Econ 5322 - MS Macroeconomics Spring 2023
Granados

Midterm Exam

03/06/2023

Answer the following questions. Show and explain your work instead of only writing the final answers. Read the entire exam before starting to answer it..

1. (20 points) Suppose the economy's representative household maximizes the expected intertermporal utility function,

$$\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} u(C_s)$$

subject to the constraints given by the law motion of capital,

$$K_{t+1} = (1-\delta)K_t + I_t$$

and the budget constraint:

$$C_t + I_t = \tilde{r}_t K_t + w_t$$

where $\mathbb{E}_t$ denotes the expectation operator conditional on information available at time t, $\beta \in (0,1)$ is the discount factor, C_t is consumption in period t, K_{t+1} is capital at the beginning of period $t+1$, $\delta \in (0,1)$ is the rate of capital depreciation, I_t is investment in period t, $\tilde{r}_t$ is the rental rate on capital, w_t is the real wage, and labor suppl ly has been normalized to 1. Households rent capital to firms in a perfectly competitive market. Also, the investment is given by $I_t = K_{t+1} - (1-\delta)K_t$.

The production function is $Y_t = A_t^{\alpha} K_t^{1-\alpha}$, where $\alpha \in (0,1)$ and A_t is an exogenous technology shock

(a) Denote with R_{t+1} the return to capital accumulation at $t+1$. What is the expression for R_{t+1}? Explain.

(b) Explain how to obtain the Euler equation for capital accumulation intuitively.

2. (20 points) In the same setup as in Question 1, assume that the period utility function takes the form

$$u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}, \quad \text{with } \gamma > 0$$

Assume log-normality and homoskedasticity. Log-normality implies that, given a variable X_{t+1},

$$\log[\mathbb{E}_t(X_{t+1})] = \mathbb{E}_t[\log X_{t+1}] + \frac{1}{2} Var_t[\log X_{t+1}]$$

(a) Show that log-normality and homoskedasticity imply that the log-linear version of the Euler equation you obtained in Question 1 has the form:

$$\mathbb{E}_t(c_{t+1} - c_t) = \sigma \mathbb{E}_t r_{t+1}$$

where $c_t \equiv d \log C_t$ (difference of log with respect to steady state value), $r_t = d \log R_t$, and

$\sigma = 1/\gamma$

(b) What is σ? What does it measure?

3. (20 points) Let the solution of the real business cycle model be described by the relations:

$$k_{t+1} = \eta_{kk} k_t + \eta_{ka} a_t$$
$$c_t = \eta_{ck} k_t + \eta_{ca} a_t$$
$$y_t = \eta_{yk} k_t + \eta_{ya} a_t$$

with

$$a_t = \phi a_{t-1} + \varepsilon$$

where lower-case letters denote percentage deviations of their corresponding variables from their steady-state levels, the η's are elasticities obtained with the method of undetermined coefficients, and $\phi \in (0, 1)$

(a) What is the intuition for this solution? Put differently, why do we guess that the solution for endogenous variables depends on k_t and a_t?

(b) Why can we guess that this is the unique solution of the log-linearized model?

(c) Suppose there is a technology innovation $\varepsilon_0 = 1$ at time $t = 0$, followed by no other innovation in subsequent periods. Use the relations above to compute the responses of capital, consumption, and output to the innovation in periods 0, 1, 2.

(d) What is the response of technology to the innovation if $\phi = 1$? Do capital, consumption, and output return to the initial steady state in this case? Why?

4. (25 points) The first-order condition for optimal labor supply in the real business cycle model with variable labor implies:

$$U_{1-N_t}(C_t, 1 - N_t) = w_t U_{C_t}(C_t, 1 - N_t)$$

where C_t is consumption in period t, $1 - N_t$ is the leisure, and w_t is the real wage.

(a) Explain this condition intuitively

(b) Suppose the production function is $Y_t = N_t$ and the labor market is perfectly competitive. What is the value on the real wage? Why?

Now suppose the representative household is subject to an additional constraint for its consumption: a *cash-in-advance* constraint (CIA). The constraint in terms of the model is set as "nominal consumption cannot exceed the amount of money in the household" and can be seen as a friction that, in principle, may prevent agents from making the same consumption decisions as in the frictionless case (where they could expend beyond their cash, e.g., by borrowing money).

The first-order condition for optimal labor supply becomes:

$$U_{1-N_t}(C_t, 1 - N_t) = w_t \left(1 + \frac{i_t}{1 + i_t}\right) U_{C_t}(C_t, 1 - N_t)$$

where i_t is the nominal interest rate.

(c) Suppose production and the labor market assumptions are as above, and that a benevolent central banker (social planner) is choosing monetary policy to maximize welfare. What is the optimal interest rate that the central bank would choose? What is the intuition for the result?

[Hint: in thinking about this, you don't really need to know about the CIA model itself, but to remember the main lesson about efficiency in the RBC model in an environment without frictions and what this implies for policymaking.

For the same token, do not try to set a CIA model, the only result you need from it to answer is the first-order condition above]

5. (15 points) The Stochastic Growth Model or Real Business Cycle model in its basic version is a good benchmark but can be seen as too simplistic to explain more realistic features. The approach to deal with this has consisted in enriching the model with more nuanced features, such as new variables, mechanisms and even frictions.

   (a) Briefly explain the issues that are observed in the basic version of the model.

   (b) What is the motivation behind including variable labor in the model? What is achieved in this version of the model relative to the more basic RBC model?

   (c) Does monetary policy have any meaningful role or effect in the basic version of the model? Refer to how the private equilibrium (the one achieved by decentralized agents) compares to the one that can be achieved by a benevolent social planner.