Exchange Rate Dynamics and the Central Bank's Balance Sheet*

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Abstract

In theory, nominal exchange rates are a function of the relative difference in supply and demand of money. In practice, some central banks issue debt. In this study we ask: are nominal exchange rate variations linked to these remunerated central bank liabilities? We use two measures of implied exchange rates using central bank balance sheet data: one measure is a traditional measure that includes the monetary base, while the other also includes remunerated liabilities. We provide a simple theoretical framework to put these measures in context and to shed light on the relationship between exchange rates and the balance sheet of the central bank. We then move on to the formal empirical analysis. Nonlinear cointegration techniques are used to compare these two measures with the actual exchange rate for a set of Latin American countries using monthly data for the 2004:1-2019:12 period. The nonlinear cointegration technique allows both the number and location of thresholds to be endogenously determined based on the percentage difference between the exchange rate and the implied exchange rate. The nonlinear cointegration technique will allow us to determine whether central bank debt matters for understanding exchange rate dynamics and to determine whether passthrough is symmetric.

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1 Introduction

An increase in the domestic money supply (or monetary liability), other things equal, is usually associated with a currency depreciation. Some central banks, however, choose to also issue debt instead, i.e., remunerated liabilities that offset part of the initial increase of the monetary base.[1] In this paper we show, using theory and data from seven Latin American economies, that these remunerated liabilities affect exchange rates.

To help interpret the data, let us consider a simplified T-account of a central bank with zero net worth:

Assets	Liablities
Foreign Exchange Reserves	Monetary Base
Financial Claims	Remunerated Liabilities
Domestic Credit	

Table (1) shows these components, as a fraction of GDP, for six Latin American countries in 2018 (ranked by remunerated liabilities, in the last column).[2] The stock of remunerated liabilities as a fraction of GDP averaged 10% across these countries, with considerable heterogeneity: the stock of central bank debt was around a fourth of GDP in Uruguay and almost non-existent in Colombia. As indicated in Figure (A3), this ranking has not changed considerably between 2004 and 2019.

Table 1: Balance Sheet Components as a fraction of GDP, 2018

Country	Reserves	Financial Claims	Domestic Credit	Monetary Base	Remunerated Liabilities
Uruguay	0.30	0.00	0.10	0.06	0.25
Brazil	0.22	0.01	0.26	0.10	0.17
Chile	0.14	0.00	0.00	0.06	0.10
Paraguay	0.21	0.01	0.00	0.12	0.06
Mexico	0.15	0.02	0.00	0.07	0.02
Colombia	0.16	0.01	0.01	0.10	0.00

To put it in an international perspective, Table (A3) shows the same table for all 85 countries in the IMF IFS that have these variables available, for the 2010-2019 average: Uruguay is -together with Malasia- the country with highest ratio in the full sample. Brazil and Chile also are among the top 10 countries in terms of remunerated liabilities. Mexico and Colombia, in turn, belong to the large group of countries with little to no remunerated liabilities. So in the chosen Latin American subsample we use, there is relevant heterogeneity that fits the international extremes.

[1] Explaining the reasoning motivating this unconventional form of monetary sterilization goes beyond the scope of this paper. Some literature elaborates on this point, e.g., see Sosa-Padilla and Sturzenegger [2021] for a discussion on a potential explanation based on the risk hedging properties of local debt funded reserves accumulation setups.

[2] The criteria we followed to choose these countries is: South American countries plus Mexico. Argentina and Peru are unfortunately not included in the IMF IFS data for these variables (for Argentina, we obtain the data from the UCEMA Centro de Economía Aplicada (UCEMA-CEA)). Ecuador isn't included in IMF IFS for these variables either (and is dollarized, so not relevant for this analysis). Venezuela does not have data at monthly frequency, so we omit. We also omit Bolivia due to lack of exchange rate movement (pegged exchange rate).

Our empirical exercise relies on a measure of exchange rates computed using components of the balance sheet data for these economies: the implied exchange rate is the the ratio of domestic liabilities (measured in domestic currency) to foreign assets (measured in foreign currency, US dollars). This measure is sometimes used by economists and financial market analysts, for instance as described in Ávila [2018]. The implied exchange rate thus depreciates (increases) with increases in liabilities and appreciates (decreases) with accumulation of assets. The Liabilities we consider can be either the monetary base or also include the remunerated liabilities, so we will incorporate two alternative measures, while the assets we use are the foreign exchange reserves. In Section (4) we provide a simple theoretical framework to derive and further understand these measures.

We refer to these measures as the "Conversion Exchange Rate" (CER henceforth).[3] We will first define the "CER Base" as the measure that only incorporates the monetary base in the numerator and the "CER Full" as the measure that includes both the monetary base and remunerated liabilities. So the measures are (in pesos per dollar):

$$\text{CER Base} \equiv \frac{\text{Monetary Base}}{\text{International Reserves}} \tag{1}$$

and

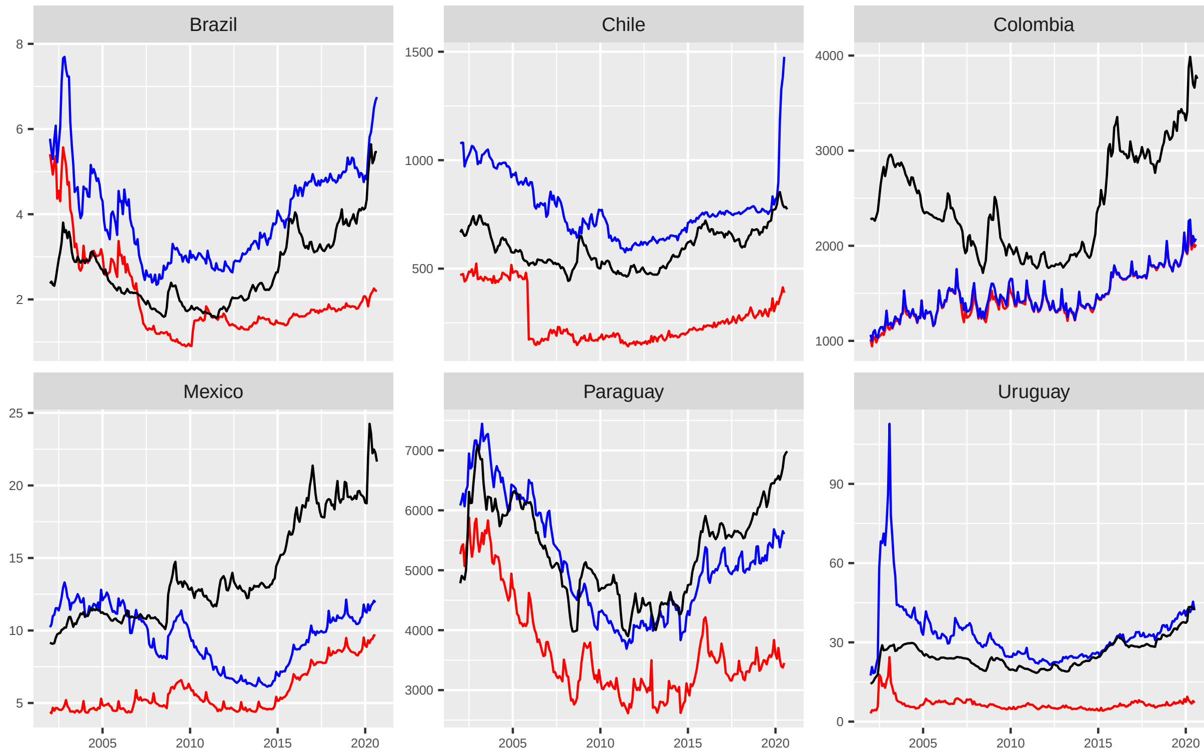
$$\text{CER Full} \equiv \frac{\text{Monetary Base+Remunerated Liabilities}}{\text{International Reserves}}. \tag{2}$$

Figure (1) depicts the exchange rate and implied exchange rates for the six Latin American countries. The implied exchange rate levels seems consistent with the actual exchange rate for four of these countries (Brazil, Chile, Paraguay and Uruguay). The difference between the CER Full and the CER Base is explained by the existence of remunerated liabilities. The larger the remunerated liabilities, the larger the difference between the two lines. The CER Full seems to track the exchange rate better than the CER Base, suggesting that remunerated liabilities matter. For Colombia and Mexico, however, the exchange rate and CER do not seem to be related. Recall from Table (1) that these two countries had little to no remunerated liabilities. A pattern seems to emerge: countries with substantial remunerated liabilities (say larger than 2% of GDP) have exchange rates that move closely with the balance sheet of the central bank, while countries without remunerated liabilities have exchange rates that are decoupled with the balance sheet of the central bank. We formally test the link between these variables using nonlinear cointegration techniques.

We also study large depreciations. For this we focus on the currency crisis in Argentina in 2018. Between April and September 2018 the argentine peso lost half its value: it went from around 20 to 40 pesos per US

[3] In spanish this is known as "Tipo de Cambio de Conversión" (Ávila [2018]).

Figure 1: Monthly Exchange Rates (black) and CER Base (red), and CER Full (blue). 2004-2019



Source: Author's calculations using IMF IFS data.

dollar. The central bank had accumulated around 8% of GDP in remunerated liabilities during the previous two years.[4] As Figure (2) shows, the magnitude of this depreciation coincided with the difference in the CER Base and CER Full. In the crisis, the actual exchange rate jumped from the former to the later. This might be suggesting that the fundamental exchange rate value is indeed given by balance sheet measures that include remunerated liabilities.[5] Through this lens, before May 2018 the currency was overvalued, relative to the fundamentals given by the Central Bank balance sheet.

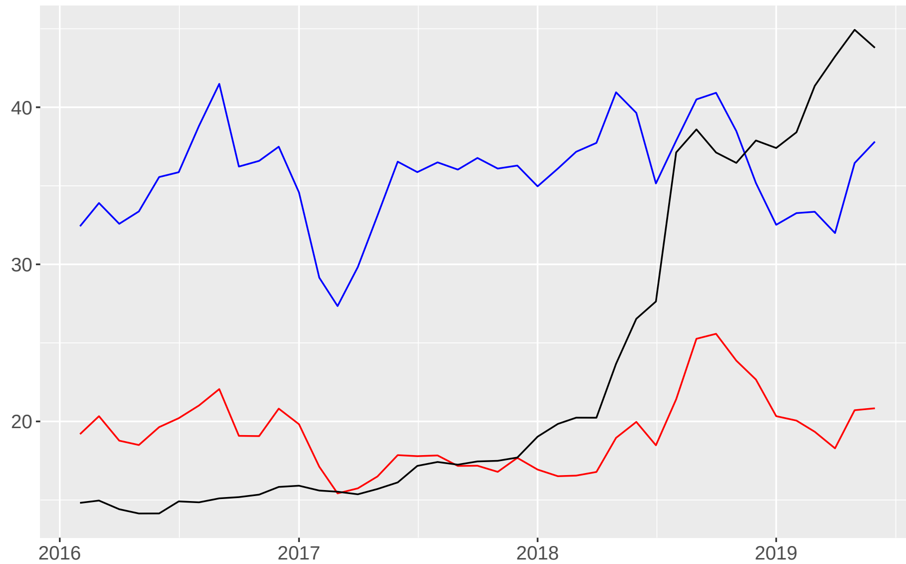
1.1 Related Literature

Monetary theory tells us that variation of the exchange rate is a function of the difference in supply and demand of money, relative to the foreign country, as exposed in for instance Obstfeld and Rogoff [1995]. An increase in the money supply (or monetary liability), other things equals, leads to a currency depreciation. These theories however mostly abstract from two features that are present in many emerging markets:

[4]For the case of Argentina we focus in the January 2016 - July 2019 period since before and after this there were tight currency controls, and thus a black foreign exchange price.

[5]For the period after the crisis (around May 2018), the CER Full is underestimated, since the IMF bailout transfers were temporarily recorded as international reserves by the Central Bank (thus the actual reserves were lower, which implies a higher CER Full). We thank Jorge Ávila for this insight.

Figure 2: Monthly Exchange Rates (black), CER Base (red), and CER Full (blue) for Argentina - Currency Crisis Case Study.



Source: Author's calculations calculation based on UCEMA-CEA.

1. Many central banks follow intermediate exchange rate regimes (neither fixed nor flexible exchange rates): Frankel [2019] proposes to define an intermediate regime, called "systematic managed floating", as an arrangement where the central bank regularly responds to changes in total exchange market pressure by allowing some fraction to be reflected as a change in the exchange rate and the remaining fraction to be absorbed as a change in foreign exchange reserves.

2. Many central banks issue remunerated liabilities, as reported before. Foreign exchange reserves are sometimes financed by issuing central bank debt. Sosa-Padilla and Sturzenegger [2021] study the link between remunerated liabilities and bond yields. We however focus on the link with exchange rates.

We contribute to the existing literature by providing evidence and a simple theoretical framework to understand exchange rate movements with the existence of remunerated liabilities. Our intention is not in predicting exchange rates: a long-standing puzzle in international economics is the difficulty of tying floating exchange rates to macroeconomic fundamentals such as money supplies, outputs, and interest rates (see Engel and West [2005] and references therein).

Our contribution can be seen as adding an extra layer of complexity to these fundamentals: remunerated liabilities can be see as another fundamental in exchange rate determination. Our findings thus might have empirical and theoretical implications for open economy macroeconomics, in particular for the Mundell-

Fleming trilemma (Mundell [1963], Fleming [1962]). Our main conclusion is that remunerated liabilities can shed light on the empirical relationship between exchange rates and the balance sheet of central banks.

The rest of the paper is organized as follows: in Section (2) we perform a VAR analysis to study the components of the balance sheet, while in Section (3) we move to the cointegration analysis. In Section (4) we provide a simple theory of the implied exchange rates used throughout the paper.

2 VAR Models for Balance Sheet Components

We estimate vector autorregresive models (VAR) for each economy based on the variables represented in (2) to investigate interactions between the balance sheet components involving the CER-Full. In each case our estimation procedure is given as follows:

$$\begin{aligned} X_t &= \Phi X_{t-1} + u_t \\ u_t &\sim N(0, \Sigma_u) \end{aligned} \tag{3}$$

Where $X_t = [er\ h\ mb\ rl]'$, and er : exchange rate, h: foreign reserves, mb: monetary base, rl: remunerated liabilities. A separate VAR model is estimated for each of the seven countries with monthly data spanning 2004:01-2019:12. We transform the data by its logarithm and model it in levels or first differences depending on the presence of cointegration in each economy, which we test for using the Johansen Multivariate cointegration test **?**.

In addition, the model in (3) denotes the (companion) VAR(1) representation of each VAR(p) model, i.e., we allow the model in each case to have a higher lag order which is assigned according to the AIC criterion with a maximum lag length of 6, as shown in table 2. Finally, in each country we check to be sure the reduced form erros do not contain autocorrelation.[6].

Table 2: Lag Selection for VAR models

Country	AIC(n)	HQ(n)	SC(n)	FPE(n)
Chile	2	2	1	2
Brazil	2	1	1	2
Paraguay	2	1	1	2
Uruguay	3	2	1	3
Colombia	2	2	2	2
Mexico	4	1	1	4
Argentina	1	1	1	1

Note: The lag length selected corresponds to the AIC from a maximum of $T^{1/3}$ (6 for our sample size).

[6]We report the results we make mention of here and leave the rest in appendix B

Table 3: Johansen cointegration test results, by country

Number of Cointegration Relationships from Johansen Test						
Country	Deterministic component			Decision by type of test		
	None	Const	Trend	Trace	Max. Eigenv.	Final Model (selected)
Chile	1 / 1	1 / 1	1 / 1	1 - None	1 - None	VEC/VAR(levels)
Brazil	1 / 0	2 / 1	0 / 0	0 - Trend	0 - None	VAR(differences)
Paraguay	1 / 1	3 / 3	1 / 0	1 - None	0 - Trend	VAR(differences)
Uruguay	1 / 1	3 / 2	1 / 1	1 - None	1 - None	VEC/VAR(levels)
Colombia	2 / 1	3 / 3	2 / 1	2 - None	1 - None	VEC/VAR(levels)
Mexico	0 / 0	1 / 1	1 / 1	0 - None	0 - None	VAR(differences)
Argentina	0 / 0	1 / 1	1 / 1	0 - None	0 - None	VAR(differences)

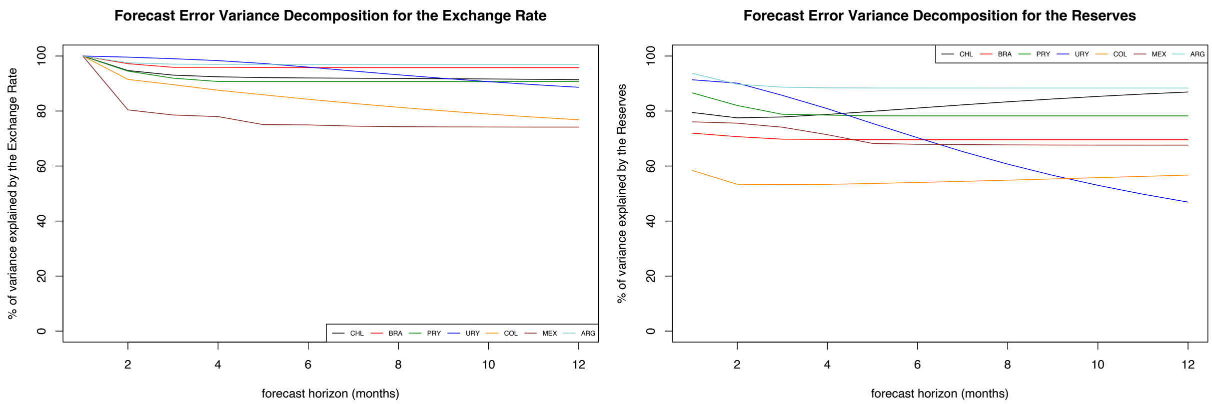
Note: each entry denotes the number of relations in each test as: "trace test / max. eigenvalue test". The deterministic component refers to additional terms included in the cointegration relationship. Decision rule: pick the simplest model (in number of relations and type of component) for which the test is rejected.

From a Balance Sheet Approach to an Implied Exchange Rates Analysis The evidence on the importance of the remunerated liabilities based on the multivariate exercises that represent the structure of the balance sheet of the central banks is mixed. First, we test for multivariate cointegration, the results are shown in table 3. The results suggest that in five out of seven countries we should proceed with a VAR model in differences, i.e., abstract from cointegration. We carry out the model indicated in the last column of the table.

The causality tests (based on **?** and a modified Wald test to consider the contemporaneous relationships) show some sings of the remunerated liabilities causing the rest of the balance sheet, it granger causes the other variables for Brazil, Paraguay, and Chile (at the 10% level). Furthermore, the forecast variance error decompositions shows that for Brazil, Paraguay, Chile, and Uruguay the remunerated liabilities explain about 2% or more (up to 10%) of the exchange rate variance. In light of these results, for all countries except Colombia the effect of remunerated liabilities on the dynamics of the currency is not necessarily trivial.

To state this we take a look at the percentage of the variance of the exchange rate that is caused by itself (see figure 3 left panel). It turns out, this variable is particularly persistent and self-determined in a more marked way than the rest of the balance sheet (e.g., see figure 3 right panel). In that sense, explaining 6% of the residual variance is still relevant. However, the high degree of self-determination in this system (for every country model) that is displayed by the exchange rate poses a challenge for establishing a meaningful relationship with other fundamentales and can be traced to the exchange rate disconnect puzzle (Obstfeld and Rogoff [2001]). Thus, exploring the relevance of the liabilities from a multivariate balance sheet perspective is likely not too promising.

Figure 3: Percentage of Variance of Exchange Rate (left panel) and the Reserves (right) explained by Itself



Nevertheless, we can take an alternative venue to verify the relevance of the liabilities for the (observed) spot rate dynamics: an exchange-to-exchange rate cointegration analysis. In particular, we can assess the co-movement (over several horizons) of the spot rate with two balance sheet implied measures of the exchange rate, the CER Full that accounts for the effect of the remunerated liabilities on the balance sheet and the CER Base that abstracts from it. If the former is more closely related to the actual spot rate, then we could say that the way reserves accumulation are funded matters (and not only the overall level of reserve assets does).

This approach is further justified by the empirical literature, e.g., Engel et al. [2015] and Verdelhan [2018] which show evidence in favor strong comovements between exchange rates in international markets against their base currency. A simplified implication of such thinking is that exchange rates, spot or implied, that are related to the same base currency will also co-move strongly.

3 Cointegration Analysis

3.1 Cointegration Framework

We first focus on data for Argentina, Brazil, Chile, Paraguay, and Uruguay using the logged values for monthly data from 2004:1-2019:12.

The econometric analysis consists of three steps. First, we investigate the order of integration of each series using the Augmented Dickey Fuller (ADF) unit root test of Dickey and Fuller [1979, 1981] and the minimum Lagrange multiplier unit root test that allows for structural breaks in level and trend of Lee and

Strazicich [2003]. When the series are found to be integrated of the same order, we employ the threshold cointegration technique of Sephton and Mann [2013] which combines the threshold selection method of Gonzalo and Pitarakis [2002] with the F test developed by Seo [2008]. The test for threshold cointegration examines the residuals from the cointegrating regression in equation (4), and conditional on selecting one or more thresholds (for expository purposes, we assume there are two in equation (5), denoted by τ_1 and τ_2) it examines the null hypothesis that ($\delta_1 = \delta_2 = \delta_3 = 0$) against the alternative they are not jointly zero. In the case of a single threshold, the testing equation (5) would delete the term involving the residual between the thresholds. In the case of zero thresholds the method collapses on the testing equations of the traditional cointegration test of Engle and Granger [1987]. Inference for threshold cointegation tests use residual block based bootstrapping and inference for Engle Granger cointegration tests follow critical values of MacKinnon [1996].

$$Y_t = \beta_1 + \beta_2 Trend_t + \beta_3 X_t + \varepsilon_t \tag{4}$$

$$\triangle\varepsilon_t = \delta_1 \varepsilon_{t-1} I\left(\varepsilon_{t-1} < \tau_1\right) + \delta_2 \varepsilon_{t-1} I\left(\tau_1 < \varepsilon_{t-1} < \tau_2\right) + \delta_3 \varepsilon_{t-1} I\left(\tau_2 < \varepsilon_{t-1}\right) + \sum_{i=1}^{p} \alpha_i \triangle\varepsilon_{t-i} + v_t \tag{5}$$

such that

- Y_t and X_t are the logged implied exchange rate (*Base* or *Full*) and the exchange rate (*er*).
- ε_t is the residual from the cointegrating regression from equation (4).
- $\tau_1 < \tau_2$ are the thresholds that divide observations in to the bottom, middle and top regimes.
- $I()$ is the Heaviside indicator.
- Δ is the difference operator.

The advantage of Sephton and Mann [2013] over the traditional approach is that it endogenously determines both the number of thresholds and their location. Most cointegration techniques, both in theory and applications, assume the absence of thresholds (i.e., Engle and Granger test); that a single threshold exists that is equal to zero (hence separating observations into two regimes with ε_{t-1} being positive in the first and negative in the second); or that two thresholds exist that are symmetric around zero. Here we allow for as many as three thresholds, and as few as zero. If the procedure chooses two thresholds, they may be symmetric about zero, but this decision is not determined *a priori* – the data determines the number of thresholds and their locations. For further details on the threshold cointegration technique and its properties please see Sephton and Mann [2013].

Finally, when we reject the null hypothesis for the cointegration tests an error correction model (ECM) can be used to determine how the variables move through time, and which of them adjusts to restore the system to the long-run relationship. The ECM also provides estimates of the speed at which the adjustment takes place. Incorporating the threshold into the ECM allows both the speed at which the adjustment takes place and the impact of the shock to vary by regime.

An ECM with two thresholds is expressed in equation system (6-7), where for convenience, we have set the lag length on changes in Y_t and X_t to be the same across equations and that which is prescribed by the AIC. The equations for zero, one and three threscholds are a slight modificaiton to the equation system (6-7).

$$\triangle Y_t = \gamma_1 \varepsilon_{t-1} I(\varepsilon_{t-1} < \tau_1) + \gamma_2 \varepsilon_{t-1} I(\tau_1 < \varepsilon_{t-1} < \tau_2) + \gamma_3 \varepsilon_{t-1} I(\tau_2 < \varepsilon_{t-1}) + \sum_{i=1}^{p} (\alpha_i \triangle Y_{t-i} + \beta_i \triangle X_{t-i}) + u_t \tag{6}$$

$$\triangle X_t = \gamma_4 \varepsilon_{t-1} I(\varepsilon_{t-1} < \tau_1) + \gamma_5 \varepsilon_{t-1} I(\tau_1 < \varepsilon_{t-1} < \tau_2) + \gamma_6 \varepsilon_{t-1} I(\tau_2 < \varepsilon_{t-1}) + \sum_{i=1}^{p} (\chi_i \triangle Y_{t-i} + \delta_i \triangle X_{t-i}) + v_t \tag{7}$$

such that

- Y_t and X_t are the logged implied exchange rate (*Base* or *Full*) and the exchange rate (*er*).
- ε_t is the residual from the cointegrating regression from equation (4).
- $\tau_1 < \tau_2$ are the thresholds that divide observations in to the bottom, middle and top regime.
- γ_i are the regime specific speeds of adjustment to the long-run relationship.
- $I()$ is the Heaviside indicator.
- Δ is the difference operator.

Estimates of the speeds of adjustment coefficients will demonstrate which variables are weakly exogenous and do not move to restore the long run relationship.

3.2 Cointegration Results

Results from the unit root tests presented in Table 2 are consistent across all seven countries; we fail to reject the null hypothesis of a unit root for all seven CER Base and CER Full. This allows us to proceed to the threshold cointegration tests for all exchange rate-implied exchange rate pairs.

Results from the cointegration tests presented in Table 4 indicate that seven of the 10 exchange rate-implied exchange rate pairs are cointegrated with either two or three thresholds. The remaining three pairs are Chile Base, Brazil Base, and Brazil Full with zero threshold being selected for Chile Base and three thresholds being selected for Brazil Base and Brazil Full.

Table 4: ADF Unit Root Test Results for Exchange Rate (*er*) and Implied Exchange Rate (*Base* and *Full*) 2004:1-2019:12

	ADF	LS
Chile Ex	-2.076	
Chile Base	-2.112	-2.063
Chile Full	-1.444	
Uruguay Ex	-0.494	-2.015
Uruguay Base	-2.549	-2.445
Uruguay Full	-1.521	-1.624
Paraguay Ex	-1.655	-4.81
Paraguay Base	-2.467	-3.882
Paraguay Full	-1.096	-4.02
Brazil Ex	-1.092	-4.085*
Brazil Base	-1.585	-3.748
Brazil Full	-1.425	-3.872
Argentina Ex	0.125	
Argentina Base	-2.292	
Argentina Full	-1.949	

Notes: The null hypothesis is a unit root. Deterministic components, number and type of structural breaks determined by ocular inspection of the time series plot; lag length for unit root tests is selected by the AIC from a maximum of $T^{1/3}$. Significance at $\alpha = 0.10$, 0.05 and 0.01 denoted by *, **, and *** respectively. A blank entry indicates the absence of a structural break.

The final version of this paper will extend the results to include the ECM from equations (6) and (7) for the seven cointegrated pairs to determine how the variables move through time, and which of them adjusts to restore the system to the long-run relationship. Care will be taken to interpret the ECM in terms of whether the level or the differenced percentage difference is used as the threshold variable. (The final version will also extend the analysis to include Mexico and Colombia.)

TBC

Table 5: Threshold Cointegration Test Results

	Test Statistic	Number of Thresholds
Chile Base		0
Chile Full	24.482***	2
Uruguay Base	17.210***	2
Uruguay Full	25.998***	3
Paraguay Base	65.215***	3
Paraguay Full	19.410**	3
Brazil Base	11.362	3
Brazil Full	6.306	3
Argentina Base	17.559*	3
Argentina Full	17.862***	1

Notes: The null hypothesis is no cointegration against an alternative of (threshold) cointegration. Input specifications for Engle-Granger test (i.e., zero thresholds): lag length for the testing equations is selected by the AIC from a maximum of $T_{1/3}$; critical values follow MacKinnon (1996). Input specifications for the threshold cointegration tests (i.e., 1, 2, or three thresholds): threshold locations include the middle 90% of observations divided into 50 increments; each regime requires a minimum of 24 observations (not binding); AIC used throughout; critical values for the threshold cointegration test simulated following the residual-based block bootstrap methodology by Seo (2008) with a block length of 6 and 9,999 replications under the null. Significance at $\alpha = 0.10$, 0.05 and 0.01 denoted by *, **, and *** respectively.

4 Theoretical Framework

What is the theoretical origins of equations (1) and (2)? How can we think of the relationships described previously? In this section we provide a simple theoretical link between exchange rates and remunerated liabilities.

We follow the main building blocks and notation in Engel and West [2005] (see references therein for earlier contributions), however we extend the framework to allow for accumulation of international reserves and remunerated liabilities.[7]

Assume that in the home country there is a money market relationship given by

$$m_t = p_t + \gamma y_t - \alpha i_t + v_{mt}. \tag{8}$$

Here, m_t is the log of the home money supply, p_t is the log of the home price level, i_t is the level of the home interest rate, y_t is the log of output, and v_{mt} is a shock to money demand. Assume that a similar equation holds in the foreign country. The analogous foreign variables are m_t^*, p_t^*, i_t^*, y_t^* and v_t^*, and the parameters of the foreign money demand are identical to the home country's parameters.

The nominal exchange rate equals its purchasing power parity (PPP) value plus the real exchange rate:

$$s_t = p_t - p_t^* + q_t. \tag{9}$$

In financial markets, the interest parity relationship is

[7] We use reduced form equations, but many of these equations can be derived from microfounded models, as in Obstfeld and Rogoff [1995] or Vegh [2013].

$$E_t s_{t+1} - s_t = i_t - i_t^* + \rho_t. \tag{10}$$

Here, ρ_t is the deviation from rational expectations uncovered interest parity. It can be interpreted as a risk premium or an expectational error. Putting these equations together and rearranging, we get

$$s_t = \frac{1}{1+\alpha}\left[m_t - m_t^* - \gamma(y_t - y_t^*) + q_t - (v_{mt} - v_{mt}^*) - \alpha\rho_t\right] + \frac{\alpha}{1+\alpha}E_t s_{t+1}. \tag{11}$$

which is the expression in Engel and West [2005]. To interpret this equation, assume $\gamma = 1$, $q_t = 1$, $v_{mt} = v_{mt}^* = 0$, $\rho_t = 0$, and let $b = \frac{1}{1+\alpha}$, so

$$s_t = b\left[m_t - m_t^* - (y_t - y_t^*)\right] + (1-b)E_t s_{t+1}. \tag{12}$$

The observable fundamentals are given by $f_{1t} = m_t - m_t^* - \gamma(y_t - y_t^*)$. Notice that equation (12) holds absent intervention of the central banks. It is thus a pure flexible exchange rate "counterfactual" equilibrium. We however consider an environment where the central bank regularly responds to changes in total exchange market pressure by allowing some fraction to be reflected as a change in the exchange rate and the remaining fraction to be absorbed as a change in foreign exchange reserve (Frankel [2019]).

A positive productivity shock, for instance, that leads to an increase in output y_t might be reflected as combination of an appreciation and reserve accumulation. We thus need to introduce international reserves into the picture.

Central Bank's Balance Sheet A simplified Central Bank's balance sheet is given by

A	L
$s_t H_t$	MB_t
	RL_t

here H are international reserves (in foreign currency, "dollars"), s is the exchange rate and MB is the monetary base and RL_t are remunerated liabilities (in domestic currency, "pesos").[8]

We consider an environment where the central bank's net worth is zero and does not vary over time (as typically assumed, see Vegh [2013] for example in different models). Thus, assets and liabilities vary, in dollar terms, proportionally. This implies that international reserves equals, to a first degree, total liabilities in dollar terms, $H = \frac{MB+RL}{s}$. Rearranging, we get the relationship used in the empirical exercise:

[8]We abstract from claims to financial institutions and from from domestic credit, mainly because we focus on foreign denominated assets). We also abstract from valuation of the assets held by the Central Bank, a more general framework could include some of these elements, in line with Ghironi et al. [2015].

$$s = \frac{MB + RL}{H} \tag{13}$$

Define total liabilities (TL) as the sum of monetary base and remunerated liabilities $TL = MB + RL$ and let $\omega \equiv \frac{MB}{MB+RL}$ be the weight of the monetary base in total liabilities (and thus $1-\omega = \frac{RL}{MB+RL}$). Then, taking logs and differentiating with respect to time, the dynamics of the exchange rate is given by

$$\frac{\Delta s}{s} = \omega \frac{\Delta MB}{MB} + (1-\omega)\frac{\Delta RL}{RL} - \frac{\Delta H}{H} \tag{14}$$

where Δ represents changes. Equations (14) and (12) have some similarity: other things equal, the exchange rate depreciates with increases in the monetary liabilities (which now also includes remunerated liabilities) and decreases with output growth (with tends to increase international reserves). [9] A framework that does not include remunerated liabilities ($\triangle RL = 0$) tends to ignore relevant sources of exchange rate variations, if the central bank indeed accumulated such remunerated liabilities. Since the expected exchange rate already incorporates the expected dynamics of remunerated liabilities, equation (12) also pins down the exchange rate in the flexible exchange rate regime case.

The central bank can decide to intervine in the foreign exchange market. Given a shock/pressure on the exchange rate, the central bank might decide to prevent a movement in the exchange rate and let international reserves fully absorb the shock. Such a central bank, at least for this period, is fixing its exchange rate in de facto dollar terms (Levy-Yeyati and Sturzenegger [2005]). In this case, during the relevant period, international reserves are explained by variations in total liabilities (in real terms, since the exchange rate does not change), $\frac{\triangle H}{H} = \omega \frac{\triangle MB}{MB} + (1-\omega)\frac{\triangle RL}{RL}$. In this case, given for instance a positive shock, international reserves will increase by the sum of total liabilities.[10]

In general terms, however, given a shock, the central bank can choose it to be reflected partially in the exchange rate and partially in international reserves. Thus, in general terms, the reflection of a shock in the balance sheet can be described by a vector $(\Delta s, \Delta H, \Delta MB, \Delta RL)$. The policy maker however cannot freely choose all of the elements of this vector.

Do remunerated liabilities give an extra degree of freedom that can be used to override the the Mundell-Fleming trilemma (Mundell [1963], Fleming [1962])? To the contrary: our framework suggests that remu-

[9] The intuition for this is in the money market: the exchange rate is the equilibrium price of supply and demand for domestic money. Output growth tends to increase money demand, which tends to appreciate the currency under flexible exchange rates. Under predetermined exchange rates however the central bank increases the supply of money -by purchasing foreign exchange reserves- and hence the exchange rate does not change but reserves do increase.

[10] Not all liabilities are the same, however. On one hand, international reserves accumulation can be explained by increases in the monetary base in real terms, which we can call as a "genuine" source of accumulation since it is endogenous under predetermined exchange rates and grows with money demand (economic growth). On the other hand, international reserves can be financed by increases in remunerated liabilities (debt accumulation). Sosa-Padilla and Sturzenegger [2021] focus on these differences.

nerated liabilities worsen the trilemma.

To see this, consider this example. A central bank can choose two of the following: the exchange rate management, autonomous monetary policy or free capital mobility. In the framework we are thinking of, an extra layer is added, since central banks can choose the amount of remunerated liabilities. To set ideas, assume that the central bank has chosen to have free capital mobility and a predetermined exchange rate regime (so the interest rate and the amount of money are endogenous) but the central bank has remunerated liabilities. From equation (13) it is clear that central banks in this context cannot freely choose the amount of remunerated liabilities, since doing so will affect exchange rates (since the expected exchange rate moves according to these liabilities, and hence so does the spot exchange rate). Remunerated liabilities are in this sense “endogenous": they can only change as long as the monetary base or international reserves move to compensate them (which also are endogenous in this context due to the trilemma). If on the other hand the central bank chooses the amount of remunerated liabilities, then it cannot have a predetermined exchange rate. The Mundell-Fleming trilemma is in full force again, albeit now with remunerated liabilities.

Thus, the theory in this section is consistent with the empirical findings previously reported: exchange rates are related to the stock of remunerated liabilities.

5 Preliminary Conclusion

TBC

References

Ávila, Jorge C, "Plan de dolarización sostenible," Technical Report, Serie Documentos de Trabajo 2018.

Balke, Nathan S and Thomas B Fomby, "Threshold cointegration," *International economic review*, 1997, pp. 627–645.

Dickey, David A and Wayne A Fuller, "Distribution of the estimators for autoregressive time series with a unit root," *Journal of the American statistical association*, 1979, *74* (366a), 427–431.

_ and _ , "Likelihood ratio statistics for autoregressive time series with a unit root," *Econometrica: journal of the Econometric Society*, 1981, pp. 1057–1072.

Enders, Walter and Pierre L Siklos, "Cointegration and threshold adjustment," *Journal of Business & Economic Statistics*, 2001, *19* (2), 166–176.

Engel, Charles and Kenneth D West, "Exchange rates and fundamentals," *Journal of political Economy*, 2005, *113* (3), 485–517.

_ , Nelson Mark, and Kenneth West, "Factor Model Forecasts of Exchange Rates," *Econometric Reviews*, 2015, *34* (1-2), 32–55.

Engle, Robert F and Clive WJ Granger, "Co-integration and error correction: representation, estimation, and testing," *Econometrica: journal of the Econometric Society*, 1987, pp. 251–276.

Fleming, J Marcus, "Domestic financial policies under fixed and under floating exchange rates," *Staff Papers*, 1962, *9* (3), 369–380.

Frankel, Jeffrey, "Systematic managed floating," *Open Economies Review*, 2019, *30* (2), 255–295.

Ghironi, Fabio, Jaewoo Lee, and Alessandro Rebucci, "The valuation channel of external adjustment," *Journal of International Money and Finance*, 2015, *57*, 86–114.

Gonzalo, Jesús and Jean-Yves Pitarakis, "Estimation and model selection based inference in single and multiple threshold models," *Journal of Econometrics*, 2002, *110* (2), 319–352.

Lee, Junsoo and Mark C Strazicich, "Minimum Lagrange multiplier unit root test with two structural breaks," *Review of economics and statistics*, 2003, *85* (4), 1082–1089.

Levy-Yeyati, Eduardo and Federico Sturzenegger, "Classifying exchange rate regimes: Deeds vs. words," *European economic review*, 2005, *49* (6), 1603–1635.

MacKinnon, James G, "Numerical distribution functions for unit root and cointegration tests," *Journal of applied econometrics*, 1996, *11* (6), 601–618.

Mundell, Robert A, "Capital mobility and stabilization policy under fixed and flexible exchange rates," *The Canadian Journal of Economics and Political Science/Revue canadienne d'Economique et de Science politique*, 1963, *29* (4), 475–485.

Obstfeld, Maurice and Kenneth Rogoff, "Exchange rate dynamics redux," *Journal of political economy*, 1995, *103* (3), 624–660.

_ and _ , "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?," in "NBER Macroeconomics Annual 2000, Volume 15" NBER Chapters, National Bureau of Economic Research, Inc, december 2001, pp. 339–412.

Seo, Myung Hwan, "Unit root test in a threshold autoregression: asymptotic theory and residual-based block bootstrap," *Econometric Theory*, 2008, pp. 1699–1716.

Sephton, Peter and Janelle Mann, "Threshold cointegration: Model selection with an application," *Journal of Economics and Econometrics*, 2013, *56* (2), 54–77.

Sosa-Padilla, César and Federico Sturzenegger, "Does It Matter How Central Banks Accumulate Reserves? Evidence from Sovereign Spreads," Technical Report, National Bureau of Economic Research 2021.

Vegh, Carlos A, *Open economy macroeconomics in developing countries*, MIT press, 2013.

Verdelhan, Adrien, "The Share of Systematic Variation in Bilateral Exchange Rates," *The Journal of Finance*, 2018, *73* (1), 375–418.

A Data Source

We use standardized central bank balance sheet data from the IMF-IFS statistics.[11] The Table below, from Sosa-Padilla and Sturzenegger [2021], summarizes the relevant balance-sheet data that we will use:

Assets	Liabilities
Claims on non-residents (1)	Liabilities to non-residents (a)
Claims on others depository corporations (2)	Monetary base (b)
Net Claims on Central Government (3)	Other Liabilities To Other Depository Corporations (c)
	Deposits and Securities other than Shares Excluded from Monetary Base (d)
	Loans (e)
	Financial Derivatives (f)
	Shares and equity (g)
	Other items (h)

Hence the CER Base is

$$\text{CER Base} \equiv \frac{\text{Monetary Base}}{\text{International Reserves}} = \frac{(b)}{(1)}$$

Following Sosa-Padilla and Sturzenegger [2021], we define the Remunerated Liabilities as the sum of (c), (d), (e) and (f). The CER Full is then

$$\text{CER Full} \equiv \frac{\text{Monetary Base+Remunerated Liabilities}}{\text{International Reserves}} = \frac{(b)+(c)+(d)+(e)+(f)}{(1)}$$

[11] As mentioned before, for Argentina we use the database in UCEMA-CEA, which provides data on monetary base and remunerated liabilities directly.

B Additional Results of the Balance Sheet based VAR estimations

Table A1: Johansen cointegration test results, by country

Number of Cointegration Relationships from Johansen Test					
Country	Deterministic component		Decision by type of test		
	Constant	Trend	Trace	Max. Eigenv.	Final Model (selected)
Chile	1 / 1	1 / 1	1 - Constant	1 - Constant	VEC/VAR(levels)
Brazil	2 / 1	0 / 0	0 - Trend	0 - Trend	VAR(differences)
Paraguay	3 / 3	1 / 0	1 - Trend	0 - Trend	VAR(differences)
Uruguay	3 / 2	1 / 1	1 - Trend	1 - Trend	VEC/VAR(levels)
Colombia	3 / 3	2 / 1	2 - Trend	1 - Trend	VEC/VAR(levels)
Mexico	1 / 1	1 / 1	1 - Constant	0 - Constant	VAR(differences)
Argentina	1 / 1	1 / 1	0 - Constant	0 - Constant	VAR(differences)

Note: each entry denotes the number of relations in each test as: "trace test / max. eigenvalue test". The deterministic component refers to additional terms included in the cointegration relationship. Decision rule: pick the simplest model (in number of relations and type of component) for which the test is rejected.

Here we replicate the cointegration test without the NONE (no deterministic component in the cointegration vector) to cover the case we include at least a constant in one or more variables. The conclusion is the same regarding whether we use a VAR in levels or differences (based on the maximum eigenvalue test).

Table A2: Causality Tests in the VAR models, by country

Instantaneous causality test					Granger causality test				
Country	ER	H	MB	RL	Country	ER	H	MB	RL
Chile	0.000	0.000	0.000	0.000	CHL	0.547	0.003	0.008	0.074
Brazil	0.000	0.000	0.000	0.000	BRA	0.427	0.038	0.005	0.031
Paraguay	0.000	0.000	0.000	0.000	PRY	0.045	0.002	0.000	0.001
Uruguay	0.000	0.000	0.009	0.000	URY	0.358	0.225	0.092	0.210
Colombia	0.000	0.000	0.805	0.562	COL	0.084	0.000	0.223	0.578
Mexico	0.000	0.000	0.012	0.005	MEX	0.016	0.000	0.051	0.502
ARG	0.003	0.000	0.000	0.000	ARG	0.058	0.290	0.048	0.479

Note: the p-value is shown. Null hyp.: the variable in the column causes (instantaneously or Granger) the rest of the system.

Figure A1: FEVD for the Exchange Rate (up-left), Reserves (up-right), and Monetary Base (bottom) (% explained by RL)

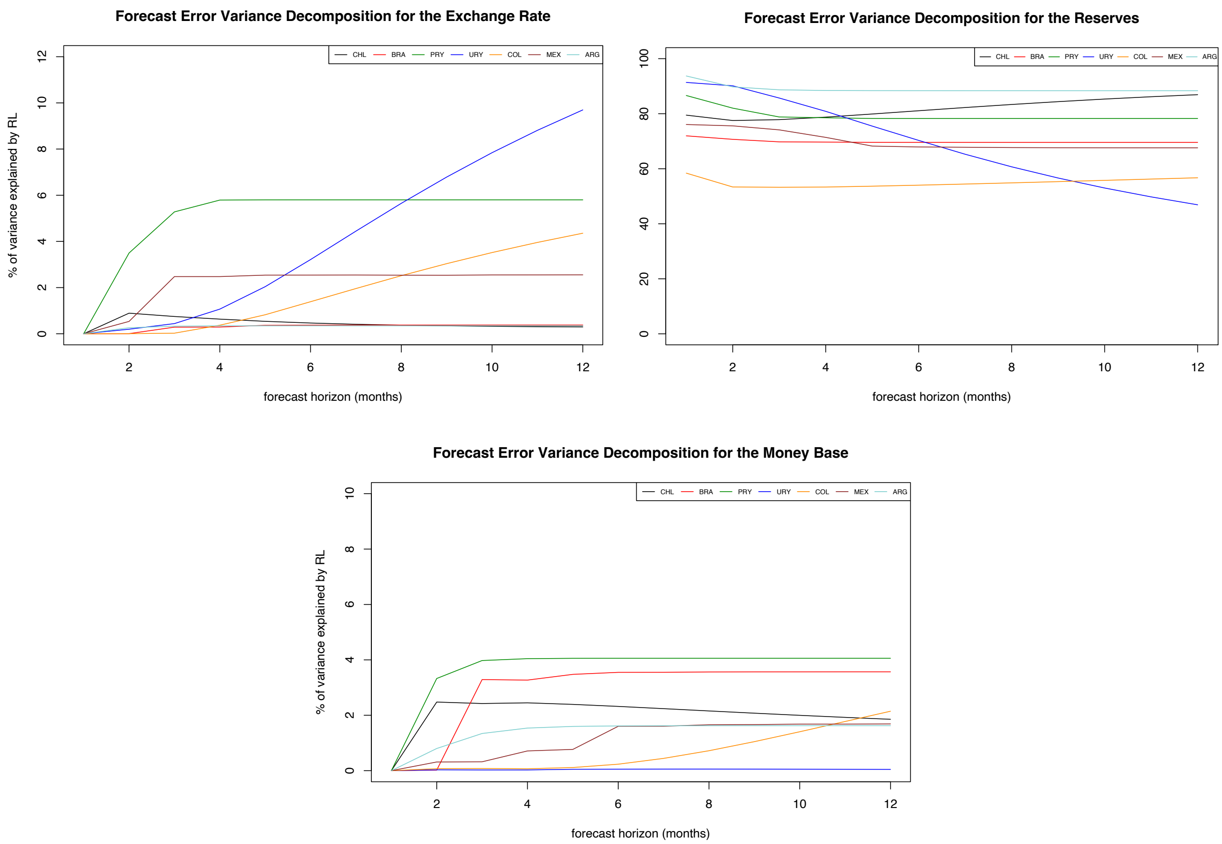
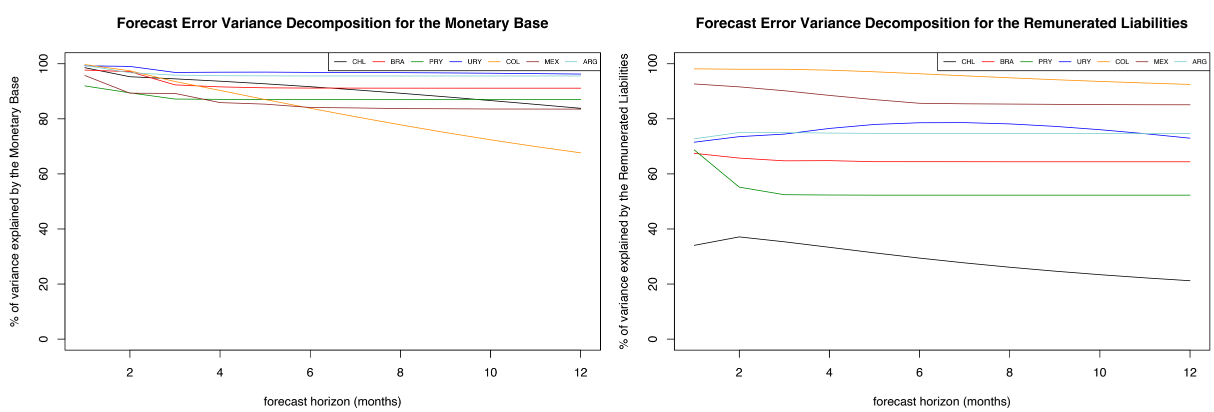
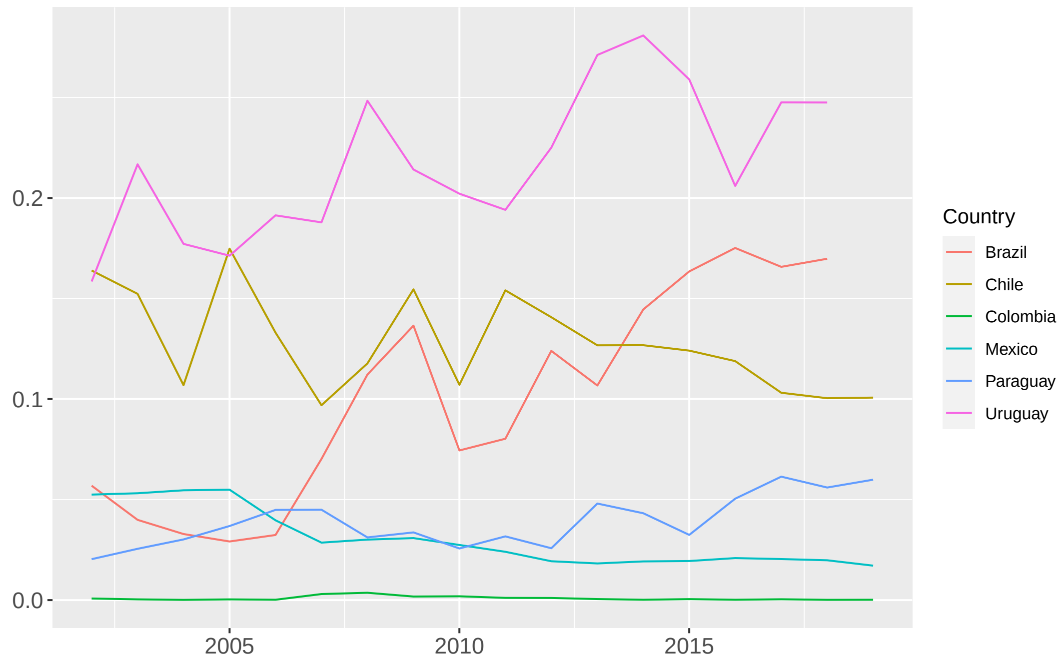


Figure A2: Percentage of Variance of Monetary Base (left) and Remunerated Liabilities (right) explained by Itself



C Plots and Tables

Figure A3: Evolution of Remunerated Liabilities as a fraction of GDP. 2004-2019



Source: Author's calculations based on IMF IFS.

Table A3: Balance Sheet Components over GDP

Country	Reserves	Financial Claims	Domestic Credit	Monetary Base	Remunerated Liabilities
Malaysia	0.40	0.03	0.00	0.10	0.22
Uruguay	0.25	0.00	0.14	0.06	0.22
China, P.R.: Macao	0.73	0.01	0.00	0.08	0.14
Jamaica	0.18	0.00	0.10	0.10	0.14
Algeria	0.76	0.00	0.03	0.23	0.13
Chile	0.15	0.00	0.02	0.07	0.13
Korea, Rep. of	0.24	0.01	0.01	0.06	0.12
Dominican Rep.	0.07	0.02	0.12	0.11	0.11
Brazil	0.14	0.01	0.18	0.10	0.10
Philippines	0.24	0.00	0.03	0.13	0.08
Bolivia	0.37	0.03	0.07	0.25	0.07
Romania	0.22	0.01	0.00	0.09	0.06
Papua New Guinea	0.14	0.00	0.02	0.07	0.06
Nigeria	0.12	0.02	0.03	0.05	0.06
Mongolia	0.18	0.05	0.02	0.11	0.05
Seychelles	0.22	0.00	0.15	0.16	0.05
Botswana	0.63	0.00	0.00	0.15	0.04
Belarus, Rep. of	0.10	0.03	0.01	0.06	0.04
Bhutan	0.52	0.01	0.00	0.28	0.04
Paraguay	0.16	0.01	0.02	0.11	0.04
Venezuela, Rep. Bolivariana de	0.17	0.00	0.04	0.12	0.04
Ghana	0.09	0.01	0.07	0.07	0.03
Mexico	0.12	0.01	0.00	0.05	0.03
Guatemala	0.14	0.01	0.07	0.14	0.03
Maldives	0.14	0.00	0.11	0.18	0.02
Sweden	0.09	0.02	0.02	0.04	0.02
Rwanda	0.14	0.00	0.02	0.04	0.02
Cambodia	0.31	0.00	0.01	0.19	0.02
Azerbaijan, Rep. of	0.15	0.02	0.01	0.13	0.02
Angola	0.17	0.01	0.02	0.07	0.02
Mozambique, Rep. of	0.17	0.00	0.02	0.09	0.02
Vanuatu	0.24	0.00	0.03	0.19	0.02
Ukraine	0.18	0.04	0.09	0.17	0.01
Namibia	0.12	0.01	0.00	0.04	0.01
Samoa	0.14	0.01	0.00	0.10	0.01
Georgia	0.15	0.02	0.04	0.12	0.01
Kyrgyz Rep.	0.28	0.01	0.03	0.18	0.01
Bangladesh	0.08	0.01	0.03	0.09	0.01
Australia	0.05	0.03	0.00	0.04	0.01
Tajikistan, Rep. of	0.08	0.02	0.04	0.13	0.01
Sri Lanka	0.10	0.00	0.03	0.06	0.01
Japan	0.01	0.05	0.35	0.42	0.01
Kenya	0.10	0.01	0.02	0.07	0.01
Tanzania, United Rep. of	0.11	0.00	0.02	0.07	0.01
Turkey	0.13	0.02	0.02	0.10	0.01
St. Kitts and Nevis	0.24	0.00	0.00	0.22	0.01
Antigua and Barbuda	0.16	0.00	0.02	0.17	0.01
Suriname	0.14	0.00	0.06	0.13	0.01
Fiji, Rep. of	0.17	0.00	0.02	0.16	0.01
Trinidad and Tobago	0.37	0.00	0.15	0.15	0.00
Uganda	0.12	0.00	0.06	0.05	0.00
Dominica	0.20	0.00	0.00	0.17	0.00
Grenada	0.17	0.00	0.00	0.15	0.00
Croatia, Rep. of	0.26	0.00	0.00	0.21	0.00
St. Lucia	0.14	0.00	0.00	0.13	0.00
Russian Federation	0.26	0.04	0.01	0.14	0.00
Armenia, Rep. of	0.18	0.03	0.00	0.15	0.00
Barbados	0.12	0.00	0.07	0.16	0.00
Guyana	0.22	0.00	0.13	0.19	0.00
Kuwait	0.19	0.00	0.00	0.11	0.00
St. Vincent and the Grenadines	0.16	0.00	0.01	0.15	0.00
Belize	0.15	0.00	0.07	0.17	0.00
Nicaragua	0.18	0.00	0.31	0.16	0.00
Anguilla	0.16	0.00	0.01	0.16	0.00
Morocco	0.24	0.03	0.01	0.24	0.00
Myanmar	0.03	0.01	0.15	0.17	0.00
Comoros, Union of the	0.16	0.00	0.03	0.12	0.00
Sierra Leone	0.12	0.00	0.14	0.07	0.00
Albania	0.21	0.01	0.07	0.25	0.00
Costa Rica	0.14	0.00	0.01	0.20	0.00
Colombia	0.13	0.01	0.01	0.08	0.00
Gambia, The	0.11	0.00	0.06	0.10	0.00
Canada	0.00	0.00	0.04	0.04	0.00
Zambia	0.09	0.00	0.11	0.07	0.00
Honduras	0.22	0.00	0.06	0.20	0.00
SÃ£o TomÃ© and PrÃncipe, Dem. Rep. of	0.00	0.00	0.00	0.00	0.00
Cameroon	0.09	0.00	0.02	0.09	0.00
Central African Rep.	0.11	0.00	0.13	0.13	0.00
Chad	0.07	0.01	0.05	0.08	0.00
Congo, Rep. of	0.23	0.00	0.05	0.15	0.00
Czech Rep.	0.31	0.00	0.00	0.28	0.00
Equatorial Guinea, Rep. of	0.14	0.00	0.02	0.08	0.00
Eswatini, Kingdom of	0.14	0.00	0.01	0.04	0.00
Gabon	0.10	0.00	0.04	0.09	0.00
Lesotho, Kingdom of	0.40	0.00	0.02	0.06	0.00