

**ECON 5322**

**Macroeconomic Theory for Applications**

**Topic 1: Course Overview and Introduction**

**Part 1: Macroeconomic data facts**

1. Course Logistics (syllabus and lectures plan)
2. Please note HW0 on course website

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3. Basic Business Cycle Facts
4. A Brief History of Modern Business Cycle Theory

## About me: Camilo Granados

- Assistant Professor of Economics – UT Dallas, EPPS
- Background: Ph.D. in Economics (U. Washington), B.S. and M.S. Economics and Postgraduate Diploma in Statistics (U. Nacional, Colombia)
- Born in Colombia
- Worked in a Central Bank for 13 years (09-22), including a leave for the Ph.D.
  - Economist, Research Economist
- Teaching experience:
  - UTD: international finance (ugrad), intermediate macroeconomics (ugrad), macroeconomics (MS), international finance and macroeconomics (PhD)
  - UW: advanced macro, international finance, business finance, intro to macroeconomics, MBA microeconomics, elements of statistics
  - Before the PhD: Advanced Econometrics, Advanced Microeconomics, Game Theory, Principles of Economics.

## General Teaching Philosophy

- Train students how to think about economics
- Ultimately, it is not just what you know but also how you think - synthesize and process information - that matters
- Lectures won't follow textbooks exactly; they draw out important concepts from some books and rely on study notes.
- Lecture slides available ahead of lecture to facilitate note-taking.
  - My advice: download them and take notes on top during class
- Slides with annotations are posted AFTER lecture
- Aim for more interaction - please "raise hands"

**Requirements:** See Syllabus (read it closely, It contains our “playing rules” and I will assume it is common knowledge to everyone)

**Textbooks:** Romer, Advanced Macroeconomics, 5ed., McGraw Hill and Chugh, Modern Macroeconomics, 2015.

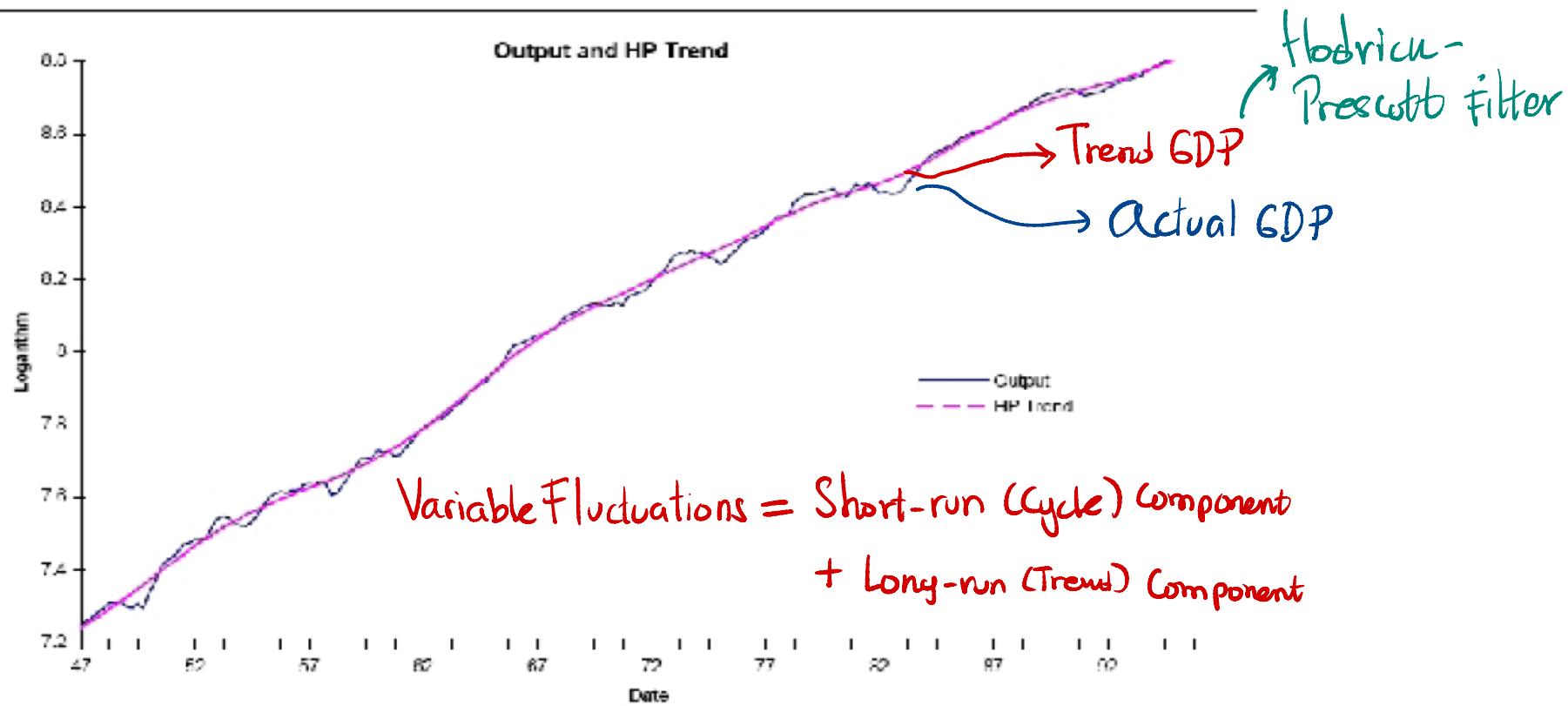
(we'll use them for some end of chapter problems but you can do the whole course with the slides and study notes only)

This course is not easy: heavy on math

Given that, I try to help:

- Most of the grading weight is put on take-home assignments rather than in-lecture tests
- The heavy lifting is put on the homework and practical projects
- If I see effort I am happy to adjust the grading weights along the way

# Some Business Cycle Facts (for the U.S. 1947:1 – 1996:4)



Decompose "Trend" vs. "Cycles" in a time series

$$y_t = y_t^T + y_t^C \text{ (e.g., with } \underline{\text{filters}}\text{)}$$

$$GDP_t = GDP^{\text{Trend}} + GDP^{\text{Cycle}}$$

Wiggles or fluctuations around trend

$$y_t^{\text{Cycle}} = y_t - y_t^{\text{Trend}}$$

HP Filter ( $\lambda=1600$ )

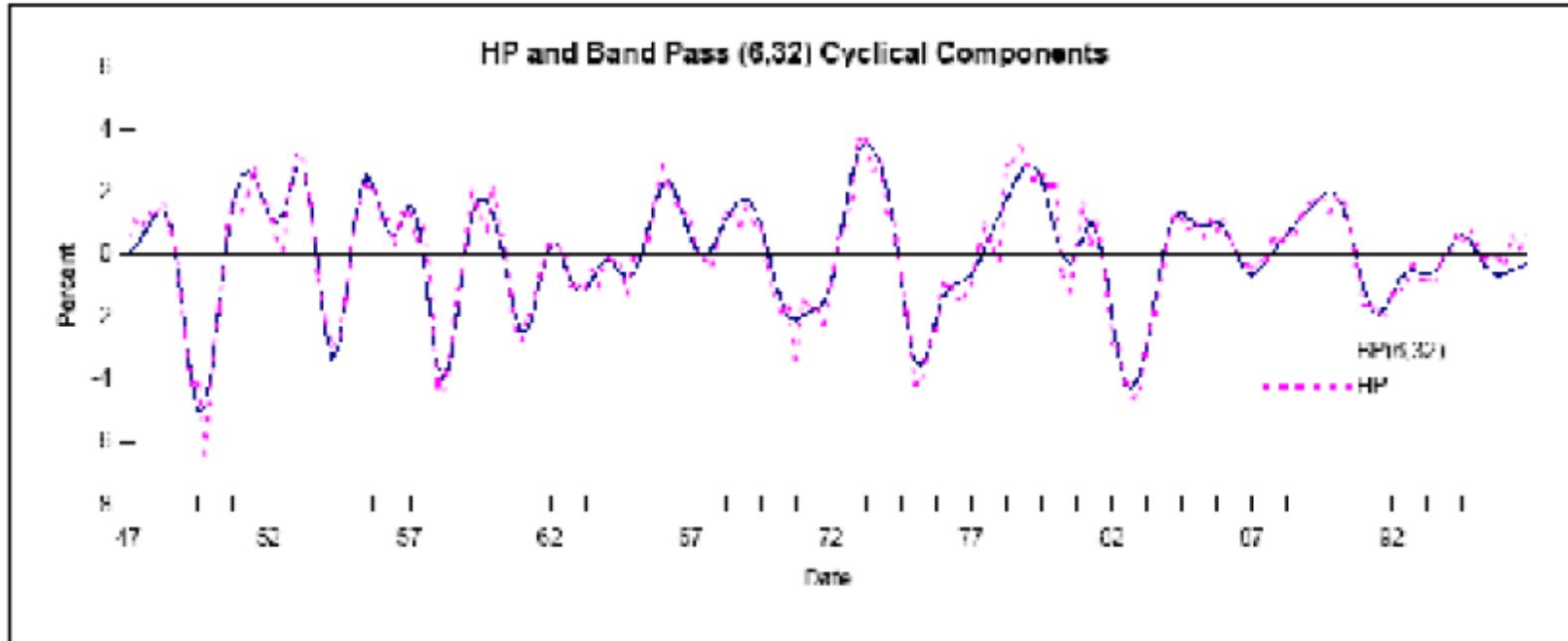
Other: Band-pass filter, Christiano-Fitzgerald filter; Linear-trend; Linear-quadratic trend

## Cyclical Component

high frequency fluctuations

$$y_t = \alpha + \beta_1 t + \beta_2 t^2 + \varepsilon_t$$

$\hookrightarrow$  Cycle



Common filters: linear, Hodrick-Prescott (HP), Bandpass

Intuitively: a way to extract the long-run growth portion or the “low frequency” components of the data

## Business Cycle Dating

NBER → determine whether we are in a recession

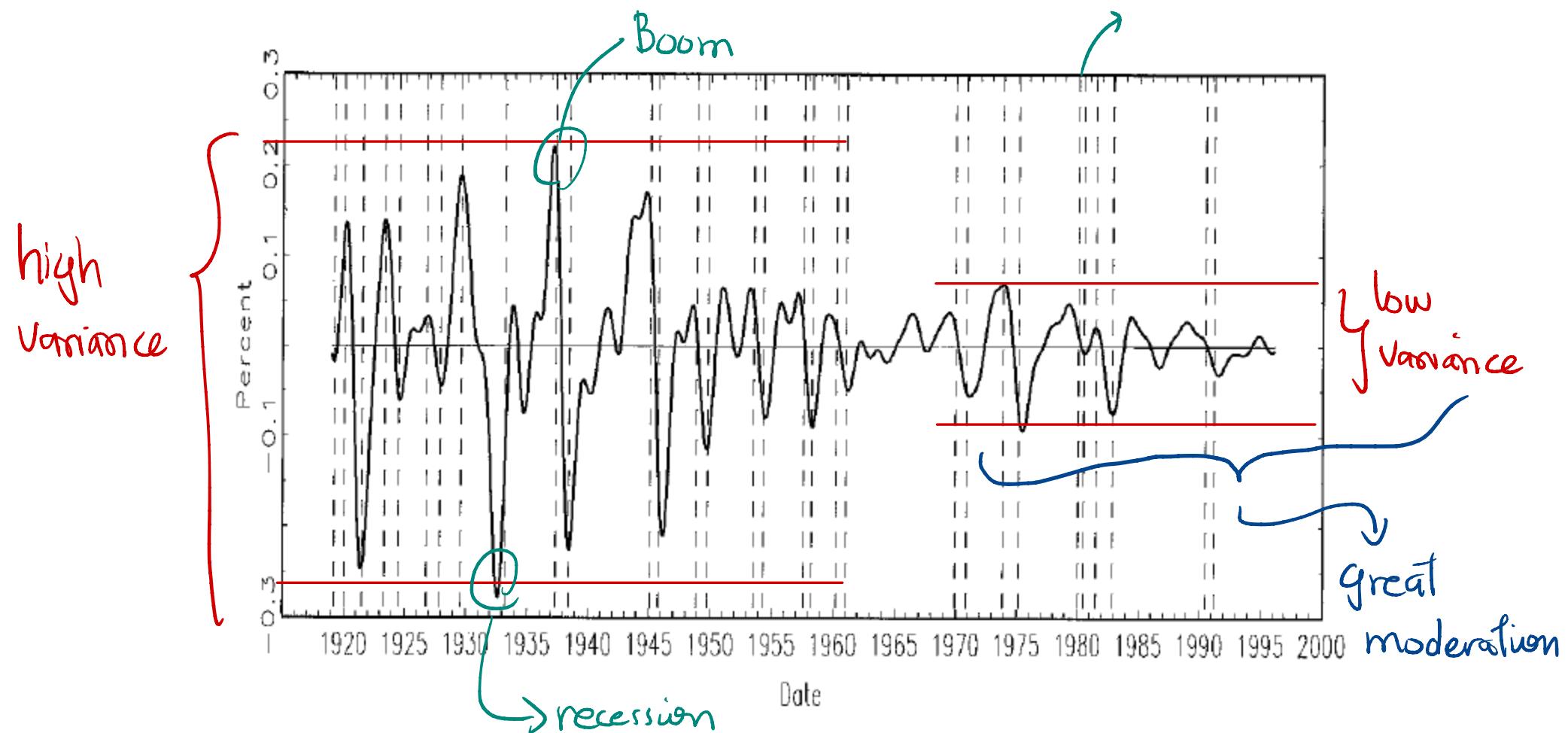


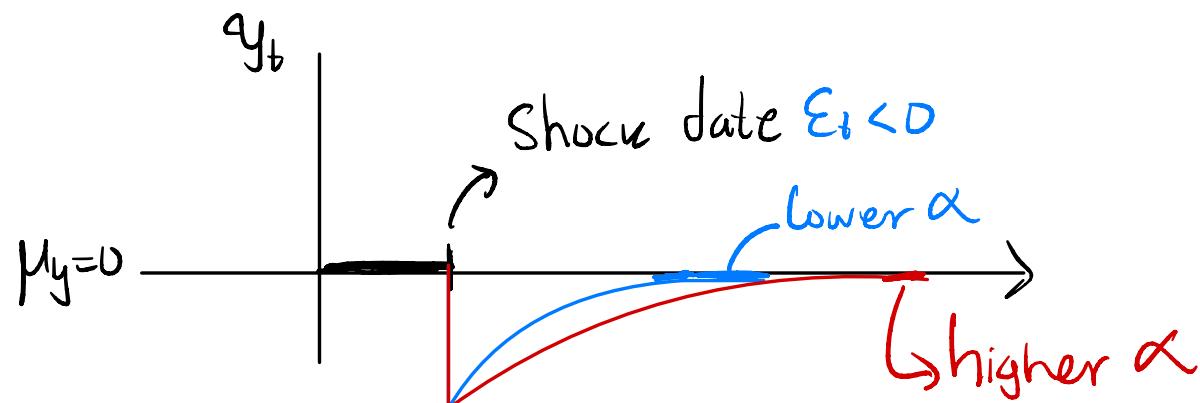
Fig. 1.2. Business cycle component of industrial production index.

## Some Stylized Facts about the U.S. Business Cycles:

What do we look for?

1. Volatility/amplitude of fluctuations → Variance of the cycle
2. Comovements → Covariances & Correlations w/ economic variables
3. Persistence/lead and lag patterns → Autoregressive patterns

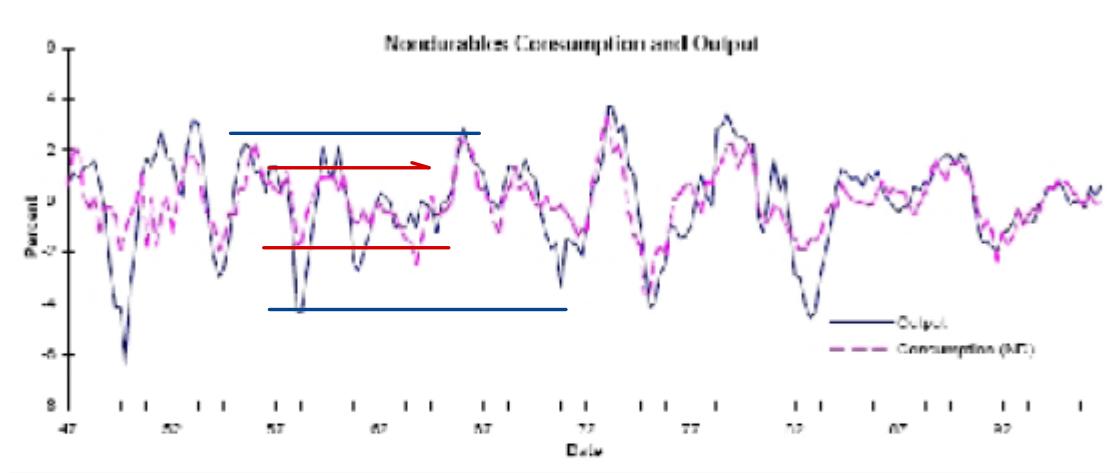
$$AD \text{ process: } Y_t = \mu_y + \alpha Y_{t-1} + \epsilon_t$$



$$Y_t = C_t + I_t + G_t + \cancel{NX}_t^0 \text{ (Closed economy)}$$

## Nondurable consumption

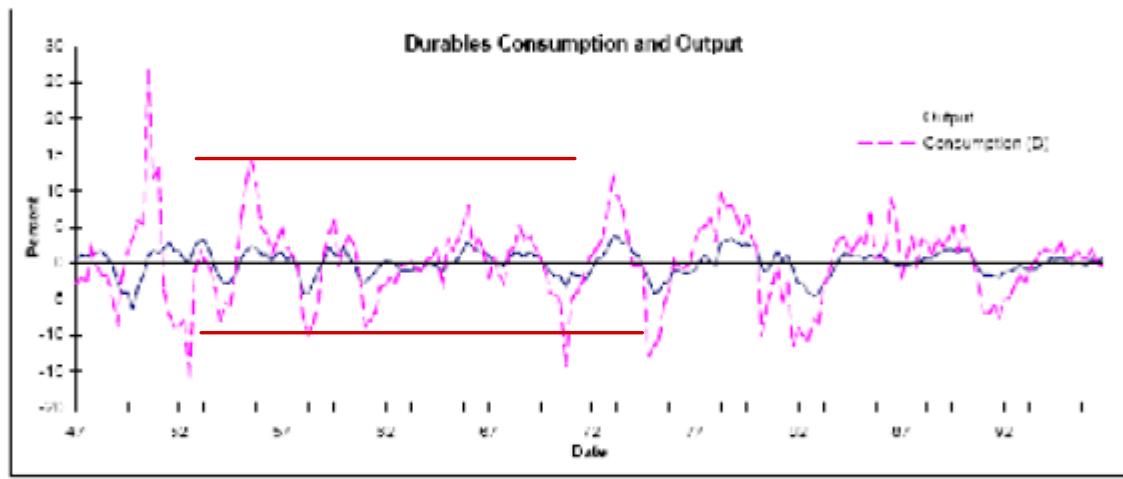
- Non-durable
- Durable



$$\sigma_{C, \text{nondurable}} < \sigma_y$$

\* Less volatile than output

# Consumer durables

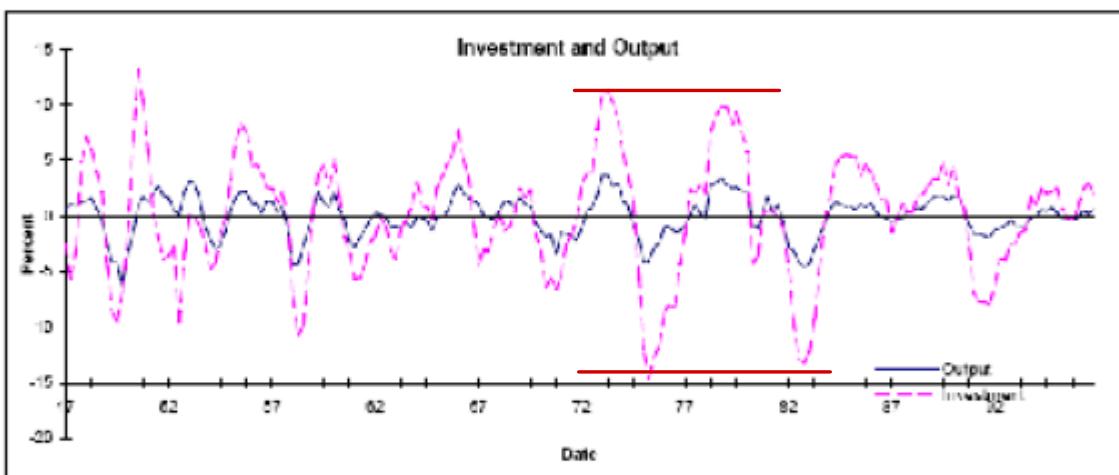


$$\sigma_{C,\text{nondurable}} < \sigma_y$$

\* More volatile than output

$$\sigma_{C,\text{durable}} > \sigma_y$$

# Investment



$$\sigma_{C,\text{non-durable}} < \sigma_y$$

\* 3 times more volatile than output

$$\sigma_{C,\text{durable}} > \sigma_y$$

$$\sigma_I > \sigma_y$$

$$Y_t = C_t + I_t + G_t$$

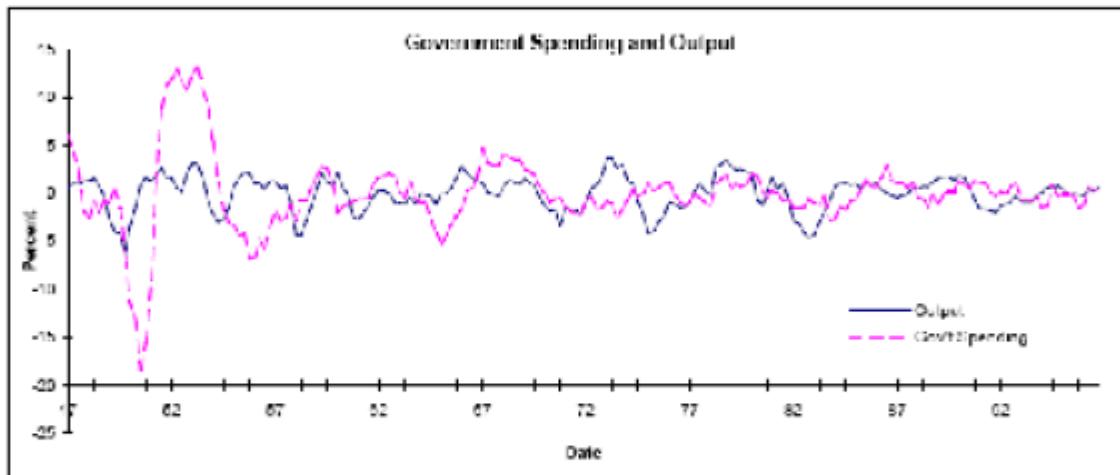
## Government expenditures

$$\sigma_{C,\text{nondurable}} < \sigma_y$$

$$\sigma_{C,\text{durable}} > \sigma_y$$

$$\sigma_I > \sigma_y$$

$$\sigma_G < \sigma_y$$

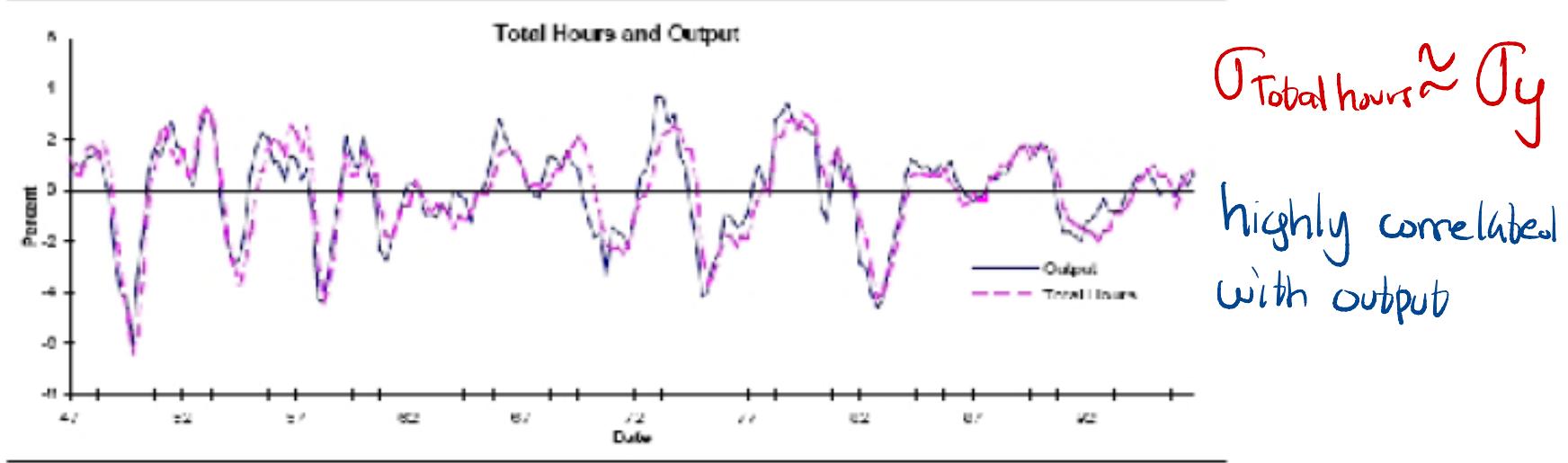


- \* Less volatile than output

## Total hours worked

( $N \times$  hrs per worker)

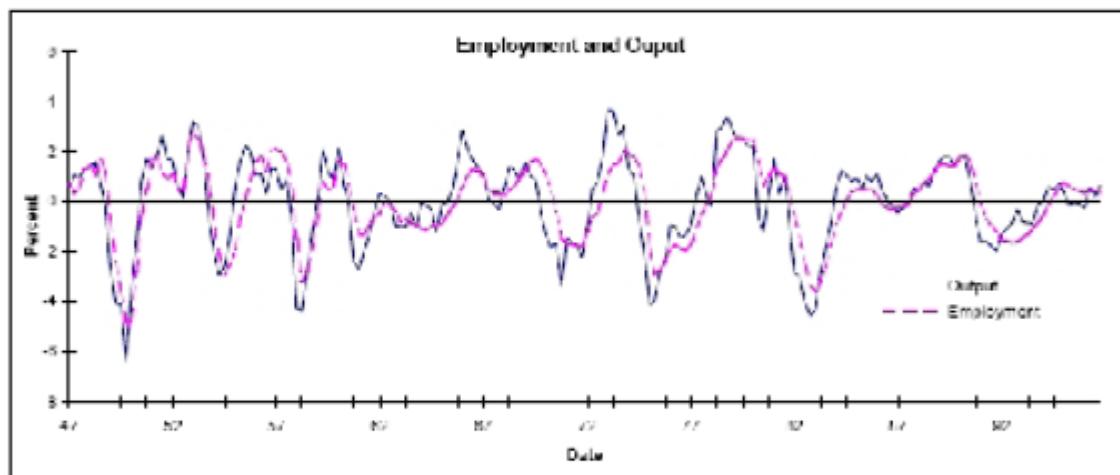
N: Number of workers



- About as volatile as output
- Business cycle is most clearly manifested in the labor market

# Employment

↪ # of workers



$$\sigma_N \approx \sigma_y$$

- as volatile as output

## Hours worked

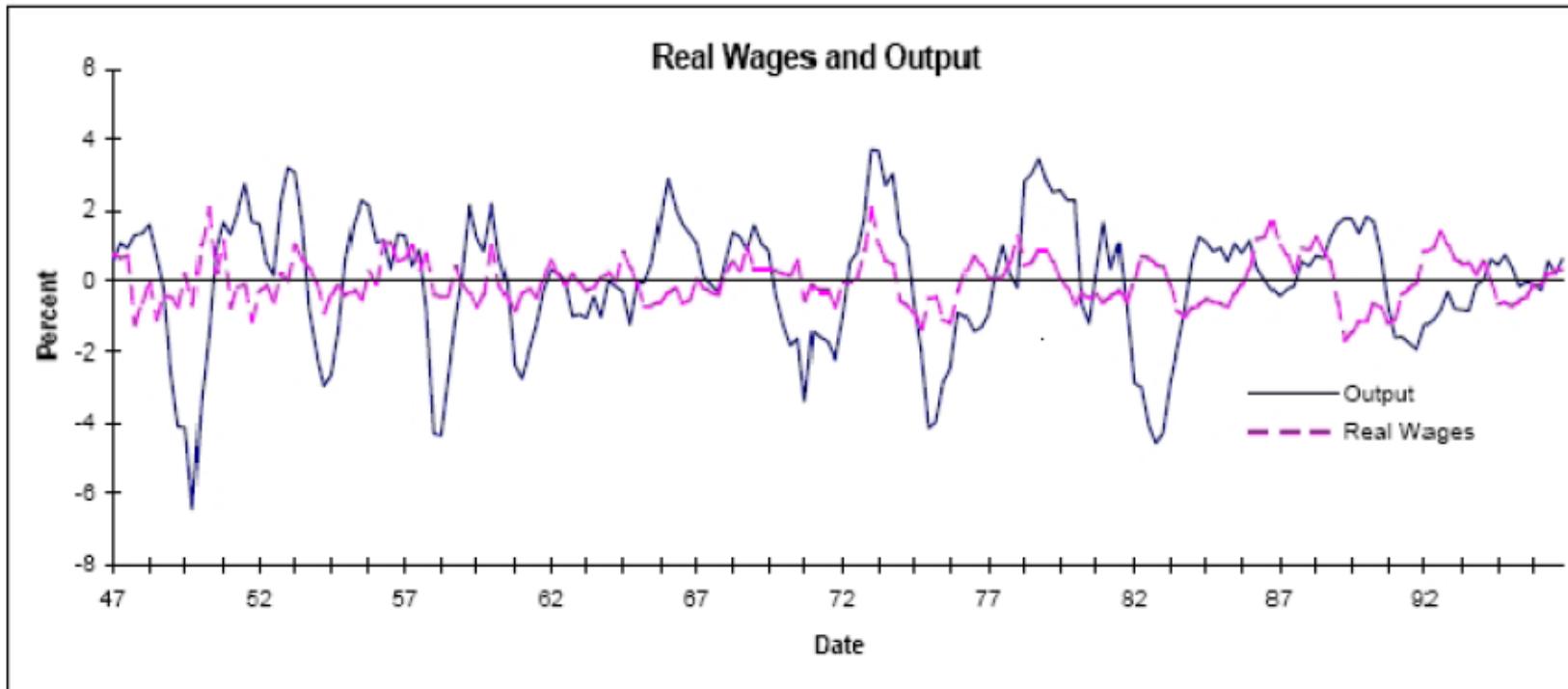
$$\sum \text{Hours} < \sum y$$



- Hours per worker: Much less volatile than output
- Most variation in total hours stems from changes in employment, rather than adjustment in hours worked per employee.

$$\text{real wage} = \frac{W}{P} \xrightarrow{\substack{\text{Nominal Wage} \\ \text{Prices level}}} \text{Real Wage} \sim MPL \left( \frac{\partial Y}{\partial L} \right)$$

$$\log \text{real wage} = \log W - \log P$$



- Much less volatile than output
- Slightly PRO-cyclical (correlation = 0.14) => important fact

④ Correlated w/ GDP cycle

(≈ barely procyclical or even acyclical)

Are we still going through a great moderation?  
(HW1)

## An Obvious Question

- What do the US business cycle statistics look like post-1999 & post-2007?  
Are the patterns described above still true? How about other countries?
- How to produce the basic business cycle statistics: discussed in the first two sections of King and Rebelo (1999) Handbook chapter

↳ alternative: chp 1 of Open Economy Macroeconomics  
by Uribe & Schmitt-Grohe (2017)

## Data Sources:

- for the US: FRED
  - <https://fred.stlouisfed.org/> (HW1)
- international macro: IMF-International Financial Statistics (IFS)
- World Bank - WDI

Can you see in the data the following view?

- Old Macro: Analyzes pre- versus post- 1984:Q4. Great moderation
  - New Macro: Analyzes pre- versus post- August 2007 Global Financial Crisis;  
Pandemic crisis/Inflation Surge
    - End of the **Great Moderation**
    - Downturn precipitated by disruption of **Financial Intermediation**
    - **Unconventional Monetary Policy** and **Zero Lower Bound**, balance sheet management, macro-prudential policy...
  - COVID-19 macro dynamics → Supply or Demand Shock? or both?  
How transitory
    - New inflation dynamics → Inflationary Consequences  
of Policy Stimulus
- 
- The diagram illustrates three types of economic cycles using black and teal oscillating lines on a grid background. The top line is labeled 'Countercyclical' and shows a smooth, wave-like pattern where peaks and troughs occur at different times. The middle line is labeled 'Acyclical' and shows a horizontal line with small, frequent oscillations around it. The bottom line is labeled 'Pro cyclical' and shows a pattern where peaks and troughs occur simultaneously for both lines.

As we go through models after models after models\*... **ALWAYS** ask yourself:

- 1.What is the Motivation behind the model?
- 2.What is the Economic intuition?
- 3.What is the relevant Technique/Tool to pick up?
- 4.What does the Data say?
- 5.What are some Alternatives to model or test the same phenomenon?

=> Learn not (just) their thoughts, but how they **THINK** (how to approach and formalize the issue at hand)

## **Key Questions (Extra Credit)**

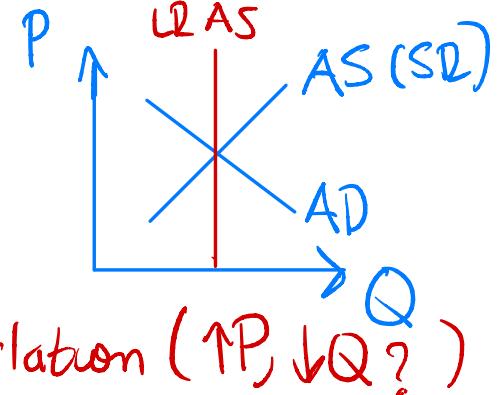
- What are some “stylized facts” about US Business cycle dynamics up?
- What is Neoclassical Synthesis?

## Part 2: From IS-LM Neo Classical Synthesis to New Synthesis

A. Some course logistics – **important dates: posted on course website!**

B. Brief History of Modern Macro

- The Neoclassical Synthesis  $\rightarrow$  IS-LM / AD-AS
- The Breakdown of the Consensus  $\rightarrow$  1970's Stagflation ( $\uparrow P, \downarrow Q?$ )
  - o Price Adjustment: Phillips-Solow-Samuelson vs. Friedman-Phelps
- Is the Long-Run Phillips Curve Really Vertical?
- Rational Expectations Revolution
- Main Approaches to modeling Aggregate Supply since then
- The New-Neoclassical Synthesis  $\rightarrow$  RBC toolkit + Keynesian market  
Frictional ideas.  
Topic 2



## Housekeeping:

- First midterm is done after Topic 2
- + - The 6 Homework assignments are submitted in ~~class~~ eLearning
  - First Assignment (HW0) due next week: Get full credits just by submitting (only for HW0)
- A Final Exam, at the end of the semester
- + • Last week of class: group project presentations
- + • Reminder: Extra credit option: submitting answers to “Key Questions” at the end of ~~each lecture (before the start of the next)~~ the course

## A VERY Brief History of Macro:

(IS-LM: AD - AS  $\rightarrow$  Intermediate Macro)

- Neoclassical Synthesis
- The Breakdown of the Consensus + the Rational Expectations Revolution
- Main approaches to modeling Aggregate Supply in the 80s & 90s
- “New” Neoclassical Synthesis → {  
    Old lessons from Initial Synthesis  
    + Rational Expectations  
    + Intertemporal decision making}
- Post-2008: Financial Friction, “Unconventional” Monetary Policy...etc.
- Now?  $\rightarrow$  DSGE     (RBC + Market Frictions)

I. **Neoclassical Synthesis**: Consensus in the 1960's (Review of intro to macro)

a. **Aggregate Demand**: from IS-LM framework: **goods market** and **money market** equilibria, and Walras Law implies **asset market clearing** → (Supply = Demand)

① Goods Market  $Y = C(Y - T) + I(r) + G + NX$

$\overset{i - \pi}{\uparrow}$  Ignore  
for now

$+$   $-$

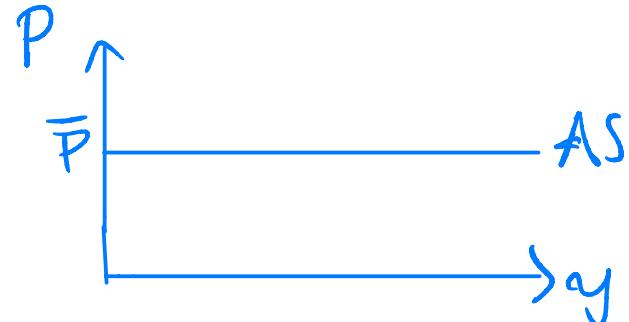
② Money Market  $\frac{M^S}{P} = L(i, Y)$

+ Walras law: If  $N-1$  markets are in equilibrium, then the  $N$ -th market is in equilibrium (assets market)

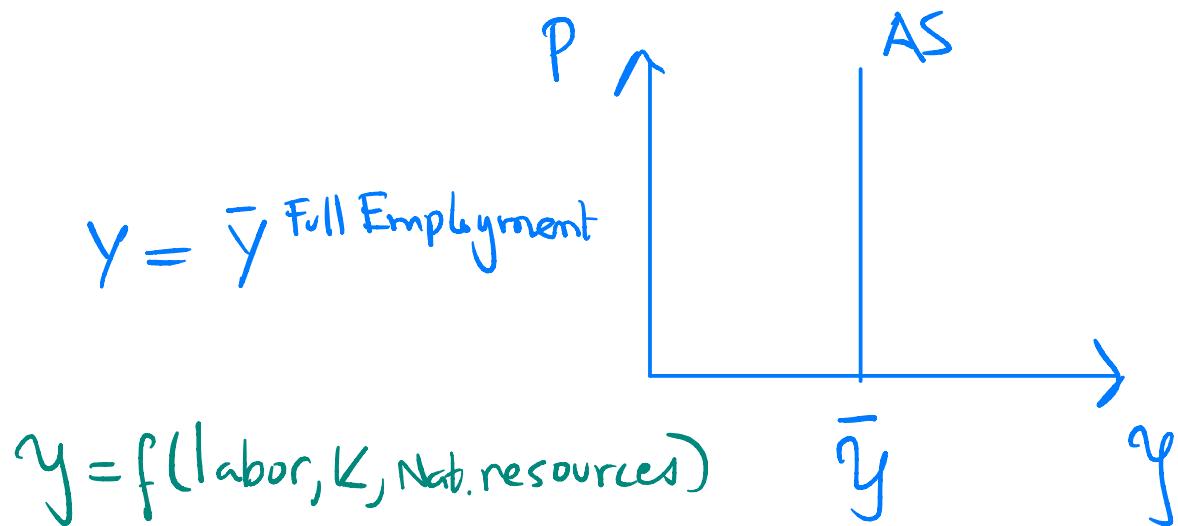
⇒ Obtain Aggregate Demand (from IS-LM equilibrium)

b. Aggregate Supply: Keynesian vs. Classicalist: Are prices sticky?

i) Keynesian Nominal rigidity (sticky prices/wages) => Short-run

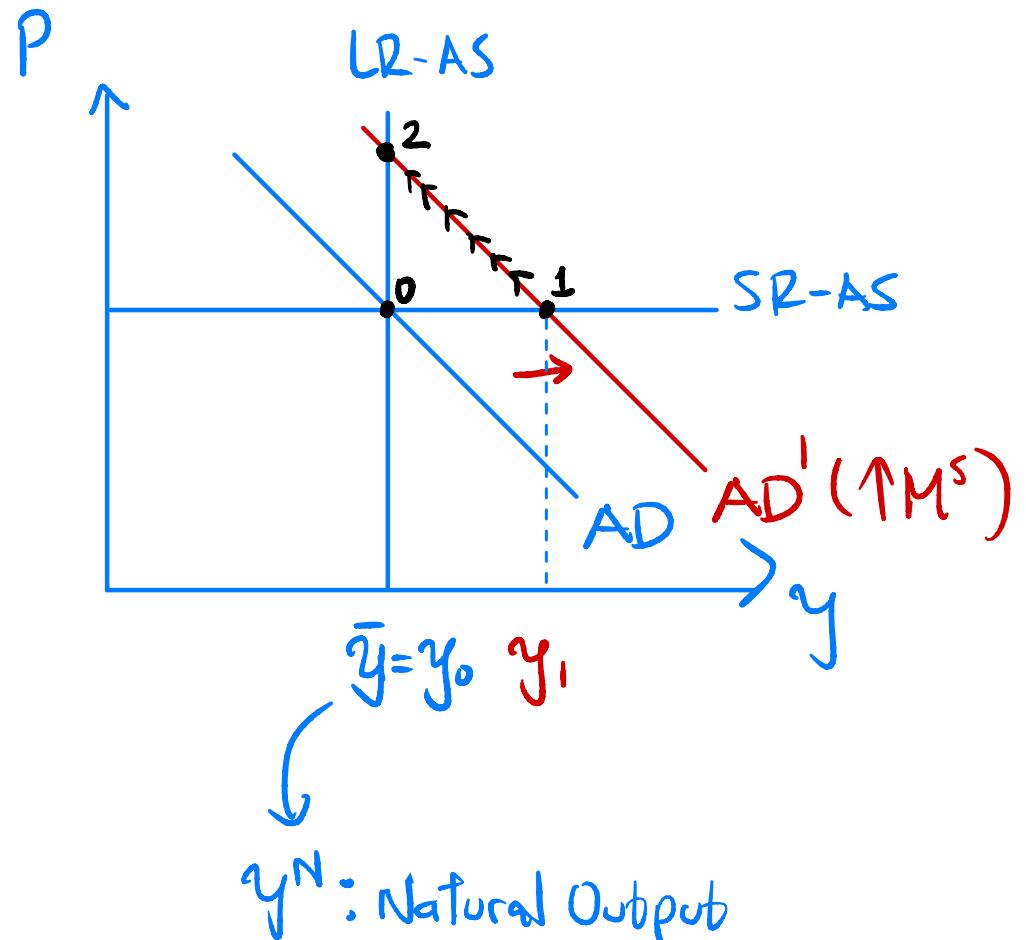


ii) Natural Rate Hypothesis (prices/wages fully flexible) & monetary neutrality => Long-run



Neoclassical Synthesis: put relationships above together,

- with 1) and 2) giving AD
- i) for Short-Run AS
- ii) for Long-Run AS



How do we adjust from SR to LR?

$$\text{SR: } \underbrace{y > y^N}_{\text{Economy is overheated}} \longrightarrow \uparrow P$$

(\uparrow \pi : \text{Inflation})

Economy is overheated

c. linking SR and LR with **price-adjustment dynamics: the Phillips**

Curve:

$$\pi = \alpha (Y - Y^N) = -\beta(u - u^N)$$

↗ Natural unemployment  
↓ Unemployment rate

$$\begin{cases} \text{corr}(y, N) > 0 \\ \text{corr}(y, u) < 0 \end{cases}$$

## How well did this “model” do?

- Data in the 50's-60s supported the above => tradeoff between  $\pi$  and  $u$ , providing **scope for policy actions**
- At the applied level: refinements of above
  - large-scaled models: “MPS”, Harvard, Fed models with several hundred of equations
  - these models are based on empirically observed relationships (between output and consumption, money demand, in unemployment... etc)
  - the aim of these models was to predict the effects of policies
  - they were pretty successful at it until the 1960s
- Bob Solow: “Macroeconomics is finished” (as in done/completed!)

# The Breakdown of the Consensus in the early 1970s

a. Empirically:

Models couldn't explain the simultaneous rising inflation AND unemployment in the 1970s: Vietnam War, G↑, M<sup>S</sup>↑ => π↑ but no u↓

b. Theoretically:

- Friedman (1968), Phelps (1968): Phillips' curve cannot be right!

Violation of the **Natural Rate Hypothesis**: Long run unemployment should NOT depend on the average rate of money growth, i.e. What if Fed changes the money growth rate?? Say from 0% to 5%, =>  $\pi = 5\%$  =>  $u \downarrow$  and  $Y \uparrow$  in the LR!

=> **Expectation-augmented Phillips Curve**

$$\pi = -\beta(u - u^N) + \gamma E\pi$$

as policy changes from 0 to 5% (money growth)  
 $E[\pi]$  from 0 to 5%  
(assumes  $\gamma \approx 1$ )

Compatible if

- Lucas Critique ('73) and the **Rational Expectation** revolution
  - When evaluating policy, need to consider the feedback with expectations: if policy maker changes the rule, public expectation will adjust as well, so the equilibrium condition for the economy will change too.

=> All of the above point to the “danger” of using ad hoc, reduced-form empirical relationships with no “micro-foundation”!

Correlations

Tom Sargent, “Macro is finished”... (as in “destroyed”)

20+ years of confusion and division to follow....

## Lucas Critique ('73) and the Rational Expectation Revolution

- Expectation-augmented Phillips curve:

$$\pi = -\beta(u - u^N) + \gamma E\pi$$

↗ Feedback of expectations into  $\pi$   
 $\gamma E\pi$   
↖ Expected Inflation

Or:

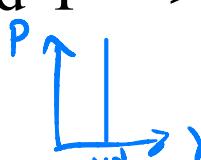
$$\pi_t = a - bu_t + \gamma E_{t-1}\pi_t$$

Note: in the long-run,  $\pi = E\pi$  (by definition, of LR)

long run :

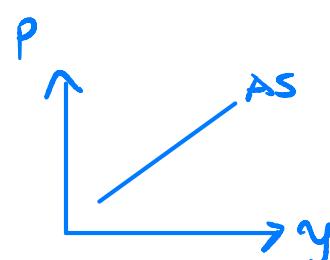
- If  $\gamma = 1$ , then  $u = u^N$ , so LR, at  $u^N$  and  $Y^N \Rightarrow$  LR-AS is vertical

$$\gamma=1 \Rightarrow \pi = -\beta(u-u^N) + \cancel{\gamma} \pi \rightarrow u=u^N (\gamma=\gamma^N)$$



- Otherwise,  $u$  will depend on both  $u^N$  and  $\pi \Rightarrow$  LR-AS is sloped ( $Y$  or  $u$  depend on prices/inflation)

$$(\gamma \neq 1, \gamma < 1) \xrightarrow{LR} \pi = -\frac{\beta}{1-\gamma} (u - u^N)$$



## How to Measure/Model $E\pi$

$$\pi_t = a - bu_t + \gamma E_{t-1}\pi_t$$

①   
→ ②

- It is therefore important to know the value of  $\gamma$ ,
- To gauge its value: look into past data of inflation and unemployment,

BUT:

- also need to know how to measure and model  $E\pi$

# Lucas Critique ('73) and the Rational Expectation Revolution

- Nobel Prize 1995
- Much of what economists were doing and the policy conclusions were **WRONG**
  - Using a model with fixed coefficients estimated from reduced-form equations and historical data to evaluate the effects of new policy would give misleading results because **expectations need to be endogenous**
  - i.e. Changes in policy will affect expectations
- True whenever expectations are forward-looking (need not be rational)

“Rational” = **model-consistent**

## Three Methodological Tenets for Rational Expectations<sup>1</sup>

- Results widely applicable; use Phillips Curve as an example below

To form “model-consistent” rational expectations:

### 1. Partial Equilibrium.

1 market + Account by stochastic behavior of  $E_t \Pi_t$

Agents form expectations appropriately given the **stochastic process** generating the variables of interest. Expectations cannot be specified without first specifying the underlying stochastic process.

e.g. Given  $\pi_t = a - bu_t + \gamma E_{t-1} \pi_t$  (\*), want to know  $\gamma$

=> first specify how  $\pi_t$  is determined

e.g.  $\Pi_t$  follows an AR(1) process

$$\Pi_t = \rho \Pi_{t-1} + \varepsilon_t$$

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$$\hookrightarrow 0 < \rho < 1$$

Shock (white noise)

$$\begin{cases} E(\varepsilon_t) = 0 \\ E(\varepsilon_t^2) = \sigma^2 \\ E(\varepsilon_t, \varepsilon_s) = 0 \end{cases}$$

<sup>1</sup> N.G. Mankiw

Explain Expectation Operator (and take  $E_{t-1}\pi_t$  given it is AR(1))

$E[X_b]$ : Expected value of  $X_b$

$E_{t-1}[X_t]$ :  $E[X_t | \text{Information up to } t-1] = E[X_t | I_{t-1}]$

## Properties

①  $E[\cdot]$  is a linear operation

②  $E[\alpha \cdot X_t] = \alpha E[X_t]$

( $\alpha$  is a constant)

$$\left. \begin{aligned} E_{t-1}[T_b] &= E_{t-1}[\rho T_{b-1} + \epsilon_b] \\ E_{t-1}[T_b] &= E_{t-1}[\rho T_{b-1}] + E_{t-1}[\epsilon_b] \\ E_{t-1}[T_b] &= \rho E_{t-1}[T_{b-1}] + E_{t-1}[\underbrace{\epsilon_b}_{=0}] \\ E_{t-1}[T_b] &= \rho T_{t-1} \end{aligned} \right\}$$

Given the stochastic process,  $E_{t-1}\pi_t = \rho\pi_{t-1}$  under Rational Expectation

Plug this into (\*), we see that

$$\pi_t = a - bu_t + \underline{\gamma\rho\pi_{t-1}}$$

Plug it into PC

1) the relation b/w inflation and unemployment depends on lagged inflation ( $\pi_{t-1}$ ) too.

2) Can we just find  $\gamma$  by looking at the correlation b/w inflation and unemployment? No. Without specifying  $\rho$  first, we cannot determine  $\gamma$

## // 2. General Equilibrium.

The stochastic process of any variable generally depends on the entire model. To solve for expectations, one must assume that agents know and solve correctly the model of the economy.

- In our case, it means endogenizing  $\pi_t$ : have it determined from within the system
- We will adopt a variant of the Phillips curve (\*) instead too.

Consider an economy described by a simple General Equilibrium (GE) model (note, all variables are in logs, allowing us to use linear equations):

$$1) m_t - p_t = y_t \quad (\text{AD})$$

$$2) y_t = \alpha(p_t - E_{t-1}p_t) \quad (\text{AS})$$

$$\hookrightarrow y_t = \alpha \left( \pi_t - E_{t-1}\pi_t \right)$$

~~$p_t - p_{t-1}$~~        ~~$E_{t-1} - E_{t-2}$~~        ~~$\pi_t - \pi_{t-1}$~~

$$① m_t - p_t = y_t \quad (\text{AS})$$

$$② y_t = \alpha(p_t - E_{t-1}p_t) \quad (\text{AD})$$

from (1)  $p_t = m_t - y_t$

Take  $E_{t-1}$ :  $E_{t-1}p_t = E_{t-1}(m_t - y_t) \quad (1')$

from (2):  $E_{t-1}[y_t] = \alpha E_{t-1}(p_t - E_{t-1}p_t)$   
 (Take  $E_{t-1}$ )  
 $= \alpha E_{t-1}p_t - \underbrace{\alpha E_{t-1}[E_{t-1}p_t]}_{E_{t-1}p_t} = 0$

$$E_{t-1}[E_{t-1}[x_t]] = E_{t-1}[x_t]$$

$\Rightarrow$  in (1'):  $E_{t-1}p_t = E_{t-1}m_t$

back in AS:  $y_t = \alpha(p_t - E_{t-1}p_t) \Rightarrow y_t = \alpha(m_t - y_t - E_{t-1}m_t)$

What does this result mean?

$\Rightarrow$  Solve for  $y_t$ :

$$y_t = \frac{\alpha}{1 + \alpha} [m_t - E_{t-1}m_t] \quad (\text{AS}^{\text{New}})$$

$\Rightarrow$  Output depends on “surprised” or unexpected money

Policy rule : Framework according to which a  
Policy is set

### 3. Policy Evaluation (where does $m_t$ come from?)

- The rules governing policy are among the equations in any complete model of the economy
- Because a change in a policy rule alters the stochastic processes generating many variables, it also changes people's expectations
- Hence, when evaluating alternative policy rules, one must take account of this feedback between policy rules and how expectations are formed.
- Any policy evaluation that fails to take account of this feedback is flawed and useless (Lucas Critique)

$$m_t - m_{t-1} = \mu_A + \varepsilon_t$$

+

(3) Consider a simple monetary policy rule:  $m_t = \mu_A + m_{t-1} + \varepsilon_t$  (3)  
where  $\mu_A$  ("mu" \_A) is a constant and  $\varepsilon_t$  is a white noise process gain

Note: since these variables are in logs,  $\mu_A = \underline{E(m_t - m_{t-1})}$

represent the **expected growth rate of money** or the **trend money growth rate** (Remember: log (small) differences = % change or growth rate.)

Combining (1)-(3) to solve the GE model:

$$E_{t-1}m_t = \mu_A + m_{t-1}$$

$$\Rightarrow y_t = \frac{\alpha}{1+\alpha} [m_t - m_{t-1} - \mu_A]$$

- This tells us that in general equilibrium, the level of output at each time depends on both money growth from the previous period ( $m_t - m_{t-1}$ ), but also on the general trend growth rate set by monetary policy ( $\mu_A$ )

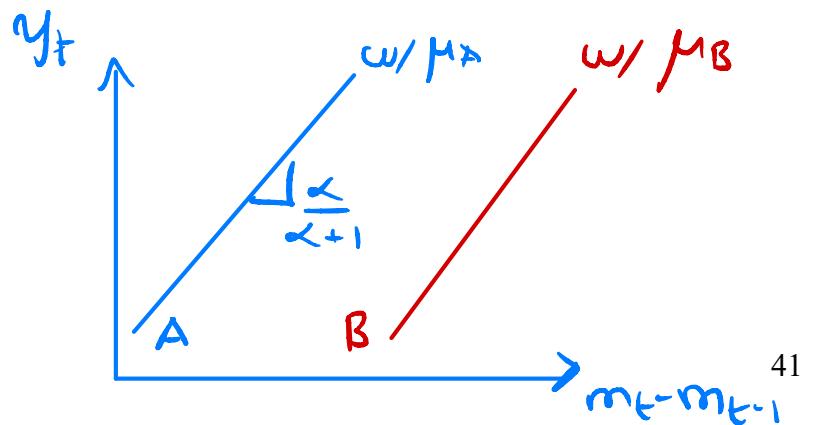
## Policy implication from this rational expectation general eq. model:

- Output level is positively correlated with money growth from period-to-period. However, this relationship does NOT imply **policy tradeoff** because the line above is only valid under the policy rule (3) with  $\mu_A$
- If policymaker increases money growth rate from point A to point B (e.g. by changing money growth from  $\mu_A$  to  $\mu_B$ ), (3) would no longer be the correct rule. The rule would have trend money growth  $\mu_B$  instead
- People forming rational expectations would adjust as well based on the new rule. Resolving the system, we see that the curve **shifts to the right** (to reflect the new output-money growth relationship)

New:  $m_t = \mu_B + m_{t-1} + \epsilon_t$  ( $\mu_B > \mu_A$ )

$\Rightarrow$  New AS should be

$$y_t = \frac{\alpha}{\alpha+1} (m_t - m_{t-1} - \mu_B)$$



## Summary:

- When evaluating policy, need to take into account the feedback with expectations.
  - If policy maker changes the rule, expectation will change so the equilibrium condition for the economy will change too.
- ⇒ All of the above point to the “danger” of using ad hoc, reduced-form empirical relationships with no “micro-foundation”!

## **Results of the Rational Expectations revolution:**

- 1) More focus on structural modeling of the economy: micro-foundation based on first principles, rather than estimating reduced-form equations. Identification: correlation does not equal causality

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- 2) Taking expectations seriously  
Lucas Critique does NOT imply policy ineffectiveness, but that policy making should focus on developing on-going strategy and long-term rules, not one-time change

⇒ **Four Main Approaches to Aggregate Supply**

[Detour] [This is a good point for checking the Technical Slides #1]

Technical Slides:

- Stochastic Processes/Time series review
- Expectations (def. and properties)
- MATLAB intro

[End of detour]

# Four Main Approaches to Aggregate Supply

Policy intervention is unnecessary

Markets are frictionless  $\Rightarrow$

		Do Markets Clear? (instantaneously)	
		Yes	No
Is Money Neutral? (in medium, Long run)	Yes	<b>1. Classical/RBC</b> Kydland & Prescott Minnesota (Topic 2)	<b>3. Real Rigidity</b> e.g. Efficiency wage theory Akerlof, Yellen
	No	<b>2. Imperfect Information</b> Friedman Lucas '77	<b>4. Nominal Rigidity</b> Nominal contracts, menu costs: Fischer, Taylor, Calvo ...

There are market frictions such as rigid prices that create inefficiencies in private market that lead to suboptimal outcomes = Role for policy intervention

## Unification Again Since the 1990's

“**New** Neoclassical Synthesis”

New Keynesians  
RBC toolkit + Market Frictions

Real Business Cycle tools (Romer Ch.5) + New Keynesian (Ch. 6) ideas:

- Dynamic Stochastic General Equilibrium Modeling → Toolkit
- Business cycle may be caused by real shocks, but nominal rigidity  
leads to inefficiency, hence role for policy ↗ From RBC ↘ Frictions

## Key Questions (Extra Credit, due next session)

- What is the Lucas Critique?
- Intuitively, how do the three perspectives of rational expectations differ (and/or improve, complement each other)?
- How is Lucas' rational expectation captured within economic models?
- What does money neutrality mean?
- What is the Neoclassical Synthesis? And what were its main problems?

## **References:**

- Mankiw, N Gregory. 1990. "A Quick Refresher Course in Macroeconomics," Journal of Economic Literature
- Woodford, Michael. 1999, "Revolution and Evolution in Twentieth-Century Macroeconomics."
- Goodfriend, M. 2002. "Monetary Policy in the New Neoclassical Synthesis: A Primer"

# Appendix

[Further look into the implications of the stochastic process and the expectation operator]

- **Note:** Given  $\pi_t = \rho\pi_{t-1} + \varepsilon_t$  where  $\varepsilon_t$  is a “white noise” process:
- $\varepsilon_t$  is a time series process that's independently and identically distributed (iid), with zero mean  $E(\varepsilon_t) = 0$ , a constant variance  $E(\varepsilon_t^2) = \sigma^2$ , and  $E(\varepsilon_t\varepsilon_s) = 0$

**Applying the Expectation operator  $E_{t-1}$  to  $\pi_t$  (taking its expectation at time t-1, implying based on information we have at time t-1):**

$$\begin{aligned} E_{t-1}\pi_t &= E_{t-1}[\pi_t] = E[\pi_t | I_{t-1}] && \text{different notations for the same thing} \\ &= E_{t-1}[\rho\pi_{t-1} + \varepsilon_t] && \text{substituting in the definition of } \pi_t \\ &= E_{t-1}[\rho\pi_{t-1}] + E_{t-1}[\varepsilon_t] && \text{“operation” is linear (can do it term by term)} \\ &= \rho\pi_{t-1} + 0 && \text{what you expect at any time given info at that time = the info itself} \end{aligned}$$

[Clarifying note: Relation btw Phillips Curves in the slides]

*Shown previously*

**Note:** The AS in the previous page is another way to express the expectation-augmented Phillips curve (\*). From either of these expressions (again, equivalent):

$$\pi_t = a - bu_t + \gamma E_{t-1} \pi_t$$

$$\pi_t = -\beta(u_t - u^N) + \gamma E_{t-1} \pi_t$$

Since output  $y_t$  is inversely related to unemployment  $u_t$ :

$$\pi_t = cy_t + \gamma E_{t-1} \pi_t$$

*log-*

Since inflation is the difference in prices:  $\pi_t = p_t - p_{t-1}$

$$p_t - p_{t-1} = cy_t + \gamma E_{t-1}[p_t - p_{t-1}]$$

Re-arrange and note:

$$E_{t-1}[p_{t-1}] = p_{t-1}$$

$$y_t = \alpha(p_t - E_{t-1}p_t)$$

[Clarification-Why we use logs or similar approximations]

### A note about **linearized equations**:

- We will be working with “linearized” equations frequently, to make the models easier to solve.
- In order to express general economic equations in linear forms, we often take the (natural) logs of the variables. For example, the quantity theory of money,  $M_t V_t = P_t Y_t$  can be expressed as  $m_t + v_t = p_t + y_t$  where each of the lower-case variables is the log of the capitalized variables.
- We will use variations of  $m_t + v_t = p_t + y_t$  to represent **aggregate demand** for a while.