# International Macroeconomics - Field Summary[1]

## Contents




[1] Camilo Granados (cagranados8@gmail.com): these notes may contain typos and errors. I appreciate any feedback on them.

# Part I.
# International Finance

## 1. Global Financial Crisis history

*Common patterns:*

(i) Asset market collapses are deep and prolongued.

(ii) Output and employment also declines profoundly.

(iii) Real value of Government Debt exploded.

**MS2011:** US homeowners borrowed against the rising value of their homes (leverage boom) possibly due to liquidity constraints and less self control. Result: home-equity based borrowing was the principal cause of ensuing financial crisis.

**RR2009:** Prior to financial crisis there is an important growth in private and public debt. Other symptoms are asset price inflation, rising leverage, sustained current account deficits, slow trajectory of economic growth.

The aftermath of the crisis shares the three characteristics mentioned above, in addition to each: (i) downturn lasts over three years. (ii) unemployment surge lasts over four years, GDP decline around two years. (iii) increase in debt is not explained by cost of bailout, rather it is due to collapse in tax revenues given the output decline as well as the cost of countercyclical fiscal policies.

**RR2014:** Part of the costs of banking crises relies in the slow pace of recovery. On average it takes about eight years to reach pre-crisis level of income. Also it is found that the subprime crisis is not an anomaly with respect to pre-war data and that it makes sense for advanced economies to rely on policies that are currently characteristic of developing countries as debt restructuring, higher inflation, capital controls and other forms of financial macroprudential policies.

## 2. Global imbalances

Recalling the low interest rate puzzle, we had that in a Consumption CAPM framework, a linearization of the Euler Equation with CRRA preferences and log normal consumption growth yields,

$$r = \gamma E_t \Delta \ln c_{t+1} - \frac{\gamma^2}{2} Var_t(\Delta \ln c_{t+1}) + \rho.$$

Using postwar data we found the EPP that implied that the risk aversion coefficient was too high $\gamma \in [20, 200]$ or we obtained the low interest rate puzzle, i.e., taking a reasonable $\gamma$ and discount rate (e.g. $\rho = 0$) then we get that the implied $r$ by the model is much higher (3.3) than the interest rate observed in reality (0.8).

Explanations mentioned for these puzzles still apply: 1. Consumption aggregation dismisses agents heterogeneity; 2. rare events lead to overestimating average returns; 3. Survivorship bias; 4. Loss aversion; others.

A new explanation can be considered: the Global Imbalances.

Basic framework and Intuition:

Consider an endowment SOE with 1 internationally traded riskless bond.

The SOE solves: $\max U(C_t) \quad s.t. \quad C_t + A_{t+1} = Y_t + (1+r)A_t$

The current account is: $CA_t = A_{t+1} - A_t = Y_t - C_t + rA_t = TB_t + rA_t$

In this setup a country runs a CA surplus/deficit wheneber its output exceeds/is below its permanent output.

**Puzzle:** Suppose an EME (China) output is rising over time, then if permanent output is larger than the current one, it should run a CA deficit instead of a surplus as observed.

**Metzler Diagram**

Consider two countries (h, f), add investment, then we need to consider endogenous output $Y_t = Z_t F(K_t)$ with $K_{t+1} = K_t + I_t$. The current account is now given by,

$$CA = Y - C - I + rA$$

The savings are $S = Y + rA - C$ as before and therefore,

$$CA = S - I$$
$$CA^* = S^* - I^*$$

Asset market clearing implies $CA + CA^* = 0$, then $S - I = -(S^* - I*)$.

To find the interest rate that clears the markets, i.e. the $r$ that equals global savings and investment we use a Metzler diagram. To do it we obtain $S(r), I(r), S(r)^*, I(r)^*$ from the FOCs.

From the FOC for K we get $Z_t F'(K_t) = r_t$ and using $K_{t+1} = K_t + I_t$ we get $I'(r) < 0$,

$$Z_t \alpha K_t^{\alpha-1} = r_t$$
$$Z_t (K_{t+1} - I_t)^{\alpha-1} = r_t$$
$$K_{t+1} - I_t = \left(\frac{r_t}{\alpha Z_t}\right)^{\frac{1}{\alpha-1}}$$

by implicit function theorem,

$$\frac{\partial I}{\partial r} = -\frac{\partial g(I,r)/\partial r}{\partial g(I,r)/\partial I} = -\frac{\frac{1}{\alpha-1}\left(\frac{r_t}{\alpha Z_t}\right)^{\frac{2-\alpha}{1-\alpha}}\frac{1}{\alpha Z_t}}{-1} < 0$$

Also, for $r$ close to autarky levels we have $C_t'(r) < 0$ and $S_t'(r) > 0$. Then we put the savings and investment together in a graph for each country and obtain the clearing market interest rate as the rate where asset market clear $CA + CA^* = 0$.

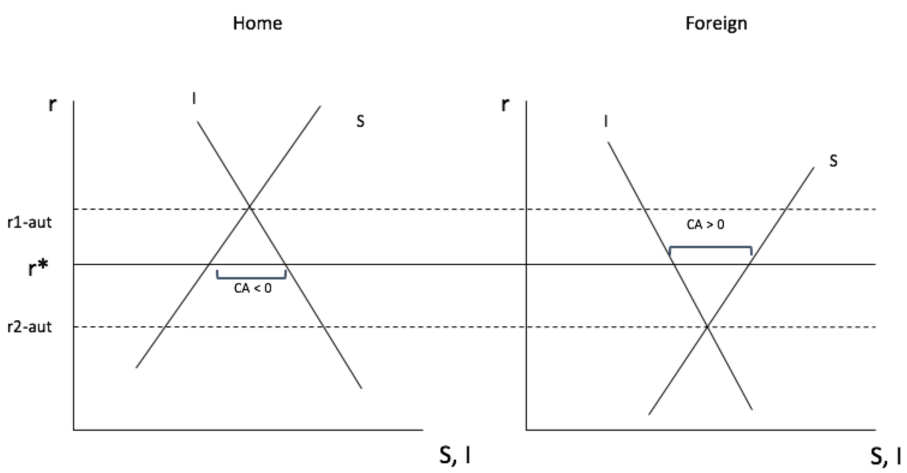



Figure 1: Metzler Diagram

Note that according to the diagram, a country will run a $CA < 0$ if its autarkic interest rate exceeds that of his trading partner and as a result, the equilibrium interest rate will be inbetween the two autarky rates.

Now, even under this setup the puzzle remains, EME still run current account surplus and capital flows towards countries that have low interest rates (and low productivity growth). Then the could be thinking that regardless of the market interest rate observed, there may be frictions that lower the autarky rates more than what they increase by the productivity dynamics. This frictions are considered in the literature recently.

Articles considering such frictions while still relaxing some of the assumptions that lead to the low interest rate puzzle are:

**MQR2009:** heterogeneous agents, incomplete markets, asymmetric information that generates a borrowing constraint.

This article focuses on the savings that are increased for every $r$ due to precautionary motive. Due to information asymmetries, associated with a lack of financial development, the SOE faces a tighter borrowing constraint that results in a failure to insure idiosyncratic risks and leads the agents to increase their precautionary savings. All of this lowers their autarky rate and can be represented as a shift in the S curve in the Metlzler diagram.

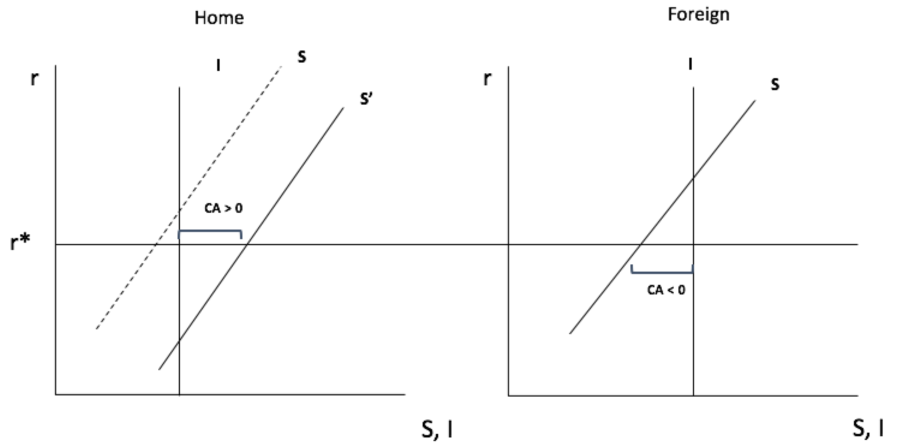



Figure 2: Metzler Diagram (shift in Savings)

**CFG2008:** OLG structure with imperfect pledgeability of future income due to lack of financial development.

The authors study the supply of assets (Investment) rather than the behaviour of savings as before. In particular, they focus on constraints in the supply of assets used to hedge risks, i.e. shortage of safe assets. Such shortage is the result of lack of income pledgeability, due to the presence of a financial friction that limits the degree of financial development ($\delta$), acts as a friction, limiting the autarky rates, and results in a reduction of the investment. The article also finds supporting evidence of a positive correlation between the interest rates and the degree of financial development, therefore, low financial development can lead to lower interest rates regarless of the other factors governing the dynamics of the returns.

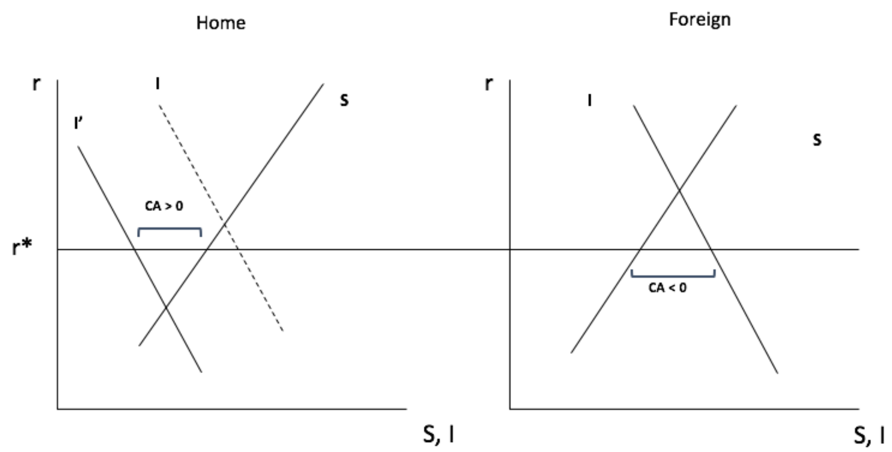



Figure 3: Metzler Diagram (decrease in Investment due to Financial Friction)

**Jin 2012:** Differences in factor productivities and intensities across countries that exacerbate the imbalances in capital flows.

This study explores the effect in assets flows of the trade specialization patterns due to factor intensities across countries. Then, in addition to the usual convergence effect, where the capital flows to where it's more scarce, it also considers the composition effect of factor proportions within a country. Countries that are more labor intensive would attract less capital flows, explaining why countries with larger returns won't neccesarilly have more capital inflows than other financially developed economies.

Then, a labor expansion in a developing country will deepen both his and the capital intensive country factor specialization, leading to a capital flow towards the developed country. In terms of the diagram this shock implies a larger increase in S than in I.

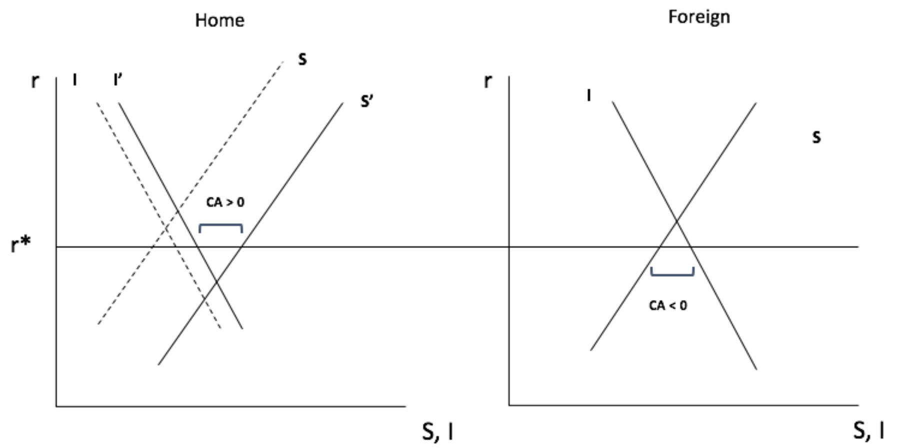



Figure 4: Metzler Diagram (positive labor shock in EME)

# 3. Low interest rate puzzle, International Risk Sharing, puzzles

- OR 285-94 (Consumption correlation puzzle)
- OR 306-19 (EPP, low interest rate puzzle)
- OR 329-32 (Gains of international risk sharing)
- BKK1992 (IM)

## 3.1. OR96 285-94 (Consumption correlation puzzle)

In the complete markets model consumption between home and the rest of the world is perfectly correlated: $corr(c, c^*) = 1$ whereas output is less correlated. Empirically we find the opposite $Corr(c, c^*) < Corr(y, y^*)$ and correlation of consumption is lower than predicted.

A possible explanation is that markets are not complete so it's not possible to fully hedge risk. In particular it's possible to think about traded and non-trade goods (Lewis, 1996). In such case what should hold is that consumption in traded goods are perfectly correlated: $Corr(c_T, c_T^*) \approx 1$ that is, there is a *Constrained Efficient Risk Sharing.*

To test that, Lewis checks the correlation between traded output and traded consumption which should be zero in a risk sharing scenario. The result are still not what predicted in theory although the condition holds better for countries without restrictions to the Exchange Rate.

Caution is warranted since the results are subject to data handling and assumptions. In particular, only output risk is examined.

## 3.2. OR1996 306-19 (EPP, low interest rate puzzle)

**Equity Premium Puzzle:** (MP1985, MZ1991) Excess return of stocks wrt to bonds is too high to be rationalizable using a CRRA:

$$E(r^m) - r = -(1+r)Cov\left[\beta\left(\frac{c_2}{c_1}\right)^{-\rho}, r^m - r\right]$$

Given that the $Cov(\cdot)$ term is very low then observed returns would imply $\rho \in [2, 200]$ which imply unrealisticly overly risk averse agents.

**Deep connection with home bias:** If agetns are so risk averse then higher effort to diversify should be observed (lower to none HB).

Explanations: • Higher transaction costs of trading stocks (AG1991), then there is a liquidity premium on bonds that partly explains equity premium. • Non diversifiable labor income (AGW1992) • Interactions of several frictions (HL1995) • Habit persistence in consumption: generates higher volatility of intertemporal MRS • Ommited high return losses, market casualties that bias upwards estimated equity returns.

**Low Risk-free Rate Puzzle**: Using the CAPM model with a CRRA utility and data on consumption (average difference and variance) yields a riskless rate of about 3.34. Whereas actual estimates are way lower (0.8).

Accommodating parameters with the estimation and data would imply a negative discount factor.

$$c_1^{-\rho} = (1+r)\beta E_1\{c_2^{-\rho}\}$$
$$1+r = \frac{1}{\beta E_1 \left(\frac{c_2}{c_1}\right)^{-\rho}}$$

approximate with logs: $r = \rho E_1\{\log\left(\frac{c_2}{c_1}\right)\} - \frac{\rho^2}{2} Var(\log \Delta c_2) - \log \beta$
with $\Delta c \approx 0.018$, $Var(\Delta c) \approx 0.0013$ also let $\rho = 2$, $\beta \approx 1 \Rightarrow r = 3.34$. However the estimated is $r = 0.8$. To allow for the estimated one we would have to assume a negative discount factor.

## 3.3. OR1996 329-32 (Gains of international risk sharing)

Risk sharing comes with Efficient Risk allocation but possibly with small welfare gains. Then, with transaction costs Home Bias can arise.

Lucas (1987): Lifetime utility with CRRA utility is approximated as:

$$U_t = \frac{\bar{c}^{1-\rho}}{1-\rho}\left[\frac{1}{1-\beta(1+g)^{1-\rho}}\right]\exp\{-\frac{1}{2}(1-\rho)\rho Var(\epsilon)\}$$

where $\epsilon$ is a normal random process. Then the lifetime utility without uncertainty, i.e., with Risk Sharing is:

$$U_t = \frac{\bar{c}^{1-\rho}}{1-\rho}\left[\frac{1}{1-\beta(1+g)^{1-\rho}}\right]$$

Therefore te proportional consumption gain of removing uncertainty is given by $\tau$ s.t.:

$$\frac{(1+\tau)\bar{c}^{1-\rho}}{1-\rho}\exp\{-\frac{1}{2}(1-\rho)\rho Var(\epsilon)\} = \frac{\bar{c}^{1-\rho}}{1-\rho}$$
$$\vdots$$
$$\tau = \left\{\exp\left[\frac{1}{2}(1-\rho)\rho Var(\epsilon)\right]\right\}^{\frac{1}{1-\rho}} - 1$$

A first order approximation about $Var(\epsilon) = 0$ yields $\tau \approx \frac{1}{2}Var(\epsilon)$.

With 1950-90 US data $Var(\epsilon) = 0.000708$ and $\rho = 10$ gives $\tau \approx 0.00354$ which implies very low gains from risk sharing.

Caution is warranted:

- It's assumed that consumption fluctuates around a time trend. This is criticized.

- Stability of US consumption is atypical.

- Calculations don't allow for individual heterogeneity. No uninsurable idiosyncratic risks.

Still CO1991 find trivial gains from risk sharing. Similar for BKK1992 where the equilibrium dynamics are no different than that of autarky after including costs of trade.

In addition, Mendoza (1995) and Tesar (1995) find a similar result: It's easier to diversify risk domestically with intertemporal resource allocation than across countries.

However these findings are sensitive to assumptions: van Wincoop (1994), BB2002, Sutherland (2004) find large gains of international risk diversification (Key: $ES_{h,f} \neq 1$).

## 3.4. Other Puzzles

### 3.4.1. OR2000 NBER Macroeconomics annual

**Six Major Puzzles in International Macroeconomics: Is there a common cause? (OR)**

**- Home bias in Trade:** Trade within a country is considerably greater than between countries, even with no substantial barriers.

**- Home bias in equity portfolio:** Agents hold disproportionate amount of domestic assets and few foreign equity despite arbitrage opportunities of diversification.

**- Fielstein-Horioka Puzzle:** After diversification savings rate should not depend on domestic investment (in the same way consumption should not depend on endowments). In reality there is a strong correlation between these two variables.

**- Low consumptions correlation puzzle:** Treated above.

**- PPP Puzzle:** Real Exchange Rate is way more volatile and persistent than models would suggest (the half-life of RER shocks is about 4 years).

**- Exchange Rate Disconnect puzzle:** Weak short term feedback between exchange rate and fundamentals (Meese-Rogoff, 1983)

(derived from the former) **Backus-Smith Puzzle:** Consumption growth and RER are highly correlated according to a complete markets model with traded and non traded goods. The intuition is that countries with cheap prices should receive transfers to take advantage of cheap consumption. Such correlation is not observed empirically (correlation is zero or even negative).

**Key friction:** International trade costs in good markets (transport costs, tariffs, other barriers).

**Approach:** Not to explore a market imperfection directly but depart from frictionless scenario and add the most plausible imperfection to see if the puzzle remains. The finding is that one trade costs are allowed into the models, i.e., most of real side of the the puzzles dissapear. The model doesn't fully explain the puzzles but aims to state the importance of including trade costs in general.

**Other puzzles:**

**Forward Rate Puzzle:** Forward interest rate doesn't predict the exchange rate. Worse, it's being found to be negatively correlated.

The test is performed by estimating $e_{t+1} - e_t = a_0 + a_1(f_t - e_t) + \epsilon_t$ and testing $a_0 = 0,\ a_1 = 1$. Those hipotheses are rejected and worse it is found that $a_1 < 0$.

Also, according to Fama (1984): $Var(F_t - E[e_{t+1}]) > Var(E[e_{t+1}] - e_t)$, i.e., variance of risk premium is larger than that of expected variation of exchange rate.

## 3.5. Basic SOE model (methodology)

One asset (bond only), non-stochastic. Later production and endogenous labor supply

Preferences can be typical CD or GHH. With GHH preferences are quasilinear and MRS between consumption and leisure is independent of contemporaneous consumption level.

Rep. agent, one final good so there is only inter-temporal trade.

Agent's UMP is:

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$\text{s.t.}$$

$$c_t + a_{t+1} = y_t + (1+r)a_t$$

$$\lim_{T\to\infty} \frac{a_{T+1}}{(1+r)^T} \geq 0, \quad a_0 \text{ given}$$

Define:

- Trade Balance (model without investment) $TB_t = y_t - c_t$

- Current Account $CA_t = a_{t+1} - a_t = TB_t + ra_t$ (Trade balance plus return on assets. Change in foreign assets.)

- Intra-temporal Budget Constraint: $c_t + a_{t+1} = y_t + (1+r)a_t$

- Lifetime BC: by recursive substitution of $a_{t+j}$:

$$\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t} = \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t} + (1+r)a_0$$

$$\sum_{t=0}^{\infty} \frac{TB_t}{(1+r)^t} = -(1+r)a_0$$

The UMP is then:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t [u(c_t) - \lambda_t(c_t + a_{t+1} - y_t - (1+r)a_t)]$$

FOC:

$$u'(c_t) = \lambda_t$$

$$\lambda_t = \beta(1+r)\lambda_{t+1}$$

$$u'(c_t) = \beta(1+r)u'(c_{t+1})$$

$$\lim_{T\to\infty} \frac{a_{T+1}}{(1+r)^T} = 0$$

From the EE it can be seen that anytime $(1+r)\beta > 1$ we will have $\frac{c_{t+1}}{c_t} > 1$, i.e., the consumption will grow forever and therefore there will not be an statiorary steady state solution. The reverse will hold with $\frac{c_{t+1}}{c_t} < 1$.

The model will be stationary only in a particular case: $1 = (1+r)\beta$. However, this is a knife edge condition similar to what is mentioned in CO1991 and in general this simplified model is not stationary.

Example: utility is CRRA $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$

$$\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} = \beta(1+r)$$
$$c_{t+j} = [\beta(1+r)]^{\frac{j}{\sigma}} c_t$$

subs. in the NPV of revenues:

$$c_t \sum_{j=0}^{\infty} \frac{[\beta(1+r)]^{\frac{j}{\sigma}}}{(1+r)^j} = \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t} + (1+r)a_0 = NPVR_t$$

when $1 = \beta(1+r)$ and given that $\sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^j = \frac{1+r}{r}$:

$$c_t = \frac{r}{1+r} NPVR_t$$

If $y$ increases by 1 permanently the NPVR increases by $\frac{1+r}{r}$ then $c$ increases by 1 $\Rightarrow$ CA, TB constant.

If $y$ increases by 1 permanently $\Delta NPVR < \frac{r}{1+r}$ then $CA = y - c > 0$ (CA is procyclical unlike in the data).

Finally if $\Delta a_0 = 1$ then $\Delta c = r$ and agents save more permanently. Also **any level of NFA is compatible with the steady state**. This can be troublesome for log-linear approximations that remain valid only about a small region of state variables values.

Technology: $Y_t = e^{Z_t} K_t^{1-\alpha} L_t^{\alpha}$

Preferences: 1. CD: $u_t = \frac{(c_t^{\gamma}(1-L_t)^{1-\gamma})^{1-\sigma}}{1-\sigma}$

2. GHH: $u_t = \frac{(c_t - \tau L_t^{\upsilon})^{1-\sigma}}{1-\sigma}$

Key difference in prefereces: labor supply

with CD: $\frac{(1-\gamma)c_t}{\gamma(1-L_t)} = \alpha\frac{Y_t}{L_t}$

with GHH: $\tau\nu L_t^{\nu-1}\alpha = \frac{Y_t}{L_t}$

In CD there is income effect that mitigates labor's response to productivity shocks. In GHH there is no effect and therefore after a productivity shock there is a sharp drop in leisure (can be more realistic).

Also in the model without investment (CA are the total savings), the TB is procyclical which contradicts empirical evidence. In a model with investment TB becomes countercyclical. Therefore, this model is improven with such addition.

With investment:

$$a_{t+1} + c_t + i_t = (1+r)a_t + z_t f(k_t)$$
$$i_t = k_{t+1} - (1-\delta)k_t$$

The definitions should be updated:

- Trade Balance: $TB_t = y_t - c_t - i_t$

The new UMP is:

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$
\begin{aligned}
\text{s.t.,} \quad c_t + a_{t+1} + i_t &= z_t f(k_t) + (1+r)a_t \\
i_t &= k_{t+1} - (1-\delta)k_t \\
\lim_{T\to\infty} \frac{a_{T+1}}{(1+r)^T} &\geq 0 \\
\lim_{T\to\infty} \frac{k_{T+1}}{(1+r)^T} &\geq 0 \\
a_0, k_0 &\text{ given}
\end{aligned}
$$

FOC:

$$
\begin{aligned}
c_t : \quad & u'(c_t) = \lambda_t \\
a_{t+1} : \quad & \lambda_t = \beta(1+r)\lambda_{t+1} \\
k_{t+1} : \quad & \lambda_t = \beta\lambda_{t+1}[z_t f'(k_{t+1}) + (1-\delta)]
\end{aligned}
$$

If we add investment adjustment costs we would have:

$$
a_{t+1} + c_t + i_t + \frac{\omega}{2}\frac{(i_t - \delta k_{ss})^2}{k_t} = (1+r)a_t + z_t f(k_t)
$$

And the new FOC is:

$$
k_{t+1}: \qquad \lambda_t\left(1 + \omega\frac{(i_t - \delta k_{ss})}{k_t}\right) = \beta\lambda_{t+1}\left[z_t f'(k_{t+1}) + (1-\delta)\left[1 + \omega\frac{(i_{t+1} - \delta k_{ss})}{k_{t+1}}\right]\omega(i_{t+1} - \delta k_{ss})^2\right]
$$

As mentioned, the model is not stationary (only in knife-edge conditions as CO1991). Then some tricks to address this are:

- SGU2001: various methods including interest rate as decreasing function of foreign assets, endogenous discount factor, portfolio adjustment costs ($(a_{t+1} - a_t)^2$).

- Mendoza 1991: Endogenous discount factor, i.e., decreasing on consumption and leisure ($\beta = (1 + c - \frac{\psi_0}{\psi} N^{\psi})^{-\chi}$).

- Ghironi 2006: OLG, newborns with no assets break Ricardian Equivalence and make assets matter in Euler Equations and steady state.

### 3.5.1. Felstein-Horioka Puzzle

Based on the SOE model above.

As seen, a way to address the TB procyclicality is to include investment. Another one is to include persistent growth in output: $\Delta y_{t+1} = \alpha + \rho\Delta y_t + \varepsilon_{t+1}$ with $\rho > 0$. If $\uparrow Y$ by 1, then with the inertia $\uparrow Y_{+1}$ but by less. Consumption increases by more (by one), therefore TB decreases, i.e., $\rho_{y,TB} < 0$ (countercyclical).

Coming back to the model with Investment we obtain the Feldstein-Horioka puzzle:

$$
\begin{aligned}
y_t &= A_t f(k_t) \\
k_{t+1} &= (1-\delta)k_t + i_t \qquad (\text{let } \delta = 0) \\
B_{t+1} - B_t &= y_t + rB_t - c_t - g_t - i_t \qquad \text{[BC]}
\end{aligned}
$$

FOC:

$$
1 = E_t\left\{\underbrace{(A_{t+1}f'(k_t) + 1)}_{R_{t+1}}\underbrace{\beta\frac{u'(c_{t+1})}{u'(c_t)}}_{Q_{t+1}}\right\}
$$

Now let $E(Q_{t+1}) = E\left(\beta\frac{u'(c_{t+1})}{u'(c_t)}\right) = \beta \cancelto{1}{E\left(\frac{u'(c_{t+1})}{u'(c_t)}\right)} = \frac{1}{1+r}$.
Also use that $E(XY) = E(X)E(Y) + Cov(X,Y)$ with $X = R_{t+1}$, $Y = Q_{t+1}$ and solve for $E(R_{t+1})$:

$$E(R_{t+1}) = (1+r)[1 - Cov(R_{t+1}, Q_{t+1})]$$

- Assume that $Cov(\cdot) = 0$:

Then $E_t(A_{t+1}f'(k_{t+1})) = r$ which is a constant, therefore: $k_{t+1}, i_t$ are uncorrelated with $U'(\cdot)$, i.e., with $\{c_t\}$. Then $s_t, i_t$ are uncorrelated.

- In general: $\uparrow A_{t+1} \to \uparrow y_{t+1} \to \uparrow \bar{y} \to \uparrow c_{t+1} \Rightarrow \downarrow u'(c_{t+1})$ then $Cov(\cdot) < 0$ (higher output shrinks marginal utility while at the same time investment moves).

Then we would have: $Corr(s_t, i_t) \neq 0$. Mostly in the part that would depend on $A_{t+1}$.

Now consider the process $A_{t+1}$ follows is: $A_{t+1} - \bar{A} = \rho(A_t - \bar{A}) + \varepsilon_{t+1}$:

- $\rho = 0,\ \varepsilon_t > 0$: $A_{t+1}, y_{t+1}$ not affected, then $i_t$ doesn't change. $y_t \uparrow, \bar{y} \uparrow \Rightarrow s_t \uparrow \Rightarrow\ CA_t = s_t - i_t > 0$

- $\rho = 1,\ \varepsilon_t > 0$: $A_{t+1} \uparrow, y_{t+1} \uparrow \to\ i_t \uparrow$ also, due to new investment: $\Delta i_{t+1} > \Delta i_t$, then $\Delta \bar{y} > \Delta y_t\ \Rightarrow c_t\ \uparrow > y_t \uparrow$ (PIH, cons. smoothing) $\Rightarrow s_t \downarrow$, therefore: $Cov(i_t, s_t) < 0$

Then with investment: $Cov(y, CA) < 0$ but $Cov(S, I)$ depends although it would be small or close to zero.

According to Feldstein-Horioka (1980) under complete markets S,I should be independent or as seen at least the covariance should be low. However, empirically $Cov(S, I)$ is high.

## 3.6. A-D asset market structure (methodology)

Complete markets setup, 2 period model, no uncertainty in $t = 1$ (obs. $Y_1$) and $N$ states in $t = 2$ each with probability $\pi(S)$ with $S \in N$ and $Y_2(S)$.

With the A-D security structure the budget constraint is:

$$Y_1 = C_1 + \sum_s \frac{P(S)}{1+r} B_2(S),$$
$$C_2(S) = Y_2(S) + B_2(S), \qquad \forall\ S,$$

where $P(S)$ is the price of bond and includes the probability of the state $S$.

$$\mathcal{L} = u(C_1) + \beta \sum_s \pi(S) u(C_2(S)) + \lambda \left[ Y_1 - C_1 - \sum_s \frac{P(S)}{1+r} \left( C_2(S) - Y_2(S) \right). \right]$$

FOC:

$$u'(C_1) = \lambda, \qquad [C_1]$$
$$\beta \pi(S) u'(C_2(S)) = \lambda \frac{P(S)}{1+r}. \qquad [C_2]$$

From these FOC we get:

$$\frac{P(S)}{1+r} u'(C_1) = \beta \pi(S) u'(C_2(S)). \qquad (1)$$

Assume CRRA and solve for $C_1 \neq C_2(S)$:

$$\frac{P(S)}{1+r} C_1^{-\rho} = \beta \pi(S) C_2(S)^{-\rho},$$

rearranging for home and foreign,

$$C_2(S) = \left(\frac{\pi(S)\beta(1+r)}{P(S)}\right)^{\frac{1}{\rho}} C_1, \tag{2}$$

$$C_2^*(S) = \left(\frac{\pi(S)\beta(1+r)}{P(S)}\right)^{\frac{1}{\rho}} C_1^*, \tag{3}$$

The Market Clearing conditions for $t = 1, 2$ are:

$$C_1 + C_1^* = Y_1 + Y_1^* = Y_1^W,$$
$$C_2(S) + C_2^*(S) = Y_2(S) + Y_2^*(S) + Y_2^W(S),$$

summing (2) and (3):

$$\underbrace{C_2(S) + C_2^*(S)}_{Y_2^W(S)} = \left[\frac{\pi(S)\beta(1+r)}{P(S)}\right]^{\frac{1}{\rho}} \underbrace{(C_1 + C_1^*)}_{Y_1^W}.$$

Solving for the price of the bond,[2]

$$\frac{P(S)}{1+r} = \pi(S)\beta\left[\frac{Y_2^W(S)}{Y_1^W}\right]^{-\rho}, \quad \forall S,$$

and by considering CRRA preferences in (1)

$$\frac{P(S)}{1+r} = \pi(S)\beta\left[\frac{C_2(S)}{C_1}\right]^{-\rho}.$$

Then, given an analogous condition for the foreign country it will hold that:

$$\frac{Y_2^W}{Y_1^W} = \frac{C_2(S)}{C_1} = \frac{C_2^*(S)}{C_1^*}. \tag{4}$$

This is the risk sharing condition.

### 3.6.1. Consumption Correlation Puzzle

According to (4) consumption growth is equalized across countries and is equal to world output growth.

also $\frac{C_2(S)}{Y_2^W(S)} = \frac{C_1}{Y_1}$, i.e., countries consume a fixed fraction of world output regardless of state.

Then consumption growth rates should be more correlated (all equal to $Y_2/Y_1$) than output growth. Empirically this does not hold (**Consumption Correlation Puzzle**).

Interpretation of the bonds: if follows that Countries will buy/sell A-D s.t. $C_2(S) = \mu Y_2(S)$. Then if $Y_2(j) < \mu Y_2^W(j)$ the country will buy $B_2(j)$.

Here $\mu = \frac{Y_1 + \sum_{S=1}^{N} \frac{P(S)}{1+r} Y_2(S)}{Y_1^W + \sum_{S=1}^{N} \frac{P(S)}{1+r} Y_2^W(S)}$

## 3.7. Home Bias

With the consumption correlation puzzle as a result in a basic framework, it was explored to consider more realistic asset markets structures in the models. A natural way to move in that direction would be to consider equities and not only bonds.

[2]also, considering another state $S'$ we can get an expression for relative prices that as expected depends on the likelihood odds: $\frac{P(S)}{P(S')} = \frac{\pi(S)}{\pi(S')}\left[\frac{Y_2^W(S)}{Y_2^W(S')}\right]^{-\rho}$.

With such modification a new puzzle emerges, the portfolio diversification puzzle. We depart from a similar model as before (OR 5.3):

2 periods, N countries, S states in $t = 2$. Assets: Riskless bond and Shares of each country's output (equities).

The budget constraint for $t = 1$ and $t = 2$:

$$
\begin{aligned}
t = 1: &\qquad Y_1^n + \overbrace{V_1^n}^{\text{Stock mkt value at t=1 of n's t=2 output}} = C_1^n + \overbrace{B_2^n}^{\text{Bond w/ return r}} + \sum_{m=1}^{N} \overbrace{\chi_m^n}^{\text{n country claims of m's output}} V_1^m, \\
t = 2: &\qquad C_2^n(S) = (1+r)B_2^n + \sum_{m=1}^{N} \chi_m^n Y_2^m(S), \quad \forall S,
\end{aligned}
$$

with $-1 \leq \chi_m^n \leq 1$.

Also, notice that net purchases made by country n to foreigners is: $(1 - \chi_n^n)V_1^n$.

UMP:

$$
\max_{c_i,\chi_i,B_2} U(\overbrace{Y_1^n + V_1^n - B_2^n - \sum_{m=1}^{N} \chi_m^n V_1^m}^{C_1}) + \beta \sum_{s=1}^{S} \pi(S) U(\overbrace{(1+r)B_2^n + \sum_{m=1}^{N} \chi_m^n Y_2^m(S)}^{C_2^n(S)})
$$

FOC:

$$
u'(C_1) = (1+r)\beta \sum_s \pi(S) u'(C_2^n(S)), \qquad [B_2]
$$

$$
V_1^m u'(C_1) = \beta \sum_s \pi(S) u'(C_2^n(S)) Y_2^m(S), \qquad [\chi_m^n]
$$

then,

$$
U'(C_1) = \beta \frac{E[U'(C_2(S)Y_2^m(S))]}{V_1^m}. \tag{5}
$$

Define: $\mu^n = \frac{Y_1^n + V_1^n}{\sum_{m=1}^{N} Y_1^m + V_1^m}$ and let $\frac{Y_2^m}{V_1^m}$ be the gross return on equity m. Then (5) becomes:

$$
u'(C_1) = \beta E\left[\left(\frac{Y_2^m}{V_1^m} u'(C_2^n)\right)\right], \qquad \forall m,
$$

and recall from C-CAPM that,

$$
u'(C_1) = \beta E\left[(1 + \tilde{R}_m) u'(C_2)\right], \qquad \forall m.
$$

We can guess the solution for consumption is of the form:

$$
\begin{aligned}
C_1^n &= \mu^n \sum_m Y_1^m = \mu^n Y_1^W, \\
C_2^n &= \mu^n \sum_m Y_2^m = \mu^n Y_2^W(S), \qquad \forall S,
\end{aligned}
$$

and then $B_2 = 0$ since consumption is already the one in which no bond is purchased/sold.

Then, the portfolio share will be $\chi_m^n = \mu^n \quad \forall m$, i.e., $n$ holds a fraction $\mu^n$ of equities from every other country and from the world output portfolio.

**Summary**

In a standard model of portfolio choice with equity and bond trade with identical investors and i.i.d. random returns: it is optimal for agents to hold shares equal to the country's initial share of world wealth in a fully diversified global portfolio (e.g. 10% of every country's output if n's share is a tenth).

Despite the former results, French and Poterba (1991) document the Home Bias. Tesar and Werner (1995) review developed countries data and find a strong home bias in the 1970-1990 period. They also provide evidence that the diversified world portfolio dominates the domestic portfolios (higher mean, lower variance).

Baxter and Jermann (1997): The puzzle is worse than thought. By including production and factors ($Y = AK^{\alpha}L^{1-\alpha}$) it follows that labor and capital returns are perfectly correlated (cash flows are $wL = \alpha Y$ and $rK = (1-\alpha)Y$).

Domestic stock market return (a proxy of domestic capital) hedges against human capital risk (non traded). People should take short positions in domestic stock to insure negative output shocks. Actually to hedge output risk it would make even more sense to hold foreign equities only.

Botazzi, Pesenti, and van Wincoop (1996) find the oposite result $Corr(\text{labor returns, stock mkt returns}) < 0$ and therefore domestic quities is a good hedge, i.e., HB would be partially explained.

Still there is a considerable lack of diversification with explanations such as: (1) Asset transaction costs, and (2) Non Traded goods.

Regarding (1) Tesar and Werner explain that it is false, the turnover of foreign to domestic transactions is high. (2) implies that there should be a lower implied consumption correlation w.r.t. a complete market, only traded goods setup.

Nonetheless, diversification implication when considering NT goods depend on traded and non traded goods $(C_T, C_{NT})$ interact in the utility function. If T,NT are separable in the utility function then $C_T, C_T^*$ should be ferfectly correlated.

For example:

$$U = \frac{C_{T,1}^{1-\rho}}{1-\rho} + G(C_{NT,1}) + \sum_{s=1}^{S} \beta\pi(S) \left[ \frac{C_{T,2}^{1-\rho}}{1-\rho} + G(C_{NT,2}) \right],$$

then:

$$C_{T,1}^{n} = \mu^{n} Y_{T,1}^{W},$$
$$C_{T,2}^{n}(S) = \mu^{n} Y_{T,2}^{W}(S),$$

but,

$$C_{NT,1}^{n} = Y_{NT,1}^{n},$$
$$C_{NT,2}^{n}(S) = Y_{NT,2}^{n}(S),$$

with,

$$\chi_{NT,m}^{n} = \begin{cases} 1 & \text{if } m = n, \\ 0 & \text{if } m \neq n, \end{cases}$$
$$\chi_{T,m^n} = \mu^n,$$
$$\mu^n = \frac{Y_{T,1}^{n} + V_{T,1}^{n}}{\sum_{m=1}^{N}(Y_{T,1}^{m} + V_{T,m})},$$

therefore, all non traded risk is held only by residents of country n.

Implication: by considering NT goods there will be some bias towards home assets that partly explains the HB. This is desirable but some problems remain:

- Size: The HB explained is not large as the observed one.

- Non separable utility cases: with non separable utility there are higher gains of diversification and then even fewer HB explained.

On the other hand, as mentioned before, according to Lucas 82, the size of the gain of diversification is trivial (about a fifth of a one percent of proportional increase in consumption), so there are not many incentives to diversify.

Possible explanation (OR2000, 6 puzzles paper): Transportation costs and other barriers to trade. In a static model Iceberg costs explain HB better than non-traded goods.

Model:

- 1 period, 2 countries, 2 goods (H,F). Symmetric preferences over the two goods. Endowments: $S = (Y_H, Y_F)$

- Cost of shipping: $\tau$

$$EU = E\left\{ \frac{1}{1-\rho} \left[ \left( C_H^{\frac{\theta-1}{\theta}} + C_F^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \right]^{1-\rho} \right\}$$

Assume complete markets A-D: therefore the relative MU of the two countries over each good = relative prices (for all S).

$$\frac{\partial U}{\partial C_H} \frac{1}{P_H} = \frac{\partial U^*}{\partial C_H^*} \frac{1}{P_H^*}$$

also no-arbitrage conditions imply prices are: $P_H = (1-\tau)P_H^*$ and $P_F = P_F^*/(1-\tau)$.

then,

$$C_H^{-1/\theta} = (1-\tau) C_H^{*-1/\theta},$$
$$C_F^{-1/\theta} = (1-\tau) C_F^{*-1/\theta},$$

Market Clearing conditions are:

$$C_H^* = (1-\tau)(Y_H - C_H),$$
$$C_F = (1-\tau)(Y_F - C_F^*),$$

four equations and four unknowns: $C_H, C_F, C_H^*, C_F^*$. Now, we assume that $\theta = 1/\rho$ and get equity shares for the home country given by:

$$C_H = \chi_H = \frac{1}{1 + (1-\tau)^{\theta-1}} Y_H,$$
$$\frac{C_F}{1-\tau} = \chi_F = \frac{(1-\tau)^{\theta-1}}{1 + (1-\tau)^{\theta-1}} Y_F.$$

Without iceberg costs: $\chi_H = \chi_F = 1/2$.

But with $\theta = 6$, $\tau - 0.25$: $\chi_H = 0.81$ i.e., there is a considerable degree of HB.

However, the assumption $\theta = 1/\rho$ is important to get this result, and in addition, the conclusions are weaker in dynamic models where there is a potential for reinvesting the dividends of the assets.

### 3.7.1. Endogenous Adjustment - CO1991

In their seminal paper CO1991 argue that ToT variation makes domestic investment more appealing since it automatically hedges against domestic output risk.

Framework: $H$ produces a fraction $n$ of $Y^W$ and $F$ produces the rest $(1-n)$. Consumption is given by a CD: $C = C_H^n C_F^{1-n}$.

The FOC imply:

$$C_H = n\frac{C}{P_H}, \qquad C_F = (1-n)\frac{C}{P_F}$$

Subs. in Market Clearing conditions:

$$\begin{aligned} nC_H + (1-n)C_H^* &= nY \\ n(nC + (1-n)C^*) &= nP_H Y \\ (1-n)(nC + (1-n)C^*) &= (1-n)P_F Y^* \end{aligned}$$

then,

$$P_H Y = P_F Y^*$$

Then if relative output change, relative prices will change to compensate and ensure that $Y/Y^* = P_F/P_H$, that is, insure risk and therefore it would not be much need to insure by buying foreign equities.

On the other hand, as mentioned before, OR1991 calculates that the gains of diversification are trivial, implying even less of a reason for departing from a HB scenario.

Critique: vW1999 explains that these result depend heavily on parametrization. In particular on assuming that the $ES_{H,F}$ is 1.

### 3.7.2. Other approaches

HP2008, CKM2009 include more assets: Bonds to hedge ER shocks and then Equities would be used mostly to hedge non tradable income risk. In that sense it is argued that interaction among assets can break the expected results.

BC1997 explore information frictions. The former authors build a model in which signal of foreign stock future performance is weaker (less precise), making the foreign stock be perceived as riskier. Such scenario would lead to HB. In a similar fashion vNV2008 consider local informational advantages, associated with a lower perceived riskiness that interacts with an endogenous information adquisition that leads to learning spillovers on domestic assets and reinforce the domestic bias.

## 4. ZLB, unconventional monetary policy

**Definition:** Liguidity trap is a situation when the real interest rate is negative $r < 0$, meaning the nominal rate is equal or close to zero.

Occurs due to very high patience (negative discount rate) or low expected consumption growth.

Stilized Facts during a liquidity trap: bank sector collapse and a deleveraging of the the private sector that altoguether leads to a decrease in the expected consumption growth.

Episodes: Great depression, Japan in the 90's, years posterior to the 2008-10 recession.

Effect on the output depends on price rigidities:

- Flexible Prices: output remains at FB level and prices adjust to deliver expected inflation to match the negative real rate.
- Sticky Prices: output lowers with respect to FB level.

Policy response (how to leave the trap):

- Commit to future expansionary Monetary Policy
- Temporary expansionary Fiscal Policy

- Unconventional Monetary Policy: Quantitative Easing and time varying taxes.

Main References: Krugman (1998), Eggertsson and Krugman (2012).

## 4.1. Krugman (1998)

- Endowment economy
- Dynamic model
- CRRA preferences and Cash in Advance constraint
- Households receive endowments and sell it for money to buy consumption good
- Bonds: pay $i_{t+1}$, if the rate is positive the agents won't hold money beyond what needed for consumption.
- Fixed long run expectations of variables (money supply, output, etc.)
- CIA constraint binds $CP = M$

Euler Equation and market clearing ($Y = C$):

$$1 + r_{t+1} = \frac{1}{\beta}\left(\frac{Y_{t+1}}{Y_t}\right)^{\rho}$$

Fisher equation (link between nominal and real rate):

$$1 + i_{t+1} = (1 + r_{t+1})\frac{P_{t+1}}{P_t}$$

assume $Y_{t+1} = \bar{Y}$ (fixed output in LR, LR here is one period forward), also fixed money $M$ and then via CIA constraint fixed prices: $\bar{P}$. Then rewrite the fisher equation and from it the Euler Equation:

$$1 + i_{t+1} = (1 + r_{t+1})\frac{\bar{P}}{P_t} = \frac{1}{\beta}\left(\frac{\bar{Y}}{Y_t}\right)^{\rho} \tag{CC}$$

CIA constraint with $M$ exogenous:

$$Y_t P_t \leq M_t \tag{MM}$$

the equilibrium will be the intersection of the curves:

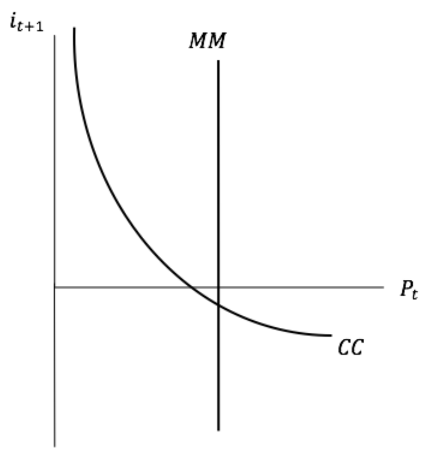


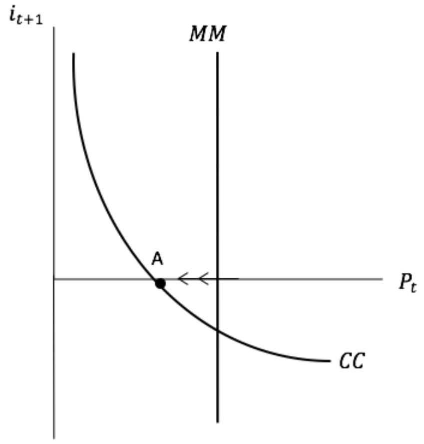

Figure 5: Source: Krugman 1998

Cases:

1. Flexible prices:

There is an excess of money at $i_{t+1}i = 0$. Money and bonds are perfect substitutes and then $P_t$ accomodates by decreasing to restore equilibrium towards point $A$ (MM moves to the left). With fixed prices in $t+1$, i.e., $\bar{P}$ we will have an expected increase in inflation between $t$ and $t+1$ ($\pi^e_{t,t+1} > 0$) s.t.,

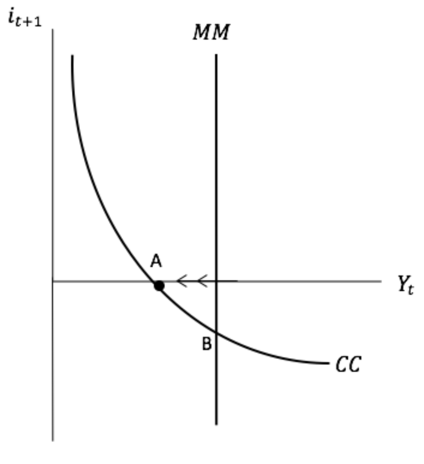
$$i_{t+1} = \pi^e_{t,t+1} + r_{t+1} = 0$$




2. Sticky prices $P_t = \bar{P}$

There cannot be an adjustment via prices since $P_t = P_{t+1} = \bar{P}$ and then the only way to equilibrate the market is by $\downarrow C_t(Y_t)$ (and then the output because of market clearing)

$i = 0$ is too high relative to the intersection equilibrium (at negative levels), then additional savings are induced s.t. $\downarrow C_t$, as an outcome there will be an inefficiently low level of Consumption/ouput relative to the potential ouput.




Conclusions can be extended to a DSGE framework:

$$x_t = -\frac{1}{\sigma}[i_t - E_t[\pi_{t+1}] - r^n_t] + E_t[x_{t+1}] \quad \text{(Dynamic IS)}$$

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa x_t \quad \text{(NKPC)}$$

$$i_t = \max\{r^n_t + \phi_\pi \pi_t + \phi_x x_t, 0\} \quad \text{(Taylor Rule)}$$

Substitute the inflation (NKPC) in the Taylor rule and obtain,

$$i_t = \max\{r_t^n + (\phi_\pi \kappa + \phi_x)x_t + \phi_\pi \beta E_t(\pi_{t+1}), 0\}$$

assume that the economy is out of the liquidity trap in $t+1$ (LR), with $E_t\pi_{t+1} = E_t x_{t+1} = 0$, then,

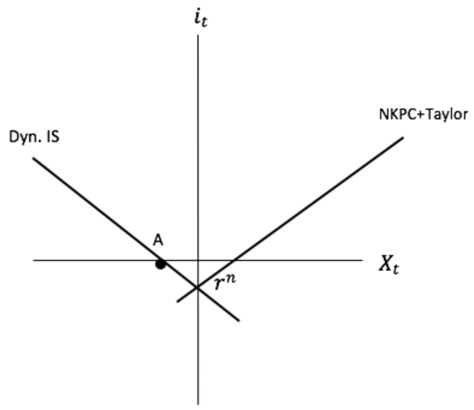



Figure 6: Source: Krugman 1998

That is, during liquidity trap, with sticky prices, output falls to A (below efficient level).

**Liquidity trap according to the Euler Equation:**

$$1 + r_{t+1} = \frac{1}{\beta}\left(\frac{C_{t+1}}{C_t}\right)^\rho$$

in summary, the key mechanism in Krugman 1998 works through the EE.

In the EE, $r < 0$ can happen due to: large $\beta$ or low expected $C$ growth.

## 4.2. Eggertsson and Krugman (2012)

Liquidity trap can also be explained by a shock in the borrowing constraints rather than more vague notions as patience or contraction in consumption growth.

**Model:**

Unit measure of borrowers, unit measure of savers.

Preferences:

$$E_t \sum_{z=t}^{\infty} \beta^{z-t}(i) \ln C_z(i) \quad \text{for } i = s, b$$

with $\beta(s) > \beta(b)$ (savers more patient than borrowers)

Endowment: $\frac{1}{2}Y$ for both borrowers and savers.

Assets Market: free trade in real riskless bond with return $r_{t+1}$ subject to a debt limit,

$$(1 + r_{t+1})D_{t+1}(i) \leq D^h$$

The solution would be obtained as an equilibrium conjecture:

- Constant consumption in steady state

- Constant consumption implies constant interest rate (EEq): $r_t = r_{t+1} = r$

- Borrowers hit the debt limit (binding) in every period:

That means that, once the SS is reached:

$$D_t(b)(1 + r_t) = D^h$$
$$D_{t+1}(b)(1 + r_{t+1}) = D^h$$

The BC for the borrowers is given by

$$C_t(b) + (1 + r_t)D_t(b) = \frac{1}{2}Y + D_{t+1}(b)$$

Interpretation: today's endowment will be used to consume, then to pay old debt plus interest and to buy debt for tomorrow ($D_{t+1}(b)$).

From the hit limit for borrowers above, substitute: $D_t(b) = \frac{D^h}{1+r}$, $D_{t+1}(b) = \frac{D^h}{1+r}$ and get:

$$C_t(b) = \frac{1}{2}Y - \frac{r}{1+r}D^h$$

and for by goods MC we have for the savers,

$$C_t(s) = \frac{1}{2}Y + \frac{r}{1+r}D^h$$

which implies that the asset market clearing condition is: $D_t(s) = -D^h$ for savers.

The savers are not constrained in any way so the EE holds for them:

$$\frac{1}{C_t(s)} = (1 + r_{t+1})\beta(s)\frac{1}{C_{t+1}(s)}$$

implying that in SS: $1 + r = \frac{1}{\beta(s)}$

For the borrowers we will have,

$$\frac{1}{C_t(b)} \geq (1 + r_{t+1})\beta(b)\frac{1}{C_{t+1}(b)}$$

with equality when not constrained, i.e., when $D_{t+1}(b) < D^h$

in SS: $1 + r \leq \frac{1}{\beta(b)}$

*Shock to the Borrowing constraint:* $D^h \to D^l$ with $D^l < D^h$.

A new steady state is reached in $t + 1$.

it will have the form:

$$C_\tau(b) = \frac{1}{2}Y - \frac{r}{1+r}D^l$$
$$C_\tau(s) = \frac{1}{2}Y + \frac{r}{1+r}D^l$$
$$1 + r_{\tau+1} = 1 + r_\tau = \frac{1}{\beta(s)} \qquad \forall \tau \geq t+1$$

Now, debt for borrowers is reduced in $t+1$ from $\frac{D^h}{1+r_t}$ to $\frac{D^l}{1+r_{t+1}}$.

The new BC in $t$ is:

$$C_t(b) \leq \frac{1}{2}Y + \frac{1}{r_{t+1}}D^l - D^h\frac{(1+r_t)}{1+r_t}$$

this is the same BC except that cannot be simplified since given the shock $r_t \neq r_{t+1}$, i.e., there will be a new equilibrium interest rate in the new SS.

the borrowing will be reduced from $\frac{D^h}{1+r_t}$ to $\frac{D^l}{1+r_{t+1}}$

More importantly the new interest rate is pinned down by the savers EE: $1 + r_{t+1} = \frac{1}{\beta(s)}\left(\frac{C_{t+1}(s)}{C_t(s)}\right)$.

Whereas for the borrowers,

$$1 + r_{t+1} < \frac{1}{\beta(b)}\frac{C_{t+1}(b)}{C_t(b)}$$

which means that the borrowers will find optimal to delever, i.e. cut down their consumption.

Market Clearing requires that real interest rate falls to induce the savers to increase consumption to offset the change induced by the borrowers.

If $D^l$ is too low with respect to the old ceiling $D^h$ then the market clearing $r$ becomes negative ($r < 0$).

Modifications: the basic intuition behind this model still works with (i) precautionary savings or (ii) endogenous output, such that, constrained agents a can work more to offset the debt based cut in consumption.

## 4.3. How to escape the Liquidity Trap

As mentioned it is possible to leave the liquidity trap with expansionary monetary policy that induces an adjustment in the expected inflation, a temporal expansionary fiscal policy or with unconventional monetary policy:

1. Monetary Policy committed to create inflation in the future:

   The central bank raises the inflation expectation to obtain a non-negative nominal interest rate that is consistent with the negative real rate. In terms of the models it corresponds to an upward shift in the CC curve.

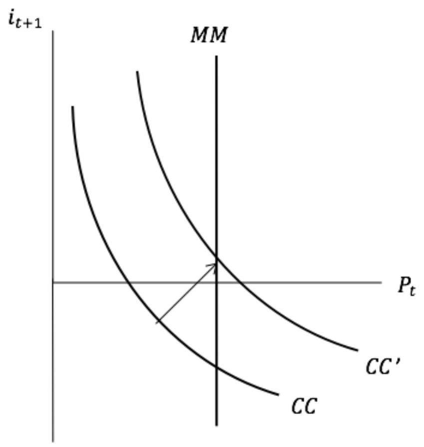



Figure 7: Source: Krugman 1998

In terms of a DSGE model the central bank would have to commit to a future expansionary monetary policy with nominal interest rate $\{i_s\}_{s=t}^{\infty}$ that gives low real interest rate in the future. The problem with this is that ex post, i.e., once the economy exits the liquidity trap, is no longer optimal to set $i_t - E_t[\pi_{t+1}] - r^n < 0$ since there would be an inneficiently high inflation and output.

2. Temporary Fiscal Stimulus:

   Add government spending into the model, then $Y_t = C_t + G_t$ and the CIA constraint is $P_t(C_t + G_t) \leq M_t^{private} + M_t^G = M_t$.

   The new CC curve is:

$$1 + i_{t+1} = \frac{\bar{P}}{\beta P_t}\left(\frac{\bar{Y} - \bar{G}}{Y_t - G_t}\right)^{\rho}$$

   Then if we change $G$ temporarily, i.e., only $G_t$ and not $\bar{G}$ then the CC curve shifts upwards and we would have an increase in the nominal rate towards non negative levels.

   Here a stimulus in the government spending crowds out the private consumption so as to raise the real interest rate.

   In the NK model the mechanism is slightly different, there, a higher G can also stimulate production and generate inflation.

3. Unconventional Monetary Policy:

   Curdia and Woodford, 2009: Quantitative Easing policies would yield portfolio balance effects, i.e., changes in the composition of assets that affect the relative spreads and the associated borrowing costs of the agents.

   CFNT 2013: Unconventional tax policies in the ZLB can be applied to use time varying taxes and engineer an increase in the expected inflation.

# 5. Exchange Rate Determination:

## 5.1. Rossi 2011: Exchange Rate Predictability (literature review)

Literature Review: post-MR1983 ER forecast methodologies. Is the ER predictable?: It could and depends on predictor an methodology.

Prediction is apparent when predictor is a Taylor Rule or NFA fundamentals, the model is linear and the model is parsimonious (small number or parameters estimated). According to Molodstova and Papell (2009) a model that works better that a RW is given by a symmetric Taylor rule without lagged interest rates: $E_t s_{t+1} - s_t = \tilde{\mu} + \tilde{\lambda}(\pi_t - \pi_t^*) + \tilde{\gamma}(y_t^{gap} - y_t^{gap*})$. On the other hand the latter, based on NFA tends to work even better than the Taylor Rule based models.

In general, the toughest benchmark is the RW without drift.

There is instability over samples for all models and then there is no systematic pattern about which horizons or sample periods work best.

PPP and monetary models have no success in the SR and MR (1-3yrs), also using typical fundamentals differential have shown to have a predictability ability that changes through time (there is substantial instability in the models' performance).

Finally, data transformations and data revisions can have substantial effects on the predictability of the models.

Summary:

TABLE 1
LITERATURE REVIEW: PREDICTORS AND ECONOMIC MODELS

| Predictors ($f_t$) | Economic fundamentals | Mnemonics |
|---|---|---|
| $i_t - i_t^*$ | Interest rate differentials | $i$ |
| $F_t - s_t$ | Forward discount | $F$ |
| $p_t - p_t^*$ | (log) price differentials | $p$ |
| $\pi_t - \pi_t^*$ | Inflation differentials | $\pi$ |
| $y_t - y_t^*$ | (log) Output differentials | $y$ |
| $m_t - m_t^*$ | (log) Money differentials | $m$ |
| $z_t$ | Productivity differentials | $z$ |
| $b_t - b_t^*$ | Asset differentials | $b$ |
| $y_t^{gap} - y_t^{gap*}$ | Output gap differentials | $y^{gap}$ |
| $nxa_t$ | Net foreign assets | $nxa$ |
| $CP_t$ | Commodity prices | $CP$ |

| Model | $f_t$ | Mnemonics |
|---|---|---|
| UIRP (CIRP) | $i_t - i_t^*, (F_t - s_t)$ | $i, F$ |
| PPP | $p_t - p_t^*$ or $\pi_t - \pi_t^*$ | $p, \pi$ |
| Monetary model with flexible prices (I) | $[(i_t - i_t^*), (y_t - y_t^*), (m_t - m_t^*)]'$ | $i, y, m$ |
| Monetary model with flexible prices (II) (or Frenkel–Bilson model) | $[(y_t - y_t^*), (m_t - m_t^*)]'$ | $y, m$ |
| Monetary model with sticky prices (I) | $[(i_t - i_t^*), (y_t - y_t^*), (m_t - m_t^*), (p_t - p_t^*)]'$ | $i, y, m, p$ |
| Monetary model with sticky prices (II) (or Dornbusch–Frankel model) | $[(i_t - i_t^*), (y_t - y_t^*), (m_t - m_t^*), (\pi_t - \pi_t^*)]'$ | $i, y, m, \pi$ |
| Model with productivity differentials (or Balassa–Samuelson (1964) model) | $[(i_t - i_t^*), (y_t - y_t^*), (m_t - m_t^*), z_t]'$ | $i, y, m, z$ |
| Portfolio balance model (or Hooper and Morton (1982) model) | $[(i_t - i_t^*), (b_t - b_t^*)]'$ | $i, b$ |
| Taylor rule model | $[(\pi_t - \pi_t^*), [(y_t^{gap} - y_t^{gap*})]'$ | $\pi, y^{gap}$ |
| Net foreign asset model | $nxa_t$ | $nxa$ |
| Commodity prices | $CP_t$ | $CP$ |

*Notes:* The table reports the name of the model, the fundamental predictors used in the model ("$f_t$"), and the mnemonics used to refer to these fundamentals in Table 2.

Figure 8: Source: Rossi 2011.

## 5.2. Overview: ER Puzzles

### 5.2.1. ER disconnect with the Fundamentals

The general approach is to look for relationship with fundamentals as well as causality direction.

In most cases the departing point is the UIP be it with respect to the nominal interest rate, or the real one that includes the price level.

Engel (2015) on Exchange Rate and Interest Parity: Depart from a UIP deviation:

$$\lambda_t \equiv i_t^* + E_t S_{t+1} - S_t - i_t$$

Apply logs and use repeated substitution:

$$s_t \equiv -\sum_{j=0}^{\infty} E_t(i_{t+j} - i_{t+j}^*) - \sum_{j=0}^{\infty} E_t \lambda_{t+j} + \lim_{j\to\infty} E_t s_{t+j+1}$$

Alternatively, in terms of the real rate:

$$s_t \approx -\sum_{j=0}^{\infty} E_t(r_{t+j} - r_{t+j^*}) - \sum_{j=0}^{\infty} E_t \lambda_{t+j} + p_t - p_t^* + \lim_{j\to\infty} E_t q_{t+j+1}$$

Then what we have is that the exchange rate is related to the future values of the fundamentals, rather than to their past values which explains the apparent disconnect between exchange rates and fundamentals found empirically as in Messe and Rogoff (1983) (MR1993).

Another way to see it is that the causality goes backwards, i.e., the exchange rate can be a predictor of future values of fundamentals. Granger causality tests, performed by EW2005.

### 5.2.2. Risk (deviations of UIP) and Forward Premium Puzzle

In the former equations there is used a deviation of the UIP. This can be associated with risk in the sense that it represents a type of risk premium or compensation when comparing $i_t$ versus $i_t^* + E_t(S_{t+1} - S_t)$

When considered the Covered Interest Parity (CIP) there is no risk premium since this condition is expected to hold. However this condition ($i_t = i_t^* + (F_t - S_t)$) stopped holding. Possibly due to very low interest rates that made unprofitable to use arbitrage conditions, given transaction costs, or due to liquidity issues in the forward instruments $F_t$ as those imposed by capital controls.

From the UIP we have: $E_t S_{t+1} - S_t = (i_t - i_t^*) - \lambda_t$.

We can test if the UIP holds, for that we simplify by assuming Rational Expectations (perfect foresight):

$$S_{t+1} - S_t = (i_t - i_t^*) - \lambda_t$$

we test this equation empirically, i.e., we test $\alpha = 0, \beta = 1$ in a linear model (Fama 1980).

However, approximating $\lambda_t$ remains a problem. A further simplyfication would pick countries with a small (as much as possible) $\lambda_t$, for example USA and Canada.

The result is that $\hat{\beta} \approx 0$ in the best cases and in others $\hat{\beta} < 0$, this is known as the **Forward Premium Puzzle (FPP)** or UIP puzzle. The result was checked with different horizons and the shorter the worse the coefficients.

Some approaches to deal with the puzzle is to not consider RE ($E_t S_{t+1} \neq S_{t+1} + \varepsilon_t$) or to try to approximate $\lambda_t$ (ignoring the risk variable can be the trouble itself if it is correlated to the interest rate differential and therefore causing an ommited variable bias in the estimated coefficients) .

## 5.3. Monetary approach (FNPV)

The idea is to identify a set of economic fundamentals that determine nominal exchange rate in the long run. A critical assumption is that of flexible prices, then the conditions for equilibrium can be focused only on the money market.

When using flexible prices, we are considering PPP as one of the building blocks. In this case, we consider that LOP holds for all goods and that if all goods are traded and preferences are symmetric, then $q_t = s_t + p_t^* - p_t$ is constant over time. This is not the case though, mainly due to non-traded goods. Then it is prefered to consider PPI as the price index instead of the CPI since it reflects tradables better. The CPI will be valid only for the LR.

Departing from a money market equilibrium condition $\frac{M_s}{P} = L(i, y)$ its possible to consider $i = f(M_s, P, Y, \dots)$. Additionally we can consider a monetary policy rule: $i = \alpha + \rho_\pi(\pi_t - \bar{\pi}) + \rho_y(y_t - y^n)$.

More specifically, following Mark (2001) we consider the money supply and demand as:

$$\begin{aligned} m_t &= \theta r_t + (1-\theta)d_t \qquad \text{w/ } \theta = E(R_t)/E(B_t) \\ m_t^d - p_t &= \phi y_t - \lambda i_t + \epsilon_t \qquad 0 < \phi < 1, 0 < \lambda, \epsilon \sim i.i.d.(0, \sigma_\epsilon^2) \end{aligned}$$

Given UIP, PPP holds and $\bar{s}$ is fixed (price is determined only by foreign price level) and then $p_t = \bar{s} + p_t^*$ and $i_t = i_t^*$.

In equilibrium $m_t = m_t^d$ and after substituting,

$$\theta r_t = \bar{s} + p_t^* + \phi y_t - \lambda i_t - (1-\theta)d_t + \epsilon_t$$

where $r_t$ denotes the log of the foreign exchange reserves, $d_t$ the domestic credit, $\bar{s}$ the exchange rate (assumed credibly fixed), $y_t$ the output and $p_t$ the log price level.

**Flexible Exchange Rate:**

Equilibrium in the domestic foreign money markets is given by,

$$\begin{aligned} m_t - p_t &= \phi y_t - \lambda i_t \\ m_t^* - p_t^* &= \phi y_t^* - \lambda i_t^* \end{aligned}$$

with $0 < \theta < 1, \lambda > 0$. Also, capital market equilibrium is given by the UIP,

$$i_t - i_t^* = E_t s_{t+1} - s_t \tag{1}$$

the relation between prices and exchange rate is given by the PPP:

$$s_t = p_t - p_t^* \tag{2}$$

let the economic fundamentals be:

$$f_t = (m_t - m_t^*) - \theta(y_t - y_t^*)$$

substitution of the the money markets, UIP and fundamental equations in (2) yields,

$$s_t = f_t + \lambda(E_t s_{t+1} - s_t) \tag{3}$$

solving for $s_t$:

$$s_t = \gamma f_t + \psi E_t s_{t+1} \tag{4}$$

in $t+1$: $s_{t+1} = \gamma f_{t+1} + \psi E_{t+1} s_{t+2}$, substitute in (4) and repeat for $t+2, t+3, \ldots$.

$$s_t = \gamma \sum_{j=0}^{k} \psi^j E_t f_{t+j} + \psi^{k+1} E_t s_{t+k+1} \tag{5}$$

with $\psi < 1$ we have $\lim_{k\to\infty} \psi^k E_t s_{t+k+1} = 0$ (TVC) and therefore:

$$s_t = \gamma \sum_{j=0}^{\infty} \psi^j E_t f_{t+j} \tag{6}$$

which means that the exchange rate is proportional to the discounted present value of the fundamentals. Similarities to asset pricing: In finance $s$ would be the price of the asset and $f$ would be the dividend.

Rational bubbles: Refers to bubbles that although explosive can be part of a valid solution of the model. In that case such solution would deviate from the fundamental based one:

Suppose TVC won't hold and therefore a bubble $b_t = (1/\psi)b_{t-1} + \eta_t$ with $\eta_t \sim i.i.d.(0, \sigma_\eta^2)$ and $1/\psi > 1$.

The solution for the exchange rate will be the sum of the fundamental driven part plus the bubble:

$$\hat{s}_t = s_t + b_t$$

we substitute in the TVC:

$$\psi^{t+k} E_t \hat{s}_{t+k} = \cancelto{0}{\psi^{t+k} E_t s_{t+k}} + \psi^{t+k} \cancelto{\frac{1}{\psi^{t+k}} b_t}{E_t b_{t+k}} = b_t$$

here $\hat{s}_t$ is a valid solution to the model but $b_t$ may lead the exchange rate to deviate from the fundamental based solution and dominate its behavior. In any case, the bubble arises in a model with rational expectations and then it is reffered as a rational bubble. This bubbles may exist but usually pop and therefore it is possible to focus mostly in the no-bubbles solution.

**Excess volatility of the Exchange Rate**

The volatility of the fundamentals deviation from the exchange rate $f_t - s_t$ is way lower than that of the returns of the exchange rates itself $\Delta s_t$, i.e., the ER displays *excess volatility* with respect to other economic variables.

The monetary model is consistent with the excess volatility in the ER if the growth rate of fundamentals is persistent:

$$\Delta f_t = \rho \Delta f_{t-1} + \epsilon_t, \qquad \epsilon_t \sim i.i.d.N(0, \sigma_\epsilon^2)$$

The implied k-step prediction formula is given by $E_t(\Delta f_{t+k}) = \rho^k \Delta f_t$. In levels, summing on the LHS until obtaining $f_t$,taking expectations and rearranging on the RHS we get:

$$\begin{aligned}
E_t f_{t+k} &= f_t + \cancelto{\rho^{k-1}\Delta f_t}{E_t(\Delta f_{t+k-1})} + \cancelto{\rho^{k-2}\Delta f_t}{E_t(\Delta f_{t+k-2})} + \cdots + \rho^k \Delta f_t \\
E_t f_{t+k} &= f_t + \sum_{i=1}^{k} \rho^i \Delta f_t \\
E_t f_{t+k} &= f_t + \frac{1-\rho^k}{1-\rho} \rho \Delta f_t
\end{aligned}$$

Substitute in (6) and use the fact that $\gamma = 1 - \psi$:

$$s_t = \gamma \sum_{j=0}^{\infty} \psi^j f_t + \gamma \sum_{j=0}^{\infty} \frac{\psi^j}{1-\rho} \rho \Delta f_t - \gamma \sum_{j=0}^{\infty} \frac{(\rho\psi)^j}{1-\rho} \rho \Delta f_t$$
$$s_t = f_t + \frac{\rho\psi}{1-\rho\psi} \Delta f_t$$

finally, after algebra we get:

$$Var(s_t) = \frac{(1-\rho\psi)^2 + 2\rho\psi(1-\rho)}{(1-\rho\psi)^2} Var(\Delta f_t) > Var(\Delta f_t)$$

Empirical tests: the net present value model has been tested empirically, mainly in terms of forecasting fit where there is a large body of research represented mainly by MR1983.

A second strategy is to test the cointegration relationship implied by the model between $s_t$ and $f_t$. This is developed by MacDonald and Taylor (1994) who find poor adjustment when testing the coefficient restrictions implied by the model.

The model implies estimates that are too smooth with respect to the reality. One of the possible shortcomings of the model is that it assumes that PPP holds even in the short run.

## 5.4. Empirical forecasting tests (MR1983 and EW2005)

With the goal to predict the nominal ER and compare structural models versus a RW in terms of prediction the authors perform forecasts evualuations based on the following general specification:

$$s = \alpha_0 + \alpha_1(m - m^*) - \alpha_2(y - y^*) + \alpha_3(i - i^*) + \alpha_4(\pi^e - \pi^{e*}) + \alpha_5 \sum (TB - TB^*) + u$$

where $s$ : log of nominal exchange rate, $m$ : log of money supply, $y$ : log of GDP, $\pi^e$ : expected inflation and $TB$ : Trade balance.

The forecast horizons span from 1 to 12 month horizon and when future values of variables are involved, the actual realized (future) values are used, which implies that the exercise is somewhat biased in favor of the model structure.

Still the results is that no structural model outperforms a random walk. According to Mark (2001) the reason for this is that it was impossible to find a time invariant relationship between the ER and the fundamentals.

#### EW2005

Out of sample tests considered are too harsh. Beating a RW in forecasting is possible a too strong criterion for accepting a model. After all ER's mainly incorporate news of future fundamentals and give few to none weight on current fundamentals, implying a near random walk behavior (the best forecast with current info (that plays no role, provides no info) is the current exchange rate value).

Actually when the future matters more, i.e., when discount factors are close to 1 as in most of the models, the exchange rate would behave more markedly as a RW.

## 5.5. Second framework: benchmark model w/ home, foreign bonds and forwards (from OR1996)

Assets:

$M_t$: home money

$B_t$: home nominal interest bearing assets at rate $i$

$B_t^*$: home nominal interest bearing assets at rate $i^*$

$F_t$: forward contract, purchase 1 unit of foreign currency for $f_t$ units of home currency ($f_t$ known in period $t$).

$e_t$: spot exchange rate (home currency per foreign)

HH UMP:

$$\max_{B_t, B_t^*, F_t, C_t} E_t \sum_{t=0}^{\infty} \beta^t U\left(C_t, \frac{M_t}{P_t}\right)$$
$$s.t$$
$$P_t Y_t + (1+i_{t-1})B_{t-1} + e_t(1+i_{t-1}^*)B_{t-1}^* + (e_t - f_{t-1})F_{t-1} + M_{t-1} = P_t C_t + B_t + e_t B_t^* + M_t$$

FOCs:

$$[B_t]: \quad \beta E_t[\lambda_{t+1}](1+i_t) = \lambda_t \tag{1}$$
$$[B_t^*]: \quad \beta E_t[e_{t+1}\lambda_{t+1}](1+i_t^*) = e_t \lambda_t \tag{2}$$
$$[F_t]: \quad E_t[(e_{t+1} - f_t)\lambda_{t+1}] = 0$$
$$E_t[e_{t+1}\lambda_{t+1}] = f_t E_t[\lambda_{t+1}] \tag{3}$$
$$[C_t]: \quad \lambda_t = \frac{U'_{c,t}}{P_t} \tag{4}$$

Covered Interest Parity: (1) over (2) yields:

$$\frac{1}{f_t}\frac{1+i_t}{1+i_t^*} = \frac{1}{e_t}$$
$$\frac{1+i_t}{1+i_t^*} = \frac{f_t}{e_t} \tag{CIP}$$

where we used (3) ($1/f_t = E_t[\lambda_{t+1}]/E_t[e_{t+1}\lambda_{t+1}]$).

Notice that in logs the CIP above can be expressed as usual: $\ln f_t - \ln e_t = i_t - i_t^*$.

Uncovered Interest Parity: Again, use (1) and (2) but replacing (4) ($\lambda = U'_c/P$ in $t$ and $t+1$),

$$e_t\left(\frac{1+i_t}{1+i_t^*}\right) = \frac{E_t\left[e_{t+1}\frac{U'_{c,t+1}}{P_{t+1}}\right]}{E_t\left[\frac{U'_{c,t+1}}{P_{t+1}}\right]}$$
$$= \frac{E_t[e_{t+1}]E_t[\frac{U'_{c,t+1}}{P_t}] + Cov\left(e_{t+1}, \frac{U'_{c,t+1}}{P_{t+1}}\right)}{E_t\left[\frac{U'_{c,t+1}}{P_{t+1}}\right]}$$
$$= E_t e_{t+1} + RP_t \tag{UIP}$$

Notice UIP can be rearranged too, in the usual way: $i_t - i_t^* = \ln E_t e_{t+1} - \ln e_t + rp_t$

The interpretation of the risk premium is straightforward: the foreign currency will have high value in bad states, i.e., when the marginal utility of consumption is high (covariance term). Also in the LHS the foreign currency assets hedge will be given by the negative sign of $i_t^*$.

As expected, it is troublesome for the equation empirical validity that the correlation implying the marginal utility of consumption does not change by much.

Market Efficiency Condition:

We can rewrite (3) as,

$$
\begin{aligned}
f_t &= \frac{E_t\left[e_{t+1}\frac{U'_{c,t+1}}{P_{t+1}}\right]}{E_t\left[\frac{U'_{c,t+1}}{P_{t+1}}\right]} \\
&= E_t[e_{t+1}] + \frac{Cov_t\left(e_{t+1}, \frac{U'_{c,t+1}}{P_{t+1}}\right)}{E_t\left[\frac{U'_{c,t+1}}{P_{t+1}}\right]} \\
&= E[e_{t+1}] + RP_t
\end{aligned}
$$

i.e., the forward rate is equal to the expected one plus the risk premium.

## 5.6. Handbook of International Economics (2014) Ch7: 5 Empirical RER facts

General stilized ER facts to be explained with the models: Co-movements or real and nominal exchange rates, high volatility of exchange rates, RER persistence (long half life).

1. Real Exchange rates for consumer prices co-move closely with nominal exchange rates at short and medium horizons. The persistence of these RERs is large with long half-lives.

   Half-life: Number of periods needed for the shocks to dissipate by half. E.g., for an AR(1) model of the form $q_t - q_0 = \rho(q_{t-1} - q_0) + \varepsilon_t$, the half life is given by $h = \ln(\frac{1}{2})/\ln(\hat{\rho})$.

   Mark (2001), using data under floating ER regimes, estimates h ranging between 2 to 5 years.

   At the micro level, several articles find large idiosyncratic movements in prices. Overgall, when disaggreagating RERs it's found that product-level RERs comove closely wiht NERs but displaying large idiosyncratic movements.

   Border effects: (Engel and Rogers 1996) cross country comparison of between and within RER's per product. With large deviation from the geographical border there is a larger deviation of PPP.

2. Movements in RERs for traded goods are roughly as large as those in overall CPI-based RERs when tradeable goods prices are measured using consumer prices or producer prices, but significantly smaller when measured using border prices (import prices, which contain a smaller non-tradeable component).

   Then traded goods would account for most of the variation of total RER, or rather said, the traded component of the RER explain most of its variation. Engel (1999) considers the following decomposition when obtaining such result:

   $$\Delta rer^{cpi}_{in,t} = \Delta rer^{T}_{in,t} + \delta rer^{NT}_{in,t}$$

   where i is the foreign country, n is the home country and,

$$\Delta rer^{T}_{in,t} = \delta s_{in,t} + \delta cpi^{T}_{i,t} - \Delta cpi^{T}_{n,t}$$
$$\Delta rer^{NT}_{in,t} = \Delta cpi_{i,t} - \Delta cpi^{T}_{i,t} - \Delta cpi_{n,t} + \Delta cpi^{T}_{n,t}$$

Still, a critique to Engel's methodology is that, as implied by the decomposition above, when using the CPI and PPI the traded component accounts all of the exchange rate account behaviour so the result itself is an assumption at the same time. Such critique is supported by the fact that when using only traded goods indexes as the import prices, the results won't hold.

3. Aggregate Exchange Rates Pass-Through (ERPT) into consumer prices is lower than into border prices. ERPT into border prices is typically incomplete in the long run, displays dynamics, and varies considerably across countries.

   Border prices, like import prices are constructed differently across countries with quite differing composition of import bundles, that can explain the differences in ERPT.

4. Micro-data border prices, in whatever invoicing currency they are set in, respond partially to exchange rate shocks at most empirically estimated horizons.

   By partial, it means that the response to the shock is incomplete.

5. There are large deviations from relative PPP for traded goods produced in a common location but sold in multiple locations. On average, these deviations co-move with exchange rates across locations.

   The deviations may come from Pricing to Market (PTM), i.e., a firm sets different prices for the same good in different markets. A particular type of PTM is the Local Currency Pricing (LCP).

   LCP: nominal ridigity in the currency of the buyer. BD1996 introduce LCP into a GE model with nominal rigidities and find that LCP affects the fraction of NER passed on to the RER. It also increases the volatility of the NER and RER.

   Basically the higher the degree of LCP the less the effect of NER in domestic price levels, less oeffect on terms of trade, i.e., LCP weakens the expenditure switching channel.

   Empirically this is noted in the fact that the TOT are smoother than the PPI-based RER.

## 5.7. Other reference articles on ER

**AAK2009:** Time varying risk, interest rate and exhange rate in general equilibrium.

The authors develop a model that delivers a NER appreciation when the risk premium falls. The problem in general is that consumption is too smooth and given it's relation with the SDF and the implied changes in the risk premium it will yield too few variability in the latter.

To deal with this it is formulated a model with cash in advance where the source of uncertainty is the money growth and the HH face a fixed cost of participating in the asset market. Also when the money growth is small the HH won't enter in the market and will consume their money holdings. As a result the consumption is an increasing, concave funtion the money growth.

The MU of the HH that enter the market will be more variable (and therefore the SDF) at lower money growth rates. The ER risk premium will be more variable too and decreasing in the money growth rate.

The model delivers an expected ER appreciation upon increases in money growth due to the movement in the risk premium for not-too-large increases in the money growth rate.

**BvW2009:** Infrequent portfolio decisions, a solution for the forward discount puzzle.

FP Puzzle: high interest rate currencies tend to appreciate. At the same time it is observed that only a small fraction of foreign currency holdings is actively managed, e.g., banks hold not many overnight positions and mutual funds are constrained to trade only certain assets.

An OLG model in which investors make a single portfolio decision every T periods is proposed. After a higher but mean reverting Foreign Interest rate shift, once the decision period passed and during intermediate periods, only newborn investors buy bonds. Investors however, foresee a lower interest rate differential in the future and will shift in foreign bonds demand, causing an expected future depreciation that will outweigh the interest rate differential and a delayed overshooting pattern arises.

**LV2007:** The Cross Section of Foreign Currency Risk Premia and Consumption Growth Risk.

To price returns Consumption Euler Equation SDF approach with Epstein-Zin preferences [3] over durable and non-durable consumption is used. The model price the returns well but suffers from EPP, requiring a risk aversion coefficient of 100. The Authors relate their findings to the BS puzzle, the correlation between ERs and Consumption growth is high.

**BvW2006:** Can Information Heterogeneity Explain the Exchange Rate Determination Puzzle?.

Macro fundamental approaches lack explanatory power wrt microstructure approaches. Then BvW2006, to reconcile these approaches, explore including investor heterogeneity into a standard monetary model.

Two types of heterogeneity: Symmetrically dispersed information about fundamentals and non-fundamentals based heterogeneity.

Trader face idiosyncratic ER exposures on their non-asset income that are private information. Then the ER has an aggregate non-observable hedging component.

There is a signal extraction confusion about ER fluctuations: private investors can't tell whether ER movements are driven by hedging or new information about the fundamentals. They rationally ascribe the change to the latter. Then the effect of hedging trades on the ER is magnified, leading to a disconnect from the fundamentals.

In the LR there is a deeper connection between ER and fundamentals since the information about the fundamentals is gradually known to all investors though price discovery.

ER is expressed as a linear combination of public information and order flow. Given the ER is sensitive to private info in the model, it is linked to order flow in the SR.

The authors find that information dispersion leads to a magnification and high persistence of non-fundamentals trade shocks on the ER. Then findings are consistent with empirical evidence where:

- Fundamentlas play little role explaining ER SR fluctuations.

- Over the LR, ER and fundamentals are closely connected.

- Exchange rates help predict weakly future fundamentals.

**Market Structure Exercises: Chen et al docs (x3)**

As seen by MR1983 and EW2005, and as implied by the NFPV model, there is an association between the exchange rate and fundamentals, but it is the current exchange rate the variable that contains information about future fundamentals. In that spirit and considering that some of these fundamentals, like the CPI include the commodity prices, it is possible to us the ER to forecast them.

The countries considered by Chen et al (2010) are NZ, CAN, AUS, Chile, South Africa. The general approach is to consider $CP_t^W = f(\Delta s_{t-i})$. Also in the countries with fixed exchange rate, it is used the real exchange rate instead.

**Term Structure of interest rates:** exploring the UIP puzzle.

[3] EZ preferences refers to a recursive utility of the form $U_t = [(1-\beta)c_t^\rho + \beta\mu_t(U_{t+1})^\rho]^{\frac{1}{\rho}}$ where $\mu_t$ is a real valued certainty equivalent operator.

The UIP puzzle: when checking empirically UIP doesn't hold and even the relation between expected appreciation and interest rates has the opposite sign ($\hat{\beta} < 0$ in the regression given by $\Delta s_{t+1} = \alpha + \beta(i_t - i_t^*) + \epsilon_{t+1}$).

One of the potential causes is the omission of the risk in the estimation. Then a possible solution is to estimate the risk and including it in the UIP.

This can be done by looking at the term structures of the interest rates, e.g., of the yield curves to correct the risk component. Arguably, a risk assesment of the market should be included in the returns of the curves.

A good property of the YC is that it takes into account the fundamentals and the relation to UIP as well as the SDF and returns of the asset (term structure), i.e., the YC captures the expected future fundamentals and the perceived risks.

The correctly specified UIP is:

$$i_t^m - i_t^{m*} = E_t \Delta s_{t+m} - \rho_t^{FX,m}, \quad \forall m$$

The YC captures the expected average return for a given maturity as well as the risk:

$$i_t^m = \frac{1}{m} \sum_{j=0}^{m-1} E_t[i_{t+j}^1] + \rho_t^{B_m}$$

where $\rho_t^{B_m}$ is the time-varying risk premia. This component should price the same risks as in the FX market, i.e.:

$$\rho_t^{FX,m} = g(\rho_t^{B_m,*}, \rho_t^{B_m})$$

On the other hand, the NS1987 approach to summarize the YC is:

$$i_t^m = L_t + S_t \left( \frac{1 - \exp(-\lambda m)}{\lambda m} \right) + C_t \left( \frac{1 - \exp(-\lambda m)}{\lambda m} - \exp(-\lambda m) \right) + \epsilon_t^m$$

where L,S,C refer to the Level, Slope and Curvature of the YC. L shifts the whole curve, S moves the short end of the YC, C moves the middle part of the YC.

Therefore, what we can do is to express the difference in the interest rates as the relative YC, i.e., the relative factors for several $m$:

$$i_t^m - i_t^{m*} = L_t^R + S_t^R \left( \frac{1 - \exp(-\lambda m)}{\lambda m} \right) + C_t^R \left( \frac{1 - \exp(-\lambda m)}{\lambda m} - \exp(-\lambda m) \right) + \epsilon_t^m$$

then we fit $\Delta s_t \approx f(\text{Relative Factors})$ and check for both fit of the estimation as well as negative signs (these will be indicative of UIP puzzle).

The gain here is that in the YC it is clearer the role of risk (appreciation arises from higher risk that offsets interest rate differential, as it should be in the UIP).

Econometrically it is estimated as: $1200 \frac{(s_{t+m} - s_t)}{m} = \beta_{0,m} + \beta_{Lm} L_t^R + \beta_{Sm} S_t^R + \beta_{Cm} C_t^R + \varepsilon_{t+m}$

for m: $1, 3, 6, 12, 18, 24$ months.

Additionally, with the yield curve factors (relative, to approximate the int. rate differential) and assuming no systematic expectation errors it is possible to argue that the excess returns capture the risk premium:

$$XR_{t+k} = i_t^{k*} - i_t^k + \Delta s_{t+k} = \rho_t^{FX,k}$$

then to check this, the risk premium is empirically approximated as:

$$XR_{t+m} = \gamma_{m,0} + \gamma_{m,1} L_t^R + \gamma_{m,2} S_t^R + \gamma_{m,3} C_t^R + \nu_{t+m}$$

Most of the coefficients from this set of regressions (one for each country) were significant, implying that risk is related to the factors.

Finally, after correcting for risk, more accurate predictions for depreciation can be done based on the UIP. Implying that there is not a disconnect between macro fundamentals and ER but a risk disregard and that the YC helps approximate the risk premium, therefore, providing a feasible explanation for the UIP puzzle.

**FX derivatives Term Structure predictive power on ER**

In a similar fashion, the UIP puzzle can be explored with the FX options term structure instead than with the YC. Here the approach of Chen and Gwati (2016) is explained.

The results are similar with the main difference being the scope of this article, that is more empirical. In summary, more moments of the expectation of depreciation (SD, Kurtosis, Asymmetry) are added to the UIP regressions to account for risk (which can be thought as non linear components of the utility and UIP itself) and a better fit is obtained.

The idea is to correct for the correct but still poor approximation of first order linear equations, also, having into account that FX options payoffs are not linear in $s_{t+\tau}$.

The justification for including higher momentsis that the investors want to maximize their expected future utility $E[U_{t+1}]$ that with feasible preferences as CARA and log-normality of returns would imply a mean-variance optimization.

In summary, what the authors do is to obtain data for Daily OTC European Options, compare 6 currency pairs with 7 tenors (horizons), back out the probabilty distribution of the future spot rates and then compute the moments that will capture the expected risk. Finally this moments are used as regressors in ER forecasting equations.

# 6. Sovereign Debt Crisis

**Overview:** until now we have assumed that international debt contracts are fully enforceable. However, in real life, foreign debt is harder to enforce and a country may decide to default on its debt service. Such lack of enforceability explain departures of efficiency and limited risk sharing in the debt markets and is mainly caused by the mismatch between the willingness to pay and the ability to do so, where the key factor is the comparison a country does between the benefit of paying a debt versus the cost of reneging it.

The two main approaches explored on this subject are the reputation based models, where a coutry is cutoff from the international financial markets and then prevented from taking future borrowing, and the direct sanction models where there is a punishment once a default has benn observed.

## 6.1. Sovereign Default Models:

Sovereign country is the borrower, normally assumed to be a SOE. Lenders are not subject to enforceability constraints.

**One period models: basic setup**

**Sanction:**

- Country faces risk income: $Income = \bar{Y} + \varepsilon$, $\varepsilon$ stochastic with N states

- Consumption insurance can be purchased in advance form international market

- Risk neutral lenders (lend as long as they receive the same expected value)

Borrowing country UMP: (borrower chooses repayment to maximize utility)

$$
\begin{aligned}
&\max_{p(\varepsilon)} EU(C) \\
&s.t. \\
&C = \bar{Y} + \varepsilon - p(\varepsilon) \\
&\sum_{i=1}^{N} \pi(\varepsilon_i) p(\varepsilon_i) = 0
\end{aligned}
$$

where $\varepsilon$ is a R.V. with $\varepsilon \in [\varepsilon_l, \varepsilon_h]$ and $E[\varepsilon] = 0$, $p(\varepsilon)$ is a debt payment, contingent on the shock.

1. No enforcement problem (First Best): always pay if can

   in this case it is optimal to set $p(\varepsilon) = \varepsilon$ s.t. $C = \bar{Y}$ (full insurance, risk sharing)

2. Country will pay only if profitable: Second Best

   - Without sanction: country never repays if $p(\varepsilon) > 0$ then no insurance is possible.

   - Sanction/Penalty case:

   Sanction: $\eta$ times GDP, then country pays if,

$$
p(\varepsilon) < \eta(\bar{Y} + \varepsilon) \tag{ICC}
$$

then the UMP becomes,

$$
\begin{aligned}
&\max_{p(\varepsilon)} EU(C) \\
&s.t. \quad \sum_{i=1}^{N} \pi(\varepsilon_i) p(\varepsilon_i) = 0 \\
&\qquad\; p(\varepsilon) \leq \eta(\bar{Y} + \varepsilon)
\end{aligned}
$$

then,

$$
\mathcal{L} = \sum_{i=1}^{N} \pi(\varepsilon_i) U(\bar{Y} + \varepsilon_i - p(\varepsilon_i)) + \mu \sum_{i}^{N} \pi(\varepsilon_i) p(\varepsilon_i) + \sum_{i}^{N} \lambda(\varepsilon_i)(\eta(\bar{Y} + \varepsilon_i) - p(\varepsilon_i))
$$

FOCs:

$$
\begin{aligned}
[p(\varepsilon_i)]: &\qquad \pi_i U'(C) + \lambda_i = \pi_i \mu \\
\lambda(\varepsilon_i)]: &\qquad \lambda_i [p(\varepsilon_i) - \eta(\bar{Y} + \varepsilon_i)] = 0
\end{aligned}
$$

Cases:

- If the ICC doesn't bind ($\lambda_i = 0$) then contract is honored and $U'(C) = \mu$ independently of the state, i.e., full insurance

- If the ICC binds ($\lambda_i > 0$) then there is no full insurance, consumption changes with the state.

In general, for high values of the shock $\varepsilon$, associated with high output, the ICC will bind and there will be harder to sustain full insurance.

Let the contract be contingent on $\varepsilon$ and have the following form:

$$p(\varepsilon) = P_0 + \varepsilon \qquad \text{for } \varepsilon \leq e$$
$$p(\varepsilon) = \eta(\bar{Y} + \varepsilon) \qquad \text{for } \varepsilon > e$$

then we constraint the high states with limited liability. Thus we need to add some constant $P_0$ to low states to preserve the expected value of payments $Ep(\varepsilon) = 0$.

At $\varepsilon = e$ the country is indifferent, that is, Utility of repaying = Utility of defaulting and assuming sanction.

from the contract we can solve for $P_0$ at the indifference threshold: $P_0 = \eta\bar{Y} - (1-\eta)e$ and then we can rewrite payment schedule (contract) as:

$$p(\varepsilon) = \eta(\bar{Y} + e) + (\varepsilon - e) \qquad \text{for } \varepsilon \leq e$$
$$p(\varepsilon) = \eta(\bar{Y} + e) + \eta(\varepsilon - e) \qquad \text{for } \varepsilon > e.$$

finally $e$ can be computed from the zero profit condition and it will depend on the distribution of $\varepsilon$.

**Results:**

- The consumption is smoothed for $\varepsilon < e$

- Some extra cost $P_0$ is required to make sure lenders break even

- A higher sanction (higher $\eta$ or $e$) makes the contract more enforceable.

- $E(C) = \bar{Y}$ but there is still no full insurance, that is, country is unable to commit to repay in high output states and therefore, in such cases, $C = f(\varepsilon)$

## 6.2. Reputation: (Eaton and Gersovitz, 1981)

- There is no output sanction, rather the defaulting country will lose access to future lending.

- Infinite horizon model

- Risk free rate: $(1+r)\beta = 1$

- Output: $Y + \varepsilon$

Gain from default: $U(\bar{Y} + \varepsilon) - U(\bar{Y})$

Given absense of future lending:

Loss from default: $\frac{\beta}{1-\beta}[U(\bar{Y}) - EU(\bar{Y} + \varepsilon)]$

Notice the loss is positive if risk averse.

The condition for full insurance is that for all $\varepsilon$ the gains from default are lower than the associated cost:

$$U(\bar{Y} + \varepsilon_{max}) - U(\bar{Y}) \leq \tfrac{\beta}{1-\beta}[U(\bar{Y}) - EU(\bar{Y} + \varepsilon)]$$

This inequality will always hold for $\beta \to 1$. Also the more risk averse, the higher the RHS and therefore the more this condition will hold.

Conclusion: Reputation equilibrium supports international lending if country has low discount rate or if it is very risk averse.

Partial Insurance: consumption will be insured if ICC is slack, which will happen for some but not all states. Consider the non-default condition (in its general form, i.e., depending of both $\varepsilon, P(\varepsilon)$) in each period,

$$U(\bar{Y} + \varepsilon_t) - U(\bar{Y} + \varepsilon_t - p(\varepsilon_t)) \leq \frac{\beta}{1-\beta} E[U(\bar{Y} + \varepsilon - p(\varepsilon)) - U(\bar{Y} + \varepsilon)]$$

Now consider the UMP subject to this ICC and zero profit, again, solving for $p(\varepsilon)$,

FOC:

$$\pi(\varepsilon_i) + \lambda(\varepsilon_i) + \frac{\beta}{1-\beta}\pi(\varepsilon_i)\sum_{j=1}^{N}\lambda(\varepsilon_j)U'(C) = \pi(\varepsilon_i)\mu$$

Now let $\lambda(\varepsilon_i) = 0$ (not binding ICC), then,

$$\begin{aligned} U'(C) &= \frac{\pi(\varepsilon_i)\mu}{\pi(\varepsilon_i) + \frac{\beta}{1-\beta}\pi(\varepsilon_i)\sum_{j=1}^{N}\lambda(\varepsilon_j)} \\ &= \frac{\mu}{1 + [\beta/(1-\beta)]\sum_j \lambda(\varepsilon_j)} \end{aligned}$$

then $C \perp \varepsilon_i$

However, in high states, ICC won't be binding and consumption will increase with $\varepsilon$. Actually, we will have $\frac{\partial p(\varepsilon)}{\partial \varepsilon} = 1 - \frac{U'(\bar{Y}+\varepsilon)}{U'(\bar{Y}+\varepsilon-p(\varepsilon))} < 1$, which means that repayment won't offset the shock as before and then consumption will change.

Note: The EG reputation equilibrium model assumes that the country is excluded from borrowing but also from saving. If savings are allowed then they are used as collateral and ICC will never bind. The SOE will find better to default always and the reputational contract is not sustainable.

That is, the threat of being cut-off from the international markets in minor if country has other investment opportunities.

**Bullow and Rogoff:** If the SOE is allowed to engage in other type of savings, for example by adquiring T-bills or other stocks then the reputational equilibrium is not sustained.

The SOE defaults eventually and uses the otherwise resources for repayment as collateral to finance other savings, then it uses the interests to finance future consumption.

Any candidate reputation contract must have a state of nature where the country prefers to default. Reputation alone cannot enforce repayment.

Reason: in any debt contract there will be a point where the value of debt reaches a maximum. There the country prefers to default and save in a way that replicates the debt contract but also generates extra income (interest not repaid). such sequence of savings is possible as long as the market offers assets indexed to the same contingencies.

## 6.3. Investment model with Borrowing Constraints (debt ceiling)

- Two periods: in $t = 1$ invests and in $t = 2$ produces

- SOE borrows but cannot commit to full repayment, only to a fraction $\eta$ of output in period 2 (limited commitment: $\eta < 1$)

The general UMP is:

$$\begin{aligned} \max_{C_1,C_2,K,D} \; & U = u(C_1) + \beta u(C_2) \\ s.t \; & C_1 + K \leq Y_1 + D \\ & C_2 \leq F(K) + K - R \\ & R = \min\{(1+r)D, \eta(F(K)+K)\} \end{aligned}$$

where:

D: borrowing from foreigner

K: capital stock

R: repayment (minimum between face value of debt and lost fraction of output)

Three outcomes:

1. *First Best:* Commitment to pay as possible $\eta = 1$

$$\max_{K,D} U = u(Y_1 + D - K) + \beta u(F(K) + K - (1+r)D)$$
$$s.t. \quad F(K) + K \geq (1+r)D \quad \text{(no default)}$$

FOCs:

$$\begin{aligned} [D]: & \qquad u'(C_1) - (1+r)\beta u'(C_2) - \lambda(1+r) = 0 \\ [K]: & \qquad -u'(C_1) + \beta u'(C_2)[F'(K)+1] + \lambda(F'(K)+1) = 0 \end{aligned}$$

- Under repayment $\lambda = 0$ then $F'(K^{FB}) + 1 = 1 + r$. That is, $MPL =$cost of capital (benchmark result)

- If constraint binds $F(K) + K = (1+r)D$ and there is in increase in marginal utility of consumption of period one (implying higher capital stock - investment). Additionally by substituting in $C_2$:

$C_2 = F(K) + K - (1+r)D = 0$ that is not optimal (corner solution).

2. *Commitment under borrowing friction/constraint:* $\eta < 1$

Change in ICC: $\eta(F(K) + K) \geq (1+r)D$

same UMP as before,

FOCs:

$$\begin{aligned} [D]: & \qquad u'(C_1) - (1+r)\beta u'(C_2) - \lambda(1+r) = 0 \\ [K]: & \qquad -u'(C_1) + \beta u'(C_2)[F'(K)+1] + \lambda\boldsymbol{\eta}(F'(K)+1) = 0 \end{aligned}$$

Cases:

- $\lambda = 0$ (not binding): $\eta(F(K) + K) > (1+r)D$

SOE finds better not to default and we obtain the same FB allocation: $F'(K) + 1 = 1 + r$, i.e., $F'(K) = r$.

This case is likely if $\eta$ or $Y_1$ is too high.

- $\lambda > 0$ constraint binds, then the country i (SOE) face a higher interest rate $r^W$.

It will find better to invest instead or repaying and then meet ICC but also overinvest the excedent.

There will be a corresponding increase in the capital w.r.t. the FB that will go beyond: $MPK =$ Intertemporal MRS.

The constraint would be $\eta(F(K) + K) = (1+r)D$.

We can solve for $u'(C_1)$ in each FOC:

From [D]: $u'(C_1) = (1+r)[\beta u'(C_2) + \lambda]$, from [K]: $u'(C_1) = (\beta u'(C_2) + \lambda\eta)[F'(K) + 1]$

Then,

$$(1+r)[\beta u'(C_2) + \lambda] = [\beta u'(C_2) + \lambda\eta][F'(K) + 1]$$

since $\beta u'(C_2) + \lambda > \beta u'(C_2) + \lambda\eta \;\Rightarrow\; F'(K) > r$, therefore, $K < K^{FB}$.

3. *No commitment under borrowing constraint*

No asymmetric or moral hazard problem. Outcome is similar to former cases.

Lenders worry that K to be chosen once he lends D will be such that it will be optimal to default. Then they try to take into account the decision process of the SOE, i.e., the investment level given D and whether it will lead to default (as with a typical ICC but per case).

Backward solution:

i. Calculate optimal $K$ given D, under repayment and default: $K^{*n}(D), K^{*d}(D)$

ii. Lenders choose $D$ constrained by a debt ceiling: $D < \bar{D}$, where $\bar{D}$ is the maximum amount sustainable without default.

Given D, default if $U^d(D) > U^n(D)$, where:

$$
\begin{aligned}
U^n(D) &= \max_{K^n} u(Y_1 + D - K^n) + \beta u(F(K^n) + K^n - (1+r)D) \\
U^d(D) &= \max_{K^d} u(Y_1 + D - K^d) + \beta u((1-\eta)(F(K^d) + K^d)) \\
&s.t. \quad D < \bar{D}
\end{aligned}
$$

and $\bar{D}$ determined from $U^d(\bar{D}) = u^n(\bar{D})$

FOCs for $K$:

$$
\begin{aligned}
u'(C_1^n) &= \beta u'(C_2)(F'(K^{*n}) + 1) \\
u'(C_1^d) &= \beta u'(C_2)(1-\eta)(F'(K^{*d}) + 1)
\end{aligned}
$$

Then substitute $C_1$, $C_2$:

$$
\begin{aligned}
u'(Y_1 + D - K^{n*}) &= \beta u'(F(K^{n*}) + K^{n*} - (1+r)D)[F'(K^{n*}) + 1] \\
u'(Y_1 + D - K^{d*}) &= \beta u'((1-\eta)(F(K^{d*}) + K^{*d}))(1-\eta)[F'(K^{d*}) + 1]
\end{aligned}
$$

finally, assume $u = \ln c, F(K) = K^\alpha$.

$$
\begin{aligned}
\frac{1}{(Y_1 + D - K^{n*})} &= \frac{\beta}{(K^{n*})^\alpha + K^{n*} - (1+r)D}(\alpha(K^{n*})^{\alpha-1} + 1) \\
\frac{1}{(Y_1 + D - K^{d*})} &= \frac{\beta}{(K^{n*})^\alpha + K^{n*}}(\alpha(K^{d*})^{\alpha-1} + 1)
\end{aligned}
$$

Now, since given D the marginal cost of additional capital is increasing, we guess that the marginal benefit (the marginal utility in the RHS of each of these two equations) is larger under no default (n). $MB^n > MB^D$, i.e.,

$$
\frac{1}{F(K^{n*}) - (1+r)D} > \frac{1}{F(K^{d*})}
$$

Then given D, the investment $K$ is higher under Non default than under Default.

Also, relative to the FB, the consumption will tilt to the first period. Effective domestic interest rate rises given that $K$ is financed with debt and the investment decreases ($K < K^{FB}$).

## 6.4. Debt Overhang model

CA deficits may cause less growth since higher debt may hurt incentives to invest:

↑ debt, ↑ prob. of default, ↑ implicit tax on investment, ↓ K.

$D$ : Exogenous Debt

$R$: Repayment, with $R = \min\{\eta AF(K), (1+r)D\}$ and A is stochastic

$V(K, D)$: Debt's value (Equals the expected value of repayment $E(R)$)

Notice that $V(K, D) \neq D$ (face value of debt) unless no default is certain.

Countries can buy back their own debts:

- Average price: $p = \frac{V(K(D),D)}{D}$
- Marginal price: $p = \frac{dV(K(D),D)}{dD}$

There will be a gain by paying back $q$ but also a possible loss given that tere is a reduction in the debt given by the slope of the laffer curve $q = dV/dD$

Output: Exogenous, Random $Y \sim f(y)$ support: $[Y_l, Y_h]$

For simplicity, let the repayment be $R = min\{\eta Y, D\}$ and $x = 1$ if SOE defaults and $x = 0$ otherwise.

Expected value of repayment (debt's value):

Since the support on Y: $[Y_l, Y_h]$ will yield a corresponding support on $R : [R, l, R^h]$,

$$\begin{aligned} E(R) = \int_{R_l}^{R^h} Rf_R(r)dr &= \int_{Y_l}^{Y^h} Rf_y(y)dy \\ &= \int_{Y_l}^{Y_h} \min\{\eta Y, D\}f_y(y)dy \\ &= \int_{Y_l}^{D/\eta} \eta Y f(y)dy + \int_{D/\eta}^{Y_h} Df(y)dy \end{aligned}$$

then,

$$V(D) = E(R|x=1) + E(R|x=0) = \int_{Y_l}^{D/\eta} \eta Y f(y)dy + \int_{D/\eta}^{Y_h} Df(y)dy$$

by Leibniz rule[4] the marginal value of the debt (derivative w.r.t. $D$) is

$$V'(D) = \int_{D/\eta}^{Y_h} f(y)dy + \frac{1}{\eta}\eta\frac{D}{\eta}f\left(\frac{D}{\eta}\right) - \frac{1}{\eta}Df\left(\frac{D}{\eta}\right) = \int_{\frac{D}{\eta}}^{Y_h} f(y)dy$$

i.e., the marginal value of debt, $V'(D)$ is equal to the probability to repay.

Notice how the probability tends to 0 as $D$ increases towards the sanction value: $\lim_{D \to \eta Y_h} V'(D) = 0$

Dynamics in prices (pre and post buy prices):

The average price is decreasing in D,

$$\frac{\partial(V/D)}{\partial D} = \frac{V'D - V}{D^2} < 0$$

[4]Let $F(c) = \int_{a(c)}^{b(c)} f(c,t)$ then $F'(c) = \int_{a(c)}^{b(c)} f_c(c,t)dt + b'(c)f(c,b(c)) - a'(c)f(c,a(c))$

since $DV'(D) = E(R|x=0) < E(R|x=1) + E(R|x=0) = V(D)$.

Assume that the debt decreases from $D_1$ to $D_2$ ($D_2 < D_1$), it follows that,

$$p_2 = \frac{V(D_2)}{D_2} > \frac{V(D_1)}{D_1} = p_1$$

where $p_1$: pre buyback price, $p_2$: post buyback price.

$p_2 > p_1$ denotes that prices will increase after D falls which will happens because a lower debt makes the payment more likely, leading to an increase in the value of debt.

## 6.5. Moral Hazard model (Gertler and Rogoff 1990)

Agents are risk neutral and the model has one period, then: $U = C$

$r$ : Interest rate

Endowment: $Y$ invested at a riskless rate or used to finance risky investment.

Investment: $I$ yields $Z$ with probability $\pi(I)$, with $\pi' > 0, , \pi'' < 0, \pi(0) = 0$

Assumptions: (i) $\pi'(0)Z > 1 + r$: investment always dominates savings (risk free rate), (ii) $I > Y$ SOE must borrow to finance investment.

Budget constraint:

$$I + L = Y + D$$

with Lending $L \geq 0$, debt $D \geq 0$

Net capital flows:

$$B = D - L$$

where $L$ yields a return $r$.

Foreigners are risk neutral and competitive, i.e., earn $r$ in expectation.

Contract: $\{D, P_1, P_0\}$

with Inflows D, repayment in goods sate ($p_1$), repayment in bad state ($p_2$).

$I$ is unobservable by lenders and hence non-contractible. $Y, D, Z$ are observed.

- *First Best:* Investment is observed, then contractible.

The contract should maximize SOE's consumption:

$$\begin{aligned}
\max_{D,L,p_1,p_0,I} \quad & \pi(I)(Z - p_1) + (1 - \pi(I))(0 - p_0) + (1 + r)L \\
s.t. \quad & L + I = Y + D \\
& \pi(I)p_1 + (1 - \pi(I))p_0 = (1 + r)D
\end{aligned}$$

the last constraint denotes that the expected payoff for lenders must be equal to market return (otherwise there would be an arbitrage oportunity in equilibrium).

Substitute the constraints in the objective function:

$$\begin{aligned}
\pi(I)Z - (\pi(I)p_1 + (1 + \pi(I)p_0) + (1 + r)L &= \pi(I)Z - (1 + r)D + (1 + r)L \\
&= \pi(I)Z + (1 + r)(Y - I)
\end{aligned}$$

take FOC,

$$[I]: \qquad \pi'(I)Z = (1 + r)$$

now, given $\pi(\cdot)$ we pin down $I^*$ and thus,

$$D^* - L^* = I^* - Y > 0$$

with $L^* \in \mathbb{R}_+$ and where $p_0^* \leq (1+r)L^*$, $p_1^*$ are set such that the zero profits condition holds (e.g. $p_0^* = 0, p_1^* = (1+r)\frac{L^*+I^*-Y^*}{\pi(I)} > 0$).

- *Second best:* unobserved $I$

$$\begin{aligned}
&\max_{D,L,p_1,p_0,I} \pi(I)(Z-p_1) + (1-\pi(I))(0-p_0) + (1+r)L \\
&\quad s.t. \quad L + I = Y + D \\
&\qquad\qquad \pi(I)p_1 + (1-\pi(I))p_0 = (1+r)D \\
&(ICC) \qquad (I, L) \in \arg\max \pi(I)(Z-p_1) - (1-\pi(I))p_0 + (1+r)L \quad \text{with } D, p_1, p_0 \text{ observed}
\end{aligned}$$

here $I, L$ non contractible, then must be incentive compatible, i.e., it holds that (taking derivative w.r.t. I):

$$\pi'(\tilde{I})(Z - (p_1 - p_0)) = 1 + r$$

compare with the condition under the FB: $\pi'(I^{FB})Z = 1 + r$

We will have that if $p_1 > p_0$: $\quad \tilde{I} < I^{FB}$

Which follows because the lender requires a higher repayment in good state in this case, i.e., $p_1 > p_0$ is a risk sharing mechanism (the lender agrees to lower the repayment in the bad state, offsetting it at some extent).

Given risk neutrality we can assume $p_0 = 0$ and make the guess that $L = 0$ that leads to the 3x3 system of equations:

IC: $p_1 = Z - (1+r)/\pi'(I)$

Zero profits: $p_1 = (1+r)D/\pi(I)$

Feasibility: $I + L = D + Y$

After solving GR1990 obtain that $\frac{\partial I}{\partial Y} > 0$. Also that if higher income raises investment, then the interest rate will increase w.r.t. to the reiskless rate and then it could explain why capital doesn't necessarily fly from rich to poor countries.

The intuition is that poorer countries are borrowed-constrained, requiring external financing. Such financing (higher debt) hurts investment, increasing the required marginal product of capital that compensates the lenders. From the point of view of the richer country, it saves more but also invests more, receiving higher capital inflows.

## 6.6. Arellano (2008), Gopinath and Aguiar (2006)

part of the sanction models, here a defaulting country is cut-off from future borrowing and will be received back after some time with a positive probability and zero net foreign assets. These dynamics will be reflected in the continuation value of the country.

- SOE with stochastic endowment $\{y_t\}_{t=0}^{\infty}$
- Single good, single asset, 1 period bond
- $u = c^{1-\gamma}/(1-\gamma)$ CRRA
- $y_t = e^{z_t}$ with $z_t = \rho_z z_{t-1} + \varepsilon_t^z$, $\varepsilon_t^z \sim N(0, \sigma_z^2)$
- $a_t$: NFA, price $q_t$ assumed and $t$ and giving 1 unit of good in $t+1$.

We want to endogeneize $q_t = f(a_t, z_t)$.

The SOE decides, after observing $a_t, z_t$, whether to default on $a_t$,

$$V(a_t, z_t) = \max\{V^G, V^B\}$$

where $V^B$: value in default state (bad state), $V^G$: value in good credit state.

Under the bad state the SOE will be cutoff from international credit market and would be able to consume only the endowment net of default cost: $y^{def}$.

There is an exogenous probability of re-entry $\lambda$ in $t+1$ with $a_{t+1} = 0$.

The value functions are,

$$\begin{aligned} V^B(z_t) &= u(y^{def}) + \beta\left((1-\lambda)E_t V^B(z_{t+1}) + \lambda E_t V(a_{t+1} = 0, z_t + 1)\right) \\ V^G(a_t, z_t) &= \max_{c_t}\left[u(c_t) + \beta E_t V(a_{t+1}, z_{t+1})\right] \\ &\quad s.t. \qquad c_t = y_t + a_t - q_t a_{t+1} \end{aligned}$$

International Capital Market:

- Risk neutral lender with opportunity cost $r^*$

let $D(a_t, z_t) = 1$ if the country defaults and 0 otherwise. Then in equilibrium,

$$q_t(a_{t+1}, z_t) = \frac{E_t\{1 - D(a_{t+1}, z_{t+1})\}}{1 + r^*}$$

Finally, the types of $y^{def}$ considered are:

- $y^{def} = (1-\delta)y$ where $\delta$: proportional cost of defaulting (Gopinath and Aguiar, 2006)
- $y^{def} = \min\{y, \hat{y}\}$, kinked output (Arellano, 2008)

**Comparison:**

Regardless of the size of the productivity shock the value function follows the same pattern if the sanction is proportional to the income: for low asset positions is better to default as the sanction cost, a fraction of the output, is low. For larger asset positions the value functions increase, and the good credit conditions one dominates, i.e., it's better no repay and the intuition is the same, the output increased significantly and therefore is much more costly to default.

Under the kinked output case we have a similar patter for low productivity shocks, however, when a high productivity shock is observed, the cost of defaulting is prohibitivetly large for all asset positions and therefore it's better to repay. In this case the punishment for defaulting is perceived as more severe.

The prices will reflect such dynamics in the value function, assets will not be highly valued if the position (and therefore the output) is low and the price will be low and close to zero. However, once it's better to repay the asset prices will surge, reflecting that the more assets the agents hold the higher their value function will be.

Finally, case 2 (kinked output) captures better the stilized facts of sovereign debt defaults, for example, the probabilities of defaulting (conditional on having default or unconditional) and also the negative correlation between the assets position and the interest rate. It could be said that the Arellano approximation captures better the trade-off faced by agents when deciding whether to default.

## 7. Financial Crisis models-Speculative Attacks, Global Games

Speculative Attacks to currency models can be summarized in three generations of models: (1) fundamental based: Focused on timing (Krugman, 1979), (2) Multiple equilibria strategic coordination models (Obstfeld 1996), and (3) Global Games and lack of common knowledge (Morris-Shin 1998).

The first approach is based entirely in a Rational Expectations framework with no strategic complementarities or behavior between agents. The second is focused on the strategic interactions of the traders that can coordinate to attack a currency and will lead to multiple equilibrium outcomes. Finally, the third one is offers a bridge between (1) and (2) by allowing information imperfections to limit the effects of the strategic behavior of the agents and therefore to recover a unique equilibrium as outcome.

## 7.1. First Generation

Here the ER peg is unsustainable because fiscal and monetary policy are inconsistent, i.e., there is a permanent fiscal deficit that is not backed up by money growth, due to a fixed exchange rate policy. Then eventually the peg has to be broken to deal with the imbalance. The focus in this approach is about timing of the speculative attack of the fiscal deficit.

**Model:**

Continuous time, perfect foresight. PPP and UIP are assumed to hold.

Money market equilbrium is given by:

$$m_t - p_t = -\eta i_t + \delta y_t$$

with continuous time it is possible to set: $y^* = i^* = 0$, also $y = 0$. The PPP is assumed to hold, then $p_t = e_t + p_t^*$

However, with $p^*$ and $y = 0$ we have,

$$m_t - e_t = -\eta i_t = -\eta \dot{e}_t$$

since by UIP: $\dot{e}_t = i_t - \cancel{i_t^*}$

with fixed ER $\bar{e}$: $\dot{e} = 0$ and therefore,

$$m_t = e_t = \bar{e} = \bar{m}$$

The Central Bank balance sheet in levels is given by:

$$M_t = \bar{M} = B_{h,t} + \bar{E} B_{f,t}$$

where $M_t$ are the liabilities and the assets are the domestic bonds $B_h$ plus the foreign ones in domestic currency $EB_f$.

Now we assume that the dynamics for the domestic bond are:

$$\frac{\dot{B}_{h,t}}{B_{h,t}} = \dot{b}_{h,t} = \mu > 0$$

Implying a fiscal deterioration, i.e., the CB gets bonds to finance fiscal deficit. Also with a pegged ER $\bar{E}$:

$$\dot{M}_t = 0 = \dot{B}_{h,t} + \bar{E}\dot{B}_{f,t} \quad \Rightarrow \quad \dot{B}_{h,t} = \underbrace{-\bar{E}\dot{B}_{f,t}}_{\text{Reserves}}$$

Now if $B_{f,t} = 0$ then $\dot{M}_t = \dot{B}_{h,t} > 0$ and therefore the peg is abandoned.

Let $\tilde{e}$: be the log shadow ER. Also, with $B_{f,t} = 0$ we had $M_t = B_{h,t}$. In logs $m_t = b_{h,t} = b_{h0} + \mu t$.

The shadow ER will be given by the average of future (expected but perfectly foreseen) ER:

$$\tilde{e}_t = \frac{1}{\eta}\int_t^{\infty} e^{-\frac{-(s-t)}{\eta}}(b_{h0} + \mu S)dS = b_{h,t} + \eta\mu$$

Once the peg is expected to break (i.e. an increase in ER is anticipated), the agents will anticipate such trend and will drop $B_f$ (they attack the currency), the money supply will fall due to the attack, and then after the peg is abandoned the money stock starts growing again.

The speculative attack occurs when the shadow rate equals the peg, that is when the level of reserves is positive.

Actually after the collapse $m_t = b_{h,t}$ that is, $\dot{b}_{h,t} = \mu, \dot{\tilde{e}}_t = \mu$ and since the speculative attack happens at $T$ s.t. $\tilde{e}_T = \bar{e}$ then we can solve for the period of attack T:

$$b_{h0} + \mu T + \eta\mu = \bar{e} \quad \rightarrow \quad T = \frac{\ln(B_{h0} + \bar{E}B_{f0}) - \eta\mu - b_{h0}}{\mu}$$

It should be noticed that this model is not speculative, the agents are rational and fully informed of when to attack the currency.

Then the model is good to explain rational currency attacks when a country has large holdings of reserves. But it is ill-suited to address strategic behavior problems under uncertainty.

## 7.2. Second Generation

A second approach allows for speculative behavior about a depreciation, this depreciation expectations can be self-fulfilling. Here Monetary and Fiscal policy will not be inconsistent, instead, there is some degree of lack of credibility in the policies, partly due to the fact that defending the peg is costly.

Therefore, there is space for strategic complementarities in the behavior of the agents and the speculative attack may occur regarless of fundamentals and due to the agents coordination.

A trader may want to attack if other agents are likely to attack too. The expectations will determine the possible equilibrium outcomes that will be multiple. For example, consider an scenario of modest reserves and normal macroeconomic fundamentals outlook, then if the peg is credible, the central banck can increase the money growth without breaking the regime. On the other hand, if agents attack the banck will not be able to sustain the peg due to low reserves.

## 7.3. Third Generation

This model recovers aspect from the former two generations. The main idea is that with an additional small amount of uncertainty about fundamentals is possible to restore the uniqueness of the equilibrium, regardless of a multiplicity of possible results.

Here we will have both strategic complementarities but also strategic uncertainty. Then we have an equilibrium that is driven by fundamentals (1st Gen) but with strategic complementarities and coordination (2nd Gen).

Agents will follow cutoff strategies according to their signals and the success of the attack will be determined by fundamentals.

The dispersion of information is important for uniqueness, and actually, adding an informative enougth public signal about fundamentals can re introduce multiplicity of solutions.

The framework in which this works follows Carlsson and van Damme (1993) approach: noise is added to a game theory framework, leading to strategic uncertainty. Then with private information, the coordination among agents gets complicated enough that a unique equilibrium arises.

**Model:**

Three periods. Continuum of traders in $(0, 1]$, each with a unit of home currency.

$t = 0$

$\theta$ : amount of reserves, randomly determined.

$t = 1$

Each agent observes a private signal $x_i$ about $\theta$: $x_i = \theta + \varepsilon_i$, with $\varepsilon \sim N(0, \sigma_\varepsilon^2)$, let the precision of the signal be given by $\lambda_\varepsilon = \frac{1}{\lambda_\varepsilon^2}$.

An action is taken by each agent after observing the signal:

$$I_i = \begin{cases} 1 & Attack \\ 0 & Not \end{cases}$$

Attack refers to sending the unit of home currency to the CB in exchange for its value in foreign currency. The payoff of the actions $\Pi(I_i)$ is:

$$\Pi(1) = \begin{cases} -c & \text{if } R = 0 \\ 1 - c & \text{if } R = 1 \end{cases}$$

$$\Pi(0) = 0$$

where $c$ is the cost of attacking and $R = 1$ denotes that the ER peg is abandoned.

$t = 2$

CB observes $\theta$ and $\{I_i\}$ and chooses $R$.

Let $A$: size of the attack. The peg is abandoned if $A$ exceeds the reserves amount:

$$\begin{aligned} &R = 1 \iff A = \int_0^1 I_j d_j > \theta \\ &R = 0 \quad \text{O.W.} \end{aligned}$$

We solve the game by Backwards Induction. The agents will choose an action such that,

$$I_i(x_i) = \arg\max E[u(I_i(x_i), I_{-i}(x_{-i}), \theta | x_i]$$

Notice that $I_{-i}(x_{-i})$ indicates the presence of strategic complementarities: agents' decisions affect each other. with:

$$u = \begin{cases} I_i(x_i)[-c] & \text{if } A \leq \theta \quad (R = 0) \\ I_i(x_i)[1 - c] & \text{if } A > \theta \quad (R = 1) \end{cases}$$

**(i) Perfect Information Bechmark**

$x_i = \theta \quad \Rightarrow$ Multiple Equilibrium (Obstfeld 1996) if $\theta \in (0, 1)$.

Two equilibria:

No attack: $I_j = 0 \quad \forall j \quad A = 0 < \theta$

All attack: $I_j = 1 \quad \forall j \quad A = 1 > \theta$

**(ii) Imperfect Information case**: threshold rule ($x^*$ cutoff)

$$I(x_i) = \begin{cases} 1 & \text{if } x_i < x^* \\ 0 & \text{if } x_i \geq x^* \end{cases}$$

Given $\theta$, the probability of attack is given by (we drop $i$ guessing the eq. is symmetric):

$$
\begin{aligned}
Pr(x < x^*|\theta) &= Pr(\theta + \varepsilon < x^*|\theta) \\
&= Pr(\varepsilon < x^* - \theta|\theta) \\
&= \Phi\left(\frac{x^* - \theta}{\sigma_\varepsilon}\right) \\
&= \Phi(\sqrt{\lambda_\varepsilon}(x^* - \theta))
\end{aligned}
$$

By LLN we can pin down the size of the attack (all agents' signal is i.i.d., and the size of the attack in a strictly monotonic decreasing function of $\theta$): $A(\theta) = \Phi(\sqrt{\lambda_\varepsilon}(x^* - \theta))$.

Moreover, $A(\theta) - \theta$ is also decreasing, with $A(0) - 0 \geq 0$ and $A(1) - 1 \leq 0$, then we can find $\theta$ by considering that there is a $\theta^* \in [0, 1]$ s.t., $A(\theta^*) = \theta^*$, i.e., we will find $\theta$ as:

$$
\theta^* = \Phi(\sqrt{\lambda_\varepsilon}(x^* - \theta^*)) \tag{1}
$$

The agents observe $x_i$ and consider $I_i = 1$, i.e.,

$$
\begin{aligned}
E(u) &= -cPr(A(\theta) \leq \theta|x_i) + (1-c)Pr(A(\theta) \geq \theta|x_i) \\
&= Pr(A(\theta) \geq \theta|x_i) - c \\
&= Pr(\theta^* > \theta|x_i) - c \qquad \text{given } \theta^* = A(\theta^*)
\end{aligned}
$$

we can assume a flat distribution (posterior, i.e., given $x_i$) for $\theta$: $\theta|x_i \sim N(x_i, \lambda_\varepsilon^{-1})$. Therefore

$$
E(u) = 1 - \Phi(\sqrt{\lambda_\varepsilon}(x_i - \theta^*)) - c
$$

where we flipped the probability argument.

The indifference condition $x_i = x^*$ is given by:

$$
\begin{aligned}
E(u|I = 1) &= \cancelto{0}{E(u|I = 0)} \\
1 - \Phi(\sqrt{\lambda_\varepsilon}(x_i - \theta^*)) - c &= 0 \\
\Phi(\sqrt{\lambda_\varepsilon}(x_i - \theta^*)) &= 1 - c
\end{aligned} \tag{2}
$$

by (1): $\theta^* = 1 - c$. Substitute in (2):

$$
\Phi(\sqrt{\lambda_\varepsilon}(x_i - (1 - c))) = 1 - c
$$

then we can solve for $x_i$:

$$
x_i = \frac{\Phi^{-1}(1 - c)}{\sqrt{\lambda_\varepsilon}} + 1 - c
$$

Here the key message is that no matter how precise the signal is, private information is different from public, and just a little uncertainty can reduce the outcome to a unique equilibrium.

## 7.4. Summary: Handbook of Int. Econ. Chapter 12

**G. Lorenzoni's Summary on International Financial Crises (first sections of Ch. 12)**

Financial crisis are broadly defined as a sudden outflow of financial resources in an economy. It is usually driven by expectations of large devualuations in fixed ER regimes or by changes in the expected fundamentals.

The literature has explored first the *Currency Crisis* (Krugman 1979) which refers to a situation in which the agents have doubts about the capacity of a central bank to back a fixed exchange regime with enough reserves and therefore a subsequent attack on the currency occurs. Here the NER is viewed as a monetary phenomenon and the focus is mainly on fixed rate regimes.

The initial approach (called first generation of models) is based entirely on rational expectations and abstract from strategic complementarities between agents' expectations and actions (attack or not the currency).

The key element in this approach is that of unsustainable economic policies, for example inconsistent fiscal and monetary policies, that lead to discontinuous adjustment processes in the exchange rate due to forward looking behavior by the agents.

In this case, without CB independence, and given that fiscal and monetary policy are connected by the government budget constraint, certain fiscal policies can lead to ER instability, and in general, a successful fixed ER regime requires commitment to fiscal discipline and MP independence.

A second generation of models study the coordination strategic behaviour between agents and how, depending on the expectations and for intermediate levels of the fundamentals (where is not entirely clear what is the willingness of the CB to defend the ER peg), multiple equilibrium outcomes can arise.

Here the focus in on overvaluation of the exchange rate, regardless of the fundamentals, once they take some intermediate values. Here, the main element is a lack of commitment from the CB to a peg that is self-fulfilling and leads to a recessionary shock that pushes the CB to choose a devaluation.

The intermediate values that the fundamentals take are what lead to possible outcomes under different plausible expectations about the expected benefit of abandoning the peg from the perspective of a CB that compares defending the peg versus a welfare maximizing allocation net of the cost of breaking the regime:

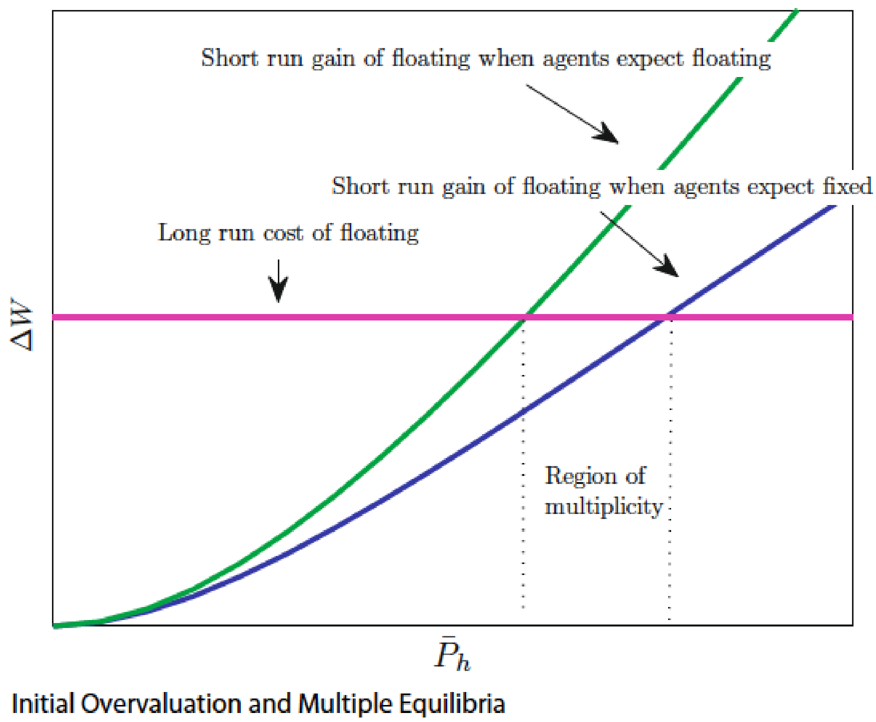



Figure 9: Source: Handbook of Int. Macroeconomics, Chapter 12.

The third strand of literature on ER regime attacks is denominated the Global Game and is based on MS1998 as mentioned in previous sections. This approach arises from noticing that the possibility of multiple equilibria is not entirely independent of the fundamentals but quite the opposite, they take values such that several scenarios about ER are possible. Then MS1998 add a small amount of information uncertainty to difficult strategic coordination among agents and then discard most of the outcomes.

The new degree of idyosincratic uncertainty has an effect on the agents guesses about his and others' expectations on the value of the ER that reduces the outcome to an unique equilibrium as shown in the previous section.

Finally, a second block of literature, departed from examining the ER regime and have explored the *Current Account Reversals* as a whole. The emphasis is put on financial flows and suddent stop of capital flows, i.e., to a loss of access to international borrowing and therefore to a forced reduction on the CA deficit (Dornbusch et al 1995, Calvo 1998).

The sudden stops can be taken either as exogenous in the literature, and then their effect is studied, or can be considered as endogenous, leading to models of defaultable debt where the fear of default triggers self-fulfilling capital fligths.

In this approach the mechanism about how an expectation of a devaluation can lead to a capital flight, forcing the CB to tighten the domestic policy and leading to a recession are also at work undera flexible ER regime as long as the CB tries to dampen the effect of swings in expectations of ER changes.

Such swings are the real reason behind the so called fear of floating of policy makers.

$$c_t^i + (1 - q_t\theta)i_t = r_t n_t + (\phi_t q_t + (1 - \phi_t)q_t^R)(1 - \delta)n_t + p_t m_t$$

# A Simple Two Period Open Economy Model

*Courtesy of Ippei Fujiwara*

For simplicity, we assume that the exchange rate is fixed at 1. This will be similar to a two agents model in the closed economy.

## Domestic Country

### Household

The representative household in the home country maximizes welfare:

$$u(C_1) - v(h_1) + \beta[u(C_2) - v(h_2)]$$

subject to the budget constraints:

$$B_1 + C_1 = W_1 h_1 + \Pi_1$$

and

$$C_2 = W_2 h_2 + R_2 B_1 + \Pi_2$$

### Firm

A representative firm in the home country maximizes the profit:

$$\Pi_t = Y_t - W_t h_t$$

subject to the production function:

$$Y_t = f(z_t h_t) \tag{1}$$

## Foreign country

## household

The representative household in the foreign country maximizes welfare:

$$u(C_1^*) - v(h_1^*) + \beta[u(C_2^*) - v(h_2^*)]$$

subject to the budget constraints:

$$B_1^* + C_1^* = W_1^* h_1^* + \Pi_1^*$$

and

$$C_2^* = W_2^* h_2^* + R_2^* B_1^* + \Pi_2^*$$

### Firm

A representative firm in the home country maximizes the profit:

$$\Pi_t^* = Y_t^* - W_t^* h_t^*$$

subject to the production function:

$$Y_t^* = f(z_t^* h_t^*) \tag{2}$$

## Clearing conditions

The financial market clearing condition is given by:

$$B_t + B_t^* = 0 \tag{3}$$

By substituting the profits into the budget constraints, we have:

$$B_1 + C_1 = Y_1 \tag{4}$$
$$C_2 = Y_2 + R_2 B_1 \tag{5}$$
$$B_1^* + C_1^* = Y_1^* \tag{6}$$
$$C_2 = Y_2 + R_2 B_1 \tag{7}$$

Together with the financial market clearing conditions, we can derive the goods market clearing conditions, i.e., we can derive the resource constraints:

$$C_1 + C_1^* = Y_1 + Y_1^* \tag{8}$$
$$C_2 + C_2^* = Y_2 + Y_2^* \tag{9}$$

## Optimality conditions

From households' problems we have:

$$v'(h_1) = u'(C_1)W_1 \tag{10}$$
$$v'(h_2) = u'(C_2)W_2 \tag{11}$$
$$v'(h_1^*) = u'(C_1^*)W_1^* \tag{12}$$
$$v'(h_2^*) = u'(C_2^*)W_2^* \tag{13}$$
$$u'(C_1) = \beta R_2 u'(C_2) \tag{14}$$
$$u'(C_1^*) = \beta R_2 u'(C_2^*) \tag{15}$$

From the firms' problem we have:

$$W_1 = z_1 f'(z_1 h_1) \tag{16}$$
$$W_2 = z_2 f'(z_2 h_2) \tag{17}$$
$$W_1^* = z_1^* f'(z_1^* h_1^*) \tag{18}$$
$$W_2^* = z_2^* f'(z_2^* h_2^*) \tag{19}$$

## System of equations

Equations (1) to (19) solve for the 19 endogenous variables: $C_1, C_2, C_1^*, C_2^*, h_1, h_2, h_1^*, h_2^*, Y_1, Y_2, Y_1^*, Y_2^*, B_1, B_1^*, W_1, W_2, W_1^*, W_2^*$ and $R_2$.

### Reduction

After substituting some variables, the 9 equations below solve for the 9 endogenous variables: $C_1, C_2, C_1^*, C_2^*, h_1, h_2, h_1^*, h_2^*$ and $R_2$.

$$
\begin{aligned}
v'(h_1) &= u'(C_1) z_1 f'(z_1 h_1) \\
v'(h_2) &= u'(C_2) z_2 f'(z_2 h_2) \\
v'(h_1^*) &= u'(C_1^*) z_1^* f'(z_1^* h_1^*) \\
v'(h_2^*) &= u'(C_2^*) z_2^* f'(z_2^* h_2^*) \\
U'(C_1) &= \beta R_2 U'(C_2) \\
\frac{u'(C_2)}{u'(C_1)} &= \frac{u'(C_2^*)}{u'(C_1^*)} \\
C_1 + C_1^* &= f(z_1 h_1) + f(z_1^* h_1^*) \\
C_2 + C_2^* &= f(z_2 h_2) + f(z_2^* h_2^*) \\
C_1 + \frac{C_2}{R_2} &= f(z_1 h_1) + \frac{f(z_2 h_2)}{R_2}
\end{aligned}
$$

Note that the last equation implies a similar condition must hold in the foreign country as well, but is becomes redundant.

# B Risk Sharing - Model with Contingent Claims

## One Bond, two states and two periods

With non-contingent bonds: temporary negative output shocks lead to CA deficits (borrowing).

With State contingent bonds: No change in consumption. Risk insurance.

$$\max_{C_1, B_2(1), B_2(2)} u(C_1) + \beta E_1[u(C_2)]$$

s.t.

$$C_1 + \frac{P(1)}{1+r} B_2(1) + \frac{P(2)}{1+r} B_2(2) = Y_1$$

and,

$$C_2(S) = Y_2(S) + B_2(S), \quad \text{for } S = \{1, 2\}$$

The UMP can be expressed as:

$$\max_{B_2(1), B_2(2)} u\left(Y_1 - \frac{P(1)}{1+r} B_2(1) - \frac{P(2)}{1+r} B_2(2)\right) + \beta \sum_{S=1}^{2} \pi(S) u\left(Y_2(S) + B_2(S)\right)$$

FOC:

$$[B_2(S)]: \qquad \frac{P(S)}{1+r} u'(C_1) = \beta \pi(S) u'(C_2(S))$$

By taking ratios it can be seen the how the relative price depends is equal to the MRS in expected terms (meaning that the more likely a state the higher the relative price of a consumption claim in it),

$$\frac{P(1)}{P(2)} = \frac{\pi(1) u'(C_2(1))}{\pi(2) u'(C_2(2))}$$

Notice that if $\frac{P(1)}{P(2)} = \frac{\pi(1)}{\pi(2)} \Rightarrow C_2(1) = C_2(2)$, i.e., we have Full Insurance.

## Risk Sharing Conditions

The conditions above work analogously for the foreign agents. Then we have the following risk sharing conditions:

### Intertemporal

$$\begin{aligned} \pi(S)\beta \frac{u'(C_2(S))}{u'(C_1)} &= \frac{P(S)}{1+r} = \pi(S)\beta \frac{u'(C_2(^*S))}{u'(C_1^*)} \\ \Rightarrow \quad \frac{u'(C_2(S)}{u'(C_1)} &= \frac{u'(C_2^*(S))}{u'(C_1^*)} \end{aligned} \tag{1}$$

### Across states

$$\begin{aligned} \frac{u'(C_2(S))}{u'(C_2(S'))} &= \frac{P(S)}{P(S')} = \frac{u'(C_2(^*S))}{u'(C_2^*(S'))} \\ \Rightarrow \quad \frac{u'(C_2(S)}{u'(C_2(S'))} &= \frac{u'(C_2^*(S))}{u'(C_2^*(S'))} \end{aligned} \tag{2}$$

Therefore, in a complete markets set up we have equal MRS across countries. **Date by date and state by state.**

With CRRA:

$$C_2(S) = \left[\frac{\beta\pi(S)(1+r)}{P(S)}\right]^{\frac{1}{\rho}} C_1$$
$$C_2^*(S) = \left[\frac{\beta\pi(S)(1+r)}{P(S)}\right]^{\frac{1}{\rho}} C_1^*$$

Market clearing conditions:

$$C_1 + C_1^* = Y_1 + Y_1^*$$
$$C_2(S) + C_2(S)^* = Y_2(S) + Y_2(S)^*$$

By adding the first order conditions and plugging the market clearing constraints:

$$Y_2^w(S) = \left[\frac{\beta\pi(S)(1+r)}{P(S)}\right]^{\frac{1}{\rho}} Y_1^w$$

$$\Rightarrow \quad \beta\pi(S)\left[\frac{Y_2^w(S)}{Y_1^w}\right]^{-\rho} = \frac{P(S)}{1+r}$$

and by taking ratios across states:

$$\frac{P(S)}{P(S')} = \frac{\pi(S)}{\pi(S')}\left[\frac{Y_2^w(S)}{Y_2^w(S')}\right]^{-\rho}$$

Then, by (1):

$$\frac{C_2(S)}{C_1} = \frac{Y_2^w(S)}{Y_1^w} = \frac{C_2^*(S)}{C_1^*}$$

Constant shares or the world output across dates

Analogously, by (2):

$$\frac{C_2(S)}{C_2(S')} = \frac{Y_2^w(S)}{Y_2^w(S')} = \frac{C_2^*(S)}{C_2^*(S')}$$

Constant shares across states

# Part II.
# International Macroeconomics

## 8. Introduction - Complete Markets benchmark to Sequential Incomplete Markets basic setup

### 8.1. Complete Markets

Benchmark: A-D structure with complete markets, traded at $t = 0$.

Exogenous state: $s_t \in S$; storty up to $t$: $s^t = [s_0, \ldots, s_t]$

Probability: $\pi_t(s^t)$, conditional on $s^\tau$: $\pi_t(s^t|s^\tau)$

$I$ countries, each with a representative HH: $i = 1, \ldots, I$

$HH_i$ owns a stochastic endowment $y_t^i(s^t)$, its consumption is given by $c^i = \{c_t^i(s^t)\}_{t=0}^{\infty}$

$$U(c^i) = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t u(c_t^i(s^t))\pi_t(s^t) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^i)$$

$$\sum_{t=0}^{\infty} \sum_{s^t} q_t^0(s^t) c_t^i(s^t) \leq \sum_{t=0}^{\infty} \sum_{s^t} q_t^0(s^t) y_t^i(s^t) \tag{BC}$$

where: $q_t^0(s^t)$ is the price of contingent period $t$-consumption claims, with a date of trade 0 and contingent to $s^t$.

Result: The equilibrium allocation only depends on the aggregate endowment (*Perfect Risk Insurance*) i.e., the consumption won't depend on Country-level endowments/shocks or on preceding history.

UMP:

$$\max U(c^i) \quad s.t. \text{ (BC)}$$

FOC:

$$\beta^t u'(c_t^i(s^t))\pi_t(s^t) = \mu^i q_t^0(s^t)$$

$$\Rightarrow \frac{u'(c_t^i(s^t))}{u'(c_t^j(s^t))} = \frac{\mu^i}{\mu^j} \quad \text{(constant ratios of marginal utilities across histories)}$$

Risk Sharing: Consumption levels will move in the same direction regardless of endowment dynamics

Rearrange:

$$c_t^i(s^t) = u'^{-1}\left(u'(c_t^1(s^t))\frac{\mu_i}{\mu_1}\right)$$

Substitute in MC Cond. ($\sum_i c_t^i(s^t) \leq \sum_i y_t^i(s^t)$):

$$\sum_i u'^{-1}\left(u'(c_t^1(s^t))\frac{\mu_i}{\mu_1}\right) = \underbrace{\sum_i y_t^i(s^t)}_{\text{Aggregate Endowment}}$$

Therefore, consumption is not history dependent (only depends on aggregate $y$)

**Proposition:** *The equilibrium allocation is a function of the world endowment.*

$$c_t^i(s^t) = c_\tau^i(\tilde{s}^\tau) \text{ for } s^t,\ \tilde{s}^\tau \text{ if } \sum_i y_t^i(s^t) = \sum_i y_\tau^i(\tilde{s}^\tau)$$

with power utility (CRRA): $c_t^i(s^t) = c_t^j(s^t)\left(\frac{\mu_i}{\mu_j}\right)^{-1/\gamma}$ in this case the fractions of world endowment are independent of the realization of $s^t$

## 8.2. Incomplete Markets

Risk sharing condition doesn't hold anymore, the equilibrium allocation is no longer history independent.

- Two countries

- HH supplies L and trades a 1 period bond. The bond is NOT contingent: Markets are incomplete.

- Two consumption goods. Full specialization by country. Competitive firms, distributed in $[0,1]$, $[0,a]$ for home, and $[a,1]$ for foreign firms.

Preferences: $U_{t_0} = E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[\rho\log C_t^i + (1-\rho)\log(1-L_t^j)\right]$

Consumption: $C_t^j = \left[a^{\frac{1}{\omega}}(C_{H,t}^j)^{\frac{\omega-1}{\omega}} + (1-a)^{\frac{1}{\omega}}(C_{F,t}^j)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}$ $\omega > 0$: IES bw $C_H$ and $C_F$

BC: $B_{t+1}^j = (1+r_t)B_t^j + w_t L_t^j - C_t^j$

FOC:

$$L_t = 1 - \frac{1-\rho}{\rho}\frac{C_t}{w_t} \quad \text{(same for i,j)}$$
$$C_t^{-1} = \beta(1+r_{t+1})E_t(C_{t+1}^{-1})$$

Firms

Technology: $Y_t^{S_i} = Z_t L_t^i$ (supply)

Demand: $Y_t^{D_i} = RP_t^{-\omega}C_t^W$ with $C_t^W = aC_t + (1-a)C_t^*$ with $RP$ : price of home goods in units of consumption (measure of relative prices).

in Perfect Competition: Price = Marginal Cost:

$$RP_t = \frac{w_t}{Z_t}$$

Now consider Monopolistic Competition:

$$Y_t^{S_i} = Y_t^{D_i}, \quad C_t^W = Y_t^{S_W} = Y_t^{D_W} = Y_t$$
$$Z_t L_t = (RP_t)^{-\omega}Y_t^W$$

Then:

$$L_t = (RP_t)^{-\omega}\frac{Y_t^W}{Z_t}$$

Aggregation:

International Bond MC: $aB_t + (1-a)B_t^* = 0$

Aggregate per-capita output: $y_t = RP_t Z_t L_t$ This is obtained by expressing each firm's output in terms of the world consumption basket, multiplying by the number of firms and dividing by population.

In perfect competition we also have $y_t = w_t L_t$

### 8.2.1. Indeterminancy of the Steady State

The Steady State equation for the asset accumulation can be expressed as[5]

$$\bar{B}[1-\beta(1+\bar{r})] = \bar{w}[1-(1-\beta)\frac{1+\bar{r}}{\bar{r}}]$$

Additionally from the Euler Equation from consumption we have $\beta(1+\bar{r}) = 1$, i.e., $\bar{r} = \frac{1-\beta}{\beta}$.

Substitute $\bar{r}$ and you obtain an indetermination: $0 = 0$

The reason for the indetermination is that the Euler Equation doesn't depend on the assets.

Now, take the ratio of the Euler Equations:

$$\frac{C_t^{-1}}{C_t^{*-1}} = \frac{C_{t+1}^{-1}}{C_{t+1}^{*-1}}$$

Log-linearize:

$$\mathsf{C_t} - \mathsf{C_t^*} = E_t(\mathsf{C_{t+1}} - \mathsf{C^*_{t+1}})$$

The Consumption differential will be a random walk.

The model still has a solution but it is path dependent (e.g. OR1995).

*Problem:* Log-linear approximation is valid in a neighborhood of the steady state but with non-stationarity we may depart from such region after any shock.

*Implication:* Since all shocks are permanent (even the temporary ones), we will have infinite unconditional variances of endogenous variables.

**Solutions**

1. *Back to complete markets:* Risk Sharing removes non stationarity (Since $\mathsf{C_t} = \mathsf{C_t^*}$ always). The problem is the poor empirical support of this option.
2. *Add the Real Exchange Rate (RER):* The risk sharing condition becomes: $\frac{u'(C_t)}{u'(C_t^*)} = \kappa Q_t$. Then the consumption baskets are allowed to be different (e.g., home bias in consumption). However, the result is that consumption differential and RER are perfectly correlated which doesn't happen in reality (B-S puzzle).
3. *Knife Edge parameter values:*

   Let the superscript D denote the cross country differential across variables in deviations from the SS with $\bar{B} = \bar{B}^* = 0$. We would have the following expressions for human wealth and assets.

$$\mathsf{h_t^D} = \beta \mathsf{h_{t+1}^D} + (1-\beta)\mathsf{w_t^D}$$
$$\mathsf{B_{t+1}} = \frac{1}{\beta}\mathsf{B_t} + (1-a)(\mathsf{y_t^D} - \mathsf{C_t^D})$$

   This system can be rewritten as:

$$\begin{bmatrix} \mathsf{h_{t+1}^D} \\ \mathsf{B_{t+1}} \end{bmatrix} = M_1 \begin{bmatrix} \mathsf{h_t^D} \\ \mathsf{B_t} \end{bmatrix} + M_2 \mathsf{Z_t^D}$$

[5] substitute the optimal labor from the FOC in the budget constraint: $B_{t+1} = (1+r_t)B_t + w_t + \frac{1}{\rho}C_t$. Now let human wealth be defined as $h_t = \sum_{s=t}^{\infty} R_{t,s} w_s$ with $R_{t,s} = \Pi_{u=t+1}^{s}[(1+r_u)]^{-1}$. Consider the deterministic case, then $w_t = w_{t+j}$ $\forall j$ and $h_t = w_t + \beta w_t + \beta^2 w_t + \cdots$ can be expressed as $h_t = \frac{1}{1-\beta}w_t = \frac{1+r}{r}w_t$. Finally, in such case it also holds that $B_{t+1} = (1+r_t)B_t$. After these substitutions consumption will be given by $C_t = \rho(1-\beta)[(1+r_t)B_t + h_t]$ and the asset accumulation will be $B_{t+1} = \beta(1+r_t)B_t + w_t - (1-\beta)h_t$.

with

$$M_1 = \begin{bmatrix} \frac{\rho\omega+\beta(1-\rho)}{\beta[1+\rho(\omega-1)]} & -\frac{\rho(1-\rho)(1-\beta)^2}{\beta^2(1-a)[1+\rho(\omega-1)]} \\ -\frac{\omega(1-a)}{1+\rho(\omega-1)} & \frac{1+\rho(\omega\beta-1)}{\beta[1+\rho(\omega-1)]} \end{bmatrix}, \quad M_2 = \begin{bmatrix} -\frac{\rho(1-\beta)(\omega-1)}{\beta[1+\rho(\omega-1)]} \\ \frac{(\omega-1)(1-a)}{1+\rho(\omega-1)} \end{bmatrix}$$

The eigenvalues of $M_1$ are $1/\beta$ and 1. Since there is one non-predetermined variable and one eigen-value outside of the unit circle the Blahchard-Kahn conditions hold and this system has a determinate solution. However, the eigenvalue equal to 1 is consistent with the non-stationarity of the assets.

Now, if we let $\omega = 1$ then $M_2 = 0$. We would have a unique solution given by $\mathsf{h}_\mathsf{t}^\mathsf{D} = \mathsf{B}_\mathsf{t} = 0\ \forall t$ (Cole and Obstfeld 1991).

This means that for initial $NFA = 0$, $\omega = 1$ (consumption basket in CD form with unitary elasticity between home and foreign goods) we get as solution that $NFA_t = 0 \forall t$ and that there will not be a consumption differential.

This result is driven by the ToT. With $\omega = 1$ the ToT will move in a way that $\mathsf{RP}_\mathsf{t}^\mathsf{D} = -\mathsf{Z}_\mathsf{t}^\mathsf{D}$. Then, the price and quantity effects of the ToT will exactly offset.

4. *Make the Euler Equation depend on Asset Holdings:* for example, include convex cost of adjustment of bond holdings.

   With convex cost the BC becomes:

$$B_{t+1}^j + \frac{\mu}{2}(B_{t+1}^j - B)^2 = (1 + r_t)B_t^j + w_t L_t^j + T_t^j - C_t^j$$

   The EEq becomes:

$$C_t^{-1}[1 + \mu(B_{t+1} - B)] = \beta(1 + r_{t+1})E_t(C_{t+1}^{-1})$$

   in the SS:

$$1 + \mu(\bar{B} - B) = \beta(1 + \bar{r})$$
$$1 + \mu(\bar{B}^* - B^*) = \beta(1 + \bar{r})$$

   then,

$$1 + \mu\underbrace{[a\bar{B} + (1 - a)\bar{B}^* - aB - (1 - a)B^*]}_{0} = \beta(1 + \bar{r})$$

   then $1 = \beta(1 + \bar{r})$, i.e., in SS: $\bar{B} = B,\ \bar{B}^* = B^*$ Meaning that the NFA will be equal to an exogenous reference level. This method is popular but is an ad-hoc solution.

   Other solutions within this framework: Exogenous Discount Factor, Debt elastic Interest rate premium, bonds in utility (S-GU2003) (all link Consumption growth and assets)

   Alternative framework solution: OLG (Breaking Ricardian Equivalence)

   - 2 Countries H,F

   - Each period there are $N_t^W$ agents (infinitely lived). The population growth is given by $N_{t+1} = (1 + n)N_t,\ N_0^W = 1$.

   - HH's are born at different periods with no assets.

   Key changes:

   The EE now depends on $C_{t+1}^{t+1}$ (newborns consumption at $t + 1$)

$$c_t = \frac{1 + n}{\beta(1 + r_{t+1})}\left(c_{t+1} - \frac{n}{1 + n}C_{t+1}^{t+1}\right) \qquad \text{if} n > 0 \text{the model is stationary}$$

   with $C_{t+1}^{t+1} = \rho(1 - \beta)h_{t+1}$

   Dynamics of the NFA:

$$(1 + n)B_{t+1} = (1 + r_t)B_t + w_t L_t - c_t$$

   There will be a discrepancy bw average asset holdings of old and newborns. Then aggregate percapita consumption growth depends on the asset holdings.

Stationarity: A given shock affects assets of alive population but newborns enter and wipe the effect away over time (temporary shocks won't have permanent effect anymore).

Then endogenously: $\bar{B} = \bar{B}^* = 0$, $\bar{r} = (1-\beta)/\beta$ (if an agent is a debtor or creditor will depend on whether $\beta(1+\bar{r}) \gtrless 1$

# 9. SOE - IRBC model

A first step into modeling an open economy is to consider economies that interact with the rest of the world but consider foreign variables as exogenous. In that sense, the structure looks similar to the closed economy setups, except that we have NFA, a trade balance, and current account fluctuations.

Initial versions consider frictionless setups with homogeneous goods and preferences across locations (home and foreign). The seminal SOE-IRBC contribution is made in Mendoza (1991). After that, the setup has been complemented features such as multiple country-specific goods, and non-traded goods, which respectively, imply introducing new relative prices that become relevant for open economies such as the terms-of-trade and real-exchange-rate.

## 9.1. BKK 1992 - SOE-IRBC with one good

Context/Benchmark: Mendoza 1991, IRBC-SOE with incomplete markets and endogenous Discount Factor.

- Two countries (analogous), one final good. No role for RER in BKK 1992

- Immobile labor

- Production subject to country specific shocks

- Setup: Campbell 1994 with: (i) Leisure habits, (ii) inventories, (iii) time to build. These are all eliminated in BKK 1994

$l_t^i$ : Leisure

Labor follows a distributed lag:

$$\begin{aligned} l_t &= 1 - \alpha n_t - (1-\alpha)\eta a_t \\ a_{t+1} &= (1-\eta)a_t + n_t \end{aligned} \tag{3}$$

Where $n_t$ : working time and $a_t$: effect of past leisure    past leisure choices affect current utility

UMP:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[(c_t^i)^{\mu}(l_t^i)^{1-\mu}\right]^{\gamma}}{\gamma} \qquad i = h, f$$

Technology:

$$y_t^i = \left\{[\lambda_t^i (k_t^i)^{\theta}(n_t^i)^{1-\theta}]^{-\nu} + \sigma(z_t^i)^{-\nu}\right\}^{-1/\nu} \qquad \nu > -1,\ \sigma > 0 \tag{4}$$

$\lambda_t^i$: technology shock

Market Clearing Feasibility: Global output is equal to sum of aggregate consumption, fixed investment and inventory accumulation.

$$\sum_i (c_t^i + x_t^i + z_{t+1}^i - z_t^i) = \sum_i y_t^i \tag{5}$$

Net exports:

$$nx_t^i = y_t^i - (c_t^i + \underbrace{x_t^i}_{\text{investment}} + \underbrace{z_{t+1}^i - z_t^i}_{\text{Inventories change}}) \tag{6}$$

Capital dynamics:

$$k_{t+1} = (1-\delta)k_t^i + \underset{\substack{\text{inv. projects 1 period} \\ \text{from completion}}}{s_{1,t}^i} \tag{7}$$

Fixed investment:

$$x_t^i = \sum_{j=1}^{J} \phi_j s_{j,t}^i \tag{8}$$

Technology Shocks: Country specific (idiosyncratic) $\lambda_t = [\lambda_t^h, \lambda_t^f]' \sim VAR(1)$, i.e.,

$$\boldsymbol{\lambda}_{t+1} = \mathbf{A}\boldsymbol{\lambda}_t + \boldsymbol{\epsilon}_{t+1}, \quad \text{with} \quad \boldsymbol{\epsilon}_t \sim MVN(0, \mathbf{V}) \tag{9}$$

Equilibrium: Pinned down by solving a SPP,

$$\max \phi E(U^h) + (1-\phi)E(U^f)$$
$$s.t. \;\; (3) - (9)$$

Deterministic Steady State:

Because of symmetry, the SS is the same of a closed economy but replicated twice.

Implicit Assumption: Countries start with zero initial net wealth.

Consumption, labor and inventories are constant. Then: $r = \frac{1-\beta}{\beta}$

Investment $= \delta K$

then the resource constraint becomes: $c + \delta k = y$

Rental price of inventories: $r$

Rental price of capital: $q(r+\delta)$, with $q = \sum_i^J \phi_j (1+r)^{j-1}$

FOCs:

$$\begin{aligned} r &= \sigma(y/z)^{1+\nu} \\ q(r+\delta) &= \theta(y/k)[1-\sigma(y/z)^{\nu}] \\ w &= (1-\theta)(y/n)[1-\sigma(y/z)^{\nu}] \end{aligned}$$

Consumers's FOC for leisure ($U_l/U_c = w$):

$$\frac{(1-\mu)c(\alpha r + n)}{r+\nu} = \mu w(1-n)$$

Calibration: $\nu = 3, \;\; J = 4, \;\; \alpha = 1, \;\; \mu = .34, \;\; \gamma = -1$

$$A = \begin{bmatrix} 0.91 & 0.088 \\ 0.088 & 0.91 \end{bmatrix}$$

**Results:**

Relative to data:

- Consumption is too smooth
- Investment is too volatile
- S shaped profile of consumptions cross-correlation is found but way weaker than in the data (also too high consumption correlation)
- Trade Balance not countercyclical
- Negative output correlation
- Too high consumption correlation - far more than output (Feldstein-Horioka puzzle)

IRFs: home investment boom, i.e., shock to $\epsilon^h$.

At home:

- $\uparrow$ productivity that decays
- $\uparrow$ Investment, consumption and output
- Given large $\uparrow$ Investment (larger than output) $\Rightarrow$ Trade deficit

Foreign:

- Positive spillover in productivity
- Initially $\downarrow$ y, $\downarrow$ x due to resources shifting to home. No assets interaction b/w countries but solution is given by a SPP, then policy maker shifts resources. Furthermore, added to technology there is also a high interaction via trade of the final good.

**Intuition:** The discrepancies with the data appear because agents have greater ability to shift resources across countries. That could explain the found flaws: Excessive consumption smoothing, excessive volatility of investment, negative output correlation.

**Introduction of trade costs**: The goal is to make harder for agents to shift resources.

Trade costs: after including convex costs of trade feasibility becomes

$$\sum_i (c_t^i + x_t^i + z_{t+1}^i - z_t^i) = \sum_t y_t^i - \sum_i \tau (nx_t^i)^2$$

Under autarky: spillover effects come only from technology shocks correlation (only connection between countries).

With the trade cost: $Corr(y^h, y^f) = 0.02$ (before it was -0.18).

but still $Corr(c^h, c^f) = 0.88$ (too high although improved since before it was 0.91).

**Conclusion:** New puzzle (Consumption anomaly puzzle)

$Corr(y, y^*) > Corr(c, c^*)$ in the data

$Corr(y, y^*) < Corr(c, c^*)$ in the model

with negative correlation of output.

## 9.2. BKK1994 - SOE-IRBC with Two Goods and TOT

- Two country IRBC to analyze two features of the data:

1. Countercyclical Trade Balance
2. S shaped correlation between Trade Balance (TB) and Terms of Trade (ToT). Negative correlation with future $\Delta ToT$, positive correlation with past $\Delta ToT$

- Fully specialized production $\Rightarrow$ Role for ToT. main difference with BKK1992

Good $a$ is produced in country 1 and good $b$ in country 2.

- Government spending shocks (more sources of fluctuations).

- **Complete Assets Markets** also in BKK1992. Then the equilibrium is solvable by freely transferring resources between markets via a SPP.

- No habits of leisure nor inventories.

UMP:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[(c_{i,t})^{\mu}(1-n_{i,t})^{1-\mu}\right]^{\gamma}}{\gamma}, \quad i = 1, 2 \quad \gamma < 1$$

Technology:

$$y_{i,t} = Z_{i,t} k_{i,t}^{\theta} n_{i,t}^{1-\theta}, \quad i = 1, 2$$

where $Z_{i,t}$ is an aggregate technology shock in the economy $i$.

Resource constraints:

$$a_{1,t} + a_{2,t} = y_{1,t}$$
$$b_{1,t} + b_{2,t} = y_{2,t}$$

with $a_i, b_i$: usage of each good by country $i$.

Then, $a_{2,t}$: exports from country 1 to country 2, $b_{1,t}$ : imports from 2 into 1.

**New:** Consumption, investment and government purchases are CES composites of $a$ and $b$:

$$c_{1,t} + x_{1,t} + g_{1,t} = \left[\omega_1(a_{1,t})^{-\rho} + \omega_2(b_{1,t})^{-\rho}\right]^{-\frac{1}{\rho}}$$
$$c_{2,t} + x_{2,t} + g_{2,t} = \left[\omega_1(b_{2,t})^{-\rho} + \omega_2(a_{2,t})^{-\rho}\right]^{-\frac{1}{\rho}}$$

This representation allows for Home Bias in consumption (the RER plays a role here).

Notice that the goods are crossed in the function, i.e., for each country the first coefficient represents the share of local good in consumption.

- $\sigma = \frac{1}{1+\rho}$: Elasticity of Subst. between domestic and foreign good.

- $g$ is exogenous and stochastic

Capital dynamics:

$$k_{i,t+1} = (1-\delta)k_{i,t} \quad + \underbrace{s_{i,t-J+1}}_{\substack{\text{planned additions}\\\text{to capital stock in}\\t+J}}$$

Investment: sum of expenditures on active projects $x_{i,t} = \frac{1}{J}\sum_{j=0}^{J-1} s_{i,t-j}$

$J = 1$ used for the calibration.

Shocks: technology and government spending

$$\mathbf{z}_{t+1} = \mathbf{A}\mathbf{z_t} + \boldsymbol{\epsilon}^z_{t+1},\ \mathbf{z_t} = [z_{1,t}, z_{2,t}]', \boldsymbol{\epsilon}^z_t = [\epsilon^z_{1,t}, \epsilon^z_{2,t}]$$
$$\mathbf{g}_{t+1} = \mathbf{A}\mathbf{g_t} + \boldsymbol{\epsilon}^g_{t+1},\ \mathbf{g_t} = [g_{1,t}, g_{2,t}]', \boldsymbol{\epsilon}^g_t = [\epsilon^g_{1,t}, \epsilon^g_{2,t}]$$

Resource constraint by country:

$q_{1,t}$ : Price of a good $a$ in units of the consumption basket.
$q_{2,t}$ : Price of a good $b$ in units of the consumption basket.

Then for country 1:

$$c_{it} + x_{it} + g_{it} = q_{1t}a_{1t} + q_{2t}b_{1t}$$

The ToT are defined as: $p_t = q_{2t}/q_{1t}$ (relative price of imports, reverse of OR1995 definition.)

The ToT is obtained as the MRT between the two goods in country 1:

$$p_t = \frac{q_{2t}}{q_{1t}} = \frac{\omega_2}{\omega_1}\left(\frac{a_{1t}}{b_{1t}}\right)^{1/\sigma}$$

Results:

NX and ToT are highly correlated Here inclusion of K is key. Otherwise the agents have very high incentives to smooth consumption.

NX are counter cyclical: by adding K the model displays procyclical investment, then, $\uparrow I, \uparrow C \Rightarrow Corr(Y, NX) < 0$

Strong positive correlation between ToT and Y. (This is still a flaw since it's negative in the data).

The correlation is negative, but with very high substitutability ($\sigma$) it can become positive. Also the S shaped pattern remains for the different values of $\sigma$.

Withouth Capital: No S curve, TB becomes procyclical and $Corr(TB, ToT) > 0$. The reason is that there's not an investment boom upon the technology shock.

Problem: Too little ToT volatility.
New Puzzle: Toot little reltive price volatility.
Same unresolved puzzle: Consumption-ouput anomaly (BKK Puzzle)

## 9.3. BS1993 - SOE-IRBC with Non-Traded Goods and RER fluctuations

Before, the ToT are relevant. Still, with all goods were traded, LOP holds and symmetric preferences the PPP holds. With $P_t = P_t^*$ we have a constant RER ($RER_t = 1 \; \forall t$).

**Non Traded Goods** (NT) are added so that the consumption baskets differ, hence, the PPP won't hold that the RER fluctuates.
the traded good is the same so that the $ToT_t = 1$ and the LOP hold. Still the consumption baskets differ.

The set up assumes Complete Markets.

Prediction: $\frac{u'(c)}{u'(c^*)} \approx RER_t$ (Consumption ratio and the RER are perfectly correlated).

**BS Puzzle:** Data shows 0 or negative sign.

**Model:**

$I$ countries, $I+1$ goods (I non tradables, 1 in each country).

States: $z_t$, history: $z^t$, prob. of $z^t$: $\pi(z^t)$
$q_0$: date $t$ price of traded good, $Q_0$: date 0 price.
$q_i$: date $t$ price of $i$'s non traded good, $Q_i$: date 0 price.
$a_i$: country $i$'s consumption of traded good
$b_i$: country $i$'s consumption of non-traded good

UMP:

$$\max U_i = \sum \beta^t \sum_{z^t \in Z^t} \pi(z^t) u[a_i(z^t), b_i(z^t)] \tag{1}$$

Price index: $p_i(q_0, q_i)$

Quantity index: $c_i(a_i, b_i)$, s.t., $u[a_i, b_i] = v[c_i(a_i, b_i)]$, $v'(\cdot) > 0$

With homothetic preferences: $\exists\ p)i, c_i$, s.t.:

$$q_0 a_i + q_i b_i = p_i(q_0, q_i) c_i(a_i, b_i)$$

Utility is assumed time separable, but period utility is not separable between traded and non-traded goods $a_i, b_i$.

Example of an aggregator:

$$c(a_i, b_i) = [\alpha a_i^\rho + (1-\alpha) b_i^\rho]^{\frac{1}{\rho}}$$
$$p_i(q_0, q_i) = \left[\alpha^{\frac{1}{1-\rho}} q_0^{\frac{\rho}{\rho-1}} + (1-\alpha)^{\frac{1}{1-\rho}} q_i^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}}$$

$\frac{1}{1-\rho}$: elasticity of substitution between traded and non-traded goods ($\rho = 1$: perfect substitutes, $\rho = 0$: Cobb-Douglas basket)

RER:

$$e_{ij} = \frac{p_j(q_0, q_j)}{p_i(q_0, q_i)}$$

Risk free bond (A-D securities):

$$s_i(z^t) = \sum_{z_{t+1}} \frac{p_i[Q_0(z^{t+1}), Q_i(z^{t+1})]}{p_i[Q_0(z^t), Q_i(z^t)]}$$

Real interest rate:

$$r_i(z^t) = s_i(z^t)^{-1} - 1, \quad r_i(z^t) \approx -\log[s_i(z^t)]$$

Date 0, present value, Budget Constraint:

$$\sum_{t=0}^{T} \sum_{z^t} \left[Q_0(z^t)a_i(z^t) + Q_i(z^t)b_i(z^t)\right] = Q_0(z_0)n_i(z_0) + \sum_{t=0}^{T} \sum_{z_t} \left[Q_0(z^t)w_i(z^t) + Q_i(z^t)x_i(z^t)\right] \tag{2}$$

where $n_i(z_0)$ is the net foreign asset position of country $i$ at the start of the period 0.

The consumer's UMP is to choose $\{a_i, b_i\}_{t=0}^{\infty}$ to maximize (1) s.t. (2)

Additionally, a solution must satisfy the market clearing condition in all states:

$$\sum_{i=1}^{I} a_i(z^t) = \sum_{i=1}^{I} w_i(z^t) \equiv W(z^t)$$

$$b_i(z^t) = x_i(z^t), \quad \forall i, z^t$$

Countries cannot hedge each other against idiosyncratic risk on the endowment of the non-traded good.

Equilibrium:

without NT good:

PPP holds: $e_{ij} = 1$

If u(c) is CRRA: $p(z^t) = W(z^t)^{-\gamma}$

Consumption rtios are constant (then consumption of $i$ and $j$ are perfectly correlated)

Real interest rates are equalized:

$$s_i(z^t) = \sum_{z_{t+1}} \beta\pi(z_{t+1}|z^t)\frac{p(z^{t+1})}{p(z^t)}$$

Intuition: Since there is only one good, the real rate of return on the risk free consumption based bond is the same to all countries.

With NT good:

The equilibrium is characterized by solving a SPP:

$$\max \sum_{i=1}^{I} \lambda_i U_i \quad s.t. \quad \sum_{i=1}^{I} a_i(z^t) \leq W(z^t)$$
$$b_i(z^t) \leq x_i(z^t)$$

the respective lagrange multipliers are: $\beta^t\pi(z^t)q_0(z^t)$, $\beta^t\pi(z^t)q_i(z^t)$.

FOCs:

$$[a_i]: \quad \lambda_i \frac{\partial u(a_i, b_i)}{\partial a_i} = q_0, \qquad [b_i]: \quad \lambda_i \frac{\partial u(a_i, b_i)}{\partial b_i} = q_i$$

i.) PPP won't hold since the endowments and prices of NT goods differ

ii.) Consumption indexes are NOT perfectly correlated

iii.) Monotonic relation between $\frac{c_i}{c_j}$ and RER $e_{ij}$. The FOC (with CRRA) implies $(\lambda_j/\lambda_i)\left(\frac{c_i}{c_j}\right)^{\gamma} = \frac{p_j}{p_i} = e_{ij}$

Consider the log-linearized model:

$$\gamma(\mathsf{c_i} - \mathsf{c_j}) = \mathsf{e_{ij}}$$

then:

$$\gamma^2 Var(\mathsf{c_i} - \mathsf{c_j}) = Var(\mathsf{e_{ij}})$$

i.e., the variances will be equal only if $\gamma = 1$.

Summary:

PPP won't hold because of the presence of NT goods

Real interest rate will vary across $i$.

There will be restrictions on the moments of real interest rate differential.

**BS Puzzle**: Model implies perfect correlation between consumption differential and RER. Obvious candidate to deal with this puzzle: Market incompleteness.

# 10. Deviations of the PPP

Let $\epsilon$: nominal exchange rate, $Q$: real exchange rate ($\uparrow Q$: depreciation)

Log-linearizing:

$$\mathsf{Q} = \mathsf{e}_t + \mathsf{P_t^*} - \mathsf{P_t}$$

If PPP holds we have that: $\mathsf{Q_t} = 0$ (i.e., $\varepsilon P^* = P$)

However, there are several sources of PPP fluctuations:

1. Deviations from LOP: frictions in trade costs, Pricing to Market (PTM), Local Currency Pricing (LCP)
2. Home Bias in preferences
3. Non-traded goods

## 10.1. Deviations from the LOP

### Frictions in trade costs

Weights in the consumption basket (2 goods, 2 countries setup): $a$: home good, $(1 - a)$: foreign good

Assume an identical consumption basket. The prices level in each location are:

$$\mathsf{P_t} = a\mathsf{P_{H,t}} + (1 - a)\mathsf{P_{F,t}},$$
$$\mathsf{P_t^*} = a\mathsf{P_{H,t}^*} + (1 - a)\mathsf{P_{F,t}^*},$$

with $\mathsf{P_{H,t}}$: home currency price of home good, $\mathsf{P_{F,t}^*}$: foreign currency price of foreign good.

If LOP holds:

$$\mathsf{P_{F,t}} = \mathsf{e}_t + \mathsf{P_{F,t}^*},$$
$$\mathsf{P_{H,t}} = \mathsf{e}_t + \mathsf{P_{H,t}^*},$$

We can substitute these expressions in the equations above and get: $\mathsf{P_t} = \mathsf{P_t^*} + \mathsf{e}_t \quad \Rightarrow \ \mathsf{Q_t} = 0$ (i.e., PPP holds)

Now, suppose LOP does not hold: there is a friction in international trade, e.g., a tariff $\tau_t$ s.t.:

$$\mathsf{P_{F,t}} = \tau_t + \mathsf{e}_t + \mathsf{P_{F,t}^*},$$
$$\mathsf{P_{H,t}} = \tau_t + \mathsf{e}_t + \mathsf{P_{H,t}^*},$$

Substituting: $\mathsf{P_t} = \mathsf{e}_t + \mathsf{P_t^*} + \tau_t$ (PPP will not hold) $\rightarrow \ \mathsf{Q_t} = -\tau_t$ That is, the fluctuations in the trade costs will translate into the RER fluctuations

### PTM: Pricing to Market

Firms charge different prices in different markets (locations)

For example, as a result of price discrimination under flexible prices (strand of literature in the 80's)

#### LCP: Local Currency Pricing

Firms set prices in currency of consumers.

Implies deviations from PPP if prices are sticky: with 1 period stickiness $P_t, P_t^*$ won't move in $t$, then $\mathsf{P_t} = \mathsf{P_t^*} = 0$.

Then, there is no pass-through of the exchange rate into the prices.

Thus, in $t$: $\mathsf{Q_t} = \mathsf{e}_t$ (in $t$ the RER tracks the ER)

## 10.2. Home Bias in preferences

Change in weights: $a \in (0,1)$ for Domestic good (own good), and $(1-a) \in (0,1)$ for foreign good.

$a = 1/2$ implies no Home Bias (same consumption baskets)

$a > 1/2$: Home Bias in preferences

Basket prices now are:

$$\begin{aligned} \mathsf{P_t} &= a\mathsf{P_{H,t}} + (1-a)\mathsf{P_{F,t}}, \qquad \text{Before: } \mathsf{P_t} = a\mathsf{P_{H,t}} + (1-a)\mathsf{P_{F,t}},\ \mathsf{P_t^*} = a\mathsf{P_{H,t}^*} + (1-a)\mathsf{P_{F,t}^*} \\ \mathsf{P_t^*} &= a\mathsf{P_{F,t}^*} + (1-a)\mathsf{P_{H,t}^*}, \end{aligned}$$

Assume trade is frictionless and prices are flexible (or are set in currency of producers). Then: LOP holds

$$\begin{aligned} \mathsf{P_{F,t}} &= \mathsf{e}_t + \mathsf{P_{F,t}^*} \\ \mathsf{P_{H,t}} &= \mathsf{e}_t + \mathsf{P_{H,t}^*} \end{aligned}$$

Define the terms of trade (relative price of exports):

$$\mathsf{TOT}_t = \mathsf{P_{H,t}} - \mathsf{e}_t - \mathsf{P_{F,t}^*} \qquad (\text{TOT improvement/appreciation: } \uparrow \mathsf{TOT_t})$$

Then, use algebra to link the RER and the TOT:

$$\begin{aligned} \mathsf{Q}_t &= \mathsf{e}_t + \mathsf{P}_t^* - \mathsf{P}_t \\ &= \mathsf{e}_t + a\mathsf{P}_{F,t}^* + (1-a)\mathsf{P}_{H,t}^* - a\mathsf{P}_{H,t} - (1-a)\mathsf{P}_{F,t} \\ &= \mathsf{e}_t + a\mathsf{P}_{F,t}^* + (1-a)(\mathsf{P}_{H,t} - \mathsf{e}_t) - a\mathsf{P}_{H,t} - (1-a)(\mathsf{e}_t + \mathsf{P}_{F,t}^*) \\ &= (1-2a)(-\mathsf{e}_t) + (1-2a)\mathsf{P}_{H,t} + (1-2a)(-\mathsf{P}_{F,t}^*) \\ &= (1-2a)[\mathsf{P}_{H,t} - \mathsf{e}_t - \mathsf{P}_{F,t}^*] \\ &= -(2a-1)\mathsf{TOT}_t \qquad \Rightarrow \text{RER is negatively proportional to TOT} \end{aligned}$$

If $a = 1/2$ (No HB) then $\mathsf{Q_t} = 0$ and PPP holds; if $a > 1/2$ (HB): $\uparrow \mathsf{TOT_t} = \downarrow \mathsf{Q_t}$.

## 10.3. Non-Traded goods

Assume there is non-traded good, the consumption weights are: $b$ for the traded, $(1-b)$ for the non-traded good.

The prices are:

$$\mathsf{P}_t = b\mathsf{P}_{T,t} + (1-b)\mathsf{P}_{NT,t} \tag{1}$$

$$\mathsf{P}_t^* = b\mathsf{P}_{T,t}^* + (1-b)\mathsf{P}_{NT,t}^* \tag{2}$$

Then,

$$\mathsf{Q}_t = \mathsf{Q}_{T,t} + (1-b)\overbrace{[\mathsf{P}_{T,t} - \mathsf{P}_{NT,t} - (\mathsf{P}_{T,t}^* - \mathsf{P}_{NT,t}^*)]}^{\text{Diff in Relative prices b/w countries}},$$

with, $\mathsf{Q}_{T,t} = \mathsf{e}_t + \mathsf{P}_{T,t}^* - \mathsf{P}_{T,t}$

RER=RER traded+Difference in relative prices' deviations

If PPP holds for traded goods: $\mathsf{Q_T} = 0$

If PPP does not hold for T goods: (e.g., because of HB, or due to deviations of LOP) $\uparrow \mathsf{Q}_T \rightarrow \mathsf{Q}$

$\mathsf{Q}_t$ also moves if $\mathsf{P}_T - \mathsf{P}_{NT}$ moves: $\downarrow (\mathsf{P}_T - \mathsf{P}_{NT}) \Rightarrow \downarrow \mathsf{Q}$

Engel 1999: Relative importance of Q-traded vs. Relative T/NT prices.

- Most of RER (Q) variation comes from $\Delta \mathsf{Q}_T$

- Variation in $\mathsf{Q}_T$ is driven by deviations of LOP

Engel's results led to adopt LCP as underlying theory of PPP deviations.

#### Relation to Risk Sharing

With complete markets: ratio of $MU_c \propto$ RER

e.g., with CRRA: $\gamma(\mathsf{C}_t - \mathsf{C}_t^*) = \mathsf{Q}_t$

With PPP deviations:

- LCP + sticky prices: $\gamma(\mathsf{C}_t - \mathsf{C}_t^*) = \mathsf{e}_t$ (in period in which prices are fixed)
- HB in preferences: $\gamma(\mathsf{C}_t - \mathsf{C}_t^*) = -(2a-1)\mathsf{TOT_t}$
- Non-traded goods: $\gamma(\mathsf{C}_t - \mathsf{C}_t^*) = \mathsf{Q}_{T,t} + (1-b)\mathsf{P}_{T,t} - \mathsf{P}_{NT,t} - (\mathsf{P}_{T,t}^* - \mathsf{P}_{NT,t}^*)$

Problem here: Risk sharing is not supported by evidence.

## 11. OR1995 - LOE model with Nominal Rigidities

(For an example of a LOE with flexible prices (LOE-RBC) and its UIP implications see section 5.5.)

Microfounded international macro model **with nominal rigidities**. Made to replace Mundell-Fleming, Dornbusch setup with a microfounded one, among other reasons, to perform welfare (normative) analyses.

**Model:**

- Perfect foresight, two countries, **preset prices** (prices are set the period before —1 period price stickiness).

- No capital included, but it is not an endowment economy as labor is elastic and determines production.

- Continuum of goods $z \in [0, 1]$, produced by monopolistically competitive firms (each one good).

- Home are producers in the interval $[0, n)$; Foreign in $(n, 1]$.

(Identical) **preferences**:

$$\text{HHs will maximize:} \quad U_t^i = \sum_{s=t}^{\infty} \beta^{s-t} \left[ \log C_s + \chi \log \frac{M_s}{P_s} - \frac{\kappa}{2} y_s(i)^2 \right]$$

where $y_t(i)$ is the output of good $i$ in period $t$ (disutility of labor supply used for production, assumes labor supply of household $i$ is used for production of good $i$)

**Consumption**: given by an aggregate of variety goods

$$C^i = \left[ \int_0^1 (C^i(z))^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \quad \theta > 1$$

where $\theta$ is the elasticity of substition.

Note that variety counter goes up to 1 (a constant) and thus there is a **constant number of firms**, there is no firm entry in this model.

The agents obtain utility from the money they carry to the next period ("cash when I'm done") and not from the money they arrive with. The timing matters for equilibrium properties, in particular in finite horizon models to make final period money to give utility.

**Prices**:

$$P = \left[\int_0^1 p(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}}$$

the LOP will hold for each individual good, i.e., $p(z) = \varepsilon p^*(z)$, this allows us to rewrite the CPIs:

$$P = \left[\int_0^n p(z)^{1-\theta} dz + \int_n^1 \varepsilon p^*(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}},$$

$$P^* = \left[\int_0^n \frac{1}{\varepsilon} p^*(z)^{1-\theta} dz + \int_n^1 \frac{1}{\varepsilon} p^*(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}};$$

as a result the PPP holds: $P = \varepsilon P^*$ (not surprising given the LOP holds and households have identical preferences).

**Demands**:

Each producer faces local and foreign demand.

Thus the demand for good z by the representative home household is

$$c^i(z) = \left(\frac{p(z)}{P}\right)^{-\theta} C^i,$$

and by the foreign household,

$$c^{*i}(z) = \left(\frac{p(z)}{P}\right)^{-\theta} C^{*i},$$

This implies (done by integrating demand for good $z$ across all agents, and making use of the LOP, and PPP) that the Total world demand for good $z$ is

$$y^d(z) = \left(\frac{p(z)}{P}\right)^{-\theta} C^W,$$

where total world consumption is $C^W \equiv \int_0^n C^i di + \int_n^1 C^{*i} = nC + (1-n)C^*$

**Budget Constraint:**

- Asset markets are incomplete: Here the only traded asset is a non-contingent nominal bond denominated in each country's currency (i.e., these are nominal bonds). This is a departure —for the sake of clarity of some derivations— of the original model that only has one bond given in units of consumption.

- The agents also hold units of their domestic currency.

$$[\text{BC(home)}]: \quad B^i_{t+1} + \varepsilon_t B^{*i}_{t+1} + M^i_t = (1+i_t)B^i_t + \varepsilon_t(1+i^*_t)B^{*i}_t + M^i_{t-1} + p_t(i)y_t(i) - P_tC^i_t - P_tT_t,$$

where $i_t$ is the nominal interest rate between $t-1$ and $t$ which was known at $t-1$ (this is why it's risk free), and $M^i_{t-1}$ are the agent $i$'s holdings of nominal money balances entering period $t$.

FOCs: dropping superscript $i$ for simplicity (also notice that when optimizing agents take $C^W$ and $T$ as given).

$$[\text{Euler Equation}]: \quad C_t = [\beta(1+r_{t+1})]^{-1} C_{t+1}$$

$$[M_t]: \quad \frac{M_t}{P_t} = \chi C_t \left(\frac{1+i_{t+1}}{i_{t+1}}\right)$$

Intratemporal condition, indiff. between consuming or holding more balances

$$[y_t(h)]: \quad y_t(h) = \frac{\theta-1}{\kappa\theta}\frac{p_t(h)}{P_t}\frac{1}{C_t}$$

Intratemporal condition, indiff. between leisure and consuming (working)

above $r_{t+1}$ denotes the real interest rate, and we substitute the output and price variety index for $h$ to denote the output and price of the representative home good (the variety they generate when supplying labor).

Notice we have two intratemporal conditions, one for the decision of holding money and another for the labor supply decision.[6]

Additionally the Fisher parity defines the link between real and nominal interest rates:

$$1 + r_{t+1} = \frac{P_t}{P_{t+1}}(1 + i_{t+1}),$$

with Home and Foreign assets a no arbitrage condition is implied by the associated Euler Equations,

$$[UIP]: \quad 1 + i_{t+1} = \frac{\varepsilon_{t+1}}{\varepsilon_t}(1 + i^*_{t+1})$$

**Government**: Changes the money supply. Operates a balance budget with lump-sum transfers: $T_t = \frac{M_t - M_{t-1}}{P_t}$

**World Equilibrium**

**Asset Markets**: Demand of assets must equal supply (zero net supply), for each location this implies,

$$[\text{home}] \qquad nB_{t+1} + (1-n)B_{*t+1} = 0$$
$$[\text{foreign}] \qquad nB^*_{t+1} + (1-n)B^*_{*t+1} = 0$$

where $B_{*t+1}$ denotes foreign holdings of the home currency bond (subscript location: holder, superscript: type of asset —home, foreign).

Multiply the foreign condition by $\varepsilon_t$ and add the two equations:

$$n(B_{t+1} + \varepsilon_t B^*_{t+1}) + (1-n)(B_{*t+1} + \varepsilon_t B^*_{*t+1}) = 0$$

that is,

$$nA_{t+1} + (1-n)A^*_{t+1} = 0$$

home NFA + Foreign Net NFA = 0
NFA: Net foreign assets

**Goods Markets:** Given this global asset-market clearing conditions, we can derive an aggregate global goods-market clearing condition. For this, divide both sides of the home and foreign Budget Constraints by their corresponding prices levels ($P_t$ for Home and $P^*_t$ for Foreign). Then take a population weighted average of the two budget constraints. Impose the asset market clearing condition above, as well as the government balance budgets for each country, then we get

$$C^W \equiv nC_t + (1-n)C^*_t = n\frac{p_t(h)}{P_t}y_t(h) + (1-n)\frac{p^*_t(f)}{P^*_t}y^*_t(f) \equiv Y^W_t,$$

where $y^*_t(f)$ is the output of the representative good in the Foreign country.

**Current Account**: The relationships above can also lead to a definition of the trade balance.

[6] In the intra-temporal condition between labor an leisure, we have that the output is inefficiently low because of the misalignment of the mark-up with the endogenous labor. The misalignment occurs in terms of the relative prices of the items the consumer cares about, consumption goods (priced at a markup), and leisure (priced at no markup). As a result agents with a mark-up give up consumption to increase leisure and end up under producing. A solution for this can be a tax on leisure to the extent of the mark-up.

Rewrite the budget constraint of the Home country in terms of the NFA,

$$A_{t+1} = (1+i_t)A_t + p_t(h)y_t(h) - P_tC_t,$$

Set in real terms (divide by $P_t$)

$$\begin{aligned}
\frac{A_{t+1}}{P_t} &= (1+i_t)\frac{P_{t-1}}{P_t}\frac{A_t}{P_{t-1}} + \frac{p_t(h)}{P_t}y_t(h) - P_tC_t \\
a_{t+1} &= (1+r_t) + \frac{p_t(h)}{P_t}y_t(h) - C_t \Rightarrow \underbrace{a_{t+1} - a_t}_{ca_t} = \underbrace{r_t a_t}_{\text{Income balance}} + \underbrace{\frac{p_t(h)}{P_t}y_t(h) - C_t}_{\text{Trade Balance}},
\end{aligned}$$

where $ca_t$ denotes the current account in real terms. From the current account equations of both locations we also have

$$nca_t + (1-n)ca_t^* = 0$$

**A Symmetric Steady State**

The initial steady-state levels of the variables are denoted with a subscript 0 and an overbar.

Nothing pis down the steady-state level of bonds in the model as the Euler Equations don't depend on the assets. Thus, for now, we will assume that the bonds are zero in the initial steady-state.

For obtaining the steady-state consumption levels at Home and Foreign, we can use the real current account equation above:

$$\begin{aligned}
\bar{C}_0 &= \bar{r}_0\bar{a}_0 + \frac{\bar{p}_0(h)}{\bar{P}_0}\bar{y}_0(h) \\
\bar{C}_0^* &= -\left(\frac{n}{1-n}\right)\bar{r}_0\bar{a}_0 + \frac{\bar{p}_0^*(f)}{\bar{P}_0^*}\bar{y}_0^*(f)
\end{aligned} \tag{1}$$

For the last one we considered the same equation but for Foreign, and replaced the Foreign NFAs from the global asset market clearing condition ($a_{t+1}^* = -\frac{n}{1-n}a_{t+1}$).

With zero bond holdings (as we are assuming) the steady-state is symmetric:

$$\bar{p}_0(h)/\bar{P}_0 = \bar{p}_0^*(f)/\bar{P}_0^* = 1, \quad \bar{C}_0 = \bar{y}_0(h), \quad \bar{C}_0^* = \bar{y}_0^*(f)$$

The first equalities come from the fact that producers are symmetric in both countries and at the cross-country level via the LOP ($\bar{p}_0(h) = \varepsilon_0\bar{p}_0^*(f)$).

Then, from the FOC with respect to the output (the intratemporal leisure/consumption trade-off) we can replace these results and get

$$\bar{y}_0(h) = \bar{y}_0(f) = \bar{y}_0 = \left(\frac{\theta-1}{\kappa\theta}\right)^{\frac{1}{2}},$$

here we see the inefficiency generated by the combination of a mark-up with an endogenous leisure choice. Output ends up being too low as agents contract labor to enjoy more leisure. A fix for this is to tax the leisure.

On the other hand, from the Euler Equation we can obtain the steady-state interest rate:

$$\bar{r}_0 = \delta = \frac{1-\beta}{\beta}$$

The real money balances steady-state is also symmetric across countries and can be obtained from the intratemporal condition that trades-off consumption and money holding

$$\frac{\bar{M}_0}{\bar{P}_0} = \frac{\bar{M}_0^*}{\bar{P}_0^*} = \frac{\chi(1-\delta)}{\delta}\bar{y}_0$$

**Log-linearized model** Now we can obtain the log-linear version of the model (approximated around the steady-state). The main equations of the log-linearized model are:

Price indexes
$$P_t = n p_t(h) + (1-n)(e_t + p_t^*(f)) \tag{2}$$
$$P_t^* = n(p_t(h) - e_t) + (1-n)p_t^*(f) \tag{3}$$
PPP
$$P_t - P_t^* = e_t \tag{4}$$
Output-demand schedules
$$y_t(h) = \theta(P_t - p_t(h)) + C_t^W \tag{5}$$
$$y_t^*(f) = \theta(P_t^* - p_t^*(f)) + C_t^W \tag{6}$$
World output
$$Y_t^W \equiv n y_t(h) + (1-n) y_t^*(f) = n C_t + (1-n) C_t^* \equiv C_t^W \tag{7}$$
Output supply
$$y_t(h) = p_t(h) - P_t - C_t \tag{8}$$
$$y_t^*(f) = p_t^*(f) - P_t^* - C_t^* \tag{9}$$
Consumption Euler equations
$$C_t = -\frac{\delta}{1+\delta} r_{t+1} + C_{t+1} \tag{10}$$
$$C_t^* = -\frac{\delta}{1+\delta} r_{t+1} + C_{t+1}^* \tag{11}$$
Money demand
$$M_t - P_t = C_t - \frac{r_{t+1}}{1+\delta} - \frac{P_{t+1} - P_t}{\delta} \tag{12}$$
$$M_t^* - P_t^* = C_t^* - \frac{r_{t+1}}{1+\delta} - \frac{P_{t+1}^* - P_t^*}{\delta} \tag{13}$$

Take (12) subtract (13) and substitute $P_{t+s} - P_{t+s}^* = e_{t+s}$ for $s = \{0, 1\}$ from (4)

$$e_t = M_t - M_t^* - (C_t - C_t^*) + \frac{1}{\delta}(e_{t+1} - e_t). \quad \text{ER} = f(\text{fundamentals, expected change in ER}) \tag{14}$$

Now, given the incomplete markets environment, we don't have a stationarity inducing device. Then the shocks have permanent effects.

**No stationarity:** take (10) − (11)

$$C_t - C_t^* = C_{t+1} - C_{t+1}^*$$

The shocks in current consumption will become permanent.

Given this we can inquire into the long-run effect of shocks.

With fully flexible prices, and with permanent shocks, the economy jumps from one steady state to the new one instantly since there is no reason to smooth consumption (i.e., no reason to smooth impact of a shock over time). With sticky prices the new steady state will be reached too but only in the long-run. The shock considered here is one in the wealth of Home households ($a$).

To approximate the change in steady-states we can log-linearize the steady-state consumption around the initial steady-state. That is log-linearize the equations[7] in (1)

$$\begin{aligned} \bar{\mathsf{C}} &= \delta \bar{\mathsf{a}} + \bar{\mathsf{p}}(h) - \bar{\mathsf{P}} + \bar{\mathsf{y}}(h), \\ \bar{\mathsf{C}}^* &= -\left(\frac{n}{1-n}\right)\delta \bar{\mathsf{a}} + \bar{\mathsf{p}}^*(f) - \bar{\mathsf{P}}^* + \bar{\mathsf{y}}^*(f), \end{aligned} \tag{15}$$

here the barred variables with no time subscript refer to the percentual change between the new steady-state value and the initial one ($\bar{\mathsf{C}} = d\bar{C}/\bar{C}_0 = (\bar{C} - \bar{C}_0)/\bar{C}_0$). Additionally, the initial steady-state for the NFA is zero, thus, the log-linear approximation of the assets would not be well defined and its definition is therefore adjusted as $\bar{\mathsf{a}} = d\bar{a}/\bar{C}_0^W$ (with $\bar{C}_0^W = \bar{C}_0 = \bar{y}_0$).

The equations (2)-(13) hold at all points in time, including when they reach the new steady-state. Therefore,

[7] The log-linearization works as usual, except that variables are not indexed because these equations are not valid over time but only for the steady-states. To give an example of how these are derived, the first equation is obtained by taking a log-linearization standard shortcut as $\bar{C}_0\bar{\mathsf{C}} = \delta \bar{a}_0 \frac{d\bar{a}}{\bar{a}_0} + \frac{\bar{p}_0(h)\bar{y}_0(h)}{P_0}(\bar{\mathsf{p}}(h) - \bar{\mathsf{P}} + \bar{\mathsf{y}}(h))$, we then divide both sides by $\bar{\mathsf{C}}_0$, and finally we substitute the definition of $\bar{\mathsf{a}}$, simplify the coefficient terms, e.g., reminding that $y_0(h) = \bar{C}_0$.

the change in steady-state is described by barred versions of these equations together with (15).

Obstfeld and Rogoff proceed to solve the system following Aoki(1981) technique: Solve for cross country differences and population weighted world averages first.

Difference of demands: (5)-(6)

$$\mathsf{y}_t(h) - \mathsf{y}_t^*(f) = -\theta(\mathsf{p}_t(h) - \mathsf{p}_t^*(f) - \mathsf{e}_t) \tag{16}$$

This equation states that the relative demand is a positive function of the terms-of-trade.

Now, take the relative supply from

solve for $\mathsf{p}_t(h) - \mathsf{p}_t^*(f)$ from the relative supply (8)-(9), and substitute in the relative demand above to obtain

$$\mathsf{y}_t(h) - \mathsf{y}_t^*(f) = -\frac{\theta}{1+\theta}(\mathsf{C}_t - \mathsf{C}_t^*). \tag{17}$$

Now, given PPP holds ($\mathsf{P}_t - \mathsf{P}_t^* - \mathsf{e}_t = 0$), we get from subtracting the second equation from the first in (15)

$$\bar{\mathsf{C}} - \bar{\mathsf{C}}^* = \frac{1}{1-n}\delta\bar{\mathsf{a}} + \bar{\mathsf{y}}(h) - \bar{\mathsf{y}}^* + [\bar{\mathsf{p}}(h) - \bar{\mathsf{p}}^*(f) - \bar{\mathsf{e}}]$$

Substitute from (barred versions of) (16) and (17):

$$\bar{\mathsf{C}} - \bar{\mathsf{C}}^* = \left(\frac{1}{1-n}\right)\left(\frac{1+\theta}{2\theta}\right)\delta\bar{\mathsf{a}} \quad \text{Home NFA} \uparrow \Rightarrow \text{H Consumption} \uparrow$$

Notice the change in consumption is lower than $\left(\frac{1}{1-n}\right)$ because with higher income agents shift labor out of production (increase leisure).

It also follows that

$$\bar{\mathsf{p}}(h) - \bar{\mathsf{p}}^*(f) - \bar{\mathsf{e}} = \left(\frac{1}{1-n}\right)\left(\frac{1+\theta}{2\theta}\right)\delta\bar{\mathsf{a}},$$

then, the steady-state improve if the Home's NFA increase.

### World Aggregates

We can take a population weighted average of the supply equations (8), (9) while taking into account the pricing equations (2), (3) yields $\bar{\mathsf{Y}}^W = -\bar{\mathsf{C}}^W$. But the equilibrium in goods markets requires $\bar{\mathsf{Y}}^W = \bar{\mathsf{C}}^W$. Thus, $\bar{\mathsf{Y}}^W = \bar{\mathsf{C}}^W = 0$.. Small asset changes have no first-order effect on world aggregates.

### One-period Price Stickyness

In the short-run, at time 1 prices are fixed at the initial steady state ten: $p_1(h) = \bar{p}_0(h)$, $p_1^*(f) = \bar{p}_0^*(f)$.

For the LOP to hold $p^*(h), p(f)$ must be allowed to fluctuate (i.e., $p_1(h) = \varepsilon_1 p_1^*(h)$ implies the price movement since $\varepsilon$ may move and $p_1(h)$ is fixed). Then, this model assumes Producer Currency Pricing (PCP) where the firms set prices in their currency and the LOP dictates the price in other locations.

Consider the law of motion of Home NFA:

$$a_{t+1} = (1 + r_t) + \frac{p_t(h)}{P_t}y_t(h) - C_t.$$

Log-linearizing this equation, using the price index equations, the assumption of preset prices and the fact that assets do not change on impact:

$$\bar{\mathsf{a}} = \mathsf{y}(h) - \mathsf{C} - (1-n)\mathsf{e}, \qquad \text{Check in OR96}$$
$$\bar{\mathsf{a}}^* = -\left(\frac{n}{1-n}\right)\bar{\mathsf{a}} = \mathsf{y}^*(f) - \mathsf{C}^* + n\mathsf{e}.$$

These equations provide the link between the long run solution and the short-run dynamics in the model (the SR dynamics are reflected in NFA assets changes, or in $\bar{\mathsf{a}}$, which affect the LR solution).

**Shock effects:** a monetary shock will affect $\bar{a}$ only in the period of the shock, that changes the long-run levels of the rest of the variables. Thus, there is monetary non-neutrality (it appears due to the non-stationarity of the model).

### Overshooting

From Dornbusch (1976): the money supply increases, the interest rate lowers, and the RER increases by more than in the long-run because of the UIP plus a liquidity effect with sticky prices.

OR1995: No overshooting in the basic model.

To see this, assume a permanent shock such that

$$\bar{M} - \bar{M}^* = M - M^*$$

. The convergence to the new steady-state by the end of the second period (when prices adjust) implies by (14)

$$e = M - M^* - (C - C^*) + \frac{1}{\delta}(\bar{e} - e), \quad \text{1 period convergence} \tag{18}$$

also

$$\bar{e} = \bar{M} - \bar{M}^* - (\bar{C} - \bar{C}^*), \text{ and } \quad C - C^* = \bar{C} - \bar{C}^*, \quad \text{no stationarity condition}$$

then $\bar{e} = \bar{M} - \bar{M}^* - (C - C^*)$, implying

$$e = \bar{e} + \frac{1}{\delta}(\bar{e} - e)$$

that is, $e = \bar{e}$ ($e_{t+1} = e_t$ No overshooting)

### Equilibrium effects of money shocks

From before the ER equation becomes,

$$[\text{MM}] \qquad e = M - M^* - (C - C^*). \tag{19}$$

Equation (19) represents a schedule on the ER that is downward sloping in consumption because an increase in relative home consumption raises money demand, and therefore, the home's relative price level must fall, which implies an ER appreciation.

A second schedule can be derived using the current account equation as follows. Taking the home current account and subtracting the home one (in terms of $\bar{a}$) we get

$$\bar{a} = (1 - n)[y(h) - y^*(f) - (C - C^*) - e]$$

Alternatively, this equation can be obtained by rearranging and subtracting the equation (7) from the current account equation with one-period price stickyness derived for $\bar{a}$.

Now, we can substitute $y(h) - y^*(f)$ from the difference in the demand equations (and using the assumption of price stickyness with symmetric steady state between home and foreign)

$$\bar{a} = (1 - n)[\theta e - (C - C^*) - e]$$

Next, we can substitute this expression for $\bar{a}$ into $\bar{C} - \bar{C}^* = \left(\frac{1}{1-n}\right)\left(\frac{1+\theta}{2\theta}\right)\delta\bar{a}$ (with $\bar{C} - \bar{C}^*$)

$$C - C^* = \left(\frac{1}{1-n}\right)\left(\frac{1+\theta}{2\theta}\right)\delta\left[(1 - n)[\theta e - (C - C^*) - e]\right],$$

where solving for $e$ we get

$$[\text{GG}] \qquad e = \frac{2\theta + (1+\theta)\delta}{\delta(\theta^2 - 1)}(C - C^*). \tag{20}$$

This schedule is upward sloping because home consumption can rise relative to foreign when the home currency experiences a depreciation (making home output rise relative to foreign).

Given the solution for $\mathsf{e}$ we can recover $\mathsf{y}(h) - \mathsf{y}^*(f)$ and $\bar{\mathsf{a}}$

$$\bar{\mathsf{a}} = \frac{2(1-n)(\theta-1)}{2+\delta(1+\theta)}(\mathsf{M}-\mathsf{M}^*).$$

The larger is home $(n)$ the smaller the effect of a monetary shock on its current account. Intuition: as $n$ grows the foreign economy shrinks and foreign asset accumulation has a smaller effect on the domestic current account.

From these two-equations system we can solve for $\mathsf{e}$ and $\mathsf{C}-\mathsf{C}^*$ in terms of the relative money supplly

$$\mathsf{e} = \frac{2\theta+\delta(1+\theta)}{2\theta+\theta\delta(1+\theta)}(\mathsf{M}-\mathsf{M}^*), \tag{21}$$

$$\mathsf{C}-\mathsf{C}^* = \frac{\delta(\theta^2-1)}{2\theta+\theta\delta(1+\theta)}(\mathsf{M}-\mathsf{M}^*). \tag{22}$$

Similarly, from the intersection of these two schedules we can get graphically the solution for $\mathsf{e}$.

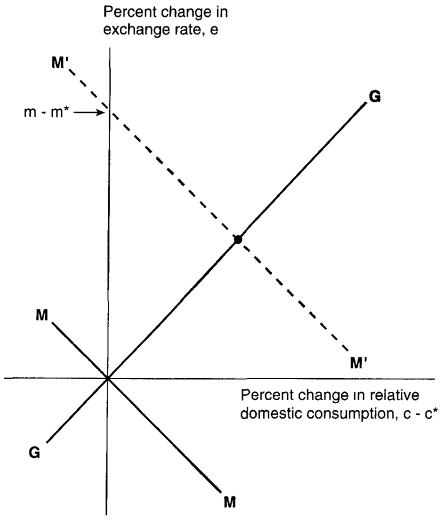



Figure 10: Source: Obstfeld and Rogoff (1996), Chapter 10.

Going back to the effect of the shock, above we can see that if $\mathsf{M}-\mathsf{M}^*$ increases the MM curve shifts outward to MM' and GG remains unchanged. As a result, we get an unambiguous ER depreciation that raises relative consumption.

$\mathsf{e}\uparrow\rightarrow$ real relative income : $\mathsf{y}(h)-\mathsf{y}^*(f)\uparrow\rightarrow$ *ca* surplus due to consumption smoothing. In LR: higher wealth + endogenous labor $\Rightarrow$ HH shift some labor into leisure $\Rightarrow$ home supply lowers $\Rightarrow$ $\mathsf{p}(h)\downarrow\Rightarrow$ *TOT* $\uparrow$

Note: if the initial steady-state asset holdings would not be zero, then GG would react (shift to the left) to $\mathsf{M}-\mathsf{M}^*$ and the effect on consumption would be ambiguous.

### Short-Run World Aggregates

To complete the solution of the model, we can obtain the world aggregates.

From a population weighted average of the Euler equations

$$\mathsf{C}^W = \bar{\mathsf{C}}^W - \frac{\delta}{1-\delta}\mathsf{r},$$

but we had that $\bar{\mathsf{C}}^W = 0$. Hence, $\mathsf{C}^W = \frac{\delta}{1-\delta}\mathsf{r}$. Now use the log-linear aggregate money demands, price index equations, the assumption of short run price rigidity, and $\bar{\mathsf{C}}^W = 0$ to write:

$$\mathsf{M}^W = \mathsf{C}^W - \frac{1}{1+\delta}\mathsf{r} - \frac{1}{\delta}\mathsf{M}^W.$$

Combine this equation with $\mathsf{C}^W = -\frac{\delta}{1+\delta}\mathsf{r}$,

$$\mathsf{C}^W = \mathsf{Y}^W = \mathsf{M}^W, \qquad \mathsf{r} = -\frac{1+\delta}{\delta}\mathsf{M}^W.$$

A monetary expansion at home or abroad lowers the world real interest rate in proportion to the size of the expanding country, and therefore, $\mathsf{C}^W$ increases. In the long-run $\mathsf{C}^W$ and $\mathsf{r}$ return to their initial levels (e.g., $\bar{\mathsf{C}} = 0$).

Then the long-run non neutrality of money showed only in the country differentials, but not on the world aggregate. The reason is that the former changed acted through changes in net asset positions in each location, but at the world level the net assets are zero.

Note: Given the solutions above, we can recover the short-run changes in country specific variables form the world aggregates and cross country differences as

$$\mathsf{x}_t = \mathsf{x}_t^W + (1-n)(\mathsf{x}_t - \mathsf{x}_t^*), \qquad \mathsf{x}_t^* = \mathsf{x}_t^W - n(\mathsf{x}_t - \mathsf{x}_t^*)$$

**Welfare**

One of the main points of the micro-foundation is the ability of performing welfare analysis.

If $\mathsf{M}$ and $\mathsf{M}^*$ both increase such that $\mathsf{M} - \mathsf{M}^* = 0$ the welfare increases in both countries.

Reason: Country differentials won't change, hence neither TOT, ER do. But the world aggregates increase ($\mathsf{Y}^W \uparrow$, $\mathsf{C}^W \uparrow$, $\mathsf{M}^W \uparrow$). (This refers to an unexpected shock, not a systematic increase policy)

For asymmetric money shocks: We focus on the real part of utility[8]

$$U_t^R = \sum_{s=t}^{\infty} \beta^{s-t}\left[\log C_s - \frac{\kappa}{2} y_s(h)^2.\right]$$

Total differentiate $U_t^R$, and together with the fact that the economy converges to new steady-state in one period yields:

$$dU^R = \mathsf{C} - \kappa \bar{y}_0(h)^2 \mathsf{y}(h) + \frac{1}{\delta}\left[\bar{\mathsf{C}} - \kappa \bar{y}_0(h)^2 \bar{\mathsf{y}}(h)\right],$$

and substituting the expression for $\bar{y}_0(h)$,

$$dU^R = \mathsf{C} - \frac{\theta-1}{\theta}\mathsf{y}(h) + \frac{1}{\delta}\left[\bar{\mathsf{C}} - \frac{\theta-1}{\theta}\bar{\mathsf{y}}(h)\right].$$

It can be shown that

$$dU^R = \frac{\mathsf{C}^W}{\theta} = \frac{\mathsf{M}^W}{\theta}.$$

Thus, a change in utility is proportional to the world money supply, regardless of its origin.

Reason: Small changes in relative prices induced by UR changes have no first order effects in equilibrium (since in the initial equilibrium all firms were setting prices at their optimal levels). Similarly, as agents are optimally smoothing consumption over time, small changes in the current account have only second order effects ($ca_t$).

Note: It should be noted that the OR model it's only valid around a small neighborhood of the steady state (given its approximation technique), and assumes perfect foresight due to the non-stationary nature of the model.

**Summary:**

[8] This practice is a bechmark in MIU models. It can be subject to criticism, because biases the policy evaluation against the optimality of the Friedman rule (zero nominal interest rate to remove opportunity cost of holding money) by assuming utility effect of money is negligible.

Bechmark OR: No overshooting, and money expansions are welfare improving regardless or shock origin (prosper-thy-neighbor effects of monetary expansions).

What follows is to address the absence of overshooting by including non-traded goods.

## 12. CP2001 - Welfare And Macroeconomic Interdependence (WAMI)

The OR95 approach made important process in providing the microfoundations for the behaviour of the exchange rate and current account. However, it made use of restrictive assumptions (perfect foresight, non-stationarity) that limited its results.

Cosetti and Pesenti (2001), building on the approach of Cole and Obstfeld (1991)[9] try to build a version of the OR95 model that can be solved in closed form and can deal with the stochastic analysis.

First, they observe that in OR95 there is no distinction between the elasticity of substitution (ES) between goods produced inside the home economy and the ES between home and foreign goods. They are both $\theta > 1$. This assumption makes impossible to disentangle between the two forms of monopoly power and their effects.

Then, CP add monopolistic power of: (two sources of monopolistic power)

- Firm over its good
- Country over its basket of goods

Both being a single parameter $\theta$ has no empirical support: $\text{ES}_{\text{within}} > 1$, $\text{ES}_{\text{between}} \approx 1$

CP apply such elasticities with competitive goods but differentiated labor inputs (analogous to monopolistic competition wiht competitive labor).

**Model**

Two countries, each produces a traded good. Population size is normalized to 1.

**Home agent preferences:**

The home agent $j \in [0,1]$ has an expected intertemporal utility

$$U_t(j) = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[ \frac{C_\tau(j)^{1-\rho}}{1-\rho} + \chi \ln \frac{M_\tau(j)}{P_\tau} + V(G_\tau) - \frac{\kappa}{2} l_\tau(j)^2 \right], \quad \beta = \frac{1}{1+\delta},$$

where $G$ is government spending, $V(\cdot)$ is a function, $C$ is consumption, $l$ is labor, $M/P$ are real money balances.

The representative foreign agents $j^* \in [0,1]$ maximizes an analogous function but the model allows for different specifications of the parameters $\chi^*,\ \kappa^*,\ V^*(\cdot)$.

**Consumption**

The consumption basket is a CD aggregate of home and foreign goods

$$C_t(j) = C_{H,t}(j)^\gamma C_{F,t}(j)^{1-\gamma}, \quad C_t^*(j^*) = C_{H,t}^*(j)^\gamma C_{F,t}^*(j)^{1-\gamma}, \quad 0 < \gamma < 1.$$

the CD aggregate implies an $\text{ES}_{H,F} = 1$. Also, $\gamma$ can differ from $1/2$, but since the weight associated to the home good is equal ($\gamma$) in both locations, there is no home bias in consumption.

**Prices**

Let $\gamma_W \equiv \gamma^\gamma (1-\gamma)^{1-\gamma}$, then the home and foreign CPI's are

$$P_t \equiv \frac{1}{\gamma_W} (P_{H,t})^\gamma (P_{F,t})^{1-\gamma}, \quad P_t^* \equiv \frac{1}{\gamma_W} (P_{H,t}^*)^\gamma (P_{F,t}^*)^{1-\gamma},$$

[9] CO1991 central result implied that a unitarity of elasticity of substitution between home and foreign goods in consumption implies no movement in the net foreign assets.

where $P_{H,t}$ and $P_{F,t}$ are the prices of the home and foreign goods in the home country (in home currency), and $P^*_{H,t}$ and $P^*_{F,t}$ are the prices of the home and foreign goods in the foreign country (in foreign currency).

### Technology

$$Y_t = \left( \int_0^1 l_t(j)^{\frac{\phi-1}{\phi}} \right)^{\frac{\phi}{\phi-1}}, \ \phi > 1 \qquad \text{differentiated labor inputs, } \mathsf{ES}_{\mathsf{within}} = \phi > 1.$$

Thus, the households are monopolistically competitive suppliers of specific labor inputs.[10]

The firms, in contrast, will act competitively.

The FOC of the firm's PMP with respect to labor demand is:

$$l_t(j) = \left( \frac{W_t(j)}{P_{H,t}} \right)^{-\phi} Y_t.$$

A similar condition holds for the foreign firms (the ES in that case can differ and be $\phi^* > 1$).

There are no impediments to trade, then the LOP holds,

$$P_{F,t} = \varepsilon_t P^*_{F,t}, \quad P_{H,t} = \frac{1}{\varepsilon_t} P^*_{H,t}$$

The consumption basket (and preferences) are the same in both locations, an so the PPP holds too: $P_t = \varepsilon_t P^*_t$.

Additionally, as in OR95, the TOT, $P_{H,t}/(\varepsilon_t P^*_{F,t})$, will move in response to shocks.

### Budget and Resource Constraints

Incomplete asset markets: Only one non-contingent bond is traded. It is denominated in terms of home currency. The bond pays a risk-free nominal interest rate $i_t$, and the Fisher parity hold: $1 + r_t = (1 + i_t)P_{t-1}/P_t$.

The budget contraints are:

$$B_{t+1}(j) + M_t(j) \leq (1 + i_t)B_t(j) + M_{t-1}(j) + W_t(j)l_t(j) - P_t T_t(j) - P_{H,t}C_{H,t}(j) - P_{F,t}(j)C_{F,t}(j)$$
$$B^*_{t+1}(j^*) + M^*_t(j^*) \leq (1 + i_t)B_t(j^*)^* + M^*_{t-1}(j^*) + W^*_t(j^*)l^*_t(j^*) - P^*_t T^*_t(j^*) - P^*_{H,t}C^*_{H,t}(j^*) - P^*_{F,t}(j^*)C^*_{F,t}(j^*)$$

where $B$ is the nominal bond position of home households ($B^*$ that of the foreign household), $T$ ($T^*$) a lump-sum tax. Also, notice the foreign return for buying the international bond is $(1 + i_t)\varepsilon_{t-1}/\varepsilon_t$.

### Government

The government has full Home Bias and only consumes goods produced domestically (simplification). In addition it sets $G_t$, $T_t(j)$, and $M_t = \int_0^1 M_t(j)dj$ s.t.

$$M_t - M_{t-1} + P_t \int_0^1 B^*_t(j)dj \geq P_{H,t}G_t.$$

An analogous condition holds for the foreign government.

### Market Clearing

Asset Markets: Zero net supply

$$\int_0^1 B_t(j)dj + \int_0^1 B^*_t(j^*)dj^* = 0.$$

[10] This model yields exactly the same result taht one in which the labor market is competitive and there are monopolistically competitive firms at home and aborad, with single goods produec by home and foreign replaced by consumption sub-baskets defined over a continuum of good varieties in each location: $C_{H,t} = \left( \int_0^1 C_{H,t}(j)^{\frac{\phi-1}{\phi}} dj \right)$, and $C_{F,t} = \left( \int_0^1 C_{F,t}(j)^{\frac{\phi-1}{\phi}} dj \right)$.

Final goods in each location:

$$Y_t \geq G_t + \int_0^1 C_{H,t}(j)dj + \int_0^1 C^*_{H,t}(j^*)dj^*$$
$$Y^*_t \geq G^*_t + \int_0^1 C_{F,t}(j)dj + \int_0^1 C^*_{F,t}(j^*)dj^*$$

**Nominal Rigidities**

Wages rigidity: Predetermined 1-period nominal wages. $W_t(j)$ is set in $t-1$

Household $j$ is a monopolistic supplier (of labor). It takes demand ($l_t(j) = (W_t(j)/P_{H,t})^{-\phi} Y_t$) into account when setting wage.

FOC for wage setting problem:

$$\mathbb{E}_{t-1}\left[\kappa l_t(j)^2\right] = \frac{\phi - 1}{\phi} W_t(j) \mathbb{E}_{t-1}\left[\frac{1}{P_t}\frac{l_t(j)}{C_t(j)^\rho}\right].$$

expected disutility of working = exp. utility from extra wage by increasing labor

This FOC can be rewritten as

$$W_t(j) = \underbrace{\frac{\phi}{\phi - 1}}_{\text{mark-up}} \frac{\mathbb{E}_{t-1}[\kappa l_t(j)^2]}{\mathbb{E}_{t-1}\left[\frac{1}{P_t}\frac{l_t(j)}{C_t(j)^\rho}\right]}$$

Wage is set as a mark-up over the expected cost of labor adjusted for utility value of labor income.

Once the wages are set agents are willing to meet unanticipated changes in labor demand (shocks) as long as: Real Wage $\geq MRS_{\text{cons,leisure}}$ (MRS: marginal rate of substitution).

(Before: in a competitive world $\frac{W}{P}$ = MRS = MRT, but empirically $\frac{W}{P}$ is counter-cyclical, and thus the equality is not realistic. With a wedge, such as the mark-up, we obtain a more feasible condition $\frac{W}{P} = \gamma$MRS $= \gamma$MRT)

Thus, the participation constraint (such that the worker will provide labor) is:

$$\frac{W_t(j)}{P_t} \geq \kappa l_t(j) C_t(j)^\rho$$

CP assume shocks are given in a way that this constraint holds.

**Equilibrium**

As usual, due to symmetry we can drop the indexes $j$, $j^*$.

Also, we interpret the variables in per-capita terms.

In an equilibrium where $l_t(j) = l_t$, and $l_t(j^*) = l^*_t$, the output is a linear product of labor: $Y_t = l_t$, and $Y^*_t = l^*_t$.

Then, the labor demand becomes: $l_t = \left(\frac{W_t}{P_{H,t}}\right)^{-\phi} Y_t$ at home. From this output and its foreign analog it follows that the product prices are equal to the nominal wages: $W_t = P_{H,t}$, and $W^*_t = P_{F,t}$. ($\cancel{l_t} = (W_t/P_{H,t})^{-\phi}\cancel{Y_t}$, and then $W_t = P_{H,t}$, from this we also have that wages and prices share the 1-period ridigity.)

**Perfect foresight exercise as in Dornbusch-OR**

As an illustration CP perform the same perfect-foresight exercise as in Dornbush (1976) and OR95: An unanticipated permanent increase in the money supply.

Start from the steady state with zero assets. Drop $t$ subscripts, and denote the initial steady state with a 0 subscript, the variables in the new steady-state (long run) with upperbars, and variables in the short run without subindexes. Define the short-run as the duration of wage contracts (1-period).

Since all shocks are unanticipated, wages in the short run are set at the initial steady-state level: $W = W_0 = P_{H,0}$, and $W^* = W^*_0 = P^*_{F,0}$.

Money shocks:

$$\bar{M} = M \geq M_0, \quad \bar{M}^* = M^* > M_0^*$$

Fiscal policy shocks: Let $g \equiv Y/(Y+G)$, $\bar{g} \equiv \bar{Y}/(\bar{Y}+\bar{G})$. A fiscal shock is an unanticipated change in $\bar{g}$ such that

$$g \geq g_0.$$

Similar definitions hold abroad.

The structural solution of the model is given as follows,

STRUCTURAL FORM OF THE MODEL

| | | |
|---|---|---|
| (17) | $C^{-\rho} = \beta(1+r)\bar{C}^{-\rho}$ | $(C^*)^{-\rho} = \beta(1+r)(\bar{C}^*)^{-\rho}$ |
| (18) | $\frac{\bar{M}}{P} = \chi \frac{1+i}{i} C^{\rho}$ | $\frac{\bar{M}^*}{P^*} = \chi^* \frac{(1+i)\mathscr{E}}{(1+i)\mathscr{E} - \bar{\mathscr{E}}} (C^*)^{\rho}$ |
| (19) | $\frac{\bar{M}}{\bar{P}} = \chi \frac{1+\delta}{\delta} \bar{C}^{\rho}$ | $\frac{\bar{M}^*}{\bar{P}^*} = \chi^* \frac{1+\delta}{\delta} (\bar{C}^*)^{\rho}$ |
| (20) | $B = P_{\mathrm{H}} \frac{Y}{g} - PC$ | $-\frac{B}{\mathscr{E}} = P_{\mathrm{F}}^* \frac{Y^*}{g^*} - P^*C^*$ |
| (21) | $\bar{P}\bar{C} = \bar{P}_{\mathrm{H}} \frac{\bar{Y}}{\bar{g}} + \delta\bar{B}$ | $\bar{P}^*\bar{C}^* = \bar{P}_{\mathrm{F}}^* \frac{\bar{Y}^*}{\bar{g}^*} - \delta \frac{\bar{B}}{\bar{\mathscr{E}}}$ |
| (22) | $\frac{P_{\mathrm{H}}Y}{Pg} = \gamma(C + C^*)$ | $\frac{P_{\mathrm{F}}^*Y^*}{P^*g^*} = (1-\gamma)(C + C^*)$ |
| (23) | $\frac{\bar{P}_{\mathrm{H}}\bar{Y}}{\bar{P}\bar{g}} = \gamma(\bar{C} + \bar{C}^*)$ | $\frac{\bar{P}_{\mathrm{F}}^*\bar{Y}^*}{\bar{P}^*\bar{g}^*} = (1-\gamma)(\bar{C} + \bar{C}^*)$ |
| (24) | $\bar{Y} = \Phi \frac{\bar{P}_{\mathrm{H}}}{\bar{P}} \bar{C}^{-\rho}$ | $\bar{Y}^* = \Phi^* \frac{\bar{P}_{\mathrm{F}}^*}{\bar{P}^*} (\bar{C}^*)^{-\rho}$ |
| | $\Phi \equiv \frac{\phi - 1}{\kappa\phi}$ | $\Phi^* \equiv \frac{\phi^* - 1}{\kappa^*\phi^*}$ |

Figure 11: Source: Corsetti and Pesenti (2001).

In this table, the equations numbering is the original one used in the paper. In there, equations (17) are the Euler Equations, equations (18), (19) describe the equilibrium in money markets in the short and long run. The steady-state nominal interest rate is $\delta$. Equations (20) are the short-run current account identities (RHS: output minus absorption; LHS: Net asset position change —that considers the initial zero net assets position $B_0$), while (21) are the long-run current account identities. Equations (22), (23) are the short-run and long-run goods market clearing conditions, and (24) describe the trade-off between labor supply and leisure in the steady state.

From this solution, an (expected, given the previous results) outcome is that policy shocks do not lead to any asset redistribution through current account movements (in the long run: consumption is equal to output without government spending plus income from interests).

From the short-run (20) and long-run current accounts (21), together with (22), and the PPP ($P_t = \varepsilon_t P_t^*$) we have

$$\frac{C + B/P}{C^* - B/P} = \frac{\gamma}{1-\gamma}, \quad \text{and,} \quad \frac{\bar{C} - \delta\bar{B}/\bar{P}}{\bar{C}^* + \delta\bar{B}/\bar{P}} = \frac{\gamma}{1-\gamma} \tag{+}$$

Now, the Euler equations for home and foreign consumption imply,

$$\frac{C}{C^*} = \frac{\bar{C}}{\bar{C}^*}$$

These equations, together with the observation that $(1+i)B = (1+\delta)\bar{B}$ imply that $B = \bar{B} = 0$ (the shock implies no movements in the NFA).[11]

This implies a constant consumption ratio $C/C^* = \bar{C}/\bar{C}^* = \gamma/(1-\gamma)$ (or a zero non-consumption differential in log-linear terms).(thus mimicking a complete markets outcome and removing the non-stationarity problem.)

Implications: The adjustment to the shock occurs only through TOT changes.

Intuition: (Price and quantity effects offset.) in the CP model the elasticity of relative net output demand $(Y-G)/(Y^*-G^*)$ to the TOT $(P_H/(\varepsilon P_F))$ is 1, which is also the elasticity of substitution between home and foreign goods. When TOT decreases due to the ER depreciation resulting from the monetary expansion, relative demand for the home good increases in a proportional fashion. Home agents' nominal income increases relative to foreign agents', but their purchasing power declines proportionally, and therefore, there is no incentive to borrow or lend internationally. If the elasticity of substitution were larger than 1, home agents' real income would increase relative to foreign and they would want to lend abroad (and the opposite is it were lower).

The absence of net foreign asset changes makes it possible to solve the model without any approximation. The model goes back to a unit root if initial assets are not zero.

**Macroeconomic effects of a money expansion**

$M\uparrow$ (by 1) $\rightarrow \varepsilon \uparrow$ (by 1 too —there no overshooting) $\rightarrow (\varepsilon P_F^*)\uparrow$ (home imports price $\uparrow$, TOT $\downarrow$) $\rightarrow P_t \uparrow$ (inflation), but $P_H$ is fixed (due to rigidity) and thus the price level change is $(1-\gamma) < 1$, then $\frac{M}{P}\uparrow \Rightarrow Y\uparrow$.

Intuition: $P$ increases only by a fraction of the money supply increase, but the world interest rate falls (due to higher money supply at the world level), and consumption increases symmetrically in all locations. HIgher world consumption and higher relative price of foreign goods unambiguously increase demand for home goods and short-run output increases.

Here there are no current account effects, and then the money is long-run neutral: $\bar{P}_H$ moves 1:1 with $\bar{M}$ (consumption, output, real moeny and TOT returns to initial levels).

**Welfare and Macroeconomic Interdependence** (WAMI)

To analyze how does welfare change after the policy shocks (permanent change in $M$ or $g$) CP use the fact that the economy reaches its new long-run position by the end of the position after the shock, and then, the lifetime utility (in present value) can be written as

$$U = \frac{C^{1-\rho}}{1-\rho} + \chi \ln \frac{\bar{M}}{P} + V(G) - \frac{\kappa}{2}Y^2 + \frac{1}{\delta}\left(\frac{\bar{C}^{1-\rho}}{1-\rho} + \chi \ln \frac{\bar{M}}{\bar{P}} + V(\bar{G}) - \frac{\kappa}{2}\bar{Y}^2\right).$$

To see the effect of $\Delta M$ we can check[12]

$$\frac{\partial U}{\partial \bar{M}} = \frac{\gamma}{\rho \bar{M}}\left[C^{1-\rho} + \chi\rho - \kappa\frac{Y^2}{g}\left(1+\rho\frac{1-\gamma}{\gamma}\right).\right]$$

Small shocks: To gauge the effect of a small shock, evaluate $\frac{\partial U}{\partial \bar{M}}$ at $\bar{M} = M_0$. Abstracting from the government spending ($g = g_0 = 1$) and using the solution of the model we have

$$\text{sign}\left(\frac{\partial U}{\partial \bar{M}}\right)|_{\bar{M}=M_0} = \text{sign}\left\{1 + \chi\rho\left[\gamma\gamma_W^{(1-\rho)/(1+\rho)}\Phi_W^{1/(1+\rho)}\right]^{\rho-1} - \frac{\phi-1}{\phi}\left(1-\rho\frac{1-\gamma}{\gamma}\right)\right\},$$

where $\Phi_W = \left(\frac{\phi-1}{\kappa\phi}\right)^{\gamma}\left(\frac{\phi^*-1}{\kappa\phi^*}\right)^{1-\gamma}$.

This expression shows the key difference between a closed an an open economy analysis: In a closed economy $\gamma = 1$ and then the second term is zero and we would always get $\partial U/\partial \bar{M} > 0$, which would yield a welfare gain

[11] To obtain that $B = \bar{B} = 0$, obtain expressions for $C/C^*$, $\bar{C}/\bar{C}^*$ from the (+) equations, then subtract the equations, taking into account $\frac{C}{C^*} = \frac{\bar{C}}{\bar{C}^*}$. Finally replace $\bar{B} = B(1+i)/(1+\delta)$ to get $(B/P)(1/C^*) = -(1+i)/(1+\delta) * (B/\bar{P})(1/\bar{C}^*)$ which can only hold if $B = 0$. Finally, the relationship between $B$ and $\bar{B}$ is obtained from the current account the period after the shock $\bar{B} = (1+i)B + \bar{P}_H\bar{Y}/\bar{g}$ and (21).

[12] To obtain this expression, take a derivative of $U$ while taking into account how the variables —$C, \bar{M}/P, Y$— depend on $\bar{M}$ in the final solution given by "Table II" on the paper. For example for the first term see that $\frac{\partial U}{\partial \bar{M}} = \frac{1-\rho}{1-\rho}C^{-\rho}\frac{\partial C}{\partial \bar{M}} + \ldots$, with $C = a_1(\bar{M}_W)^{1/\rho}$, and then $\frac{\partial C}{\partial \bar{M}} = \frac{\gamma}{\rho \bar{M}}C$, where $\bar{M}_W$ is a weighted geometric average of $\bar{M}$ and $\bar{M}^*$ with weights $\gamma$ and $1-\gamma$.

that only reflects the fact that extra utility of consumption effect dominates over the extra disutility of effort.

Due to a monopolistic power, wage setting yields inefficiently high real wages and low output. Then a small inflationary shock that lowers real wage, and increases output, consumption and employment is welfare-improving.

**Open Economy:** $\gamma < 1$

$\bar{M} \uparrow \rightarrow \varepsilon \uparrow \rightarrow TOT \downarrow \Rightarrow$ Purchasing power of home households $\downarrow$

The shock, then, delivers a positve (+) Aggregate demand effect — erodes monopoly power, and a negative (−) TOT effect. This second effect may overtake on the welfare effect of the money expansion (and deliver welfare losses).

Then, $\frac{\partial U}{\partial M} < 0$ when: $\begin{cases} \gamma \text{ is NOT very large (Foreign good matters a lot in CPI)} \\ \phi \text{ is NOT too small (not very large distortion from monopoly power and endogenous labor)} \end{cases}$

**WAMI vs. OR:** In OR there is no distinction between firm level and country level monopoly power. Then $\bar{M} \uparrow$ is always good (different to WAMI.)

**Optimal Policy**

Now, to define optimal level of money $\tilde{M}$ take $\frac{\partial U}{\partial M} = 0$:

$$C^{1-\rho} - \kappa \frac{Y^2}{g} = -\chi\rho + \frac{1-\gamma}{\gamma}\rho\frac{Y^2}{g} \tag{1}$$

and solve for $C^{1-\delta} - \kappa\frac{Y^2}{g}$.[13] The associated level of $M$ with this solution to (1) is the optimal policy ($\tilde{M}$).

Now, recalling the participation constraint is $C^{1-\rho} \geq \kappa\frac{Y^2}{g}$, then we have that in a closed economy the optimal policy is not feasible (with $\gamma = 1$ the RHS of (1) is $-\chi\rho$).

Optimal policy in closed economy: Set $\bar{M} \leq \tilde{M}$ such that the participation constraint holds.

That is, bring employment, and output as close to the potential as feasible. (potential here: output without monopolistic distortion where $W/P = \text{MRS}_{\text{cons,leisure}}$.)

In the closed economy case, as long as the participation constraint holds, it is beneficial to raise the money supply (non-systematically). In fact it has a monotonically relation with utility.

In Open Economy: With $\gamma < 1$ the relation between money increases and utility has an inverted U shape.

The home M innovation that maximizes welfare is lower than the one closing the output gap $\bar{M} \leq \tilde{M}$

Only if increase in money is done in sync, such that, $\bar{M} - \bar{M}^* = 0$ (both $\uparrow$), the ER and TOT effect will disappear and each country benefits from expanding their money supply to the point output reaches their potential. (intuition: expansion mitigates distortion without the TOT and expenditure switching cost.)

If the increase is not synchronized in this fashion, we have that the gains from appreciating the TOT at the margin offset the efficiency losses from setting a lower output than the potential.

**International Monetary Spillover** (increase in foreign money that appreciates ER)

Comparison to OR:

In OR: $M^* \uparrow \rightarrow M^W \uparrow \Rightarrow$ Welfare increases for home

In CP: $M^* \uparrow \rightarrow TOT \uparrow \rightarrow$ effect on home output depends on whether H and F goods are complements/substitutes.

The goods are complements if $U_{C_H,C_F} > 0$; with CRRA + CES: if $1/\rho > 1$ (or if IES > Intra IES; there is a positive effect on home output (from the higher demand for the foreign good).

[13] This is the only variable in the FOC, which we have a solution for in terms of money (table II in paper), thus after solving for the endogenous part of the equation, and replacing the solution we just need to rearrange to get the optimal money supply.

Effect on Welfare:

$$\frac{\partial U}{\partial \bar{M}^*} = \frac{1-\gamma}{\rho \bar{M}^*}\left[C^{1-\rho} + \chi\rho - \kappa\frac{Y^2}{g}(1-\rho)\right] > 0$$

This expression is positive unambiguously as long as the participation constraint holds. (monetary shocks have a prosper-thy-neighbor effect.) (hence, OR and CP contradict usual beggar-thy-neighbor argument of money expansions.)

Intuition: Foreign money expansion raises home welfare due to the TOT improvement which allows home households to finance higher consumption for any level of labor supply.

In OR there is also a prosper-thy-neighbor effect but for a different reason. In CP is due to the TOT effect rather than the mitigation of the distortion in the labor market.

Key policy implication: No incentive to engage in competitive devaluations.

**International Policy Links**: Rewriting the system of equations $\partial U/\partial \bar{M} = 0$, $\partial U^*/\partial \bar{M}^* = 0$

$$\left(\frac{\gamma}{\rho} + 2 - \gamma\right)\ln\frac{\bar{M}}{M_0} = -\ln\left[\frac{\phi-1}{\phi}\left(1+\rho\frac{1-\gamma}{\gamma}\right)\right] - \overbrace{(1-\gamma)\left(\frac{1-\rho}{\rho}\right)\ln\frac{\bar{M}^*}{M_0^*}}^{\text{policy interdependence due to trans. of monet. shocks}} - \ln\frac{g}{g_0}$$

$$\left(\frac{1-\gamma}{\rho} + 1 + \gamma\right)\ln\frac{\bar{M}^*}{M_0^*} = \underbrace{-\ln\left[\frac{\phi^*-1}{\phi^*}\left(1+\rho\frac{1-\gamma}{\gamma}\right)\right]}_{\substack{\text{Internal distortions (firm/HH mon. power) vs.}\\ \text{External distortions (country mon. power on TOT)}}} - \gamma\left(\frac{1-\rho}{\rho}\right)\ln\frac{\bar{M}}{M_0} - \ln\frac{g^*}{g_0^*}$$

From the first term in the RHS: The more open the economy the lower the incentives to shock inflation due to the effect on TOT (the square brackets term will be larger than one leading to a deflationary bias). (Unlike in a closed economy, domestic distortions will not necessarily impart an inflationary bias to optimal policies.)

Second term in the RHS: Policy interdependence due to international transmission of monetary shocks. As discussed, if the goods are complements, then $M^* \uparrow$ improves welfare. If in addition $M \downarrow$ and $\varepsilon$ (appreciation), then welfare will improve even more due to reduced disutility of home work effort associated with the output expansion (prosper-thy-neighbor, and the opposite happens if $\rho > 1$).

Foreign fiscal policy has indirect effect on home monetary policy via its effect on foreign money.

**Summary:**

CP build on the approach of Cole and Obstfeld (1991) to provide a framework in similar spirit as OR95 but with less restrictive assumptions, that can still be solved in closed form. Their work makes a key distinction between firm-level monopoly on individual good prices (or households on wages) and country-level monopoly power on the TOT (basket of goods).

Their results suggest important policy spillovers across borders and explain that a monetary expansion can be a prosper-thy-neighbor policy under some circumstances. Thus, they argue against the usual competitive devaluation beggar-thy-neighbor perspective.

On the other hand, CP and OR assume relative frictionless markets and producer currency pricing (PCP), which implies a complete pass-through from the exchange rate fluctuations to prices. This assumption, however, has less empirical support in the data, and also can affect drastically the main implications of their setups.

To explore this issue further, a number of subsequent studies have explored the role of local-currency-pricing (LCP) in shaping these international policy spillovers and the cross-border macroeconomic interdependence.

# 13. Local Currency Pricing

## 13.1. BD2000 - Partial Local Currency Pricing

Betts and Devereux (2000) develop a version of the OR95 model that allows for pricing to market in the form of firms setting prices in the currency of consumers: local currency pricing (LCP).[14]

They assume firms segment markets by country. Now, they also must assume stricky prices, otherwise, LCP is equivalent to PCP. With these two additions the PPP won't hold ($\bar{P} \neq \varepsilon \bar{P}^*$, where a bar denotes a constant).

**Implications of LCP (in terms of previous results):**

- One consequence of the LCP setup is that it increases the ER volatility.

- LCP affects the transmission of policy shocks and their welfare implications. Monetary policy becomes Beggar Thy Neighbor

  - The higher the degree of LCP pricing the lower the comovment in consumption across countries, but the higher the comovement of output.

**Model**

- Two countries

- Continuum of goods $z \in [0, 1]$

- Monopolistic competition; $n$ firms in home, $(1 - n)$ firms abroad.

**Preferences:**

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \left[ \log C_s + \frac{\chi}{1-\lambda} \left( \frac{M_s}{P_s}^{1-\lambda} \right) + \eta \log(1 - L_s) \right],$$

where $1 - L_s$ denotes leisure, $L_t$ denotes hours worked and the consumption basket $C_t$ is given by a bundle of consumed varieties

$$C_t = \left[ \int_0^1 (c_t(z))^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \quad \theta > 1,$$

where there is only one meaningful elasticity of substitution $\theta$; thus, as in OR95 the elasticity of substitution between country-specific varieties is the same as that within country varieties. $\text{ES}_{\text{within}} = \text{ES}_{\text{within}} > 1$.

**Prices:** a key difference is that a fraction $\alpha$ of the firms can segment markets by country (LCP) while the remainder goods can be freely traded by consumers (LOP)

$$P_t = \Bigg[ \overbrace{\int_0^1 (p_t(z))^{1-\theta} dz}^{\text{home varieties}} + \overbrace{\int_n^{(1-n)\alpha} (p_t^*(z))^{1-\theta}}^{\text{foreign varieties with LCP}} + \overbrace{\int_{n+(1-n)\alpha}^1 (\varepsilon_t q_t^*(z))^{1-\theta} dz}^{\text{foreign varieties with LOP/PCP}} \Bigg],$$

where $p_t^*(z)$ is the home currency price of a foreign LCP good $z$ and $q_t^*(z)$ is the foreign currency price of a foreign PCP good $z$.

An analogous price basket expression would hold for the foreign households ($P_t^*$).

**Optimal demands for each type of good:**

$$c_t(z) = \begin{cases} \left( \frac{p_t(z)}{P_t} \right)^{-\theta} C_t & \text{if } z \in [0, n) \\ \left( \frac{p_t^*(z)}{P_t} \right)^{-\theta} C_t & \text{if } z \in (n, n + (1-n)\alpha) \\ \left( \frac{\varepsilon_t q_t(z)^*}{P_t} \right)^{-\theta} C_t & \text{if } z \in (n + (1-n)\alpha, 1] \end{cases}$$

[14]The authors refer to the pricing as Pricing To Market which is a more general concept, used more traditionally to refer to situations where setting different prices in different markets generates deviations from the law of one price under flexible prices; for example in the case where different elasticity of substitutions among goods differ between home and foreign consumers resulting in a structural reason for different prices of the same good in different locations. LCP in contrast will generate segregation too but will generate deviations from the LOP only under sticky prices.

**Budget constraint:**

$$P_tC_t + M_t + d_tB_t = W_th_t + \Pi_t + M_{t-1} + T_t + B_{t-1},$$

where $B_{t-1}$ is holdings of a bond denominated in home currency, $d_t$ is the price of the bond (inverse of one plus interest rate), and $\Pi_t$ are the profits from the firm ownership.

The home-currency denominated bond is the only asset traded internationally.[15]

$T_t$ is a lump-sum tax transfer and satisfy a government balanced budget constraint given by: $P_tG_t + T_t = M_t - M_{t-1}$ where $G_t$ is a wasteful government consumption, with the same composition as $C_t$ and similar government demands for individual goods' varieties.

### Production and pricing by firms

The production is linear in labor and price setting, under flexible prices by a LCP home firm $z$ choosing $p_t(z)$ and $q_t(z)$ to maximize profit yields:

$$p_t(z) = \varepsilon q_t(z) = \frac{\theta}{\theta - 1}W_t.$$

And clearly, a PCP home firm $z'$ will choose the same markup over marginal cost and have the price abroad determined by the LOP.

Hence, PPP holds under flexible prices in BD2000. No deviation of LOP under flexible prices and LCP

**Export price index:**

$$\Gamma_t = \left[\int_0^{(1-\alpha)n} (p_t(z))^{1-\theta} + \int_{(1-\alpha)n}^{n} (\varepsilon_t q_t(z))^{1-\theta} dz\right]^{\frac{1}{1-\theta}}.$$

This index reflects that of the $n$ goods produced and exported by home, $(1-\alpha)n$ are priced in home currency and $\alpha n$ are priced in foreign currency.

An analogous definition for $\Gamma_t^*$ follows and the terms-of-trade are

$$TOT_t \equiv \frac{\Gamma}{\varepsilon\Gamma^*}$$

### Sticky prices

The same 1-period price stickyness as in OR95 is assumed. This is combined with a partial degree of LCP.

For PCP firms the LOP holds; for LCP firms shocks in the ER cause deviations from LOP in the short run.

Output is then demand-determined during the period in which pre-set prices are in place. As in the two previous models, the economy converges to its new long-run equilibrium in the period $t+1$ after a shock in $t$.

This model shares the same problem of steady-state indeterminacy and non-stationary with the OR setup. Thus, we have a transition between steady states after a shock.

Use as before sans serif fonts to denote percentage deviations from the initial, symmetric, steady state with zero assets.

Let: $\mathsf{B}_t \equiv \mathsf{d}B_t/(\bar{P}\bar{C}^W),\ \mathsf{G}_{t+1} \equiv \mathsf{d}G_t/(\bar{C}^W),\ \mathsf{G}^*_{t+1} \equiv \mathsf{d}G^*_t/(\bar{C}^W)$

Long-run responses:

$t+1$ responses of the ER and consumption differential to shocks are,

$$\mathsf{e}_{t+1} = \mathsf{M}_{t+1} - \mathsf{M}^*_{t+1} - \frac{1}{\lambda}(\mathsf{C}_{t+1} - \mathsf{C}^*_{t-1}), \tag{1}$$

$$\mathsf{C}_{t+1} - \mathsf{C}^*_{t+1} = \frac{\theta - 1 + \eta}{\theta - 1 + \theta\eta}\left[\frac{1-\beta}{1-n}\mathsf{B}_t - \mathsf{G}_{t+1} - \mathsf{G}^*_{t+1}\right]. \tag{2}$$

[15] Unlike before, here it is important that for the bond not to be denominated in units of consumption since LCP will result in PPP deviations and a real interest rate differential across countries, which, given identical consumption baskets could not manifest if real bods are denominated in units of consumption.

The first equation has the standard form and follows from equilibrium in money markets. The second one, states that for a given government expenditure, a trade surplus in the short run ($\mathsf{d}B_t > 0$) generates a positive consumption differential in the long-run (as in OR95).

Short-run responses:

$$\mathsf{e}_t(1-\alpha) = \mathsf{M}_t - \mathsf{M}_t^* - \frac{1}{\lambda}(\mathsf{C}_t - \mathsf{C}_t^*) - \frac{1}{\lambda\delta}(\mathsf{e}_t - \mathsf{e}_{t+1}),$$

where $\delta$ is the discount rate as in OR95. We can note that the short-run response in OR is the same if $\lambda = 1$ —log disutility— and $\alpha = 0$ —no LCP. (in OR SR response is $\mathsf{e} = \mathsf{M} - \mathsf{M}^* - (\mathsf{C} - \mathsf{C}^*) + (1/\delta)(\bar{\mathsf{e}} - \mathsf{e})$)

The Euler equations yield,

$$\mathsf{C}_t - \mathsf{C}_t^* = \mathsf{C}_{t+1} - \mathsf{C}_{t+1}^* + \alpha\mathsf{e}_t \tag{3}$$

Intuition: PPP does not hold as long as $\alpha > 0$. Then, the real interest rates will differ across countries, and in consequence, there will be a corresponding difference in rates of growth of consumption, despite the presence of free trade of assets (nominal bond).

Replacing $\mathsf{e}_{t+1}$ from (1) in the short-run equation for $\mathsf{e}_t$

$$\mathsf{e}_t(1-\alpha) = \mathsf{M}_t - \mathsf{M}_t^* - \frac{1}{\lambda}(\mathsf{C}_t - \mathsf{C}_t^*) - \frac{1}{\lambda\delta}\left(\mathsf{e}_t - \left(\mathsf{M}_{t+1} - \mathsf{M}_{t+1}^* - \frac{1}{\lambda}(\mathsf{C}_{t+1} - \mathsf{C}_{t-1}^*)\right)\right),$$

Now substitute $\mathsf{C}_{t+1} - \mathsf{C}_{t+1}^*$ from (3) and solve for $\mathsf{e}_t$

$$\mathsf{e}_t\left(1 - \alpha + \frac{\lambda - \alpha}{\lambda^2\delta}\right) = \left(1 + \frac{1}{\lambda\delta}\right)(\mathsf{M}_t - \mathsf{M}_t^*) - \frac{1}{\lambda}\left(1 + \frac{1}{\lambda\delta}\right)(\mathsf{C}_t - \mathsf{C}_t^*).$$

From the price index formula we have that when $\alpha = 1$ (full LCP) the ER does not affect the price level. (full LCP & stickiness $\Rightarrow ER \perp P$)

If in addition, $\lambda = 1$ then the ER does not affect the nominal interest rate differentials and the ER will not have any effect altogether.

> Disgression: UIP in the BD2000 model
>
> We can recall that the nominal interest rate at home is: $1 + i_{t+1} = \frac{1}{d_t}$. Analogously, the foreign nominal interest rate (on bonds deminonated in home currency) is: $1 + i_{t+1}^* = \frac{\varepsilon_t}{\varepsilon_{t+1}}\frac{1}{d_t}$. Taking the ration,
>
> $$\frac{1 + i_{t+1}}{1 + i_{t+1}^*} = \frac{1/d_t}{(\varepsilon_t/dt)/\varepsilon_{t+1}} = \frac{\varepsilon_{t+1}}{\varepsilon_t}$$
>
> Then, we have that the UIP holds in the BD model, and it can be verified that, if $\alpha = \lambda = 1$, the equilibrium interest rate differential is not affected by the ER movements.

Taking the log-linear versions of the balance of payment equations and using the market clearing conditions,

$$\mathsf{e}_t = \frac{\mathsf{C}_t - \mathsf{C}_t^* + \frac{\beta}{1-n}\mathsf{B}_t + \mathsf{G}_t - \mathsf{G}_t^*}{(1-\alpha)(\theta - 1) + \alpha}. \tag{4}$$

**Intuition:**

| | |
|---|---|
| $\alpha = 0$ (PCP) | $\alpha = 1$ (LCP) |
| $\uparrow\varepsilon$ (depreciation) $\rightarrow$ price of foreign goods ($\uparrow \varepsilon_t P_t^*$) | $\uparrow \varepsilon \rightarrow$ No effect on prices |
| Demand shifts toward home | But relative incomes are still affected: income redistribution towards home country |
| $\Rightarrow \uparrow$ production, $\uparrow C_t$ | $\Rightarrow \uparrow$ production, $\uparrow C_t$ |

Thus, we get the same effect even without (relative) price changes. The LCP only turns off the relative price effect but the relative income effect is present in both.

Finally, the solutions for the short-run ER and relative consumption are:

$$\mathsf{e}_t = \frac{\varphi\lambda(\mathsf{M}_t - \mathsf{M}_t^*) + \mathsf{G}_t - \mathsf{G}_t^* + \frac{1}{\delta}(\mathsf{G}_{t+1} - \mathsf{G}_{t+1}^*)}{(1-\alpha)(\lambda\varphi + \theta - 1) + \alpha\varphi\frac{1+1/\delta}{1+1/(\delta\lambda)}},$$

$$\mathsf{C}_t - \mathsf{C}_t^* = \left[\frac{(1-\alpha)(\theta-1)}{\varphi} + \alpha\right]\mathsf{e}_t + \frac{1}{\varphi}[\mathsf{G}_t - \mathsf{G}_t^* + \frac{1}{\delta}(\mathsf{G}_{t+1} - \mathsf{G}_{t+1}^*)],$$

with $\varphi \equiv 1 + \frac{\theta - 1 + \theta\eta}{\delta(\theta - 1 + \eta)}$.

For any $\alpha$, unanticipated domestic money or government spending expansions generate a depreciation. However, the intuition under full-LCP is different from the standard OR95 PCP case.

ER responses to shocks with full LCP: ($\uparrow \alpha$ will magnify the response (volatility) of the ER)

If PPP holds (full PCP, or $\alpha = 0$) the ER depreciates by less, the higher the elasticity of substitution $\theta$. (Reason: $\uparrow$ER $\Rightarrow$ Expenditure switching.) The higher the substitability effect between goods the lower the ER change required to keep the markets in equilibrium.

With LCP: high substitability plays no role in mitigating the ER response since relative changes (cross differential variables) will not change by moving the ER (no expenditure switching). We end up with a more volatile ER as a result.[16]

### Monetary Shock and Overshooting

Here we focus on a monetary shock at home relative to foreign ($\mathsf{M}_{t+1} - \mathsf{M}_{t+1}^* = \mathsf{M}_t - \mathsf{M}_t^* > 0$). This analysis abstracts from fiscal shocks (i.e., assumes $\mathsf{G}_t = \mathsf{G}_t^* = \mathsf{G}_{t+1} = \mathsf{G}_{t+1}^* = 0$).

In $t+1$ we have equation (1):

$$\mathsf{e}_{t+1} = \mathsf{M}_t - \mathsf{M}_t^* - \frac{1}{\lambda}(\mathsf{C}_{t+1} - \mathsf{C}_{t+1}^*),$$

and from the Euler equations we obtain equation (3) stating,

$$\mathsf{C}_{t+1} - \mathsf{C}_{t+1}^* = \mathsf{C}_t - \mathsf{C}_t^* - \alpha\mathsf{e}_t.$$

Then, by replacing $\mathsf{C}_{t+1} - \mathsf{C}_{t+1}^*$ from (2),

$$\mathsf{e}_t = \frac{\varphi(\mathsf{C}_t - \mathsf{C}_t^*)}{(1-\alpha)(\theta-1) + \alpha\varphi} \tag{5}$$

which is obtained by replacing $\mathsf{B}_t$ from (4) and simplifying the coefficient in terms of the definition of $\varphi$.

From equations (1), (3), and (5) we have the implied future response of the ER ($\mathsf{e}_{t+1}$):

$$\mathsf{e}_{t+1} = \mathsf{M}_t - \mathsf{M}_t^* - \frac{(1-\alpha)(\theta-1)}{\varphi}\mathsf{e}_t. \tag{6}$$

From this expression we can see that as long as $\alpha < 1$ the long-run ER movement is smaller than the change in money supply (i.e., the second term has a negative coefficient and lowers the impact).

[16] In general, a rise in $\alpha$ will magnify the responde of th ER to policy shocks as long as $\theta - 1 + (\lambda - 1)\frac{\varphi}{1+1/(\delta\lambda)} > 0$. Given $\theta > 1$, a sufficient condition for this is just $\lambda \geq 1$ which is consistent with the empirical evidence.

The reason is that the shock causes a trade surplus in the short run. Then, there is a higher consumption differential in the long run ($\mathsf{C}_{t+1} - \mathsf{C}^*_{t+1} > 0$). This mitigates the ER depreciation (remember than from equation for ER: $\mathsf{e}_t \propto (\mathsf{C}_{t+1} - \mathsf{C}^*_{t+1})$).

In contrast, when $\alpha = 1$ (full LCP) $\mathsf{e}_{t+1} = \mathsf{M}_t - \mathsf{M}^*_t$ (same response as the money shock). In this case there are no changes in the consumption differential, and thus, the response of the long-run ER is the same than under flexible prices.[17]

Going back to (6) and combining it with the money market equilibrium conditions,

$$\mathsf{e}_{t+1} - \mathsf{e}_t = -\frac{\alpha}{\varphi}\frac{\lambda - 1}{\lambda}\mathsf{e}_t.$$

Hence, absent LCP ($\alpha = 0$) there is no overshooting. (expected since the model boils down to the OR95 setup)

When there's any extent of LCP ($\alpha > 0$) there will be overshooting as long as the consumption elasticity of money demand is below one ($1/\lambda < 1$). (the same condition ensuring LCP increases ER volatility)

In the full LCP case ($\alpha = 1$), while the long-run ER is proportional to the money shock, we have that the short-run ER solution is higher:

$$\mathsf{e}_t = \frac{\lambda + 1/\delta}{1 + 1/\delta}(\mathsf{M}_t - \mathsf{M}^*_t), \tag{7}$$

which exceeds the long-run response (coefficient larger than one) as long as $\lambda > 1$ (same empirically plausible condition as before).

**Response to money shocks**

**RER response**: Consider the log linear price index equations:

$$\mathsf{P}_t = (1 - n)(1 - \alpha)\mathsf{e}_t, \text{ and } \mathsf{P}^*_t = -m(1 - \alpha)\mathsf{e}_t.$$

Using them to obtain the real exchange rate,

$$\mathsf{e}_t + \mathsf{P}^*_t - \mathsf{P}_t = \alpha\mathsf{e}_t.$$

Then, with full PCP ($\alpha = 0$) the PPP holds, and with full LCP the RER becomes identical to the nominal ER.

Intuitively, with full LCP, all prices in each country's CPI are fixed in the short run. Hence, the RER will move just as much as the NER.

**TOT response**: The short-run TOT response to money shocks is

$$\mathsf{TOT}_t = (2\alpha - 1)\mathsf{e}_t.$$

The direction of the TOT movement depends on the degree of LCP. With full PCP (no LCP or $\alpha = 0$) the TOT deteriorate (as obtained in previous analyses). Here, since prices are set in the exporter's currency, the home currency price of imports increases, while the price of exports is unchanged.

When $\alpha = 1$ (full LCP), the TOT improve. Here prices are set in the (buyer) importer's currency. Then a depreciation raises the home currency price of exports, while the price of imports is unchanged.

Finally, when $\alpha = 1/2$ the rise in export prices just cancels out with the rise in import prices, and the there is no TOT change. Then, the TOT after a ER depreciation is $\alpha > 1/2$.

Disgression: The TOT and Relative Consumer Prices

Consider a representative good $h$ and a representative foreign good $f$. Let $p_t(h)$ be the home currency price of $h$, $p_t(f)$ the home currency price of $f$m $q_t(h)$ the foreign currency price of $h$, and $q_t(f)$ the foreign currency price of $f$.

[17] The implication is that, by removing the expenditure switching, full LCP removes the long-run monetary non-neutrality of the PCP model. However, this does not imply the model becomes stationary. The model with full LCP still would have permanent fiscal shocks.

We can tell that under full PCP all that matters are the relative prices:

$$\frac{p_t(h)}{p_t(f)} = \frac{p_t(h)}{\varepsilon_t q_t(f)} = \frac{\varepsilon_t q_t(h)}{\varepsilon_t q_t(f)} = \frac{\varepsilon_t q_t(h)}{p_t(f)} = \frac{q_t(h)}{q_t(f)}.$$

These equalities hold due to the LOP, and show that the relative price that consumers care about and the TOT coincide (it is analogous whether TOT is defined here as $p_t(h)/(\varepsilon_t q_t(f))$ or $\varepsilon_t q_t(h)/p_t(f)$). We have that an ER depreciation that causes the TOT to deteriorate leads consumers to relocate spending away from foreign goods.

In contrast, under full LCP, both $p_t(h)$ and $p_t(f)$ are set in advance, and the relative price that affects consumers does not move. As a consequence, the equalities above are broken.
There is no expenditure switching in consumption with LCP

**Real interest rates response**: Absence of PPP implies that the real interest rates will differ across countries.

The rates are defined in the model as $\mathsf{r}_{t+1} = \mathsf{d}r_{t+1}/\delta$ and $\mathsf{r}^*_{t+1} = \mathsf{d}r^*_{t+1}/\delta$. Given these, BD2000 obtain,

$$\mathsf{r}_{t+1} = -\frac{1+\delta}{\delta}[(1-n)\alpha\mathsf{e}_t + \mathsf{C}^W_t], \qquad \mathsf{r}^*_{t+1} = \frac{1+\delta}{\delta}(n\alpha\mathsf{e}_t - \mathsf{C}^W_t),$$

with $\mathsf{C}^W_t = \frac{\lambda+1/\delta}{1+1/\delta}[n\mathsf{M}_t + (1-n)\mathsf{M}^*_t]$.

The Euler equations imply

$$\mathsf{C}_{t+1} - \mathsf{C}_t = \frac{\delta}{1+\delta}\mathsf{r}_{t+1}, \qquad \mathsf{C}^*_{t+1} - \mathsf{C}^*_t = \frac{\delta}{1+\delta}\mathsf{r}^*_{t+1},$$

We also had

$$\mathsf{C}_t - \mathsf{C}^*_t = \mathsf{C}_{t+1} - \mathsf{C}^*_{t+1} + \alpha\mathsf{e}_t.$$

Then, we can write

$$\mathsf{C}_{t+1} - \mathsf{C}_t - (\mathsf{C}^*_{t+1} - \mathsf{C}^*_t) = -\alpha\mathsf{e}_t = \frac{\delta}{1+\delta}(\mathsf{r}_{t+1} - \mathsf{r}_{t+1}).$$

From here we can see that, after a positive money shock that leads to a depreciation ($\uparrow \mathsf{e}_t$) we have a lower interest rate differential. The home rate will unambiguosly fall, whereas the effect on the foreign one is ambiguous — it will not change if $\alpha = \lambda = 1$.

**Consumption response**: In the short run solutions for consumption are

$$\mathsf{C}_t = \mathsf{C}^W_t + (1-n)\left[\frac{(1-\alpha)(\theta-1)}{\varphi} + \alpha\right]\mathsf{e}_t,$$
$$\mathsf{C}^*_t = \mathsf{C}^W_t - n\left[\frac{(1-\alpha)(\theta-1)}{\varphi} + \alpha\right]\mathsf{e}_t,$$

If $\alpha$ is small (low extent of LCP) consumption rises in both countries (as in OR95; intuition: with more passthrough foreign CPI falls and consumption rises via money market equilibrium).

With more LCP the extent of passthrough is lower, and the effect of a home money shock on foreign consumption is smaller, while the effect on home consumption is larger. Thus, a increase in the degree of LCP reduces international transmission of money shocks to consumption.

The real interest will move to reflect this. With $\lambda = 1$ and full LCP, $C_t$ rises, $C_{t+1}$ is unchanged, and thus, $r_{t+1}$ falls (see Euler equations); at the same time, $r^*_{t+1}$ is unaffected as foreign consumption does not move.

**Output response**: The solution for the short-run output in each location are (linear in labor):

$$\mathsf{L}_t = \mathsf{C}^W_t + (1-n)(1-\alpha)\theta\mathsf{e}_t$$
$$\mathsf{L}^*_t = \mathsf{C}^W_t - n(1-\alpha)\theta\mathsf{e}_t$$

If $\alpha$ is lower, the effects of money are smaller (as there is also a depreciation). With $\alpha = 0$ and $\lambda = 1$ we have the OR95 result: home money shock raises output, and foreign money shock lowers it ($\uparrow M_t \to\uparrow L_t$, $\uparrow M_t^* \to\downarrow L_t$). Thus, even if consumption transmission is positive, output transmission is negative.

**Intuition:** with expenditure switching ($\alpha = 0$) a depreciation raises home output and lowers the foreign one.

As there is more LCP ($\uparrow \alpha$) the effect of home shocks on foreign output increases. There is a lower effect in relative prices (given lower ER passthrough). Output effect comes only primarily from switch in demand from domestic consumers.

In the full LCP case there is an identical increase in demand for both locations (given there is no expenditure switching) and output increases by the same amount $\mathsf{C}_t^W$ (intuition: with PCP the demand increase is mitigated by changes in relative prices).[18]

Because there is no mitigation in the effects from expenditure switching (i.e., from changes in relative prices that allow consumers to smooth the effect of the money shock on output via cheaper consumption abroad): LCP reduces the consumption correlation and increases the output correlation.

LCP Helps to ameliorate BKK puzzle (output-consumption anomaly).

**Current account response**: The current account effect is

$$B_t = \frac{(1-n)(\varphi-1)(\theta-1)(1-\alpha)}{\beta\varphi}\mathsf{e}_t.$$

A domestic money shock improves the CA. But the effect is smaller with more LCP ($\uparrow \alpha$).

With full LCP: No effect.

**Intuition:** With full LCP (and $\lambda = 1$ or with elasticity of interest rate to consumption $1/\lambda = 1$) money and consumption increase by $\mathsf{M}_t$ and so agents would like to save income to smooth consumption. However a lower real interest (and lower rate differential as the foreign rate does not change) prevents them from it and induces them to consume more, and thus, nothing induces foreign agents to borrow more. No current account effect.

Long run effect: the effect of the $\mathsf{M}$ shock in $\mathsf{C}_{t+1}$, $\mathsf{L}_{t+1}$ decreases with $\alpha$.

In OR95: $\uparrow \mathsf{M} \to\uparrow \mathsf{C}$, $\downarrow \mathsf{C}^*$ by improving the NFA position.

Here: With full LCP the effect of the shock is felt only in the short run. So long-run non-neutrality is removed.

### Welfare effects

As before we focus only on the real part:

$$U_t^R = \sum_{s=t}^{\infty} \beta^{s-t}\left[\log C_s + \eta \log(1-L_s)\right],$$

with an analogous function for the foreign household.

It can be obtained that

$$\mathsf{d}U_t^R = \frac{\mathsf{C}_t^W}{\theta} + (1-n)\alpha\mathsf{e}_t, \qquad \mathsf{d}U_t^{R*} = \frac{\mathsf{C}_t^W}{\theta} - n\alpha\mathsf{e}_t.$$

(in OR95 both are $\mathsf{C}_t^W/\theta$ and money expansions are welfare improving regardless of origin —*prosper-thy-neighbor*—)

Here, money helps as in OR95 but as $\uparrow \alpha$ (with more LCP): Home shock increases home welfare at the expense of foreign welfare (policy can become *beggar-thy-neighbor*)

And full LCP will depict *beggar-thy-neighbor* effects as long as $(1-\alpha)\left(\lambda + \frac{\theta-1}{\varphi}\right) < \left(\theta - \frac{1+1/\delta}{1+1/(\lambda\delta)}\right)\alpha$, which is plausible for $\alpha = 1$ and always satisfied if $\alpha = \lambda = 1$.

**Intuition:** Foreign agents will not benefit as much from increased world output (as the policy improves home TOT, but it worsens foreign TOT) and still will have to work more to meet increased home demand from (higher income) home agents.

[18] With full LCP and $\lambda = 1$ $\uparrow \mathsf{M}$ raises $\mathsf{L}_t^*$ by $n\mathsf{M}_t$, deteriorates by $n\mathsf{M}_t$, and then the changes offset; the foreign output does not change. In home, output rises by $m\mathsf{M}_t$, TOT appreciates by $(1-n)\mathsf{M}_t$, and thus, output and consumption increase (by $\mathsf{M}_t$).

**Conclusion**: BD-LCP setup restores the *beggar-thy-neighbor* effects of monetary policy. It also has potential for explaining puzzles such as the consumption-output anomaly.

## 13.2. CKM2002 - Full Local Currency Pricing

Chari, Kehoe, and McGrattan (CKM, 2002, ReStud) evaluate the quantitative properties of a sticky-price model with LCP. They are interested in assessing the capacity of the framework for reproducing the observed volatility and persistence of the RER.

Stilized fact (used to test the model): volatility of RER and NER ($\sigma_{RER}$, $\sigma_{NER}$) are high; RER, NER are highly persistent while price ratio ($P^*/P$) across locations is less volatile.

Decompositions of data confirm Engel (1999) and Burstein et al (2005) results: Movements in RER for traded goods are the main source of RER fluctuations. PPP deviations for traded goods $\rightarrow$ key source of PPP deviations.

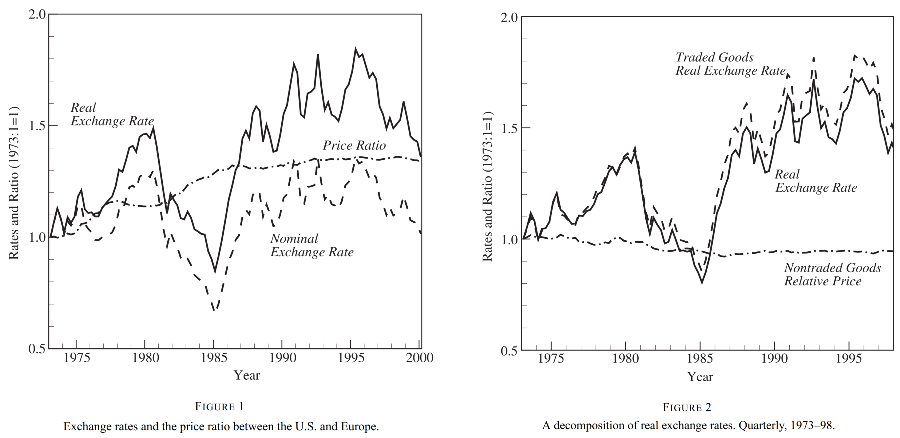



Figure 12: Real Exchange Rates - Different decompositions

Source: CKM2002.
Note: RER is $eP^*/P$, where the nominal exchange rate is $e$ is the U.S. dollar price of a basket of European currencies, $P^*$ is an aggregate of European CPIs, and $P$ is the U.S. CPI. The price ratio is $P^*/P$.

### Model

- Two countries, complete international assets market, Local Currency Pricing (LCP).

- Consumption: CES aggregate of home basket and foreign basket of intermediary goods (aggregation is made by competitive firms).[19]

- The domestic and foreign intermediate goods are produced by monopolistically competitive firms (that use labor and capital).

- The intermediary producers engage in LCP.

- Staggered prices as in Taylor (1980): N period stickyness by firm.

Each period $1/N$ firms set prices at home $P_H(z, s^{t-1})$ and abroad $P_H^*(z, s^{t-1})$ prices are fixed until all firms change. Thus $P_H(z, s^{t+\tau-1}) = P_H(z, s^{t-1})$ for $\tau = 0, 1, \cdots, N-1$ (and the same with $P_H^*$); $z$ is the firm index.

- Same condition as in BS1993: $Q(s^t) = \kappa \frac{U_c^*(s^t)}{U_c(s^t)}$

- Monetary policy: assumed persistent and to follow a stochastic process $M^s \sim AR(1)$

**Results**:

The model can account for the RER volatility with reasonable parameters (e.g., $N = 4$, risk aversion of 5, etc.). However, the model cannot account for the persistence of the RER. (CKM call this the *Persistence anomaly*).

[19] Results would be identical if we assume monopolistically competitive producers of final goods aggregated by preferences in a CES consumption basket.

The model also yields the BS1993 puzzle: It generates a condition that leads to a high correlation between the RER and relative consumption. There is no such pattern in the data. *consumption-real exchange rate anomaly*.

Proposed modifications: Incomplete markets, habit persistence, wage stickyness, Taylor Rule. These still do not solve the issues.[20]

**Conclusion**: Sticky price models are subject to a robust Consumption-RER anomaly (BS puzzle).

- On the other hand, Imbs, Mumtaz, Ravn, and Rey (QJE, 2005) argue that the CKM model does not give the sticky price model a fair change to match the persistence of the RER. The key insight is that, given sectoral heterogeneity, to match the persistence of the RER, a model with multiple sectors should be used. Or analogously, if we use a one-sector model without heterogeneity, we should be trying to match the persistence obtained with techniques that purge the effect of heterogeneity. If this is done, the CKM can capture the persistence.

- On the consumption-RER anomaly, important contributions have also been made by Benigno and Thoenissen (2008), and Corsetti, Dedola, and Leduc (2008).

# 14. Exchange Rate Dynamics Determination

All models until now have assumed monetary policy to be exogenous (or set based on rules). However, given the meaningful role that monetary policy can have on the determination of the ER dynamics, a parallel literature started to develop to explore the implications of endogenous interest rate setting for the ER and welfare in open economies. In this line, we first move to Benigno and Benigno (2008) to examine the role of interest rate decisions on the ER. Afterwards, we compare how the monetary policy prescriptions differ in open economies, relative to closed economies where price stability is the optimal policy prescription, or to setups that yielded an isomorphism between the optimal prescriptions of closed and open economies under very specific conditions.

## 14.1. BB2008 - Endogenous Interest Rate Setting and Exchange Rate Dynamics

Until now, we considered monetary shocks to be exogenous or part of a Taylor Rule setup. Now, we consider Benigno and Benigno (2008, JIMF), a framework with endogenous interest rate setting for open economies.

Central insight: Role for money can be de-emphasized —money demand plays no role in the ER determination.

**Model**

- Continuum of HH and goods in $[0, 1]$: $n$ in home, $(1 - n)$ in the foreign country.

- Each firm produces a differentiated good under monopolistic competition.

- Firms (under monopolistic competition) set prices in PCP, no home bias (same consumption basket), and LOP holds $\Rightarrow$ PPP holds.

- Complete asset markets.

- Price rigidity as in Calvo (1983) - Yun (1996).

The model will have three blocks of equations, the aggregate demand (AD) block, the aggregate supply (AS), and the policy block.

**Aggregate demand block**

**Consumption aggregate**:

$$C^j = \frac{(C_H^j)^n (C_F^j)^{1-n}}{n^n (1-n)^{1-n}},$$

[20] Market incompleteness would be an ideal candidate here as it breaks the risk-sharing-condition. However, in their model, the trade of a non-contingent asset starting from a zero-asset steady state is capable of bringing the allocation close to the complete markets outcome. On the other hand, the Taylor rule, wit persistence and responses to inflation and output deviations, worsens the performance of the model in capturing the RER persistence.

with,

$$C_H^j = \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n c^j(h)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{\theta}{\theta-1}}, \quad C_F^j = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\theta}} \int_n^1 c^j(f)^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta}{\theta-1}}$$

**Prices**:

$$P = (P_H)^n (P_F)^{1-n}$$

with,

$$P_H = \left(\frac{1}{n}\int_0^n p(h)^{1-\theta} dh\right)^{\frac{1}{1-\theta}}, \quad P_F = \left(\frac{1}{1-n}\int_n^1 p(f)^{1-\theta} df\right)^{\frac{1}{1-\theta}}$$

TOT are defined as $TOT \equiv P_F/P_H = P_F^*/P_H^*$ (this is the reciprocal of the usual definition, and the second equality follows from the law of one price). Given the reciprocal definition, a TOT decrease is an improvement or appreciation ot the terms-of-trade.

**Demands**:

For individual goods and by household $j$,

$$\begin{aligned} c^j(h) &= \left(\frac{p(h)}{P_H}\right)^{-\theta} TOT^{1-n} C^j \\ c^j(f) &= \left(\frac{p(f)}{P_F}\right)^{-\theta} TOT^{-n} C^j \\ g(h) &= \left(\frac{p(h)}{P_H}\right) G_H \end{aligned}$$

Total demands of homegood $h$ and foreign good $f$ after aggregating demands by households (of each location) and governments,

$$y^d(h) = \left(\frac{p(h)}{P_H}\right)^{-\theta} \left(TOT^{1-n} C^W + G_H\right), \quad y^d(f) = \left(\frac{p(f)}{P_H}\right)^{-\theta} \left(TOT^{-n} C^W + G_H\right),$$

where $C^W \equiv nC + (1-n)C^*$.

Aggregate demand for home output is obtained by aggregating quantities of individual goods across home producers (after putting them in comparable units —home sub-basket),

$$Y_{AG}^H = \int_0^n \frac{p(h)}{P_H} y^d(h) dh = \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n y^d(h)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{\theta}{\theta-1}},$$

where the last equality follows after substituting the expression for $P_H$ and $p(h) = [y^d(h)/(TOT^{1-n}C^W + G_H)]^{-1/\theta} P_H$.

Substitute $y^d(h)$ to obtain,

$$\begin{aligned} Y_{AG}^H &= n(TOT^{1-n} C^W + G_H), \\ Y_{AG}^F &= (1-n)(TOT^{-n} C^W + G_F^*). \end{aligned}$$

In per capita terms:

$$\begin{aligned} Y^H &= \frac{Y_{AG}^H}{n} = \left[\frac{1}{n}\int_0^n y^d(h)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{\theta}{\theta-1}}, \\ Y^F &= \frac{Y_{AG}^F}{1-n} = \left[\frac{1}{1-n}\int_n^1 y^d(f)^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta}{\theta-1}}, \end{aligned}$$

and then, $Y^H = TOT^{1-n} C^W + G_H$, and $Y^F = TOT^{-n} C + G_F^*$.

Now, Complete markets + Symmetric initial NFA position implies: $C = C^* + C^W$.

This occurs due to full insurance even in presence of potentially different output levels (e.g., after TOT and gov. spending shocks).

Then,

$$Y^H = TOT^{1-n}C + G_H, \quad Y^F = TOT^{-n}C + G_F^*.$$

Log-linearizing (and adding back the time indexes):

$$\mathsf{Y}_t^H = (1-n)\mathsf{TOT}_t + \mathsf{C}_t + \mathsf{G}_{H,t}, \quad \mathsf{Y}_t^F = n\mathsf{TOT}_t + \mathsf{C} + \mathsf{G}_F^*,$$

where $\mathsf{G}_{H,t}$ and $\mathsf{G}_F^*$ are country-specific demand shocks (from government spending).

These equations imply a global market clearing condition for the final goods market:

$$\mathsf{Y}_t^W = \mathsf{C}_t + \mathsf{G}_t^W,$$

where the $\mathsf{Y}_t^W \equiv n\mathsf{Y}_t^H + (1-n)\mathsf{Y}_t^F$ and $\mathsf{G}_t^W \equiv n\mathsf{G}_{H,t} + (1-n)\mathsf{G}_{F,t}^*$.

**Euler equation:** Depart from the individual Euler equations: $\mathbb{E}_t\mathsf{C}_{t+1} = \mathsf{C}_t + \rho^{-1}(\mathsf{i}_{t+1} - \mathbb{E}_t\hat{\pi}_{t+1})$, and $\mathbb{E}_t\mathsf{C}_{t+1}^* = \mathsf{C}_t^* + \rho^{-1}(\mathsf{i}_{t+1}^* - \mathbb{E}_t\hat{\pi}_{t+1}^*)$, where $\hat{\pi}_t$ is the percent deviation of home consumer inflation from the steady state. Also, $i_{t+1}$ denotes the interest agreed on $t$ and payable on $t+1$ so it is a known quantity at $t$.

Taking a population size weighted average of the Euler equations:

$$n\mathbb{E}_t\mathsf{C}_{t+1} + (1-n)\mathbb{E}_t\mathsf{C}_{t+1}^* = n\mathsf{C}_t + (1-n)\mathsf{C}_t^* + n\rho^{-1}(\mathsf{i}_{t+1} - \mathbb{E}_t\hat{\pi}_{t+1}) + (1-n)\rho^{-1}(\mathsf{i}_{t+1}^* - \mathbb{E}_t\hat{\pi}_{t+1}^*),$$

and recalling that under complete markets $\mathsf{C}_t = \mathsf{C}_t^*$, and noting that, given identical consumption baskets and the LOP we would have the following link between producer prices and consumer prices:[21]

$$n\hat{\pi}_t + (1-n)\hat{\pi}_t^* = n\hat{\pi}_t^p + (1-n)\hat{\pi}_t^{p*},$$

where $\hat{\pi}_t^p$ ($\hat{\pi}_t^{p*}$) is the percent deviation of the home (foreign) *producer* inflation rate from the steady state. In addition, BB2003 assume that the steady state features zero producer price inflation in each location.

Given the last expression, and the initial Euler equations, the average of Euler equations above is equivalent to

$$\mathbb{E}_t\mathsf{C}_{t+1} = \mathsf{C}_t + \rho^{-1}n(\mathsf{i}_{t+1} - \mathbb{E}_t\hat{\pi}_{t+1}^p) + \rho^{-1}(1-n)(\mathsf{i}_{t+1}^* - \mathbb{E}_t\hat{\pi}_{t+1}^{p*}). \tag{1}$$

Let the world output gap with respect to the flexible prices allocation be $\mathsf{y}_t^W = \mathsf{Y}_t^W - \tilde{\mathsf{Y}}_t^W$; then, use the condition $\mathsf{Y}_t^t = \mathsf{C}_t^W + \mathsf{G}_t^W$ to rewrite the Euler equation,

$$\mathbb{E}_t\mathsf{Y}_{t+1}^W = \mathsf{Y}_t^W + \rho^{-1}n(\mathsf{i}_{t+1} - \mathbb{E}_t\hat{\pi}_{t+1}^p - \tilde{\mathsf{R}}_{t+1}^W) + \rho^{-1}(1-n)(\mathsf{i}_{t+1}^* - \mathbb{E}_t\hat{\pi}_{t+1}^{p*} - \tilde{\mathsf{R}}_{t+1}^W), \tag{2}$$

where $\tilde{\mathsf{R}}_{t+1}^W$ denotes the perfurpations to the world natural real interest rate (natural interest rate in log-linear terms, or *Wicksellian* rate of Woodford, 2003).[22] In the paper it will also be a combination of world supply and demand shocks.

Equation (2) is the intertemporal, microfounded IS expression in the BB2008 model.

**Terms of Trade**:

The definition of TOT implies,

$$\mathsf{TOT}_t = \mathsf{TOT}_{t-1} + \mathsf{e}_t + \bar{\pi}_t^{p*} - \bar{\pi}_t^p, \tag{3}$$

where $\mathsf{e}_t$ is the percentage deviation of gross nominal exchange rate depreciation ($\varepsilon_t/\varepsilon_{t-1}$). Notice this implies a change of notation with respect to papers discussed in previous sections. Unlike before, here the log-linear version (percentage deviation relative to the steady state) of the exchange rate is $\epsilon$ (and not $\mathsf{e}$), and $\mathsf{e}_t = \epsilon_t - \epsilon_{t-1}$.

[21] In more detail: $n\hat{\pi}_t + (1-n)\hat{\pi}_t^* = n[n\hat{\pi}^p + (1-n)(\mathsf{e}_t + \hat{\pi}_t^{p*})] + (1-n)[n(\hat{\pi}_t^p - \mathsf{e}_t) + (1-n)\pi_t^{p*}] = n^2\hat{\pi}_t^p + n(1-n)\mathsf{e}_t + n(1-n)\hat{\pi}_t^{p*} + n(1-n)\hat{\pi}_t^p - n(1-n)\mathsf{e}_t + (1-n)^2\hat{\pi}_t^{p*} = n\hat{\pi}_t^p + (1-n)\hat{\pi}_t^{p*}$, where $\mathsf{e}_t$ is the percentage deviation of gross nominal exchange rate depreciation ($\varepsilon_t/\varepsilon_{t-1}$) from the steady state (with zero depreciation).

[22] $\tilde{\mathsf{R}}_{t+1}^W$ is the real rate that would arise if prices were perfectly flexible (or the world nominal rate if each country's producer price inflation is zero under flexible prices)

Therefore, the model will have a state variable: Lagged TOT the model displays persistence additional to that of the exogenous shocks. (good feature as) In closed economy models this is modified in an ad-hoc manner to generate hump shaped IRFs

**Interest rates**: On the other hand, the UIP holds $\mathsf{i}_{t+1} - \mathsf{i}^*_{t+1} = \mathbb{E}_t \mathsf{e}_{t+1}$.

### Aggregate Supply Block

The AS block of the model consists of New Keynesian Phillips curves for each location's producer price inflation (as implied by the type of nominal rigidity used),

**Phillips curves**:

$$\hat{\pi}^p_t = \lambda \mathsf{mc}_t + \beta \mathbb{E}_t \hat{\pi}^p_{t+1}, \tag{4}$$

$$\hat{\pi}^{*p}_t = \lambda \mathsf{mc}^*_t + \beta \mathbb{E}_t \hat{\pi}^{*p}_{t+1}, \tag{5}$$

where $\mathsf{mc}_t(\mathsf{mc}^*_t)$ is the percentage deviation of home (foreign) marginal cost from the steady state, $\lambda \equiv [(1-\alpha\beta)(1-\alpha)]/[\alpha(1+\theta\eta)]$ and $\lambda^* \equiv [(1-\alpha^*\beta)(1-\alpha^*)]/[\alpha^*(1+\theta\eta)]$. In these latter terms, $1-\alpha$ is the probability of price adjustment for home firms, and $\eta$ the elasticity of labor disutility.

**Marginal costs**: The FOC for output supply make it possible to obtain the marginal costs as (a function of the MRS between consumption and production):

$$\mathsf{mc}_t = (1-n)(1+\eta)\left(\mathsf{TOT}_t - \tilde{\mathsf{TOT}}_t\right) + (\rho+\eta)\mathsf{y}^W_t, \tag{6}$$

$$\mathsf{mc}^*_t = -n(1+\eta)\left(\mathsf{TOT}_t - \tilde{\mathsf{TOT}}_t\right) + (\rho+\eta)\mathsf{y}^W_t. \tag{7}$$

Intuition for $\mathsf{mc} \propto \mathsf{y}$: the marginal cost can usually be expressed as a function of total output, as the input prices are derived from FOC implying the production function. For an example, think of the log-utility (of consumption) case, with quadratic cost of effort $l(j)^2/2$ and a linear production function $y(j) = l(j)$. This latter type of case also can help to see why the expression involves the elasticity of labor disutility.

Importantly, we can see that the marginal costs will depend on the movements in the TOT (with opposite signs for different locations). To see this even more clearly, note that when $\rho = \eta = 1$ the expression becomes $\mathsf{mc}_t = 2[(1-n)(\mathsf{TOT}_t - \tilde{\mathsf{TOT}}_t) + \mathsf{y}^W_t]$.

Using (6) and (7) we can reweite the NK Phillips curves as

$$\hat{\pi}^p_t = k_C \mathsf{y}^W_t + (1-n)k_T\left(\mathsf{TOT}_t - \tilde{\mathsf{TOT}}_t\right) + \beta \mathbb{E}_t \hat{\pi}^p_{t+1}, \tag{8}$$

$$\hat{\pi}^{*p}_t = k^*_C \mathsf{y}^W_t - n k^*_T\left(\mathsf{TOT}_t - \tilde{\mathsf{TOT}}_t\right) + \beta \mathbb{E}_t \hat{\pi}^{*p}_{t+1}, \tag{9}$$

where $k_C \equiv \lambda(\rho+\eta)$, $k^C \equiv \lambda^*(\rho+\eta)$, $k_T \equiv k_C \frac{1+n}{\rho+\eta}$, and $k^*_T \equiv k^*_C \frac{1+n}{\rho+\eta}$.

**Implications**: Real marginal costs are **not** just proportional to the output gap anymore due to cross-country interdependence via TOT movements. (before $\mathsf{mc} \propto \mathsf{y}$, now $\mathsf{mc} = f(\mathsf{y}, TOT)$ .)

In fact, the smaller and more open the economy, the more the TOT affects the marginal cost and inflation. $\uparrow$ TOT $\rightarrow$ NKPC shifts and $\uparrow \pi$: home goods become relatively cheaper, then demand increases (and lowers the merginal utility of nominal income).

On the other hand, the relation between TOT and the marginal cost creates inertia in the resulting marginal costs, and then on inflation (as TOT depend on its lags). This is an improvement with respect to closed economy models making unnecessary to include lags of the variables ad-hoc (e.g., hybrid NKPCs).

### Monetary Policy

Credible commitment is assumed. Three cases are considered:

- Fixed Exchange Rates: Follower economy sets interest rate to peg domestic currency to foreign one. The leader follows an interest rate rule without responses to the ER.
- Flexible Exchange Rate: Interest rate w/o ER

- Managed Exchange Rate: One of the two countries responds to ER in addition to inflation and output.

Digression: A common interpretation to the fact that a fixed ER implies the country to shadow the leader's interest rate $\mathsf{i}_{t+1} = \mathsf{i}^*_{t+1}$ is that the follower country follows a policy rule of the form: $\mathsf{i}_{t+1} = \mathsf{i}^*_{t+1}$. This is a wrong misconception.

If that were the case, the ER would become indeterminate: (using the UIP, $\mathsf{i}_{t+1} - \mathsf{i}^*_{t+1} = \mathbb{E}_t \mathsf{e}_t$) $\mathbb{E}_t \epsilon_{t+1} = \epsilon_t$ (the ER would be a random walk, or any $\epsilon_t$ such that it equals the expected value of tomorrow's ER would work). Such kind of (indeterminate) rule can actually be very costly in terms of welfare.

Instead, imagine the foreign country (trying to implement a peg to the other country) follows a rule given by: $\mathsf{i}^*_{t+1} = \mathsf{i}_{t+1} + \tau\epsilon_t$. Such rule states that the bank commits to increasing the rate if the foreign currency depreciates.

Combining this proposed rule with the UIP we obtain $(1+\tau)\epsilon_t = \mathbb{E}_t \epsilon_{t+1}$. This equation has only one solution $\epsilon_t = 0$:

$$\epsilon_t = \frac{1}{1+\tau}\mathbb{E}_t \epsilon_{t+1} = \left(\frac{1}{1+\tau}\right)^2 \mathbb{E}_t \varepsilon_{t+2} = \cdots = \left(\frac{1}{1+\tau}\right)^T \mathbb{E}_t \varepsilon_{t+T} \xrightarrow[T\to\infty]{} 0$$

Then, we get a peg as a result of such rule ($\epsilon_t = 0$).

Thus: $\mathsf{i}^*_{t+1} = \mathsf{i}_{t+1}$ is a consequence, and not a policy rule. Here a credible threat to increase the interest rate if ER moves implies zero movements by the ER and yields an endogenous interest rate equalization.

How this is reflected in the literature:

- Sargent and Wallace (1975, JPE): Interest rate pegging results in indeterminacy (policy irrelevance result).

- Woodford (2003): Indeterminacy appears because the interest rate cannot pin down any endogenous variables. In such case the problem originates from using a rule that sets the policy as a function of exogenous shocks (or variables) only. Instead, the rate should be set in terms of endogenous variables too (e.g, the exchange rate).

We have that to pin down the dynamics of the exchange rate, a policy rule must set the interest rate differential as a function of the ER (or depreciation). A similar indeterminacy issue arises in closed economies.[23]

In BB2008, some possible rules considered are:

$$\mathsf{i}^*_{t+1} = \mathsf{i}_{t+1} - \tau\epsilon_t, \text{with } \tau > 0 \ \Rightarrow \epsilon_t = 0$$
$$\mathsf{i}^*_{t+1} = \mathsf{i}_{t+1} - \tau_e \mathsf{e}_t, \text{with } \tau_e > 1 \ \Rightarrow \mathsf{e}_t = 0$$

where these rules are either going to generate a peg, or a zero depreciation if the steady state that the variables are approximated around are constant (and with an initial ER of zero), which are the baseline assumptions —for each type of rule— in Benigno and Benigno). Additionally, these rules will induce an equality of rates in equilibrium.

Notice also the parameters values. They imply that responding to the ER is necessary but not sufficient. The reaction must be intense enough.

In Benigno, Benigno, Ghironi, 2007 explore the design of a rule that implements a determinate, fixed ER in a non-linear stochastic setting and show that it requires combining the rule with credible commitment.

### Determinacy of the rest of the economy

The rule $\mathsf{i}^*_{t+1} = \mathsf{i}_{t+1} - \tau\epsilon_t,\ \tau > 0$ is sufficient for yielding determinacy of the ER but not for that of the rest of the economy (other variables): It is the leader interest rate the variable that pins down the equilibrium for all variables in the model (in this paper the home country is the leader).

[23] To see this, think of an Euler equation, with flexible prices, and no real shocks. In that case $\mathsf{C}_t = \mathbb{E}_t \mathsf{C}_{t+1}$, and in by replacing in the Euler equation we have $\mathsf{i}_{t+1} - \mathbb{E}_t \hat{\pi}_{t+1}$. Supposing an interest rate peg rule $\mathsf{i}_{t+1} = 0$ (the case of Sargent and Wallace) yields $\mathbb{E}\hat{\pi} = 0$, or $\mathbb{E}_t \mathsf{P}_{t+1} = \mathsf{P}_t$. In this case, the price level and inflation are indeterminate, and instead, to have a policy capable to pin down inflation we need to impose a policy rule that is an actual function of inflation (e.g., $\mathsf{i}_{t+1} = \tau_\pi \hat{\pi}_t$ such that it implies $\hat{\pi}$=0).

Suppose the home economy follows the rule

$$\mathsf{i}_{t+1} = \alpha_1 \mathsf{y}_t^H + \alpha_2 \hat{\pi}_t^p, \quad \alpha_1 \geq 0,\ \alpha_2 \geq 0,$$

with $\mathsf{y}_t^H = \mathsf{Y}_t^H - \tilde{\mathsf{Y}}_t^H$.

Here, a multiplicity of parameters may ensure determinacy. In fact, the condition for determinacy is

$$(\alpha_2 - 1)k_C + \alpha_1(1-\beta) > 0$$

This is the same condition as for a closed economy (with the same type of rule). Intuition: Once the foreign country pins down a determined fixed ER, the home central bank sets the interest rate for the whole world economy, which is a type of closed economy.

This restriction reduces to the Taylor principle ($\alpha_2 > 1$) if the leader central bank reacts only to inflation ($\alpha_1 = 0$).

**Flexible exchange rate**

Benigno and Benigno assume that, under a flexible ER regime, home and foreign nominal interest rates are set following rules as

$$\begin{aligned}\mathsf{i}_{t+1} &= \alpha_1 \mathsf{y}_t^H + \alpha_2 \hat{\pi}_t^p + \alpha_3 \mathsf{i}_t,\ \alpha_i \leq 0,\\ \mathsf{i}_{t+1}^* &= \alpha_1^* \mathsf{y}_t^{*H} + \alpha_2^* \hat{\pi}_t^{*p} + \alpha_3 \mathsf{i}_t^*,\ \alpha_i \leq 0,\end{aligned}$$

for $i = \{1, 2, 3\}$. In the paper, a restriction of $\alpha_i = \alpha_i^*$ is imposed to simplify the analysis. An equal degree (parameters) of nominal rigidity across countries is also assumed.

Thus, the system becomes symmetric and for the economy to feature determinacy it becomes necessary that both cross-country differences and aggregates are determinate.

The conditions for determinacy are:

$$\begin{aligned}(\alpha_2 + \alpha_3 - 1)k_T + \alpha_1(1-\beta) > 0,\\ (\alpha_2 + \alpha_3 - 1)k_C + \alpha_1(1-\beta) > 0,\end{aligned}$$

where $k_C$ is the coefficient on $\mathsf{y}_t^W$ in the NKPC for home and foreign producer price inflation and $k_T$ the coefficient on the $\mathsf{TOT}$ term.

Both rules (home and foreign) need to be "aggressive" enough with respect to endogenous variables. In fact, if only one central bank follows an interest rule of this type, the system becomes indeterminate at both locations.

**TOT dynamics under fixed ER**: We have that the definition of TOT in equation (3) can be rearranged as,

$$\hat{\pi}_t^p - \hat{\pi}_t^{p*} = -\left(\mathsf{TOT}_t - \mathsf{TOT}_{t-1}\right) + \epsilon_t - \epsilon_{t-1}. \tag{10}$$

After substituting (10) and its time $t+1$ version into the resulting expression from subtracting the NKPCs —equation (9) minus (8) (and after assuming symmetric nominal rigidity parameters),

$$\mathsf{TOT}_t - \mathsf{TOT}_{t-1} = -k_T \underbrace{\left(\mathsf{TOT}_t - \tilde{\mathsf{TOT}}_t\right)}_{\text{Dev. wrt. flexible price TOT}} + \beta \underbrace{\mathbb{E}_t \left(\mathsf{TOT}_{t+1} - \mathsf{TOT}_t\right)}_{\text{Forward looking component}} \tag{11}$$

**Exchange rate dynamics**

First we depart from,

$$\mathsf{y}_t^H - \mathsf{y}_t^F = \mathsf{TOT}_t - \tilde{\mathsf{TOT}}_t$$

where we used that $\mathsf{Y}_t^H = (1-n)\mathsf{TOT}_t + \mathsf{C}_t + \mathsf{G}_{H,t}$, $\mathsf{Y}_t^F = -n\mathsf{TOT}_t + \mathsf{C}_t + \mathsf{G}_{F,t}^*$, and $\mathsf{y}_t^H - \mathsf{y}_t^F = \mathsf{Y}_t^H - \mathsf{Y}_t^F - (\tilde{\mathsf{Y}}_t^H - \tilde{\mathsf{Y}}_t^F)$.

Then, take the different of policy rules without inertia (e.g., $\mathsf{i}_{t+1} = \alpha_1 \mathsf{y}_t^H + \alpha_2 \hat{\pi}_t^p$):

$$\mathsf{i}_{t+1} - \mathsf{i}_{t+1}^* = \alpha_1 \left(\mathsf{TOT}_t - \mathsf{TOT}_t\right) + \alpha_2 \left(\hat{\pi}_t^p - \hat{\pi}_t^{*p}\right)$$

Finally, we can replace the UIP ($\mathsf{i}_{t+1} - \mathsf{i}_{t+1}^* = \mathbb{E}_t \mathsf{e}_{t+1}$):

$$\mathbb{E}_t \mathsf{e}_{t+1} = \alpha_1 \left(\mathsf{TOT}_t - \tilde{\mathsf{TOT}}_t\right) + \alpha_2 \left(\hat{\pi}_t^p - \hat{\pi}_t^{*p}\right).$$

From this expression we can see that it's possible to characterize a full determination of the ER without resorting to money demand.

**Conclusion**

This paper gives insights into the ER dynamics with sticky prices and endogenous interest rates.

Importantly, this paper makes some simplifying assumptions that, once relaxed could lead to other important explorations. For example, when NFA changes play a role in the transmissions of shocks, there may be important consequences for the ER dynamics (Cavallo and Ghironi, 2002, JME).

Now, in a similar vein, we turn to optimality frameworks for monetary policies in large open economy setups. We will see whether an open economy setup with potentially interdependent policy choices (in presence of meaningful cross border effects of policy) leads to different policy prescriptions relative to the standard price flexibility results of closed (and small open) economies.

## 14.2. GG2002 - The role of NFA in the ER dynamics under an endogenous interest rate setting

Cavallo and Ghironi (2002, JEDC) explore the role of NFA in a model in which their dynamics contribute to the ER under endogenous interest rate setting. In previous models (e.g., BB2008) the endogenous interest rates are explores but the NFA do not play any role in shock transmissions. Similarly, previous papers ignore the role of NFA due to the non-stationarity issues that could arise due to the indetermination of the NFA position in the steady state (see Ghironi, 2006, JIE).

The idea is to reconcile the stilized fact that the positive productivity shocks experienced by the U.S. in the 1990's led to run a negative current account and to increase borrowing from the rest of the world that led to a marked ER appreciation (lower CA, higher capital inflows, and higher demand of dollars to buy USD denominated assets).

Such role of assets is overseen in post-2000 models or when stationarity is induced by implementing knife-edge conditions such as in OR1995, CP2001 with specific elasticity of substitution values.

**Method**: OLG framework as Ghironi (2006) with price stickyness and endogenous monetary policy rule.

Breaking down Ricardian equivalence is sufficient to ensure the existence of a determinate steady state and stationarity of real variables.

The Benchmark model with PPP can be solved analytically. The solution for the ER exhibits a unit root —consistent with Meese and Rogoff (1983). However, the ER also depends on real net foreign assets (accumulated in previous period).

Price stickyness is induces by Monopolistic Competition.

The rule for monetary policy is similar as in BB2008: $\mathsf{i}_{t+1} = \alpha_1 \mathsf{y}_t + \alpha_2 \hat{\pi}_t^{CPI} + \xi_t$, with $\alpha_1 \geq -,\ \alpha_2 > 1$, where $\xi_t$ ($\xi_t^*$) is an exogenous home (foreign) interest rate shock. Notice here the rules are set with respect to the CPI inflation since this paper does not care about the role of the mark-up.

A conjecture solution for the ER with flexible is tested:

$$\epsilon_t = \eta_{\epsilon\epsilon} \epsilon_{t-1} + \eta_{\epsilon B} B_t + \eta_{\epsilon Z^D} Z_t^D + \eta_{\epsilon \xi^D} \xi_t^D,$$

where $B$ denotes the assets position, $Z$ the aggregate productivity, and the superindex $D$ denotes the differencial of the variable between home and foreign.

Findings: $\eta_{\epsilon\epsilon} = 1$ (unit root result), $\eta_{\epsilon B} > 1$ (intuition: Inflows of capital or lower current account spur an ER appreciation).

On the other hand, with sticky prices the solution includes a lagged output term,

$$\epsilon_t = \eta_{\epsilon\epsilon}\epsilon_{t-1} + \eta_{\epsilon B}B_t + \eta_{\epsilon y^D}\mathrm{y}_{t-1}^D + \eta_{\epsilon Z^D}Z_t^D + \eta_{\epsilon\xi^D}\xi_t^D$$

where due to $\eta_{\epsilon y^D}$, higher NFA induces people to work less (to decrease disutility of labor) which lowers output.

Additionally, overshooting is obtained; and finally, as BB2008 money is de-emphasized (the ER solution does not depend on money supply) too but a role is assigned to the NFA.

## 14.3. BB2003 - Price Stability in Open Economies and Cross-Border Policy Interdependence

Price stability is regarded as a desirable policy outcome. The general notion is that it facilitates an efficient allocation of resources across time and the case for its optimality is robust in closed economy models.

Closed economy outcome: Under commitment it is optimal to implement price stability as it mimics the flexible price equilibrium.

In a scenario where the only market friction comes from monopolistic power setting (with price stickyness), achieving the flex-price allocation is equivalent to reaching a first-best outcome. With this in mind, even under discretion —where a policymaker has the incentive to inflate the economy to undo a monopolistic distortive effect on output— and together with subsidies to remove the "inflation bias" or flexible-price markup, the optimal equilibrium consists on mimicking the flexible price allocation.

Open economy outcome: Obstfeld and Rogoff (2002) emphasize this result, explaining that the benefits of pursuing a nationally-oriented policy of price stability outweighs the gains from international monetary policy coordination. More concretely, they find that there are gains from policy coordination but the these are trivially small and thus it may not be worth to incur in substantial regulatory efforts too coordinate.

The OR2002 result hinges critically on a knife-edge condition for the elasticity of substitution across country goods: it is 1.

Benigno and Benigno (2003, ReStud) revisit this topic, allowing for an elasticity of substitution between home and foreign goods **different from 1** (unlike CP2001, OR2002, Devereux and Engel, 2003, and others). this difference makes gains non-trivial.

Other features of this setup: Complete markets, 1-period price stickyness, PCP.

**Result**: The conditions in which flexible price equilibrium is optimal are very restrictive. It requires either of:

- Perfectly correlated shocks across countries
- Same level of monopolistic distortion across countries
- Unitary elasticity of substitution (ES) $\omega = 1$

The conditions under it would be optimal for independent (non-centralized) planners to implement the flexible price allocation (in each location) are even more restrictive (no longer sufficient for distortions to be equal in both locations).

Reason: in the Nash equilibrium, policymakers face the externality that they can manipulate the TOT to their own country's advantage.

Hence, nationally-oriented policy makers have the incentive to manipulate the TOT in a welfare-improving manner (except under very specific preference specifications that remove that incentive, such as the unitary ES).

The new optimal policy can have both **inflationary and deflationary bias**. Only if these biases cancel out exactly, the pursuit of price stability becomes optimal.

**Biases:**

- To inflate: Monopolistic competition with endogenous labor supply induces an inefficiently low output level due to lower work effort by agents that try to mitigate their disutility of labor (e.g., CP2001).
- To deflate: Manipulation of TOT to increase output (by making use of expenditure switching patterns).

Even under discretion, and in contrast to the closed economy outcome, policymakers will not generally choose to implement flexible prices allocations. They will only do it in presence of the right subsidies and when the inflationary biases cancel out.

**Model**

- Two countries: home ($H$) and foreign ($F$), producing a continuum of goods indexed on the intervals $[0, n)$ and $[n, 1]$, respectively.

- In each country, there is a continuum of agents, with population size equal to the ranges of goods' varieties.

- Each agent is a monopolist producing a single differentiated good.

**Preferences and UMP**: each household $j$ maximizes

$$U_t^j = \mathbb{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left[ U(C_s^j) - V(y_s^j, Z_s) \right] \right\},$$

where $C^j$ is a consumption bundle, $U$ is increasing and concave, $V$ is increasing and convex, $y^j$ is the production of the household good variety, and $Z$ is a country specific, aggregate productivity shock.

The foreign households have identical preferences.

**Consumption**: standard CES aggregator. Aggregation takes place first between individual varieties, and then between country specific goods' baskets.

$$C_t^j = \left[ n^{\frac{1}{\omega}} \left( C_{Ht} \right)^{\frac{\omega-1}{\omega}} + (1-n)^{\frac{1}{\omega}} \left( C_{Ft} \right)^{\frac{\omega-1}{\omega}} \right],\ \omega > 0.$$

$$C_{Ht}^j = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\theta}} \int_0^n c_t^j(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}},\ \text{and}\ C_{Ft}^j = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\theta}} \int_n^1 c_t^j(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}},\ \theta > 1.$$

There are no impediments to trade and pricing is done in a PCP fashion. Thus, the LOP holds: $p_t(h) = \varepsilon_t p_t^*(h)$. Given identical preferences, the PPP also holds: $P_t = \varepsilon_t P_t^*$. A similar parity holds at the country basket level: $P_{Ht} = \varepsilon_t P_{Ht}^*,\ P_{Ft} = \varepsilon_t P_{Ft}^*$.

Optimal demands: Given the structure above the demands for $h$ and $f$ goods are,

$$y_t^d(h) = \left( \frac{p_t(h)}{P_{Ht}} \right)^{-\theta} \left( \frac{P_{Ht}}{P_t} \right)^{-\omega} C_t^W,\ \text{and}\ y_t^d(f) = \left( \frac{p_t(f)}{P_{Ft}} \right)^{-\theta} \left( \frac{P_{Ft}}{P_t} \right)^{-\omega} C_t^W, \tag{1}$$

where $C_t^W \equiv nC_t + (1-n)C_t^*$.

- Complete asset markets (in each location): Agents trade a set of contingent assets denominated in units of the world consumption basket.

Additionally, at time $-1$ agents in both countries commit to trade state contingent financial wealth that their lifetime budget constraints are the same at time 0. This assumption is relevant to ensure perfect risk sharing regardless of the 1-period stickyness.

- PPP holds, and together with complete markets this implies perfect consumption risk sharing: $C_t = C_t^* = C_t^W$

- Money is not included in the model (cashless setup as Woodford, 2003)

**Flexible price allocation**

A generic seller $h$ in the home economy chooses prices $p_t(h)$ to maximize,

$$d_t(h) = (1-\tau)\lambda_t p_t(h) y_t^d(h) - V(y_t^d(h), Z_t), \tag{2}$$

where $\tau$ is a proportional revenue tax (rebated via lump sum transfers), $\lambda_t$ is the marginal utility of nominal income at time $t$ ($\lambda_t \equiv U_C(C_t)/P_t$), and $V(y_t^d(h), Z_t)$ is the utility cost of production. We can notice this expression is given in utility units.

Given the demand equations (1), risk sharing, and symmetry of price setting across all producers in each location, optimal price setting at home and abroad implies:

$$(1-\Phi)U_C(C_t)\frac{P_{Ht}}{P_t} = V_y\left(\left(\frac{P_{Ht}}{P_t}\right)^{-\omega} C_t, Z_t\right), \tag{3}$$

$$(1-\Phi^*)U_C(C_t)\frac{P_{Ft}}{P_t} = V_y\left(\left(\frac{P_{Ft}}{P_t}\right)^{-\omega} C_t, Z_t^*\right), \tag{4}$$

where its also used that $P^*_{Ft}/P^*_t = P_{Ft}/P_t$ (in the second expression).

On the other hand, the price index definition implies,

$$1 = n(P_{Ht}/P_t)^{1-\omega} + (1-n)(P_{Ft}/P_t)^{1-\omega}.$$

This expression, together with the equations (3), and (4) are used to determine the level of consumption and relative prices in the flexible-price allocation. (Note this is a system of three equations and three unknowns: $C_t$, $P_{Ht}/P_t$, $P_{Ft}/P_t$.)

$\Phi$ and $\Phi^*$ in (3), and (4) capture the level of monopolistic distortion —after the extent of correction by distortionary taxes. Hence, they denote an after-tax mark-up:

$$1-\Phi \equiv \frac{\theta-1}{\theta}(1-\tau), \quad \text{and} \quad 1-\Phi^* \equiv \frac{\theta-1}{\theta}(1-\tau^*),$$

wjere $\theta/(\theta-1)$ is the flexible-price markup.

The real marginal costs can be defined in terms of the units of each country's good, or in units of consumption. As a consequence, the marginal cost will be constant and tied to the level implied by the distortion,

$$1 = \frac{1}{1-\Phi}mc_t, \quad \text{and} \quad 1 = \frac{1}{1-\Phi^*}mc_t^*$$

or alternatively, related to the relative prices through the mark-ups,

$$\frac{P_{Ht}}{P_t} = \frac{1}{1-\Phi}mc_t^C, \quad \text{and} \quad \frac{P_{Ft}}{P_t} = \frac{1}{1-\Phi^*}mc_t^{C*}.$$

In any case, when $\Phi = \Phi^* = 0$ the resulting allocation reproduces the competitive one.

**Welfare**

The monetary authorities maximize the households' expected utility,

$$\mathcal{W}_t \equiv \mathbb{E}_t\left\{\sum_{s=t}^{\infty}\beta^{s-t}\left[U(C_s) - \frac{\int_0^n V(y_s(h), Z_s)dh}{n}\right]\right\},$$

$$\mathcal{W}_t \equiv \mathbb{E}_t\left\{\sum_{s=t}^{\infty}\beta^{s-t}\left[U(C_s) - \frac{\int_n^1 V(y_s(f), Z_s^*)df}{1-n}\right]\right\},$$

where each period criteria are the instantaneous average utility among the households in each location.

**Preferences**

Preferences are assumed isoelastic,

$$U(C_t^j) \equiv \frac{(C_t^j)^{1-\rho}}{1-\rho},$$

$$V(y_t^j, Z_t) \equiv \frac{Z_t(y_t^j)^{\nu}}{\nu} \text{ if } j \in H \quad \text{and } V(y_t^j, Z_t^*) \equiv \frac{Z_t^*(y_t^j)^{\nu}}{\nu} \text{ if } j \in F,$$

where $\rho^{-1}$ is the elasticity of intertemporal substitution in consumption, with $\rho > 0$, and $\eta \equiv \nu - 1$ is the elasticity of labor supply, with $\nu \geq 1$.

Note: in this particular setup an increase in $Z_t$ is an unfavorable home productivity shock (and similar with $Z^*$ for the foreign country).

Corsetti and Pesenti (2005, JME) focus on the case $\rho = \nu = \omega = 1$, while Devereux and Engel (2003) and Obstfeld and Rogoff (2002, QJE) assume $\nu = \omega = 1$ (and also assume non-traded goods).

**Closed economy case**

To start, Benigno and Benigno analyze the case of a closed economy (i.e., with $n = 1$).

Prices are sticky and set one period in advance. The optimal price setting by the generic producer $j$ that sets $p_t^j$ implies:

$$\mathbb{E}_{t-1}\left\{\left[(1-\Phi)U_C(C_t)\frac{p_t^j}{P_t} - V_y(y_t^j, Z_t)\right] y_t^j\right\} = 0 \ \forall j, t, \tag{5}$$

where $y_t^j = \left(\frac{p_t^j}{P_t}\right)^{-\theta} C_t$.

Since (given symmetry) all producers set the same price, $p_t^j = P_t$, and $y_t^j = Y_t = C_t$ (given symmetry and the fact that the economy is closed), and we can write (5) as,

$$\mathbb{E}_t\left\{\left[(1-\Phi)U_c(Y_t) - V_y(Y_t, Z_t)\right] Y_t\right\} = 0 \ \forall t. \tag{6}$$

Equilibrium under policy commitment: Under ex ante commitment, the policymaker maximizes $\mathcal{W}_t$ with the information set of time $t-1$, subject to the sequence of constraints (6), for each period $t$ onwards.

To characterize the optimal policy is useful to introduce the definition of *notional price*.

Notional price: Price a supplier would choose if it were free to choose a price in $t$ independent of past and future prices. The notional price is denoted as $p_t^N$ and satisfies,

$$(1-\Phi)U_C(Y_t)\frac{p_t^N}{P_t} = V_y\left(\left(\frac{p_t^N}{P_t}\right)^{-\theta} Y_t, Z_t\right). \tag{7}$$

In this expression we can see that since $p_t^N$ is not necessarily equal to $P_t$, then the output level supplied by this producer is $(p_t^N/P_t)Y_t$.

With the assumed isoelastic utility of consumption and disutility of effort, (7) implies:

$$\frac{Y_t}{Y_t^n} = \left(\frac{p_t^N}{P_t}\right)^{\frac{1+\theta\eta}{\rho+\eta}}, \tag{8}$$

where $Y_t^N$ is the natural rate of output that would arise under flexible prices: $Y_t^n \equiv [(1-\Phi)Z_t]^{1/(\rho+\eta)}$.

Thus, in $t$, output can deviate from its natural rate if the notional price differs from the average price level in $t$.

We have that a policy specified in terms of notional prices can determine the average price level at each time.[24]

**Price stability**: Benigno and Benigno define Price Stability as a situation of zero notional inflation, itself defined as the equivalence between the notional price and the average actual price: $p_t^N = P_t$.

**Proposition 1:** (Closed economy result: Zero notional inflation —price stability— is optimal) Under commitment, the policymaker will want to apply a policy of zero notional inflation, that is, price stability. The allocation will coincide with the flexible-price allocation.

This means that monetary policy binds itself not to inflate to try undo the monopolistic distortion. The equilibrium achieved still will be constrained-efficient since output is inefficiently low due to the monopolistic distortion.

[24] Substituting the expression for $Y_t$ from (8) into (6) shows that prices $P_t$, present at $t-1$, depend only on the joint distribution of $\{p_t^N, Y_t^n\}$. Additionally, once $P_t$ is determined, the actual realization of $p_t^N$ determines the actual level of output $Y_t$; and the interest rate adjustment necessary to control the notional price can be retrieved form the consumers' Euler equation.

Now, since there is no monetary frictions to undo (e.g., cash-in-advance constraints), and given the type of price stickyness assumed, it follows that although price stability is the optimal policy allocation, it will not pin down the optimal inflation rate. In this particular environment there can be a positive inflation at no cost.[25]

Under discretion, the policymaker maximizes welfare at a generic period $t$, subject to the incentive compatibility constraints given by (6) from period $t+1$ onwards. The optimality condition at time $t$ is now:

$$U_C(Y_t) = V_y(Y_t, Z_t). \tag{9}$$

Once prices are fixed, a policymaker acting under discretion has an incentive to inflate the economy and push output toward the competitive level. This creates an inflationary bias (Kydland-Prescott/Barro-Gordon argument).

Intuition: In general the flexible price allocation is not optimal or efficient. The optimality of this allocation holds under discretion only when the monopolistic distortion is offset with mark-up removing taxation.

**Open Economies case**

With 1-period price stickyness, the optimal price choice for period $t$ maximizes the expected value of (2) based on information available at $t-1 : \mathbb{E}_{t-1}(d_t(h))$.

Optimal price setting at home and abroad implies,

$$\mathbb{E}_{t-1}\left\{\left[(1-\Phi)U_C(C_t)\frac{P_{Ht}}{P_t} - V_y\left(\left(\frac{P_{Ht}}{P_t}\right)^{-\omega} C_t, Z_t\right)\right]\left(\frac{P_{Ht}}{P_t}\right)^{-\omega} C_t\right\} = 0, \tag{10}$$

$$\mathbb{E}_{t-1}\left\{\left[(1-\Phi^*)U_C(C_t)\frac{P^*_{Ft}}{P^*_t} - V_y\left(\left(\frac{P^*_{Ft}}{P^*_t}\right)^{-\omega} C_t, Z^*_t\right)\right]\left(\frac{P^*_{Ft}}{P^*_t}\right)^{-\omega} C_t\right\} = 0, \tag{11}$$

where $P^*_{Ft}/P^*_t = P_{Ft}/P_t$.

BB2003 begin by examining the conditions under which the flexible-price allocation is the constrained-efficient policy. The case of focus is that of centralized planner that maximizes global expected welfare:

$$\mathbb{E}_{t-1}[n\mathcal{W}_t + (1-n)\mathcal{W}^*_t] \tag{12}$$

Under ex ante commitment, the constrained-efficient allocation is obtained by maximizing (12) subject to (10), and (11), and the price index constraint $1 = n(P_{Ht}/P_t)^{1-\omega} + (1+n)(P^*_{Ft}/P^*_t)^{1-\omega}$. From this, the result below follows.

**Proposition 2:** (Conditions for price stability optimality result in open economy cooperative equilibrium) If shocks are symmetric $Z_t = Z^*_t$ (perfectly correlated) the flexible price allocation is constrained efficient. If shocks are asymmetric, price stability is constrained efficient if $\phi = \phi^*$ (same distortions). Otherwise, it should hold $\omega = 1$, or $\omega = \rho^{-1}$.

Previous literature had focused on the case of $\omega = 1$ where the flexible-price allocation is optimal regardless of the level of symmetry across shocks or the extent of monopolistic distortions. By relaxing this assumption, Benigno and Benigno show that the flexible-price allocation is constrained efficient when the monopolistic distortions equal across countries. Now, on the other specific case regarding $\omega$ ($\omega = \rho^{-1}$, i.e., EIS equals the ES between country goods) we have that it is optimal to have price stability around the flexible-price allocation and the ER will move to accommodate asymmetric shocks.

To understand the case of $\omega = 1$ notice that, by making use of the isoelastic-function assumptions, $Y_{Ht} = (P_{Ht}/P_t)^{-\omega}$, and $Y_{Ft} = (P^*_{Ft}/P^*_t)^{-\omega}$, the equations (10) and (11) can be rewritten as,

$$\mathbb{E}_{t-1}[V_y(Y_{H,t}, Z_t)] = \frac{1-\Phi}{v}\mathbb{E}_{t-1}\left[U_C(C_t)C_t\left(\frac{P_{Ht}}{P_t}\right)^{1-\omega}\right], \tag{13}$$

[25] The monopolistic distortion induced an inefficient level of output (below optimal). Thus, the policy equilibrium that commits to not inflate —and generate a higher output— is still constrained-efficient, even if optimal from a policymaker perspective.

$$\mathbb{E}_{t-1}[V_y(Y_{F,t}, Z_t^*)] = \frac{1-\Phi^*}{v}\mathbb{E}_{t-1}\left[U_C(C_t)C_t\left(\frac{P_{Ft}^*}{P_t^*}\right)^{1-\omega}\right]. \tag{14}$$

With $\Phi \neq \Phi^*$ and in absence of shocks, the steady-state levels of output are different across locations. Then, there is a distortion to cope with —as well as there are national incentives to manipulate the TOT. The cooperative planner will make use (more efficiently than individual planners) of the TOT fluctuations to correct for the different expected level of disutility from producing goods across countries (keep in mind these should equalize, in a pareto optimal context, as the consumption baskets are symmetric).

In these expressions we can see that if $\omega = 1$ the policymaker becomes unable to achieve a better allocation than the flexible-prices one (as $(P_{Ht}/P_t)^{1-\omega} = 1$). In such case there is no role for the TOT in correcting the structural distortion.

If $\omega = \rho^{-1}$ the conditions become,

$$E_{t-1}[V_y(Y_{Ht}, Z_t)] = \frac{1-\Phi}{\nu}\mathbb{E}_{t-1}\left[(Y_{Ht})^{\frac{\omega-1}{\omega}}\right],$$
$$E_{t-1}[V_y(Y_{Ft}, Z_t^*)] = \frac{1-\Phi^*}{\nu}\mathbb{E}_{t-1}\left[(Y_{Ft})^{\frac{\omega-1}{\omega}}\right],$$

and then, the domestic output (in each location) is the only relevant variable for the analysis of the stabilization problem in each economy. The utility with respect to consumption becomes separable in this case and the social welfare function also changes, the real marginal costs are also reduced to a proportion of the output gaps, and thus the planner no longer sets prices (these become irrelevant) and instead will pick each output directly.

If these conditions are not met, the efficient equilibrium requires variable mark-ups at the country level (something we have under sticky prices). Then, the optimal allocation under sticky prices can improve upon the flexible-price allocation, which remains feasible but is no longer optimal.

In general: Policy of state-contingent notional price is optimal.

Crucially, notice how in open economies departures from optimality of price stability arise without assuming features such as: Transaction frictions, government spending shocks, more general preferences.

### Non-cooperative case (Nash equilibrium)

Now Benigno and Benigno want to understand the conditions under which price stability is optimal in a decentralized equilibrium (Nash).

Strategy space (decision variable): relative notional price in each location $p_{Ht}^N/P_{Ht}$, $p_{Ft}^N/P_{Ft}$

The home policymaker sets $p_{Ht}^N/P_{Ht}$, and the foreign one the other relative price. Once the prices are determined we can use the Euler equations to back out the corresponding interest rate decisions.

Each planner will set the tool optimally to maximize national welfare, subject to the incentive compatibility constraint (equation (10) or (11) depending on the location), and the price index constraint, while taking the policy response of the other policymaker as given.[26]

**Proposition 3:** (Conditions price stability optimality in open economy Nash equilibrium) If shocks are symmetric, i.e. $Z_t = Z_t^*$ the flexible price allocation is a Nash equilibrium under ex ante commitment. Otherwise, price stability is a Nash equilibrium in both countries under commitment if either $\omega = 1$, or $\omega = \rho^{-1}$ for any $\Phi$, $\Phi^*$.

Here, the conditions are even more restrictive. $\Phi = \Phi^*$ is no longer sufficient for optimality of price stability. The intuition for this is that now the policy makers exploit the TOT volatility to decrease their domestic utility of producing goods, without internalizing the negative externality of their policy decision on the other country.

Importantly, even if both planners make use of the TOT volatility. Their incentives to move the TOT differ since the cross-border negative externality of each policy tool is not trivial (and only the cooperative planner internalizes it). Thus, the optimal allocations don't coincide.

[26] For testing whether price stability is optimal, Benigno and Benigno assume the international policymaker's policy is given at the level yielding price-stability in their country, and check how the resulting optimal policy compares with the the price-stability associated one. On the other hand, for checking the optimal Nash allocation in general, we can instead consider the Ramsey planner problem and maximize each national welfare subject to both price setters' FOCs. The difference with respect to the cooperative planner in the latter case, will be the objective function.

The other conditions are analogous to the cooperative case because in such cases either the TOT don't play any role, or the economies are insular with respect to the TOT movements, and then, the domestic policymakers do not care about the policy of their international counterpart.

Policies under discretion: If policymakers are allowed to re-optimize in each period, taking as given the constraint implies bu the optimal price setting, we have, as in the closed economy model, that the optimal allocation (in this case Nash) is not the flexible-prices one.

Unlike in the closed economy case, the discretionary open economy policy enforcing the flexible-price allocation involves a positive degree of monopolistic distortion with knife-edge conditions for the parameters:

**Proposition 4:** (Additional condition price stability optimality in open economy Nash equilibrium) Within the class of preferences assumed, when $\omega = 1$, the strategy of price stability is a time-consistent Nash equilibrium if and only if $\Phi = \bar{\Phi}$ and $\Phi^* = \bar{\Phi}^*$ with:

$$\bar{\Phi} = \frac{(1-n)n^{-1}(\rho+\eta)/(1+\eta)}{1+(1-n)n^{-1}(\rho+\eta)/(1+\eta)}, \quad \text{and} \quad \bar{\Phi}^* = \frac{(1-n)^{-1}n(\rho+\eta)/(1+\eta)}{1+(1-n)n^{-1}(\rho+\eta)/(1+\eta)}.$$

In the particular case $\omega = \rho^{-1}$ the price-stability allocation is a time-consistent Nash equilibrium if and only if if $\Phi = \tilde{\Phi}$ and $\Phi^* = \tilde{\Phi}^*$ with:

$$\tilde{\Phi} = 1-n, \quad \text{and} \quad \tilde{\Phi}^* = n.$$

Intuition: As in the closed economy case, the distortion induced with monopolistic competition with endogenous output induces an inflationary bias in policy. At the same time, in open economies, each policymaker also faces a deflationary bias, given the incentive to manipulate the TOT (Corsetti and Pesenti, 2001; Tille, 2001).

There is a point, with positive monopolistic distortions, at which the inflationary and deflationary incentives balance exactly. At this point, policymakers acting independently find optimal to implement the optimal price allocation.

Finally, if $\omega = \rho^{-1}$, the price stability allocation is the dominant strategy. In that case, if $\Phi \neq \tilde{\Phi}$, the home policymaker has an incentive to inflate or deflate, depending on $\Phi$ being above or below the cutoff, respectively.

### Other findings in the literature

Caveat: BB2003 findings are specific to their setup assumptions, including the type of policy tool considered.

Corsetti and Pesenti, 2005: There are gains from cooperation if PT is imperfect (e.g., some degree of LCP)

Obstfeld and Rogoff, 2002: Even with imperfect PT there are no gains from cooperation.

Devereux and Engel, 2003: No gains of cooperation with full or zero PT (LCP case).

Sutherland, 2004: Significant gains if $\omega \neq 1$ that change depending on the structure of financial markets.

### Conclusions

Benigno and Benigno obtain relevant results on the conditions under which price stability (that mimic a flexible-price equilibrium) arise as an optimal equilibrium in open economies.

The flexible-price allocation is not efficient unless very specific conditions are met (even in cooperative settings).

In general, for cooperative policymakers, monopolistic distortions need to be equalized across countries. Otherwise, knife-edge conditions for the elasticity of substitution between home and foreign goods are required.

In the non-cooperative case the equalization of distortions is no longer sufficient to implement price stability as the decentralized equilibrium.

Under discretion, there is even less scope for an equilibrium to exist.

Importantly, the non-cooperative and cooperative equilibria don't generally coincide, and gains from international policy cooperation may be possible, even if asset markets are complete, goods markets are fully integrated, and PCP holds.