

International Finance 4832

Lecture 2: Exchange Rates

Camilo Granados
The University of Texas at Dallas
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Reminder

- ▶ **Course Clearing House:** <https://cagranados.github.io/intfinance.html>
You will find there the Syllabus, updates, Lecture Slides, Problem Sets. (Syllabus: [\[Link\]](#))
- ▶ **Textbook:** International Economics by Feenstra and Taylor - Edition 4 (or 3)
- ▶ **Workload Expectations:** Next HW: Prob. Set 1 2/9
 - 5 problem sets, 3 exams, and a final group presentation module (optional but good bonus)
- ▶ **Attendance:** Not required, but recommended and gives bonus.
- ▶ **Office Hours:** M 4:00-6:00PM or by appointment
(if door's not open during my OH I'm likely with an student already so feel free to knock)
- ▶ **Email:** camilo.granados@utdallas.edu

An Overview of this course

1. Exchange Rates *today!*
2. Balance of Payments (Int. Borrowing and Lending)
3. Open Economy Macroeconomics (applications of theory in Policy)
4. Recent Research (optional presentations if time allows)

This Lecture: Exchange Rates

This lecture - Exchange Rates Fundamentals (Chapter 13)

1. Foreign Exchange Definitions

- ▶ Definitions
- ▶ Cross-country prices
- ▶ Exchange Rate Regimes
- ▶ Contracts in FX (Forex) markets

2. No-arbitrage conditions (definition of Market Equilibrium)

- ▶ Triangular Arbitrage, vehicle currencies
- ▶ (CIP) Covered Interest Rate Parity → Forward Rate
- ▶ (UIP) Uncovered Interest Rate Parity → Spot Rate (ER)

3. Later: Price levels and Exchange Rates (in the long run - Chapter 14)

- ▶ Price Parity Conditions
- ▶ Money and the Exchange Rate

Exchange Rates

Exchange Rate (E): Price of one currency in terms of another

- + "Another": Usually a "home" currency (e.g., 1.19 dollars $\frac{\$}{USD}$ per canadian dollar CAD)

This one (ER between any two currencies) is also called "bilateral ER": $E_{\$/\text{€}} = 1.15$ or

$$\underline{E_{\$/CAD} = 0.8}$$

Must be very careful about the units

$E_{\$/\text{£}}$

What is being priced or valued
↳ Units used to define price

Those above are the prices of € and CAD!

Should be read as 1.15 Dollars per Euro or 0.8 Dollars per Canadian Dollars

This quote (price) works exactly as with any other good (e.g. 3.5 dollars for a cup of coffee)

Key: The units give away that we are pricing the other currency (like with other goods)

Exchange Rates (cont.)

Given $E_{\$/\epsilon} = 1.15$ or $E_{\$/CAD} = 0.8$

We can get the price of the other currency:

$$\text{Euro Price of Dollar: } E_{\epsilon/\$} = \frac{1}{E_{\$/\epsilon}} = \frac{1}{1.15} = 0.87$$

$$\text{CAD price of US Dollar: } E_{CAD/\$} = \frac{1}{E_{\$/CAD}} = \frac{1}{0.8} = 1.25$$

See how the units "split" or "revert" ... again, we could do this with any good, like finding how many coffee cups are equivalent to a dollar: $\frac{1}{3.5} = 0.28$ (a bit more than a quarter of a cup)

Exchange Rates (cont.)

When $E_{\$/\text{€}}$ decreases:

Now you pay fewer \$ for each € \Rightarrow USD (\$) became valuable

The Dollar appreciates (strengthens) (USD gains value or a Euro is cheaper)

The Euro: Depreciates (weakens)

Price of €: $E_{\$/\text{€}} \downarrow$

Price of \$: $E_{\text{€}/\$} = \left(\frac{1}{E_{\$/\text{€}}} \right) \uparrow$

Exchange Rate Growth Rate: Calculated like any % rate (also called depreciation rate)

Example: $E_{\$/\text{€}, 2020} = 1.22$ and $E_{\$/\text{€}, 2021} = 1.13$

$$\text{Growth Rate} = \frac{\underbrace{E_{\$/\text{€}, 2021}}_{E_{\$/\text{€}, 20} - E_{\$/\text{€}, 20}} - 1}{\underbrace{E_{\$/\text{€}, 20}}_{E_{\$/\text{€}, 20}}} = \frac{1.13}{1.22} - 1 = -0.0734 \rightarrow \text{the Euro depreciated 7.34\% against the Dollar}$$

(if you want to know how much the Dollar appreciated against the Euro you need to compute $E_{\$/\text{€}, 2020}$, $E_{\$/\text{€}, 2021}$ and find the growth rate)

Note: If you want another step-by-step example, check the textbook (3rd edition), page 439

Exchange rate

Let's find the appreciation of the dollar for this example:

$$E_{\text{€}/\$,\text{2020}} = \frac{1}{1.22} = 0.82$$

$$E_{\text{€}/\$,\text{2021}} = \frac{1}{1.13} = 0.88$$

How much did the Dollar Appreciate or Depreciate against the Euro?

$$\left(\frac{0.88}{0.82} - 1 \right) \times 100 = 7.31\%$$

Note: The Euro depreciation is not exactly equal (in magnitude) to the appreciation of the Dollar.

(the lower the variation the more similar but they are not "equal" in general)

EUR deprec. by 7.34%.

USD apprec. by 7.31%.

Exchange Rate Changes (cont.)

Known property of growth rates:

$$\text{Depreciation of a currency} \neq -\text{Appreciation of the other currency}$$

i.e., if the USD appreciates 15% against the Euro, the Euro is not depreciating 15% relative to the USD.

An example:

$$\left(\frac{10}{5} - 1\right) \times 100 = 100\% \quad \text{compared to} \quad \left(\frac{5}{10} - 1\right) \times 100 = -50\%$$

When the growth rates are closer to zero (lower in absolute value) the rates are more similar in magnitude (as in the example in the slide before).

Multilateral Exchange Rates USD : Appreciated : { CNY wrt EUR , Depreciated : { GBP (£) wrt MXP } Did the value of the USD increase or decrease ?]

In principle any country has a bilateral ER with every other country in the world (e.g., US-Thailand or US-Argentina)

The Dollar may appreciate against some and depreciate against others

how to know whether the Dollar gained/lost value? → By checking the Effective ER

answer: hard to say; but can be determined by Effect. ER

Effective Exchange Rate: trade-weighted average of bilateral Exchange Rates

We can average the bilateral ER growth rates and weight by their trade share:

$$\frac{E_{t+1}}{E_t} - 1 = \sum_i \left[\left(\frac{E_{\$/i,t+1}}{E_{\$/i,t}} - 1 \right) \frac{\text{trade}_i}{\text{total trade}} \right] \quad (i: \text{country index})$$

For example,

growth rate of bilateral ER
(USD per i's currency)

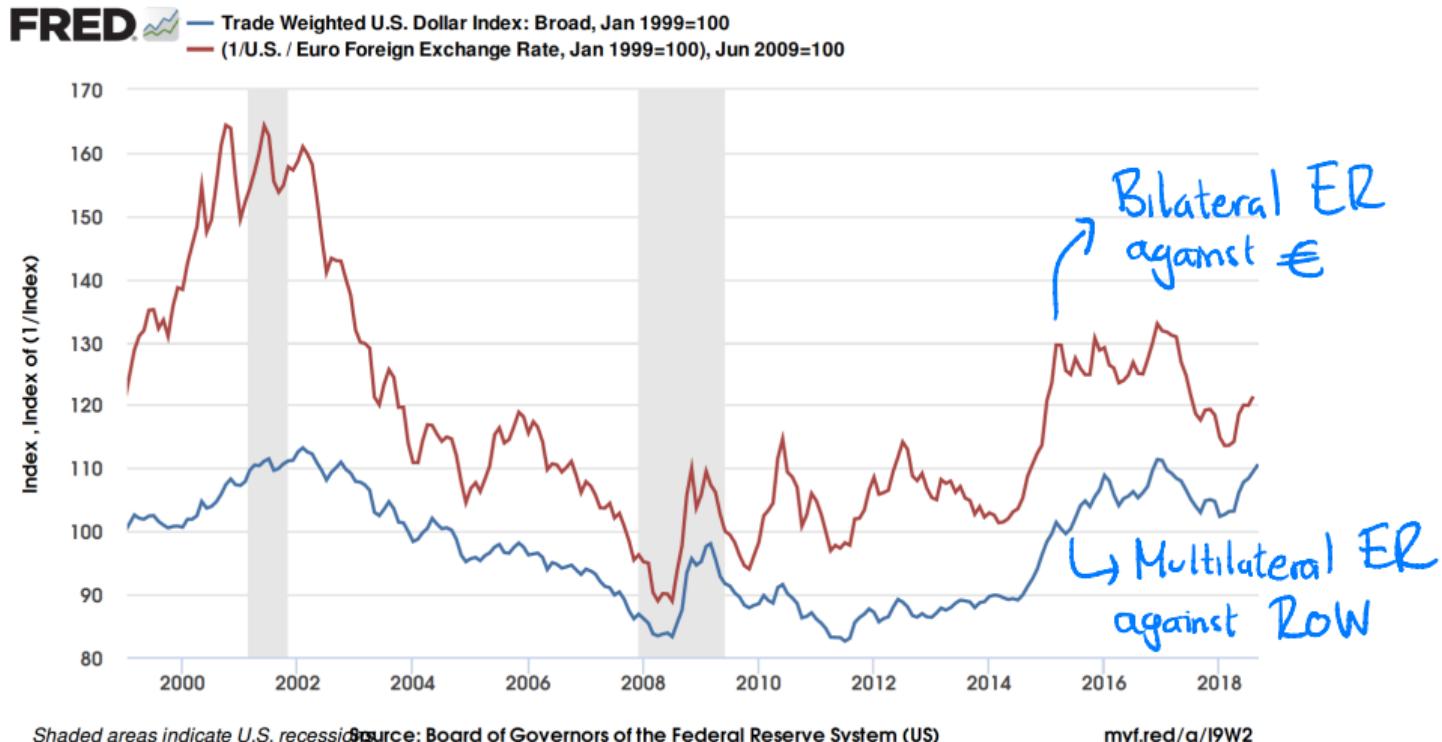
for the US vs Thailand (when i denotes Thailand):

trade_i : US Imports from Thailand + US Exports to Thailand

total trade : US Imports to all countries + US Exports to all countries

{ Trade_i : US Exports to i + US Imports to i
Total Trade : Total US Exports + Total US Imports

Bilateral USD/Euro ER vs Multilateral US ER



Exchange Rates and Prices

We mentioned that ERs do change relative prices of goods (and assets) across countries

How can we state this?

→ Because we assume ERs (and financial assets' prices in general) change much more frequently than Goods and Services prices

Not unreasonable: we see this empirically (e.g., is the price of a coffee changing by the day?)

Assumption: Local-currency goods prices are sticky (rigid) in the short run

With that, changes in the ERs can change the prices levels and relative price of goods when expressed in other countries

Then, even if local prices won't change, changes in ER induce changes in the prices of goods (in foreign countries)

(why we need the assumption? ... because, otherwise it could happen that the local price also changes, offsetting the change induced by the change in the ER)

Exchange Rates and Prices (cont.)

Let's see an example with an actual good (from the book, table 13-2)

Cost of suit in local currency:

£2000 in London, HK \$ 30000 in Hong Kong, \$ 4000 in New York City

- ▶ If $E_{HK\$/\text{£}} = 15, E_{\$/\text{£}} = 2$ the prices in pounds are (pay close attention to the units cancelling):

$$p_{hk} = \frac{30000}{15} = \text{£}2000 \quad \text{and} \quad p_{ny} = \frac{4000}{2} = \text{£}2000$$

$\cancel{\text{HK\$}}$ $\cancel{\$}$
 $\cancel{\text{HK\$}/\text{£}}$ $\cancel{\$/\text{£}}$

- ▶ If $E_{HK\$/\text{£}} = 16, E_{\$/\text{£}} = 1.9$ (£ appreciated vs. the HK\$ and depreciated vs. the USD):

$$p_{hk} = \frac{30000}{16} = \text{£}1875 \quad \text{and} \quad p_{ny} = \frac{4000}{1.9} = \text{£}2105$$

Where to buy? → Hong Kong

Exchange Rates and Prices

When the home country's currency **depreciates**:

Foreign currency becomes more valuable (or home currency less valuable)

Prices of home's exports in foreign currency? (or in the foreign location)

Become cheaper (e.g. same € buys more US goods if $\uparrow E_{\$/\text{€}}$)

Prices of foreign imports in the home currency?

More expensive now (costlier to buy abroad with home currency)

Does this depend on Price Stickiness?

Yes and no: If prices are sticky locally (don't change) then ERs will matter more (will be the sole drivers of the price)

If they are not sticky the price paid by locals (of foreign goods) will change both by the change in the ER and by the change in local prices.

Exchange Rate Regimes

By this point is clear that changes in the ER can affect the livelihood of people

Thus, governments usually take actions regarding how much to allow the ER to move:

Fixed (government sets the ER) vs. Flexible Exchange Rates (floating ERs):

What is the trade-off?

► Fixing the ER

- Pros: Stable prices for Exports, Imports, and Investments (foreign assets)
- Cons: Costly, loss of Monetary Autonomy

Implies using Fed rates to stabilize ER
rather than to pursue other goals (e.g. ↓ Inflation)

Types of Fixed ER Regimes: Pegs, currency boards, no domestic currency at all (dollarization)

Types of Floating ERs: Bands, Managed Floating, Free Floats (market is allowed to determine the ER)

Problem of floating: can generate too volatile ERs → volatile economic outcomes

We see both reality. Why? → Depends on country's goals (e.g., China does this often), Ecuador too

Currency Markets (Forex market) - How are currencies traded?

- ▶ Over the counter market (dealers vs. central exchange)
- ▶ Global market but taking place at key financial locations: Mostly US, UK, Japan
- ▶ Participants are **mostly banks**
- ▶ Trade is done against major currencies (vehicle currencies)

\$, €, £, ¥



What gets traded?

- ▶ Contracts denominated in a currency
- ▶ Spot contracts: amounts of money/deposits in the Current ER (or "on the spot") → $\text{Spot Date} = \text{Current ER}$
- ▶ Derivative contracts: forwards, swaps, futures, options
Prices "derived" partially from Spot price. Imply buying the deposits at future ER values
What changes is how to deal with uncertainty about future value of ER
- ▶ Forwards: two parties agree on price and quantity to exchange in the future.
example: agree to exchange \$2000 for €2400 one year from today. Forward rate: $F_{\$/\text{€}} = 1.2$

Although not all currency pairs are exchanged, based on the few traded we can deduce all bilateral ERs

Outline (again)

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Arbitrage

Arbitrage: (1) Exploit price differences to profit, (2-Finance) Costless profit opportunity

In this context: Prices \equiv Exchange Rates

Definition of Equilibrium we use: No Arbitrage opportunities (or absence of arbitrage)

i.e., in an equilibrium ERs will not move further because no one has further incentives to buy one or sell another. This can only happen remaining arbitrage opportunities have been used up.

Arbitrage (example)

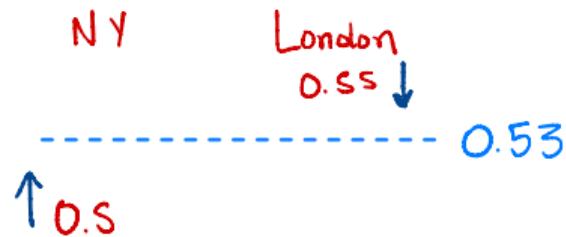
Example: Suppose $E_{£/\$} = 0.55$ in London and $E_{£/\$} = 0.50$ in New York

You could sell pounds in London and buy in New York (would win £0.05 for each unit sold)

How would markets respond to these prices?

More buying (\uparrow demand) in NY $\Rightarrow \uparrow E_{£/\$}$ in NY

More selling (\uparrow supply) in London $\Rightarrow \downarrow E_{£/\$}$ in London



This goes on until no one has incentives to trade ... that is when $E_{£/\$}$ is equal in both locations

What might prevent this process (and profiting):

Fees, market regulations, low liquidity in markets, poor ability to borrow/lend (more on these frictions later)

Triangular Arbitrage

Arbitrage with 3 currencies: exchange for one currency, then for another and make a profit when compared to direct (bilateral) exchange.

Not expected in equilibrium

(by the very same logic as bilateral arbitrage; neither expected with N currencies)

Example: $E_{\text{€}/\$} = 0.8$, $E_{\text{£}/\text{€}} = 0.7$, $E_{\text{£}/\$} = 0.5$

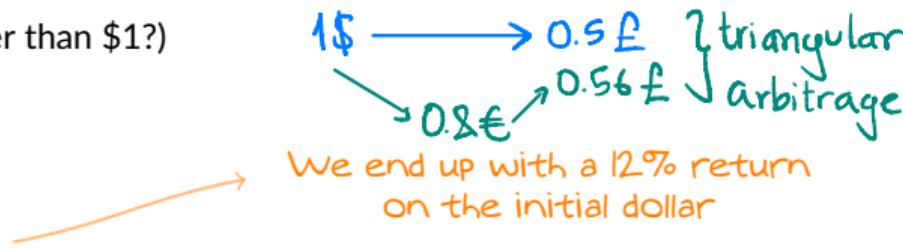
Assume: No transaction costs (small fees for repeated transactions) ... is there arbitrage? ... Yes:

Start with \$1 (can we end with anything greater than \$1?)

1. Buy €: $\$1 \times 0.8\text{€}/\$ = 0.8\text{€}$

2. Buy £: $0.8\text{€} \times 0.7\text{£}/\text{€} = 0.56\text{£}$

3. Buy \$ again: $0.56\text{£} \times \frac{1}{0.5\text{£}/\$} = 1.12\text{\$}$



Notice: For converting pounds to dollars we divide by ER of "pounds per dollar". Can just find price of dollar in pounds first and then multiply but the result is identical as ER of dollars per pound is $E_{\$/\text{£}} = \frac{1}{E_{\text{£}/\$}}$

In reality we don't see this. Agents in FX market realize and extinguish arbitrage opportunity quickly

Triangular Arbitrage (cont.)

Absence of Arbitrage implies that we expect to see:

$$E_{\text{£}/\$} = E_{\text{£}/\text{€}} \times E_{\text{€}/\$}$$



That is, we should be indifferent between exchanging pounds for dollars directly than in several transactions with another currency (changing by Euros first).

Given $E_{\text{€}/\$} = 0.8$, $E_{\text{£}/\text{€}} = 0.7$, $E_{\text{£}/\$} = \cancel{0.5}$, the no-arbitrage pound-dollar rate should be:

$$E_{\text{£}/\$} = E_{\text{£}/\text{€}} \times E_{\text{€}/\$} = 0.7 \times 0.8 = \underline{0.56}$$

This is what tends to hold and that is why if we know two of the ERs we can always compute the third one. That's how many bilateral ERs are obtained in practice

(as I mentioned there are no transactions between every pair of ER always)

Arbitrage and Interest Rates

Alternatives to compare → {
(1) Buy asset at home (US)
(2) Buy asset abroad (EU)}

Similar results can be obtained if we consider buying and holding assets (e.g., bonds, money deposits) rather than just exchanging currencies directly.

Here we compare investing in one asset at home vs. investing in one abroad

For investing abroad: Must exchange home currency for foreign before buying the asset there

There are two moving parts here:

(2) implies → Exchange USD for €,
Buy asset in EU,
Exchange back to USD

► The Return of Assets: Given by the Interest Rate they pay

► The Future Exchange Rate after the investment period: Big source of uncertainty

We can cover the risk coming from the second part with **forward** contracts

Doing so leads to the: Covered Interest Rate Parity CIP: Future ER risk is covered

Or ... we can assume that risk using Spot contracts: Uncovered Interest Rate Parity : UIP

↳ No hedge → use future spot ER

The Covered Interest Rate Parity (CIP)

Here we tie our definition of equilibrium (no arbitrage) to interest rates across the world

We are going to cover the Exchange Rate Risk (risk of ending with different ER than expected in the future) with a Forward.

If I have 1\$ I should be indifferent between depositing (investing) it in one location or another (or better said, the ERs and Interest rate should be such that I'm indifferent)

Option 1: Buy a US bond (T-bill) that yields an interest rate $i_{\$}$ → get $\$1(1+i_{\$})$

Option 2: Buy a Euro bond yielding $i_{\text{€}}$

Going with Option 2 usually implies assuming Foreign Exchange Risk

But this is covered here with a **forward contract**:

The parties trading agree on a Future ER from now

$$\text{get } \$1 \cdot \frac{1}{E_{\$/\text{€}}} \cdot (1+i_{\text{€}}) \cdot E_{\$/\text{€}}^{\text{Future}}$$

Unknown; guess (forecast)
or forward rate

We will see how much an investor gets for each dollar in each Option

Note: FT make the CIP, UIP examples with deposits in banks (or money market assets). Here it's done with assets ... why? → it's equivalent. Bonds yield a low riskless return just as in the money market (or deposits do).

Covered Interest Rate Parity (cont.)

An Investor has \$1.

Covered \Rightarrow Use Forward ER


Option 1: Investing at home \rightarrow gets $\underline{\$1(1 + i_{\$})}$ after a year

Going with **Option 2** implies (for a US investor):

Taking the 1\$ exchanging it for Euros for $\frac{1}{E_{\$/\epsilon}}$

Buying the foreign asset and getting a return i_{ϵ} , i.e., getting in total: $\frac{1}{E_{\$/\epsilon}}(1 + i_{\epsilon})$

Taking that amount and exchanging it for dollars at a **forward rate** $F_{\$/\epsilon}$ \rightarrow getting: $\underline{\frac{F_{\$/\epsilon}}{E_{\$/\epsilon}}(1 + i_{\epsilon})}$

Then you compare:

$$\$1(1 + i_{\$}) \begin{matrix} > \\ \text{v.s.} \\ < \end{matrix} \$1 \frac{F_{\$/\epsilon}}{E_{\$/\epsilon}}(1 + i_{\epsilon})$$

With absence of arbitrage these two should be equivalent: $(1 + i_{\$}) = \frac{F_{\$/\epsilon}}{E_{\$/\epsilon}}(1 + i_{\epsilon})$

That is the CIP

Covered Interest Rate Parity (cont.)

Moreover, this expression tells us what a Forward Rate depends on (given the other variables are known today):

$$\underbrace{F_{\$/\epsilon}}_{\text{Forward ER}} = \frac{(1 + i_{\$})}{1 + i_{\epsilon}} \underbrace{E_{\$/\epsilon}}_{\text{Spot ER}}$$

Here it's easy to see how the Forward value is derived from the Spot ER value

That's why the Forward is a "Derivative"

Covered Interest Rate Parity (cont.)

This parity generally holds in the data ...

But there have been important periods of deviation (i.e. where the parity breaks) particularly since the Global Financial Crisis of 2008

We can plot the Profit of arbitrage given by the LHS minus the RHS of the CIP equation (profit should be about zero if CIP holds)

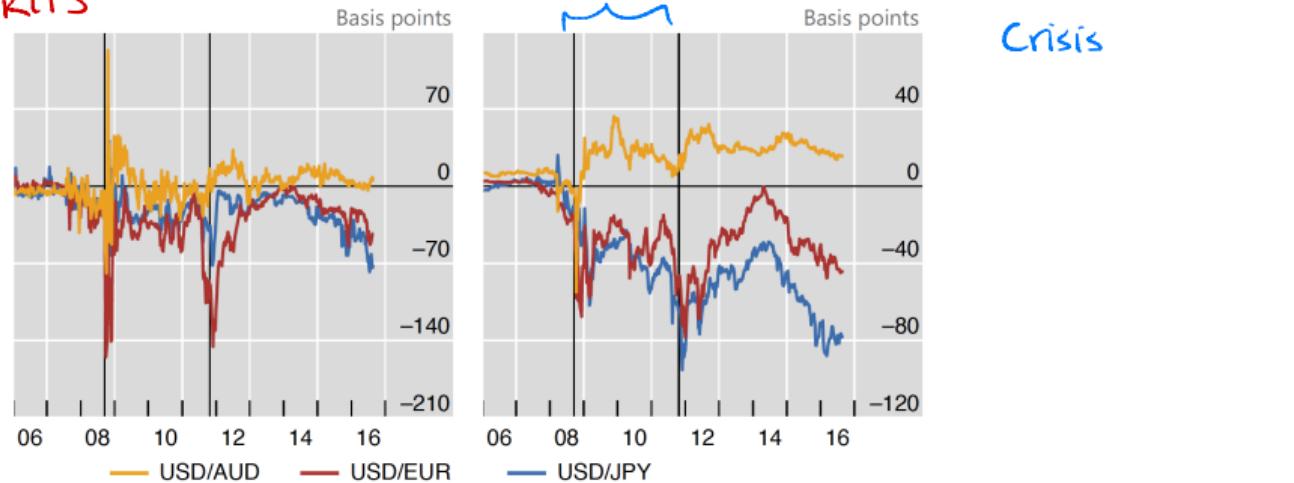
Figure: CIP Deviations (left: 3-month basis; right: 3-year basis)

$$\text{CIP deviation} = \text{LHS} - \text{RHS}$$

(of CIP equation)



Should be about zero
if CIP holds



Why the CIP can break?

Note: This is different from what the book says ... by its publication it was not as clear the CIP could break!
This is why we have a final optional module (presentations on recent research) ... not all is said and done on this field

Why there is no absence of arbitrage here (CIP is not holding):

- Role of Risk after the Crisis is more important
- Using an Arbitrage opportunity implies high volumes of financial trading
 - requires borrowing/lending: High leverage

Borrowing is risky

Stricter Regulation: After the crisis, liquidity is lower, and access to borrowing is more difficult

Idea of regulators: To discourage Risky Behavior (e.g., borrowing with low collateral or high leverage)

Thus, borrowing (funds) required to use up arbitrage opportunities has become harder to achieve

Uncovered Interest Rate Parity (UIP)

Riskier option:

investor of foreign asset opts to "wait and see" what the future Spot ER will be and use that ER to trade

He still makes an assessment (guess) based on what he expects the future spot ER to be

He does it by using the **expected** ER in lieu of the future ER → Expected ER: $E_{\$/\epsilon}^e$

Let's see the options of the investor holding 1\$:

" e ": *Expected*

Option 1: Invest at home (US bond) and get $\$1(1 + i_\$)$

Option 2: Buy an Euro bond that yields i_ϵ wait a year and exchange back to dollars

The second option implies foreign exchange risk which is NOT covered as before. Instead, the investor buys at the spot price in t and sells at the spot price in $t + 1$ (e.g. the next year)

Uncovered: like not buying an insurance & instead accept any future outcome (ER)

Uncovered Interest Rate Parity - UIP (cont.)

An Investor has \$1

Option 1: Investing at home → gets $\$1(1 + i_{\$})$ after a year

Going with **Option 2** implies (for a US investor):

Taking the 1\$ exchanging it for Euros for $\frac{1}{E_{\$/\epsilon}}$

Buying the foreign asset and getting a return i_{ϵ} , i.e., getting in total: $\frac{1}{E_{\$/\epsilon}}(1 + i_{\epsilon})$

Taking that amount and exchanging it for dollars **in the future**. He does not know yet the future rate. So for his investment comparison he uses his **guess** of the future rate: The Expected ER:

$E_{\$/\epsilon}^e \rightarrow$ getting, **expectedly**, in total: $\frac{E_{\$/\epsilon}^e}{E_{\$/\epsilon}}(1 + i_{\epsilon})$

Then you compare: $\$1(1 + i_{\$})$ v.s. $\$1 \frac{E_{\$/\epsilon}^e}{E_{\$/\epsilon}}(1 + i_{\epsilon})$.

With absence of arbitrage these two should be equivalent: $(1 + i_{\$}) = \frac{E_{\$/\epsilon}^e}{E_{\$/\epsilon}}(1 + i_{\epsilon})$

That is the UIP

The UIP and the Spot ER

$$(1+i_{\$}) = \frac{E^e_{\$/\epsilon} (1+i_{\epsilon})}{E_{\$/\epsilon}}$$

$$E_{\$/\epsilon} (1+i_{\$}) = \cancel{E_{\$/\epsilon}} E^e_{\$/\epsilon} (1+i_{\epsilon})$$

Similar to before, this parity allows us to understand what an ER depends on.

Here we can rearrange it and solve for the Spot ER:

$$\underbrace{E_{\$/\epsilon}}_{\text{Spot ER}} = \frac{(1 + i_{\epsilon})}{(1 + i_{\$})} \quad \underbrace{E^e_{\$/\epsilon}}_{\text{Expected ER}}$$

$$E_{\$/\epsilon} \frac{(1+i_{\$})}{(1+i_{\$})} = \cancel{E_{\$/\epsilon}} \frac{E^e_{\$/\epsilon} (1+i_{\epsilon})}{(1+i_{\$})}$$

More caveats to this parity though: The expectations have to "be correct", i.e., a good guess ...

Usually the expectations are measured using surveys to market participants

The Carry Trade

How does arbitrage look like in this case?

You would **borrow in a low-interest yielding currency (and location) and lend (or invest) in the high-interest currency (location)** without a forward cover (remember the UIP, U: Uncovered)

This type of investment is known as **carry trade**

An investor would expect the difference in rates to be higher than what the ER appreciates to his disadvantage

Example: $E_{\text{¥}} = 111$, $i_{\text{¥}} = 0.12\%$, and $i_{\$} = 2.83\%$

If the investor expects ER not to change what does he do with \$1? → borrow from Japan and lend in the US

$$\text{Profit} = (1 + 0.0283) \frac{111 \text{¥}/\$}{111 \text{¥}/\$} - (1 + 0.0012) = 0.027$$

Earns 2.7% return on his dollar (or ends up with \$1.027 dollars)

The Carry Trade (cont.)

For what value of the Exchange Rate the investor breaks even?

$$\underline{\text{Profit}} = 0 = (1 + 0.0283) \frac{E_{\text{¥}/\$}^e}{111 \text{¥}/\$} - (1 + 0.0012) \quad \Rightarrow \quad E_{\text{¥}/\$}^e = 108.74$$

i.e., if the dollar depreciates (or yen appreciates) s.t. it takes less than 108 yen to buy a dollar (or more than 1/108 dollars to buy a yen) he will have losses

Such investing strategy **requires arbitrage**. Which we would not have if the UIP holds.

Does the UIP hold?

For the UIP there is less evidence that it holds ... however it's a useful benchmark
(and efforts are made in understanding the extend to which it won't hold)

Useful Approximations to the Parities

The parities involve multiplications of terms

This is less intuitive than thinking about summation of terms

We can approximate them in terms of summations:

- Use natural logs

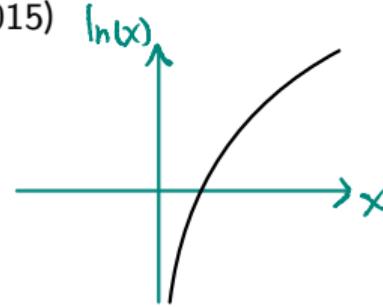
Property 1: if x is small (close to zero) then $\ln(1 + x) \approx x$

Applied to a gross rate as in the UIP/CIP: $\ln(1 + i) \approx i$ (e.g. $\ln(1.015) \approx 0.015$)

Property 2: $\ln(AB) = \ln(A) + \ln(B)$

$$\hookrightarrow \ln(2.3) = \ln(2) + \ln(3)$$

$$\begin{aligned}\ln(1.002) &\approx 0.002 \\ \ln(0.97) &\approx -0.03\end{aligned}$$



Useful Approximations (cont.)

Let's apply this to the UIP:

$$\ln(1 + i_{\$}) = \ln \left(\frac{E_{\$, \epsilon, t+1}^e}{E_{\$, \epsilon, t}} (1 + i_{\epsilon}) \right)$$

Prop 2: $\ln(A \cdot B) = \ln(A) + \ln(B)$

Using property 2:

$$\ln(1 + i_{\$}) = \ln \left(\frac{E_{\$, \epsilon, t+1}^e}{E_{\$, \epsilon, t}} \right) + \ln(1 + i_{\epsilon})$$

Prop 1: $\ln(1+x) \approx x$ (x small)

Using property 1 and the fact that $\frac{E_{\$, \epsilon}^e}{E_{\$, \epsilon}} \approx 1 + d_{\$, \epsilon}^e$ (1 + expected ER depreciation):

$$\ln(1 + d_{\$, \epsilon}^e) \approx d_{\$, \epsilon}^e$$

$$i_{\$} \approx d_{\$, \epsilon}^e + i_{\epsilon}$$

Approximate UIP (simplified)

Now just by having the rates and depreciation in mind we can more or less formulate the UIP

With the CIP is the same but with the depreciation of the forward rate

Useful Approximations (cont.)

FT book: Similar approximation without using as many mathematical tools, just plain algebra:

Similarly, we use that $\frac{E_{\$/\epsilon}^e}{E_{\$/\epsilon}} \approx 1 + d_{\$/\epsilon}^e$,

The UIP becomes:

$$1 + i_{\$} = (1 + i_{\epsilon}) (1 + d_{\$/\epsilon}^e)$$

We apply the product on the RHS:

$$1 + i_{\$} = 1 + i_{\epsilon} + d_{\$/\epsilon}^e + i_{\epsilon}d_{\$/\epsilon}^e$$

≈ 0 (approximately)

Now assume that $i_{\epsilon}d_{\$/\epsilon}^e$ is very small (close to 0, as it tends to be), yielding an approximate UIP:

$$\cancel{1 + i_{\$}} \approx \cancel{1 + i_{\epsilon}} + d_{\$/\epsilon}^e$$

Cancel out the 1 on each side:

$$i_{\$} \approx i_{\epsilon} + d_{\$/\epsilon}^e$$

Result: Same but we had to be a bit more lax with the math (assuming something is close to zero, etc.)