Exchange Rate Dynamics and the Central Bank's Balance Sheet*

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Abstract

In theory, nominal exchange rates are a function of the relative difference in supply and demand of money. In practice, some central banks issue debt. In this study we ask: are nominal exchange rate variations linked to these remunerated central bank liabilities? We use two measures of implied exchange rates using central bank balance sheet data: one measure is a traditional measure that includes the monetary base, while the other also includes remunerated liabilities. We provide a simple theoretical framework to put these measures in context and to shed light on the relationship between exchange rates and the balance sheet of the central bank. We then move on to the formal empirical analysis. Nonlinear cointegration techniques are used to compare these two measures with the actual exchange rate for a set of Latin American countries using monthly data for the 2004:1-2019:12 period. The nonlinear cointegration technique allows both the number and location of thresholds to be endogenously determined based on the percentage difference between the exchange rate and the implied exchange rate. The nonlinear cointegration technique will allow us to determine whether central bank debt matters for understanding exchange rate dynamics and to determine whether passthrough is symmetric.

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1 Introduction

An increase in the domestic money supply (or monetary liability), other things equals, is associated with a currency depreciation. Some central banks however also issue debt. Do these remunerated liabilities affect exchange rates? In this paper we tackle this question. We provide a simple theoretical framework to understand the link between exchange rate dynamics and the balance sheet of the central bank. We then empirically test the link between changes in remunerated liabilities and exchange rates, focusing on a set of Latin American countries.

To help interpret the data, consider a simplified T-account of a central bank with zero net worth:

Assets	Liablities
Foreign Exchange Reserves	Monetary Base
Financial Claims	Remunerated Liabilities
Domestic Credit	

Table (1) shows these components, as a fraction of GDP, for six Latin American countries in 2018 (ranked by remunerated liabilities, in the last column).[1] The stock of remunerated liabilities as a fraction of GDP averaged 10% across these countries, with considerable heterogeneity: the stock of central bank debt was around a fourth of GDP in Uruguay and almost non-existent in Colombia. The source of this data is the International Monetary Fund's International Financial Statistics (IMF IFS) (see Appendix (A) for details on the construction of these variables).

Table 1: Balance Sheet Components over GDP, 2018

Country	Reserves	Financial Claims	Domestic Credit	Monetary Base	Remunerated Liabilities
Uruguay	0.30	0.00	0.10	0.06	0.25
Brazil	0.22	0.01	0.26	0.10	0.17
Chile	0.14	0.00	0.00	0.06	0.10
Paraguay	0.21	0.01	0.00	0.12	0.06
Mexico	0.15	0.02	0.00	0.07	0.02
Colombia	0.16	0.01	0.01	0.10	0.00

To put in international perspective, Table (A1) shows the same table for all 85 countries in the IMF IFS that have these variables available, for the 2010-2019 average: Uruguay is -together with Malasia- the country with highest ratio in the full sample. Brazil and Chile also are among the top 10 countries in terms of remunerated liabilities. Mexico and Colombia, in turn, belong to the large group of countries with little

[1]The criteria we followed to choose these countries is: South American countries plus Mexico. Argentina and Peru are unfortunately not included in the IMF IFS data for these variables (for Argentina, we obtain the data from a different source, as we describe below). Ecuador isn't included in IMF IFS for these variables either (and is dollarized, so not relevant for this analysis). Venezuela does not have data at monthly frequency, so we omit. We also omit Bolivia due to lack of exchange rate movement (pegged exchange rate).

to no remunerated liabilities. So in this Latin American subsample we have relevant heterogeneity that fits the international extremes.

Our empirical exercise relies on a measure of exchange rates computed from this balance sheet data: this implied exchange rate is the the ratio of domestic liabilities (measured in domestic currency) to foreign assets (measured in foreign currency). This measure is sometimes used by economists and financial market analysts, for instance as described in Ávila [2018]. The implied exchange rate thus depreciates (increases) with increases in liabilities and appreciates (decreases) with accumulation of assets. The assets we include are foreign exchange reserves. The liabilities however can be either the monetary base or also include remunerated liabilities, so we will use two alternative measures. In Section (3) we provide a simple theoretical framework to derive and further understand this measure.

We refer to these measures as the "Conversion Exchange Rate" (CER henceforth).[2] We will first define the "CER Base" as the measure that only incorporates the monetary base in the numerator and the "CER Full" as the measure that includes both the monetary base and remunerated liabilities. So the measures are:

$$\text{CER Base} \equiv \frac{\text{Monetary Base}}{\text{International Reserves}} \tag{1}$$

and

$$\text{CER Full} \equiv \frac{\text{Monetary Base+Remunerated Liabilities}}{\text{International Reserves}}. \tag{2}$$

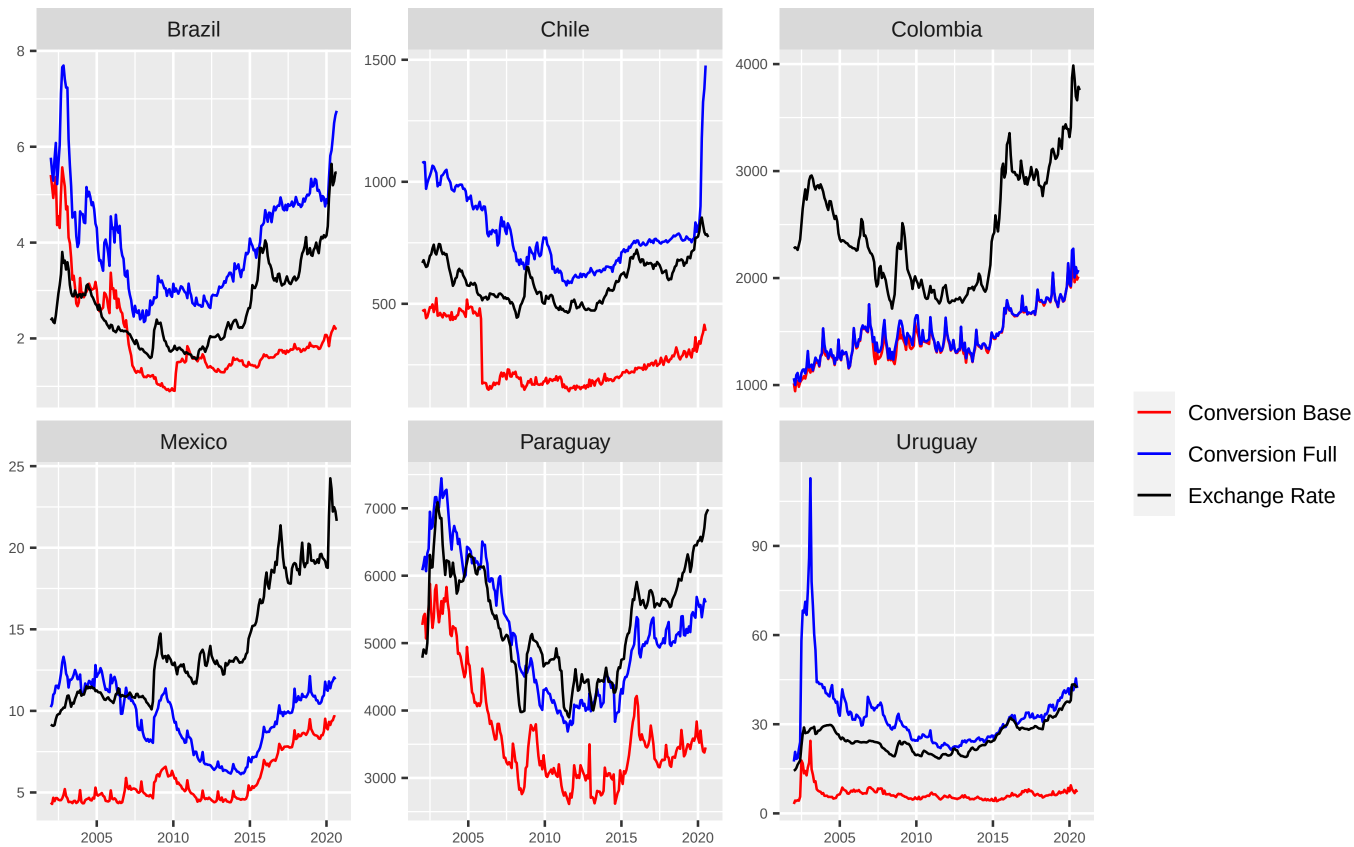
Figure (1) depicts the exchange rate (in black) and implied ones for these countries. The implied exchange rate levels are visually consistent with the actual exchange rate for four of these countries (Brazil, Chile, Paraguay and Uruguay). The red line is the CER Base and the blue line is the CER Full. The difference between the blue line and the red line is explained by the existence of remunerated liabilities. The larger the remunerated liabilities, the larger the difference between the two lines.

The CER Full seems to track the exchange rate better than the CER Base, suggesting that remunerated liabilities matter. For Colombia and Mexico, however, the exchange rate and CER do not seem to be related. Recall from Table (1) that these two countries had little to none remunerated liabilities. A pattern seems to emerge: countries with substantial remunerated liabilities (say larger than 2% of GDP) have exchange rates that move closely with the balance sheet of the central bank, while countries without remunerated liabilities have exchange rates that are decoupled with the balance sheet of the central bank. We formally test the link between these variables.

We also study large depreciations. For this we focus on the currency crisis in Argentina in 2018. Between

[2] In spanish this is known as "Tipo de Cambio de Conversión" (Ávila [2018]).

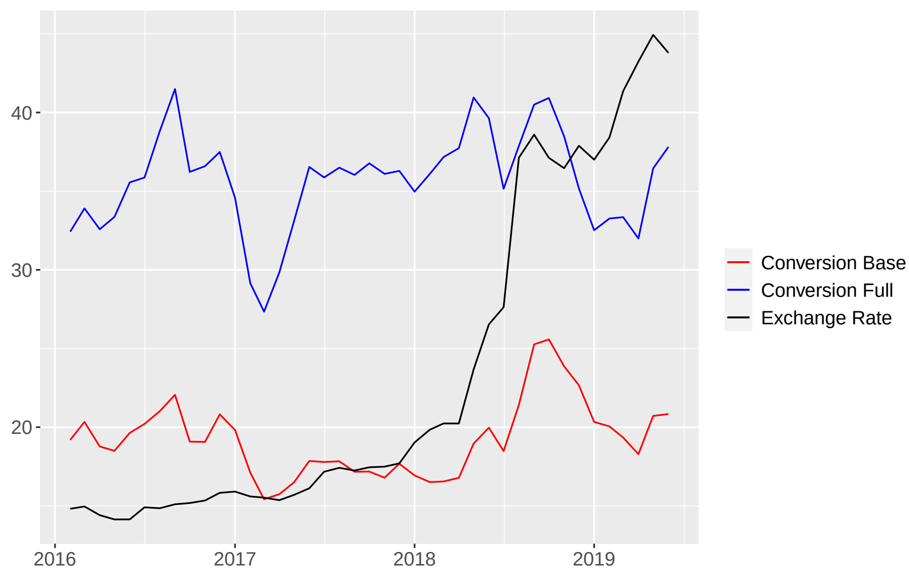
Figure 1: Exchange Rates and CER



Source: Author's calculations based on IMF IFS.

April and September 2018 the argentine peso lost half its value: it went from around 20 to 40 pesos per US dollar. The central bank had accumulated around 8% of GDP in remunerated liabilities during the previous two years.[3] As Figure (2) shows, the magnitude of this depreciation coincided with the difference in the CER Base and CER Full. In the crisis, the actual exchange rate jumped from the former to the later. This might be suggesting that the fundamental exchange rate value is indeed given by balance sheet measures that include remunerated liabilities.[4] Through this lens, before May 2018 the currency was overvalued, relative to the fundamentals given by the Central Bank balance sheet.

Figure 2: Argentina (currency crisis case study): Exchange Rate and Conversion Exchange Rate



Source: Author's calculations calculation based on UCEMA-CEA.

In Section (2) we provide a literature review and describe our contribution. In Section (3) we provide a simple theory of the implied exchange rates used throughout the paper. In Section (4) we move to the formal econometric analysis and then conclude.

[3]Since there is no such data for Argentina in the IMF IFS, we constructed the measures using data from UCEMA Centro de Economía Aplicada (UCEMA-CEA), which provides data from the central bank and other sources. For the case of Argentina we focus in the January 2016 - July 2019 period since before and after this there were tight currency controls, and thus a black foreign exchange price.

[4]For the period after the crisis (around May 2018), the CER Full is underestimated, since the IMF bailout transfers were temporarily recorded as international reserves by the Central Bank (thus the actual reserves were lower, which implies a higher CER Full). We thank Jorge Ávila for this insight.

2 Related Literature

Monetary theory tells us that variation of the exchange rate is a function of the difference in supply and demand of money, relative to the foreign country, as exposed in for instance Obstfeld and Rogoff [1995]. An increase in the money supply (or monetary liability), other things equals, leads to a currency depreciation. These theories however mostly abstract from two features that are present in many emerging markets:

1. Many central banks follow intermediate exchange rate regimes (neither fixed nor flexible exchange rates): Frankel [2019] proposes to define an intermediate regime, called "systematic managed floating", as an arrangement where the central bank regularly responds to changes in total exchange market pressure by allowing some fraction to be reflected as a change in the exchange rate and the remaining fraction to be absorbed as a change in foreign exchange reserves.

2. Many central banks issue remunerated liabilities, as reported before. Foreign exchange reserves are sometimes financed by issuing central bank debt. Sosa-Padilla and Sturzenegger [2021] study the link between remunerated liabilities and bond yields. We however focus on the link with exchange rates.

We contribute to the existing literature by providing evidence and a simple theoretical framework to understand exchange rate movements with the existence of remunerated liabilities. Our intention is not in predicting exchange rates: a long-standing puzzle in international economics is the difficulty of tying floating exchange rates to macroeconomic fundamentals such as money supplies, outputs, and interest rates (see Engel and West [2005] and references therein).

Our contribution can be seen as adding an extra layer of complexity to these fundamentals: remunerated liabilities can be see as another fundamental in exchange rate determination. Our findings thus might have empirical and theoretical implications for open economy macroeconomics, in particular for the Mundell-Fleming trilemma (Mundell [1963], Fleming [1962]). Our main conclusion is that remunerated liabilities can shed light on the empirical relationship between exchange rates and the balance sheet of central banks.

3 Theoretical Framework

How can we think of the relationship described previously? In this section we provide a simple theoretical link between exchange rates and remunerated liabilities. We follow the main building blocks and notation in Engel and West [2005] (see references therein for earlier contributions), however we extend the framework to allow for accumulation of international reserves and remunerated liabilities.[5]

Assume that in the home country there is a money market relationship given by

$$m_t = p_t + \gamma y_t - \alpha i_t + v_{mt}. \tag{3}$$

Here, m_t is the log of the home money supply, p_t is the log of the home price level, i_t is the level of the home interest rate, y_t is the log of output, and v_{mt} is a shock to money demand. Assume that a similar equation holds in the foreign country. The analogous foreign variables are m_t^*, p_t^*, i_t^*, y_t^* and v_t^*, and the parameters of the foreign money demand are identical to the home country's parameters.

The nominal exchange rate equals its purchasing power parity (PPP) value plus the real exchange rate:

$$s_t = p_t - p_t^* + q_t. \tag{4}$$

In financial markets, the interest parity relationship is

$$E_t s_{t+1} - s_t = i_t - i_t^* + \rho_t. \tag{5}$$

Here, ρ_t is the deviation from rational expectations uncovered interest parity. It can be interpreted as a risk premium or an expectational error. Putting these equations together and rearranging, we get

$$s_t = \frac{1}{1+\alpha}\left[m_t - m_t^* - \gamma\left(y_t - y_t^*\right) + q_t - \left(v_{mt} - v_{mt}^*\right) - \alpha\rho_t\right] + \frac{\alpha}{1+\alpha} E_t s_{t+1}. \tag{6}$$

which is the expression in Engel and West [2005]. To interpret this equation, assume $\gamma = 1$, $q_t = 1$, $v_{mt} = v_{mt}^* = 0$, $\rho_t = 0$, and let $b = \frac{1}{1+\alpha}$, so

$$s_t = b\left[m_t - m_t^* - \left(y_t - y_t^*\right)\right] + (1-b) E_t s_{t+1}. \tag{7}$$

The observable fundamentals are given by $f_{1t} = m_t - m_t^* - \gamma\left(y_t - y_t^*\right)$. Notice that equation (7) holds absent intervention of the central banks. It is thus a pure flexible exchange rate "counterfactual" equilibrium.

[5] We use reduced form equations, but many of these equations can be derived from microfounded models, as in Obstfeld and Rogoff [1995] or Vegh [2013].

We however consider an environment where the central bank regularly responds to changes in total exchange market pressure by allowing some fraction to be reflected as a change in the exchange rate and the remaining fraction to be absorbed as a change in foreign exchange reserve (Frankel [2019]).

A positive productivity shock, for instance, that leads to an increase in output y_t might be reflected as combination of an appreciation and reserve accumulation. We thus need to introduce international reserves into the picture.

Central Bank's Balance Sheet A simplified Central Bank's balance sheet is given by

A	L
$s_t H_t$	MB_t
	RL_t

here H are international reserves (in foreign currency, "dollars"), s is the exchange rate and MB is the monetary base and RL_t are remunerated liabilities (in domestic currency, "pesos").[6]

We consider an environment where the central bank's net worth does not vary over time (as typically assumed, see Vegh [2013] for example in different models). Thus, assets and liabilities vary, in real terms, proportionally. This implies that international reserves equals, to a first degree, total liabilities in real terms (dollars), $H = \frac{MB+RL}{s}$. Rearranging, we get the relationship used in the empirical exercise:

$$s = \frac{MB + RL}{H} \tag{8}$$

Define total liabilities (TL) as the sum of monetary base and remunerated liabilities $TL = MB + RL$ and let $\omega \equiv \frac{MB}{MB+RL}$ be the weight of the monetary base in total liabilities (and thus $1 - \omega = \frac{RL}{MB+RL}$). Then, taking logs and differentiating with respect to time, the dynamics of the exchange rate is given by

$$\frac{\Delta s}{s} = \omega \frac{\Delta MB}{MB} + (1 - \omega) \frac{\Delta RL}{RL} - \frac{\Delta H}{H} \tag{9}$$

where Δ represents changes. Equations (9) and (7) have some similarity: other things equal, the exchange rate depreciates with increases in the monetary liabilities (which now also includes remunerated liabilities) and decreases with output growth (with tends to increase international reserves). [7] A framework that does not include remunerated liabilities ($\triangle RL = 0$) tends to ignore relevant sources of exchange rate variations,

[6] We abstract from claims to financial institutions and from from domestic credit, mainly because we focus on foreign denominated assets). We also abstract from valuation of the assets held by the Central Bank, a more general framework could include some of these elements, in line with Ghironi et al. [2015].

[7] The intuition for this is in the money market: the exchange rate is the equilibrium price of supply and demand for domestic money. Output growth tends to increase money demand, which tends to appreciate the currency under flexible exchange rates. Under predetermined exchange rates however the central bank increases the supply of money -by purchasing foreign exchange reserves- and hence the exchange rate does not change but reserves do increase.

if the central bank indeed accumulated such remunerated liabilities. Since the expected exchange rate already incorporates the expected dynamics of remunerated liabilities, equation (7) also pins down the exchange rate in the flexible exchange rate regime case.

The central bank can decide to intervine in the foreign exchange market. Given a shock/pressure on the exchange rate, the central bank might decide to prevent a movement in the exchange rate and let international reserves fully absorb the shock. Such a central bank, at least for this period, is fixing its exchange rate in de facto dollar terms (Levy-Yeyati and Sturzenegger [2005]). In this case, during the relevant period, international reserves are explained by variations in total liabilities (in real terms, since the exchange rate does not change), $\frac{\Delta H}{H} = \omega \frac{\Delta MB}{MB} + (1-\omega)\frac{\Delta RL}{RL}$. In this case, given for instance a positive shock, international reserves will increase by the sum of total liabilities.[8]

In general terms, however, given a shock, the central bank can choose it to be reflected partially in the exchange rate and partially in international reserves. Thus, in general terms, the reflection of a shock in the balance sheet can be described by a vector $(\Delta s, \Delta H, \Delta MB, \Delta RL)$. The policy maker however cannot freely choose all of the elements of this vector.

Do remunerated liabilities give an extra degree of freedom that can be used to override the the Mundell-Fleming trilemma (Mundell [1963], Fleming [1962])? To the contrary: our framework suggests that remunerated liabilities worsen the trilemma.

To see this, consider this example. A central bank can choose two of the following: the exchange rate management, autonomous monetary policy or free capital mobility. In the framework we are thinking of, an extra layer is added, since central banks can choose the amount of remunerated liabilities. To set ideas, assume that the central bank has chosen to have free capital mobility and a predetermined exchange rate regime (so the interest rate and the amount of money are endogenous) but the central bank has remunerated liabilities. From equation (8) it is clear that central banks in this context cannot freely choose the amount of remunerated liabilities, since doing so will affect exchange rates (since the expected exchange rate moves according to these liabilities, and hence so does the spot exchange rate). Remunerated liabilities are in this sense "endogenous": they can only change as long as the monetary base or international reserves move to compensate them (which also are endogenous in this context due to the trilemma). If on the other hand the central bank chooses the amount of remunerated liabilities, then it cannot have a predetermined exchange rate. The Mundell-Fleming trilemma is in full force again, albeit now with remunerated liabilities. [9]

[8] Not all liabilities are the same, however. On one hand, international reserves accumulation can be explained by increases in the monetary base in real terms, which we can call as a "genuine" source of accumulation since it is endogenous under predetermined exchange rates and grows with money demand (economic growth). On the other hand, international reserves can be financed by increases in remunerated liabilities (debt accumulation). Sosa-Padilla and Sturzenegger [2021] focus on these differences.

[9] TBC (ALSO CHECK OLDER DRAFT AND CAMILO'S COMMENTS): A related discussion is whether stockpiling remunerated liabilities can affect exchange rate stability or even foster a currency crisis. According to our framework, the mismatch in the balance sheet might have created the currency crisis that Argentina suffered in 2018 (and thus the debt crisis, since a large devaluation

Exchange rates are thus related to the stock of remunerated liabilities. We now move to the empirical analysis.

increases the debt to GDP significantly). Remunerated liabilities are not a hedge in a crisis, as Sosa-Padilla and Sturzenegger [2021] argue. Remunerated liabilities created the crisis itself: from our framework the exchange rate simply jumped from CER Base to CER Full during the crisis. Macroprudential policy would have narrowed the gap between these measures (reducing remunerated liabilities).

4 VAR Exercise

VAR exercises for Latam economies

Vector Autorregresive model:

$$X_t = \Phi X_{t-1} + u_t$$

where $X_t = [er\ h\ mb\ rl]'$

With: er : exchange rate, h: foreign reserves, mb: monetary base, rl: remunerated liabilities.

All variables are included as the first difference of the logarithm of their levels.

Countries included: Chile, Brazil, Paraguay, Uruguay, Colombia

Sample: 2002.01 - 2020.07

Sample size: 222 (each VAR), 1110 total.

Lag Selection for VAR

Country	AIC(n)	HQ(n)	SC(n)	FPE(n)
CHL	1	1	1	1
BRA	1	1	1	1
PRY	1	1	1	1
URY	2	1	1	2
COL	5	1	1	5

Causality Tests

Instantaneous causality test				
Country	ER	H	MB	RL
CHL	0.000	0.000	0.000	0.000
BRA	0.000	0.000	0.000	0.000
PRY	0.000	0.000	0.000	0.000
URY	0.017	0.000	0.092	0.000
COL	0.000	0.000	0.855	0.161

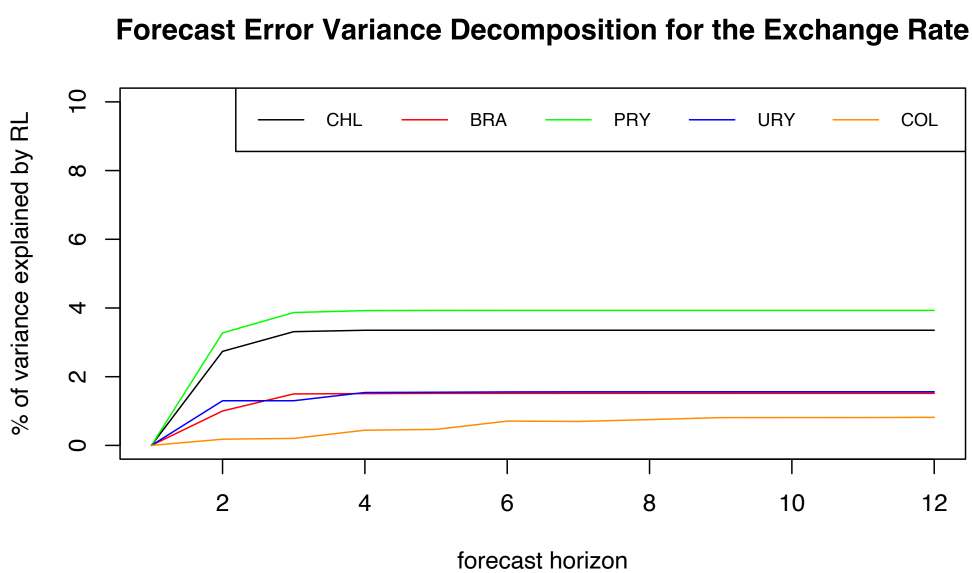
Note: the p-value is shown. Null hyp. for the test: the variable in the column Granger causes the rest of the system.

Granger causality test				
Country	ER	H	MB	RL
CHL	0.129	0.000	0.000	0.003
BRA	0.037	0.000	0.721	0.182
PRY	0.218	0.002	0.021	0.014
URY	0.119	0.031	0.075	0.813
COL	0.024	0.000	0.185	0.937

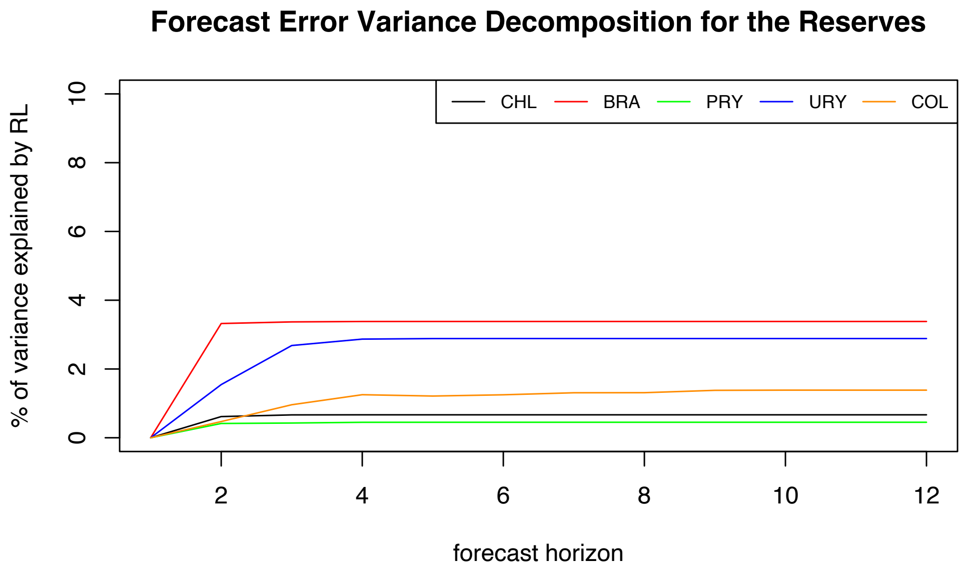
Note: the p-value is shown. Null hyp. for the test: the variable in the column Granger causes the rest of the system.

Forecast Error Variance Decomposition Plots

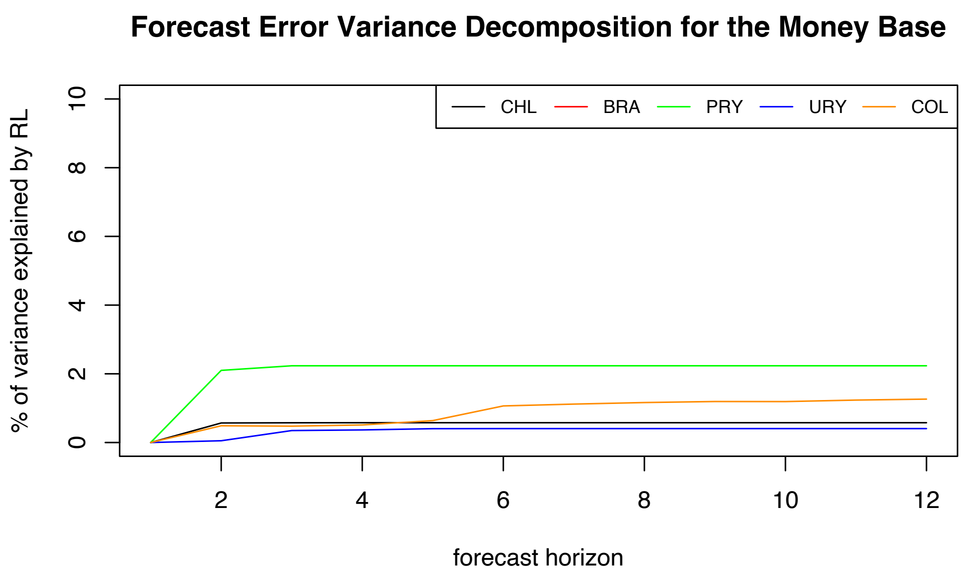
FEVD for the Exchange Rate (% explained by RL)



FEVD for the Reserves (% explained by RL)

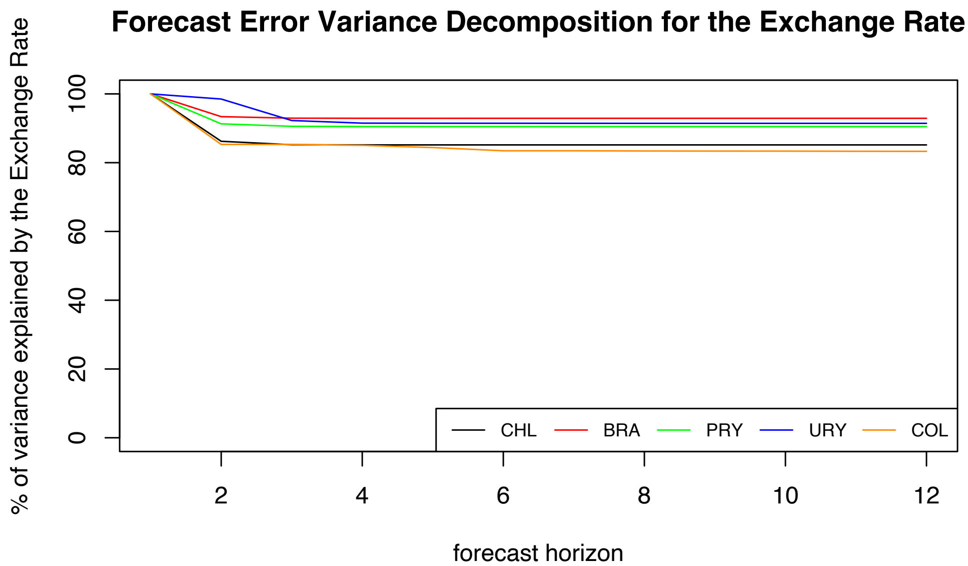


FEVD for the Monetary Base (% explained by RL)

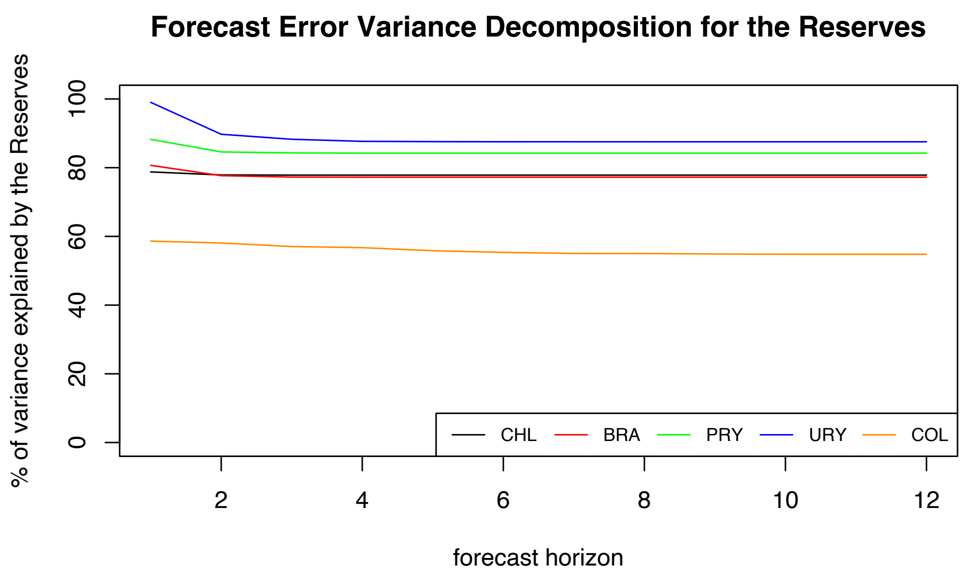


How persistent are these processes:

FEVD for the Exchange Rate (% explained by itself)



FEVD for the Reserves (% explained by itself)



Forecast Error Variance Decomposition from Panel VAR model

Exchange Rate:

Horizon	ER	H	MB	RL
1	100.00	0.00	0.00	0.00
2	97.42	2.46	0.10	0.02
3	96.47	2.90	0.61	0.02
4	96.45	2.90	0.62	0.03
5	96.44	2.90	0.62	0.03
6	96.44	2.90	0.63	0.03
7	96.44	2.90	0.63	0.03
8	96.44	2.90	0.63	0.03
9	96.44	2.90	0.63	0.03
10	96.44	2.90	0.63	0.03
11	96.44	2.90	0.63	0.03
12	96.44	2.90	0.63	0.03

Reserves:

Horizon	ER	H	MB	RL
1	12.92	87.08	0.00	0.00
2	13.23	86.25	0.27	0.25
3	13.50	85.92	0.29	0.29
4	13.51	85.86	0.33	0.30
5	13.51	85.85	0.33	0.31
6	13.51	85.85	0.33	0.31
7	13.51	85.85	0.33	0.31
8	13.51	85.85	0.33	0.31
9	13.51	85.85	0.33	0.31
10	13.51	85.85	0.33	0.31
11	13.51	85.85	0.33	0.31
12	13.51	85.85	0.33	0.31

Monetary base:

Horizon	ER	H	MB	RL
1	0.19	0.00	99.81	0.00
2	0.32	0.53	99.13	0.02
3	0.42	0.55	99.01	0.02
4	0.42	0.58	98.97	0.03
5	0.43	0.58	98.97	0.03
6	0.43	0.58	98.97	0.03
7	0.43	0.58	98.97	0.03
8	0.43	0.58	98.97	0.03
9	0.43	0.58	98.97	0.03
10	0.43	0.58	98.97	0.03
11	0.43	0.58	98.97	0.03
12	0.43	0.58	98.97	0.03

Remunerated liabilities:

Horizon	ER	H	MB	RL
1	0.60	0.50	0.95	97.95
2	0.53	0.50	1.20	97.77
3	0.53	0.51	1.30	97.67
4	0.52	0.57	1.39	97.51
5	0.52	0.58	1.39	97.50
6	0.53	0.58	1.40	97.50
7	0.53	0.58	1.40	97.50
8	0.53	0.58	1.40	97.50
9	0.53	0.58	1.40	97.50
10	0.53	0.58	1.40	97.50
11	0.53	0.58	1.40	97.50
12	0.53	0.58	1.40	97.50

Camilo, email:

- Para Chile y Paraguay los RL causan al resto de variables (ER, H, MB) pero no para Colombia, Uruguay y Brasil. - Para todos los países menos Colombia los RL causan de forma instantanea a las otras variables.

Las reservas por otro lado si causan a el resto de variables para todos los países.

- En la descomposición varianza la mayor contribución de RL a el ER es para Chile y Paraguay, llegando casi a un 4% de varianza explicada. - El ER tiende a ser explicado por si mismo en mayor medida que las reservas y otras variables. Por lo mismo, el porcentaje en que RL lo explica no es trivial con respecto a otras variables. - En un Panel VAR la contribución cruzada entre variables (por ej de RL a el ER en lugar de cada variable en si misma) es menor que lo encontrado para países en particular.

Todos estos tests sugieren que hay países en que los RL son muy relevantes mientras que en el resto no es tan claro que lo sean. Esto puede ser interesante en la medida que se estudie en que caso importa la composición de las Reservas y en cuales es no y porqué.

TBC: interpretation, plots, tables, Argentina

5 Cointegration Analysis

5.1 Cointegration Framework

We first focus on data for Argentina, Brazil, Chile, Paraguay, and Uruguay using the logged values for monthly data from 2004:1-2019:12.

The econometric analysis consists of three steps. First, we investigate the order of integration of each series using the Augmented Dickey Fuller (ADF) unit root test of Dickey and Fuller [1979, 1981] and the minimum Lagrange multiplier unit root test that allows for structural breaks in level and trend of Lee and Strazicich [2003]. When the series are found to be integrated of the same order, we employ the threshold cointegration technique of Sephton and Mann [2013] which combines the threshold selection method of Gonzalo and Pitarakis [2002] with the F test developed by Seo [2008]. The test for threshold cointegration examines the residuals from the cointegrating regression in equation (10), and conditional on selecting one or more thresholds (for expository purposes, we assume there are two in equation (11), denoted by τ_1 and τ_2) it examines the null hypothesis that ($\delta_1 = \delta_2 = \delta_3 = 0$) against the alternative they are not jointly zero. In the case of a single threshold, the testing equation (11) would delete the term involving the residual between the thresholds. In the case of zero thresholds the method collapses on the testing equations of the traditional cointegration test of Engle and Granger [1987]. Inference for threshold cointegation tests use residual block based bootstrapping and inference for Engle Granger cointegration tests follow critical values of MacKinnon [1996].

$$Y_t = \beta_1 + \beta_2 Trend_t + \beta_3 X_t + \varepsilon_t \tag{10}$$

$$\triangle\varepsilon_t = \delta_1\varepsilon_{t-1} I\left(P_{t-1} < \tau_1\right) + \delta_2\varepsilon_{t-1} I\left(\tau_1 < P_{t-1} < \tau_2\right) + \delta_3\varepsilon_{t-1} I\left(\tau_2 < P_{t-1}\right) + \sum_{i=1}^{p} \alpha_i \triangle\varepsilon_{t-i} + v_t \tag{11}$$

such that

- Y_t and X_t are the logged implied exchange rate and the exchange rate.
- P_t is the percentage difference between the exchange rate and the implied exchange rate.
- $\tau_1 < \tau_2$ are the thresholds that divide observations in to the bottom, middle and top regimes.
- $I\,()$ is the Heaviside indicator.
- Δ is the difference operator.

The advantage of Sephton and Mann [2013] over the traditional approach is that it endogenously determines both the number of thresholds and their locations. Many other cointegration techniques assume the absence of thresholds (i.e., Engle and Granger test); that a single threshold exists that is equal to zero (hence separating observations into two regimes with ε_{t-1} being positive in the first and negative in the second); or that two thresholds exist that are symmetric around zero. Here we allow for as many as three thresholds, and as few as zero. The method allows the threshold variable to be exogenous to the system, but requires that it is stationary. We use the percentage difference between the exchange rate and the implied exchange rate as the threshold variable. If it is not stationary in levels, we use its first difference. This parallels TAR and MTAR models by Enders and Siklos [2001]. If the procedure chooses two thresholds, they may be symmetric about zero, but this decision is not determined *a priori* – the data determines the number of thresholds and their locations. While this approach could choose a specification similar to the traditional band-TAR model described by Balke and Fomby [1997], it is very flexible, and has been shown to work well in practice. For further details on the threshold cointegration technique and its properties please see Sephton and Mann [2013].

Finally, when we reject the null hypothesis for the cointegration tests an error correction model (ECM) can be used to determine how the variables move through time, and which of them adjusts to restore the system to the long-run relationship. The ECM also provides estimates of the speed at which the adjustment takes place, and the system can be used to simulate the impact of shocks to the series. Incorporating the threshold into the ECM allow both the speed at which the adjustment takes place and the impact of the shock to vary by regime.

An ECM with two thresholds is expressed in equation system (12-13), where for convenience, we have set the lag length on changes in the dependent variables to be the same across equations.

$$\triangle Y_t = \gamma_1 \varepsilon_{t-1} I\left(P_{t-1} < \tau_1\right) + \gamma_2 \varepsilon_{t-1} I\left(\tau_1 < P_{t-1} < \tau_2\right) + \gamma_3 \varepsilon_{t-1} I\left(\tau_2 < P_{t-1}\right) + \sum_{i=1}^{p} \left(\alpha_i \triangle Y_{t-i} + \beta_i \triangle X_{t-i}\right) + u_t \tag{12}$$

$$\triangle X_t = \gamma_4 \varepsilon_{t-1} I\left(P_{t-1} < \tau_1\right) + \gamma_5 \varepsilon_{t-1} I\left(\tau_1 < P_{t-1} < \tau_2\right) + \gamma_6 \varepsilon_{t-1} I\left(\tau_2 < P_{t-1}\right) + \sum_{i=1}^{p} \left(\chi_i \triangle Y_{t-i} + \delta_i \triangle X_{t-i}\right) + v_t \tag{13}$$

such that

- Y_t and X_t are the logged implied exchange rate and the exchange rate.
- P_t is the percentage difference between the exchange rate and the implied exchange rate.

- $\tau_1 < \tau_2$ are the thresholds that divide observations in to the bottom, middle and top regime.
- γ_i are the speeds of adjustment to the long-run relationship.
- $I()$ is the Heaviside indicator.
- Δ is the difference operator.

The AIC criteria has been used throughout the paper to determine the optimal lag length. Estimates of the speeds of adjustment coefficients will demonstrate which variables are weakly exogenous and do not move to restore the long run relationship.

TBC

5.2 Cointegration Results

Results from the unit root tests presented in Table 2 are consistent across all five countries; the exchange rate and the implied exchange rate (both Full and Base) are $I(1)$. This allows us to proceed to the threshold cointegration tests for all 10 exchange rate-implied exchange rate pairs. The unit root test results presented in Table 3 show that the percentage difference between the exchange rate and the implied exchange rate can be used for the threshold variable for all pairs except for the exchange rate-Base pair for both Brazil and Argentina and the exchange rate-Full pair for Argentina. For these three pairs the differenced percentage difference between the exchange rate and the implied exchange rate is used.

Table 2: ADF Unit Root Test Results for Exchange Rate (Ex) and Implied Exchange Rate (Base and Full) for 2004:1-2019:12

	ADF	LS
Chile Ex	-2.076	
Chile Base	-2.112	-2.063
Chile Full	-1.444	
Uruguay Ex	-0.494	-2.015
Uruguay Base	-2.549	-2.445
Uruguay Full	-1.521	-1.624
Paraguay Ex	-1.655	-4.81
Paraguay Base	-2.467	-3.882
Paraguay Full	-1.096	-4.02
Brazil Ex	-1.092	-4.085*
Brazil Base	-1.585	-3.748
Brazil Full	-1.425	-3.872
Argentina Ex	0.125	
Argentina Base	-2.292	
Argentina Full	-1.949	

Notes: The null hypothesis is a unit root. Deterministic components, number and type of structural breaks determined by ocular inspection of the time series plot; lag length for the unit root tests is selected by the AIC from a maximum of $T^{1/3}$. Significance at $\alpha = 0.10$, 0.05 and 0.01 denoted by *, **, and *** respectively. A blank entry indicates the absence of a structural break.

Table 3: Unit Root Test Results for Threshold Variable for 2004:1-2019:12

	ADF	LS
Chile Margin Base	2.764	-4.399**
Chile Margin Full	-4.083***	-4.711***
Uruguay Margin Base	-2.689*	
Uruguay Margin Full	-3.446 **	
Paraguay Margin Base	-4.644***	
Paraguay Margin Full	-3.362**	-4.210**
Brazil Margin Base	-2.156	
Brazil Margin Full	-4.130***	
Argentina Margin Base	-1.329	-4.94
Argentina Margin Full	-1.335	-3.966

Notes: Ibid, Table 2.

Results from the cointegration tests presented in Table 4 indicate that seven of the 10 exchange rate-implied exchange rate pairs are cointegrated with either two or three thresholds. The remaining three pairs are Chile Base, Brazil Base, and Brazil Full with zero threshold being selected for Chile Base and three thresholds being selected for Brazil Base and Brazil Full.

The final version of this paper will extend the results to include the ECM from equations (12) and (13) for the seven cointegrated pairs to determine how the variables move through time, and which of them adjusts to restore the system to the long-run relationship. Care will be taken to interpret the ECM in terms of whether the level or the differenced percentage difference is used as the threshold variable. (The final version will also extend the analysis to include Mexico and Colombia.)

Table 4: Threshold Cointegration Test Results

	Test Statistic	Number of Thresholds
Chile Base		0
Chile Full	24.482***	2
Uruguay Base	17.210***	2
Uruguay Full	25.998***	3
Paraguay Base	65.215***	3
Paraguay Full	19.410**	3
Brazil Base	11.362	3
Brazil Full	6.306	3
Argentina Base	17.559*	3
Argentina Full	17.862***	1

Notes: The null hypothesis is no cointegration against an alternative of (threshold) cointegration. Input specifications for Engle-Granger test (i.e., zero thresholds): lag length for the testing equations is selected by the AIC from a maximum of $T_{1/3}$; critical values follow MacKinnon (1996). Input specifications for the threshold cointegration tests (i.e., 1, 2, or three thresholds): threshold locations include the middle 90% of observations divided into 50 increments; each regime requires a minimum of 24 observations (not binding); AIC used throughout; critical values for the threshold cointegration test simulated following the residual-based block bootstrap methodology by Seo (2008) with a block length of 6 and 9,999 replications under the null. Significance at $\alpha = 0.10$, 0.05 and 0.01 denoted by *, **, and *** respectively.

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6 Preliminary Conclusion

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OLDER: We found evidence of a long-run relationship between exchange rate and implied exchange rate for seven of the 10 pairs.

In Brazil, there is no cointegration between the exchange rate and implied exchange rates. The reason is beyond the scope of this paper, but it might be related to the fact that Brazil is the country with highest domestic credit component, as shown in Table 1. Future research can shed light on this outlier.

In Argentina the series are cointegrated, which might be unexpected from the graphs. In future versions of this paper we plan to analyze the case study of the 2018 currency crisis in the context of a structural break.

We think that our findings have relevance for international macroeconomic theory and policy. In particular, have the remunerated liabilities made the Mundell-Fleming trilemma different? Or is it really a multilemma? These seem like promising questions for future research.

References

Ávila, Jorge C, "Plan de dolarización sostenible," Technical Report, Serie Documentos de Trabajo 2018.

Balke, Nathan S and Thomas B Fomby, "Threshold cointegration," *International economic review*, 1997, pp. 627–645.

Dickey, David A and Wayne A Fuller, "Distribution of the estimators for autoregressive time series with a unit root," *Journal of the American statistical association*, 1979, *74* (366a), 427–431.

_ and _ , "Likelihood ratio statistics for autoregressive time series with a unit root," *Econometrica: journal of the Econometric Society*, 1981, pp. 1057–1072.

Enders, Walter and Pierre L Siklos, "Cointegration and threshold adjustment," *Journal of Business & Economic Statistics*, 2001, *19* (2), 166–176.

Engel, Charles and Kenneth D West, "Exchange rates and fundamentals," *Journal of political Economy*, 2005, *113* (3), 485–517.

Engle, Robert F and Clive WJ Granger, "Co-integration and error correction: representation, estimation, and testing," *Econometrica: journal of the Econometric Society*, 1987, pp. 251–276.

Fleming, J Marcus, "Domestic financial policies under fixed and under floating exchange rates," *Staff Papers*, 1962, *9* (3), 369–380.

Frankel, Jeffrey, "Systematic managed floating," *Open Economies Review*, 2019, *30* (2), 255–295.

Ghironi, Fabio, Jaewoo Lee, and Alessandro Rebucci, "The valuation channel of external adjustment," *Journal of International Money and Finance*, 2015, *57*, 86–114.

Gonzalo, Jesús and Jean-Yves Pitarakis, "Estimation and model selection based inference in single and multiple threshold models," *Journal of Econometrics*, 2002, *110* (2), 319–352.

Lee, Junsoo and Mark C Strazicich, "Minimum Lagrange multiplier unit root test with two structural breaks," *Review of economics and statistics*, 2003, *85* (4), 1082–1089.

Levy-Yeyati, Eduardo and Federico Sturzenegger, "Classifying exchange rate regimes: Deeds vs. words," *European economic review*, 2005, *49* (6), 1603–1635.

MacKinnon, James G, "Numerical distribution functions for unit root and cointegration tests," *Journal of applied econometrics*, 1996, *11* (6), 601–618.

Mundell, Robert A, "Capital mobility and stabilization policy under fixed and flexible exchange rates," *The Canadian Journal of Economics and Political Science/Revue canadienne d'Economique et de Science politique*, 1963, *29* (4), 475–485.

Obstfeld, Maurice and Kenneth Rogoff, "Exchange rate dynamics redux," *Journal of political economy*, 1995, *103* (3), 624–660.

Seo, Myung Hwan, "Unit root test in a threshold autoregression: asymptotic theory and residual-based block bootstrap," *Econometric Theory*, 2008, pp. 1699–1716.

Sephton, Peter and Janelle Mann, "Threshold cointegration: Model selection with an application," *Journal of Economics and Econometrics*, 2013, *56* (2), 54–77.

Sosa-Padilla, César and Federico Sturzenegger, "Does It Matter How Central Banks Accumulate Reserves? Evidence from Sovereign Spreads," Technical Report, National Bureau of Economic Research 2021.

Vegh, Carlos A, *Open economy macroeconomics in developing countries*, MIT press, 2013.

A Data Source

We use standardized central bank balance sheet data from the IMF IFS statistics.[10] The Table below, from Sosa-Padilla and Sturzenegger [2021], summarizes the relevant balance-sheet data that we will use:

Assets	Liabilities
Claims on non-residents (1)	Liabilities to non-residents (a)
Claims on others depository corporations (2)	Monetary base (b)
Net Claims on Central Government (3)	Other Liabilities To Other Depository Corporations (c)
	Deposits and Securities other than Shares Excluded from Monetary Base (d)
	Loans (e)
	Financial Derivatives (f)
	Shares and equity (g)
	Other items (h)

Hence the CER Base is

$$\text{CER Base} \equiv \frac{\text{Monetary Base}}{\text{International Reserves}} = \frac{(b)}{(1)}$$

Following Sosa-Padilla and Sturzenegger [2021], we define the Remunerated Liabilities as the sum of (c), (d), (e) and (f). The CER Full is then

$$\text{CER Full} \equiv \frac{\text{Monetary Base+Remunerated Liabilities}}{\text{International Reserves}} = \frac{(b)+(c)+(d)+(e)+(f)}{(1)}$$

[10] As mentioned before, for Argentina we use the database in UCEMA-CEA, which provides data on monetary base and remunerated liabilities directly.

B Plots and Tables

Table A1: Balance Sheet Components over GDP

Country	Reserves	Financial Claims	Domestic Credit	Monetary Base	Remunerated Liabilities
Malaysia	0.40	0.03	0.00	0.10	0.22
Uruguay	0.25	0.00	0.14	0.06	0.22
China, P.R.: Macao	0.73	0.01	0.00	0.08	0.14
Jamaica	0.18	0.00	0.10	0.10	0.14
Algeria	0.76	0.00	0.03	0.23	0.13
Chile	0.15	0.00	0.02	0.07	0.13
Korea, Rep. of	0.24	0.01	0.01	0.06	0.12
Dominican Rep.	0.07	0.02	0.12	0.11	0.11
Brazil	0.14	0.01	0.18	0.10	0.10
Philippines	0.24	0.00	0.03	0.13	0.08
Bolivia	0.37	0.03	0.07	0.25	0.07
Romania	0.22	0.01	0.00	0.09	0.06
Papua New Guinea	0.14	0.00	0.02	0.07	0.06
Nigeria	0.12	0.02	0.03	0.05	0.06
Mongolia	0.18	0.05	0.02	0.11	0.05
Seychelles	0.22	0.00	0.15	0.16	0.05
Botswana	0.63	0.00	0.00	0.15	0.04
Belarus, Rep. of	0.10	0.03	0.01	0.06	0.04
Bhutan	0.52	0.01	0.00	0.28	0.04
Paraguay	0.16	0.01	0.02	0.11	0.04
Venezuela, Rep. Bolivariana de	0.17	0.00	0.04	0.12	0.04
Ghana	0.09	0.01	0.07	0.07	0.03
Mexico	0.12	0.01	0.00	0.05	0.03
Guatemala	0.14	0.01	0.07	0.14	0.03
Maldives	0.14	0.00	0.11	0.18	0.02
Sweden	0.09	0.02	0.02	0.04	0.02
Rwanda	0.14	0.00	0.02	0.04	0.02
Cambodia	0.31	0.00	0.01	0.19	0.02
Azerbaijan, Rep. of	0.15	0.02	0.01	0.13	0.02
Angola	0.17	0.01	0.02	0.07	0.02
Mozambique, Rep. of	0.17	0.00	0.02	0.09	0.02
Vanuatu	0.24	0.00	0.03	0.19	0.02
Ukraine	0.18	0.04	0.09	0.17	0.01
Namibia	0.12	0.01	0.00	0.04	0.01
Samoa	0.14	0.01	0.00	0.10	0.01
Georgia	0.15	0.02	0.04	0.12	0.01
Kyrgyz Rep.	0.28	0.01	0.03	0.18	0.01
Bangladesh	0.08	0.01	0.03	0.09	0.01
Australia	0.05	0.03	0.00	0.04	0.01
Tajikistan, Rep. of	0.08	0.02	0.04	0.13	0.01
Sri Lanka	0.10	0.00	0.03	0.06	0.01
Japan	0.01	0.05	0.35	0.42	0.01
Kenya	0.10	0.01	0.02	0.07	0.01
Tanzania, United Rep. of	0.11	0.00	0.02	0.07	0.01
Turkey	0.13	0.02	0.02	0.10	0.01
St. Kitts and Nevis	0.24	0.00	0.00	0.22	0.01
Antigua and Barbuda	0.16	0.00	0.02	0.17	0.01
Suriname	0.14	0.00	0.06	0.13	0.01
Fiji, Rep. of	0.17	0.00	0.02	0.16	0.01
Trinidad and Tobago	0.37	0.00	0.15	0.15	0.00
Uganda	0.12	0.00	0.06	0.05	0.00
Dominica	0.20	0.00	0.00	0.17	0.00
Grenada	0.17	0.00	0.00	0.15	0.00
Croatia, Rep. of	0.26	0.00	0.00	0.21	0.00
St. Lucia	0.14	0.00	0.00	0.13	0.00
Russian Federation	0.26	0.04	0.01	0.14	0.00
Armenia, Rep. of	0.18	0.03	0.00	0.15	0.00
Barbados	0.12	0.00	0.07	0.16	0.00
Guyana	0.22	0.00	0.13	0.19	0.00
Kuwait	0.19	0.00	0.00	0.11	0.00
St. Vincent and the Grenadines	0.16	0.00	0.01	0.15	0.00
Belize	0.15	0.00	0.07	0.17	0.00
Nicaragua	0.18	0.00	0.31	0.16	0.00
Anguilla	0.16	0.00	0.01	0.16	0.00
Morocco	0.24	0.03	0.01	0.24	0.00
Myanmar	0.03	0.01	0.15	0.17	0.00
Comoros, Union of the	0.16	0.00	0.03	0.12	0.00
Sierra Leone	0.12	0.00	0.14	0.07	0.00
Albania	0.21	0.01	0.07	0.25	0.00
Costa Rica	0.14	0.00	0.01	0.20	0.00
Colombia	0.13	0.01	0.01	0.08	0.00
Gambia, The	0.11	0.00	0.06	0.10	0.00
Canada	0.00	0.00	0.04	0.04	0.00
Zambia	0.09	0.00	0.11	0.07	0.00
Honduras	0.22	0.00	0.06	0.20	0.00
SÃ£o TomÃ© and PrÃncipe, Dem. Rep. of	0.00	0.00	0.00	0.00	0.00
Cameroon	0.09	0.00	0.02	0.09	0.00
Central African Rep.	0.11	0.00	0.13	0.13	0.00
Chad	0.07	0.01	0.05	0.08	0.00
Congo, Rep. of	0.23	0.00	0.05	0.15	0.00
Czech Rep.	0.31	0.00	0.00	0.28	0.00
Equatorial Guinea, Rep. of	0.14	0.00	0.02	0.08	0.00
Eswatini, Kingdom of	0.14	0.00	0.01	0.04	0.00
Gabon	0.10	0.00	0.04	0.09	0.00
Lesotho, Kingdom of	0.40	0.00	0.02	0.06	0.00