Strategic Macroprudential Policymaking: When Does Cooperation Pay Off? *

Camilo Granados †

September 24, 2021

Abstract

I study whether emerging economies can navigate the global financial cycle more successfully by resorting to internationally coordinated macroprudential policies. For this, I set an open economy model with banking frictions in a center-periphery environment with multiple emerging economies. Then, I evaluate the performance of several policy arrangements that differ by the degree and type of cooperation. I find that cooperation can generate welfare gains but is not always beneficial relative to nationally-oriented policies. Instead, only regimes where the financial center acts cooperatively generate welfare gains. When present, two mechanisms generate the gains: a cancellation effect of national incentives to manipulate the global interest rate and a motive for steering capital flows to emerging economies. The first mechanism eliminates unnecessary policy fluctuations and the second helps prevent capital retrenchments in the center. These effects can be quantitatively relevant as good cooperation regimes can reduce the welfare losses induced by a financial friction between 60% and 80%.

IEL Codes: F38, F42, E44, G18

Key words: Macroprudential Policies, International Policy Coordination, Banking Frictions.

^{*}This paper has been benefited by the guidance and advising of Ippei Fujiwara and Yu-chin Chen. I am grateful for their continuous feedback and support. I also want to thank the comments and suggestions of Fabio Ghironi, Scott Davis, Martin Bodenstein, Giovanni Lombardo, Andrew Levin, Phil Brock, Brian Greaney, Dmitry Brizhatyuk, Sergio Ocampo and participants at the EEA Annual Congress (2021), the University of Surrey CIMS conference, the LACEA-LAMES annual meetings, and at the IFM Brownbag seminar and other graduate workshops at the University of Washington (UW) Economics department. Finally, I gratefully acknowledge the financial support of the Ensley fellowship at UW and of the Lauchlin Currie Scholarship at the Central Bank of Colombia (Banco de la República). The opinions in this paper are exclusively those of the author and do not necessarily represent that of the Central Bank of Colombia.

[†]Banco de la República, Carrera 7 No. 14-78, Bogotá, Colombia. Email: jgranaca@banrep.gov.co

1 Introduction

The emerging economies' fragility to the global financial cycle has become a core concern in international finance in the last decade.¹ As these economies started to attract more capital flows, they have become a new source of (global) financial risk, presenting new challenges to policymakers. On the one hand, the local regulators would like to facilitate the participation of emerging economies in international financial markets while still protecting their economies from adverse external shocks. On the other hand, financial centers and multilateral institutions prioritize the mitigation of new sources of risk.

As a result, we have witnessed a general increase in the usage of macroprudential policy regulations in the form of stricter balance sheet requirements at the banking level (e.g., leverage caps, loan-to-value ratios, or taxes). Crucially, these regulations affect domestic and international agents as the balance sheets' links of these banks extend beyond national borders, raising immediate questions over the potential gains from international policy cooperation.

With this in mind, I study whether international macroprudential policy cooperation is beneficial for emerging economies and could be used to improve their economic performance. In particular, I address two specific questions: (i) is macroprudential cooperation beneficial for emerging economies in general, i.e., over the business cycle?, (ii) If so, what are the mechanisms driving the welfare gains?.

To answer these questions, I extend an open economy model with banking frictions to include two smaller economies that depend financially on a Center but that still have general equilibrium effects. Based on this setup, I use a simplified model to outline the welfare effects and policy mechanisms under cooperation and relative to nationally-oriented regulations. Afterward, I set a quantitative model and obtain the optimal macroprudential policies of several regimes that vary by their degree of international cooperation. Based on this framework, I perform a comprehensive welfare comparison and study the regulatory arrangements' relative performance and properties.

The macroeconomic framework I use for modeling the banking sector follows Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), extended to an open economy environment. Unlike other open economy studies that consider the banking sector explicitly (e.g., Banerjee, Devereux, and Lombardo (2016) and Aoki, Benigno, and Kiyotaki (2018)) I abstract from the role of monetary policy. This assumption allows me to focus solely on financial regulators' interactions while easily extending the framework to one of multiple peripheral

¹See Rey (2013, 2016).

economies that interact with a Center (three countries environment). In that regard, and to the best of my knowledge, this is the first paper that studies the macroprudential policy coordination of emerging economies featuring general equilibrium effects, but that are still considerably fragile to a financial center. ² ³

The addition of the third economy is meaningful since it allows me to consider a wider array of policy regimes with varying degrees and types of cooperation. Based on these results, I can identify when coordination is beneficial or even counterproductive. Just as importantly, I can consider the potential regional cooperation of peripheral economies in the presence of considerable policy spillovers by a center that may want to adjust its policies in response.

In this setup, I account for agency frictions in the banking lending relationships that create a costly enforcement distortion due to the possibility of default by financial intermediaries. Consequently, the interest rates are adjusted to reflect the higher risk, which ultimately translates in higher credit spreads and financially augmented credit cycles in the same spirit as Bernanke, Gertler, and Gilchrist (1999) and Kiyotaki and Moore (1997). This distortion opens the scope for policy interventions in the form of macroprudential taxes on the banking revenue rates. Simultaneously, the rates affected by these policy tools shape the balance sheet dynamics of these intermediaries and the international links between banks, which potentially opens a scope for coordinated financial regulations.

Using the simplified model, I set a modified social planner problem following Davis and Devereux (2019) to study the mechanisms driving these policies' welfare effects. I find that the effects are substantially stronger when using forward-looking policies because their effect builds on into the future through retained profits and net worth dynamics of the banking sector. This feature underscores the relevance of accounting for these policies' persistency and total effect over time when assessing different policy regimes.

Similarly, the magnitude of the policy effects will increase with the extent of financial frictions. Implying these policies can be more effective for more distorted economies, something consistent with the fact that the financial distortion is the feature that creates a role for policy in this environment.

²Here an emerging economy is defined as an economy with an underdeveloped financial sector (in the spirit of Céspedes, Chang, and Velasco (2017)) and that in consequence relies on the funding from a center.

³See Jin and Shen (2020) for a setup with small open economies interactions and an exogenous center.

⁴This policy is chosen for its analytical and interpretation convenience, but also because it encompasses several types of actual macroprudential instruments.

⁵Other papers that study the policy coordination in presence of financial frictions have focused on revisiting the monetary policy case (Sutherland (2004), Fujiwara and Teranishi (2017)) or in the interaction between different types of policy makers (De Paoli and Paustian (2017) and an application of Bodenstein, Guerrieri, and LaBriola (2019))

I identify the formation of two fundamental mechanisms that shape macroprudential policy incentives under cooperation: a portfolio cancellation effect and a capital relocation motive. The portfolio cancellation effect consists of the elimination of national (individual) incentives for manipulating the global interest rate to improve the net foreign assets position. The relocation of capital motive refers to an incentive for increasing capital inflows to peripheries at the expense of the capital accumulation at the center. The first mechanism arises from the fact that a cooperative planner pools the conflicting national incentives of savers and borrowers of foreign assets affected by the same interest rate. In contrast, the second is a byproduct of the centralized planner's new policy aim, namely boosting global welfare rather than any national economic performance.

Based on these insights, I set a larger scale dynamic general equilibrium model to perform a comprehensive assessment of an array of policy regimes. Each policy problem is set in a timeless perspective formulation under commitment and relies on the open-loop Nash equilibrium as the solution criterion.⁶

I find that there are sizable social welfare gains from international cooperation that range between 12% to 15% of equivalent world consumption increases relative to a non-cooperative framework. Crucially, however, the gains are only present for policy frameworks where the center acts cooperatively. The gains are maximized under worldwide cooperation, followed by the cooperation between the center and a subset of the peripheries, then by the non-cooperative (Nash) regime, and finally by the regional (emerging) cooperation where the peripheries form a coalition and play against an independent center. Thus, another salient implication of this ranking is that the emerging cooperative efforts can be counterproductive (leading to a 6% consumption loss).

The primary sources of welfare gains are the two policy mechanisms arising under cooperation (portfolio cancelation effect and relocation of capital). The first removes sources of variation from the policy tools, thereby generating smoother policy and capital accumulation dynamics. The second facilitates a more efficient allocation of the international capital flows towards the most productive destinations.

Furthermore, both mechanisms work better when the peripheric block's welfare weights become more comparable to that of the center. I use the relative economic population size as the weight, which implies the social gains increase for regimes where more peripheries participate in the cooperative arrangement with the center.

The intuition behind this result is that the cooperative planner relies on the welfare effects

⁶See Bodenstein et al. (2019) for a discussion on macroeconomic games in this setup and the open loop Nash equilibrium framework.

from the increased intermediation with the peripheries to boost its objective quantity, the global welfare (a weighted average of the national welfares). Naturally, the strategy of implementing welfare increases at the peripheries despite the welfare cost from limiting the investment flows at the center is worth pursuing if the net social welfare gain is positive. The former only occurs when the planner cares comparably enough about the welfare effects originated in the emerging economies. Consequently, for small open economy environments, the gains of cooperation will be absent.

These mechanisms, or lack thereof, also help understand why the emerging economies' regional cooperation regime fails to yield gains. The first channel is not present as all involved national incentives (in the coalition) to manipulate the interest rate go in the same direction; that is, for the cancellation to take effect, we need both global creditors (Center) and debtors (EMEs) to cooperate. In contrast, the debtor's portfolio incentives are pooled for the emerging economies, strengthening the peripheral capacity to manipulate the interest rates. As a result, the center exerts a strong retaliative effort, engaging in a marked regulatory struggle that creates salient social and individual welfare losses. On the other hand, the second welfare inducing mechanism will not be present either, as it relates only to coordinated policy efforts with a global intermediator (center).

The first mechanism (portfolio cancelation effect) is similar to the source of cooperation gains in Davis and Devereux (2019) and Korinek (2020) when studying the case of capital controls with collateral constraints, or Bengui (2014) for the case of liquidity regulations in endowment economies. Here, however, I prove that a similar result holds for regulated banks with agency frictions, which is less straightforward, as I do not rely on a direct off-setting of the variables targeted by the policies.

Simultaneously, this source of welfare gains contrasts with Jin and Shen (2020), where regional cooperation is welfare improving because the peripheric block internalizes and uses their increased capacity to manipulate the global interest rates. Here, conversely, the same reasoning delivers welfare losses. The explanation behind the difference is that I allow the financial center regulator to engage in retaliative policy actions.

The first mechanism also contrasts with Kara (2016), where decentralized planning leads to welfare losses due to inefficiently low levels of policy actions. Here on the contrary, eliminating regulatory reactions that are not aligned with the financial stability goals is welfare improving, i.e., the mechanism depicted in this paper generates welfare by preventing planners from intervening too much and from attempting to manipulate the fundamentals with yield-seeking purposes. Thus, from a regulatory standpoint, in the framework of this study, the cooperative planners have a comparative regulatory advantage relative to nationally-oriented policymakers because they can focus solely on financial

stabilitation goals.

Notably, the national distribution of welfare across regimes shows that enforcing the best policy framework can be challenging. On the one hand, cooperation seems likely as the peripheries always want to collaborate with the center; actually, they would prefer to have an exclusive partnership with that economy. However, the center prefers to cooperate with a subset of peripheries rather than with the rest of the world. In the former case, the coalition participants, i.e., the center and one periphery, will join efforts and improve their outcomes substantially at the expense of the second, left out periphery that ends up worse than under any other regime.

The rest of this paper is structured as follows. Section 2 describes the recent empirical trend of capital flows and associated policy responses. In section 3, I set a simplified model used to describe the main policy mechanisms. Sections 4 and 5 describe the main model and policy regimes under consideration. In Section 6, I show the results and address the research questions. Finally, Section 7 concludes.

2 Capital Flows Dynamics After the Crisis and Policy Response

The period before the global financial crisis was characterized by a strong flow of capitals towards advanced economies (see figure 1), such phenomenon, denoted as the global savings glut⁷, was partly explained by a financial deregulation process in the largest advanced economies after the termination of the main banking separation Acts put in place as a response to the financial crises of the early 1900s,⁸ and contributed to the downward trend of the interest rates of traditional assets in the main economies (Bernanke, Demarco, Bertaut, and Kamin, 2011).

Rather than a change in the direction of the capital flows, given the lower average returns, the observed response of the markets in the 2000's was a reliance on high leveraged intermediation, together with financial innovation efforts (e.g., securitization of assets) to continue attracting investments with competitive returns but at the expense of a substantial build-up of risk.

Once the bubble burst and the crisis ensued there was a strong institutional effort towards strengthening the financial regulation, and a higher recognition of the threat posed by the

⁷See Justiniano et al. (2013) and Bernanke (2005) for a discussion on this topic.

⁸In the USA the Glass-Steagal Act of 1933.

risk of financial contagion prompted an urgent revision of the Basel accords. The G-20 met for the first time in history to deal with an economic matter and as result founded the Financial Stability Board, an institution that has as one of its objectives to promote the coordination of financial regulations.

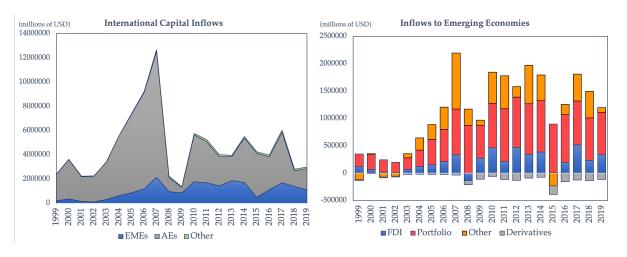


Figure 1: Global Capital Inflows: 1999-2019

Source: IMF-IFS and BOP Statistics.

Note: the countries in each group follow the IMF definitions. That is, 23 advanced economies, 58 emerging economies and 199 developing countries (other in the graph).

After that, the financial markets have featured stricter regulations and a decrease in the level of interbank connectedness in advanced economies. Simultaneously, and as a byproduct, the international investment flows have shifted their direction towards the emerging economies. Furthermore, the main type of flows entering these economies were the portfolio and banking flows (Other in the figure 1, right panel). These items, that take place within the financial intermediation sector, represent the most volatile types of capital flows. Thus, the banking sector in the emerging economies happens to be at the core of the post-global financial crisis potential sources of risk.

The observed policy response consisted in stricter macroprudential regulations with respect to pre-crisis times, both globally, and specially in the emerging and developing economies. This can be seen in the figure 2 that shows the policy stance by type of economy. There, a tightening of a macroprudential instrument is counted as (+1) and a loosening as (-1), this is computed and aggregated for 17 policy instruments and then by country groups. It can be seen that globally, and by the end of 2018, there were more than 200 tightenings in the instruments (e.g. an increase in the Loan-to-Value requirements or in the banking taxes).

In addition to the observed increase in the usage of these policies in emerging, and

eventually in advanced economies, it can also be suggested, from the overall and compositional policy stance dynamics in figure 2, that there may be potential comovement patterns between the instruments, both at the cross-country level and with the business cycles.

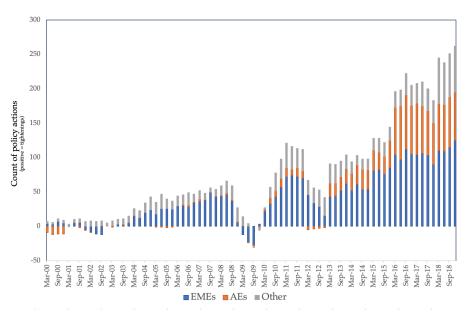


Figure 2: Macroprudential policies stance by type of economy

Source: IMF - Integrated Macroprudential Policy Database (iMaPP) by Alam et al. (2019). Note: the countries in each group follow the IMF definitions. The figure includes information for 23 advanced economies, 52 emerging economies and 60 developing countries (other in the graph). The figure shows a four-quarter rolling window sum of the stances in each date.

In that regard, several papers document the presence of significant external policy effects, for example, Forbes, Reinhardt, and Wieladek (2017) study the UK case and show that these policies can have large spillovers in the international capital flows. Buch and Goldberg (2017) document how the macroprudential policies generate significant cross-border credit effects that spill over through the interbank lending, and Aiyar, Calomiris, and Wieladek (2014) show how stricter capital requirements on UK-owned banks made the foreign banks operating in UK to increase their lending, in a regulatory arbitrage attempt to substitute the curtailed intermediation generated by the policy change, and thereby, (partially) off-setting the intended effect of the new regulations. Similarly, but finding international spillovers in a Center-periphery environment, Tripathy (2020) studies the spillover of banking regulations from Spain to Mexico through Mexican subsidiaries of Spanish banks and explains how the borderless nature of the banking business, operated by large global banks can imply significant cross-country spillovers.

Judging from these findings, and as explained by Forbes (2020), it can be thought that the presence of these leakages could mitigate the effectiveness of the macroprudential policies

or generate new vulnerabilities and risks. In that vein, it is interesting to determine from a theoretical perspective if these spillovers may open some scope for cooperative policy schemes, or if instead, they just represent efficient adjustment effects that would render the cooperation redundant.

To contribute to the understanding of these policy effects, in the next section I show in a modelling framework the direct and cross-border spillovers of a macroprudential instrument, and explore whether the cross-border policy effects have the same mechanisms at work under cooperation.

3 Simple Three-Period Model

Before analyzing the main dynamic model of this paper, I lay out a simplified setup in finite horizon for building intuition about the main mechanisms at work. In that spirit, I consider the simplest possible model that still features a dynamic decision making by banks and macroprudential regulators. This model shares the essential features of the main one, and can be thought of as a small scale version of it, with the advantage of allowing to analytically disentangle the welfare effects of different types of policies, for example, tools that are forward looking, static, nationally-oriented or cooperative. Clearly, there is a trade-off between the improved tractability, and the potential uses of a more quantitatively involved model, e.g., the smaller scale model would not allow for a complete study of the response of the economy to shocks or a comprehensive welfare accounting comparison between models. I leave such additional applications for subsequent sections of the paper that are based on the larger-scale model.

Similarly, when a sector is completely analogous to that of the main model explained in section 4 I review it more briefly here, and instead focus more in the sectors with meaningful differences, the banks and the households.

3.1 Setup

General economic environment. Time is discrete and there are three periods, $t = \{1, 2, 3\}$. The world economy is populated by three countries, two emerging economies or periferies, labeled as a and b, and a financial center c. The relative population size of each economy is given by n_i with $i = \{a, b, c\}$ and these sizes are such that the sum of the periferies is never

⁹For reference an even simpler finite time horizon version of this model, with static banks and one-shot policies can be found in Granados (2021).

larger than the population size at the center, that is, $n_c \ge \frac{1}{2}$, with $n_c = 1 - n_a - n_b$. Each economy is populated by five types of agents: households, final goods firms, investors, the government and a representative bank.

The households own the firms (final good, capital and banks) and there is a production technology that transforms the predetermined capital into a final consumption good with a Cobb-Douglas agregator: $Y_t^i = A_t^i K_{t-1}^i$. This good is identical across countries.

The economies are endowed with a predetermined level of capital in the first period (K_0), after that, a bank intermediates the physical capital acquisition for production. For this, at the end of each period, the firm will take its input and indebtedness decisions, the bank will provide the funds and will be repaid the next period after production takes place.

This implies there are two periods of financial intermediation, the first at the end of the first period, and one more a period later. Importantly, as long as there are intermediation activities in the future, the banks may continue in business and in that case retain profits, thus the banking decisions are dynamic, or forward looking, in t=1, while in t=2 the banking problem is static. I focus on the differences in the decision making of the bankers and policy-makers between these two periods.

The households will have standard preferences over consumption and their welfare is given by: $W^i = u(C_1^i) + \beta u(C_2^i) + \beta^2 u(C_3^i)$, with $u(C) = C^{1-\sigma}/(1-\sigma)$.

Additionally, given the homogeneous good assumption, and the identical preferences at the world level, we have that the law of one price and purchasing power parity will hold. Consequently, we can abstract from the real exchange rate. Finally, for this simple model I work with a perfect foresight assumption.

3.2 Banks

Each economy has a representative bank that aims to maximize the present value of its franchise. There are two important features that distinguish emerging economies (EME) banks from that of the Center: First, the EME banks will be subject to a financial friction in the form of agency costs, and second, the Center bank will act as creditor of the EME banks in the interbank market. The latter feature will appear due to the limited capacity of local intermediation in the peripheries.

EME-Banks. The banks in the emerging economies will intermediate funds in order to provide resources to local firms for capital acquisition. These banks will be financially

constrained and depict a lower level of financial development, in the spirit of Chang and Velasco (2001). As a consequence, two features arise that characterize these banks. First, these firms will have a lower capacity of financial intermediation at the local level, and to compensate, they rely on borrowing money from the Center in an international interbank market. Second, their lending relationships are subject to a costly-enforcement agency friction where the banks could divert a portion κ of the assets they intermediate.

The friction creates a distortion in the credit spread of these banks that takes the form of a default risk premium. This feature is modelled following the structure of Gertler and Kiyotaki (2010) and Céspedes, Chang, and Velasco (2017).

In the first period of intermediation (end of t=1) the bank aims to maximize its expected franchise value, given by J_1 and solves:

$$\begin{split} J_1^e &= \max_{F_1^e, L_1^e} \mathbb{E}_1 \left\{ (1-\theta) \Lambda_{1,2}^e (R_{k,2}^e L_1^e - R_{b,1}^e F_1^e) + \Lambda_{1,3}^e \theta (R_{k,3}^e L_2^e - R_{b,2}^e F_2^e) \right\} \\ s.t \quad L_1^e &= F_1^e + \delta_B Q_1^e K_0^e \\ L_2^e &= F_2^e + \delta_B Q_2^e K_1^e + \theta [R_{k,2}^e L_1^e - R_{b,1}^e F_1^e] \end{split} \qquad \qquad \text{[Balance sheet in t=1]} \\ J_1^e &\geq \kappa Q_1^e K_1^e \end{split} \qquad \qquad \text{[ICC, t=1]}$$

Where the country index for emerging economies is e with $e = \{a, b\}$, $L_t = Q_t K_t$ is the total lending intermediated with the local firms, F_t is the cross-border borrowing they obtain from the Center, $R_{k,t}$ is the gross revenue rate of the banking services, paid by the firms, $R_{b,t}$ is the interbank borrowing rate for the banks that they pay to the Center intermediary, Q_t is the price of capital, $\delta_B Q_t K_{t-1}$ represents a start-up capital the bankers get from their owner households, and $\Lambda_{t,t+j}$ is the stochastic discount factor between periods t and t+j.

Also, notice I highlight (in a different color) the terms that correspond to future periods for a clearer exposision of the dynamic nature of the banking problem.

The present value of the bank, will be given by the expected profits in the next period. For this, I include the posibility of exit from the banking business, with an associated probability of survival θ . ¹⁰ Thus, with probability $(1 - \theta)$ the bank will fail and transfer back its profits to the household, and with probability θ the bank will be able to continue its business and pursue future profits.

The constraints are given by the balance sheets of the bank for each period in which

¹⁰This feature is critical in the main model framework as it allows the incentive compatibility constraint to bind and will prevent the presence of Ponzi schemes in the model

they operate and an incentive compatibility constraint. These balance sheets have, on the asset side, the loans that are intermediated, and on the liabilities side, the interbank foreign borrowing and their net worth. The latter in the initial period is only a bequest or start-up capital that they receive from their household owners, while later also accounts for previously retained earnings. That is, I assume the bank will retain its earnings as long as it operates.¹¹

Finally, the incentive compatibility constraint (ICC) reflects the imposition that the value of the franchise has to be equal or larger than the value the bank could divert after defaulting its creditors, which is given by a fraction κ of the intermediated assets.¹² For simplicity, this divertable fraction will be constant across locations and time.

In the second period, the banks solve a simpler problem, as their objective will not depict a continuation value (making their decisions static):

$$\begin{split} J_2^e &= \max_{F_2^e, L_2^e} \mathbb{E}_2 \left\{ \Lambda_{2,3}^e (R_{k,3}^e L_2^e - R_{b,2}^e F_2^e) \right\} \\ s.t. \quad L_2^e &= F_2^e + \delta_B Q_2^e K_1^e + \theta [R_{k,2}^e L_1^e - R_{b,1}^e F_1^e] \\ J_2^e &\geq \kappa Q_2^e K_2^e \end{split}$$

It can be noticed the problem they solve is affected by their previous intermediation decisions as the balance sheet constraint includes the retained profits from last period (as highlighted).

From these two problems, we can obtain the following first order conditions:

$$[F_1^e]: \quad \mathbb{E}_1\Omega_1^e(1+\mu_1^e)(R_{k,2}^e-R_{b,1}^e) = \kappa\mu_1^e \qquad [F_2^e]: \quad \mathbb{E}_2(1+\mu_2^e)(R_{k,3}^e-R_{b,2}^e) = \kappa\mu_2^e$$

Where μ_t^e is the lagrange multiplier of the ICC of e country bank in each period and $\Omega_1^e = (1-\theta)\Lambda_{1,2}^e + \theta^2 R_{k,3}^e \Lambda_{1,3}^e$ is the effective stochastic discount factor of the bankers that accounts for the probability of a bank failure in the future.

With these conditions an initial result can be stated:

Proposition 1: *If the ICC binds the credit spread is positive in each period and increases in* κ

¹¹This assumption is common in the literature and also particularly reasonable in this model environment as, given the friction, the returns from banking tend to be higher than those of other assets.

¹²I follow Gertler and Karadi (2011) closely in the formulation of the ICC and assume the bank only considers to divert assets as soon as they obtain the funds. Other formulations are also possible, e.g., in Granados (2021) I explore a stricter ICC case where the potential diversion occurs the next period, after the firms repay their debt.

Proof: See appendix A.

Since the friction is embodied in a positive spread, this result implies we can talk about κ and the extent of the distortion as analogous concepts.

Center-Banks. The Center representative intermeriary will solve a similar problem but without being subject to frictions. Therefore, the only constraints it faces are the balance sheets in each period. These reflect that the Center-Bank acts as the creditor of the EME-Banks.

In t = 1 the Center-Bank solves:

$$\begin{split} J_1^c &= \max_{F_1^a, F_1^b, L_1^c, D_1} \mathbb{E}_1 \left\{ (1-\theta) \Lambda_{1,2}^c (R_{k,2}^c L_1^c + R_{b,1}^a F_1^a + R_{b,1}^b F_1^b - R_{D,1} D_1) \right. \\ &\qquad \qquad + \Lambda_{1,3}^c \theta (R_{k,3}^c L_2^c + R_{b,2}^a F_2^a + R_{b,2}^b F_2^b - R_{D,2} D_2) \right\} \\ &s.t \quad L_1^c + F_1^a + F_1^b = D_1 + \delta_B Q_1^c K_0^c \qquad \qquad \text{[Balance sheet in t=1]} \\ &\qquad \qquad L_2^c + F_2^a + F_2^b = D_2 + \delta_B Q_2^c K_1^c \\ &\qquad \qquad + \theta [R_{k,2}^c L_1^c + R_{b,1}^a F_1^a + R_{b,1}^b F_1^b - R_{D,1} D_1] \qquad \text{[Balance sheet in t=2]} \end{split}$$

This problem is dynamic, as it accounts for the potential profits and balance sheets of every intermediation period.

In contrast, in the next period the bank will solve a simpler (static) problem, consisting of maximizing the profits of a single term.

$$\begin{split} J_2^c &= \max_{F_2^a, F_2^b, L_2^c, D_2} \mathbb{E}_2 \left\{ \Lambda_{2,3}^c (R_{k,3}^c L_2^c + R_{b,2}^a F_2^a + R_{b,2}^b F_2^b - R_{D,2} D_2) \right\} \\ s.t \\ L_2^c + F_2^a + F_2^b &= D_2 + \delta_B Q_2^c K_1^c + \theta [R_{k,2}^c L_1^c + R_{b,1}^a F_1^a + R_{b,1}^b F_1^b - R_{D,1} D_1] \end{split}$$

The resulting first order conditions will just reflect that the expected credit spread is zero for all of the assets considered by the center (F_2, L_2, D_2) . By using that result and the perfect foresight assumption, we can drop the borrowing cross border rates $(R_{b,t})$ as they are all equal to the rate for deposits at the Center $(R_{D,t})$. Furthermore, the Euler equations for the Center households with respect to the bonds and deposits can be used to simplify further and replace the deposits rate with that of the bonds.

3.3 Production Sectors

There are two types of firms. Here I describe them briefly as the structure is analogous to the main model and the detailed formulation is explained in subsequent sections.

Final Good Firm. There is a firm that maximizes their profits, given by the value of the production, plus the sales of undepreciated capital after production, minus the payment of banking loans. The only constraint it faces is the production technology. From the first order condition with respect to the capital, we can pin down the gross rate of return paid to the banks as $R_{k,t} = \frac{r_t + (1-\delta)Q_t}{Q_{t-1}}$ with $t = \{2,3\}$. Here , $r_t = \frac{\alpha Y_t}{K_{t-1}}$ is the marginal product of capital and Q_t is the price of capital in period t.

Capital Producers. There is a firm carrying out the investments in each economy. Their job will be to buy any remaining undepreciated capital from the final good firms and to produce the new physical capital for future production. Moreover, the investment is subject to a cost of adjustment that depends on the investment growth with respect to the previous period.

3.4 Households

The households own the three types of firms (final goods, capital and banks) and use their profits for consumption, saving, and for supplying the bequests to their banks. They don't pay the banking taxes directly, instead, these are paid by the banks before distributing profits. However, they receive a lump sum transfer from the government once the latter levies the financial intermediaries.

Since the capital is already predetermined in the initial period, there is no intermediation for K_0 . Instead, and only for that period, the households rent the capital to the firms directly.

EME-households. The households maximize the present value of their life-stream of utility by solving:

$$\max_{\{C_t^e\}_{t=1}^3, \{B_t^e\}_{t=1}^2} u(C_1^e) + \beta u(C_2^e) + \beta^2 u(C_3^e)$$

s.t.

$$C_{1}^{e} + \frac{B_{1}^{e}}{R_{1}^{e}} = r_{1}^{e} K_{0}^{e} + \pi_{f,1}^{e} + \pi_{inv,1}^{e} - \delta_{B} Q_{1}^{e} K_{0}^{e}$$

$$C_{2}^{e} + \frac{B_{2}^{e}}{R_{2}^{e}} = \pi_{f,2}^{e} + \pi_{inv,2}^{e} + \pi_{bank,2}^{e} - \delta_{B} Q_{2}^{e} K_{1}^{e} + B_{1}^{e} - T_{2}^{e}$$

$$C_{3}^{e} = \pi_{f,3}^{e} + \pi_{bank,3}^{e} + B_{2}^{e} - T_{3}^{e} \qquad for \ e = \{a, b\}$$

Here B_t denotes the bonds or net foreign assets position, R_t the interest rate on bonds, and T_t the lump sum taxes. As for the profits terms, $\pi_{f,t}$ corresponds to the final goods firms profits, $\pi_{inv,t}$ to the capital firms profits, and $\pi_{bank,t}$ to the banking profits.

I also assume that the household does not have access to deposits. This is a simplification that reflects the lower financial development in the periphery and that generates the financial dependency from EME-Banks on Center-Banks. It should be noted that this assumption is inconsequential for the saving decisions of the households as they can freely access the bonds market.

Center-households. The households at the Center solve a similar problem. The only difference is that they do have access to local deposits and that their banking profits account for the fact that their banks act as creditors of the EMEs:

$$\max_{\{C_t^c\}_{t=1}^3, \{B_t^c\}_{t=1}^2} u(C_1^c) + \beta u(C_2^c) + \beta^2 u(C_3^c)$$
s.t.
$$C_1^c + \frac{B_1^c}{R_1^c} + D_1 = r_1^c K_0^c + \pi_{f,1}^c + \pi_{inv,1}^c - \delta_B Q_1^c K_0^c$$

$$C_2^c + \frac{B_2^c}{R_2^c} + D_2 = \pi_{f,2}^c + \pi_{inv}^c + \pi_{bank,2}^c - \delta_B Q_2^c K_1^c + B_1^c + R_{D,1} D_1 - T_2^c$$

$$C_3^c = \pi_{f,3}^c + \pi_{bank,3}^c + B_2^c + R_{D,2} D_2 - T_3^c$$

3.5 Macroprudential Policy

The frictions in the banking sector generate a role for policymaking. This policy takes the form of a macroprudential policy tax that targets the banks. A government will tax the rate of return of the bankers in each period, and afterwards, rebates the tax income back to the households.

As a result, the effective revenue rate perceived by the banks after paying their taxes is: $R_{k,t} = \frac{(1-\tau_t)r_t + (1-\delta)Q_t}{Q_{t-1}}$, where τ_t is the macroprudential tax.

With such structure, the following proposition holds:

Proposition 2: An increase in the macroprudential tax decreases the leverage ratio of banks and its effect grows with the friction

This result suggests that, in addition to the direct effect in decreasing the credit spread of a distorted economy, the macroprudential tax lowers the leverage of the banking sector. Furthermore, the extent at which this occurs increases with the financial friction (κ).

In addition, notice that since τ_2 affects the first intermediation period, when the banks' decisions are forward looking, and τ_3 the terminal period, where the decisions are static, it also follows that τ_2 and τ_3 are, respectively, a forward-looking and a static policy tool.

3.6 Equilibrium

Market Clearing and International Links. The bonds market depicts a zero-net-supply in the first two periods:

$$n_a B_t^a + n_b B_t^a + n_c B_t^c = 0$$
, for $t = \{1, 2\}$

In addition, I assume the uncovered parity holds which allows us to equate the interest rate of the bonds in each country:

$$R_t^a = R_t^b = R_t^c = R_t$$

Furthermore, I make use of the Euler equation for the deposits and bonds from the first order conditions of the Center, according to which $C_t^{c - \sigma} = \beta R_{D,t} C_{t+1}^{c - \sigma}$ and $C_t^{c - \sigma} = \beta R_t C_{t+1}^{c - \sigma}$, to determine that $R_{D,t} = R_t$ for $t = \{1, 2\}$.

Equilibrium. Given the policies $\tau_t = \{\tau_t^a, \tau_t^b, \tau_t^c\}_{t=2,3}$, the equilibrium consists on the prices $\{Q_t^i\}$, rates $\{R_1, R_2, R_{k,2}^i, R_{k,3}^i\}$ and quantities $\{B_1^i, B_2^i, K_1^i, K_2^i, F_1^e, F_2^e, D_1, D_2\}$ and $\{C_t^i\}$ for $t = \{1, 2, 3\}$, with $i = \{a, b, c\}$ and $e = \{a, b\}$ such that: in each period, the households solve their utility maximization problem, the firms solve their profit maximization problems, the banks maximize their franchise value, the government runs a balanced budget, and the goods and bonds markets clear.

A summary of the final set of equilibrium conditions used for solving the model can be found in table 5. I solve this system of equations non-linearly and using a perfect foresight

approximation.

3.7 Welfare Effects of Policy

Based on the 3-period model we can approximate the welfare effects of policy at the national and cross-border level.

Numerical solution. I solve the model private equilibrium using the parameters shown in table 6. The agents take the taxes as given, and hence, I have to provide them exogenously when solving for the private equilibrium. I solve the model with zero taxes and compare it with the solution for different levels of the policy tools. The results are shown in table 1.

The table shows the numerical approximation to the derivative in welfare with respect to a change in a tax. The results indicate that the welfare effect of forward-looking taxes (τ_2) is stronger than that of the terminal (static) tax (τ_3). This is particularly true for the cross-border effects of the taxes in both the Center and peripheral countries. This is consistent with studies such as Davis and Devereux (2019) and Gertler et al. (2020) where the taxes that are pre-emptive and prudential in nature are potentially more effective than crisis-management policies.

I also obtain that for most of the taxes' increases, the direct effect of the Center tax, i.e., on its own welfare, is weaker than its cross-border effects. This is consistent with, and rationalizes, literature findings indicating that the macroprudential policies are less effective at financial Centers (relative to emerging economies, see Cerutti et al. (2017)) but also that these countries tend to exert stronger international spillovers (e.g., Rey (2013), Aizenman et al. (2020), and Agénor and da Silva (2018))

Table 1: Welfare effects in 3-period model

Effect		Change in tax			
		1%	3%	5%	8%
Direct effect	$ au_2^a o W^a$	0.146	0.144	0.142	0.138
of $ au_2$	$ au_2^b o W^b$	0.146	0.144	0.142	0.138
	$\tau_2^c \to W^c$	-0.242	-0.457	-0.179	-0.027
Cross-border	$ au_2^a o W^b$	-0.047	-0.047	-0.047	-0.048
effect	$\tau_2^a \to W^c$	-0.016	-0.017	-0.017	-0.017
	$\tau_2^b \to W^a$	-0.047	-0.047	-0.047	-0.048
	$\tau_2^b \to W^c$	-0.016	-0.017	-0.017	-0.017
	$\tau_2^c \to W^a$	-0.162	-0.226	-0.180	-0.155
	$\tau_2^c \to W^b$	-0.162	-0.226	-0.180	-0.155
Direct effect	$ au_3^a o W^a$	0.057	0.057	0.056	0.056
of $ au_3$	$\tau_3^b \to W^b$	0.057	0.057	0.056	0.056
	$\tau_3^c \to W^c$	-0.087	-0.122	-0.243	-0.134
Cross-border	$ au_3^a o W^b$	-0.018	-0.018	-0.018	-0.018
effect	$\tau_3^a \to W^c$	0.006	0.005	0.004	0.003
	$\tau_3^b \to W^a$	-0.018	-0.018	-0.018	-0.018
	$\tau_3^b \to W^c$	0.006	0.005	0.004	0.003
	$\tau^c_3 \to W^a$	-0.051	-0.059	-0.087	-0.074
	$\tau^c_3 \to W^b$	-0.051	-0.059	-0.087	-0.074

Note: Each column denotes a different size of the change in taxes. The specific tax changed is indicated in the second column, as well as the welfare affected. The effect is obtained by the numerical approximation to the derivative of welfare with respect to a change in the tax $(\frac{\Delta W}{\Delta \tau})$. The superindexes refer to the countries with a: EME-A, b: EME-B and c: Center.

In terms of international policy effects, these results indicate there is a negative cross-border policy spillover from setting taxes set in the EMEs as the local and international welfare responses to a change in the emerging taxes have opposite signs. Finally, the spillovers from the Center tax are positive, suggesting potential policy free-riding incentives by the peripheries that may want to rely on the Center macroprudential taxes.

Analytical Welfare Effects In order to understand the mechanisms that generate these spillovers I set a Social Planner Problem and obtain the analytical welfare effects, following the methodology of Davis and Devereux (2019). For this, I set the welfare expressions associated to a social planner problem and simplify them using the private equilibrium conditions. Afterwards, I obtain the welfare effects via implicit differentiation.

A social planner will consider the following welfare expressions.

$$W_0^a = u\left(C_1^a\right) + \beta u\left(C_2^a\right) + \beta^2 u\left(C_3^a\right) + \lambda_1^a \left\{ A_1^a K_0^{a \alpha} + Q_1^a I_1^a - C(I_1^a, I_0^a) - \delta_B Q_1^a K_0^a - C_1^a - \frac{B_1^a}{R_1} \right\}$$

$$+ \beta \lambda_2^a \left\{ \varphi(\tau_2^a) A_2^a K_1^{a \alpha} + Q_2^a I_2^a - C(I_2^a, I_1^a) - \delta_B Q_2^a K_1^a + \kappa \left(\frac{Q_1^a K_1^a}{\Lambda_{12}} - \Lambda_{23} \theta Q_2^a K_2^a \right) + B_1^a - C_2^a - \frac{B_2^a}{R_2} \right\}$$

$$+ \beta^2 \lambda_3^a \left\{ \left(1 - \alpha \left(1 - \tau_3^a \right) \right) A_3^a K_2^{a \alpha} + \kappa \frac{Q_2^a K_2^a}{\Lambda_{12}} + B_2^a - C_3^a \right\}$$

$$(1)$$

with
$$\varphi(\tau) = (1 - \alpha (1 - \tau))$$

$$W_{0}^{c} = u\left(C_{1}^{c}\right) + \beta u\left(C_{2}^{c}\right) + \beta^{2}u\left(C_{3}^{c}\right) + \lambda_{1}^{c}\left\{A_{1}^{c}K_{0}^{c} + Q_{1}^{c}I_{1}^{c} - C\left(I_{1}^{c}, I_{0}^{c}\right) - \delta_{B}Q_{1}^{c}K_{0}^{c} - C_{1}^{c} - \frac{B_{1}^{c}}{R_{1}} - D_{1}\right\}$$

$$+\beta\lambda_{2}^{c}\left\{\left(1 - \alpha\theta\left(1 - \tau_{2}^{c}\right)\right)A_{2}^{c}K_{1}^{c} + Q_{2}^{c}I_{2}^{c} - C\left(I_{2}^{c}, I_{1}^{c}\right) + \left(1 - \theta\right)\left(\left(1 - \delta\right)Q_{2}^{c}K_{1}^{c} + R_{b1}^{a}F_{1}^{a} + R_{b1}^{b}F_{1}^{b}\right) - \theta R_{1}D_{1} - \delta_{B}Q_{2}^{c}K_{1}^{c} + B_{1}^{c} - C_{2}^{c} - \frac{B_{2}^{c}}{R_{2}} - D_{2}\right\}$$

$$+\beta^{2}\lambda_{3}^{c}\left\{A_{3}^{c}K_{2}^{c} + \left(1 - \delta\right)Q_{3}K_{2}^{c} + R_{b2}^{a}F_{2}^{a} + R_{b2}^{b}F_{2}^{b} + B_{2} - C_{3}^{c}\right\}$$

$$(2)$$

To obtain these expressions I set the welfare as the present value of the life-stream of utility plus a budget constraints in each period (times their Lagrange multipliers). Then, I replace the profits and tax rebates. Notice that these expressions are correct given the constraints are binding, and hence sum to zero, leaving the usual definition of welfare as result.

Setting the welfare in this fashion is convenient, given the algebra is simplified because we can ignore the effect of the decision variables of the households as their first order conditions (zero in equilibrium) become a factor of the associated differential terms.

Next, I obtain the welfare effects from changing each type of tax. We should remember that a planner setting the tax in the last period,¹³ will take the taxes and variables from the previous period as given, hence, we just need to differentiate with respect to R_2 , Q_2 , I_2 . K_2 for both types of countries plus $R_{b,2}$, F_2 for the center. In contrast, for the first period we must also consider the lagged versions of these variables.

The welfare effects of the taxes are:

For the EMEs:

$$\frac{dW_0^a}{d\tau_2^a} = \beta \lambda_2^a \bigg\{ \overbrace{\alpha_1(\kappa) \frac{dK_1^a}{d\tau_2^a} + \alpha_2(\kappa) \frac{dQ_1^a}{d\tau_2^a} + \frac{B_1^a}{R_1} \frac{dR_1}{d\tau_2^a} + \alpha Y_2^a}^{\text{dklamic effects}} + \underbrace{\alpha_3(\kappa) \frac{dK_2^a}{d\tau_2^a} + \alpha_4(\kappa) \frac{dQ_2^a}{d\tau_2^a} + \frac{B_2^a}{(R_2)^2} \frac{dR_2}{d\tau_2^a}}_{\text{dklamic effects}} \bigg\}$$

¹³The time index of the tax corresponds to the period in which the banks pay it, i.e., the initial tax is τ_2 and the one for the final intermediation period is τ_3 .

$$\frac{dW_0^a}{d\tau_3^a} = \beta \lambda_2^a \left\{ \overbrace{\alpha_5(\kappa) \frac{dK_2^a}{d\tau_3^a} + \alpha_4(\kappa) \frac{dQ_2^a}{d\tau_3^a} + \frac{B_2^a}{(R_2)^2} \frac{dR_2}{d\tau_3^a} + \alpha \frac{Y_3^a}{R_2}}^{\text{(only) static effects}} \right\}$$

$$\text{with } \alpha_1(\kappa) = \kappa R_1 Q_1^a + \varphi\left(\tau_2^a\right) r_2^a \text{, } \alpha_2(\kappa) = R_1\left(I_1^a + \kappa K_1^a\right) \text{, } \alpha_3(\kappa) = \kappa\left(1 - \theta\Lambda_{23}\right) Q_2^a + \varphi\left(\tau_3^a\right) \Lambda_{12} r_3^a \text{,} \\ \alpha_4(\kappa) = I_2^a + \kappa\left(1 - \theta\Lambda_{23}\right) K_2^a \text{, } \alpha_5(\kappa) = \kappa\left(1 - \theta\Lambda_{23}\right) Q_2^a + \varphi\left(\tau_3^a\right) \Lambda_{23} r_3^a \text{, and } \frac{\partial \alpha_s}{\partial \kappa} > 0 \text{ for } s = \{1, \dots, 5\}.$$

and for the Center:

$$\frac{dW_{0}^{c}}{d\tau_{2}^{c}} = \beta \lambda_{2}^{c} \left\{ \gamma_{1} \frac{dK_{1}^{c}}{d\tau_{2}^{c}} + \left(\frac{B_{1}^{c}}{R_{1}} - \theta D_{1} \right) \frac{dR_{1}}{d\tau_{2}^{c}} + \frac{K_{1}^{c}}{R_{1}} \frac{dQ_{1}^{c}}{d\tau_{2}^{c}} + \alpha \theta Y_{2}^{c} + (1 - \theta) \left(F_{1}^{ab} \frac{dR_{b,1}^{eme}}{d\tau_{2}^{c}} + R_{b,1}^{eme} \frac{dF_{1}^{ab}}{d\tau_{2}^{c}} \right) \right\}$$

$$+ \beta^{2} \lambda_{3}^{c} \left\{ \gamma_{2} \frac{dK_{2}^{c}}{d\tau_{2}^{c}} + \frac{B_{2}^{c}}{R_{2}} \frac{dR_{2}}{d\tau_{2}^{c}} + \gamma_{3} \frac{dQ_{2}^{c}}{d\tau_{2}^{c}} + F_{2}^{ab} \frac{dR_{b,2}^{eme}}{d\tau_{2}^{c}} + R_{b,2}^{eme} \frac{dF_{2}^{ab}}{d\tau_{2}^{c}} \right\}$$

$$\text{dynamic effects}$$

$$\frac{dW_0^c}{d\tau_3^c} = \beta^2 \lambda_3^c \left\{ \gamma_2 \frac{dK_2^c}{d\tau_3^c} + \frac{B_2^c}{R_2} \frac{dR_2}{d\tau_3^c} + \gamma_3 \frac{dQ_2^c}{d\tau_3^c} + F_2^{ab} \frac{dR_{b,2}^{eme}}{d\tau_3^c} + R_{b,2}^{eme} \frac{dF_2^{ab}}{d\tau_3^c} \right\}$$

$$\text{With } \gamma_1 = \left(1 - \alpha\theta \left(1 - \tau_2^c\right)\right) r_2^c + (1 - \theta)(1 - \delta)Q_2^c, \\ \gamma_2 = \left(r_3^c + (1 - \delta)Q_3\right), \\ \gamma_3 = R_2 \left(I_2^c + (1 - \theta)(1 - \delta)K_1^c\right), \\ \text{and } F_t^{ab} = F_t^a + F_t^b.$$

The interpretation of these effects goes as follows: First, we can see that there are more sources of variations for taxes that are forward-looking in nature (τ_2), whereas for the terminal taxes we only get the static effects. This explains why the effects of the former are stronger.

On the other hand, there are four drivers of the static welfare effects of the tax: (i) effect from hindering the capital accumulation, (ii) effect from changes in the global interest rate, which are proportional to the net foreign asset position, (iii) effect from changes in the prices of capital, and in addition, for the Center, (iv) changes in the cross-border lending rates and quantities. The welfare effects (i) and (iv) are negative and capture a halting in banking intermediation, while the sign of (ii) and (iii) depends, respectively, on whether an economy is a net creditor or on the investment growth, in that sense, we expect (ii) to be positive for an emerging economy and negative for the Center. Finally, assuming that the investment in these economies is still growing after the tax change, (iii) is negative.

The dynamic effects will have similar drivers. However, in all cases it will refer to the effect in future variables, for instance, (i) would refer to the effect on future capital accumulation and (ii) on the future net assets position. The signs for the dynamic effects will not be as straightforward. Then, we may expect similar signs but with potential corrections, for example, when tighter initial taxes imply delaying investment or capital acumulation plans for future periods.

Noticeably, the aforementioned negative effects are reflective of the potential negative growth consequences of setting these taxes that are akin to putting sand in the wheels of the financial sector. That is what the literature refers to when pointing out the potential immiserizing growth effects of these tools.¹⁴ Of course, the policy trade-off here is that mitigating the financial friction may be well worth such cost.

A critical feature that can be observed is that the welfare effects from changes in capital accumulation and capital prices are augmented by the degree of the financial friction (κ). This indicates that these taxes are potentially more effective for highly distorted economies.

Optimal taxes. From the analytical welfare effects it's possible to solve for the optimal taxes. The derivation procedure and other expressions are shown in appendix A. Here, to describe their structure, I show the optimal nationally-oriented tax for the Center:

$$\tau_3^c = \frac{Q_2^c}{r_3^c} \left\{ \gamma_2 \frac{dK_2^c}{dF_2^{ab}} + \Lambda_{23} B_2^c \frac{dR_2}{dF_2^{ab}} + \gamma_3 \frac{dQ_2^c}{dF_2^{ab}} + F_2^{ab} \frac{dR_{b2}^{\text{eme}}}{dF_2^{ab}} \right\} + \frac{(1 - \delta)Q_3}{r_3^c} + 1 \tag{3}$$

with
$$\gamma_2=(r_3^c+(1-\delta)Q_3)$$
, $\gamma_3=R_2\left(I_2^c+(1-\theta)(1-\delta)K_1^c\right)$, and $F_2^{ab}=F_2^a+F_2^b$

Two relevant features can be found in both types of taxes (forward-looking and static), first, the peripheral taxes grow in scale with the financial distortion, and second, the center depicts a substitution effect motive between local and foreign intermediation ($\frac{\partial K^c}{\partial F}$ terms in (3)). This latter effect helps to understand how the tax setting of the Center differs from that of the periphery given its role of international creditor. In a nutshell, the regulators at the Center trade-off local intermediation for global lending, a feature that matters for understanding the importance of the Center in generating gains from policy cooperation.

Finally, in terms of the dynamic effects of the forward-looking taxes (set in non-terminal periods), I obtain that these reflect the effects on future variables from a change in the capital accumulation of the economy where the instrument is being set.

On the other hand, when considering the cross-border effects of these policies I obtain

¹⁴See Boar, Gambacorta, Lombardo, and da Silva (2017) and Belkhir, Naceur, Candelon, and Wijnandts (2020) for a discussion on the growth effects of macroprudential policies

similar expressions, with the difference that there will be no direct welfare effects from changing the taxes, i.e., any welfare change will come only from variations in the endogenous economic variables, i.e. from their general equilibrium effects. Simultaneously, the variable driving the changes in the differentials will be that of a foreign country.

Welfare effects and Policy in Cooperative Settings. I have analyzed the spillover effects of these policies and optimal taxes for individual policy makers (non-cooperative) that maximize their national welfare. In contrast, in cooperative settings the planners join efforts and act as one with the objective of maximizing the aggregate welfare of their coalition.¹⁵ As a result, the global welfare effects are given by weighted averages of the individual expressions shown previously.

With these new welfare effect expressions we can find the optimal cooperative taxes as well. Here I show the resulting optimal tax for the Center in the last period. In the appendix A I show how to obtain this expression from the average of the individual welfare effects.

substitution of Center capital accumulation for foreign intermediation (EMEs) motive

$$\tau_3^{c,coop} = \tau_3^{c,nash} + \overbrace{\frac{Q_2^c}{\Lambda_{23}r_3^c}\frac{\lambda_2^a}{\lambda_2^c}}^{\text{substitution of Center capital accumulation for foreign intermediation (EMEs) motive} - \underbrace{\frac{\lambda_2^c}{\lambda_2^c}\frac{\lambda_2^c}{\lambda_2^c}\frac{\lambda_2^c}{\lambda_2^c}}_{\text{manipulation motive}}^{\text{NFA-led interest rate manipulation motive}}_{\text{manipulation motive}}$$
(4)

Where $\tau_3^{c,nash}$ is the optimal tax for the non-cooperative planner as in (3).

As previously:
$$\alpha_4(\kappa) = (I_2^a + \kappa (1 - \theta \Lambda_{23}) K_2^a)$$
 and $\alpha_5(\kappa) = (\kappa (1 - \theta \Lambda_{23}) Q_2^a + \varphi (\tau_3^a) \Lambda_{23} r_3^a)$, with $\frac{\partial \alpha_s(\kappa)}{\partial \kappa} > 0$ for $s = \{4, 5\}$.

Crucially, the welfare effects from changes in the global interest rates, that are proportional to the net foreign assets positions of the economies, cancel out between creditors and debtors that cooperate (i.e., the last term in (4) cancels with an analogous term, but with opposite sign within $\tau_3^{c,nash}$). Additionally, a motive for increasing the Center taxes emerges, which is proportional to the increase in capital accumulation at the EMEs after a change in global intermediation, and increases with the extent of the financial friction κ .

These two features, the first one present in every country, and the second in the Center, are the main factors explaining welfare differences between cooperative and non-cooperative policy settings as we will see in the results section.

As for the presence of welfare gains from cooperation and, if these exist, their distribution between economies, I set a more comprehensive model that accounts for the entire path

¹⁵All the policy cases I consider are shown in detail in table 2

of the taxes and the persistency of their effects in a stochastic environment. For that, I endogeneize the taxes by formulating a Ramsey policy problem. I present the model and policy problems in the following two sections.

4 The Main Model

In this section I set the main model of this study and analyze how the perfect-foresight results hold in a stochastic environment. The model borrows standard elements from the literature for representing each agent. In particular, I take elements from Banerjee, Devereux, and Lombardo (2016), Agénor, Kharroubi, Gambacorta, Lombardo, and da Silva (2017) and Gertler and Karadi (2011) and incorporate them into a three country center-periphery framework with incomplete markets.

The world economy consists of three countries, one financial center (C) with population size $1 - n_a - n_b$ and two periferies, A and B, with population sizes n_a and n_b , with $n_a + n_b \le \frac{1}{2}$.

The agents have access to an international bonds market where they can trade non-contingent bonds. There is a single consumption good in the world which is freely traded. The model is set in real terms. Also, the preferences are identical between agents in each country and the law of one price holds. Thus, the purchasing power parity holds and the real exchange rate is one. In addition, the uncovered interest rate parity holds.

This implies that the only friction present in this model is the financial agency friction in borrower-lending relationships. In that regard, this is a costly-enforcement framework like Gertler and Kiyotaki (2010).

To analyze the banking incentives in different types of economies I incorporate distinct levels of financial development across countries, with the emerging economies featuring lower financial development, which makes necessary for their banks to rely on funding from financial centers, in order to fulfill their role as intermediaries with the local firms.

Throughout this section, the superindex i will be used when the expression applies to each country $i = \{a, b, c\}$, otherwise I use the corresponding specific superindex.

4.1 Households

The households in each economy choose consumption, savings (with bonds or deposits) and leisure to maximize their welfare, given by the present value of their life-stream utility:

$$\max_{\{C_t, H_t, B_t, D_t\}_{t=0}^{\infty}} W_0^i = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{i(1-\sigma)}}{1-\sigma} - \frac{H_t^{i(1+\psi)}}{1+\psi} \right)$$
 (1)

s.t.,

$$C_t^i + B_t^i + \frac{\eta}{2}(B_t^i)^2 + D_t^i + \frac{\eta}{2}(D_t^i - \bar{D}^i)^2 = R_{t-1}^i B_{t-1}^i + R_{D,t-1}^i D_{t-1}^i + w_t^i H_t^i + \Pi_t^i$$
 (2)

With $i = \{a, b, c\}$ and where B_t^i : non-contingent international bonds, D_t^i : domestic deposits, $w_t^i H_t^i$: labor income (wages times hours), R^i the interest rate on bonds, R_D^i the interest rate on deposits, Π_t^i : profits from banks and other firms net of lump-sum taxes.

In addition, adjustment costs from changes in assets positions are included to prevent non-stationarity of the model in an incomplete markets setup (see Schmitt-Grohe and Uribe (2003)).

The consumption of the final good by the home household in the country i is C^i . Since only one good is produced, that is, there are no country-specific commodities, a retail and intermediate goods sector is not included. That implies there is no home bias in consumption generated by the asymmetric size of the countries. Furthermore, since no departure from the law of one price is assumed, the relative prices across countries and real exchange rates are abstracted from.

Financial Center. The F.O.C. for the households of the Center are:

$$\mathbb{E}_{t} \left[R_{t} \Lambda_{t,t+1}^{c} \right] = 1 + \eta(B_{t}^{c})$$

$$\mathbb{E}_{t} \left[R_{D,t}^{c} \Lambda_{t,t+1}^{c} \right] = 1 + \eta(D_{t}^{c} - \bar{D}^{c})$$

$$C_{t}^{c - \sigma} = \frac{H_{t}^{c \psi}}{w_{t}^{c}}$$

Where $\Lambda_{t,t+1} = \beta \lambda_{t+1}/\lambda_t$ is the stochastic discount factor, λ_t is the marginal utility of consumption, and the interest rate on bonds takes into account that their return is equalized across economies (via UIP).

Emerging Economy Households. One difference between the advanced economy and the emerging ones is that, at the former, households are able to freely purchase deposits from the Center banks while the emerging economy banks will have a limited local intermediation capacity. This implies the banks in these countries hold less deposits. As a simplification, I drop the deposits for these countries altogether (i.e., D_t^a and D_t^b are zero). Note that this feature is not explicitly reflected in the household budget constraint above.

The F.O.C. of the emerging economy A are:

$$\mathbb{E}_t \left[R_t \Lambda_{t,t+1}^a \right] = 1 + \eta(B_t^a)$$

$$C_t^{a - \sigma} = \frac{H_t^{a \psi}}{w_t^a}$$

The F.O.C. of the emerging economy B are analogous.

4.2 Final Goods Firms

A single final good is produced with a CD technology:

$$Y_t^i = A_t^i \left(\xi_t^i K_{t-1}^i \right)^\alpha H_t^{i(1-\alpha)} \tag{3}$$

 H^i, K^i are labor and capital, A^i is a productivity shock, and ξ^i is a capital-quality shock (both are first-order AR processes).

The capital quality shock implies the depreciation rate is given by $\delta^i_t(\xi^i_t) = 1 - (1 - \delta)\xi^i_t$.

Each period, the firms choose labor and capital inputs to maximize the profits obtained from producing and from the sales of undepreciated capital to investors, while paying wages and the banking loan with which they funded the acquisition of physical capital:

$$\max_{K_{t-1}^i, H_t^i} \Pi_t^{i,prod} = Y_t^i + (1 - \delta) \xi_t^i Q_t^i K_{t-1}^i - w_t^i H_t^i - \tilde{R}_{k,t}^i Q_{t-1}^i$$
 s.t. (3)

I define the marginal product of capital as $r_t^i \equiv \alpha A_t^i \xi_t^i \, {}^{\alpha} K_{t-1}^i \, H_t^{i \, 1-\alpha}$, and obtain the wages and gross rate of returns paid to the banking sector from the FOCs with respect to labor and capital:

$$w_t^i = (1 - \alpha) A_t^i H_t^{i(-\alpha)} \xi_t^i {}^{\alpha} K_{t-1}^{i(\alpha)}$$

$$\tilde{R}_{k,t}^{i} = \frac{r_t^i + (1 - \delta)\xi_t^i Q_{t-1}^i}{Q_{t-1}^i}$$

The physical capital is funded by selling company securities to domestic banks in a one to one relationship, i.e., $Z_t^i = K_t^i$, where Z_t^i is the stock of securities from the representative final goods firm in the country i. In that spirit, the marginal product of capital r_t^i can also be interpreted as the return from the firm securities. ¹⁶

4.3 Capital Goods Firms

Physical capital is produced in a competitive market by using old capital and investment. The depreciation rate of capital is $1 - (1 - \delta)\xi_t^i$, also the investment is subject to convex adjustment costs, i.e., the total cost of investing I_t^i is:

$$C(I_t^i) = I_t^i \left(1 + \frac{\zeta}{2} \left(\frac{I_t^i}{I_{t-1}^i} - 1 \right)^2 \right)$$

The capital dynamics are: 17

$$K_t^i = I_t^i + (1 - \delta)\xi_t^i K_{t-1}^i \tag{4}$$

After production takes place, these firms buy the old capital stock from the final goods firms at price Q_t^i and produce new capital subject to the adjustment cost.

Finally, the problem of the capital goods firm choosing their investment level is:

$$\max_{\{I_t^i\}_{t=0}^{\infty}} E_0 \sum_{s=0}^{\infty} \Lambda_{t,t+s}^i \left\{ Q_{t+s}^i I_{t+s}^i - I_{t+s}^i \left(1 + \frac{\zeta}{2} \left(\frac{I_{t+s}^i}{I_{t+s-1}^i - 1} \right)^2 \right) \right\}$$

From the first order condition we can pin down the dynamics for the price of capital:

$$Q_t^i = 1 + \frac{\zeta}{2} \left(\frac{I_t^i}{I_{t-1}^i} - 1 \right)^2 + \zeta \left(\frac{I_t^i}{I_{t-1}^i} - 1 \right) \frac{I_t^i}{I_{t-1}^i} - \mathbb{E}_t \left[\Lambda_{t,t+1}^i \zeta \left(\frac{I_{t+1}^i}{I_t^i} \right)^2 \left(\frac{I_{t+1}^i}{I_t^i} - 1 \right) \right]$$
 (5)

 $^{^{16}}$ For simplicity, when solving the model, I replace $\tilde{R}_{k,t}$ back in the profit function so that I can drop it as a variable and work only with the effective (after tax) revenue rate perceived by banks. When doing such substitution a standard expression for the profits is obtained: $\Pi_t^{i,prod} = Y_t^i - r_t^i K_t^i + W_t^i H_t^i.$

¹⁷The time index used for capital denotes the period in which it was determined, rather than the period when it is used for production.

4.4 Banking Sector

The set-up for this sector is based on Gertler and Karadi (2011). Each economy has a financial firm that intermediates funds between savers and firms. It borrows funds from either the depositors or the interbank market and lends them to local firms that use them for acquiring capital. The spread in the interest rates of lending and borrowing generates the profits for this sector.

I consider a setup with a continuum of symmetric banks that are subject to entry and exit to their business with a survival rate θ . This prevents the banks from engaging in self-funding schemes that would prevent the agency frictions constraints to bind. The entering banks receive a start-up capital from their household owners that is proportional to the scale of the banking assets in the preceding period. At each date, the continuing banks re-invest their proceeds back in its business. However, when the bank fails and exits the market, it gives back its net worth as profits to its owners.

In each case, I consider an incentive compatibility constraint (ICC) that reflects the agency problem in the lending relationships of the bank. I assume this constraint is binding.

The structure of the sector in each country and the decisions they face are explained in detail in the following subsections. However, it can be said that in general, the problem of the j-th bank in t consists in maximizing a financial intermediation value function $J(N_{j,t}) = \mathbb{E}_t \max \Lambda_{t,t+1}[(1-\theta)N_{j,t+1} + \theta J(N_{j,t+1})]$ subject to the dynamics of the net worth of the bank (N), its balance sheet and the ICC.

The emerging markets' banks also have the additional constraint of having a limited intermediation capacity. This eventually implies funding flows from the Center economy to the peripheries that results in balance sheet effects at the cross country level.

EME Banks. The banks start with a bequest from the households and continue their activities with probability θ . The index e refers to either emerging market with $e = \{a, b\}$.

Let N_{jt}^e be the net worth and F_{jt}^e the amount borrowed from center banks at a real rate $R_{b,t}^e$. The balance sheet of the bank j is given by:

$$Q_t^e Z_{jt}^e = N_{jt}^e + F_{jt}^e (6)$$

We also have that there is a one to one relationship between the securities of the bank and the physical capital units, i.e., $Z^e = K^e$.

The aggregate net worth of the banking system is:

$$N_t^e = \overbrace{\theta N_{j,t}^e}^{\text{surviving banks'}} + \overbrace{\delta_T Q_t^e K_{t-1}^e}^{\text{new banks'}}$$

Here, the bequests provided by the households to the banks are proportional to the preexisting level of intermediation (capital) times the current price of capital. At the same time, $N_{i,t}^e$ is the net-worth of surviving banks and have the following dynamics:

$$N_{j,t}^e = R_{k,t}^e Q_{t-1}^e K_{j,t-1}^e - R_{b,t-1}^e F_{j,t-1}^e \tag{7}$$

The gross return on capital, $R_{k,t}^e$, accounts for the payment of the macroprudential tax:

$$R_{k,t}^e = \frac{(1 - \tau_t^e)r_t^e + (1 - \delta)\xi_t^e Q_t^e}{Q_{t-1}^e}$$

with $\tau_t^e \geq 0$ representing a tax/subsidy.

The contracts between savers and banks are subject to limited enforceability, i.e., a bank can default, in which case, the savers take it to court but can recover only a portion $(1 - \kappa^e)$ of their payment. In practice, this implies the bank can divert a portion κ^e of the assets.

The problem of the j-th banker is to maximize the franchise value of the bank:¹⁸

$$J_{j,t}^e(N_{j,t}^e) = \max_{N_{j,t}^e, Z_{j,t}^e, F_{j,t}^e} \mathbb{E}_t \Lambda_{t,t+1}^e \left[(1-\theta) N_{j,t+1+s}^e + \theta J_{j,t+1}^e(N_{j,t+1}^e) \right]$$

subject to the net worth dynamics (7), their balance sheet (6) and associated ICC:

$$J_{i,t}^e \ge \kappa^e Q_t^e K_{i,t}^e \tag{8}$$

This incentive compatibility constraint states that the continuation value of the bank is larger than the potential profit of defaulting.¹⁹

The bank's problem yields the following optimality conditions:

F.O.C. with respect to intermediated capital:

$$[K_{i,t}^e]: \qquad \mathbb{E}_t \Omega_{t+1|t}^e \left(R_{k,t+1}^e - R_{b,t}^e \right) = \mu_t^e \kappa^e$$
 (9)

¹⁸An analogous sequential problem is: $J^e(N^e_{j,t}) = \max_{\{N_t, Z^e_t, F^e_{j,t}\}_{t=0}^{\infty}} \mathbb{E}_t(1-\theta) \sum_{s=0}^{\infty} \Lambda^e_{t,t+1+s} [\theta^s N^e_{j,t+1+s}]$ ¹⁹There are several feasible choices for the right hand side term depending on the timing of the assets

¹⁹There are several feasible choices for the right hand side term depending on the timing of the assets absconding. Here I assume they compare the value of the bank to diverting assets as soon as they obtain them, i.e., before these yield returns.

and envelope condition:

$$[N_{j,t}^e]: J^{e'}(N_{j,t}^e)(1-\mu_t^e) = \mathbb{E}_t \Omega_{t+1|t}^e R_{b,t}^e (10)$$

Where μ^e_t is the lagrange multiplier associated with the ICC and $\Omega^e_{t+1|t} = \Lambda^e_{t,t+1} \left(1 - \theta + \theta J^{e'}_{t+1}\right)$ is the effective stochastic discount factor of the bank.

Center Economy Banks. The structure of the center economy banks is similar. We only need to be careful when setting the balance sheet and net worth dynamics. Both need to reflect the foreign claims intermediated and the proceeds from being a global creditor.

The balance sheet of the global country bank j is:

$$F_{i,t}^a + F_{i,t}^b + Q_t^c Z_{i,t}^c = N_{it}^c + D_t^c$$
(11)

where D^c are the deposits from the households, $F_{j,t}^e$ are the (international) claims on the $e = \{a, b\}$ representative periphery banks (EMEs), and $Q_t^c Z_{j,t}^c$ are (local) claims on the Center country capital stock with $Z_{j,t}^c = K_{j,t}^c$.

Their net (after taxes) return on intermediated capital is:

$$R_{k,t}^c = \frac{(1 - \tau_t^c)r_t^c + (1 - \delta)\xi_t^c Q_t^c}{Q_{t-1}^c}$$

The bank *j* value function is:

$$J_{j,t}^c(N_{j,t}^c) = \max_{N_{j,t}^c, Z_t^c, F_{j,t}^c, D_t^c} \mathbb{E}_t \Lambda_{t,t+1}^c \Big[(1-\theta) (\overbrace{R_{k,t+1}^c Q_t^c Z_{j,t}^c + R_{b,t}^a F_{j,t}^a + R_{b,t}^b F_{j,t}^b}^{\text{deposits' repayment}} - \overbrace{R_{D,t}^c D_t^c}^{\text{deposits' repayment}} \Big) \\ + \theta J_{j,t+1}^c (N_{j,t+1}^c) \Big]$$

The bank maximizes such value while being subject to the balance sheet constraint (11) and to an incentive compatibility constraint given by:

$$J_{j,t}^c \ge \kappa_{F_1}^c F_{jt}^a + \kappa_{F_2}^c F_{jt}^b + \kappa^c Q_t^c Z_{j,t}^c \tag{12}$$

The optimality Conditions are:

$$[Z_{j,t}^c]: \quad \mathbb{E}_t \Omega_{t+1|t}^c (R_{k,t+1}^c - R_{D,t}^c) = \kappa^c \mu_t^c \tag{13}$$

$$[F_{j,t}^a]: \quad \mathbb{E}_t \Omega_{t+1|t}^c \left(R_{b,t}^a - R_{D,t}^c \right) = \kappa_{F_1}^c \mu_t^c \tag{14}$$

$$[F_{j,t}^b]: \quad \mathbb{E}_t \Omega_{t+1|t}^c \left(R_{b,t}^b - R_{D,t}^c \right) = \kappa_{F_2}^c \mu_t^c \tag{15}$$

and the envelope condition,

$$[N_{i,t}^c]: \quad J^{c'}(N_{i,t}^c)(1-\mu_t^c) = \mathbb{E}_t \Omega_{t+1|t}^c R_{D,t}^c$$
(16)

4.5 Macroprudential Policy

The policy tool I consider is a tax on the return to capital. This is a general enough tool that encompasses several varieties of macroprudential instrumets. For example, and as I showed in the proposition 2, it has leverage-ratio implications.

Furthermore, setting the tool as a tax on the revenue rate of banking has the advantage of affecting directly the wedge between return on capital and deposit rate (credit spread). Therefore, policy actions can be applied right at the source of inefficiencies.

$$\tau_t^i r_t^i K_{t-1}^i + T_t^i = 0$$
 $i = \{a, b, c\}$

The regulators rebate the tax proceeds to their households citizens as a lump-sum tax.

Effect of the macroprudential tool in the model. In the appendix B I explain the effect of the policy in the infinite horizon setup. However, the implications are analogous to those of propositions 1 and 2 in section 3. In a nutshell, with a binding ICC, an increase in the prudential tax will lower the credit spread and leverage, and the extent at which these effects occur is larger for a higher degree of the financial friction.

4.6 Market Clearing Conditions

The corresponding market clearing conditions of the model, for the final goods market and bonds, are:

Goods market:
$$\sum_{i} n_{i} Y_{t}^{i} = \sum_{i} n_{i} \left(C_{t}^{i} + I_{t}^{i} \left(1 + \frac{\zeta}{2} \left(\frac{I_{t}^{i}}{I_{t-1}^{i}} - 1 \right)^{2} \right) + \frac{\eta}{2} (B_{t}^{i})^{2} + \frac{\eta}{2} \left(D_{t}^{i} - \bar{D}^{i} \right)^{2} \right)$$
 Bonds market:
$$\sum_{i} n_{i} B_{t}^{i} = 0, \qquad \forall t$$

where i denotes a country index, i.e., $i = \{a, b, c\}$.

Notice that the market clearing condition for the final goods reflects, both, the adjustment cost of executing investment projects, and that the final good is fully tradable and produced in each economy (no home bias).

Equilibrium. For a given path of macroprudential policies $\tau_t = \{\tau_t^a, \tau_t^b, \tau_t^c\}$ a tax-distorted competitive equilibrium is given by the prices $\{w_t^i, Q_t^i\}$, rates $\{R_t, R_{D,t}, R_{k,t}^i, R_{b,t}^e\}$ and quantities $\{C_t^i, H_t^i, B_t^i, D_t^c, K_t^i, I_t^i, N_t^i, F_t^e, Y_t^i\}$ with $i = \{a, b, c\}$ and $e = \{a, b\}$ such that,

Given $\{w_t^i, R_t, R_{D,t}\}$, the sequences $\{C_t^i, B_t^i, D_t^c, H_t^i\}$ solve the households utility maximization problem for each t.

Given $\{Q_t^i, w_t^i, R_{k,t}^i\}$ and the technological constraint $\{Y_t^i\}$, $\{K_t^i, H_t^i\}$ solve the final goods firms profit maximization problem for each t.

Given $\{Q_t^i\}$ and the expected path of prices $\{\mathbb{E}_t Q_{t+s}\}_{t=0}^{\infty}$, $\{I_t^i\}$ solves the capital producer profit maximization problem.

Given $\{Q_t^i, R_{k,t}^i, R_{b,t}^e, R_{D,t}\}$, $\{N_t^i, Z_t^i, F_t^e\}$, with $Z_t^i = K_t^i$ solves the franchise value maximization problem of the banks.

In addition, capital dynamics are given by (4), and the goods and bonds market clearing conditions hold for each t.

In the table 11 in the appendix B, I show the final system of equations that characterizes the equilibrium. These structural equations will be used as the set of constraints for the policy makers that decide the optimal level of the tools in each of the regimes considered.

5 Ramsey Policy Problem

So far I have characterized the private equilibrium for this economy. In that context, the policy tools are exogenous to the agents (they take them as given). However, I am interested in the endogenous determination of these tools for a set of regimes that vary by the degree of international cooperation. For that, I use the Ramsey Planner Problem, consisting on choosing the optimal level of the policy tools, and the rest of variables, subject to the private equilibrium conditions.

Table 2: Policy Cases Considered

	Planners/Players	Objective Function	Decision variables
Cooperation (all countries)	World	$W_0^{Coop} = n_a W_0^a + n_b W_0^b + n_c W_0^c$	$\mathbf{x_t}, oldsymbol{ au}_t$
Semi-Cooperation (EMEs vs. Center)	Periphery block A+B	$W_0^{ab} = n_a W_0^a + n_b W_0^b$	$\mathbf{x_t}, \tau_t^a, \tau_t^b$
	Center	W_0^c	$\mathbf{x_t}, \tau^c_t$
Semi-Cooperation (EME-A + C vs. EME-B)	Cooperative A+C	$W_0^{ac} = n_a W_0^a + n_c W_0^c$	$\mathbf{x_t}, \tau_t^a, \tau_t^c$
	EME-B	W_0^b	$\mathbf{x_t}, au_t^b$
Nash (non-cooperative) One planner per country	EME-A	W_0^a	$\mathbf{x_t}, \tau^a_t$
	EME-B	W_0^b	$\mathbf{x_t}, \tau_t^b$
	Center	W^c_0	$\mathbf{x_t}, \tau^c_t$

Note: $\boldsymbol{\tau}_t = (\tau_t^a, \tau_t^b, \tau_t^c)'$

The idea is to respect the private equilibrium structure while still shaping the final resulting allocation by setting the policy instruments optimally. I consider four policy schemes that range from no-cooperation (Nash) to world cooperation while allowing for semi-cooperative cases where subsets of countries form regulatory coalitions.

As shown in table 2, two features are critical for differentiating the cases: first, the objective funtion of the planner is the weighted welfare of the countries that belong to a coalition (in the non-cooperative case each economy has an individual planner whose objetive function will be the local welfare), and second, the cooperative planners, by joining efforts and acting as one, will have a larger menu of policy tools available.

5.1 Planning Problems

In every case I consider the planning problem under commitment with a timeless perspective. ²⁰ As explained by King and Wolman (1999) this implies to assume the planners were making optimal decisions in the past in a time consistent manner. This setup is standard in the literature given its property of avoiding indeterminacy issues in the model solution.²¹

²⁰See Woodford (2003) and Benigno and Woodford (2004) for a detailed discussion on the timeless perspective and time consistency in the policy problem.

²¹In the appendix B I discuss the solution of this model without time consistency and its welfare properties in regards to policy cooperation.

In addition, I solve for the *open-loop Nash* equilibrium for the cases where there are two or more players interacting simultaneously.

Definition 1. Open-loop Nash equilibrium

An open-loop Nash equilibrium is a sequence of tools $\{\tau_t^i\,^*\}_{t=0}^\infty$ such that for all t^* , $\tau_{t^*}^i$ maximizes the player i's objective function subject of the structural equations of the economy characterizing the private equilibrium for given sequences $\{\tau_{-t^*}^i\}_{t=0}^\infty$ and $\{\tau_t^{-i\,*}\}_{t=0}^\infty$, where $\{\tau_{-t^*}^i\}_{t=0}^\infty$ denotes the policy instruments of player i in other periods than t^* and $\{\tau_t^{-i\,*}\}_{t=0}^\infty$ the policy moves by all other players. In this sense, each player's action is the best response to the other players' best responses.

Given that the policymakers specify a contingent plan at time 0 for the complete path of their instruments $\{\tau_t^i\}_{t=0}^\infty$ for $i=\{a,b,c\}$, the problem they solve can be interpreted as a static game, which allows me to recast their maximization problems as an optimal control problem where the instruments of the other planners are taken as given.

In that vein and as in the static Nash equilibrium concept, the player i focuses on his own objective function. Having said this, the key difference across regimes is whether the planners maximize their national welfare or, jointly, that of a coalition.

World Cooperation. Under commitment, a single planner whose objective function is the worldwide welfare, chooses the vector of endogenous variables and policy instruments to solve:

$$W_0^{coop} = \max_{\mathbf{x}_t, \mathbf{\tau}_t} [n_a W_0^a + n_b W_0^b + (1 - n_a - n_b) W_0^c]$$
(17)

subject to the system of equations that characterize the private equilibrium (private FOCs, budget constraints and market clearing conditions):

$$\mathbb{E}_t F(\mathbf{x}_{t-1}, \mathbf{x}_t, \mathbf{x}_{t+1}, \boldsymbol{\tau}_{t-1}, \boldsymbol{\tau}_t, \boldsymbol{\tau}_{t+1}; \boldsymbol{\varphi}_t) = 0$$

where W_0^i denotes the welfare of the country i as in (1), \mathbf{x}_t is the vector of endogenous variables, $\boldsymbol{\tau}_t = (\tau_t^a, \tau_t^b, \tau_t^c)'$ is the vector of instruments and $\boldsymbol{\varphi}_t$ is a vector of exogenous variables and shocks.

Semi-cooperative case 1 - cooperation between the Center and the EME-A. The planners of the C and A economies form a coalition, acting as one and solve:

$$W_0^{coop(C+A)} = \max_{\mathbf{x}_t, \tau_t^a, \tau_t^c} [n_a W_0^a + n_c W_0^c]$$
(18)

s.t.,
$$\mathbb{E}_t F(\mathbf{x}_{t-1}, \mathbf{x}_t, \mathbf{x}_{t+1}, \boldsymbol{\tau}_{t-1}, \boldsymbol{\tau}_t, \boldsymbol{\tau}_{t+1}; \boldsymbol{\varphi}_t) = 0$$

where $F(\cdot)$ denotes the private equilibrium conditions. Notice that these system of constraints will be the same for every planner across all the policy frameworks.

The remaining country (B) solves the same problem as in the Nash case.

Semi-cooperative case 2 - cooperation between Emerging Economies. The planners of the A and B economies form a coalition and solve:

$$W_0^{coopEME} = \max_{\mathbf{x}_t, \tau_t^a, \tau_t^b} [n_a W_0^a + n_b W_0^b]$$
 (19)

s.t.,
$$\mathbb{E}_t F(\mathbf{x}_{t-1}, \mathbf{x}_t, \mathbf{x}_{t+1}, \boldsymbol{\tau}_{t-1}, \boldsymbol{\tau}_t, \boldsymbol{\tau}_{t+1}; \boldsymbol{\varphi}_t) = 0$$

The remaining country (C) solves the same problem as in the Nash case.

Nash (no cooperation). Finally, a non-cooperative policy-maker of the country $i = \{a, b, c\}$, with the domestic welfare as objective function, solves:

$$W_0^{i,nash} = \max_{\mathbf{x}_t, \tau_t^i} W_0^i \tag{20}$$

s.t.,
$$\mathbb{E}_t F(\mathbf{x}_{t-1}, \mathbf{x}_t, \mathbf{x}_{t+1}, \boldsymbol{\tau}_{t-1}, \boldsymbol{\tau}_t, \boldsymbol{\tau}_{t+1}; \boldsymbol{\varphi}_t) = 0$$

5.2 Gains From Cooperation

To compare the performance of the models, I compute the global expected conditional welfare and compute the welfare gains with respect to a benchmark. For example, the welfare gain of world cooperation relative to the non-cooperative (Nash) model is:

$$Gain_{Coop/Nash} \equiv W_0^{coop} - (n_a W_0^{a,nash} + n_b W_0^{b,nash} + (1 - n_a - n_b) W_0^{c,nash})$$

The gain is approximated at the second order around the non-stochastic steady state. Moreover, as it is, this welfare gain is given in utility units which makes difficult to assess the magnitude of the relative performance of each model. Then, for a better comparison, we can look for the consumption equivalent variation that would make the private agents indifferent between the models. For this case, that quantity is given by λ , the proportional

increase in the steady-state consumption of the world cooperation model that would deliver the same welfare as the Nash case:

$$W_0^{i,coop}(\lambda) = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{((1+\lambda)C_t^{i,coop})^{1-\sigma}}{1-\sigma} - \frac{(H_t^{i,coop})^{(1+\psi)}}{1+\psi} \right) = W_0^{i,nash}$$

For each economy $i = \{a, b, c\}$. Similarly, the global consumption equivalent gain (cost) is the weighted average of the national ones.

Clearly, an overperforming model, or in this example a model with gains from cooperation, would depict a negative λ . In that case, a negative λ would indicate that consumption in the cooperative regime would have to decrease in order to match the non-cooperative equilibrium, implying the cooperative regime is better from a welfare perspective. I approximate λ by normalizing the welfare gain (in utility units) by the increase in steady-state welfare that would be obtained from a 1% increment in consumption. ²²

6 Results

In this section, I discuss the solution of the main model under different policy schemes and how it helps to answer whether (i) the international cooperation of macroprudential policies is convenient for emerging economies in general?, and (ii) if so, what are the drivers of the associated welfare gains. For that, I first compare the expected long run welfare that the policy frameworks in table 2 deliver. Then, I discuss the features of each type of optimal policy to obtain the main welfare-inducing mechanisms.

Steady State of the Policy Instruments. The table 3 shows the steady states of the policy taxes for each policy regime considered. The algorithm used implies computing an instrument conditional steady state and follows the steps outlined in Christiano, Motto, and Rostagno (2007) and Bodenstein, Guerrieri, and LaBriola (2019). A detailed explanation can be found in the appendix A. I obtain that the Center always applies subsidies to its banking sector in the long run, while planners of the EMEs subsidize its banking sector only when cooperating with the Center, and instead, set a tax to the financial intermediaries in the non-cooperative case or under the emerging coalition. Thus, at least in the long-run, cooperation with the center consists on setting higher subsidies (lower taxes).

²²In the results (table 4) I show the consumption compensation variation that agents in the First Best model (with no frictions) would undergo in order to match each one of the considered regimes.

Table 3: Steady State values for the policy tools

	Nash	Cooperation (Center+EME-A)	Cooperation (EMEs)	Cooperation (All)
$ au^c$	-0.850	-0.530	-0.806	-0.864
$ au^a$	0.319	-0.164	0.348	-0.697
$ au^b$	0.319	0.328	0.348	-0.697

6.1 Welfare Accounting Comparison

We can compare the regimes in terms of the welfare they deliver. For this, I compute the conditional welfare that yields comparable quantities across models (see Bilbiie, Fujiwara, and Ghironi (2014) for details). I condition all the models on the same initial state given by the average of the steady state values of a subset of the regimes. Based on the resulting welfare quantities I rank the regimes and verify the existence of cooperation gains.

Table 4: Welfare cost in consumption equivalent compensation relative to the First Best

	Consumption Equivalent Compensation				
	Nash	Cooperation (Center+EME-A)	Cooperation (EMEs)	Cooperation (All)	
\overline{C}	-11.7	2.9	-13.2	-3.9	
A	-19.5	0.4	-27.4	-2.4	
B	-19.5	-28.3	-27.4	-2.4	
World	<i>-</i> 15.6	-5.5	-20.4	-3.2	
EMEs	-19.5	-13.9	-27.4	-2.4	

Notes: Compensation using the First Best as benchmark. The numbers in bold denote the departure from the FB model, in terms of steady state consumption, i.e., the equivalent variation in consumption agents undergo if they transition from the FB to the column's regime.

The table 4 shows the expected conditional welfare obtained at a second order of approximation. The associated welfare levels are shown in table 10 in the appendix B. I compute the consumption equivalent compensation by normalizing the welfare wedge between each policy model and a reference model (the First Best) by the increase in welfare that would be obtained if consumption were to increase by 1%.²³ These numbers can be interpreted as the

In Cooperation symmetry between instruments rules is assumed for EMEs

²³The increase in consumption is applied to the consumption and utility levels used as the initial state for

equivalent consumption cost derived from transitioning from the first best model to each of the models in the table columns. For example, the world Cooperation model implies a welfare cost equivalent to a decrease of 3.2% in the consumption.

Using the global welfare in the fifth row as the criterion for ranking the regimes, I find that the best policy framework is the worldwide cooperation, followed by the cooperation between the Center and one periphery (A in Center+EME-A), the third best policy would be the non-cooperative one (Nash) and, finally, the worst performing one is the regional cooperation between peripheries (EMEs).

These results suggest that not every type of cooperation is welfare improving relative to the nationally-oriented regime (Nash). On the contrary, the cooperation arrangements that are beneficial, globally and to the EMEs, are those involving a cooperative Center. This helps us answer the main question of this study: The emerging economies will not be better off under every type of cooperation, instead, they will only benefit when cooperating with a financial center.

At the same time, when looking at the national distribution of the welfare gains, we can see that sustaining the global cooperation would be challenging as the coalition participants are better-off in the semi-cooperative arrangement (Center+EME-A in the table or Coop(A+C) in the model notation). In that case, the gains for the EME-A and the Center are such that they can even overcome the first best allocation, although at the expense of the periphery that is left out of the coalition (EME-B).

Gains relative to non-cooperative regime The table 4 compares the welfare of each regime with respect to the first best. Similarly, we can also compare each regime's performance relative to the non-cooperative policies. In that case, the social welfare gains for global cooperation amount to a 15% in equivalent consumption units, 12% for the Coop(A+C) regime, and a 6% loss for the emerging cooperation framework (EMEs). The result by country (and region) are shown in table 8 in appendix B.

Sources of Welfare Gains From Cooperation For identifying the origins and mechanisms that generate the welfare gains, we can resort to the analytical expression for the optimal tax in the Center under cooperation derived before and keeping in mind that the optimal taxes have a similar structure in this version of the model.

all models. As an alternative, the consumption equivalent cost is computed using a log-utility in consumption approximation, in Lucas (1987). The approximation is relatively valid as our CRRA parameter is close to one and the results are qualitatively the same. The table is reported in the table 9 in the appendix B.

I find that the optimal tax in the financial center has the following form:

national Int. rate manipulation motive local capital for foreign (EME) intermediation substitution motive
$$\tau_3^{c,coop} = \tau_3^{c,nash} - \overbrace{\varphi_3^{c,NFA}}^{c,NFA} + \overbrace{\psi_3^{eme}(\kappa)}^{eme}(\kappa)$$
 (21)

This equation is obtained in the appendix A, NFA stands for net foreign assets and $\tau_3^{c,nash}$ corresponds to the optimal tax for a nationally-oriented (non-cooperative) Center in the equation (3).

Here $\varphi_3^{NFA} < 0$ and $\psi_3^{eme}(\kappa) > 0$, thus, cooperative planners tend to set higher taxes to local intermediation in order to favor the peripheral capital accumulation. Furthermore, $\varphi_3^{eme'}(\kappa) > 0$, i.e., the strength of this effect increases with the financial distortions.

The welfare enhancing mechanisms, explained by each of the last two terms in the right hand side of (21) work as follows:

Higher Smoothness of Cooperative Taxes: A Cooperative planner that can set the policy tools of the Center and of some or all peripheries (Coop and Coop(A+C)) finds that the incentives to manipulate the global interest rate, in order to improve the net foreign assets position, dissapear $(-\varphi_3^{NFA}$ cancels out with the same positive term in $\tau_3^{c,nash}$). This happens because in the cooperative welfare expressions, the net foreign assets terms of debtor (EMEs) and creditor (Center) countries go in opposite directions and cancel out, partially or completely, with each other. As a result, there is one fewer source of fluctuations in the cooperative taxes which will make these instruments more stable.

The cancellation effect works better with more peripheries in the policy coalition, and even perfectly, when the sum of the welfare weights of the participating EMEs equals that one of the Center, as in the global cooperation regime.

This mechanism is also present in the literature on cooperative capital controls, e.g., Davis and Devereux (2019) describe this effect as the absence of terms of trade manipulation motives for cooperative planners. Here, however, I obtain an analogous mechanism holds when regulating the banking sector, rather than when taxing the net foreign assets directly.

Substitution Motive of Local Capital for Foreign Intermediation: The cooperative planner has an additional motive for increasing taxes at the Center. By doing so, it will discourage the local capital accumulation, which in turn protects the capital inflows at the EMEs.

This incentive, represented by φ_3^{NFA} in the equation (21), increases with the financial friction (κ), and is proportional to the scale of the increase in the EMEs capital accumulation after a change in global intermediation.

In summary, there are two main mechanisms at work: first, a cancellation motive that lowers the volatility of the taxes under cooperation, something that is welfare increasing and favors a more efficient pursuit of financial stability goals, as other potentially conflicting policy goals become absent, and second, a new policy motive towards encouraging capital flows to the peripheries, even if it comes at the expense of the local capital accumulation at the Center.

Both motives add to the overall financial stability of the world economy. The first prevents unnecessary fluctuations in the taxes and even in the global interest rate, leading to lower volatility in the international capital fluctuations, as the yield-seeking reaction of non-cooperative regulators are muted. The second one, on the other hand, encourages capital flows to the peripheries which can be useful in preventing Capital retrenchment episodes, e.g., in presence of external shocks at the Center.

Simultaneously, the second motive boosts the efficiency of the capital flows as these are allocated in the more productive destinations. In that spirit, the welfare gains from (center-led) cooperation are magnified as these regimes feature both a higher financial stability and increased efficiency in the use of capital.

Furthermore, it is important to remark that both motives are present only under cooperative frameworks that include the Center. The first is a cancellation effect between global debtors and creditors incentives, and will be absent if all the countries in the cooperative coalition are debtors as in the emerging regional cooperation (CoopEMEs).

The second one, on the other hand, is an effect that is unique to the Center given its role as global creditor and recognizes the fact that the cooperative planner acting on behalf of the Center will now internalize the unique capacity she has for boosting the global welfare. This means the tax is not set with the aim of maximizing the domestic welfare, something that tentatively implies increasing the local accumulation of capital. Instead, the planner focuses on increasing the global welfare by allocating the capital flows optimally.

Role of the Welfare Weights. Both of the mechanisms generating welfare gains work better for regimes assigning higher weights of the emerging economies' welfare in the objective of the cooperative planner. Here, I use the relative sizes of the economies (n_i for $i = \{a, b, c\}$) as the welfare weights for cooperative regimes. Furthermore, in the baseline calibration I assume the sum of the peripheral sizes amount to that of the Center ($n_a + n_b = n_c$). With this, first, individual incentives to manipulate the interest rate cancel out more evenly under cooperation, and second, there is a stronger motive for facilitating the intermediation in the peripheries as these have a stronger effect in the global welfare.

By the same token, as the environment converges to that of a small open economy $(n_a, n_b \to 0)$ the cancellation of incentives to manipulate the interest rate would no longer work as the planner would be biased to favor the Center. Also, the regulator would not find worthwhile to sacrifice capital accumulation at the Center to encourage peripheral intermediation as the latter, even if more efficient, would not contribute substantially to the global GDP.

Finally, it is relevant to remark that the difference in the welfare weights in favor of the Center is the reason explaining why the semi-cooperative model Coop(A+C) does not perform as well as the global cooperation regime. Having a cooperative planner relatively biased to increase the welfare of the Center does not allow for a strong enough offsetting of the national interest rate manipulation motives.

Given these features, the inclusion of additional peripheral countries in the cooperative interactions, represents a way to balance the incentives of these economies in a welfare improving fashion. That is, to boost the social gains, we can either consider Center interactions with larger peripheries, or include more smaller economies into the cooperative arrangement to prevent national biases in the cooperative planners.

7 Conclusions

I study whether the international macroprudential policy cooperation is beneficial for emerging economies and can be used to improve their macroeconomic performance and financial stability. I formulate two specific questions: (i) is macroprudential cooperation beneficial for these and other economies? (ii) If so, what are the mechanisms driving the welfare effects of cooperation? I answer to this from a normative perspective.

First, in a simplified framework I characterize the structure of the cross-border policy effects and optimal macroprudential policies. As a result, I obtain that two new policy mechanisms appear for cooperative policymakers setting the macroprudential tools. These features translate in improved financial stability and an enhanced interbank intermediation towards the emerging economies, which in turn, can generate welfare gains in policy coordination frameworks. Noticeably, these welfare-inducing features are absent in regimes where only emerging economies engage in cooperation.

I perform a welfare evaluation in an stochastic environment and confirm the existence of welfare gains for frameworks where peripheries collaborate with a Center, thereby concluding that: cooperation is indeed useful, however, not every type of cooperation pays off, and the inclusion of a financial center in the coordinated arrangement is crucial.

I also obtain that the socially optimal regime will be the worldwide cooperation, followed by the cooperation between the Center and a subset of the peripheries. This is explained by the fact that the welfare-increasing mechanisms work better when more peripheries are included in the coalitions. Therefore, the policy recommendation for a periphery would be that conditional on a participating Center, it is advised to engage in cooperation.

However, the results also suggest there can exist distributional challenges to the implementation of the best social outcome as the second best regime will be more beneficial for its participants and at the expense of the peripheries outside the cooperative coalition.

It should also be noted that an advantage of this study with respect to the rest of the literature is that it provides a clear identification of the two main sources of the welfare gains while also accounting for different types of cooperative and semi-cooperative policies. This allowed me to determine when cooperation works and when it does not, and to generate a clear and innovative policy recomendation.

Finally, while I think this framework represents a contribution in understanding the international role of the macroprudential policies, I acknowledge it still corresponds to a simplified framework that abstracts from other relevant features, such as additional sources of risk (e.g., currency fluctuations) or the presence of regulatory arbitrage and shadowbanking, a core concern for financial regulators. I leave the inclusion of these elements for future work.

References

- Agénor, P.-R., E. Kharroubi, L. Gambacorta, G. Lombardo, and L. A. P. da Silva (2017, June). The international dimensions of macroprudential policies. BIS Working Papers 643, Bank for International Settlements.
- Agénor, P.-R. and L. A. P. da Silva (2018, June). *Financial spillovers, spillbacks, and the scope for international macroprudential policy coordination*. Number 97 in BIS Papers. Bank for International Settlements.
- Aiyar, S., C. W. Calomiris, and T. Wieladek (2014, February). Does Macro-Prudential Regulation Leak? Evidence from a UK Policy Experiment. *Journal of Money, Credit and Banking* 46(s1), 181–214.
- Aizenman, J., M. D. Chinn, and H. Ito (2020, July). Financial Spillovers and Macroprudential Policies. *Open Economies Review 31*(3), 529–563.

- Alam, Z., A. Alter, J. Eiseman, R. G. Gelos, H. Kang, M. Narita, E. Nier, and N. Wang (2019, March). Digging Deeper–Evidence on the Effects of Macroprudential Policies from a New Database. IMF Working Papers 19/66, International Monetary Fund.
- Aoki, K., G. Benigno, and N. Kiyotaki (2018). Monetary and Financial Policies in Emerging Markets. Working paper.
- Banerjee, R., M. B. Devereux, and G. Lombardo (2016). Self-oriented monetary policy, global financial markets and excess volatility of international capital flows. *Journal of International Money and Finance* 68(C), 275–297.
- Belkhir, M., S. B. Naceur, B. Candelon, and J.-C. Wijnandts (2020, May). Macroprudential Policies, Economic Growth, and Banking Crises. IMF Working Papers 20/65, International Monetary Fund.
- Bengui, J. (2014). Macro-prudential policy coordination. Technical report, University of Montreal.
- Benigno, P. and M. Woodford (2004, July). Optimal Monetary and Fiscal Policy: A Linear-Quadratic Approach. In *NBER Macroeconomics Annual 2003, Volume 18*, NBER Chapters, pp. 271–364. National Bureau of Economic Research, Inc.
- Bernanke, B. S. (2005). The global saving glut and the U.S. current account deficit. Speech 77, Board of Governors of the Federal Reserve System (U.S.).
- Bernanke, B. S., L. P. Demarco, C. C. Bertaut, and S. B. Kamin (2011). International capital flows and the returns to safe assets in the United States, 2003-2007. International Finance Discussion Papers 1014, Board of Governors of the Federal Reserve System (U.S.).
- Bernanke, B. S., M. Gertler, and S. Gilchrist (1999). The financial accelerator in a quantitative business cycle framework. In J. B. Taylor and M. Woodford (Eds.), *Handbook of Macroeconomics*, Volume 1 of *Handbook of Macroeconomics*, Chapter 21, pp. 1341–1393. Elsevier.
- Bianchi, J. (2011, December). Overborrowing and Systemic Externalities in the Business Cycle. *American Economic Review 101*(7), 3400–3426.
- Bilbiie, F. O., I. Fujiwara, and F. Ghironi (2014). Optimal monetary policy with endogenous entry and product variety. *Journal of Monetary Economics* 64(C), 1–20.
- Boar, C., L. Gambacorta, G. Lombardo, and L. A. P. da Silva (2017, September). What are the effects of macroprudential policies on macroeconomic performance? *BIS Quarterly Review*.

- Bodenstein, M., L. Guerrieri, and J. LaBriola (2019). Macroeconomic policy games. *Journal of Monetary Economics* 101, 64 81.
- Buch, C. M. and L. S. Goldberg (2017, March). Cross-Border Prudential Policy Spillovers: How Much? How Important? Evidence from the International Banking Research Network. *International Journal of Central Banking* 13(2), 505–558.
- Cerutti, E., S. Claessens, and L. Laeven (2017). The use and effectiveness of macroprudential policies: New evidence. *Journal of Financial Stability* 28(C), 203–224.
- Chang, R. and A. Velasco (2001). A Model of Financial Crises in Emerging Markets. *The Quarterly Journal of Economics* 116(2), 489–517.
- Christiano, L., R. Motto, and M. Rostagno (2007). Notes on ramsey-optimal monetary policy.
- Céspedes, L. F., R. Chang, and A. Velasco (2017). Financial intermediation, real exchange rates, and unconventional policies in an open economy. *Journal of International Economics* 108(S1), 76–86.
- Davis, J. S. and M. B. Devereux (2019, March). Capital Controls as Macro-prudential Policy in a Large Open Economy. NBER Working Papers 25710, National Bureau of Economic Research, Inc.
- De Paoli, B. and M. Paustian (2017). Coordinating Monetary and Macroprudential Policies. *Journal of Money, Credit and Banking* 49(2-3), 319–349.
- Evans, G. W. and S. Honkapohja (2003). Expectations and the Stability Problem for Optimal Monetary Policies. *Review of Economic Studies* 70(4), 807–824.
- Evans, G. W. and S. Honkapohja (2006, March). Monetary Policy, Expectations and Commitment. *Scandinavian Journal of Economics* 108(1), 15–38.
- Fernández, A., A. Rebucci, and M. Uribe (2015). Are capital controls countercyclical? *Journal of Monetary Economics* 76(C), 1–14.
- Forbes, K., D. Reinhardt, and T. Wieladek (2017). The spillovers, interactions, and (un)intended consequences of monetary and regulatory policies. *Journal of Monetary Economics* 85(C), 1–22.
- Forbes, K. J. (2020, August). The International Aspects of Macroprudential Policy. NBER Working Papers 27698, National Bureau of Economic Research, Inc.

- Fujiwara, I., Y. Hirose, and M. Shintani (2011, February). Can News Be a Major Source of Aggregate Fluctuations? A Bayesian DSGE Approach. *Journal of Money, Credit and Banking* 43(1), 1–29.
- Fujiwara, I. and Y. Teranishi (2017). Financial frictions and policy cooperation: A case with monopolistic banking and staggered loan contracts. *Journal of International Economics* 104(C), 19–43.
- Gertler, M. and P. Karadi (2011, January). A model of unconventional monetary policy. *Journal of Monetary Economics* 58(1), 17–34.
- Gertler, M. and N. Kiyotaki (2010). Financial Intermediation and Credit Policy in Business Cycle Analysis. In B. M. Friedman and M. Woodford (Eds.), *Handbook of Monetary Economics*, Volume 3 of *Handbook of Monetary Economics*, Chapter 11, pp. 547–599. Elsevier.
- Gertler, M., N. Kiyotaki, and A. Prestipino (2020, August). Credit Booms, Financial Crises, and Macroprudential Policy. *Review of Economic Dynamics* 37, 8–33.
- Ghironi, F. and G. K. Ozhan (2020, May). Interest Rate Uncertainty as a Policy Tool. NBER Working Papers 27084, National Bureau of Economic Research, Inc.
- Granados, C. (2021, 6). *International Coordination of Macroprudential Policies: A Multicountry Framework*. Ph. D. thesis, University of Washington, Savery Hall, Chelan Ln, Seattle, WA 98105. Department of Economics, Chapter 1.
- Jeanne, O. and A. Korinek (2010, May). Excessive Volatility in Capital Flows: A Pigouvian Taxation Approach. *American Economic Review* 100(2), 403–407.
- Jeanne, O. and A. Korinek (2019). Managing credit booms and busts: A Pigouvian taxation approach. *Journal of Monetary Economics* 107(C), 2–17.
- Jin, H. and H. Shen (2020, January). Foreign Asset Accumulation among Emerging Market Economies: a Case for Coordination. *Review of Economic Dynamics* 35, 54–73.
- Justiniano, A., G. E. Primiceri, and A. Tambalotti (2013). The Effects of the Saving and Banking Glut on the U.S. Economy. In *NBER International Seminar on Macroeconomics* 2013, NBER Chapters, pp. 52–67. National Bureau of Economic Research, Inc.
- Kara, G. I. (2016). Systemic risk, international regulation, and the limits of coordination. *Journal of International Economics* 99(C), 192–222.
- King, R. and A. L. Wolman (1999, July). What Should the Monetary Authority Do When Prices Are Sticky? In *Monetary Policy Rules*, NBER Chapters, pp. 349–404. National Bureau of Economic Research, Inc.

- Kiyotaki, N. and J. Moore (1997, April). Credit Cycles. *Journal of Political Economy* 105(2), 211–248.
- Korinek, A. (2020). Currency Wars or Efficient Spillovers? A General Theory of International Policy Cooperation. *Review of Economic Studies, Forthcoming*.
- Lucas, R. J. (1987). Models of Business Cycles, Volume 1. Basil Blackwell, New York.
- Rey, H. (2013). Dilemma not trilemma: the global cycle and monetary policy independence. *Proceedings Economic Policy Symposium Jackson Hole*, 1–2.
- Rey, H. (2016, May). International Channels of Transmission of Monetary Policy and the Mundellian Trilemma. *IMF Economic Review* 64(1), 6–35.
- Richter, B., M. Schularick, and I. Shim (2019). The costs of macroprudential policy. *Journal of International Economics* 118, 263 282.
- Schmitt-Grohe, S. and M. Uribe (2003, October). Closing small open economy models. *Journal of International Economics* 61(1), 163–185.
- Steinsson, J. (2008, March). The Dynamic Behavior of the Real Exchange Rate in Sticky Price Models. *American Economic Review 98*(1), 519–533.
- Sutherland, A. (2004, February). International Monetary Policy Coordination and Financial Market Integration. CEPR Discussion Papers 4251, C.E.P.R. Discussion Papers.
- Tripathy, J. (2020). Cross-border effects of regulatory spillovers: Evidence from mexico. *Journal of International Economics* 126, 103350.
- Uribe, M. and S. Schmith-Grohe (2017). *Open Economy Macroeconomics* (1 ed.). 41 William Street, Princeton, New Jersey 08540: Princeton University Press.
- Woodford, M. (2003). *Interest and prices : foundations of a theory of monetary policy / Michael Woodford*. Princeton University Press Princeton, N.J. : Woodstock.

A Results from the Simple Three Periods Model

Proof of proposition 1.

Proof. W.L.O.G. I will work in a perfect foresight setup, otherwise the same result applies to the expected credit spread.

The time index of the spread is given by the time in which the revenue rate is paid. We can obtain the credit spreads from the EME-Banks F.O.C. with respect to F_1 and F_2 .

For t = 2, 3 the spreads are given by:

$$Spr_{2} = R_{k,2} - R_{b,1} = \frac{\mu_{1}\kappa}{(1 + \mu_{1})\Omega_{1}}$$
$$Spr_{3} = R_{k,3} - R_{b,2} = \frac{\mu_{2}\kappa}{(1 + \mu_{2})\Lambda_{2,3}}$$

if the ICCs bind we have $\mu_t > 0$ and it follows that:

$$\begin{split} \frac{\partial Spr_2}{\partial \kappa} &= \frac{\mu_1}{(1+\mu_1)\Omega_1} > 0 \\ \frac{\partial Spr_3}{\partial \kappa} &= \frac{\mu_2}{(1+\mu_2)\Lambda_{2,3}} > 0 \end{split}$$

Proof of proposition 2.

Proof: W.L.O.G. I will work in a perfect foresight setup, otherwise the same result applies to the expected value of the leverage.

From the ICC of the EME-Banks for each period I obtain the leverage, defined as total assets over net worth. Then I differentiate the resulting expression with respect to the tax.

For the last period:

The ICC is:
$$J_2 = \Lambda_{2,3}(R_{k,3}L_2 - R_{b,2}F_2) = \kappa_2 L_2$$

By substituting the foreign lending $F_2 = L_2 - N_2$, where N_2 is the net worth in the last period (bequests plus retained previous profits) and solving for L_2 :

$$L_2 = \overbrace{\frac{-\Lambda_{2,3}R_{b,2}}{\Lambda_{2,3}(R_{k,3} - R_{b,2}) - \kappa}}^{\phi_2} N_2$$

where ϕ_2 denotes the leverage. Now, I substitute $R_{k,3}(\tau_3) = [(1-\tau_3)r_3 + (1-\delta)Q_3]/Q_2$ and differentiate with respect to the policy instrument:

$$\frac{\partial \phi_2}{\partial \tau_3} = -\frac{(\Lambda_{2,3})^2 R_{b,2} \cdot r_3}{(\Lambda_{2,3}(R_{k,3} - R_{b,2}) - \kappa)^2 Q_2} < 0$$

For the first period:

The procedure is the same but the algebra is a bit lengthier as I substitute both balance sheets ($F_1 = L_1 - \delta_B Q_1 K_0$, and $F_2 = Q_2 K_2 - N_2$) in the value of the bank in the right hand side of the ICC for the first intermediation period $J_1 = \kappa L_1$.

After substitutions and some algebra the ICC becomes:

$$[\tilde{\Omega}_1(R_{k,2} - R_{b,1}) - \kappa]L_1 + [\tilde{\Omega}_1 R_{b,1}]\delta_B Q_1 K_0 + \Lambda_{1,3}\delta[(R_{k,3} - R_{b,2})L_2 + R_{b,2}\delta_B Q_2 K_1] = 0$$

With
$$\tilde{\Omega}_1 = (1 - \theta)\Lambda_{1,2} + \Lambda_{1,3}\theta^2 R_{b,2}$$

The leverage is given by:

$$\phi_1 = \frac{L_1}{\delta_B Q_0 K_1} = \frac{-[\tilde{\Omega}_1 R_{b,1}] - \Lambda_{1,3} \theta[(R_{k,3} - R_{b,2}) L_2 + R_{b,2} \delta_B Q_2 K_1] / (\delta_B Q_0 K_1)}{[\tilde{\Omega}_1 (R_{k,2} - R_{b,1}) - \kappa]}$$

Then,

$$\frac{\partial \phi_1}{\partial \tau_2} = -\frac{\tilde{\Omega}_1 R_{b,1} + \Lambda_{1,3} \theta [(R_{k,3} - R_{b,2}) L_2 + R_{b,2} \delta_B Q_2 K_1] / (\delta_B Q_0 K_1)}{[\tilde{\Omega}_1 (R_{k,2} - R_{b,1}) - \kappa]^2} \cdot \left(\frac{r_2(\tau_2)}{Q_1}\right) < 0$$

Finally, notice how in the expressions $\frac{\partial \phi_1}{\partial \tau_2}$ and $\frac{\partial \phi_2}{\partial \tau_3}$ the denominator implies that the derivatives grow with the friction parameter κ .

Table 5: Summary of equilibrium equations of the three-period model

Common to all countries:

$$\begin{aligned} Q_t &= 1 + \frac{\zeta}{2} \left(\frac{I_t}{I_t - 1} - 1 \right)^2 + \zeta \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} - \Lambda_{t,t+1} \zeta \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \end{aligned} \qquad \begin{aligned} & \text{[Price of Capital, t=\{1,2\}]} \\ K_t &= I_t + (1 - \delta) K_{t-1} \\ R_{k,t} &= \frac{(1 - \tau_t) \alpha A_t K_{t-1}^{\alpha - 1} + (1 - \delta) Q_t}{Q_{t-1}} \end{aligned} \end{aligned} \end{aligned} \end{aligned} \end{aligned} \end{aligned} \end{aligned} \end{aligned} \end{aligned} \end{aligned}$$

for EMEs:

$$\begin{array}{lll} Q_1K_1 = F_1 + \delta_B Q_1K_0 & \text{[bal. sheet of banks, t=1]} \\ Q_2K_2 = F_2 + \delta_B Q_2K_1 + \theta \left[R_{k,2}Q_1K_1 - R_{b,1}F_1 \right] & \text{[bal. sheet of banks, t=2]} \\ (1-\theta)\Lambda_{1,2} \left(R_{k,2}Q_1K_1 - R_1F_1 \right) + \Lambda_{1,3}\theta \left(R_{k,3}Q_2K_2 - R_2F_2 \right) = kQ_1K_1 & \text{[ICC, t=1]} \\ \Omega_1 \left(1 + \mu_1 \right) \left(R_{k,2} - R_1 \right) = \mu_1\kappa & \text{[Credit spread, t=2]} \\ \Lambda_{2,3} \left(R_{k,3}Q_2K_2 - R_2F_2 \right) = kQ_2K_2 & \text{[ICC, t=2]} \\ (1+\mu_2) \Lambda_{2,3} \left(R_{k,3} - R_2 \right) = \mu_2\kappa & \text{[Credit spread, t=3]} \\ C_1 + \frac{B_1}{R_1} = r_1K_0 + \pi_{f,1} + \pi_{inv,1} - \delta_BQ_1K_0 & \text{[BC for t=1]} \\ C_2 + \frac{B_2}{R_2} = \pi_{f,2} + \pi_{inv,2} + \pi_{b,2} - \delta_BQ_2K_1 + B_1 - T_2 & \text{[BC for t=2]} \\ C_3 = \pi_{f3} + T_3 + B_2 - T_3 & \text{[BC for t=3]} \end{array}$$

for the Center:

$$\begin{split} Q_1^c K_1^c + F_1^a + F_1^b &= D_1 + \delta_B Q_1^c K_0^c & \text{[Bal. sheet of banks, t=1]} \\ Q_2^c K_2^c + F_2^a + F_2^b &= D_2 + \delta_B Q_2^c K_1^c + \theta \left[R_{k,2}^c Q_1^c K_1^c + R_1^a F_1^a + R_1^b F_1^b - R_1 D_1 \right] & \text{[Bal. sheet of banks, t=2]} \\ C_1^c + \frac{B_1^c}{R_1} + D_1 &= r_1^c K_0^c + \pi_{f,1}^c + \pi_{1nv,1}^c - \delta_B Q_1^c K_0^c & \text{[BC for t=2]} \\ C_2^c + \frac{B_2^c}{R_1} + D_2 &= \pi_{f,2}^c + \pi_{inv,2}^c + \pi_{b,2}^c - \delta_B Q_2^c K_1^c + R_1 D_1 + B_1^c - T_2^c & \text{[BC for t=2]} \\ C_3^c &= \pi_{f,3}^c + \pi_{b,3}^c + B_2^c + R_2 D_2 - T_3^c & \text{[BC for t=3]} \end{split}$$

International Links:

$$n_a B_t^a + n_b B_t^b + n_c B_t^c = 0$$
 [Net Supply of Bonds, t = {1,2}]

Note: when solving the model normalize the initial world capital to 1 and distribute it across countries according to their population sizes. Initial investment is set as $I_0 = \delta K_0$, and since $I_3 = 0$ the price Q_3 is a constant.

Auxiliary definitions:

Stochastic discount factor:
$$\Lambda_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}$$

Effective discount factor of banks: $\Omega_1 = (1 - \theta)\Lambda_{1,2} + \theta^2 R_{k,3}\Lambda_{1,3}$

Taxes:
$$T_t = -\tau_t r_t K_{t-1}$$

Marginal product of capital: $r_t = \alpha A_t K_{t-1}^{\alpha-1}$

Profits of firms:
$$\pi_{f,t} = (1 - \alpha)A_t K_{t-1}^{\alpha}$$

Profits of investors:
$$\pi_{inv,t} = Q_t I_t - C(I_t, I_{t-1}) = Q_t I_t - I_t \left(1 + \frac{\zeta}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2\right)$$

Profits of bankers in EMEs, t=2: $\pi_{b,2}^e = (1-\theta) \left(R_{k,2} Q_1^e K_1^e - R_1 F_1^e \right)$ Profits of bankers in EMEs, t=3: $\pi_{b,3}^e = R_{k,3}^e Q_2^e K_2^e - R_2 F_2^e$, $e = \{a,b\}$ Profits of bankers in Center, t=2: $\pi_{b,2}^c = (1-\theta) \left(R_{k,2}^c Q_1^c K_1^c + R_1^a F_1^a + R_1^b F_1^b - R_1 D_1 \right)$ Profits of bankers in Center, t=3: $\pi_{b,3}^c = R_{k,3}^c Q_2^c K_2^c + R_{b2}^a F_2^a + R_2^b F_2^b - R_2 D_2$

Table 6: Parameters in the 3-period model

Parameter		Value	Comment/Source
Adjustment costs of investment	ζ	4.65	Cespedes, Chang and Velasco (2017)
Start-up transfer rate to banks	δ_b	0.005	Gertler and Karadi (2011), Gertler and Kiyotaki (2010)
Divertable fraction of capital	$\kappa^a = \kappa^b$	0.399	Aoki, Benigno and Kiyotaki (2018)
Discount factor	β	0.99	Standard
Risk Aversion parameter	σ	2	Standard
Country size	$n_a = n_b$	0.25	Captures large open economy effects in all countries
Depreciation rate	δ	0.6	Targets a longer period duration than quarterly
Capital share	α	0.333	Standard

A.1 Optimal Taxes

Individual optimal taxes. The procedure for obtaining the optimal taxes consists in equating the welfare effects $\frac{dW}{d\tau}$ to zero and then solving for the tax. This is done via backwards induction. First, I solve the last period case for τ_3 , and afterwards in the first period for $\tau_2(\tau_3,\cdot)$. Afterwards, I replace the solution found in the first step to obtain τ_2 .

In the case of the Center and for the last period, there is no explicit τ_3^c terms in the welfare effect. Then, to pintpoint the tax I use the fact that banking returns show the tax explicitely $(R_{k,3}(\tau_3))$ to back out the tax after substituting it for one of the rates it equates.

$$\tau_2^a = \overbrace{\frac{\alpha - 1}{\alpha} - \frac{1}{\alpha r_2^a} \left\{ (I_1 + \kappa K_1) \frac{dQ_1^a}{dK_1^a} + \frac{B_1^a}{R_1} \frac{dR_1}{dK_1^a} + \kappa R_1 Q_1^a \right\}}^{\text{contemporaneous component}}$$

$$+\left(1-\frac{\Lambda_{1,2}}{\Lambda_{2,3}}\right)\alpha_{4}(\kappa)\frac{dQ_{2}^{a}}{dK_{1}^{a}}+\left(1-\Lambda_{1,2}\right)\frac{B_{2}^{a}}{R_{2}}\frac{dR_{2}}{dK_{1}^{a}}+\kappa\left(1+\theta\left(\Lambda_{1,2}-\Lambda_{2,3}\right)-\frac{\Lambda_{1,2}}{\Lambda_{2,3}}\right)Q_{2}^{a}\frac{dK_{2}^{a}}{dK_{1}^{a}}\right\}$$

forward-looking component

$$\tau_{3}^{a} = -\frac{1}{\Lambda_{2,3}\alpha r_{3}^{a}} \left\{ \alpha_{4}(\kappa) \frac{dQ_{2}^{a}}{dK_{2}^{a}} + \Lambda_{2,3} \frac{B_{2}^{a}}{R_{2}} \frac{dR_{2}}{dK_{2}^{a}} + \kappa \left(1 - \theta \Lambda_{2,3}\right) Q_{2}^{a} \right\} + 1 - \frac{1}{\alpha}$$

contemporaneous component

$$\tau_{2}^{c} = -\frac{1}{\theta \alpha r_{2}^{c}} \left\{ (1 - \theta)(1 - \delta)Q_{2}^{c} + \left(\frac{B_{1}^{c}}{R_{1}} - \theta D_{1}\right) \frac{dR_{1}}{dK_{1}^{c}} + R_{1}K_{1}^{c} \frac{dQ_{1}^{c}}{dK_{1}^{c}} + (1 - \theta) \left(\frac{dR_{b,1}^{eme}}{dK_{1}^{c}} F_{1}^{ab} + R_{b1}^{eme} \frac{dF_{1}^{ab}}{dK_{1}^{c}}\right) \right\} + \frac{1}{R_{2}} \left[\gamma_{2} \frac{dK_{2}^{c}}{dK_{1}^{c}} + \frac{B_{2}^{c}}{R_{2}} \frac{dR_{2}}{dK_{1}^{c}} + \gamma_{3} \frac{dQ_{2}^{c}}{dK_{1}^{c}} + \left(\frac{dR_{b2}^{eme}}{dK_{1}^{2}} F_{2}^{ab} + R_{b2}^{eme} \frac{dF_{2}^{ab}}{dK_{1}^{c}}\right) \right] \right\} + \frac{\alpha \theta - 1}{\alpha \theta}$$

forward looking component

$$\tau_3^c = \frac{Q_2^c}{r_3^c} \left\{ \gamma_2 \frac{dK_2^c}{dF_2^{ab}} + \Lambda_{2,3} B_2^c \frac{dR_2}{dF_2^{ab}} + \gamma_3 \frac{dQ_2^c}{dF_2^{ab}} + \left(F_2^{ab}\right) \frac{dR_{b2}^{\rm eme}}{dF_2^{ab}} \right\} + \frac{(1-\delta)Q_3}{r_3^c} + 1$$

With
$$\alpha_4(\kappa) = I_2^a + \kappa (1 - \theta \Lambda_{2,3}) K_2^a$$
, $\gamma_2 = r_3^c + (1 - \delta) Q_3$, $\gamma_3 = R_2 (I_2^c + (1 - \theta)(1 - \delta) K_1^c)$, $F_t^{ab} = F_t^a + F_t^b$, and $\frac{\partial \alpha_4(\kappa)}{\partial \kappa} > 0$.

Optimal Taxes Under Cooperation. This section shows how to get the optimal Center tax under cooperation and the equation (21).

The procedure is analogous to the individual welfare case (non-cooperative), I will find the welfare effect of setting τ_3^c for the cooperative planner, i.e. $\frac{dW^{coop}}{d\tau_3^c}$, set it equal to zero and solve for the optimal policy $\tau_3^{c,coop}$.

$$\frac{dW_0^{coop}}{d\tau_3^c} = n_a \frac{dW_0^a}{d\tau_3^c} + n_b \frac{dW_0^b}{d\tau_3^c} + (1 - n_a - n_c) \frac{dW_0^c}{d\tau_3^c}$$

Now, given the perfect foresight assumption, the equilibrium allocation and welfare is symmetric between peripheries:

$$\frac{dW_0^{coop}}{d\tau_3^c} = (n_a + n_b)\frac{dW_0^a}{d\tau_3^c} + (1 - n_a - n_c)\frac{dW_0^c}{d\tau_3^c}$$

Furthermore, I simplify further by using the parameter values $n_a = n_b = \frac{1}{4}$. That is, the summation of the sizes of the peripheral economies equals that of the Center,

$$\frac{dW_0^{coop}}{d\tau_3^c} = \frac{dW_0^a}{d\tau_3^c} + \frac{dW_0^c}{d\tau_3^c}$$

By substituting each of the individual welfare effects in the right hand side:

$$\begin{split} \frac{dW_0^{coop}}{d\tau_3^c} &= \left[\beta \lambda_2^a \left(\kappa \left(1 - \theta \Lambda_{2,3}\right) Q_2^a + \varphi \left(\tau_3^c\right) \Lambda_{2,3} r_3^a\right) \frac{dK_2^a}{d\tau_3^c} + \beta \lambda_2^a \left(I_2^a + \kappa \left(1 - \theta \Lambda_{2,3}\right) K_2^a\right) \frac{dQ_2^a}{d\tau_3^c} \right. \\ &+ \beta^2 \lambda_3^a \frac{B_2^a}{R_2} \frac{dR_2}{d\tau_3^c} \right] + \left[\beta^2 \lambda_3^c \left(r_3^c + (1 - \delta)Q_3\right) \frac{dK_2^c}{d\tau_3^c} + \beta^2 \lambda_3^c \frac{B_2^c}{R_2} \frac{dR_2}{d\tau_3^c} + \beta \lambda_2^c \left(I_2^c + (1 - \theta)(1 - \delta)K_1^c\right) \frac{dQ_2^c}{d\tau_3^c} \right. \\ &+ \beta^2 \lambda_3^c \left(\frac{dR_{b2}^{eme}}{d\tau_3^c} \left(F_2^a + F_2^b\right) + R_{b2}^{eme} \left(\frac{dF_2^a}{d\tau_3^c} + \frac{dF_2^b}{d\tau_3^c}\right)\right) \right] \end{split}$$

Or in simpler terms and with $F_2^{ab} = F_3^a + F_3^b$:

$$\frac{dW_0^{coop}}{d\tau_3^c} = \left[\alpha_1 \frac{dK_2^a}{d\tau_3^c} + \alpha_2 \frac{dQ_2^a}{d\tau_3^c} + \beta^2 \lambda_3^a \frac{B_2^a}{R_2} \frac{dR_2}{d\tau_3^c}\right] + \left[\beta^2 \lambda_3^c \alpha_3 \frac{dK_2^c}{d\tau_3^c} + \beta^2 \lambda_3^c \frac{B_2^c}{R_2} \frac{dR_2}{d\tau_3^c} + \alpha_4 \frac{dQ_2^c}{d\tau_3^c} + \beta^2 \lambda_3^c \frac{dR_{b2}^c}{R_2} \frac{dR_2}{d\tau_3^c} + \beta^2 \lambda_3^c \frac{dR_{b2}^c}{d\tau_3^c} + \beta^2 \lambda_3^c \frac{dR_{b2}^c}{d\tau_3$$

The first term in square brackets corresponds to the welfare effects for the peripheric block and the second to that of the Center. Now I use the UIP assumption and absence of a spread in the center to replace: $R_{b,2}^{eme} = R_{k,3}^c = \frac{(1-\tau_3^c)r_3^c+(1-\delta)Q_3}{Q_2^c}$ and equate $\frac{dW^a}{d\tau_3^c}$ to zero, meaning that τ_3^c in the expression becomes the optimal one $\tau_3^{c,coop}$:

$$\begin{split} \frac{dW_0^{coop}}{d\tau_3^c} &= \left[\alpha_1 \frac{dK_2^a}{d\tau_3^c} + \alpha_2 \frac{dQ_2^a}{d\tau_3^c} \right. \\ \left. + \beta^2 \lambda_3^a \frac{B_2^a}{d\tau_3^c} \frac{dR_2}{d\tau_3^c} \right] + \left[\beta^2 \lambda_3^c \alpha_3 \frac{dK_2^c}{d\tau_3^c} + \beta^2 \lambda_3^c \frac{B_2^c}{R_2} \frac{dR_2}{d\tau_3^c} + \alpha_4 \frac{dQ_2^c}{d\tau_3^c} \right. \\ &\left. + \beta^2 \lambda_3^c \frac{dR_{b2}^{eme}}{d\tau_3^c} F_2^{ab} + \beta^2 \lambda_3^c \frac{(1 - \tau_3^{c,coop}) r_3^c + (1 - \delta)Q_3}{Q_2^c} \frac{dF_2^{ab}}{d\tau_3^c} \right] = 0 \end{split}$$

Solving for $\tau_3^{c,coop}$, and replacing $\alpha_1, \alpha_2, \alpha_3, \alpha_4$, yields:

$$\begin{split} \tau_{3}^{c,coop} &= \frac{Q_{2}^{c}}{\Lambda_{2,3}r_{3}^{c}} \frac{\lambda_{2}^{a}}{\lambda_{2}^{c}} \left\{ (\kappa(1-\theta\Lambda_{2,3})Q_{2} + \varphi(\tau_{3}^{a})\Lambda_{2,3}r_{3}^{a}) \frac{dK_{2}^{a}}{dF_{2}^{ab}} + (I_{2}^{a} + \kappa(1-\theta\Lambda_{2,3}K_{2}^{a})) \frac{dQ_{2}^{a}}{dF_{2}^{ab}} \right\} \\ &+ \frac{Q_{2}^{c}}{\Lambda_{2,3}r_{3}^{c}} \left(\Lambda_{2,3} \left(r_{3}^{c} + (1-\delta)Q_{3} \right) \frac{dK_{2}^{c}}{\partial F_{2}^{ab}} + (I_{2}^{c} + (1-\theta)(1-\delta)K_{1}^{c}) \frac{dQ_{2}^{c}}{dF_{2}^{ab}} + \Lambda_{2,3}F_{2}^{ab} \frac{dR_{b2}^{eme}}{dF_{2}^{ab}} \right) \\ &+ \frac{(1-\delta)Q_{3}^{c}}{r_{3}^{c}} + 1 + \frac{Q_{2}^{c}}{r_{3}^{c}} \left(\frac{B_{2}^{c}}{R_{2}} \frac{dR_{2}}{dF_{2}^{ab}} - \frac{\lambda_{2}^{a}}{\lambda_{2}^{c}} \frac{B_{2}^{c}}{R_{2}} \frac{dR_{2}}{dF_{2}^{ab}} \right) \end{split}$$

In this expression I substituted $B_2^a = -B_2^c$ for the last term.

We can notice the last two lines in the expression are equal to $\tau_3^{c,nash} - \frac{Q_2^c}{r_3^c} \frac{\lambda_2^a}{\lambda_2^c} \frac{B_2^c}{R_2} \frac{dR_2}{dF_2^{ab}}$ where $\tau_3^{c,nash}$ is the optimal individual planner tax given by the equation 3. Thus the optimal cooperative tax can be expressed as:

New substitution of Center capital accumulation for foreign intermediation (EMEs) motive under cooperation

$$\tau_3^{c,coop} = \overbrace{\frac{Q_2^c}{\Lambda_{2,3}r_3^c}}^{\lambda_2^a} \underbrace{\frac{\lambda_2^a}{\lambda_2^c}}_{\left\{ (\kappa(1-\theta\Lambda_{2,3})Q_2 + \varphi(\tau_3^a)\Lambda_{2,3}r_3^a) \frac{dK_2^a}{dF_2^{ab}} + (I_2^a + \kappa(1-\theta\Lambda_{2,3}K_2^a)) \frac{dQ_2^a}{dF_2^{ab}} \right\} \\ + \tau_3^{c,nash} - \underbrace{\frac{\lambda_2^a}{\lambda_2^c}}_{NFA-led interest rate} \underbrace{\frac{Q_2^c}{R_2} \frac{B_2^c}{dF_2^{ab}}}_{NFA-led interest rate}$$

The first right hand side term will represent a new motive for pushing up the taxes in order to lower local Center capital accumulation in favor of emerging economies capital accumulation and intermediation. This term is unambiguously positive for the considered parameter values (as long as the taxes at the periphery is larger than -2).

On the other hand, the last term represents a cancelation term that offsets the policy incentives of the Center for manipulating the global interest rate to take benefit of their net foreign assets (bonds) position. This manipulation incentive is canceled out because the welfare effects of movements in the net foreign assets of the countries engaging in the cooperative arrangement will go in opposite directions between debtors and creditors.

We can make a further simplification²⁴, for a clearer argument and assume the $\lambda_2^a = \lambda_2^c$ which leads to the equation (21).

An analogous procedure can be carried out with the welfare effects of the peripheral taxes under cooperation which would generate the following optimal tax:

$$\tau_{3}^{a, rosop} = \overbrace{\frac{\alpha - 1}{\alpha} - \frac{1}{\alpha \Lambda_{2,3} r_{3}^{a}} \left\{ \left(\alpha_{4}(\kappa) \frac{dQ_{2}^{a}}{dK_{2}^{a}} + \kappa \left(1 - \theta \Lambda_{2,3} \right) Q_{2}^{a} \right) + \left(\frac{B_{2}^{a}}{(R_{2})^{2}} - \frac{\lambda_{2}^{c}}{\lambda_{2}^{a}} \frac{B_{2}^{a}}{(R_{2})^{2}} \right) \frac{dR_{2}}{dK_{2}^{a}} + \left(\gamma_{2} \Lambda_{2,3} \frac{dK_{2}^{c}}{dK_{2}^{a}} + \gamma_{3} \frac{dQ_{2}^{c}}{dK_{2}^{a}} + \Lambda_{2,3} F_{2}^{ab} \frac{dR_{b,2}^{eme}}{dK_{2}^{a}} + R_{b,2}^{eme} \frac{dF_{2}^{ab}}{dK_{2}^{a}} \right) \right\}$$

with
$$\alpha_4 = I_2^a + \kappa (1 - \theta \Lambda_{2,3}) K_2$$
, $\gamma_2 = r_3^c + (1 - \delta) Q_3$, and $\gamma_3 = I_2^c + (1 - \theta) (1 - \delta) K_1^c$

In terms of the interpretation in section 6 we can express the tax in terms of a wedge with

²⁴Otherwise, and in general with $\lambda_2^a \neq \lambda_2^c$, the compensation effect acts even stronger and in favor of the peripheries as $\lambda_2^a > \lambda_2^c$.

respect to the non-cooperative one as:

$$\tau_3^{a,coop} = \tau_3^{a,nash} - \varphi_3^{a,NFA} - \omega_3$$

Although not referred to explicitly in the main sections, it can be noticed ω_3 is consistent the fact a cooperative planner sets higher subsidies with the EMEs instruments.

B Results from the Main Model

B.1 Effect of the macroprudential tool in the model

In the finite horizon version of this model with simple dynamics, I obtained that leverage is a function of the macroprudential instrument and that their relation is negative, i.e., an increase in the tax decreases the leverage ratio of banks. As a result, by implementing a tax, the planner would also enforce a leverage ratio in the banking sector, a commonly used prudential policy.

In the infinite horizon setup of this section, proving such result is less straightforward because the future effects of the policies show up only implicitly in the continuation values of the recursive expressions for the value of the bank.

Nevertheless, it is still possible to describe the way leverage responds to an increase in the tax. I do it by following Gertler and Karadi (2011) and setting the value of the bank in terms of current lending, net worth, and two dynamic coefficients. Here I present the expressions for the emerging economies, but the same results hold for the advanced one that intermediates more types of assets. The value of the bank can be expressed as:

$$J_{it}^e = \nu_t Q_t^e K_{it}^e + \eta_t N_{it}^e$$

with,

$$\nu_{t} = \mathbb{E}_{t} \{ (1 - \theta) \beta \Lambda_{t,t+1}^{e} (R_{k,t+1}^{e} - R_{b,t}^{e}) + \beta \Lambda_{t,t+1}^{e} \theta x_{t,t+1} \nu_{t+1} \}$$

$$\eta_{t} = \mathbb{E}_{t} \{ (1 - \theta) + \beta \Lambda_{t,t+1}^{e} \theta z_{t,t+1} \eta_{t+1} \}$$

Where
$$x_{t,t+i} = Q_{t+i}^e K_{j,t+i}^e / Q_t^e K_{j,t}^e$$
 and $z_{t,t+i} = N_{j,t+i}^e / N_{j,t}^e$

Now, I substitue J_{jt}^e from (8) when it binds and obtain the leverage as ϕ_t^e :

$$\frac{Q_t^e K_t^e}{N_t^e} = \phi_t^e = \frac{\eta_t}{\kappa^e - \nu_t} \tag{22}$$

Where I removed the j sub-index as the components of the leverage will not depend on firm-specific factors. It also follows that $z_{t,t+1} = [(R_{k,t+1}^e - R_{b,t})\phi_t^e + R_{b,t}^e]$ and $x_{t,t+1} = (\phi_{t+1}^e/\phi_t^e)z_{t,t+1}$.

With this, we can see that as the tax increases and the spread goes down, η_t and ν_t will decrease. The overall effect on leverage would be negative. However, even if we can indicate the direction of the changes in the leverage expression, i.e., in the equation (22), it is difficult to pinpoint the actual change in leverage as the tax increases as in the simpler setup because the terms in the right hand side of the equations will depend on current and future values of the leverage themselves.

At the same time, notice that the effects on leverage occur only for values of κ^e that keep the ratio positive. This is the infinite horizon counterpart to the result indicating that the strength of these effects grow with the financial friction.

B.2 Steady State of the Policy Models

The Ramsey model works with a instrument conditional steady state, i.e., a value for the policy tools $\bar{\tau}$ is set and the steady state for the rest of the variables is obtained. A related question of utmost importance is: how to determine the instrument ($\bar{\tau}$) for conditioning?.

For that, I follow an algorithm outlined in Christiano, Motto and Rostagno (2007):

- 1. set any value for $\bar{\tau}$ and solve, using the static private FOCs, for the steady state of private variables: x
- 2. replace x in the remaining N + k equations, the policy FOC w.r.t. the N endogenous variables and k tools: get a linear system of N + k equations for N unknowns (policy multipliers)
- 3. With more equations than unknowns the solution is subject to an error u:
 - (i) set the N+k static equations in vector form as: $U_1 + \bar{\lambda}[1/\beta F_3 + F_2 + \beta F_1] = 0$
 - (ii) let $Y=U_1', X=[1/\beta F_3+F_2+\beta F_1]$ and $\beta=\bar{\lambda}'$
 - (iii) get the tools as: $\beta = (X'X)^{-1}X'Y$ with error $\mathbf{u} = Y X\beta$
 - (iv) repeat for several values of the tools and choose $\bar{\tau}$ such that: $\bar{\tau} = \arg\min_{\tau} \mathbf{u}$

B.3 Parameters of the Model

Table 7: Parameters used in the baseline model

Parameter		Value	Comment/Source
Adjustment costs of investment	ζ	3.456	Banerjee et al. (2016)
Adjustment costs of assets	η	0.0025	Ghironi and Ozhan (2020)
Start-up transfer rate to banks	δ_b	0.003	Gertler and Karadi (2011), Gertler and Kiyotaki (2010)
Survival rate of banking sector	heta	0.95	Gertler and Karadi (2011), Gertler and Kiyotaki (2010)
Divertable fraction of capital	$\kappa^a, \kappa^b, \kappa^c, \kappa^c_{F_1}, \kappa^c_{F_2}$	0.38	Banerjee et al. (2016) Aoki, Benigno and Kiyotaki (2018)
Discount factor	β	0.99	Standard
Risk Aversion parameter	σ	1.02	Standard
Inverse Frisch elasticity of labor supply	ψ	0.276	Standard
Country size	$n_a = n_b$	0.25	
Depreciation rate	δ	0.025	Standard
Capital share	α	0.333	Standard
Persistency of productivity shocks	$ ho_A$	0.85	Standard
Persistency of capital shock	$ ho_{xi}$	0.85	Standard
Std. Dev. of productivity shocks	σ_A	0.007	Standard
Std. Dev. of capital shock	σ_{xi}	0.005	Standard

B.4 Welfare Accounting Supplementary Exercises

Table 8: Welfare cost in consumption equivalent compensation relative to no cooperation

Consumption Equivalent Compensation					
	Cooperation (Center+EME-A)	Cooperation (EMEs)	Cooperation (All)		
\overline{C}	16.5	-1.7	8.8		
A	24.7	-9.8	21.2		
B	-10.9	-9.8	21.2		
World	12.0	-5.7	14.7		
EMEs	6.9	-9.8	21.4		

Notes: Compensation using the Nash (no cooperation) as benchmark. The numbers in bold denote the departure from the benchmark, in terms of equivalent consumption variation.

In Cooperation symmetry between instruments rules is assumed for EMEs

Table 9: Welfare in consumption equivalent compensation units (alternative method)

Consumption Equivalent % Compensation					
	Nash Cooperation Cooperation Cooperation Cooperation (Center+EME-A) (EMEs) (All) (Time Variant)				1
C	-10.8	2.9	-12.1	-3.8	-93.9
A	-17.5	-0.4	-23.7	-2.3	-97.6
B	-17.5	-24.3	-23.7	-2.3	-97.6
World	-14.2	-5.3	-18.1	-3.0	-96.1
EMEs	-17.5	-12.8	-23.7	-2.3	-97.6

Notes: Compensation using the First Best as benchmark.

In Cooperation symmetry between instruments rules is assumed for EMEs

On Time Consistency. As part of the auxiliary exercises I solved a time variant version of this model to explore whether time consistency is relevant in this environment from a welfare perspective. I obtained potentially interesting results (see table 10). On one hand, it is more difficult to solve the models, something relatively expected as a well known property of time inconsistent models is the presence of underterminacy and sunspots equilibria (Evans and Honkapohja (2003), Evans and Honkapohja (2006)). In fact, it is not possible to obtain a solution for every policy framework. However, the world Cooperation and one of the semicooperative models does yield a solution. This can point to another advantage of cooperation, namely, overriding undeterminacy and non fundamental driven solutions. This may be relevant as the non-fundamental equilibria tend to be welfare decreasing.

Finally, even in the cooperative models that yield a solution, there is a substantial welfare loss with respect to every model I compute under the time consistent framework (timeless perspective). With this, I confirm the conveniency of working with the timeless perspective approximation for the main simulations of this study.

Table 10: Welfare levels and consumption equivalent compensation (includes Time Variant Model)

	Nash	Cooperation (Center+EME-A)	Cooperation (EMEs)	Cooperation (All)	Cooperation (Time Variant)
Welfare	levels				
W^c	-4975.8	-4961.6	-4977.4	-4968.3	-5243.6
W^a	-5036.2	-5016.6	-5044.0	-5019.4	-5388.6
W^b	-5036.2	-5044.9	-5044.0	-5019.4	-5388.6
W	-5006.0	-4996.2	-5010.7	-4993.8	-5316.1
W^{ab}	-5036.2	-5030.7	-5044.0	-5019.4	-5388.6
Consumption Equivalent Compensation					
C	-11.7	2.9	-13.2	3.9	-286.1
A	-19.5	0.4	-27.4	-2.4	-377.5
B	-19.5	-28.3	-27.4	-2.4	-377.5
World	-15.6	-5.5	-20.4	-3.2	-332.2
EMEs	-19.5	-13.9	-27.4	-2.4	-377.1

Notes: Compensation using the First Best as benchmark. In Cooperation symmetry between instruments rules is assumed for EMEs

Summary of final model equations. To obtain a summarized version of the model equations I substitute the marginal product of capital, wages, tax rebates and the interest rates that are equalized due to the uncovered interest rate parity. The result is:

Table 11: Summary of private equilibrium equations of the baseline model

Common to all countries:	
$Q_t^i = 1 + \frac{\zeta}{2} \left(\frac{I_t^i}{I_t^i} - 1 \right)^2 + \zeta \left(\frac{I_t^i}{I_t^i} - 1 \right) \frac{I_t^i}{I_t^i} - \Lambda_{t,t+1}^i \zeta \left(\frac{I_{t+1}^i}{I_t^i} \right)^2 \left(\frac{I_{t+1}^i}{I_t^i} - 1 \right)$	[Price of Capital]
$K_t^i = I_t^i + (1 - \delta)\xi_t^i K_{t-1}^i$	[Capital Dynamics]
$R_{k,t}^i = \frac{\left(1 - \tau_t^i\right) \alpha A_t^i H_t^i \stackrel{(1 - \alpha)}{\leftarrow} \xi_t^{i\alpha} K_{t-1}^i \stackrel{(\alpha - 1)}{\leftarrow} + (1 - \delta) \xi_t^i Q_t^i}{Q_{t-1}^i}$	[Banks rate of return]
$R_t \Lambda^i_{t,t+1} = 1 + \eta\left(B^i_t ight)$	[Euler Equation, bonds]
$C_t^{i - \sigma} = \frac{H_t^{i \psi}}{(1 - \alpha) A_t^i (\xi_t^i K_{t-1}^i)^{\alpha} H_t^{i (-\alpha)}}$	[Intra-temporal Euler Equation, labor]
$Y_t^i = A_t^i \left(\xi_t^i K_{t-1}^i \right)^{\alpha} H_t^{i \ 1-\alpha}$	[Output]
$\Lambda_{t,t+1}^i = eta \left(rac{C_{t+1}^i}{C_t^i} ight)^{-\sigma}$	[Stochastic Discount Factor]
$A_t^i = \rho_A A_{t-1}^i + \sigma_A \epsilon_{A,t}^i$	[Aggregate Productivity]
$\xi_t^i = ho_\xi \xi_{t-1}^i + \sigma_\xi \epsilon_{k,t}^i$	[Capital Quality]
for EMEs:	
$Q_t^e K_t^e = N_t^e + F_t^e$	[Bal. sheet of banks]
$\mathbb{E}_t \Omega^i_{t+1 t} \left(R^i_{k,t+1} - R^i_{b,t} \right) = \mu^i_t \kappa^i$	[Credit Spread]
$j_t^e N_t^e = \kappa^e Q_t^e K_t^e$	[ICC]
$N_t^a = \theta \left[R_{k,t}^a Q_{t-1}^a K_{t-1}^a - R_{b,t-1}^a F_{t-1}^a \right] + \delta_B Q_t^a K_{t-1}^a \kappa$	[Net Worth Dynamics]
$j_t^e \left(1 - \mu_t^e\right) = \mathbb{E}_t \left[\Omega_{t+1 t}^e R_{b,t}^e\right]$	[Envelope Condition for Net Worth]
$C_t^e + B_t^e + \frac{\eta}{2} (B_t^e)^2 = R_{t-1} B_{t-1}^e + (1 - \alpha) A_t^e (\xi_t^e K_{t-1}^e)^\alpha H_t^e^{(1-\alpha)} + \Pi_t^a$	[Budget Constraint, households]
for the Center:	
$Q_t^c K_t^c + F_t^a + F_t^b = N_t^c + D_t^c$	[Bal. sheet of banks]
$\mathbb{E}_t \Omega_{t+1 t}^c \left(R_{k,t+1}^c - R_{D,t}^c \right) = \mu_t^c \kappa^c$	[Credit Spread for Local Intermediation]
$\mathbb{E}_t \Omega_{t+1 t}^c \left(R_{b,t}^a - R_{D,t}^c \right) = \mu_t^c \kappa_{F_a}^c$	[Spread for Foreign Lending to EME-A]
$\mathbb{E}_t \Omega_{t+1 t}^c \left(R_{b,t}^b - R_{D,t}^c \right) = \mu_t^c \kappa_{F_b}^c$	[Spread for Foreign Lending to EME-B]
$j^c_t N^c_t = \kappa^c Q^c_t K^c_t + \kappa^c_{F_a} F^a_t + \kappa^c_{F_b} F^b_t$	[ICC]
$N_t^c = \theta \left[R_{k,t}^c Q_{t-1}^c K_{t-1}^c + R_{b,t-1}^a F_{t-1}^a + R_{b,t-1}^b F_{t-1}^b - R_{D,t-1}^c D_{t-1}^c \right] + \delta_B Q_t^c K_{t-1}^c$	1 [Net Worth Dynamics]
$j_{t}^{c}\left(1-\mu_{t}^{c}\right)=\mathbb{E}_{t}\left[\Omega_{t+1\mid t}^{c}R_{D,t}^{c}\right]$	[Envelope Condition for Net Worth]
$C_t^c + B_t^c + \frac{\eta}{2} (B_t^c)^2 + D_t^c + \frac{\eta}{2} \left(D_t^c - \bar{D}^c \right)^2 = R_{t-1}^c B_{t-1}^c + R_{D,t-1}^c D_{t-1}^c + w_t^c H_t^c + \frac{\eta}{2} \left(D_t^c - \bar{D}^c \right)^2 = R_{t-1}^c B_{t-1}^c + R_{D,t-1}^c D_{t-1}^c + w_t^c H_t^c + \frac{\eta}{2} \left(D_t^c - \bar{D}^c \right)^2 = R_{t-1}^c B_{t-1}^c + R_{D,t-1}^c D_{t-1}^c + w_t^c H_t^c + \frac{\eta}{2} \left(D_t^c - \bar{D}^c \right)^2 = R_{t-1}^c B_{t-1}^c + R_{D,t-1}^c D_{t-1}^c + w_t^c H_t^c + \frac{\eta}{2} \left(D_t^c - \bar{D}^c \right)^2 = R_{t-1}^c B_{t-1}^c + R_{D,t-1}^c D_{t-1}^c + w_t^c H_t^c + \frac{\eta}{2} \left(D_t^c - \bar{D}^c \right)^2 + \frac{\eta}{2} \left(D_t^c $	$-\Pi_t^c$ [Budget Constraint, households]
$R_{D,t}^c \Lambda_{t+1}^c = 1$	[Euler Equation, deposits]
T	

International Links:

$$n_a B_t^a + n_b B_t^b + n_c B_t^c = 0$$
 [Net Supply of Bonds]

Note: $i=\{a,b,c\}, e=\{a,b\}$ and $w^c_t=(1-\alpha)Y^c_t/H^c_t$ corresponds to the wages.

In this system of equations I use the following auxiliary definitions:

$$\begin{split} \Pi_t^c &= (1-\theta) \left[Q_{t-1}^c R_{k,t}^c K_{t-1}^c + R_{b,t-1}^a F_{t-1}^a + R_{b,t-1}^b F_{t-1}^b - R_{D,t-1}^c D_{t-1}^c \right] - \delta_B Q_t^c K_{t-1}^c + Q_t^c I_t^c \\ &- I_t^c \left(1 + \frac{\zeta}{2} \left(\frac{I_t^c}{I_{t-1}^c} - 1 \right)^2 \right) + \tau_t^c \alpha A_t^c H_t^c \stackrel{(1-\alpha)}{\leftarrow} \xi_t^c \stackrel{\alpha}{\sim} K_{t-1}^c \stackrel{(\alpha)}{\leftarrow} K_{t-1}^c \right) \\ \Pi_t^a &= (1-\theta) \left[Q_{t-1}^a R_{k,t}^a K_{t-1}^a - R_{b,t-1}^a F_{t-1}^a \right] - \delta_B Q_t^a K_{t-1}^a + Q_t^a I_t^a - I_t^a \left(1 + \frac{\zeta}{2} \left(\frac{I_t^a}{I_{t-1}^a} - 1 \right)^2 \right) \right. \\ &+ \tau_t^a \alpha A_t^a H_t^a \stackrel{(1-\alpha)}{\leftarrow} \xi_t^a \stackrel{\alpha}{\sim} K_{t-1}^a \right) \\ \Pi_t^b &= (1-\theta) \left[Q_{t-1}^b R_{k,t}^b K_{t-1}^b - R_{b,t-1}^b F_{t-1}^b \right] - \delta_B Q_t^b K_{t-1}^b + Q_t^b I_t^b - I_t^b \left(1 + \frac{\zeta}{2} \left(\frac{I_t^b}{I_{t-1}^b} - 1 \right)^2 \right) \right. \\ &+ \tau_t^b \alpha A_t^b H_t^b \stackrel{(1-\alpha)}{\leftarrow} \xi_t^b \stackrel{\alpha}{\sim} K_{t-1}^b \right) \end{split}$$