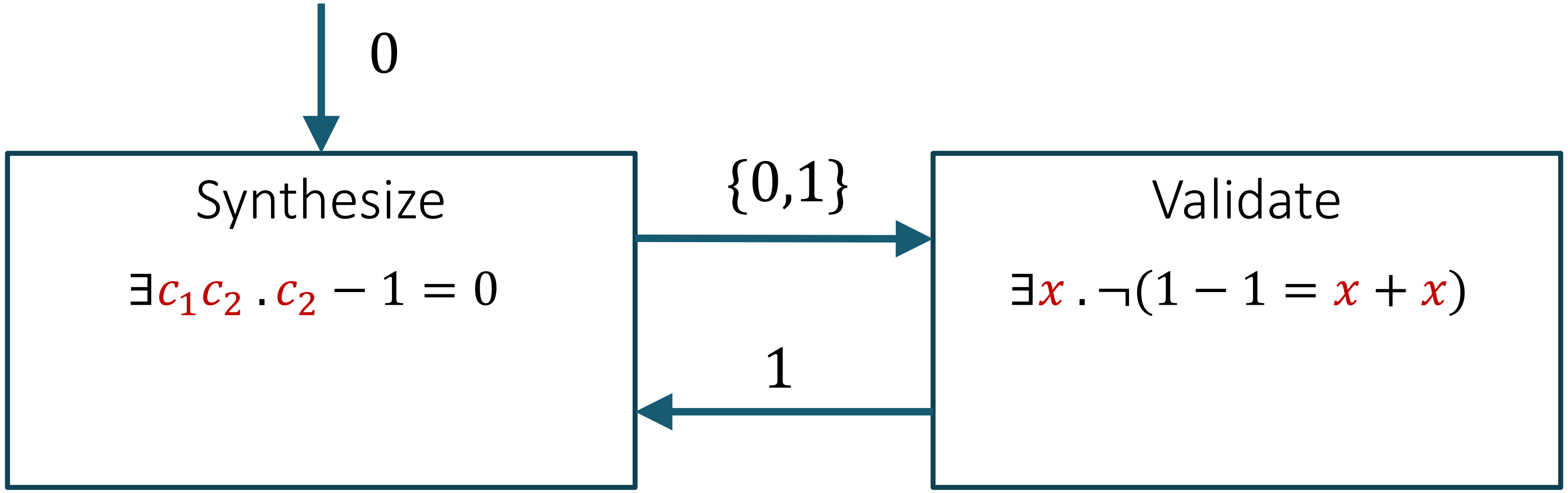
$$\exists c \,.\, \forall x \,.\, Q(c, x)$$

Idea 2: Rely on a validation oracle to generate counterexamples



Example

$$\exists c_1 c_2 \,.\, \forall x \,.\, c_1 * x + c_2 - 1 = x + x$$



Example

$$\exists c_1 c_2 \,.\, \forall x \,.\, c_1 * x + c_2 - 1 = x + x$$

0

Synthesize

$\exists c_1 c_2 \,.\, c_2 - 1 = 0$

$\wedge\ c_1 + c_2 - 1 = 2$

$\{2,1\}$

Validate

$\exists x \,.\, \neg(2 * x + 1 - 1 = x + x)$

Unsat

$\{2,1\}$ is the solution!