ECON 6356
International Finance and Macroeconomics

Lecture 2: A Small Open Endowment Economy Model

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slides

chapter 2

an open endowment economy

These slides are an adjusted version of the materials for Chapter 2 of the OEM book provided by the authors

Motivation

The purpose of this chapter is to build a canonical dynamic, general equilibrium model of the open economy and contrast its predictions with some of the empirical regularities documented in chapter 1.

The model economy developed in this chapter is simple enough to allow for a full characterization of its equilibrium dynamics using pen and paper.

- The economy is inhabited by households that receive an exogenous but stochastic endowment of perishable goods each period.

- Households have access to an internationally traded bond, which they use to smooth consumption in response to random income disturbances.

- In turn, consumption smoothing gives rise to equilibrium movements in the trade balance and current account.

The framework is called the **intertemporal approach to the current account**.

The Model

The problem of the representative household is to choose processes for consumption in period t, denoted c_t, and debt, denoted d_t, to maximize

Choose c_t, d_t to

$$\max_{\{c_t, d_t\}} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t) \tag{2.1}$$

subject to the sequential budget constraint

$$c_t + (1+r)d_{t-1} = y_t + d_t, \tag{2.2}$$

here d_t denotes debt & not savings (as usual)

and to the no-Ponzi-game constraint

$$\lim_{j\to\infty} E_t \frac{d_{t+j}}{(1+r)^j} \leq 0, \tag{2.3}$$

given d_{-1}, where y_t is an exogenous endowment and r is a constant interest rate. At the optimal allocation, the no-Ponzi-game constraint must hold with equality.

Comment: In the closed economy context, this is the permanent income model.

Optimality Conditions

The Lagrangian associated with the household's problem is

$$\mathcal{L}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \{U(c_t) + \lambda_t [d_t + y_t - (1+r)d_{t-1} - c_t]\},$$

where λ_t is a Lagrange multiplier. The optimality conditions with respect to c_t and d_t are, respectively

[c_t]: $$U'(c_t) = \lambda_t,$$

and

[d_t]: $$\lambda_t = \beta(1+r)E_t\lambda_{t+1}.$$

Combining these two expression, yields the Euler Equation

$$U'(c_t) = \beta(1+r)E_t U'(c_{t+1}). \tag{2.4}$$

Interpretation: at the margin, the household is indifferent between consuming a unit of good today or saving it and consuming it the next period along with the interest.

The Intertemporal Resource Constraint

Writing the sequential budget constraint (2.2) for period $t+j$, dividing by $(1+r)^j$, and taking expected values conditional on information available in period t yields:

BC for t+j :
(divided by $(1+r)^j$)

$$\frac{E_t c_{t+j}}{(1+r)^j} + \frac{E_t d_{t-1+j}}{(1+r)^{j-1}} = \frac{E_t y_{t+j}}{(1+r)^j} + \frac{E_t d_{t+j}}{(1+r)^j}.$$

Then sum for $j = 0$ to $j = J$:

=0 (TVC)
when $J \to \infty$

$$\sum_{j=0}^{J} \frac{E_t c_{t+j}}{(1+r)^j} + (1+r)d_{t-1} = \sum_{j=0}^{J} \frac{E_t y_{t+j}}{(1+r)^j} + \cancel{\frac{E_t d_{t+J}}{(1+r)^J}}.$$

Taking limit for $J \to \infty$ and using the transversality condition (equation (2.3) holding with equality) yields the intertemporal resource constraint

Debt Repayment ≈ PV of Savings

$$(1+r)d_{t-1} = \sum_{j=0}^{\infty} \frac{E_t(y_{t+j} - c_{t+j})}{(1+r)^j}. \quad (2.5)$$

Interpretation: at every point in time, the economy must be expected to put aside a stream of resources, $\{E_t y_{t+j} - E_t c_{t+j}\}_{j=0}^{\infty}$, large enough in present discounted value to cover the outstanding external debt.

Can An Economy Run A Perpetual Trade Deficit?

Because in this economy there is only one good, the trade balance, denoted tb_t, is given by the difference between output and consumption, that is,

$$tb_t = y_t - c_t. \tag{2.6}$$

Using this definition, we can rewrite the intertemporal resource constraint (2.5) as

$$(1+r)d_{t-1} = \sum_{j=0}^{\infty} \frac{E_t tb_{t+j}}{(1+r)^j}.$$

If economy is a Net Debtor $d_{t-1} > 0$
$\Rightarrow$ at least once it must be that $tb_{t+j} > 0$

The answer to the above question depends on the country's initial debt position:

If the country is a net debtor ($d_{t-1} > 0$), then it must generate an expected trade surplus in at least one period, $E_t tb_{t+j} > 0$ for some j. That is, if the country starts out as a net debtor, then it cannot run perpetual trade deficits.

Two Simplifying Assumptions

The following two assumptions are handy because they allow for a closed-form solution of the model.

(1) The subjective and market discount rates are equal

$$\beta = \frac{1}{1+r}.$$

(2) The period utility function is quadratic

w/ this model becomes Hall ('78) permanent model of consumption

$$U(c) = -\frac{1}{2}(c - \bar{c})^2, \tag{2.7}$$

where $\bar{c}$ is a large enough number so that $c < \bar{c}$ in equilibrium. Under these assumptions, the Euler equation (2.4) becomes

$$c_t = E_t c_{t+1}, \tag{2.8}$$

that is, consumption follows a **random walk**. Households expect to maintain their current level of consumption forever

Substitute forward & use law of iterated expectations

$$E_t c_{t+j} = c_t$$

for all $j > 0$.

Before presenting the closed form solutions to the model we introduce two variables that will be convenient in their characterization and interpretation

— nonfinancial permanent income, y_t^p

— difference between current income and permanent income, $y_t - y_t^p$

Nonfinancial permanent income, y_t^p

Define nonfinancial permanent income, denoted y_t^p, as the constant level of income that, if received in all future periods $t+j$ for $j \geq 0$, is equal to the expected present discounted value of the stochastic income process as of time t. That is, define y_t^p as the solution to

$$\sum_{j=0}^{\infty} \frac{y_t^p}{(1+r)^j} = \sum_{j=0}^{\infty} \frac{E_t y_{t+j}}{(1+r)^j}$$

Solving for y_t^p we obtain

$$y_t^p = \frac{r}{1+r} \sum_{j=0}^{\infty} \frac{E_t y_{t+j}}{(1+r)^j}. \tag{2.10}$$

This equation says that y_t^p is a weighted average of expected future income levels, with weights adding up to one.

The difference between current and permanent income, $y_t - y_t^p$

$$
\begin{aligned}
y_t - y_t^p &= y_t + \frac{1}{r} y_t^p - \left(\frac{1+r}{r}\right) y_t^p \\
&= y_t + \sum_{j=0}^{\infty} \frac{E_t y_{t+j}}{(1+r)^{j+1}} - \sum_{j=0}^{\infty} \frac{E_t y_{t+j}}{(1+r)^j} \\
&= \sum_{j=1}^{\infty} \frac{E_t y_{t+j-1}}{(1+r)^j} - \sum_{j=1}^{\infty} \frac{E_t y_{t+j}}{(1+r)^j} \\
&= -\sum_{j=1}^{\infty} \frac{E_t y_{t+j} - E_t y_{t+j-1}}{(1+r)^j}
\end{aligned}
$$

Lag j terms so that
Counter starts at t=1

take out first term

If income is expected to
grow then permanent
income exceeds current income

$\Rightarrow$

$$
y_t - y_t^p = -\sum_{j=1}^{\infty} \frac{E_t \Delta y_{t+j}}{(1+r)^j} \tag{2.27}
$$

Interpretation: Permanent income exceeds current income when income is expected to grow in the future.

replace $E_t c_{t+j} = c_t$ in (2.5) and solve for c_t
(to get a closed form solution)

$(1+r)d_{t-1} = \sum_{j=1}^{\infty} \frac{E_t y_{t+j}}{(1+r)^j} - c_t \cancel{\sum_{j=1}^{\infty} \frac{1}{(1+r)^j}}^{\frac{1+r}{r}} \Rightarrow c_t = \underbrace{\frac{r}{1+r} \sum_{j=1}^{\infty} \frac{y_{t+i}}{(1+r)^j}}_{y_t^p} - r.d_{t-1}$

↳ in 'p' form this term is exogenous ⇒ This eq. is already the solution for c_t

The Closed-Form Equilibrium Solution

replace in 2.5 & Solve for $c_t \Rightarrow$ (2.11)

Use the fact that $E_t c_{t+j} = c_t$ to eliminate $E_t c_{t+j}$ from the intertemporal resource constraint (2.5):

$$c_t = y_t^p - r d_{t-1}, \tag{2.11}$$

Because d_{t-1} is predetermined in t and y_t^p is exogenous, equation (2.11) represents the closed-form solution for c_t.

Interpretation: in equilibrium consumption is equal to nonfinancial permanent income minus interest on existing initial debt.

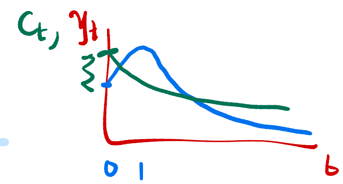
The Equilibrium Behavior of the Trade Balance

Using the equilibrium level of consumption given in (2.11), we obtain the closed-form solution for the trade balance, $tb_t = y_t - c_t$, as

↳ Subst from (2.11):

depends on what happens w/ y_t^p ? ↓ or ↑

$$tb_t = y_t - y_t^p + rd_{t-1} \tag{2.16}$$



⇒ the trade balance responds countercyclically to changes in current income if permanent income increases by more than current income in response to increases in current income.

This is an important result, because we saw in chapter 1 that the trade balance is countercyclical. To capture this fact in the context of the present model, the endowment process must be such that permanent income increases by more than one for one with current income. What type of endowment process satisfies this requirement? We will return to this point soon.

(2.2) $c_t + (1+r)d_{t-1} = y_t + d_t$

Subs $c_t = y_t^p - rd_{t-1}$

The Equilibrium Behavior of External Debt

Use (2.11) again to eliminate c_t from the sequential budget constraint (2.2) to get

$$d_t - d_{t-1} = y_t^p - y_t. \tag{2.12}$$

which says that the economy borrows to cover deviations of current income from permanent income. Like consumption, external debt has a unit root.

The Equilibrium Behavior of the Current Account

The current account is defined as the trade balance minus interest payments on the external debt

$$ca_t \equiv tb_t - rd_{t-1}. \tag{2.13}$$

Using the closed-form solution for tb_t given in (2.16), we get

For the CA to be countercyc. we need a larger than proportional response

$$ca_t = y_t - y_t^p. \tag{2.15}$$

From chapter 1: current account is countercyclical. Thus, as with the trade balance, permanent income must increase more than one for one with current income (for this model to explain the empirical regularity).

in y_t^p (than increase in y_t)

Comparing the above expression with (2.12) , we have that

$$ca_t = -(d_t - d_{t-1}). \tag{2.14}$$

This is known as the **fundamental balance of payments identity**: the current account equals the change in the country's net asset position.

Let's collect the results obtained thus far: When $\beta(1+r) = 1$ and $U(c) = -1/2(c-\bar{c})^2$, then the model has the following closed-form solution:

$$c_t = y_t^p - r d_{t-1} \tag{2.11}$$

$$d_t = d_{t-1} + y_t^p - y_t \tag{2.12}$$

$$ca_t = y_t - y_t^p \tag{2.15}$$

$$tb_t = y_t - y_t^p + r d_{t-1} \tag{2.16}$$

where

$$y_t^p \equiv \frac{r}{1+r} \sum_{j=0}^{\infty} \frac{E_t y_{t+j}}{(1+r)^j} \tag{2.10}$$

and d_{-1} and the stochastic process for y_t are exogenously given.

endowment

Memo item:

$$ca_t = y_t - y_t^p = -\sum_{j=1}^{\infty} \frac{E_t \Delta y_{t+j}}{(1+r)^j} \tag{2.27}$$

A General Principle

Take another look at the expression $ca_t = y_t - y_t^p$. It says that the current account is used whenever the economy experiences temporary deviations of output from permanent income, that is, whenever $y_t - y_t^p \neq 0$. By contrast, permanent output shocks, that is movements in y_t that leave $y_t - y_t^p$ unchanged do not produce movements in the current account. Thus, the following principle emerges:

> *Finance temporary shocks (by running current account surpluses or deficits with little change in consumption) and adjust to permanent shocks (by changing consumption but not the current account).*

1. Temporary shocks are financed w/ movements in ca_t
2. There is adjustment to permanent shocks via c_t so that ca_t does not change

The Income Process

Countercyclicality depends on behavior of y_t $\Rightarrow$ Consider 3 stochastic Processes

We have established that in order for the present model to account for the observed countercyclicality of the current account and the trade balance, it is required that $y_t - y_t^p$ be countercyclical. In turn, the cyclicality of $y_t - y_t^p$ depends on the particular process assumed for the endowment y_t. For this reason, we now characterize the equilibrium under three representative income processes:

- Two Stationary Processes
  - y_t follows an AR(1) process
  - y_t follows an AR(2) process

- A nonstationary process, Δy_t follows an AR(1) process.

2.2 An AR(1) Income Process

$\tilde{y}_t = y_t - \bar{y} \Rightarrow$

$\tilde{y}_t = \rho \tilde{y}_{t-1} + \epsilon_t$

Suppose y_t follows the law of motion

$$y_t - \bar{y} = \rho(y_{t-1} - \bar{y}) + \epsilon_t,$$

where ϵ_t is a white noise with mean zero and variance σ_ϵ^2 and $\bar{y}$ is a positive constant. The parameter $\rho \in (-1, 1)$ measures persistence. Then, the j-step-ahead forecast of y_t as of period t is

$$E_t y_{t+j} = \bar{y} + \rho^j (y_t - \bar{y}).$$

Thus, if $\rho > 0$, which is the case of greatest empirical interest, the forecast converges monotonically to the unconditional mean of the process, namely $\bar{y}$. The more persistent the process (the higher ρ) is, the slower the speed of convergence of the forecast to the unconditional mean will be.

(2.10) $y_t^p = \frac{r}{1+r}\sum_{j=0}^{\infty}\frac{E_t y_{t+j}}{(1+r)^j}$

$= \frac{r}{1+r}\sum \frac{\rho^j(y_t-\bar{y})}{(1+r)^j} + \bar{y}\,\frac{r}{1+r}\sum \frac{1}{(1+r)^j}$ $\quad \frac{1+r}{r}$

$\Rightarrow y_t^p - \bar{y} = (y_t-\bar{y})\,\frac{r}{1+r}\sum\left(\frac{\rho}{1+r}\right)^j \Rightarrow (2.17)$ $\quad \frac{1+r}{1+r-\rho}$

Permanent Income with AR(1) Endowment

Use $E_t y_{t+j} - \bar{y} = \rho^j(y_t - \bar{y})$ to eliminate $E_t y_{t+j}$ from (2.10) to obtain

$$y_t^p - \bar{y} = \underbrace{\frac{r}{1+r-\rho}}(y_t - \bar{y}). \tag{2.17}$$

$\begin{cases} \approx \frac{r}{1+r} & \text{if } \rho \to 0 \\ \approx 1 & \text{if } \rho \to 1 \end{cases}$

This expression is quite intuitive:

• if the endowment process is temporary ($\rho \to 0$), then only a small fraction, $r/(1+r)$, of movements in the endowment are incorporated into permanent income.

• if the endowment process is highly persistent ($\rho \to 1$), most of movements in the current endowment are reflected in movements in permanent income.

A key variable is $y_t - y_t^p$, which under the AR(1) process is given by

$$y_t - y_t^p = \underbrace{\frac{1-\rho}{1+r-\rho}}(y_t - \bar{y}). \tag{*}$$

'Ideally < 0 so $y_t - y_t^p$ is countercyclical

The coefficient

But the problem is that: $\dfrac{1-\rho}{1+r-\rho} > 0$ for all $\rho \in (-1, 1)$

is positive for all possible values of ρ, implying that $y_t - y_t^p$ is procyclical. Therefore the trade balance and the current account are predicted to be procyclical — which according to the evidence shown in chapter 1 is counterfactual.

(2.11) $c_t = y_t^p - rd_{t-1}$
(2.17) $y_t^p - \bar{y} = \frac{r}{1+r-\rho}(y_t - \bar{y})$

Consumption Adjustment with AR(1) Income

By (2.11) and (2.17)

$$c_t = y_t^p - rd_{t-1} = \bar{y} + \frac{r}{1+r-\rho}(y_t - \bar{y}) - rd_{t-1} \qquad (2.18)$$

• if $\rho = 0$ consumption increases less than one for one with current income. Why? Because current income is higher than permanent income.

• if $\rho \approx 1$, consumption adjusts one-for-one with current output, as current income is equal to permanent income.

(2.12): $d_t = d_{t-1} + y_t^p - y_t$

(*): $y_t - y_t^p = \frac{1-\rho}{1+r-\rho}(y_t - \bar{y})$

$\rho = 1 \rightarrow \varphi = 0$

External Debt Adjustment with AR(1) Income

φ $\quad \rho = 0 \rightarrow \varphi = \frac{1}{1+r} \approx 1$

By (2.12) and (*)

$$d_t = d_{t-1} + (y_t^p - y_t) = d_{t-1} - \frac{1-\rho}{1+r-\rho}(y_t - \bar{y}) \tag{2.21}$$

• if $\rho = 0$ debt decreases almost one-for-one with income. Why? The country saves most of the temporary income increase to be able to consume not only more in the current period but also in all future periods.

• if $\rho \approx 1$, debt is unchanged. Why? Because income will be also higher in the future and hence there is no need to save.

Current Account Adjustment with AR(1) Income

By (2.15) and (*)

$$ca_t = y_t - y_t^p = \frac{1-\rho}{1+r-\rho}(y_t - \bar{y}) \tag{2.20}$$

• if $\rho = 0$ the change in the current account is close to the change in current income. Why? The country saves most of the temporary income increase to be able to consume not only more in the current period but also in all future periods. $\rho \approx 0 \Rightarrow \uparrow ca_t \rightarrow$ household $\uparrow$ savings

• if $\rho \approx 1$, the current account is unchanged (and equal to zero). Why? Because income will be also higher in the future and hence there is no need to save.

$\rho \approx 1 \Rightarrow ca_t$ won't change after change in y_t (shock)

$$(2.16) \quad tb_t = y_t - y_t^p + rd_{t-1}$$

Trade Balance Adjustment with AR(1) Income

$$tb_t = \frac{1-\rho}{1+r-\rho}(y_t - \bar{y}) + rd_{t-1}$$

By (2.16) and (*)

$$tb_t = y_t - y_t^p + rd_{t-1} \tag{1}$$

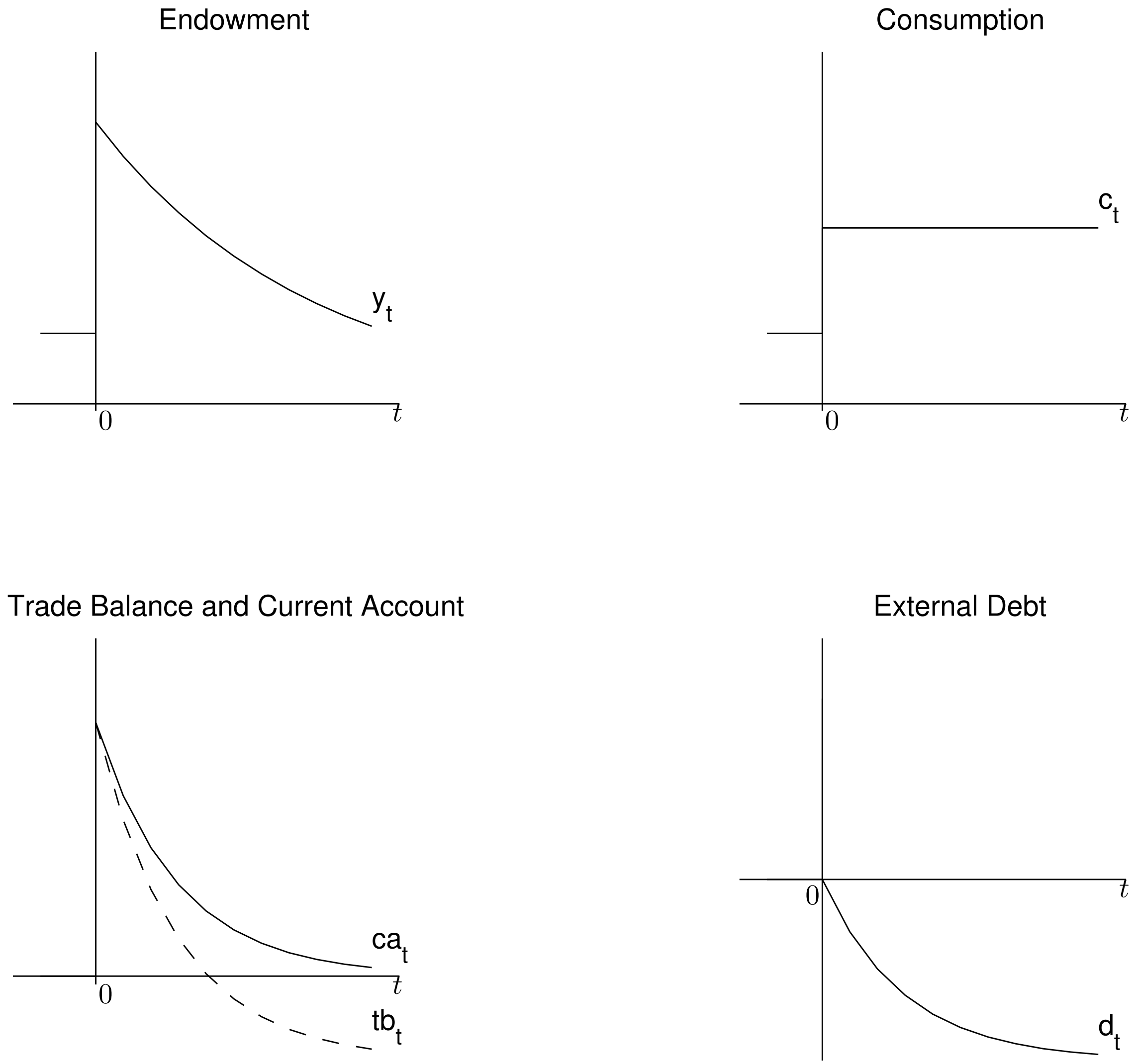
• if $\rho = 0$, then most of the change in current income is exported resulting in a trade balance improvement.

• if $\rho \approx 1$, then none of the change in current income is exported and the trade balance is unchanged.

The next graph displays the impulse response of y_t, c_t, d_t, tb_t, and ca_t to a unit increase in y_t assuming zero initial debt, $d_{t-1} = 0$ for a value of $\rho \in (0, 1)$.

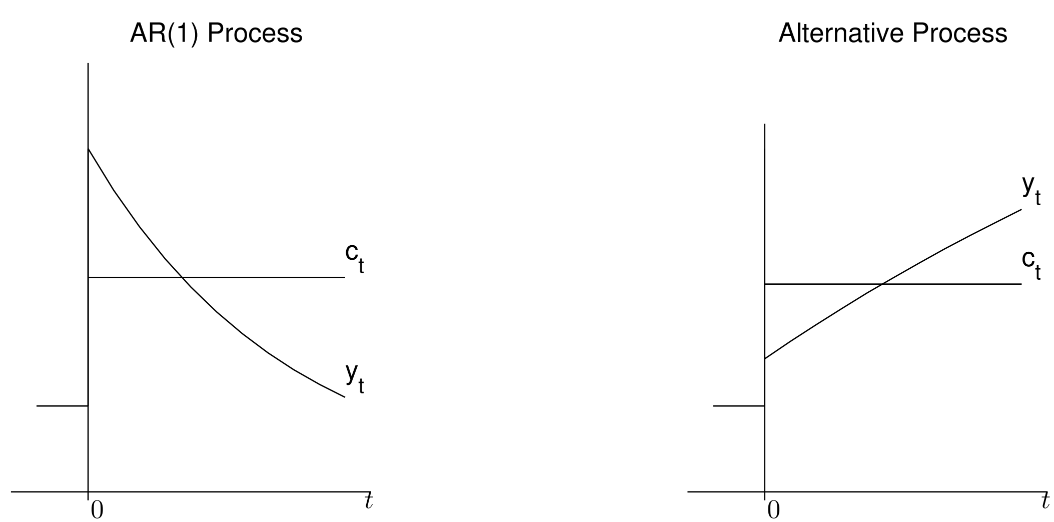
$\hookrightarrow$ Income shock

Response to a positive and persistent endowment shock: AR(1) process



Counterfactual Predictions with AR(1) Endowment

The figure on the previous slide illustrates that a positive endowment shock causes an improvement in the trade balance and the current account. This is counterfactual. In chapter 1, we documented that both of these variables are countercyclical. The intuition behind the model's prediction is provided by the left panel of the following figure



Under the assumed AR(1) process, an increase in output in the current period is expected to die out over time. So future output is expected to be lower than current output. As a result, consumption smoothing households save part of the current increase in output for future consumption.

Now suppose we could come up with an output process like the one shown on the right panel of the figure. There, an increase in output today creates expectations of even higher output in the future. As a result, consumption today increases by more than output, as consumption-smoothing households borrow against future income. In this case, an output shock would tend to worsen the trade balance and the current account, which is more in line with the data.

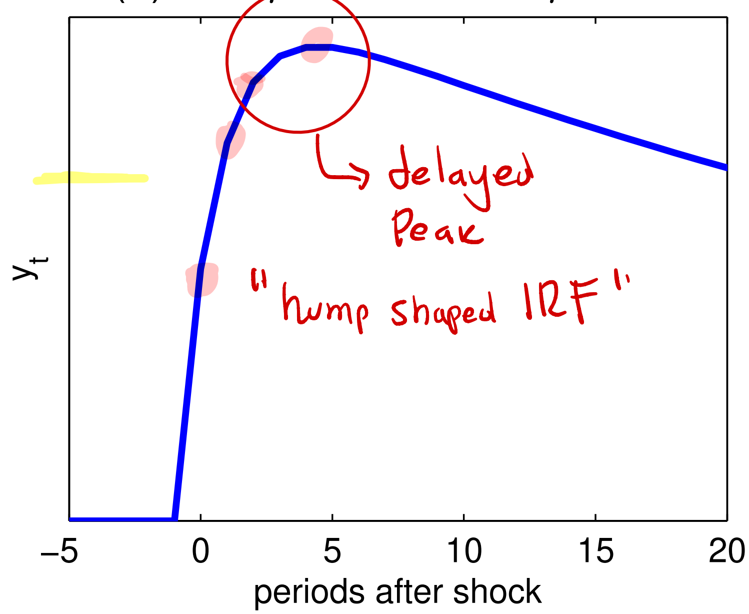
What type of endowment processes can give rise to an upward sloping impulse response for output? We turn to this issue next.

2.3 An AR(2) Income Process

$$y_t = \bar{y} + \rho_1(y_{t-1} - \bar{y}) + \rho_2(y_{t-2} - \bar{y}) + \epsilon_t \tag{2.22}$$

Impulse Response of Endowment

AR(2) with $\rho_1 = 1.5$ and $\rho_2 = -0.51$.



• impulse response is hump-shaped, that is, the peak output response occurs several periods after the shock occurs.

• current level of output **may** rise by less than permanent income, that is, the change in y_t may be less than the change in y_t^p.

• If so, the trade balance and the current account will deteriorate in response to an increase in output, bringing the model closer to the data.

Restrictions on ρ_1 and ρ_2 to ensure stationarity

• income process should be mean reverting (stationary), i.e., $E_t y_{t+j}$ exists for all $j \geq 0$ and

$$\lim_{j \to \infty} E_t y_{t+j} = \bar{y} \quad \forall t \geq 0$$

Let $Y_t = \begin{bmatrix} y_t - \bar{y} \\ y_{t-1} - \bar{y} \end{bmatrix}$ Then write (2.22) as:

$$Y_t = R\,Y_{t-1} + \begin{bmatrix} \epsilon_t \\ 0 \end{bmatrix} \quad \text{with } R = \begin{bmatrix} \rho_1 & \rho_2 \\ 1 & 0 \end{bmatrix}$$

This implies that

$$E_t Y_{t+j} = R^j Y_t \tag{2.23}$$

Thus $\lim_{j \to \infty} E_t y_{t+j} = \bar{y}$ holds for any initial conditions y_t, y_{t-1}, if and only if both eigenvalues of $\mathbf{R}$ lie inside the unit circle.

The eigenvalues of any 2x2 matrix $\mathbf{M}$ lie inside the unit circle if and only if

$$|det(M)| < 1$$
$$|tr(M)| < 1 + det(M)$$

In our application we have,

$$det(R) = -\rho_2 \quad \text{and} \quad tr(M) = \rho_1$$

It follows that the AR(2) process is mean reverting iff

$$\rho_2 < 1 - \rho_1$$
$$\rho_2 < 1 + \rho_1$$
$$\rho_2 > -1$$

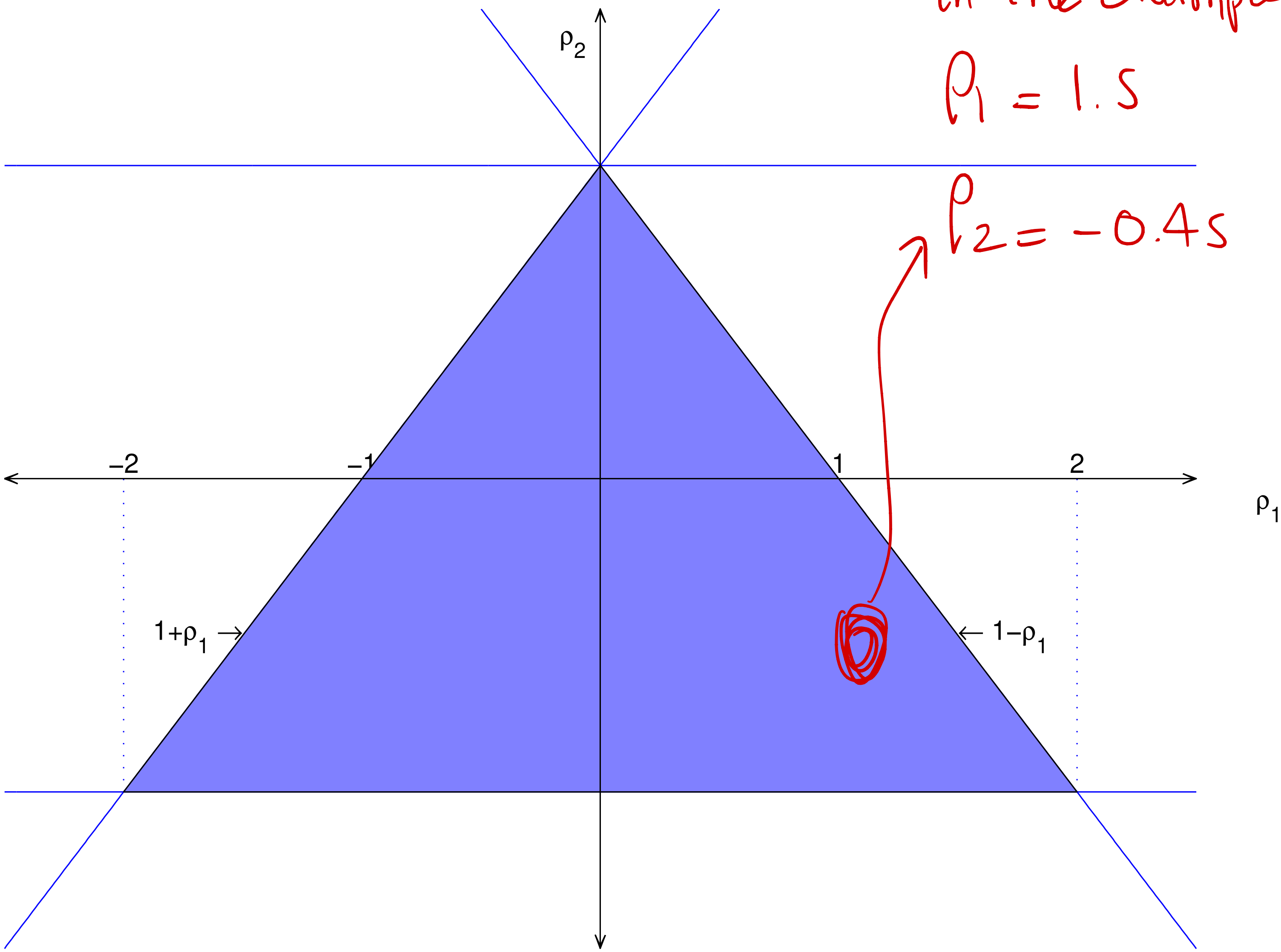
Take a look at the graph on the next slide. The set of allowable pairs (ρ_1, ρ_2) are those inside the triangle.

Restrictions on ρ_1 and ρ_2 to ensure stationarity

In the example

$\rho_1 = 1.5$

$\rho_2 = -0.45$



Restrictions on ρ_1 and ρ_2 to ensure countercyclical impact response of trade balance

Recall that the key variable determining the response of the trade balance and the current account is: $y_t - y_t^p$, the difference between current income and permanent income.

In particular, we want to know for which pairs (ρ_1, ρ_2) current income increases by less than permanent income in response to a positive income shock in the AR(2) case.

Permanent Income with AR(2) Endowment

Recall the definition of permanent income, y_t^p, given in (2.10):

$$y_t^p - \bar{y} = \frac{r}{1+r}\sum_{j=0}^{\infty}(1+r)^{-j}E_t(y_{t+j} - \bar{y})$$

Use (2.23)

$$\begin{aligned}
E_t Y_{t+j} &= \mathbf{R}^j Y_t \\
(1+r)^{-j}E_t Y_{t+j} &= (\mathbf{R}/(1+r))^j Y_t \\
\sum_{j=0}^{\infty}(1+r)^{-j}E_t Y_{t+j} &= \sum_{j=0}^{\infty}\underbrace{(\mathbf{R}/(1+r))^j}_{\text{Geometric series equivalent}} Y_t \\
\frac{r}{1+r}\sum_{j=0}^{\infty}(1+r)^{-j}E_t Y_{t+j} &= \frac{r}{1+r}[\mathbf{I} - \mathbf{R}/(1+r)]^{-1} Y_t \\
&= \frac{r}{(1+r)(1+r-\rho_1)-\rho_2}\begin{bmatrix} 1+r & \rho_2 \\ 1 & 1+r-\rho_1 \end{bmatrix} Y_t
\end{aligned}$$

From here we obtain

$$y_t^p - \bar{y} = \frac{r}{(1+r)(1+r-\rho_1)-\rho_2}\left[(1+r)(\underline{y_t} - \bar{y}) + \rho_2(\underline{y_{t-1}} - \bar{y})\right] \qquad (2.24)$$

$\Rightarrow$ The impact response of permanent income, y_t^p, to an increase in y_t is always positive. (Can you show this?)

The key variable $y_t - y_t^p$ with AR(2) Endowment

From (2.24) it follows that

$$y_t - y_t^p = \gamma_0(y_t - \bar{y}) + \gamma_1(y_{t-1} - \bar{y})$$

with

$$\gamma_0 = \frac{(1-\rho_1-\rho_2) + r(1-\rho_1)}{(1-\rho_1-\rho_2) + r(1-\rho_1) + r(1+r)} \quad \text{and} \quad \gamma_1 = \ldots$$

The model therefore predicts a counterycyclical response on impact of $y_t - y_t^p$ iff

$$\gamma_0 < 0.$$

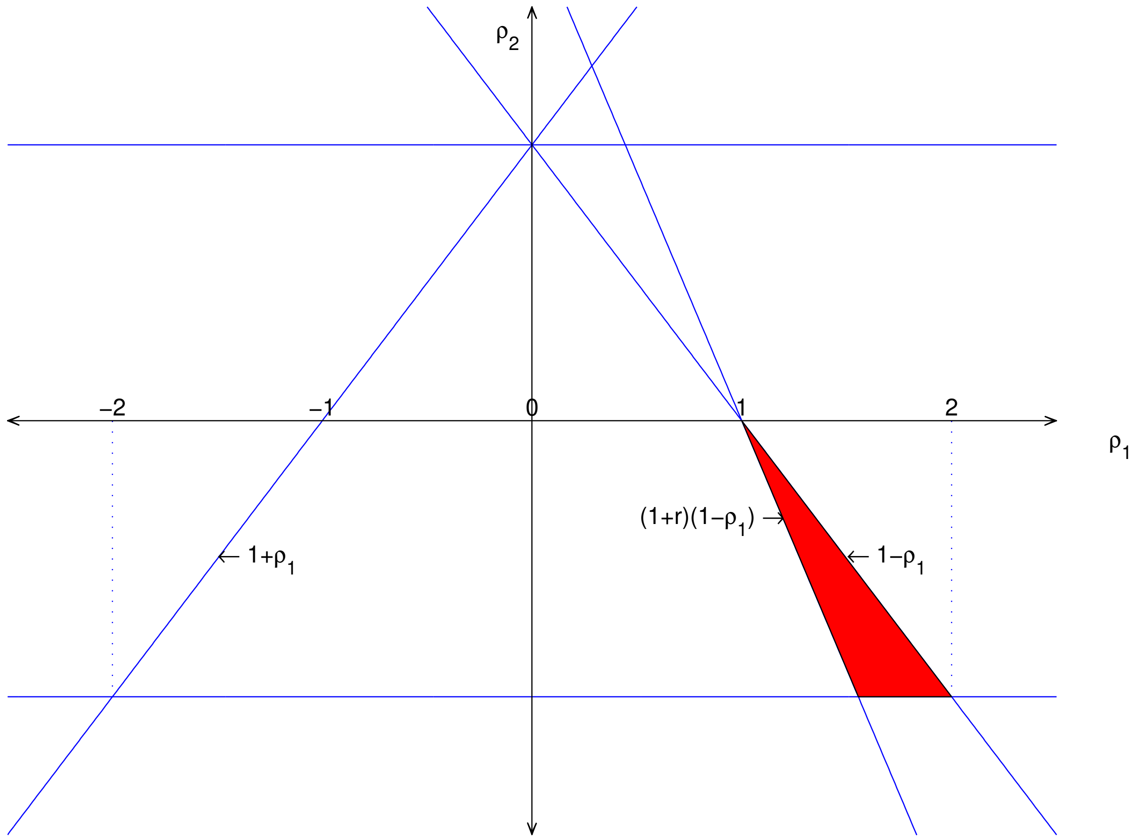
This requires that the numerator and the denominator of γ_0 have different signs, which we can ensure by adding the requirement

$$\rho_2 > (1+r)(1-\rho_1) \tag{**}$$

to the requirements of stationarity.

The graph on the next slide shows there exist combinations of (ρ_1, ρ_2) that satisfy this criterion and the stationarity requirements. Thus in principle the model with an AR(2) endowment process can predict a countercyclical response of the trade balance and the current account. It is then an empirical question whether output conforms to those restrictions. (See Exercise 2.8)

Restrictions on ρ_1 and ρ_2 to ensure countercyclical impact response of trade balance with AR(2) income



We need $\rho_1 > 1$. Why? To have future income to be higher than current income. And we need $\rho_2 < 0$ for stationarity, but not too negative to avoid that the initial increase in output is followed by a future decline.

Summary of adjustment with AR(2) income

$\uparrow y_t$

In response to a positive income shock in period t:

- y_t^p increases
- By (2.11): $c_t = y_t^p - rd_{t-1}$, hence c_t procyclical on impact.

and provided $\rho_2 > (1+r)(1-\rho_1)$

- $y_t^p - y_t$ increases,
- By (2.12): $d_t = d_{t-1} + y_t^p - y_t$, hence d_t increases.
- By (2.15): $ca_t = y_t - y_t^p$, hence ca_t countercyclical
- By (2.16): $tb_t = y_t - y_t^p + rd_{t-1}$, hence tb_t countercyclical

We have shown that it is possible for the model to predict a counter cyclical trade balance adjustment even under a stationary income process.

Thus non-stationarity is not a necessary assumption for a countercyclical adjustment. Next we show that a non-stationary income process can give rise to a countercyclical trade balance adjustment as well.

2.4 A Nonstationary Income Process

$$\Delta y_t \equiv y_t - y_{t-1} \tag{2.25}$$

$$\Delta y_t = \rho \Delta y_{t-1} + \epsilon_t \tag{2.26}$$

$\Delta y_t \sim I(0)$; $y_t \sim I(1)$

The key variable $y_t - y_t^p$ with Nonstationary Income

In general (i.e., not using any particular stochastic for the income process)

$$y_t - y_t^p = -\sum_{j=1}^{\infty} \frac{E_t \Delta y_{t+j}}{(1+r)^j} \tag{2.27}$$

When income follows (2.26), then

$$E_t \Delta y_{t+j} = \rho^j \Delta y_t$$

and hence (2.27) can be expressed as

$$y_t - y_t^p = -\frac{\rho}{1+r-\rho} \Delta y_t \tag{***}$$

This equation says that permanent income increases by more than current income if $\rho > 0$, that is, then the growth rate shock is persistent. In this case, the level of output is expected to rise in the future.

Current Account Adjustment with Nonstationary Income

By (2.16) and (***)

$$tb_t = y_t - y_t^p + rd_{t-1} = -\frac{\rho}{1+r-\rho}\Delta y_t + rd_{t-1}$$

and by (2.15) and (***)

$$ca_t = y_t - y_t^p = -\frac{\rho}{1+r-\rho}\Delta y_t$$

Hence, as long as, $\rho > 0$, the economy with nonstationary income predicts a countercyclical impact response of the trade balance and of the current account.

Why? Agents increase consumption more than income in anticipation of future income increases.

Excess Consumption Volatility with Nonstationary Income

Can the model account for the empirical regularity that consumption changes are more volatile than output changes, as documented in Chapter 1.

Let

$$\triangle c_t \equiv c_t - c_{t-1}$$

and $\sigma_{\triangle c}$ and $\sigma_{\triangle y}$ denote the standard deviation of consumption and output changes, respectively.

Use

$$ca_t = y_t - c_t - rd_{t-1}$$

Take differences

$$ca_t - ca_{t-1} = \triangle y_t - \triangle c_t - r(d_{t-1} - d_{t-2}).$$

Noting that $d_{t-1} - d_{t-2} = -ca_{t-1}$ and solving for Δc_t, we obtain:

$$\begin{aligned}
\Delta c_t &= \Delta y_t - ca_t + (1+r)ca_{t-1} \\
&= \Delta y_t + \frac{\rho}{1+r-\rho}\Delta y_t - \frac{\rho(1+r)}{1+r-\rho}\Delta y_{t-1} \\
&= \frac{1+r}{1+r-\rho}\Delta y_t - \frac{\rho(1+r)}{1+r-\rho}\Delta y_{t-1} \\
&= \frac{1+r}{1+r-\rho}\,\epsilon_t. \qquad (2.29)
\end{aligned}$$

Subs $\Delta y_t = \rho \Delta y_{t-1} + \epsilon_t$

The change in consumption is a white noise. Why? By the Euler equation (2.8).

By (2.29)

$$\sigma_{\triangle c} = \frac{1+r}{1+r-\rho}\sigma_\epsilon.$$

and by equation (2.26)

$$\sigma_{\triangle y}\sqrt{1-\rho^2} = \sigma_\epsilon$$

so that

$$\frac{\sigma_{\triangle c}}{\sigma_{\triangle y}} = \left[\frac{1+r}{1+r-\rho}\right]\sqrt{1-\rho^2} \tag{2.30}$$

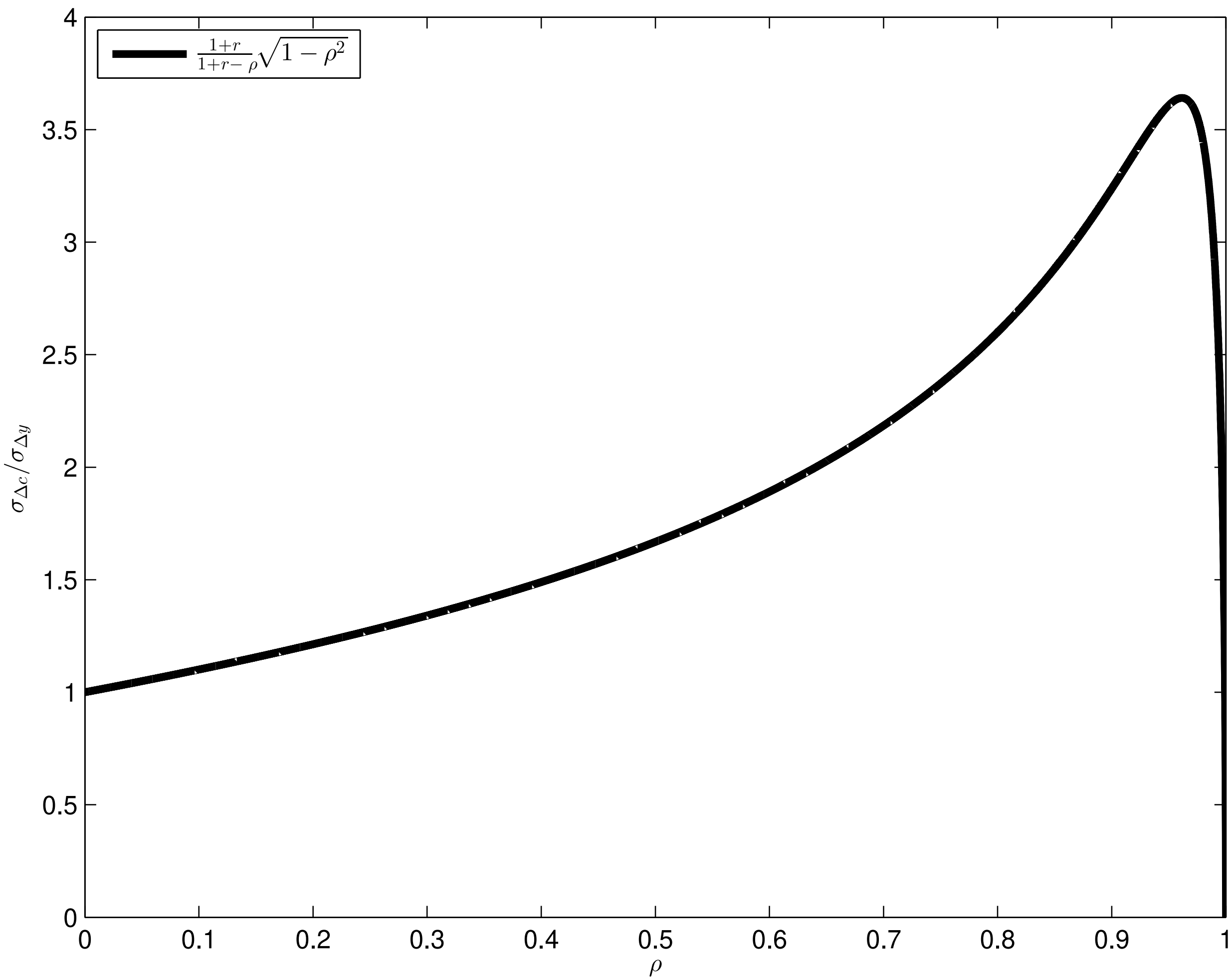
When $\rho = 0$, consumption and output changes are equally volatile.

When $\rho > 0$, consumption changes can become more volatile than output changes. To see this: RHS of (2.30) is increasing in ρ at $\rho = 0$. Since consumption and output changes are equally volatile at $\rho = 0$, it follows that there are values of ρ in the interval $(0, 1)$ for which the volatility of consumption changes is higher than that of output changes.

This property ceases to hold as Δy_t becomes highly persistent. This is because as $\rho \to 1$, the variance of Δy_t becomes infinitely large as changes in income become a random walk, whereas, as expression (2.29) shows, Δc_t follows an i.i.d. process with finite variance for all values of $\rho \in [0, 1]$.

Excess Volatility of Consumption Changes and the Persistence of Output Changes

Note. This figure plots equation (2.30) as a function of ρ for an interest rate of 4 percent ($r = 0.04$).



2.5 Testing the Intertemporal Approach to the Current Account

• Hall (1978) initiates a large empirical literature testing the random walk hypothesis for consumption implied by the permanent income hypothesis.

• Campbell (1987) tests the predictions of the permanent income model for savings.

• Nason and Rogers (2006) test the predictions regarding the current account.

In the present model savings and the current accounts are equal (because we are abstracting from investment).

(2.15) $ca_t = y_t - y_t^p$

(2.27) $y_t - y_t^p = -\sum_{j=1}^{\infty} \frac{E_t \Delta y_{t+j}}{(1+r)^j}$

Combining (2.15) and (2.27) yields

$$ca_t = y_t - y_t^p = -\underbrace{\sum_{j=1}^{\infty} (1+r)^{-j} E_t \Delta y_{t+j}}^{\text{PV of Expected Income Changes}} \tag{2.28}$$

This equation says that a country runs a current account deficit (i.e., borrows from the rest of the world), when the present discounted value of future income changes is positive. And the country runs a current account surplus (i.e., lends to the rest of the world), when current income exceeds permanent income. It also says that current account data should forecast income changes.

Testable Restrictions of the Intertemporal Approach to the Current Account

Test $ca_t \simeq -PV_t$ Income Changes

The testable restriction we focus on is:

$$ca_t = -\sum_{j=1}^{\infty} \frac{E_t \Delta y_{t+j}}{(1+r)^j} \tag{2.28 R}$$

Approximate it w/ VAR in Δy_t, ca_t

We have observations on the LHS of this equality because we have current account data. But we do not have data on the RHS. How can we get an estimate of the RHS? The idea is to use a vector autoregression (VAR) model in output changes (Δy_t) and the current account (ca_t) to construct an estimate of the RHS given a calibrated value of r. Given the estimate of the RHS, we can test the hypothesis that the LHS is equal to the RHS.

To construct an estimate of the RHS, proceed as follows.

Estimate the VAR model:

$$x_t = Dx_{t-1} + \epsilon_t \tag{2.31}$$

where

$$x_t = \begin{bmatrix} \Delta y_t \\ ca_t \end{bmatrix}$$

and D is a 2x2 matrix of coefficients and ϵ_t is a 2x1 mean-zero i.i.d. process.

Note: The model may or may not imply that x_t has a VAR representation of this form. [An example in which such a representation exists is when Δy_t itself is a univariate AR(1) process like the one assumed in Section 2.4].

$H_t =$ information contained in the vector x_t.

$$\sum_{j=1}^{\infty} a^j = \frac{a}{1-a}$$

Forecast of x_{t+j} given H_t:

$$E_t[x_{t+j}|H_t] = D^j x_t$$

$$a = \frac{D}{1+r} \; ; \; \frac{1}{1-a} \approx \left[I - \frac{D}{1+r}\right]^{-1}$$

Sum the forecasts:

$$\sum_{j=1}^{\infty} (1+r)^{-j} E_t[x_{t+j}|H_t] = \left[I - \frac{D}{1+r}\right]^{-1} \frac{D}{1+r} \begin{bmatrix} \Delta y_t \\ ca_t \end{bmatrix}.$$

Pre-multiplying this expression by $\begin{bmatrix} 1 & 0 \end{bmatrix}$ we obtain: $\rightsquigarrow$ the PV of Income changes

$$\sum_{j=1}^{\infty} (1+r)^{-j} E_t[\Delta y_{t+j}|H_t] = \begin{bmatrix} 1 & 0 \end{bmatrix} \left[I - \frac{D}{1+r}\right]^{-1} \frac{D}{1+r} \begin{bmatrix} \Delta y_t \\ ca_t \end{bmatrix}.$$

Let

$$F \equiv - \begin{bmatrix} 1 & 0 \end{bmatrix} \left[I - \frac{D}{1+r}\right]^{-1} \frac{D}{1+r}.$$

Use this expression as the estimate of the RHS of (2.28)

$$\widehat{RHS} = F \begin{bmatrix} \Delta y_t \\ ca_t \end{bmatrix}$$

The left-hand-side of (2.28) can be written as:

Are they equal?

$$LHS = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta y_t \\ ca_t \end{bmatrix}$$

The Null hypothesis we wish to test is:

$$H_0 : F = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

The null hypothesis thus consists of 2 restrictions imposed on (a function of) the coefficients of the matrix D.

Nason and Rogers (2006) estimate a VAR with 4 lags using Canadian data on the current account and GDP net of investment and government spending. (The above VAR assumes 1 lag; in the next slide we show how to modify the test to 4 lags.) The sample period is 1963:Q1-1997:Q4. They assume that $r = 0.037$ per year.

Their data strongly **reject the null hypothesis**. They find that the Wald statistic, which reflects the distance between F and $\begin{bmatrix} 0 & 1 \end{bmatrix}$, is 16.1 with an asymptotic p-value of 0.04.
Thus, if H_0 were true, then the Wald statistic would take a value of 16.1 or higher only 4 out of 100 times.

The VAR of Nason and Rogers has 4 lags, but our example above assumed a VAR with one lag. Here we show how to perform the test for a VAR with 4 lags.

Estimate the following VAR(4):

$$\Delta y_t = \sum_{j=1}^{4} a_j \Delta y_{t-j} + \sum_{j=1}^{4} b_j \Delta ca_{t-j} + \epsilon_t^1$$

and

$$ca_t = \sum_{j=1}^{4} c_j \Delta y_{t-j} + \sum_{j=1}^{4} d_j \Delta ca_{t-j} + \epsilon_t^2$$

Then let:

$$x_t = \begin{bmatrix} \Delta y_t & \Delta y_{t-1} & \Delta y_{t-2} & \Delta y_{t-3} & ca_t & ca_{t-1} & ca_{t-2} & ca_{t-3} \end{bmatrix}'$$

The vector x_t evolves over time as

$$x_t = Dx_{t-1} + u_t$$

where D is a 8x8 matrix of coefficients

$$D = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & b_1 & b_2 & b_3 & b_4 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ c_1 & c_2 & c_3 & c_4 & d_1 & d_2 & d_3 & d_4 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

and u_t is the 8x1 vector :

$$u_t = [\epsilon_t^1 \quad 0 \quad 0 \quad 0 \quad \epsilon_t^2 \quad 0 \quad 0 \quad 0]'$$

Following the same steps as in the AR(1) case, we obtain.

$$\sum_{j=1}^{\infty}(1+r)^{-j}E_t[\Delta y_{t+j}|H_t] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\left[I - \frac{D}{1+r}\right]^{-1}\frac{D}{1+r}x_t$$

Let:

$$F \equiv -\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\left[I - \frac{D}{1+r}\right]^{-1}\frac{D}{1+r}.$$

Then the RHS of 2.28 can be expressed as

$$RHS = Fx_t$$

The LHS of (2.28) can be written as:

$$LHS = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}x_t$$

For the VAR(4) the Null hypothesis becomes:

$$H_0 : F = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

A second testable restriction.

Start again with the following implication of the theoretical model

$$ca_t = -\sum_{j=1}^{\infty} \frac{E_t \Delta y_{t+j}}{(1+r)^j} \tag{2.28 R}$$

and express it as

$$ca_t = \frac{-E_t \Delta y_{t+1}}{1+r} - \sum_{j=2}^{\infty} \frac{E_t \Delta y_{t+j}}{(1+r)^j} = \frac{-E_t \Delta y_{t+1}}{1+r} + \frac{1}{1+r} E_t ca_{t+1}$$

Rearranging yields:

$$E_t \Delta y_{t+1} - E_t ca_{t+1} + (1+r) ca_t = 0$$

Let

$$Z_{t+1} \equiv \Delta y_{t+1} - ca_{t+1} + (1+r) ca_t$$

Note that Z_{t+1} is observable. Regress Z_{t+1} on x_t (or past x_t , the information contained in H_t). If the theoretical model is true, all coefficients on x_t should be zero, that is, is should be true that

$$\left[\begin{bmatrix} 1 & -1 \end{bmatrix} D + (1+r) \begin{bmatrix} 0 & 1 \end{bmatrix} \right] = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

Nason and Rogers find that this hypothesis is rejected with a p-value of 0.06.

Intertemporal model of CA: Not Good for explaining CA

Conclusions

(delivers procyclicality for AR(1) income; Implies PV predictions that are not in line w/ the data)

• propagation mechanism invoked by canonical intertemporal model of the current account does not provide a satisfactory account of the observed current account dynamics.

AR(1) ⟶ Procyclicality

• for AR(1) output specifications that model predicts the current account to be procyclical, whereas it is countercyclical in the data.

still for AR(2) ⟶ Can generate countercyclicality

• AR(2) or nonstationary specifications for the income process can in principle imply a countercyclical adjustment of the current account. Yet, empirical tests rejects the basic mechanism of the intertemporal model of the current account namely that current accounts are equal to future expected income changes.

But empirical tests reject the model

• to bring observed and predicted behavior of the current account closer together, in the following chapters we will enrich both the model's sources of fluctuations and its propagation mechanism.

⇒ Need to enrich i) Sources of fluctuations; ii) Model's propagation mechanism