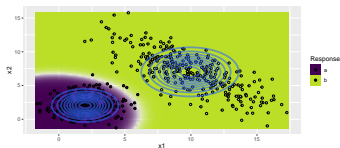# Introduction to Machine Learning

Classification
Naive Bayes


Learning goals

- Understand the idea of Naive Bayes
- Understand in which sense Naive Bayes is a special QDA model

NAIVE BAYES CLASSIFIER

NB is a generative multiclass technique. Remember: We use Bayes'
theorem and only need $p(\mathbf{x}|y = k)$ to compute the posterior as:

$$\pi_k(\mathbf{x}) = \mathbb{P}(y = k \mid \mathbf{x}) = \frac{\mathbb{P}(\mathbf{x}|y = k)\mathbb{P}(y = k)}{\mathbb{P}(\mathbf{x})} = \frac{p(\mathbf{x}|y = k)\pi_k}{\sum\limits_{j=1}^{g} p(\mathbf{x}|y = j)\pi_j}$$

NB is based on a simple **conditional independence assumption**: the
features are conditionally independent given class y.

$$p(\mathbf{x}|y = k) = p((x_1, x_2, ..., x_p)|y = k) = \prod_{j=1}^{p} p(x_j|y = k).$$

So we only need to specify and estimate the distribution $p(x_j|y = k)$,
which is considerably simpler as this is univariate.

NB: NUMERICAL FEATURES

We use a univariate Gaussian for $p(x_j|y = k)$, and estimate (μ_j, σ_j^2) in the standard manner. Because of $p(\mathbf{x}|y = k) = \prod\limits_{j=1}^{p} p(x_j|y = k)$, the joint conditional density is Gaussian with diagonal but non-isotropic covariance structure, and potentially different across classes. Hence, NB is a (specific) QDA model, with quadratic decision boundary.
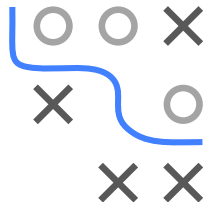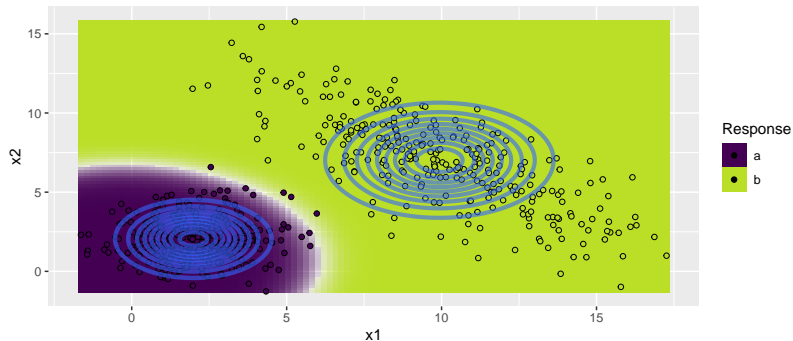
NB: CATEGORICAL FEATURES

We use a categorical distribution for $p(x_j|y = k)$ and estimate the probabilities p_{kjm} that, in class k, our j-th feature has value m, $x_j = m$, simply by counting the frequencies.

$$p(x_j|y = k) = \prod_m p_{kjm}^{[x_j=m]}$$

Because of the simple conditional independence structure it is also very easy to deal with mixed numerical / categorical feature spaces.

LAPLACE SMOOTHING

If a given class and feature value never occur together in the training data, then the frequency-based probability estimate will be zero.

This is problematic because it will wipe out all information in the other probabilities when they are multiplied.

A simple numerical correction is to set these zero probabilities to a small value to regularize against this case.

NAIVE BAYES: APPLICATION AS SPAM FILTER

- In the late 90s, Naive Bayes became popular for e-mail spam filter programs
- Word counts were used as features to detect spam mails (e.g., "Viagra" often occurs in spam mail)
- Independence assumption implies: occurrence of two words in mail is not correlated
- Seems naive ("Viagra" more likely to occur in context with "Buy now" than "flower"), but leads to less required parameters and therefore better generalization, and often works well in practice.