ECON 6356
International Finance and Macroeconomics

Lecture 9: Exchange Rate Dynamics Determination

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Introduction

All models until now have assumed monetary policy to be exogenous (or set based on rules).

However, given the meaningful role that monetary policy can have on the determination of the ER dynamics, a parallel literature developed to explore the implications of endogenous interest rate setting for the ER and welfare in open economies.

In this line, we first move to Benigno and Benigno (2008) to examine the role of interest rate decisions on the ER. We then discuss the role of NFA fluctuations in driving the ER fluctuations with Cavallo and Ghironi (2012).

Finally, we compare how the monetary policy prescriptions differ in open economies, relative to closed economies where price stability is the optimal policy prescription, or to setups that yielded an isomorphism between the optimal prescriptions of closed and open economies under very specific conditions.

Endogenous Interest Rate Setting and ER Dynamics

Until now, we considered monetary shocks to be exogenous or part of a Taylor Rule setup. Now, we consider Benigno and Benigno (2008, JIMF), a framework with endogenous interest rate setting for open economies.

Central insight: Role for money can be de-emphasized —money demand plays no role in the ER determination.

Model

- Continuum of HH and goods in $[0, 1]$: n in home, $(1 - n)$ in the foreign country.

- Each firm produces a differentiated good under monopolistic competition.

- Firms (under monopolistic competition) set prices in PCP, no home bias (same consumption basket), and LOP holds $\Rightarrow$ PPP holds.

- Complete asset markets.

- Price rigidity as in Calvo (1983) - Yun (1996).

The model will have three blocks of equations, the aggregate demand (AD) block, the aggregate supply (AS), and the policy block.

Aggregate demand block

Consumption aggregate:

$$C^j = \frac{(C^j_H)^n (C^j_F)^{1-n}}{n^n (1-n)^{1-n}},$$

with,

$$C^j_H = \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n c^j(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}}, \quad C^j_F = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\theta}} \int_n^1 c^j(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}$$

Prices:

$$P = (P_H)^n (P_F)^{1-n}$$

with,

$$P_H = \left(\frac{1}{n} \int_0^n p(h)^{1-\theta} dh\right)^{\frac{1}{1-\theta}}, \quad P_F = \left(\frac{1}{1-n} \int_n^1 p(f)^{1-\theta} df\right)^{\frac{1}{1-\theta}}$$

TOT are defined as $TOT \equiv P_F/P_H = P^*_F/P^*_H$ (this is the reciprocal of the usual definition, and the second equality follows from the law of one price). Given the reciprocal definition, a TOT decrease is an improvement or appreciation ot the terms-of-trade.

Demands:

For individual goods and by household j,

$$c^j(h) = \left(\frac{p(h)}{P_H}\right)^{-\theta} TOT^{1-n} C^j$$

$$c^j(f) = \left(\frac{p(f)}{P_F}\right)^{-\theta} TOT^{-n} C^j$$

$$g(h) = \left(\frac{p(h)}{P_H}\right) G_H$$

Total demands of homegood h and foreign good f after aggregating demands by households (of each location) and governments,

$$y^d(h) = \left(\frac{p(h)}{P_H}\right)^{-\theta} \left(TOT^{1-n} C^W + G_H\right), \quad y^d(f) = \left(\frac{p(f)}{P_F}\right)^{-\theta} \left(TOT^{-n} C^W + G_F^*\right),$$

where $C^W \equiv nC + (1-n)C^*$.

Aggregate demand block(cont.)

Aggregate demand for home output is obtained by aggregating quantities of individual goods across home producers (after putting them in comparable units —home sub-basket),

$$Y_{AG}^{H} = \int_{0}^{n} \frac{p(h)}{P_H} y^d(h)dh = \left[\left(\frac{1}{n} \right)^{\frac{1}{\theta}} \int_{0}^{n} y^d(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}},$$

where the last equality follows after substituting the expression for P_H and $p(h) = [y^d(h)/(TOT^{1-n}C^W + G_H)]^{-1/\theta} P_H$.

Substitute $y^d(h)$ to obtain,

$$Y_{AG}^{H} = n(TOT^{1-n}C^W + G_H),$$
$$Y_{AG}^{F} = (1-n)(TOT^{-n}C^W + G_F^*).$$

Aggregate demand block (cont.)

In per capita terms:

$$Y^H = \frac{Y^H_{AG}}{n} = \left[\frac{1}{n}\int_0^n y^d(h)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{\theta}{\theta-1}},$$

$$Y^F = \frac{Y^F_{AG}}{1-n} = \left[\frac{1}{1-n}\int_n^1 y^d(f)^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta}{\theta-1}},$$

and then, $Y^H = TOT^{1-n}C^W + G_H$, and $Y^F = TOT^{-n}C + G^*_F$.

Now, recall Complete markets + Symmetric initial NFA implies: $C = C^* = C^W$.

Then,

$$Y^H = TOT^{1-n}C + G_H, \quad Y^F = TOT^{-n}C + G^*_F.$$

There is full insurance even (at consumption level) but also potentially different output levels (e.g., after TOT and gov. spending shocks).

Aggregate demand block (cont.)

Log-linearizing (and adding back the time indexes):

$$\mathsf{Y}_t^H = (1-n)\mathsf{TOT}_t + \mathsf{C}_t + \mathsf{G}_{H,t}, \quad \mathsf{Y}_t^F = n\mathsf{TOT}_t + \mathsf{C} + \mathsf{G}_F^*,$$

where $\mathsf{G}_{H,t}$ and G_F^* are country-specific demand shocks (from government spending).

These equations imply a global market clearing condition for the final goods market:

$$\mathsf{Y}_t^W = \mathsf{C}_t + \mathsf{G}_t^W,$$

where the $\mathsf{Y}_t^W \equiv n\mathsf{Y}_t^H + (1-n)\mathsf{Y}_t^F$ and $\mathsf{G}_t^W \equiv n\mathsf{G}_{H,t} + (1-n)\mathsf{G}_{F,t}^*$.

Euler equation:

Depart from the individual Euler equations:

$$\mathbb{E}_t \mathsf{C}_{t+1} = \mathsf{C}_t + \rho^{-1}(\mathsf{i}_{t+1} - \mathbb{E}_t \widehat{\pi}_{t+1}), \text{and } \mathbb{E}_t \mathsf{C}^*_{t+1} = \mathsf{C}^*_t + \rho^{-1}(\mathsf{i}^*_{t+1} - \mathbb{E}_t \widehat{\pi}^*_{t+1}),$$

where $\widehat{\pi}_t$ is the percent deviation of home consumer inflation from the steady state. Also, i_{t+1} denotes the interest agreed on t and payable on $t+1$ so it is a known quantity at t.

Taking a population size weighted average of the Euler equations:

$$n\mathbb{E}_t \mathsf{C}_{t+1} + (1-n)\mathbb{E}_t \mathsf{C}^*_{t+1} = n\mathsf{C}_t + (1-n)\mathsf{C}^*_t + n\rho^{-1}(\mathsf{i}_{t+1} - \mathbb{E}_t \widehat{\pi}_{t+1}) + (1-n)\rho^{-1}(\mathsf{i}^*_{t+1} - \mathbb{E}_t \widehat{\pi}^*_{t+1}),$$

and recalling that under complete markets $\mathsf{C}_t = \mathsf{C}^*_t$, and noting that, given identical consumption baskets and the LOP we would have the following link between producer prices and consumer prices:

$$n\widehat{\pi}_t + (1-n)\widehat{\pi}^*_t = n\widehat{\pi}^p_t + (1-n)\widehat{\pi}^{p*}_t,$$

where $\widehat{\pi}^p_t$ ($\widehat{\pi}^{p*}_t$) is the percent deviation of the home (foreign) *producer* inflation rate from the steady state. (BB2008 assume the steady state features zero producer price inflation).

Euler equation (cont.)

Given the last expression, and the initial Euler equations, the average of Euler equations above is equivalent to

$$\mathbb{E}_t \mathsf{C}_{t+1} = \mathsf{C}_t + \rho^{-1} n(\mathsf{i}_{t+1} - \mathbb{E}_t \widehat{\pi}^{p}_{t+1}) + \rho^{-1}(1-n)(\mathsf{i}^*_{t+1} - \mathbb{E}_t \widehat{\pi}^{p*}_{t+1}). \quad (1)$$

Let the world output gap with respect to the flexible prices allocation be $\mathsf{y}^W_t = \mathsf{Y}^W_t - \tilde{\mathsf{Y}}^W_t$. Then, use $\mathsf{Y}^t_W = \mathsf{C}^W_t + \mathsf{G}^W_t$ to rewrite the Euler equation,

$$\mathbb{E}_t \mathsf{y}^W_{t+1} = \mathsf{y}^W_t + \rho^{-1} n(\mathsf{i}_{t+1} - \mathbb{E}_t \widehat{\pi}^{p}_{t+1} - \tilde{\mathsf{R}}^W_{t+1}) + \rho^{-1}(1-n)(\mathsf{i}^*_{t+1} - \mathbb{E}_t \widehat{\pi}^{p*}_{t+1} - \tilde{\mathsf{R}}^W_{t+1}), \quad (2)$$

where $\tilde{\mathsf{R}}^W_{t+1} \equiv n\tilde{i}_{t+1} + (1-n)\tilde{i}^*_{t+1}$ is the real rate that would arise if prices were perfectly flexible (or the world nominal rate if each country's producer price inflation is zero under flexible prices).

Eq. (2) is the intertemporal, microfounded IS expression in the BB2008 model.

Note: $\tilde{\mathsf{R}}^W_{t+1}$ also denotes the perturbations to the world natural real interest rate (natural *Wicksellian* interest rate in log-linear terms as in Woodford, 2003).

Terms of Trade:
The definition of TOT implies,

$$\mathsf{TOT}_t = \mathsf{TOT}_{t-1} + \mathsf{e}_t + \bar{\pi}_t^{p*} - \bar{\pi}_t^{p}, \tag{3}$$

where e_t is the percentage deviation of gross nominal exchange rate depreciation $(\varepsilon_t/\varepsilon_{t-1})$.

Notice this implies a change of notation with respect to papers discussed in previous sections. Unlike before, here the log-linear version (percentage deviation relative to the steady state) of the exchange rate is ϵ (and not e), and $\mathsf{e}_t = \epsilon_t - \epsilon_{t-1}$.

Therefore, the model will have a state variable: Lagged TOT

The model displays persistence additional to that of the exogenous shocks. Good feature as in closed economy models this is modified in an ad-hoc manner to generate hump shaped IRFs.

Interest rates: On the other hand, the UIP holds $\mathsf{i}_{t+1} - \mathsf{i}^*_{t+1} = \mathbb{E}_t \mathsf{e}_{t+1}$.

Aggregate Supply Block

The AS block of the model consists of New Keynesian Phillips curves for each location's producer price inflation (as implied by the type of nominal rigidity),

Phillips curves:

$$\widehat{\pi}_t^p = \lambda \mathsf{mc}_t + \beta \mathbb{E}_t \widehat{\pi}_{t+1}^p, \tag{4}$$
$$\widehat{\pi}_t^{*p} = \lambda \mathsf{mc}_t^* + \beta \mathbb{E}_t \widehat{\pi}_{t+1}^{*p}, \tag{5}$$

where $\mathsf{mc}_t(\mathsf{mc}_t^*)$ is the percentage deviation of home (foreign) marginal cost from the steady state, $\lambda \equiv [(1-\alpha\beta)(1-\alpha)]/[\alpha(1+\theta\eta)]$ and $\lambda^* \equiv [(1-\alpha^*\beta)(1-\alpha^*)]/[\alpha^*(1+\theta\eta)]$. In these terms, $1-\alpha$ is the probability of price adjustment for home firms, and η the elasticity of labor disutility.

Marginal costs: FOCs for output supply make it possible to obtain the marginal costs as (a function of the MRS between consumption and production):

$$\mathsf{mc}_t = (1-n)(1+\eta)\left(\mathsf{TOT}_t - \tilde{\mathsf{TOT}}_t\right) + (\rho+\eta)\mathsf{y}_t^W, \tag{6}$$
$$\mathsf{mc}_t^* = -n(1+\eta)\left(\mathsf{TOT}_t - \tilde{\mathsf{TOT}}_t\right) + (\rho+\eta)\mathsf{y}_t^W. \tag{7}$$

Aggregate Supply Block (cont.)

Intuition for $mc \propto y$: the marginal cost can usually be expressed as a function of total output, as the input prices are derived from FOC implying the production function. For an example, think of the log-utility (of consumption) case, with quadratic cost of effort $l(j)^2/2$ and a linear production function $y(j) = l(j)$.
This specific case also shows how the expression involves the elasticity of labor disutility.

Importantly, we can see the marginal costs will depend on the movements in the TOT (with opposite signs for different locations).

To see this even more clearly, note that when $\rho = \eta = 1$ the expression becomes $\text{mc}_t = 2[(1-n)(\text{TOT}_t - \tilde{\text{TOT}}_t) + \text{y}_t^W]$.

Using (6) and (7) we can rewrite the NK Phillips curves as

$$\widehat{\pi}_t^p = k_C \text{y}_t^W + (1-n)k_T\left(\text{TOT}_t - \tilde{\text{TOT}}_t\right) + \beta\mathbb{E}_t\widehat{\pi}_{t+1}^p, \quad (8)$$

$$\widehat{\pi}_t^{*p} = k_C^* \text{y}_t^W - nk_T^*\left(\text{TOT}_t - \tilde{\text{TOT}}_t\right) + \beta\mathbb{E}_t\widehat{\pi}_{t+1}^{*p}, \quad (9)$$

where $k_C \equiv \lambda(\rho+\eta)$, $k^C \equiv \lambda^*(\rho+\eta)$, $k_T \equiv k_C\frac{1+n}{\rho+\eta}$, and $k_T^* \equiv k_C^*\frac{1+n}{\rho+\eta}$.

Aggregate Supply Block (cont.)

Implications: Real marginal costs are **not** only proportional to the output gap anymore due to cross-country interdependence via TOT movements.
(before $\mathrm{mc} \propto \mathrm{y}$, now $\mathrm{mc} = f(\mathrm{y}, TOT)$.)

In fact, the smaller and more open the economy, the more the TOT affects the marginal cost and inflation.

$\uparrow$ TOT $\rightarrow$ NKPC shifts and $\uparrow \pi$: home goods become relatively cheaper, then demand increases (and lowers the marginal utility of nominal income).

On the other hand, the relation between TOT and the marginal cost creates inertia in the resulting marginal costs, and then on inflation (as TOT depend on its lags). This is an improvement with respect to closed economy models making unnecessary to include lags of the variables ad-hoc (e.g., hybrid NKPCs).

Monetary Policy

Credible commitment is assumed. Three cases are considered:

- **Fixed Exchange Rates**: Follower economy sets interest rate to peg domestic currency to foreign one. The leader follows an interest rate rule without responses to the ER.

- **Flexible Exchange Rate**: Interest rate rule without ER (as argument).

- **Managed Exchange Rate**: One of the two countries responds to ER in addition to inflation and output.

Digression: policy rules under a peg and equilibrium determinacy

A common interpretation to the fact that a fixed ER implies the country to shadow the leader's interest rate $\mathrm{i}_{t+1} = \mathrm{i}^*_{t+1}$ is that the follower country follows a policy rule of the form: $\mathrm{i}_{t+1} = \mathrm{i}^*_{t+1}$. This is a common misconception.

If that were the case, the ER would become indeterminate: (using the UIP, $\mathrm{i}_{t+1} - \mathrm{i}^*_{t+1} = \mathbb{E}_t \mathrm{e}_t$)

$$\mathbb{E}_t \epsilon_{t+1} = \epsilon_t$$

The ER would be a random walk, or any ϵ_t such that it equals the expected value of tomorrow's ER would work. Such kind of (indeterminate) rule can actually be very costly in terms of welfare.

Instead, imagine the foreign country (trying to implement a peg to the other country) follows a rule given by:

$$\mathrm{i}^*_{t+1} = \mathrm{i}_{t+1} + \tau \epsilon_t.$$

Such rule states that the bank commits to increasing the rate if the foreign currency depreciates.

Combining this proposed rule with the UIP we obtain $(1+\tau)\epsilon_t = \mathbb{E}_t \epsilon_{t+1}$. This equation has only one solution $\epsilon_t = 0$.

Digression: policy rules under a peg and equilibrium determinacy (cont.)

Combining this proposed rule with the UIP we obtain $(1+\tau)\epsilon_t = \mathbb{E}_t\epsilon_{t+1}$. This equation has only one solution $\epsilon_t = 0$:

$$\epsilon_t = \frac{1}{1+\tau}\mathbb{E}_t\epsilon_{t+1} = \left(\frac{1}{1+\tau}\right)^2 \mathbb{E}_t\varepsilon_{t+2} = \cdots = \left(\frac{1}{1+\tau}\right)^T \mathbb{E}_t\varepsilon_{t+T} \xrightarrow[T\to\infty]{} 0$$

Then, we get a peg as a result of such rule ($\epsilon_t = 0$).

Thus: $\mathrm{i}^*_{t+1} = \mathrm{i}_{t+1}$ is a consequence, and not a policy rule.

Here a credible threat to increase the interest rate if ER moves implies zero movements by the ER and yields an endogenous interest rate equalization.

How this is reflected in the literature:

- **Sargent and Wallace (1975, JPE)**: Interest rate pegging results in indeterminacy (policy irrelevance result).

- **Woodford (2003)**: Indeterminacy appears because the interest rate cannot pin down any endogenous variables. In such case the problem originates from using a rule that sets the policy as a function of exogenous shocks (or variables) only. Instead, the rate should be set in terms of endogenous variables too (e.g, the exchange rate).

Monetary Policy (cont.)

If we have that to pin down the dynamics of the exchange rate, a policy rule must set the interest rate differential as a function of the ER (or depreciation).

A similar indeterminacy issue arises in closed economies.

In BB2008, some possible rules considered are:

$$\begin{aligned} \mathrm{i}^*_{t+1} &= \mathrm{i}_{t+1} - \tau\epsilon_t, \qquad \text{with } \tau > 0 \;\Rightarrow \epsilon_t = 0 \\ \mathrm{i}^*_{t+1} &= \mathrm{i}_{t+1} - \tau_e \mathrm{e}_t, \qquad \text{with } \tau_e > 1 \;\Rightarrow \mathrm{e}_t = 0. \end{aligned}$$

These rules are either going to generate a peg, or zero depreciation if the steady state that the variables are approximated around are constant (and with an initial ER of zero).

Such steady states are the baseline assumptions —for each type of rule— in Benigno and Benigno). Additionally, these rules will induce an equality of rates in equilibrium.

Notice also the parameters values. They imply that responding to the ER is necessary but not sufficient. The reaction must be intense enough.

Benigno, Benigno, Ghironi (2007) explore the design of a rule that implements a determinate, fixed ER in a non-linear stochastic setting and show that it requires combining the rule with credible commitment.

Determinacy of the rest of the economy

The rule $\mathrm{i}^*_{t+1} = \mathrm{i}_{t+1} - \tau\epsilon_t,\ \tau > 0$ is sufficient for yielding determinacy of the ER but not for that of the rest of the economy: The leader's interest rate is the variable that pins down the equilibrium for all variables in the model.
(in this paper the home country is the leader.)

Suppose the home follows the rule: $\mathrm{i}_{t+1} = \alpha_1 \mathrm{y}^H_t + \alpha_2 \widehat{\pi}^p_t, \quad \alpha_1 \geq 0,\ \alpha_2 \geq 0$, with $\mathrm{y}^H_t = \mathrm{Y}^H_t - \tilde{\mathrm{Y}}^H_t$.

Here, a multiplicity of parameters may ensure determinacy. In fact, the condition for determinacy is: $(\alpha_2 - 1)k_C + \alpha_1(1-\beta) > 0$

This is the same condition as for a closed economy. Intuition: Once the foreign country pins down a determined fixed ER, the home central bank sets the interest rate for the whole world economy, which is a type of closed economy.

This restriction reduces to the Taylor principle ($\alpha_2 > 1$) if the leader central bank reacts only to inflation ($\alpha_1 = 0$).

Flexible exchange rate

Under a flexible ER regime, the interest rates are set following rules as

$$\begin{aligned} \mathrm{i}_{t+1} &= \alpha_1 \mathrm{y}_t^H + \alpha_2 \widehat{\pi}_t^p + \alpha_3 \mathrm{i}_t,\ \alpha_i \geq 0, \\ \mathrm{i}^*_{t+1} &= \alpha^*_1 \mathrm{y}_t^{*F} + \alpha^*_2 \widehat{\pi}_t^{*p} + \alpha_3 \mathrm{i}^*_t,\ \alpha^*_i \geq 0,\ \text{ for } i = \{1,2,3\}. \end{aligned}$$

In the paper $\alpha_i = \alpha_i^*$ is imposed to simplify the analysis. An equal degree (parameters) of nominal rigidity across countries is also assumed.

The system becomes symmetric and both cross-country differences and aggregates must be determinate. The conditions for determinacy are:

$$\begin{aligned} (\alpha_2 + \alpha_3 - 1)k_T + \alpha_1(1-\beta) > 0, \\ (\alpha_2 + \alpha_3 - 1)k_C + \alpha_1(1-\beta) > 0, \end{aligned}$$

where k_C is the coefficient on y_t^W in the NKPC for home and foreign producer price inflation and k_T the coefficient on the TOT term.

Both rules (home and foreign) need to be ”aggressive” enough with respect to endogenous variables.

TOT dynamics under fixed ER

We have that the definition of TOT in equation (3) can be rearranged as,

$$\widehat{\pi}_t^p - \widehat{\pi}_t^{p*} = -\left(\mathrm{TOT}_t - \mathrm{TOT}_{t-1}\right) + \epsilon_t - \epsilon_{t-1}. \tag{10}$$

After substituting (10) and its time $t+1$ version into the resulting expression from subtracting the NKPCs —equation (9) minus (8) (and after assuming symmetric nominal rigidity parameters),

$$\mathrm{TOT}_t - \mathrm{TOT}_{t-1} = -k_T \underbrace{\left(\mathrm{TOT}_t - \tilde{\mathrm{TOT}}_t\right)}_{\text{Dev. wrt. flexible price TOT}} + \beta\, \mathbb{E}_t \underbrace{\left(\mathrm{TOT}_{t+1} - \mathrm{TOT}_t\right)}_{\text{Forward looking component}} \tag{11}$$

Exchange rate dynamics

First we depart from,

$$\mathsf{y}_t^H - \mathsf{y}_t^F = \mathsf{TOT}_t - \tilde{\mathsf{TOT}}_t$$

where we used that $\mathsf{Y}_t^H = (1-n)\mathsf{TOT}_t + \mathsf{C}_t + \mathsf{G}_{H,t}$, $\mathsf{Y}_t^F = -n\mathsf{TOT}_t + \mathsf{C}_t + \mathsf{G}_{F,t}^*$, and $\mathsf{y}_t^H - \mathsf{y}_t^F = \mathsf{Y}_t^H - \mathsf{Y}_t^F - (\tilde{\mathsf{Y}}_t^H - \tilde{\mathsf{Y}}_t^F)$.

Then, take the different of policy rules without inertia (e.g., $\mathsf{i}_{t+1} = \alpha_1 \mathsf{y}_t^H + \alpha_2 \widehat{\pi}_t^p$):

$$\mathsf{i}_{t+1} - \mathsf{i}_{t+1}^* = \alpha_1 \left(\mathsf{TOT}_t - \mathsf{TOT}_t\right) + \alpha_2 \left(\widehat{\pi}_t^p - \widehat{\pi}_t^{*p}\right)$$

Finally, we can replace the UIP ($\mathsf{i}_{t+1} - \mathsf{i}_{t+1}^* = \mathbb{E}_t \mathsf{e}_{t+1}$):

$$\mathbb{E}_t \mathsf{e}_{t+1} = \alpha_1 \left(\mathsf{TOT}_t - \tilde{\mathsf{TOT}}_t\right) + \alpha_2 \left(\widehat{\pi}_t^p - \widehat{\pi}_t^{*p}\right).$$

From this expression we can see that it's possible to characterize a full determination of the ER without resorting to money demand.

Conclusion

This paper gives insights into the ER dynamics with sticky prices and endogenous interest rates.

Importantly, this paper makes some simplifying assumptions that, once relaxed could lead to other important explorations.

For example, when NFA changes play a role in the transmissions of shocks, there may be important consequences for the ER dynamics (Cavallo and Ghironi, 2002, JME). We discuss their insights briefly next.

The role of NFA in the ER dynamics under an endogenous interest rate setting

Cavallo and Ghironi (2002, JEDC) explore the role of NFA in a model in which their dynamics contribute to the ER under endogenous interest rate setting.

In previous models (e.g., BB2008) the endogenous interest rates are explored but the NFA do not play any role in shock transmissions.

Similarly, previous papers ignore the role of NFA due to the non-stationarity issues that could arise due to the indetermination of the steady-state NFA.

The idea is to reconcile the stilized fact that the positive productivity shocks experienced by the U.S. in the 1990's led to run a negative CA and to increase borrowing from the rest of the world that led to a marked ER appreciation.

($\downarrow$ CA, higher capital inflows, and higher demand of dollars to buy USD denominated assets.)

Such role of assets is overseen when stationarity is induced by implementing knife-edge conditions such as in OR1995 or CP2001 with specific elasticity of substitution values.

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Method

OLG framework as Ghironi (2006) with price stickyness and endogenous monetary policy rule.

Breaking down Ricardian equivalence is sufficient to ensure the existence of a determinate steady state and stationarity of real variables.

The Benchmark model with PPP can be solved analytically. The solution for the ER exhibits a unit root —consistent with Meese and Rogoff (1983).

However, the ER also depends on real net foreign assets (accumulated in previous period).

Method (cont.)

Price stickyness is induced by Monopolistic Competition.

The rule for monetary policy is similar as in BB2008:

$$\mathrm{i}_{t+1} = \alpha_1 \mathrm{y}_t + \alpha_2 \widehat{\pi}_t^{CPI} + \xi_t,$$

with $\alpha_1 \geq 0,\ \alpha_2 > 1$, where ξ_t (ξ_t^*) is an exogenous home (foreign) interest rate shock.

Notice that here the rules are set with respect to the CPI inflation since this paper does not care about the role of the mark-up.

A conjecture solution for the ER with flexible prices is tested:

$$\epsilon_t = \eta_{\epsilon\epsilon}\epsilon_{t-1} + \eta_{\epsilon B} B_t + \eta_{\epsilon Z^D} Z_t^D + \eta_{\epsilon\xi^D} \xi_t^D,$$

where B denotes the assets position, Z the aggregate productivity, and the superindex D denotes the differencial of the variable between home and foreign.

Findings

$\eta_{\epsilon\epsilon} = 1$ (unit root result), $\eta_{\epsilon B} > 1$.

Intuition: Inflows of capital or lower current account spur an ER appreciation

On the other hand, with sticky prices the solution includes a lagged output term,

$$\epsilon_t = \eta_{\epsilon\epsilon}\epsilon_{t-1} + \eta_{\epsilon B}B_t + \eta_{\epsilon y^D}\mathsf{y}_{t-1}^D + \eta_{\epsilon Z^D}Z_t^D + \eta_{\epsilon\xi^D}\xi_t^D.$$

Due to $\eta_{\epsilon y^D}$, higher NFA induces people to work less (to decrease disutility of labor) which lowers output.

Additionally, **overshooting is obtained**; and finally, as in BB2008, money is de-emphasized too but a role is assigned to the NFA.
(the ER solution does not depend on money supply)

Now we turn to optimality frameworks for monetary policies in large open economy setups. We will see whether an open economy setup with potentially interdependent policy choices leads to different policy prescriptions relative to the standard price flexibility results for closed economies.

Price Stability in Open Economies and Cross-Border Policy Interdependence

Price stability is regarded as a desirable policy outcome. The general notion is that it facilitates an efficient allocation of resources across time and the case for its optimality is robust in closed economy models.

Closed economy outcome: Under commitment it is optimal to implement price stability as it mimics the flexible price equilibrium.

When the only market friction comes from monopolistic power setting (with price stickyness), achieving the flex-price allocation is equivalent to reaching a first-best outcome.

With this in mind, even under discretion —where a policymaker has the incentive to inflate the economy to undo a monopolistic distortive effect on output— and together with subsidies to remove the ”inflation bias” or flexible-price markup, the optimal equilibrium consists on mimicking the flexible price allocation.

Open economy outcome: Obstfeld and Rogoff (2002) emphasize this result, explaining that the benefits of pursuing a nationally-oriented policy of price stability outweighs the gains from international monetary policy coordination.

More concretely, they find that there are gains from policy coordination but the these are trivially small and thus it may not be worth the effort.

The OR2002 result hinges critically on a knife-edge condition for the elasticity of substitution across country goods: it is 1.

Benigno and Benigno (2003, ReStud) revisit this topic, allowing for an elasticity of substitution between home and foreign goods **different from 1** (unlike CP2001, OR2002, Devereux and Engel, 2003, and others).
This difference makes gains non-trivial.

Other features of this setup: Complete markets, 1-period price stickyness, PCP.

Results: The conditions in which flexible price equilibrium is optimal are very restrictive. It requires either of:

- Perfectly correlated shocks across countries.
- Same level of monopolistic distortion across countries.
- Unitary elasticity of substitution (ES) $\omega = 1$.

Conditions making optimal for independent (non-centralized) planners to implement the flexible price allocation (in each location) are even more restrictive: No longer sufficient for distortions to be equal in both locations.

Reason: In the Nash equilibrium, policymakers face the externality that they can manipulate the TOT to their own country's advantage.

Hence, nationally-oriented policy makers have the incentive to manipulate the TOT in a welfare-improving manner (except under very specific preference specifications that remove that incentive, such as the unitary ES).

Result (cont.)

The new optimal policy can have both **inflationary and deflationary bias**. Only if these biases cancel out exactly, the pursuit of price stability becomes optimal.

Biases:

- To inflate: Monopolistic competition with endogenous labor supply induces an inefficiently low output level due to lower work effort by agents that try to mitigate their disutility of labor (e.g., CP2001).

- To deflate: Manipulation of TOT to increase output (by making use of expenditure switching patterns).

Even under discretion, and in contrast to the closed economy outcome, policymakers will not choose to implement flexible prices allocations. They will only do it in presence of the right subsidies and when the biases cancel out.

Model

- Two countries: home (H) and foreign (F), producing a continuum of goods indexed on the intervals $[0, n)$ and $[n, 1]$, respectively.

- In each country, there is a continuum of agents, with population size equal to the ranges of goods' varieties.

- Each agent is a monopolist producing a single differentiated good.

Preferences and UMP: each household j maximizes

$$U_t^j = \mathbb{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left[U(C_s^j) - V(y_s^j, Z_s) \right] \right\},$$

where C^j is a consumption bundle, U is increasing and concave, V is increasing and convex, y^j is the production of the household good variety, and Z is a country specific, aggregate productivity shock.

The foreign households have identical preferences.

Consumption:

Standard CES aggregator. Aggregation takes place first between individual varieties, and then between country specific goods' baskets.

$$C_t^j = \left[n^{\frac{1}{\omega}} \left(C_{Ht} \right)^{\frac{\omega-1}{\omega}} + (1-n)^{\frac{1}{\omega}} \left(C_{Ft} \right)^{\frac{\omega-1}{\omega}} \right], \ \omega > 0.$$

$$C_{Ht}^j = \left[\left(\frac{1}{n} \right)^{\frac{1}{\theta}} \int_0^n c_t^j(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}}, \text{ and } C_{Ft}^j = \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\theta}} \int_n^1 c_t^j(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}, \ \theta > 1.$$

There are no impediments to trade and pricing is done in a PCP fashion. Thus, the LOP holds: $p_t(h) = \varepsilon_t p_t^*(h)$.

Given identical preferences, the PPP also holds: $P_t = \varepsilon_t P_t^*$.

A similar parity holds at the country basket level: $P_{Ht} = \varepsilon_t P_{Ht}^*, \ P_{Ft} = \varepsilon_t P_{Ft}^*$.

Optimal demands:

Given the structure above the demands for h and f goods are,

$$y_t^d(h) = \left(\frac{p_t(h)}{P_{Ht}}\right)^{-\theta} \left(\frac{P_{Ht}}{P_t}\right)^{-\omega} C_t^W, \qquad y_t^d(f) = \left(\frac{p_t(f)}{P_{Ft}}\right)^{-\theta} \left(\frac{P_{Ft}}{P_t}\right)^{-\omega} C_t^W, \tag{12}$$

where $C_t^W \equiv nC_t + (1-n)C_t^*$.

- Complete asset markets (in each location): Agents trade a set of contingent assets denominated in units of the world consumption basket.

Additionally, at time -1 agents in both countries commit to trade state-contingent financial wealth so that their lifetime budget constraints are the same at time 0. This assumption is relevant to ensure perfect risk sharing regardless of the 1-period stickyness.

- PPP holds, and together with complete markets this implies perfect consumption risk sharing:

$$C_t = C_t^* = C_t^W$$

- Money is not included in the model (cashless setup as Woodford, 2003)

Flexible price allocation

A generic seller h in the home economy chooses prices $p_t(h)$ to maximize,

$$d_t(h) = (1-\tau)\lambda_t p_t(h) y_t^d(h) - V(y_t^d(h), Z_t), \tag{13}$$

τ is a proportional revenue tax (rebated via lump sum transfers), λ_t is the marginal utility of nominal income at time t ($\lambda_t \equiv U_C(C_t)/P_t$), and $V(y_t^d(h), Z_t)$ is the utility cost of production. This expression is given in utility units.

Given the demands in (12), risk sharing, and symmetry of price setting across all producers in each location, optimal price setting (at H and F) implies:

$$(1-\Phi)U_C(C_t)\frac{P_{Ht}}{P_t} = V_y\left(\left(\frac{P_{Ht}}{P_t}\right)^{-\omega} C_t, Z_t\right), \tag{14}$$

$$(1-\Phi^*)U_C(C_t)\frac{P_{Ft}}{P_t} = V_y\left(\left(\frac{P_{Ft}}{P_t}\right)^{-\omega} C_t, Z_t^*\right), \tag{15}$$

where its also used that $P_{Ft}^*/P_t^* = P_{Ft}/P_t$ (in the second expression).

Flexible price allocation (cont.)

On the other hand, the price index definition implies,

$$1 = n(P_{Ht}/P_t)^{1-\omega} + (1-n)(P_{Ft}/P_t)^{1-\omega}.$$

This expression, together with the equations (14), and (15) are used to determine the level of consumption and relative prices in the flexible-price allocation.

(Note this is a system of three equations and three unknowns: $C_t,\ P_{Ht}/P_t,\ P_{Ft}/P_t$.)

Φ and Φ^* in (14), and (15) capture the level of monopolistic distortion —after the extent of correction by distortionary taxes. Hence, they denote an after-tax mark-up:

$$1-\Phi \equiv \frac{\theta-1}{\theta}(1-\tau), \quad \text{and} \quad 1-\Phi^* \equiv \frac{\theta-1}{\theta}(1-\tau^*),$$

where $\theta/(\theta-1)$ is the flexible-price markup.

Flexible price allocation (cont.)

The real marginal costs can be defined in terms of the units of each country's good, or in units of consumption. As a consequence, the marginal cost will be constant and tied to the level implied by the distortion,

$$1 = \frac{1}{1-\Phi} mc_t, \quad \text{and} \quad 1 = \frac{1}{1-\Phi^*} mc_t^*$$

or alternatively, related to the relative prices through the mark-ups,

$$\frac{P_{Ht}}{P_t} = \frac{1}{1-\Phi} mc_t^C, \quad \text{and} \quad \frac{P_{Ft}}{P_t} = \frac{1}{1-\Phi^*} mc_t^{C*}.$$

In any case, when $\Phi = \Phi^* = 0$ the resulting allocation reproduces the competitive one.

Welfare

The monetary authorities maximize the households' expected utility,

$$\mathcal{W}_t \equiv \mathbb{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left[U(C_s) - \frac{\int_0^n V(y_s(h), Z_s) dh}{n} \right] \right\},$$

$$\mathcal{W}_t \equiv \mathbb{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left[U(C_s) - \frac{\int_n^1 V(y_s(f), Z_s^*) df}{1-n} \right] \right\},$$

where each period criteria are the instantaneous average utility among the households in each location.

Preferences

Preferences are assumed isoelastic,

$$U(C_t^j) \equiv \frac{(C_t^j)^{1-\rho}}{1-\rho},$$

$$V(y_t^j, Z_t) \equiv \frac{Z_t (y_t^j)^\nu}{\nu} \text{ if } j \in H \quad \text{and } V(y_t^j, Z_t^*) \equiv \frac{Z_t^* (y_t^j)^\nu}{\nu} \text{ if } j \in F,$$

where ρ^{-1} is the elasticity of intertemporal substitution in consumption, with $\rho > 0$, and $\eta \equiv \nu - 1$ is the elasticity of labor supply, with $\nu \geq 1$.

Note: in this particular setup an increase in Z_t is an unfavorable home productivity shock (and similar with Z^* for the foreign country). (they generate disutility.)

Corsetti and Pesenti (2005, JME) focus on the case $\rho = \nu = \omega = 1$, while Devereux and Engel (2003) and Obstfeld and Rogoff (2002, QJE) assume $\nu = \omega = 1$ (and also assume non-traded goods).

Closed economy case

To start, Benigno and Benigno analyze the case of a closed economy (i.e., with $n = 1$).

Prices are sticky and set one period in advance. The optimal price setting by the generic producer j that sets p_t^j implies:

$$\mathbb{E}_{t-1}\left\{\left[(1-\Phi)U_C(C_t)\frac{p_t^j}{P_t} - V_y(y_t^j, Z_t)\right] y_t^j\right\} = 0 \; \forall j, t, \tag{16}$$

where $y_t^j = \left(\frac{p_t^j}{P_t}\right)^{-\theta} C_t$.

Since (given symmetry) all producers set the same price, $p_t^j = P_t$, and $y_t^j = Y_t = C_t$ (given symmetry and the fact that the economy is closed), and we can write (16) as,

$$\mathbb{E}_t \left\{\left[(1-\Phi)U_c(Y_t) - V_y(Y_t, Z_t)\right] Y_t\right\} = 0 \; \forall t. \tag{17}$$

Equilibrium under policy commitment: Under ex ante commitment, the policymaker maximizes $\mathcal{W}_t$ with the information set of time $t-1$, subject to the sequence of constraints (17).

To characterize the optimal policy is useful to introduce the definition of *notional price*.

Closed economy case (cont.)

Notional price: Price a supplier would choose if it were free to choose a price in t independent of past and future prices. The notional price is denoted as p_t^N and satisfies,

$$(1-\Phi)U_C(Y_t)\frac{p_t^N}{P_t} = V_y\left(\left(\frac{p_t^N}{P_t}\right)^{-\theta} Y_t, Z_t\right). \tag{18}$$

In this expression we can see that since p_t^N is not necessarily equal to P_t, then the output level supplied by this producer is $(p_t^N/P_t)Y_t$.

With the assumed isoelastic utility of consumption and disutility of effort, (18) implies:

$$\frac{Y_t}{Y_t^n} = \left(\frac{p_t^N}{P_t}\right)^{\frac{1+\theta\eta}{\rho+\eta}}, \tag{19}$$

where Y_t^N is the natural rate of output that would arise under flexible prices: $Y_t^n \equiv [(1-\Phi)Z_t]^{1/(\rho+\eta)}$.

Thus, in t, output can deviate from its natural rate if the notional price differs from the average price level in t.

We have that a policy specified in terms of notional prices can determine the average price level at each time.

Closed economy case (cont.)

Price stability: Benigno and Benigno define Price Stability as a situation of zero notional inflation, itself defined as the equivalence between the notional price and the average actual price: $p_t^N = P_t$.

Proposition 1: (Closed economy result: Zero notional inflation ?price stability? is optimal) Under commitment, the policymaker will want to apply a policy of zero notional inflation, that is, price stability. The allocation will coincide with the flexible-price allocation.

This means that monetary policy binds itself not to inflate to try undo the monopolistic distortion. The equilibrium achieved still will be constrained-efficient since output is inefficiently low due to the monopolistic distortion.

Now, since there is no monetary frictions to undo (e.g., cash-in-advance constraints), and given the type of price stickyness assumed, it follows that although price stability is the optimal policy allocation, it will not pin down the optimal inflation rate. In this particular environment there can be a positive inflation at no cost.*

Under discretion, the policymaker maximizes welfare at a generic period t, subject to the incentive compatibility constraints given by (17) from period $t+1$ onwards. The optimality condition at time t is now:

$$U_C(Y_t) = V_y(Y_t, Z_t). \tag{20}$$

*The monopolistic distortion induced an inefficient level of output (below optimal). Thus, the policy equilibrium that commits to not inflate —and generate a higher output— is still constrained-efficient, even if optimal from a policymaker perspective.

Once prices are fixed, a policymaker acting under discretion has an incentive to inflate the economy and push output toward the competitive level. This creates an inflationary bias (Kydland-Prescott/Barro-Gordon argument).

Intuition: In general the flexible price allocation is not optimal or efficient. The optimality of this allocation holds under discretion only when the monopolistic distortion is offset with mark-up removing taxation.