International Macroeconomics - Field Summary

Contents

Part I

International Finance

1 Global Financial Crisis history

Common patterns:

(i) Asset market collapses are deep and prolongued.

(ii) Output and employment also declines profoundly.

(iii) Real value of Government Debt exploded.

MS2011: US homeowners borrowed against the rising value of their homes (leverage boom) possibly due to liquidity constraints and less self control. Result: home-equity based borrowing was the principal cause of ensuing financial crisis.

RR2009: Prior to financial crisis there is an important growth in private and public debt. Other symptoms are asset price inflation, rising leverage, sustained current account deficits, slow trajectory of economic growth.

The aftermath of the crisis shares the three characteristics mentioned above, in addition to each: (i) downturn lasts over three years. (ii) unemployment surge lasts over four years, GDP decline around two years. (iii) increase in debt is not explained by cost of bailout, rather it is due to collapse in tax revenues given the output decline as well as the cost of countercyclical fiscal policies.

RR2014: Part of the costs of banking crises relies in the slow pace of recovery. On average it takes about eight years to reach pre crisis level of income. Also it is found that the subprime crisis is not an anomaly with respect to pre-war data and that it makes sense for advanced economies to rely on policies that are currently characteristic of developing countries as debt restructuration, higher inflation, capital controls and other forms of financial macroprudential policies.

2 Global imbalances

Recalling the low interest rate puzzle, we had that in a Consumption CAPM framework, a linearization of the Euler Equation with CRRA preferences and log normal consumption growth yields,

$$r = \gamma E_t \Delta \ln c_{t+1} - \frac{\gamma^2}{2} Var_t(\Delta \ln c_{t+1}) + \rho$$

using postwar data we found the EPP that implied that the risk aversion coefficient was too high $\gamma \in [20, 200]$ or we obtained the low interest rate puzzle, i.e., taking a reasonable γ and discount rate (e.g. $\rho = 0$) then we get that the implied r by the model is much higher (3.3) than the interest rate observed in reality (0.8).

Explanations mentioned for these puzzles still apply: 1. Consumption aggregation dismisses agents heterogeneity, 2. rare events lead to overestimating the average returns, 3. Survivorship bias, 4. Loss aversion, and others.

A new explanation can be considered: the Global Imbalances.

Basic framework and Intuition:

Consider an endowment SOE with 1 internationally traded riskless bond.

The SOE solves: $\max U(C_t) \quad s.t. \quad C_t + A_{t+1} = Y_t + (1 + r)A_t$

The current account is: $CA_t = A_{t+1} - A_t = Y_t - C_t + rA_t = TB_t + rA_t$

In this setup a country runs a CA surplus/deficit wheneber its output exceeds/is below its permanent output.

Puzzle: Suppose an EME (China) output is rising over time, then if permanent output is larger than the current one, it should run a CA deficit instead of a surplus as observed.

Metzler Diagram

Consider two countries (h, f), add investment, then we need to consider endogenous output $Y_t = Z_t F(K_t)$ with $K_{t+1} = K_t + I_t$. The current account is now given by,

$$CA = Y - C - I + rA$$

The savings are $S = Y + rA - C$ as before and therefore,

$$CA = S - I$$
$$CA^* = S^* - I^*$$

Asset market clearing implies $CA + CA^* = 0$, then $S - I = -(S^* - I*)$.

To find the interest rate that clears the markets, i.e. the r that equals global savings and investment we use a Metzler diagram. To do it we obtain $S(r), I(r), S(r)^*, I(r)^*$ from the FOCs.

From the FOC for K we get $Z_t F'(K_t) = r_t$ and using $K_{t+1} = K_t + I_t$ we get $I'(r) < 0$,

$$Z_t \alpha K_t^{\alpha-1} = r_t$$
$$Z_t (K_{t+1} - I_t)^{\alpha-1} = r_t$$
$$K_{t+1} - I_t = \left(\frac{r_t}{\alpha Z_t}\right)^{\frac{1}{\alpha-1}}$$

by implicit function theorem,

$$\frac{\partial I}{\partial r} = -\frac{\partial g(I,r)/\partial r}{\partial g(I,r)/\partial I} = -\frac{\frac{1}{\alpha-1}\left(\frac{r_t}{\alpha Z_t}\right)^{\frac{2-\alpha}{1-\alpha}}\frac{1}{\alpha Z_t}}{-1} < 0$$

Also, for r close to autarky levels we have $C_t'(r) < 0$ and $S_t'(r) > 0$. Then we put the savings and investment together in a graph for each country and obtain the clearing market interest rate as the rate where asset market clear $CA + CA^* = 0$.

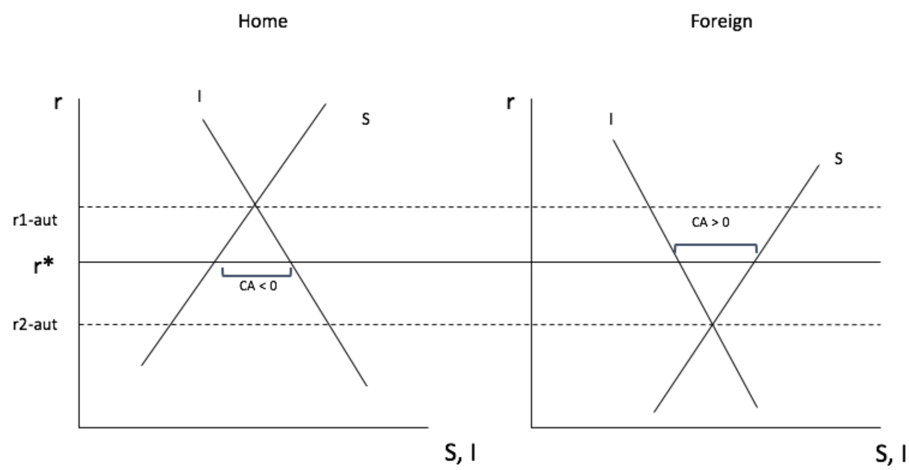


Figure 1: Metzler Diagram

Note that according to the diagram, a country will run a $CA < 0$ if its autarkic interest rate exceeds that of his trading partner and as a result, the equilibrium interest rate will be inbetween the two autarky rates.

Now, even under this setup the puzzle remains, EME still run current account surplus and capital flows towards countries that have low interest rates (and low productivity growth). Then the could be thinking that regardless of the market interest rate observed, there may be frictions that lower the autarky rates more than what they increase by the productivity dynamics. This frictions are considered in the literature recently.

Articles considering such frictions while still relaxing some of the assumptions that lead to the low interest rate puzzle are:

MQR2009: heterogeneous agents, incomplete markets, asymmetric information that generates a borrowing constraint.

This article focuses on the savings that are increased for every r due to precautionary motive. Due to information asymmetries, associated with a lack of financial development, the SOE faces a tighter borrowing constraint that results in a failure to insure idiosyncratic risks and leads the agents to increase their precautionary savings. All of this lowers their autarky rate and can be represented as a shift in the S curve in the Metlzler diagram.

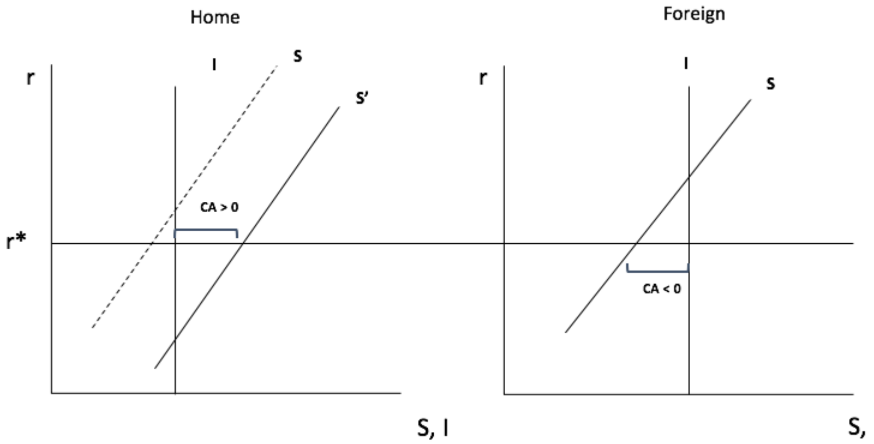


Figure 2: Metzler Diagram (shift in Savings)

CFG2008: OLG structure with imperfect pledgeability of future income due to lack of financial development.

The authors study the supply of assets (Investment) rather than the behaviour of savings as before. In particular, they focus on constraints in the supply of assets used to hedge risks, i.e. shortage of safe assets. Such shortage is the result of lack of income pledgeability, due to the presence of a financial friction that limits the degree of financial development (δ), acts as a friction, limiting the autarky rates, and results in a reduction of the investment. The article also finds supporting evidence of a positive correlation between the interest rates and the degree of financial development, therefore, low financial development can lead to lower interest rates regarless of the other factors governing the dynamics of the returns.

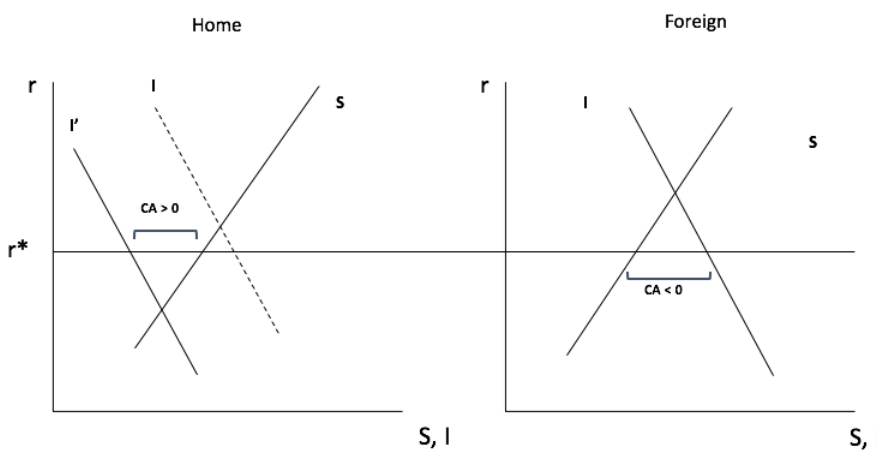


Figure 3: Metzler Diagram (decrease in Investment due to Financial Friction)

Jin 2012: Differences in factor productivities and intensities across countries that exacerbate the imbalances in capital flows.

This study explores the effect in assets flows of the trade specialization patterns due to factor intensities across countries. Then, in addition to the usual convergence effect, where the capital flows to where it's more scarce, it also considers the composition effect of factor proportions within a country. Countries that are more labor intensive would attract less capital flows, explaining why countries with larger returns won't neccesarilly have more capital inflows than other financially developed economies.

Then, a labor expansion in a developing country will deepen both his and the capital intensive country factor specialization, leading to a capital flow towards the developed country. In terms of the diagram this shock implies a larger increase in S than in I.

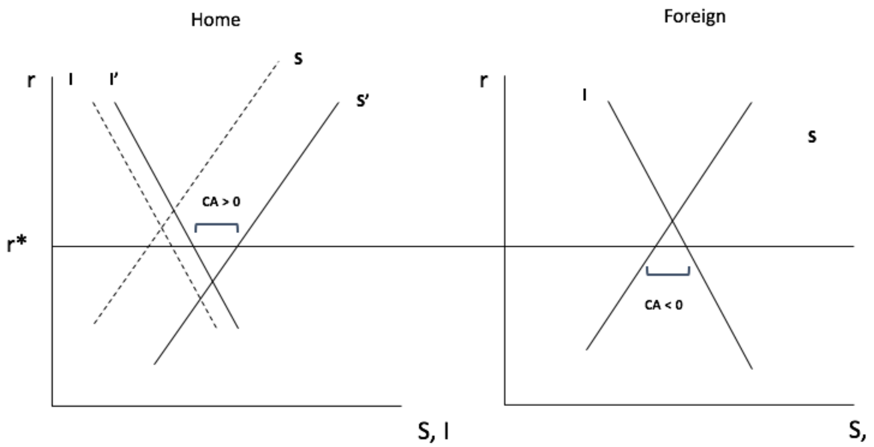


Figure 4: Metzler Diagram (positive labor shock in EME)

3 Low interest rate puzzle, International Risk Sharing, puzzles

- OR 285-94 (Consumption correlation puzzle)
- OR 306-19 (EPP, low interest rate puzzle)
- OR 329-32 (Gains of international risk sharing)
- BKK1992 (IM)

3.1 OR96 285-94 (Consumption correlation puzzle)

In the complete markets model consumption between home and the rest of the world is perfectly correlated: $corr(c, c^*) = 1$ whereas output is less correlated. Empirically we find the opposite $Corr(c, c^*) < Corr(y, y^*)$ and correlation of consumption is lower than predicted.

A possible explanation is that markets are not complete so it's not possible to fully hedge risk. In particular it's possible to think about traded and non-trade goods (Lewis, 1996). In such case what should hold is that consumption in traded goods are perfectly correlated: $Corr(c_T, c_T^*) \approx 1$ that is, there is a *Constrained Efficient Risk Sharing.*

To test that, Lewis checks the correlation between traded output and traded consumption which should be zero in a risk sharing scenario. The result are still not what predicted in theory although the condition holds better for countries without restrictions to the Exchange Rate.

Caution is warranted since the results are subject to data handling and assumptions. In particular, only output risk is examined.

3.2 OR1996 306-19 (EPP, low interest rate puzzle)

Equity Premium Puzzle: (MP1985, MZ1991) Excess return of stocks wrt to bonds is too high to be rationalizable using a CRRA:

$$E(r^m) - r = -(1+r)Cov\left[\beta\left(\frac{c_2}{c_1}\right)^{-\rho}, r^m - r\right]$$

Given that the $Cov(\cdot)$ term is very low then observed returns would imply $\rho \in [2, 200]$ which imply unrealisticly overly risk averse agents.

Deep connection with home bias: If agetns are so risk averse then higher effort to diversify should be observed (lower to none HB).

Explanations: • Higher transaction costs of trading stocks (AG1991), then there is a liquidity premium on bonds that partly explains equity premium. • Non diversifiable labor income (AGW1992) • Interactions of several frictions (HL1995) • Habit persistence in consumption: generates higher volatility of intertemporal MRS • Ommited high return losses, market casualties that bias upwards estimated equity returns.

Low Risk-free Rate Puzzle: Using the CAPM model with a CRRA utility and data on consumption (average difference and variance) yields a riskless rate of about 3.34. Whereas actual estimates are way lower (0.8).

Accomodating parameters with the estimation and data would imply a negative discount factor.

$$c_1^{-\rho} = (1+r)\beta E_1\{c_2^{-\rho}\}$$
$$1 + r = \frac{1}{\beta E_1 \left(\frac{c_2}{c_1}\right)^{-\rho}}$$

approximate with logs: $r = \rho E_1\{\log\left(\frac{c_2}{c_1}\right)\} - \frac{\rho^2}{2} Var(\log \Delta c_2) - \log \beta$
with $\Delta c \approx 0.018$, $Var(\Delta c) \approx 0.0013$ also let $\rho = 2,\ \beta \approx 1 \ \Rightarrow\ r = 3.34$. However the estimated is $r = 0.8$. To allow for the estimated one we would have to assume a negative discount factor.

3.3 OR1996 329-32 (Gains of international risk sharing)

Risk sharing comes with Efficient Risk allocation but possibly with small welfare gains. Then, with transaction costs Home Bias can arise.

Lucas (1987): Lifetime utility with CRRA utility is approximated as:

$$U_t = \frac{\bar{c}^{1-\rho}}{1-\rho}\left[\frac{1}{1-\beta(1+g)^{1-\rho}}\right]\exp\{-\frac{1}{2}(1-\rho)\rho Var(\epsilon)\}$$

where ϵ is a normal random process. Then the lifetime utility without uncertainty, i.e., with Risk Sharing is:

$$U_t = \frac{\bar{c}^{1-\rho}}{1-\rho}\left[\frac{1}{1-\beta(1+g)^{1-\rho}}\right]$$

Therefore te proportional consumption gain of removing uncertainty is given by τ s.t.:

$$\frac{(1+\tau)\bar{c}^{1-\rho}}{1-\rho}\exp\{-\frac{1}{2}(1-\rho)\rho Var(\epsilon)\} = \frac{\bar{c}^{1-\rho}}{1-\rho}$$
$$\vdots$$
$$\tau = \left\{\exp\left[\frac{1}{2}(1-\rho)\rho Var(\epsilon)\right]\right\}^{\frac{1}{1-\rho}} - 1$$

A first order approximation about $Var(\epsilon) = 0$ yields $\tau \approx \frac{1}{2}Var(\epsilon)$.

With 1950-90 US data $Var(\epsilon) = 0.000708$ and $\rho = 10$ gives $\tau \approx 0.00354$ which implies very low gains from risk sharing.

Caution is warranted:

- It's assumed that consumption fluctuates around a time trend. This is criticized.
- Stability of US consumption is atypical.
- Calculations don't allow for individual heterogeneity. No uninsurable idiosyncratic risks.

Still CO1991 find trivial gains from risk sharing. Similar for BKK1992 where the equilibrium dynamics are no different than that of autarky after including costs of trade.

In addition, Mendoza (1995) and Tesar (1995) find a similar result: It's easier to diversify risk domestically with intertemporal resource allocation than across countries.

However these findings are sensitive to assumptions: van Wincoop (1994), BB2002, Sutherland (2004) find large gains of international risk diversification (Key: $ES_{h,f} \neq 1$).

3.4 Other Puzzles

3.4.1 OR2000 NBER Macroeconomics annual

Six Major Puzzles in International Macroeconomics: Is there a common cause? (OR)

- Home bias in Trade: Trade within a country is considerably greater than between countries, even with no substantial barriers.

- Home bias in equity portfolio: Agents hold disproportionate amount of domestic assets and few foreign equity despite arbitrage opportunities of diversification.

- Fielstein-Horioka Puzzle: After diversification savings rate should not depend on domestic investment (in the same way consumption should not depend on endowments). In reality there is a strong correlation between these two variables.

- Low consumptions correlation puzzle: Treated above.

- PPP Puzzle: Real Exchange Rate is way more volatile and persistent than models would suggest (the half-life of RER shocks is about 4 years).

- Exchange Rate Disconnect puzzle: Weak short term feedback between exchange rate and fundamentals (Meese-Rogoff, 1983)

(derived from the former) **Backus-Smith Puzzle:** Consumption growth and RER are highly correlated according to a complete markets model with traded and non traded goods. The intuition is that countries with cheap prices should receive transfers to take advantage of cheap consumption. Such correlation is not observed empirically (correlation is zero or even negative).

Key friction: International trade costs in good markets (transport costs, tariffs, other barriers).

Approach: Not to explore a market imperfection directly but depart from frictionless scenario and add the most plausible imperfection to see if the puzzle remains. The finding is that one trade costs are allowed into the models, i.e., most of real side of the the puzzles dissapear. The model doesn't fully explain the puzzles but aims to state the importance of including trade costs in general.

Other puzzles:

Forward Rate Puzzle: Forward interest rate doesn't predict the exchange rate. Worse, it's being found to be negatively correlated.

The test is performed by estimating $e_{t+1}-e_t = a_0+a_1(f_t-e_t)+\epsilon_t$ and testing $a_0 = 0,\ a_1 = 1$. Those hipotheses are rejected and worse it is found that $a_1 < 0$.

Also, according to Fama (1984): $Var(F_t - E[e_{t+1}]) > Var(E[e_{t+1}] - e_t)$, i.e., variance of risk premium is larger than that of expected variation of exchange rate.

3.5 Basic SOE model (methodology)

One asset (bond only), non-stochastic. Later production and endogenous labor supply

Preferences can be typical CD or GHH. With GHH preferences are quasilinear and MRS between consumption and leisure is independent of contemporaneous consumption level.

Rep. agent, one final good so there is only inter-temporal trade.

Agent's UMP is:

$$
\begin{aligned}
&\max_{\{c_t,a_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t) \\
&\text{s.t.} \\
&c_t + a_{t+1} = y_t + (1+r)a_t \\
&\lim_{T\to\infty} \frac{a_{T+1}}{(1+r)^T} \geq 0, \quad a_0 \text{ given}
\end{aligned}
$$

Define:

- Trade Balance (model without investment) $TB_t = y_t - c_t$

- Current Account $CA_t = a_{t+1} - a_t = TB_t + ra_t$ (Trade balance plus return on assets. Also variation in foreign assets)

- Intra-temporal Budget Constraint: $c_t + a_{t+1} = y_t + (1+r)a_t$

- Lifetime BC: by recursive substitution of a_{t+j}:

$$
\begin{aligned}
\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t} &= \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t} + (1+r)a_0 \\
\sum_{t=0}^{\infty} \frac{TB_t}{(1+r)^t} &= -(1+r)a_0
\end{aligned}
$$

The UMP is then:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t [u(c_t) - \lambda_t (c_t + a_{t+1} - y_t - (1+r)a_t)]$$

FOC:

$$\begin{aligned}
u'(c_t) &= \lambda_t \\
\lambda_t &= \beta(1+r)\lambda_{t+1} \\
u'(c_t) &= \beta(1+r)u'(c_{t+1}) \\
\lim_{T\to\infty} \frac{a_{T+1}}{(1+r)^T} &= 0
\end{aligned}$$

From the EE it can be seen that anytime $(1+r)\beta > 1$ we will have $\frac{c_{t+1}}{c_t} > 1$, i.e., the consumption will grow forever and therefore there will not be an statiorary steady state solution. The reverse will hold with $\frac{c_{t+1}}{c_t} < 1$.

The model will be stationary only in a particular case: $1 = (1+r)\beta$. However, this is a knife edge condition similar to what is mentioned in CO1991 and in general this simplified model is not stationary.

Example: utility is CRRA $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$

$$\begin{aligned}
\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} &= \beta(1+r) \\
c_{t+j} &= [\beta(1+r)]^{\frac{j}{\sigma}} c_t
\end{aligned}$$

subs. in the NPV of revenues:

$$c_t \sum_{j=0}^{\infty} \frac{[\beta(1+r)]^{\frac{j}{\sigma}}}{(1+r)^j} = \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t} + (1+r)a_0 = NPVR_t$$

when $1 = \beta(1+r)$ and given that $\sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^j = \frac{1+r}{r}$:

$$c_t = \frac{r}{1+r} NPVR_t$$

If y increases by 1 permanently the NPVR increases by $\frac{1+r}{r}$ then c increases by 1 $\Rightarrow$ CA, TB constant.

If y increases by 1 permanently $\Delta NPVR < \frac{r}{1+r}$ then $CA = y - c > 0$ (CA is procyclical, poor empirical support).

Finally if $\Delta a_0 = 1$ then $\Delta c = r$ and agents save more permanently. Also **any level of NFA is compatible with the steady state**. This can be troublesome for log-linear approximations that remain valid only about a small region of state variables values.

Technology: $Y_t = e^{Z_t} K_t^{1-\alpha} L_t^{\alpha}$

Preferences: 1. CD: $u_t = \frac{(c_t^{\gamma}(1-L_t)^{1-\gamma})^{1-\sigma}}{1-\sigma}$

2. GHH: $u_t = \frac{(c_t - \tau L_t^{v})^{1-\sigma}}{1-\sigma}$

Key difference in prefereces: labor supply

with CD: $\frac{(1-\gamma)c_t}{\gamma(1-L_t)} = \alpha \frac{Y_t}{L_t}$
with GHH: $\tau \nu L_t^{\nu-1} \alpha = \frac{Y_t}{L_t}$

In CD there is income effect that mitigates labor's response to productivity shocks. In GHH there is no effect and therefore after a productivity shock there is a sharp drop in leisure (can be more realistic).

Also in the model without investment (CA are the total savings), the TB is procyclical which contradicts empirical evidence. In a model with investment TB becomes countercyclical. Therefore, this model is improven with such addition.

With investment:

$$a_{t+1} + c_t + i_t = (1+r)a_t + z_t f(k_t)$$
$$i_t = k_{t+1} - (1-\delta)k_t$$

The definitions should be updated:

- Trade Balance: $TB_t = y_t - c_t - i_t$

The new UMP is:

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$
$$\text{s.t.}$$
$$c_t + a_{t+1} + i_t = z_t f(k_t) + (1+r)a_t$$
$$i_t = k_{t+1} - (1-\delta)k_t$$
$$\lim_{T\to\infty} \frac{a_{T+1}}{(1+r)^T} \geq 0$$
$$\lim_{T\to\infty} \frac{k_{T+1}}{(1+r)^T} \geq 0$$
$$a_0, k_0 \text{ given}$$

FOC:

$$\begin{aligned} c_t:& \quad u'(c_t) = \lambda_t \\ a_{t+1}:& \quad \lambda_t = \beta(1+r)\lambda_{t+1} \\ k_{t+1}:& \quad \lambda_t = \beta\lambda_{t+1}[z_t f'(k_{t+1}) + (1-\delta)] \end{aligned}$$

If we add investment adjustment costs we would have:

$$a_{t+1} + c_t + i_t + \frac{\omega}{2}\frac{(i_t - \delta k_{ss})^2}{k_t} = (1+r)a_t + z_t f(k_t)$$

And the new FOC is:

$$k_{t+1}: \qquad \lambda_t \left(1 + \omega \frac{(i_t - \delta k_{ss})}{k_t}\right) = \beta\lambda_{t+1}\left[z_t f'(k_{t+1}) + (1-\delta)\left[1 + \omega \frac{(i_{t+1} - \delta k_{ss})}{k_{t+1}}\right] \omega (i_{t+1} - \delta k_{ss})^2\right]$$

As mentioned, the model is not stationary (only in knife-edge conditions as CO1991). Then some tricks to address this are:

- SGU2001: various methods including interest rate as decreasing function of foreign assets, endogenous discount factor, portfolio adjustment costs ($(a_{t+1} - a_t)^2$).

- Mendoza 1991: Endogenous discount factor, i.e., decreasing on consumption and leisure ($\beta = (1+c-\frac{\psi_0}{\psi}N^{\psi})^{-\chi}$).

- Ghironi 2006: OLG, newborns with no assets break Ricardian Equivalence and make assets matter in Euler Equations and steady state.

3.5.1 Felstein-Horioka Puzzle

Based on the SOE model above.

As seen, a way to address the TB procyclicality is to include investment. Another one is to include persistent growth in output: $\Delta y_{t+1} = \alpha + \rho \Delta y_t + \varepsilon_{t+1}$ with $\rho > 0$. If $\uparrow Y$ by 1, then with the inertia $\uparrow Y_{+1}$ but by less. Consumption increases by more (by one), therefore TB decreases, i.e., $\rho_{y,TB} < 0$ (countercyclical).

Coming back to the model with Investment we obtain the Feldstein-Horioka puzzle:

$$\begin{aligned} y_t &= A_t f(k_t) \\ k_{t+1} &= (1-\delta)k_t + i_t \qquad (\text{let } \delta = 0) \\ B_{t+1} - B_t &= y_t + rB_t - c_t - g_t - i_t \end{aligned} \tag{BC}$$

FOC:

$$1 = E_t \left\{ \underbrace{(A_{t+1} f'(k_t) + 1)}_{R_{t+1}} \underbrace{\beta \frac{u'(c_{t+1})}{u'(c_t)}}_{Q_{t+1}} \right\}$$

Now let $E(Q_{t+1}) = E\left(\beta \frac{u'(c_{t+1})}{u'(c_t)}\right) = \beta E \cancelto{1}{\left(\frac{u'(c_{t+1})}{u'(c_t)}\right)} = \frac{1}{1+r}$.
Also use that $E(XY) = E(X)E(Y) + Cov(X,Y)$ with $X = R_{t+1}$, $Y = Q_{t+1}$ and solve for $E(R_{t+1})$:

$$E(R_{t+1}) = (1+r)[1 - Cov(R_{t+1}, Q_{t+1})]$$

- Assume that $Cov(\cdot) = 0$:

Then $E_t(A_{t+1} f'(k_{t+1})) = r$ which is a constant, therefore: k_{t+1}, i_t are uncorrelated with $U'(\cdot)$, i.e., with $\{c_t\}$. Then s_t, i_t are uncorrelated.

- In general: $\uparrow A_{t+1} \rightarrow \uparrow y_{t+1} \rightarrow \uparrow \bar{y} \rightarrow \uparrow c_{t+1} \Rightarrow \downarrow u'(c_{t+1})$ then $Cov(\cdot) < 0$ (higher output shrinks marginal utility while at the same time investment moves).

Then we would have: $Corr(s_t, i_t) \neq 0$. Mostly in the part that would depend on A_{t+1}.

Now consider the process A_{t+1} follows is: $A_{t+1} - \bar{A} = \rho(A_t - \bar{A}) + \varepsilon_{t+1}$:

- $\rho = 0,\ \varepsilon_t > 0$: A_{t+1}, y_{t+1} not affected, then i_t doesn't change. $y_t \uparrow, \bar{y} \uparrow \Rightarrow s_t \uparrow \Rightarrow\ CA_t = s_t - i_t > 0$

- $\rho = 1,\ \varepsilon_t > 0$: $A_{t+1} \uparrow, y_{t+1} \uparrow \rightarrow\ i_t \uparrow$ also, due to new investment: $\Delta i_{t+1} > \Delta i_t$, then $\Delta \bar{y} > \Delta y_t \ \Rightarrow c_t\ \uparrow > y_t \uparrow$ (PIH, cons. smoothing) $\Rightarrow s_t \downarrow$, therefore: $Cov(i_t, s_t) < 0$

Then with investment: $Cov(y, CA) < 0$ but $Cov(S, I)$ depends although it would be small or close to zero.

According to Feldstein-Horioka (1980) under complete markets S,I should be independent or as seen at least the covariance should be low. However, empirically $Cov(S, I)$ is high.

3.6 A-D asset market structure (methodology)

Complete markets setup, 2 period model, no uncertainty in $t = 1$ (obs. Y_1) and N states in $t = 2$ each with probability $\pi(S)$ with $S \in N$ and $Y_2(S)$.

With the A-D security structure the budget constraint is:

$$\begin{aligned} Y_1 &= C_1 + \sum_s \frac{P(S)}{1+r} B_2(S) \\ C_2(S) &= Y_2(S) + B_2(S) \qquad \forall\ S \end{aligned}$$

where $P(S)$ is the price of bond and includes the probability of the state S.

$$\mathcal{L} = u(C_1) + \beta \sum_s \pi(S) u(C_2(S)) + \lambda \left[Y_1 - C_1 - \sum_s \frac{P(S)}{1+r} \left(C_2(S) - Y_2(S) \right) \right]$$

FOC:

$$u'(C_1) = \lambda \tag{C_1}$$

$$\beta \pi(S) u'(C_2(S)) = \lambda \frac{P(S)}{1+r} \tag{C_2}$$

from these FOC we get:

$$\frac{P(S)}{1+r} u'(C_1) = \beta \pi(S) u'(C_2(S)) \tag{1}$$

Assume CRRA and solve for $C_1 \neq C_2(S)$:

$$\frac{P(S)}{1+r} C_1^{-\rho} = \beta \pi(S) C_2(S)^{-\rho}$$

rearranging for home and foreign,

$$C_2(S) = \left(\frac{\pi(S)\beta(1+r)}{P(S)} \right)^{\frac{1}{\rho}} C_1 \tag{2}$$

$$C_2^*(S) = \left(\frac{\pi(S)\beta(1+r)}{P(S)} \right)^{\frac{1}{\rho}} C_1^* \tag{3}$$

The Market Clearing conditions for $t = 1, 2$ are:

$$C_1 + C_1^* = Y_1 + Y_1^* = Y_1^W$$
$$C_2(S) + C_2^*(S) = Y_2(S) + Y_2^*(S) + Y_2^W(S)$$

summing (2) and (3):

$$\underbrace{C_2(S) + C_2^*(S)}_{Y_2^W(S)} = \left[\frac{\pi(S)\beta(1+r)}{P(S)}\right]^{\frac{1}{\rho}} \underbrace{(C_1 + C_1^*)}_{Y_1^W}$$

solving for the price of the bond,[1]

$$\frac{P(S)}{1+r} = \pi(S)\beta\left[\frac{Y_2^W(S)}{Y_1^W}\right]^{-\rho} \qquad \forall S$$

consider CRRA preferences in (1):

$$\frac{P(S)}{1+r} = \pi(S)\beta\left[\frac{C_2(S)}{C_1}\right]^{-\rho}$$

then, considering the analogous condition for the foreing country it will hold that:

$$\frac{Y_2^W}{Y_1^W} = \frac{C_2(S)}{C_1} = \frac{C_2^*(S)}{C_1^*} \tag{4}$$

that is the risk sharing condition.

3.6.1 Consumption Correlation Puzzle

According to (4) consumption growth is equalized across countries and is equal to world output growth.

also $\frac{C_2(S)}{Y_2^W(S)} = \frac{C_1}{Y_1}$, i.e., countries consume a fixed fraction of world output regardless of state.

Then consumption growth rates should be more correlated (all equal to Y_2/Y_1) than output growth. Empirically this does not hold (**Consumption Correlation Puzzle**).

Interpretation of the bonds: if follows that Countries will buy/sell A-D s.t. $C_2(S) = \mu Y_2(S)$. Then if $Y_2(j) < \mu Y_2^W(j)$ the country will buy $B_2(j)$.

Here $\mu = \frac{Y_1 + \sum_{S=1}^{N} \frac{P(S)}{1+r} Y_2(S)}{Y_1^W + \sum_{S=1}^{N} \frac{P(S)}{1+r} Y_2^W(S)}$

[1] also, considering another state S' we can get an expression for relative prices that as expected depends on the likelihood odds,

$$\frac{P(S)}{P(S')} = \frac{\pi(S)}{\pi(S')}\left[\frac{Y_2^W(S)}{Y_2^W(S')}\right]^{-\rho}$$

3.7 Home Bias

With the consumption correlation puzzle as a result in a basic framework, it was explored to consider more realistic asset markets structures in the models. A natural way to move in that direction would be to consider equities and not only bonds.

With such modification a new puzzle emerges, the portfolio diversification puzzle. We depart from a similar model as before (OR 5.3):

2 periods, N countries, S states in $t = 2$. Assets: Riskless bond and Shares of each country's output (equities).

The budget constraint for $t = 1$ and $t = 2$:

$$t=1: \qquad Y_1^n + \overbrace{V_1^n}^{\text{Stock mkt value at t=1 of n's t=2 output}} = C_1^n + \overbrace{B_2^n}^{\text{Bond w/ return r}} + \sum_{m=1}^{N} \overbrace{\chi_m^n}^{\text{n country claims of m's output}} V_1^m$$

$$t=2: \qquad C_2^n(S) = (1+r)B_2^n + \sum_{m=1}^{N} \chi_m^n Y_2^m(S) \quad \forall S$$

with $-1 \leq \chi_m^n \leq 1$

Also notice that net purchases made by country n to foreigners is: $(1-\chi_n^n)V_1^n$

UMP:

$$\max_{c_i,\chi_i,B_2} U(\overbrace{Y_1^n + V_1^n - B_2^n - \sum_{m=1}^{N} \chi_m^n V_1^m}^{C_1}) + \beta \sum_{s=1}^{S} \pi(S) U(\overbrace{(1+r)B_2^n + \sum_{m=1}^{N} \chi_m^n Y_2^m(S)}^{C_2^n(S)})$$

FOC:

$$u'(C_1) = (1+r)\beta \sum_s \pi(S) u'(C_2^n(S)) \qquad [B_2]$$

$$V_1^m u'(C_1) = \beta \sum_s \pi(S) u'(C_2^n(S)) Y_2^m(S) \qquad [\chi_m^n]$$

then,

$$U'(C_1) = \beta \frac{E[U'(C_2(S) Y_2^m(S))]}{V_1^m} \tag{5}$$

Define: $\mu^n = \frac{Y_1^n + V_1^n}{\sum_{m=1}^{N} Y_1^m + V_1^m}$ and let $\frac{Y_2^m}{V_1^m}$ be the gross return on equity m. Then (5) becomes:

$$u'(C_1) = \beta E\left[\left(\frac{Y_2^m}{V_1^m} u'(C_2^n)\right)\right], \qquad \forall m$$

recall from C-CAPM:

$$u'(C_1) = \beta E\left[(1+\tilde{R}_m)u'(C_2)\right], \qquad \forall m$$

we guess the solution for consumption is of the form:

$$C_1^n = \mu^n \sum_m Y_1^m = \mu^n Y_1^W$$
$$C_2^n = \mu^n \sum_m Y_2^m = \mu^n Y_2^W(S), \qquad \forall S$$

and then $B_2 = 0$ since consumption is already the one in which no bond is purchased/sold.

Then the portfolio share will be $\chi_m^n = \mu^n \quad \forall m$, i.e., n holds a fraction μ^n of equities from every other country and from the world output portfolio.

Summary

In a standard model of portfolio choice with equity and bond trade with identical investors and i.i.d. random returns: it is optimal for agents to hold shares equal to the country's initial share of world wealth in a fully diversified global portfolio (e.g. 10% of every country's output if n's share is a tenth).

Despite the former results, French and Poterba (1991) document the Home Bias. Tesar and Werner (1995) review developed countries data and find a strong home bias in the 1970-1990 period. They also provide evidence that the diversified world portfolio dominates the domestic portfolios (higher mean, lower variance).

Baxter and Jermann (1997): The puzzle is worse than thought. By including production and factors ($Y = AK^\alpha L^{1-\alpha}$) it follows that labor and capital returns are perfectly correlated (cash flows are $wL = \alpha Y$ and $rK = (1-\alpha)Y$).

Domestic stock market return (a proxy of domestic capital) hedges against human capital risk (non traded). People should take short positions in domestic stock to insure negative output shocks. Actually to hedge output risk it would make even more sense to hold foreign equities only.

Botazzi, Pesenti, and van Wincoop (1996) find the oposite result $Corr$(labor returns, stock mkt returns) < 0 and therefore domestic quities is a good hedge, i.e., HB would be partially explained.

Still there is a considerable lack of diversification with explanations such as: (1) Asset transaction costs, and (2) Non Traded goods.

Regarding (1) Tesar and Werner explain that it is false, the turnover of foreign to domestic transactions is high. (2) implies that there should be a lower implied consumption correlation w.r.t. a complete market, only traded goods setup.

Nonetheless, diversification implication when considering NT goods depend on traded and non traded goods (C_T, C_{NT}) interact in the utility function. If T,NT are separable in the utility function then C_T, C_T^* should be ferfectly correlated.

For example:

$$U = \frac{C_{T,1}^{1-\rho}}{1-\rho} + G(C_{NT,1}) + \sum_{s=1}^{S} \beta\pi(S)\left[\frac{C_{T,2}^{1-\rho}}{1-\rho} + G(C_{NT,2})\right]$$

then:

$$
\begin{aligned}
C^n_{T,1} &= \mu^n Y^W_{T,1} \\
C^n_{T,2}(S) &= \mu^n Y^W_{T,2}(S)
\end{aligned}
$$

but,

$$
\begin{aligned}
C^n_{NT,1} &= Y^n_{NT,1} \\
C^n_{NT,2}(S) &= Y^n_{NT,2}(S)
\end{aligned}
$$

with,

$$
\begin{aligned}
\chi^n_{NT,m} &= \begin{cases} 1 & \text{if } m = n \\ 0 & \text{if } m \neq n \end{cases} \\
\chi_{T,m^n} &= \mu^n \\
\mu^n &= \frac{Y^n_{T,1} + V^n_{T,1}}{\sum_{m=1}^N (Y^m_{T,1} + V_{T,m})}
\end{aligned}
$$

therefore all non traded risk is held only by residents of country n.

Implication: by considering NT goods there will be some bias towards home assets that partly explains the HB. This is desirable but some problems remain:

- Size: The HB explained is not large as the observed one.

- Non separable utility cases: with non separable utility there are higher gains of diversification and then even fewer HB explained.

On the other hand, as mentioned before, according to Lucas 82, the size of the gain of diversification is trivial (about a fifth of a one percent of proportional increase in consumption), so there are not many incentives to diversify.

Possible explanation (OR2000, 6 puzzles paper): Transportation costs and other barriers to trade. In a static model Iceberg costs explain HB better than non-traded goods.

Model:

- 1 period, 2 countries, 2 goods (H,F). Symmetric preferences over the two goods. Endowments: $S = (Y_H, Y_F)$

- Cost of shipping: τ

$$
EU = E\left\{ \frac{1}{1-\rho} \left[\left(C_H^{\frac{\theta-1}{\theta}} + C_F^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \right]^{1-\rho} \right\}
$$

Assume complete markets A-D: therefore the relative MU of the two countries over each good = relative prices (for all S).

$$
\frac{\partial U}{\partial C_H} \frac{1}{P_H} = \frac{\partial U^*}{\partial C^*_H} \frac{1}{P^*_H}
$$

also no-arbitrage conditions imply prices are: $P_H = (1-\tau)P_H^*$ and $P_F = P_F^*/(1-\tau)$.

then,

$$
\begin{aligned}
C_H^{-1/\theta} &= (1-\tau)C_H^{*-1/\theta} \\
C_F^{-1/\theta} &= (1-\tau)C_F^{*-1/\theta}
\end{aligned}
$$

Market Clearing conditions are:

$$
\begin{aligned}
C_H^* &= (1-\tau)(Y_H - C_H) \\
C_F &= (1-\tau)(Y_F - C_F^*)
\end{aligned}
$$

four equations and four unknowns: C_H, C_F, C_H^*, C_F^*. Now, we assume that $\theta = 1/\rho$ and get equity shares for the home country given by:

$$
\begin{aligned}
C_H = \chi_H &= \frac{1}{1+(1-\tau)^{\theta-1}} Y_H \\
\frac{C_F}{1-\tau} = \chi_F &= \frac{(1-\tau)^{\theta-1}}{1+(1-\tau)^{\theta-1}} Y_F
\end{aligned}
$$

Without iceberg costs: $\chi_H = \chi_F = 1/2$.

But with $\theta = 6$, $\tau - 0.25$: $\chi_H = 0.81$ i.e., there is a considerable degree of HB.

However, the assumption $\theta = 1/\rho$ is important to get this result, and in addition, the conclusions are weaker in dynamic models where there is a potential for reinvesting the dividends of the assets.

3.7.1 Endogenous Adjustment - CO1991

In their seminal paper CO1991 argue that ToT variation makes domestic investment more appealing since it automatically hedges against domestic output risk.

Framework: H produces a fraction n of Y^W and F produces the rest $(1-n)$. Consumption is given by a CD: $C = C_H^n C_F^{1-n}$.

The FOC imply:

$$
C_H = n\frac{C}{P_H}, \qquad C_F = (1-n)\frac{C}{P_F}
$$

Subs. in Market Clearing conditions:

$$
\begin{aligned}
nC_H + (1-n)C_H^* &= nY \\
n(nC + (1-n)C^*) &= nP_H Y \\
(1-n)(nC + (1-n)C^*) &= (1-n)P_F Y^*
\end{aligned}
$$

then,

$$P_H Y = P_F Y^*$$

Then if relative output change, relative prices will change to compensate and ensure that $Y/Y^* = P_F/P_H$, that is, insure risk and therefore it would not be much need to insure by buying foreign equities.

On the other hand, as mentioned before, OR1991 calculates that the gains of diversification are trivial, implying even less of a reason for departing from a HB scenario.

Critique: vW1999 explains that these result depend heavily on parametrization. In particular on assuming that the $ES_{H,F}$ is 1.

3.7.2 Other approaches

HP2008, CKM2009 include more assets: Bonds to hedge ER shocks and then Equities would be used mostly to hedge non tradable income risk. In that sense it is argued that interaction among assets can break the expected results.

BC1997 explore information frictions. The former authors build a model in which signal of foreign stock future performance is weaker (less precise), making the foreign stock be perceived as riskier. Such scenario would lead to HB. In a similar fashion vNV2008 consider local informational advantages, associated with a lower perceived riskiness that interacts with an endogenous information adquisition that leads to learning spillovers on domestic assets and reinforce the domestic bias.

4 ZLB, unconventional monetary policy

Definition: Liguidity trap is a situation when the real interest rate is negative $r < 0$, meaning the nominal rate is equal or close to zero.

Occurs due to very high patience (negative discount rate) or low expected consumption growth.

Stilized Facts during a liquidity trap: bank sector collapse and a deleveraging of the the private sector that altoguether leads to a decrease in the expected consumption growth.

Episodes: Great depression, Japan in the 90's, years posterior to the 2008-10 recession.

Effect on the output depends on price rigidities:

- Flexible Prices: output remains at FB level and prices adjust to deliver expected inflation to match the negative real rate.
- Sticky Prices: output lowers with respect to FB level.

Policy response (how to leave the trap):

- Commit to future expansionary Monetary Policy
- Temporary expansionary Fiscal Policy
- Unconventional Monetary Policy: Quantitative Easing and time varying taxes.

Main References: Krugman (1998), Eggertsson and Krugman (2012).

4.1 Krugman (1998)

- Endowment economy
- Dynamic model
- CRRA preferences and Cash in Advance constraint
- Households receive endowments and sell it for money to buy consumption good
- Bonds: pay i_{t+1}, if the rate is positive the agents won't hold money beyond what needed for consumption.
- Fixed long run expectations of variables (money supply, output, etc.)
- CIA constraint binds $CP = M$

Euler Equation and market clearing ($Y = C$):

$$1 + r_{t+1} = \frac{1}{\beta}\left(\frac{Y_{t+1}}{Y_t}\right)^{\rho}$$

Fisher equation (link between nominal and real rate):

$$1 + i_{t+1} = (1 + r_{t+1})\frac{P_{t+1}}{P_t}$$

assume $Y_{t+1} = \bar{Y}$ (fixed output in LR, LR here is one period forward), also fixed money M and then via CIA constraint fixed prices: $\bar{P}$. Then rewrite the fisher equation and from it the Euler Equation:

$$1 + i_{t+1} = (1 + r_{t+1})\frac{\bar{P}}{P_t} = \frac{1}{\beta}\left(\frac{\bar{Y}}{Y_t}\right)^{\rho} \tag{CC}$$

CIA constraint with M exogenous:

$$Y_t P_t \leq M_t \tag{MM}$$

the equilibrium will be the intersection of the curves:

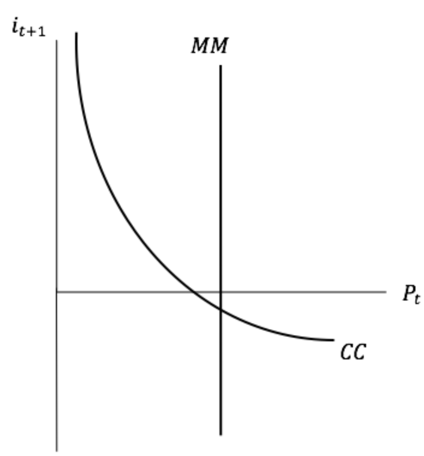


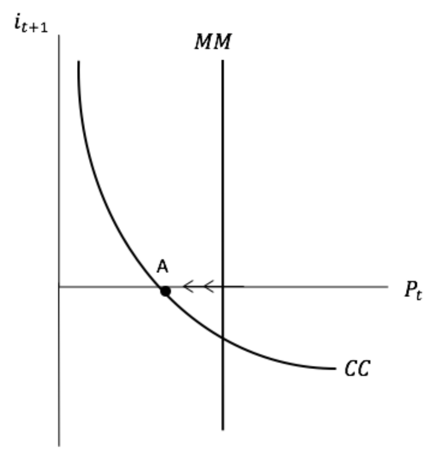
Figure 5: Source: Krugman 1998

Cases:

1. Flexible prices:

There is an excess of money at $i_{t+1}i = 0$. Money and bonds are perfect substitutes and then P_t accomodates by decreasing to restore equilibrium towards point A (MM moves to the left). With fixed prices in $t+1$, i.e., $\bar{P}$ we will have an expected increase in inflation between t and $t+1$ ($\pi^e_{t,t+1} > 0$) s.t.,

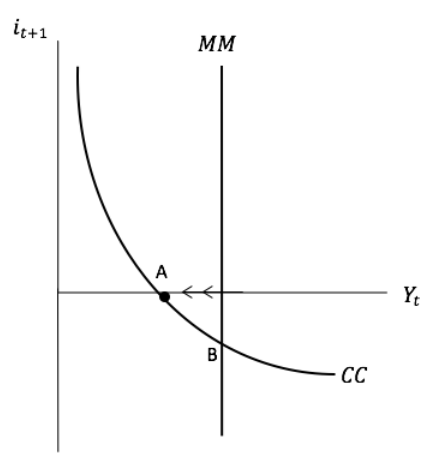
$$i_{t+1} = \pi^e_{t,t+1} + r_{t+1} = 0$$



2. Sticky prices $P_t = \bar{P}$

There cannot be an adjustment via prices since $P_t = P_{t+1} = \bar{P}$ and then the only way to equilibrate the market is by $\downarrow C_t(Y_t)$ (and then the output because of market clearing)

$i = 0$ is too high relative to the intersection equilibrium (at negative levels), then additional savings are induced s.t. $\downarrow C_t$, as an outcome there will be an inefficiently low level of Consumption/ouput relative to the potential ouput.



Conclusions can be extended to a DSGE framework:

$$x_t = -\frac{1}{\sigma}[i_t - E_t[\pi_{t+1}] - r_t^n] + E_t[x_{t+1}] \qquad \text{(Dynamic IS)}$$
$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa x_t \qquad \text{(NKPC)}$$
$$i_t = \max\{r_t^n + \phi_\pi \pi_t + \phi_x x_t, 0\} \qquad \text{(Taylor Rule)}$$

Substitute the inflation (NKPC) in the Taylor rule and obtain,

$$i_t = \max\{r_t^n + (\phi_\pi \kappa + \phi_x)x_t + \phi_\pi \beta E_t(\pi_{t+1}), 0\}$$

assume that the economy is out of the liquidity trap in $t+1$ (LR), with $E_t \pi_{t+1} = E_t x_{t+1} = 0$, then,

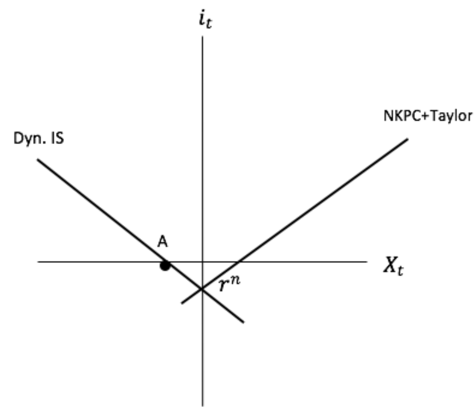


Figure 6: Source: Krugman 1998

That is, during liquidity trap, with sticky prices, output falls to A (below efficient level).

Liquidity trap according to the Euler Equation:

$$1 + r_{t+1} = \frac{1}{\beta}\left(\frac{C_{t+1}}{C_t}\right)^\rho$$

in summary, the key mechanism in Krugman 1998 works through the EE.

In the EE, $r < 0$ can happen due to: large β or low expected C growth.

4.2 Eggertsson and Krugman (2012)

Liquidity trap can also be explained by a shock in the borrowing constraints rather than more vague notions as patience or contraction in consumption growth.

Model:

Unit measure of borrowers, unit measure of savers.

Preferences:

$$E_t \sum_{z=t}^{\infty} \beta^{z-t}(i) \ln C_z(i) \quad \text{for } i = s, b$$

with $\beta(s) > \beta(b)$ (savers more patient than borrowers)

Endowment: $\frac{1}{2}Y$ for both borrowers and savers.

Assets Market: free trade in real riskless bond with return r_{t+1} subject to a debt limit,

$$(1 + r_{t+1})D_{t+1}(i) \leq D^h$$

The solution would be obtained as an equilibrium conjecture:

- Constant consumption in steady state

- Constant consumption implies constant interest rate (EEq): $r_t = r_{t+1} = r$

- Borrowers hit the debt limit (binding) in every period:

That means that, once the SS is reached:

$$D_t(b)(1 + r_t) = D^h$$
$$D_{t+1}(b)(1 + r_{t+1}) = D^h$$

The BC for the borrowers is given by

$$C_t(b) + (1 + r_t)D_t(b) = \frac{1}{2}Y + D_{t+1}(b)$$

Interpretation: today's endowment will be used to consume, then to pay old debt plus interest and to buy debt for tomorrow ($D_{t+1}(b)$).

From the hit limit for borrowers above, substitute: $D_t(b) = \frac{D^h}{1+r}$, $D_{t+1}(b) = \frac{D^h}{1+r}$ and get:

$$C_t(b) = \frac{1}{2}Y - \frac{r}{1 + r}D^h$$

and for by goods MC we have for the savers,

$$C_t(s) = \frac{1}{2}Y + \frac{r}{1 + r}D^h$$

which implies that the asset market clearing condition is: $D_t(s) = -D^h$ for savers.

The savers are not constrained in any way so the EE holds for them:

$$\frac{1}{C_t(s)} = (1 + r_{t+1})\beta(s)\frac{1}{C_{t+1}(s)}$$

implying that in SS: $1 + r = \frac{1}{\beta(s)}$

For the borrowers we will have,

$$\frac{1}{C_t(b)} \geq (1+r_{t+1})\beta(b)\frac{1}{C_{t+1}(b)}$$

with equality when not constrained, i.e., when $D_{t+1}(b) < D^h$

in SS: $1+r \leq \frac{1}{\beta(b)}$

Shock to the Borrowing constraint: $D^h \to D^l$ with $D^l < D^h$.

A new steady state is reached in $t+1$.

it will have the form:

$$\begin{aligned} C_\tau(b) &= \frac{1}{2}Y - \frac{r}{1+r}D^l \\ C_\tau(s) &= \frac{1}{2}Y + \frac{r}{1+r}D^l \\ 1+r_{\tau+1} = 1+r_\tau &= \frac{1}{\beta(s)} \qquad \forall \tau \geq t+1 \end{aligned}$$

Now, debt for borrowers is reduced in $t+1$ from $\frac{D^h}{1+r_t}$ to $\frac{D^l}{1+r_{t+1}}$.

The new BC in t is:

$$C_t(b) \leq \frac{1}{2}Y + \frac{1}{r_{t+1}}D^l - D^h\frac{(1+r_t)}{1+r_t}$$

this is the same BC except that cannot be simplified since given the shock $r_t \neq r_{t+1}$, i.e., there will be a new equilibrium interest rate in the new SS.

the borrowing will be reduced from $\frac{D^h}{1+r_t}$ to $\frac{D^l}{1+r_{t+1}}$

More importantly the new interest rate is pinned down by the savers EE: $1+r_{t+1} = \frac{1}{\beta(s)}\left(\frac{C_{t+1}(s)}{C_t(s)}\right)$.

Whereas for the borrowers,

$$1+r_{t+1} < \frac{1}{\beta(b)}\frac{C_{t+1}(b)}{C_t(b)}$$

which means that the borrowers will find optimal to delever, i.e. cut down their consumption.

Market Clearing requires that real interest rate falls to induce the savers to increase consumption to offset the change induced by the borrowers.

If D^l is too low with respect to the old ceiling D^h then the market clearing r becomes negative ($r < 0$).

Modifications: the basic intuition behind this model still works with (i) precautionary savings or (ii) endogenous output, such that, constrained agents a can work more to offset the debt based cut in consumption.

4.3 How to escape the Liquidity Trap

As mentioned it is possible to leave the liquidity trap with expansionary monetary policy that induces an adjustment in the expected inflation, a temporal expansionary fiscal policy or with unconventional monetary policy:

1. Monetary Policy committed to create inflation in the future:

   The central bank raises the inflation expectation to obtain a non-negative nominal interest rate that is consistent with the negative real rate. In terms of the models it corresponds to an upward shift in the CC curve.

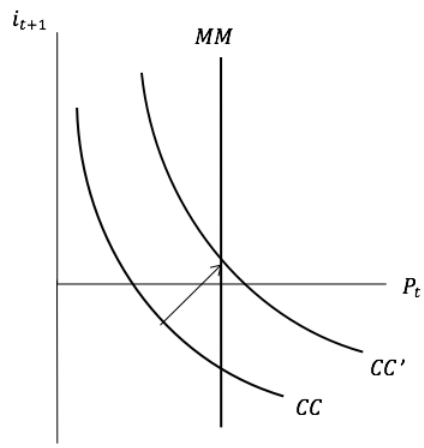
   

   Figure 7: Source: Krugman 1998

   In terms of a DSGE model the central bank would have to commit to a future expansionary monetary policy with nominal interest rate $\{i_s\}_{s=t}^{\infty}$ that gives low real interest rate in the future. The problem with this is that ex post, i.e., once the economy exits the liquidity trap, is no longer optimal to set $i_t - E_t[\pi_{t+1}] - r^n < 0$ since there would be an inneficiently high inflation and output.

2. Temporary Fiscal Stimulus:

   Add government spending into the model, then $Y_t = C_t + G_t$ and the CIA constraint is $P_t(C_t + G_t) \leq M_t^{private} + M_t^G = M_t$.

   The new CC curve is:

   $$1 + i_{t+1} = \frac{\bar{P}}{\beta P_t}\left(\frac{\bar{Y} - \bar{G}}{Y_t - G_t}\right)^{\rho}$$

   Then if we change G temporarily, i.e., only G_t and not $\bar{G}$ then the CC curve shifts upwards and we would have an increase in the nominal rate towards non negative levels.

   Here a stimulus in the government spending crowds out the private consumption so as to raise the real interest rate.

   In the NK model the mechanism is slightly different, there, a higher G can also stimulate production and generate inflation.

3. Unconventional Monetary Policy:

   Curdia and Woodford, 2009: Quantitative Easing policies would yield portfolio balance effects, i.e., changes in the composition of assets that affect the relative spreads and the associated borrowing costs of the agents.

CFNT 2013: Unconventional tax policies in the ZLB can be applied to use time varying taxes and engineer an increase in the expected inflation.

5 Exchange Rate Determination:

5.1 Rossi 2011: Exchange Rate Predictability (literature review)

Literature Review: post-MR1983 ER forecast methodologies. Is the ER predictable?: It could and depends on predictor an methodology.

Prediction is apparent when predictor is a Taylor Rule or NFA fundamentals, the model is linear and the model is parsimonious (small number or parameters estimated). According to Molodstova and Papell (2009) a model that works better that a RW is given by a symmetric Taylor rule without lagged interest rates: $E_t s_{t+1} - s_t = \tilde{\mu} + \tilde{\lambda}(\pi_t - \pi_t^*) + \tilde{\gamma}(y_t^{gap} - y_t^{gap*})$. On the other hand the latter, based on NFA tends to work even better than the Taylor Rule based models.

In general, the toughest benchmark is the RW without drift.

There is instability over samples for all models and then there is no systematic pattern about which horizons or sample periods work best.

PPP and monetary models have no success in the SR and MR (1-3yrs), also using typical fundamentals differential have shown to have a predictability ability that changes through time (there is substantial instability in the models' performance).

Finally, data transformations and data revisions can have substantial effects on the predictability of the models.

Summary:

TABLE 1
LITERATURE REVIEW: PREDICTORS AND ECONOMIC MODELS

Predictors (f_t)	Economic fundamentals	Mnemonics
$i_t - i_t^*$	Interest rate differentials	i
$F_t - s_t$	Forward discount	F
$p_t - p_t^*$	(log) price differentials	p
$\pi_t - \pi_t^*$	Inflation differentials	π
$y_t - y_t^*$	(log) Output differentials	y
$m_t - m_t^*$	(log) Money differentials	m
z_t	Productivity differentials	z
$b_t - b_t^*$	Asset differentials	b
$y_t^{gap} - y_t^{gap*}$	Output gap differentials	y^{gap}
nxa_t	Net foreign assets	nxa
CP_t	Commodity prices	CP

Model	f_t	Mnemonics
UIRP (CIRP)	$i_t - i_t^*, (F_t - s_t)$	i, F
PPP	$p_t - p_t^*$ or $\pi_t - \pi_t^*$	p, π
Monetary model with flexible prices (I)	$[(i_t - i_t^*), (y_t - y_t^*), (m_t - m_t^*)]'$	i, y, m
Monetary model with flexible prices (II) (or Frenkel–Bilson model)	$[(y_t - y_t^*), (m_t - m_t^*)]'$	y, m
Monetary model with sticky prices (I)	$[(i_t - i_t^*), (y_t - y_t^*), (m_t - m_t^*), (p_t - p_t^*)]'$	i, y, m, p
Monetary model with sticky prices (II) (or Dornbusch–Frankel model)	$[(i_t - i_t^*), (y_t - y_t^*), (m_t - m_t^*), (\pi_t - \pi_t^*)]'$	i, y, m, π
Model with productivity differentials (or Balassa–Samuelson (1964) model)	$[(i_t - i_t^*), (y_t - y_t^*), (m_t - m_t^*), z_t]'$	i, y, m, z
Portfolio balance model (or Hooper and Morton (1982) model)	$[(i_t - i_t^*), (b_t - b_t^*)]'$	i, b
Taylor rule model	$[(\pi_t - \pi_t^*), [(y_t^{gap} - y_t^{gap*})]'$	π, y^{gap}
Net foreign asset model	nxa_t	nxa
Commodity prices	CP_t	CP

Notes: The table reports the name of the model, the fundamental predictors used in the model ("f_t"), and the mnemonics used to refer to these fundamentals in Table 2.

Figure 8: Source: Rossi 2011.

5.2 Overview: ER Puzzles

5.2.1 ER disconnect with the Fundamentals

The general approach is to look for relationship with fundamentals as well as causality direction.

In most cases the departing point is the UIP be it with respect to the nominal interest rate, or the real one that includes the price level.

Engel (2015) on Exchange Rate and Interest Parity: Depart from a UIP deviation:

$$\lambda_t \equiv i_t^* + E_t S_{t+1} - S_t - i_t$$

Apply logs and use repeated substitution:

$$s_t \equiv -\sum_{j=0}^{\infty} E_t(i_{t+j} - i_{t+j}^*) - \sum_{j=0}^{\infty} E_t \lambda_{t+j} + \lim_{j\to\infty} E_t s_{t+j+1}$$

Alternatively, in terms of the real rate:

$$s_t \approx -\sum_{j=0}^{\infty} E_t(r_{t+j} - r_{t+j^*}) - \sum_{j=0}^{\infty} E_t \lambda_{t+j} + p_t - p_t^* + \lim_{j\to\infty} E_t q_{t+j+1}$$

Then what we have is that the exchange rate is related to the future values of the fundamentals, rather than to their past values which explains the apparent disconnect between exchange rates and fundamentals found empirically as in Messe and Rogoff (1983) (MR1993).

Another way to see it is that the causality goes backwards, i.e., the exchange rate can be a predictor of future values of fundamentals. Granger causality tests, performed by EW2005.

5.2.2 Risk (deviations of UIP) and Forward Premium Puzzle

In the former equations there is used a deviation of the UIP. This can be associated with risk in the sense that it represents a type of risk premium or compensation when comparing i_t versus $i_t^* + E_t(S_{t+1} - S_t)$

When considered the Covered Interest Parity (CIP) there is no risk premium since this condition is expected to hold. However this condition ($i_t = i_t^* + (F_t - S_t)$) stopped holding. Possibly due to very low interest rates that made unprofitable to use arbitrage conditions, given transaction costs, or due to liquidity issues in the forward instruments F_t as those imposed by capital controls.

From the UIP we have: $E_t S_{t+1} - S_t = (i_t - i_t^*) - \lambda_t$.

We can test if the UIP holds, for that we simplify by assuming Rational Expectations (perfect foresight):

$$S_{t+1} - S_t = (i_t - i_t^*) - \lambda_t$$

we test this equation empirically, i.e., we test $\alpha = 0, \beta = 1$ in a linear model (Fama 1980).

However, approximating λ_t remains a problem. A further simplyfication would pick countries with a small (as much as possible) λ_t, for example USA and Canada.

The result is that $\hat{\beta} \approx 0$ in the best cases and in others $\hat{\beta} < 0$, this is known as the **Forward Premium Puzzle (FPP)** or UIP puzzle. The result was checked with different horizons and the shorter the worse the coefficients.

Some approaches to deal with the puzzle is to not consider RE ($E_t S_{t+1} \neq S_{t+1} + \varepsilon_t$) or to try to approximate λ_t (ignoring the risk variable can be the trouble itself if it is correlated to the interest rate differential and therefore causing an ommited variable bias in the estimated coefficients) .

5.3 Monetary approach (FNPV)

The idea is to identify a set of economic fundamentals that determine nominal exchange rate in the long run. A critical assumption is that of flexible prices, then the conditions for equilibrium can be focused only on the money market.

When using flexible prices, we are considering PPP as one of the building blocks. In this case, we consider that LOP holds for all goods and that if all goods are traded and preferences are symmetric, then $q_t = s_t + p_t^* - p_t$ is constant over time. This is not the case though, mainly due to non-traded goods. Then it is prefered to consider PPI as the price index instead of the CPI since it reflects tradables better. The CPI will be valid only for the LR.

Departing from a money market equilibrium condition $\frac{M_s}{P} = L(i, y)$ its possible to consider $i = f(M_s, P, Y, \dots)$. Additionally we can consider a monetary policy rule: $i = \alpha + \rho_\pi(\pi_t - \bar{\pi}) + \rho_y(y_t - y^n)$.

More specifically, following Mark (2001) we consider the money supply and demand as:

$$\begin{aligned} m_t &= \theta r_t + (1-\theta) d_t \qquad \text{w/ } \theta = E(R_t)/E(B_t) \\ m_t^d - p_t &= \phi y_t - \lambda i_t + \epsilon_t \qquad 0 < \phi < 1, 0 < \lambda, \epsilon \sim i.i.d.(0, \sigma_\epsilon^2) \end{aligned}$$

Given UIP, PPP holds and $\bar{s}$ is fixed (price is determined only by foreign price level) and then $p_t = \bar{s} + p_t^*$ and $i_t = i_t^*$.

In equilibrium $m_t = m_t^d$ and after substituting,

$$\theta r_t = \bar{s} + p_t^* + \phi y_t - \lambda i_t - (1-\theta) d_t + \epsilon_t$$

where r_t denotes the log of the foreign exchange reserves, d_t the domestic credit, $\bar{s}$ the exchange rate (assumed credibly fixed), y_t the output and p_t the log price level.

Flexible Exchange Rate:

Equilibrium in the domestic foreign money markets is given by,

$$\begin{aligned} m_t - p_t &= \phi y_t - \lambda i_t \\ m_t^* - p_t^* &= \phi y_t^* - \lambda i_t^* \end{aligned}$$

with $0 < \theta < 1, \lambda > 0$. Also, capital market equilibrium is given by the UIP,

$$i_t - i_t^* = E_t s_{t+1} - s_t \tag{1}$$

the relation between prices and exchange rate is given by the PPP:

$$s_t = p_t - p_t^* \tag{2}$$

let the economic fundamentals be:

$$f_t = (m_t - m_t^*) - \theta(y_t - y_t^*)$$

substitution of the the money markets, UIP and fundamental equations in (2) yields,

$$s_t = f_t + \lambda(E_t s_{t+1} - s_t) \tag{3}$$

solving for s_t:

$$s_t = \gamma f_t + \psi E_t s_{t+1} \tag{4}$$

in $t+1$: $s_{t+1} = \gamma f_{t+1} + \psi E_{t+1} s_{t+2}$, substitute in (4) and repeat for $t+2, t+3, \ldots$.

$$s_t = \gamma \sum_{j=0}^{k} \psi^j E_t f_{t+j} + \psi^{k+1} E_t s_{t+k+1} \tag{5}$$

with $\psi < 1$ we have $\lim_{k\to\infty} \psi^k E_t s_{t+k+1} = 0$ (TVC) and therefore:

$$s_t = \gamma \sum_{j=0}^{\infty} \psi^j E_t f_{t+j} \tag{6}$$

which means that the exchange rate is proportional to the discounted present value of the fundamentals. Similarities to asset pricing: In finance s would be the price of the asset and f would be the dividend.

Rational bubbles: Refers to bubbles that although explosive can be part of a valid solution of the model. In that case such solution would deviate from the fundamental based one:

Suppose TVC won't hold and therefore a bubble $b_t = (1/\psi)b_{t-1} + \eta_t$ with $\eta_t \sim i.i.d.(0, \sigma_\eta^2)$ and $1/\psi > 1$.

The solution for the exchange rate will be the sum of the fundamental driven part plus the bubble:

$$\hat{s}_t = s_t + b_t$$

we substitute in the TVC:

$$\psi^{t+k} E_t \hat{s}_{t+k} = \cancelto{0}{\psi^{t+k} E_t s_{t+k}} + \psi^{t+k} \cancelto{\frac{1}{\psi^{t+k}} b_t}{E_t b_{t+k}} = b_t$$

here $\hat{s}_t$ is a valid solution to the model but b_t may lead the exchange rate to deviate from the fundamental based solution and dominate its behavior. In any case, the bubble arises in a model with rational expectations and then it is reffered as a rational bubble. This bubbles may exist but usually pop and therefore it is possible to focus mostly in the no-bubbles solution.

Excess volatility of the Exchange Rate

The volatility of the fundamentals deviation from the exchange rate $f_t - s_t$ is way lower than that of the returns of the exchange rates itself Δs_t, i.e., the ER displays *excess volatility* with respect to other economic variables.

The monetary model is consistent with the excess volatility in the ER if the growth rate of fundamentals is persistent:

$$\Delta f_t = \rho \Delta f_{t-1} + \epsilon_t, \qquad \epsilon_t \sim i.i.d.N(0, \sigma_\epsilon^2)$$

The implied k-step prediction formula is given by $E_t(\Delta f_{t+k}) = \rho^k \Delta f_t$. In levels, summing on the LHS until obtaining f_t,taking expectations and rearranging on the RHS we get:

$$\begin{aligned}
E_t f_{t+k} &= f_t + \cancelto{\rho^{k-1}\Delta f_t}{E_t(\Delta f_{t+k-1})} + \cancelto{\rho^{k-2}\Delta f_t}{E_t(\Delta f_{t+k-2})} + \cdots + \rho^k \Delta f_t \\
E_t f_{t+k} &= f_t + \sum_{i=1}^{k} \rho^i \Delta f_t \\
E_t f_{t+k} &= f_t + \frac{1-\rho^k}{1-\rho} \rho \Delta f_t
\end{aligned}$$

Substitute in (6) and use the fact that $\gamma = 1 - \psi$:

$$\begin{aligned}
s_t &= \gamma \sum_{j=0}^{\infty} \psi^j f_t + \gamma \sum_{j=0}^{\infty} \frac{\psi^j}{1-\rho} \rho \Delta f_t - \gamma \sum_{j=0}^{\infty} \frac{(\rho\psi)^j}{1-\rho} \rho \Delta f_t \\
s_t &= f_t + \frac{\rho\psi}{1-\rho\psi} \Delta f_t
\end{aligned}$$

finally, after algebra we get:

$$Var(s_t) = \frac{(1-\rho\psi)^2 + 2\rho\psi(1-\rho)}{(1-\rho\psi)^2} Var(\Delta f_t) > Var(\Delta f_t)$$

Empirical tests: the net present value model has been tested empirically, mainly in terms of forecasting fit where there is a large body of research represented mainly by MR1983.

A second strategy is to test the cointegration relationship implied by the model between s_t and f_t. This is developed by MacDonald and Taylor (1994) who find poor adjustment when testing the coefficient restrictions implied by the model.

The model implies estimates that are too smooth with respect to the reality. One of the possible shortcomings of the model is that it assumes that PPP holds even in the short run.

5.4 Empirical forecasting tests (MR1983 and EW2005)

With the goal to predict the nominal ER and compare structural models versus a RW in terms of prediction the authors perform forecasts evualuations based on the following general specification:

$$s = \alpha_0 + \alpha_1(m - m^*) - \alpha_2(y - y^*) + \alpha_3(i - i^*) + \alpha_4(\pi^e - \pi^{e*}) + \alpha_5 \sum (TB - TB^*) + u$$

where s : log of nominal exchange rate, m : log of money supply, y : log of GDP, π^e : expected inflation and TB : Trade balance.

The forecast horizons span from 1 to 12 month horizon and when future values of variables are involved, the actual realized (future) values are used, which implies that the exercise is somewhat biased in favor of the model structure.

Still the results is that no structural model outperforms a random walk. According to Mark (2001) the reason for this is that it was impossible to find a time invariant relationship between the ER and the fundamentals.

EW2005

Out of sample tests considered are too harsh. Beating a RW in forecasting is possible a too strong criterion for accepting a model. After all ER's mainly incorporate news of future fundamentals and give few to none weight on current fundamentals, implying a near random walk behavior (the best forecast with current info (that plays no role, provides no info) is the current exchange rate value).

Actually when the future matters more, i.e., when discount factors are close to 1 as in most of the models, the exchange rate would behave more markedly as a RW.

5.5 Second framework: benchmark model w/ home, foreign bonds and forwards (from OR1996)

Assets:

M_t: home money

B_t: home nominal interest bearing assets at rate i

B_t^*: home nominal interest bearing assets at rate i^*

F_t: forward contract, purchase 1 unit of foreign currency for f_t units of home currency

e_t: spot exchange rate (home currency per foreign)

HH UMP:

$$\max_{B_t, B_t^*, F_t, C_t} E_t \sum_{t=0}^{\infty} \beta^t U\left(C_t, \frac{M_t}{P_t}\right)$$
$$s.t$$
$$P_t Y_t + (1+i_{t-1})B_{t-1} + e_t(1+i_{t-1}^*)B_{t-1}^* + (e_t - f_t)F_{t-1} + M_{t-1} = P_t C_t + B_t + e_t B_t^* + M_t$$

FOCs:

$$[B_t]: \qquad \beta E_t[\lambda_{t+1}](1+i_t) = \lambda_t \tag{1}$$
$$[B_t^*]: \qquad \beta E_t[e_{t+1}\lambda_{t+1}](1+i_t^*) = e_t\lambda_t \tag{2}$$
$$[F_t]: \qquad E_t[(e_{t+1} - f_{t+1})\lambda_{t+1}] = 0$$
$$E_t[e_{t+1}\lambda_{t+1}] = f_t E_t[\lambda_{t+1}] \tag{3}$$
$$[C_t]: \qquad \lambda_t = \frac{U'_{c,t}}{P_t} \tag{4}$$

Covered Interest Parity: (1) over (2) yields:

$$\frac{1}{f_t}\frac{1+i_t}{1+i_t^*} = \frac{1}{e_t}$$
$$\frac{1+i_t}{1+i_t^*} = \frac{f_t}{e_t} \tag{CIP}$$

where we used (3) ($1/f_t = E_t[\lambda_{t+1}]/E_t[e_{t+1}\lambda_{t+1}]$).

Notice that in logs the CIP above can be expressed as usual: $\ln f_t - \ln e_t = i_t - i_t^*$.

Uncovered Interest Parity: Again, use (1) and (2) but replacing (4) ($\lambda = U_c'/P$ in t and $t+1$),

$$\begin{aligned}
e_t\left(\frac{1+i_t}{1+i_t^*}\right) &= \frac{E_t\left[e_{t+1}\frac{U'_{c,t+1}}{P_{t+1}}\right]}{E_t\left[\frac{U'_{c,t+1}}{P_{t+1}}\right]} \\
&= \frac{E_t[e_{t+1}]E_t[\frac{U'_{c,t+1}}{P_t}] + Cov\left(e_{t+1}, \frac{U'_{c,t+1}}{P_{t+1}}\right)}{E_t\left[\frac{U'_{c,t+1}}{P_{t+1}}\right]} \\
&= E_t e_{t+1} + RP_t
\end{aligned} \tag{UIP}$$

Notice UIP can be rearranged too, in the usual way: $i_t - i_t^* = \ln E_t e_{t+1} - \ln e_t + rp_t$

The interpretation of the risk premium is straightforward: the foreign currency will have high value in bad states, i.e., when the marginal utility of consumption is high (covariance term). Also in the LHS the foreign currency assets hedge will be given by the negative sign of i_t^*.

As expected, it is troublesome for the equation empirical validity that the correlation implying the marginal utility of consumption does not change by much.

Market Efficiency Condition:

We can rewrite (3) as,

$$\begin{aligned}
f_t &= \frac{E_t\left[e_{t+1}\frac{U'_{c,t+1}}{P_{t+1}}\right]}{E_t\left[\frac{U'_{c,t+1}}{P_{t+1}}\right]} \\
&= E_t[e_{t+1}] + \frac{Cov_t\left(e_{t+1}, \frac{U'_{c,t+1}}{P_{t+1}}\right)}{E_t\left[\frac{U'_{c,t+1}}{P_{t+1}}\right]} \\
&= E[e_{t+1}] + RP_t
\end{aligned}$$

i.e., the forward rate is equal to the expected one plus the risk premium.

5.6 Handbook of International Economics (2014) Ch7: 5 Empirical RER facts

General stilized ER facts to be explained with the models: Co-movements or real and nominal exchange rates, high volatility of exchange rates, RER persistence (long half life).

1. Real Exchange rates for consumer prices co-move closely with nominal exchange rates at short and medium horizons. The persistence of these RERs is large with long half-lives.

   Half-life: Number of periods needed for the shocks to dissipate by half. E.g., for an AR(1) model of the form $q_t - q_0 = \rho(q_{t-1} - q_0) + \varepsilon_t$, the half life is given by $h = \ln(\frac{1}{2})/\ln(\hat{\rho})$.

   Mark (2001), using data under floating ER regimes, estimates h ranging between 2 to 5 years.

   At the micro level, several articles find large idiosyncratic movements in prices. Overgall, when disaggreagating RERs it's found that product-level RERs comove closely wiht NERs but displaying large idiosyncratic movements.

   Border effects: (Engel and Rogers 1996) cross country comparison of between and within RER's per product. With large deviation from the geographical border there is a larger deviation of PPP.

2. Movements in RERs for traded goods are roughly as large as those in overall CPI-based RERs when tradeable goods prices are measured using consumer prices or producer prices, but significantly smaller when measured using border prices (import prices, which contain a smaller non-tradeable component).

   Then traded goods would account for most of the variation of total RER, or rather said, the traded component of the RER explain most of its variation. Engel (1999) considers the following decomposition when obtaining such result:

$$\Delta rer_{in,t}^{cpi} = \Delta rer_{in,t}^{T} + \delta rer_{in,t}^{NT}$$

   where i is the foreign country, n is the home country and,

$$\begin{aligned}
\Delta rer_{in,t}^{T} &= \delta s_{in,t} + \delta cpi_{i,t}^{T} - \Delta cpi_{n,t}^{T} \\
\Delta rer_{in,t}^{NT} &= \Delta cpi_{i,t} - \Delta cpi_{i,t}^{T} - \Delta cpi_{n,t} + \Delta cpi_{n,t}^{T}
\end{aligned}$$

   Still, a critique to Engel's methodology is that, as implied by the decomposition above, when using the CPI and PPI the traded component accounts all of the exchange rate account behaviour so the result itself is an assumption at the same time. Such critique is supported by the fact that when using only traded goods indexes as the import prices, the results won't hold.

3. Aggregate Exchange Rates Pass-Through (ERPT) into consumer prices is lower than into border prices. ERPT into border prices is typically incomplete in the long run, displays dynamics, and varies considerably across countries.

   Border prices, like import prices are constructed differently across countries with quite differing composition of import bundles, that can explain the differences in ERPT.

4. Micro-data border prices, in whatever invoicing currency they are set in, respond partially to exchange rate shocks at most empirically estimated horizons.

By partial, it means that the response to the shock is incomplete.

5. There are large deviations from relative PPP for traded goods produced in a common location but sold in multiple locations. On average, these deviations co-move with exchange rates across locations.

   The deviations may come from Pricing to Market (PTM), i.e., a firm sets different prices for the same good in different markets. A particular type of PTM is the Local Currency Pricing (LCP).

   LCP: nominal ridigity in the currency of the buyer. BD1996 introduce LCP into a GE model with nominal rigidities and find that LCP affects the fraction of NER passed on to the RER. It also increases the volatility of the NER and RER.

   Basically the higher the degree of LCP the less the effect of NER in domestic price levels, less oeffect on terms of trade, i.e., LCP weakens the expenditure switching channel.

   Empirically this is noted in the fact that the TOT are smoother than the PPI-based RER.

5.7 Other reference articles on ER

AAK2009: Time varying risk, interest rate and exhange rate in general equilibrium.

The authors develop a model that delivers a NER appreciation when the risk premium falls. The problem in general is that consumption is too smooth and given it's relation with the SDF and the implied changes in the risk premium it will yield too few variability in the latter.

To deal with this it is formulated a model with cash in advance where the source of uncertainty is the money growth and the HH face a fixed cost of participating in the asset market. Also when the money growth is small the HH won't enter in the market and will consume their money holdings. As a result the consumption is an increasing, concave funtion the money growth.

The MU of the HH that enter the market will be more variable (and therefore the SDF) at lower money growth rates. The ER risk premium will be more variable too and decreasing in the money growth rate.

The model delivers an expected ER appreciation upon increases in money growth due to the movement in the risk premium for not-too-large increases in the money growth rate.

BvW2009: Infrequent portfolio decisions, a solution for the forward discount puzzle.

FP Puzzle: high interest rate currencies tend to appreciate. At the same time it is observed that only a small fraction of foreign currency holdings is actively managed, e.g., banks hold not many overnight positions and mutual funds are constrained to trade only certain assets.

An OLG model in which investors make a single portfolio decision every T periods is proposed. After a higher but mean reverting Foreign Interest rate shift, once the decision period passed and during intermediate periods, only newborn investors buy bonds. Investors however, foresee a lower interest rate differential in the future and will shift in foreign bonds demand, causing an expected future depreciation that will outweigh the interest rate differential and a delayed overshooting pattern arises.

LV2007: The Cross Section of Foreign Currency Risk Premia and Consumption Growth Risk.

To price returns Consumption Euler Equation SDF approach with Epstein-Zin preferences [2] over durable and non-durable consumption is used. The model price the returns well but suffers from EPP, requiring a risk

[2] EZ preferences refers to a recursive utility of the form $U_t = [(1-\beta)c_t^{\rho} + \beta\mu_t(U_{t+1})^{\rho}]^{\frac{1}{\rho}}$ where μ_t is a real valued certainty equivalent operator.

aversion coefficient of 100. The Authors relate their findings to the BS puzzle, the correlation between ERs and Consumption growth is high.

BvW2006: Can Information Heterogeneity Explain the Exchange Rate Determination Puzzle?.

Macro fundamental approaches lack explanatory power wrt microstructure approaches. Then BvW2006, to reconcile these approaches, explore including investor heterogeneity into a standard monetary model.

Two types of heterogeneity: Symmetrically dispersed information about fundamentals and non-fundamentals based heterogeneity.

Trader face idiosyncratic ER exposures on their non-asset income that are private information. Then the ER has an aggregate non-observable hedging component.

There is a signal extraction confusion about ER fluctuations: private investors can't tell whether ER movements are driven by hedging or new information about the fundamentals. They rationally ascribe the change to the latter. Then the effect of hedging trades on the ER is magnified, leading to a disconnect from the fundamentals.

In the LR there is a deeper connection between ER and fundamentals since the information about the fundamentals is gradually known to all investors though price discovery.

ER is expressed as a linear combination of public information and order flow. Given the ER is sensitive to private info in the model, it is linked to order flow in the SR.

The authors find that information dispersion leads to a magnification and high persistence of non-fundamentals trade shocks on the ER. Then findings are consistent with empirical evidence where:

- Fundamentlas play little role explaining ER SR fluctuations.

- Over the LR, ER and fundamentals are closely connected.

- Exchange rates help predict weakly future fundamentals.

Market Structure Exercises: Chen et al docs (x3)

As seen by MR1983 and EW2005, and as implied by the NFPV model, there is an association between the exchange rate and fundamentals, but it is the current exchange rate the variable that contains information about future fundamentals. In that spirit and considering that some of these fundamentals, like the CPI include the commodity prices, it is possible to us the ER to forecast them.

The countries considered by Chen et al (2010) are NZ, CAN, AUS, Chile, South Africa. The general approach is to consider $CP_t^W = f(\Delta s_{t-i})$. Also in the countries with fixed exchange rate, it is used the real exchange rate instead.

Term Structure of interest rates: exploring the UIP puzzle.

The UIP puzzle: when checking empirically UIP doesn't hold and even the relation between expected appreciation and interest rates has the opposite sign ($\hat{\beta} < 0$ in the regression given by $\Delta s_{t+1} = \alpha + \beta(i_t - i_t^*) + \epsilon_{t+1}$).

One of the potential causes is the omission of the risk in the estimation. Then a possible solution is to estimate the risk and including it in the UIP.

This can be done by looking at the term structures of the interest rates, e.g., of the yield curves to correct the risk component. Arguably, a risk assesment of the market should be included in the returns of the curves.

A good property of the YC is that it takes into account the fundamentals and the relation to UIP as well as the SDF and returns of the asset (term structure), i.e., the YC captures the expected future fundamentals and the perceived risks.

The correctly specified UIP is:

$$i_t^m - i_t^{m*} = E_t \Delta s_{t+m} - \rho_t^{FX,m}, \quad \forall m$$

The YC captures the expected average return for a given maturity as well as the risk:

$$i_t^m = \frac{1}{m} \sum_{j=0}^{m-1} E_t[i_{t+j}^1] + \rho_t^{B_m}$$

where $\rho_t^{B_m}$ is the time-varying risk premia. This component should price the same risks as in the FX market, i.e.:

$$\rho_t^{FX,m} = g(\rho_t^{B_m,*}, \rho_t^{B_m})$$

On the other hand, the NS1987 approach to summarize the YC is:

$$i_t^m = L_t + S_t \left(\frac{1 - \exp(-\lambda m)}{\lambda m} \right) + C_t \left(\frac{1 - \exp(-\lambda m)}{\lambda m} - \exp(-\lambda m) \right) + \epsilon_t^m$$

where L,S,C refer to the Level, Slope and Curvature of the YC. L shifts the whole curve, S moves the short end of the YC, C moves the middle part of the YC.

Therefore, what we can do is to express the difference in the interest rates as the relative YC, i.e., the relative factors for several m:

$$i_t^m - i_t^{m*} = L_t^R + S_t^R \left(\frac{1 - \exp(-\lambda m)}{\lambda m} \right) + C_t^R \left(\frac{1 - \exp(-\lambda m)}{\lambda m} - \exp(-\lambda m) \right) + \epsilon_t^m$$

then we fit $\Delta s_t \approx f(\text{Relative Factors})$ and check for both fit of the estimation as well as negative signs (these will be indicative of UIP puzzle).

The gain here is that in the YC it is clearer the role of risk (appreciation arises from higher risk that offsets interest rate differential, as it should be in the UIP).

Econometrically it is estimated as: $1200 \frac{(s_{t+m} - s_t)}{m} = \beta_{0,m} + \beta_{Lm} L_t^R + \beta_{Sm} S_t^R + \beta_{Cm} C_t^R + \varepsilon_{t+m}$

for m: $1, 3, 6, 12, 18, 24$ months.

Additionally, with the yield curve factors (relative, to approximate the int. rate differential) and assuming no systematic expectation errors it is possible to argue that the excess returns capture the risk premium:

$$XR_{t+k} = i_t^{k*} - i_t^k + \Delta s_{t+k} = \rho_t^{FX,k}$$

then to check this, the risk premium is empirically approximated as:

$$XR_{t+m} = \gamma_{m,0} + \gamma_{m,1} L_t^R + \gamma_{m,2} S_t^R + \gamma_{m,3} C_t^R + \nu_{t+m}$$

Most of the coefficients from this set of regressions (one for each country) were significant, implying that risk is related to the factors.

Finally, after correcting for risk, more accurate predictions for depreciation can be done based on the UIP. Implying that there is not a disconnect between macro fundamentals and ER but a risk disregard and that the YC helps approximate the risk premium, therefore, providing a feasible explanation for the UIP puzzle.

FX derivatives Term Structure predictive power on ER

In a similar fashion, the UIP puzzle can be explored with the FX options term structure instead than with the YC. Here the approach of Chen and Gwati (2016) is explained.

The results are similar with the main difference being the scope of this article, that is more empirical. In summary, more moments of the expectation of depreciation (SD, Kurtosis, Asymmetry) are added to the UIP regressions to account for risk (which can be thought as non linear components of the utility and UIP itself) and a better fit is obtained.

The idea is to correct for the correct but still poor approximation of first order linear equations, also, having into account that FX options payoffs are not linear in $s_{t+\tau}$.

The justification for including higher momentsis that the investors want to maximize their expected future utility $E[U_{t+1}]$ that with feasible preferences as CARA and log-normality of returns would imply a mean-variance optimization.

In summary, what the authors do is to obtain data for Daily OTC European Options, compare 6 currency pairs with 7 tenors (horizons), back out the probabilty distribution of the future spot rates and then compute the moments that will capture the expected risk. Finally this moments are used as regressors in ER forecasting equations.

6 Sovereign Debt Crisis

Overview: until now we have assumed that international debt contracts are fully enforceable. However, in real life, foreign debt is harder to enforce and a country may decide to default on its debt service. Such lack of enforceability explain departures of efficiency and limited risk sharing in the debt markets and is mainly caused by the mismatch between the willingness to pay and the ability to do so, where the key factor is the comparison a country does between the benefit of paying a debt versus the cost of reneging it.

The two main approaches explored on this subject are the reputation based models, where a coutry is cutoff from the international financial markets and then prevented from taking future borrowing, and the direct sanction models where there is a punishment once a default has benn observed.

6.1 Sovereign Default Models:

Sovereign country is the borrower, normally assumed to be a SOE. Lenders are not subject to enforceability constraints.

One period models: basic setup

Sanction:

- Country faces risk income: $Income = \bar{Y} + \varepsilon$, ε stochastic with N states

- Consumption insurance can be purchased in advance form international market

- Risk neutral lenders (lend as long as they receive the same expected value)

Borrowing country UMP: (borrower chooses repayment to maximize utility)

$$\max_{p(\varepsilon)} EU(C)$$
$$s.t.$$
$$C = \bar{Y} + \varepsilon - p(\varepsilon)$$
$$\sum_{i=1}^{N} \pi(\varepsilon_i)p(\varepsilon_i) = 0$$

where ε is a R.V. with $\varepsilon \in [\varepsilon_l, \varepsilon_h]$ and $E[\varepsilon] = 0$, $p(\varepsilon)$ is a debt payment, contingent on the shock.

1. No enforcement problem (First Best): always pay if can

   in this case it is optimal to set $p(\varepsilon) = \varepsilon$ s.t. $C = \bar{Y}$ (full insurance, risk sharing)

2. Country will pay only if profitable: Second Best

   - Without sanction: country never repays if $p(\varepsilon) > 0$ then no insurance is possible.

   - Sanction/Penalty case:

   Sanction: η times GDP, then country pays if,

$$p(\varepsilon) < \eta(\bar{Y} + \varepsilon) \tag{ICC}$$

then the UMP becomes,

$$\max_{p(\varepsilon)} EU(C)$$
$$s.t. \quad \sum_{i=1}^{N} \pi(\varepsilon_i)p(\varepsilon_i) = 0$$
$$p(\varepsilon) \leq \eta(\bar{Y} + \varepsilon)$$

then,

$$\mathcal{L} = \sum_{i=1}^{N} \pi(\varepsilon_i)U(\bar{Y} + \varepsilon_i - p(\varepsilon_i)) + \mu \sum_{i}^{N} \pi(\varepsilon_i)p(\varepsilon_i) + \sum_{i}^{N} \lambda(\varepsilon_i)(\eta(\bar{Y} + \varepsilon_i) - p(\varepsilon_i))$$

FOCs:

$$[p(\varepsilon_i)] : \qquad \pi_i U'(C) + \lambda_i = \pi_i \mu$$
$$\lambda(\varepsilon_i)] : \qquad \lambda_i[p(\varepsilon_i) - \eta(\bar{Y} + \varepsilon_i)] = 0$$

Cases:

- If the ICC doesn't bind ($\lambda_i = 0$) then contract is honored and $U'(C) = \mu$ independently of the state, i.e., full insurance

- If the ICC binds ($\lambda_i > 0$) then there is no full insurance, consumption changes with the state.

In general, for high values of the shock ε, associated with high output, the ICC will bind and there will be harder to sustain full insurance.

Let the contract be contingent on ε and have the following form:

$$p(\varepsilon) = P_0 + \varepsilon \qquad \text{for } \varepsilon \le e$$
$$p(\varepsilon) = \eta(\bar{Y} + \varepsilon) \qquad \text{for } \varepsilon > e$$

then we constraint the high states with limited liability. Thus we need to add some constant P_0 to low states to preserve the expected value of payments $Ep(\varepsilon) = 0$.

At $\varepsilon = e$ the country is indifferent, that is, Utility of repaying = Utility of defaulting and assuming sanction.

from the contract we can solve for P_0 at the indifference threshold: $P_0 = \eta\bar{Y} - (1-\eta)e$ and then we can rewrite payment schedule (contract) as:

$$p(\varepsilon) = \eta(\bar{Y} + e) + (\varepsilon - e) \qquad \text{for } \varepsilon \le e$$
$$p(\varepsilon) = \eta(\bar{Y} + e) + \eta(\varepsilon - e) \qquad \text{for } \varepsilon > e.$$

finally e can be computed from the zero profit condition and it will depend on the distribution of ε.

Results:

- The consumption is smoothed for $\varepsilon < e$

- Some extra cost P_0 is required to make sure lenders break even

- A higher sanction (higher η or e) makes the contract more enforceable.

- $E(C) = \bar{Y}$ but there is still no full insurance, that is, country is unable to commit to repay in high output states and therefore, in such cases, $C = f(\varepsilon)$

6.2 Reputation: (Eaton and Gersovitz, 1981)

- There is no output sanction, rather the defaulting country will lose access to future lending.

- Infinite horizon model

- Risk free rate: $(1+r)\beta = 1$

- Output: $Y + \varepsilon$

Gain from default: $U(\bar{Y} + \varepsilon) - U(\bar{Y})$

Given absense of future lending:

Loss from default: $\frac{\beta}{1-\beta}[U(\bar{Y}) - EU(\bar{Y} + \varepsilon)]$

Notice the loss is positive if risk averse.

The condition for full insurance is that for all ε the gains from default are lower than the associated cost:

$$U(\bar{Y} + \varepsilon_{max}) - U(\bar{Y}) \le \frac{\beta}{1-\beta}[U(\bar{Y}) - EU(\bar{Y} + \varepsilon)]$$

This inequality will always hold for $\beta \to 1$. Also the more risk averse, the higher the RHS and therefore the more this condition will hold.

Conclusion: Reputation equilibrium supports international lending if country has low discount rate or if it is very risk averse.

Partial Insurance: consumption will be insured if ICC is slack, which will happen for some but not all states. Consider the non-default condition (in its general form, i.e., depending of both $\varepsilon, P(\varepsilon)$) in each period,

$$U(\bar{Y}+\varepsilon_t) - U(\bar{Y}+\varepsilon_t - p(\varepsilon_t)) \leq \frac{\beta}{1-\beta} E[U(\bar{Y}+\varepsilon - p(\varepsilon)) - U(\bar{Y}+\varepsilon)]$$

Now consider the UMP subject to this ICC and zero profit, again, solving for $p(\varepsilon)$,

FOC:

$$\pi(\varepsilon_i) + \lambda(\varepsilon_i) + \frac{\beta}{1-\beta}\pi(\varepsilon_i)\sum_{j=1}^{N}\lambda(\varepsilon_j)U'(C) = \pi(\varepsilon_i)\mu$$

Now let $\lambda(\varepsilon_i) = 0$ (not binding ICC), then,

$$\begin{aligned} U'(C) &= \frac{\pi(\varepsilon_i)\mu}{\pi(\varepsilon_i) + \frac{\beta}{1-\beta}\pi(\varepsilon_i)\sum_{j=1}^{N}\lambda(\varepsilon_j)} \\ &= \frac{\mu}{1 + [\beta/(1-\beta)]\sum_j \lambda(\varepsilon_j)} \end{aligned}$$

then $C \perp \varepsilon_i$

However, in high states, ICC won't be binding and consumption will increase with ε. Actually, we will have $\frac{\partial p(\varepsilon)}{\partial \varepsilon} = 1 - \frac{U'(\bar{Y}+\varepsilon)}{U'(\bar{Y}+\varepsilon-p(\varepsilon))} < 1$, which means that repayment won't offset the shock as before and then consumption will change.

Note: The EG reputation equilibrium model assumes that the country is excluded from borrowing but also from saving. If savings are allowed then they are used as collateral and ICC will never bind. The SOE will find better to default always and the reputational contract is not sustainable.

That is, the threat of being cut-off from the international markets in minor if country has other investment opportunities.

Bullow and Rogoff: If the SOE is allowed to engage in other type of savings, for example by adquiring T-bills or other stocks then the reputational equilibrium is not sustained.

The SOE defaults eventually and uses the otherwise resources for repayment as collateral to finance other savings, then it uses the interests to finance future consumption.

Any candidate reputation contract must have a state of nature where the country prefers to default. Reputation alone cannot enforce repayment.

Reason: in any debt contract there will be a point where the value of debt reaches a maximum. There the country prefers to default and save in a way that replicates the debt contract but also generates extra income (interest not repaid). such sequence of savings is possible as long as the market offers assets indexed to the same contingencies.

6.3 Investment model with Borrowing Constraints (debt ceiling)

- Two periods: in $t = 1$ invests and in $t = 2$ produces

- SOE borrows but cannot commit to full repayment, only to a fraction η of output in period 2 (limited commitment: $\eta < 1$)

The general UMP is:

$$\begin{aligned}\max_{C_1,C_2,K,D}\ & U = u(C_1) + \beta u(C_2)\\ s.t\ \ & C_1 + K \leq Y_1 + D\\ & C_2 \leq F(K) + K - R\\ & R = \min\{(1+r)D, \eta(F(K)+K)\}\end{aligned}$$

where:

D: borrowing from foreigner

K: capital stock

R: repayment (minimum between face value of debt and lost fraction of output)

Three outcomes:

1. *First Best:* Commitment to pay as possible $\eta = 1$

$$\begin{aligned}\max_{K,D} U = u(Y_1 + D - K) + \beta u(F(K) + K - (1+r)D)\\ s.t.\quad F(K) + K \geq (1+r)D \quad \text{(no default)}\end{aligned}$$

FOCs:

$$\begin{aligned}[D]: &\qquad u'(C_1) - (1+r)\beta u'(C_2) - \lambda(1+r) = 0\\ [K]: &\qquad -u'(C_1) + \beta u'(C_2)[F'(K)+1] + \lambda(F'(K)+1) = 0\end{aligned}$$

- Under repayment $\lambda = 0$ then $F'(K^{FB}) + 1 = 1 + r$. That is, $MPL =$cost of capital (benchmark result)

- If constraint binds $F(K) + K = (1+r)D$ and there is in increase in marginal utility of consumption of period one (implying higher capital stock - investment). Additionally by substituting in C_2:

$C_2 = F(K) + K - (1+r)D = 0$ that is not optimal (corner solution).

2. *Commitment under borrowing friction/constraint:* $\eta < 1$

Change in ICC: $\eta(F(K) + K) \geq (1+r)D$

same UMP as before,

FOCs:

$$\begin{aligned}[D]: &\qquad u'(C_1) - (1+r)\beta u'(C_2) - \lambda(1+r) = 0\\ [K]: &\qquad -u'(C_1) + \beta u'(C_2)[F'(K)+1] + \lambda\boldsymbol{\eta}(F'(K)+1) = 0\end{aligned}$$

Cases:

- $\lambda = 0$ (not binding): $\eta(F(K) + K) > (1+r)D$

SOE finds better not to default and we obtain the same FB allocation: $F'(K) + 1 = 1 + r$, i.e., $F'(K) = r$.

This case is likely if η or Y_1 is too high.

- $\lambda > 0$ constraint binds, then the country i (SOE) face a higher interest rate r^W.

It will find better to invest instead or repaying and then meet ICC but also overinvest the excedent.

There will be a corresponding increase in the capital w.r.t. the FB that will go beyond: $MPK =$ Intertemporal MRS.

The constraint would be $\eta(F(K) + K) = (1 + r)D$.

We can solve for $u'(C_1)$ in each FOC:

From [D]: $u'(C_1) = (1 + r)[\beta u'(C_2) + \lambda]$, from [K]: $u'(C_1) = (\beta u'(C_2) + \lambda\eta)[F'(K) + 1]$

Then,

$$(1 + r)[\beta u'(C_2) + \lambda] = [\beta u'(C_2) + \lambda\eta][F'(K) + 1]$$

since $\beta u'(C_2) + \lambda > \beta u'(C_2) + \lambda\eta \;\Rightarrow\; F'(K) > r$, therefore, $K < K^{FB}$.

3. *No commitment under borrowing constraint*

No asymmetric or moral hazard problem. Outcome is similar to former cases.

Lenders worry that K to be chosen once he lends D will be such that it will be optimal to default. Then they try to take into account the decision process of the SOE, i.e., the investment level given D and whether it will lead to default (as with a typical ICC but per case).

Backward solution:

i. Calculate optimal K given D, under repayment and default: $K^{*n}(D), K^{*d}(D)$

ii. Lenders choose D constrained by a debt ceiling: $D < \bar{D}$, where $\bar{D}$ is the maximum amount sustainable without default.

Given D, default if $U^d(D) > U^n(D)$, where:

$$\begin{aligned}
U^n(D) &= \max_{K^n} u(Y_1 + D - K^n) + \beta u(F(K^n) + K^n - (1 + r)D) \\
U^d(D) &= \max_{K^d} u(Y_1 + D - K^d) + \beta u((1 - \eta)(F(K^d) + K^d)) \\
&\quad s.t. \quad D < \bar{D}
\end{aligned}$$

and $\bar{D}$ determined from $U^d(\bar{D}) = u^n(\bar{D})$

FOCs for K:

$$\begin{aligned}
u'(C_1^n) &= \beta u'(C_2)(F'(K^{*n}) + 1) \\
u'(C_1^d) &= \beta u'(C_2)(1 - \eta)(F'(K^{*d}) + 1)
\end{aligned}$$

Then substitute C_1, C_2:

$$\begin{aligned}
u'(Y_1 + D - K^{n*}) &= \beta u'(F(K^{n*}) + K^{n*} - (1 + r)D)[F'(K^{n*}) + 1] \\
u'(Y_1 + D - K^{d*}) &= \beta u'((1 - \eta)(F(K^{d*}) + K^{*d}))(1 - \eta)[F'(K^{d*}) + 1]
\end{aligned}$$

finally, assume $u = \ln c, F(K) = K^{\alpha}$.

$$\begin{aligned}\frac{1}{(Y_1 + D - K^{n*})} &= \frac{\beta}{(K^{n*})^{\alpha} + K^{n*} - (1+r)D}(\alpha(K^{n*})^{\alpha-1} + 1)\\ \frac{1}{(Y_1 + D - K^{d*})} &= \frac{\beta}{(K^{n*})^{\alpha} + K^{n*}}(\alpha(K^{d*})^{\alpha-1} + 1)\end{aligned}$$

Now, since given D the marginal cost of additional capital is increasing, we guess that the marginal benefit (the marginal utility in the RHS of each of these two equations) is larger under no default (n). $MB^n > MB^D$, i.e.,

$$\frac{1}{F(K^{n*}) - (1+r)D} > \frac{1}{F(K^{d*})}$$

Then given D, the investment K is higher under Non default than under Default.

Also, relative to the FB, the consumption will tilt to the first period. Effective domestic interest rate rises given that K is financed with debt and the investment decreases ($K < K^{FB}$).

6.4 Debt Overhang model

CA deficits may cause less growth since higher debt may hurt incentives to invest:

↑ debt, ↑ prob. of default, ↑ implicit tax on investment, ↓ K.

D : Exogenous Debt

R: Repayment, with $R = \min\{\eta AF(K), (1+r)D\}$ and A is stochastic

$V(K, D)$: Debt's value (Equals the expected value of repayment $E(R)$)

Notice that $V(K, D) \neq D$ (face value of debt) unless no default is certain.

Countries can buy back their own debts:

- Average price: $p = \frac{V(K(D),D)}{D}$

- Marginal price: $p = \frac{dV(K(D),D)}{dD}$

There will be a gain by paying back q but also a possible loss given that tere is a reduction in the debt given by the slope of the laffer curve $q = dV/dD$

Output: Exogenous, Random $Y \sim f(y)$ support: $[Y_l, Y_h]$

For simplicity, let the repayment be $R = min\{\eta Y, D\}$ and $x = 1$ if SOE defaults and $x = 0$ otherwise.

Expected value of repayment (debt's value):

Since the support on Y: $[Y_l, Y_h]$ will yield a corresponding support on $R : [R, l, R^h]$,

$$
\begin{aligned}
E(R) = \int_{R_l}^{R^h} R f_R(r) dr &= \int_{Y_l}^{Y^h} R f_y(y) dy \\
&= \int_{Y_l}^{Y_h} \min\{\eta Y, D\} f_y(y) dy \\
&= \int_{Y_l}^{D/\eta} \eta Y f(y) dy + \int_{D/\eta}^{Y_h} D f(y) dy
\end{aligned}
$$

then,

$$
V(D) = E(R|x=1) + E(R|x=0) = \int_{Y_l}^{D/\eta} \eta Y f(y) dy + \int_{D/\eta}^{Y_h} D f(y) dy
$$

by Leibniz rule[3] the marginal value of the debt (derivative w.r.t. D) is

$$
V'(D) = \int_{D/\eta}^{Y_h} f(y) dy + \frac{1}{\eta} \eta \frac{D}{\eta} f\left(\frac{D}{\eta}\right) - \frac{1}{\eta} D f\left(\frac{D}{\eta}\right) = \int_{\frac{D}{\eta}}^{Y_h} f(y) dy
$$

i.e., the marginal value of debt, $V'(D)$ is equal to the probability to repay.

Notice how the probability tends to 0 as D increases towards the sanction value: $\lim_{D \to \eta Y_h} V'(D) = 0$

Dynamics in prices (pre and post buy prices):

The average price is decreasing in D,

$$
\frac{\partial (V/D)}{\partial D} = \frac{V'D - V}{D^2} < 0
$$

since $DV'(D) = E(R|x=0) < E(R|x=1) + E(R|x=0) = V(D)$.

Assume that the debt decreases from D_1 to D_2 ($D_2 < D_1$), it follows that,

$$
p_2 = \frac{V(D_2)}{D_2} > \frac{V(D_1)}{D_1} = p_1
$$

where p_1: pre buyback price, p_2: post buyback price.

$p_2 > p_1$ denotes that prices will increase after D falls which will happens because a lower debt makes the payment more likely, leading to an increase in the value of debt.

6.5 Moral Hazard model (Gertler and Rogoff 1990)

Agents are risk neutral and the model has one period, then: $U = C$

r : Interest rate

Endowment: Y invested at a riskless rate or used to finance risky investment.

[3] Let $F(c) = \int_{a(c)}^{b(c)} f(c,t)$ then $F'(c) = \int_{a(c)}^{b(c)} f_c(c,t) dt + b'(c) f(c, b(c)) - a'(c) f(c, a(c))$

Investment: I yields Z with probability $\pi(I)$, with $\pi' > 0, , \pi'' < 0, \pi(0) = 0$

Assumptions: (i) $\pi'(0)Z > 1 + r$: investment always dominates savings (risk free rate), (ii) $I > Y$ SOE must borrow to finance investment.

Budget constraint:

$$I + L = Y + D$$

with Lending $L \geq 0$, debt $D \geq 0$

Net capital flows:

$$B = D - L$$

where L yields a return r.

Foreigners are risk neutral and competitive, i.e., earn r in expectation.

Contract: $\{D, P_1, P_0\}$

with Inflows D, repayment in goods sate (p_1), repayment in bad state (p_2).

I is unobservable by lenders and hence non-contractible. Y, D, Z are observed.

- *First Best:* Investment is observed, then contractible.

The contract should maximize SOE's consumption:

$$\begin{aligned} \max_{D,L,p_1,p_0,I} \quad & \pi(I)(Z - p_1) + (1 - \pi(I))(0 - p_0) + (1 + r)L \\ s.t. \quad & L + I = Y + D \\ & \pi(I)p_1 + (1 - \pi(I))p_0 = (1 + r)D \end{aligned}$$

the last constraint denotes that the expected payoff for lenders must be equal to market return (otherwise there would be an arbitrage oportunity in equilibrium).

Substitute the constraints in the objective function:

$$\begin{aligned} \pi(I)Z - (\pi(I)p_1 + (1 + \pi(I)p_0) + (1 + r)L &= \pi(I)Z - (1 + r)D + (1 + r)L \\ &= \pi(I)Z + (1 + r)(Y - I) \end{aligned}$$

take FOC,

$$[I] : \qquad \pi'(I)Z = (1 + r)$$

now, given $\pi(\cdot)$ we pin down I^* and thus,

$$D^* - L^* = I^* - Y > 0$$

with $L^* \in \mathbb{R}_+$ and where $p_0^* \leq (1 + r)L^*$, p_1^* are set such that the zero profits condition holds (e.g. $p_0^* = 0, p_1^* = (1 + r)\frac{L^* + I^* - Y^*}{\pi(I)} > 0$).

- *Second best:* unobserved I

$$
\begin{aligned}
&\max_{D,L,p_1,p_0,I} \pi(I)(Z-p_1)+(1-\pi(I))(0-p_0)+(1+r)L \\
&\quad s.t. \quad L+I=Y+D \\
&\qquad\qquad \pi(I)p_1+(1-\pi(I))p_0=(1+r)D \\
&(ICC) \qquad (I,L)\in \arg\max \pi(I)(Z-p_1)-(1-\pi(I))p_0+(1+r)L \quad \text{with } D,p_1,p_0 \text{ observed}
\end{aligned}
$$

here I, L non contractible, then must be incentive compatible, i.e., it holds that (taking derivative w.r.t. I):

$$\pi'(\tilde{I})(Z-(p_1-p_0))=1+r$$

compare with the condition under the FB: $\pi'(I^{FB})Z=1+r$

We will have that if $p_1>p_0$: $\quad \tilde{I}<I^{FB}$

Which follows because the lender requires a higher repayment in good state in this case, i.e., $p_1>p_0$ is a risk sharing mechanism (the lender agrees to lower the repayment in the bad state, offsetting it at some extent).

Given risk neutrality we can assume $p_0=0$ and make the guess that $L=0$ that leads to the 3x3 system of equations:

IC: $p_1=Z-(1+r)/\pi'(I)$

Zero profits: $p_1=(1+r)D/\pi(I)$

Feasibility: $I+L=D+Y$

After solving GR1990 obtain that $\frac{\partial I}{\partial Y}>0$. Also that if higher income raises investment, then the interest rate will increase w.r.t. to the reiskless rate and then it could explain why capital doesn't necessarily fly from rich to poor countries.

The intuition is that poorer countries are borrowed-constrained, requiring external financing. Such financing (higher debt) hurts investment, increasing the required marginal product of capital that compensates the lenders. From the point of view of the richer country, it saves more but also invests more, receiving higher capital inflows.

6.6 Arellano (2008), Gopinath and Aguiar (2006)

part of the sanction models, here a defaulting country is cut-off from future borrowing and will be received back after some time with a positive probability and zero net foreign assets. These dynamics will be reflected in the continuation value of the country.

- SOE with stochastic endowment $\{y_t\}_{t=0}^{\infty}$

- Single good, single asset, 1 period bond

- $u=c^{1-\gamma}/(1-\gamma)$ CRRA

- $y_t=e^{z_t}$ with $z_t=\rho_z z_{t-1}+\varepsilon_t^z$, $\varepsilon_t^z\sim N(0,\sigma_z^2)$

- a_t: NFA, price q_t assumed and t and giving 1 unit of good in $t+1$.

We want to endogeneize $q_t=f(a_t,z_t)$.

The SOE decides, after observing a_t, z_t, whether to default on a_t,

$$V(a_t,z_t)=\max\{V^G,V^B\}$$

where V^B: value in default state (bad state), V^G: value in good credit state.

Under the bad state the SOE will be cutoff from international credit market and would be able to consume only the endowment net of default cost: y^{def}.

There is an exogenous probability of re-entry λ in $t+1$ with $a_{t+1} = 0$.

The value functions are,

$$
\begin{aligned}
V^B(z_t) &= u(y^{def}) + \beta\left((1-\lambda)E_t V^B(z_{t+1}) + \lambda E_t V(a_{t+1} = 0, z_t + 1)\right) \\
V^G(a_t, z_t) &= \max_{c_t}\left[u(c_t) + \beta E_t V(a_{t+1}, z_{t+1})\right] \\
s.t. \qquad & c_t = y_t + a_t - q_t a_{t+1}
\end{aligned}
$$

International Capital Market:

- Risk neutral lender with opportunity cost r^*

let $D(a_t, z_t) = 1$ if the country defaults and 0 otherwise. Then in equilibrium,

$$
q_t(a_{t+1}, z_t) = \frac{E_t\{1 - D(a_{t+1}, z_{t+1})\}}{1 + r^*}
$$

Finally, the types of y^{def} considered are:

- $y^{def} = (1-\delta)y$ where δ: proportional cost of defaulting (Gopinath and Aguiar, 2006)

- $y^{def} = \min\{y, \hat{y}\}$, kinked output (Arellano, 2008)

Comparison:

Regardless of the size of the productivity shock the value function follows the same pattern if the sanction is proportional to the income: for low asset positions is better to default as the sanction cost, a fraction of the output, is low. For larger asset positions the value functions increase, and the good credit conditions one dominates, i.e., it's better no repay and the intuition is the same, the output increased significantly and therefore is much more costly to default.

Under the kinked output case we have a similar patter for low productivity shocks, however, when a high productivity shock is observed, the cost of defaulting is prohibitivetly large for all asset positions and therefore it's better to repay. In this case the punishment for defaulting is perceived as more severe.

The prices will reflect such dynamics in the value function, assets will not be highly valued if the position (and therefore the output) is low and the price will be low and close to zero. However, once it's better to repay the asset prices will surge, reflecting that the more assets the agents hold the higher their value function will be.

Finally, case 2 (kinked output) captures better the stilized facts of sovereign debt defaults, for example, the probabilities of defaulting (conditional on having default or unconditional) and also the negative correlation between the assets position and the interest rate. It could be said that the Arellano approximation captures better the trade-off faced by agents when deciding whether to default.

7 Financial Crisis models-Speculative Attacks, Global Games

Speculative Attacks to currency models can be summarized in three generations of models: (1) fundamental based: Focused on timing (Krugman, 1979), (2) Multiple equilibria strategic coordination models (Obstfeld 1996), and (3) Global Games and lack of common knowledge (Morris-Shin 1998).

The first approach is based entirely in a Rational Expectations framework with no strategic complementarities or behavior between agents. The second is focused on the strategic interactions of the traders that can coordinate to attack a currency and will lead to multiple equilibrium outcomes. Finally, the third one is offers a bridge between (1) and (2) by allowing information imperfections to limit the effects of the strategic behavior of the agents and therefore to recover a unique equilibrium as outcome.

7.1 First Generation

Here the ER peg is unsustainable because fiscal and monetary policy are inconsistent, i.e., there is a permanent fiscal deficit that is not backed up by money growth, due to a fixed exchange rate policy. Then eventually the peg has to be broken to deal with the imbalance. The focus in this approach is about timing of the speculative attack of the fiscal deficit.

Model:

Continuous time, perfect foresight. PPP and UIP are assumed to hold.

Money market equilbrium is given by:

$$m_t - p_t = -\eta i_t + \delta y_t$$

with continuous time it is possible to set: $y^* = i^* = 0$, also $y = 0$. The PPP is assumed to hold, then $p_t = e_t + p_t^*$

However, with p^* and $y = 0$ we have,

$$m_t - e_t = -\eta i_t = -\eta \dot{e}_t$$

since by UIP: $\dot{e}_t = i_t - \cancel{i_t^*}$

with fixed ER $\bar{e}$: $\dot{e} = 0$ and therefore,

$$m_t = e_t = \bar{e} = \bar{m}$$

The Central Bank balance sheet in levels is given by:

$$M_t = \bar{M} = B_{h,t} + \bar{E} B_{f,t}$$

where M_t are the liabilities and the assets are the domestic bonds B_h plus the foreign ones in domestic currency EB_f.

Now we assume that the dynamics for the domestic bond are:

$$\frac{\dot{B}_{h,t}}{B_{h,t}} = \dot{b}_{h,t} = \mu > 0$$

Implying a fiscal deterioration, i.e., the CB gets bonds to finance fiscal deficit. Also with a pegged ER $\bar{E}$:

$$\dot{M}_t = 0 = \dot{B}_{h,t} + \bar{E}\dot{B}_{f,t} \quad \Rightarrow \quad \dot{B}_{h,t} = \underbrace{-\bar{E}\dot{B}_{f,t}}_{\text{Reserves}}$$

Now if $B_{f,t} = 0$ then $\dot{M}_t = \dot{B}_{h,t} > 0$ and therefore the peg is abandoned.

Let $\tilde{e}$: be the log shadow ER. Also, with $B_{f,t} = 0$ we had $M_t = B_{h,t}$. In logs $m_t = b_{h,t} = b_{h0} + \mu t$.

The shadow ER will be given by the average of future (expected but perfectly foreseen) ER:

$$\tilde{e}_t = \frac{1}{\eta}\int_t^{\infty} e^{-\frac{-(s-t)}{\eta}}(b_{h0} + \mu S)dS = b_{h,t} + \eta\mu$$

Once the peg is expected to break (i.e. an increase in ER is anticipated), the agents will anticipate such trend and will drop B_f (they attack the currency), the money supply will fall due to the attack, and then after the peg is abandoned the money stock starts growing again.

The speculative attack occurs when the shadow rate equals the peg, that is when the level of reserves is positive.

Actually after the collapse $m_t = b_{h,t}$ that is, $\dot{b}_{h,t} = \mu, \dot{\tilde{e}}_t = \mu$ and since the speculative attack happens at T s.t. $\tilde{e}_T = \bar{e}$ then we can solve for the period of attack T:

$$b_{h0} + \mu T + \eta\mu = \bar{e} \quad \rightarrow \quad T = \frac{\ln(B_{h0} + \bar{E}B_{f0}) - \eta\mu - b_{h0}}{\mu}$$

It should be noticed that this model is not speculative, the agents are rational and fully informed of when to attack the currency.

Then the model is good to explain rational currency attacks when a country has large holdings of reserves. But it is ill-suited to address strategic behavior problems under uncertainty.

7.2 Second Generation

A second approach allows for speculative behavior about a depreciation, this depreciation expectations can be self-fulfilling. Here Monetary and Fiscal policy will not be inconsistent, instead, there is some degree of lack of credibility in the policies, partly due to the fact that defending the peg is costly.

Therefore, there is space for strategic complementarities in the behavior of the agents and the speculative attack may occur regarless of fundamentals and due to the agents coordination.

A trader may want to attack if other agents are likely to attack too. The expectations will determine the possible equilibrium outcomes that will be multiple. For example, consider an scenario of modest reserves and normal macroeconomic fundamentals outlook, then if the peg is credible, the central banck can increase the money growth without breaking the regime. On the other hand, if agents attack the banck will not be able to sustain the peg due to low reserves.

7.3 Third Generation

This model recovers aspect from the former two generations. The main idea is that with an additional small amount of uncertainty about fundamentals is possible to restore the uniqueness of the equilibrium, regardless of a multiplicity of possible results.

Here we will have both strategic complementarities but also strategic uncertainty. Then we have an equilibrium that is driven by fundamentals (1st Gen) but with strategic complementarities and coordination (2nd Gen).

Agents will follow cutoff strategies according to their signals and the success of the attack will be determined by fundamentals.

The dispersion of information is important for uniqueness, and actually, adding an informative enougth public signal about fundamentals can re introduce multiplicity of solutions.

The framework in which this works follows Carlsson and van Damme (1993) approach: noise is added to a game theory framework, leading to strategic uncertainty. Then with private information, the coordination among agents gets complicated enough that a unique equilibrium arises.

Model:

Three periods. Continuum of traders in $(0, 1]$, each with a unit of home currency.

$t = 0$

θ : amount of reserves, randomly determined.

$t = 1$

Each agent observes a private signal x_i about θ: $x_i = \theta + \varepsilon_i$, with $\varepsilon \sim N(0, \sigma_\varepsilon^2)$, let the precision of the signal be given by $\lambda_\varepsilon = \frac{1}{\lambda_\varepsilon^2}$.

An action is taken by each agent after observing the signal:

$$I_i = \begin{cases} 1 & Attack \\ 0 & Not \end{cases}$$

Attack refers to sending the unit of home currency to the CB in exchange for its value in foreign currency.

The payoff of the actions $\Pi(I_i)$ is:

$$\Pi(1) = \begin{cases} -c & \text{if } R = 0 \\ 1 - c & \text{if } R = 1 \end{cases}$$

$$\Pi(0) = 0$$

where c is the cost of attacking and $R = 1$ denotes that the ER peg is abandoned.

$t = 2$

CB observes θ and $\{I_i\}$ and chooses R.

Let A: size of the attack. The peg is abandoned if A exceeds the reserves amount:

$$R = 1 \iff A = \int_0^1 I_j d_j > \theta$$
$$R = 0 \quad \text{O.W.}$$

We solve the game by Backwards Induction. The agents will choose an action such that,

$$I_i(x_i) = \arg\max E[u(I_i(x_i), I_{-i}(x_{-i}), \theta | x_i]$$

Notice that $I_{-i}(x_{-i})$ indicates the presence of strategic complementarities: agents' decisions affect each other.

with:

$$u = \begin{cases} I_i(x_i)[-c] & \text{if } A \leq \theta \quad (R = 0) \\ I_i(x_i)[1 - c] & \text{if } A > \theta \quad (R = 1) \end{cases}$$

(i) Perfect Information Bechmark

$x_i = \theta \quad \Rightarrow$ Multiple Equilibrium (Obstfeld 1996) if $\theta \in (0,1)$.

Two equilibria:

No attack: $I_j = 0 \quad \forall j \quad A = 0 < \theta$

All attack: $I_j = 1 \quad \forall j \quad A = 1 > \theta$

(ii) Imperfect Information case: threshold rule (x^* cutoff)

$$I(x_i) = \begin{cases} 1 & \text{if } x_i < x^* \\ 0 & \text{if } x_i \geq x^* \end{cases}$$

Given θ, the probability of attack is given by (we drop i guessing the eq. is symmetric):

$$\begin{aligned} Pr(x < x^*|\theta) &= Pr(\theta + \varepsilon < x^*|\theta) \\ &= Pr(\varepsilon < x^* - \theta|\theta) \\ &= \Phi\left(\frac{x^* - \theta}{\sigma_\varepsilon}\right) \\ &= \Phi(\sqrt{\lambda_\varepsilon}(x^* - \theta)) \end{aligned}$$

By LLN we can pin down the size of the attack (all agents' signal is i.i.d., and the size of the attack in a strictly monotonic decreasing function of θ): $A(\theta) = \Phi(\sqrt{\lambda_\varepsilon}(x^* - \theta))$.

Moreover, $A(\theta) - \theta$ is also decreasing, with $A(0) - 0 \geq 0$ and $A(1) - 1 \leq 0$, then we can find θ by considering that there is a $\theta^* \in [0,1]$ s.t., $A(\theta^*) = \theta^*$, i.e., we will find θ as:

$$\theta^* = \Phi(\sqrt{\lambda_\varepsilon}(x^* - \theta^*)) \tag{1}$$

The agents observe x_i and consider $I_i = 1$, i.e.,

$$\begin{aligned} E(u) &= -cPr(A(\theta) \leq \theta|x_i) + (1-c)Pr(A(\theta) \geq \theta|x_i) \\ &= Pr(A(\theta) \geq \theta|x_i) - c \\ &= Pr(\theta^* > \theta|x_i) - c \qquad \text{given } \theta^* = A(\theta^*) \end{aligned}$$

we can assume a flat distribution (posterior, i.e., given x_i) for θ: $\theta|x_i \sim N(x_i, \lambda_\varepsilon^{-1})$. Therefore

$$E(u) = 1 - \Phi(\sqrt{\lambda_\varepsilon}(x_i - \theta^*)) - c$$

where we flipped the probability argument.

The indifference condition $x_i = x^*$ is given by:

$$\begin{aligned} E(u|I=1) &= \cancelto{0}{E(u|I=0)} \\ 1 - \Phi(\sqrt{\lambda_\varepsilon}(x_i - \theta^*)) - c &= 0 \\ \Phi(\sqrt{\lambda_\varepsilon}(x_i - \theta^*)) &= 1 - c \end{aligned} \tag{2}$$

by (1): $\theta^* = 1 - c$. Substitute in (2):

$$\Phi(\sqrt{\lambda_\varepsilon}(x_i - (1 - c))) = 1 - c$$

then we can solve for x_i:

$$x_i = \frac{\Phi^{-1}(1 - c)}{\sqrt{\lambda_\varepsilon}} + 1 - c$$

Here the key message is that no matter how precise the signal is, private information is different from public, and just a little uncertainty can reduce the outcome to a unique equilibrium.

7.4 Summary: Handbook of Int. Econ. Chapter 12

G. Lorenzoni's Summary on International Financial Crises (first sections of Ch. 12)

Financial crisis are broadly defined as a sudden outflow of financial resources in an economy. It is usually driven by expectations of large devualuations in fixed ER regimes or by changes in the expected fundamentals.

The literature has explored first the *Currency Crisis* (Krugman 1979) which refers to a situation in which the agents have doubts about the capacity of a central bank to back a fixed exchange regime with enough reserves and therefore a subsequent attack on the currency occurs. Here the NER is viewed as a monetary phenomenon and the focus is mainly on fixed rate regimes.

The initial approach (called first generation of models) is based entirely on rational expectations and abstract from strategic complementarities between agents' expectations and actions (attack or not the currency).

The key element in this approach is that of unsustainable economic policies, for example inconsistent fiscal and monetary policies, that lead to discontinuous adjustment processes in the exchange rate due to forward looking behavior by the agents.

In this case, without CB independence, and given that fiscal and monetary policy are connected by the government budget constraint, certain fiscal policies can lead to ER instability, and in general, a successful fixed ER regime requires commitment to fiscal discipline and MP independence.

A second generation of models study the coordination strategic behaviour between agents and how, depending on the expectations and for intermediate levels of the fundamentals (where is not entirely clear what is the willingness of the CB to defend the ER peg), multiple equilibrium outcomes can arise.

Here the focus in on overvaluation of the exchange rate, regardless of the fundamentals, once they take some intermediate values. Here, the main element is a lack of commitment from the CB to a peg that is self-fulfilling and leads to a recessionary shock that pushes the CB to choose a devaluation.

The intermediate values that the fundamentals take are what lead to possible outcomes under different plausible expectations about the expected benefit of abandoning the peg from the perspective of a CB that compares defending the peg versus a welfare maximizing allocation net of the cost of breaking the regime:

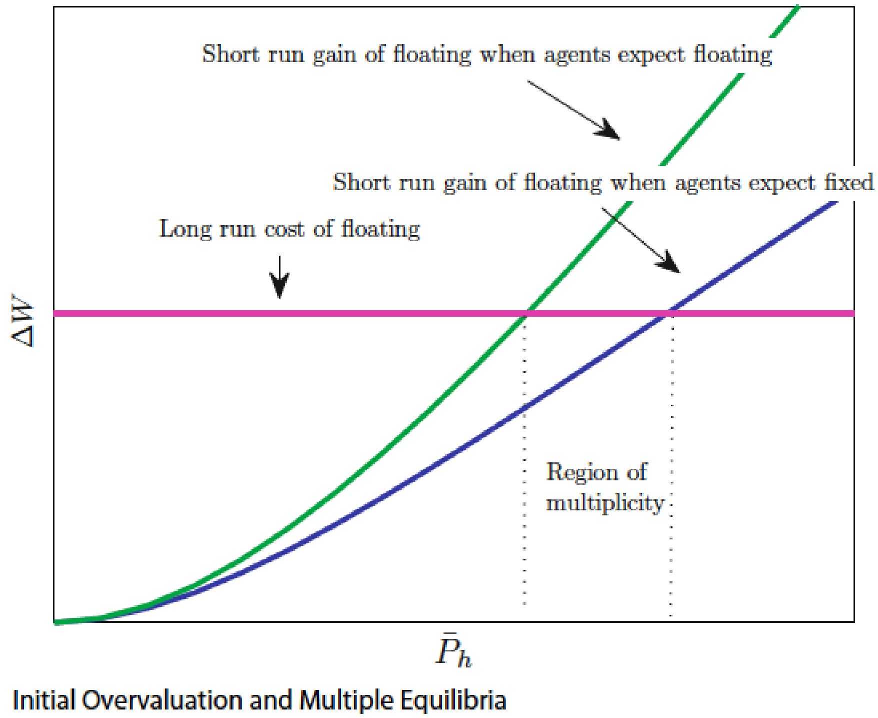


Figure 9: Source: Handbook of Int. Macroeconomics, Chapter 12.

The third strand of literature on ER regime attacks is denominated the Global Game and is based on MS1998 as mentioned in previous sections. This approach arises from noticing that the possibility of multiple equilibria is not entirely independent of the fundamentals but quite the opposite, they take values such that several scenarios about ER are possible. Then MS1998 add a small amount of information uncertainty to difficult strategic coordination among agents and then discard most of the outcomes.

The new degree of idyosincratic uncertainty has an effect on the agents guesses about his and others' expectations on the value of the ER that reduces the outcome to an unique equilibrium as shown in the previous section.

Finally, a second block of literature, departed from examining the ER regime and have explored the *Current Account Reversals* as a whole. The emphasis is put on financial flows and suddent stop of capital flows, i.e., to a loss of access to international borrowing and therefore to a forced reduction on the CA deficit (Dornbusch et al 1995, Calvo 1998).

The sudden stops can be taken either as exogenous in the literature, and then their effect is studied, or can be considered as endogenous, leading to models of defaultable debt where the fear of default triggers self-fulfilling capital fligths.

In this approach the mechanism about how an expectation of a devaluation can lead to a capital flight, forcing the CB to tighten the domestic policy and leading to a recession are also at work undera flexible ER regime as long as the CB tries to dampen the effect of swings in expectations of ER changes.

Such swings are the real reason behind the so called fear of floating of policy makers.

$$c_t^i + (1 - q_t\theta)i_t = r_t n_t + (\phi_t q_t + (1 - \phi_t)q_t^R)(1 - \delta)n_t + p_t m_t$$

A Simple Two Period Open Economy Model

Courtesy of Ippei Fujiwara

For simplicity, we assume that the exchange rate is fixed at 1. This will be similar to a two agents model in the closed economy.

Domestic Country

Household

The representative household in the home country maximizes welfare:

$$u(C_1) - v(h_1) + \beta[u(C_2) - v(h_2)]$$

subject to the budget constraints:

$$B_1 + C_1 = W_1 h_1 + \Pi_1$$

and

$$C_2 = W_2 h_2 + R_2 B_1 + \Pi_2$$

Firm

A representative firm in the home country maximizes the profit:

$$\Pi_t = Y_t - W_t h_t$$

subject to the production function:

$$Y_t = f(z_t h_t) \tag{1}$$

Foreign country

household

The representative household in the foreign country maximizes welfare:

$$u(C_1^*) - v(h_1^*) + \beta[u(C_2^*) - v(h_2^*)]$$

subject to the budget constraints:

$$B_1^* + C_1^* = W_1^* h_1^* + \Pi_1^*$$

and

$$C_2^* = W_2^* h_2^* + R_2^* B_1^* + \Pi_2^*$$

Firm

A representative firm in the home country maximizes the profit:

$$\Pi_t^* = Y_t^* - W_t^* h_t^*$$

subject to the production function:

$$Y_t^* = f(z_t^* h_t^*) \tag{2}$$

Clearing conditions

The financial market clearing condition is given by:

$$B_t + B_t^* = 0 \tag{3}$$

By substituting the profits into the budget constraints, we have:

$$\begin{aligned} B_1 + C_1 &= Y_1 &(4)\\ C_2 &= Y_2 + R_2 B_1 &(5)\\ B_1^* + C_1^* &= Y_1^* &(6)\\ C_2 &= Y_2 + R_2 B_1 &(7) \end{aligned}$$

Together with the financial market clearing conditions, we can derive the goods market clearing conditions, i.e., we can derive the resource constraints:

$$\begin{aligned} C_1 + C_1^* &= Y_1 + Y_1^* &(8)\\ C_2 + C_2^* &= Y_2 + Y_2^* &(9) \end{aligned}$$

Optimality conditions

From households' problems we have:

$$\begin{aligned} v'(h_1) &= u'(C_1)W_1 &(10)\\ v'(h_2) &= u'(C_2)W_2 &(11)\\ v'(h_1^*) &= u'(C_1^*)W_1^* &(12)\\ v'(h_2^*) &= u'(C_2^*)W_2^* &(13)\\ u'(C_1) &= \beta R_2 u'(C_2) &(14)\\ u'(C_1^*) &= \beta R_2 u'(C_2^*) &(15) \end{aligned}$$

From the firms' problem we have:

$$\begin{aligned} W_1 &= z_1 f'(z_1 h_1) &(16)\\ W_2 &= z_2 f'(z_2 h_2) &(17)\\ W_1^* &= z_1^* f'(z_1^* h_1^*) &(18)\\ W_2^* &= z_2^* f'(z_2^* h_2^*) &(19) \end{aligned}$$

System of equations

Equations (1) to (19) solve for the 19 endogenous variables: $C_1, C_2, C_1^*, C_2^*, h_1, h_2, h_1^*, h_2^*, Y_1, Y_2, Y_1^*, Y_2^*, B_1, B_1^*$, W_1, W_2, W_1^*, W_2^* and R_2.

Reduction

After substituting some variables, the 9 equations below solve for the 9 endogenous variables: $C_1, C_2, C_1^*, C_2^*, h_1, h_2, h_1^*, h_2^*$ and R_2.

$$
\begin{aligned}
v'(h_1) &= u'(C_1) z_1 f'(z_1 h_1) \\
v'(h_2) &= u'(C_2) z_2 f'(z_2 h_2) \\
v'(h_1^*) &= u'(C_1^*) z_1^* f'(z_1^* h_1^*) \\
v'(h_2^*) &= u'(C_2^*) z_2^* f'(z_2^* h_2^*) \\
U'(C_1) &= \beta R_2 U'(C_2) \\
\frac{u'(C_2)}{u'(C_1)} &= \frac{u'(C_2^*)}{u'(C_1^*)} \\
C_1 + C_1^* &= f(z_1 h_1) + f(z_1^* h_1^*) \\
C_2 + C_2^* &= f(z_2 h_2) + f(z_2^* h_2^*) \\
C_1 + \frac{C_2}{R_2} &= f(z_1 h_1) + \frac{f(z_2 h_2)}{R_2}
\end{aligned}
$$

Note that the last equation implies a similar condition must hold in the foreign country as well, but is becomes redundant.

B Risk Sharing - Model with Contingent Claims

One Bond, two states and two periods

With non-contingent bonds: temporary negative output shocks lead to CA deficits (borrowing).

With State contingent bonds: No change in consumption. Risk insurance.

$$\max_{C_1, B_2(1), B_2(2)} u(C_1) + \beta E_1[u(C_2)]$$

s.t.

$$C_1 + \frac{P(1)}{1+r} B_2(1) + \frac{P(2)}{1+r} B_2(2) = Y_1$$

and,

$$C_2(S) = Y_2(S) + B_2(S), \quad \text{for } S = \{1, 2\}$$

The UMP can be expressed as:

$$\max_{B_2(1), B_2(2)} u\left(Y_1 - \frac{P(1)}{1+r} B_2(1) - \frac{P(2)}{1+r} B_2(2)\right) + \beta \sum_{S=1}^{2} \pi(S) u\left(Y_2(S) + B_2(S)\right)$$

FOC:

$$[B_2(S)]: \qquad \frac{P(S)}{1+r} u'(C_1) = \beta \pi(S) u'(C_2(S))$$

By taking ratios it can be seen the how the relative price depends is equal to the MRS in expected terms (meaning that the more likely a state the higher the relative price of a consumption claim in it),

$$\frac{P(1)}{P(2)} = \frac{\pi(1) u'(C_2(1))}{\pi(2) u'(C_2(2))}$$

Notice that if $\frac{P(1)}{P(2)} = \frac{\pi(1)}{\pi(2)} \Rightarrow C_2(1) = C_2(2)$, i.e., we have Full Insurance.

Risk Sharing Conditions

The conditions above work analogously for the foreign agents. Then we have the following risk sharing conditions:

Intertemporal

$$\begin{aligned} \pi(S)\beta\frac{u'(C_2(S))}{u'(C_1)} &= \frac{P(S)}{1+r} = \pi(S)\beta\frac{u'(C_2(^*S))}{u'(C_1^*)} \\ \Rightarrow \quad \frac{u'(C_2(S)}{u'(C_1)} &= \frac{u'(C_2^*(S))}{u'(C_1^*)} \end{aligned} \tag{1}$$

Across states

$$\begin{aligned} \frac{u'(C_2(S))}{u'(C_2(S'))} &= \frac{P(S)}{P(S')} = \frac{u'(C_2(^*S))}{u'(C_2^*(S'))} \\ \Rightarrow \quad \frac{u'(C_2(S)}{u'(C_2(S'))} &= \frac{u'(C_2^*(S))}{u'(C_2^*(S'))} \end{aligned} \tag{2}$$

Therefore, in a complete markets set up we have equal MRS across countries. **Date by date and state by state.**

With CRRA:

$$C_2(S) = \left[\frac{\beta\pi(S)(1+r)}{P(S)}\right]^{\frac{1}{\rho}} C_1$$

$$C_2^*(S) = \left[\frac{\beta\pi(S)(1+r)}{P(S)}\right]^{\frac{1}{\rho}} C_1^*$$

Market clearing conditions:

$$C_1 + C_1^* = Y_1 + Y_1^*$$
$$C_2(S) + C_2(S)^* = Y_2(S) + Y_2(S)^*$$

By adding the first order conditions and plugging the market clearing constraints:

$$Y_2^w(S) = \left[\frac{\beta\pi(S)(1+r)}{P(S)}\right]^{\frac{1}{\rho}} Y_1^w$$

$$\Rightarrow \quad \beta\pi(S)\left[\frac{Y_2^w(S)}{Y_1^w}\right]^{-\rho} = \frac{P(S)}{1+r}$$

and by taking ratios across states:

$$\frac{P(S)}{P(S')} = \frac{\pi(S)}{\pi(S')}\left[\frac{Y_2^w(S)}{Y_2^w(S')}\right]^{-\rho}$$

Then, by (1):

$$\frac{C_2(S)}{C_1} = \frac{Y_2^w(S)}{Y_1^w} = \frac{C_2^*(S)}{C_1^*}$$

Constant shares or the world output across dates

Analogously, by (2):

$$\frac{C_2(S)}{C_2(S')} = \frac{Y_2^w(S)}{Y_2^w(S')} = \frac{C_2^*(S)}{C_2^*(S')}$$

Constant shares across states

Part II

International Macroeconomics

8 Introduction - Complete Markets benchmark to Sequential Incomplete Markets basic setup

8.1 Complete Markets

Benchmark: A-D structure with complete markets, traded at $t = 0$.

Exogenous state: $s_t \in S$; storty up to t: $s^t = [s_0, \dots, s_t]$

Probability: $\pi_t(s^t)$, conditional on s^τ: $\pi_t(s^t|s^\tau)$

I countries, each with a representative HH: $i = 1, \dots, I$

HH_i owns a stochastic endowment $y^i_t(s^t)$, its consumption is given by $c^i = \{c^i_t(s^t)\}^\infty_{t=0}$

$$U(c^i) = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t u(c^i_t(s^t))\pi_t(s^t) = E_0 \sum_{t=0}^{\infty} \beta^t u(c^i_t)$$

$$\sum_{t=0}^{\infty} \sum_{s^t} q^0_t(s^t) c^i_t(s^t) \leq \sum_{t=0}^{\infty} \sum_{s^t} q^0_t(s^t) y^i_t(s^t) \tag{BC}$$

where: $q^0_t(s^t)$ is the price of contingent period t-consumption claims, with a date of trade 0 and contingent to s^t.

Result: The equilibrium allocation only depends on the aggregate endowment (*Perfect Risk Insurance*) i.e., the consumption won't depend on Country-level endowments/shocks or on preceding history.

UMP:

$$\max U(c^i) \quad s.t. \text{ (BC)}$$

FOC:

$$\beta^t u'(c^i_t(s^t))\pi_t(s^t) = \mu^i q^0_t(s^t)$$

$$\Rightarrow \frac{u'(c^i_t(s^t))}{u'(c^j_t(s^t))} = \frac{\mu^i}{\mu^j} \quad \text{(constant ratios of marginal utilities across histories)}$$

Risk Sharing: Consumption levels will move in the same direction regardless of endowment dynamics

Rearrange:

$$c^i_t(s^t) = u'^{-1}\left(u'(c^1_t(s^t)\frac{\mu_i}{\mu_1}\right)$$

Substitute in MC Cond. ($\sum_i c^i_t(s^t) \leq \sum_i y^i_t(s^t)$):

$$\sum_i u'^{-1}\left(u'(c^1_t(s^t)\frac{\mu_i}{\mu_1}\right) = \underbrace{\sum_i y^i_t(s^t)}_{\text{Aggregate Endowment}}$$

Therefore, consumption is not history dependent (only depends on aggregate y)

Proposition: *The equilibrium allocation is a function of the world endowment.*

$$c_t^i(s^t) = c_\tau^i(\tilde{s}^\tau) \text{ for } s^t,\ \tilde{s}^\tau \text{ if } \sum_i y_t^i(s^t) = \sum_i y_\tau^i(\tilde{s}^\tau)$$

with power utility (CRRA): $c_t^i(s^t) = c_t^j(s^t)\left(\frac{\mu_i}{\mu_j}\right)^{-1/\gamma}$ in this case the fractions of world endowment are independent of the realization of s^t

8.2 Incomplete Markets

Risk sharing condition doesn't hold anymore, the equilibrium allocation is no longer history independent.

- Two countries

- HH supplies L and trades a 1 period bond. The bond is NOT contingent: Markets are incomplete.

- Two consumption goods. Full specialization by country. Competitive firms, distributed in $[0,1]$, $[0,a]$ for home, and $[a,1]$ for foreign firms.

Preferences: $U_{t_0} = E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[\rho\log C_t^i + (1-\rho)\log(1-L_t^j)\right]$

Consumption: $C_t^j = \left[a^{\frac{1}{\omega}}(C_{H,t}^j)^{\frac{\omega-1}{\omega}} + (1-a)^{\frac{1}{\omega}}(C_{F,t}^j)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}$ $\quad$ $\omega>0$: IES bw C_H and C_F

BC: $B_{t+1}^j = (1+r_t)B_t^j + w_tL_t^j - C_t^j$

FOC:

$$L_t = 1 - \frac{1-\rho}{\rho}\frac{C_t}{w_t} \quad \text{(same for i,j)}$$
$$C_t^{-1} = \beta(1+r_{t+1})E_t(C_{t+1}^{-1})$$

Firms

Technology: $Y_t^{S_i} = Z_tL_t^i$ (supply)

Demand: $Y_t^{D_i} = RP_t^{-\omega}C_t^W$ with $C_t^W = aC_t + (1-a)C_t^*$ with RP : price of home goods in units of consumption (measure of relative prices).

in Perfect Competition: Price = Marginal Cost:

$$RP_t = \frac{w_t}{Z_t}$$

Now consider Monopolistic Competition:

$$Y_t^{S_i} = Y_t^{D_i}, \quad C_t^W = Y_t^{S_W} = Y_t^{D_W} = Y_t$$
$$Z_tL_t = (RP_t)^{-\omega}Y_t^W$$

Then:

$$L_t = (RP_t)^{-\omega}\frac{Y_t^W}{Z_t}$$

Aggregation:

International Bond MC: $aB_t + (1-a)B_t^* = 0$

Aggregate per-capita output: $y_t = RP_tZ_tL_t$ This is obtained by expressing each firm's output in terms of the world consumption basket, multiplying by the number of firms and dividing by population.

In perfect competition we also have $y_t = w_tL_t$

8.2.1 Indeterminancy of the Steady State

The Steady State equation for the asset accumulation can be expressed as[4]

$$\bar{B}[1-\beta(1+\bar{r})] = \bar{w}[1-(1-\beta)\frac{1+\bar{r}}{\bar{r}}]$$

Additionally from the Euler Equation from consumption we have $\beta(1+\bar{r}) = 1$, i.e., $\bar{r} = \frac{1-\beta}{\beta}$.

Substitute $\bar{r}$ and you obtain an indetermination: $0 = 0$

The reason for the indetermination is that the Euler Equation doesn't depend on the assets.

Now, take the ratio of the Euler Equations:

$$\frac{C_t^{-1}}{C_t^{*-1}} = \frac{C_{t+1}^{-1}}{C_{t+1}^{*-1}}$$

Log-linearize:

$$\mathsf{C_t} - \mathsf{C_t^*} = E_t(\mathsf{C_{t+1}} - \mathsf{C_{t+1}^*})$$

The Consumption differential will be a random walk.

The model still has a solution but it is path dependent (e.g. OR1995).

Problem: Log-linear approximation is valid in a neighborhood of the steady state but with non-stationarity we may depart from such region after any shock.

Implication: Since all shocks are permanent (even the temporary ones), we will have infinite unconditional variances of endogenous variables.

Solutions

1. *Back to complete markets:* Risk Sharing removes non stationarity (Since $\mathsf{C_t} = \mathsf{C_t^*}$ always). The problem is the poor empirical support of this option.

2. *Add the Real Exchange Rate (RER):* The risk sharing condition becomes: $\frac{u'(C_t)}{u'(C_t^*)} = \kappa Q_t$. Then the consumption baskets are allowed to be different (e.g., home bias in consumption). However, the result is that consumption differential and RER are perfectly correlated which doesn't happen in reality (B-S puzzle).

3. *Knife Edge parameter values:*

   Let the superscript D denote the cross country differential across variables in deviations from the SS with $\bar{B} = \bar{B}^* = 0$. We would have the following expressions for human wealth and assets.

$$\mathsf{h_t^D} = \beta \mathsf{h_{t+1}^D} + (1-\beta)\mathsf{w_t^D}$$
$$\mathsf{B_{t+1}} = \frac{1}{\beta}\mathsf{B_t} + (1-a)(\mathsf{y_t^D} - \mathsf{C_t^D})$$

   This system can be rewritten as:

$$\begin{bmatrix} \mathsf{h_{t+1}^D} \\ \mathsf{B_{t+1}} \end{bmatrix} = M_1 \begin{bmatrix} \mathsf{h_t^D} \\ \mathsf{B_t} \end{bmatrix} + M_2 \mathsf{Z_t^D}$$

[4] substitute the optimal labor from the FOC in the budget constraint: $B_{t+1} = (1+r_t)B_t + w_t + \frac{1}{\rho}C_t$. Now let human wealth be defined as $h_t = \sum_{s=t}^{\infty} R_{t,s} w_s$ with $R_{t,s} = \Pi_{u=t+1}^{s}[(1+r_u)]^{-1}$. Consider the deterministic case, then $w_t = w_{t+j}\ \forall j$ and $h_t = w_t + \beta w_t + \beta^2 w_t + \cdots$ can be expressed as $h_t = \frac{1}{1-\beta} w_t = \frac{1+r}{r} w_t$. Finally, in such case it also holds that $B_{t+1} = (1+r_t)B_t$. After these substitutions consumption will be given by $C_t = \rho(1-\beta)[(1+r_t)B_t + h_t]$ and the asset accumulation will be $B_{t+1} = \beta(1+r_t)B_t + w_t - (1-\beta)h_t$.

with

$$M_1 = \begin{bmatrix} \frac{\rho\omega+\beta(1-\rho)}{\beta[1+\rho(\omega-1)]} & -\frac{\rho(1-\rho)(1-\beta)^2}{\beta^2(1-a)[1+\rho(\omega-1)]} \\ -\frac{\omega(1-a)}{1+\rho(\omega-1)} & \frac{1+\rho(\omega\beta-1)}{\beta[1+\rho(\omega-1)]} \end{bmatrix}, \quad M_2 = \begin{bmatrix} -\frac{\rho(1-\beta)(\omega-1)}{\beta[1+\rho(\omega-1)]} \\ \frac{(\omega-1)(1-a)}{1+\rho(\omega-1)} \end{bmatrix}$$

The eigenvalues of M_1 are $1/\beta$ and 1. Since there is one non-predetermined variable and one eigen-value outside of the unit circle the Blahchard-Kahn conditions hold and this system has a determinate solution. However, the eigenvalue equal to 1 is consistent with the non-stationarity of the assets.

Now, if we let $\omega = 1$ then $M_2 = 0$. We would have a unique solution given by $\mathsf{h}_\mathsf{t}^\mathsf{D} = \mathsf{B}_\mathsf{t} = 0\ \forall t$ (Cole and Obstfeld 1991).

This means that for initial $NFA = 0$, $\omega = 1$ (consumption basket in CD form with unitary elasticity between home and foreign goods) we get as solution that $NFA_t = 0 \forall t$ and that there will not be a consumption differential.

This result is driven by the ToT. With $\omega = 1$ the ToT will move in a way that $\mathsf{RP}_\mathsf{t}^\mathsf{D} = -\mathsf{Z}_\mathsf{t}^\mathsf{D}$. Then, the price and quantity effects of the ToT will exactly offset.

4. *Make the Euler Equation depend on Asset Holdings:* for example, include convex cost of adjustment of bond holdings.

   With convex cost the BC becomes:

$$B_{t+1}^j + \frac{\mu}{2}(B_{t+1}^j - B)^2 = (1+r_t)B_t^j + w_t L_t^j + T_t^j - C_t^j$$

   The EEq becomes:

$$C_t^{-1}[1 + \mu(B_{t+1} - B)] = \beta(1 + r_{t+1})E_t(C_{t+1}^{-1})$$

   in the SS:

$$1 + \mu(\bar{B} - B) = \beta(1 + \bar{r})$$
$$1 + \mu(\bar{B}^* - B^*) = \beta(1 + \bar{r})$$

   then,

$$1 + \mu\underbrace{[a\bar{B} + (1-a)\bar{B}^* - aB - (1-a)B^*]}_{0} = \beta(1 + \bar{r})$$

   then $1 = \beta(1 + \bar{r})$, i.e., in SS: $\bar{B} = B$, $\bar{B}^* = B^*$ Meaning that the NFA will be equal to an exogenous reference level. This method is popular but is an ad-hoc solution.

   Other solutions within this framework: Exogenous Discount Factor, Debt elastic Interest rate premium, bonds in utility (S-GU2003) (all link Consumption growth and assets)

   Alternative framework solution: OLG (Breaking Ricardian Equivalence)

   - 2 Countries H,F

   - Each period there are N_t^W agents (infinitely lived). The population growth is given by $N_{t+1} = (1+n)N_t$, $N_0^W = 1$.

   - HH's are born at different periods with no assets.

   Key changes:

   The EE now depends on C_{t+1}^{t+1} (newborns consumption at $t+1$)

$$c_t = \frac{1+n}{\beta(1+r_{t+1})}\left(c_{t+1} - \frac{n}{1+n}C_{t+1}^{t+1}\right) \qquad \text{if} n > 0 \text{the model is stationary}$$

with $C_{t+1}^{t+1} = \rho(1-\beta)h_{t+1}$

Dynamics of the NFA:

$$(1+n)B_{t+1} = (1+r_t)B_t + w_t L_t - c_t$$

There will be a discrepancy bw average asset holdings of old and newborns. Then aggregate percapita consumption growth depends on the asset holdings.

Stationarity: A given shock affects assets of alive population but newborns enter and wipe the effect away over time (temporary shocks won't have permanent effect anymore).

Then endogenously: $\bar{B} = \bar{B}^* = 0$, $\bar{r} = (1-\beta)/\beta$ (if an agent is a debtor or creditor will depend on whether $\beta(1+\bar{r}) \gtrless 1$

9 BKK 1992

Context/Benchmark: Mendoza 1991, IRBC-SOE with incomplete markets and endogenous Discount Factor.

- Two countries (analogous), one final good. No role for RER in BKK 1992

- Immobile labor

- Production subject to country specific shocks

- Setup: Campbell 1994 with: (i) Leisure habits, (ii) inventories, (iii) time to build. These are all eliminated in BKK 1994

l_t^i : Leisure

Labor follows a distributed lag:

$$\begin{aligned} l_t &= 1 - \alpha n_t - (1-\alpha)\eta a_t \\ a_{t+1} &= (1-\eta)a_t + n_t \end{aligned} \tag{3}$$

Where n_t : working time and a_t: effect of past leisure past leisure choices affect current utility.

UMP:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[(c_t^i)^{\mu}(l_t^i)^{1-\mu}\right]^{\gamma}}{\gamma} \qquad i = h, f$$

Technology:

$$y_t^i = \left\{[\lambda_t^i (k_t^i)^{\theta}(n_t^i)^{1-\theta}]^{-\nu} + \sigma(z_t^i)^{-\nu}\right\}^{-1/\nu} \qquad \nu > -1,\ \sigma > 0 \tag{4}$$

λ_t^i: technology shock

Market Clearing Feasibility: Global output is equal to sum of aggregate consumption, fixed investment and inventory accumulation.

$$\sum_i (c_t^i + x_t^i + z_{t+1}^i - z_t^i) = \sum_i y_t^i \tag{5}$$

Net exports:

$$nx_t^i = y_t^i - (c_t^i + \underbrace{x_t^i}_{\text{investment}} + \underbrace{z_{t+1}^i - z_t^i}_{\text{Inventories change}}) \tag{6}$$

Capital dynamics:

$$k_{t+1} = (1-\delta)k_t^i + \underset{\substack{\text{inv. projects 1 period} \\ \text{from completion}}}{s_{1,t}^i} \tag{7}$$

Fixed investment:

$$x_t^i = \sum_{j=1}^{J} \phi_j s_{j,t}^i \tag{8}$$

Technology Shocks: Country specific (idiosyncratic) $\lambda_t = [\lambda_t^h, \lambda_t^f]' \sim VAR(1)$, i.e.,

$$\boldsymbol{\lambda}_{t+1} = \mathbf{A}\boldsymbol{\lambda}_t + \boldsymbol{\epsilon}_{t+1}, \quad \text{with} \quad \boldsymbol{\epsilon}_t \sim MVN(0, \mathbf{V}) \tag{9}$$

Equilibrium: Pinned down by solving a SPP,

$$\max \phi E(U^h) + (1-\phi)E(U^f)$$
$$s.t. \;\; (3) - (9)$$

Deterministic Steady State:

Because of symmetry, the SS is the same of a closed economy but replicated twice.

Implicit Assumption: Countries start with zero initial net wealth.

Consumption, labor and inventories are constant. Then: $r = \frac{1-\beta}{\beta}$

Investment $= \delta K$

then the resource constraint becomes: $c + \delta k = y$

Rental price of inventories: r

Rental price of capital: $q(r+\delta)$, with $q = \sum_i^J \phi_j (1+r)^{j-1}$

FOCs:

$$\begin{aligned} r &= \sigma(y/z)^{1+\nu} \\ q(r+\delta) &= \theta(y/k)[1-\sigma(y/z)^{\nu}] \\ w &= (1-\theta)(y/n)[1-\sigma(y/z)^{\nu}] \end{aligned}$$

Consumers's FOC for leisure ($U_l/U_c = w$):

$$\frac{(1-\mu)c(\alpha r + n)}{r+\nu} = \mu w(1-n)$$

Calibration: $\nu = 3,\; J = 4,\; \alpha = 1,\; \mu = .34,\; \gamma = -1$

$$A = \begin{bmatrix} 0.91 & 0.088 \\ 0.088 & 0.91 \end{bmatrix}$$

Results:

Relative to data:

- Consumption is too smooth
- Investment is too volatile
- S shaped profile of consumptions cross-correlation is found but way weaker than in the data (also too high consumption correlation)
- Trade Balance not countercyclical

- Negative output correlation
- Too high consumption correlation - far more than output (Feldstein-Horioka puzzle)

IRFs: home investment boom, i.e., shock to ϵ^h.

At home:

- $\uparrow$ productivity that decays
- $\uparrow$ Investment, consumption and output
- Given large $\uparrow$ Investment (larger than output) $\Rightarrow$ Trade deficit

Foreign:

- Positive spillover in productivity
- Initially $\downarrow$ y, $\downarrow$ x due to resources shifting to home. No assets interaction b/w countries but solution is given by a SPP, then policy maker shifts resources. Furthermore, added to technology there is also a high interaction via trade of the final good.

Intuition: The discrepancies with the data appear because agents have greater ability to shift resources across countries. That could explain the found flaws: Excessive consumption smoothing, excessive volatility of investment, negative output correlation.

Introduction of trade costs: The goal is to make harder for agents to shift resources.

Trade costs: after including convex costs of trade feasibility becomes

$$\sum_i (c_t^i + x_t^i + z_{t+1}^i - z_t^i) = \sum_t y_t^i - \sum_i \tau (nx_t^i)^2$$

Under autarky: spillover effects come only from technology shocks correlation (only connection between countries).

With the trade cost: $Corr(y^h, y^f) = 0.02$ (before it was -0.18).

but still $Corr(c^h, c^f) = 0.88$ (too high although improved since before it was 0.91).

Conclusion: New puzzle (Consumption anomaly puzzle)

$Corr(y, y^*) > Corr(c, c^*)$ in the data

$Corr(y, y^*) < Corr(c, c^*)$ in the model

with negative correlation of output.

10 BKK1994

- Two country IRBC to analyze two features of the data:

1. Countercyclical Trade Balance
2. S shaped correlation between Trade Balance (TB) and Terms of Trade (ToT). Negative correlation with future ΔToT, positive correlation with past ΔToT

- Fully specialized production $\Rightarrow$ Role for ToT. main difference with BKK1992

Good a is produced in country 1 and good b in country 2.

- Government spending shocks (more sources of fluctuations).

- **Complete Assets Markets** also in BKK1992. Then the equilibrium is solvable by freely transferring resources between markets via a SPP.

- No habits of leisure nor inventories.

UMP:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[(c_{i,t})^{\mu}(1-n_{i,t})^{1-\mu}\right]^{\gamma}}{\gamma}, \quad i=1,2 \quad \gamma<1$$

Technology:

$$y_{i,t} = Z_{i,t} k_{i,t}^{\theta} n_{i,t}^{1-\theta}, \quad i=1,2$$

where $Z_{i,t}$ is an aggregate technology shock in the economy i.

Resource constraints:

$$a_{1,t} + a_{2,t} = y_{1,t}$$
$$b_{1,t} + b_{2,t} = y_{2,t}$$

with a_i, b_i: usage of each good by country i.

Then, $a_{2,t}$: exports from country 1 to country 2, $b_{1,t}$: imports from 2 into 1.

New: Consumption, investment and government purchases are CES composites of a and b:

$$c_{1,t} + x_{1,t} + g_{1,t} = \left[\omega_1 (a_{1,t})^{-\rho} + \omega_2 (b_{1,t})^{-\rho}\right]^{-\frac{1}{\rho}}$$
$$c_{2,t} + x_{2,t} + g_{2,t} = \left[\omega_1 (b_{2,t})^{-\rho} + \omega_2 (a_{2,t})^{-\rho}\right]^{-\frac{1}{\rho}}$$

This representation allows for Home Bias in consumption (the RER plays a role here).

Notice that the goods are crossed in the function, i.e., for each country the first coefficient represents the share of local good in consumption.

- $\sigma = \frac{1}{1+\rho}$: Elasticity of Subst. between domestic and foreign good.

- g is exogenous and stochastic

Capital dynamics:

$$k_{i,t+1} = (1-\delta)k_{i,t} + \underbrace{s_{i,t-J+1}}_{\substack{\text{planned additions} \\ \text{to capital stock in} \\ t+J}}$$

Investment: sum of expenditures on active projects $x_{i,t} = \frac{1}{J}\sum_{j=0}^{J-1} s_{i,t-j}$

$J = 1$ used for the calibration.

Shocks: technology and government spending

$$\mathbf{z}_{t+1} = \mathbf{A}\mathbf{z_t} + \boldsymbol{\epsilon}^z_{t+1}, \; \mathbf{z_t} = [z_{1,t}, z_{2,t}]', \boldsymbol{\epsilon}^z_t = [\epsilon^z_{1,t}, \epsilon^z_{2,t}]$$
$$\mathbf{g}_{t+1} = \mathbf{A}\mathbf{g_t} + \boldsymbol{\epsilon}^g_{t+1}, \; \mathbf{g_t} = [g_{1,t}, g_{2,t}]', \boldsymbol{\epsilon}^g_t = [\epsilon^g_{1,t}, \epsilon^g_{2,t}]$$

Resource constraint by country:

$q_{1,t}$: Price of a good a in units of the consumption basket.
$q_{2,t}$: Price of a good b in units of the consumption basket.

Then for country 1:

$$c_{it} + x_{it} + g_{it} = q_{1t}a_{1t} + q_{2t}b_{1t}$$

The ToT are defined as: $p_t = q_{2t}/q_{1t}$ (relative price of imports, reverse of OR1995 definition.)

The ToT is obtained as the MRT between the two goods in country 1:

$$p_t = \frac{q_{2t}}{q_{1t}} = \frac{\omega_2}{\omega_1}\left(\frac{a_{1t}}{b_{1t}}\right)^{1/\sigma}$$

Results:

NX and ToT are highly correlated Here inclusion of K is key. Otherwise the agents have very high incentives to smooth consumption.

NX are counter cyclical: by adding K the model displays procyclical investment, then, $\uparrow I, \uparrow C \Rightarrow Corr(Y, NX) < 0$

Strong positive correlation between ToT and Y. (This is still a flaw since it's negative in the data).

The correlation is negative, but with very high substitutability (σ) it can become positive. Also the S shaped pattern remains for the different values of σ.

Withouth Capital: No S curve, TB becomes procyclical and $Corr(TB, ToT) > 0$. The reason is that there's not an investment boom upon the technology shock.

Problem: Too little ToT volatility.
New Puzzle: Toot little reltive price volatility.
Same unresolved puzzle: Consumption-ouput anomaly (BKK Puzzle)

11 BS1993

Before, the ToT are relevant. Still, with all goods were traded, LOP holds and symmetric preferences the PPP holds. With $P_t = P_t^*$ we have a constant RER ($RER_t = 1\ \forall t$).

Non Traded Goods (NT) are added so that the consumption baskets differ, hence, the PPP won't hold that the RER fluctuates.
the traded good is the same so that the $ToT_t = 1$ and the LOP hold. Still the consumption baskets differ.

The set up assumes Complete Markets.

Prediction: $\frac{u'(c)}{u'(c^*)} \approx RER_t$ (Consumption ratio and the RER are perfectly correlated).

BS Puzzle: Data shows 0 or negative sign.

Model:

I countries, $I+1$ goods (I non tradables, 1 in each country).

States: z_t, history: z^t, prob. of z^t: $\pi(z^t)$
q_0: date t price of traded good, Q_0: date 0 price.
q_i: date t price of i's non traded good, Q_i: date 0 price.
a_i: country i's consumption of traded good
b_i: country i's consumption of non-traded good

UMP:

$$\max U_i = \sum \beta^t \sum_{z^t \in Z^t} \pi(z^t) u[a_i(z^t), b_i(z^t)] \tag{1}$$

Price index: $p_i(q_0, q_i)$

Quantity index: $c_i(a_i, b_i)$, s.t., $u[a_i, b_i] = v[c_i(a_i, b_i)]$, $v'(\cdot) > 0$

With homothetic preferences: $\exists\ p)i, c_i$, s.t.:

$$q_0 a_i + q_i b_i = p_i(q_0, q_i) c_i(a_i, b_i)$$

Utility is assumed time separable, but period utility is not separable between traded and non-traded goods a_i, b_i.

Example of an aggregator:

$$c(a_i, b_i) = [\alpha a_i^{\rho} + (1-\alpha) b_i^{\rho}]^{\frac{1}{\rho}}$$
$$p_i(q_0, q_i) = \left[\alpha^{\frac{1}{1-\rho}} q_0^{\frac{\rho}{\rho-1}} + (1-\alpha)^{\frac{1}{1-\rho}} q_i^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}}$$

$\frac{1}{1-\rho}$: elasticity of substitution between traded and non-traded goods ($\rho = 1$: perfect substitutes, $\rho = 0$: Cobb-Douglas basket)

RER:

$$e_{ij} = \frac{p_j(q_0, q_j)}{p_i(q_0, q_i)}$$

Risk free bond (A-D securities):

$$s_i(z^t) = \sum_{z_{t+1}} \frac{p_i[Q_0(z^{t+1}), Q_i(z^{t+1})]}{p_i[Q_0(z^t), Q_i(z^t)]}$$

Real interest rate:

$$r_i(z^t) = s_i(z^t)^{-1} - 1, \quad r_i(z^t) \approx -\log[s_i(z^t)]$$

Date 0, present value, Budget Constraint:

$$\sum_{t=0}^{T} \sum_{z^t} \left[Q_0(z^t) a_i(z^t) + Q_i(z^t) b_i(z^t)\right] = Q_0(z_0) n_i(z_0) + \sum_{t=0}^{T} \sum_{z_t} \left[Q_0(z^t) w_i(z^t) + Q_i(z^t) x_i(z^t)\right] \tag{2}$$

where $n_i(z_0)$ is the net foreign asset position of country i at the start of the period 0.

The consumer's UMP is to choose $\{a_i, b_i\}_{t=0}^{\infty}$ to maximize (1) s.t. (2)

Additionally, a solution must satisfy the market clearing condition in all states:

$$\sum_{i=1}^{I} a_i(z^t) = \sum_{i=1}^{I} w_i(z^t) \equiv W(z^t)$$
$$b_i(z^t) = x_i(z^t), \quad \forall i, z^t$$

Countries cannot hedge each other against idiosyncratic risk on the endowment of the non-traded good.

Equilibrium:

without NT good:

PPP holds: $e_{ij} = 1$

If u(c) is CRRA: $p(z^t) = W(z^t)^{-\gamma}$

Consumption rtios are constant (then consumption of i and j are perfectly correlated)

Real interest rates are equalized:

$$s_i(z^t) = \sum_{z_{t+1}} \beta \pi(z_{t+1}|z^t) \frac{p(z^{t+1})}{p(z^t)}$$

Intuition: Since there is only one good, the real rate of return on the risk free consumption based bond is the same to all countries.

With NT good:

The equilibrium is characterized by solving a SPP:

$$\max \sum_{i=1}^{I} \lambda_i U_i \quad s.t. \quad \sum_{i=1}^{I} a_i(z^t) \leq W(z^t)$$
$$b_i(z^t) \leq x_i(z^t)$$

the respective lagrange multipliers are: $\beta^t \pi(z^t) q_0(z^t)$, $\beta^t \pi(z^t) q_i(z^t)$.

FOCs:

$$[a_i]: \quad \lambda_i \frac{\partial u(a_i, b_i)}{\partial a_i} = q_0, \qquad [b_i]: \quad \lambda_i \frac{\partial u(a_i, b_i)}{\partial b_i} = q_i$$

i.) PPP won't hold since the endowments and prices of NT goods differ

ii.) Consumption indexes are NOT perfectly correlated

iii.) Monotonic relation between $\frac{c_i}{c_j}$ and RER e_{ij}. The FOC (with CRRA) implies $(\lambda_j/\lambda_i)\left(\frac{c_i}{c_j}\right)^{\gamma} = \frac{p_j}{p_i} = e_{ij}$

Consider the log-linearized model:

$$\gamma(\mathsf{c_i} - \mathsf{c_j}) = \mathsf{e_{ij}}$$

then:

$$\gamma^2 Var(\mathsf{c_i} - \mathsf{c_j}) = Var(\mathsf{e_{ij}})$$

i.e., the variances will be equal only if $\gamma = 1$.

Summary:

PPP won't hold because of the presence of NT goods

Real interest rate will vary across i.

There will be restrictions on the moments of real interest rate differential.

BS Puzzle: Model implies perfect correlation between consumption differential and RER. Obvious candidate to deal with this puzzle: Market incompleteness.

12 Deviations of the PPP

TBA