Intermediate Macroeconomics

The IS Curve

ECON 3311 – Spring 2025

UT Dallas

Short-run model: Output vs. Inflation (AS-AD)

Aggregate Demand part: IS → Investment/Savings

Introduction

based on the negative relationship between the Investment & interest rates

In this chapter we will go over:

- The foundation of the short-run model which is the **IS curve**
  - The IS curve depicts the **inverse relationship** between the **real interest rate** and **short-run output**
- How **shocks** to consumption, investment, government purchases, or net exports—'aggregate demand shocks'—can **shift the IS curve**.
- The life-cycle/permanent-income hypothesis, which describes consumption behavior
- How **investment is the key channel** through which changes in real interest rates affect GDP in the short run

→ Shocks that affect aggregate demand ⟶ They shift the IS curve: higher/lower Investment & Output for the same interest rate.

Fed → FFR → Short term nominal interest rate
(Policy rate)

Introduction

↑ Int. rate → ↓ Investment → ↓ Output
(↓ Demand in general)

The Federal Reserve influences the level of economic activity in the short run.

- The Fed **targets the federal funds rate** – interest rate at which banks can borrow from each other
- This rate is **highly correlated with the short-term nominal interest rate** at which people borrow and lend in financial markets

$$\uparrow \text{interest rate} \Rightarrow \downarrow \text{investment} \Rightarrow \downarrow \text{output}$$

The importance of this rate is: An **increase in the interest rate** leads to a **decrease in investment**, which leads to **decrease in output**, and vice versa

IS curve: Illustrates the negative relationship between interest rates and short-run output

IS goes for "Investment-Savings"

IS Curve: int. rate vs. output plot

Downward sloping

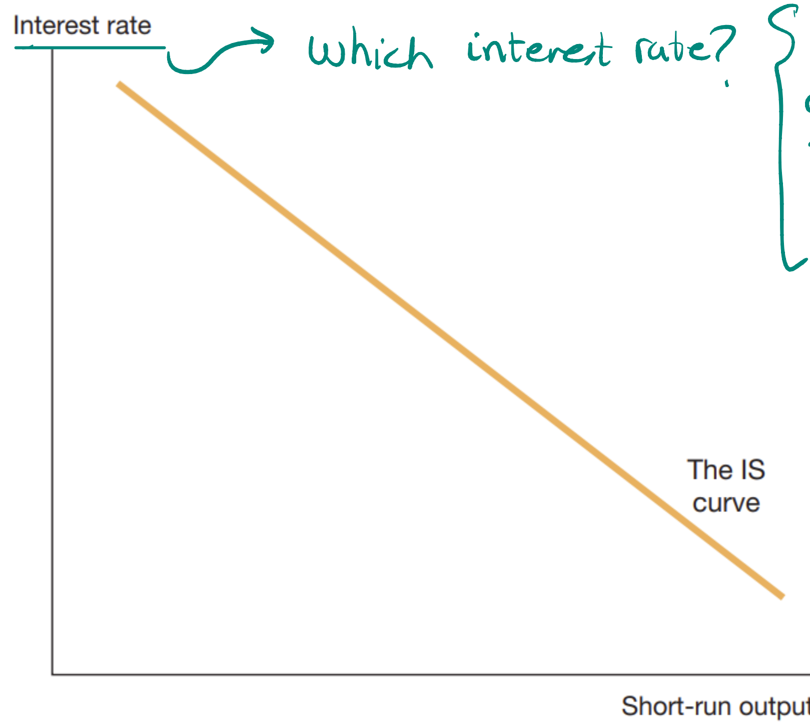
The IS curve

Why downward sloping?

↑ Int. rate → ↑ Cost of lending ⇒ firms don't borrow $ to invest.

The IS curve is graphed with the interest rate on the vertical axis and short-run output on the horizontal axis

Introducing the IS Curve



which interest rate? { Real Rate
also nominal given Prices don't change by much in the short-run

Downward sloping:

A higher interest rate leads to lower output, operating through investment (I)

Mechanism: Higher cost of borrowing leads firms to get fewer funds from banks for Investing, and households to cut their consumption (e.g., from credit cards spending).

All of this lowers GDP (remember the national accounting formula!)

Solving for the IS curve

1.) Depart from GDP definition

2.) Assume $C, G, EX, IM \propto \bar{Y}$ → Proportional

3.) Assume $I \propto \bar{Y}$ but also that I lowers with the interest rate

To solve for the IS curve, we begin with the national income identity:

$$Y_t = C_t + I_t + G_t + EX_t - IM_t$$

We will assume the following:

- Consumption is a fraction of potential output:
- Government purchases behave similarly:
- Exports are a fraction of potential
- Imports are a fraction of potential output:

$$\boxed{\begin{aligned} C_t &= \bar{a}_c \bar{Y}_t \\ G_t &= \bar{a}_g \bar{Y}_t \\ EX_t &= \bar{a}_{ex} \bar{Y}_t \\ IM_t &= \bar{a}_{im} \bar{Y}_t \end{aligned}}$$

→ assumptions

Then the equation becomes:

Replace these

$$Y_t = \bar{a}_c \bar{Y}_t + I_t + \bar{a}_g \bar{Y}_t + \bar{a}_{ex} \bar{Y}_t - \bar{a}_{im} \bar{Y}_t$$

4.) Divide by $\bar{Y}_t$ & solve for $I_t/\bar{Y}_t$

Assumption: $I_t = \bar{a}_i \bar{Y}_t - \bar{b}(R_t - \bar{r})\bar{Y}_t$

Solving for the IS curve

For investment, let's assume it's given by a fraction of the potential output **plus** an amount that depends on the gap between Real interest rate and the MPK, then:

rearrange

$$\frac{I_t}{\bar{Y}_t} = \bar{a}_i - \bar{b}(\underbrace{R_t}_{\text{Real interest rate}} - \overbrace{\bar{r}}^{\text{MPK}}).$$

$\bar{a}_i$ =Share of investment from potential output

$\bar{b}$ = Parameter weighting difference between the real interest rate and MPK

R_t = Real interest rate: Rate at which firms can save or borrow

$\bar{r}$ = Marginal product of capital (MPK): How much a firm can make from investing in one more unit of capital

Investment share of potential output: Similar to what we saw with C, G, EX, and IM

Extra term: If extra output from physical investment is low, then a firm can instead put the money in the bank and get R_t, but otherwise: Extra incentives to Invest.

Intuition: If it is much better to ↑ Capital than to buy a financial asset ($R_t < \bar{r}$)
$\Rightarrow$ firms want to invest more

Alternative yet similar view: R_t as Cost of Debt → Compare: Cost of Debt vs. Gain from Debt*

The Marginal Product of Capital (MPK)

*from taking on debt to ↑ Investment

We can see that a key component of the investment equation is the relationship between R_t (real interest rate) and r_t (marginal product of capital)

- If the MPK is lower than the cost of borrowing, then firms should save more
- If the MPK is higher than the cost of borrowing then firms should borrow and buy capital (e.g. expand their factories)

MPK > Cost of Debt (R_t) ⇒ ↑ I

For example, suppose R = 5 and r = MPK = 10, and the firm borrows 1000 units of capital:

MPK = 0.1, R = 0.05

To pay back: (0.05*$1000)=50

⇒ Can take a $1000 loan

Can be obtained from Investing: (0.1*$1000)=100

Invest & make $1100,

Yielding a potential profit of $50

Pay back (−1000) & pay interest (−50) ⇒ Profits = 50

Short run versus long run

In the short-run the Interest Rate is NOT Necessarily equal to MPK

$(R_t - \bar{r}) \neq 0$ in the short-run

Note that in this example, the marginal product of capital is **not equal** to the real interest rate like it was in the long-run model

The idea behind a possible discrepancy between the two is that it takes time for the marginal product of capital to adjust, while a change in the real interest rate is instantaneous

- As **more capital is invested, the MPK (*r*) will decrease** and if less capital is invested then MPK (*r*) will increase
- In the long-run: We are back to the real interest rate equaling the marginal product of capital and then the investment is just a share a_i of potential output

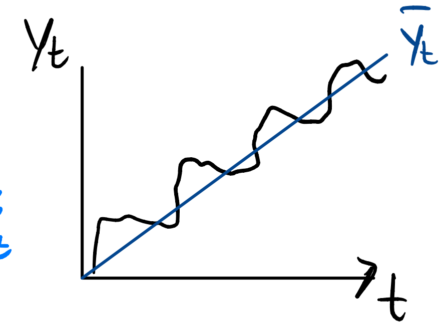
→ Adjustment of MPK towards R → It happens but takes time

How the real interest rate is determined will be the focus later

For now: Notice we are using the principle of **decreasing marginal productivity**

Deriving the IS curve

1.) GDP definition
2.) Replace assumptions
3.) Divide by $\bar{Y}_t$
4.) Group terms & replace $\tilde{Y}_t$



We start with the national income identity:

$\rightarrow \bar{a}_i \bar{Y}_t - \bar{b}(R_t - \bar{r})\bar{Y}_t$

$$Y_t = C_t + I_t + G_t + EX_t - IM_t$$

$\hookrightarrow \bar{a}_c \bar{Y}_t$

We can divide by $\bar{Y}_t$ and then substitute in the earlier equations for *Ct, It, Gt, EXt,* and *IMt* :

$$\left(\frac{1}{\bar{Y}_t}\right): \quad \frac{Y_t}{\bar{Y}_t} = \bar{a}_c + \bar{a}_i - \bar{b}(R_t - \bar{r}) + \bar{a}_g + \bar{a}_{ex} - \bar{a}_{im}$$

To get from this equation to short-run output, subtract 1 from both sides:

$$\frac{Y_t}{Y_{t-1}} - 1 \neq$$

GDP growth
(Not the same as $\searrow$)

$$\underbrace{\frac{Y_t}{\bar{Y}_t} - 1}_{\tilde{Y}_t} = \underbrace{\bar{a}_c + \bar{a}_i + \bar{a}_g + \bar{a}_{ex} - \bar{a}_{im} - 1}_{\bar{a}} - \bar{b}(R_t - \bar{r})$$

$\bar{a} \hookrightarrow$ gather all constant terms into a single term ($\bar{a}$)

Recall: $\tilde{Y}_t = \frac{Y_t - \bar{Y}_t}{\bar{Y}_t} = \frac{Y_t}{\bar{Y}_t} - 1$

IS: $\tilde{Y}_t = \bar{a} - \bar{b}(R_t - \bar{r})$

Deriving the IS curve

In IS:

$\tilde{Y}$ changes if

- $(R_t - \bar{r})$ changes
- $\bar{a}$ changes

↳ Shocks: can affect $\bar{a} \Rightarrow$ IS shifts

After some simplification we are left with:

+ +

$$\tilde{Y}_t = \bar{a} + \bar{b}(R_t - \cancel{\bar{b}}^{\,\bar{r}})$$

We can see that there are two factors that determine fluctuations:

1) Gap between the real interest rate (R) and the marginal product of capital (r)
2) Shocks to the economy which are represented by changes in $\bar{a}$

Using the IS curve

IS: $\tilde{Y}_t = \bar{a} - \bar{b}(R_t - \bar{r})$

Usually $\bar{a}=0$ (baseline), but this can change if a shock affects $\bar{a}$

($\bar{a}$ ~ aggregate demand shock)

Given the IS curve is plotted with short-run output and the real interest rate on the two axes, it has a negative slope.

In LR: $\tilde{Y}_t = \frac{\bar{Y}_t}{\bar{Y}_t} - 1 = 0$; $R_t = \bar{r}$

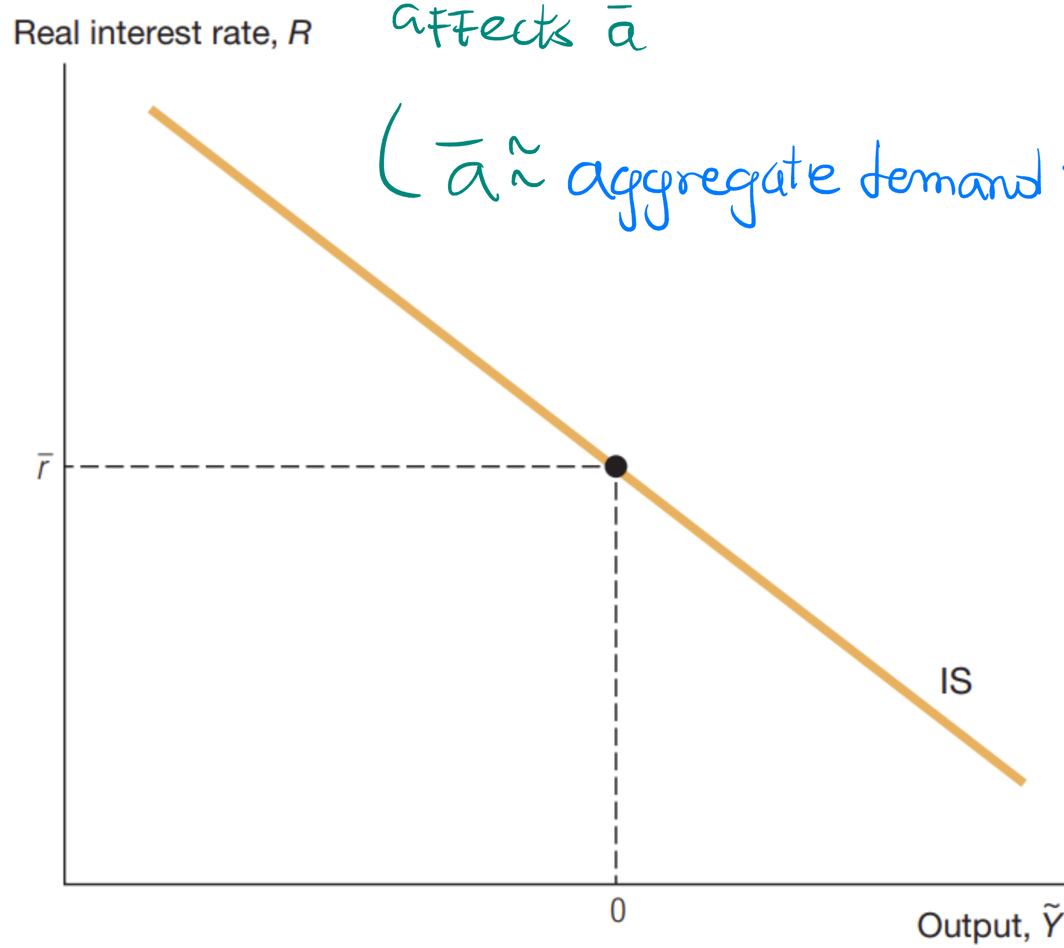
Note that $\bar{a}$ = 0 $\Rightarrow 0 = \bar{a} - \bar{b}\cdot 0$ (IS)

You can see this by considering the equation in the long-run when $Y_t = \bar{Y}_t$

$\bar{a}$ can also be thought as an **aggregate demand shock** and by **default we assume it's zero**

Therefore: $\tilde{Y}_t = 0$ when $\bar{r} = R$ (baseline case)

The IS Curve



Key here and in general: If something included on the axes changes we move along the curve. In contrast, we shift the curve if what changes is not on the axes.

Using the IS curve

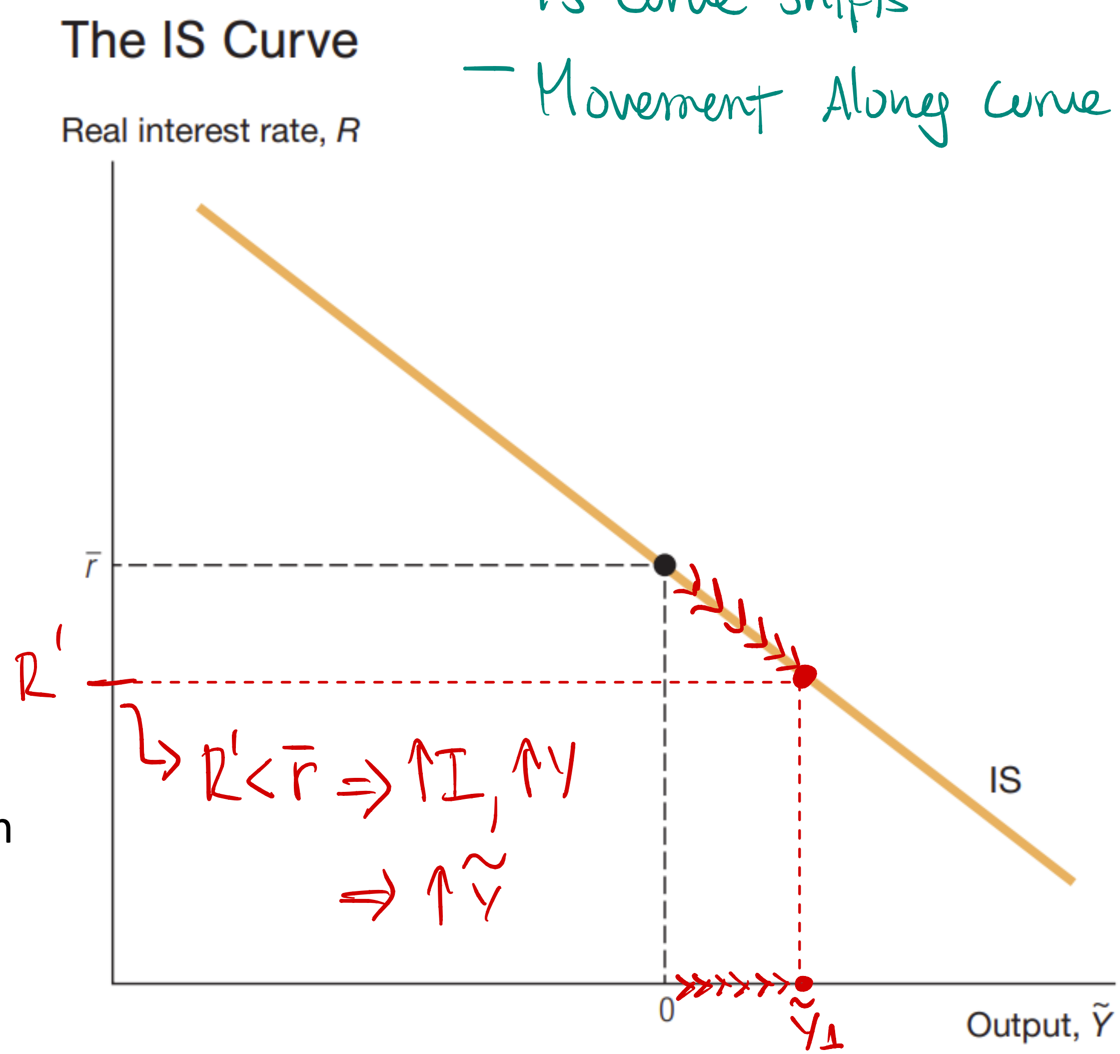
If R changes
- ~~IS curve shifts~~
- Movement Along curve ✓

What can cause the economy to move from the point where $\tilde{Y}_t = 0$?

One is a change in the real interest rate (R) so that it is no longer at its long-run trend of $\bar{r} = R$

This will cause a **movement along the IS curve**

$\downarrow R$ to R'



How much depends on the change in R and $\bar{b}$

Using the IS curve

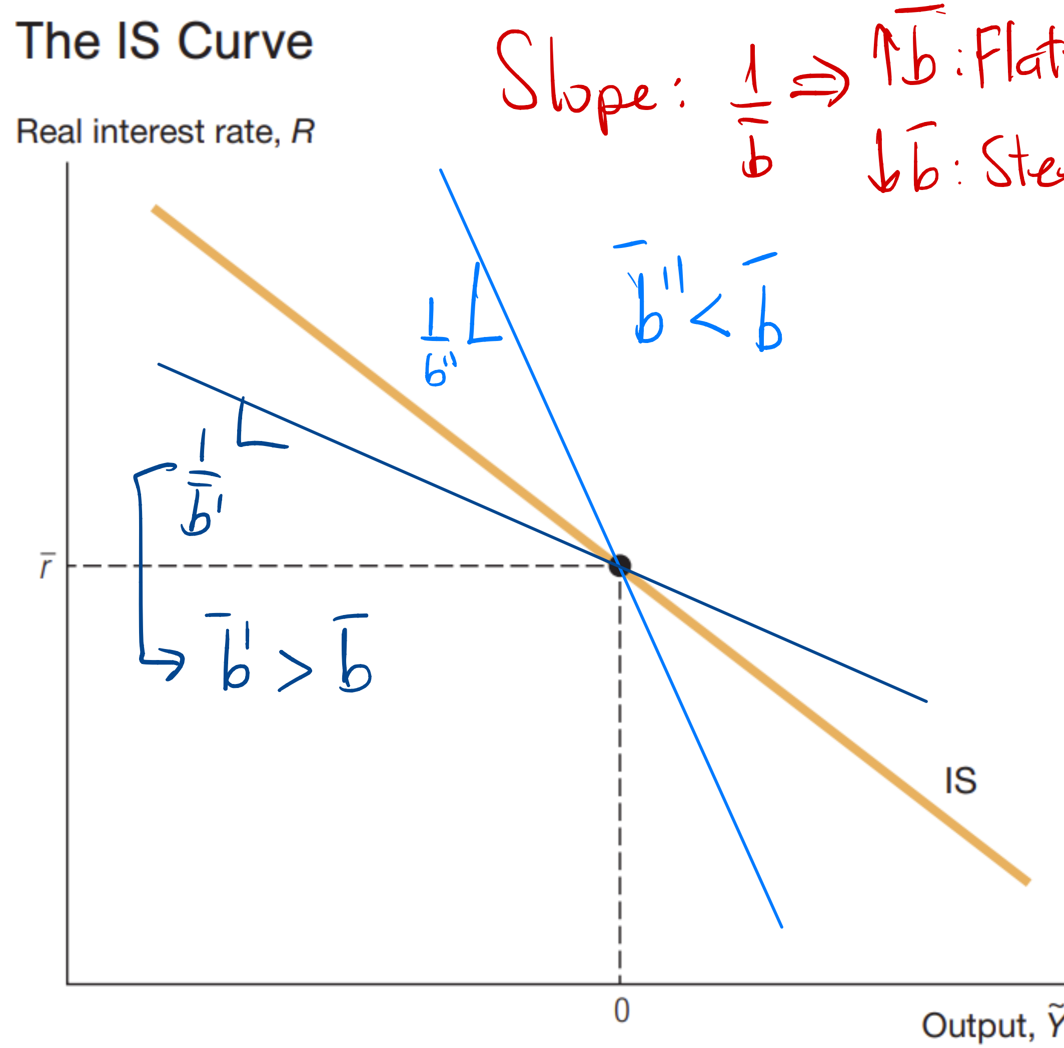
Changes in $\bar{b}$: Make the curve Steeper / Flatter

Slope: $\frac{1}{\bar{b}} \Rightarrow$ $\uparrow \bar{b}$: Flatter, $\downarrow \bar{b}$: Steeper

The slope of the IS curve:

How does $\bar{b}$ affect the IS curve?

If there is an increase in $\bar{b}$ then the IS curve becomes **flatter** and each change in the interest rate will associate to a larger change in short-run output



This is easier to see if we convert the equation of the IS curve to slope-intercept format:

IS:

$$\tilde{Y}_t = \bar{a} - \bar{b}(R_t - \bar{r}) \quad \rightarrow \quad \underbrace{R_t = \frac{\bar{a}}{\bar{b}} + \bar{r} - \boxed{\frac{1}{\bar{b}}}\tilde{Y}_t}$$

what we really plot

(Slope: $\frac{1}{\bar{b}}$)

Shocks: Change $\bar{a}$: $\uparrow \bar{a}$: $\uparrow$ higher $\tilde{Y}$ for any R $\Rightarrow$ IS Shifts up
$\downarrow \bar{a} \rightarrow$ IS Shifts down

Using the IS curve

What about the macroeconomic shocks we've mentioned before

These are represented by changes in $\bar{a}$

Examples:

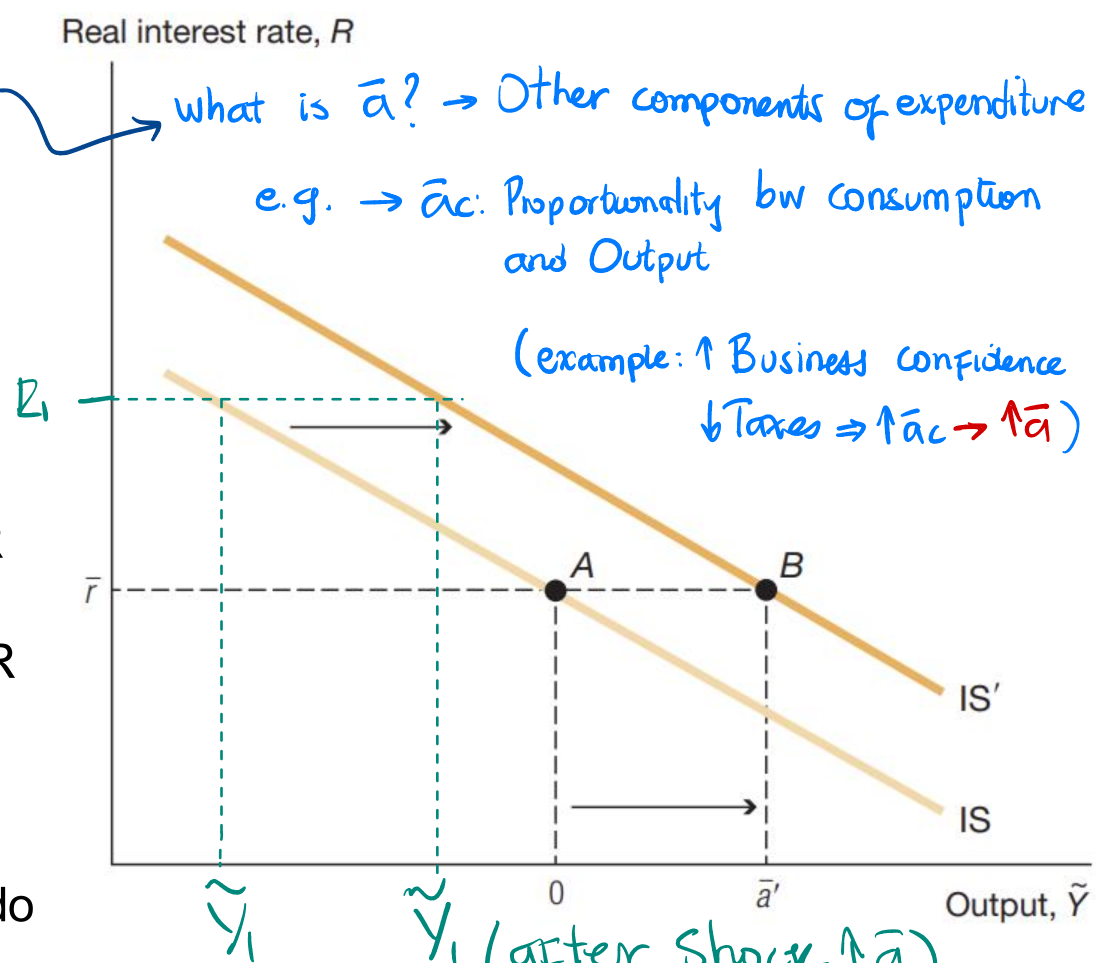
Changes in consumer confidence, business optimism, etc. can **shift** the IS curve

Positive shock: Higher Output for any R

Negative shock: Lower Output for any R

Again, these are relevant changes in Variables that are **not on the axes** but do affect the economy (and output)

An Aggregate Demand Shock



what is $\bar{a}$? → Other components of expenditure

e.g. → $\bar{a}_c$: Proportionality b/w consumption and Output

(example: ↑ Business confidence ↓ Taxes ⇒ $\uparrow \bar{a}_c \rightarrow \uparrow \bar{a}$)

R_1 $\tilde{Y}_1$ $\tilde{Y}_1$ (after Shock $\uparrow \bar{a}$)

In a nutshell: Anything relevant for GDP that is NOT on the axes can shift the IS

Using the IS curve

If a shock changes $\bar{Y}$: Nothing would change in the IS plot

What would happen if potential output changes?

Short answer: Nothing changes (this has some nuances)

$\uparrow Y_t \qquad\qquad \uparrow \bar{Y}_t$

Output = Potential Output + Cycle

$$\tilde{Y}_t = \frac{\uparrow Y_t}{\uparrow \bar{Y}_t} - 1$$

First, note that potential output does not enter the IS equation

$$\tilde{Y}_t = \frac{1.1\, Y_t}{1.1\, \bar{Y}_t} - 1$$

But, actually, it is in there through the definition of $\tilde{Y}_t = \frac{Y_t - \bar{Y}_t}{\bar{Y}_t} = \frac{Y_t}{\bar{Y}_t} - 1$

However, **a shock affecting $\bar{Y}_t$ is likely to affect Y_t in the same fashion**.

And in the definition of $\tilde{Y}_t$ **these effects would offset**: $\frac{\uparrow Y_t}{\uparrow \bar{Y}_t} = \tilde{Y}_t - 1$

For instance, an increase in technology would increase potential output, but also actual short run output

Of course, there could be a change affecting more/less $\bar{Y}_t$ than Y_t which would change $\tilde{Y}_t$ but for simplicity we abstract from those cases

But if it would happen the IS would not change: We would move along the curve

If confused, check the IS equation (plotted one): $R = \frac{\bar{a}}{\bar{b}} + r - \frac{1}{\bar{b}} \tilde{Y}_t$

Microfoundations of the components of the IS curve

Consumption $C_t = \bar{a}_c \bar{Y}_t$

People consume in proportion to their average income

What determines the different levels of C, I, G, EX, and IM?

There are two theories that seek to explain consumption (C) behavior:

1) Permanent-income hypothesis → People won't drastically change consumption patterns in presence of isolated income shocks
2) Life-cycle model of consumption

Examples: – Finding \$100 only once
– Sudden car repair

Key idea here: Consumers **smooth their consumption** (this follows from the idea of diminishing marginal utility)

Permanent-income hypothesis: Consumers base consumption decisions on average income over time and not current income

Are this ideas consistent? Yes! If you get some surprise income (let's say 5000) you will likely not increase your typical consumption by that much next month.

Microfoundations of the components of the IS curve

rather eat 1-2 eggs a day than a dozen but only on monday

Life-cycle model of consumption: People base their consumption based on lifetime average income.

This also follows the same idea of **consumption smoothing**: People won't change drastically their habits according to their age

(even if income is much lower at the beginning and end of life)

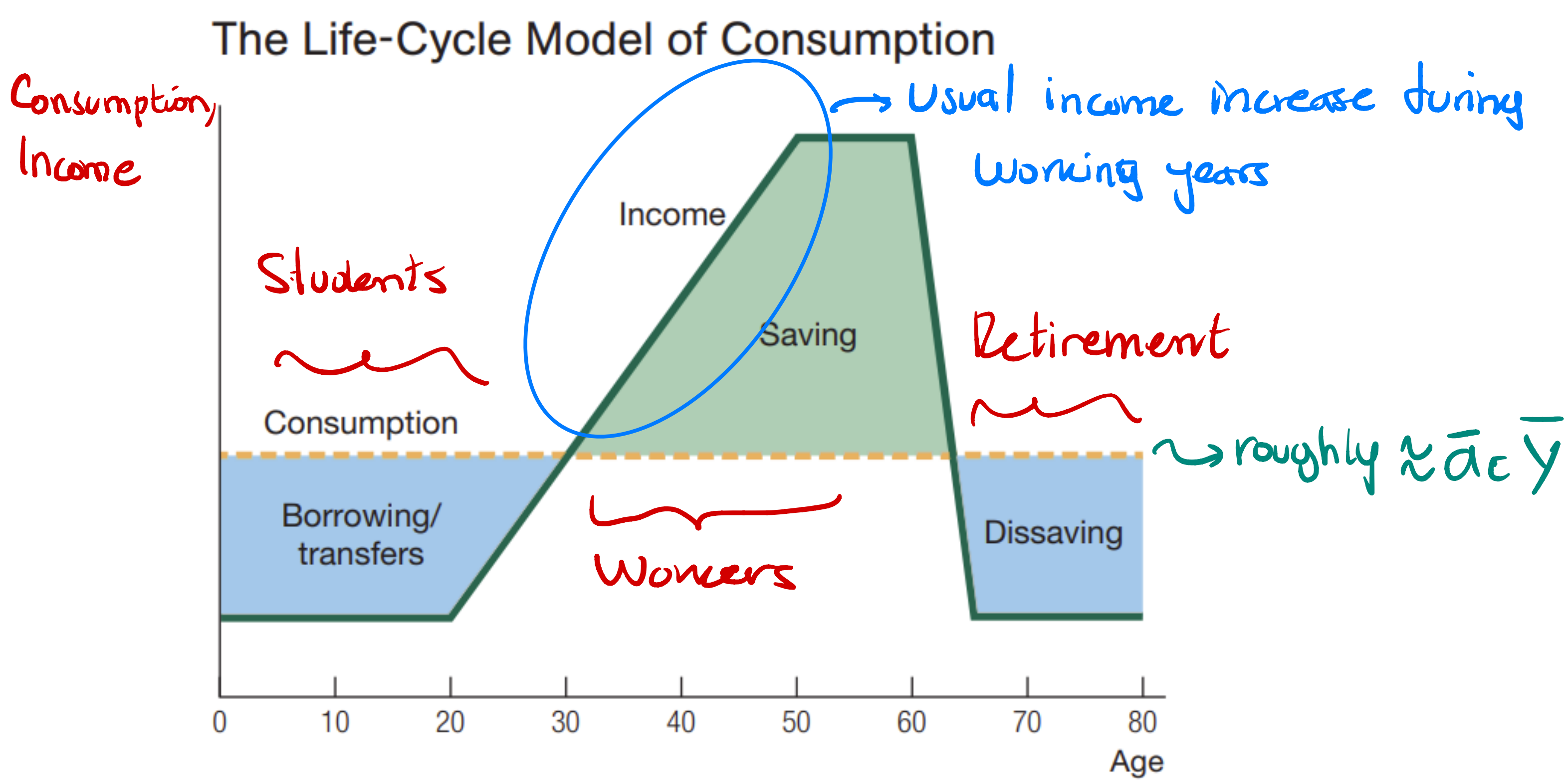
- For most students consumption is higher than income – they take out loans or get support.
- During working years, earnings are likely higher than spending —saving for retirement.
- When one retires, their income falls drastically, but consumption's decrease is smooth.

Similar Idea: Life Cycle

- Students take on loans $C > Y$
- Workers save $C < Y$
- Elders no longer work $C > Y$

Microfoundations of the components of the IS curve

This changes in consumption by age are usually depicted graphically:



Microfoundations of the components of the IS curve

(an alternative formulation with current income is:

$C_t = \bar{a}_c Y_t$)

The **LC/PI hypotheses** is the reason we can write: $C_t = \bar{a}_c \bar{Y}_t$

Consumption is based on potential output and not actual output because potential output is smoother and thus represents better the average income

This theory does not always hold up in practice:

Examining the Alaska Permanent Fund (Hsieh, 2003), it was found that consumers smooth their permanent fund payments, but not their tax refunds

This can be explained by the permanent fund being more predictable, while people are not always clear on what their tax refund/payment is going to be

Not a perfect theory though: Exceptions have been documented

Sometimes C can react to (short-run) income changes

(Changes that would not affect $\bar{Y}$)

Extension to allow C_t to change with $\tilde{Y}$ too

Multiplier Effects

If we expect consumption to also react to changes in actual output (like in the tax refund finding), then it can be written as:

$C_t = \bar{a}_c \bar{Y}_t + \bar{x} \bar{Y}_t \tilde{Y}_t$

$$\frac{C_t}{\bar{Y}_t} = \bar{a}_c + \underbrace{\bar{x}}_{\approx \text{ MPC: Marginal Propensity to Consume}}\tilde{Y}_t$$

If there is an increase in GDP/income then consumption will increase

The IS curve will change to:

Derivation of IS works as before but now yields:

$$\tilde{Y}_t = \underbrace{\boxed{\frac{1}{1-\bar{x}}}}_{\text{Multiplier}} \overbrace{(\bar{a} - \bar{b}(R_t - \bar{r}))}^{\text{RHS of original IS}}$$

$\Rightarrow \bar{x}$ amplifies the effects captured in the IS curve

with $\bar{x} = 0.9 \Rightarrow \frac{1}{1-0.9} = 10$

A 'multiplier' has been added to the equation

For example, if $\bar{a}$ increases by 2% and $\bar{x}$ is equal to 1/2, then output will rise by 4% - the change in $\bar{a}$ is multiplied by a number that is greater than 1

Multiplier Effects

Intuition: Change in Int. Rates → Changes: Investment, (Cons) Output

Multiplier that leads to further output changes

This changes Income & Expenditure

- Suppose that a change in the interest rate increases investment by 1%
- This extra investment leads to more construction jobs which leads to higher income to construction workers.
- This **extra income is going to increase spending** (we assume that $\bar{x} > 0$ (elsewhere, increasing the demand for workers in those areas
- As more workers are hired, income and spending increase
- The original change in investment is being 'multiplied'

amplification of initial effect

Some noteworthy details here:

The multiplier won't change the "form" of the IS curve: It just implies an adjustment to the intercept and slope.

This captures the notion that economic shocks may have larger than proportional effects.

This does not contradict the idea that consumption is smooth or depending mainly on permanent income ($\bar{x}$ may still be small)

With Multiplier $\bar{x} \neq 0$
- IS retains the same shape
- Consumption is still relatively smooth (even if less than before)

Investment

I depends on – Trade-off between R & MPK
• Cost of Debt vs. Gain from Debt
• or where to put $ { – Real Production / – Financial Sector }

The equation for investment that we derived earlier was:

Real rate

$$\frac{I_t}{\bar{Y}_t} = \bar{a}_i - \bar{b}(R_t - \bar{r})$$

MPK

There are two main determinants of investment:

1) Gap between the real interest rate (R_t) and the Marginal Product of Capital ($\bar{r}$

2) Cash flow – this refers to how much resources the firm has: A firm with more resources can find cheaper borrowing or may just be able to invest without borrowing.

The equation above can be rewritten in the following way:

rewrite equation above: $I_t = \bar{a}_i\bar{Y}_t - \bar{b}(R_t - \bar{r})\bar{Y}_t$ → Cashflows

Cash flow can be seen as being directly related to $\bar{Y}_t$

It grows with income and that facilitates investing regardless of cost of funding conditions.

Intuition for $G = \bar{a}_g \bar{Y}$ → The higher the average income the higher G can be

- Direct Spending: G
- Taxes / Subsidies

Government Purchases

The effect of **government purchases** on short-run output can be viewed in a couple of ways:

1) A sharp change in government spending can directly cause a fluctuation (example: defense spending)
2) Government spending can be used as a policy to reduce fluctuations (example: relief packages)

A change in government spending appears as a change in $\bar{a}_g$

↳ $\uparrow \bar{a}_g \Rightarrow \uparrow G \rightarrow \uparrow$ aggregate Demand $\Rightarrow$ IS shifts

Another important instrument: **Taxes**

These are transfers (payments, sometimes subsidies) imposed by the government that **can push up/down expenditure** by all agents (consumption, investment)

If set as rates proportional to the expenditure they function as **automatic stabilizers** by mitigating fluctuations of GDP automatically

$\uparrow G \rightarrow$ boosts Y

$\uparrow$ Taxes $\rightarrow \downarrow Y$

Net Exports (NX)

$NX_t = EX_t - IM_t$
$= \bar{a}_{ex} \bar{Y}_t - \bar{a}_{im} \bar{Y}_t$

The expressions for exports and imports are respectively $\bar{a}_{ex}\bar{Y}_t$ and $\bar{a}_{im}\bar{Y}_t$

Net exports (NX) are equal to exports minus imports

- If **net exports** are positive there is a **trade surplus**
- If **net exports** are negative there is a **trade deficit**

$NX > 0$ surplus $\uparrow Y$
$NX < 0$ deficit $\downarrow Y$

Changing exports/imports and the IS curve:

When exports increase relative to imports then the IS curve shifts out
When exports decrease relative to imports then the IS curve shifts in

(this follows the same principle mentioned before: Are NX or its parts included on the axes? No. Then they act as shifters —when changing.)

If a shock $\uparrow$ NX (or $\uparrow$ EX): IS shifts to the right $\rightarrow$ higher $\tilde{Y}$ for any level of R
$\downarrow$ NX (or $\uparrow$ IM): " " " Left $\rightarrow$ lower $\tilde{Y}$ " " " " "

Conclusion

IS : Relation between Y (via I) and the Interest rates in the short run.

The IS curve depicts how changes in the real interest rate affect economic activity in the short run

Higher interest rates lead both firms and households to reduce their investment spending (business investment and housing investment)

Mechanism: Cost of Debt

Changes in other variables act as **aggregate demand shocks** which can lead to **shifts in the IS curve**

→ Other variables: Aggregate Demand (Shift IS)

Through the **multiplier effect**, the change in investment can then lead to a larger change in short-run output

IS curve: Mainly about the relationship between **Investment/Savings and the cost of borrowing** but it also embeds changes in other parts of GDP

Next: The role of policy and how the central bank controls the interest rate that plays such a crucial role in the IS curve

Review Questions

What would we expect to happen to the economy in each of the following cases? Use the factors we used to derive the IS curve in your answer

a) There is a deep recession in Europe ($\downarrow$ Exports, $\downarrow \bar{a}_{ex}$)

b) Housing prices rise above their trend ($\uparrow$ Increase regardless of $R-r$: $\uparrow \bar{a}_i$)*

c) Mortgages lenders raise interest rates ($\uparrow R$)

d) The government decides to close 20 percent of their domestic military bases ($\downarrow G$)

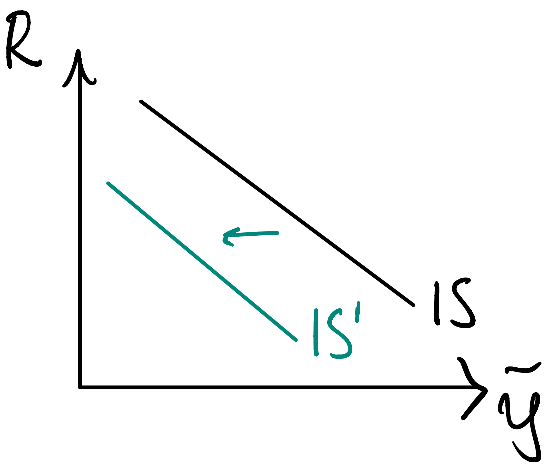
($\downarrow \bar{a}_g$)

* also $\uparrow$ wealth $\Rightarrow \uparrow \bar{a}_c$

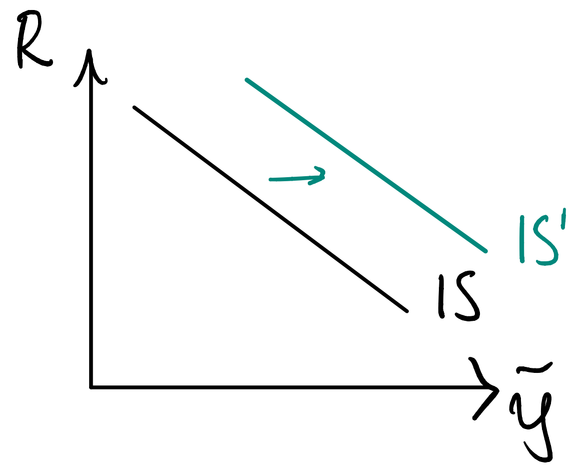
($\uparrow$ Consumption)

$$IS: \quad R_t = \frac{\bar{a}}{\bar{b}} + \bar{r} - \frac{1}{\bar{b}}\tilde{y}_t \quad \text{or} \quad \tilde{Y}_t = \bar{a} - \bar{b}(R_t - \bar{r})$$

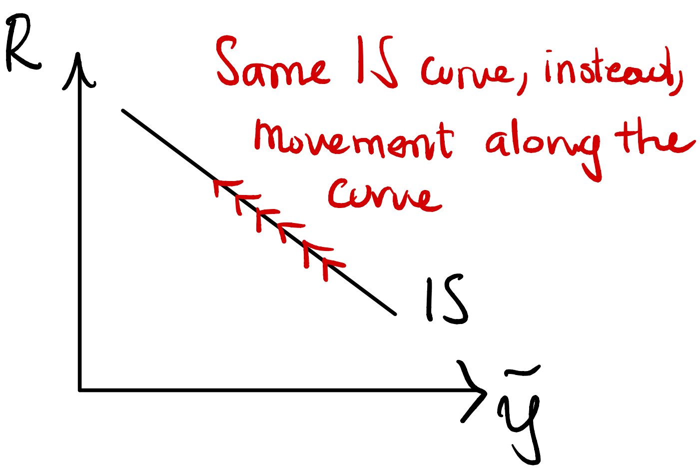
a-) ($\downarrow$ Exports, $\downarrow \bar{a}_{ex}$)



b-) ($\uparrow$ Increase regardless of $R - r$: $\uparrow \bar{a}_i$)*



c-) ($\uparrow R$)



d-) ($\downarrow G$) ($\downarrow \bar{a}_g$)