Lecture 4
Weighted Enumerative Search

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Plan for this week

Today:

- Weighted enumerative search

Thursday:

- Discuss the Euphony paper
- Synthesis frameworks + suggested projects

Project:

- Proposals due in ten days
- Talk to me about the topic

Scaling enumerative search

Prune

Discard useless subprograms

`m * N`^{2} `m * (N - 1)`^{2}

Prioritize

Explore more promising candidates first

```
P = { [0][N..N] ,
      x[N..N]  ,
      ... }
```

dequeue this first

Order of search

Enumerative search explores programs by depth / size

- Good default bias: small solution is likely to generalize
- But far from perfect

Result:

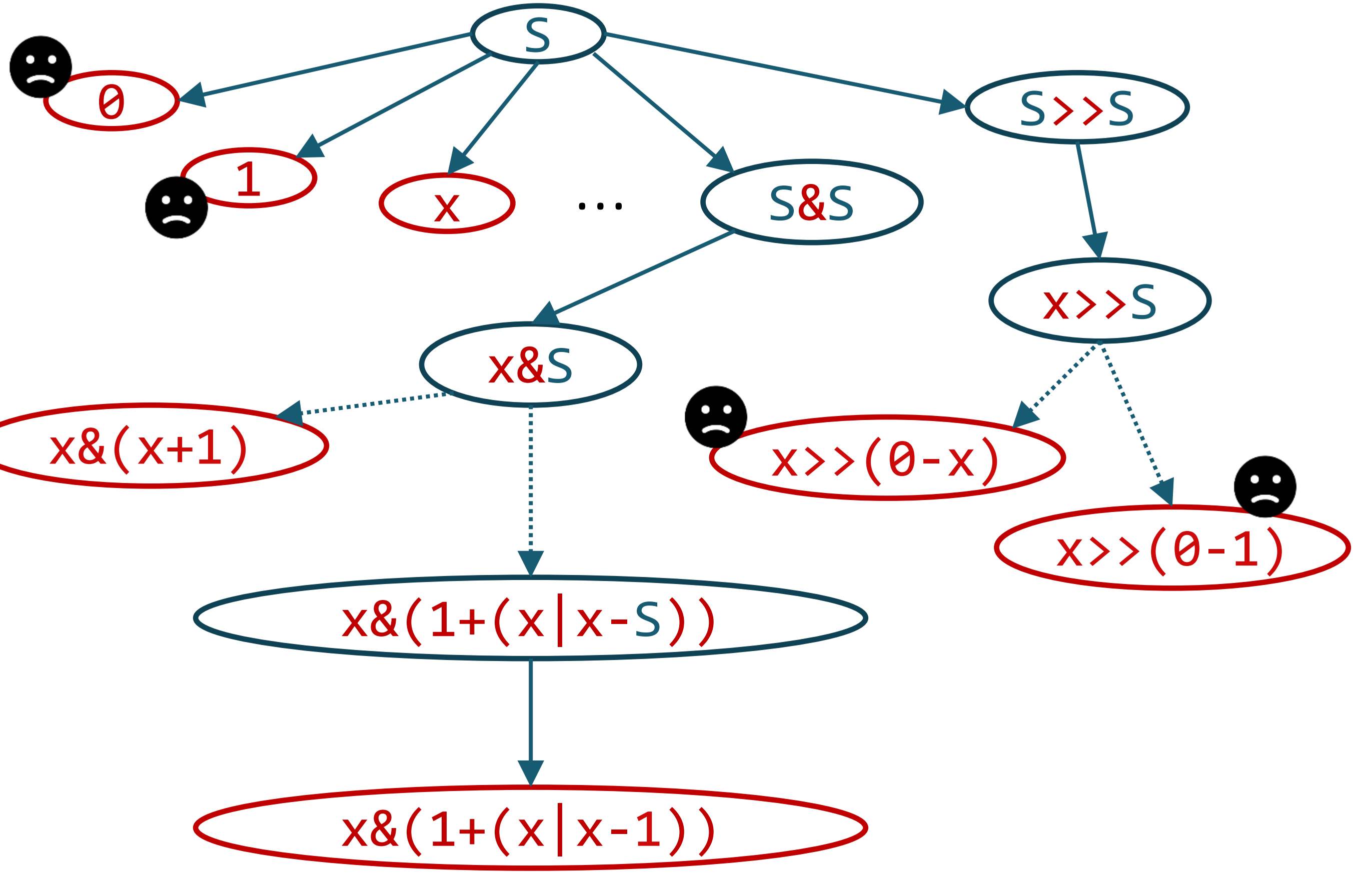
- Scales poorly with the size of the smallest solution to a given spec

Top-down search (revisited)

Turn off the rightmost sequence of **1**s:

```
00101 → 00100
01010 → 01000
10110 → 10000
```

```
S ->  0 | 1 | x |
      S + S     |
      S - S     |
      S & S     |
      S | S     |
      S << S    |
      S >> S
```



Explores many unlikely programs!

Biasing the search

Idea: explore programs in the order of lieklihood, not size

Q1: how do we know which programs are likely?

- hard-code domain knowledge
- learn from a corpus of programs
- learn on the fly

Q2: how do we use this information to guide search?

- our focus today!

Weighted enumerative search

Example: DeepCoder

Balog et al. DeepCoder: Learning to Write Programs. ICLR'17

Probabilistic Grammars

Weighted top-down search

Weighted bottom-up search

DeepCoder

Input: IO-examples

```
[-17 -3 4 11 0 -5 -9 13 6 6 -8 11]
→ [-12 -20 -32 -36 -68]
```

↓ DeepCoder

Output: Program in a list DSL

```
a <- [int]
b <- Filter (<0) a
c <- Map (*4) b
d <- Sort c
e <- Reverse d
```

DeepCoder

Input: IO-examples

[-17 -3 4 11 0 -5 -9 13 6 6 -8 11]
→ [-12 -20 -32 -36 -68]

↓ neural network

component weights

(+1)	(-1)	(*2)	(/2)	(*-1)	(**2)	(*3)	(/3)	(*4)	(/4)	(>0)	(>0)	(%2==1)	(%2==0)	HEAD	LAST	MAP
.0	.0	.1	.0	.0	.0	.0	.0	1.0	.0	.0	1.0	.0	.2	.0	.0	1.0

FILTER	SORT	REVERSE	TAKE	DROP	ACCESS	ZIPWITH	SCANL1	+	-	*	MIN	MAX	COUNT	MINIMUM	MAXIMUM	SUM
1.0	1.0	.7	.0	.1	.0	.4	.0	.0	.1	.0	.2	.1	.0	.0	.0	.0

↓ weighted search

Output: Program in a list DSL
Goal: Minimize sum of component weights

DeepCoder: search strategies

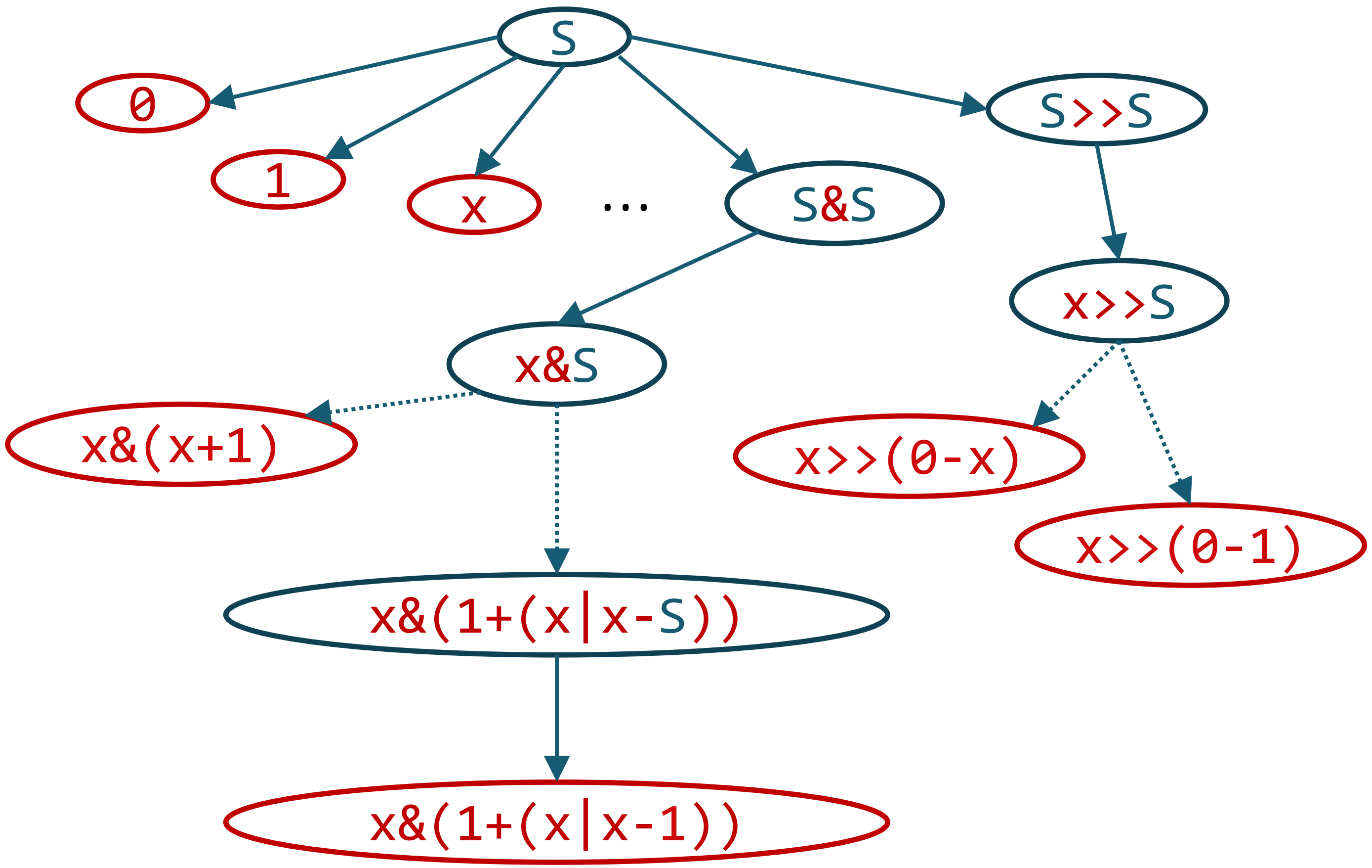
Top-down DFS

- Picks expansions for the current non-terminal in the order of probability

Sort-and-add

- start with N most probable functions
- when search fails, add next N functions

Pros and cons?



Recall: goal is to explore programs in the order of total weight!

Weighted enumerative search

DeepCoder

Probabilistic Grammars

Weighted top-down search

Weighted bottom-up search

Probabilistic Language Models

Originated in Natural Language Processing

In general: a probability distribution over sentences in a language

- $P(s)$ for $s \in L$

In practice:

- must be in a form that can be used to guide search
- for enumerative search: probabilistic (or weighted) grammars

Probabilistic (Tree) Grammar

regular tree grammar

production probability (given context)

$$\langle G, \wp \rangle$$

Production probability: $\wp: R \times T_\Sigma(N) \to [0,1]$

- for example: $\wp(S \to x \mid S) = 0.3$ $\wp(S \to x \mid x - S) = 0.0001$
- only defined for contexts where rule's LHS is the leftmost non-terminal
- probabilities of all productions in the same context add up to 1:

$$\forall \tau. S \to^* \tau \wedge \tau \notin T_\Sigma \Rightarrow \sum_{r \in dom(P(.|\tau))} P(r \mid \tau) = 1$$

Term probability:

- let $S = \tau_0 \to^{r_1} \tau_1 \to^{r_2} \dots \to^{r_n} \tau_n = \tau$ be the unique derivation of partial program τ

$$\wp(\tau) = \prod_{i=1}^{n} \wp(r_i \mid \tau_i)$$

Types of context

$$\wp : \mathrm{R} \times T_{\Sigma}(N) \rightarrow [0,1]$$

In general, can depend on any part of the context term!

But this is unwieldy

- bad for learning
- bad for (some) search algorithms

In practice we want to restrict the context

- PCFG
- n-grams
- PHOG

Probabilistic Context-Free Grammars (PCFG)

	$\wp(R)$
`S -> 0`	0.13
`S -> 1`	0.13
`S -> x`	0.18
`S -> S + S`	0.11
`S -> S - S`	0.11
`S -> S & S`	0.12
`S -> S | S`	0.12
`S -> S << S`	0.05
`S -> S >> S`	0.05

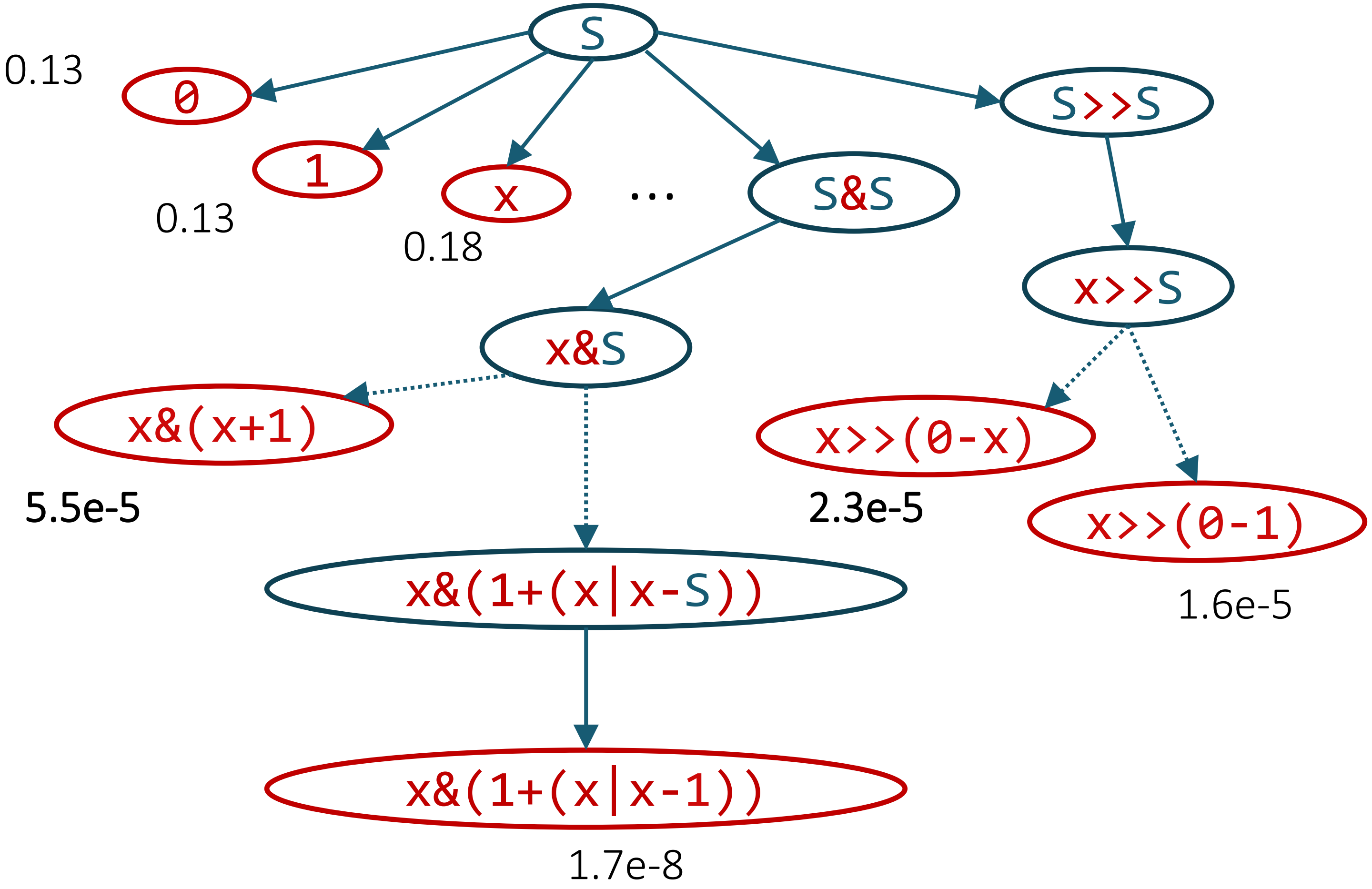
$$\wp: \mathrm{R} \rightarrow [0,1]$$

Encodes the popularity of each production (operation)

- here: variable more likely than constant, plus more likely than shift

Probabilistic Context-Free Grammars (PCFG)

Rule	$\wp(R)$
S -> 0	0.13
S -> 1	0.13
S -> x	0.18
S -> S + S	0.11
S -> S - S	0.11
S -> S & S	0.12
S -> S | S	0.12
S -> S << S	0.05
S -> S >> S	0.05



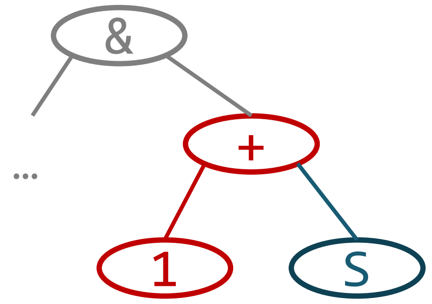
N-grams

`N[left sibling, parent] -> rhs`

	$\wp$
`S[x,-] -> 1`	0.72
`S[x,-] -> x`	0.02
`S[x,-] -> S + S`	0.12
`S[x,-] -> S - S`	0.12
`...`	
`S[1,+] -> 1`	0.26
`S[1,+] -> x`	0.25
`S[1,+] -> S + S`	0.19
`S[1,+] -> S - S`	0.08

Encodes likelihood of a production in a fixed context

- fixed set of AST nodes determined relative to the focus nonterminal
- e.g. left sibling and parent

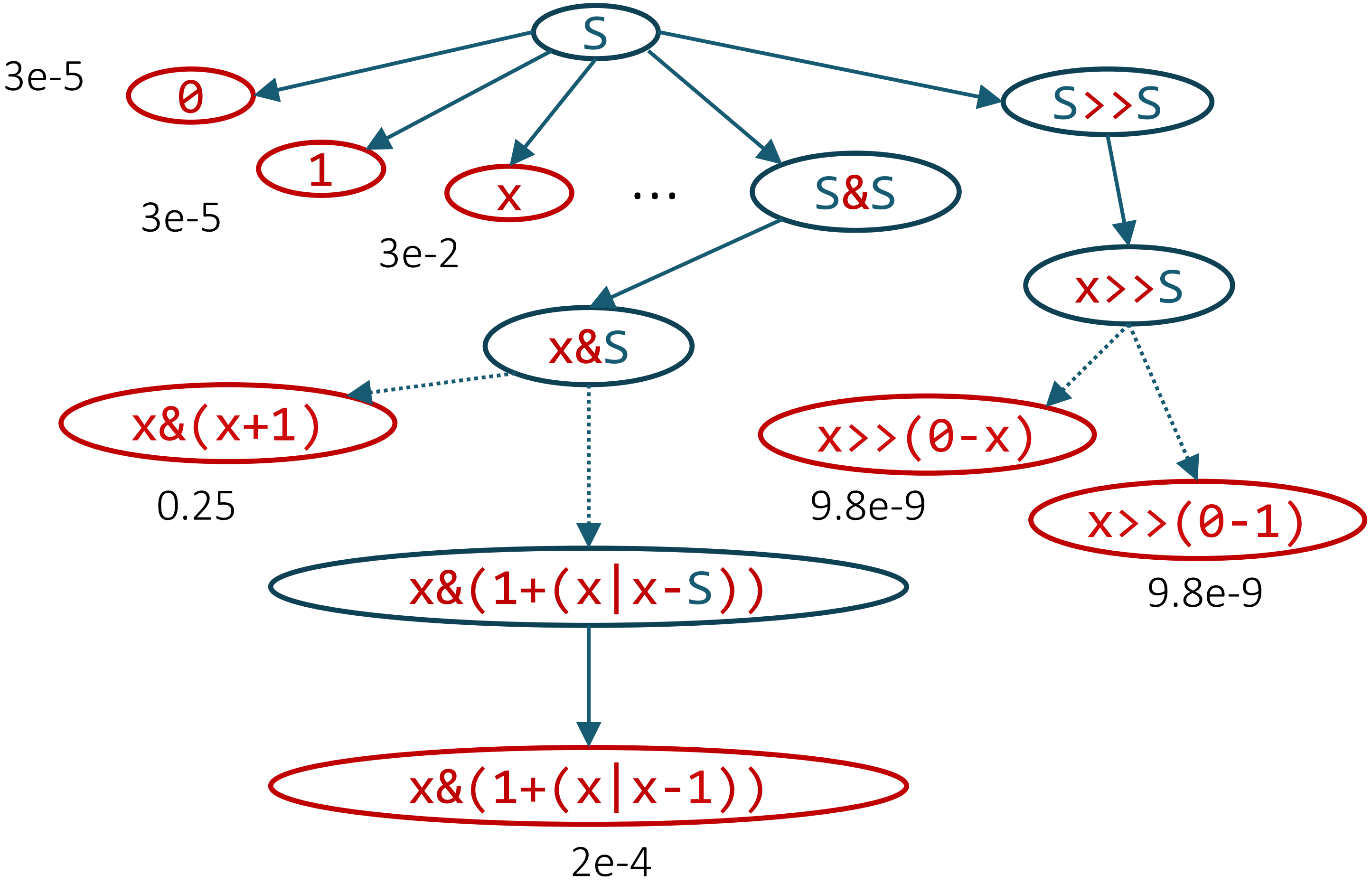


- here: `x` is not likely in `x - S`? but likely in `1 + S`?

N-grams

```
N[left sibling, parent] -> rhs
```

		$\wp$
S[x,-] ->	1	**0.72**
S[x,-] ->	x	0.02
S[x,-] ->	S + S	0.12
S[x,-] ->	S - S	0.12
...		
S[1,+] ->	1	0.26
S[1,+] ->	x	0.25
S[1,+] ->	S + S	0.19
S[1,+] ->	S - S	0.08



Probabilistic Higher-Order Grammar (PHOG)

The same fixed context might not work for every problem

Idea:

1. define context as a program that traverses the AST
2. learn the best context together with probabilities

Bielik, Raychev, Vechev. PHOG: Probabilistic Model for Code. ICML'16

Conditional models

Unconditional model

Which programs are more natural in this DSL?

+ easier to get data / learn

- need more context to capture interesting properties

Conditional model

Which programs are more likely to solve a given spec?

- harder to get data / learn

- can get away with less context

Weighted enumerative search

- DeepCoder
- Probabilistic Grammars
- Weighted top-down search
  - Lee, et al: Accelerating Search-Based Program Synthesis using Learned Probabilistic Models. PLDI'18
- Weighted bottom-up search
  - Barke, Peleg, Polikarpova. Just-in-Time Learning for Bottom-Up Enumerative Synthesis. OOPSLA'20
  - Shi, Bieber, Singh. TF-Coder: Program Synthesis for Tensor Manipulations. arXiv

Weighted top-down search

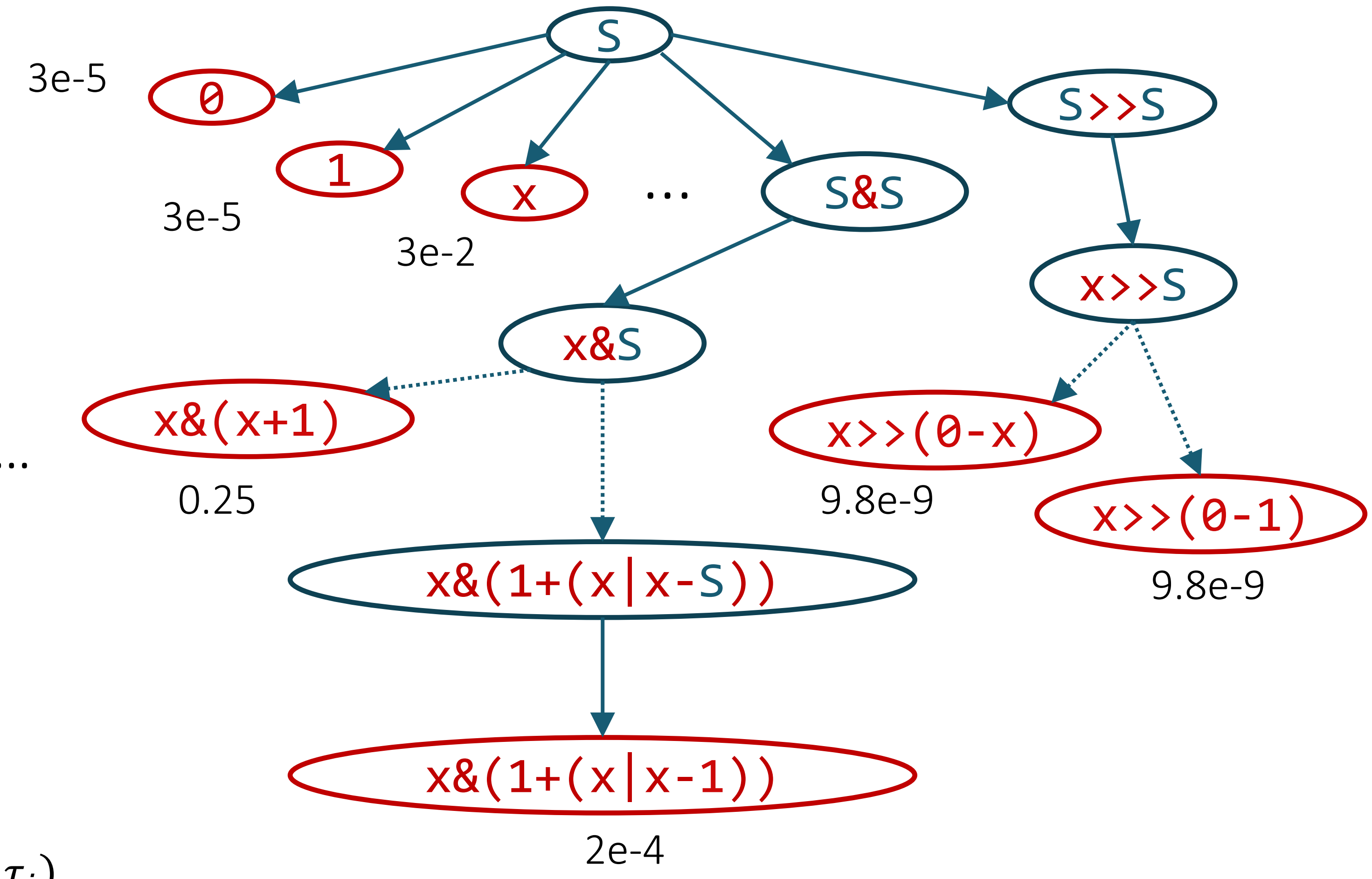
Wanted: explore programs in the order of probability

$$\wp(t) = \prod_{(r_i,\tau_i)\in S\to^* t} \wp(r_i \mid \tau_i)$$

Hard to maximize multiplicative cost…
but easy to minimize additive cost!
= shortest path

$$cost(t) = \sum_{(r_i,\tau_i)\in S\to^* t} weight(r_i \mid \tau_i)$$

$$-\log_2 \wp(t) = \sum_{(r_i,\tau_i)\in S\to^* t} -\log_2 \wp(r_i \mid \tau_i)$$



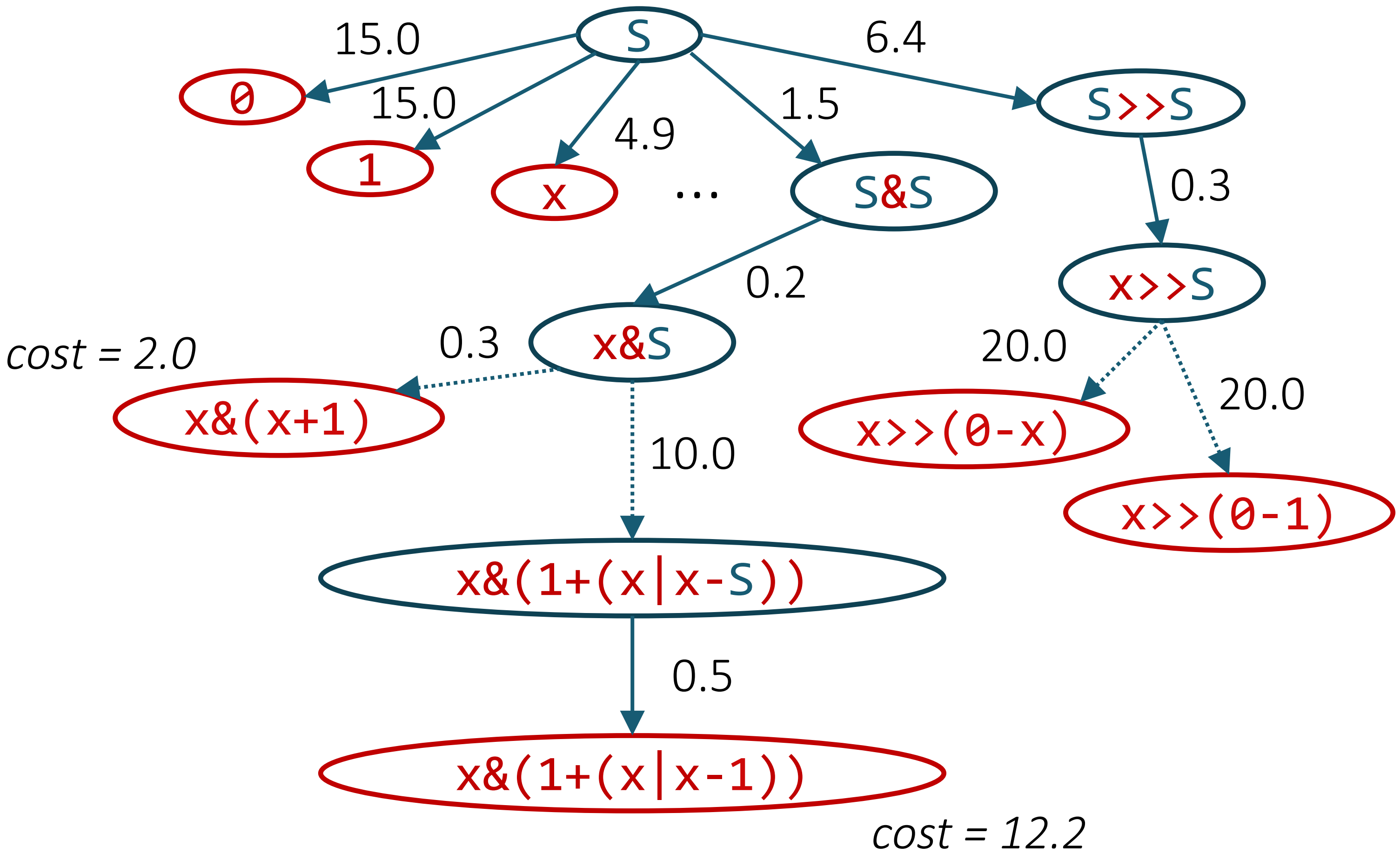
Weighted top-down search

Assigns weights to edges:

$$weight(r_i \mid \tau_i) = -\log_2 \wp(r_i \mid \tau_i)$$

Now *cost(t) < cost(t')*
iff *t* is more likely than *t'*!

We can use shortest path algo
(e.g. Dijkstra) to search by cost!



Weighted top-down search (Dijkstra)

```
top-down(<Σ, N, R, S>, [i → o]) {
  wl := [<S,0>]
  while (wl != [])
    <τ,c> := wl.dequeue_min(c);
    if (complete(τ) && τ([i]) = [o])
      return τ;
    wl.enqueue(unroll(τ,c));
}

unroll(τ,c) {
  wl’ := []
  A := left-most nonterminal in τ
  forall (A → rhs) in R:
    wl’ += <τ[A -> rhs], c + w(A → rhs|τ)>
  return wl’;
}
```

`wl` now stores candidates (nodes) together with their costs

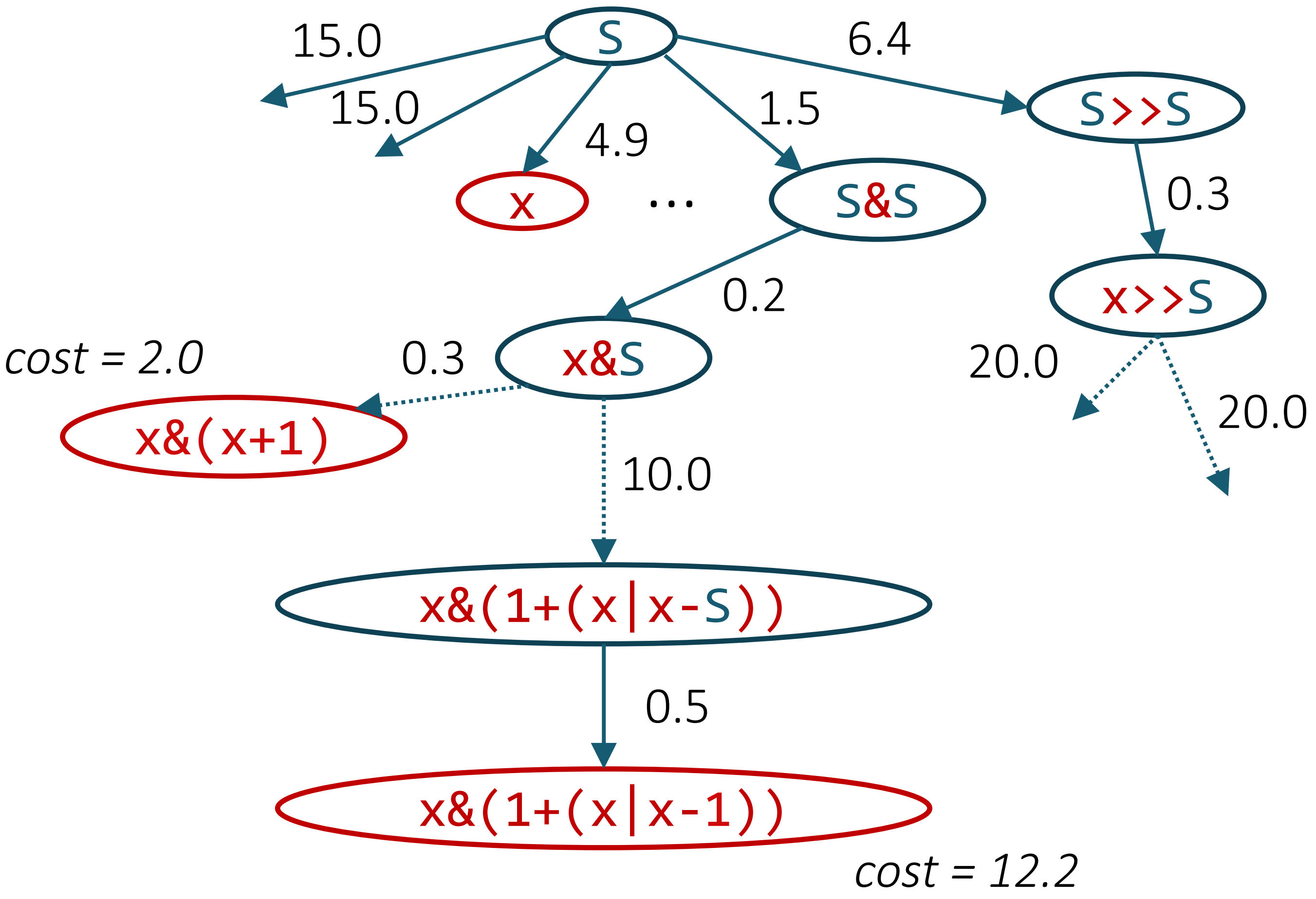
Dequeue the node with minimal cost

Distance to a new node: add the *w(R)*

Can we do better?

Dijkstra: explores a lot of intermediate nodes that don't lead to any cheap leaves

A*: introduce heuristic function *h(p)* that estimates how close we are to the closest leaf



Weighted top-down search (A*)

```
top-down(<Σ, N, R, S>, [i → o]) {
  wl := [<S,0,h(S)>]
  while (wl != [])
    <τ,c,h> := wl.dequeue_min(c + h);
    if (complete(τ) && τ([i]) = [o])
      return τ;
    wl.enqueue(unroll(τ,c));
}

unroll(τ,c) {
  wl' := []
  A := leftmost nonterminal in τ
  forall (A → rhs) in R:
    wl' += <τ[A -> rhs], c + w(A → rhs|τ),
                          h(τ[A -> rhs])>
  return wl';
}
```

Roughly how close is this program to the closest leaf

Weighted enumerative search

DeepCoder

Balog et al. DeepCoder: Learning to Write Programs. ICLR’17

Weighted top-down search

Lee, et al: Accelerating Search-Based Program Synthesis using Learned Probabilistic Models. PLDI'18

Weighted bottom-up search

Barke, Peleg, Polikarpova. Just-in-Time Learning for Bottom-Up Enumerative Synthesis. OOPSLA’20

Shi, Bieber, Singh. TF-Coder: Program Synthesis for Tensor Manipulations. TOPLAS’22

Bottom-up search (revisited)

```
bottom-up (<Σ, N, R, S>, [i → o]):
  bank[A,d] := {} forall A, d
  for d in [0..]:
    forall (A → rhs) in R:
      forall p in new-terms(A→rhs, d, bank):
        if (A = S ∧ p([i]) = [o]):
          return p
        bank[A,d] += p;

new-terms(A → σ(A1…An), d, bank):
 if (d = 0 ∧ n = 0) yield σ
 else forall <d1,…,dn> in [0..d-1]^n s.t. max(d1,…,dn) = d-1:
       forall <p1,…,pn> in bank[A1,d1] × … × bank[An,dn]:
         yield σ(p1,…,pn)
```

Search by depth

Bottom-up variations

```
new-terms(A → σ(A_1…A_n), d, bank):
 if (d = 0 ∧ n = 0) yield σ
 else forall <d_1,…,d_n> in [0..d-1]^n s.t. max(d_1,…,d_n) = d-1:
       forall <p_1,…,p_n> in bank[A_1,d_1] × … × bank[A_n,d_n]:
         yield σ(p_1,…,p_n)
```

by depth

```
new-terms(A → σ(A_1…A_n), s, bank):
 if (s = 1 ∧ n = 0) yield σ
 else forall <s_1,…,s_n> in [0..s-1]^n s.t. sum(s_1,…,s_n) = s-1:
       forall <p_1,…,p_n> in bank[A_1,s_1] × … × bank[A_n,s_n]:
         yield σ(p_1,…,p_n)
```

by size

```
new-terms(A → σ(A_1…A_n), c, bank):
 budget = c - w(A → σ(A_1…A_n))
 if (budget = 0 ∧ n = 0) yield σ
 else forall <c_1,…,c_n> in [0.. budget]^n s.t. sum(c_1,…,c_n) = budget:
       forall <p_1,…,p_n> in bank[A_1,c_1] × … × bank[A_n,c_n]:
         yield σ(p_1,…,p_n)
```

by cost!

Bottom-up by cost: discussion

What kind of cost functions are supported?

- positive
- integer
- context-free

Bottom-up: example

cost

```
L ::= sort(L)   | 10
      L + L     | 3
      x           1
```

by depth

d= 0: `x`

d =1: `sort(x)`
`x + x`

d = 2: `sort(sort(x))`
`sort(x + x)`
`x + sort(x)`
`sort(x) + x`
`x + (x + x)`
`(x + x) + x`

d = 3: …

by size

s= 1: `x`

s =2: `sort(x)`

s = 3: `x + x`
`sort(sort(x))`

s = 4: `sort(x + x)`
`sort(sort(sort(x)))`
`x + sort(x)`
`sort(x) + x`

s = 5: …

by cost

c= 1: `x`

c =2,3,4:

c = 5: `x + x`

c =6,7,8:

c = 9: `x + (x + x)`
`(x + x) + x`

c = 10:

c = 11: `sort(x)`

c = 12:

c = 13: `x + (x + (x + x))`
`(x + x) + (x + x)`
`(x + (x + x)) + x`

Weighted search

Top-down

+ Supports real-valued weights: optimal enumeration order

+ Supports context-dependent weights

Bottom-up

+ Inherits benefits of bottom up: dynamic programming, OE

Euphony

Q1: What does Euphony use as behavioral constraints? Structural constraint? Search strategy?

- IO Examples (or first-order formula via CEGIS)
- PHOG
- Weighted enumerative search via A*

Euphony

Rep x “-” S

Q2: What would these productions look like if we replaced the PHOG with a PCFG? With 3-grams?

PHOG:

```
S[“-”,Rep] ->  “.”    0.72
S[“-”,Rep] ->  “-”    0.001
S[“-”,Rep] ->  x      0.12
S[“-”,Rep] ->  S + S  0.02
…
```

PCFG:

```
S ->  “.”    0.2
S ->  “-”    0.2
S ->  x      0.3
S ->  S + S  0.2
…
```

3-grams:

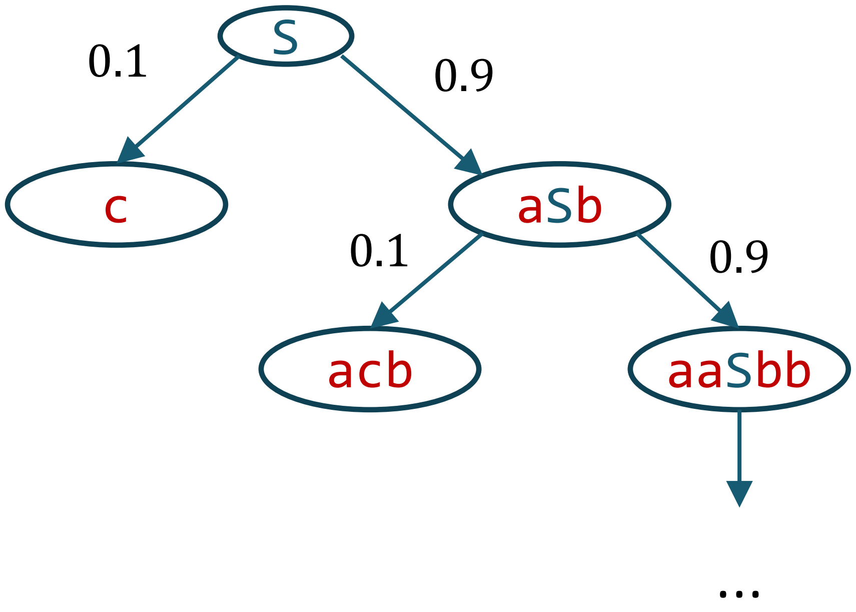
```
S[x,“-”] ->  “.”    0.72
S[x,“-”] ->  “-”    0.001
S[x,“-”] ->  x      0.12
S[x,“-”] ->  S + S  0.02
…
```

Do you think these other probabilistic models would work as well as a PHOG?

Euphony

Q3: What does h(S) = 0.1 mean? Why is it the case?

```
S ->  a S b  0.9
S ->  c      0.1
```



Euphony

Q4: Give an example of sentential forms n_i, n_j and set of points *pts* such that n_i and n_j are equivalent on *pts* but not weakly equivalent

```
pts = []
```

```
n1 = x + “-” n2 = “-” + x
pts = [“-”, “--”]
```

```
n1 = Rep(x,x,S) n2 = S
```

```
n1 = Rep(S+x,”.”,”-”) n2 = Rep(S,”.”,”-”) + Rep(x,”.”,”-”)
```

Euphony: strengths

Efficient way to guide search by a probabilistic grammar

- Much better than DeepCoder’s sort-and-add
- First to use A* and propose a sound heuristic

Transfer learning for PHOGs

- Abstraction is key to learning models of code!

Extend observational equivalence to top-down search

Euphony: weaknesses

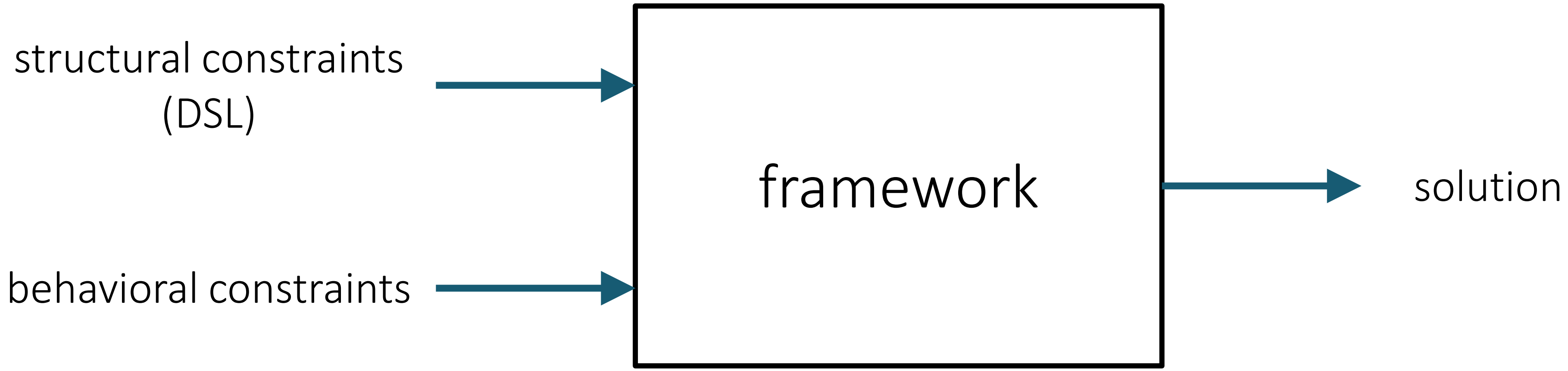
Requires high-quality training data

- for each problem domain!

Transfer learning requires manually designed features

Synthesis frameworks

synthesis framework = a highly-configurable synthesizer



Synthesis frameworks

Sketch (https://people.csail.mit.edu/asolar/)

Rosette (https://emina.github.io/rosette/)

- see also: https://www.cs.utexas.edu/~bornholt/post/building-synthesizer.html

PROSE (https://www.microsoft.com/en-us/research/project/prose-framework/)

Sketch

[Solar-Lezama 2013]

Problem: isolate the least significant zero bit in a word

- example: 0010 0101 → 0000 0010

Easy to implement with a loop

```
int W = 32;

bit[W] isolate0 (bit[W] x) {       // W: word size
        bit[W] ret = 0;
        for (int i = 0; i < W; i++)
                if (!x[i]) { ret[i] = 1; return ret; }
}
```

Can this be done more efficiently with bit manipulation?

- Trick: adding 1 to a string of ones turns the next zero to a 1
- i.e. 000111 + 1 = 001000

Sketch: space of possible implementations

```
/**
 * Generate the set of all bit-vector expressions
 * involving +, &, xor and bitwise negation (~).
*/

generator bit[W] gen(bit[W] x){
    if(??) return x;
    if(??) return ??;
    if(??) return ~gen(x);
    if(??){
        return {| gen(x) (+ | & | ^) gen(x) |};
    }
}
```

Sketch: synthesis goal

```
generator bit[W] gen(bit[W] x, int depth){
    assert depth > 0;
    if(??) return x;
    if(??) return ??;
    if(??) return ~gen(x, depth-1);
    if(??){
        return {| gen(x, depth-1) (+ | & | ^) gen(x, depth-1) |};
    }
}

bit[W] isolate0fast (bit[W] x) implements isolate0 {
     return gen(x, 3);
}
```

Sketch: output

```
bit[W] isolate0fast (bit[W] x) {
  return (~x) & (x + 1);
}
```

Rosette

[Torlak, Bodik 2014]

A solver-aided language on top of Racket

- Racket's metaprogramming + symbolic variables + solver queries
- Can define full-fledged SDSLs (Solver-aided DSLs)

Let's see how to solver the same problem in Rosette

PROSE

[Polozov, Gulwani '15]

Ranking function

```
r :: Program -> Int
```

Grammar

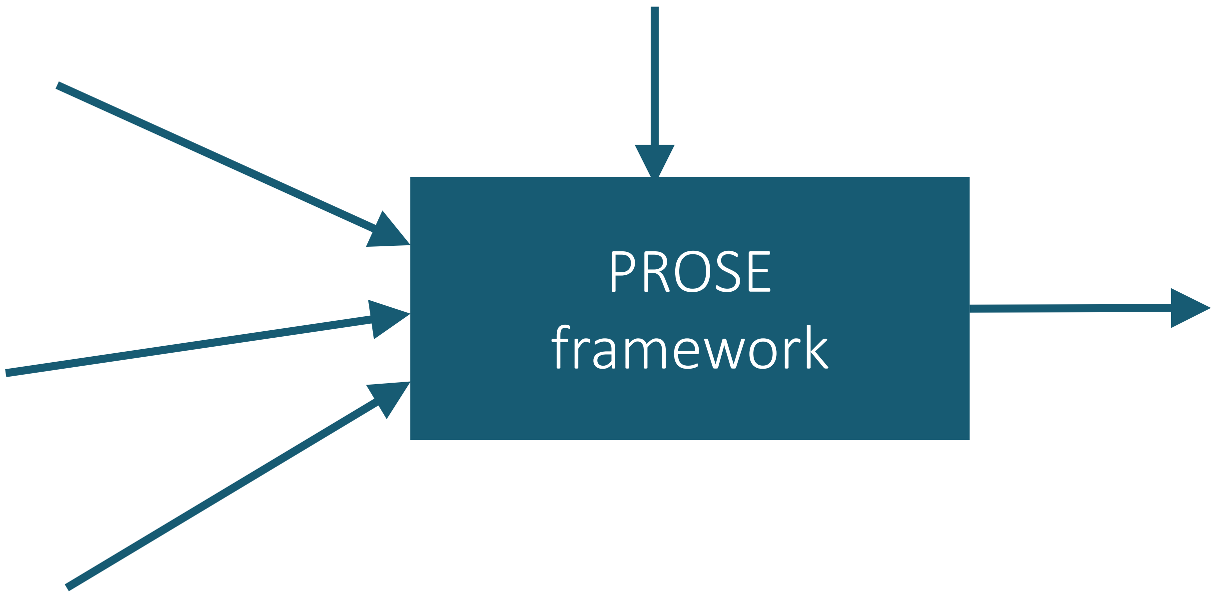
```
A ::= f(B, C) | ...
B ::= g(D, E) | ...
```

Semantics

```
f :: (B, C) -> A
f (b, c) = ...
```

Inverse semantics

```
f⁻¹ :: A -> {(B, C)}
f⁻¹ a = ...
```



VSA learning algorithm

https://microsoft.github.io/prose/

Next week

Topics:

- Representation-based search
- Stochastic search

Paper: Rishabh Singh: BlinkFill: Semisupervised Programming By Example for Syntactic String Transformations. VLDB'16

Projects:

- Once you have decided on the topic, put it on the Google sheet next to any of the team members
- If you haven't decided, talk to me after class or in OH