Intermediate Macroeconomics

The IS Curve

ECON 3311 – Fall 2024

UT Dallas

Short run model : Output & Inflation relationship

↳ Mechanism: Demand-driven { Investment & Interest rate (IS curve)

Introduction

In this chapter we will go over:

- The foundation of the short-run model which is the **IS curve**
    - The IS curve depicts the **inverse relationship** between the **real interest rate** and **short-run output**
- How **shocks** to consumption, investment, government purchases, or net exports—'aggregate demand shocks'—can **shift the IS curve**.
- The life-cycle/permanent-income hypothesis, which describes consumption behavior
- How **investment is the key channel** through which changes in real interest rates affect GDP in the short run

Demand Shocks Shift the IS Curve

Introduction

Fed → FFR → Short term nominal Interest rate
(Policy rate)

↑ Int. Rate → ↓ Investment → ↓ Output
(↓ Demand in general)

The Federal Reserve influences the level of economic activity in the short run.

- The Fed **targets the federal funds rate** – interest rate at which banks can borrow from each other
- This rate is **highly correlated with the short-term nominal interest rate** at which people borrow and lend in financial markets

$$\uparrow \text{interest rate} \Rightarrow \downarrow \text{investment} \Rightarrow \downarrow \text{output}$$

The importance of this rate is: An **increase in the interest rate** leads to a **decrease in investment**, which leads to **decrease in output**, and vice versa

IS curve: Illustrates the negative relationship between interest rates and short-run output

IS goes for "Investment-Savings"

IS curve: "Investment-Savings"
int. rate vs. output plot
(Downward sloping)

The IS curve

Why downward sloping?

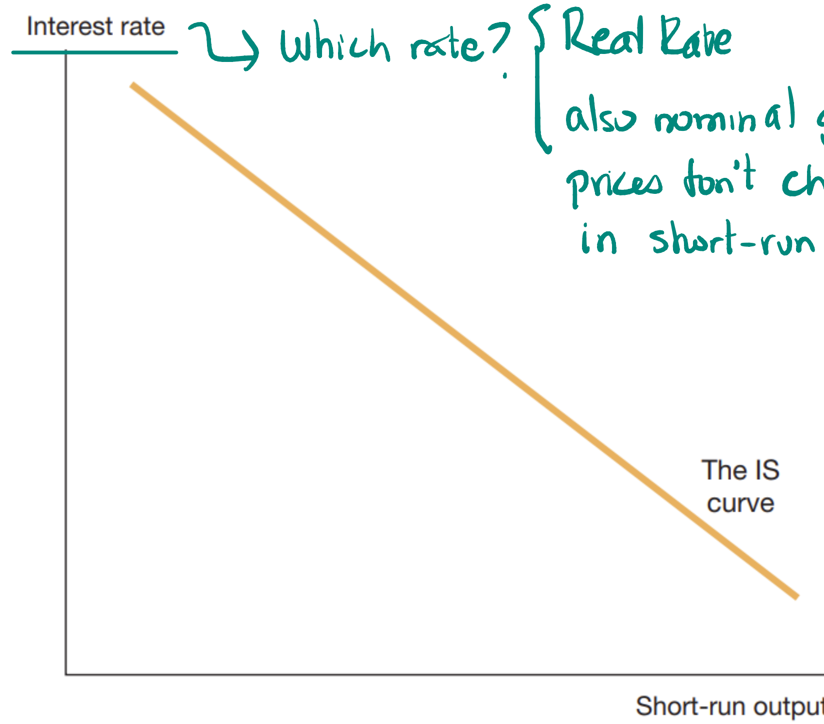
↑ Int Rate ⇒ ↑ Cost of lending ⇒ firms don't borrow \$ to Invest

The IS curve is graphed with the interest rate on the vertical axis and short-run output on the horizontal axis

Downward sloping:

A higher interest rate leads to lower output, operating through investment (I)

Introducing the IS Curve



which rate? Real Rate

also nominal given prices don't change much in short-run

Mechanism: Higher cost of borrowing leads firms to get fewer funds from banks for Investing, and households to cut their consumption (e.g., from credit cards spending).

All of this lowers GDP (remember the national accounting formula!)

Solving for the IS curve

1.) Depart from GDP definition

2.) Assume C, G, EX, IM $\propto \bar{Y}$

3.) Assume $I \propto \bar{Y}$ but also that it lowers with the Interest rate

To solve for the IS curve, we begin with the national income identity:

$$Y_t = C_t + I_t + G_t + EX_t - IM_t$$

We will assume the following:

- Consumption is a fraction of potential output: $C_t = \boxed{\bar{a}_c \bar{Y}_t}$
- Government purchases behave similarly: $G_t = \boxed{\bar{a}_g \bar{Y}_t}$
- Exports are a fraction of potential $EX_t = \boxed{\bar{a}_{ex} \bar{Y}_t}$
- Imports are a fraction of potential output: $IM_t = \boxed{\bar{a}_{im} \bar{Y}_t}$

Replace:

Then the equation becomes:

$$Y_t = \bar{a}_c \bar{Y}_t + I_t + \bar{a}_g \bar{Y}_t + \bar{a}_{ex} \bar{Y}_t - \bar{a}_{im} \bar{Y}_t$$

4.) Divide by $\bar{Y}_t$, and solve for $I_t / \bar{Y}_t$: $\frac{I_t}{\bar{Y}_t} = \bar{a}_c + \bar{a}_g + \bar{a}_{ex} - \bar{a}_{im}$

$I_t = \bar{a}_i \bar{Y}_t - \bar{b}(R_t - \bar{r})\bar{Y}_t$

Solving for the IS curve

For investment, let's assume it's given by a fraction of the potential output **plus** an amount that depends on the gap between Real interest rate and the MPK, then:

$$\frac{I_t}{\bar{Y}_t} = \bar{a}_i - \bar{b}(\underbrace{R_t}_{\text{Real Interest rate}} - \bar{r}).$$

(rearrange)

$\bar{a}_i$ = Share of investment from potential output

$\bar{b}$ = Parameter weighting difference between the real interest rate and MPK

R_t = Real interest rate: Rate at which firms can save or borrow

$\bar{r}$ = Marginal product of capital (MPK): How much a firm can make from investing in one more unit of capital

Intuition: If $R >$ MPK $\Rightarrow$ it is better to invest is producing & instead can put money in the bank...

$\uparrow I$ if R is lower relative to the MPK

Investment share of potential output: Similar to what we saw with C, G, EX, and IM

Extra term: If extra output from physical investment is low, then a firm can instead put the money in the bank and get R_t, but otherwise: Extra incentives to Invest.

... Works both ways: If it is much better to $\uparrow$ Capital than to buy a financial asset ($R_t < \bar{r}$)

$\Rightarrow$ Firms want to invest more

Alternative yet similar view: R_t as Cost of Debt → Compare: Cost of Debt vs. Gain from Debt*

The Marginal Product of Capital (MPK)

* from taking on debt to ↑ Investment

We can see that a key component of the investment equation is the relationship between R_t (real interest rate) and r_t (marginal product of capital)

- If the MPK is lower than the cost of borrowing, then firms should save more
- If the MPK is higher than the cost of borrowing then firms should borrow and buy capital (e.g. expand their factories)

MPK > Cost of Debt (R_t) ⇒ ↑I

For example, suppose R = 5 and r = MPK = 10, and the firm borrows 1000 units of capital:

MPK = 0.1, R = 0.05

To pay back: (0.05*$1000)=50

Can be obtained from Investing: (0.1*$1000)=100

⇒ Can take a $1000 loan,

Yielding a potential profit of $50

Invest and make $1100, pay back (−1000) & Pay Interests (50)

⇒ Profits = 50

Note: In short run the interest rate is NOT equal to MPK ($(R_b - \bar{r}) \neq 0$, short run)

Short run versus long run

Note that in this example, the marginal product of capital is **not equal** to the real interest rate like it was in the long-run model

The idea behind a possible discrepancy between the two is that it takes time for the marginal product of capital to adjust, while a change in the real interest rate is instantaneous

- As **more capital is invested, the MPK (*r*) will decrease** and if less capital is invested then MPK (*r*) will increase
- In the long-run: We are back to the real interest rate equaling the marginal product of capital and then the investment is just a share a_i of potential output

→ Adjustment of MPK toward R → It happens BUT it takes time

How the real interest rate is determined will be the focus later

For now: Notice we are using the principle of **decreasing marginal productivity**

Then: last slides' mechanism does not contradict the earlier models (growth) we reviewed → These models are for the "long-run"

Deriving the IS curve

1.) GDP definition

2.) Replace assumptions

3.) Divide by $\bar{Y}_t$

4.) Group terms & replace $\tilde{Y}_t$

We start with the national income identity:

$\rightarrow \bar{a}_i \bar{Y}_t - b(R_t - \bar{r})\bar{Y}_t$

$$Y_t = C_t + I_t + G_t + EX_t - IM_t$$

$\hookrightarrow \bar{a}_c \bar{Y}_t$

We can divide by $\bar{Y}_t$ and then substitute in the earlier equations for C_t, I_t, G_t, EX_t, and IM_t :

$\left(\frac{1}{\bar{Y}_t}\right)$;

$$\frac{Y_t}{\bar{Y}_t} = \bar{a}_c + \bar{a}_i - \bar{b}(R_t - \bar{r}) + \bar{a}_g + \bar{a}_{ex} - \bar{a}_{im}$$

To get from this equation to short-run output, subtract 1 from both sides:

$$\underbrace{\frac{Y_t}{\bar{Y}_t} - 1}_{\tilde{Y}_t} = \underbrace{\bar{a}_c + \bar{a}_i + \bar{a}_g + \bar{a}_{ex} - \bar{a}_{im} - 1}_{\bar{a}} - \bar{b}(R_t - \bar{r})$$

$\rightarrow$ gather all constant terms into a single term $(\bar{a})$

Recall: $\tilde{Y}_t = \frac{Y_t - \bar{Y}_t}{\bar{Y}_t} = \frac{Y_t}{\bar{Y}_t} - 1$

IS: $\tilde{Y}_t = \bar{a} - \bar{b}(R_t - \bar{r})$

Deriving the IS curve

IS as below

$\Rightarrow \tilde{Y}$ changes if

- $(R_t - \bar{r})$ changes
- $\bar{a}$ changes

After some simplification we are left with:

Shocks: Can affect $\bar{a}$ (less commonly $\bar{r}$)

$\Rightarrow$ IS shifts

+ +

$$\tilde{Y}_t = \bar{a} \cancel{+} \bar{b}(R_t - \cancel{\bar{b}}\,\bar{r})$$

$$\tilde{Y}_t = \bar{a} - \bar{b}R_t + b\bar{r}$$

$\downarrow$ $\downarrow$

We can see that there are two factors that determine fluctuations:

1) Gap between the real interest rate (R) and the marginal product of capital (r)
2) Shocks to the economy which are represented by changes in $\bar{a}$

Using the IS curve

IS: $\tilde{Y}_t = \bar{a} - \bar{b}(R_t - \bar{r})$

Given the IS curve is plotted with short-run output and the real interest rate on the two axes, it has a negative slope.

Note that $\bar{a}$ = 0
You can see this by considering the equation in the long-run when $Y_t = \bar{Y}_t$

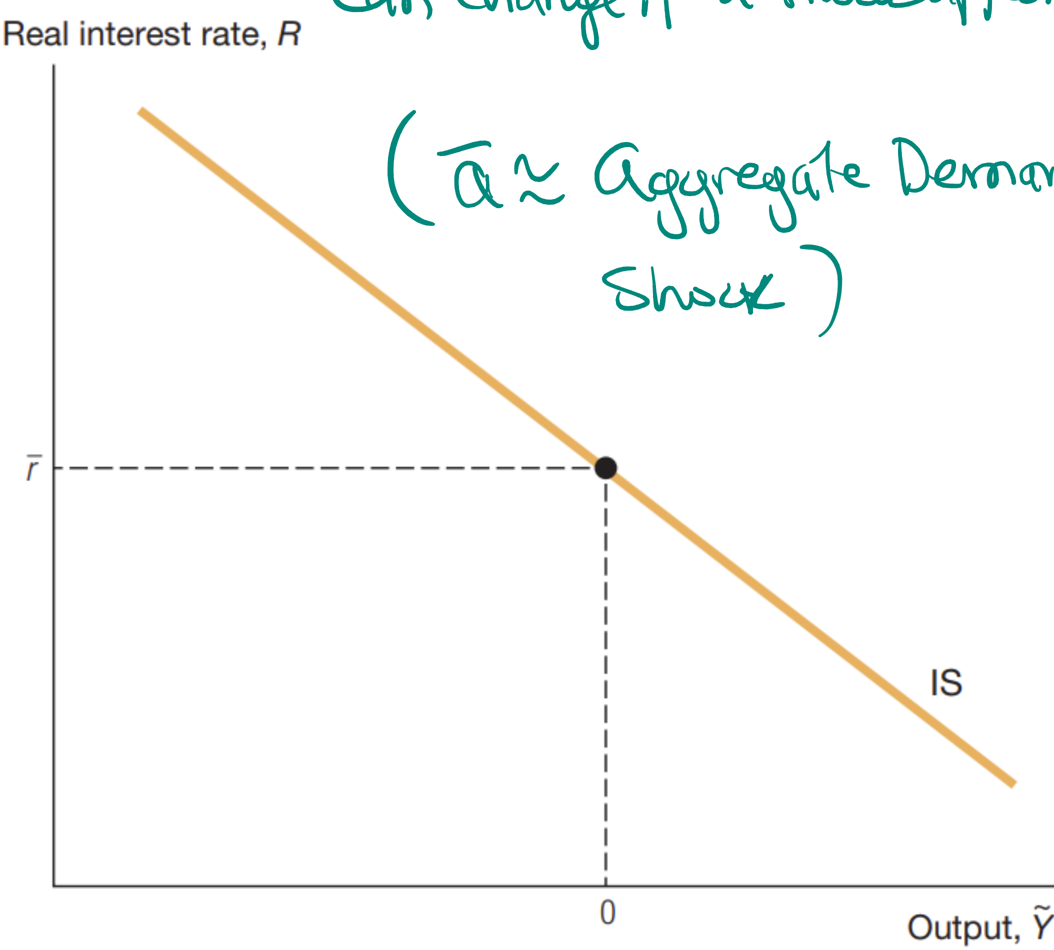
$\bar{a}$ can also be thought as an **aggregate demand shock** and by **default we assume it's zero**

Therefore: $\tilde{Y}_t = 0$ when $\bar{r} = R$
(baseline case)

Usually $\bar{a}=0$, but this can change if a shock affects $\bar{a}$

($\bar{a} \simeq$ Aggregate Demand Shock)

The IS Curve



Key here and in general: If something included on the axes changes we move along the curve. In contrast, we shift the curve if what changes is not on the axes.

Using the IS curve

If R Changes

- ~~IS curve shifts~~ ×
- Movement Along Curve ✓

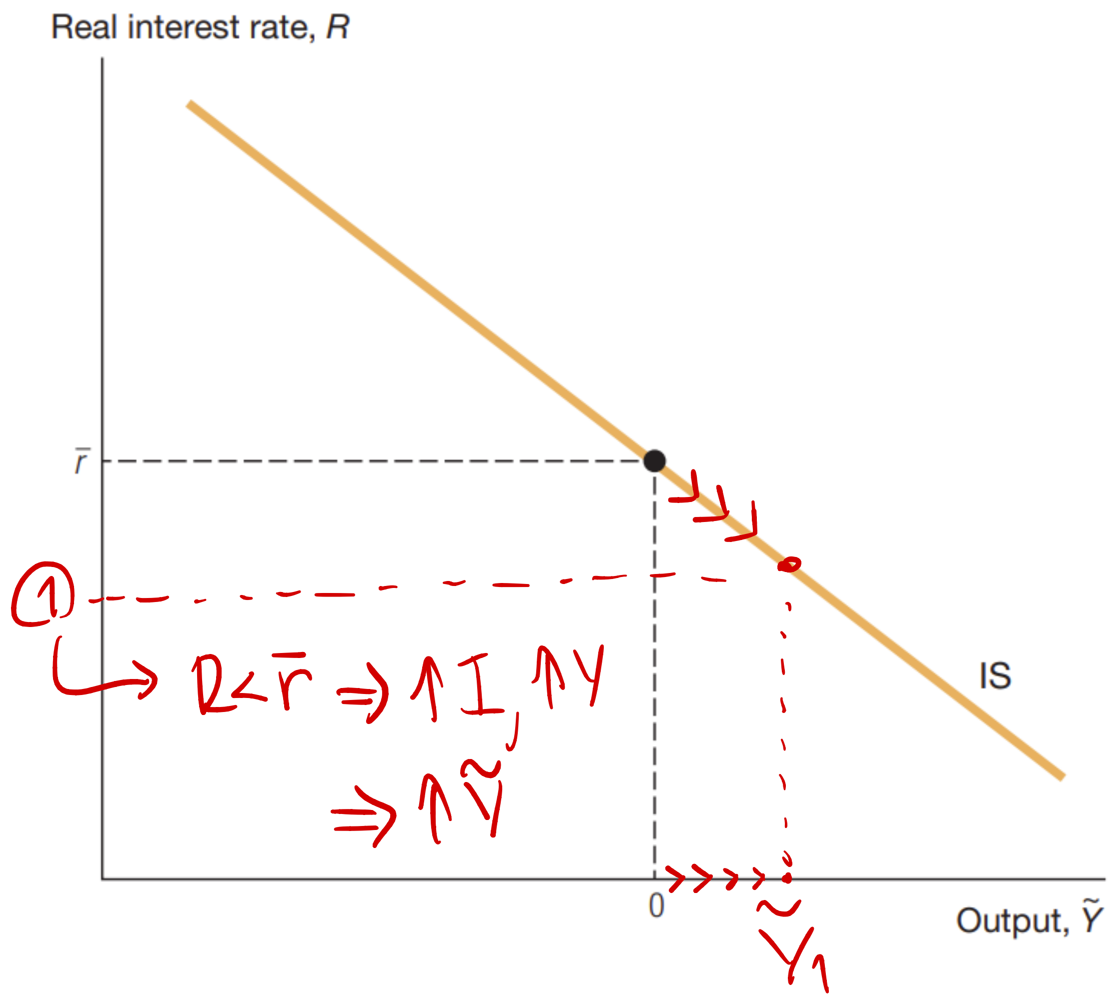
What can cause the economy to move from the point where $\tilde{Y}_t = 0$?

One is a change in the real interest rate (R) so that it is no longer at its long-run trend of $\bar{r} = R$

This will cause a **movement along the IS curve**

How much depends on the change in R and $\bar{b}$

The IS Curve



Using the IS curve

Changes in $\bar{b}$: Makes the curve steeper/flatter

Slope: $\frac{1}{\bar{b}} \Rightarrow$ $\uparrow \bar{b}$: Flatter

$\downarrow \bar{b}$: Steeper

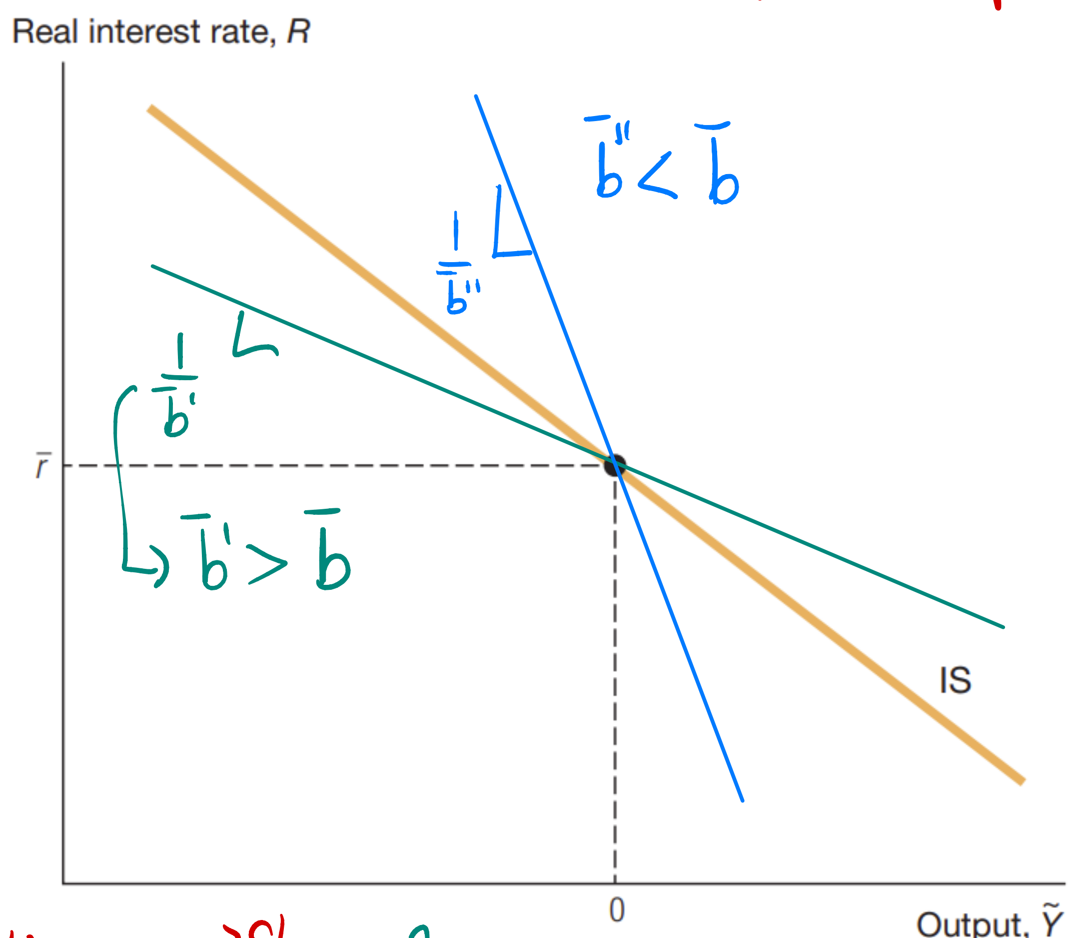
The slope of the IS curve:

How does $\bar{b}$ affect the IS curve?

If there is an increase in $\bar{b}$ then the IS curve becomes **flatter** and each change in the interest rate will associate to a larger change in short-run output

This is easier to see if we convert the equation of the IS curve to slope-intercept format:

The IS Curve



What is plotted:

$$\tilde{Y}_t = \bar{a} - \bar{b}(R_t - \bar{r}) \qquad \rightarrow \qquad R_t = \frac{\bar{a}}{\bar{b}} + \bar{r} - \underbrace{\frac{1}{\bar{b}}}_{\text{Slope}}\tilde{Y}_t$$

We are plotting R in the vertical axis

Shocks: Change $\bar{a}$ $\begin{cases} \uparrow \bar{a} \to \text{higher } \tilde{Y} \text{ for any } R \\ \Rightarrow \text{Shifts IS up} \\ \downarrow \bar{a} \to \text{IS shifts down} \end{cases}$

Using the IS curve

What about the macroeconomic shocks we've mentioned before

These are represented by changes in $\bar{a}$

→ What is $\bar{a}$? → Other components of expenditure

e.g. → $\bar{a}_c$: Proportionality b/w consumption and output

(example: ↑ business confidence, ↓ taxes ⇒ ↑ $\bar{a}_c$ ⇒ ↑ $\bar{a}$)

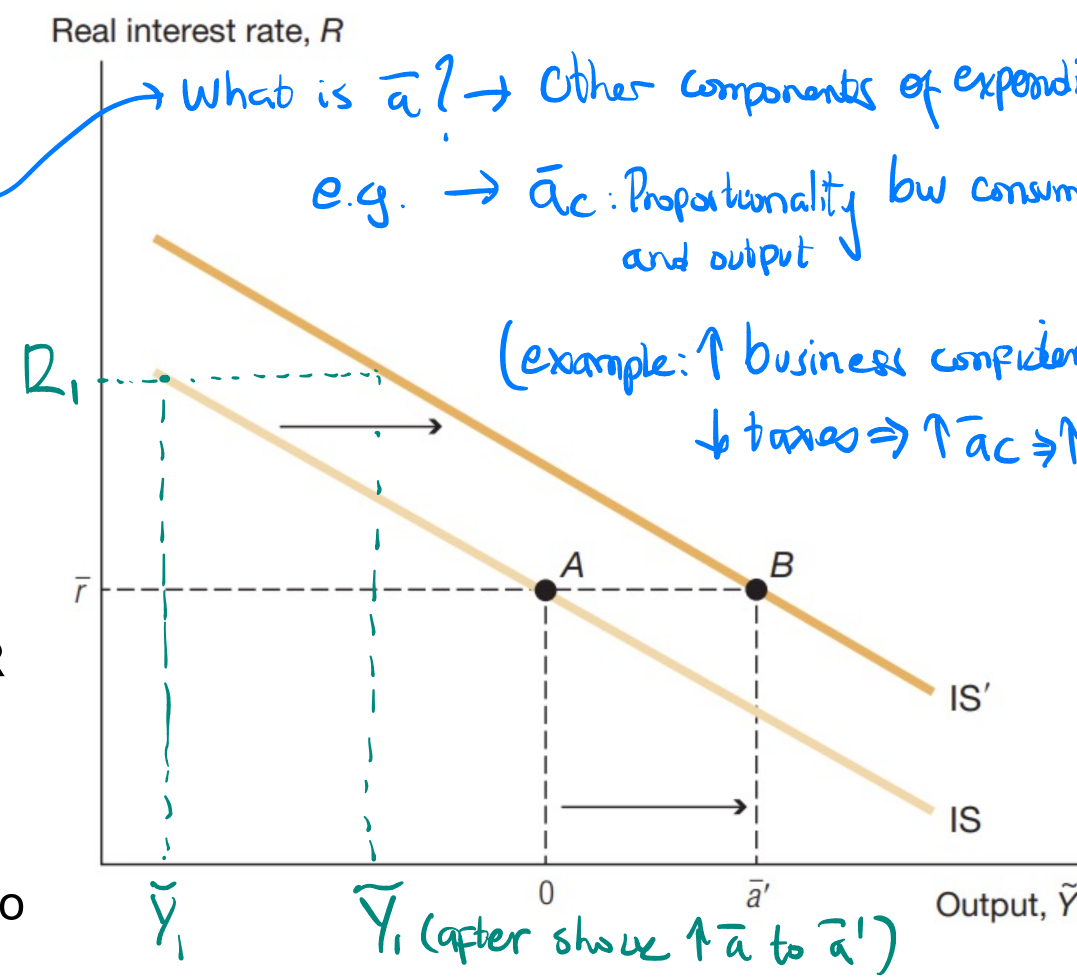
Examples:

Changes in consumer confidence, business optimism, etc. can **shift** the IS curve

Positive shock: Higher Output for any R

Negative shock: Lower Output for any R

An Aggregate Demand Shock



Again, these are relevant changes in Variables that are **not on the axes** but do affect the economy (and output)

↳ In a nutshell: anything relevant for GDP that is not on the axes can shift the IS

If a shock changes $\bar{Y}$: Nothing would change in the IS plot

Using the IS curve

Intuition: Such a shock changes $\bar{Y}$ but also changes Y (actual output) and the effects cancel out

What would happen if potential output changes?

$$\tilde{Y}_t = \frac{\uparrow Y_t}{\uparrow \bar{Y}_t} - 1$$

Short answer: Nothing changes (this has some nuances)

Output = Potential output + Short run fluctuations

First, note that potential output does not enter the IS equation

$$\frac{\cancel{1.1}\, Y_t}{\cancel{1.1}\, \bar{Y}_t} = \underbrace{\tilde{Y}_t - 1}_{\text{unchanged}}$$

But, actually, it is in there through the definition of $\tilde{Y}_t = \frac{Y_t - \bar{Y}_t}{\bar{Y}_t} = \frac{Y_t}{\bar{Y}_t} - 1$

However, **a shock affecting $\bar{Y}_t$ is likely to affect Y_t in the same fashion**.

And in the definition of $\tilde{Y}_t$ **these effects would offset**: $\frac{\uparrow Y_t}{\uparrow \bar{Y}_t} = \tilde{Y}_t - 1$

For instance, an increase in technology would increase potential output, but also actual short run output

Of course, there could be a change affecting more/less $\bar{Y}_t$ than Y_t which would change $\tilde{Y}_t$ but for simplicity we abstract from those cases

and see if something changes

But if it would happen the IS would not change: We would move along the curve

If confused: Check the IS equation (plotted one): $R = \frac{\bar{a}}{b} + \bar{r} - \frac{1}{b}\tilde{Y}_t$

Microfoundations of the components of the IS curve

Consumption: $C_t = \bar{a}_c \bar{Y}_t$

People consume in proportion to average income

What determines the different levels of C, I, G, EX, and IM?

There are two theories that seek to explain consumption (C) behavior:

1) Permanent-income hypothesis → People won't drastically change consumption patterns if they go through an isolated shock

2) Life-cycle model of consumption

Income

Examples: – Finding a $100 only once
– Sudden car repair

Key idea here: Consumers **smooth their consumption** (this follows from the idea of diminishing marginal utility)

Permanent-income hypothesis: Consumers base consumption decisions on average income over time and not current income

Are this ideas consistent? Yes! If you get some surprise income (let's say 5000) you will likely not increase your typical consumption by that much next month.

Microfoundations of the components of the IS curve

Key Principle: Consumption Smoothing for (1), (2)

rather eat 1 or 2 eggs a day than a dozen but only on Mondays

Life-cycle model of consumption: People base their consumption based on lifetime average income.

This also follows the same idea of **consumption smoothing**: People won't change drastically their habits according to their age

(even if income is much lower at the beginning and end of life)

- For most students consumption is higher than income – they take out loans or get support.
- During working years, earnings are likely higher than spending —saving for retirement.
- When one retires, their income falls drastically, but consumption's decrease is smooth.

Similar Idea: Life Cycle

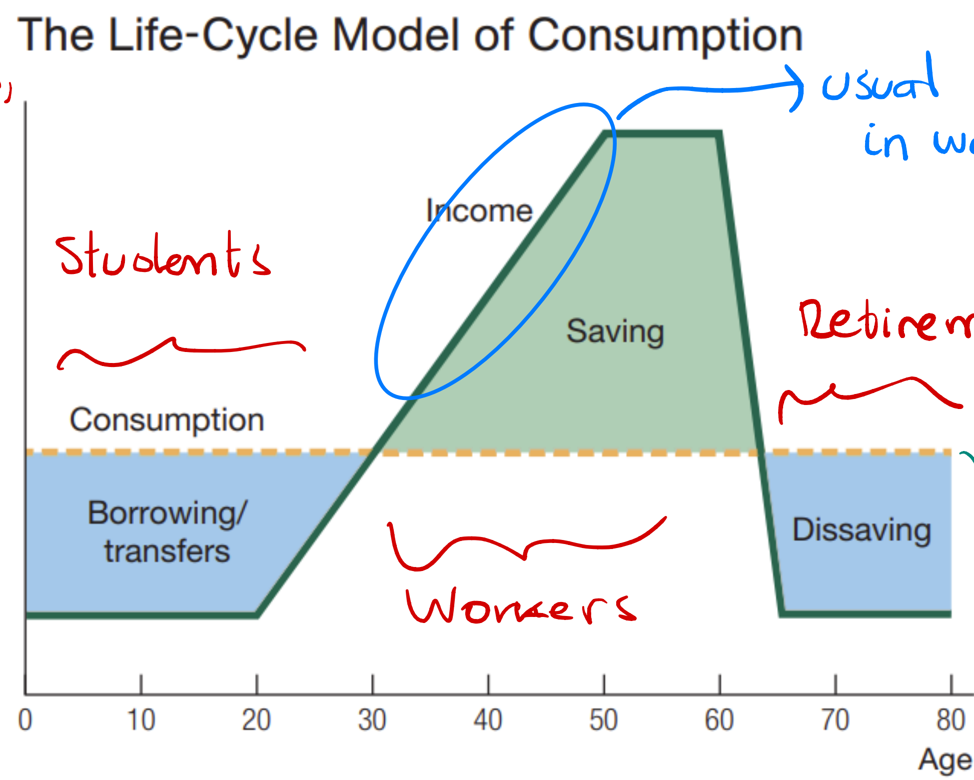
- Students take on loans $C > Y$
- Workers save $C < Y$
- Elders no longer work (but have savings) $C > Y$

Y of reference is the average income

Microfoundations of the components of the IS curve

This changes in consumption by age are usually depicted graphically:

Consumption, Income



Students

Workers

Retirement

usual income increase in working years

roughly: $\approx \bar{a}_c \cdot \bar{Y}$

Microfoundations of the components of the IS curve

Based on
Cons Smoothing, P.I., Lifecycle: $C_t = \bar{a}_c \bar{Y}_t$

(as opposed to alternatives such as $C_t = \bar{a}_c Y_t$)

The **LC/PI hypotheses** is the reason we can write: $C_t = \bar{a}_c \bar{Y}_t$

Consumption is based on potential output and not actual output because potential output is smoother and thus represents better the average income

This theory does not always hold up in practice:

Examining the Alaska Permanent Fund (Hsieh, 2003), it was found that consumers smooth their permanent fund payments, but not their tax refunds

This can be explained by the permanent fund being more predictable, while people are not always clear on what their tax refund/payment is going to be

However: Exceptions have been documented in some studies

in some cases C can react to (short-run) income changes

(changes that would not affect the permanent or average income)

to simplify we still use $C_t = \bar{a} \bar{Y}_t$ but we can extend framework to account for this (IS won't change)

Extension to allow C_t to change w/ $\tilde{Y}$ too

Multiplier Effects

If we expect consumption to also react to changes in actual output (like in the tax refund finding), then it can be written as:

MPC: marginal Propensity to Consume

$$\frac{C_t}{\bar{Y}_t} = \bar{a}_c + \bar{x}\tilde{Y}_t$$

If there is an increase in GDP/income then consumption will increase

The IS curve will change to:

Derivation of the IS works as before but now yields:

$$\tilde{Y}_t = \boxed{\frac{1}{1-\bar{x}}}\underbrace{(\bar{a} - \bar{b}(R_t - \bar{r}))}_{\text{RHS of original IS}}$$

$\Rightarrow \bar{x}$ amplifies the effects captured in the IS curve

A 'multiplier' has been added to the equation

with $\frac{1}{2}$: $\frac{1}{1-\frac{1}{2}} = 2$

For example, if $\bar{a}$ increases by 2% and $\bar{x}$ is equal to 1/2, then output will rise by 4% - the change in $\bar{a}$ is multiplied by a number that is greater than 1

Multiplier Effects

- Suppose that a change in the interest rate increases investment by 1%
- This extra investment leads to more construction jobs which leads to higher income to construction workers.
- This **extra income is going to increase spending** (we assume that $\bar{x} > 0$) elsewhere, increasing the demand for workers in those areas
- As more workers are hired, income and spending increase
- The original change in investment is being ‘multiplied’

Some noteworthy details here:

The multiplier won’t change the “form” of the IS curve: It just implies an adjustment to the intercept and slope.

This captures the notion that economic shocks may have larger than proportional effects.

This does not contradict the idea that consumption is smooth or depending mainly on permanent income ($\bar{x}$ may still be small)

Investment

The equation for investment that we derived earlier was:

$$\frac{I_t}{\bar{Y}_t} = \bar{a}_i - \bar{b}(R_t - \bar{r})$$

There are two main determinants of investment:

1) Gap between the real interest rate (R_t) and the Marginal Product of Capital ($\bar{r}$

2) Cash flow – this refers to how much resources the firm has: A firm with more resources can find cheaper borrowing or may just be able to invest without borrowing.

The equation above can be rewritten in the following way:

$$I_t = \bar{a}_i\bar{Y}_t - \bar{b}(R_t - \bar{r})\bar{Y}_t$$

Cash flow can be seen as being directly related to $\bar{Y}_t$

It grows with income and that facilitates investing regardless of cost of funding conditions.

Government Purchases

The effect of **government purchases** on short-run output can be viewed in a couple of ways:

1) A sharp change in government spending can directly cause a fluctuation (example: defense spending)
2) Government spending can be used as a policy to reduce fluctuations (example: relief packages)

A change in government spending appears as a change in $\bar{a}_g$

Another important instrument: **Taxes**

These are transfers (payments, sometimes subsidies) imposed by the government that **can push up/down expenditure** by all agents (consumption, investment)

If set as rates proportional to the expenditure they function as **automatic stabilizers** by mitigating fluctuations of GDP automatically

Net Exports (NX)

The expressions for exports and imports are respectively $\bar{a}_{ex}\bar{Y}_t$ and $\bar{a}_{im}\bar{Y}_t$

Net exports (NX) are equal to exports minus imports

- If **net exports** are positive there is a **trade surplus**
- If **net exports** are negative there is a **trade deficit**

Changing exports/imports and the IS curve:

When exports increase relative to imports then the IS curve shifts out
When exports decrease relative to imports then the IS curve shifts in

(this follows the same principle mentioned before: Are NX or its parts included on the axes? No. Then they act as shifters —when changing.)

Conclusion

The IS curve depicts how changes in the real interest rate affect economic activity in the short run

Higher interest rates lead both firms and households to reduce their investment spending (business investment and housing investment)

Changes in other variables act as **aggregate demand shocks** which can lead to **shifts in the IS curve**

Through the **multiplier effect**, the change in investment can then lead to a larger change in short-run output

IS curve: Mainly about the relationship between **Investment/Savings and the cost of borrowing** but it also embeds changes in other parts of GDP

Next: The role of policy and how the central bank controls the interest rate that plays such a crucial role in the IS curve

Review Questions

What would we expect to happen to the economy in each of the following cases? Use the factors we used to derive the IS curve in your answer

a) There is a deep recession in Europe

b) Housing prices rise above their trend

c) Mortgages lenders raise interest rates

d) The government decides to close 20 percent of their domestic military bases