Summary Ch 4 Open Economy Real Business Cycles Model

Before: w/ physical capital and persistent shocks the open economy model can explain the Countercyclical trade balance

Now: Can it also explain the other business cycles properties?

Examples: $\sigma_{yc} > 0$, Serial Corr., $\sigma_i > \sigma_y$

to give a better chance to the model add: Elastic Labor Supply, Uncertainty in technology shock, Capital depreciation $\Rightarrow$ the model becomes the SOE-RBC

Model

$$\text{Max} \quad E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) \tag{4.1}$$

subject to

$$c_t + i_t + \Phi(k_{t+1} - k_t) + (1 + r_{t-1})d_{t-1} = y_t + d_t \tag{4.2}$$

$$y_t = A_t F(k_t, h_t) \tag{4.3}$$

$$k_{t+1} = (1 - \delta)k_t + i_t \tag{4.4}$$

$$\lim_{j \to \infty} E_t \frac{d_{t+j}}{\Pi_{s=0}^{j}(1 + r_s)} \leq 0 \tag{4.5}$$

Adjustment costs; 1st rate no longer constant

Stationarity: $C_t \sim RW$

In the previous model we have that Consumption is a Random Walk (it is non-stationary). This is troublesome as the Steady State becomes history dependent. That is, the model is valid but after a shock there is no guarantee of Convergence to the SS it was approximated around.

Furthermore, the approximated solution is valid around one Steady State.

Then we must adjust the model to induce stationarity. Strategy used here: time varying risk premium in int. rate of debt.

drop i_t by substituting, then take FOCs

Household's Optimality Conditions

$[\lambda_t]$: $$c_t + k_{t+1} - (1 - \delta)k_t + \Phi(k_{t+1} - k_t) + (1 + r_{t-1})d_{t-1} = A_t F(k_t, h_t) + d_t \tag{4.6}$$

$[d_t]$: $$\lambda_t = \beta(1 + r_t)E_t\lambda_{t+1} \tag{4.7}$$

$[c_t]$: $$U_c(c_t, h_t) = \lambda_t \tag{4.8}$$

$[h_t]$: $$-U_h(c_t, h_t) = \lambda_t A_t F_h(k_t, h_t) \tag{4.9}$$

$[k_{t+1}]$: $$1 + \Phi'(k_{t+1} - k_t) = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[A_{t+1}F_k(k_{t+1}, h_{t+1}) + 1 - \delta + \Phi'(k_{t+2} - k_{t+1})\right] \tag{4.10}$$

add $r_t = r^* + p(\tilde{d}_t)$ w/ $\tilde{d}_t = d_t$ to induce stationarity and a unique solution, then drop λ_t (by substituting): $\tilde{d}_t$ is taken as given when taking FOCs then replaced by d_t

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = A_t F_h(k_t, h_t) \tag{4.11}$$

$$c_t + k_{t+1} - (1 - \delta)k_t + \Phi(k_{t+1} - k_t) + [1 + r^* + p(d_{t-1})]d_{t-1} = A_t F(k_t, h_t) + d_t \tag{4.16}$$

$$U_c(c_t, h_t) = \beta(1 + r^* + p(d_t))E_t U_c(c_{t+1}, h_{t+1}) \tag{4.17}$$

$$1 = \beta E_t \left\{ \frac{U_c(c_{t+1}, h_{t+1})}{U_c(c_t, h_t)} \frac{[A_{t+1}F_k(k_{t+1}, h_{t+1}) + 1 - \delta + \Phi'(k_{t+2} - k_{t+1})]}{1 + \Phi'(k_{t+1} - k_t)} \right\} \tag{4.18}$$

This is a second order difference system (since it has k_{t+2}, k_{t+1}, k_t)

To make it a First order difference system add one equation: $k^f_t = k_{t+1}$ ($\Rightarrow k_{t+2} = k^f_{t+1}$) (This is what Dynare always does)

Now we have a 1st Order difference system of eqs. in $k^f_t, k_t, c_t, d_{t-1}, y_t$

Solution: $y_t = g(x_t, \sigma)$, $x_{t+1} = h(x_t, \sigma) + \sigma\epsilon_{t+1}$

y_t: Non predetermined variables

x_t: States: Predetermined & Exogenous variables

(w/ $\sigma = 0$: deterministic)

Solution will be obtained w/ a 1st order Taylor Exp. approximation around SS of $y_t, x_t, \sigma = 1$: Solution (Approx). $\hat{y}_t = g_x \hat{x}_t$, $\hat{x}_{t+1} = h_x \hat{x}_t + \eta\epsilon_{t+1}$

To continue we specify functional forms:

CRRA GHH: $U(c,h) = \dfrac{G(c,h)^{1-\sigma} - 1}{1-\sigma}$, $G(c,h) = c - \dfrac{h^\omega}{\omega}$, $\omega > 1, \sigma > 0$

Debt Elast. Int. Rate: $p(d) = \psi(e^{d-\bar{d}} - 1)$, $\psi > 0$

P/n fn: $F(k,h) = k^\alpha h^{1-\alpha}$, $\alpha > 0$

Adj. Cost: $\Phi(x) = \frac{\phi}{2}x^2$, $\phi > 0$ Params: $\omega, \sigma, \alpha, \phi, \bar{d}, \psi$

(Note: w/ GHH the wage won't depend on $C_t \Rightarrow$ No wealth effect in labor decisions)

Calibration

we need to calibrate $[\omega \ \alpha \ \delta \ r^* \ \beta \ \sigma \ \phi \ \rho \ \tilde{\eta} \ \bar{d} \ \psi]$

We use 3 types of restrictions (strategies)

A. Give values to parameters from external sources (Not the particular data that is being explained) (from Lit: $\sigma = 2, \delta = 0.1, r^* = 0.04, \beta = 1/(1+r^*)$)

B. Target First Moments. Here: labor share $= 0.68$, $\frac{tb}{y} = 0.02$

C. Give values to target Second Moments. Here $\sigma_y, \sigma_h, \sigma_i, \sigma_{tb/y}, \rho_{\ln y_t, \ln y_{t-1}}$

Some values in B imply a parameter value in a straightforward way (e.g. $\alpha = 0.32$ since labor share $= 0.68$)

For the rest:

Step 1: Let $\Theta = [\omega \ \bar{d} \ \phi \ \psi \ \rho \ \tilde{\eta}]$ be the remaining parameters to define

Guess values for all but 1 parameters in Θ (here $\bar{d}$)

Steps 2-4: Solve analitically for the remaining parameter and Steady State

2: Given guess for ω find h (ss of labor)

First get k/h from Euler Eq: $1 = \beta[\alpha(k/h)^{\alpha-1} + 1 - \delta]$

plug it into labor market eq: $h^{\omega-1} = (1-\alpha)(k/h)^\alpha$, solve for h

w/ h, solve for $k = (k/h) \cdot h$, $y = A(k/h)^\alpha h$

3: let S_{tb} be the average trade balance-to-output ratio (that we found in B as 0.02)

in SS: $S_{tb} = \dfrac{r^*\bar{d}}{y}$ (uses the fact that $d = \bar{d}$, or $p(d) = 0$), get $\bar{d} = S_{tb} y / r^*$

4: Find c from resource constraint $c + \delta k + r^* + \psi(e^{d-\bar{d}} - 1)d = y$

Step 5: w/ SS (c, k, h, d) and parameters at hand compute the second moments predicted by the model

(theoretical moments, e.g., from var matrix) $X(\Theta) = [\sigma_y \ \sigma_h \ \sigma_i \ \sigma_{tb/y} \ \text{Corr}(\ln y_t, \ln y_{t-1})]$

Step 6: Compute distance between model's implied moments and targeted moments $D(\Theta) = [X(\Theta) - X^*]$

Step 7: Repeat by adjusting Θ (guess) until D is small: $D < D^*$ (threshold)

After calibration we compare the model and data

	Canadian Data 1946 to 1985 σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,y_t}	Canadian Data 1960 to 2011 σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,y_t}	Model σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,y_t}
y	2.8	0.6	1	3.7	0.9	1	$\boxed{3.1}$	$\boxed{0.6}$	1
c	2.5	0.7	0.6	2.2	0.7	0.6	2.7	0.8	0.8
i	9.8	0.3	0.6	10.3	0.7	0.8	$\boxed{9.0}$	0.1	0.7
h	2.0	0.5	0.8	3.6	0.7	0.8	$\boxed{2.1}$	0.6	1
$\frac{tb}{y}$	1.9	0.7	-0.1	1.7	0.8	0.1	$\boxed{1.8}$	0.5	-0.04
$\frac{ca}{y}$							1.4	0.3	0.05

$\square \rightarrow$ Calibration targets $\therefore$ it's normal fit is good, can't be used to test the model

test 1: Model places σ_c well $\sigma_{tb/y} < \sigma_h < \sigma_c < \sigma_y < \sigma_i$

test 2: tb/y is countercyclical

Problem: Model overestimates Corr of h, c w/ y (ρ_{hy}, ρ_{cy})

- $\rho_{h_t, y_t} = 1$ Due to GHH preferences and log-linearized intra-temporal Euler Equation: $\hat{w h_t} = \hat{y}_t$

Other Stationarity inducing methods

Complete Markets in Assets

we add State-Consumption Smoothing (before only time)

Main change: Budget Constrait ($\Rightarrow$ Euler Eqs)

$$\mathbb{E}_t \, q_{t,t+1} b_{t+1} = b_t + A_t F(k_t, h_t) - c_t - i_t - \Phi(k_{t+1} - k_t)$$

w/ complete markets the future assets keep the expectation operator it denotes the aggregation of state contingent assets, times the probability of the states

notice the timing

	In LHS	in RHS
	$(1 + r_{t-1})d_{t-1}$	d_t
Incomplete Mkts	what you get in t (Proceeds from your investment in $t-1$)	what you buy/invest in t so that you get $(1+r_t)b_t$ in $t+1$
Complete Mkts	$\mathbb{E}_t \, q_{t+1} b_{t+1}$, SDF $= q_{t,t+1}$	b_t
	what you buy in t for future periods (many state contingent assets s.t. in expectation you get b_{t+1})	what you get in t

Use open economy assumption: Frictionless int. markets $\Rightarrow$ UIP holds $r_{t+1} = r^*_{t+1}$

then for foreign agents $[b_{t+1}]$: $\lambda^*_t r_{t+1} = \beta\lambda_{t+1}$

for domestic agents $[b_{t+1}]$: $\lambda_t r_{t+1} = \beta\lambda_{t+1}$

$\Big\} \ \lambda_t = \underbrace{\xi \lambda^*_t}$ Constant for SOE

$\therefore$ $u'(c_t)$ or c_t is no longer a R.W.; Now it's Constant! ($\Rightarrow$ Stationary)

(in non SOE environments similar conclusions hold but in terms of Cons. differential)

to recap: Incomplete markets: $U_c(c_t, h_t) = E_t U_c(c_{t+1}, h_{t+1})$ (w/ $\beta(1+r) = 1$)

Complete markets: $U_c(c_t, h_t) = \text{Constant}$

Current account and Trade balance:

$CA_t = Tb_t + \underbrace{b_t - E_{t-1} q_{t-1} b_t}_{\text{Net invest. income}}$

$Tb_t = y_t - c_t - i_t - \Phi(k_{t+1} - k_t)$ (or $S_t = b_t + Tb_t$, w/ $S_t = E_t q_{t+1} b_{t+1}$)

Internal Debt Elastic Interest Rate

$\hookrightarrow$ the debt in the premium is a decision variable

$r_t = r + p(d_t)$ (before: $\tilde{d}_t$ (average - exog))

E Eq. changes: $\lambda_t = \beta[1 + r + p(d_t) + \underline{p'(d_t)d_t}]\mathbb{E}_t\lambda_{t+1}$

Calibration is the same as EDEIR but SS value of d_t changes

Other stationarity Inducing Devices (Cont.)

Portfolio Adjustment Cost Model (PAC)

Include adj. costs of debt in BC:

$$d_t = (1+r_{t-1}) d_{t-1} - y_t + c_t + i_t + \phi(k_{t+1} - k_t) + \underline{\frac{\psi_3}{2}(d_t - \bar{d})^2}$$

Effect: EEq. depends on assets

$$\lambda_t [1 - \psi_3 (d_t - \bar{d})] = \beta(1+r_t) \mathbb{E}_t \lambda_{t+1}$$

External Discount Factor

UMP: $\max \mathbb{E}_0 \sum_{t=0}^{\infty} \Theta_t U(c_t, h_t)$

$$\Theta_{t+1} = \beta(\underbrace{\tilde{c}_t, \tilde{h}_t}_{\text{taken as given}})\Theta_t, \quad \Theta_0 = 1$$

(aggregate)

$$\lambda_t = \beta(\tilde{c}_t, \tilde{h}_t)(1+r_{t-1}) \mathbb{E}_t \lambda_{t+1}$$

in equilibrium: $c_t = \tilde{c}_t$, $h_t = \tilde{h}_t$

Internal Discount Factor

$$\Theta_{t+1} = \beta(c_t, h_t)\Theta_t, \quad \Theta_0 = 1$$

Θ_{t+1} is now a choice variable