# Strategic Macro-prudential Policy Setting in Emerging Economies: The Role of Coordination in International Markets*

**[Draft]**

Camilo Granados †

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**Abstract**

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## 1. Introduction

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## 2. Literature Review

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## 3. The Model

In this section we set an infinite horizon model analogous to the finite version of the main paper. The reader interested in the finite horizon model only can skip the

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†Department of Economics, University of Washington, Seattle. Email: jcgc@uw.edu

remaining sections.

As mentioned, the model borrows standard elements from the literature for representing each agent. In particular, we take elements from Banerjee et al. (2016), Agénor et al. (2017) and Gertler and Karadi (2011) and incorporate them into a three country center-periphery framework (with incomplete markets).

## Households

The households in each economy maximize their welfare, given by their present value of their life-stream utility:

$$\max_{\{C_t, B_t, D_t\}_{t=0}^{\infty}} W_0^i = E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{i(1-\sigma)}}{1-\sigma} - \frac{H_t^{i(1+\psi)}}{1+\psi} \right) \tag{1}$$

s.t.,

$$C_t^i + B_t^i + \frac{\eta}{2}(B_t^i)^2 + D_t^i + \frac{\eta}{2}(D_t^i - \bar{D}^i)^2 = R_{t-1}^i B_{t-1}^i + R_{D,t-1}^i D_{t-1}^i + W_t^i H_t^i + \Pi_t^i \tag{2}$$

With $i = \{a, b, c\}$ and where $B_t^i$: non-contingent international bond, $D_t^i$: domestic deposits, $W_t^i H_t^i$ : labor income, $\Pi_t^i$ : profits from banks and capital firms net of lump-sum taxes.

In addition, adjustment costs from changes in assets positions are included to prevent non-stationarity of the model in an incomplete markets setup (see Schmitt-Grohe and Uribe (2003)).

Only one good is produced worldwide and $C^i$ is the corresponding consumption of it by the home household in the emerging country $i$.

Since only one good is produced, a retail and intermediate goods sector is not included. That implies there is no home bias in consumption generated by the asymmetric size of the countries. Furthermore, for every economy the consumption baskets are expressed in terms of the same good and no departure from the LOP is assumed, meaning that the relative prices and Real Exchange Rate are abstracted from.

In line with these assumption, the corresponding market clearing conditions are:

$$\text{Goods market:} \qquad \sum_i n_i Y_t^i = \sum_i n_i \left( C_t^i + I_t^i \left( 1 + \frac{\zeta}{2} \left( \frac{I_t^i}{I_{t-1}^i} - 1 \right)^2 \right) + \frac{\eta}{2} (B_t^i)^2 + \frac{\eta}{2} (D_t^i - \bar{D}^i)^2 \right)$$

$$\text{Net supply of bonds:} \qquad \sum_i n_i B_t^i = 0, \qquad \forall t$$

where $i$ denotes a country index, i.e., $i = \{a, b, c\}$.

Notice that the market clearing condition for the final goods reflects, first, the adjustment cost of executing investment projects, and second, the fact that the final good is fully tradable and produced in each economy (no home bias).

One difference between the households of the advanced economy and the emerging one is that the former will be able to freely purchase deposits from the center country banks (i.e., without limitations as in the periphery). Nevertheless, note that this feature is not reflected in the household budget constraint but in the bank's intermediation capacity constraint.

## Final goods firms

There is one single good produced in the world that is obtained from a CD technology:

$$Y_t^i = A_t^i \left( \xi_t^i K_{t-1}^i \right)^{\alpha} H_t^{i(1-\alpha)} \tag{3}$$

$H^i, K^i$ are labor and capital, $A^i$ is a labor productivity shock, and $\xi^i$ is a capital-quality shock (both are first-order AR processes).

Each period, the firms will choose labor and capital inputs to maximize the profits obtained from producing and from the sale of undepreciated physical capital to investors:

$$\max_{K_{t-1}, H_t} \Pi_t^{i,prod} = Y_t + (1-\delta)\xi_t Q_t K_{t-1} - W_t H_t - \tilde{R}_{k,t} Q_{t-1}$$
$$s.t. \quad (3)$$

We define the marginal product of capital as $r_t \equiv \alpha A_t^i \xi_t^{\alpha} K_{t-1}^{i\ \alpha-1} H_t^{i\ 1-\alpha}$ and obtain from the FOCs with respect to labor and capital the wages and gross rate of returns paid to the banking sector:

$$W_t^i = (1-\alpha)A_t^i H_t^{i(-\alpha)} \xi_t^{i\ \alpha} K_{t-1}^{i(\alpha)}$$

$$\tilde{R}_{k,t} = \frac{r_t^i + (1-\delta)\xi_t^i Q_{t-1}^i}{Q_{t-1}^i}$$

As we will see when describing the banking sector, the capital is funded by selling company securities to domestic banks in a one to one relationship, i.e., $Z_t^i = K_t^i$, where $Z_t^i$ is the stock of securities from the representative firm in the country $i$. In that spirit. the marginal product of capital $r_t^i$ can also be interpreted as the return from the firm securities.[1]

## Capital goods firms

Physical capital is produced in a competitive market by using old capital and investment. The depreciation rate of capital is $1-(1-\delta)\xi_t^i$. The investment will be subject to convex adjustment costs, i.e., the total cost of investing $I_t^i$ is:

$$C(I_t^i) = I_t^i \left(1 + \frac{\zeta}{2}\left(\frac{I_t^i}{I_{t-1}^i} - 1\right)^2\right)$$

The firms will buy back the old capital stock from the banks at price $Q_t^i$ and produce new capital subject to the adjustment cost.

The problem of the capital goods firm choosing the investment level is given by:

$$\max_{\{I_t\}_{t=0}^{\infty}} E_0 \sum_{s=0}^{\infty} \Lambda_{t,t+s} \left\{ Q_{t+s}^c I_{t+s}^c - I_{t+s}^c \left(1 + \frac{\zeta}{2}\left(\frac{I_{t+s}^c}{I_{t+s-1}^c - 1}\right)^2\right)\right\}$$

From the first order condition we can derive the dynamics for the price of capital:

[1]For simplicity, when solving the model, we will replace $\tilde{R}_{k,t}$ back in the profit function so that we can drop $\tilde{R}$ as a variable work only with the effective (after tax) revenue rate perceived by banks. When we do such substitution we obtain the standard expression for the profits: $\Pi_t^{i,prod} = Y_t^i - r_t^i K_t^i + W_t^i H_t^i$.

$$Q_t^c = 1 + \frac{\zeta}{2}\left(\frac{I_t^c}{I_{t-1}^c} - 1\right)^2 + \zeta\left(\frac{I_t^c}{I_{t-1}^c} - 1\right)\frac{I_t^c}{I_{t-1}^c} - \Lambda_{t+1}^c \zeta \left(\frac{I_{t+1}^c}{I_t^c}\right)^2 \left(\frac{I_{t+1}^c}{I_t^c} - 1\right)$$

Finally, the capital stock dynamics are given by,

$$K_t^i = I_t^i + (1-\delta)\xi_t^i K_{t-1}^i$$

## Banking sector

The set-up is based on Gertler and Karadi (2011). In every economy the banks will be subject to an incentive compatibility constraint (ICC) that reflects the agency problem between creditors and debtors in their loans. To prevent self-funding strategies and guarantee a binding ICC, we allow for exit of banks at a rate $1-\theta$. At the same time, a number banks will enter each period. These banks will be given a start-up bequest by the household that depends on the scale of the banking assets in the preceding period. In exchange, when a bank exits the intermediation market, it gives their net worth as profits to the households.

The structure of the sector in each country and the decisions they face are explained in detail in the following subsections. However, it can be said that in general, the problem of the bank in $t$ consists in maximizing the financial intermediation value function $J(N_{j,t}) = E_t \max \Lambda_{t,t+1}[(1-\theta)N_{j,t+1} + \theta J(N_{j,t+1})]$ subject to the dynamics of the net worth of the bank ($N$), the balance sheet and the ICC.

The emerging market banks will also have an additional constraint that reflects their limited intermediation capacity. This eventually implies funding from the core economy to the peripheries that results in balance sheet effects at the cross country level.

### EME Banks:

The banks start with a bequest from the households and continue their activities with probability $\theta$. The index $e$ refers to either emerging market with $e = \{a, b\}$.

Let: $N_{jt}^e$ : net worth, $F_{jt}^e$ : amount borrowed from center banks at a real rate $R_{b,t}$ and

$D^e_{jt}$: deposits from domestic households. The balance sheet of the bank $j$ is given by:

$$Q^e_t Z^e_{jt} = N^e_{jt} + F^e_{jt} + D^e_{jt}$$

The deposits are borrowed at a rate $R^e_{D,t}$ (risk free rate).

To include financial dependency from the Advanced Economy, we assume that the deposits are limited to a fraction of the global bank's borrowing $(\psi_{e,D} - 1)$ with $\psi_{e,D} > 1$:

$$D^e_{jt} \leq (\psi_{e,D} - 1) F^e_{jt}$$

It should be noticed that since the deposits are priced at a risk-free rate, this inequality always binds when there is a positive credit spread (cheaper local funding).

Therefore, the balance sheet of the representative bank is:

$$Q^e_t K^e_{jt} = N^e_{jt} + \psi_{e,D} F^e_{jt}$$

Where we also accounted by the fact that $Z^e = K^e$. The aggregate net worth of the banking system is:

$$N^e_t = \underbrace{\theta N^e_{j,t}}_{\text{surviving banks}} + \underbrace{\delta_T Q^e_t K^e_t}_{\substack{\text{new banks} \\ \text{start-up K}}}$$

Where, $N^e_{j,t}$ is the net-worth of the surviving banks which displays the following dynamics:

$$N^e_{j,t} = R^e_{k,t} Q^e_{t-1} K^e_{j,t-1} - \tilde{R}^e_{b,t-1} F^e_{j,t-1}$$

where: $\tilde{R}^e_{b,t} = (R_{b,t} - R^e_{D,t}) + \psi_{e,D} R^e_{D,t}$ is the borrowing rate total debt, adjusted by the fact that a portion $(\psi_{e,D} - 1) F_{jt}$ can be loaned at the lower deposits risk free rate.

The gross return on capital will account for the payment of the macroprudential instrument:

$$R^e_{k,t} = \frac{(1 - \tau^e_t) r^e_t + (1 - \delta) \xi^e_t Q^e_t}{Q^e_{t-1}}$$

$\tau^e_t$ is a tax/subsidy.

The contracts between savers and banks will be subject to limited enrfoceability (a bank can run away with a portion $\kappa^e$ of the assets).

The problem of the $j$ banker is to maximize the value of the bank:[2]

$$J^e_{j,t}(N^e_{j,t}) = \mathbb{E}_t \max_{N^e_{j,t}, Z^e_{j,t}, V^e_{j,t}} \Lambda^e_{t+1}\left[(1-\theta)N^e_{j,t+1+s} + \theta J^e_{j,t+1}(N^e_{j,t+1})\right]$$

subject to the net worth ($N^e_{j,t}$) dynamics, the balance sheet constraint and the associated Incentive Compatibility Constraint:

$$J^e_{j,t} \geq \kappa^e \mathbb{E}_t\left[\Lambda^e_{t+1}\left(R^e_{k,t+1} Q^e_t K^e_{j,t}\right)\right]$$

This ICC condition states that the continuation value of the bank is larger than the potential profit of defaulting.[3]

The bank problem yields the following optimality conditions:

F.O.C. with respect to intermediated capital:

$$[K^e_{j,t}]: \qquad \mathbb{E}_t \Omega^e_{t+1|t}\left(R^e_{k,t+1} - \frac{1}{\psi_{e,D}}\tilde{R}^e_{b,t}\right) = \mu^e_t \kappa^e \mathbb{E}_t\left[\Lambda^e_{t+1} R^e_{k,t+1}\right]$$

and envelope condition:

$$[N^e_{j,t}]: \qquad J^{e'}(N^e_{j,t})(1-\mu^e_t) = \mathbb{E}_t \Omega^e_{t+1|t}\frac{1}{\psi_{e,D}}\tilde{R}^e_{b,t}$$

where $\mu^e_t$ is the lagrange multiplier associated with the ICC and $\Omega^e_{t+1|t} = \Lambda^e_{t+1}\Big(1-\theta+\theta J^{e'}_{t+1}\Big)$ is the effective pricing kernel of the bank. In addition, $\frac{1}{\psi_{e,D}}\tilde{R}^e_{b,t}$ represents a weighted average of the borrowing rates the bank has access and can be interpreted as the average cost of an additional unit of funds for the bank.

### Core Economy Banks:

The structure of the center economy banks is similar. We only need to be careful when setting the balance sheet and net worth dynamics. Both need to reflect the

[2]An analogous problem is given by maximizing: $J^e(N^e_{j,t}) = E_t \max_{\{N_t, Z^e_t, V^e_{j,t}\}^{\infty}_{t=0}}(1-\theta)\sum^{\infty}_{s=0}\Lambda^e_{t+1+s}[\theta^s N^e_{j,t+1+s}]$

[3]There are several feasible choices for the right hand side term depending on the timing of the assets absconding. Here we assume they compare the value of the bank to absconding right after the assets yield returns.

foreign claims intermediated and proceeds.

The balance sheet of the global country bank $j$ is:

$$F^a_{j,t} + F^b_{j,t} + Q^c_t Z^c_{j,t} = N^c_{jt} + D^c_t$$

where $D^c$ are the deposits from the households and $F^e_{j,t}$ are the claims on the $e = \{a, b\}$ representative periphery banks (EMEs) and $Q^c_t Z^c_{j,t}$ are claims on the core country capital stock with $Z^c_{j,t} = K^c_{j,t}$.

Their return on intermediated capital is:

$$R^c_{k,t} = \frac{(1-\tau^c_t) r^c_t + (1-\delta)\xi^c_t Q^c_t}{Q^c_{t-1}}$$

The bank $j$ value function is:

$$J^c_{j,t}(N^c_{j,t}) = E_t \max_{N^c_{j,t},\, Z^c_t,\, V^e_{j,t},\, D^c_t} \Lambda^c_{t+1}\Big[(1-\theta)(\underbrace{R^c_{k,t+1} Q^c_t Z^c_{j,t} + R^a_{b,t} V^a_{j,t} + R^b_{b,t} V^b_{j,t}}_{\text{gross return on assets}} - \underbrace{R^c_{D,t} D^c_t}_{\substack{\text{deposits}\\ \text{repayment}}}) + \theta J^c_{j,t+1}(N^c_{j,t+1})\Big]$$

The bank determines such value while being subject to the balance sheet constraint and to an incentive compatibility constraint given by:

$$J^c_{j,t} \geq \mathbb{E}_t \left[\Lambda^c_{t+1} \left(\kappa^c_{F_1} R^a_{b,t} F^a_{jt} + \kappa^c_{F_2} R^b_{b,t} F^b_{jt} + \kappa^c R^c_{k,t+1} Q^c_t Z^c_{j,t}\right)\right] \tag{ICC-C}$$

with $\kappa^c_{V_i}, \kappa^c > 0$, i.e., the pledgeable fraction can be asymmetric across assets.

The optimality Conditions are:

$$\begin{aligned}
[Z_{j,t}]: &\quad \mathbb{E}_t \Omega^c_{t+1|t}(R^c_{k,t+1} - R^c_{D,t}) = \kappa^c \mu^c_t \mathbb{E}_t R^c_{k,t+1} \\
[V^a_{j,t}]: &\quad \mathbb{E}_t \Omega^c_{t+1|t}\left(R^a_{b,t} - R^c_{D,t}\right) = \kappa^c_{V_1} \mu^c_t \mathbb{E}_t R^a_{b,t} \\
[V^b_{j,t}]: &\quad \mathbb{E}_t \Omega^c_{t+1|t}\left(R^b_{b,t} - R^c_{D,t}\right) = \kappa^c_{V_2} \mu^c_t \mathbb{E}_t R^b_{b,t}
\end{aligned}$$

and the envelope condition,

$$[N^c_{j,t}]: \qquad J^{c'}(N^c_{j,t})(1-\mu^c_t) - \mathbb{E}_t\Omega^c_{t+1|t}R^c_{D,t} = 0$$

### 3.1. Macroprudential Policy

The macroprudential tool considered is a tax on the return to capital.

Advantage: affects directly wedge between return on capital and deposit rate (credit spread). Therefore, policy actions can be applied directly to the source of inefficiencies.

$$\tau^i_t r^i_t K^i_{t-1} + T^i_t = 0 \qquad i = \{a, b, c\}$$

The welfare objective of each policy maker is given by: $W^i_0$ as in (1).

In addition, each social planner could consider whether to coordinate or not with the planners of other economies. Clearly, its choice depends on which arrangement implies larger welfare gains for its economy.

## 4. Results

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## 5. Conclusions

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## References

Agénor, P.-R., Kharroubi, E., Gambacorta, L., Lombardo, G., and da Silva, L. A. P. (2017). The international dimensions of macroprudential policies. BIS Working Papers 643, Bank for International Settlements.

Banerjee, R., Devereux, M. B., and Lombardo, G. (2016). Self-oriented monetary policy, global financial markets and excess volatility of international capital flows. *Journal of International Money and Finance*, 68(C):275–297.

Bodenstein, M., Guerrieri, L., and LaBriola, J. (2019). Macroeconomic policy games. *Journal of Monetary Economics*, 101:64 – 81.

Gertler, M. and Karadi, P. (2011). A model of unconventional monetary policy. *Journal of Monetary Economics*, 58(1):17–34.

Schmitt-Grohe, S. and Uribe, M. (2003). Closing small open economy models. *Journal of International Economics*, 61(1):163–185.

# A. Model Equations and Variables

In this section I list the variables and equations used to solve the infinite horizon model. The equations numbered are the ones actually used. Unlabeled equations are auxiliary or redundant.

In all cases: $j = \{a, b, c\}$ where $c$: center, $a$: emerging economy A, $b$: emerging economy B.

**Variables:**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| `Cc` | `Rc` | `Ac` | `Ic` | `Qc` | `Va` | `Lambdac` | `tauc` |
| `Ca` | `Ra` | `Aa` | `Ia` | `Qa` | `Vb` | `Lambdaa` | `taua` |
| `Cb` | `Rb` | `Ab` | `Ib` | `Qb` | | `Lambdab` | `taub` |
| `Hc` | `Rdc` | `xic` | `rc` | `Rkc` | `Dc` | `jc` | `Tc` |
| `Ha` | `Rda` | `xia` | `ra` | `Rka` | `Da` | `ja` | `Ta` |
| `Hb` | `Rdb` | `xib` | `rb` | `Rkb` | `Db` | `jb` | `Tb` |
| `Bc` | `Yc` | `Kc` | `Wc` | `Nc` | `Rba` | `gammac` | |
| `Ba` | `Ya` | `Ka` | `Wa` | `Na` | `Rbb` | `gammaa` | |
| `Bb` | `Yb` | `Kb` | `Wb` | `Nb` | | `gammab` | |

Count: 67 (64 without macroprudential tax rate)

**Advanced Economy**

Household:

$$R_t^c \Lambda_{t+1}^c = 1 + \eta(B_t^c) \tag{1}$$

$$R_{D,t}^c \Lambda_{t+1}^c = 1 + \eta(D_t^c - \bar{D}^c) \tag{2}$$

$$C_t^{c\ -\sigma} = \frac{H_t^{c\ \psi}}{W_t^c} \tag{3}$$

$$C_t^c + B_t^c + \frac{\eta}{2}(B_t^c)^2 + D_t^c + \frac{\eta}{2}(D_t^c - \bar{D}^c)^2 = R_{t-1} B_{t-1}^c + R_{D,t-1}^c D_{t-1}^c + W_t^c H_t^c + \Pi_t^c$$

Firms:

Final Goods:

$$r_t^c = \alpha A_t^c \xi_t^{c\ \alpha} K_{t-1}^{c\ (\alpha-1)} H_t^{c\ (1-\alpha)} \tag{4}$$
$$W_t^c = (1-\alpha) A_t^c \xi_t^{c\ \alpha} K_{t-1}^{c\ (\alpha)} H_t^{c\ (-\alpha)} \tag{5}$$

Capital Goods:

$$Q_t^c = 1 + \frac{\zeta}{2}\left(\frac{I_t^c}{I_{t-1}^c} - 1\right)^2 + \zeta\left(\frac{I_t^c}{I_{t-1}^c} - 1\right)\frac{I_t^c}{I_{t-1}^c} - \Lambda_{t+1}^c \zeta \left(\frac{I_{t+1}^c}{I_t^c}\right)^2 \left(\frac{I_{t+1}^c}{I_t^c} - 1\right) \tag{6}$$
$$K_t^c = I_t^c + (1-\delta)\xi_t^c K_{t-1}^c \tag{7}$$

Government:

$$\tau_t^c r_t^c K_{t-1}^c + T_t^c = 0 \tag{8}$$

Banks:

when solving the model we can omit the individual bank subscript (we only need to be careful when distinguishing between aggregate networth and representative bank's net worth because of the bequests).

$$R_{k,t}^c = \frac{(1-\tau_t^c) r_t^c + (1-\delta)\xi_t^c Q_t^c}{Q_{t-1}^c} \tag{9}$$

$$\mathbb{E}_t \Omega_{t+1|t}^c \left(R_{k,t+1}^c - R_{D,t}^c\right) = \mu_t^c \kappa^c \tag{10}$$
$$\mathbb{E}_t \Omega_{t+1|t}^c \left(R_{b,t}^a - R_{D,t}^c\right) = \mu_t^c \kappa_{F_a}^c \tag{11}$$
$$\mathbb{E}_t \Omega_{t+1|t}^c \left(R_{b,t}^b - R_{D,t}^c\right) = \mu_t^c \kappa_{F_b}^c \tag{12}$$
$$j_t^c (1-\mu_t^c) = \mathbb{E}_t \Omega_{t+1|t}^c R_{D,t}^c \tag{13}$$

$$j_t^c N_t^c = \kappa^c Q_t^c K_t^c + \kappa_{F_a}^c F_t^a + \kappa_{F_b}^c F_t^b \tag{14}$$
$$N_t^c = \theta\left[R_{k,t}^c Q_{t-1}^c K_{t-1}^c + R_{b,t-1}^a F_{t-1}^a + R_{b,t-1}^b F_{t-1}^b - R_{D,t-1}^c D_{t-1}^c\right] + \delta^T Q_t^c K_{t-1}^c \tag{15}$$
$$Q_t^c K_t^c + F_t^a + F_t^b = N_t^c + D_t^c \tag{16}$$

The ICC makes use of the auxiliar variable defining J as $J_{j,t}^a = j_t^a N_t^a$.

**Emerging Market 1**

Household:

$$R_t^a \Lambda_{t+1}^a = 1 + \eta(B_t^a) \tag{17}$$

$$R_{D,t}^a \Lambda_{t+1}^a = 1 + \eta(D_t^a - \bar{D}^a) \tag{18}$$

$$C_t^{a\ -\sigma} = \frac{H_t^{a\ \psi}}{W_t^a} \tag{19}$$

$$C_t^a + B_t^a + \frac{\eta}{2}(B_t^a)^2 + D_t^a + \frac{\eta}{2}(D_t^a - \bar{D}^a)^2 = R_{t-1}B_{t-1}^a + R_{D,t-1}^a D_{t-1}^a + W_t^a H_t^a + \Pi_t^a \tag{20}$$

Firms:

Final Goods:

$$r_t^a = \alpha A_t^a H_t^{a\ (1-\alpha)} \xi_t^{a\ \alpha} K_{t-1}^{a\ (\alpha-1)} \tag{21}$$

$$W_t^a = (1-\alpha) A_t^a \xi_t^{a\ \alpha} K_{t-1}^{a\ (\alpha)} H_t^{i\ (-\alpha)} \tag{22}$$

Capital Goods:

$$Q_t^a = 1 + \frac{\zeta}{2}\left(\frac{I_t^a}{I_{t-1}^a} - 1\right)^2 + \zeta\left(\frac{I_t^a}{I_{t-1}^a} - 1\right)\frac{I_t^a}{I_{t-1}^a} - \Lambda_{t+1}^a \zeta \left(\frac{I_{t+1}^a}{I_t^a}\right)^2 \left(\frac{I_{t+1}^a}{I_t^a} - 1\right) \tag{23}$$

$$K_t^a = I_t^a + (1-\delta)\xi_t^a K_{t-1}^a \tag{24}$$

Government:

$$\tau_{k,t}^a r_t^a K_{t-1}^a + T_t^a = 0 \tag{25}$$

Banks:

$$R_{k,t}^a = \frac{(1-\tau_t^a) r_t^a + (1-\delta)\xi_t^a Q_t^a}{Q_{t-1}^a} \tag{26}$$

$$\mathbb{E}_t \Omega_{t+1|t}^a \left(R_{k,t+1}^a - \frac{1}{\psi_{a,D}} \tilde{R}_{b,t}^a\right) = \mu_t^a \kappa^a \tag{27}$$

$$j_t^a(1-\mu_t^a) = \mathbb{E}_t\left[\Omega^a_{t+1|t}\frac{1}{\psi_{a,D}}\tilde{R}^a_{b,t}\right] \tag{28}$$

$$j_t^a N_t^a = \kappa^a Q_t^a K_t^a \tag{29}$$

$$N_t^a = \theta\left[R^a_{k,t}Q^a_{t-1}K^a_{t-1} - \tilde{R}^a_{b,t-1}F^a_{t-1}\right] + \delta^T Q_t^a K^a_{t-1} \tag{30}$$

$$Q_t^a K_t^a = N_t^a + \psi_{a,D}F_t^a \tag{31}$$

$$D_t^a = (\psi_{a,D}-1)F_t^a \tag{32}$$

The ICC makes use of the auxiliar variable defining J as $J^a_{j,t} = j^a_t N^a_t$.

**Emerging Market 2**

Household:

$$R_t^b\Lambda^b_{t+1} = 1+\eta(B_t^b) \tag{33}$$

$$R^b_{D,t}\Lambda^b_{t+1} = 1+\eta(D_t^b - \bar{D}^b) \tag{34}$$

$$C_t^{b\ -\sigma} = \frac{H_t^{b\ \psi}}{W_t^b} \tag{35}$$

$$C_t^b + B_t^b + \frac{\eta}{2}(B_t^b)^2 + D_t^b + \frac{\eta}{2}(D_t^b-\bar{D}^b)^2 = R_{t-1}B^b_{t-1} + R^b_{D,t-1}D^b_{t-1} + W_t^b H_t^b + \Pi_t^b \tag{36}$$

Firms:

Final Goods:

$$r_t^b = \alpha A_t^b H_t^{b\ (1-\alpha)}\xi_t^{b\ \alpha}K_{t-1}^{b\ (\alpha-1)} \tag{37}$$

$$W_t^b = (1-\alpha)A_t^b\xi_t^{b\ \alpha}K_{t-1}^{b\ (\alpha)}H_t^{b\ (-\alpha)} \tag{38}$$

Capital Goods:

$$Q_t^b = 1+\frac{\zeta}{2}\left(\frac{I_t^b}{I_{t-1}^b}-1\right)^2 + \zeta\left(\frac{I_t^b}{I_{t-1}^b}-1\right)\frac{I_t^b}{I_{t-1}^b} - \Lambda^b_{t+1}\zeta\left(\frac{I^b_{t+1}}{I_t^b}\right)^2\left(\frac{I^b_{t+1}}{I^b_t}-1\right) \tag{39}$$

$$K_t^b = I_t^b + (1-\delta)\xi_t^b K_{t-1}^b \tag{40}$$

Government:

$$\tau_{k,t}^b r_t^b K_{t-1}^b + T_t^b = 0 \tag{41}$$

Banks:

$$R_{k,t}^b = \frac{(1-\tau_t^b)r_t^b + (1-\delta)\xi_t^b Q_t^b}{Q_{t-1}^b} \tag{42}$$

$$\mathbb{E}_t \Omega_{t+1|t}^b \left( R_{k,t+1}^b - \frac{1}{\psi_{b,D}} \tilde{R}_{b,t}^b \right) = \mu_t^b \kappa^b \tag{43}$$

$$j_t^b (1-\mu_t^b) = \mathbb{E}_t \left[ \Omega_{t+1|t}^b \frac{1}{\psi_{b,D}} \tilde{R}_{b,t}^b \right] \tag{44}$$

$$j_t^b N_t^b = \kappa^b Q_t^b K_t^b \tag{45}$$

$$N_t^b = \theta \left[ R_{k,t}^b Q_{t-1}^b K_{t-1}^b - \tilde{R}_{b,t-1}^b F_{t-1}^b \right] + \delta^T Q_t^b K_{t-1}^b \tag{46}$$

$$Q_t^b K_t^b = N_t^b + \psi_{b,D} F_t^b \tag{47}$$

$$D_t^b = (\psi_{b,D} - 1) F_t^b \tag{48}$$

The ICC makes use of the auxiliar variable defining J as $J_{j,t}^b = j_t^b N_t^b$.

Market Clearing Conditions:

At the individual country level:

$$Y_t^i = A_t^i (\xi_t^i K_{t-1}^i)^\alpha H_t^{i\ 1-\alpha} \tag{49-51}$$

At the world level:

$$n_c B_t^c + n_a B_t^a + n_b B_t^b = 0 \tag{52}$$

$$n_c Y_t^c + n_a Y_t^a + n_b Y_t^b = n_c \left[ C_t^c + I_t^c \left( 1 + \frac{\zeta}{2} \left( \frac{I_t^c}{I_{t-1}^c} - 1 \right)^2 \right) + \frac{\eta}{2} (B_t^c)^2 + \frac{\eta}{2} (D_t^c - \bar{D}^c)^2 \right] \tag{53}$$

$$
\begin{aligned}
&+ n_a \left[ C_t^a + I_t^a \left( 1 + \frac{\zeta}{2} \left( \frac{I_t^a}{I_{t-1}^a} - 1 \right)^2 \right) + \frac{\eta}{2} (B_t^a)^2 + \frac{\eta}{2} (D_t^a - \bar{D}^a)^2 \right] \\
&+ n_b \left[ C_t^b + I_t^b \left( 1 + \frac{\zeta}{2} \left( \frac{I_t^b}{I_{t-1}^b} - 1 \right)^2 \right) + \frac{\eta}{2} (B_t^b)^2 + \frac{\eta}{2} (D_t^b - \bar{D}^b)^2 \right]
\end{aligned}
$$

To close the model:

$$A_t^i = \rho_A A_{t-1}^i + \sigma_A \epsilon_{A,t}^i \tag{54-56}$$

$$\xi_t^i = \rho_\xi \xi_{t-1}^i + \sigma_\xi \epsilon_{k,t}^i \tag{57-59}$$

Risk Sharing Conditions:

$$
\begin{aligned}
\mathbb{E}_t \left( \frac{C_{t+1}^a}{C_t^a} \right)^{-\sigma} &= \frac{1 + \eta (B_t^a)}{1 + \eta (B_t^b)} \mathbb{E}_t \left( \frac{C_{t+1}^b}{C_t^b} \right)^{-\sigma} \\
\mathbb{E}_t \left( \frac{C_{t+1}^a}{C_t^a} \right)^{-\sigma} &= \frac{1 + \eta (B_t^a)}{1 + \eta (B_t^c)} \mathbb{E}_t \left( \frac{C_{t+1}^c}{C_t^c} \right)^{-\sigma}
\end{aligned}
$$

Uncovered interest partity for bonds rates (Real Exchange Rate = 1 due to LOP and PPP).

$$R_t^c = R_t^a \tag{60}$$

$$R_t^a = R_t^b \tag{61}$$

**Auxiliary variables**: $i$-labeled equations count thrice.

$$
\begin{aligned}
\Lambda_{t+1}^i &= \beta \left( \frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} && (62\text{-}64) \\
\Omega_{t+1|t}^i &= \Lambda_{t+1}^i \Big( 1 - \theta + \theta j_{t+1}^i \Big) \\
\tilde{R}_{b,t}^e &= (R_{b,t}^e - R_t^e) + \psi_{e,D} R_t^e \qquad \text{for } e = \{a, b\} \\
J_t^i &= j_t^i N_t^i = J_t^{i'} N_t \\
\Pi_t^c &= (1 - \theta) \left[ Q_{t-1}^c R_{k,t}^c K_{t-1}^c + R_{b,t-1}^a F_{t-1}^a + R_{b,t-1}^b F_{t-1}^b - R_{t-1}^c D_{t-1}^c \right]
\end{aligned}
$$

$$- \delta^T Q_t^c K_{t-1}^c + Q_t^c I_t^c - I_t^c \left(1 + \frac{\zeta}{2}\left(\frac{I_t^c}{I_{t-1}^c} - 1\right)^2\right) - T_t^c$$

$$\Pi_t^a = (1-\theta)\left[Q_{t-1}^a R_{k,t}^a K_{t-1}^a - \tilde{R}_{b,t-1}^a F_{t-1}^a\right] - \delta^T Q_t^a K_{t-1}^a + Q_t^a I_t^a - I_t^a \left(1 + \frac{\zeta}{2}\left(\frac{I_t^a}{I_{t-1}^a} - 1\right)^2\right) - T_t^a$$

$$\Pi_t^b = (1-\theta)\left[Q_{t-1}^b R_{k,t}^b K_{t-1}^b - \tilde{R}_{b,t-1}^b F_{t-1}^b\right] - \delta^T Q_t^b K_{t-1}^b + Q_t^b I_t^b - I_t^b \left(1 + \frac{\zeta}{2}\left(\frac{I_t^b}{I_{t-1}^b} - 1\right)^2\right) - T_t^b$$

Notes: for the private agents equilibrium system we should have 64 equations only since the taxes are taken as exogenous. If we don't solve for the Ramsey problem solution then we consider a simple policy rule to set the taxes:

$$\tau_t^i = (1-\phi_\tau)\bar{\tau}^i + \phi_\tau \tau_{t-1}^i + \phi_z (K_t^i - K_{t-1}^i) \tag{65-67}$$

This rule is based on Bodenstein et al. (2019). In our benchmark case we consider $\phi_\tau = 0.7$ and $\phi_z = 1$. $\bar{\tau}^i$ is the steady state value, set in line with Agénor et al. (2017) and the model solution for each single economy case.

# B. Simplification of the model

Here I simplify the model and lean towards the minimum state variables model in order to facilitate the estimation of the Ramsey model

Original System of Equations: (numbering follows the code structure)

$$R^c_t \Lambda^c_{t+1} = 1 + \eta(B^c_t) \tag{1}$$
$$R^a_t \Lambda^a_{t+1} = 1 + \eta(B^a_t) \tag{2}$$
$$R^b_t \Lambda^b_{t+1} = 1 + \eta(B^b_t) \tag{3}$$
$$R^c_{D,t} \Lambda^c_{t+1} = 1 + \eta(D^c_t - \bar{D}^c) \tag{4}$$
$$R^a_{D,t} \Lambda^a_{t+1} = 1 + \eta(D^a_t - \bar{D}^a) \tag{5}$$
$$R^b_{D,t} \Lambda^b_{t+1} = 1 + \eta(D^b_t - \bar{D}^b) \tag{6}$$
$$C^{c\ -\sigma}_t = \frac{H^{c\ \psi}_t}{W^c_t} \tag{7}$$
$$C^{a\ -\sigma}_t = \frac{H^{a\ \psi}_t}{W^a_t} \tag{8}$$
$$C^{b\ -\sigma}_t = \frac{H^{b\ \psi}_t}{W^b_t} \tag{9}$$
$$C^a_t + B^a_t + \frac{\eta}{2}(B^a_t)^2 + D^a_t + \frac{\eta}{2}(D^a_t - \bar{D}^a)^2 = R_{t-1}B^a_{t-1} + R^a_{D,t-1}D^a_{t-1} + W^a_t H^a_t + \Pi^a_t \tag{10}$$
$$C^b_t + B^b_t + \frac{\eta}{2}(B^b_t)^2 + D^b_t + \frac{\eta}{2}(D^b_t - \bar{D}^b)^2 = R_{t-1}B^b_{t-1} + R^b_{D,t-1}D^b_{t-1} + W^b_t H^b_t + \Pi^b_t \tag{11}$$
$$r^c_t = \alpha A^c_t \xi^{c\ \alpha}_t K^{c\ (\alpha-1)}_{t-1} H^{c\ (1-\alpha)}_t \tag{12}$$
$$r^a_t = \alpha A^a_t H^{a\ (1-\alpha)}_t \xi^{a\ \alpha}_t K^{a\ (\alpha-1)}_{t-1} \tag{13}$$
$$r^b_t = \alpha A^b_t H^{b\ (1-\alpha)}_t \xi^{b\ \alpha}_t K^{b\ (\alpha-1)}_{t-1} \tag{14}$$
$$W^c_t = (1-\alpha) A^c_t \xi^{c\ \alpha}_t K^{c\ (\alpha)}_{t-1} H^{c\ (-\alpha)}_t \tag{15}$$
$$W^a_t = (1-\alpha) A^a_t \xi^{a\ \alpha}_t K^{a\ (\alpha)}_{t-1} H^{i\ (-\alpha)}_t \tag{16}$$
$$W^b_t = (1-\alpha) A^b_t \xi^{b\ \alpha}_t K^{b\ (\alpha)}_{t-1} H^{b\ (-\alpha)}_t \tag{17}$$
$$Q^c_t = 1 + \frac{\zeta}{2}\left(\frac{I^c_t}{I^c_{t-1}} - 1\right)^2 + \zeta\left(\frac{I^c_t}{I^c_{t-1}} - 1\right)\frac{I^c_t}{I^c_{t-1}} - \Lambda^c_{t+1}\zeta\left(\frac{I^c_{t+1}}{I^c_t}\right)^2\left(\frac{I^c_{t+1}}{I^c_t} - 1\right) \tag{18}$$
$$Q^a_t = 1 + \frac{\zeta}{2}\left(\frac{I^a_t}{I^a_{t-1}} - 1\right)^2 + \zeta\left(\frac{I^a_t}{I^a_{t-1}} - 1\right)\frac{I^a_t}{I^a_{t-1}} - \Lambda^a_{t+1}\zeta\left(\frac{I^a_{t+1}}{I^a_t}\right)^2\left(\frac{I^a_{t+1}}{I^a_t} - 1\right) \tag{19}$$
$$Q^b_t = 1 + \frac{\zeta}{2}\left(\frac{I^b_t}{I^b_{t-1}} - 1\right)^2 + \zeta\left(\frac{I^b_t}{I^b_{t-1}} - 1\right)\frac{I^b_t}{I^b_{t-1}} - \Lambda^b_{t+1}\zeta\left(\frac{I^b_{t+1}}{I^b_t}\right)^2\left(\frac{I^b_{t+1}}{I^b_t} - 1\right) \tag{20}$$
$$K^c_t = I^c_t + (1-\delta)\xi^c_t K^c_{t-1} \tag{21}$$
$$K^a_t = I^a_t + (1-\delta)\xi^a_t K^a_{t-1} \tag{22}$$
$$K^b_t = I^b_t + (1-\delta)\xi^b_t K^b_{t-1} \tag{23}$$
$$\tau^c_t r^c_t K^c_{t-1} + T^c_t = 0 \tag{24}$$
$$\tau^a_{k,t} r^a_t K^a_{t-1} + T^a_t = 0 \tag{25}$$

$$\tau^b_{k,t} r^b_t K^b_{t-1} + T^b_t = 0 \tag{26}$$

$$R^c_{k,t} = \frac{(1-\tau^c_t)r^c_t + (1-\delta)\xi^c_t Q^c_t}{Q^c_{t-1}} \tag{27}$$

$$R^a_{k,t} = \frac{(1-\tau^a_t)r^a_t + (1-\delta)\xi^a_t Q^a_t}{Q^a_{t-1}} \tag{28}$$

$$R^b_{k,t} = \frac{(1-\tau^b_t)r^b_t + (1-\delta)\xi^b_t Q^b_t}{Q^b_{t-1}} \tag{29}$$

$$\mathbb{E}_t \Omega^c_{t+1|t} \left( R^c_{k,t+1} - R^c_{D,t} \right) = \mu^c_t \kappa^c \tag{30}$$

$$\mathbb{E}_t \Omega^a_{t+1|t} \left( R^a_{k,t+1} - \frac{1}{\psi_{a,D}} \tilde{R}^a_{b,t} \right) = \mu^a_t \kappa^a \tag{31}$$

$$\mathbb{E}_t \Omega^b_{t+1|t} \left( R^b_{k,t+1} - \frac{1}{\psi_{b,D}} \tilde{R}^b_{b,t} \right) = \mu^b_t \kappa^b \tag{32}$$

$$\mathbb{E}_t \Omega^c_{t+1|t} \left( R^a_{b,t} - R^c_{D,t} \right) = \mu^c_t \kappa^c_{F_a} \tag{33}$$

$$\mathbb{E}_t \Omega^c_{t+1|t} \left( R^b_{b,t} - R^c_{D,t} \right) = \mu^c_t \kappa^c_{F_b} \tag{34}$$

$$j^c_t(1-\mu^c_t) = \mathbb{E}_t \Omega^c_{t+1|t} R^c_{D,t} \tag{35}$$

$$j^a_t(1-\mu^a_t) = \mathbb{E}_t \left[ \Omega^a_{t+1|t} \frac{1}{\psi_{a,D}} \tilde{R}^a_{b,t} \right] \tag{36}$$

$$j^b_t(1-\mu^b_t) = \mathbb{E}_t \left[ \Omega^b_{t+1|t} \frac{1}{\psi_{b,D}} \tilde{R}^b_{b,t} \right] \tag{37}$$

$$N^c_t = \theta \left[ R^c_{k,t} Q^c_{t-1} K^c_{t-1} + R^a_{b,t-1} F^a_{t-1} + R^b_{b,t-1} F^b_{t-1} - R^c_{D,t-1} D^c_{t-1} \right] + \delta^T Q^c_t K^c_{t-1} \tag{38}$$

$$N^a_t = \theta \left[ R^a_{k,t} Q^a_{t-1} K^a_{t-1} - \tilde{R}^a_{b,t-1} F^a_{t-1} \right] + \delta^T Q^a_t K^a_{t-1} \tag{39}$$

$$N^b_t = \theta \left[ R^b_{k,t} Q^b_{t-1} K^b_{t-1} - \tilde{R}^b_{b,t-1} F^b_{t-1} \right] + \delta^T Q^b_t K^b_{t-1} \tag{40}$$

$$j^c_t N^c_t = \kappa^c Q^c_t K^c_t + \kappa^c_{F_a} F^a_t + \kappa^c_{F_b} F^b_t \tag{41}$$

$$j^a_t N^a_t = \kappa^a Q^a_t K^a_t \tag{42}$$

$$j^b_t N^b_t = \kappa^b Q^b_t K^b_t \tag{43}$$

$$Q^c_t K^c_t + F^a_t + F^b_t = N^c_t + D^c_t \tag{44}$$

$$Q^a_t K^a_t = N^a_t + \psi_{a,D} F^a_t \tag{45}$$

$$Q^b_t K^b_t = N^b_t + \psi_{b,D} F^b_t \tag{46}$$

$$D^a_t = (\psi_{a,D} - 1) F^a_t \tag{47}$$

$$D^b_t = (\psi_{b,D} - 1) F^b_t \tag{48}$$

$$Y^c_t = A^c_t (\xi^c_t K^c_{t-1})^\alpha H^{c\ 1-\alpha}_t \tag{49}$$

$$Y^a_t = A^a_t (\xi^a_t K^a_{t-1})^\alpha H^{a\ 1-\alpha}_t \tag{50}$$

$$Y^b_t = A^b_t (\xi^b_t K^b_{t-1})^\alpha H^{b\ 1-\alpha}_t \tag{51}$$

$$n_c B^c_t + n_a B^a_t + n_b B^b_t = 0 \tag{52}$$

$$R^c_t = R^a_t \tag{53}$$

$$R^a_t = R^b_t \tag{54}$$

$$n_c Y^c_t + n_a Y^a_t + n_b Y^b_t = n_c \left[ C^c_t + I^c_t \left( 1 + \frac{\zeta}{2} \left( \frac{I^c_t}{I^c_{t-1}} - 1 \right)^2 \right) + \frac{\eta}{2} (B^c_t)^2 + \frac{\eta}{2} (D^c_t - \bar{D}^c)^2 \right]$$

$$+n_a\left[C_t^a + I_t^a\left(1+\frac{\zeta}{2}\left(\frac{I_t^a}{I_{t-1}^a}-1\right)^2\right)+\frac{\eta}{2}(B_t^a)^2+\frac{\eta}{2}(D_t^a-\bar{D}^a)^2\right]$$
$$+n_b\left[C_t^b + I_t^b\left(1+\frac{\zeta}{2}\left(\frac{I_t^b}{I_{t-1}^b}-1\right)^2\right)+\frac{\eta}{2}(B_t^b)^2+\frac{\eta}{2}(D_t^b-\bar{D}^b)^2\right] \tag{55}$$
$$A_t^c = \rho_A A_{t-1}^c + \sigma_A \epsilon_{A,t}^c \tag{56}$$
$$A_t^a = \rho_A A_{t-1}^a + \sigma_A \epsilon_{A,t}^a \tag{57}$$
$$A_t^b = \rho_A A_{t-1}^b + \sigma_A \epsilon_{A,t}^b \tag{58}$$
$$\xi_t^c \rho_\xi \xi_{t-1}^c + \sigma_\xi \epsilon_{k,t}^c \tag{59}$$
$$\xi_t^a \rho_\xi \xi_{t-1}^a + \sigma_\xi \epsilon_{k,t}^a \tag{60}$$
$$\xi_t^b \rho_\xi \xi_{t-1}^b + \sigma_\xi \epsilon_{k,t}^b \tag{61}$$
$$\Lambda_{t+1}^c = \beta\left(\frac{C_{t+1}^c}{C_t^c}\right)^{-\sigma} \tag{62}$$
$$\Lambda_{t+1}^a = \beta\left(\frac{C_{t+1}^a}{C_t^a}\right)^{-\sigma} \tag{63}$$
$$\Lambda_{t+1}^b = \beta\left(\frac{C_{t+1}^a}{C_t^b}\right)^{-\sigma} \tag{64}$$
$$\tau_t^c = (1-\phi_\tau)\bar{\tau}^c + \phi_\tau \tau_{t-1}^c + \phi_z(K_t^c - K_{t-1}^c) \tag{65}$$
$$\tau_t^a = (1-\phi_\tau)\bar{\tau}^a + \phi_\tau \tau_{t-1}^a + \phi_z(K_t^a - K_{t-1}^a) \tag{66}$$
$$\tau_t^b = (1-\phi_\tau)\bar{\tau}^b + \phi_\tau \tau_{t-1}^b + \phi_z(K_t^b - K_{t-1}^b) \tag{67}$$

Simplifications: Subs: $r_t$, $W_t$, $T$; $R = R^a = R^b = R^c$

$$R_t \Lambda_{t+1}^c = 1 + \eta(B_t^c) \tag{1}$$
$$R_t \Lambda_{t+1}^a = 1 + \eta(B_t^a) \tag{2}$$
$$R_t \Lambda_{t+1}^b = 1 + \eta(B_t^b) \tag{3}$$
$$R_{D,t}^c \Lambda_{t+1}^c = 1 + \eta(D_t^c - \bar{D}^c) \tag{4}$$
$$R_{D,t}^a \Lambda_{t+1}^a = 1 + \eta(D_t^a - \bar{D}^a) \tag{5}$$
$$R_{D,t}^b \Lambda_{t+1}^b = 1 + \eta(D_t^b - \bar{D}^b) \tag{6}$$
$$C_t^{c\ -\sigma} = \frac{H_t^{c\ \psi}}{(1-\alpha)A_t^c \xi_t^{c\ \alpha} K_{t-1}^{c\ (\alpha)} H_t^{c\ (-\alpha)}} \tag{7}$$
$$C_t^{a\ -\sigma} = \frac{H_t^{a\ \psi}}{(1-\alpha)A_t^a \xi_t^{a\ \alpha} K_{t-1}^{a\ (\alpha)} H_t^{i\ (-\alpha)}} \tag{8}$$
$$C_t^{b\ -\sigma} = \frac{H_t^{b\ \psi}}{(1-\alpha)A_t^b \xi_t^{b\ \alpha} K_{t-1}^{b\ (\alpha)} H_t^{b\ (-\alpha)}} \tag{9}$$
$$C_t^a + B_t^a + \frac{\eta}{2}(B_t^a)^2 + D_t^a + \frac{\eta}{2}(D_t^a - \bar{D}^a)^2 = R_{t-1}B_{t-1}^a + R_{D,t-1}^a D_{t-1}^a + (1-\alpha)A_t^a \xi_t^{a\ \alpha} K_{t-1}^{a\ (\alpha)} H_t^{i\ (1-\alpha)} + \Pi_t^a \tag{10}$$
$$C_t^b + B_t^b + \frac{\eta}{2}(B_t^b)^2 + D_t^b + \frac{\eta}{2}(D_t^b - \bar{D}^b)^2 = R_{t-1}B_{t-1}^b + R_{D,t-1}^b D_{t-1}^b + (1-\alpha)A_t^b \xi_t^{b\ \alpha} K_{t-1}^{b\ (\alpha)} H_t^{b\ (1-\alpha)} + \Pi_t^b \tag{11}$$

$$Q_t^c = 1 + \frac{\zeta}{2}\left(\frac{I_t^c}{I_{t-1}^c} - 1\right)^2 + \zeta\left(\frac{I_t^c}{I_{t-1}^c} - 1\right)\frac{I_t^c}{I_{t-1}^c} - \Lambda_{t+1}^c \zeta\left(\frac{I_{t+1}^c}{I_t^c}\right)^2\left(\frac{I_{t+1}^c}{I_t^c} - 1\right) \tag{12}$$

$$Q_t^a = 1 + \frac{\zeta}{2}\left(\frac{I_t^a}{I_{t-1}^a} - 1\right)^2 + \zeta\left(\frac{I_t^a}{I_{t-1}^a} - 1\right)\frac{I_t^a}{I_{t-1}^a} - \Lambda_{t+1}^a \zeta\left(\frac{I_{t+1}^a}{I_t^a}\right)^2\left(\frac{I_{t+1}^a}{I_t^a} - 1\right) \tag{13}$$

$$Q_t^b = 1 + \frac{\zeta}{2}\left(\frac{I_t^b}{I_{t-1}^b} - 1\right)^2 + \zeta\left(\frac{I_t^b}{I_{t-1}^b} - 1\right)\frac{I_t^b}{I_{t-1}^b} - \Lambda_{t+1}^b \zeta\left(\frac{I_{t+1}^b}{I_t^b}\right)^2\left(\frac{I_{t+1}^b}{I_t^b} - 1\right) \tag{14}$$

$$K_t^c = I_t^c + (1-\delta)\xi_t^c K_{t-1}^c \tag{15}$$

$$K_t^a = I_t^a + (1-\delta)\xi_t^a K_{t-1}^a \tag{16}$$

$$K_t^b = I_t^b + (1-\delta)\xi_t^b K_{t-1}^b \tag{17}$$

$$R_{k,t}^c = \frac{(1-\tau_t^c)\alpha A_t^c H_t^{c\ (1-\alpha)} \xi_t^{c\ \alpha} K_{t-1}^{c\ (\alpha-1)} + (1-\delta)\xi_t^c Q_t^c}{Q_{t-1}^c} \tag{18}$$

$$R_{k,t}^a = \frac{(1-\tau_t^a)\alpha A_t^a H_t^{a\ (1-\alpha)} \xi_t^{a\ \alpha} K_{t-1}^{a\ (\alpha-1)} + (1-\delta)\xi_t^a Q_t^a}{Q_{t-1}^a} \tag{19}$$

$$R_{k,t}^b = \frac{(1-\tau_t^b)\alpha A_t^b H_t^{b\ (1-\alpha)} \xi_t^{b\ \alpha} K_{t-1}^{b\ (\alpha-1)} + (1-\delta)\xi_t^b Q_t^b}{Q_{t-1}^b} \tag{20}$$

$$\mathbb{E}_t \Omega_{t+1|t}^c \left(R_{k,t+1}^c - R_{D,t}^c\right) = \mu_t^c \kappa^c \tag{21}$$

$$\mathbb{E}_t \Omega_{t+1|t}^a \left(R_{k,t+1}^a - \frac{1}{\psi_{a,D}}\tilde{R}_{b,t}^a\right) = \mu_t^a \kappa^a \tag{22}$$

$$\mathbb{E}_t \Omega_{t+1|t}^b \left(R_{k,t+1}^b - \frac{1}{\psi_{b,D}}\tilde{R}_{b,t}^b\right) = \mu_t^b \kappa^b \tag{23}$$

$$\mathbb{E}_t \Omega_{t+1|t}^c \left(R_{b,t}^a - R_{D,t}^c\right) = \mu_t^c \kappa_{F_a}^c \tag{24}$$

$$\mathbb{E}_t \Omega_{t+1|t}^c \left(R_{b,t}^b - R_{D,t}^c\right) = \mu_t^c \kappa_{F_b}^c \tag{25}$$

$$j_t^c(1-\mu_t^c) = \mathbb{E}_t \Omega_{t+1|t}^c R_{D,t}^c \tag{26}$$

$$j_t^a(1-\mu_t^a) = \mathbb{E}_t\left[\Omega_{t+1|t}^a \frac{1}{\psi_{a,D}}\tilde{R}_{b,t}^a\right] \tag{27}$$

$$j_t^b(1-\mu_t^b) = \mathbb{E}_t\left[\Omega_{t+1|t}^b \frac{1}{\psi_{b,D}}\tilde{R}_{b,t}^b\right] \tag{28}$$

$$N_t^c = \theta\left[R_{k,t}^c Q_{t-1}^c K_{t-1}^c + R_{b,t-1}^a F_{t-1}^a + R_{b,t-1}^b F_{t-1}^b - R_{D,t-1}^c D_{t-1}^c\right] + \delta^T Q_t^c K_{t-1}^c \tag{29}$$

$$N_t^a = \theta\left[R_{k,t}^a Q_{t-1}^a K_{t-1}^a - \tilde{R}_{b,t-1}^a F_{t-1}^a\right] + \delta^T Q_t^a K_{t-1}^a \tag{30}$$

$$N_t^b = \theta\left[R_{k,t}^b Q_{t-1}^b K_{t-1}^b - \tilde{R}_{b,t-1}^b F_{t-1}^b\right] + \delta^T Q_t^b K_{t-1}^b \tag{31}$$

$$j_t^c N_t^c = \kappa^c Q_t^c K_t^c + \kappa_{F_a}^c F_t^a + \kappa_{F_b}^c F_t^b \tag{32}$$

$$j_t^a N_t^a = \kappa^a Q_t^a K_t^a \tag{33}$$

$$j_t^b N_t^b = \kappa^b Q_t^b K_t^b \tag{34}$$

$$Q_t^c K_t^c + F_t^a + F_t^b = N_t^c + D_t^c \tag{35}$$

$$Q_t^a K_t^a = N_t^a + \psi_{a,D} F_t^a \tag{36}$$

$$Q_t^b K_t^b = N_t^b + \psi_{b,D} F_t^b \tag{37}$$

$$D_t^a = (\psi_{a,D} - 1)F_t^a \tag{38}$$

$$D_t^b = (\psi_{b,D} - 1)F_t^b \tag{39}$$

$$Y_t^c = A_t^c(\xi_t^c K_{t-1}^c)^\alpha H_t^{c\ 1-\alpha} \tag{40}$$

$$Y_t^a = A_t^a(\xi_t^a K_{t-1}^a)^\alpha H_t^{a\ 1-\alpha} \tag{41}$$

$$Y_t^b = A_t^b(\xi_t^b K_{t-1}^b)^\alpha H_t^{b\ 1-\alpha} \tag{42}$$

$$n_c B_t^c + n_a B_t^a + n_b B_t^b = 0 \tag{43}$$

$$\begin{aligned} n_c Y_t^c + n_a Y_t^a + n_b Y_t^b = n_c &\left[ C_t^c + I_t^c \left( 1 + \frac{\zeta}{2} \left( \frac{I_t^c}{I_{t-1}^c} - 1 \right)^2 \right) + \frac{\eta}{2}(B_t^c)^2 + \frac{\eta}{2}(D_t^c - \bar{D}^c)^2 \right] \\ +n_a &\left[ C_t^a + I_t^a \left( 1 + \frac{\zeta}{2} \left( \frac{I_t^a}{I_{t-1}^a} - 1 \right)^2 \right) + \frac{\eta}{2}(B_t^a)^2 + \frac{\eta}{2}(D_t^a - \bar{D}^a)^2 \right] \\ +n_b &\left[ C_t^b + I_t^b \left( 1 + \frac{\zeta}{2} \left( \frac{I_t^b}{I_{t-1}^b} - 1 \right)^2 \right) + \frac{\eta}{2}(B_t^b)^2 + \frac{\eta}{2}(D_t^b - \bar{D}^b)^2 \right] \end{aligned} \tag{44}$$

$$A_t^c = \rho_A A_{t-1}^c + \sigma_A \epsilon_{A,t}^c \tag{45}$$

$$A_t^a = \rho_A A_{t-1}^a + \sigma_A \epsilon_{A,t}^a \tag{46}$$

$$A_t^b = \rho_A A_{t-1}^b + \sigma_A \epsilon_{A,t}^b \tag{47}$$

$$\xi_t^c \rho_\xi \xi_{t-1}^c + \sigma_\xi \epsilon_{k,t}^c \tag{48}$$

$$\xi_t^a \rho_\xi \xi_{t-1}^a + \sigma_\xi \epsilon_{k,t}^a \tag{49}$$

$$\xi_t^b \rho_\xi \xi_{t-1}^b + \sigma_\xi \epsilon_{k,t}^b \tag{50}$$

$$\Lambda_{t+1}^c = \beta \left( \frac{C_{t+1}^c}{C_t^c} \right)^{-\sigma} \tag{51}$$

$$\Lambda_{t+1}^a = \beta \left( \frac{C_{t+1}^a}{C_t^a} \right)^{-\sigma} \tag{52}$$

$$\Lambda_{t+1}^b = \beta \left( \frac{C_{t+1}^a}{C_t^b} \right)^{-\sigma} \tag{53}$$

$$\tau_t^c = (1-\phi_\tau)\bar{\tau}^c + \phi_\tau \tau_{t-1}^c + \phi_z(K_t^c - K_{t-1}^c) \tag{54}$$

$$\tau_t^a = (1-\phi_\tau)\bar{\tau}^a + \phi_\tau \tau_{t-1}^a + \phi_z(K_t^a - K_{t-1}^a) \tag{55}$$

$$\tau_t^b = (1-\phi_\tau)\bar{\tau}^b + \phi_\tau \tau_{t-1}^b + \phi_z(K_t^b - K_{t-1}^b) \tag{56}$$

$$\begin{aligned} \Pi_t^c = (1-\theta)\left[ Q_{t-1}^c R_{k,t}^c K_{t-1}^c + R_{b,t-1}^a F_{t-1}^a + R_{b,t-1}^b F_{t-1}^b - R_{D,t-1}^c D_{t-1}^c \right] - \delta^T Q_t^c K_{t-1}^c + Q_t^c I_t^c \\ -I_t^c \left( 1 + \frac{\zeta}{2} \left( \frac{I_t^c}{I_{t-1}^c} - 1 \right)^2 \right) + \tau_t^c \alpha A_t^c H_t^{c\ (1-\alpha)} \xi_t^{c\ \alpha} K_{t-1}^{c\ (\alpha)} \end{aligned}$$

$$\Pi_t^a = (1-\theta)\left[ Q_{t-1}^a R_{k,t}^a K_{t-1}^a - \tilde{R}_{b,t-1}^a F_{t-1}^a \right] - \delta^T Q_t^a K_{t-1}^a + Q_t^a I_t^a - I_t^a \left( 1 + \frac{\zeta}{2} \left( \frac{I_t^a}{I_{t-1}^a} - 1 \right)^2 \right) + \tau_t^a \alpha A_t^a H_t^{a\ (1-\alpha)} \xi_t^{a\ \alpha} K_{t-1}^{a\ (\alpha)}$$

$$\Pi_t^b = (1-\theta)\left[ Q_{t-1}^b R_{k,t}^b K_{t-1}^b - \tilde{R}_{b,t-1}^b F_{t-1}^b \right] - \delta^T Q_t^b K_{t-1}^b + Q_t^b I_t^b - I_t^b \left( 1 + \frac{\zeta}{2} \left( \frac{I_t^b}{I_{t-1}^b} - 1 \right)^2 \right) + \tau_t^b \alpha A_t^b H_t^{b\ (1-\alpha)} \xi_t^{b\ \alpha} K_{t-1}^{b\ (\alpha)}$$

Simplifications (cont.): Subs: $D_t^{eme} = (\psi_D - 1)F_t^{eme}$

$$R_t \Lambda_{t+1}^c = 1 + \eta(B_t^c) \tag{1}$$

$$R_t \Lambda_{t+1}^a = 1 + \eta(B_t^a) \tag{2}$$

$$R_t \Lambda_{t+1}^b = 1 + \eta(B_t^b) \tag{3}$$

$$R_{D,t}^c \Lambda_{t+1}^c = 1 + \eta(D_t^c - \bar{D}^c) \tag{4}$$

$$R_{D,t}^a \Lambda_{t+1}^a = 1 + \eta((\psi_{a,D} - 1)F_t^a - \bar{D}^a) \tag{5}$$

$$R_{D,t}^b \Lambda_{t+1}^b = 1 + \eta((\psi_{b,D} - 1)F_t^b - \bar{D}^b) \tag{6}$$

$$C_t^{c\ -\sigma} = \frac{H_t^{c\ \psi}}{(1-\alpha)A_t^c \xi_t^{c\ \alpha} K_{t-1}^{c\ (\alpha)} H_t^{c\ (-\alpha)}} \tag{7}$$

$$C_t^{a\ -\sigma} = \frac{H_t^{a\ \psi}}{(1-\alpha)A_t^a \xi_t^{a\ \alpha} K_{t-1}^{a\ (\alpha)} H_t^{i\ (-\alpha)}} \tag{8}$$

$$C_t^{b\ -\sigma} = \frac{H_t^{b\ \psi}}{(1-\alpha)A_t^b \xi_t^{b\ \alpha} K_{t-1}^{b\ (\alpha)} H_t^{b\ (-\alpha)}} \tag{9}$$

$$C_t^a + B_t^a + \frac{\eta}{2}(B_t^a)^2 + (\psi_{a,D} - 1)F_t^a + \frac{\eta}{2}((\psi_{a,D} - 1)F_t^a - \bar{D}^a)^2 = R_{t-1}B_{t-1}^a + R_{D,t-1}^a(\psi_{a,D} - 1)F_{t-1}^a + (1-\alpha)A_t^a \xi_t^{a\ \alpha} K_{t-1}^{a\ (\alpha} \tag{10}$$

$$C_t^b + B_t^b + \frac{\eta}{2}(B_t^b)^2 + (\psi_{b,D} - 1)F_t^b + \frac{\eta}{2}((\psi_{b,D} - 1)F_t^b - \bar{D}^b)^2 = R_{t-1}B_{t-1}^b + R_{D,t-1}^b(\psi_{b,D} - 1)F_{t-1}^b + (1-\alpha)A_t^b \xi_t^{b\ \alpha} K_{t-1}^{b\ (\alpha)} H \tag{11}$$

$$Q_t^c = 1 + \frac{\zeta}{2}\left(\frac{I_t^c}{I_{t-1}^c} - 1\right)^2 + \zeta\left(\frac{I_t^c}{I_{t-1}^c} - 1\right)\frac{I_t^c}{I_{t-1}^c} - \Lambda_{t+1}^c \zeta \left(\frac{I_{t+1}^c}{I_t^c}\right)^2 \left(\frac{I_{t+1}^c}{I_t^c} - 1\right) \tag{12}$$

$$Q_t^a = 1 + \frac{\zeta}{2}\left(\frac{I_t^a}{I_{t-1}^a} - 1\right)^2 + \zeta\left(\frac{I_t^a}{I_{t-1}^a} - 1\right)\frac{I_t^a}{I_{t-1}^a} - \Lambda_{t+1}^a \zeta \left(\frac{I_{t+1}^a}{I_t^a}\right)^2 \left(\frac{I_{t+1}^a}{I_t^a} - 1\right) \tag{13}$$

$$Q_t^b = 1 + \frac{\zeta}{2}\left(\frac{I_t^b}{I_{t-1}^b} - 1\right)^2 + \zeta\left(\frac{I_t^b}{I_{t-1}^b} - 1\right)\frac{I_t^b}{I_{t-1}^b} - \Lambda_{t+1}^b \zeta \left(\frac{I_{t+1}^b}{I_t^b}\right)^2 \left(\frac{I_{t+1}^b}{I_t^b} - 1\right) \tag{14}$$

$$K_t^c = I_t^c + (1-\delta)\xi_t^c K_{t-1}^c \tag{15}$$

$$K_t^a = I_t^a + (1-\delta)\xi_t^a K_{t-1}^a \tag{16}$$

$$K_t^b = I_t^b + (1-\delta)\xi_t^b K_{t-1}^b \tag{17}$$

$$R_{k,t}^c = \frac{(1-\tau_t^c)\alpha A_t^c H_t^{c\ (1-\alpha)} \xi_t^{c\ \alpha} K_{t-1}^{c\ (\alpha-1)} + (1-\delta)\xi_t^c Q_t^c}{Q_{t-1}^c} \tag{18}$$

$$R_{k,t}^a = \frac{(1-\tau_t^a)\alpha A_t^a H_t^{a\ (1-\alpha)} \xi_t^{a\ \alpha} K_{t-1}^{a\ (\alpha-1)} + (1-\delta)\xi_t^a Q_t^a}{Q_{t-1}^a} \tag{19}$$

$$R_{k,t}^b = \frac{(1-\tau_t^b)\alpha A_t^b H_t^{b\ (1-\alpha)} \xi_t^{b\ \alpha} K_{t-1}^{b\ (\alpha-1)} + (1-\delta)\xi_t^b Q_t^b}{Q_{t-1}^b} \tag{20}$$

$$\mathbb{E}_t \Omega_{t+1|t}^c \left(R_{k,t+1}^c - R_{D,t}^c\right) = \mu_t^c \kappa^c \tag{21}$$

$$\mathbb{E}_t \Omega_{t+1|t}^a \left(R_{k,t+1}^a - \frac{1}{\psi_{a,D}}\tilde{R}_{b,t}^a\right) = \mu_t^a \kappa^a \tag{22}$$

$$\mathbb{E}_t \Omega_{t+1|t}^b \left(R_{k,t+1}^b - \frac{1}{\psi_{b,D}}\tilde{R}_{b,t}^b\right) = \mu_t^b \kappa^b \tag{23}$$

$$\mathbb{E}_t \Omega_{t+1|t}^c \left(R_{b,t}^a - R_{D,t}^c\right) = \mu_t^c \kappa_{F_a}^c \tag{24}$$

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$$j_t^c(1-\mu_t^c) = \mathbb{E}_t \Omega_{t+1|t}^c R_{D,t}^c \tag{26}$$

$$j_t^a(1-\mu_t^a) = \mathbb{E}_t\left[\Omega_{t+1|t}^a \frac{1}{\psi_{a,D}} \tilde{R}_{b,t}^a\right] \tag{27}$$

$$j_t^b(1-\mu_t^b) = \mathbb{E}_t\left[\Omega_{t+1|t}^b \frac{1}{\psi_{b,D}} \tilde{R}_{b,t}^b\right] \tag{28}$$

$$N_t^c = \theta\left[R_{k,t}^c Q_{t-1}^c K_{t-1}^c + R_{b,t-1}^a F_{t-1}^a + R_{b,t-1}^b F_{t-1}^b - R_{D,t-1}^c D_{t-1}^c\right] + \delta^T Q_t^c K_{t-1}^c \tag{29}$$

$$N_t^a = \theta\left[R_{k,t}^a Q_{t-1}^a K_{t-1}^a - \tilde{R}_{b,t-1}^a F_{t-1}^a\right] + \delta^T Q_t^a K_{t-1}^a \tag{30}$$

$$N_t^b = \theta\left[R_{k,t}^b Q_{t-1}^b K_{t-1}^b - \tilde{R}_{b,t-1}^b F_{t-1}^b\right] + \delta^T Q_t^b K_{t-1}^b \tag{31}$$

$$j_t^c N_t^c = \kappa^c Q_t^c K_t^c + \kappa_{F_a}^c F_t^a + \kappa_{F_b}^c F_t^b \tag{32}$$

$$j_t^a N_t^a = \kappa^a Q_t^a K_t^a \tag{33}$$

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$$Q_t^c K_t^c + F_t^a + F_t^b = N_t^c + D_t^c \tag{35}$$

$$Q_t^a K_t^a = N_t^a + \psi_{a,D} F_t^a \tag{36}$$

$$Q_t^b K_t^b = N_t^b + \psi_{b,D} F_t^b \tag{37}$$

$$Y_t^c = A_t^c(\xi_t^c K_{t-1}^c)^\alpha H_t^{c\ 1-\alpha} \tag{38}$$

$$Y_t^a = A_t^a(\xi_t^a K_{t-1}^a)^\alpha H_t^{a\ 1-\alpha} \tag{39}$$

$$Y_t^b = A_t^b(\xi_t^b K_{t-1}^b)^\alpha H_t^{b\ 1-\alpha} \tag{40}$$

$$n_c B_t^c + n_a B_t^a + n_b B_t^b = 0 \tag{41}$$

$$\begin{aligned} n_c Y_t^c + n_a Y_t^a + n_b Y_t^b = n_c &\left[C_t^c + I_t^c\left(1 + \frac{\zeta}{2}\left(\frac{I_t^c}{I_{t-1}^c} - 1\right)^2\right) + \frac{\eta}{2}(B_t^c)^2 + \frac{\eta}{2}(D_t^c - \bar{D}^c)^2\right] \\ +n_a &\left[C_t^a + I_t^a\left(1 + \frac{\zeta}{2}\left(\frac{I_t^a}{I_{t-1}^a} - 1\right)^2\right) + \frac{\eta}{2}(B_t^a)^2 + \frac{\eta}{2}((\psi_{a,D} - 1)F_t^a - \bar{D}^a)^2\right] \\ +n_b &\left[C_t^b + I_t^b\left(1 + \frac{\zeta}{2}\left(\frac{I_t^b}{I_{t-1}^b} - 1\right)^2\right) + \frac{\eta}{2}(B_t^b)^2 + \frac{\eta}{2}((\psi_{b,D} - 1)F_t^b - \bar{D}^b)^2\right] \end{aligned} \tag{42}$$

$$A_t^c = \rho_A A_{t-1}^c + \sigma_A \epsilon_{A,t}^c \tag{43}$$

$$A_t^a = \rho_A A_{t-1}^a + \sigma_A \epsilon_{A,t}^a \tag{44}$$

$$A_t^b = \rho_A A_{t-1}^b + \sigma_A \epsilon_{A,t}^b \tag{45}$$

$$\xi_t^c \rho_\xi \xi_{t-1}^c + \sigma_\xi \epsilon_{k,t}^c \tag{46}$$

$$\xi_t^a \rho_\xi \xi_{t-1}^a + \sigma_\xi \epsilon_{k,t}^a \tag{47}$$

$$\xi_t^b \rho_\xi \xi_{t-1}^b + \sigma_\xi \epsilon_{k,t}^b \tag{48}$$

$$\Lambda_{t+1}^c = \beta\left(\frac{C_{t+1}^c}{C_t^c}\right)^{-\sigma} \tag{49}$$

$$\Lambda_{t+1}^a = \beta\left(\frac{C_{t+1}^a}{C_t^a}\right)^{-\sigma} \tag{50}$$

$$\Lambda_{t+1}^b = \beta\left(\frac{C_{t+1}^a}{C_t^b}\right)^{-\sigma} \tag{51}$$

$$\tau_t^c = (1-\phi_\tau)\bar{\tau}^c + \phi_\tau \tau_{t-1}^c + \phi_z(K_t^c - K_{t-1}^c) \tag{52}$$

$$\tau_t^a = (1-\phi_\tau)\bar{\tau}^a + \phi_\tau \tau_{t-1}^a + \phi_z(K_t^a - K_{t-1}^a) \tag{53}$$

$$\tau_t^b = (1-\phi_\tau)\bar{\tau}^b + \phi_\tau \tau_{t-1}^b + \phi_z(K_t^b - K_{t-1}^b) \tag{54}$$

$$\Pi^c_t = (1-\theta)\left[Q^c_{t-1}R^c_{k,t}K^c_{t-1} + R^a_{b,t-1}F^a_{t-1} + R^b_{b,t-1}F^b_{t-1} - R^c_{D,t-1}D^c_{t-1}\right] - \delta^T Q^c_t K^c_{t-1} + Q^c_t I^c_t$$
$$-I^c_t\left(1+\frac{\zeta}{2}\left(\frac{I^c_t}{I^c_{t-1}}-1\right)^2\right) + \tau^c_t \alpha A^c_t H^{c\ (1-\alpha)}_t \xi^{c\ \alpha}_t K^{c\ (\alpha)}_{t-1}$$

$$\Pi^a_t = (1-\theta)\left[Q^a_{t-1}R^a_{k,t}K^a_{t-1} - \tilde{R}^a_{b,t-1}F^a_{t-1}\right] - \delta^T Q^a_t K^a_{t-1} + Q^a_t I^a_t - I^a_t\left(1+\frac{\zeta}{2}\left(\frac{I^a_t}{I^a_{t-1}}-1\right)^2\right) + \tau^a_t \alpha A^a_t H^{a\ (1-\alpha)}_t \xi^{a\ \alpha}_t K^{a\ (\alpha)}_{t-1}$$

$$\Pi^b_t = (1-\theta)\left[Q^b_{t-1}R^b_{k,t}K^b_{t-1} - \tilde{R}^b_{b,t-1}F^b_{t-1}\right] - \delta^T Q^b_t K^b_{t-1} + Q^b_t I^b_t - I^b_t\left(1+\frac{\zeta}{2}\left(\frac{I^b_t}{I^b_{t-1}}-1\right)^2\right) + \tau^b_t \alpha A^b_t H^{b\ (1-\alpha)}_t \xi^{b\ \alpha}_t K^{b\ (\alpha)}_{t-1}$$