ECON 6356
International Finance and Macroeconomics

Lecture 6a: The Large Open Economies Model with Nominal Rigidities

Camilo Granados
University of Texas at Dallas
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Introduction

Until now we considered open economies that were small. That setup was a convenient departing point because, after adding some exogenous international variables, we could analyze the outlook of an open economy but using the intuition and techniques from closed economy models.

Even if simplified, we could analyze the dynamics of the current account, net foreign assets, and even introduce a number of relative prices that become relevant for open economies (TOT, RER, ER).

However, the SOE setup is mute about the behavior of foreign agents. Hence, we cannot say anything with that framework about potential interdependencies between agents and macroeconomic outcomes in different locations. We turn to those issues in the remainder of the course.

the Large Open Economies Model with Nominal Rigidities

Microfounded international macro model **with nominal rigidities**. Made to replace Mundell-Fleming, Dornbusch setup with a microfounded one, among other reasons, to perform welfare (normative) analyses.

Model:

- Perfect foresight, two countries, **preset prices** (prices are set the period before —1 period price stickiness).

- No capital included, but it is not an endowment economy as labor is elastic and determines production.

- Continuum of goods $z \in [0, 1]$, produced by monopolistically competitive firms (each one good).

- Home are producers in the interval $[0, n)$; Foreign in $(n, 1]$.

Preferences:(Identical)

HHs will maximize: $$U_t^i = \sum_{s=t}^{\infty} \beta^{s-t} \left[\log C_s + \chi \log \frac{M_s}{P_s} - \frac{\kappa}{2} y_s(i)^2 \right]$$

where $y_t(i)$ is the output of good i in period t (disutility of labor supply used for production, assumes labor of household i is used for production of good i).

Consumption: given by an aggregate of variety goods

$$C^i = \left[\int_0^1 (C^i(z))^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \quad \theta > 1$$

where θ is the elasticity of substition.

Note that variety counter goes up to 1 (a constant), and thus, there is a **constant number of firms**, there is no firm entry in this model.

Agents obtain utility from the money they carry to the next period ("cash when I'm done") and not from the money they arrive with.

Timing matters for equilibrium properties, in particular in finite horizon models.

Prices:

$$P = \left[\int_0^1 p(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}$$

LOP will hold, $p(z) = \varepsilon p^*(z)$, this allows us to rewrite the CPIs:

$$P = \left[\int_0^n p(z)^{1-\theta} dz + \int_n^1 \varepsilon p^*(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}},$$

$$P^* = \left[\int_0^n \frac{1}{\varepsilon} p^*(z)^{1-\theta} dz + \int_n^1 \frac{1}{\varepsilon} p^*(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}};$$

as a result the PPP holds: $P = \varepsilon P^*$

This is not surprising given the LOP holds and households have identical preferences.

Demands:

Each producer faces local and foreign demand.

Thus the demand for good z by the representative home household is

$$c^i(z) = \left(\frac{p(z)}{P}\right)^{-\theta} C^i,$$

and by the foreign household,

$$c^{*i}(z) = \left(\frac{p(z)}{P}\right)^{-\theta} C^{*i},$$

This implies (done by integrating demand for good z across all agents, and making use of the LOP, and PPP) that the Total world demand for good z is

$$y^d(z) = \left(\frac{p(z)}{P}\right)^{-\theta} C^W,$$

where total world consumption is $C^W \equiv \int_0^n C^i di + \int_n^1 C^{*i} = nC + (1-n)C^*$

Budget constraint:

- **Asset markets are incomplete:** Non-contingent nominal bond denominated in each country's currency (departure from original article with real bond —done to derive UIP more clearly).

- The agents also hold units of their domestic currency.

$$\text{[BC(home)]}: \quad B^i_{t+1}+\varepsilon_t B^{*i}_{t+1}+M^i_t = (1+i_t)B^i_t+\varepsilon_t(1+i^*_t)B^{*i}_t+M^i_{t-1}+p_t(i)y_t(i)-P_tC^i_t-P_tT_t,$$

where i_t is the nominal interest rate between $t-1$ and t which was known at $t-1$ (this is why it's risk free), and M^i_{t-1} are the agent i's holdings of nominal money balances entering t.

FOCs: dropping superscript i for simplicity (also notice that when optimizing agents take C^W and T as given).

$$\text{[Euler Equation]}: \quad C_t = [\beta(1+r_{t+1})]^{-1} C_{t+1},$$

$$[M_t]: \quad \frac{M_t}{P_t} = \chi C_t \left(\frac{1+i_{t+1}}{i_{t+1}}\right),$$

Intratemporal condition, indiff. between consuming or holding more balances

$$[y_t(h)]: \quad y_t(h) = \frac{\theta-1}{\kappa\theta}\frac{p_t(h)}{P_t}\frac{1}{C_t};$$

Intratemporal condition, indiff. between leisure and consuming (working)

r_{t+1} denotes the real interest rate, and we substitute the output and price variety index for h to denote the output and price of the representative home good (the variety they generate).

Notice we have two intratemporal conditions, one for the decision of holding money and another for the labor supply decision.

Additionally the Fisher parity defines the link between real and nominal interest rates:

$$1 + r_{t+1} = \frac{P_t}{P_{t+1}}(1 + i_{t+1}).$$

With Home and Foreign assets a no arbitrage condition is implied by the associated Euler Equations,

$$[UIP]: \quad 1 + i_{t+1} = \frac{\varepsilon_{t+1}}{\varepsilon_t}(1 + i^*_{t+1})$$

Government: Changes the money supply. Operates a balance budget with lump-sum transfers: $T_t = \frac{M_t - M_{t-1}}{P_t}$

World Equilibrium

Asset Markets: Demand of assets must equal supply (zero net), for each location this implies,

$$\text{[home]} \qquad nB_{t+1} + (1-n)B_{*t+1} = 0$$
$$\text{[foreign]} \qquad nB^*_{t+1} + (1-n)B^*_{*t+1} = 0$$

where B_{*t+1} denotes foreign holdings of the home currency bond (subscript location: holder, superscript: type of asset —home, foreign).

Multiply the foreign condition by ε_t and add the two equations:

$$n(B_{t+1} + \varepsilon_t B^*_{t+1}) + (1-n)(B_{*t+1} + \varepsilon_t B^*_{*t+1}) = 0$$

that is,

$$nA_{t+1} + (1-n)A^*_{t+1} = 0$$

home NFA + Foreign Net NFA = 0
NFA: Net foreign assets

Goods Markets: Given this global asset-market clearing conditions, we can derive an aggregate global goods-market clearing condition.

Divide both sides of the home and foreign Budget Constraints by their prices levels (P_t for Home and P^*_t for Foreign), take a population weighted average of the two budget constraints. Impose the asset market clearing condition above, and the government balance budget,

$$C^W \equiv nC_t + (1-n)C^*_t = n\frac{p_t(h)}{P_t}y_t(h) + (1-n)\frac{p^*_t(f)}{P^*_t}y^*_t(f) \equiv Y^W_t,$$

where $y^*_t(f)$ is the output of the representative good in the Foreign country.

Current Account: The relationships above can also lead to a definition of the trade balance.

Rewrite the budget constraint of the Home country in terms of the NFA,

$$A_{t+1} = (1 + i_t)A_t + p_t(h)y_t(h) - P_t C_t,$$

Set in real terms (divide by P_t)

$$\begin{aligned}
\frac{A_{t+1}}{P_t} &= (1 + i_t)\frac{P_{t-1}}{P_t}\frac{A_t}{P_{t-1}} + \frac{p_t(h)}{P_t}y_t(h) - P_t C_t \\
a_{t+1} &= (1 + r_t)a_t + \frac{p_t(h)}{P_t}y_t(h) - C_t \\
\Rightarrow \underbrace{a_{t+1} - a_t}_{ca_t} &= \underbrace{r_t a_t}_{\text{Income balance}} + \underbrace{\frac{p_t(h)}{P_t}y_t(h) - C_t}_{\text{Trade Balance}},
\end{aligned}$$

where ca_t denotes the current account in real terms. From the current account equations of both locations we also have

$$nca_t + (1 - n)ca_t^* = 0$$

A Symmetric Steady State

The initial steady-state levels of the variables are denoted with a subscript 0 and an overbar.

Nothing pis down the steady-state level of bonds in the model as the Euler Equations don't depend on the assets. Thus, for now, we will assume that the bonds are zero in the initial steady-state.

For obtaining the steady-state consumption levels at Home and Foreign, we can use the real current account equation above:

$$\begin{aligned}\bar{C}_0 &= \bar{r}_0\bar{a}_0 + \frac{\bar{p}_0(h)}{\bar{P}_0}\bar{y}_0(h) \\ \bar{C}_0^* &= -\left(\frac{n}{1-n}\right)\bar{r}_0\bar{a}_0 + \frac{\bar{p}_0^*(f)}{\bar{P}_0^*}\bar{y}_0^*(f)\end{aligned} \tag{1}$$

For the last one we considered the same equation but for Foreign, and replaced the Foreign NFAs from the global asset market clearing condition ($a_{t+1}^* = -\frac{n}{1-n}a_{t+1}$).

With zero bond holdings (as we are assuming) the steady-state is symmetric:

$$\bar{p}_0(h)/\bar{P}_0 = \bar{p}_0^*(f)/\bar{P}_0^* = 1, \quad \bar{C}_0 = \bar{y}_0(h), \quad \bar{C}_0^* = \bar{y}_0^*(f)$$

The first equalities come from the fact that producers are symmetric in both countries and at the cross-country level via the LOP ($\bar{p}_0(h) = \varepsilon_0\bar{p}_0^*(f)$).

Then, from the FOC with respect to the output (the intratemporal leisure/consumption trade-off) we can replace these results and get

$$\bar{y}_0(h) = \bar{y}_0(f) = \bar{y}_0 = \left(\frac{\theta - 1}{\kappa\theta}\right)^{\frac{1}{2}},$$

here we see the inefficiency generated by the combination of a mark-up with an endogenous leisure choice. Output ends up being too low as agents contract labor to enjoy more leisure. A fix for this is to tax the leisure.

On the other hand, from the Euler Equation we can obtain the steady-state interest rate:

$$\bar{r}_0 = \delta = \frac{1 - \beta}{\beta}$$

The real money balances steady-state is also symmetric across countries and can be obtained from the intra-temporal condition that trades-off consumption and money holding

$$\frac{\bar{M}_0}{\bar{P}_0} = \frac{\bar{M}_0^*}{\bar{P}_0^*} = \frac{\chi(1 - \delta)}{\delta}\bar{y}_0$$

Log-linearized model Now we can obtain the log-linear version of the model (approximated around the steady-state). The main equations of the log-linearized model are:

Price indexes

$$\mathsf{P}_t = n\mathsf{p}_t(h) + (1-n)(\mathsf{e}_t + \mathsf{p}^*_t(f)) \tag{2}$$

$$\mathsf{P}^*_t = n(\mathsf{p}_t(h) - \mathsf{e}_t) + (1-n)\mathsf{p}^*_t(f) \tag{3}$$

PPP

$$\mathsf{P}_t - \mathsf{P}^*_t = \mathsf{e}_t \tag{4}$$

Output-demand schedules

$$\mathsf{y}_t(h) = \theta(\mathsf{P}_t - \mathsf{p}_t(h)) + \mathsf{C}^W_t \tag{5}$$

$$\mathsf{y}^*_t(f) = \theta(\mathsf{P}^*_t - \mathsf{p}^*_t(f)) + \mathsf{C}^W_t \tag{6}$$

World output

$$\mathsf{Y}^W_t \equiv n\mathsf{y}_t(h) + (1-n)\mathsf{y}^*_t(f) = n\mathsf{C}_t + (1-n)\mathsf{C}^*_t \equiv \mathsf{C}^W_t \tag{7}$$

Output supply

$$\mathsf{y}_t(h) = \mathsf{p}_t(h) - \mathsf{P}_t - \mathsf{C}_t \tag{8}$$

$$\mathsf{y}^*_t(f) = \mathsf{p}^*_t(f) - \mathsf{P}^*_t - \mathsf{C}^*_t \tag{9}$$

Consumption Euler equations

$$\mathsf{C}_t = -\frac{\delta}{1+\delta}\mathsf{r}_{t+1} + \mathsf{C}_{t+1} \tag{10}$$

$$\mathsf{C}^*_t = -\frac{\delta}{1+\delta}\mathsf{r}_{t+1} + \mathsf{C}^*_{t+1} \tag{11}$$

Money demand

$$\mathsf{M}_t - \mathsf{P}_t = \mathsf{C}_t - \frac{\mathsf{r}_{t+1}}{1+\delta} - \frac{\mathsf{P}_{t+1} - \mathsf{P}_t}{\delta} \tag{12}$$

$$\mathsf{M}^*_t - \mathsf{P}^*_t = \mathsf{C}^*_t - \frac{\mathsf{r}_{t+1}}{1+\delta} - \frac{\mathsf{P}^*_{t+1} - \mathsf{P}^*_t}{\delta} \tag{13}$$

Log-linearized Model (continued)

Take (12) subtract (13) and subs. $\mathsf{P}_{t+s} - \mathsf{P}^*_{t+s} = \mathsf{e}_{t+s}$ for $s = \{0, 1\}$ from (4)

$$\mathsf{e}_t = \mathsf{M}_t - \mathsf{M}^*_t - (\mathsf{C}_t - \mathsf{C}^*_t) + \frac{1}{\delta}(\mathsf{e}_{t+1} - \mathsf{e}_t). \quad \text{ER} = f(\text{fundamentals, exp. change in ER}) \tag{14}$$

Now, given the incomplete markets environment, we don't have a stationarity inducing device. Then the shocks have permanent effects.

No stationarity: take (10) $-$ (11)

$$\mathsf{C}_t - \mathsf{C}^*_t = \mathsf{C}_{t+1} - \mathsf{C}^*_{t+1}.$$

Shocks in consumption are permanent $\Rightarrow$ can look into long-run effect of shocks.

With fully flexible prices and permanent shocks, the economy jumps from one steady state to the new one instantly. (no reason to smooth impact of a shock)

With sticky prices the new steady state will be reached too but only in the long run. The shock considered here is one in the wealth of Home households (a).

To approximate the change in steady-states we can log-linearize the steady-state consumption around the initial steady-state. That is log-linearize the equations in (1)

$$\begin{aligned}\bar{\mathsf{C}} &= \delta\bar{\mathsf{a}} + \bar{\mathsf{p}}(h) - \bar{\mathsf{P}} + \bar{\mathsf{y}}(h),\\ \bar{\mathsf{C}}^* &= -\left(\frac{n}{1-n}\right)\delta\bar{\mathsf{a}} + \bar{\mathsf{p}}^*(f) - \bar{\mathsf{P}}^* + \bar{\mathsf{y}}^*(f),\end{aligned} \tag{15}$$

here the barred variables with no time subscript refer to the percentual change between the new steady-state value and the initial one ($\bar{\mathsf{C}} = d\bar{C}/\bar{C}_0 = (\bar{C} - \bar{C}_0)/\bar{C}_0$).

Additionally, the initial steady-state for the NFA is zero, thus, the log-linear approximation of the assets would not be well defined and its definition is therefore adjusted as $\bar{\mathsf{a}} = d\bar{a}/\bar{C}_0^W$ (with $\bar{C}_0^W = \bar{C}_0 = \bar{y}_0$).

The equations (2)-(13) hold at all points in time, including when they reach the new steady-state. Therefore, the change in steady-state is described by barred versions of these equations together with (15).

Obstfeld and Rogoff proceed to solve the system following Aoki(1981) technique: Solve for cross country differences and population weighted world averages first.

Difference of demands: (5)-(6)

$$\mathsf{y}_t(h) - \mathsf{y}_t^*(f) = -\theta(\mathsf{p}_t(h) - \mathsf{p}_t^*(f) - \mathsf{e}_t) \tag{16}$$

This equation states that the relative demand is a positive function of the terms-of-trade.

Now, solve for $\mathsf{p}_t(h) - \mathsf{p}_t^*(f)$ from the relative supply (8)-(9), and substitute in the relative demand above to obtain

$$\mathsf{y}_t(h) - \mathsf{y}_t^*(f) = -\frac{\theta}{1+\theta}(\mathsf{C}_t - \mathsf{C}_t^*). \tag{17}$$

Given PPP holds ($\mathsf{P}_t - \mathsf{P}_t^* - \mathsf{e}_t = 0$), by subtracting the second equation from the first in (15) we get

$$\bar{\mathsf{C}} - \bar{\mathsf{C}}^* = \frac{1}{1-n}\delta\bar{\mathsf{a}} + \bar{\mathsf{y}}(h) - \bar{\mathsf{y}}^*(f) + [\bar{\mathsf{p}}(h) - \bar{\mathsf{p}}^*(f) - \bar{\mathsf{e}}]$$

Substitute from (barred versions of) (16) and (17):

$$\bar{\mathsf{C}} - \bar{\mathsf{C}}^* = \left(\frac{1}{1-n}\right)\left(\frac{1+\theta}{2\theta}\right)\delta\bar{\mathsf{a}}$$

Home NFA $\uparrow \Rightarrow$ H Consumption $\uparrow$

Notice the change in consumption is lower than $\left(\frac{1}{1-n}\right)$ because with higher income agents shift labor out of production (increase leisure).

It also follows that

$$\bar{\mathsf{p}}(h) - \bar{\mathsf{p}}^*(f) - \bar{\mathsf{e}} = \left(\frac{1}{1-n}\right)\left(\frac{1}{2\theta}\right)\delta\bar{\mathsf{a}},$$

then, the steady-state TOT improve if the Home's NFA increase.

World Aggregates

We can take a population weighted average of the supply equations (8), (9) while taking into account the pricing equations (2), (3) yields $\bar{\mathsf{Y}}^W = -\bar{\mathsf{C}}^W$. But the equilibrium in goods markets requires $\bar{\mathsf{Y}}^W = \bar{\mathsf{C}}^W$.

Thus, $\bar{\mathsf{Y}}^W = \bar{\mathsf{C}}^W = 0 \rightarrow$ Small asset changes have no first-order effect on world aggregates.

One-period Price Stickyness

In the short-run, at time 1 prices are fixed at the initial steady state ten: $p_1(h) = \bar{p}_0(h), p_1^*(f) = \bar{p}_0^*(f)$.

For the LOP to hold $p^*(h), p(f)$ must be allowed to fluctuate (i.e., $p_1(h) = \varepsilon_1 p_1^*(h)$ implies the price movement since ε may move and $p_1(h)$ is fixed).

Then, this model assumes Producer Currency Pricing (PCP) where the firms set prices in their currency and the LOP dictates the price in other locations.

One-period Price Stickyness (cont.)

Consider the law of motion of Home NFA:

$$a_{t+1} = (1 + r_t) + \frac{p_t(h)}{P_t} y_t(h) - C_t.$$

Log-linearizing this equation, using the price index equations, the assumption of preset prices and the fact that assets do not change on impact:

$$\bar{\mathsf{a}} = \mathsf{y}(h) - \mathsf{C} - (1 - n)\mathsf{e},$$
$$\bar{\mathsf{a}}^* = -\left(\frac{n}{1-n}\right)\bar{\mathsf{a}} = \mathsf{y}^*(f) - \mathsf{C}^* + n\mathsf{e}.$$

These equations provide the link between the long run solution and the short-run dynamics in the model (the SR dynamics are reflected in NFA assets changes, or in $\bar{\mathsf{a}}$, which affect the LR solution).

Shock effects: a monetary shock will affect $\bar{\mathsf{a}}$ only in the period of the shock, that changes the long-run levels of the rest of the variables. Thus, there is monetary non-neutrality (it appears due to the non-stationarity of the model).

Overshooting

Dornbusch (1976): the money supply increases, the interest rate lowers, and the ER increases by more than in the long-run because of the UIP plus a liquidity effect with sticky prices.

OR1995: No overshooting in the basic model.

To see this, assume a permanent shock such that

$$\bar{\mathsf{M}} - \bar{\mathsf{M}}^* = \mathsf{M} - \mathsf{M}^*.$$

The convergence to the new steady-state by the end of the second period (when prices adjust) implies by (14):

$$\mathsf{e} = \mathsf{M} - \mathsf{M}^* - (\mathsf{C} - \mathsf{C}^*) + \frac{1}{\delta}(\bar{\mathsf{e}} - \mathsf{e}), \quad \text{1 period convergence} \tag{18}$$

also,

$$\bar{\mathsf{e}} = \bar{\mathsf{M}} - \bar{\mathsf{M}}^* - (\bar{\mathsf{C}} - \bar{\mathsf{C}}^*), \text{ and } \quad \mathsf{C} - \mathsf{C}^* = \bar{\mathsf{C}} - \bar{\mathsf{C}}^*, \quad \text{no stationarity condition}$$

then $\bar{\mathsf{e}} = \bar{\mathsf{M}} - \bar{\mathsf{M}}^* - (\mathsf{C} - \mathsf{C}^*)$, implying

$$\mathsf{e} = \bar{\mathsf{e}} + \frac{1}{\delta}(\bar{\mathsf{e}} - \mathsf{e})$$

that is, $\mathsf{e} = \bar{\mathsf{e}}$ $\quad$ ($\mathsf{e}_{t+1} = \mathsf{e}_t$ No overshooting)

Equilibrium effects of money shocks

From before the ER equation becomes,

$$[\text{MM}] \qquad \mathsf{e} = \mathsf{M} - \mathsf{M}^* - (\mathsf{C} - \mathsf{C}^*). \tag{19}$$

Equation (19) represents a schedule on the ER that is downward sloping in consumption because an increase in relative home consumption raises money demand, and therefore, the home's relative price level must fall, which implies an ER appreciation.

A second schedule can be derived using the current account equation as follows. Taking the home current account and subtracting the home one (in terms of $\bar{\mathsf{a}}$) we get

$$\bar{\mathsf{a}} = (1-n)[\mathsf{y}(h) - \mathsf{y}^*(f) - (\mathsf{C} - \mathsf{C}^*) - \mathsf{e}]$$

Alternatively, this equation can be obtained by rearranging and subtracting the equation (7) from the current account equation with one-period price stickyness derived for $\bar{\mathsf{a}}$.

Now, we can substitute $\mathsf{y}(h) - \mathsf{y}^*(f)$ from the difference in the demand equations (and using the assumption of price stickyness with symmetric steady state between home and foreign)

$$\bar{\mathsf{a}} = (1-n)[\theta\mathsf{e} - (\mathsf{C} - \mathsf{C}^*) - \mathsf{e}]$$

Next, we can substitute this expression for $\bar{\mathsf{a}}$ into $\bar{\mathsf{C}} - \bar{\mathsf{C}}^* = \left(\frac{1}{1-n}\right)\left(\frac{1+\theta}{2\theta}\right)\delta\bar{\mathsf{a}}$ (with $\bar{\mathsf{C}} - \bar{\mathsf{C}}^*$)

$$\mathsf{C} - \mathsf{C}^* = \left(\frac{1}{1-n}\right)\left(\frac{1+\theta}{2\theta}\right)\delta\left[(1-n)[\theta\mathsf{e} - (\mathsf{C} - \mathsf{C}^*) - \mathsf{e}]\right].$$

Equilibrium effects of money shocks (cont.)

Solving for e from the last expression yields:

$$\text{[GG]} \qquad \mathsf{e} = \frac{2\theta + (1+\theta)\delta}{\delta(\theta^2 - 1)}(\mathsf{C} - \mathsf{C}^*). \tag{20}$$

This schedule is upward sloping because home consumption can rise relative to foreign when the home currency experiences a depreciation (making home output rise relative to foreign).

Given the solution for e we can recover $\mathsf{y}(h) - \mathsf{y}^*(f)$ and $\bar{\mathsf{a}}$

$$\bar{\mathsf{a}} = \frac{2(1-n)(\theta-1)}{2+\delta(1+\theta)}(\mathsf{M} - \mathsf{M}^*).$$

The larger is home (n) the smaller the effect of a monetary shock on its current account.

Intuition: As n grows the foreign economy shrinks and foreign asset accumulation has a smaller effect on the domestic current account.

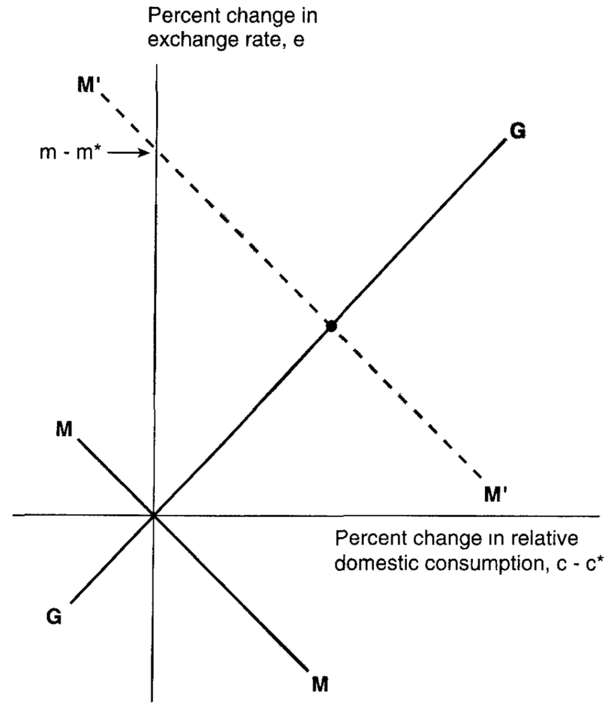
From these two-equations system we can solve for e and $\mathsf{C} - \mathsf{C}^*$ in terms of the relative money supplly

$$\mathsf{e} = \frac{2\theta + \delta(1+\theta)}{2\theta + \theta\delta(1+\theta)}(\mathsf{M} - \mathsf{M}^*), \tag{21}$$

$$\mathsf{C} - \mathsf{C}^* = \frac{\delta(\theta^2 - 1)}{2\theta + \theta\delta(1+\theta)}(\mathsf{M} - \mathsf{M}^*). \tag{22}$$

Equilibrium effects of money shocks (cont.)

Similarly, from the intersection of these two schedules we can get graphically the solution for e.



Source: Obstfeld and Rogoff (1996), Chapter 10.

Going back to the effect of the shock, above we can see that if $\mathrm{M} - \mathrm{M}^*$ increases the MM curve shifts outward to MM' and GG remains unchanged. As a result, we get an unambiguous ER depreciation that raises relative consumption.

e $\uparrow\rightarrow$ real relative income : $\mathrm{y}(h) - \mathrm{y}^*(f) \uparrow\rightarrow ca$ surplus due to consumption smoothing. In LR: $\uparrow$ wealth $+$ endog. labor $\Rightarrow$ HH shift labor into leisure $\Rightarrow\downarrow$ home supply $\Rightarrow$ $\mathrm{p}(h) \uparrow\Rightarrow TOT \uparrow$

Short-Run World Aggregates

To complete the solution of the model, we can obtain the world aggregates.

From a population weighted average of the Euler equations,

$$\mathsf{C}^W = \bar{\mathsf{C}}^W - \frac{\delta}{1-\delta}\mathsf{r},$$

but we had that $\bar{\mathsf{C}}^W = 0$. Hence, $\mathsf{C}^W = \frac{\delta}{1-\delta}\mathsf{r}$. Now use the log-linear aggregate money demands, price index equations, the assumption of short run price rigidity, and $\bar{\mathsf{C}}^W = 0$ to write:

$$\mathsf{M}^W = \mathsf{C}^W - \frac{1}{1+\delta}\mathsf{r} - \frac{1}{\delta}\mathsf{M}^W.$$

Combine this equation with $\mathsf{C}^W = -\frac{\delta}{1+\delta}\mathsf{r}$,

$$\mathsf{C}^W = \mathsf{Y}^W = \mathsf{M}^W, \qquad \mathsf{r} = -\frac{1+\delta}{\delta}\mathsf{M}^W.$$

Short-Run World Aggregates (cont.)

$$\mathsf{C}^W = \mathsf{Y}^W = \mathsf{M}^W, \qquad \mathsf{r} = -\frac{1+\delta}{\delta}\mathsf{M}^W.$$

A monetary expansion at home or abroad lowers the world real interest rate in proportion to the size of the expanding country, and therefore, C^W increases. In the long-run C^W and r return to their initial levels (e.g., $\bar{\mathsf{C}} = 0$).

Then, the long-run non neutrality of money showed up only in the country differentials, but not on the world aggregate. The reason is that the neutrality stem from changes in net asset positions in each location, but at the world level the net assets are zero.

Note: Given the solutions above, we can recover short-run changes in country specific variables from the world aggregates and cross country differences:

$$\mathsf{x}_t = \mathsf{x}_t^W + (1-n)(\mathsf{x}_t - \mathsf{x}_t^*), \qquad \mathsf{x}_t^* = \mathsf{x}_t^W - n(\mathsf{x}_t - \mathsf{x}_t^*)$$

Welfare

One of the main points of the micro-foundation is the ability of performing welfare analysis.

If M and M^* both increase such that $M - M^* = 0$ the welfare increases in both countries.

Reason: Country differentials won't change, hence neither TOT, ER do. But the world aggregates increase ($Y^W \uparrow$, $C^W \uparrow$, $M^W \uparrow$).

(This refers to an unexpected shock, not a systematic increase policy)

For asymmetric money shocks: We focus on the real part of utility

$$U_t^R = \sum_{s=t}^{\infty} \beta^{s-t} \left[\log C_s - \frac{\kappa}{2} y_s(h)^2 . \right]$$

Welfare (cont.)

Total differentiate U_t^R, and together with the fact that the economy converges to new steady-state in one period yields:

$$dU^R = \mathsf{C} - \kappa \bar{y}_0(h)^2 \mathsf{y}(h) + \frac{1}{\delta}\left[\bar{\mathsf{C}} - \kappa \bar{y}_0(h)^2 \bar{\mathsf{y}}(h)\right],$$

and substituting the expression for $\bar{y}_0(h)$,

$$dU^R = \mathsf{C} - \frac{\theta - 1}{\theta}\mathsf{y}(h) + \frac{1}{\delta}\left[\bar{\mathsf{C}} - \frac{\theta - 1}{\theta}\bar{\mathsf{y}}(h)\right].$$

It can be shown that

$$dU^R = \frac{\mathsf{C}^W}{\theta} = \frac{\mathsf{M}^W}{\theta}.$$

$\triangle$Utility: proportional to new world money supply [regardless of its origin]

Reason: Small changes in relative prices induced by ER changes have no first order effects in equilibrium (since in initial eq. all firms set prices at optimal levels). Similarly, as agents are optimally smoothing consumption over time, small changes in ca_t have only 2nd order effects.

Summary:

Bechmark OR1995: No overshooting, and money expansions are welfare improving regardless or shock origin.

[*prosper-thy-neighbo*r effects of monetary expansions]

Caveat: The OR model it's only valid around a small neighborhood of the steady state (given its approximation technique), and assumes perfect foresight due to its non-stationary nature.

Next: Addressing the absence of overshooting by including non-traded goods.