

## International Finance 4832

### Lecture 6: Long-run budget constraint and Gains from Financial Globalization

Camilo Granados  
The University of Texas at Dallas  
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# Outline

Before: the BOP (FT Chapter 16)

1. (Accounting) Measure of International Transactions
  - ▶ Balanced by construction but its accounts (CA, FA, KA) can be unbalanced
2. Unbalance trade implies borrowing/lending with ROW
3. BOP-FA: *Flow* → (is linked with) NFA /Wealth: Stock

Now: (Chapter 17, part 1) Long run budget constraint:

how much borrowing and lending is possible and sustainable (over time)

Next: (more of Chapter 17) Benefits from Financial Globalization (international borrowing/lending)

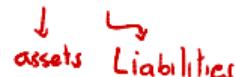
- ▶ Consumption smoothing → lowers volatility of Consumption
- ▶ Efficient investment → Borrow to Invest when good opportunities arise
- ▶ Risk diversification

## External Wealth (recap)

Country's external wealth: Assets minus Debt (liabilities)

$$W = A - L$$

Earn interest on  $A$  and pay interest on  $L$



$W > 0$ : net creditor (lender) to ROW

$W < 0$ : net debtor (borrower) to ROW

Changes in external Wealth:

$$W_t - W_{t-1} = CA_t + KA_t + \text{valuation effects}_t$$



Wealth is a stock, we measure it at the end of a period (once flows have been observed)

## Simple long run budget constraint

Some simplifying assumptions

1. Flexible Prices
2. Small Open Economy: Home country cannot influence world prices (for goods and services)
3. Constant world (real) interest rate:  $r^*$
4. All debt carries an interest rate  $r^*$ : home is paid  $r^*$  on assets and pays  $r^*$  on liabilities
5. No unilateral transfers ( $NUT = 0$ ), no capital transfers ( $KA = 0$ ), no valuation effects
6. No expatriate workers, then  $NFIA = r^* \underline{W_{t-1}}$  (factor income is capital income, the interest on wealth position)

$$r_{\text{Assets}} = r_{\text{Liabilities}} = r^*$$

## Change in wealth with simplifying assumptions

Change in wealth:

$$W_t - W_{t-1} = CA_t + KA_t - \text{valuation effects}_t$$

$\stackrel{=0}{=} \quad \stackrel{=0}{=}$

(assumption 5) No unilateral transfers, no capital transfers, no valuation effects:

$$\begin{aligned} W_t - W_{t-1} &= CA_t \\ W_t - W_{t-1} &= TB_t + NFI_A_t \quad (\text{NFI}_A = 0) \end{aligned}$$

(assumption 3 & 4) Constant interest rate, the same for assets and liabilities

(assumption 6) no labor factor income  $\Rightarrow NFI_A = r^* A_{t-1} - r^* L_{t-1}$

$$\underbrace{W_t - W_{t-1}}_{\text{change in external wealth in } t} = \underbrace{TB_t}_{\text{trade balance in } t} + \underbrace{r^* W_{t-1}}_{\text{interest paid/received on last period debt}}$$

(NFI<sub>A</sub>)

$$W_t = TB_t + r^* W_{t-1} + W_{t-1}$$

$$W_t = TB_t + (1+r^*) W_{t-1}$$

## Wealth dynamics over time

Given these assumptions we can get the wealth stock over time:

$$W_t = TB_t + (1 + r^*)W_{t-1}$$

Wealth in period t is equal to previous wealth plus the trade balance plus the new interest payments

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Intuition: New wealth is coming from ...

- ▶ Addition to wealth from net exports (selling more than buying from abroad → builds up savings)
- ▶ Interest income (or payments) on previous wealth (debt) stock

These extra resources are used to acquire assets (or pay debt)

## Example: two period world

Start in year 0, finish at the end of year 1 ("the World ends then")

There is some initial wealth from the previous period ( $W_{-1}$ )

Wealth at the end of year 1 must be zero → if positive must be spent, if negative (debt) must be paid

$$\begin{aligned} \cancel{W_0} &= TB_0 + (1 + r^*) W_{-1} \\ \circledast W_1 &= TB_1 + (1 + r^*) W_0 \end{aligned}$$

Since  $W_1 = 0$

$$-\frac{TB_1}{1 + r^*} = W_0 \quad \text{PV: Present Value}$$

Substitute into period zero wealth equation:

$$\begin{aligned} -\frac{TB_1}{1 + r^*} &= TB_0 + (1 + r^*) W_{-1} \quad (\text{here: PV at time 0}) \\ \underbrace{TB_0 + \frac{TB_1}{1 + r^*}}_{\text{PV of present and future trade balances}} &= \underbrace{-(1 + r^*) W_{-1}}_{\text{minus PV of previous wealth}} \end{aligned}$$

(PV: Present value)

## Example: two period world (cont.)

$$\underbrace{-S + \frac{TB_1}{1+r^*}}_{\text{PV of present and future trade balances}} = \underbrace{-(1+r^*)W_{-1} + 10}_{\text{minus PV of previous wealth}}$$

Intuition: (present) Value of trade balances are compensated by the country's assets (wealth)

Then a country with positive wealth can afford to run trade deficits for some periods

or ...an indebted country cannot spend more than its income from abroad

## Example: two period world (cont.)

Two-period budget constraint:

$$\underbrace{TB_0 + \frac{TB_1}{1+r^*}}_{\text{PV of present and future trade balances}} = \underbrace{-(1+r^*)W_{-1}}_{\text{minus PV of previous wealth}}$$

$W_{-1} < 0$  (debtor): average trade balance must be positive

$W_{-1} > 0$  (creditor): average trade balance must be negative

Thus, TB surpluses compensate the debt of a country ...

or, a country with (positive) assets can afford to run TB deficits

Note: the budget constraint is in *present-value* form:

- ▶  $(1+r^*)W_{-1}$  is the value of your debt at the end of period zero  
*↳ debt held in -1:  $W_{-1}$*
- ▶  $TB_0$  is the trade balance in period 0
- ▶  $\frac{TB_1}{1+r^*}$  is the period-1 trade balance in period-0 value (i.e. present value during period 0)  
*↳ PV of  $TB_1$  in Period 0*

## Detour: Present Value

Present Value (PV): Current value of cash flows that can be observed over time

It assumes that:

- Money can be invested at any point at a real interest rate (for simplicity let's assume it constant:  $r$ )
- Money also loses value due to inflation → the real rate already accounts for this

Examples:

- today's value of \$10 you found in your pocket: \$10
- Today's value of \$10 you were paid a year ago:  $(1 + r)\$10$
- Today's value of \$10 you were paid three years ago:  $(1 + r)(1 + r)(1 + r)\$10 = (1 + r)^3\$10$
- Today's value of \$10 you will be paid in one year:  $\frac{\$10}{(1+r)}$

*↳ Future Cash Flow*

Intuition for the last one:

how much money to set aside such that in 1 year you end up with exactly \$10 (10 future dollars)

## Detour: Present Value (cont.)

Present Value of a perpetual flow:

Assume we get  $x$  every period starting in one year ... what is the present value of these cashflows?

The present value is the sum of the present value of each future flow:

$$PV = \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \frac{x}{(1+r)^4} + \frac{x}{(1+r)^5} + \dots$$

Lets multiply the whole equation by  $1+r$ :

$$(1+r)PV = \frac{(1+r)x}{(1+r)} + \frac{(1+r)x}{(1+r)^2} + \frac{(1+r)x}{(1+r)^3} + \frac{(1+r)x}{(1+r)^4} + \frac{(1+r)x}{(1+r)^5} + \dots$$

$$(1+r)PV = x \left[ 1 + \frac{1}{(1+r)} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} + \frac{1}{(1+r)^4} + \dots \right]$$

First equation minus the previous one:

$$PV - (1+r)PV = -x$$

$$PV = \frac{x}{r}$$

Running a TB = 5M forever

(w/  $r=0.1$ ) yields a PV of 55:

Then the present value of running a trade balance of 5M today and onwards is:

$$PV = 5 + \left( \frac{5}{r} \right)$$

$$PV = 55 = 5 + \left( \frac{5}{0.1} \right) = 55$$

## Example: two period world (cont.)

Two-period budget constraint:

$$\begin{array}{c} 110 \\ \uparrow \quad \uparrow \\ TB_0 + \frac{TB_1}{1+r^*} = -(1+r^*)W_{-1} \\ \text{PV of present and future trade balances} \qquad \text{minus PV of previous wealth} \end{array}$$

or:  $0 + \frac{121}{1.1} = 110$

Putting in some numbers:

$W_{-1} = -\$100$  (Debt stock) and  $r^* = 0.1$  (10%) ... what TBs are feasible?

- ▶  $TB_0 = \$110$  and  $TB_1 = 0$
- ▶  $TB_0 = 0$  and  $TB_1 = \$121$
- ▶  $TB_0 = -5\$$  and  $TB_1 = \$126.5$
- ▶ ... or any other combination such that  $LHS = RHS = 110$  in the budget constraint above

the larger the TB deficit in one period, the larger  
the TB surplus (in at least one other period) later

## The Long Run Budget Constraint

Extending this analysis to a model with many periods:

$$\underbrace{-(1 + r^*) W_{-1}}_{\text{Minus PV of wealth from last period}} = \underbrace{TB_0 + \frac{TB_1}{(1 + r^*)} + \frac{TB_2}{(1 + r^*)^2} + \frac{TB_3}{(1 + r^*)^3} + \dots}_{\text{PV of current and future trade balances}}$$

Debt Sustainability:

LRBC is important → is a condition countries should meet in order to avoid exploding levels of debt  
(i.e., get more and more indebted over time in a non-sustainable way)

## LRBC and NIPAs

Using that  $GDP = GNE + TB$

$$-(1+r^*)W_{-1} = TB_0 + \frac{TB_1}{(1+r^*)} + \frac{TB_2}{(1+r^*)^2} + \frac{TB_3}{(1+r^*)^3} + \dots$$
$$-(1+r^*)W_{-1} = (GDP_0 - GNE_0) + \frac{GDP_1 - GNE_1}{(1+r^*)} + \frac{GDP_2 - GNE_2}{(1+r^*)^2} + \frac{GDP_3 - GNE_3}{(1+r^*)^3} + \dots$$

Rearranging terms:

$$GNE_0 + \frac{GNE_1}{1+r^*} + \frac{GNE_2}{(1+r^*)^2} + \dots = (1+r^*)W_{-1} + GDP_0 + \frac{GDP_1}{1+r^*} + \frac{GDP_2}{(1+r^*)^2} + \dots$$

or

$$\text{PV of spending} = \text{PV of resources}$$

*↳ Consumption, Investment, G*  
*↳ Wealth, Production, Income*

where "resources" refer to your pre-existing wealth plus what you produce.

Takeaway: The present value of your spending cannot exceed the present value of your resources ...

That is, you can spend more than what you have here and there but not all the time

## Open Economy vs. Closed Economy

Closed Economy:  $TB = 0$  at all times. Budget must balance every period

$$\underline{GNE = GDP}$$

Open Economy:  $TB \neq 0$  can run deficits/surpluses at any point. Budget must balance in the long run

$$GNE_0 + \frac{GNE_1}{1+r^*} + \frac{GNE_2}{(1+r^*)^2} + \dots = (1+r^*)W_{-1} + GDP_0 + \frac{GDP_1}{1+r^*} + \frac{GDP_2}{(1+r^*)^2} + \dots$$

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Key here: The budget condition for closed economies is much stricter.

An open economy is less restricted in the use of its resources ... making openness beneficial  
(more on these benefits soon)



## LRBC and rates of return in the US

Assumption 4:  $r_A = r_L = r^*$

This is NOT true for the United States:

The US borrows cheap and lends at higher rates ( $r_A - r_L \approx 0.015$ )

We see this in the data:  $W < 0$  but  $\overbrace{r_A A - r_L L}^{\text{capital factor income}} > 0$

$\rightarrow$  adds to NFIA

- ▶ A net debtor, but earns positive interest income
  - US invests high risk, high return assets
- ▶ Largely due to low return on foreign direct investment in US
  - or the fact that other countries buy "safe" US assets such as bonds (that have lower interest rates)



## LRBC and valuation effects in the US

Assumption 5: no valuation effects

This is not true for the United States:

prices of US external assets have increased faster than prices of external liabilities ( $\approx 2\%$  per year)

Not obvious why this is the case

Maybe the US is better at picking investments, maybe not ... it may be due to statistical errors too

Still, asset prices may go either way and become unfavorable for the US, but so far it's been favorable

In fact, some recent studies have pointed that this trend is likely changing:

The end of privilege: re-examination of the NFA of the US [[link](#)]

*"the US NFA has deteriorated in the post-crisis period, and the decline is mostly driven by valuation effects"*

*"the market value of US foreign liabilities has risen more quickly than the market value of US foreign assets"*

Another summary of this study by the NBER: [[NBER Digest link](#)]

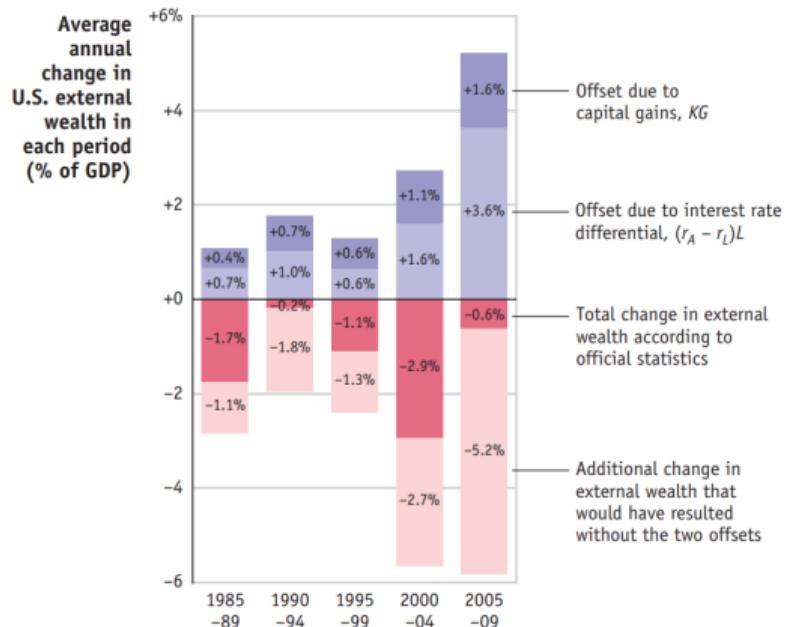
# The US LRBC

Different rates ( $r_A > r_L$ ) and valuation effects may partially offset the effect of trade deficits on wealth

$$W_t - W_{t-1} = TB_t + (r_A A - r_L L) + \text{valuation effects}$$

Up to the financial crisis this made somehow more sustainable the US Trade Balance deficit approach:

Figure: USA NFA 1989-2009 (source: BEA)



## Should the LRBC hold?

LRBC: today's debt stock must be balanced by future trade surpluses → payments to the ROW

The larger the debt, the larger the future surpluses (and lower GNE)

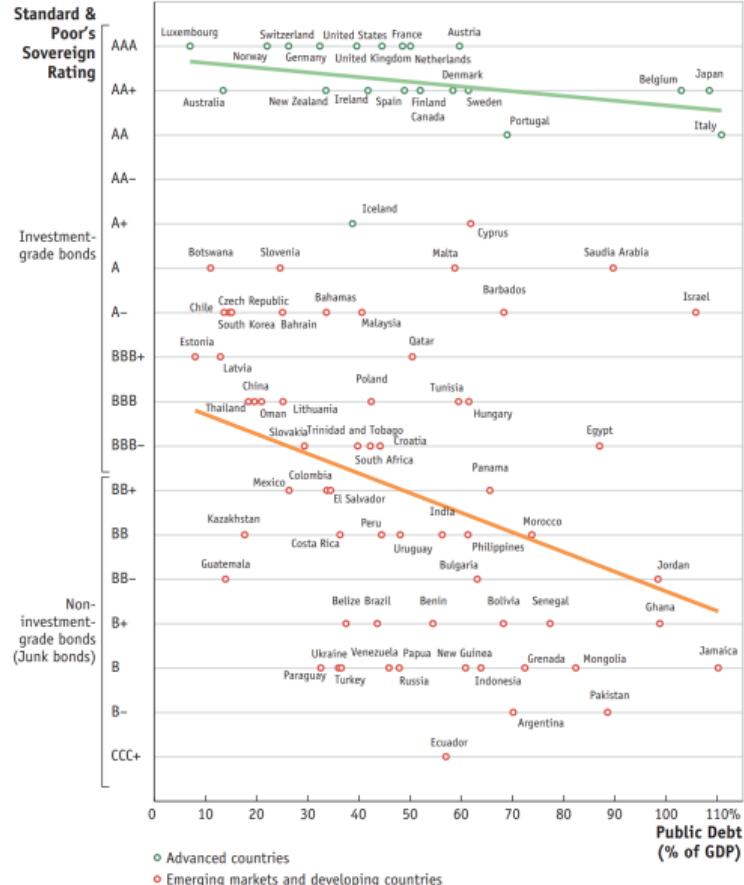
Agents in the country declare bankruptcy and not make payments ... but ...

Reputational cost: this can affect government debt ratings

- ▶ larger debt levels → have to pay higher interest rates for future borrowing
- ▶ If reputation is low the risk is higher (for a lender) pushing debt rates up even more

Relation between unsustainable levels of debt and costs of funding → rationale for a binding LRBC

# Government Debt Ratings vs. Debt Level



In general:

high rating is associated to a higher debt

AE: true ("ish")

EMEs and Low Income: true and quite strongly  
(much steeper trend)

## Benefits of Financial Globalization

## Outline

Before:

1. BOP: measure of external transaction flows → **flow** (CA: flow of resources toward savings)
2. Net Foreign Assets (external wealth) —→ **stock** (assets and liabilities with ROW)
3. Unbalance trade (or CA surplus/deficit) means borrowing or lending from the ROW
4. Long Run Budget Constraint: Debt and TB/CA offset over time (in PV, not necessarily in each period)

Now: (Chapter 17, part 2) The Gains from Financial Globalization (international borrowing/lending)

- ▶ Consumption smoothing
- ▶ Efficient investment
- ▶ Risk diversification

## Open Economy vs. Closed Economy (recap)

Closed Economy:  $TB = 0$  at all times. Budget must balance every period

$$GNE = GDP$$

Open Economy:  $TB \neq 0$  can run deficits/surpluses at any point. Budget must balance in the long run

$$GNE_0 + \frac{GNE_1}{1+r^*} + \frac{GNE_2}{(1+r^*)^2} + \dots = (1+r^*)W_{-1} + GDP_0 + \frac{GDP_1}{1+r^*} + \frac{GDP_2}{(1+r^*)^2} + \dots$$

Condition for Open Economy is less restrictive

Closed economy must lower expenditure during a recession

Instead, Open economy can maintain expenditure ( $C, I$ ) → can soften the blow of a recession

Pretty much like how a household can rely on a banking loan when needed

## Gains from Intertemporal Trade

*Key Idea:* (open economy) can make up for deficits with some periods of surpluses later

In a Trade Class you learn about the "Gains from Trade"

- ▶ Comparative advantage, Hecksher-Ohlin, higher variety of goods
- ▶ These gains are present regardless of whether trade is balanced (usually we assume it is)

Now, with potentially unbalanced trade ( $\text{Exports} \neq \text{Imports}$ )

- ▶ Trading over time: intertemporal trade
- ▶ Linked with international borrowing and lending (financial globalization)

Gains from Intertemporal Trade (or from being able to save/borrow with ROW)

- ▶ Consumption smoothing → can lower volatility of consumption
- ▶ Efficient investment → can invest whenever better opportunities become available
- ▶ Risk diversification → can hedge country-specific risk by leaning on ROW savings

## Consumption Smoothing

Simplifying assumptions:

1. Identical households → can consider single a representative household in lieu of all consumers
2. The household wants to smooth consumption → dislike volatility
3. Consumption ( $C$ ) is the only source of demand ( $G = 0, I = 0$ )
4. Zero initial wealth →  $W_{-1} = 0$
5. Small open economy: country's variables don't affect world real interest rate  $r^*$  (assumed constant)

Two period world, the LRBC is:

$$C_0 + \frac{C_1}{1+r^*} = Q_0 + \frac{Q_1}{1+r^*}$$

Then,  $GNE = C$  and  $Q = GDP$

## Consumption Smoothing (cont.)

Rewrite the LRBC:

$$C_0 + \frac{C_1}{1+r^*} = Q_0 + \frac{Q_1}{1+r^*}$$
$$C_1 - Q_1 = (1+r^*)(Q_0 - C_0)$$

If  $Q_0 - C_0 < 0$ :

Consume *more* than output in period 0  $\Rightarrow$  Consume *less* than output in period 1

The opposite is true if  $Q_0 - C_0 > 0$

Same lesson as before: If there's a deficit in some period it has to be compensated with a surplus later

Now, how would a household like set its consumption schedule? i.e.,  $C_0$  and  $C_1$ ?

## Consumption Smoothing (cont.)

For picking  $C_0$ ,  $C_1$  optimally we need to model the intertemporal utility of consumption

All we know is that households dislike consumption volatility (like to have a stable consumption profile)

We can consider any function whose optimal choice respects that feature, for example:

$$U(C_0, C_1) = \min\{C_0, C_1\}$$

The solution (optimal choice) is:  $C_0 = C_1$

If  $Q_0 = Q_1$  then just set:  $C_0 = Q_0$  and  $C_1 = Q_1$  ...that is, consume what produced

But what about times when  $Q_0 \neq Q_1$ ?

- ▶ Recessions/booms (output is generally not the same over time)
- ▶ War/peace times
- ▶ Natural disasters
- ▶ Other shocks

Where we are going here:

Intl. lending allows to make up for these output differences so that a stable consumption is achieved

## Consumption Smoothing (cont.)

Numerical example:  $Q_0 = 100$ ,  $Q_1 = 105$ , and  $r = 0.05$

Closed economy:  $C_0 = 100$ ,  $C_1 = 105$  and  $U = \min\{100, 105\} = 100$  (consumption is not smooth)

Open Economy:

$$C_0 + \frac{C_1}{1.05} = 100 + \frac{105}{1.05} = 200$$

We want  $C_0 = C_1$ , thus substitute  $C = C_0 = C_1$ :

$$C \left(1 + \frac{1}{1.05}\right) = 200$$

Solving for  $C$ :

$C = 102.44$ ,  $U = \min\{102.44, 102.44\} = 102.44 > 100$  (better off relative to closed economy!)

What's going on?

In a Closed Economy  $GNE_t = GDP_t$  which is more restrictive than an Open Economy where  
PV of GNE flows = PV of GDP flows  $\Rightarrow$  Open Econ is more capable of achieving a smooth consumption

## Consumption Smoothing (cont.)

Household is better off in the open economy → achieves a smoother consumption path

BOP accounting:

$$TB_0 = Q_0 - C_0 = 100 - 102.44 = -2.44 \quad (\text{runs a TB deficit})$$

$$CA_0 = -2.44 + 0 = -2.44 \quad (NFIA = 0)$$

$$FA_0 = 2.44 \quad (\text{export an asset: the IOU for the loan})$$

Notice:  $TB = Q - C$  is another (consistent) way to look at the trade balance. It only says that any extra output (not consumed) is exported

Thus: Borrow 2.44 in period 0, pay back with interest in period 1

$$TB_1 = Q_1 - C_1 = 105 - 102.44 = 2.56 \quad (\text{runs a TB surplus})$$

$$CA_1 = 2.44 - 0.1215 = -2.44 \quad (NFIA = -2.44 \times 0.05 = -0.1215)$$

$$FA_1 = -2.44 \quad (\text{asset is imported back home})$$

## Consumption Smoothing (cont.)

The previous example can be generalized to many periods

Example: Output is 79 in period 0, then 100 forever

t:	0	1	2	...	Present Value
Q	79	100	100	...	2709
C	99	99	99	...	2709
TB	-20	1	1	...	0
CA	-20	0	0	...	
NFIA	0	-1	-1	...	

They debt (principle) is not really paid, instead interest payments on it are made forever

Closed econ household:  $C_0 = 79$  then  $C_1 = 100$  any other period

Open Economy household:  $C = 99$  always (better off)

## Consumption Smoothing (cont.)

Less developed countries worry about access to international borrowing

Harder for these to access international borrowing during recessions

They build a buffer stock of foreign assets ( $W >> 0$ ) to spend during recessions, rather than borrow it

These savings take two forms:

1. Central bank foreign reserves (reserve assets: cash or relatively liquid assets like bonds or SDRs)
2. Sovereign wealth funds (buy assets in other countries)

Still, these economies are not displaying as much consumption smoothing as expected

This, in part, is explained by their reluctance to use these reserve assets in harder times.

More info if you're interested (optional linked readings):

- What are FX Reserves and can they help combat the global economic crisis? (WEF)
- Why countries stockpile foreign cash

## Gains from Financial Globalization

Financial Globalization implies (allows) intertemporal trade:

1. Consumption Smoothing
2. Efficient Investing
3. Risk Diversification

## Efficient Investment

Add investment to our previous model ( $GNE = C + I$ )

Now labor and capital generate the output (before it was only labor)

$$C_0 + I_0 + \frac{C_1}{1+r^*} = Q_0 + \frac{Q_1}{1+r^*}$$

No need to invest in projects in the last project if the world is ending ( $I_1 = 0$ )

The more it's invested the higher the GDP, but you trade off Consumption:

With  $I_0 = 0$ :  $Q_0 = Q_1 = 100$

With  $I_0 = 5$ :  $Q_0 = 100$ ,  $Q_1 = 100$

Great business opportunity: Invest 5, get 10 → hard to miss on that only to keep a smooth consumption

In a Closed Economy, either:

- ▶  $I_0 = 0 \Rightarrow C_0 = 100$  and  $C_1 = 100$
- ▶  $I_0 = 5 \Rightarrow C_0 = 95$ , and  $C_1 = 110$

Making the investment adds large volatility to consumption

(investing like this makes a heavy toll on a single period consumption, such impact cannot be smoothed over time in this case)

## Efficient Investment (cont.)

In an Open Economy:

$$C_0 + I_0 + \frac{C_1}{1+r^*} = Q_0 + \frac{Q_1}{1+r^*}$$

$$C_0 + 5 + \frac{C_1}{1.05} = 100 + \frac{110}{1.05}$$

As before, set  $C_0 = C_1 = C$ :

$$C \left(1 + \frac{1}{1.05}\right) = 95 + \frac{110}{1.05}$$

Solution:  $C_0 = C_1 = C = 102.32$

$$TB_0 = Q_0 - C_0 - I_0 = 100 - 102.32 - 5 = -7.32$$

Run a deficit by 7.32 and borrow the amount to invest & more

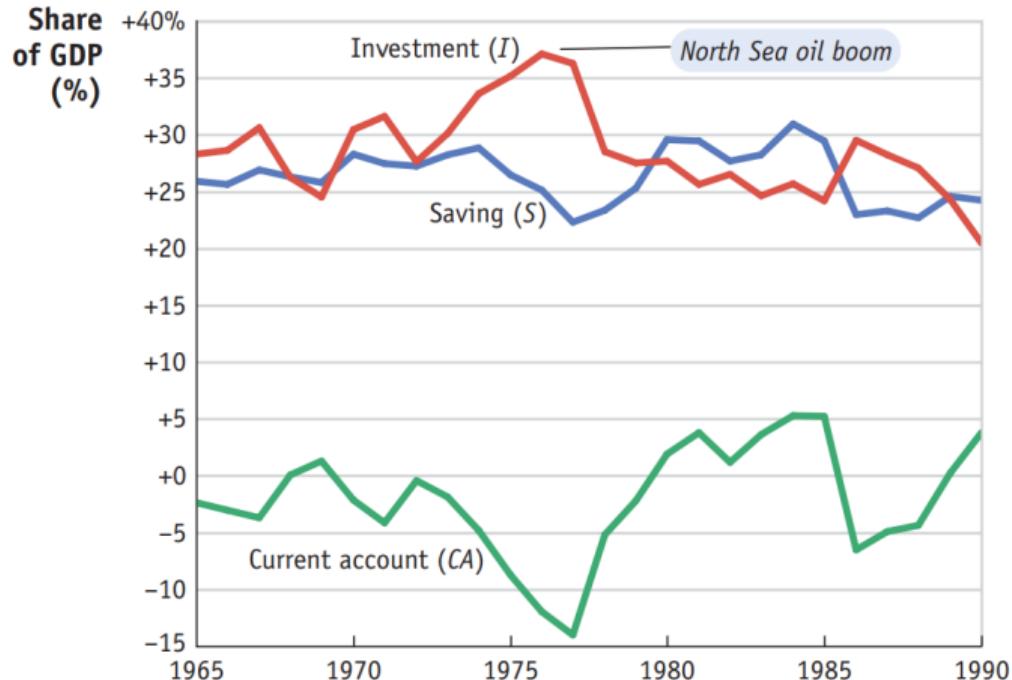
Outcome: Consume a bit more today, anticipating the proceeds from tomorrow (better result than in closed economy)

Open Economy can Invest when an opportunity arises without jeopardizing the rest of the economy  
(or at least softening the impact of doing such investments)

Now, imagine the economy could not have covered the investment at all (even if willing) → Int. Lending may help

# The Oil Boom of Norway

Figure: Investment, Savings and Current Account in Norway



Running CA deficits allowed Norway to make large investments even if its savings level did not allow it

## Gains from Financial Globalization

Financial Globalization implies (allows) intertemporal trade:

1. Consumption Smoothing
2. Efficient Investing
  - ▶ Smoothing cost of investment over time with lending
  - ▶ Move capital across countries (seeking higher returns)
3. Risk Diversification

## Efficient Investment (cont.)

Before: International lending allows to smooth costs of investment

Similar to the consumption smoothing benefit

Now: Investing abroad → allows to move capitals across countries → help equalize returns across locations

This is a long run idea ... sounds familiar? → yes: like the real interest parity from part 1 (exchange rates)

We needed flexible prices and capital mobility then which we assume here too.

## Optimal Capital Investment

Production function: technology for transforming inputs into output

$$Q = AL^{1-\theta}K^\theta$$

Where  $A$ : productivity,  $K$ : Capital,  $L$ : Labor

$\theta$ : Participation of  $K$  in output production ( $\approx 1/3$ )

In per-worker terms (divide everything by  $L$ ):

$$q = Ak^\theta$$

With  $k$ : capital per worker,  $q$  : output per worker

How much capital to choose to maximize output?

$$\max_k Ak^\theta - rk$$

$r$ : rental rate of capital

The first order condition is:

$$[k] : \quad \theta Ak^{\theta-1} = r \tag{1}$$

In the optimum: choose a level of  $k$  such that (1) holds

(FOC: derivative wrt  $k = 0$ )

## The marginal product of capital (MPK)

The first order condition says that  $MPK = r$

$$\theta A k^{\theta-1} = r$$

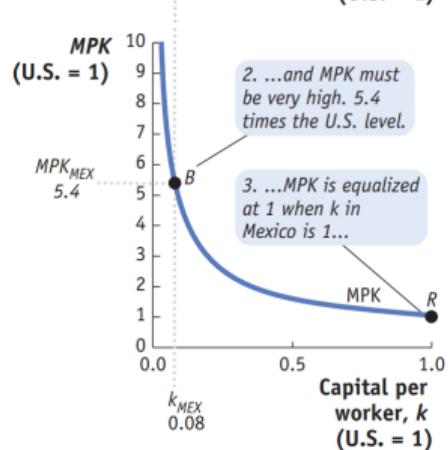
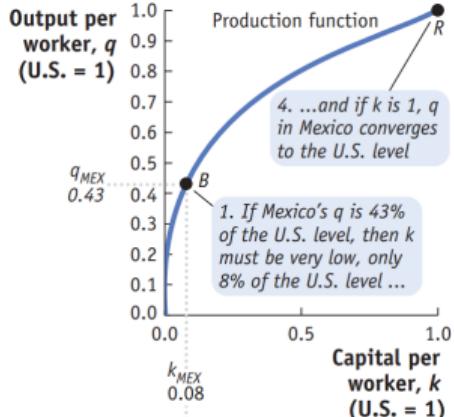
The marginal product of capital is the additional output you can generate with an extra unit of capital  
 $r$ : marginal cost of capital → if not investing in capital, could be lending resources to someone

$MPK$  is falling as  $k$  grows →  $(MPK = \frac{\theta A}{k^{1-\theta}})$

- ▶ diminishing returns to capital      (extra output created by one more unit of input is lower as we add more of the input)
- ▶ When  $k$  is small,  $MPK$  is high; when  $k$  is large,  $MPK$  is low

$A = 1$  and  $\theta = 1/3$  ... let's see what  $q$  and  $MPK$  look like

## Output function (top) and MPK (bottom)



The higher the capital the more output

But notice the slope, it's decreasing as  $k$  increases

The *MPK* (blow) is just the slope  $\frac{\partial q}{\partial k}$

We see how a country with more capital has higher production but lower MPK

(lower marginal returns for each extra unit of input)

For example:

MX has lower capital per worker and GDP per capita than the US, but investing new capital there would be more profitable ...

... 5.4 times as profitable as in the US

## Output function (top) and MPK (bottom)

We can draw it and check the slope:

## MPK in rich and poor countries

Two countries: US and Mexico

Assumption:  $A$  and  $\theta$  are the same in both countries

$$k^{us} = 1, k^{mx} = 0.08$$

$$q^{us} = 1, q^{mx} = 0.43$$

Mexico is poor relative to the US because it does not have enough factories, machinery, etc ...

But as we saw, Investing in Mexico should be a great opportunity

(given how lesser capital they have ... new capital should be very productive ...)

$$MPK^{us} = 0.333, MPK^{mx} = 1.79 \text{ that is } 5.4 \text{ times higher } (\frac{MPK^{us}}{MPK^{mx}} = 5.4)$$

Due to this: Capital should flow out of the US and to Mexico

Capital should flow out of rich countries and into poor countries (capital flows seeking higher returns)

Eventually all countries converge to the same level of  $k$  and  $r$  equalizes across countries

**The implication:** poor countries will catch up with rich countries

This transition is socially desirable and could even be sped up with foreign aid and donations

## The Lucas Paradox

We don't observe capital flowing out of rich countries and into poor ones (It even happens the opposite)

From Lucas (1990): "...If world capital markets were anywhere close to being free and complete, it is clear that, in the face of return differentials of this magnitude investment would flow to poor countries ...the assumptions on technology and trade conditions must be drastically wrong, what assumptions should we replace then?"

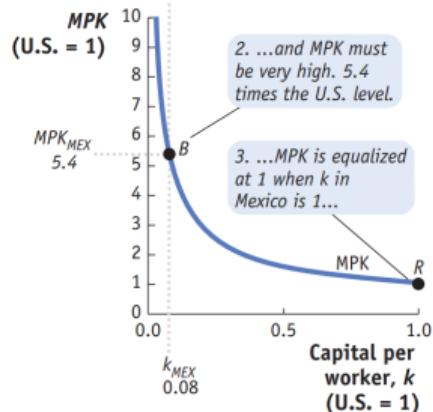
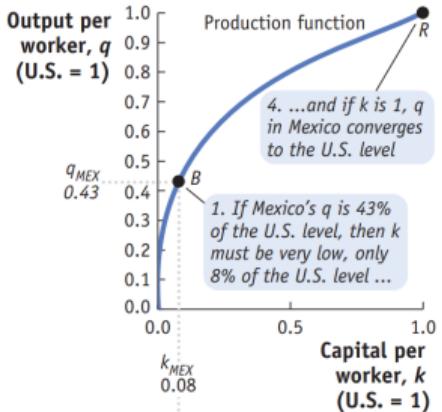
Wrong assumption: Identical productivity levels Same  $A$  in each economy

Suppose  $A^{mx} = 0.63$  and  $A^{us} = 1$  Same inputs combination is 63% as productive in Mexico as in the US

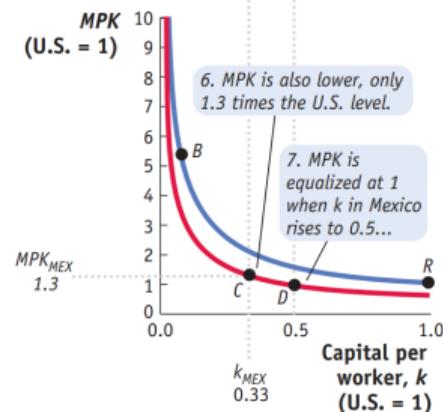
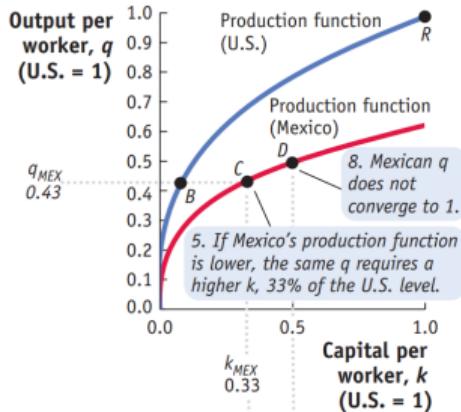
Then Mexico needs  $k^{mx} = 0.33$  to have the same output as before (and not 0.08 as with  $A = 1$ )

# Does Capital Flow to Poor Countries?

(a) Identical Production Functions in Rich and Poor Countries



(b) Different Production Functions in Rich and Poor Countries



## Technology in rich and poor countries

Assume:

$$k^{us} = 1, k^{mx} = 0.33, q^{us} = 1, q^{mx} = 0.43$$

Same technology:  $q^{us} = A^{us} k^{us}$ ,  $q^{mx} = A^{mx} k^{mx}$

But different productivities:  $A^{us} \neq A^{mx}$

Mexico is poor relative to the US because it does not have enough capital ...

...but also because it cannot produce as much output per unit of capital

In this case, the *MPK* difference we gauged before falls drastically:

$$MPK^{us} = 0.333, MPK^{mx} = 0.44, \text{ then } \frac{MPK^{us}}{MPK^{mx}} = 1.33 \text{ (way lower than the 5x time difference before)}$$

The returns are not much different anymore

This partly explains why:

- ▶ We don't see drastic capital flows to poor economies
- ▶ We don't see convergence

## What is A

A: Total Factor Productivity (TFP)      How more/less productive a given amount of factors are in generating output

It is an unobserved "residual" —→ if you know  $K$ ,  $L$ ,  $Q$  you can compute  $A$  (i.e.,  $A = Q/(L^{1-\alpha} K^\alpha)$ )

It denotes technological efficiency

- ▶ Do poor countries use worse technology? To some extent
- ▶ But not thought of as the big difference across countries

It also reflects the ability to implement technologies in general:

- ▶ Institutional quality: How good is the government, how much red tape, legal protection for investors
- ▶ the World Bank Doing Business index is inspired by this idea [\[Wikipedia link\]](#)

# Why doesn't capital flow to poor countries?

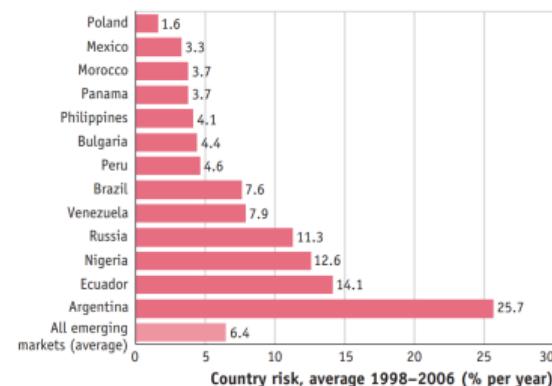
The rate of return differential is much lower than what theory predicts

In example: returns of Mexico are not 5 times those of the US but only 1.3 times

Reasons (some):

- ▶ TFP differences across countries (e.g.,  $A^{us} > A^{mx}$ )
- ▶ Risk Premiums: Poor countries default more  $\implies$  have higher cost of debt

Figure: Risk premia in Emerging Markets



Source: EMBI indices from cbonds.info

Partial solution: Foreign Aid (not a strong evidence that it's helpful)

## Gains from Financial Globalization

Financial Globalization implies (allows) intertemporal trade:

1. Consumption Smoothing
  - ▶ Smoothing cost of investment over time with lending
  - ▶ Move capital across countries (seeking higher returns)
2. Efficient Investing
3. Risk Diversification

## Risk Diversification

Business cycles are driven by shocks to income → income goes up and down (cyclically around a trend)

Problem: households would like to smooth consumption

One way to smooth consumption: Debt (as explained before)

Another way to smooth consumption: by smoothing income → hold equity in other economies

During bad times (recessions) maybe you can get higher profits from countries that are going through good times

Business cycles are not perfectly synchronized across countries → this allows for diversification of (income) risk

The more out of sync the business cycles (between countries) are → the more room for risk-sharing

## Risk Diversification (cont.)

Simplifying assumptions:

1. Labor and capital are used to produce output
2. No borrowing or lending
3. No investment, no government
4. Split between labor and capital income is 60-40
5. Two countries are subject to equal and opposite shocks to income (main assumption here)

Income Shocks: the world is in "State 1" in even years and in "State 2" in odd years

- ▶ State 1:  $Q^A = 90, Q^B = 110$
- ▶ State 2:  $Q^A = 110, Q^B = 90$

## Closed Economy

No cross-border (foreign) borrowing or equity

Each country owns all of its capital stock

	Country A			Country B			World
	K Income	L Income	GNI	K Income	L Income	GNI	GNI
State 1	36	54	90	44	66	110	200
State 2	44	66	110	36	54	90	200

Consumption alternates between 90 and 110 → not smooth

World output (income) is always the same! (constant at 200)

And yet, Closed economies cannot take advantage of this stability in world income

## Open Economy

(Assume) Countries can own some of the other country's capital stock  
buy equity stock on the other country's K

They receive income payments from their capital stock in the other country

Suppose each country owns 50% of the capital stock in the other economy (via equity investments)

	Country A					Country B			World
	K Income	L Income	GNI	TB	NFIA	K Income	L Income	GNI	GNI
State 1	40	54	94	-4	+4	40	66	106	200
State 2	40	66	106	+4	-4	40	54	94	200

In State 1:

Capital income generated by A: 36 (18 goes to each country)

Capital income generated by B: 44 (22 goes to each country) → each country ends up with 40 total

In State 2: same, 40 to each country

Capital income becomes constant (no volatility)

Income and consumption volatility fall as a result (better!)

# Limits to Risk Sharing

The extent at which risk-sharing is possible depends on two factors:

## 1. The correlation of country income

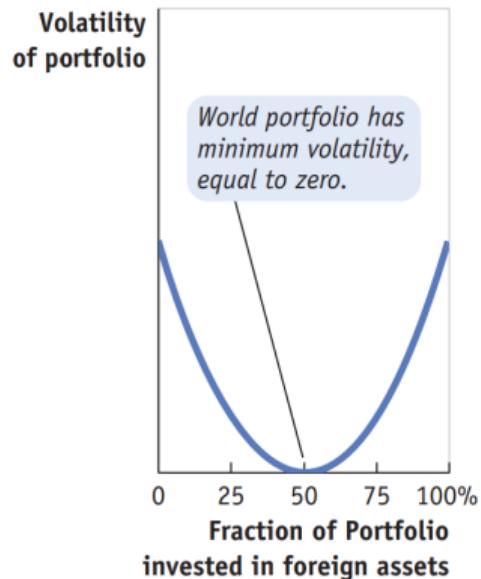
- ▶ Income shocks that are negatively correlated can be diversified
  - Bad times in one country coincides with good times in the other
- ▶ Income shocks that are positively correlated cannot
  - Example: Global recession → C, Q falls dramatically everywhere no matter what

## 2. How much income can be traded

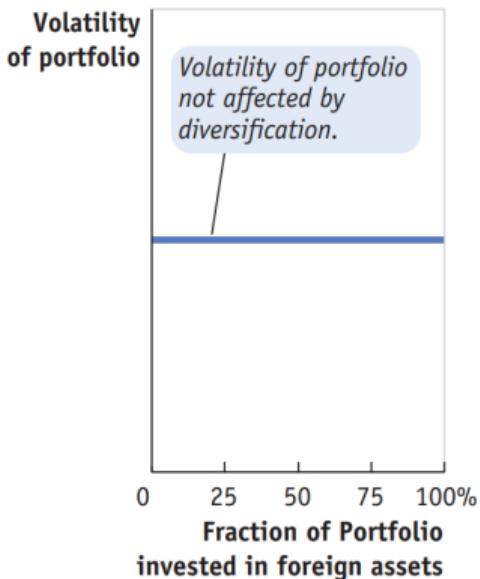
- ▶ How easy or feasible it is to own capital in a foreign country
  - Depends on capital mobility, legal system, institutions
- ▶ Not usually feasible to own someone else's labor income
  - Stocks to labor productivity of other people: slavery! Not possible
  - Thus, the whole diversification mechanism is more limited in locations where labor share in output is higher

# Diversification and Income Shocks Correlation

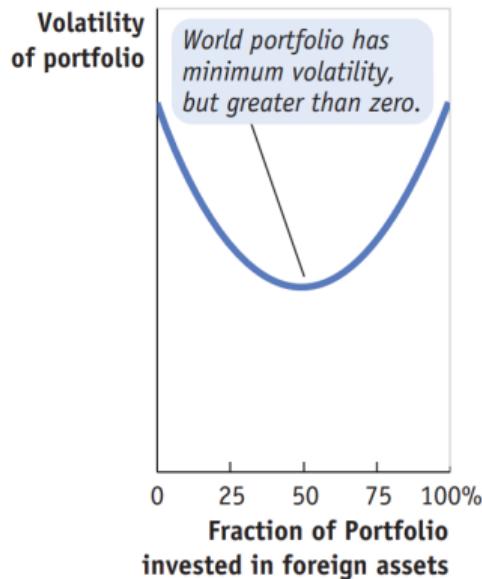
(a) Asymmetric Output  
Shocks: Perfect Negative  
Correlation (-1)



(b) Symmetric Output  
Shocks: Perfect Positive  
Correlation (+1)



(c) Other Cases:  
Combined Symmetric and  
Asymmetric Shocks



## Gains from Financial Globalization

Financial Globalization implies (allows) intertemporal trade leading in theory to:

1. Consumption Smoothing
2. Efficient Investing
  - ▶ Smoothing cost of investment over time with lending
  - ▶ Move capital across countries (seeking higher returns)
3. Risk Diversification

Do we see evidence of these gains?

Not as much as predicted by theory:

Consumption is not very smooth

Cross border investment is low

Home bias in investment: Portfolios have a disproportionate share of domestic assets

These features (lack of diversification) is more marked in poorer countries

## Limitations to International Finance

A possible take on why we still don't see much of the gains is that the world is not Financially Global enough

Maybe the international financial markets scope is not as wide as it should (or could to generate the gains)

Some possible explanations that we abstracted from:

- ▶ Regulation (limits to foreign investments)
- ▶ Capital controls
- ▶ Transaction costs
- ▶ Institutional risk (default, expropriation)
- ▶ Undiversifiable risk (global shocks)

Some of these are institutional factors —> low and heterogeneous quality of institutions

Still, this does not mean that Financial Globalization does not work:

First, the benefits we laid out are still there (even if in theory)

Second, although it can be argued that the world has not been very financially globalized so far ...

... many countries (emerging ones mainly) are in the process of becoming more financially open

Financial globalization proxied by size of cross-border investments which has increased in recent decade