ECON 6356
International Finance and Macroeconomics

Lecture 3 (part 1): the Small Open Economy Real Business Cycle model

Camilo Granados
University of Texas at Dallas
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slides

chapter 4

the open economy

real-business-cycle model

These slides are an adjusted version of the materials for Chapter 4 of the OEM book provided by the authors

Before: added k

Motivation: $\Rightarrow$ ca, tb $\to$ Countercyclical ; Focus on σ_{yx} ($x = \{c, i, tb, ca\}$) Serial Correlations

Previously, we built a model of the small open economy (with capital) driven by productivity shocks and argued that it can capture the observed countercyclicality of the trade balance.

We also established that two features of the model are important for making this prediction possible. First, productivity shocks must be sufficiently persistent. Second, capital adjustment costs must not be too strong.

Now, we explore the ability of that model to explain observed business cycles: We ask whether it can explain the sign and magnitude of business-cycle indicators, such as the standard deviation, serial correlation, and cyclicality of output, consumption, investment, the trade balance, and the current account.

The Small Open Economy RBC Model

To make the models studied in chapters 2 and 3 more empirically realistic and to give them a better chance to account for observed business-cycle regularities add:

1. endogenous labor supply and demand
2. uncertainty in the technology shock process (only TFP shocks for now)
3. capital depreciation. $\delta > 0$

The resulting theoretical framework is known as the Small Open Economy Real-Business-Cycle model, or, succinctly, the SOE-RBC model.

The Household's Maximization Problem

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) \tag{4.1}$$

subject to

$$c_t + i_t + \overbrace{\Phi(k_{t+1} - k_t)}^{\text{adjustment costs}} + (1 + \overbrace{\underline{r_{t-1}}}^{\text{Int. Rate no longer Constant}})d_{t-1} = y_t + d_t \tag{4.2}$$

$$y_t = \underline{A_t} F(k_t, \underline{h_t}) \tag{4.3}$$

$$k_{t+1} = (\underline{1 - \delta})k_t + i_t \tag{4.4}$$

$$\lim_{j \to \infty} E_t \frac{d_{t+j}}{\prod_{s=0}^{j}(1 + r_s)} \leq 0 \tag{4.5}$$

Capital adjustment cost, $\Phi(0) = \Phi'(0) = 0$; $\Phi''(0) > 0$

Notice the additions relative to the previous model (of Chapter 3 in SGU17):

- endogenous labor supply, $U(c_t, h_t)$
- endogenous labor demand, $F(k_t, h_t)$
- uncertainty, A_t is stochastic
- the interest rate is no longer constant, $r_t \neq r$
- depreciation, δ no longer 0

With this setup the model becomes the SOE-RBC model as in Mendoza (1991)

Before getting FOCs : get rid of i_t → Substitute it in BC

Household's Optimality Conditions

[λ_t]: $$c_t + k_{t+1} - (1-\delta)k_t + \Phi(k_{t+1} - k_t) + (1 + r_{t-1})d_{t-1} = A_t F(k_t, h_t) + d_t \tag{4.6}$$

[d_t]: $$\lambda_t = \beta(1 + r_t)E_t \lambda_{t+1} \tag{4.7}$$

[c_t]: $$U_c(c_t, h_t) = \lambda_t \tag{4.8}$$

[h_t]: $$-U_h(c_t, h_t) = \lambda_t A_t F_h(k_t, h_t) \tag{4.9}$$

[k_{t+1}]: $$1 + \Phi'(k_{t+1} - k_t) = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[A_{t+1} F_k(k_{t+1}, h_{t+1}) + 1 - \delta + \Phi'(k_{t+2} - k_{t+1})\right] \tag{4.10}$$

Remember: here the choice or decision variables are $\lambda_t,\ d_t,\ c_t,\ h_t,\ k_{t+1}$ (these are the variables we have to take FOCs with respect to)

Inducing Stationarity: External debt-Elastic Interest Rate (EDEIR)

$$r_t = r^* + p(\widetilde{d}_t) \tag{4.14}$$

r^* = constant world interest rate
$p(\tilde{d}_t)$ = country interest-rate premium
$\tilde{d}_t$ = cross-sectional average of debt

In equilibrium cross-sectional average of debt must equal individual debt

$$\widetilde{d}_t = d_t \tag{4.15}$$

Evolution of Total Factor Productivity, AR(1) process

$$\ln A_{t+1} = \rho \ln A_t + \widetilde{\eta}\epsilon_{t+1} \tag{4.12}$$

The Trade Balance

$$tb_t = y_t - c_t - i_t - \Phi(k_{t+1} - k_t) \tag{4.20}$$

The Current Account

$$ca_t = tb_t - r_{t-1}d_{t-1} \tag{4.21}$$

Equilibrium Conditions

$$-\frac{U_h(c_t,h_t)}{U_c(c_t,h_t)} = A_t F_h(k_t,h_t) \tag{4.11}$$

$$c_t + k_{t+1} - (1-\delta)k_t + \Phi(k_{t+1}-k_t) + [1+r^* + p(d_{t-1})]d_{t-1} = A_t F(k_t,h_t) + d_t \tag{4.16}$$

$$U_c(c_t,h_t) = \beta(1+r^*+p(d_t))E_t U_c(c_{t+1},h_{t+1}) \tag{4.17}$$

$$1 = \beta E_t \left\{ \frac{U_c(c_{t+1},h_{t+1})}{U_c(c_t,h_t)} \frac{[A_{t+1}F_k(k_{t+1},h_{t+1}) + 1 - \delta + \Phi'(k_{t+2}-k_{t+1})]}{1+\Phi'(k_{t+1}-k_t)} \right\} \tag{4.18}$$

This is a system of non-linear stochastic difference equations. It does not have a closed form solution. We will use numerical techniques to find a first-order accurate approximate solution around the nonstochastic steady state. This is a local approximation.

[For capital, the system is a second-order difference equation as it features k_t, k_{t+1} and k_{t+2}. We would like to have a system of first-order difference equations. To this end, introduce the auxiliary variable k_t^f and impose

$$k_t^f = k_{t+1}$$

Note that k_t^f is in the information set of period t.

This equation together with the four above equations forms a system of stochastic first-order difference equations in the 5 unknowns: c_t, h_t, d_{t-1}, k_t, and k_t^f.]

First-order accurate approximation of equilibrium dynamics around the non-stochastic steady state

Solution codes:

From SGU17 website (manual Matlab procedure using the Symbolic toolbox):

```
edeir_ss.m
edeir_model.m
edeir_run.m
```

Alternative (Dynare):

```
mend_91.mod
Main_Mend.m
```

[detour] **Derivation of a first-order accurate approximation**

Based on Schmitt-Grohé and Uribe JEDC 2004.

The EDEIR model developed above gives rise to equilibrium conditions of the form

$$E_t f(y_{t+1}, y_t, x_{t+1}, x_t) = 0 \tag{1}$$

where
$x_t = n_x \times 1$ vector of predetermined (or state) variables
$y_t = n_y \times 1$ vector of nonpredetermined (or control) variables
x_0 is an $n_x \times 1$ vector of initial conditions

Terminal condition: $\lim_{j\to\infty} E_t \begin{bmatrix} x_{t+j} & y_{t+j} \end{bmatrix}' \to \begin{bmatrix} \bar{x} & \bar{y} \end{bmatrix}'$

Let:

$$n = n_x + n_y$$

Then we have that,

$$f : R^{n_y} \times R^{n_y} \times R^{n_x} \times R^{n_x} \to R^n$$

A large class of dynamic stochastic general equilibrium models can be written in the form given in (1). And most studies in real and monetary business cycle analysis use models belonging to this class. Of course, there are also many types of models that do not fit into that class. For example, models with occasionally binding constraints.

Partition state vector x_t

$$x_t = \begin{bmatrix} x_t^1 \\ x_t^2 \end{bmatrix} \begin{array}{l} \longrightarrow \text{Endogenous states (Predetermined)} \\ \longrightarrow \text{Exogenous states (Shocks)} \end{array}$$

$x_t^1 =$ vector of endogenous predetermined state variables
$x_t^2 =$ vector of exogenous state variables

We assume that the exogenous state evolves as:

$$x_{t+1}^2 = \tilde{h}(x_t^2, \sigma) + \sigma \tilde{\eta} \epsilon_{t+1}, \tag{2}$$

$\sigma =$ parameter scaling the amount of uncertainty. ($\sigma = 0$ is perfect foresight.)

Solution to models that are described by (1) and (2) can then be expressed as:

$$\underline{y_t = \widehat{g}(x_t)} \tag{3}$$

$$\underline{x_{t+1} = \widehat{h}(x_t) + \sigma\eta\epsilon_{t+1}} \tag{4}$$

where

$$\eta = \begin{bmatrix} \emptyset \\ \tilde{\eta} \end{bmatrix}.$$

The shape of the functions $\widehat{h}$ and $\widehat{g}$ will in general depend on the amount of uncertainty in the economy.

Key idea of perturbation: parameterize the amount of uncertainty as follows

$$\widehat{g}(x_t) = g(x_t, \sigma) \quad \text{where} \quad g : R^{n_x} \times R^{+} \to R^{n_y}$$

$$\widehat{h}(x_t) = h(x_t, \sigma) \quad \text{where} \quad h : R^{n_x} \times R^{+} \to R^{n_x}$$

Then we can write the solution to the model described by (1) and (2) as

$$\boxed{\begin{aligned} y_t &= g(x_t, \sigma) \\ x_{t+1} &= h(x_t, \sigma) + \sigma \eta \epsilon_{t+1} \end{aligned}}$$

(5)

(6)

Perturbation methods perform a *local* approximation of $g(x, \sigma)$ and $h(x, \sigma)$ around a particular point $(\bar{x}, \bar{\sigma})$

First-order Taylor series expansion of g and h around $(x, \sigma) = (\bar{x}, \bar{\sigma})$

$$\begin{aligned} g(x, \sigma) &= g(\bar{x}, \bar{\sigma}) + g_x(\bar{x}, \bar{\sigma})(x - \bar{x}) + g_\sigma(\bar{x}, \bar{\sigma})(\sigma - \bar{\sigma}) + h.o.t. \\ h(x, \sigma) &= h(\bar{x}, \bar{\sigma}) + h_x(\bar{x}, \bar{\sigma})(x - \bar{x}) + h_\sigma(\bar{x}, \bar{\sigma})(\sigma - \bar{\sigma}) + h.o.t. \end{aligned}$$

$h.o.t.$ = higher order terms

Unknowns: $g(\bar{x}, \bar{\sigma}), g_x(\bar{x}, \bar{\sigma}), g_\sigma(\bar{x}, \bar{\sigma}), h(\bar{x}, \bar{\sigma}), h_x(\bar{x}, \bar{\sigma}), h_\sigma(\bar{x}, \bar{\sigma})$

To identify these terms, substitute the proposed solution given by equations (5) and (6) into equation (1), and define

$$\begin{aligned} F(x, \sigma) &\equiv E_t f(\overbrace{g(h(x, \sigma) + \eta\sigma\epsilon', \sigma)}^{y_{t+1}}, g(x, \sigma), \underbrace{h(x, \sigma) + \eta\sigma\epsilon'}_{x_{t+1}}, x) \\ &= 0. \end{aligned} \tag{7}$$

Here we are dropping time subscripts, and use a prime to indicate variables dated in period $t + 1$.

Because $F(x, \sigma)$ must be equal to zero for any possible values of x and σ, it must be the case that the derivatives of any order of F must also be equal to zero. Formally,

$$F_{x^k\sigma^j}(x, \sigma) = 0 \quad \forall x, \sigma, j, k, \tag{8}$$

where $F_{x^k\sigma^j}(x, \sigma)$ denotes the derivative of F with respect to x taken k times and with respect to σ taken j times.

What point to approximate around?

We need to evaluate the derivatives of $F(x, \sigma)$, $F_{x^k \sigma^j}(x, \sigma)$, at the point we are approximating the equilibrium around. In general this is difficult if not impossible. But there are some points for which evaluation of those derivatives is possible.

One such point is the non-stochastic steady state, $(x, \sigma) = (\bar{x}, 0)$, where $\bar{x}$ denotes the non-stochastic steady state value of x_t. For this point we know: $y_t = \bar{y}$, $y_{t+1} = \bar{y}$, and $x_{t+1} = \bar{x}$, where $\bar{y}$ denotes the non-stochastic steady state of y_t.

For the remainder of this chapter we will focus on approximation around the non-stochastic steady state $(x, \sigma) = (\bar{x}, 0)$.

Another point one can evaluate the derivatives of $F(x, \sigma)$ at is $x_t \neq \bar{x}$ and $\sigma = 0$. This works in cases in which one can find the exact deterministic solution of a model. In that case one can find y_t, y_{t+1} and x_{t+1} for $(x_t, \sigma) = (x_t, 0)$ but needs to resort to approximation techniques to characterize the solution to the stochastic version of the economy.

Let's write again the first-order Taylor series expansion of g and h but this time around the non-stochastic steady state, $(x, \sigma) = (\bar{x}, 0)$

$$\left.\begin{aligned} g(x, \sigma) &= g(\bar{x}, 0) + g_x(\bar{x}, 0)(x - \bar{x}) + g_\sigma(\bar{x}, 0)(\sigma - 0) \\ h(x, \sigma) &= h(\bar{x}, 0) + h_x(\bar{x}, 0)(x - \bar{x}) + h_\sigma(\bar{x}, 0)(\sigma - 0) \end{aligned}\right\} \text{Approximation around non-stochastic Steady State}$$

We wish to find:

$g(\bar{x}, 0)$

$g_x(\bar{x}, 0)$

$g_\sigma(\bar{x}, 0)$

$h(\bar{x}, 0)$

$h_x(\bar{x}, 0)$

$h_\sigma(\bar{x}, 0)$

We will see that at first order $h_\sigma = 0$, $g_\sigma = 0$

(Certainty equivalence of 1st order approximation)

Policy functions are the same as under perfect foresight except for the additive (shocks) stochastic terms

$\Rightarrow$ Up to 1st order solution is

$$y_t = \bar{y} + g_x(\bar{x}, 0)(x - \bar{x})$$

$$x_{t+1} = \bar{x} + h_x(\bar{x}, 0)(x - \bar{x}) + \sigma \eta \epsilon_t$$

Find $g(\bar{x}, 0)$ and $h(\bar{x}, 0)$

From (5)

$$g(\bar{x}, 0) = \bar{y}$$

From (6)

$$h(\bar{x}, 0) = \bar{x}$$

Find h_σ and g_σ

Recall (7)

$$\begin{aligned} 0 &= F(x,\sigma) \\ &= E_t f(g(h(x,\sigma)+\eta\sigma\epsilon',\sigma), g(x,\sigma), h(x,\sigma)+\eta\sigma\epsilon', x) \end{aligned}$$

The first derivative of $F(x,\sigma)$ with respect to σ evaluated at $(x,\sigma)=(\bar{x},0)$

$$\begin{aligned} 0 &= F_\sigma(\bar{x},0) \\ &= \underline{f_{y'}}(\bar{y},\bar{y},\bar{x},\bar{x}) \left[g_x(\bar{x},0)h_\sigma(\bar{x},0)+g_\sigma(\bar{x},0)\right] \\ &\quad + \underline{f_y}(\bar{y},\bar{y},\bar{x},\bar{x}) g_\sigma(\bar{x},0) \\ &\quad + \underline{f_{x'}}(\bar{y},\bar{y},\bar{x},\bar{x}) h_\sigma(\bar{x},0) \end{aligned}$$

Let $f_i \equiv f_i(\bar{y},\bar{y},\bar{x},\bar{x})$ for $i = y',' y, x', x$

Note that we can evaluate f_i because we know the function f and we know the steady state $(\bar{y},\bar{x})$

Rearrange to obtain

$$\begin{bmatrix} f_{y'}g_x + f_{x'} & f_{y'} + f_y \end{bmatrix} \begin{bmatrix} h_\sigma \\ g_\sigma \end{bmatrix} = 0$$

This is a linear homogenous equation in n unknowns. For it to have a unique solution it must be that

$$\begin{bmatrix} h_\sigma \\ g_\sigma \end{bmatrix} = \begin{bmatrix} \emptyset \\ \emptyset \end{bmatrix} \quad (9)$$

Certainty equivalence Principle of 1st order solution

This is an important result. It says that up to first-order accuracy one need not correct the constant term or the slope term of the approximation for the presence of uncertainty. The policy function is the same as under perfect foresight but for the additive stochastic error term. (the solution displays the certainty equivalence principle)

Up to first order accuracy the solution is:

$$\boxed{\begin{aligned} y_t &= \bar{y} + g_x(\bar{x}, 0)(x - \bar{x}) \\ x_{t+1} &= \bar{x} + h_x(\bar{x}, 0)(x - \bar{x}) + \sigma\eta\epsilon_{t+1} \end{aligned}}$$

Consider the unconditional expectations of x_t of the first-order accurate approximation:

$$\begin{aligned} E(x_t) &= E\{\bar{x} + h_x(\bar{x},0)(x_t - \bar{x}) + h_\sigma(\bar{x},0)(\sigma - 0)\} \\ &= \bar{x} + h_x(\bar{x},0)(E(x_t) - \bar{x}) + 0 \end{aligned}$$

for first order approximations the mean is the same as the unconditional expectation

$\Rightarrow$ 1st Order solution is not helpful to approximate avg. risk premia or to compute Welfare differences between policies that yield the same non-stochastic SS

It follows that up to first order accuracy:

$$Ex_t = \bar{x} \quad \text{and} \quad Ey_t = \bar{y}$$

or in words the unconditional expectation is the same as the mean. Hence first-order accurate approximations will not be helpful to approximate average risk premia (they would all be zero) or the average welfare associated with different monetary or fiscal policy that all give rise to the same nonstochastic steady state (all policies give the same welfare in the steady state).

Find $h_x(\bar{x}, 0)$ and $g_x(\bar{x}, 0)$

Start again from (7)

$$\begin{aligned} 0 &= F(x,\sigma) \\ &= E_t f(g(h(x,\sigma)+\eta\sigma\epsilon',\sigma), g(x,\sigma), h(x,\sigma)+\eta\sigma\epsilon', x) \end{aligned}$$

The first derivative of $F(x,\sigma)$ with respect to x evaluated at $(x,\sigma)=(\bar{x},0)$

$$\begin{aligned} 0 &= F_x(\bar{x},0) \\ &= f_{y'} g_x h_x + f_y g_x + f_{x'} h_x + f_x \end{aligned}$$

Rearrange to

$$\begin{bmatrix} f_{x'} & f_{y'} \end{bmatrix} \begin{bmatrix} I \\ g_x \end{bmatrix} h_x = - \begin{bmatrix} f_x & f_y \end{bmatrix} \begin{bmatrix} I \\ g_x \end{bmatrix}$$

To solve this expression for h_x and g_x use a Schur decomposition. We describe this in detail in Appendix 4.14 of the Chapter. The Matlab program `gx_hx.m` posted on our website with the materials for Chapter 4 performs this step.

Taking stock:

Thus far we have presented a first-order accurate approximation technique.

Now we can discuss how to implement this in the case of the EDEIR model.

It should be clear that an important element of the implementation is finding numerical values for the derivatives of the function $f(\cdot)$ at the non-stochastic steady state.

Our approach is to use the Symbolic Math Toolbox of Matlab to do most of the work. This has several advantages. One is that the room for error is much smaller and the other is that it eliminates any tedious linearization by hand.

To allow a Symbolic Math toolbox to implement the linearization it is convenient to specify functional forms for the utility, production, country premium, and adjustment cost functions. What will matter, given that we perform a first-order approximation to the equilibrium conditions, is at most the first and second derivatives of those functions.

[end of detour]

Utility chosen: GHH preferences

Makes labor decision separable from marginal utility of consumption

(Then labor supply is independent of C_t)

Which implies removing wealth effect of labor supply

Functional Forms

Period utility function

$$U(c,h) = \frac{\left(c - \omega^{-1}h^{\omega}\right)^{1-\sigma} - 1}{1-\sigma}; \quad \omega > 1; \sigma > 0$$

Debt-elastic interest rate

$$p(d) = \psi\left(e^{d-\bar{d}} - 1\right); \quad \psi > 0$$

Production function

$$F(k,h) = k^{\alpha}h^{1-\alpha}; \quad \alpha \in (0,1)$$

Adjustment cost function

$$\Phi(x) = \frac{\phi}{2}x^2; \quad \phi > 0$$

6 structural parameters: σ, ω, ψ, $\bar{d}$, α, ϕ

SS : (d, k, c, h)

Solution to static system

Characterizing the Deterministic Steady State

The steady state is the quadruple (d, k, c, h) satisfying

$$-\frac{U_h(c,h)}{U_c(c,h)} = AF_h(k,h) \tag{4.11'}$$

$$c + \delta k + (r^* + p(d))d = AF(k,h) \tag{4.16'}$$

$$1 = \beta(1 + r^* + p(d)) \tag{4.17'}$$

$$1 = \beta\left[AF_k(k,h) + 1 - \delta\right] \tag{4.18'}$$

Using the assumed functional forms the steady state becomes

$$h^{\omega-1} = A(1-\alpha)(k/h)^{\alpha} \tag{4.11”}$$

$$c + \delta k + (r^* + \psi(e^{d-\bar{d}}))d = A(k/h)^{\alpha} h \tag{4.16”}$$

$$1 = \beta(1 + r^* + \psi(e^{d-\bar{d}} - 1)) \tag{4.17”}$$

$$1 = \beta \left[A\alpha(k/h)^{\alpha-1} + 1 - \delta \right] \tag{4.18”}$$

This is a system of 4 equations in 4 unknown endogenous variables, (c, d, h, k) and 7 unknown parameters, $\omega, \alpha, \delta, r^*, \psi, \bar{d}, \beta$. (From (4.12), we know that in steady state $A = 1$).

The model has 4 additional structural parameters, σ, ϕ, ρ, $\tilde{\eta}$, which do not enter the steady state but which also need to be assigned values to. In sum, there are 11 structural parameters to be calibrated. They are:

$$\left[\begin{array}{ccccccccccc} \omega & \alpha & \delta & r^* & \beta & \sigma & \phi & \rho & \tilde{\eta} & \bar{d} & \psi \end{array} \right]$$

We assume that the time unit is one year and calibrate the model to the Canadian economy. This is (almost) the same calibration as Mendoza (1991).

σ	$1+r^*=1/\beta$	δ	α	ω	ϕ	ρ	σ_ϵ	$\bar{d}$
2	1.04	0.1	0.32	1.455	0.028	0.42	0.0129	0.7442

Comment: Mendoza's model uses a different stationarity inducing device (an internal discount factor (IDF) model, which we discuss in detail in section 4.10.4) and hence that calibration does not assign a value to ψ. As in Schmitt-Grohé and Uribe (2003), we set ψ to ensure that the EDEIR model predicts the same volatility of the current-account-to-output ratio as the IDF model. The value that achieves that is

$$\psi = 0.000742$$

ψ picked to match $\sigma^2_{tb/y}$ in Canada

Given values for the structural parameters, the steady state can be computed using the Matlab program `edeir_ss.m` available on the book's Website. This yields

c	d	h	k
1.1170	0.7442	1.0074	3.3977

Three calibration strategies: I. Parameters from external sources (literature)
II, III. Parameters defined by matching moments of the data
II. First moments, III. Second Moments
related to the exact data we aim to study

The Calibration Strategy

To obtain the values of the structural parameters shown in the previous slide (and Table 4.1 in the book), three types of restrictions were imposed:

Category a: restrictions using sources unrelated to the data that the model aims to explain, **4** parameters: $\sigma = 2$, $\delta = 0.1$, $r^* = 0.04$, $\beta = 1/(1 + r^*)$.

Category b: restrictions to match first moments of the data that the model aims to explain, **2** parameters: α, $\bar{d}$

$$\text{labor share } = 0.68$$
$$\text{trade-balance-to-output ratio } = 0.02$$

Category c: restrictions to match second moments of the data that the model aims to explain, **5** parameters: ω, ϕ, ψ, ρ, $\tilde{\eta}$. The second moments to be matched are: σ_y, σ_h, σ_i, $\sigma_{tb/y}$, corr$(\ln y_t, \ln y_{t-1})$

How to implement this calibration strategy? The restrictions in category a translate immediately into values for structural parameters. To go from the restrictions in categories b and c to the values of the structural parameters shown in Table 4.1, one proceeds as follows:

The labor share, s_h, is defined as

Some parameters obtained directly from the moments

$$s_h = \frac{wh}{y}$$

In the decentralized economy we have

$$A_t F_2(k_t, h_t) = w_t$$

Thus, in the steady state:

$$s_h = \frac{AF_2(k,h)h}{AF(k,h)}$$

Using the assumed functional form for $F(\cdot)$ yields

$$\underline{s_h = (1-\alpha)}$$

Hence we have that $\alpha = 1 - s_h = 1 - 0.68$, that is,

$$\alpha = 0.32$$

Let θ denote the vector of structural parameters we still need to assign numerical values to, that is, let

$$\theta \equiv \begin{bmatrix} \omega & \bar{d} & \phi & \psi & \rho & \tilde{\eta} \end{bmatrix}$$

The calibration strategy described on the previous slide consists of the following steps:

Strategy:

- Guess values for all but 1 undefined parameter in θ
- Solve analytically for the remaining parameter and steady state of model (given the parameter values and targeted 1st moments)

Step 1: guess a value for each element of θ

Step 2: Given the guess for ω find h using (4.18”)

$$\frac{k}{h} = \left(\frac{r^* + \delta}{A\alpha}\right)^{\frac{1}{\alpha - 1}}$$

- Compute the second moments based on the steady state and all structural parameters
- Compute distance between computed moments and targeted moments $D = |X(\theta) - X^*|$
- Adjust guess for θ and repeat until distance is small

in (4.11”)

$$h = ((1 - \alpha)A(k/h)^{\alpha})^{1/(\omega - 1)}$$

With h in hand, find k and y, as $k = (k/h)h$ and $y = A(k/h)^{\alpha}h$, respectively.

Step 3: Let s_{tb} denote the trade-balance-to-output ratio. In the steady state,

$$s_{tb} = \frac{r^* d}{y}$$

Solve this expression for d

$$d = \frac{s_{tb} y}{r^*}$$

Then use (4.17”) and the restriction that $\beta(1 + r^*) = 1$ to obtain

$$\bar{d} = d$$

Step 4: Find c from (4.16”)

$$c = y - \delta k - r^* d$$

Step 5: With the steady state values of (c, k, d, h) and all structural parameters in hand compute the model's predictions for

$$x(\theta) \equiv \left[\begin{array}{ccccc} \sigma_y & \sigma_h & \sigma_i & \sigma_{tb/y} & corr(\ln y_t, \ln y_{t-1}) \end{array} \right]$$

Step 6: Find the distance

$$D = |x(\theta) - x^*|$$

where x^* denotes the vector of targeted moments observed in Canadian data

Step 7: Keep adjusting θ until D is less than some threshold D^*.

Comment: In general there does not exist a θ that makes the distance D exactly equal to zero. Hence one has to pick some threshold for the distance, D^*.

Before analyzing to which extend the SOE-RBC model can account for the observed Canadian business cycle, let's first study the predictions of this model regarding the prediction for which we build intuition in Chapters 2 and 3.

Before

w/ more persistent shocks

- TBs deteriorates more after a positive shock
- σ_c/σ_y increases

w/ Adjustment costs: tbs deterioration is mitigated

Now (SOE-RBC)

These results still hold

In particular, there we showed that

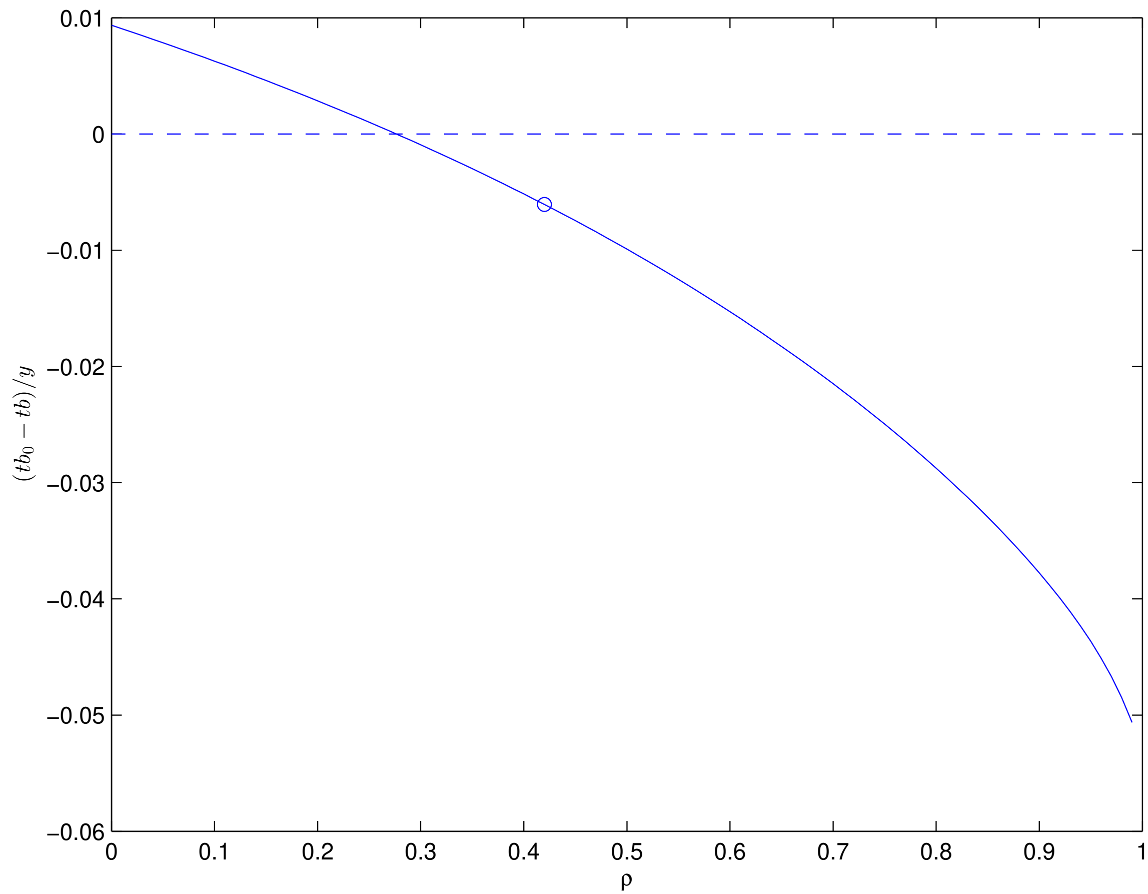
• the more persistent productivity shocks are, the more likely an initial deterioration of the trade balance will be.

• the more pronounced are capital adjustment costs, the smaller will be the initial trade balance deterioration in response to a positive and persistent productivity shock.

• the more persistent the technology shock is, the higher the volatility of consumption relative to output will be.

The next three figures show that these analytical results do indeed hold in the fully-fledged stochastic dynamic open economy real-business-cycle model.

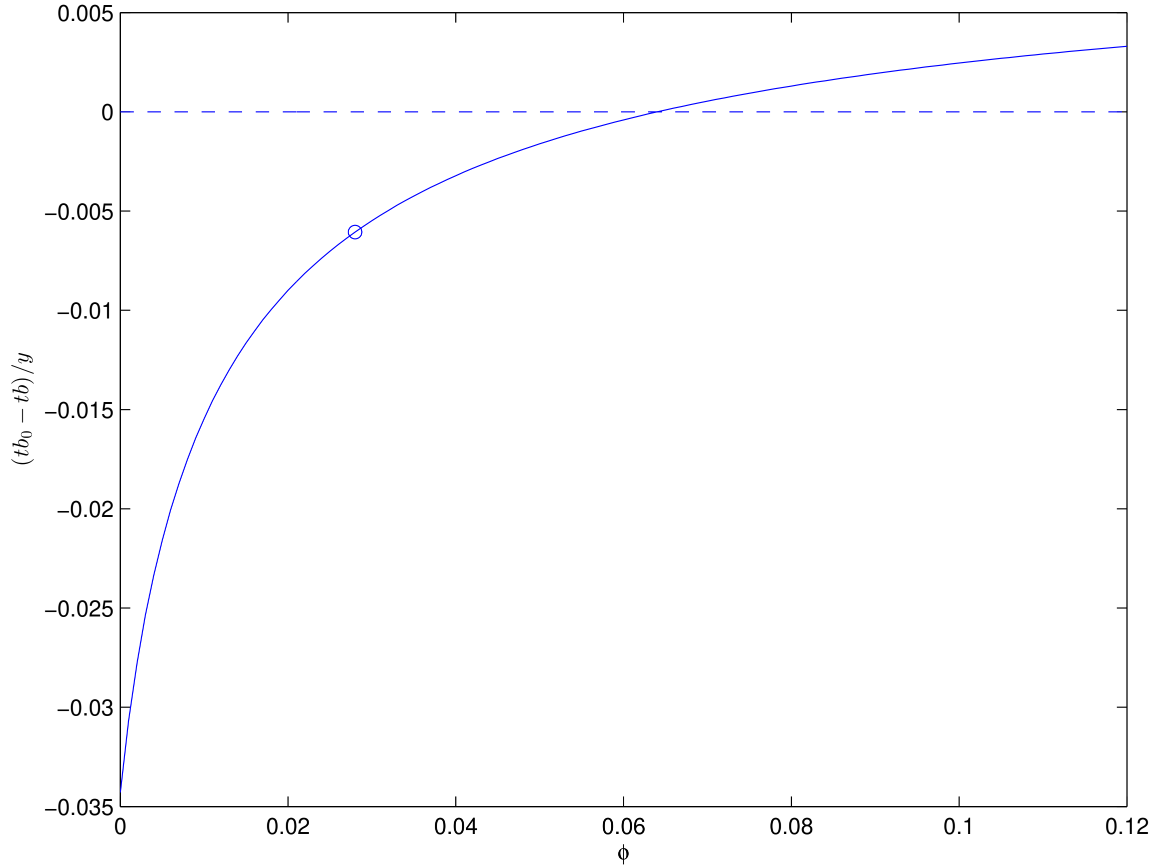
Impact response of the trade balance as a function of the persistence of the technology shock



Notes. The figure shows the impact response of the trade balance in response to a one percent positive innovation in productivity predicted by the EDEIR model presented in Chapter 4. The response of the trade balance is measured in units of steady-state output. All parameters other than ρ take the values shown in Table 4.1. The open circle indicates the baseline value of ρ.

Comments: The figure shows that the more persistent the productivity shock is the smaller the impact response of the trade balance will be. For $\rho > 0.3$, the response of the trade balance is negative, confirming the analytical results of chapters 2 and 3.

Impact response of the trade balance as a function of capital adjustment costs

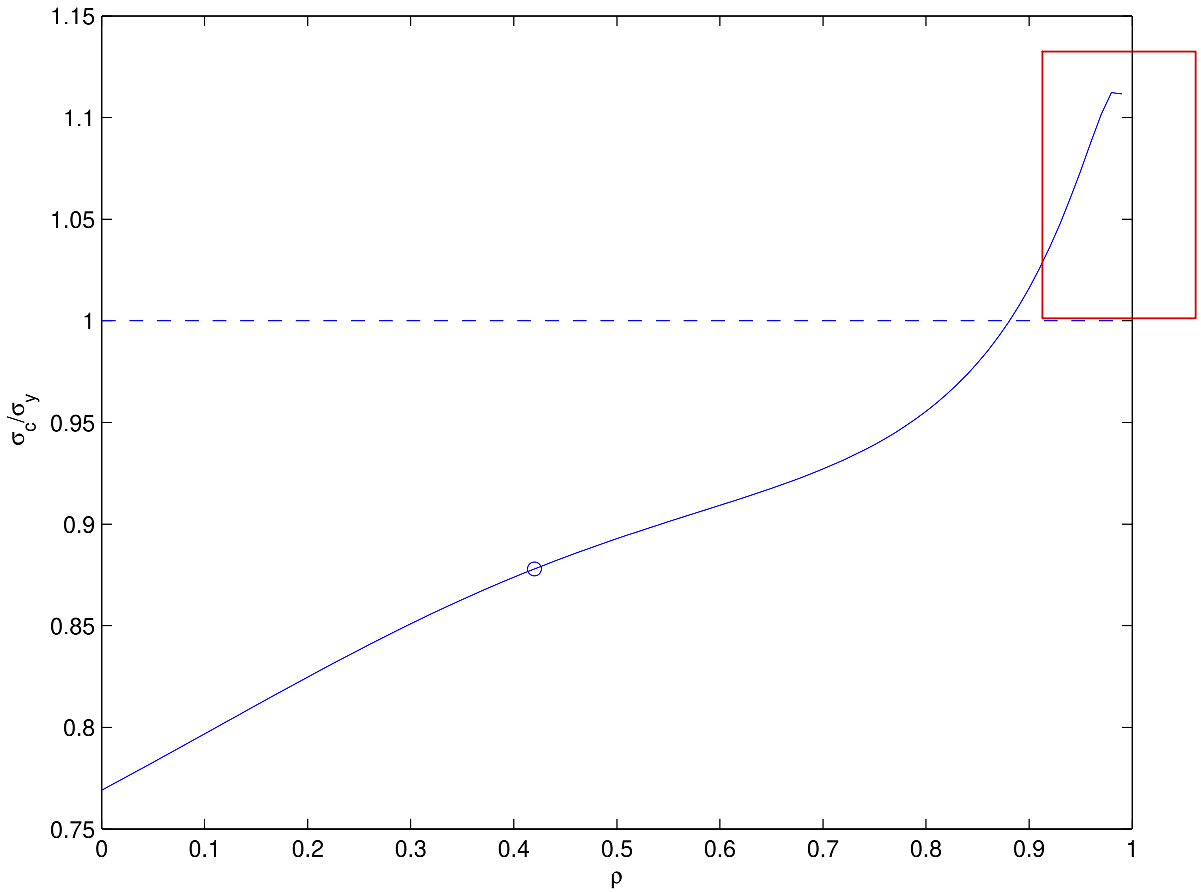


Countercyclicality of tb dampens if adjustment costs increase

Notes. The figure shows the impact response of the trade balance in response to a one percent positive innovation in productivity as a function of the size of capital adjustment costs, ϕ, predicted by the EDEIR model presented in Chapter 4. The response of the trade balance is measured in units of steady-state output. All parameters other than ρ take the values shown in Table 4.1. The open circle indicates the baseline ϕ value.

Comments: The figure shows that the higher capital adjustment costs are the larger the impact response of the trade balance will be. For $\phi > 0.06$, the response of the trade balance turns positive, confirming the analytical results of chapters 2 and 3.

Relative volatility of consumption as a function of the persistence of the stationary technology shock



for persistent productivity we obtain a larger volatility of consumption (wrt output)

Notes. The relative standard deviation shown is that implied by the EDEIR model presented in Chapter 4. All parameters other than ρ take the values shown in Table 4.1. The open circle indicates the baseline value of ρ.

Comments: The figure shows that the more persistent stationary productivity shocks are, the higher the standard deviation of consumption relative to the standard deviation of output will be, just as derived analytically in the permanent income model of Chapter 2.

We now turn an analysis of second moments predicted by the SOE-RBC model and compare them to the Canadian data.

Some Empirical Regularities of the Canadian Economy

Why Canada? Because it is a small open economy and it is the economy studied in Mendoza (1991).

Variable	Canadian Data		
	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,GDP_t}
y	2.8	0.61	1
c	2.5	0.7	0.59
i	9.8	0.31	0.64
h	2	0.54	0.8
$\frac{tb}{y}$	1.9	0.66	-0.13

Source: Mendoza AER, 1991. Annual data. Log-quadratically detrended.

$y_t = \ln Y_t$
$y_t = \underbrace{a + bt + ct^2}_{\text{trend}} + \epsilon_t, \; y_t^c = \epsilon_t$

Comments

- Volatility ranking: $\sigma_{tb/y} < \sigma_c < \sigma_y < \sigma_i$.
- Consumption, investment, and hours are procyclical.
- The trade-balance-to-output ratios is countercyclical.
- All variables considered are positively serially correlated.
- Similar stylized facts emerge from other small developed countries (see, e.g., chapter 1).

Empirical and Theoretical Second Moments

	Canadian Data						Model		
	1946 to 1985			1960 to 2011					
	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,y_t}	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,y_t}	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,y_t}
y	2.8	0.6	1	3.7	0.9	1	3.1	0.6	1
c	2.5	0.7	0.6	2.2	0.7	0.6	2.7	0.8	0.8
i	9.8	0.3	0.6	10.3	0.7	0.8	9.0	0.1	0.7
h	2.0	0.5	0.8	3.6	0.7	0.8	2.1	0.6	1
$\frac{tb}{y}$	1.9	0.7	-0.1	1.7	0.8	0.1	1.8	0.5	-0.04
$\frac{ca}{y}$							1.4	0.3	0.05

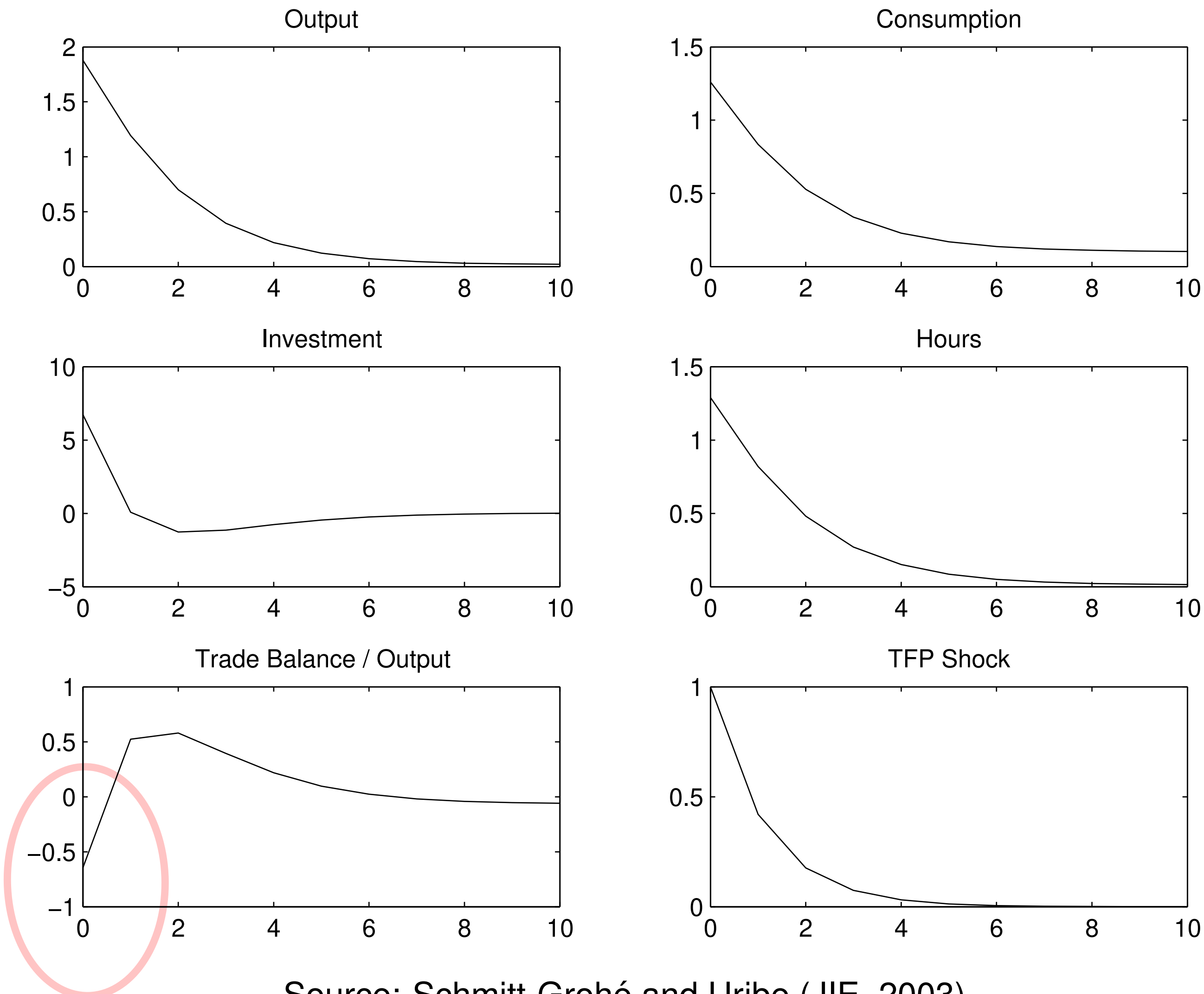
Comments:

- σ_h, σ_i, σ_y, $\sigma_{tb/y}$, and $\rho_{y_t,y_{t-1}}$ were targeted by calibration, so no real test here.
- model correctly places σ_c below σ_y and σ_i and above σ_h and $\sigma_{tb/y}$.
- model correctly makes tb/y countercyclical.
- model overestimates the correlations of hours and consumption with output.

(this is due to the GHH preferences where in the log-linearized model we get $\omega\widehat{h}_t = \widehat{y}_t$)

Since σ_c was not targeted by the calibration (like the other volatilities) it can be used to test the model
model captures well the position of σ_c in the volatility ranking
Tb_t is countercyclical
model overestimates $\rho_{h,y}$ and $\rho_{c,y}$

Response to a Positive Technology Shock



Source: Schmitt-Grohé and Uribe (JIE, 2003)

Comments:

- Output, consumption, investment, and hours expand.
- The trade balance deteriorates.

4.9 The Complete Asset Markets (CAM) Model

W/ Complete markets the future assets keep the expectations $E_t[\cdot]$

It denotes the aggregation of state contingent assets times the probability of the states

$$\underline{E_t q_{t,t+1} b_{t+1}} = b_t + y_t - c_t - i_t - \Phi(k_{t+1} - k_t),$$

$$\lim_{j\to\infty} E_t q_{t,t+j} b_{t+j} \geq 0,$$

$$q_{t,t+j} = q_{t,t+1} q_{t+1,t+2} \cdots q_{t+j-1,t+j},$$

$$\lambda_t q_{t,t+1} = \beta\lambda_{t+1}. \quad \longmapsto [b_{t+1}]$$

FOC wrt b_{t+1}

generates the same Euler eq. than in incomplete markets as $1+r_t = \frac{1}{E_t q_{t,t+1}}$

$(\lambda_t = (1+r_t) E_t \lambda_{t+1})$

$$\lambda_t^* q_{t,t+1} = \beta\lambda_{t+1}^*. \quad \longmapsto [b_{t+1}^*]$$

FOC wrt b_{t+1}^* for foreign HH

Rates are the same given UIP and no ER: $r_{t+1} = r_{t+1}^*$

Also notice the timing

	In LHS	in RHS
Incomplete Mkts	$(1+r_{t-1})d_{t-1}$ — What you get in t (Proceeds from investment in t-1)	d_t — What you buy in t so that you get $(1+r_t)d_t$ in t+1
Complete Mkts	$E_t \overset{\text{SDF}}{q_{t+1}} b_{t+1}$ — What you buy in t for future periods (many state contingent assets s.t. in expectation you get b_{t+1} in t+1)	b_t — What you get in t

$$\boxed{\frac{\lambda_{t+1}}{\lambda_t} = \frac{\lambda_{t+1}^*}{\lambda_t^*}. \qquad \lambda_t = \xi\lambda_t^*,}$$

$\longmapsto$ Perfect Risk Sharing (Won't hold only in expectation as normally but always -w/ certainty-)

$$\lambda_t = \psi_4,$$

How this achieves stationarity?
Complete markets removes the expectations operator and we end up w/ $C_t = C_{t+1} =$ Constant which is: stationary and very different from a non-stationary RW: $C_t = E_t C_{t+1}$

Calibration: Set ψ_4 so that steady-state consumption equals steady-state consumption in the model with Uzawa preferences.

The SOE-RBC Model With Complete Asset Markets: Predicted Second Moments

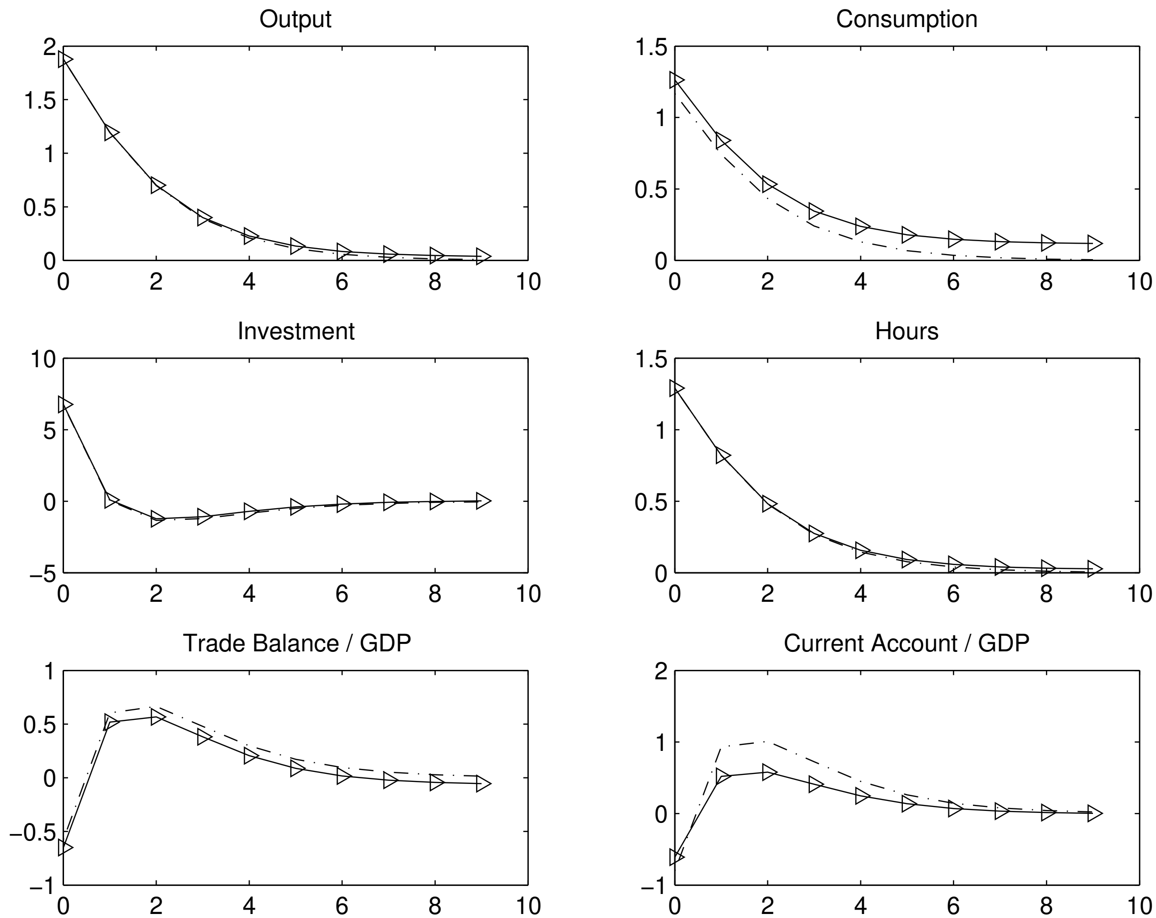
	σ_{x_t}		$\rho_{x_t,x_{t-1}}$		ρ_{x_t,GDP_t}	
variable	CAM	EDEIR	CAM	EDEIR	CAM	EDEIR
y	3.1	3.1	0.61	0.62	1.00	1.00
c	1.9	2.71	0.61	0.78	1.00	0.84
i	9.1	9.0	0.07	0.07	0.66	0.67
h	2.1	2.1	0.61	0.62	1.00	1.00
$\frac{tb}{y}$	1.6	1.78	0.39	0.51	0.13	-0.04
$\frac{ca}{y}$	3.1	1.45	-0.07	0.32	-0.49	0.05

Note. Standard deviations are measured in percentage points. The columns labeled CAM are produced with the Matlab program `cam_run.m` available at

`http://www.columbia.edu/~mu2166/closing.htm`.

Impulse Response to a Unit Technology Shock

One-Bond Versus Complete Asset Market Models



Dash-diamond, EDEIR model. Dash-dotted, complete-asset-market model.

4.10 Alternative Ways to Induce Stationarity

4.10.1 The Internal Debt-Elastic Interest Rate (IDEIR) Model

$$r_t = r + p(d_t),$$

agents internalize the impact of their debt choice in the risk premium

The Euler equation becomes

$$\lambda_t = \beta[1 + \overbrace{r + p(d_t)}^{r_t} + p'(d_t)d_t]E_t\lambda_{t+1}$$

New term in Euler Eq. from internalizing effects of d_t in cost of debt.

$$p(d) = \psi_2\left(e^{d-\bar{d}} - 1\right),$$

Calibration: Same as in the external case. Note that the steady-state value of debt is no longer equal to $\bar{d}$. Instead, d solves

$$(1 + d)e^{d-\bar{d}} = 1 \Rightarrow d = 0.4045212.$$

Internal Debt Elastic Interest rate

→ debt in the premium is a decision variable (rather than taken as given)

$r_t = r + p(d_t)$ (before: $\tilde{d}_t$ – average, "exogenous" –)

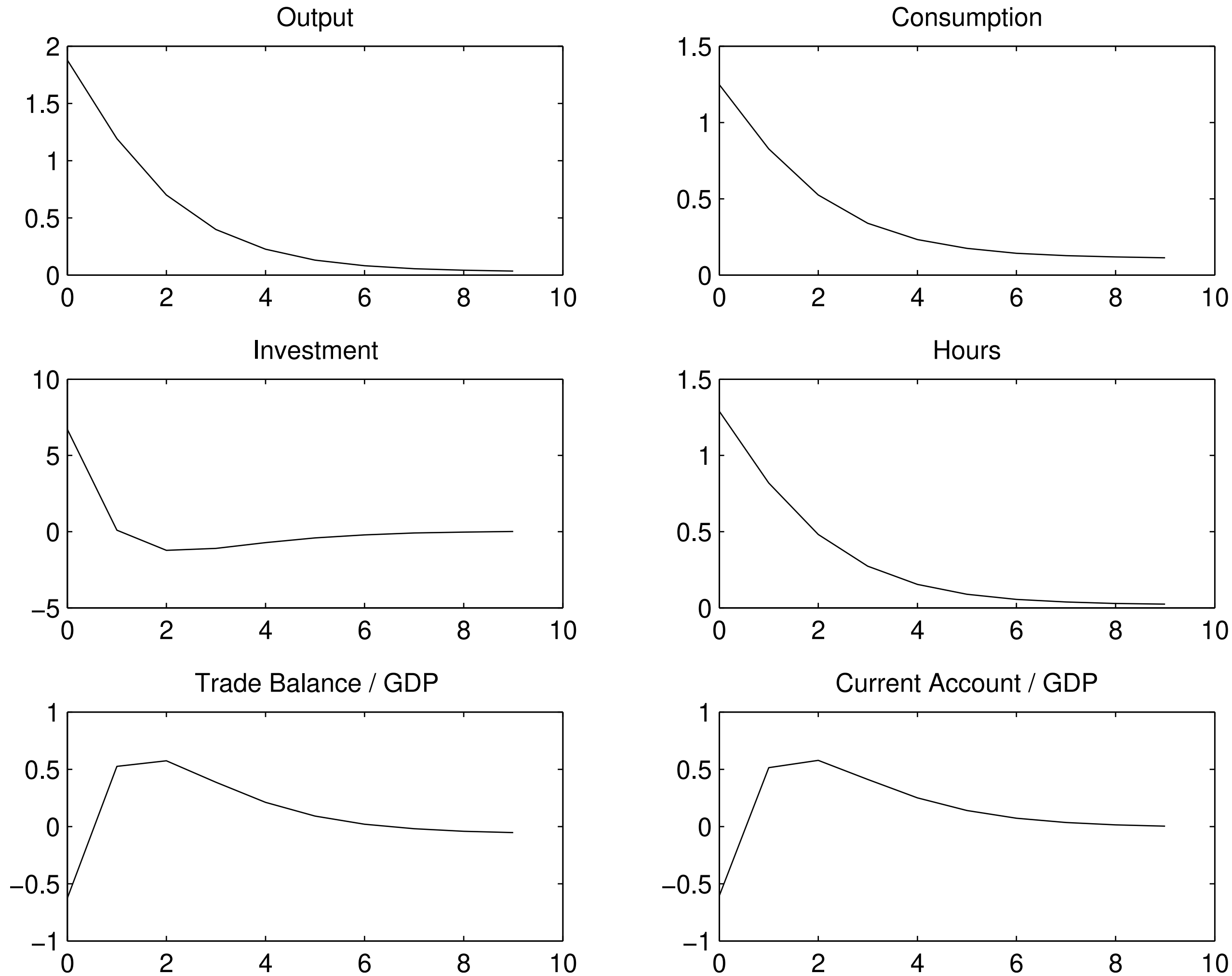
Euler equation changes: $\lambda_t = \beta[1+r+p(d_t)+p'(d_t)d_t]E_t\lambda_{t+1}$

Calibration is the same as EDEIR but SS value of d_t changes

Internal Debt-Elastic Interest-Rate

Variable	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,GDP_t}
y	3.1	0.62	1
c	2.5	0.76	0.89
i	9	0.068	0.68
h	2.1	0.62	1
tb/y	1.6	0.43	-0.036
ca/y	1.4	0.31	0.041

Internal Debt-Elastic Interest Rate Premium Response to a Positive Technology Shock



Comment: The economy with internal debt-elastic interest rate premium behaves very simula-rly to the economies featuring other stationarity inducing devices.

Portfolio Adjustment Cost Model (PAC)
Include adj. costs of debt in BC.
$d_t = (1+r_{t-1}) d_{t-1} - y_t + c_t + i_t + \phi(k_{t+1} - k_t) + \underline{\frac{\psi_3}{2}(d_t - \bar{d})^2}$

Effect: E Eq. depends on the value of assets
$\lambda_t [1 + \psi_3 (d_t - \bar{d})] = \beta (1 + r_t) \mathbb{E}_t \lambda_{t+1}$

4.10.2 The Portfolio Adjustment Cost (PAC) Model

$$d_t = (1 + r_{t-1})d_{t-1} - y_t + c_t + i_t + \Phi(k_{t+1} - k_t) + \frac{\psi_3}{2}(d_t - \bar{d})^2$$

$$\lambda_t[1 - \boxed{\psi_3(d_t - \bar{d})]} = \beta(1 + r_t)E_t\lambda_{t+1}$$

Calibration

β	$\bar{d}$	ψ_3	r
0.96	0.7442	0.00074	$\beta^{-1} - 1$

4.10.3 The External Discount Factor (EDF) Model

Endogenous Discount Factor:

use $\beta(\tilde{c}_t, \tilde{h}_t)$ instead of β

The DF changes w/ the economy (c_t, h_t)
but the agents still do not internalize
the effects of their decisions in shaping $\beta(\tilde{c}_t, \tilde{h}_t)$

$$\theta_{t+1} = \beta(\tilde{c}_t, \tilde{h}_t)\theta_t \qquad t \geq 0,$$

$$\theta_0 = 1,$$

where $\tilde{c}_t$ and $\tilde{h}_t$ denote per capita consumption and hours worked.

$$\lambda_t = \beta(\tilde{c}_t, \tilde{h}_t)(1 + r_t)E_t\lambda_{t+1}$$

$$\lambda_t = U_c(c_t, h_t)$$

$$-U_h(c_t, h_t) = \lambda_t A_t F_h(k_t, h_t)$$

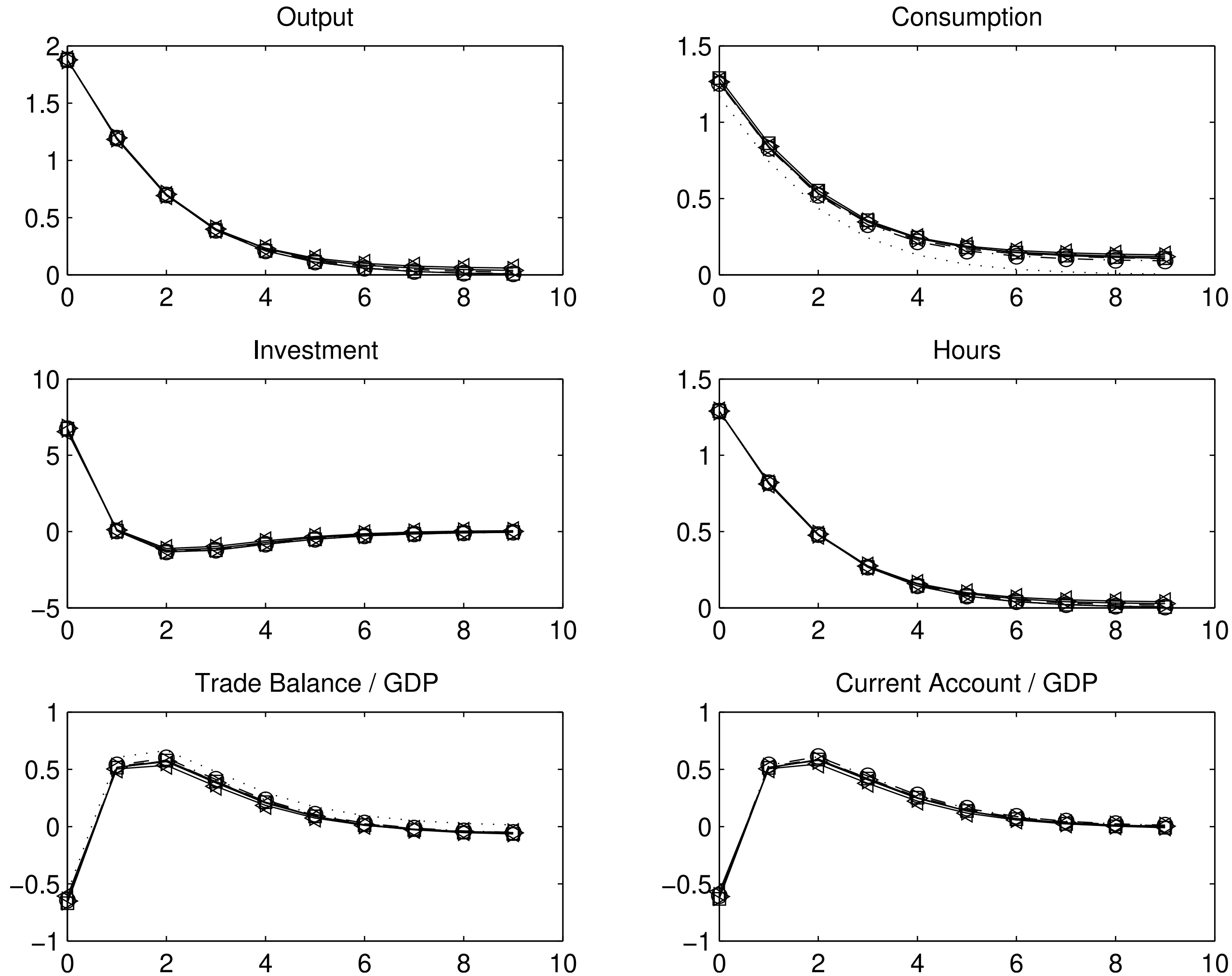
$$\begin{aligned} \lambda_t[1 + \Phi'_t] &= \boxed{\beta(\tilde{c}_t, \tilde{h}_t)} E_t\lambda_{t+1}[A_{t+1}F_k(k_{t+1}, h_{t+1}) \\ &+ 1 - \delta + \Phi'_{t+1}] \end{aligned}$$

In Equilibrium

$$c_t = \tilde{c}_t \text{ and } h_t = \tilde{h}_t$$

4.12 Inducing Stationarity: Quantitative Comparison of Alternative Methods

Impulse Response to a Unit Technology Shock in Models 1 Through 5



Source: Schmitt-Grohé and Uribe (JIE, 2003). Note. Solid line, endogenous discount factor. Squares, endogenous discount factor without internalization. Dashed line, Debt-elastic interest rate. Dash-dotted line, Portfolio adjustment cost. Dotted line, complete asset markets. Circles, No stationarity inducing elements.

Observed and Implied Second Moments

	Data	Model 1	Model 1a	Model 2	Model 3	Model 4
Standard Deviations						
y	2.8	3.1	3.1	3.1	3.1	3.1
c	2.5	2.3	2.3	2.7	2.7	1.9
i	9.8	9.1	9.1	9	9	9.1
h	2	2.1	2.1	2.1	2.1	2.1
tb/y	1.9	1.5	1.5	1.8	1.8	1.6
ca/y		1.5	1.5	1.5	1.5	
Serial Correlations						
y	0.61	0.61	0.61	0.62	0.62	0.61
c	0.7	0.7	0.7	0.78	0.78	0.61
i	0.31	0.07	0.07	0.069	0.069	0.07
h	0.54	0.61	0.61	0.62	0.62	0.61
tb/y	0.66	0.33	0.32	0.51	0.5	0.39
ca/y		0.3	0.3	0.32	0.32	
Correlations with Output						
c	0.59	0.94	0.94	0.84	0.85	1
i	0.64	0.66	0.66	0.67	0.67	0.66
h	0.8	1	1	1	1	1
tb/y	-0.13	-0.012	-0.013	-0.044	-0.043	0.13
ca/y		0.026	0.025	0.05	0.051	

Source: Schmitt-Grohé and Uribe (JIE, 2003)

Note. Standard deviations are measured in percent per year.