

ECON 5322

Macroeconomic Theory for Applications

Topic 1: Course Overview and Introduction

Part 1: Macroeconomic data facts

1. Course Logistics (syllabus and lectures plan)
2. Please note HW0 on course website
3. Basic Business Cycle Facts
4. A Brief History of Modern Business Cycle Theory

About me: Camilo Granados

- Assistant Professor of Economics – UT Dallas, EPPS
- Background: Ph.D. in Economics (U. Washington), M.S. Data Science (UT Austin), B.S. and M.S. Economics and Post. Diploma in Statistics (U. Nacional, Colombia)
- Born in Colombia
- Worked in a Central Bank for 13 years (09-22), including a leave for the Ph.D.
 - Economist, Research Economist
- Teaching experience:
 - UTD: international finance (ugrad), intermediate macroeconomics (ugrad), macroeconomics (MS), international finance and macroeconomics (PhD)
 - UW: advanced macro, international finance, business finance, intro to macroeconomics, MBA microeconomics, elements of statistics
 - Before the PhD: Advanced Econometrics, Advanced Microeconomics, Game Theory, Principles of Economics.

Office hours: Mondays 4PM-6PM (GR 2.820)

General Teaching Philosophy

- Train students how to think about economics
- Ultimately, it is not just what you know but also how you think - synthesize and process information - that matters
- Lectures won't follow textbooks exactly; they draw out important concepts from some books and rely on study notes.
- Lecture slides available ahead of lecture to facilitate note-taking.
 - My advice: download them and take notes on top during class
- Slides with annotations are posted AFTER lecture
- Aim for more interaction - please "raise hands"

Requirements: See Syllabus (read it closely, It contains our “playing rules” and I will assume it is common knowledge to everyone)

Textbooks: Romer, Advanced Macroeconomics, 5ed., McGraw Hill and Chugh, Modern Macroeconomics, 2015.

(we'll use them for some end of chapter problems but you can do the whole course with the slides and study notes only)

This course is not easy: heavy on math

Given that, I try to help:

- Most of the grading weight is put on take-home assignments rather than in-lecture tests
- The heavy lifting is put on the homework and practical projects
- If I see effort I am happy to adjust the grading weights along the way

Problem Sets: 6 (35%)

Exams: 2 (45% ; 20% Midterm
25% Final)

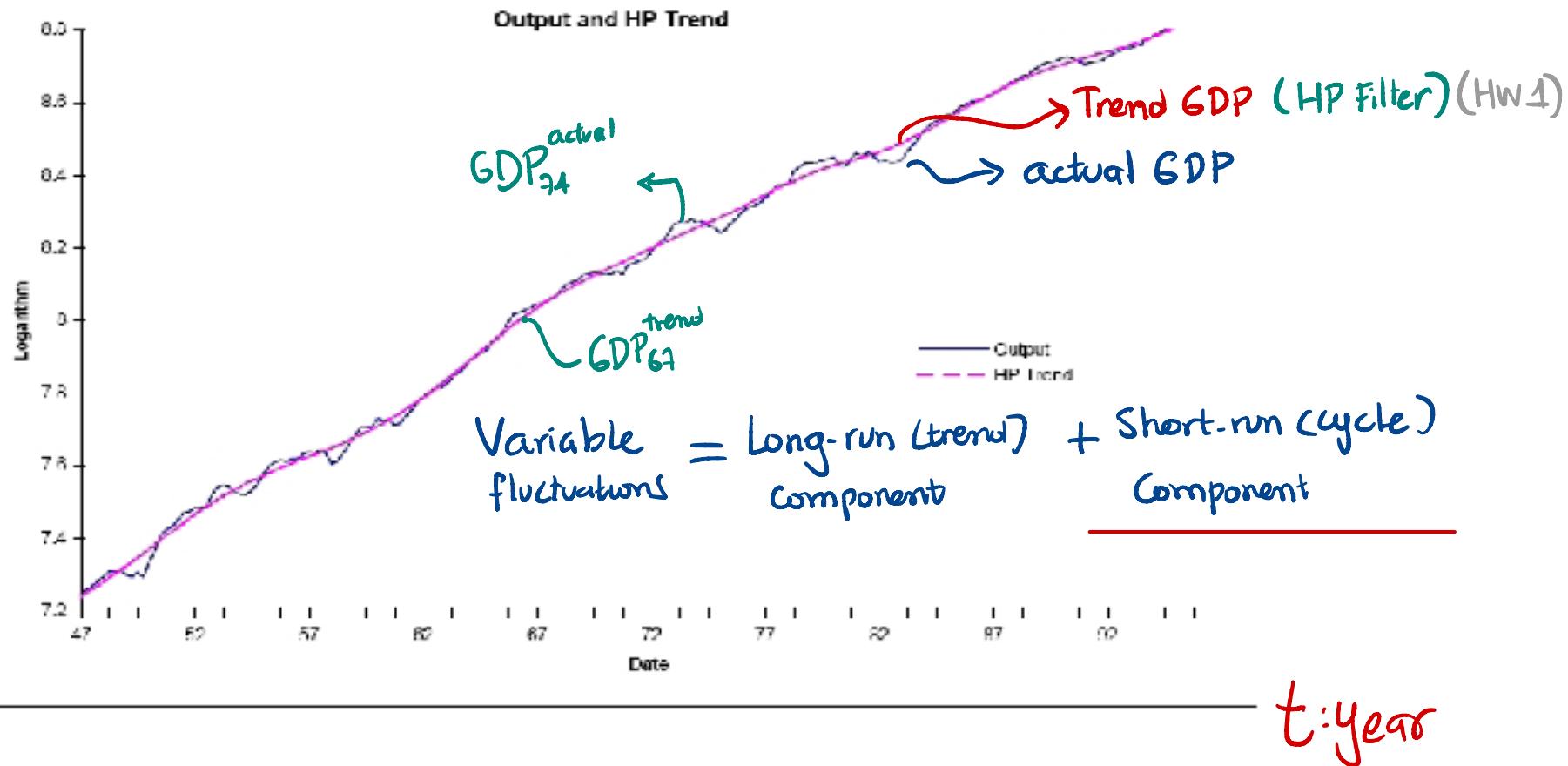
Final Presentation : 20%

Bonus: Extra Questions

Midterm: 3/9 ; Final: 5/4

These dates can change (Check website)⁴

Some Business Cycle Facts (for the U.S. 1947:1 – 1996:4)



Decompose "Trend" vs. "Cycles" in a time series
 $y_t = y_t^T + y_t^C$ (e.g., with filters)

$$GDP_t = GDP_t^{\text{Trend}} + GDP_t^{\text{cycle}}$$

Wiggles of the variable around trend

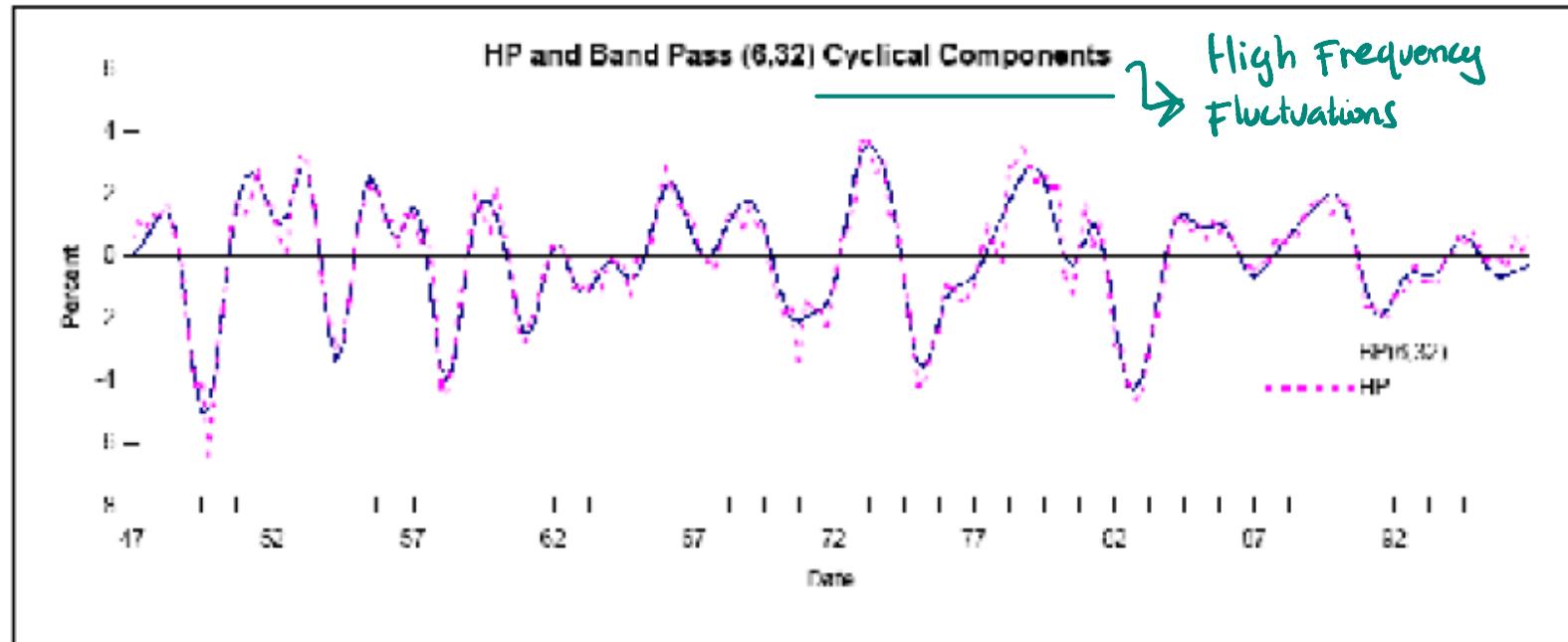
$$y_t^{\text{cycle}} = y_t - y_t^{\text{Trend}}$$

HP Filter ($\lambda=1600$)
 (Many other options: Band Pass filter; Linear trend, Linear-quadratic trend)

Cyclical Component

$$y_t = \alpha + \beta_1 t + \beta_2 t^2 + \epsilon_t$$

Cycle



Common filters: linear, Hodrick-Prescott (HP), Bandpass

Intuitively: a way to extract the long-run growth portion or the “low frequency” components of the data

Business Cycle Dating

NBER : Determine whether we are in a recession

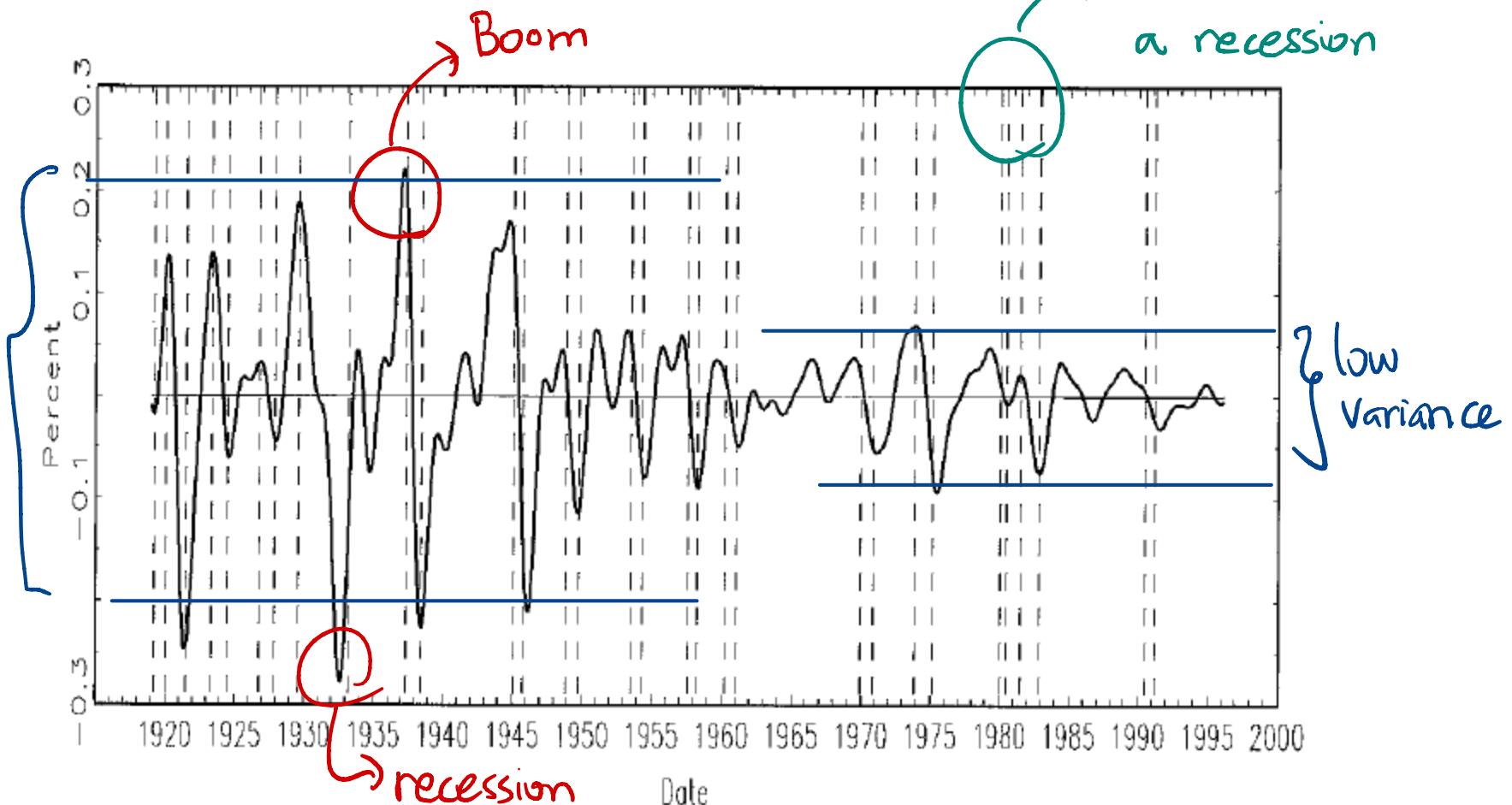


Fig. 1.2. Business cycle component of industrial production index.

high to low variance: Great Moderation

Some Stylized Facts about the U.S. Business Cycles:

What do we look for?

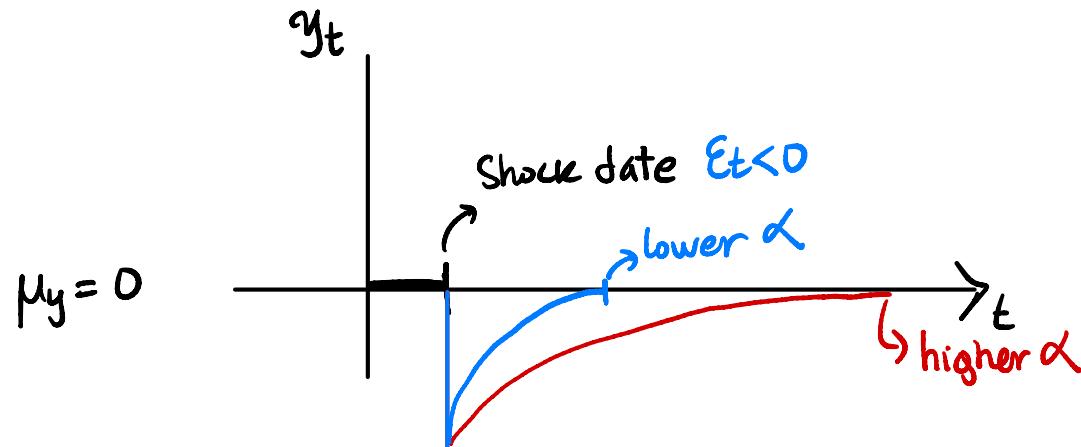
↳ Wiggles around trend

1. Volatility/amplitude of fluctuations → Variance of the cycle
2. Comovements → Covariance & Correlations (among economic variable)
3. Persistence/lead and lag patterns → Autocorrelations
Autoregressive patterns

AR Process:

$$y_t = \mu_y + \alpha \cdot y_{t-1} + \epsilon_t$$

Constant Past value error

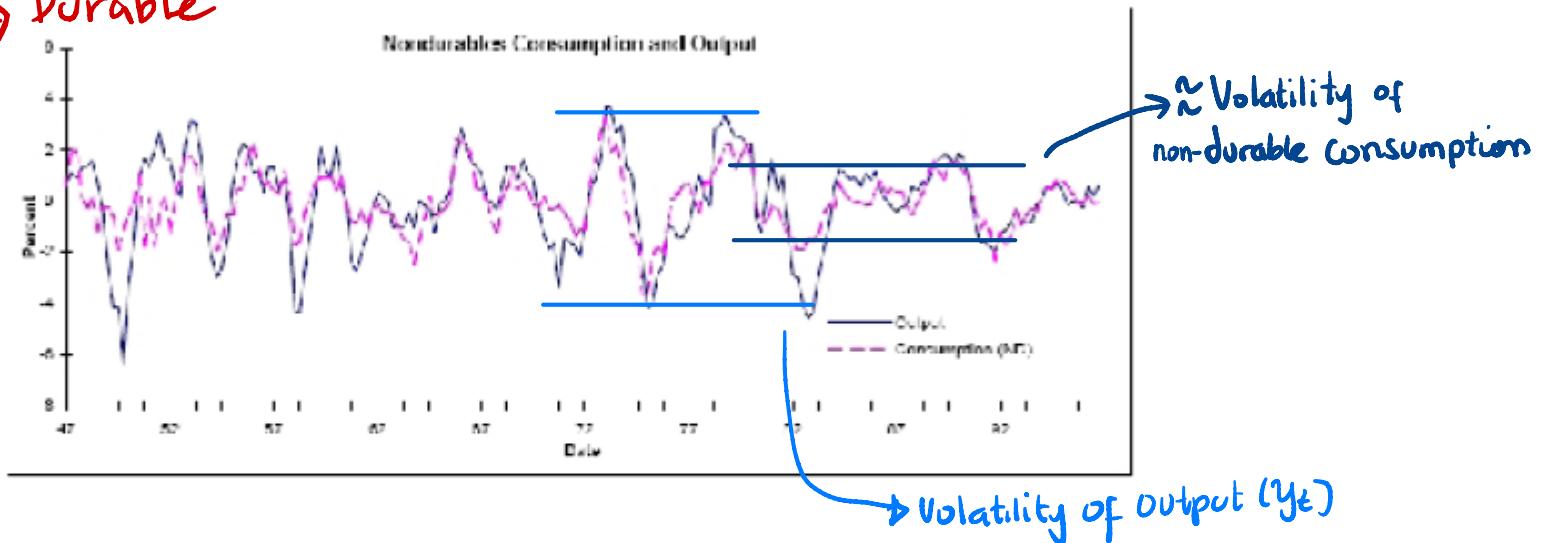


$$Y_t = \underline{C_t + I_t + G_t} + \cancel{N X_t}^0 \text{ (Closed economy)}$$

Nondurable consumption

→ Non-durable (goods that don't last for long)

→ Durable



* Less volatile than output

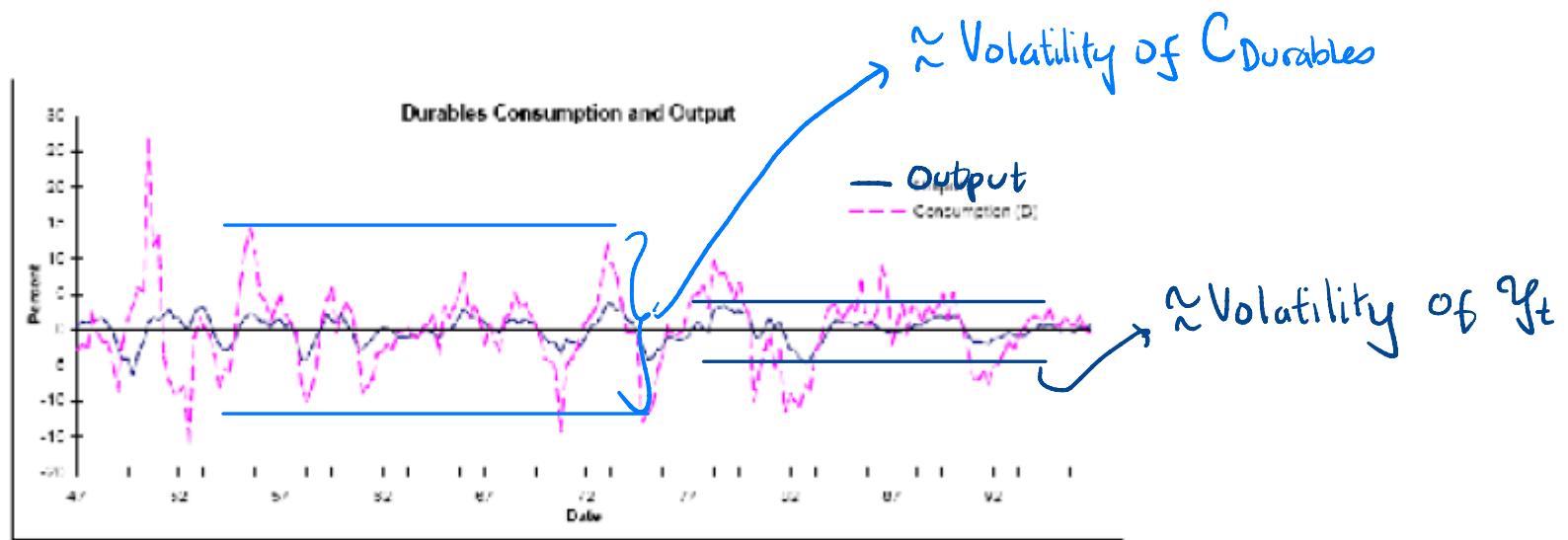
$$\sigma_{C, \text{nondurable}} < \sigma_y$$

$$Y_t = C_t + I_t + G$$

durables
Component

Consumer durables

Goods that Last
(e.g. cars, housing, etc)



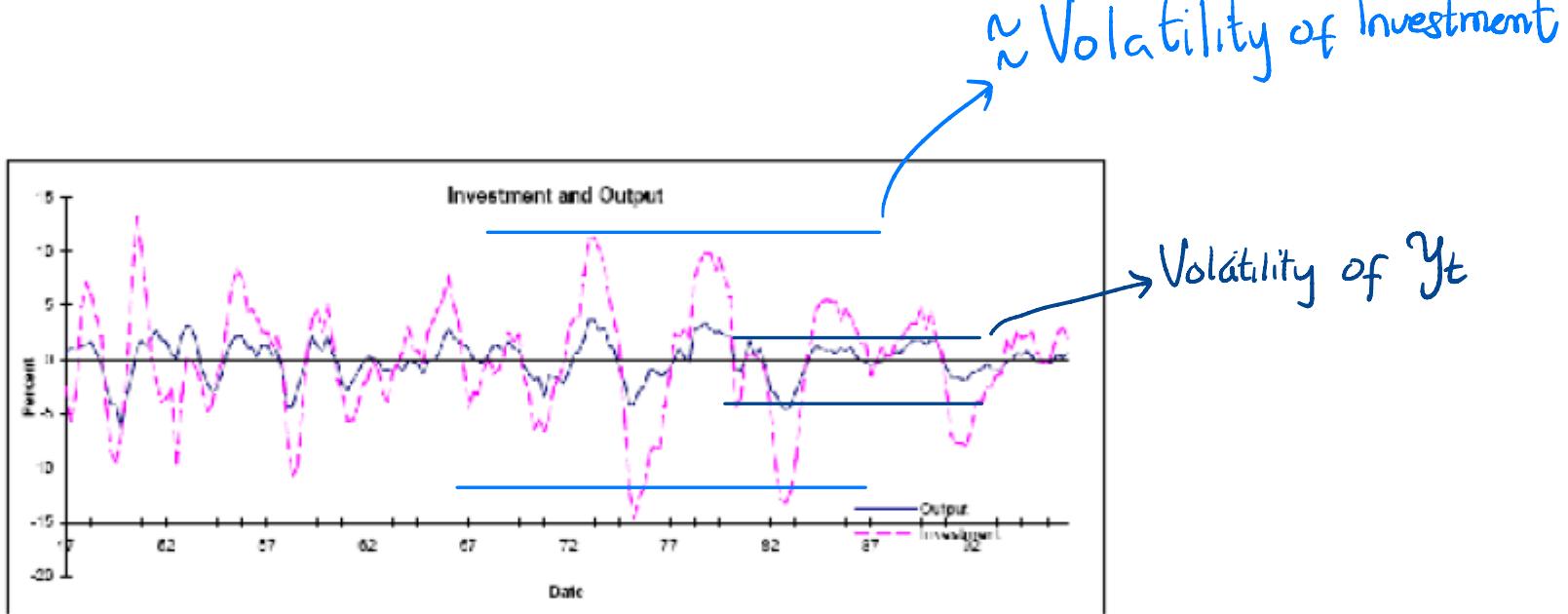
$$\sigma_{C,\text{nondurable}} < \sigma_y$$

* More volatile than output

$$\sigma_{C,\text{durable}} > \sigma_y$$

$$Y_t = C_t + I_t + G_t$$

Investment



$$\sigma_{C, \text{non durable}} < \sigma_y$$

* 3 times more volatile than output

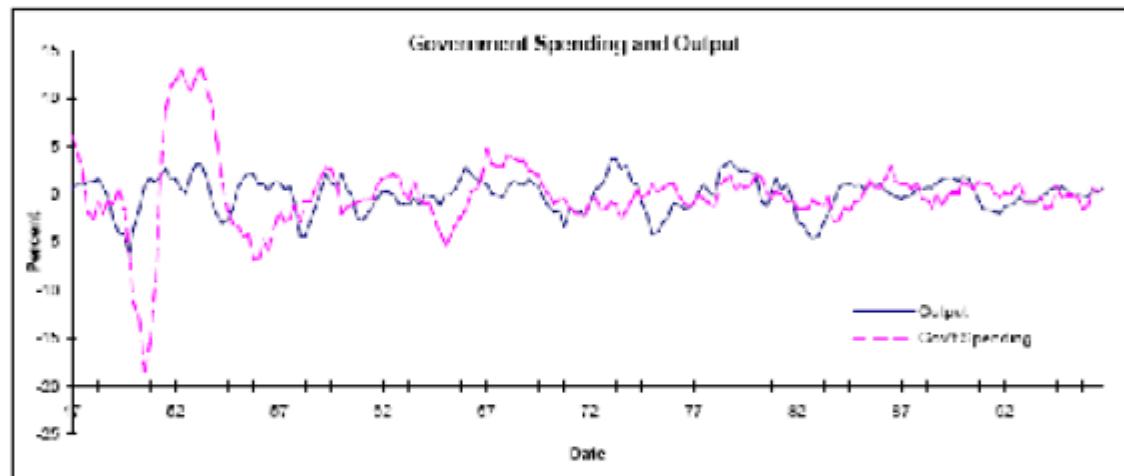
$$(\sigma_I \approx 3\sigma_y)$$

$$\sigma_{C, \text{durable}} > \sigma_y$$

$$\sigma_{\text{Investment}} > \sigma_y$$

$$Y_t = C_t + I_t + G_t$$

Government expenditures



$$\sigma_{C,\text{nondurable}} < \sigma_y$$

$$\sigma_{C,\text{durable}} > \sigma_y$$

$$\sigma_{\text{Investment}} > \sigma_y$$

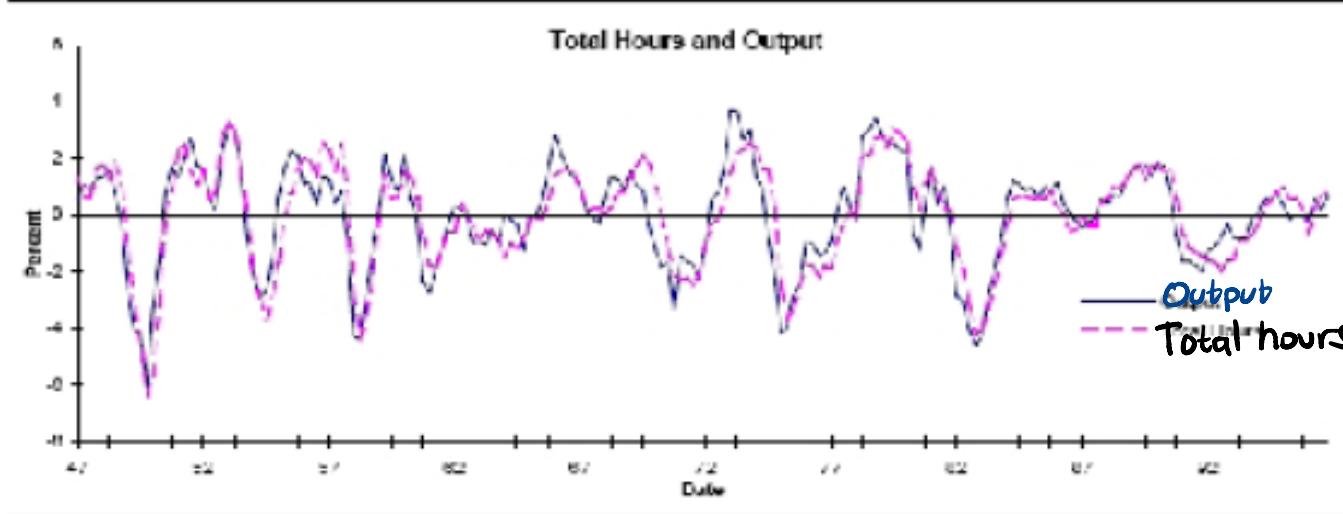
$$\sigma_G < \sigma_y$$

* Less volatile than output

Total hours worked

($N \times$ hrs per worker)

N: Number of workers



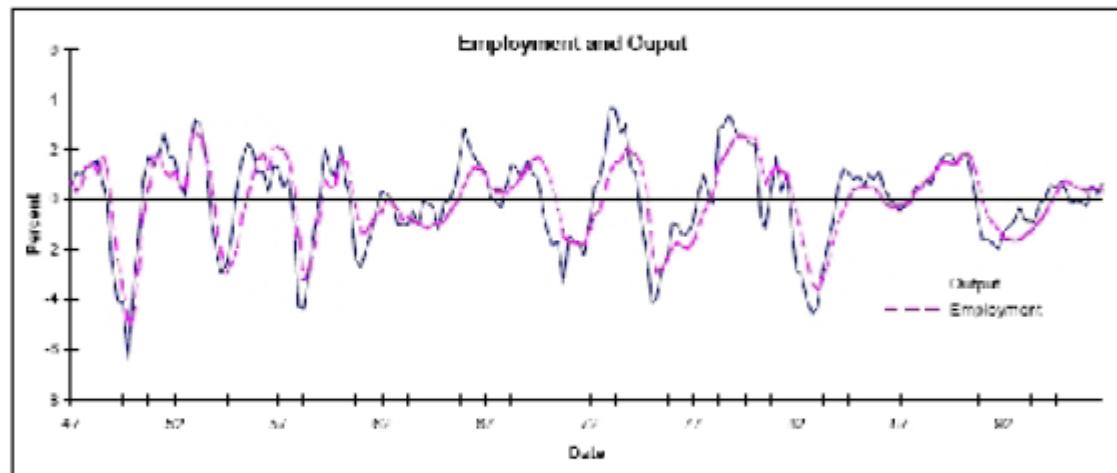
$\sigma_{\text{Total hours}} \approx \sigma_y$

highly correlated with
Output

- About as volatile as output
- Business cycle is most clearly manifested in the labor market

Employment

↳ Number of workers

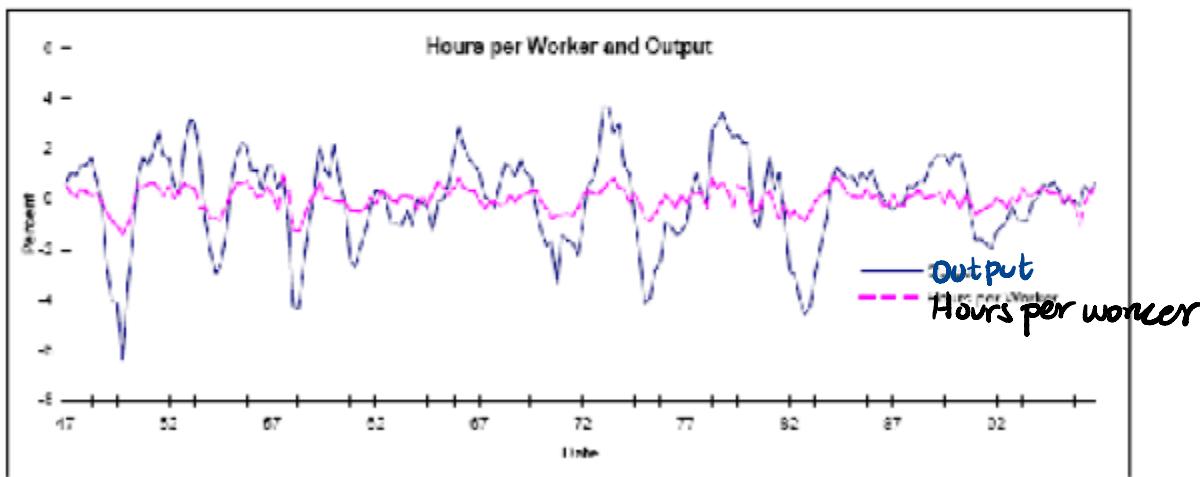


$$\sigma_N \approx \sigma_y$$

- as volatile as output

Hours worked

$$\sigma_{\text{Hours}} < \sigma_y$$



- Hours per worker: Much less volatile than output
- Most variation in total hours stems from changes in employment,
rather than adjustment in hours worked per employee.

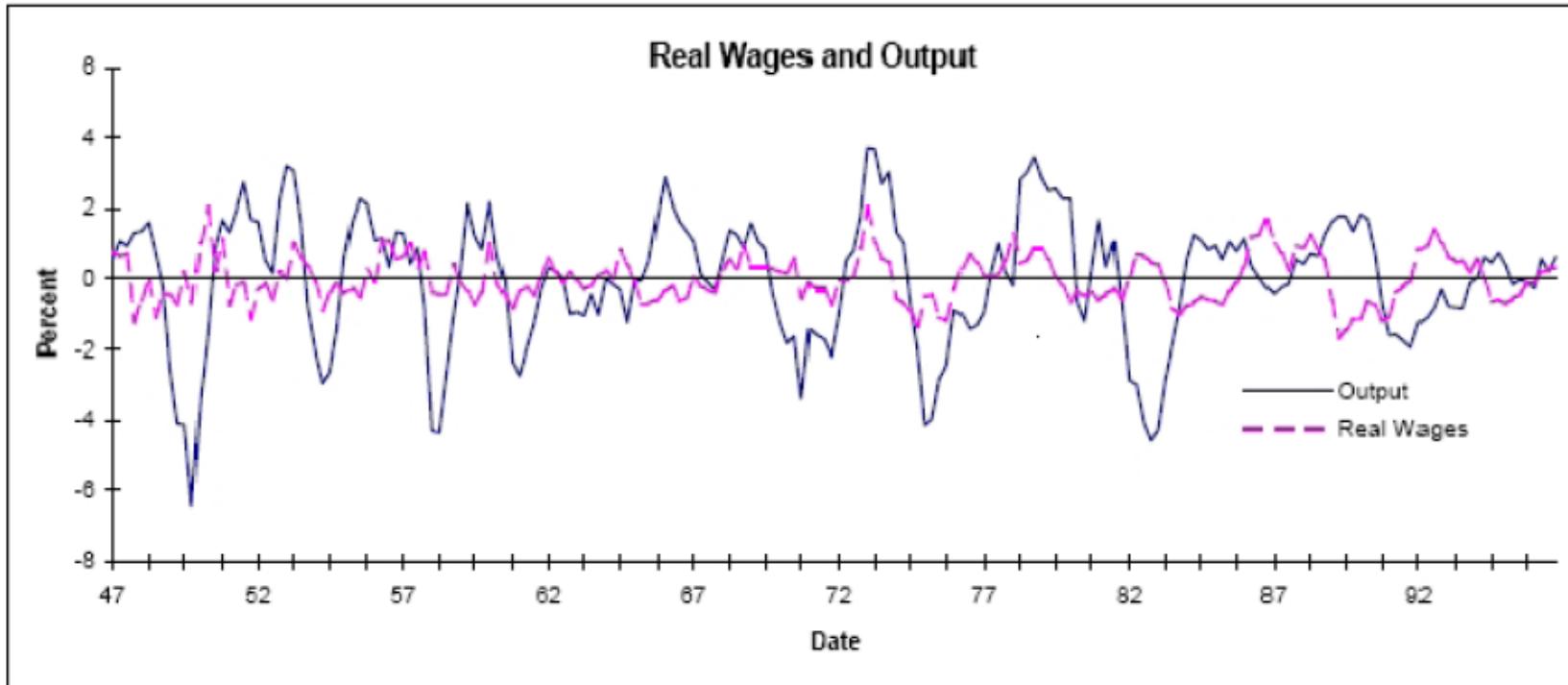
of workers
↗

$$\text{real wage} = \frac{W}{P}$$

Nominal Wage
 Prices level

Real Wage

$$\log \text{real wage} = \log W - \log P$$



- Much less volatile than output $\frac{\sigma_w}{\sigma_p} << \sigma_y$
- Slightly **PRO-cyclical** (correlation = 0.14) \Rightarrow important fact

\hookrightarrow \oplus Correlated w/ GDP cycle
 \hookrightarrow (\approx barely procyclical or even acyclical)

Are we still going through a great moderation? (HW1)

An Obvious Question

- What do the US business cycle statistics look like post-1999 & post-2007?
Are the patterns described above still true? How about other countries?
- How to produce the basic business cycle statistics: discussed in the first two sections of King and Rebelo (1999) Handbook chapter

↳ Alternative: Chp 1 of "Open Economy Macroeconomics" by
Uribe & Schmitt-Grohe (2017)

Data Sources:

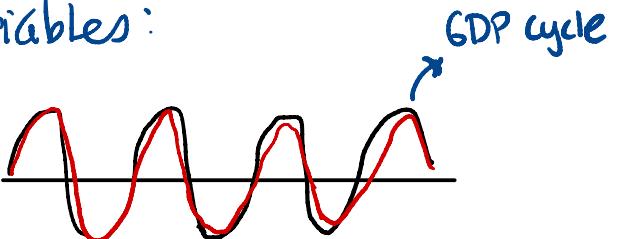
- for the US: FRED ↳ Fed. Res. Economic Data
 - <https://fred.stlouisfed.org/>
- international macro: IMF-International Financial Statistics (IFS)
- World Bank - WDI

Can you see in the data the following view?

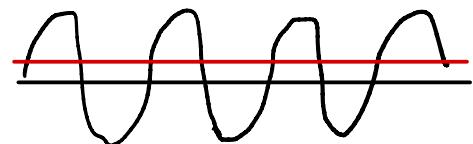
- Old Macro: Analyzes pre- versus post- 1984:Q4.
Great Moderation
- New Macro: Analyzes pre- versus post- August 2007
Global Financial Crisis ;
Pandemic Crisis ; 2020's Inflation
Surge, etc.
- End of the **Great Moderation**
- Downturn precipitated by disruption of **Financial Intermediation**
- **Unconventional Monetary Policy** and **Zero Lower Bound**, balance sheet management, macro-prudential policy...
- COVID-19 macro dynamics
- New inflation dynamics

On the variables:

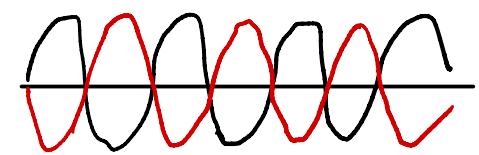
Pro cyclical
variable :



Acyclical
Variable :



Countercyclical
Variable :



As we go through models after models after models*... **ALWAYS** ask yourself:

- 1.What is the Motivation behind the model?
- 2.What is the Economic intuition?
- 3.What is the relevant Technique/Tool to pick up?
- 4.What does the Data say?
- 5.What are some Alternatives to model or test the same phenomenon?

=> Learn not (just) their thoughts, but how they **THINK** (how to approach and formalize the issue at hand)

Key Questions (Extra Credit)

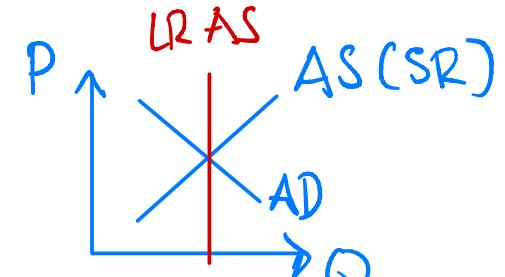
- What are some “stylized facts” about US Business cycle dynamics up?
- What is Neoclassical Synthesis?

Part 2: From IS-LM Neo Classical Synthesis to New Synthesis

A. Some course logistics – **important dates: posted on course website!**

B. Brief History of Modern Macro

- The Neoclassical Synthesis \longrightarrow IS-LM / AD-AS
- The Breakdown of the Consensus \rightarrow 1970's Stagflation ($\uparrow P, \downarrow Q?$)
 - Price Adjustment: Phillips-Solow-Samuelson vs. Friedman-Phelps
- Is the Long-Run Phillips Curve Really Vertical?
- Rational Expectations Revolution
- Main Approaches to modeling Aggregate Supply since then
- The New-Neoclassical Synthesis \rightarrow RBC toolkit + Keynesian Ideas
(about market frictions)
Topic 2



Housekeeping:

- First midterm is done after Topic 2
- The 6 Homework assignments are submitted via eLearning
 - First Assignment (HW0) due next week: Get full credits just by

submitting (only for HW0)
- A Final Exam, at the end of the semester
- Last week of class: group project presentations
- Reminder: Extra credit option: submitting answers to “Key Questions” in a single PDF file at the end of the course (via eLearning, too)



A VERY Brief History of Macro:

- Neoclassical Synthesis \longrightarrow IS-LM / AS-AD
- The Breakdown of the Consensus + the Rational Expectations Revolution
- Main approaches to modeling Aggregate Supply in the 80s & 90s \rightarrow RBC
(topic 2)
- “New” Neoclassical Synthesis
- Post-2008: Financial Friction, “Unconventional” Monetary Policy...etc.
- Now?
 - RBCs
 - Rational Expect.
 - Intertemporal decisions
 - General Equilibrium
 - + - Keynesian ideas on market Failures/Frictions

} DSGEs

I. **Neoclassical Synthesis**: Consensus in the 1960's (Review of intro to macro)

a. **Aggregate Demand**: from IS-LM framework: **goods market** and **money market** equilibria, and Walras Law implies **asset market clearing** → *Supply = Demand*

① Goods market $y = C(Y-T) + I(r) + G + NX$ $i - \pi$ \uparrow \circ (closed economy)

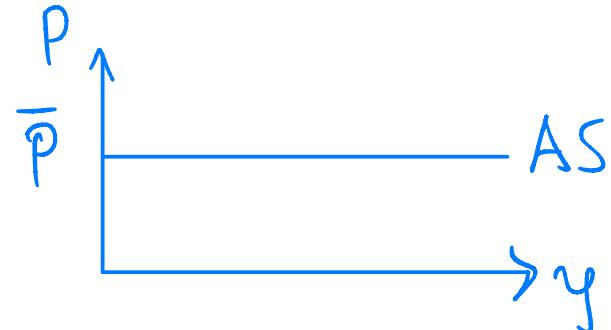
② Money market $\frac{M^s}{P} = L(i, y)$

+ Walras Law: remaining market is in equilibrium (Assets)

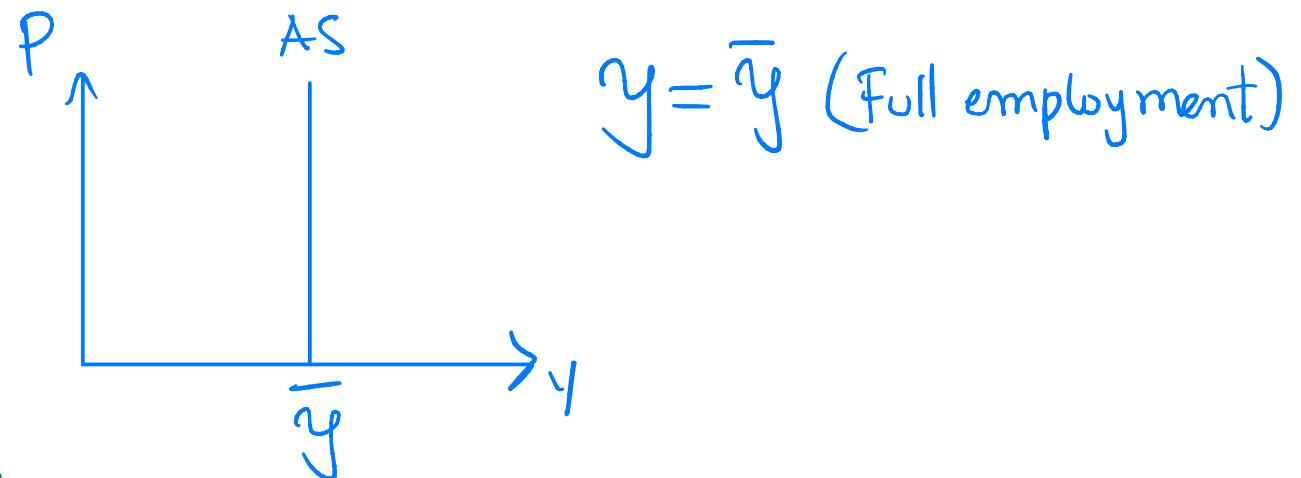
⇒ Obtain Aggregate Demand from IS-LM equil.

b. Aggregate Supply: Keynesian vs. Classicalist: Are prices sticky?

i) Keynesian Nominal rigidity (sticky prices/wages) => Short-run



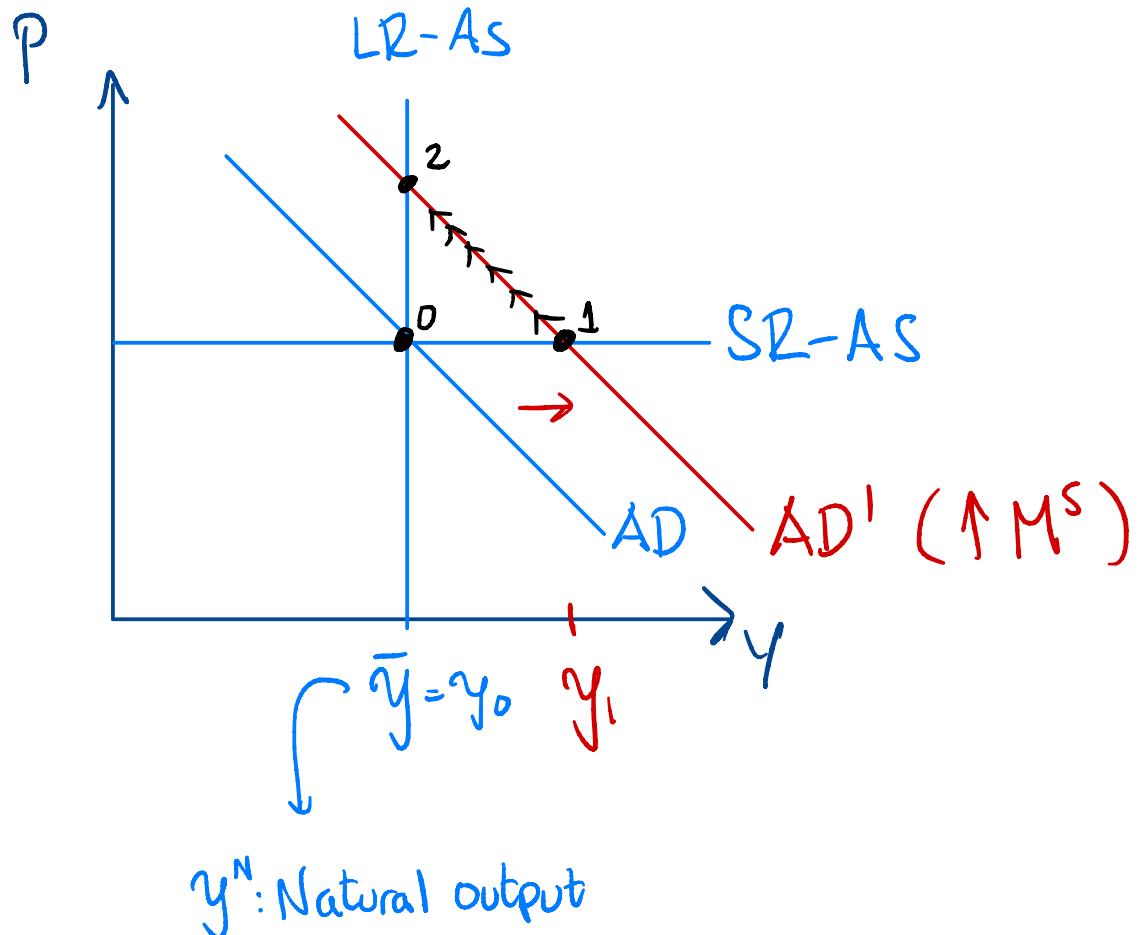
ii) Natural Rate Hypothesis (prices/wages fully flexible) & monetary neutrality => Long-run



y determined by
technology only: $y = f(\text{labor}, K, \text{nat. resources})$

Neoclassical Synthesis: put relationships above together,

- with 1) and 2) giving AD
- i) for Short-Run AS
- ii) for Long-Run AS



How do we adjust from SR to LR?

$$SR: y > y^N \Rightarrow \uparrow P \text{ (TL inflation)}$$

↳

Economy is overheated

c. linking SR and LR with **price-adjustment dynamics: the Phillips**

Curve:

$$\pi = \alpha (Y - Y^N) = -\beta(u - u^N)$$

↳ $\alpha > 0$ $\beta > 0$

Natural unemployment
↳ u^N
↳ Unemployment rate
↳ u

$$\text{Corr}(y, N) > 0$$

$$\text{Corr}(y, u) < 0$$

How well did this “model” do?

- Data in the 50's-60s supported the above => tradeoff between π and u , providing **scope for policy actions**
- At the applied level: refinements of above
 - large-scaled models: “MPS”, Harvard, Fed models with several hundred of equations
 - these models are based on empirically observed relationships (between output and consumption, money demand, in unemployment... etc)
 - the aim of these models was to predict the effects of policies
 - they were pretty successful at it until the 1960s
- Bob Solow: “Macroeconomics is finished” (as in done/completed!)

The Breakdown of the Consensus in the early 1970s

a. Empirically:

Models couldn't explain the simultaneous rising inflation AND unemployment in the 1970s: Vietnam War, G↑, M^S↑ => π↑ but no u↓

Stagflation ↑

b. Theoretically:

- Friedman (1968), Phelps (1968): Phillips' curve cannot be right!

Violation of the **Natural Rate Hypothesis**: Long run unemployment should NOT depend on the average rate of money growth, i.e. What if Fed changes the money growth rate?? Say from 0% to 5%, => π = 5% => u↓ and Y↑ in the LR!

=> **Expectation-augmented Phillips Curve**

$$\pi = -\beta(u - u^N) + \gamma E\pi$$

↑ ↑
Comparable if ↑

Consistent w/ ↑ E[π]
not with ↑ Y (↓ u) in LR

- Lucas Critique ('73) and the **Rational Expectation** revolution
 - When evaluating policy, need to consider the feedback with expectations: if policy maker changes the rule, public expectation will adjust as well, so the equilibrium condition for the economy will change too.

=> All of the above point to the “danger” of using ad hoc, reduced-form empirical relationships with no “micro-foundation”!

Tom Sargent, “Macro is finished”... (as in “destroyed”)

20+ years of confusion and division to follow....

↳ Can't use only correlations
(Use theory to speak to causality too)

Lucas Critique ('73) and the Rational Expectation Revolution

- Expectation-augmented Phillips curve:

$$\pi = -\beta(u - u^N) + \gamma E\pi$$

↗ Feedback of expectations into π
 ↳ Expected inflation

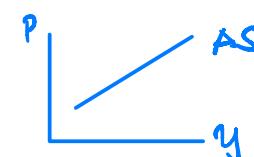
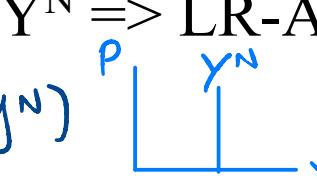
Or:

$$\pi_t = a - bu_t + \gamma E_{t-1}\pi_t$$

Note: in the long-run, $\pi = E\pi$ (by definition, of LR)

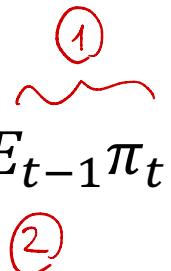
- If $\gamma = 1$, then $u = u^N$, so LR, at u^N and $Y^N \Rightarrow$ LR-AS is vertical
 $\gamma=1 \rightarrow \pi = -\beta(u-u^N) + \cancel{\gamma\pi} \Rightarrow u=u^N (\gamma=\gamma^N)$
- Otherwise, u will depend on both u^N and $\pi \Rightarrow$ LR-AS is sloped (Y or u depend on prices/inflation)

$$(\gamma \neq 1, \gamma < 1) \rightarrow \pi = \frac{-\beta}{1-\gamma} (u - u^N)$$



TS1 here (opt. 1)

How to Measure/Model $E\pi$

$$\pi_t = a - bu_t + \gamma E_{t-1}\pi_t$$


- It is therefore important to know the value of γ ,
- To gauge its value: look into past data of inflation and unemployment,

BUT:

- also need to know how to measure and model $E\pi$

Lucas Critique ('73) and the Rational Expectation Revolution

- Nobel Prize 1995
- Much of what economists were doing and the policy conclusions were **WRONG**
 - Using a model with fixed coefficients estimated from reduced-form equations and historical data to evaluate the effects of new policy would give misleading results because **expectations need to be endogenous**
 - i.e. Changes in policy will affect expectations
- True whenever expectations are forward-looking (need not be rational)

“Rational” = model-consistent

Three Methodological Tenets for Rational Expectations¹

- Results widely applicable; use Phillips Curve as an example below

To form “model-consistent” rational expectations:

1. Partial Equilibrium.

1 market + Account for stochastic behavior of $E_{t-1}T_t$

Agents form expectations appropriately given the **stochastic process** generating the variables of interest. Expectations cannot be specified without first specifying the underlying stochastic process.

e.g. Given $\pi_t = a - bu_t + \gamma E_{t-1}\pi_t$ (*), want to know γ

=> first specify how π_t is determined

e.g. T_t follows an AR(1) process

$$T_t = \rho T_{t-1} + \varepsilon_t$$

$\hookrightarrow 0 < \rho < 1$

white noise shock

$$\begin{cases} E[\varepsilon] = 0 \\ E[\varepsilon^2] = \sigma^2 \\ E[\varepsilon_t, \varepsilon_s] = 0 \end{cases}$$

¹ N.G. Mankiw

Explain Expectation Operator (and take $E_{t-1}\pi_t$ given it is AR(1))

$E[X_t]$: (Unconditional) Expected value of X_t

$E_{t-1}[X_t] = E[X_t | \text{Information up to } t-1] = E[X_t | I_{t-1}]$

Properties

- ① $E[\cdot]$ is a linear operator
- ② $E[\alpha \cdot X_t] = \alpha \cdot E[X_t]$

α is a constant

$$\begin{aligned}E_{t-1}[Tl_t] &= E_{t-1}[p \cdot Tl_{t-1} + \varepsilon_t] \\&= E_{t-1}[p Tl_{t-1}] + E_{t-1}[\varepsilon_t] \\&= p E_{t-1}[Tl_{t-1}] + E_{t-1}[\varepsilon_t]\end{aligned}$$

$$E_{t-1}[Tl_t] = p \cdot \overline{Tl}_{t-1}$$

$$\underbrace{\varepsilon_t}_{=0}$$

Given the stochastic process, $E_{t-1}\pi_t = \rho\pi_{t-1}$ under Rational Expectation

Plug this into (*), we see that

$$\pi_t = a - bu_t + \underbrace{\gamma\rho\pi_{t-1}}_{\text{red}}$$

- 1) the relation b/w inflation and unemployment depends on lagged inflation (π_{t-1}) too.
- 2) Can we just find γ by looking at the correlation b/w inflation and unemployment? No. Without specifying ρ first, we cannot determine γ

2. General Equilibrium.

The stochastic process of any variable generally depends on the entire model. To solve for expectations, one must assume that agents know and solve correctly the model of the economy.

- In our case, it means endogenizing π_t : have it determined from within the system
- We will adopt a variant of the Phillips curve (*) instead too.

Consider an economy described by a simple General Equilibrium (GE) model (note, all variables are in logs, allowing us to use linear equations):

$$1) m_t - p_t = y_t \quad (\text{AD})$$

$$2) y_t = \alpha(p_t - E_{t-1}p_t) \quad (\text{AS})$$

$$\hookrightarrow y_t = \alpha(p_t - E_{t-1}[\pi_t])$$

$\hookrightarrow p_t - p_{t-1}$ $\hookrightarrow E_{t-1}(p_t - p_{t-1})$

To Solve for Y from the system of AD and AS:

(solve for Y from the system : Eqs. 1-2)

$$\textcircled{1} \quad m_t - p_t = y_t \quad (\text{AD})$$

$$\textcircled{2} \quad y_t = \alpha(p_t - E_{t-1}p_t) \quad (\text{AS})$$

from (1) $p_t = m_t - y_t$

take $E_{t-1}(.)$: $E_{t-1}[p_t] = E_{t-1}(m_t - y_t) \quad (1')$

from (2), take E_{t-1} :

$$\begin{aligned} E_{t-1}y_t &= \alpha E_{t-1}(p_t - E_{t-1}p_t) \\ &= \alpha E_{t-1}p_t - \alpha \underbrace{E_{t-1}E_{t-1}p_t}_{E_{t-1}p_t} = 0 \end{aligned}$$

In (1'): $E_{t-1}p_t = E_{t-1}m_t$

back in AS: $y_t = \alpha(p_t - E_{t-1}p_t) \rightarrow y_t = \alpha(\underbrace{m_t - y_t - E_{t-1}m_t}_{p_t \text{ in AD}})$
 (replacing)

What does this result mean?

$$y_t = \frac{\alpha}{1 + \alpha} [m_t - E_{t-1}m_t] \quad (\text{AS new})$$

=> Output depends on “surprised” or unexpected money

Policy rule: Framework by which policy is set

3. Policy Evaluation (where does m_t come from?)

- The rules governing policy are among the equations in any complete model of the economy
- Because a change in a policy rule alters the stochastic processes generating many variables, it also changes people's expectations
- Hence, when evaluating alternative policy rules, one must take account of this feedback between policy rules and how expectations are formed.
- Any policy evaluation that fails to take account of this feedback is flawed and useless (Lucas Critique)

(3) Consider a simple monetary policy rule: $m_t = \mu_A + m_{t-1} + \varepsilon_t$ (3)

where μ_A is a constant and ε_t is a white noise process gain

Note: since these variables are in logs, $\mu_A = \underline{E(m_t - m_{t-1})}$

represent the **expected growth rate of money** or the **trend money growth rate** (Remember: log (small) differences = % change or growth rate.)

Combining (1)-(3) to solve the GE model:

$$E_{t-1}m_t = \mu_A + m_{t-1}$$

$$\Rightarrow \underline{y_t = \frac{\alpha}{1+\alpha} [m_t - m_{t-1} - \mu_A]}$$

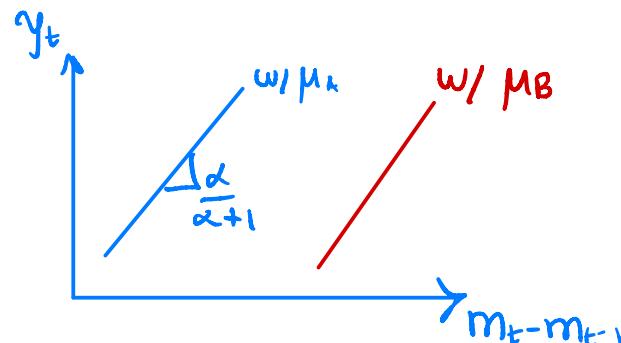
- This tells us that in general equilibrium, the level of output at each time depends on both money growth from the previous period ($m_t - m_{t-1}$), but also on the general trend growth rate set by monetary policy (μ_A)

Policy implication from this rational expectation general eq. model:

- Output level is positively correlated with money growth from period-to-period. However, this relationship does NOT imply a **policy tradeoff**. That could only be deemed valid under the policy rule (3) with μ_A
- If policymaker increases money growth rate from point (e.g. by changing money growth from μ_A to μ_B) , (3) would no longer be the correct rule. The rule would have trend money growth μ_B instead
- People forming rational expectations would adjust as well based on the new rule. Resolving the system, we see that the curve **shifts to the right** (to reflect the new output-money growth relationship)

New rule: $m_t = \mu_B + m_{t-1} + \epsilon_t$
 $(\mu_B > \mu_A)$

$$\Rightarrow \text{new AS is } y_t = \frac{\alpha}{1+\alpha} (m_t - m_{t-1} - \mu_B)$$



Summary:

- When evaluating policy, need to take into account the feedback with expectations.
 - If policy maker changes the rule, expectation will change so the equilibrium condition for the economy will change too.
- ⇒ All of the above point to the “danger” of using ad hoc, reduced-form empirical relationships with no “micro-foundation”!

Results of the Rational Expectations revolution:

- 1) More focus on structural modeling of the economy: micro-foundation based on first principles, rather than estimating reduced-form equations. Identification: correlation does not equal causality
- 2) Taking expectations seriously
Lucas Critique does NOT imply policy ineffectiveness, but that policy making should focus on developing on-going strategy and long-term rules, not one-time change

⇒ **Four Main Approaches to Aggregate Supply**

[Detour] [This is a good point for checking the Technical Slides #1]

Technical Slides:

- Stochastic Processes/Time series review
- Expectations (def. and properties)
- MATLAB intro

[End of detour]

Four Main Approaches to Aggregate Supply

		Do Markets Clear? (instantaneously)	
		Yes	No
Is Money Neutral? (in medium, Long run)	Yes	<i>Topic 2</i> 1. Classical/RBC Kydland & Prescott Minnesota	3. Real Rigidity e.g. Efficiency wage theory Akerlof, Yellen
	No	2. Imperfect Information Friedman Lucas '77	<i>Last topic</i> 4. Nominal Rigidity Nominal contracts, menu costs: Fischer, Taylor, Calvo ...

Unification Again Since the 1990's

“New Neoclassical Synthesis” RBC + Keynesians
(Market Frictions) } DS6E

Real Business Cycle tools (Romer Ch.5) + New Keynesian (Ch. 6) ideas:

- Dynamic Stochastic General Equilibrium Modeling
- Business cycle may be caused by real shocks, **but nominal rigidity**
leads to inefficiency, hence role for policy

most usual
example of friction

Key Questions (Extra Credit, due next session)

- What is the Lucas Critique?
- Intuitively, how do the three perspectives of rational expectations differ (and/or improve, complement each other)?
- How is Lucas' rational expectation captured within economic models?
- What does money neutrality mean?
- What is the Neoclassical Synthesis? And what were its main problems?

References:

- Mankiw, N Gregory. 1990. "A Quick Refresher Course in Macroeconomics," Journal of Economic Literature
- Woodford, Michael. 1999, "Revolution and Evolution in Twentieth-Century Macroeconomics."
- Goodfriend, M. 2002. "Monetary Policy in the New Neoclassical Synthesis: A Primer"

Appendix

[Further look into the implications of the stochastic process and the expectation operator]

- **Note:** Given $\pi_t = \rho\pi_{t-1} + \varepsilon_t$ where ε_t is a “white noise” process:
- ε_t is a time series process that's independently and identically distributed (iid), with zero mean $E(\varepsilon_t) = 0$, a constant variance $E(\varepsilon_t^2) = \sigma^2$, and $E(\varepsilon_t\varepsilon_s) = 0$

Applying the Expectation operator E_{t-1} to π_t (taking its expectation at time t-1, implying based on information we have at time t-1):

$$\begin{aligned} E_{t-1}\pi_t &= E_{t-1}[\pi_t] = E[\pi_t | I_{t-1}] && \text{different notations for the same thing} \\ &= E_{t-1}[\rho\pi_{t-1} + \varepsilon_t] && \text{substituting in the definition of } \pi_t \\ &= E_{t-1}[\rho\pi_{t-1}] + E_{t-1}[\varepsilon_t] && \text{“operation” is linear (can do it term by term)} \\ &= \rho\pi_{t-1} + 0 && \text{what you expect at any time given info at that time = the info itself} \end{aligned}$$

[Clarifying note: Relation btw Phillips Curves in the slides]

Note: The AS in the shown previously is another way to express the expectation-augmented Phillips curve (*). From either of these expressions (again, equivalent):

$$\pi_t = a - bu_t + \gamma E_{t-1}\pi_t$$

$$\pi_t = -\beta(u_t - u^N) + \gamma E_{t-1}\pi_t$$

Since output y_t is inversely related to unemployment u_t :

$$\pi_t = cy_t + \gamma E_{t-1}\pi_t$$

Since inflation is the log-difference in prices: $\pi_t = p_t - p_{t-1}$

$$p_t - p_{t-1} = cy_t + \gamma E_{t-1}[p_t - p_{t-1}]$$

Re-arrange and note:

$$E_{t-1}[p_{t-1}] = p_{t-1}$$

$$y_t = \alpha(p_t - E_{t-1}p_t)$$

[Clarification-Why we use logs or similar approximations]

A note about **linearized equations**:

- We will be working with “linearized” equations frequently, to make the models easier to solve.
- In order to express general economic equations in linear forms, we often take the (natural) logs of the variables. For example, the quantity theory of money, $M_t V_t = P_t Y_t$ can be expressed as $m_t + v_t = p_t + y_t$ where each of the lower-case variables is the log of the capitalized variables.
- We will use variations of $m_t + v_t = p_t + y_t$ to represent **aggregate demand** for a while.