Econ 5322 - MS Macroeconomics Spring 2023
Granados

Final Exam

Answer Key

Answer the following questions. Show and explain your work instead of only writing the final answers. Read the entire exam before starting to answer it.

This exam is open book: you can consult all materials, but do not communicate with anyone else

1. (20 points) **Short Answer/Essay questions**

   (a) In deriving the modern Aggregate Supply curve: the New Keynesian Phillips Curve, we emphasized two key ingredients: 1) monopolistic competition with CES utility, and 2) the Calvo-style probabilistic price changing probability, in addition to the general focus on rational expectations and micro-foundation. Explain why these two assumptions are needed.

> (Ans) Monopolistic competition: Gives price setting decision to firms so we can model and explain why they may choose not to change prices. The associated CES setup delivers strategic complementarity or externality in the demand functions each firm faces.
>
> Calvo probabilistic price changing: This is a variation of the Taylor pre-fixed setup where firms set pre-fixed prices in a staggered fashion. Combined with the demand function from the monopolistic competition setup, we can generate realistic output response to monetary policy observed in the data (lasting beyond the length of nominal rigidity)

   (b) At a job interview, your potential employer asks you to discuss the likely outcomes of a government stimulus program that provides a one-time transfer of $1.5K to all households.

   Hint: As with most interviews, the aim is to show off your knowledge and how you can approach the question from various perspectives and provide a synthesized analysis, not to give one short answer. You may want to discuss different variables, what their responses may depend on, in both the short versus long-run, and other aspects you can think of.

> (Ans)
> - Approach from consumption perspective: Permanent income has not gone up by $1.5K . Then, the public will smooth this temporary windfall over lifetime, so immediate consumption or aggregate demand increase would not be very large.
>
> - However, empirically there is evidence of excess smoothness: consumption tracking income. Whether it is likely to happen or by how much in this case depend on the reasons behind such high correlation (of consumption and income), e.g. buffer stock, precautionary savings, liquidity constraint.
>
> - Additionally, precautionary savings is likely to play a role. Which way buffer stock goes depends if each household is currently above or below the Buffer Stock level of

Consumption they target

- Overall, household consumption behavior is likely to cause some increase in aggregate demand. How much this translates to the actual output increase depends on the slope of the aggregate supply (e.g., classical, Keynesian..etc).

(c) (True/False/Uncertain) Explain: "According to the Calvo model of price adjustment, the higher the parameter θ, the more the COVID relief stimulus package should help boost output."

(Ans) True, since θ captures the degree of nominal rigidity. High θ implies more price stickiness and a flatter aggregate supply curve. So for each shift of the AD, output response will be larger.

(d) (True/False/Uncertain) Explain: Consider an economy where firms set prices that are fixed over one-year intervals, and the ideal price for each firm at time t is equal to the money supply at time t. If price-setting is staggered such that a quarter of the firms set prices in the beginning of each quarter, the effect of monetary policy can last beyond one year.

(Ans) False: in addition to staggering and pre-fixed prices, we also need strategic complementarity to get such a result.

2. (15 points) A consumer maximizes a life-time utility as follows (starting in period t):

$$\max_{\{C_s\}_{s=t}^{\infty}} \mathbb{E}_t \sum_{t=s}^{\infty} \beta^{s-t} \log C_s$$

subject to:

$$A_{t+1} = \tilde{R}_{t,t+1}(\tilde{Y}_t + A_t - C_t), \quad \forall\, t$$

where C_t is consumption, A_t is the value of assets (wealth) at t, $\tilde{Y}$ is the income at t, and $\tilde{R}$ is the gross return on wealth at t. Variables with a tilde ($\tilde{x}$) are stochastic.

Derive the consumer's Euler equation.

[Hint: the consumer should pick their assets and consumption optimally in every period. Also notice that the assets chosen at t, i.e., A_{t+1} will show up in more than one period, making this an inter-temporal problem]

(ans) You can tackle this problem in many ways, using your intuition, as in the basic RBC setup, using a perturbation argument as in the most recent topics, or doing the math by setting a Lagrangian.

Mathematical derivation:

Set the lagrangian:

$$\mathcal{L} = \mathbb{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left[\log C_s + \lambda_s \left(A_{s+1} - \tilde{R}_{s+1}(\tilde{Y}_s + A_s - C_s) \right) \right] \right\}$$

Taking first-order conditions with respect to the choice variables at t:

$$[C_t] : u'(C_t) + R_{t+1}\lambda_t = 0$$

with respect to the Assets value:

$$[A_{t+1}] : \lambda_t - \beta\lambda_{t+1}\tilde{R}_{t+2} = 0$$
$$\lambda_t = \beta\lambda_{t+1}\tilde{R}_{t+2} \tag{1}$$

Now, we can get rid of the λ's by replacing from (1) on (2) for t and $t+1$ (these FOCs are valid for any period, so this is a valid approach): $\lambda_t = \frac{u'(C_t)}{-\tilde{R}_{t+1}}$ and $\lambda_{t+1} = \frac{u'(C_{t+1})}{-\tilde{R}_{t+2}}$

$$\frac{u'(C_t)}{-\tilde{R}_{t+1}} = \beta \frac{u'(C_{t+1})}{-\tilde{R}_{t+2}} \tilde{R}_{t+2}$$

now we simplify, and put back the expected value conditional to t for variables that are unknown at that time:

$$u'(C_t) = \beta \mathbb{E}_t \left[u'(C_{t+1}) \tilde{R}_{t+1} \right]$$

This is the Euler equation. You could also go one step further and replace $u'(C_t) = \frac{1}{C_t}$ since $u(C_t) = \log C_t$.

Note: Alternatively, you can also solve for C_t from the budget constraint ($C_t = Y_t + A_t - \frac{A_{t+1}}{\tilde{R}_{t+1}}$), replace it in the objective function and maximize directly.

In that case you maximize: $\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \log(Y_s + A_s - \frac{A_{s+1}}{\tilde{R}_{s+1}})$. Take the FOC with respect to A_{t+1}:

$$\frac{1}{C_t}\left(-\frac{1}{\tilde{R}_{t+1}}\right) + \beta \frac{1}{C_{t+1}} = 0$$

Then rearrange to get the Euler equation: $u'(C_t) = \beta \mathbb{E}_t \left[u'(C_{t+1}) \tilde{R}_{t+1} \right]$, with $u'(C) = 1/C$

3. (10 points) Consider the following impulse responses of an economy to an exogenous shock "u". Variables x, pi, and p are as defined in the three equation New Keynesian model (output gap, inflation, prices level).

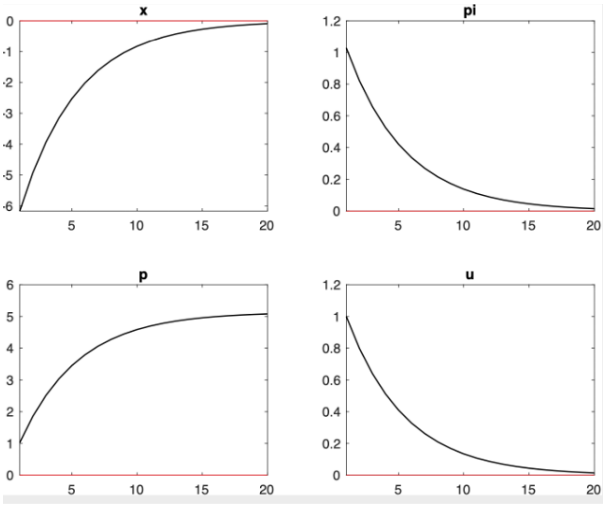

Figure 1: Impulse response of variables of the model to a 1 std. dev. "u" shock

The plots above depict the impulse responses of the standard 3 equation NK-DSGE model. What do these figures say? Is the shock permanent (random-walk) or transitory (white-noise)? What kind of shock in the model would you guess u refers to?

(ans) These plots are showing the responses in the output gap, inflation, and prices level of a positive shock on u in a 3 equations NK DSGE model. Given the model, the fact that the shock pushes inflation up instantaneously, and increases the level or prices, we could guess it is a "cost-push" shock, i.e., a shock affecting the RHS of the Phillips Curve, or Aggregate Supply equation.

The shock is clearly transitory, it starts at 1 and converges to zero after 20 quarters. For the same token, the output gap and inflation will also be shifted from their steady state values temporarily.

Notice that for the level of prices, there is a shift to a new permanent level. This is due to it not being an stationary variable, thus, even transitory shocks can change its long run level.

4. (20 points) Suppose a final consumption producer uses intermediate inputs produced by monopolistically competitive firms according to the production function:

$$Y_t = \left[\int_0^1 y_{i,t}^{\frac{\theta-1}{\theta}} d_i \right]^{\frac{\theta}{\theta-1}}$$

where $\theta > 1$. The final consumption producer operates in a perfectly competitive market and takes the price of consumption, P_t, as given. The producer chooses how much to demand of each individual input $y_{i,t}$ to maximize profits given by:

$$P_t Y_t - \int_0^1 p_{i,t} y_{i,t} d_i$$

subject to the production function above, where $p_{i,t}$ is the price the final producer pays for interme-

diate input i.

(a) Show that optimal input demand by the final good producer is determined by

$$y_{i,t} = \left(\frac{p_{i,t}}{P_t}\right)^{-\theta} Y_t$$

(ans)

We can use a Lagrangian, or analogously, substitute the production function into the profits. With this, the profit-maximization problem becomes:

$$\max_{y_{i,t}} P_t \left[\int_0^1 y_{i,t}^{\frac{\theta-1}{\theta}} d_i\right]^{\frac{\theta}{\theta-1}} - \int_0^1 p_{i,t} y_{i,t} di$$

The fist-order condition with respect to the demand of each input variety is:

$$[y_{i,t}]: \quad \frac{\theta}{\theta-1} P_t \left[\int_0^1 y_{i,t}^{\frac{\theta-1}{\theta}} d_i\right]^{\frac{\theta}{\theta-1}-1} \frac{\theta-1}{\theta} y_{i,t}^{\frac{\theta-1}{\theta}-1} - p_{i,t} = 0$$

We can simplify this expression, given that $Y_t = \left[\int_0^1 y_{i,t}^{\frac{\theta-1}{\theta}} d_i\right]^{\frac{\theta}{\theta-1}}$, which implies that $Y_t^{\frac{\theta-1}{\theta}} = \int_0^1 y_{i,t}^{\frac{\theta-1}{\theta}}$. After replacing into the first-order condition:

$$P_t \left(Y_t^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}-1} y_{i,t}^{\frac{\theta-1}{\theta}-1} - p_{i,t} = 0$$

$$P_t Y_t^{\frac{1}{\theta}} y_{i,t}^{-\frac{1}{\theta}} - p_{i,t} = 0$$

rearranging:

$$y_{i,t} = \left(\frac{p_{i,t}}{P_t}\right)^{-\theta} Y_t$$

(b) Explain this demand function

(ans) *Intuition*: the optimal demand for the input, or intermediate good i will grow when its price ($p_{i,t}$) lowers, or when the aggregate prices increase, in other words, the demand decreases in the relative price of the good i. At the same time, the demand for good i increases when the aggregate demand is higher (Y_t). The latter is very intuitive too, if total production (measures by total demand or supply in equilibrium) is higher, it's very likely that the demand of individual inputs increases.

Intermediate good producers produce goods with the production function

$$y_{i,t} = Z_t N_{i,t}$$

where Z_t is exogenous productivity and $N_{i,t}$ is labor employed by producer i. This production

function implies that the marginal cost of production for the intermediate producer is w_t/Z_t, where w_t is the real wage (in units of the final consumption good). The producer sets the price $p_{i,t}$ to maximize

$$p_{i,t}y_{i,t} - P_t\frac{w_t}{Z_t}y_{i,t}$$

subject to the demand function obtained above.

(c) Show that optimal price setting implies:

$$\frac{p_{i,t}}{P_t} = \frac{\theta}{\theta-1}\frac{w_t}{Z_t}$$

(ans) The intermediate goods firm (or wholesaler in terms of the slides) chooses a price for its product to solve:

$$\max_{p_{i,t}} = p_{i,t}y_{i,t} - P_t\frac{w_t}{Z_t}y_{i,t} \qquad s.t. \quad y_{i,t} = \left(\frac{p_{i,t}}{P_t}\right)^{-\theta}Y_t$$

We replace $y_{i,t}$ and obtain the first-order condition with respect to the price (decision variable):

$$[p_{i,t}]: \quad (1-\theta)p_{i,t}^{-\theta}P_t^{\theta}Y_t + \theta p_{i,t}^{-\theta-1}P_t\frac{w_t}{Z_t}Y_t = 0$$

rearranging:

$$(\theta-1)P_t^{\theta}Y_t = \theta p_{i,t}^{-1}P_t^{1+\theta}\frac{w_t}{Z_t}Y_t$$

$$\frac{p_{i,t}}{P_t} = \frac{\theta}{\theta-1}\frac{w_t}{Z_t}$$

Intuition: the optimal price will be set as a mark-up over the marginal cost. Notice such optimal price refers to the relative price, that marginal cost in this setup is given by the real wage adjusted by productivity w_t/Z_t and that the factor $\frac{\theta}{\theta-1} > 1$ given $\theta > 1$.

(Bonus) Optional question: Answer 1 out of the 3 choices below.

(a) Describe how the presence of financial frictions can alter the quantitative responses of an economy to, e.g. a negative productivity or recessionary shock.

(b) Describe in your own words what the dynamic inconsistency problem is. In the context of monetary policy making, how can dynamic inconsistency lead to inflation bias?

(c) What is the equity premium puzzle and what are some of the proposed solutions?

(ans)

(a) The financial frictions will have the effect of augmenting or amplifying the business cycle fluctuations. The mechanism involved is called a "financial accelerator"; in the case of a recessionary shock, the shock will decrease income, investment and consumption as expected, but in addition, the deterioration in the balance sheet of the firms will increase

the External Finance Premium which will further decrease aggregate investment. As a result, the recession will become deeper, and even more protracted than normal.

(b) Dynamic inconsistency refers to the phenomenon that the incentives of a central banker, ex-ante and ex-post to taking their policy decisions will differ (or be inconsistent). In practice, it means that the central banker ex-ante wants to impose a lower lever of money growth and inflation for a given period, but at the moment of actually executing their decision will want to increase money growth to boost the economy.

The public may be aware of that issue and may increase inflation expectations which, together with the actions of the central bank, leads to an inflationary bias. The solution to this issue could be to make the central bank to commit to a pre-announced policy decision rather than setting their toolkit in a discretionary manner.

(c) The equity premium puzzle refers to the inability of economic theory to rationalize the average return premium return of stocks over bonds (or low risk assets). Rationalizing the observed equity premium with the standard models would imply the assumption of implausible levels or risk aversion.

Several solutions and alternative explanations have been proposed, including improving the measurement of stock returns as it may suffer from survivorship bias, disconnecting the intertemporal elasticity of substitution from the risk aversion in the models (e.g., using consumption habits or Epstein-Zin preferences), improving the measurements of consumption, or recognizing the fact that stock market participation is heterogeneous and that households holding are either more risk-averse or display a higher covariance of consumption with equity returns.