Intermediate Macroeconomics

An Overview of Long-Run Economic Growth

ECON 3311 – Spring 2025
UT Dallas

Introduction

In this chapter, we will examine:

- Some facts related to **economic growth** that later chapters will seek to explain
- How economic growth has dramatically improved welfare around the world and how this growth is a relatively recent phenomenon
- Some tools used to study economic growth, including how to calculate **growth rates**
- Why a **'ratio scale'** makes plots of per capita GDP easier to understand and interpret

Why is Economic Growth Important

- Small differences in growth rates will lead to large differences in GDP <u>over time</u>
- Suppose we are looking at two countries starting with a GDP of $100 billion, one grows at a rate of 1 percent per year and the other at a rate of 0.25 percent per year

Country	Original GDP	GDP after 1 year	GDP after 10 years		
A (growth rate 1%)	$100 billion	$101 billion	$110.46 billion		
B (growth rate 0.25%)	$100 billion	$100.25 billion	$102.53 billion		

Similar GDP level after 1 year

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Country	Original GDP	GDP after 1 year	GDP after 10 years	GDP after 50 years	
A (growth rate 1%)	$100 billion	$101 billion	$110.46 billion	$164.46 billion	
B (growth rate 0.25%)	$100 billion	$100.25 billion	$102.53 billion	$113.30 billion	

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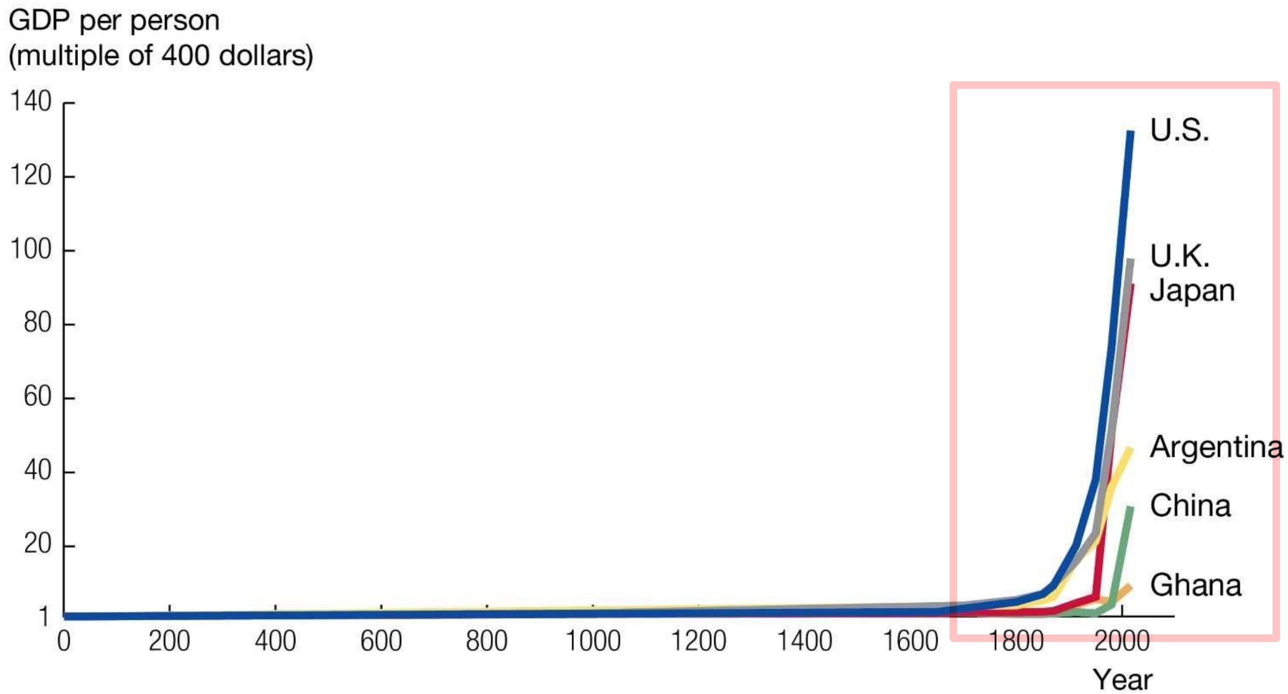
Country	Original GDP	GDP after 1 year	GDP after 10 years	GDP after 50 years	GDP after 100 years
A (growth rate 1%)	$100 billion	$101 billion	$110.46 billion	$164.46 billion	$270.48 billion
B (growth rate 0.25%)	$100 billion	$100.25 billion	$102.53 billion	$113.30 billion	$128.36 billion

The only difference between country A & B in this example is the GDP growth

Growth over the very long run

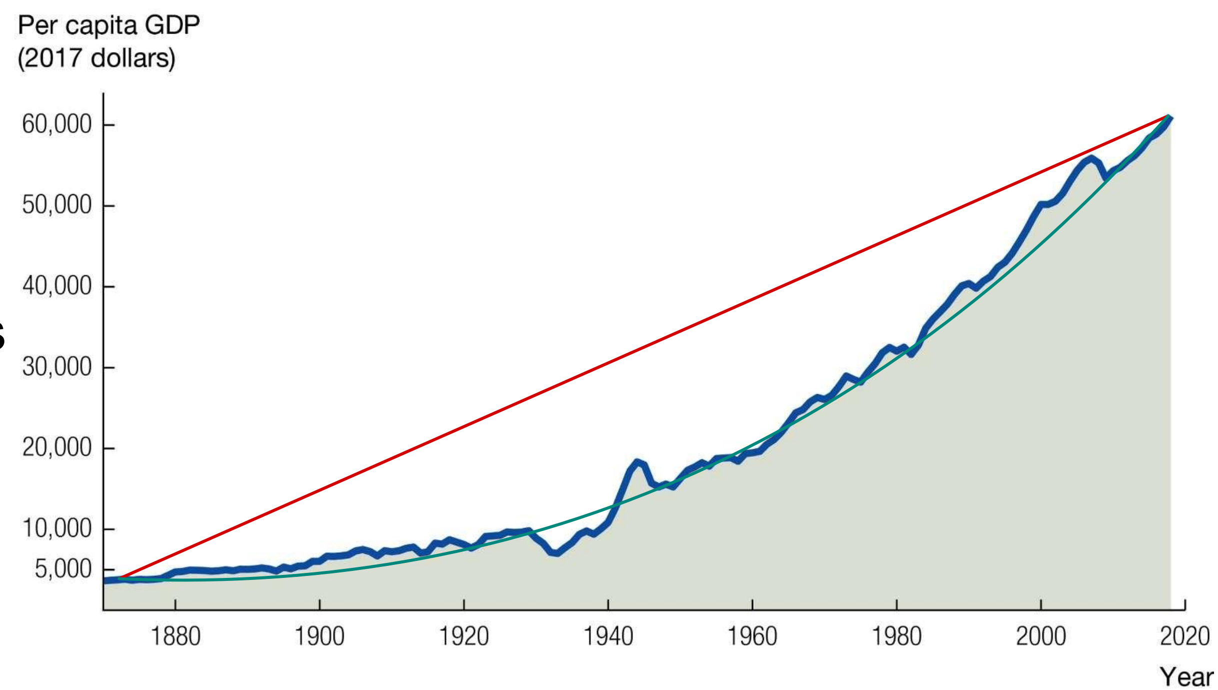
- In historical terms, economic growth is a very recent phenomenon, only beginning in the 18th century with the Industrial Revolution.
- When exactly economic growth 'started' and the reasons for it are dealt with in Economic History classes
- In this course, we focus on the causes and consequences of 'recent' economic growth



Source: The Maddison-Project, www.ggdc.net/maddison/.

Growth over the ‘recent’ long run

- Looking at the US, in 1870 per capita income was around $3,600 (in 2017 dollars)
- In 2018, per capita income was about $61,000
- This is a 17-fold increase



Source: Data from 1870 to 1928 from Barro- Ursua Macroeconomic Data, 2010. Data from 1929 to 2018 from U.S. Department of Commerce, Bureau of Economic Analysis.

why? → Previous production grows
But newly growing parts also grow
over time

GDP grows in an exponential
(non-linear) fashion with respect to
the initial point

Definition of Economic Growth

Growth of per capita GDP is the rate of change of per capita GDP

A percentage change in a variable is calculated as the change between two periods divided by the value of the variable in the initial period

- Percentage change in GDP between period t and t+1:

$$\bar{g} = \frac{y_{t+1}}{y_t} - 1$$

$$\bar{g} = \frac{y_{t+1} - y_t}{y_t} \qquad y_{t+1} = y_t(1 + \bar{g})$$

For example, what is the percentage change in per capita GDP if it increased from $12,000 to $14,000?

$$g = \frac{14000 - 12000}{12000} = \frac{14000}{12000} - 1 = 0.167$$

e.g. Annual growth in 2023: $\frac{y_{2023}}{y_{2022}} - 1 = \bar{g}$

Definition of Economic Growth

We can expand the economic growth formula to include more years

For example, if there are two years with the same growth rate

Assumption: here GDP grows at same constant rate over several years

$$y_{t+1} = y_t(1+\bar{g}) \qquad y_{t+2} = y_{t+1}(1+\bar{g})$$

- Plugging the equation on the left into the equation on the right, we obtain:

$$y_{t+2} = y_t(1+\bar{g})(1+\bar{g}) = y_t(1+\bar{g})^2$$

- Therefore, if we have a constant growth rate $\bar{g}$ for more than one year we can write:

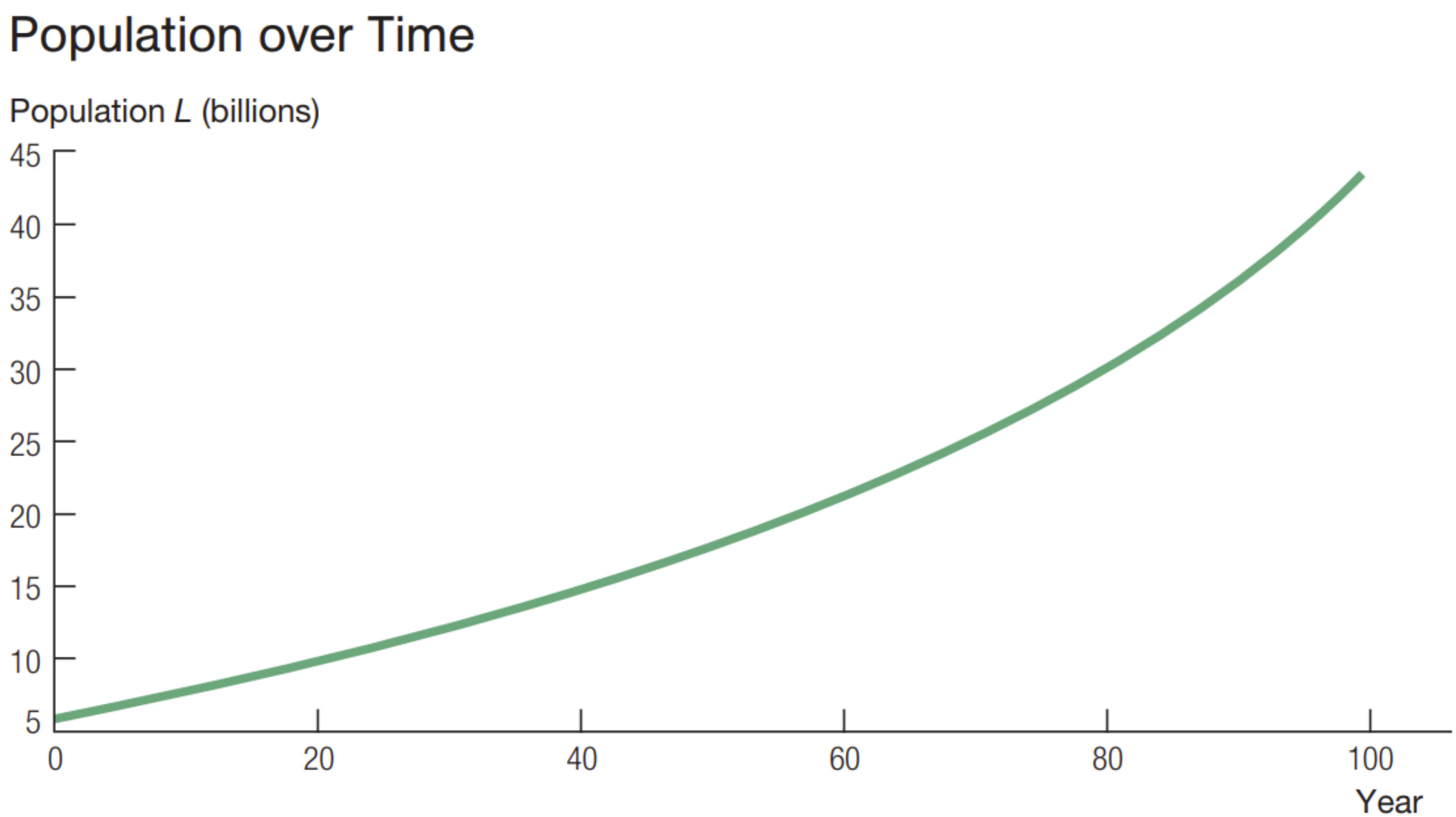
$$y_t = y_0(1+\bar{g})^t$$

$$y^{2020} = y^{2000}(1+\bar{g})^{20}$$

Where $\boldsymbol{t}$ is the number of years, y_t is the value of GDP after $\boldsymbol{t}$ years, y_0 is the original GDP value, and $\bar{g}$ is the constant growth rate

t: Years of separation between y_0 & y_t

Another Example: Population behaves in the same way



Rule of 70

$$(1+\bar{g})^t \simeq 2 \quad \begin{cases} (1+0.07)^{10} \simeq 2 \\ (1+0.1)^{7} \simeq 2 \end{cases}$$

*The rule of 70 allows us to quickly approximate how much time it takes a value (like GDP) to **double**

- If y grows at a rate of g percent per year, then the number of years it takes y to double is approximately equal to 70/g

- For instance, if GDP of a particular country grows at 2% per year, then it would take about 35 years (70/2) for GDP to double

Sometimes, the rule of 72 is used and the time to double is 72/g

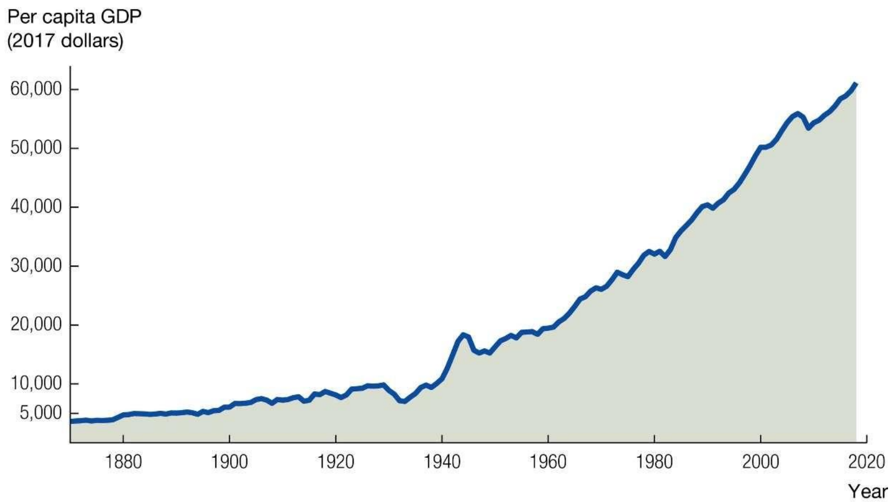
The rule of 72 is more accurate when the growth rate is 6% or higher and the rule of 70 is more accurate when the growth rate is 4% or lower

*Note that the time it takes to double only depends on the growth rate and not on the initial value

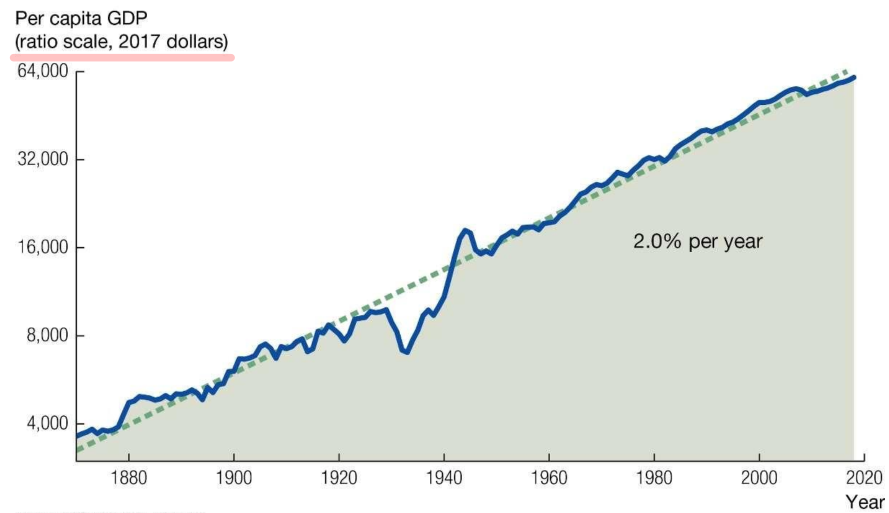
Ratio Scale

When examining GDP, we will often use the ratio scale, where equally spaced intervals on the vertical axis are rising by a constant ratio

With a ratio scale, it is easier to see how growth rates are changing Consider the following two graphs:



Source: Data from 1870 to 1928 from Barro- Ursua Macroeconomic Data, 2010. Data from 1929 to 2018 from U.S. Department of Commerce, Bureau of Economic Analysis.



Calculating Growth rates

Suppose we are given two values of GDP in different years and would like to calculate the growth rate of GDP between these two years

We can start with the growth rate formula from a previous slide and then solve for $\bar{g}$:

$$y_{t+1} = y_t(1 + \bar{g})$$

$$\frac{y_{t+1}}{y_t} - 1 = \bar{g}$$

$$\frac{y_{2015}}{y_{2005}} - 1 = \bar{g}$$

We can generalize this for any years with *t* periods of separation. For example for a the GDP *t* years after an initial period 0:

$$y_t = y_0(1 + \bar{g})^t$$

$$\frac{y_t}{y_0} = (1+\bar{g})^t$$

$$\left(\frac{y_t}{y_0}\right)^{\frac{1}{t}} - 1 = \bar{g}$$

$$\left(\frac{y_{2015}}{y_{2005}}\right)^{\frac{1}{10}} - 1 = \bar{g}$$

Growth Rate practice questions

1) If per capita GDP in 2018 was $900, in 2019 was $1,000, and in 2020 was $1,200, the growth rate of per capita GDP between 2018 and 2020 was about 33.3%.

$$\frac{y_{2020} - y_{2018}}{y_{2018}} = \frac{1200-900}{900} = \frac{1}{3} = 0.333$$

(33.3%)

2) If the population of Romania was about 22 million in 2010 and the average population growth rate was 0.2 percent, then Romania's "initial" population, in 1970, was about 20.31 million.

$\bar{g} = 0.002$

$$\bar{g} = \left(\frac{P_t}{P_0}\right)^{\frac{1}{t}} - 1 = \left(\frac{P_{2010}}{P_{1970}}\right)^{\frac{1}{40}} - 1$$

$$= \left(\frac{22}{P_{1970}}\right)^{1/40} - 1 = 0{,}002$$

$$= 1.0832 = \frac{22}{P_{1970}} \Rightarrow P_{1970} = 20.31$$

3) If a country's GDP is growing at 1% per year. How long would it take GDP to double? Answer using both the rule of 70 and also provide a precise calculation.

$$\frac{70}{\bar{g}} \approx \frac{70}{1} = 70 \text{ years}$$

$y_t = 2 \quad y_0 = 1$

$$2 = 1(1+0.01)^t$$

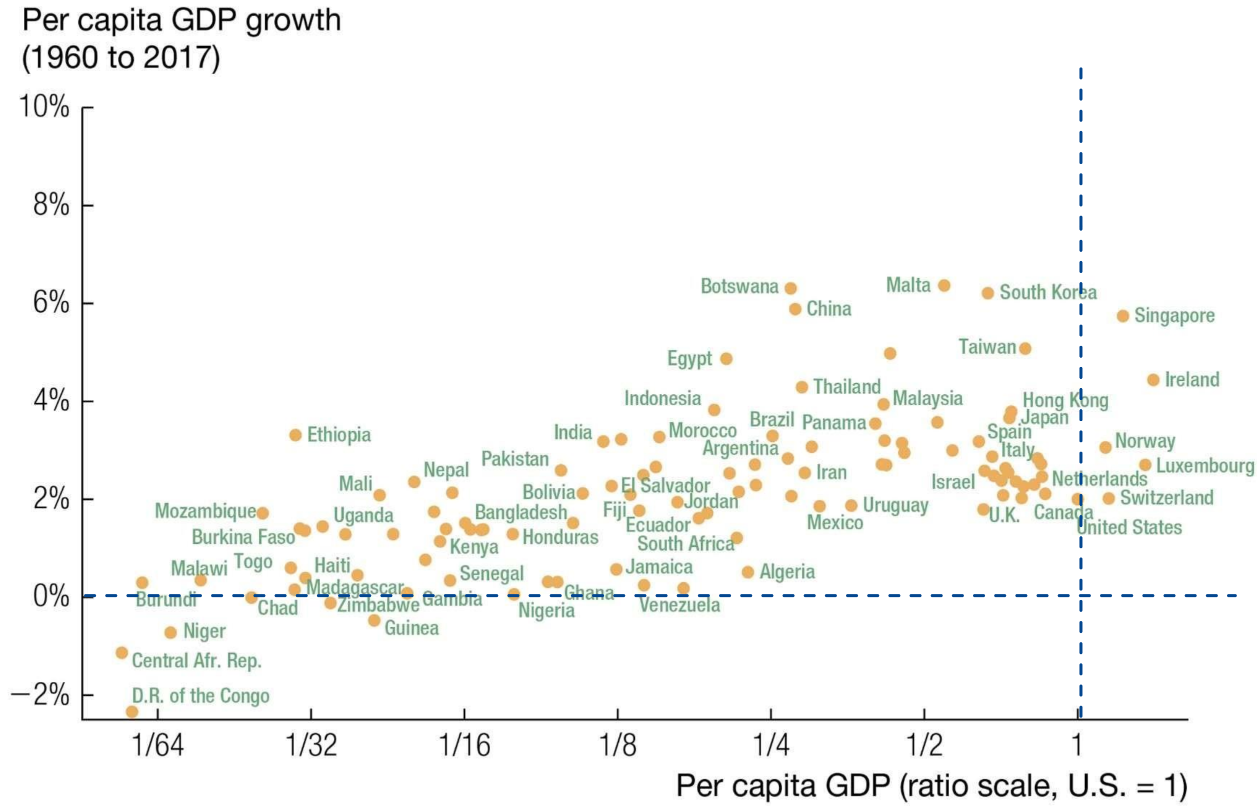
$$\ln 2 = t . \ln(1+0.01) \Rightarrow t = 69.64$$

Broad sample of countries

We can examine a broad sample of countries to get a sense of how countries compare

Note that some countries have negative growth rates over the last 60 years

Small differences in growth rates can result in large differences in GDP per capita over time



Source: Penn World Tables, Version 9.1. See the "Country Snapshots" file, snapshots.pdf, available from the author's web page for these data. The level of per capita GDP is taken from the year 2017 and is normalized so that the U.S. value equals 1.

Several rules regarding operators for growth rates

- Growth rates of ratios, products, and powers follow several rules:

For example: $z = \frac{x}{y} \quad \rightarrow \quad g_z = g_x - g_y$

For example: $z = xy \quad \rightarrow \quad g_z = g_x + g_y$

For example: $z = x^a \quad \rightarrow \quad g_z = a g_x$

Example:

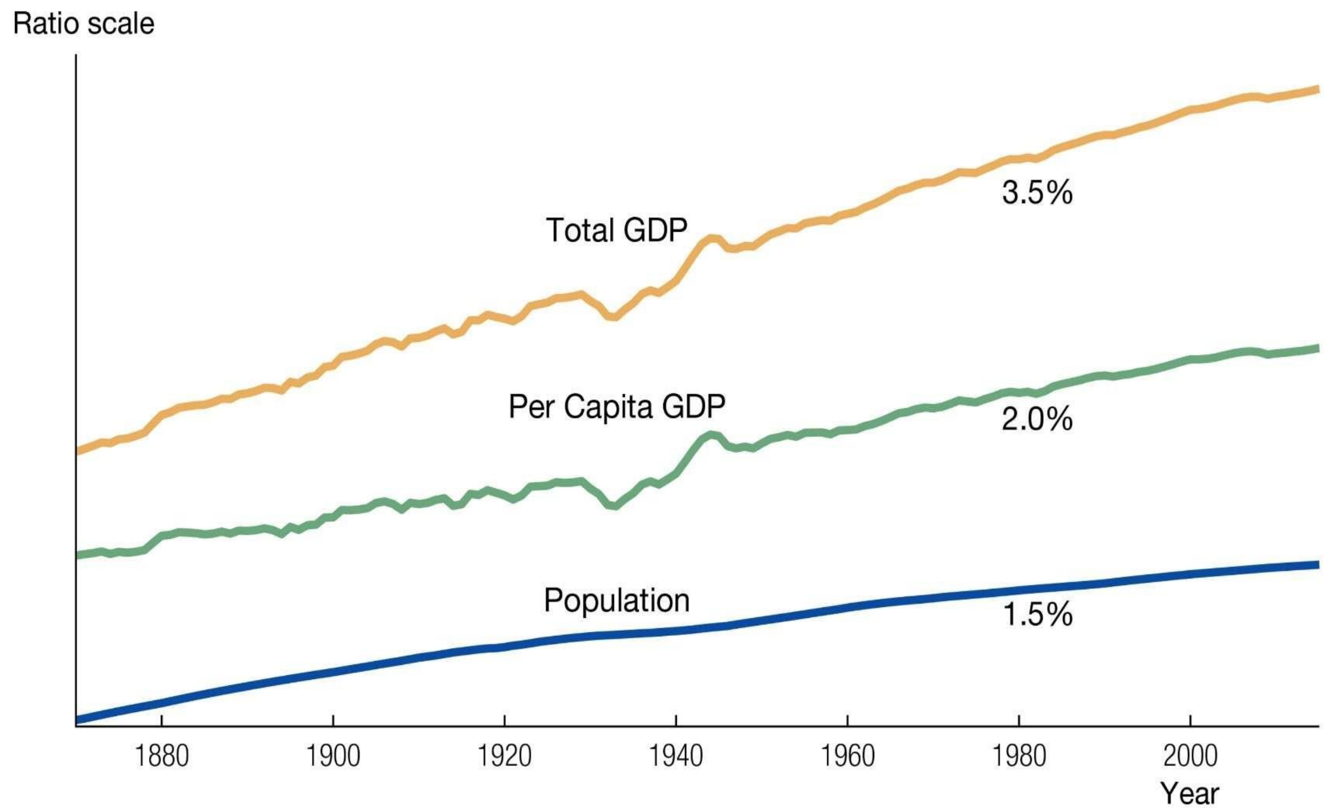
$Y = A L^{1/3} K^{2/3}$

$g_Y = g_A + \frac{1}{3} g_L + \frac{2}{3} g_K$

US GDP, Per Capita GDP, and Population

GDP per capita is equal to GDP/population

Therefore the growth rate of GDP per capita is equal to the growth rate of GDP minus the growth rate of the population



Sources: Angus Maddison, "Statistics on World Population, GDP and Per Capita GDP, 1 AD–2006 AD," and the Bureau of Economic Analysis.

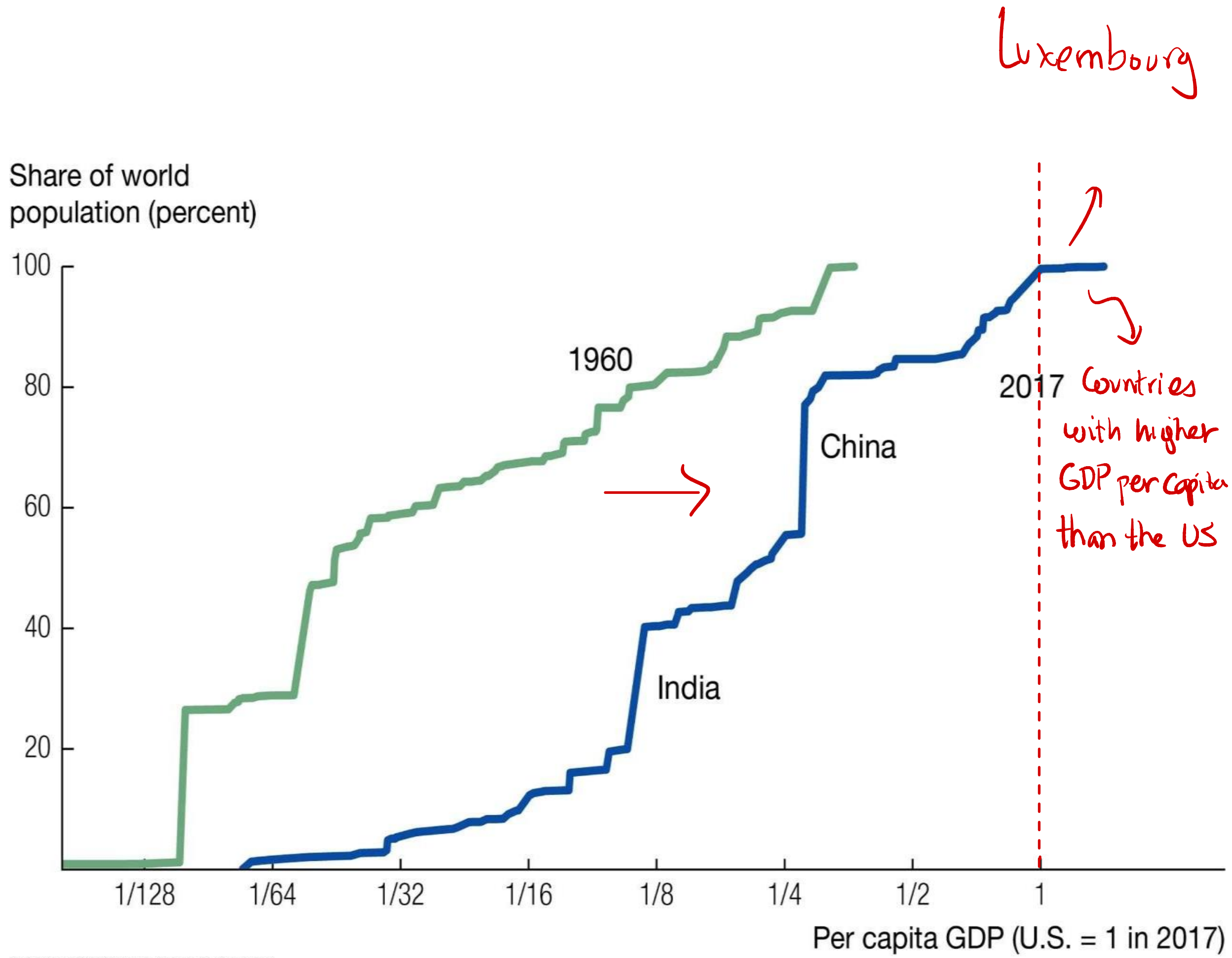
% Total GDP − % Population = % Per Capita GDP

World Economic Growth and Poverty

In the graph on the right, the share of world population is plotted on the vertical axis, and the income relative to the US on the horizontal axis

Shift to the right from 1960 to 2017 signals that more people are earning incomes closer to those of the US

Economic growth has led to a decrease in the percentage of the world population living in poverty



Practice question on growth rates

Express the following expressions in terms of growth rates:

($\bar{A}$ is a constant)

$$y_t = \bar{A} k_t^{1/3} l_t^{2/3} \qquad g_y = \cancel{g_A}^{\,0} + \frac{1}{3} g_k + \frac{2}{3} g_l$$

$$z_t = k_t^a l_t^{-a} \qquad g_z = a g_k - a g_l = a(g_k - g_l)$$

$$c_t = y_t^a w_t^b \qquad g_c = a g_y + b g_w$$

Costs of Economic Growth

There are certainly many benefits to economic growth, including:

- Improvements in health
- Higher incomes
- Increase in the variety of goods and services available

- But over time more and more emphasis has been placed on the costs as well:
  - Environmental problems – pollution, depletion of natural resources, climate change
  - Income inequality across and within countries
  - Loss of certain types of jobs – robots in assembly lines, self-checkout stands, etc.

→ Other relevant economic features Not accounted for when looking only at GDP growth.

Conclusion

- Economic growth is a relatively recent phenomenon in the course of history
- Several hundred years ago, differences in income across countries varied by a factor of 2 to 3, while today they can vary by a factor of over 100
- The reason for this is variation in growth rates of GDP

Small but persistent differences in growth can make countries differ substantially over time.

- Economic growth in highly populated countries like China and India has led to a decrease of the percentage of individuals in the world living in poverty
- While economic growth has brought about several positive changes in people's lives, there are also costs that need to be taken into account

GDP is the most important macroeconomic variable
but not the only relevant one