

ECON 5322

Macroeconomic Theory for Applications

Topic 6: Financial Frictions and the Macroeconomy

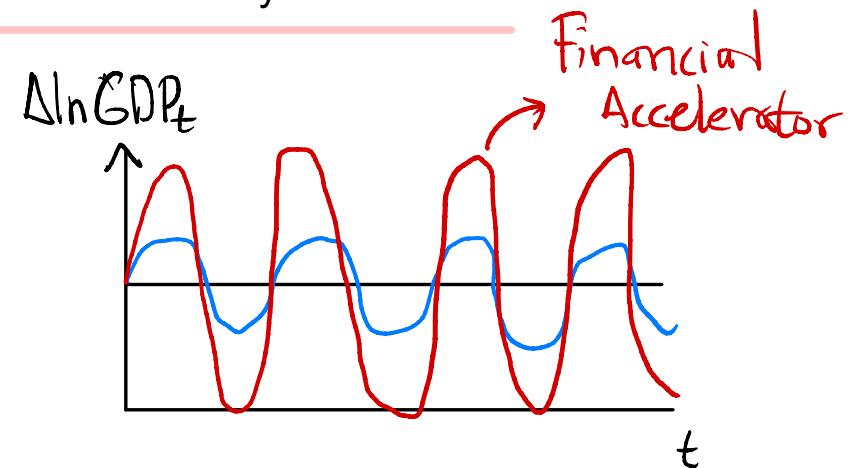
Financial Frictions: An Overview

We've covered informational frictions and nominal rigidities arising from monopolistic frictions (market power). Another frictions that have shown to be really important are the financial frictions

Just like monopolistic power generated sticky prices which were potentially damaging for the economy (i.e., generated a welfare loss relative to a frictionless case) ...

... financial frictions have the potential to generate external effects on the economy that are not desirable; in this case, the effect is that of **augmenting the cycles** of an economy

Such phenomenon is called a **financial accelerator** and we will study it in this unit



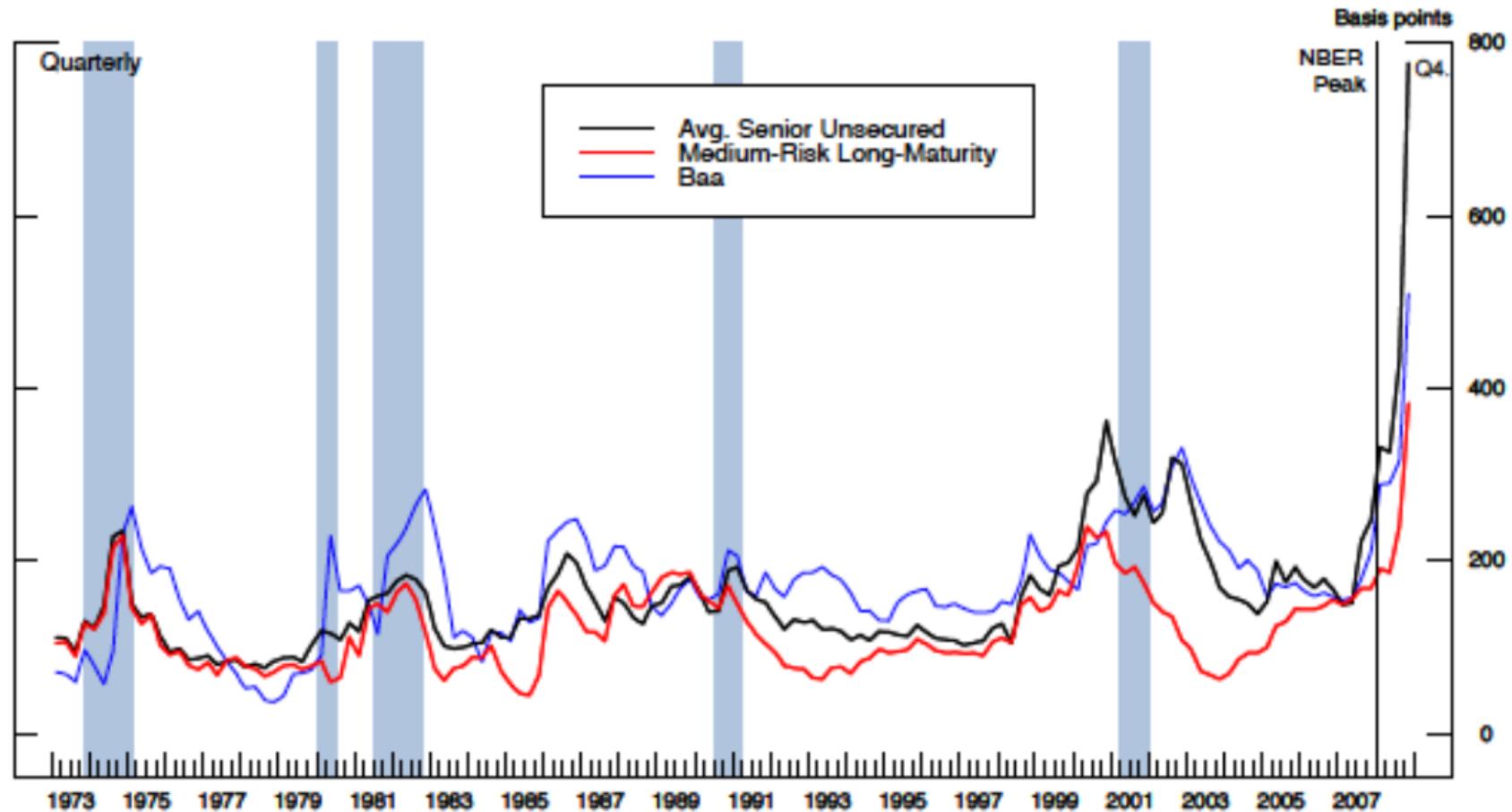
Outline

- Credit channel of monetary policy transmission
- Sources of Financial Frictions
- Models of Financial Frictions

The Grand Scheme of Things:

- Post-2007 Macro: The role of financial market frictions.
- AD side now, with focus on Monetary Policy transmission mechanism
- **Modigliani and Miller (1958):** Irrelevant “how” firms and projects are funded; no reason for financial flows to follow a cyclical pattern. Empirics say otherwise.
 - **Earlier wave:** Financial accelerator and credit cycle framework (Townsend, 1979; Diamond and Divbig, 1983, Bernanke and Gertler, 1989; Bernanke, Gertler and Gilchrist, 1999, Kiyotaki and Moore, 1997; Holstrom and Tirole, 1997)
 - **DSGE framework (w/ NK sloped-AS):** dynamics, financial shocks, credit crunches, volatility feedback, instability, multiple-equilibria, contagion, crises.
- **Emphasis:** Financial Frictions Amplify the Business Cycles
(more persistence, volatility feedback spillovers)
- This topic won the Nobel prize in economics in 2022!

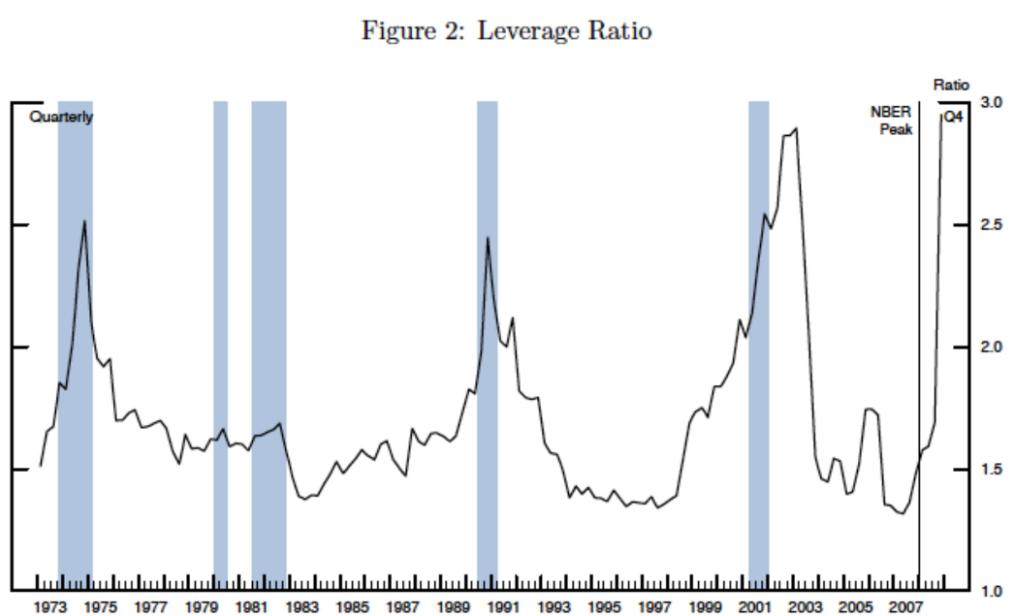
Bond Spreads and Business Cycle (Gilchrist, Ortiz, Zakrajsek (2009), "Credit Risk and the Macroeconomy: Evidence from an Estimated DSGE Model")



NOTE: The black line depicts the average credit spread for our sample of 5,269 senior unsecured corporate bonds; the red line depicts the average credit spread associated with very long maturity corporate bonds issued by firms with low to medium probability of default (see text for details); and the blue line depicts the standard Baa credit spread, measured relative to the 10-year Treasury yield. The shaded vertical bars denote NBER-dated recessions.

Credit Flows are Highly Cyclical

Figure 2: Leverage Ratio



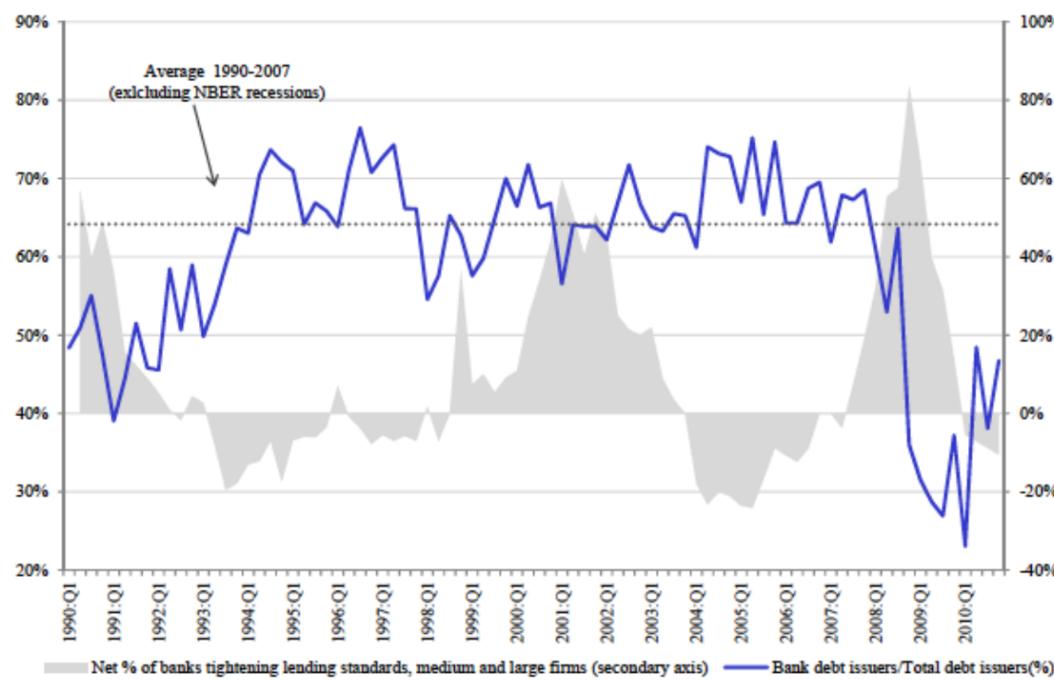
=> Financial System Augments/
Amplifies the Business Cycle

NOTE: The black line depicts the time-series of the cross-sectional averages of the leverage ratio for U.S. nonfinancial corporations. Leverage is defined as the ratio of the market-value of the firm's total assets (V) to the market-value of the firm's common equity (E), where the market-value of the firm's total assets is calculated using the Merton-DD model (see text for details). The shaded vertical bars denote NBER-dated recessions.

Bank Debt and Lending Standards:
(Becker and Ivashina, JME, 2014)

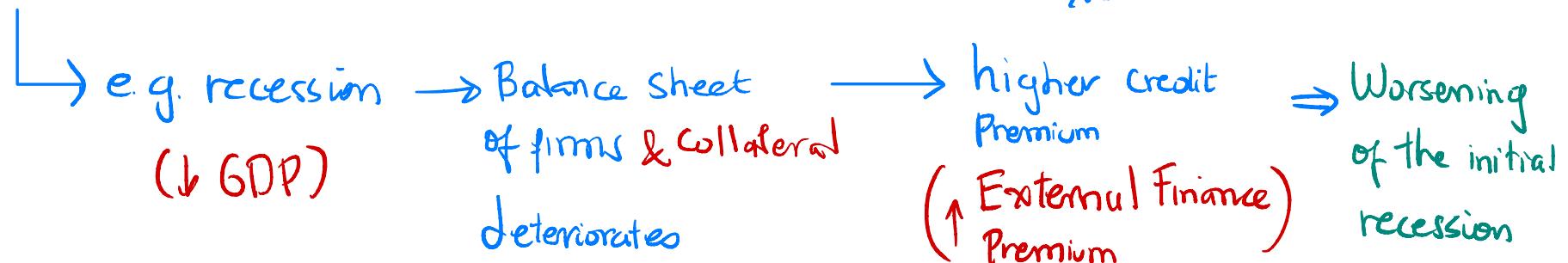
Bank tightens lending standards during recession

(for investment: when it rains, it pours)



Conceptual Overview

- Not all sources of financing are the same: bank vs. non-bank, internal vs. external (so Modigliani and Miller's irrelevance theorem does not hold)
- Heterogeneity: borrowers vs. lenders; borrowers have different vulnerability to credit conditions.
- **Frictions:** Agency Costs from imperfect information or limited pledgeability
=> distorted difficulty of access to funding: borrowing constraint, price wedges
- Concepts of multiple equilibria, liquidity constraints, credit rationing, collateral constraints
- Investment (real economy) sensitive to “accelerator” variables: net worth, cash flow
 - rationalizes data feature shown in Intro: I is more volatile than Y and very pro-cyclical.
- Agency Costs vary counter-cyclically.



Friction in Financing of Physical Capital

- Separation between savers vs. Investors or Entrepreneurs
 - The first have funds (but no ideas), the latter have ideas (and no money)
- Borrowers and Lenders have conflicting interests: How to mitigate? *Costly State Verification*
 - In the lending process, how do we make sure investors “behave”?
 - for example, how to guarantee that a lender does not default if he can?
 - How to factor in the cost of debt the information about potential default?
 - If so, what if such information is not observed by whoever is lending?
- What is the role of financial intermediaries? (banks) → *Gertler & Karadi (2011)*
 - *Costly enforcement*
- Macroeconomic implications: such frictions affect the cost of debt which in turn affects aggregate investment and output

Credit Channel Mechanism:

How Monetary Policy, via interest rates, affects the economy and output

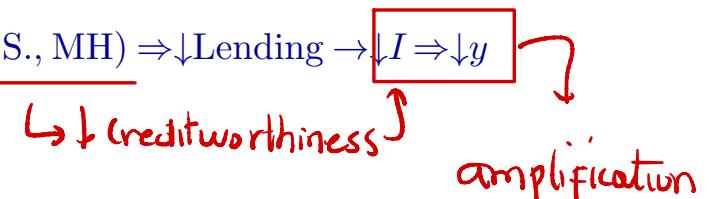
- Credit market imperfection (friction) magnifies the typical transmission mechanism of finance (interest and asset prices' changes) to the economy.
- Co-movements of External Finance Premium (EFP) with Monetary Policy.

Standard MP transmission channel: $\downarrow m \rightarrow \uparrow i \rightarrow \downarrow (I, C, ER) \Rightarrow \downarrow y$ Standard mechanism

Amplification:

$\downarrow m \rightarrow \downarrow (\pi^e, \text{cash flow}) \rightarrow \Delta \text{borrowers balance sheet} + \text{asym. info (Adv.S., MH)} \Rightarrow \downarrow \text{Lending} \rightarrow \boxed{\downarrow I} \Rightarrow \downarrow y$

Policy implications: Justification for financial stability policies
(e.g., capital requirements, FX reserve requirements among others.)



What are the Frictions?: Dead weight (extra) cost of external borrowing

- Imperfect information (asymmetric info b/w lenders and borrowers
 - lemon's premium, moral hazard
- Costly contract enforcement
- Result: imperfectly collateralized debt, cost of monitoring by lender.
- These frictions lead to change of EFP (External Finance Premium)

Credit Market Imperfections

- Amplify and propagate traditional mechanism by endogenous changes in the EFP.
(enhancement, not a separate channel)
- EFP: extra cost of raising external funds (by issuance of equity or debt) relative to the cost of internal funds
 - Size of EFP reflects imperfections in the credit market
- Wedge between expected return received by lenders and cost faced by potential borrowers.
- Monetary Policy change of the open-market interest rates changes the EFP, magnifying the impact of MP on cost of borrowing
- Financial Accelerator (amplification of cycles mechanism)

Why Should Monetary Policy affect EFP in Credit Markets?

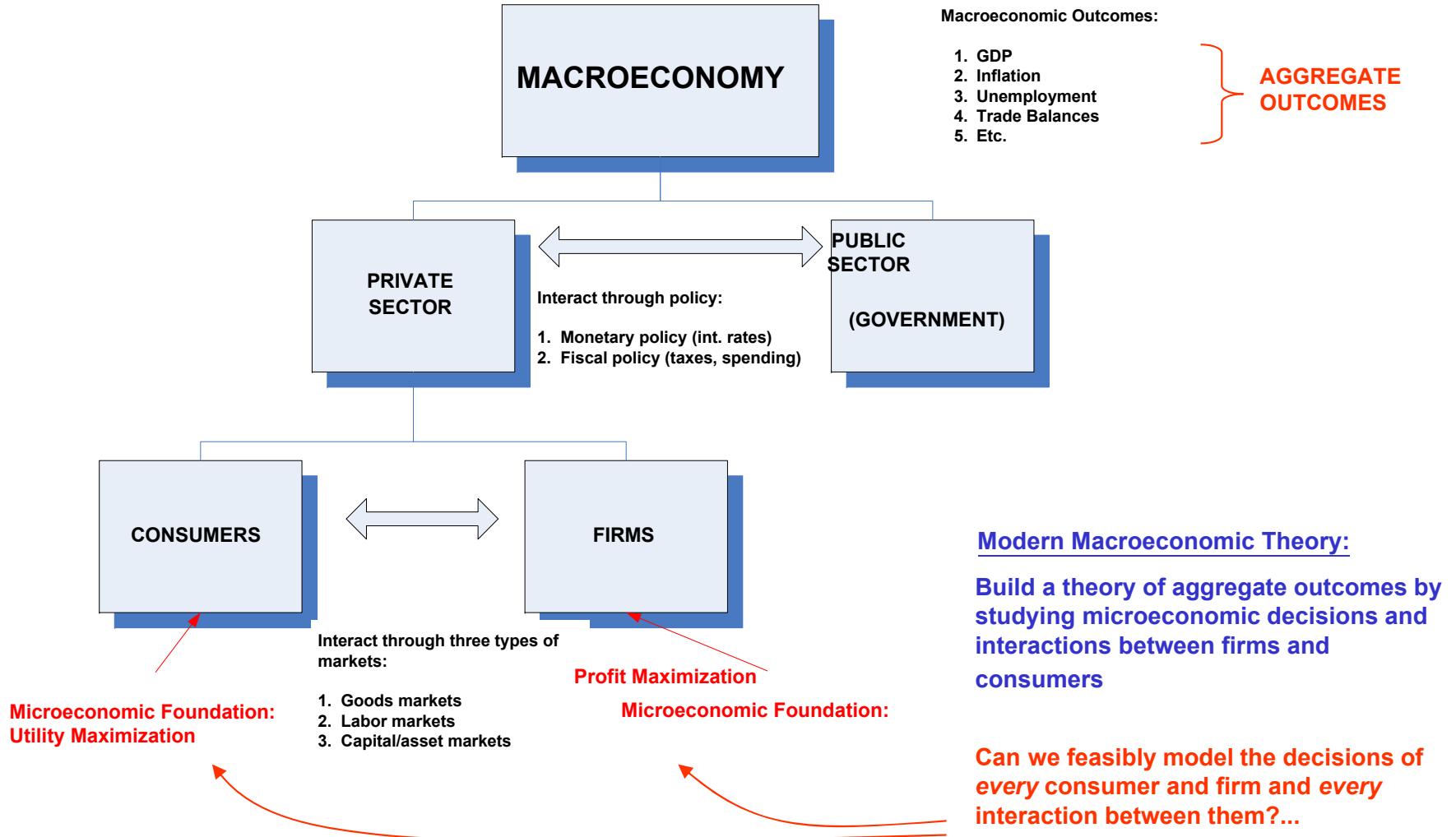
Models emphasizing deviation from perfect costless financial intermediaries:

- Asymmetric information, costly contract enforcement
- Bank lending channel: Monetary Policy effect on the supply of loans by depository institutions
- Consequences: Collateral constraints, credit rationing, liquidity shortages
- Balance Sheet Channel: MP's effect on borrower's balance sheets and income statements, e.g., net worth, cashflows, liquid assets

Financial Accelerator Framework

- “Financial accelerator” framework
 - The most widely-used and applied framework in macroeconomic theory and policy for thinking about financial markets
 - Developed in series of studies by Bernanke, Gertler, and Gilchrist in 1980’s and 1990’s
- Popular-press language
 - “Financial accelerator”
 - “Financial feedback loops”
 - “Loan spirals”
- Describes well many of the financial-macroeconomic linkages underpinning the dynamics of
 - Great Depression
 - Great Recession
- Will develop idea in context of firm theory
- Can also develop idea in context of consumer theory.
 - “Credit constraint” analysis of consumption/savings decisions.

Building Blocks of an Economy



Outline of Framework

Major ideas underlying Financial Accelerator Framework

1. Firms' **financial** assets (i.e., stocks and bonds) matter for their ability to purchase **physical** assets (i.e., machines and equipment)
2. Market **prices** of financial assets matter for **firm financing constraints**
3. Government regulation affects the linkage between financial markets and real (i.e., goods and physical capital) markets through financing constraints

Outline of Framework (cont.)

Four Building Blocks of the Financial Accelerator Framework

1. Two-Period Model of Firm Profit Maximization

- Enriched to allow for both **physical** assets (machines and equipment) and **financial** assets (stocks and bonds)

New

2. Financing Constraint – conceptually, the key building block

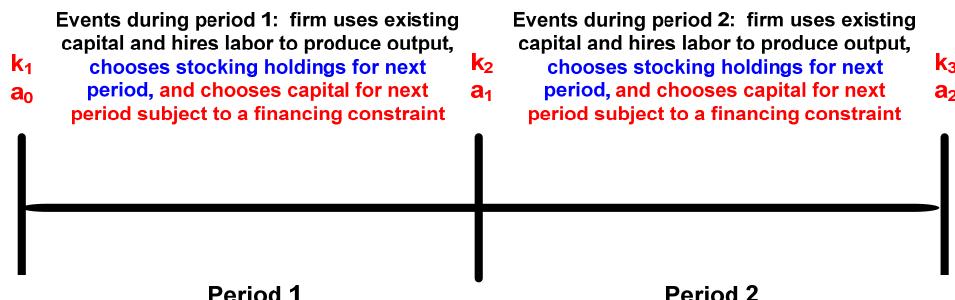
- Quantity of **physical capital** firms can purchase depends on the market value (i.e., price x quantity) of their financial assets
- Reflects market and regulatory structures designed to mitigate **informational asymmetries**
- (Basic theory of firms features no constraints of this type on firm profit maximization)

3. Government Regulation/Oversight of Financial Relationships

4. Relationship between Firm Profits and Dividends

Enriching the Basic Firm Theory

- **Timeline of events**



- Notation

- k_2 : capital used for production in period 2 (decided upon in period 1)
 - n_2 : labor used for production in period 2
 - w_2 : real wage rate for labor in period 2 ($w_2 = W_2/P_2$)
 - i : nominal interest rate (between period 1 and period 2)
 - P_2 : nominal price of output produced and sold by firm in period 2
AND nominal price of one unit of capital bought by the firm in period 2 for use in period 3
 - a_1 : real wealth (stock) holdings at beginning of period 2/end of period 1
 - S_2 : nominal price of a unit of stock in period 2
 - D_2 : nominal dividend paid in period 2 by each unit of stock held at the start of period 2
 - π_2 : net inflation rate between period 1 and period 2 (recall: $\pi_2 = P_2/P_1 - 1$)



π_2 : Inflation

Rates of Return

- “Interest rates” can be defined for any type of asset
 - There is no single interest rate in the economy
- Interpret/understand the two types of “interest rates” that co-exist in this richer theory of firm profit maximization

- i : nominal interest rate on bonds
 - Recall

$$1+i = \frac{1}{P_1^b} \quad \xleftarrow[\text{as}]{\text{can rewrite}} \quad i = \frac{1}{P_1^b} - 1 \quad \xleftarrow[\text{real interest rate}]{\text{express as}}$$

REAL INTEREST RATE ON
GOVERNMENT BONDS: A
“RISKLESS” ASSET

$$1+r = \frac{1+i}{1+\pi}$$

- Thus can think of bonds (one type of financial asset) as being in the background of the analysis
- i^{STOCK} : nominal return on stock – i.e., “interest rate on stock”

- Define according to

$$1+i^{STOCK} = \frac{S_2 + D_2}{S_1} \quad \xleftarrow[\text{as}]{\text{can rewrite}} \quad i^{STOCK} = \frac{S_2 + D_2}{S_1} - 1 \quad \xleftarrow[\text{real interest rate}]{\text{express as}}$$

REAL INTEREST RATE ON
STOCKS: A “RISKY” ASSET

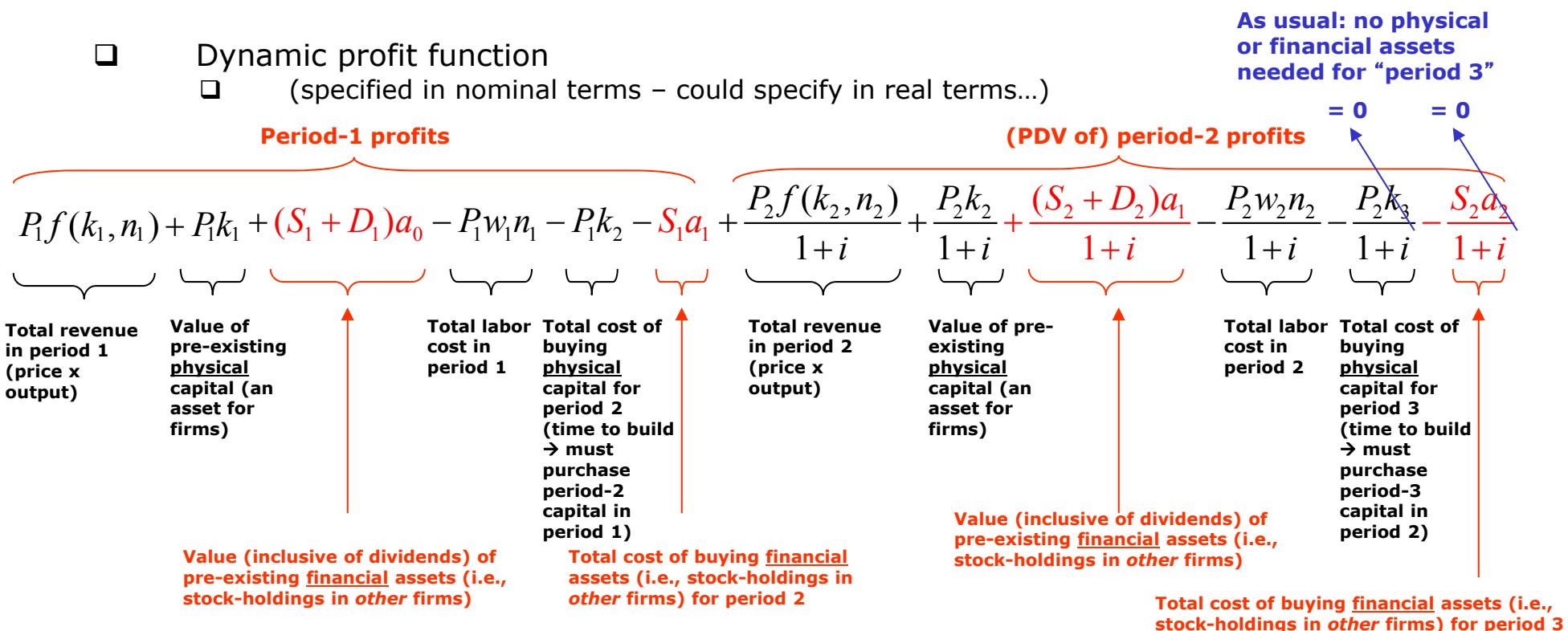
$$1+r^{STOCK} = \frac{1+i^{STOCK}}{1+\pi}$$

- Measures the net dollar return (in period 2) on one share of stock (whose purchase price was S_1 in period 1)
- Can distinguish two measures of **real** interest rates in this framework

FIRM PROFIT FUNCTION

- A dynamic profit maximization problem
 - Because firm exists for both periods
 - All analysis conducted from the perspective of the very beginning of period 1
 - → Must consider present-discounted-value (PDV) of lifetime (i.e., two-period) profits

- Dynamic profit function
 - (specified in nominal terms – could specify in real terms...)



FIRM PROFIT FUNCTION

- A dynamic profit maximization problem
 - Because firm exists for both periods
 - All analysis conducted from the perspective of the very beginning of period 1
 - → Must consider present-discounted-value (PDV) of lifetime (i.e., two-period) profits
 - Dynamic profit function
 - (specified in nominal terms – could specify in real terms...)

As usual: no physical or financial assets needed for “period 3”

INFORMATIONAL ASYMMETRIES

- “Informational asymmetries” pervasive in borrowing/lending relationships
- Borrower (whether consumer, firm, or financial institution) *much* more likely to know his own ability/willingness to repay a loan
 - Lenders only know little about the “quality” or “trustworthiness” of a borrower
 - **Asymmetry of information – cannot be eliminated**
- To mitigate **consequences** of informational asymmetries, lenders often require borrower to have a stake in “succeeding” in the project for which funds are being borrowed
 - **Consumers**
 - e.g., down payment on house purchase
 - e.g., down payment on car purchase
 - If stop making payments on house or car
 - Borrower loses down payment (in addition to the car or house...)...
 - **Affects individual’s incentives before borrowing**
 - **Total amount of loan (typically) depends on individual’s collateral**
 - **Firms**
 - Capital investment (factories, technology upgrades, etc) outlays
 - Payroll outlays
 - Financing inventories
 - **Total amount of loan (often) depends on firm’s collateral**
- Financial institutions: borrow in order to make (big) loans
 - By raising “small” quantities of funds from many different sources

“Working capital”



FINANCING CONSTRAINT

- Capture this idea through a **financing constraint** on firm's ability to purchase capital between period 1 and period 2
- Financing constraint
 - Total expenditures on period-1 physical investment must be equal to market value of firm's financial (stock) holdings
 - (Technically, smaller than or equal to, so an inequality constraint...but will only analyze constraint with equality)

$$P_1 \cdot \text{inv}_1 = S_1 \cdot a_1$$



inv₁ = k₂ - k₁ (investment is change in quantity of physical capital)

$$\underline{P_1 \cdot (k_2 - k_1) = S_1 \cdot a_1}$$

- Important: a_1 appears in the financing constraint, not a_0
 - Idea this assumption captures: firm will purposefully change the value of financial assets it holds in order to affect the quantity of physical investment in which it can engage
 - (From the perspective of beginning of period 1, a_1 has not yet been chosen, whereas a_0 is pre-determined)

GOVERNMENT OVERSIGHT OF FINANCIAL MARKETS

- Government oversight of informational asymmetries in borrower/lender relationships
 - Filing of proper documentation
 - Full disclosure (“truth-in-lending”) laws
 - Direct lending in some markets
 - ...
- Capture government **Regulation** of financial dealings in our framework in very simple way
 - Firm can borrow up to a multiple **R** of the market value of its financial assets for physical investment purposes
 - e.g., if government regulates that expenditures on investment cannot be larger than 5 times market value of financial assets, **$R = 5$ is the leverage ratio**
- Will think of **R** as government regulation...
 - ...but can and does also reflect market and institutional arrangements

GOVERNMENT OVERSIGHT OF FINANCIAL MARKETS

- ❑ Capture this idea through a **financing constraint** on firm's ability to purchase capital between period 1 and period 2
- ❑ Financing constraint
 - ❑ Total expenditures on period-1 physical investment must be equal to market value of firm's financial (stock) holdings
 - ❑ (Technically, smaller than or equal to, so an inequality constraint...but will only analyze constraint with equality)

$$P_1 \cdot inv_1 = S_1 \cdot a_1$$

↓

$$inv_1 = k_2 - k_1 \quad (\text{investment is } \underline{\text{change in}} \text{ quantity of physical capital})$$

↓

$$P_1 \cdot (k_2 - k_1) = S_1 \cdot a_1$$

↓

Government regulation R

$P_1 \cdot (k_2 - k_1) = R \cdot S_1 \cdot a_1$

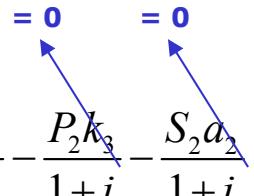
Impose this financing constraint on firm profit maximization problem

↳ financial regulation
(Policy tool)

FINANCIAL ACCELERATOR FRAMEWORK

□ Four Building Blocks of the Financial Accelerator Framework

1. Firm Profit Function

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1) a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2) a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i} - \frac{S_2 a_2}{1+i}$$


2. Financing Constraint

$$P_1 \cdot (k_2 - k_1) = S_1 \cdot a_1$$

3. Government Regulation of Financial Relationships (imposition of R on financing constraint)

$$P_1 \cdot (k_2 - k_1) = R \cdot S_1 \cdot a_1$$

4. Relationship between firm profits and dividends

LATER

FIRM PROFIT MAXIMIZATION

Maximize two-period profits

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1) a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2) a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i} - \frac{S_2 a_2}{1+i}$$

Subject to financing constraint

$$P_1 \cdot (k_2 - k_1) = R \cdot S_1 \cdot a_1$$

↳ Regulation

Construct Lagrangian

$$\mathcal{L} = P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1) a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2) a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} + \lambda [R \cdot S_1 \cdot a_1 - P_1 \cdot (k_2 - k_1)]$$

Lagrange multiplier on financing constraint

CRUCIAL OBSERVATION: in basic firm theory, value of this multiplier is....

$\lambda = 0$ i.e., there was no financing constraint!

SOON: will think about what regulatory and/or market features make the financing constraint effectively “disappear” (i.e., cause $\lambda = 0$)

FIRM PROFIT MAXIMIZATION

$$\mathcal{L} = P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1) a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2) a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} + \lambda [R \cdot S_1 \cdot a_1 - P_1 \cdot (k_2 - k_1)]$$

□ FOCs with respect to n_1, n_2

Identical
except for
time
subscripts

with respect to n_1 :

$$\cancel{P_1 f_n(k_1, n_1)} - \cancel{P_1 w_1} = 0$$

with respect to n_2 :

$$\cancel{\frac{P_2 f_n(k_2, n_2)}{1+i}} - \cancel{\frac{P_2 w_2}{1+i}} = 0$$

Equation 1

Equation 2

Standard input
FOCs

- Financing constraint does not affect profit-maximizing choices of labor hiring... ...thus same analysis from basic theory of labor demand curve, etc, applies
- GIVEN the PARTICULAR components of spending that financing constraints affect!

□ FOCs with respect to k_2, a_1

- The interesting aspects of this framework
- The heart of the accelerator mechanism

FIRM PROFIT MAXIMIZATION

$$\begin{aligned}
 & P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1) a_0 - P_1 w_1 n_1 - \cancel{P_1 k_2} - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \cancel{\frac{P_2 k_2}{1+i}} + \frac{(S_2 + D_2) a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} \\
 & + \lambda [R \cdot S_1 \cdot a_1 - P_1 \cdot (k_2 - k_1)]
 \end{aligned}$$

- FOCs with respect to k_2, a_1

with respect to k_2 :
$$-P_1 + \frac{P_2 f_k(k_2, n_2)}{1+i} + \frac{P_2}{1+i} - \lambda P_1 = 0$$
 Equation 3

with respect to a_1 :
$$-S_1 + \frac{S_2 + D_2}{1+i} + \lambda \cdot R \cdot S_1 = 0$$
 Equation 4

- Analysis of Equation 4 in isolation
 - Answers the central question: under what conditions does $\lambda = 0$?
 - Reveals how stock market returns affect financing constraints
 - Reveals how government regulation affects financing constraints
- Analysis of Equation 3 and Equation 4 jointly
 - Demonstrates how/why financial market prices (i.e., stock prices/returns) matter for macroeconomic activity
 - The financial accelerator effect

Collateral value matters for macroeconomic outcomes

WHY IS FINANCING A *CONSTRAINT*?

$$\begin{aligned} -S_1 + \frac{S_2 + D_2}{1+i} + \lambda \cdot R \cdot S_1 &= 0 && \text{Equation 4} \\ &\quad \downarrow \text{Solve for } \lambda \\ \lambda &= \left[S_1 - \frac{S_2 + D_2}{1+i} \right] \cdot \frac{1}{R \cdot S_1} \\ &\quad \downarrow \text{Pull } 1/S_1 \text{ term inside} \\ \lambda &= \left[1 - \frac{S_2 + D_2}{S_1} \cdot \frac{1}{1+i} \right] \cdot \frac{1}{R} \\ &\quad \downarrow \text{Multiply and divide second term in parentheses by } P_1 \text{ and } P_2 \\ \lambda &= \left[1 - \frac{S_2 + D_2}{S_1} \cdot \frac{P_1}{P_2} \cdot \frac{P_2}{P_1} \cdot \frac{1}{1+i} \right] \cdot \frac{1}{R} \\ &\quad \downarrow \text{Use definition of inflation, } 1 + \pi_2 = P_2 / P_1, \text{ and regroup terms} \\ \lambda &= \left[1 - \frac{S_2 + D_2}{S_1} \cdot \frac{P_1}{P_2} \cdot \frac{1 + \pi_2}{1+i} \right] \cdot \frac{1}{R} \end{aligned}$$

WHY IS FINANCING A CONSTRAINT?

$$\frac{B_2 - P_1}{P_1} = \pi_2$$

\sim

$$\frac{P_2 - 1}{P_1} = \pi_2$$

\sim

$$\frac{P_2}{P_1} = 1 + \pi_2$$

$$\lambda = \left[1 - \frac{S_2 + D_2}{S_1} \cdot \frac{P_1}{P_2} \cdot \frac{1 + \pi_2}{1 + i} \right] \cdot \frac{1}{R}$$

$$\lambda = \left[1 - \frac{1 + i^{\text{STOCK}}}{1 + \pi_2} \cdot \frac{1 + \pi_2}{1 + i} \right] \cdot \frac{1}{R}$$

$$\lambda = \left[1 - \frac{1 + r^{\text{STOCK}}}{1 + r} \right] \cdot \frac{1}{R}$$

$$\frac{\lambda + r - (\lambda + r^{\text{stock}})}{1 + r}$$

rearrange

$$\lambda = \left[\frac{r - r^{\text{STOCK}}}{1 + r} \right] \cdot \frac{1}{R}$$

The Lagrange multiplier on firm's financing constraint

The friction will not matter if:

$$\begin{cases} r - r^{\text{stock}} = 0 & (\text{No risk premium}) \\ R \text{ is huge (s.t. } \lambda \rightarrow 0) \\ \hookrightarrow \text{Policy} \end{cases}$$

$$\begin{array}{c|c} & \frac{S_2 - S_1}{S_1} & \frac{D_2 + S_2 - S_1}{S_1} \\ & \frac{S_2 + D_2}{S_1} - 1 & \frac{S_2 + D_2}{S_1} \end{array}$$

(from previous page)

Use definition of "nominal interest rate on stock", $1 + i^{\text{STOCK}} = (S_2 + D_2) / S_1$ Use definition of inflation, $1 + \pi_2 = P_2 / P_1$

Fisher equation for stock: $1 + r^{\text{STOCK}} = (1 + i^{\text{STOCK}}) / (1 + \pi_2)$

Fisher equation for bonds: $1 + r = (1 + i) / (1 + \pi_2)$

WHY IS FINANCING A CONSTRAINT?

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r} \right] \cdot \frac{1}{R}$$

The Lagrange multiplier on firm's financing constraint

- Basic firm theory:
 - No financing constraint
 - Can interpret basic firm theory analysis as featuring $\lambda = 0$
 - Interpretation: under "normal market conditions," financing constraints don't matter (much...)
 - Interpret "normal market conditions" as steady state
-

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1) a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2) a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i}$$
$$+ \cancel{R \cdot S_1 \cdot a_1 - P_1 \cdot (k_2 - k_1)} = 0$$

- If $\lambda = 0$ (i.e., "normal market conditions," aka steady state)
 - Labor demand decisions unaffected by financial market conditions
 - Capital demand decisions unaffected by financial market conditions
- Key question: what causes $\lambda = 0$?

WHY IS FINANCING A CONSTRAINT?

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r} \right] \cdot \frac{1}{R}$$

The Lagrange multiplier on firm's financing constraint

- Two conditions for $\lambda = 0$
- Market returns on risky assets **equal** returns on riskless assets

Can think of both government bonds (financial assets) and machines & equipment (physical assets) as "riskless": you (pretty much...) know what you're going to get from them.

- Risky assets: stocks
- Riskless assets
 - Bonds (financial)
 - **Machines and equipment (physical) – most directly relevant for firms' production and sales activity**
 - **Basic firm theory prediction: $r = mpk$**

$$r = r^{STOCK} \longrightarrow \lambda = 0$$

Interpretation: if returns on financial assets are aligned with returns on physical assets, financing constraints "don't matter"

- Government oversight of borrowing/lending relationships very lax

- **The larger is R , the lower is λ**
- **Financing constraint:** $P_1 \cdot (k_2 - k_1) = R \cdot S_1 \cdot a_1$

□ Holding constant market value of financial assets, higher R allows higher k_2

In practice, not literally infinity...

$$R = \infty$$

Market value of financial assets

Interpretation: if government regulations allow high borrowing with little assets, financing constraints "don't matter"

FINANCING CONSTRAINT AND CAPITAL DEMAND

- Suppose $R = 1$ in “steady state” (but keep R in rest of analysis)
 - $R > 1$ is “lax regulation” (because it lowers λ , all else constant)
 - $R < 1$ is “tight regulation” (because it increases λ , all else constant)
 - → Whether or not financing constraint matters (i.e., whether or not $\lambda = 0$) all depends on whether or not $r^{STOCK} = r$ or not

KEY IDEA:

if returns on riskless assets
= returns on risky assets

→ financing constraints
“don’t matter” for firm
production decisions

$$-P_1 + \frac{P_2 f_k(k_2, n_2)}{1+i} + \frac{P_2}{1+i} - \lambda P_1 = 0 \quad \text{Equation 3} \quad (\text{FOC on } k_2)$$

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r} \right] \cdot \frac{1}{R} \quad \text{Equation 4} \quad (\text{FOC on } a_1)$$

- Basic firm theory:
 - Capital demand function derived from Equation 3
 - Idea same as in basic theory...but now complicated by the financing constraint

$$-P_1 + \frac{P_2 f_k(k_2, n_2)}{1+i} + \frac{P_2}{1+i} - \left[\frac{r - r^{STOCK}}{1+r} \right] \frac{1}{R} P_1 = 0$$

↓ Substitute λ from Equation 4 into Equation 3

↓ Rearrange

FINANCING CONSTRAINT AND CAPITAL DEMAND

$$-P_1 + \frac{P_2 f_k(k_2, n_2)}{1+i} + \frac{P_2}{1+i} - \left[\frac{r - r^{STOCK}}{1+r} \right] \frac{1}{R} P_1 = 0 \quad (\text{from previous page})$$

↓
Divide by P_1

$$\frac{P_2 f_k(k_2, n_2)}{P_1(1+i)} + \frac{P_2}{P_1(1+i)} - \left[\frac{r - r^{STOCK}}{1+r} \right] \frac{1}{R} = 1$$

↓
Use definition of inflation, $1 + \pi_2 = P_2 / P_1$

$$\left(\frac{1 + \pi_2}{1+i} \right) f_k(k_2, n_2) + \frac{1 + \pi_2}{1+i} - \left[\frac{r - r^{STOCK}}{1+r} \right] \frac{1}{R} = 1$$

↓
Apply Fisher relation for "riskless" assets

$$\frac{f_k(k_2, n_2)}{1+r} + \frac{1}{1+r} - \left[\frac{r - r^{STOCK}}{1+r} \right] \frac{1}{R} = 1$$

↓
Multiply by $(1+r)$

Marginal product of capital, mpk

$$\underline{f_k(k_2, n_2)} + \underline{1} - \underline{\frac{r - r^{STOCK}}{R}} = \underline{1+r}$$

Suppose $R = 1$ in "steady state" but keep R in the analysis

w/o Fin. Frictions: $f_k = r$ (e.g. Topic 2)

⇒ Fin. Frictions distort equality bw MPK & r

COBB-DOUGLAS PRODUCTION FUNCTION

- Commonly-used functional form in quantitative macroeconomic analysis

$$f(k, n) = k^\alpha n^{1-\alpha}$$

- Describes the empirical relationship between aggregate GDP, aggregate capital, and aggregate labor quite well
- $\alpha \in (0, 1)$ **measures capital's share of output**
 - Hence $(1-\alpha) \in (0, 1)$ **measures labor's share of output**
 - **Interpretation**
 - Relative importance of (either) capital (or labor) in the production process
 - Estimates for U.S. economy: $\alpha \approx 0.3$
 - Estimates for Chinese economy: $\alpha \approx 0.15$ (not (yet) a very capital-rich economy)
- Cobb-Douglas form useful for illustrating factor demands
 - $mpn = f_n(k, n) = (1-\alpha)k^\alpha n^{-\alpha}$
 - $mpk = f_k(k, n) = \alpha k^{\alpha-1} n^{1-\alpha}$

FINANCING CONSTRAINT AND CAPITAL DEMAND

- Firm-level demand for capital **defined** by the relation

*MPK from Cobb-Douglas
P/n function*

Now we solve for r:

$$\begin{aligned}
 r &= \alpha k^{\alpha-1} n^{1-\alpha} - \left[\frac{r - r^{STOCK}}{R} \right] \left(= mpk - \left[\frac{r - r^{STOCK}}{R} \right] \right) \\
 &\quad \downarrow \\
 r &= \alpha k^{\alpha-1} n^{1-\alpha} - \frac{r}{R} + \frac{r^{STOCK}}{R} \\
 &\quad \downarrow \\
 \left[1 + \frac{1}{R} \right] r &= \alpha k^{\alpha-1} n^{1-\alpha} + \frac{r^{STOCK}}{R} \\
 &\quad \downarrow \\
 \left[\frac{R+1}{R} \right] r &= \alpha k^{\alpha-1} n^{1-\alpha} + \frac{r^{STOCK}}{R} \\
 &\quad \downarrow \\
 r &= \left(\frac{R}{R+1} \right) \alpha k^{\alpha-1} n^{1-\alpha} + \frac{r^{STOCK}}{R+1}
 \end{aligned}$$

Solve for r (return on "riskless" physical assets")

r vs K (standard) but with wedges → related to the presence of financial frictions

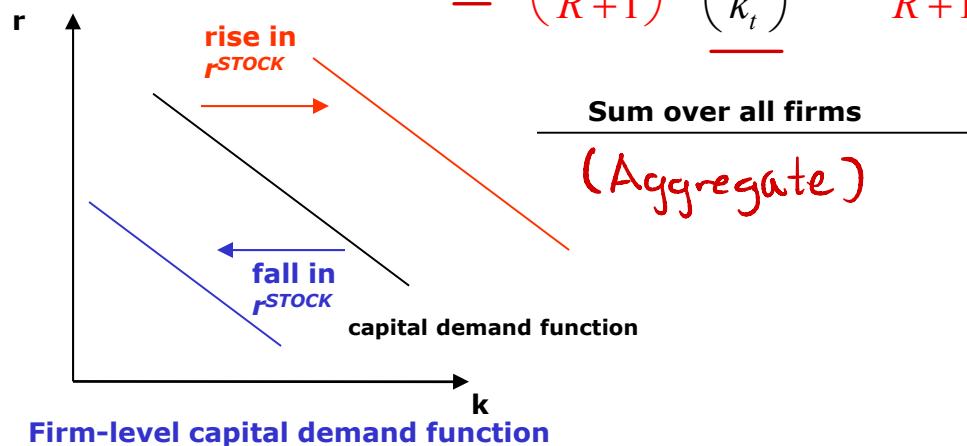
FINANCING CONSTRAINT AND CAPITAL DEMAND

- Firm-level demand for capital **defined** by the relation

$$r = \left(\frac{R}{R+1} \right) \alpha k^{\alpha-1} n^{1-\alpha} + \frac{r^{STOCK}}{R+1}$$

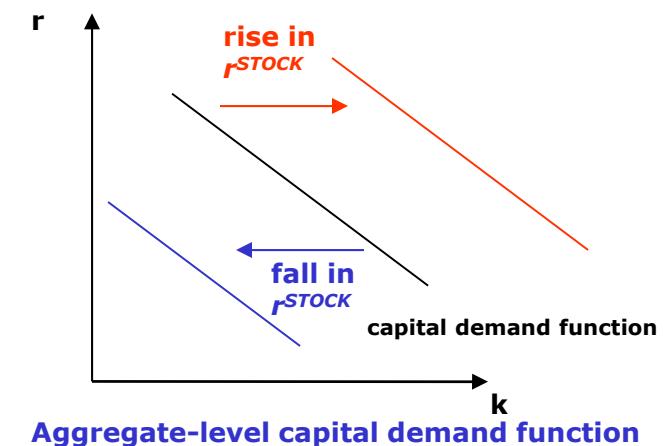
Because exponent ($\alpha - 1$) is a negative number, can move to denominator

$$r = \left(\frac{R}{R+1} \right) \alpha \left(\frac{n_t}{k_t} \right)^{1-\alpha} + \frac{r^{STOCK}}{R+1}$$



Sum over all firms
(Aggregate)

Rise (fall) in return on stock leads to shift out (in) of capital demand function



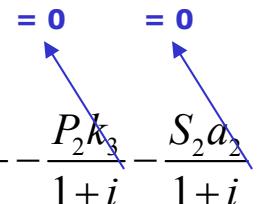
- **Important:** changes in financial market returns shift capital demand (and hence investment demand – recall $inv_t = k_{t+1} - k_t$)
- Basis for the financial accelerator effect
- Basis for understanding the role of financial oversight

FINANCIAL ACCELERATOR FRAMEWORK

□ Four Building Blocks of the Financial Accelerator Framework

1. Firm Profit Function

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1) a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2) a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i} - \frac{S_2 a_2}{1+i}$$



2. Financing Constraint *Investment (w/ Deprecation = 0)*

$$P_1 \cdot (k_2 - k_1) = S_1 \cdot a_1$$

3. Government Regulation of Financial Relationships (imposition of *R* on financing constraint)

$$\underline{P_1 \cdot (k_2 - k_1) = R \cdot S_1 \cdot a_1}$$

4. Relationship between firm profits and dividends

NOW

DIVIDENDS AND PROFITS

- Dividend: payment made by a corporation to its shareholders; the portion of corporate profits paid out to stockholders
 - Corporate dividend policies differ widely across industries and companies
 - Some companies retain most of their profits (to re-invest in ongoing projects)
 - Some industries' dividend policies subject to government regulation
 - Recent average: \approx 35 percent of profits disbursed as dividends
 - Based on recent data collected by U.S. Bureau of Economic Analysis for corporations listed on S&P 500
 - Simplifying assumption for our analytical framework
 - All (100 percent) firm profits distributed as dividends
 - In period t , $D_t = \text{nominal profits}_t$
-

□ Building Block 4: Relationship between firm profits and dividends

$$D_t = P_t \cdot \underline{\text{profit}_t}$$

Nominal
Dividends

Prices level
real profits

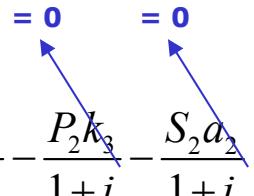
REAL profits of firm in period t

The diagram illustrates the formula $D_t = P_t \cdot \underline{\text{profit}_t}$. A blue horizontal line underlines the term profit_t . Above the formula, a red curved arrow points from the word "Nominal" to the term P_t . Another red curved arrow points from the words "Prices level" to the term P_t . A third red curved arrow points from the words "real profits" to the term profit_t . To the right of the formula, a red double-headed arrow connects the term profit_t to the text "REAL profits of firm in period t ".

FINANCIAL ACCELERATOR FRAMEWORK

□ Four Building Blocks of the Financial Accelerator Framework

1. Firm Profit Function

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1) a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2) a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i} - \frac{S_2 a_2}{1+i}$$


2. Financing Constraint

$$P_1 \cdot (k_2 - k_1) = S_1 \cdot a_1$$

3. Government Regulation of Financial Relationships (imposition of R on financing constraint)

$$P_1 \cdot (k_2 - k_1) = R \cdot S_1 \cdot a_1$$

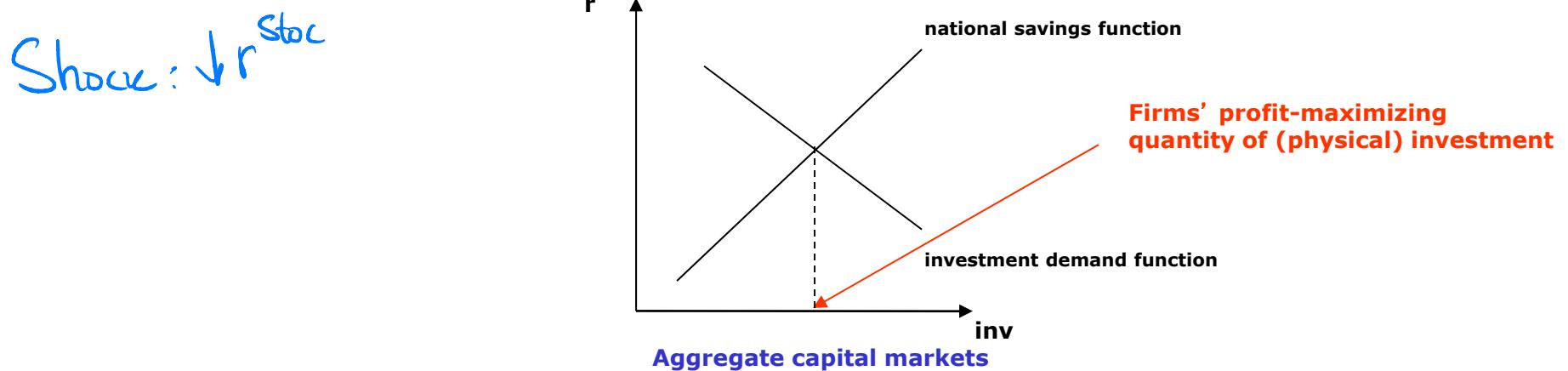
4. Relationship between firm profits and dividends

$$D_t = P_t \cdot \underline{\text{profit}_t}$$


REAL profits of firm in period t

FINANCIAL ACCELERATOR IN ACTION

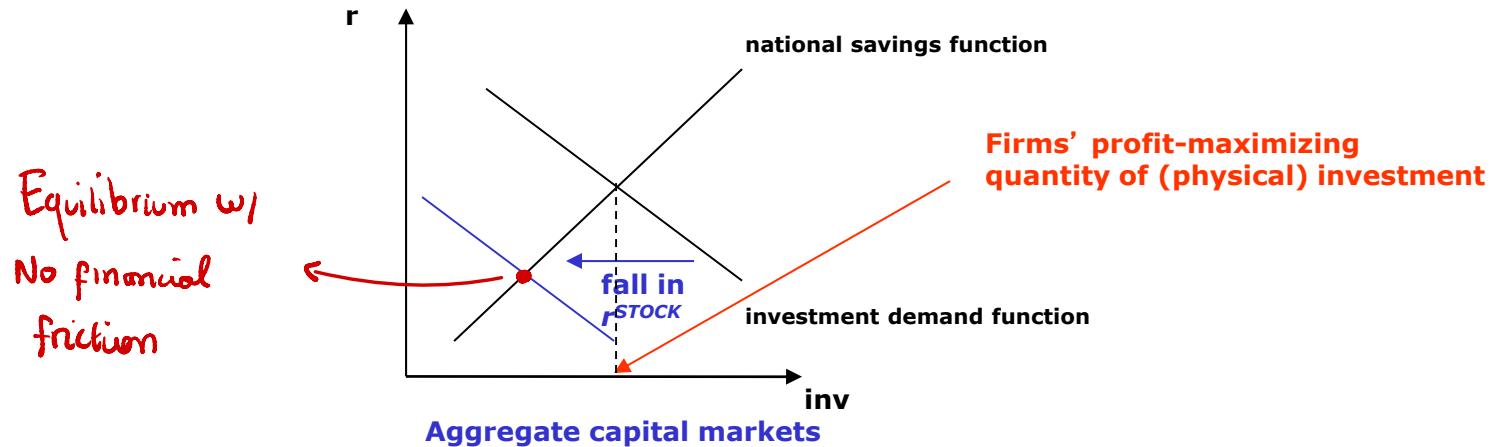
- Suppose economy is in a “steady-state” in which $r = r^{STOCK} \dots$
- ...then a **shock** causes r^{STOCK} to decline
 - i.e., broad range of financial asset returns suddenly fall...
 - ...perhaps because of problems stemming from one or a few classes of financial assets (i.e., mortgage-backed bonds)



- Technically (reminder),
 - Riskless return $1+r = \frac{1+i}{1+\pi} = \frac{1}{P_1^b} \cdot \frac{1}{1+\pi}$ and risky return $1+r^{STOCK} = \frac{1+i^{STOCK}}{1+\pi} = \frac{S_2 + D_2}{S_1} \cdot \frac{1}{1+\pi}$

FINANCIAL ACCELERATOR IN ACTION

- Suppose economy is in a “steady-state” in which $r = r^{STOCK}$...
- ...then a **shock** causes r^{STOCK} to decline
 - i.e., broad range of financial asset returns suddenly fall...
 - ...perhaps because of problems stemming from one or a few classes of financial assets (i.e., mortgage-backed bonds)



- Equilibrium quantity of (physical) investment **falls**
 - Investment $\approx 15\%$ of GDP
- Firm profits **fall** (i.e., investment no longer at profit-maximizing choice)
 - \rightarrow Dividends **fall** (Building Block 4: dividends = profits)
 - $\rightarrow r^{STOCK}$ falls even further! (because D a component of r^{STOCK})

But Profits also fall

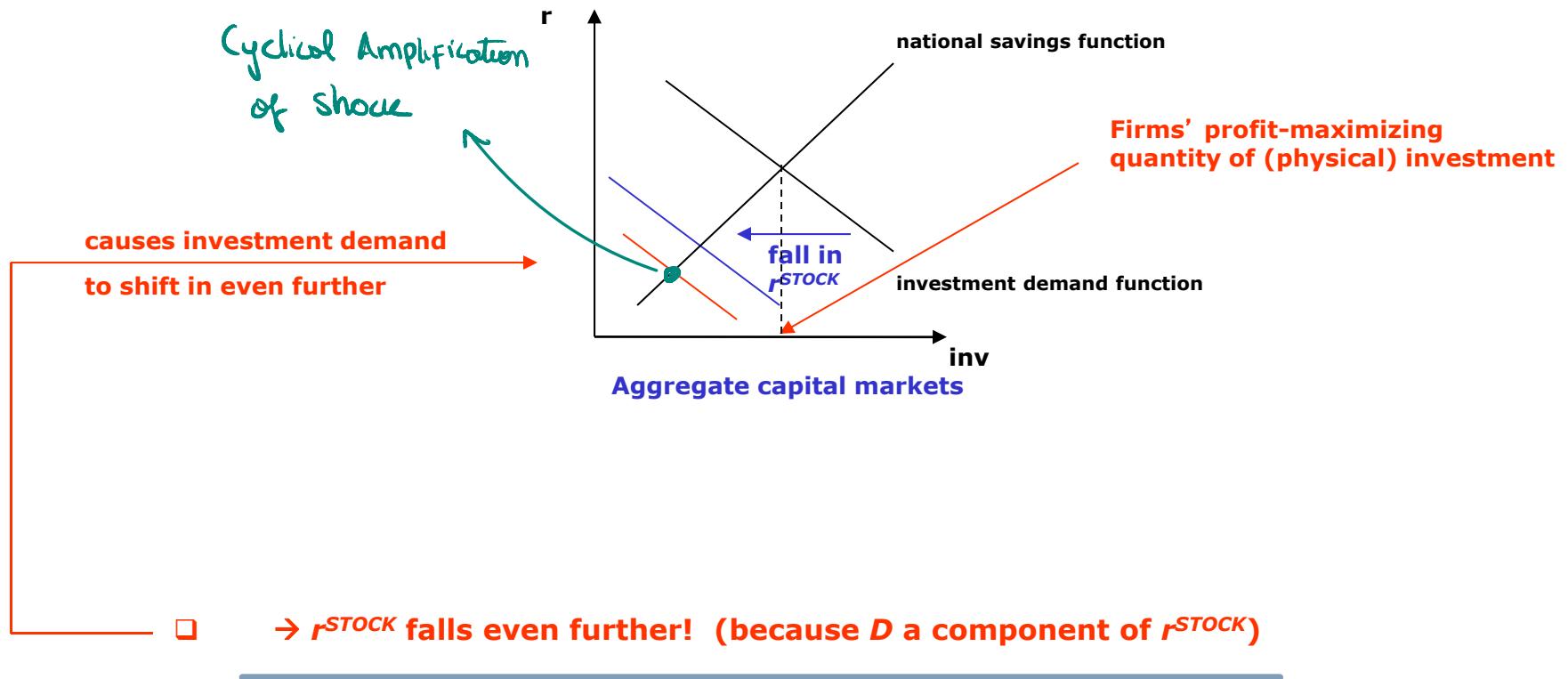
Dividends Fall

r^{STOCK} falls further

$$r^{STOCK} = \frac{S_2 + D_2}{S_1} \frac{1}{1+\pi}$$

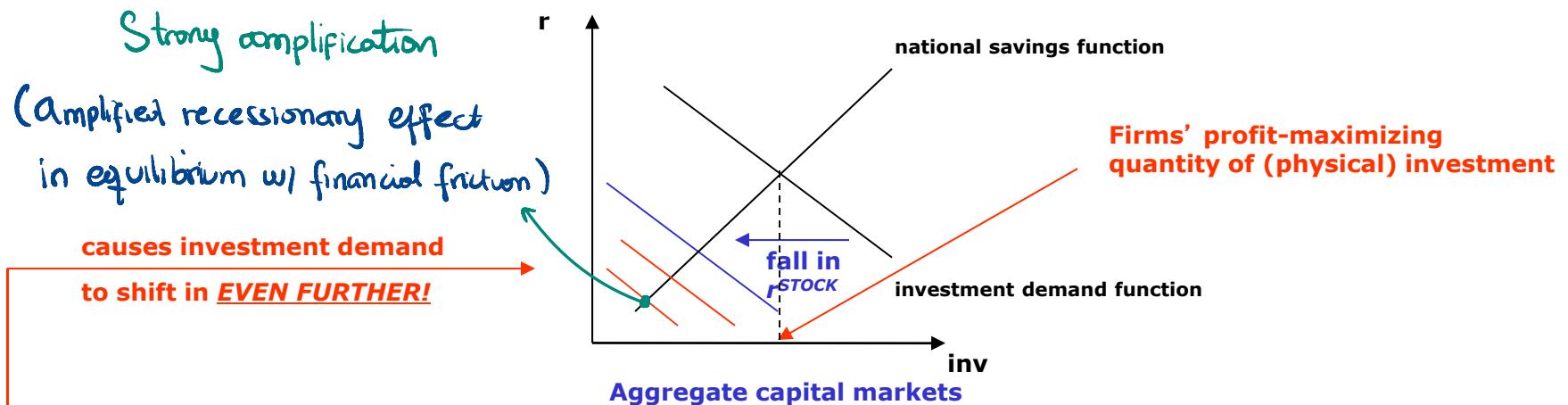
FINANCIAL ACCELERATOR IN ACTION

- Suppose economy is in a “steady-state” in which $r = r^{STOCK}$...
- ...then a **shock** causes r^{STOCK} to decline
 - i.e., broad range of financial asset returns suddenly fall...
 - ...perhaps because of problems stemming from one or a few classes of financial assets (i.e., mortgage-backed bonds)



FINANCIAL ACCELERATOR IN ACTION

- Suppose economy is in a “steady-state” in which $r = r^{STOCK}$...
- ...then a **shock** causes r^{STOCK} to decline
 - i.e., broad range of financial asset returns suddenly fall...
 - ...perhaps because of problems stemming from one or a few classes of financial assets (i.e., mortgage-backed bonds)



- Equilibrium quantity of (physical) investment falls further
 - Investment $\approx 15\%$ of GDP
- Firm profits fall further
 - \rightarrow Dividends fall further (Building Block 4: dividends = profits)
 - $\rightarrow r^{STOCK}$ falls EVEN FURTHER! (because D a component of r^{STOCK})

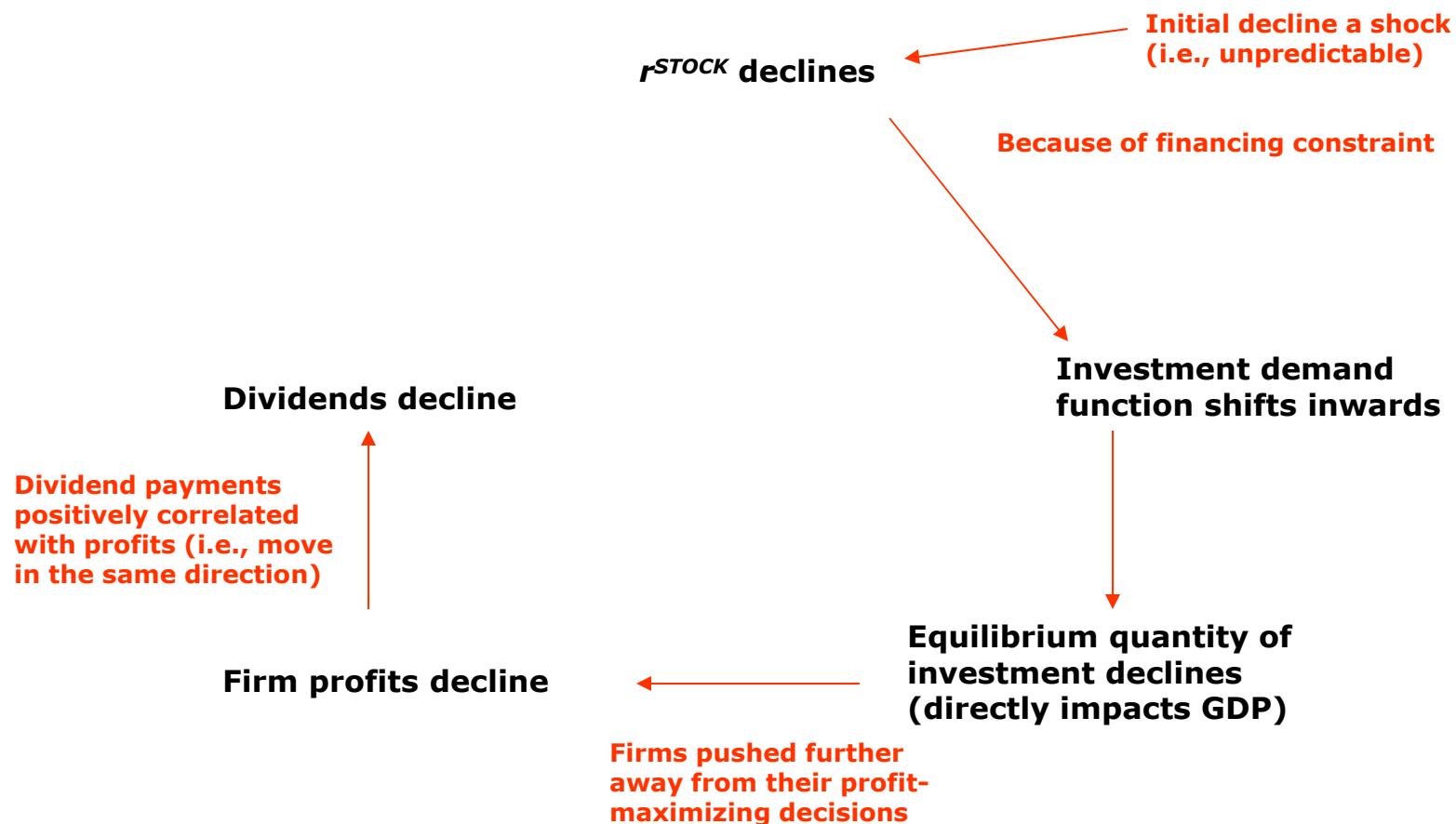
AND ON AND ON...

FINANCIAL ACCELERATOR

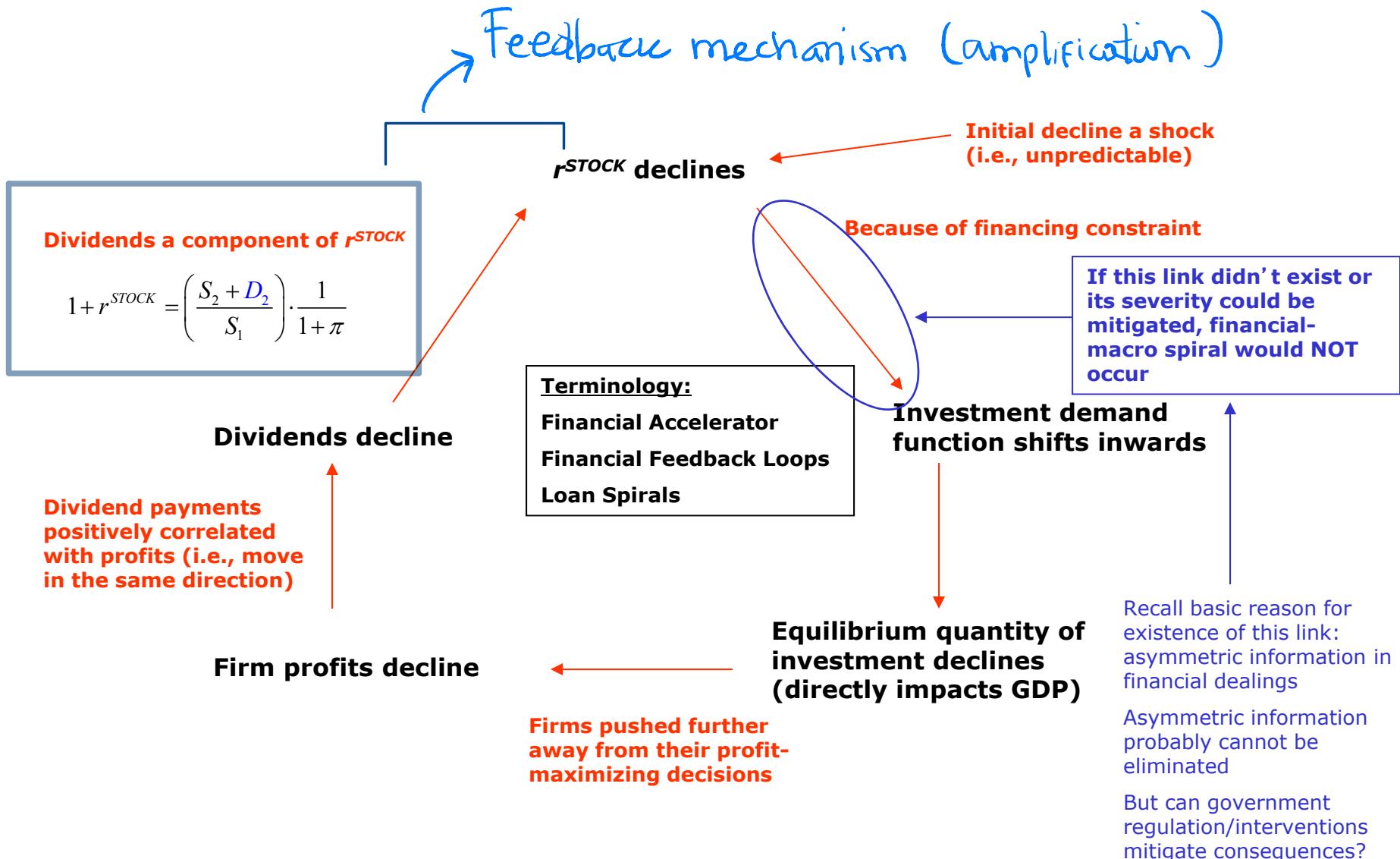
r^{STOCK} declines

Initial decline a shock
(i.e., unpredictable)

FINANCIAL ACCELERATOR



FINANCIAL ACCELERATOR



POLICY AND REGULATORY RESPONSES

- Entire accelerator mechanism due to financing constraint

$$P_1 \cdot (k_2 - k_1) = R \cdot S_1 \cdot a_1$$

- Lagrange multiplier related to asset returns and government regulation by

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r} \right] \cdot \frac{1}{R}$$

- If r^{STOCK} falls below r (which causes accelerator mechanism to begin)
 - λ increases
 - Optimal regulatory response: raise R , which would cause λ to decline!
 - If designed properly, a rise in R can perfectly offset the fall in r^{STOCK} , thus choking off the damaging effects of the accelerator
- Interpretation of rise in R
 - For a given market value of financial assets, $S_1 a_1$, a higher R allows firms to borrow more from private lenders, in turn allowing them to purchase more (physical) capital
 - One interpretation: government “guarantees” private loans
 - Allows firms to produce more for the same level of financial resources

POLICY AND REGULATORY RESPONSES

Entire accelerator mechanism due to financing constraint

$$P_1 \cdot (k_2 - k_1) = R \cdot S_1 \cdot a_1$$

- Interpretation of rise in R
 - For a given market value of financial assets, $S_1 a_1$, a higher R allows firms to borrow more in order to purchase more (physical) capital
 - Allows firms to produce more for the same exact financial resources
- Changes in R can be time-consuming to implement
 - Simultaneously controlled by Federal Reserve, Treasury, Securities and Exchange Commission (SEC), Comptroller of the Currency, and several other regulatory agencies – huge coordination delays!
- Another “policy action” that has the same effect as raising R
 - Design policies to raise financial asset prices (i.e., S_1) directly!

Have these programs work as intended?

Yes and no?...



- Exactly the intention of U.S. Troubled Asset Relief Program (TARP)
 - Direct purchases by Treasury of a wide variety of financial assets
 - The increased demand for these assets would lift their price
- Exactly the intention of Federal Reserve’s programs to buy a wide variety of financial assets – increased demand would lift prices

700B

REAL INTEREST RATE

- *r the key variable for macroeconomic analysis*
- r measures the price of period-1 consumption in terms of period- 2 consumption
- r reflects degree of impatience
- r often reflects rate of consumption growth between periods
- r measures the price/return of physical assets (i.e., machines and equipment) of firms
 - “Riskless” assets
- **Now: r also measures price/return of risky assets (i.e., stock) in “steady state”**
 - If $r = r^{STOCK}$, financing issues don't affect (very much) macroeconomic outcomes
 - If r and r^{STOCK} deviate significantly
 - Financial conditions of firms matter for investment/output
 - And can matter very importantly!
- **Can also think of λ itself as a type of real interest rate – an interest SPREAD**
 - The price of bringing funds from “outside sources” (i.e., lenders) “inside” the firm (i.e., the borrower) to finance operations
 - If $r = r^{STOCK}$, this price equals zero
 - Cost of “external funding sources” vs. “internal funding sources” due to info. asymmetry