Summary Ch 1 — Empirical facts in International Economics

Business Cycles Facts are characterized by decomposing the time series $y_t = \underbrace{y_t^c}_{\text{Cycle}} + \underbrace{y_t^s}_{\text{Trend/Secular Component}}$ (Methods: HP Filter, log-linear detrending, log-quadratic, time differences, Band Pass Filter)

Log-linear: $y_t = \ln Y_t$ (Y_t: economic time series), then let $y_t = a + bt + \epsilon_t$, cycle: $y_t^c = \epsilon_t$, trend: $y^s = a + bt$; a, b can be computed via OLS (e.g. King, Plosser, Rebelo (JME, 1988))

Log-Quadratic: $y_t = a + bt + ct^2 + \epsilon_t$, cycle: $y_t^c = \epsilon_t$, trend: $y_t^s = a + bt + ct^2$, a, b, c can be estimated via OLS (e.g. Mendoza 1991)

Business Cycles Facts w/ Annual Data

Source: WDI (1960–2011), Data included for countries that have at least 30 consecutive obs. in (all per-capita and real) log of GDP (y_t), log of real Consumption (c_t), government consumption (g_t), real investment (i_t), exports (x_t), imports (m_t)

Sample: 120 countries, 94 countries for current account

Note on Consumption: tipically studies remove durables from definition of consumption. Reason: such expenditure resembles better Investment in household physical capital. Like investment it is far more volatile than consumption in non durables and services

Results: Non durable & Services consumption is less volatile than output ($\sigma_c < \sigma_y$)
Durables consumption is more volatile than output ($\sigma_{c,durables} > \sigma_y$)

σ_c/σ_y	log-linear detrending	Quadratic detrending	HP Filter
Total	1.02	1.01	0.88
Non durables	0.87	0.84	0.64
Durables	2.47	2.53	2.95

Note on trade balance and Current account
Trade balance and current account can take on negative values then $\log(\cdot)$ cannot be used. Instead: $tb_t = \frac{X_t - M_t}{\exp(y_t^s)}$, $ca_t = \frac{CA_t}{\exp(y_t^s)}$

Ten business Cycles Facts

Fact 1: [High Global Volatility] The cross-country average standard deviation of output is about twice as large as its U.S. counterpart.

$\sigma_y^{RoW} > \sigma_y^{US}$

Fact 2: [Excess Consumption Volatility] On average across countries, private consumption including durables is more volatile than output.

w durables $\sigma_c > \sigma_y$

Fact 3: [Global Ranking of Volatilities] The ranking of cross-country average standard deviations from top to bottom is imports, investment, exports, government spending, consumption, and output.

Global Ranking: $\underset{\text{(imports)}}{\sigma_m} > \underset{\text{(Investment)}}{\sigma_i} > \sigma_x > \sigma_g > \sigma_c > \sigma_y$

Fact 4: [Procyclicality of the Components of Aggregate Demand] On average across countries, consumption, investment, exports, and imports are positively correlated with output.

C, M, I, X are procyclical

Fact 5: [Countercyclicality of the Trade Balance and the Current Account] On average across countries, the trade balance, trade-balance-to-output ratio, current account, and current-account-to-output ratio are negatively correlated with output.

TB, CA are countercyclical

Fact 6: [Acyclicality of the Share of Government Consumption in GDP] On average across countries, the share of government consumption in output is roughly uncorrelated with output.

G/Y is acyclical

Fact 7: [Persistence] The components of aggregate supply (output and imports) and aggregate demand (consumption, government spending, investment, and exports) are all positively serially correlated.

Serial Correlation in all components of Agg. Supply and agg. demand (y, m; and c, g, i, x).

Fact 8: [Excess Volatility of Poor and Emerging Countries] Business cycles in emerging or poor countries are about twice as volatile as business cycles in rich countries.

Fact 9: [Excess Consumption Volatility in Poor and Emerging Countries] The relative consumption volatility is higher in poor and emerging countries than in rich countries.

Fact 10: [The Countercyclicality of Government Spending Increases with Income] The share of government consumption is countercyclical in rich countries, but acyclical in emerging and poor countries.

Note on HP filter:

given y_t, pick y_t^c, y_t^s to solve: $\min_{\{y_t^c, y_t^s\}_{t=1}^T} \left\{ \sum_{t=1}^T (y_t^c)^2 + \lambda \sum_{t=2}^{T-1} [(y_{t+1}^s - y_t^s) - (y_t^s - y_{t-1}^s)]^2 \right\}$ s.t. $y_t^s + y_t^c = y_t$

as $\lambda \to \infty$ Δy^s become costly $\Rightarrow$ y^s converges to lineal trend
as $\lambda \to 0$ the cycle disappears ($y^c = 0$) ($y_t^s = y_t$)

in matrix form: $\min_{Y^s} (Y - Y^s)'(Y - Y^s) + \lambda (Y^{s\prime} B' B Y^s)$ $\quad B = \begin{bmatrix} 1 & -2 & 1 & 0 & \cdots & & \\ 0 & 1 & -2 & 1 & 0 & \cdots & \\ \vdots & & & & & & \\ 0 & \cdots & & 0 & 1 & -2 & 1 & 0 \\ 0 & \cdots & & & 0 & 1 & -2 & 1 \end{bmatrix}$

FOC: $-(Y - Y^s) + \lambda B'BY^s = 0 \Rightarrow Y^s = (I + \lambda B'B)^{-1} Y$ ($\Rightarrow$ HP is a linear filter)

RoW vs. US

US1: $\sigma_y^{RoW} > \sigma_y^{US}$
US4: g/y is countercyclical
US5: US is less open than RoW ($\frac{x+m}{y} \approx 40\%$ in RoW vs 19% in US)

Countries Comparison by Income

Classification by per capita GDP:

< \$3000	Poor	(40, 1/3)
\$3000–\$25000	Emerging	(58, 1/2)
> \$25000	Rich	(22, 1/6)

Fact 8: $\sigma_y^{EME}, \sigma_y^{Poor} > \sigma_y^{Rich}$

Excess volatility: 8.7% 6.1% 3.3%

Fact 9: $\frac{\sigma_c}{\sigma_y}^{Poor} > \frac{\sigma_c}{\sigma_y}^{EME} > \frac{\sigma_c}{\sigma_y}^{Rich}$ Poor & EMEs smooth consumption by less

Fact 10: Countercyclicality of Gov. Spending increases w/ income
Corr(g/y, y): Poor: 0.08, EME: −0.08, Rich: −0.39
also the government consumption is countercyclical in rich countries, but acyclical in EMEs & Poor countries

Quarterly Data

Not many long series $n^{annual} = 120$, $n^{quarterly} = 28$
Sample Period: 1980Q1–2012Q4
Facts 5, 8, 9, 10 remain to hold.

Summary Ch 3

K is introduced as endowment model fails to explain countercyc. TB$_t$ & we want to endogenize the output (& cycles)

Capital is introduced as endowment model fails to explain a countercyclical trade balance (tb_t) and because we want to endogenize the output (and cycles)

Simplifying assumptions: Perfect foresight, $\beta(1+r)=1$, No depreciation

w/ K the tb_t can become Countercyclical

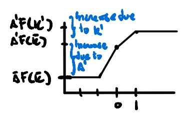
w/ an AR(1) Income: Given persistent productivity shocks the MPK is expected to be high in the future $\Rightarrow$ agents Increase Investment $\Rightarrow \uparrow$ Demand, ($\uparrow(C+I)$ relative to Output) $\downarrow$ Savings $\Rightarrow \downarrow TB_t$

now model is $\max \sum \beta^t u(c_t)$ s.t. $c_t + i_t + (1+r)d_{t-1} = y_t + d_t$ with $y_t = A_t F(k_t)$ and $k_{t+1} = k_t + i_t$ (as before $\lim_{j\to\infty} \frac{d_{t+j}}{(1+r)^j} \le 0$)

we still use intertemporal BC approach to characterize consumption $c_t = \frac{r}{1+r}\sum_{j=0}^{\infty} \frac{A_{t+j}F(k_{t+j}) - \overbrace{(k_{t+j+1}-k_{t+j})}^{i_t}}{(1+r)^j} - r d_{t-1}$ (Now cons reflects Investment expenditure)

$tb_t = y_t - c_t - i_t$ ($\Delta c_t > \Delta y_t$ No longer required to have ∇tb_t)

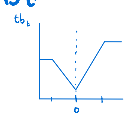
What is new that allows Tb_t to fall? : y_t is expected to increase in the future $\Rightarrow$ agents invest more:



Two Principles:

I. w/ More persistent shocks $\rightarrow$ higher deterioration of TB_t

↳ with permanent shock TB_t deteriorates at the period of the shock!
(transitory) (improves)



In contrast:
with permanent k_t doesn't increase
$\Rightarrow Tb_t$ improves by $\uparrow y$ (relative to c)

II. w/ higher K Adj. Costs $\longrightarrow$ Smaller TB_t deterioration

Agents will smooth the ΔI ($\Rightarrow TB_t$ won't be too lowered)

w/ Adj. Costs: ΔInvestment is spread out (smoothed) $\Rightarrow$ lower $t=0$ Increase in: permanent income, consumption $\Rightarrow$ Muted Instantaneous Tb_t response.

Costs $= \frac{1}{2}\frac{i_t^2}{k_t}$

Budget Constraint becomes: $c_t + i_t + \frac{1}{2}\frac{i_t^2}{k_t} + (1+r)d_{t-1} = A_t F(k_t) + d_t$

(UMP) Lagrangian: $\mathcal{L} = \sum_{t=0}^{\infty}\left\{U(c_t) + \lambda_t\left[A_tF(k_t) + d_t - c_t - i_t - \frac{1}{2}\frac{i_t^2}{k_t} + q_t(k_t + i_t - k_{t+1})\right]\right\}$

q_t: Tobin's q (LMult of K-dynamics is $\lambda_t q_t$)

New Euler Equation: Return on debt (of q_t cap. units) = Return on Physical Capital (extra output + New Price of K + Reduction in costs)

Combine FOC to get dynamics of K stock in terms of prices, quantities:

$$k_{t+1} = q_t k_t$$

$$q_t = \frac{A_{t+1}F'(q_t k_t) + (q_{t+1}-1)^2/2 + q_{t+1}}{1+r}$$

$\xrightarrow{\text{log-linearize}}$

$$\hat{k}_{t+1} = \hat{q}_t + \hat{k}_t$$

$$(1+r)\hat{q}_t = -r_{\epsilon F'}\hat{k}_{t+1} + \hat{q}_{t+1}$$

SS: $q=1$ $r=\bar{A}F'(k)$

*: Capital adj. costs dampen tb_t deterioration in response to a permanent productivity increase

New c_t dynamics with intertemp. BC now accounts for adj. costs on RHS $-\frac{1}{2}(i_{t+j}/k_{t+j})$

Still Consumption = Permanent Income = Investment Income (debt) + Non Financial Permanent Income

in matrix form: $\begin{bmatrix}\hat{k}_{t+1}\\ \hat{q}_{t+1}\end{bmatrix} = M\begin{bmatrix}\hat{k}_t\\ \hat{q}_t\end{bmatrix}$ w/ $M = \begin{bmatrix}1 & 1\\ r_{\epsilon F'} & 1+r+r_{\epsilon F'}\end{bmatrix}$

the solution will satisfy: $\lim_{t\to\infty}\begin{bmatrix}\hat{k}_t\\ \hat{q}_t\end{bmatrix} = \lim_{t\to\infty} M^t\begin{bmatrix}\hat{k}_0\\ \hat{q}_0\end{bmatrix}$

($\exists$ unique q_0 s.t. equilibrium converges to SS given k_0)

For a unique solution 1 eigenvalue of M lies outside unit circle and 1 inside

Let these eigenvalues be λ_1, λ_2. The dynamics are found as: $\hat{k}_t = \lambda_2^t \hat{k}_0$, $\hat{q}_t = -(1-\lambda_2)\lambda_2^t\hat{k}_0$

(also $\hat{q}_0 = -(1-\lambda_2)\hat{k}_0$)

$\therefore$ i) Unique saddle path stable eq. exists (in neighborhood around (q^{ss}, k^{ss}))

ii) adj. to permanent increase in productivity induces k to converge from below (k increases) and Tobin's q from above (q decreases until hitting ss)

iii) Increase in capital is spread out in many periods $\Rightarrow$ Investment is positive and because ΔK is independent of the size of adj. costs: Principle II Follows (or *)