## CHAPTER 3 TRIPRAŚNĀDHIKĀRAḤ

### (Three Questions of Direction, Place and Time)

In this chapter we discuss finding the *lagna* (ascendant), *krānti* (declination), *akṣa* (latitude of a place), *nata* (zenith distance) and *unnata* (altitude).

Ślokas 1, 2, 3 and 4: The rising (durations) at Lankā are 278,299 (and) 323 vinādīs. (are arranged) in the given and the reverse orders (alternately); the carakhaṇḍas of themselves (rāśis) are subtracted from or added to (the respective udayamānas). (These are) the rising (durations), at one's place, of the six (rāśis) from Meṣa (in the given order) and of (the six rāśis from) Tulā. The balance (degrees of the rāśis) of the sāyana Ravi, at the (given) instant, multiplied, by the rising (duration of that rāśis) and divided by 30 is the balance time of Ravi (in palas, vinādīs); (this) must be subtracted from the given time in palas. The rising (durations) of the further (succeeding) rāśis are also (subtracted); the remainder (time in palas) divided by the (rising duration of) the unsubtractable rāśi (and) multiplied by 30, in degrees etc., together with the rāśis (starting with Meṣa) prior to the unsubtractable (aśuddha rāśi), reduced by ayanāṃśa (amount of precession of the equinox) is the (nirayaṇa i.e. sidereal) ascendant (i.e. lagna).

The (given) time (if) less than the balance (time of sāyana Ravi then the same i.e. the given time in palas) multiplied by 30 and divided by the rising (duration in palas) together with (sāyana) Ravi, in degrees, is the (sāyana) ascendant (lagna).

Finding udayamāna (duration of rising) of the 12 rāśis at a given place and finding the lagna at a given time is explained in these four ślokas as follows.

(i) The durations of rising (udayamāna) of the first three sāyana rāśis viz. Meṣa, Vṛṣabha, and Mithuna at Lankā are respectively 278, 299 and 323 (in palas). Writing these numbers in the reverse order as well as in the given order successively we get the udayamānas of the remaining rāśīs. The udayamānas of the 12 rāśis at Lankā are given in Table 3.1.

Rāśi	Udayamāna palas (vighatīs)	Rāśi	Udayamāna palas (vighaṭīs)
Meṣa	278	Tulā	278
Vṛṣabha	299	Vṛṣcika	299
Mithuna	323	Dhanus	323
Karkaṭaka	323	Makara	323
Simha	299	Kumbha	299
Kanyā	278	Mīna	278

Table 3.1 Udayamānas of rāśis at Lankā

(ii) For the given place, find cara khaṇḍas (as explained under Ślokas 19 and 20 in Chapter 2). These cara khaṇḍas are added to or subtracted from the udayamānas of the 12 rāśis at Lankā to get their udayamānas at the given place. Add these khaṇḍas if the udayamānas are in the reverse order and subtract if they are in the given order.

#### (iii) Finding the lagna at a given time:

- (a) Find sāyana Ravi at the given time. Express it in rāśis, degrees etc. The number in the rāśi position gives the number of elapsed rāśis, and degrees etc. give the elapsed part of the running rāśi called bhuktāṃśas.
- (b) Subtract degrees etc. from 30°. This difference gives the gamya (balance) of the currently running rāśi called bhogyāmśas.
- (c) Multiply *bhogyāṃśas* by the *udayamāna* of the currently running *rāśi* and divide it by 30. The result gives *Ravi bhogyakāla* in *vighatīs* (or *palas*).

- (d) Express the given time in vighațīs (or palas) and subtract Ravi bhogyakāla.
- (e) Consider the difference obtained in the above step. Go on subtracting the *udayamānas* of next *rāśis* until it is possible to do so. These *rāśis* are called *śuddha* (subtractable) *rāśis*. The *rāśi* which remains as unsubtractable is called the *aśuddha rāśi*.
- (f) Now consider the remaining portion in the step (e) [i.e., the balance from which udayamāna of no more rāśi can be subtracted]. Multiply this by 30 and divide by the udayamāna of the aśuddha rāśi. The result will be in degrees etc.
- (g) To this result add the total number of rāśis prior to the aśuddha rāśi. This result which will be in rāśis, degrees etc. is the sāyana lagna at a given time. Subtracting the ayanāmśa from it we get the nirayana lagna.

**Example**: We have the given time  $11^{gh}$ . after sunrise on a certain day when

True (nirayaṇa) Sun at the sunrise =  $1^R$  3° 6′ 12″

True daily motion of the Sun = 57' 28"

Motion of the Sun in 
$$11^{gh} = \frac{57' 28'' \times 11^{gh}}{60^{gh}} = 10' 32''$$

: (Nirayaṇa) Sun at the given time =  $1^R 3^{\circ} 6' 12'' + 10' 32'' = 1^R 3^{\circ} 16' 44''$ 

$$\therefore$$
 Sāyana Ravi =  $1^R$  3° 16′ 44″ + 18° 16′ 10″ =  $1^R$  21° 32′ 54″

In the  $r\bar{a}si$  position we have 1. This means that the first  $r\bar{a}si$  is over and the  $s\bar{a}yana$  Ravi is in the second  $r\bar{a}si$  viz.  $V_rsabha$ .

The bhuktāmśa =  $21^{\circ} 32' 54''$ 

The bhogyā $m \acute{s} a = 30^{\circ} - 21^{\circ} 32' 54'' = 8^{\circ} 27' 06''$ 

Now, we have

$$Ravi\ bhogyakāla = \frac{bhogyāmśa \times udayamāna\ Vṛṣabha}{30} = \frac{8°\ 27'\ 06'' \times 255}{30}$$

≈ 71 palas

The given time =  $11^{gh}$  = 660 palas

Now, we have 660 - 71 = 589 palas

We have Mithuna rāśi next to Vṛṣabha. The udayamāna of Mithuna = 305 palas for the given place. Subtracting 305 from 589 we get 589 - 305 = 284 palas.

The duration of rising of the next rāśi namely Karkaṭaka is 341 palas which cannot be subtracted from 284 palas. Therefore, Karkaṭaka is the aśuddha rāśi.

Now, consider 
$$\frac{284 \times 30}{341} = 24^{\circ} 59' 07''$$

The total number of  $r\bar{a}sis$  prior to Karkataka is 3. Therefore adding 3  $r\bar{a}sis$  to the above result, we get the  $s\bar{a}yana\ lagna=3^R\ 24^\circ\ 59'\ 07''$ .

Now, nirayaṇa lagna = Sāyana lagna - ayanāṃśa

$$= 3^{R} 24^{\circ} 59' 07'' - 18^{\circ} 16' 10'' = 3^{R} 6^{\circ} 42' 57''$$

i.e., 6° 42′ 57″ of (nirayana) Karkataka.

Note: If the given time in palas is less than the Ravibhogyakāla, then proceed as follows to find the lagna.

Multiply the given time in palas by 30 and divide by the udayamāna of the rāśi in which the sāyana Ravi lies. This result will be in degrees etc. This is added to the sāyana Sun to get the sāyana lagna. Subtracting ayanāmśa we get the nirayana lagna at the given time.

**Example**: Suppose the given time is  $1^{gh} = 60$  palas (after sunrise).

$$\acute{S}$$
āyana Ravi =  $1^R 21^{\circ} 23' 22''$ 

We have Ravi bhogyakāla = 73 palas.

Since the given time is less than the Ravi bhogyakāla, we proceed as given below:

Śāyana Ravi is in the Vṛṣabha rāśi whose udayamāna is 255.

$$\therefore \frac{\text{given time in } palas \times 30^{\circ}}{udayam\bar{a}na} = \frac{60 \times 30^{\circ}}{255} = 7^{\circ} 3' 31''$$

$$= 1^{R} 21^{\circ} 23' 22'' + 7^{\circ} 3' 31'' = 1^{R} 28^{\circ} 26' 53''$$

Nirayana lagna = Sāyana lagna - ayanāmśa

$$= 1^{R} 28^{\circ} 26' 53'' - 18^{\circ} 16' 10'' = 1^{R} 10^{\circ} 10' 43''$$

i.e., 10°10′43″ in *Vṛṣabha*.

Ślokas 5 and 6: (The sāyana) Ravi's balance (bhogya) with elapsed part (bhukta) of lagna added to the rising times (udayamānas) of the intervening (rāśis) gives the time from the lagna (ascendant). If lagna (ascendant) and Ravi are in the same sign (rāśi) then their difference in degrees multiplied by the rising time (of the rāśi) and divided by 30 (gives the time from sunrise).

If (the ascendant and the sun being in the same sign) the *lagna* is less (than the *sāyana* Sun, the time in *ghaṭīs*) is subtracted from the (total duration of) day and night; this gives the *ghaṭikās* from (the previous) sunrise or from (the duration of) the night (giving the time from the previous sunset). The time during night can (also) be obtained by adding 6 signs (*raśis*) to the (*sāyana*) Ravi.

Obtaining the time from the given lagna is explained.

- (i) Consider the given sāyana lagna (in rāśi, degree etc.). Multiply the degrees etc. by the udayamāna of the rāśi of the sāyana lagna and divide by 30. The result gives the bhukta (elapsed) part of the sāyana lagna in vighatīs (palas).
- (ii) Similarly, the *bhogya* (balance) part of the *rāśi* occupied by the *sāyana* Sun is determined:

Subtract the degrees etc. of the sāyana Sun from 30°. Multiplying this bhogya part of the Sun by the udayamāna of the Sun's rāśi and divide by 30°. This is in vighatīs (palas).

(iii) To the sum of the results of items (i) and (ii) above, add the  $udayam\bar{a}nas$  of the intervening complete  $r\bar{a}sis$ . The resultant gives the required time (in  $vighat\bar{i}s$ ) from the local sunrise. When the same is divided by 60, we get the time in  $ghat\bar{i}s$ .

**Example**: The given (nirayaṇa)  $lagna = 3^R 6^{\circ} 42' 57''$ 

$$Ayan\bar{a}m\acute{s}a = 18^{\circ} 16' 10''$$

(i)  $S\bar{a}yana\ lagna = 3^{R}\ 24^{\circ}\ 59'\ 07''$ 

i.e., 24° 59′ 07" in Karkaṭaka whose udayamāna = 341 palas.

The bhuktāṃśa of (Sāyana) lagna =  $24^{\circ} 59' 07''$ .

The time-equivalent of this is

bhukta palas = 
$$\frac{24^{\circ} 59' 07''}{30} \times 341 \approx 284$$
 palas

(ii)  $S\bar{a}yana$  Ravi at the sunrise =  $1^R 21^{\circ} 22' 22''$ 

i.e., 21° 22′ 22" in Vṛṣabha whose udayamāna is 255 palas.

: Bhogyāmśa of sāyana Ravi = 30° - 21° 22′ 22″ = 8° 37′ 38″

The time-equivalent of the bhogyāmśa is

bhogya palas = 
$$\frac{8^{\circ} 37' 38''}{30^{\circ}} \times 255 \approx 73$$
 palas

- (iii) In between Karkaṭaka and Vṛṣabha there is only one intervening rāśi viz. Mithuna whose udayamāna is 305 palas.
- (iv) Now, adding up the results of (i), (ii) and (iii) above, we get

$$284 + 73 + 305 = 662$$
 palas

Dividing by 60, we get 11|02 gh.

This is the required time from the sunrise.

**Note**: In case the sāyana lagna and the sāyana Sun are in the same rāśi, then the required time is determined as follows:

(i) If the sāyana lagna is greater than the sāyana Sun (in the same rāśi), then (Lagna – Sun) in degrees is multiplied by the udayamāna of their common rāśi and divided by 30. The result gives the time (in palas) after the sunrise.

**Example**:  $S\bar{a}yana\ lagna = 1^R 28^\circ 26' 53''$  and  $s\bar{a}yana\ Sun = 1^R 11^\circ 23' 22''$  both in Vr; abha. Here, Lagna > Sun. We have  $udayam\bar{a}na = 255\ palas$  for the Vr; abha  $r\bar{a}si$ . Now,

$$\frac{\text{(Lagna - Sun)}}{30^{\circ}} \times 255 = \frac{17^{\circ} \ 03' \ 31'' \times 255}{30^{\circ}} = 145 \ palas$$

i.e.,  $2^{gh}$   $25^{vig}$  is the required time.

(ii) If the  $s\bar{a}yana\ lagna$  is less than the  $s\bar{a}yana$  Sun (in the same  $r\bar{a}si$ ), then (Sun – Lagna) in degrees is multiplied by the  $udayam\bar{a}na$  of their common  $r\bar{a}si$  and divided by 30°. The result gives the time (in palas) before the sunrise.

This time is subtracted from the *rātrimāna* (the duration of the night, from the prior sunset to the sunrise) to get the required time from the preceding sunset. If the *dinamāna* (duration of the day, from the sunrise to the sunset) of the previous day is added to the above time, we get the required time reckoned from the sunrise of the previous day.

The required time after the sunset can also be obtained from the astalagna (by adding  $6^R$  to the lagna at the previous sunrise).

Śloka 7: (In the northern hemisphere) adding and subtracting the ascensional difference (cara) in palas to and from 15  $n\bar{a}dik\bar{a}s$  (give) the half-day and half-night (respectively); the reverse (is the case) in the southern hemisphere. The difference between the half-day and the elapsed (time in)  $ghat\bar{t}s$  is the nata (zenith distance in time unit). The nata subtracted from the half-day becomes the unnata (altitude).

Obtaining nata and unnata is explained.

The dinārdha, duration of half of the day and the ratryardha, duration of half of the night are determined as follows:

(i) If the  $(s\bar{a}yana)$  Sun is within  $6^R$  from Meṣa (i.e., between  $0^\circ$  and  $180^\circ$ ) then adding carapalas to 15 gh. we get the  $din\bar{a}rdha$  i.e., half duration of the day time.

Similarly, subtracting the *carapala* from 15 gh., we get the half duration of the night time.

[The *day time* is from the sunrise to the sunset and the *night time* is from the sunset to the next sunrise].

(ii) If the  $s\bar{a}yana$  Sun is within  $6^R$  from  $Tul\bar{a}$  (i.e., between  $180^\circ$  and  $360^\circ$ ), then the carapala is subtracted from or added to  $15^{gh}$ . respectively to get the durations of half day time or half night time.

Example:  $Carapala = 86 \ palas = 1^{gh} \ 26^{vig}$ 

Assuming that the  $s\bar{a}yana$  Sun is in the uttaragola (northern hemisphere), we have  $din\bar{a}rdha = 15^{gh} + 1^{gh} 26^{vig} = 16^{gh} 26^{vig}$ 

$$r\bar{a}tryardha = 15^{gh} - 1^{gh} 26^{vig} = 13^{gh} 34^{vig}$$

Nata and unnata of the (sāyana) Sun are determned as follows:

- (i)  $Nata = Din\bar{a}rdha$  (in  $g\underline{h}$ ) Given time ( $g\underline{h}$ ) after sunrise.
- (ii) Unnata = Dinārdha nata = Given time (all in gh.).

If the given time is between the sunrise and the noon (i.e., within the first dinārdham) then the nata is pūrva (eastern). If the time is between the noon and the sunset (i.e., within the second dinārdham) then the nata is western.

Note: (i) Nata + unnata = Dinārdham.

(ii) In angle units, nata is the zenith distance and unnata is the altitude.

Zenith distance + altitude = 90°

Ślokas 8 to 12: The half-day (in *ghaṭīs*) reduced by 5 (*ghaṭīs*) is the *hara* of mid-heaven (*madhya kāla*). The square of zenith distance (*nata* in *gh*) is separately multiplied by 50 and (the same) added with 900 and divided (by the latter); (this) reduced from the *hara* of mid-heaven (*khahara*) is the *hara* of the desired time.

If the nati is greater than 15 (ghatis) then the hara is the nata subtracted from the half-day. The carapala divided by the first block of ascensional difference (prathma-cara-khanda), halved and squared and then reduced by 1/6th of itself, added with 10 degrees, and multiplied by hypotenuse (palakarṇa) is the hṛti. (This) hṛti divided by hara (divisor for the given time) is the (desired) karana (hypotenuse) in angulas. The hypotenuse (karna) added with and reduced by 12 (respectively), are multiplied (together); the square-root (of this product) is the shadow (dyuti, chāyā); the square root of the sum of the squares of 12 and shadow is the hypotenuse. The hrti divided by the karna is the (ista) hara; the mid-heaven (madhyakālīna) hara is reduced by iṣṭa hara, (the result) is separately multiplied by 900 and subtracted from 50; dividing (the former by the latter), its square-root is the nata. If the remainder (of subtracting ista hara from khahara) is greater than 10 then (iṣṭa) hara itself is the unnatam (altitude). Thus the determination of the shadow (of the gnomon) by the short method, devoid of sine (*jyā*) and sine-inverse (*dhanu*) of planetary operations is accomplished.

Now, obtaining the  $ch\bar{a}y\bar{a}$  (shadow) from the given time and vice versa is explained.

(i) Subtract 5 from the dinārdha ghatī to get the madhyāhna kālīna hara.

(ii) Consider 
$$\frac{(nata)^2 \times 50}{(nata^2 + 900)}$$

(iii) Subtracting the result of (ii) from that of (i), we get the *iṣṭa kālīna* hara.

Note: If nata is greater than 15 gh., then subtracting nata from the dinārdham we get the iṣṭa hara.

**Example**: Iṣṭa ghaṭ $\bar{i}$  =  $11^{gh}$ ,  $din\bar{a}rdham$  =  $16^{gh}$   $26^{vig}$  and nata =  $5^{gh}$   $26^{gh}$ . We have

Madhyāhna kālīna hara =  $16^{gh}$   $26^{vig}$   $-5^{gh}$  =  $11^{gh}$   $26^{vig}$ 

Now, 
$$\frac{(5|26)^2 \times 50}{(5|26)^2 + 900} = 1 |35 \ g\underline{h}$$

Iṣṭa kālīna hara =  $11^{gh}$   $26^{vig}$   $-1^{gh}$   $35^{vig}$  =  $9^{gh}$   $51^{vig}$ 

Suppose  $nata = 16^{gh}$ ; since it is greater than 15 we have

Iṣṭa kālīna hara = 
$$16^{gh} 26^{vig} - 16^{gh} = 0^{gh} 26^{vig}$$

- (iv) *Hṛti*: (a) Divide the *carapala* (*CP*) by the first (*prathama*) *carakhaṇḍa PCK* (obtained under Ślokas 19 and 20 in *Spaṣṭādhikāra*) and take half of it.
- (b) Square this result and subtract  $\frac{1}{6}^{th}$  of this square and add  $10^{\circ}$  i.e., obtain

$$\left(\frac{CP}{2 PCK}\right)^2 - \frac{1}{6} \left(\frac{CP}{2 PCK}\right)^2 + 10^{\circ}$$

(c) Find the palakarṇa (or akṣakarṇa) PK from the right-angled triangle in which the palakarṇa is the hypotenuse and the other two sides (mak-

ing the right-angle) are the śańku (gnomon) taken as koți and the palabhā (shadow on equinoctial midday) as the bhuja

i.e., Palakarņa 
$$PK = \sqrt{(koți)^2 + (bhuja)^2}$$
$$= \sqrt{(\acute{s}anku)^2 + (palabhā)^2}$$

(d) Multiply the result-of (b) by the palakarna PK obtained in (c) we get

$$hrti = \left[ \frac{5}{6} \left( \frac{CP}{2 PCK} \right)^2 + 10^{\circ} \right] \times PK$$

(v) Then, istkarna = hṛti / iṣṭa kālīna hara

where the numerator and the denominator are obtained in (iv) (d) and (iii).

Example: In the example considered earlier under (iii), we have  $i \not= k \bar{a} l \bar{l} n a$  hara =  $9 | 51 g \underline{h}$ .

palabhā = 5|30 ang. for the given place and  $\dot{s}anku = 12$  ang.

Now, carapalas = 86, prathama carakhaṇḍa PCK = 55 (see example under Ślokas 19 and 20 in Chapter 2).

Palakarņa PK = 
$$\sqrt{(5|30)^2 + 12^2}$$
 = 13|12 ang.

Therefore, we have

$$Hrti = \left[ \frac{5}{6} \left\{ \frac{86}{2(55)} \right\}^2 + 10^\circ \right] \times (13|12) = 138|43$$

:. Iṣṭa karṇa = 
$$\frac{138|43}{9|51}$$
 = 14|05 aṅg.

(vi) The shadow (*iṣṭa kālīna chāyā*) of the śaṅku (gnomon) for the given time on the given day is determined from the right-angled triangle with the *iṣṭa karṇa* as the hypotenuse and the śaṅku (12 aṅgulas) as the other side.

i.e., iṣṭa chāyā = 
$$[(iṣṭa karṇa)^2 - (śanku)^2]^{1/2}$$

In the example, since istakarna = 14|05 ang., we have

Iṣṭa chāyā = 
$$\sqrt{(14|05)^2 - (12)^2}$$
 = 7|22 aṅg.

From a given chāyā, we can find out the iṣṭakāla by the reverse process.

Example: We have ista chāyā = 7|22 ang. Dinārdham = 16|26 gh

Madhyāhna kālīna hara =  $16 \mid 26 \mid g\underline{h} - 5 \mid g\underline{h} = 11 \mid 26 \mid g\underline{h}$ 

∴ Palakarṇa = 
$$\sqrt{(ch\bar{a}y\bar{a})^2 + 12^2}$$
  
=  $\sqrt{(7|22)^2 + 144} = \sqrt{198|16} \approx 14|5$  ang.

Hrti = 138|43 (obtained earlier).

Now,

Istahara = 
$$\frac{Hrti}{14|5} = \frac{138|43}{14|5} = 9|51$$

S56

and Madhyāhnakālīna hara - 9 51

$$= 11|26 - 9|51 = 1|35$$

Consider 
$$\frac{(1|35) \times 900}{50 - 1|35} \approx 29|26$$

Now, we have

Nata = 
$$\sqrt{29 \mid 26} = 5 \mid 25.5 \, gh$$

 $Din\bar{a}rdham = 16|26 gh.$ 

The required time = dinārdham - nata

$$= 16 |26-5|255 = 11|0.5 gh.$$

Note: If (Madhyāhna kālina hara - Iṣṭahara) is greater than 10°, then iṣṭhara itself is unnatam and nata = dinārdham - unnatam.

Example : Suppose iṣṭa hara = 0|26, madhyhna kālina hara (khahara) = 11|26

Now, their difference, 11|26 - 0|26 = 11 > 10.

- $\therefore$  Unnatam = Işṭahara = 0 | 26 gh.
- ∴ Nata = Dinārdham Iṣṭhara

$$= 16 |26 - 0| 26 = 16 gh.$$

Ślokas 13 and 14: The blocks of declination (krāntikhaṇḍas) are 362, 341, 299, 236, 150 (and) 52 for the bhujas (in degrees) of the sāyana (i.e. with ayanāṃśa) planets. Dividing (bhuja, in degrees) by 15 and adding them (elapsed khaṇḍas) together with the product of the remainder and (the khaṇḍa) to be covered divided by 15 becomes the declination (krānti) in kalās (minutes of arc) of a sāyana planet; its directions (northern or southern) based on the hemisphere (in which the planet lies).

Obtaining the declination (krānti) of a heavenly body is explained.

- (i) The six  $kr\bar{a}nti$  khandas (at intervals of 15°) are 362, 341, 299, 236, 150 and 52.
- (ii) Find the bhuja of the sāyana planet whose krānti is required.
- (iii) Divide the *bhuja* by 15. Let q be the integer quotient and r be the remainder. The quotient q represents the number of gata (elapsed) khandas. The next khanda i.e.,  $(q+1)^{th}$  is called the *bhogya* (to be covered) khanda.
- (iv) Now, consider (sum of the gata khaṇḍas) +  $(r \times bhogya khaṇḍa)/15$ This will be in  $kal\bar{a}s$  (minutes of arc). Dividing the result by 60, we get the  $kr\bar{a}nti$  in degrees.

The  $kr\bar{a}nti$  is north or south (positive or negative) according as the  $s\bar{a}yana$  planet is in the northern  $(0^0 < \lambda < 180^0)$  or southern  $(180^0 < \lambda < 360^0)$  hemisphere (where  $\lambda = s\bar{a}yana$  longitude).

**Example**:  $S\bar{a}yana Sun = 1^R 21^{\circ} 32' 54'' = 51^{\circ} 32' 54''$ 

:.  $Bhuja = 51^{\circ} 32' 54''$ 

Dividing bhuja by 15, quotient q=3 and remainder  $r=6^{\circ}$  32' 54". This means that 3 krānti khaṇḍas viz., 362, 341 and 299 have elapsed. The next (bhogya) khaṇḍa is 236.

$$\therefore kr\bar{a}nti = (362 + 341 + 299) + \frac{6^{\circ} 32' 54''}{15^{\circ}} \times 236$$

 $= 1105.0271 \ kal\bar{a}s$ 

Dividing by 60,  $kr\bar{a}nti = 18^{\circ} 25' 01''$ .

Since  $s\bar{a}yana$  Sun < 180° (i.e., in northern hemisphere),  $kr\bar{a}nti = 18^{\circ} 25' 01'' N$ .

Remark: For a planet close to the ecliptic (i.e., its celestial latitude being ignored), its declination  $\delta$  is given by

$$\sin \delta = \sin \lambda \sin \epsilon$$

where  $\lambda$  is the sāyana longitude of the planet and the obliquity of the ecliptic with the celestial equator  $\in$  is taken as 24°.

For  $\lambda = 15^{\circ}$ ,  $30^{\circ}$ , ................................ 90° (at intervals of  $15^{\circ}$ ), the values of  $kr\bar{a}nti$  in  $kal\bar{a}s$  (minutes of arc) and the successive differences are compared with the corresponding differences given in Ślokas 13 and 14 above in the  $Karaṇa\ Kut\bar{u}halam\ (KK)$  in Table 3.1.

Anka	0	1		2		3		4		5		6
λ	0°	15	o )	30	0	45	•	60	°	75	•	90°
Krānti from (1)	0	362	.56	704.	04	1002	.88	1237	.48	1388	.02	1440
Diff. from (1)	362	2.56	34	1.48	29	8.84	23	4.60	150	0.54	51	.98
Diff. (KK)	30	32	g	341	2	299	2	236	1	50	5	52

Table 3.1 Krānti khaṇḍas

We observe that the  $kr\bar{a}nti$  values and their differences given in KK are very close to those obtained from (1).

Śloka 15: The product of *bhuja* (and the same) reduced from 180 divided by the difference of 18 *kalās* less than 443° (i.e. 442° 42′) and the product divided by 77 is *krānti* itself without (using) blocks (*khaṇḍas*).

Obtaining *krānti* by another method is explained.

Consider the bhuja B of the sāyana planet. Then the krānti is given by

$$Kr\bar{a}nti = \frac{x}{442|42 - x/77|}$$
 in degrees

where  $x = (180^{\circ} - B) B$ .

**Example**: In the example, under Ślokas 13 and 14, sāyana Sun =  $1^R 21^{\circ} 32' 54''$ . Therefore, bhuja  $B = 51^{\circ} 32' 54''$ . Using the above formula, we get  $kr\bar{a}nti = 18^{\circ} 33' 46''$  which differs by about 8' from the earlier value.

Śloka 16: The shadow (in *palas*, minutes of arc) added with 410 and (the sum), divided by 60 is added to the hypotenuse (*akṣakarṇa*).

Dividing by this, the product of the shadow ( $palabh\bar{a}$ ) and 90 is the latitude (aksa of the place) taken always as southern (for places in the northern hemisphere). The latitude (aksamsa in degrees) combined with the declination ( $kr\bar{a}nti$  in degrees) is the natamsa.

Now, obtaining of the *akṣāmśa* (terrestrial latitude) of a place and the *natāṃśa* is explained.

- (i) Mulitply the *palabhā* of the place by 90. This product is the numerator.
- (ii) Add 410 to the *palabhā* and divide the sum by 60. To the result add the *akṣakarṇa*. The sum is the denominator.
- (iii) The akṣāṃśa is given by dividing the numerator from (i) by the denominator from (ii).

i.e., 
$$Ak \cdot \bar{a} m \cdot \hat{s} a = \frac{palabh\bar{a} \times 90}{\left[\frac{palabh\bar{a} + 410}{60} + ak \cdot \hat{s} ak ar n \bar{a}\right]}$$

(iv) Natāṃśa = Krānti ± Akṣāṃśa

The akṣāṃśa i.e., the latitude of the place is always with the sign opposite to its actual one (i.e., for all places in the northern hemisphere of the earth, it is taken as -ve). The sign (positive or negative) of krānti is taken as actually it is.

**Note**: In our modern convention,

$$Nat\bar{a}m\acute{s}a = \delta - \phi$$

where  $\delta$  is the *krānti* (declination) and  $\phi$  is the latitude of the place.

(with  $\phi$  +ve in the northern hemisphere and –ve in the southern hemisphere).

Example:  $Palabh\bar{a} = 5|30$  angulas and

Numerator = 
$$(5|30) \times 90 = 495$$

Denominator (*hara*) = 
$$\frac{5|30 + 410}{60} + 13|12 \approx 20|08$$

$$\therefore Ak s \bar{a} m \hat{s} a = \frac{\text{Numerator}}{\text{Denominator}} = \frac{495}{20|08} = 24^{\circ} 35' 10''$$

$$Natam\acute{s}a = \delta - \phi = 18^{\circ} 25' 01'' - 24^{\circ} 35' 10'' = 6^{\circ} 10' 09''$$
 South.

# CHAPTER 4 CANDRAGRAHANĀDHIKĀRAH

### (Computation of Lunar eclipse)

Ślokas 1, 2, and 3: 11250 is divided by the product of nata reduced from 30° (and itself i.e. nata), (the quotient) reduced by 10° is Ravihara; reducing 1/10th (of Ravihara from itself gives) Candra's (Vidhu's hara). Division of the (manda) phala (of each) by the respective hāra (is its nataphalam). Ravi's nataphala is positive or negative according as (Ravi is in) the west or east; Candra's nataphala is positive and negative for (its position in the) east (and west). Thus (the Sun and the Moon made more accurate) by the (nata) phala reduction is (applied in the computations of) eclipses and tithi. The nata obtained by successive jyā (R sine) is acceptable to Brahmagupta (Jiṣṇu's son). The Valana-dṛk-nata-karma based on utkramajyā (R versine), (followed) by others, is not his (Brahmagupta's) opinion.

Obtaining of the nataphala of the Sun and the Moon is explained.

- (i) Subtract the *nata* from 30° and multiply the difference by the *nata*. Divide 11250 by the above result. The resultant is in *amśas* (degrees) etc. Subtracting 10° from this we get Ravi hara.
- (ii) Subtracting  $1/10^{th}$  of the Ravi hara from itself, we get Candra hara.
  - i.e., Candra  $hara = 9 \times Ravi hara/10$ .
- (iii) Dividing the mandaphalas of the Sun and the Moon respectively by the Ravi hara and Candra hara, we get the nataphalas of the Sun and the Moon.

- (iv) If the *nata*, considered in (i), is western (*paścima* when the Sun is in the western hemisphere), the *nataphala* of the Sun is taken as positive and as negative otherwise (i.e., if the Sun is in the eastern hemisphere). In the case of the Moon, its *nata phala* will have the positive or negative sign opposite to that of the Sun.
- (v) The positions of the Sun and the Moon and hence the *tithi* are corrected with their respective nata phalams.

Example: Vikrama Samvat 1677, Śālivāhana śaka year 1542, Mārgaśīrṣa śukla 15 (paurṇimā), Wednesday, corresponding to December 9, 1620 A.D.

(G). We have  $Ayan\bar{a}m\acute{s}a=18^{\circ}\ 17'\ 42''$ ,  $P\bar{u}rmim\bar{a}$  (ending):  $11|52\ gh$ . after sunset.  $R\bar{a}tryardham$  (half night duration) =  $16|49\ gh$ .

The difference = 16|49 gh. - 11|52 gh. = 4|57 gh.

i.e.,  $pr\bar{a}nnatam = 4|57 gh$ . (eastern)

(Note:  $Pr\bar{a}k = east$ ;  $pr\bar{a}k + natam = pr\bar{a}nnatam$ ).

(i) We have  $Nata \times (30^{\circ} - nata) = (4|57)(25|03) \approx 124$ 

Now, 
$$\frac{11250}{124} = 90^{\circ} 43' 33''$$

(ii) Ravi hara =  $90^{\circ} 43' 33'' - 10^{\circ} = 80^{\circ} 43' 33''$ 

Candra 
$$hara = \text{Ravi } hara - \frac{1}{10} \times \text{Ravi } hara$$

$$= 80^{\circ} 43' 32'' - 8^{\circ} 4' 21'' = 72^{\circ} 39' 11''$$

(iii) Sun's mandaphalam = 0° 38′ 04″

Candra mandaphalam = 4° 19′ 52″

Dividing the results of (iii) respectively by those of (ii), we get

Ravi nataphalam = 
$$\frac{0^{\circ} 38' 04''}{80^{\circ} 43' 33''} = 0^{\circ} 0' 28''$$

Candra nataphalam = 
$$\frac{4^{\circ} 19' 52''}{72^{\circ} 39' 11''} = 0^{\circ} 3' 34''$$

Because of the *prān nata* of Ravi his *nataphalam* is negative and hence, the Candra *nataphalam* is positive.

The nata corrected positions of the Sun and the Moon at the sunset are:

True Sun = 
$$8^R$$
 0° 04′ 17″, True Moon =  $1^R$  27° 35′ 18″.

We shall find the actual instant of ending of the  $p\bar{u}rnim\bar{a}$  tithi (i.e., the opposition of the Sun and the Moon):

We have  $(Sun + 6^R) - (Moon)$ 

$$= 2^{R} 0^{\circ} 04' 17'' - 1^{R} 27^{\circ} 35' 18'' \approx 2^{\circ} 29' = 149'$$

Moon's true daily motion - Sun's true daily motion

$$= 829'35'' - 61'21'' = 768'14''$$

.. Time required for the end of  $P\bar{u}rnim\bar{a} = \frac{149'}{768'14''} \times 60 \ gh. = 11 | 38 \ gh.$ 

after the sunset.

Śloka 4: The elapsed (gata) and that to be covered (gamya parts in ghaṭīs) are multiplied by the daily motion (in kalās) and divided by 60; this is subtracted from and added to the cara corrected positions of the Sun and the Moon to get their positions equal in kalās (for amāvāsyā).

Now, obtaining the true positions of the Sun and the Moon at the end of the full-moon day (paurnimā) is explained.

The elapsed (gata) or to be covered (gamya)  $ghat\bar{i}s$  are multiplied by the true daily motion (in  $kal\bar{a}s$ ) of a planet and divided by 60 to get the motion in  $kal\bar{a}s$ . In the case of gamya the motion is added to and in the case of the gata subtracted from the position of the planet (at the sunrise or the sunset as the case may be). In the case of the Sun and Moon, their positions are thus made equal (in  $kal\bar{a}s$ ). In the case of  $am\bar{a}v\bar{a}sy\bar{a}$  (new moon) for the solar eclipse the  $r\bar{a}sis$  etc. of the Sun and the Moon are made equal and in the case of  $paurnim\bar{a}$  these differ by  $6^R$  (i.e.,  $180^\circ$ ).

**Example**: Gamya (or eṣya, to be covered) = 11|38 gh. after the sunset.

Sun's true daily motion = 61' 21"

Moon's true daily motion = 829' 35"

Sun's motion in 
$$11|38$$
  $gh = \frac{11|38}{60} \times 61'21'' = 11'53''$ 

Since the motion is gamya, it is added to the position at sunset. Thus

Sun's position at the opposition

$$= 8^{R} 0^{\circ} 04' 17'' + 11' 53'' = 8^{R} 0^{\circ} 16' 10''$$

Moon's position at the opposition =  $2^R 0^{\circ} 16' 08''$  (the difference in seconds of arc is neglected).

 $P\bar{a}ta \text{ (Moon's node)} = 4^{R} 1^{\circ} 38' 15''$ 

[According to the modern convention, subtracting the above  $p\bar{a}ta$  from  $12^R$ , we get  $R\bar{a}hu = 7^R 28^{\circ} 21' 45''$ ].

Śloka 5: Finding the śara (latitude) of the Moon is explained.

Add the true positions of the Moon and the  $p\bar{a}ta$ , take the  $jy\bar{a}$  (i.e., 120 sin) of the *bhuja* of the sum and then multiply by 3 and divide by 4. The result gives the śara of the Moon in angulas.

The sara is northern or southern according as the (Moon + Pata) is so.

Remark: We have the Moon's latitude ß given by

$$\beta = 270 \sin (M + P) \text{ in } kal\bar{a}s \qquad \dots (1)$$

$$= \frac{270 \times 120 \sin (M + P)}{120 \times 3} \text{ angulas}$$

$$= \frac{3}{4} \text{ Jyā } (M + P) \text{ angulas}$$

Note: (i) Here,  $P = 12^R - R$  where R is the actual position of Rāhu.

- (ii) 1 angula = 3 kalās.
- (iii) The maximum latitude of the Moon is taken as 270' i.e., 4.5°. The modern known value is about 5°8'.
- (iv) Bhuja jyā is also called Dorjyā.

**Example**:  $P\bar{a}ta = 4^R 1^{\circ} 36' 15''$ , Moon =  $2^R 0^{\circ} 16' 08''$ 

$$M + P = 6^R 1^{\circ} 52' 23''$$

$$\therefore Bhuja of (M+P) = 1^{\circ} 52' 23''$$

$$\therefore \quad \acute{S}ara = \frac{3}{4} Jy\bar{a} (M+P) = \frac{3}{4} \times 120 \times \sin(1^{\circ} 52' 23'')$$

$$\approx 2|57 \text{ angulas}$$

Since  $M + P > 6^R$ , it is in the southern hemisphere. Therefore,

 $\acute{S}$ ara = 2|57 ang. south.

Ślokas 6 and 7 (first half): The six rāśis from Karka and from Makara (respectively) are the southern (dakṣiṇa) and the northern (uttara) journey (ayana of the Sun.) The blocks (of differences, khaṇḍas) of the latitude (śara of the Moon) are 70, 65, 56, 43, 27 and 9. From these the latitude can be obtained here, even like the declination (krānti) in minutes of arc (kalās and) dividing by 3 in aṅgulas etc.

The uttara (north) and dakṣiṇa (south) ayanas (solstices) are explained. Also, another method for obtaining the śara of the Moon is explained.

For a planet lying within 6 rāśis from (sāyana) Makara (Capricorn) it is uttarāyaṇa (northern course) and 6 rāśis from (sāyana) Karka (Cancer) it is dakṣiṇāyana (southern course).

The six khaṇḍas for finding the śara are 70, 65, 56, 43, 27, 9. The śara is determined from these khaṇḍas by the process similar to the one for finding the  $kr\bar{a}nti$  (Ślokas 13 and 14 in Chapter 3). The śara thus obtained is in  $kal\bar{a}s$ . Dividing the same by 3 we get it in angulas.

The *khandas* are compared with the differences of the actual *śara* values in *kalās* obtained from equation (1).

Anka 0 5 1 2 3 6 M + P $0^{\circ}$  $15^{\circ}$  $30^{\circ}$ 45° 60° 75° 90° Śara from (1) 69'.88 135' 190'.92 233'.83 260'.80 270' Diff. from (1) 69'.8865'.12 55'.92 42'.91 26'.97 9'.20Diff. from (KK) 70' 65' 56' 43' 27' 9'

Table 4.1 Śara khaṇḍas

Example: Sapāta Candra,  $M + P = 6^R 1^{\circ} 52' 23''$ 

Bhuja of 
$$(M + P) = 1^{\circ} 52' 23''$$

Dividing by  $15^{\circ}$ , quotient q = 0, remainder  $r = 1^{\circ} 52' 23''$ .

The bhogyakhanda = 70'.

$$\therefore \hat{S}ara = 0 + \frac{1^{\circ} 52' 23''}{15^{\circ}} \times 70' = 8' 44''$$

Dividing by 3, we have

$$\acute{S}$$
ara = 2 | 55 angulas.

Note: We had obtained the  $\acute{s}ara$  as 2|57 ang. by the earlier method under  $\acute{S}loka$  5.

Ślokas 7 (second half) and 8: The Moon's (angular) diameter (bimbam) is its (daily) motion divided by 74, (the diameter) of the Sun is that (its daily motion) multiplied by 2 and divided by 11. The shadow's diam-

eter ( $bh\bar{u}$ - $bh\bar{a}$ ) is the Moon's daily motion multiplied by 3 and divided by 67 reduced by one-seventh of the Sun's motion.

The node ( $p\bar{a}ta$ ) through the shadow (cone) eclipses the (full) Moon and the sphere of the Moon eclipses the Sun.

Obtaining the bimbas (diameters) of the Moon etc. in angulas is explained.

- (i) Moon's bimbam = (Moon's true daily motion/74) angulas
- (ii) Sun's  $bimbam = (Sun's true daily motion) \times \frac{2}{11}$  ang.
- (iii) Earth shadow's bimbam

$$= \left( \text{True motion of Moon} \times \frac{3}{67} \right) - \left( \frac{\text{True motion of Sun}}{7} \right) \text{ ang.}$$

In the lunar eclipse the Moon is the *chādya* (eclipsed) and the earth's shadow is the *chādaka* (eclipser). In the solar eclipse, the Sun is the *chādya* and the Moon the *chādaka*.

Example: Moon's true daily motion = 829' 35"

Sun's true daily motion = 61' 21"

Therefore, we have

(i) Moon's 
$$bimbam = \frac{829' 35''}{74} = 11|12$$
 ang.

(ii) Sun's 
$$bimbam = \frac{61' 21'' \times 2}{11} = 11|09$$
 ang.

(iii) Bhūbhā bimbam (Shadow's diameter)

= 
$$\frac{3}{67} \times 829' 35'' - \frac{1}{7} \times 61' 21'' = 28 | 22 \text{ arig.}$$

**Remark**: In Indian astronomical texts, the usual procedure for obtaining the true diameters of the Sun and the Moon is as follows:

$$True\ diameter = \frac{Mean\ diameter \times True\ daily\ motion}{Mean\ daily\ motion}$$

We have

Sun's mean daily motion = 59'08"

Moon's mean daily motion = 790' 35"

Sun's mean diameter = 32' 31"

Moon's mean diameter = 32'

∴ Sun's true diameter = 
$$\frac{32'31''}{59'08''}$$
 × (True daily motion) kalās

= 
$$\frac{32'31'' \times 2}{59'08'' \times 3 \times 2} \times \text{(True daily motion)}$$
 ang.

= 
$$\frac{2}{10.91}$$
 × (True daily motion) ang.

$$\approx \frac{2}{11} \times \text{(True daily motion of Sun)}$$
 ang.

Moon's true diameter = 
$$\frac{32'}{790'35''} \times (Moon's true daily motion) kalās$$

= 
$$\frac{32'}{790'35''\times3}$$
 × (Moon's true daily motion) ang.

$$= \frac{\text{Moon's true daily motion}}{74.117} \text{ ang.}$$

$$\approx \frac{Moon's true daily motion}{74}$$
 ang.

Śloka 9: Half of the sum of the diameters of the eclipses and the eclipsing (bodies) reduced by the (Moon's) latitude is the measure of obscuration (*channam*, *grāsamāna*). The *channam* reduced by the (diameter of) the eclipsed (body) is *khacchannam* (or *khagrāsa*).

Obtaining the grāsamāna is explained.

$$Gr\bar{a}sam\bar{a}na = \frac{(Ch\bar{a}dya\ bimbam + Ch\bar{a}daka\ bimbam)}{2} - \acute{S}ara\ in\ angulas$$

Khagrāsa (or Khachannam) = Grāsamāna - Chādya bimbam

If there is *khagrāsa* (as positive) i.e., if the *grāsamāna* is greater than the *chādya bimbam*, then there will be a total eclipse.

Example:  $Ch\bar{a}dya\ bimba\ (Moon's\ diameter) = 11|12\ ang.$ 

Chādaka bimbam (Shadow's diameter) = 28 | 22 ang.

Moon's  $\dot{s}ara = 2|57$  ang.

:. 
$$Gr\bar{a}sam\bar{a}na = \frac{11|12 + 28|28}{2} - 2|57$$
 ang. = 16|50 ang.

$$\therefore$$
 Khagrāsa = 16|50 - 11|12 = 5|38 ang.

Therefore, the lunar eclipse is total.

Śloka 10: Twice the (Moon's) latitude together with and (the sum) multiplied by the measure of obscuration (*channam*), the square-root of this multiplied by 180 and divided by the difference of the (daily) motions (of the Moon and the Sun) is the half-duration (*sthiti*) in *ghaṭikās* etc. From the *khacchannam* the half-duration of totality (*marda*) is obtained.

Sthiti and vimarda are explained. In the case of a lunar or solar eclipse, the sthiti (half-duration of the eclipse) is given by

$$Sthiti = \frac{\left[ (\acute{S}ara \times 2 + Channam) \times Channam \right]^{1/2} \times 180}{(Moon's daily motion - Sun's daily motion)}$$

In the case of *total* eclipse, replace *channam* by *khagrāsa* in the above expression to get *vimarda* i.e., the half duration of *totality* in the case of eclipse. The *sthiti* and *vimarda* obtained here are *madhya* (mean) *sthiti* and *vimarda*.

Example:  $\acute{S}$  ara = 2|57 angulas, C hannam = 16|50 ang.

Moon's daily motion – Sun's daily motion = 829'35'' - 61'21'' = 768'14''.

Half duration, 
$$sthiti = \frac{\left[ \left\{ (2|57) \times 2 + (16|50) \right\} (16|50) \right]^{\frac{1}{2}} \times 180}{768' 14''} gh.$$

i.e., Sthiti = 4 | 35 gh.

Now, Khachannam = 5|35 ang.

: Half-duration of totality (called vimarda or marda),

$$vimarda = \frac{\left[ \left\{ (2 \mid 57) \times 2 + (5 \mid 35) \right\} ((5 \mid 35)]^{1/2} \times 180}{768'14''} \ gh.$$

i.e., vimarda = 1 | 53 gh.

Ślokas 11, 12 and 13: If the sum of (the longitudes of) the Moon and its node is in the odd quadrant (I or III), the latitude (śara of the Moon) divided by 48 in nāḍīs etc. is added to and subtracted from the half-duration (sthiti), kept in two places, respectively for the commencement (sparśa) and the end (mokṣa). For the even quadrants (II or IV), the reverse (operation) is done to the half-duration (sthiti); (the case of) totality (marda) is as in sthiti. From the sunrise and the sunset (respectively for the solar and the lunar eclipses) the time is to pass (gamya) for the middle at the end of the fortnight.

The (sparśa) sthiti and the (vi) marda subtracted from this (the middle) are the commencement of the eclipse (sparśa) and of the totality (sammīlanam respectively). The addition of the (mokṣa) sthiti and the marda to that (the middle) gives the end of the eclipse (mokṣa) and of totality (unmīlanam).

Obtaining the *sthiti* and *vimarda* for the beginning and ending of an eclipse and of totality is explained.

(a) If the sapāta Candra (M + P) is in an odd quadrant (I or III), then

(i) Sparśa sthiti = 
$$\left[ Madhya Sthiti + \frac{\acute{S}ara}{48} \right] gh.$$

(ii) Mokṣa sthiti = 
$$\left[ Madhya Sthiti - \frac{\acute{S}ara}{48} \right] gh.$$

(b) If the sapāta Candra (M + P) is in even quadrant (II or IV), then

(i) Sparśa sthiti = 
$$\left[ Madhya Sthiti - \frac{\acute{S}ara}{48} \right] gh.$$

(ii) Mokṣa sthiti = 
$$\left[ Madhya Sthiti - \frac{\acute{S}ara}{48} \right] gh.$$

In the above formulae, by replacing the madhya sthiti by (madhya) vimarda (or marda) ghaṭikās, we obtain the sparśa vimarda and the mokṣa vimarda.

The same expressions hold good in the case of a solar eclipse also.

**Note**: The *madhya sthiti* is the mean half duration of the eclipse (obtained from Śloka 10 of this chapter). The results (i) and (ii) above provide the corrected first and second halves of the eclipse which are not equal.

The five timings of an eclipse are determined as follows:

(1) Beginning of the eclipse:

Sparśa kāla = Parvānta - Sparśa sthiti

(2) Beginning of the totality:

Sammilana kāla = Parvānta - Sparśa marda

(3) Middle of the eclipse:

Madhya kāla = Parvānta

(4) End of totality:

Unmīlana kāla = Parvānta + Moksa marda

(5) End of the eclipse:

Moksa kāla = Parvānta + Moksa sthiti

Here, parvānta is the ending moment of the bright fortnight (śukla pakṣa) i.e., of the full moon day (paurnimā). The beginning and the ending of the totality of a total eclipse are called respectively sammīlana and unmīlana.

Example (1): Continuing the example considered in the earlier ślokas, we have

Parvānta = 11 38 gh., Madhya sthiti = 4 35 gh.

Śara = 2|57 ang., Madhya marda = 1|55 gh.

We have, therefore, the corrected sthitis and mardas as follows:

(i) Sparśa sthiti = 
$$4|35 + \frac{2|57}{48} = 4|38 \text{ gh.}$$

(ii) Mokṣa sthiti = 
$$4|35 - \frac{2|57}{48} = 4|32 \text{ gh.}$$

(iii) Sparśa marda = 
$$1|55 + \frac{2|57}{48} = 1|58 \text{ gh.}$$

(iv) Mokṣa marda = 
$$1|55 - \frac{2|57}{48} = 1|52 \text{ gh.}$$

The five timings are as follows:

(1) Sparśa kāla = 
$$(11|38 - 4|38) gh$$
. = 7 gh.

(2) Sammīlana kāla = 
$$(11|38 - 1|58)gh. = 9|40 gh.$$

(3) Madhya kāla (middle) = 
$$11|38$$
 gh.

(4) Unmīlana kāla = 
$$(11|38 + 1|52)$$
 gh. =  $13|30$  gh.

(5) Mokṣa kāla = (11|38 + 4|32) gh. = 
$$16|10$$
 gh.

Duration of the eclipse

$$= (4|38 + 4|32) gh. = 9|10 gh.$$

Duration of totality of the eclipse

$$= (1|58 + 1|52) gh. = 3|50 gh.$$

We now consider a modern example below.

**Example (2)**: Lunar eclipse which occurred on September 27, 1996, Friday. On that day at 5.30 a.m. (IST) we have

S78

True Sun =  $160^{\circ} 21' 01''$ , True Moon =  $338^{\circ} 44' 27''$ 

 $R\bar{a}hu = 164^{\circ} 10' 14'' [Note : R\bar{a}hu = 360^{\circ} - P\bar{a}ta]$ 

Sun's true daily motion = 58' 51"

Moon's true daily motion = 861'

Rāhu's daily motion = -3'11''

(i) The instant of opposition  $(parv\bar{a}nta) = 8^h 24^m$  a.m. (IST)

At the parvanta (i.e.,  $8^h 24^m$  a.m.), we have

True Sun =  $160^{\circ} 28' 08''$ , True Moon  $M = 340^{\circ} 28' 29''$ , Rāhu,  $R = 164^{\circ} 09' 51''$ .

(ii) Diameters of the Moon and the earth's shadow:

Moon's diameter =  $\frac{861}{74} = 11|38$  ang.

Earth's shadow diameter =  $\left(\frac{861 \times 3}{67} - \frac{58|51}{7}\right)$  ang. = 30|29 ang.

(iii) Moon's latitude (śara):

$$Śara = 90 \sin (M - R)$$
 ang.  
=  $90 \sin [176^{\circ} 18' 38'']$  ang. =  $5 | 47$  ang.

[Note:  $\sin (M - R) = \sin (M + P)$  where  $P = P\bar{a}ta$ ]

#### (iv) Grāsa (obscuration):

We have

(a) 
$$Gr\bar{a}sa$$
 (Channam) =  $\frac{1}{2}(11|38+30|29) - 5|47 = 15|16$  ang.

(b) Khagrāsa = Grāsa - Moon's diameter

$$= 15|16 - 11|38 = 3|38$$
 ang.

(v) Mean half-duration of eclipse and totality:

(i) Sthiti = 
$$\frac{\sqrt{[(2(5|47) + (15|16)](15|16)} \times 180}{(861 - 58|51)} = 4|32 \text{ gh.}$$

(ii) Marda (or Vimarda)

$$= \frac{\sqrt{[2(5|47) + 3|38](3|38) \times 180}}{(861 - 58|51)} = 1|40 \text{ gh.}$$

(vi) Corrected half-durations of the beginning and the ending of the eclipse and totality:

We have 
$$\frac{\dot{S}ara}{48} = 0.1204861gh. \approx 7 \text{ vig.}$$

Here, Vyagu,  $M-R=176^{\circ}18'38''$  which is in II quadrant i.e. even quadrant.

Note: Our (M-R) is the same as Bhāskara's sapātacandra (M+P) noting that  $R=(360^{\circ}-P)$ .

(a) Sparśa sthiti = 
$$4^{gh} 32^{vig} - 7^{vig} = 4^{gh} 25^{vig} \approx 1^h 46^m$$

(b) Mokṣa sthiti = 
$$4^{gh} 32^{vig} + 7^{vig} = 4^{gh} 39^{vig} \approx 1^h 51^m 36^s$$

(c) Sparśa marda = 
$$1^{gh}40^{vig} - 7^{vig} = 1^{gh}33^{vig} \approx 0^h37^m12^s$$

(d) Makṣa marda = 
$$1^{gh} 40^{vig} + 7^{vig} = 1^{gh} 47^{vig} \approx 0^h 42^m 48^s$$

(vii) The five timings (in IST) of the eclipse:

(a) Sparśa kāla = 
$$8^h 24^m - 1^h 46^m = 6^h 38^m a.m.$$

(b) Sammilana kāla = 
$$8^h 24^m - 0^h 37^m 12^s = 7^h 46^m 48^s$$
 a.m.

(c) Madhya (middle) 
$$k\bar{a}la = 8^h 24^m a.m.$$

(d) 
$$Unm\bar{i}lana \ k\bar{a}la = 8^h \ 24^m + 0^h \ 42^m \ 48^s = 9^h \ 06^m \ 48^s$$

(e) 
$$Mokṣa kāla = 8^h 24^m + 1^h 51^m 36^s = 10^h 15^m 36^s$$

Note: According to the *Indian Astronomical Ephemeris*, the respective timings are  $6^h 42^m 3^s, 7^h 49^m 3^s, 8^h 24^m 04^s, 8^h 59^m 4^s$  and  $10^h 06^m 3^s$ .

Ślokas 14, 15 and 16: The zenith distances (nata in ghațīs) at the commencement (sparśa) and at the end (mokṣa) multiplied by 90 and divided by half of he day-length are the natāṃśas.

Its *jyā* (120 sine of the *natāṃśas*) multiplied by the latitude (*akṣa* of the place) and divided by the radius (120) are the deflections (*akṣa valanas* at *sparśa* and *mokṣa*), due to latitude, which are north for eastern *nata* and otherwise (south for the western *nata*).

The  $\bar{a}yana$  (valana) in degrees is the  $kotijy\bar{a}$  (120 cosine) of the (bhuja of the) planet (i.e. the Sun or the Moon) divided by 5. The direction of the  $\bar{a}yana$  (valana) is that of the  $s\bar{a}yana$  planet.

The  $jy\bar{a}$  (120 sine) of the sum or the difference (of the two *valanas* as the case is) multiplied by the sum of the half-diameters (of the eclipsed and the eclipsing bodies) and divided by the radius ( $trijy\bar{a}$ , 120) are the corrected deflections ( $sphuta\ valanam$ ) in angulas.

The directions (of the *sphuṭa valanam*) at *sparśa* and *mokṣa* (respectively for the eclipses of the) Sun and the Moon are of opposite directions.

Finding of akṣavalana, āyana valana and hence spaṣṭa valana at the beginning and end of an eclipse is explained.

(a) (i) The *nata* at *sparśakāla* is multiplied by 90 and divided by *rātryardha* (half duration of night) in the case of a lunar eclipse and by *dinārdham* (half duration of day) in the case of a solar eclipse.

The samething is done for the mok, a  $k\bar{a}la$  by considering the nata at the mok, a  $k\bar{a}la$ .

The result gives the *natāmśa* at the *sparśa kāla* (or *mokṣa kāla* as the case may be).

- (ii) Find the jyā (i.e. 120 sine) of the natāmśa and multiply by the akṣāmśa (terrestrial latitude) of the place and divide by the trijyā (radius) 120. The result is akṣa valana. This is determined for the sparśa and mokṣa kālas separately.
- (b) (i) Obtain the *bhuja* of  $s\bar{a}yana$  Candra at the  $spar\acute{s}a$  and  $mok \dot{s}a$   $k\bar{a}las$ . Consider the  $ko\dot{t}i$  of this *bhuja* (i.e.,  $90^{\circ} bhuja$ ) and find the  $jy\bar{a}$  of this  $ko\dot{t}i$  i.e., obtain  $120 \sin (90^{\circ} bhuja)$  or  $120 \cos (bhuja)$ . Dividing this by 5 we get  $\bar{a}yana$  valana.

$$\bar{A}yana \ valana = \frac{R \cos{(bhuja)}}{5}$$
 where  $R = 120$ .

This is determined separately for the sparśa and mokṣa kālas.

The direction of the āyana valana is the same as that of the sāyana Candra.

The direction of akṣa valanam is north or south according as the nata is eastern or western.

(c) The spasta valanam is obtained as follows.

Take the algebraic sum of the akṣa and āyana valanas. Find its jyā and multiply it by half of the sum of the diameters of the chādya (eclipsed) and chādaka (eclipser) bodies. Divide the result by 120 to get the spaṣṭa (corrected) combined valanam.

The direction of the corrected *valanam* is the same as that of the algebraic sum mentioned above.

The corrected *valanam* is found out separately for the *sparśa* and *mokṣa kālas*.

**Note**: The akṣa valanam and the āyana valanam are the angular deflections caused respectively by the latitude of the place and the obliquity of the ecliptic. The corrected combined valanam represents the directions, with respect to the east-west, of the obscuration at the beginning and the end of the eclipses.

**Example**: In Example (1) above, we have  $spar\acute{s}a \ k\bar{a}la = 7^{gh}$  (after sunset)

(a) Candra dinārdham (i.e., the actual  $r\bar{a}try$  ardham after the sunset) = 16|49|gh.

Difference of the above timings = (16|49-7) gh. = 9|49 gh.

Multiplying the difference by 90 and dividing by 16 49 we get

$$\frac{(9|49) \times 90}{16|49} = 52|32|13.$$

 $Jy\bar{a}$  (52|32|13)  $\approx$  95|02 (according to Sumatiharşa).

Multiplying by the akṣāṃśa (latitude) 24° 35′ 09" of the place, and divid-

ing by 120, we get 
$$\frac{(95|02) \times (24|35|09)}{120} = 19|28$$

i.e. Aksa valana (at the sparśa  $k\bar{a}la$ ) = 19° 28′ N.

Similarly, considering the nata at the mokṣa kāla 16|10 gh. we get

Akṣa valana (at the mokṣa kāla) =  $1^{\circ}$  29′ 44″ N

(b) At the sparśa kāla, sāyana Candra =  $2^R 17^{\circ} 19' 20''$ 

Its  $bhuja = 77^{\circ} 19' 20''$  and  $koți = 90^{\circ} - 77^{\circ} 19' 20'' = 12^{\circ} 40' 40''$ .

 $Jy\bar{a} \ (12^{\circ} \ 40' \ 40'') \approx 26|20$ . Dividing by 5, we get 5|16

i.e.,  $\bar{A}$ yana valanam (at sparśa  $k\bar{a}la$ ) = 5° 16' N.

Similarly, considering the sāyana Candra at the mokṣa kāla viz.,  $2^R 19^{\circ} 26' 26''$ , Āyana valanam (at mokṣa kāla) =  $4^{\circ} 21' 24'' N$ .

(c) At the sparśa kāla, the combined valanam is

Aksa valanam +  $\bar{A}$ yana valanam = 19° 28′ N + 5° 16′ N = 24° 44′ N

At the moksa kāla, the combined valanam is

Aksa valanam +  $\bar{A}$ yana valanam = 1° 29′ 44″ N + 4° 21′ 24″ N = 5° 51′ 08″ N.

We have,  $\frac{1}{2}$  (Sum of diameters of the *chādya* and *chādaka*)

$$= \frac{1}{2}(11|12 + 28|22) = \frac{1}{2}(39|34) = 19|47$$

At the sparśa kāla, we have

Spaṣṭa valanam = 
$$\frac{\left[Jy\bar{a}\left(24^{\circ}\ 44'\right)\right]\times\left(19\middle|47\right)}{120}=8\left|13\right|09 \text{ angulas}$$

At the moksa kāla,

$$Spaṣṭa\ valanam = \frac{[Jy\bar{a}\ (5°\ 51'\ 08'')]\ \times\ (19|47)}{120} = 2|8|05\ angulas$$

and both are northern.

Śloka 17: If the *latitude* (śara of the Moon) is obtained from (the Moon +  $P\bar{a}ta$  in) the *odd* quadrant then one-third of the half-duration (*sthiti*) is subtracted and added (respectively) for the beginning (*sparśa*) and

the mokṣa. In the case of the even quadrant, the reverse are the directions (i.e. added and subtracted respectively).

Finding the śara at the sparśa kāla and at the mokṣa kāla is explained.

If the madhya grahaṇa kālika śara is (the Moon's latitude at the middle of the eclipse) obtained from the sapāta Candra (M + P) in the odd quadrant, then

$$Sparśa śara = śara - \frac{Madhya sthiti}{3}$$

$$Mokṣa śara = śara + \frac{Madhya sthiti}{3}$$

If the sapāta Candra is in the even quadrant, then

$$Sparśa śara = śara + \frac{Madhya sthiti}{3}$$

$$Mokṣa śara = śara - \frac{Madhya sthiti}{3}$$

#### Another method (Prakārāntara):

Find the Moon (M) and the  $P\bar{a}ta$  (P) for the instants of  $spar\acute{s}a$  and  $mok \dot{s}a$  from their positions at the  $parv\bar{a}nta$  and using their true daily motions. Then

$$\acute{S}ara = \frac{3}{4} \times Jya(M+P)$$
 ang.

**Example**: Continuing from Example (1) considered earlier the mean half duration, we have

**S86** 

Madhya sthiti = 4|35 gh. and śara at the middle of the eclipse = 2|57 angulas.

Sapāta Candra,  $(M + P) = 6^R 1^{\circ} 52' 23''$  which lies in III quadrant i.e., odd quadrant.

$$\therefore$$
 Sparśa śara =  $2|57 - \frac{4|35}{3} = 1|26$  angulas

and Mokṣa śara = 
$$2|57 + \frac{4|35}{3} = 4|28$$
 angulas

By the second method:

At the sparśa kāla, Moon =  $1^R$  29° 11′ 38″ and  $P\bar{a}ta = 4^R$  1° 35′ 53″

$$M + P = 6^R 0^\circ 47' 31''$$

Bhuja of  $(M + P) = 0^{\circ} 47' 31''$ 

$$\therefore \hat{S}$$
ara =  $\frac{3}{4} Jy\bar{a} (0^{\circ} 47' 31'') = 1 | 14 \text{ angulas}$ 

At the mokṣa kāla, śara = 4|35 aṅgulas

We observe a small difference between the values obtained from the two methods.

Ślokas 18 and 19: After constructing a circle with the half-diameter of the eclipsed body as the radius, (another) circle with half the sum of

the diameters (of the eclipsing and the eclipsed bodies) as the radius is constructed. On the outer circle the desired commencement deflection (spārśika valanam) from the east and the ending deflection (mokṣa valanam) from the west are marked (for the lunar eclipse) like a chord (jyā). For the solar eclipse the same are marked from the west and the east. [The commencement (sparśa) and the end (mokṣa) latitudes (of the Moon, śara) are drawn in the shape of chords from the respective end valanas].

The geometrical projection (parilekhā) of an eclipse is described.

With a fixed point on the level ground as the centre draw a circle with radius equal to half the sum of the diameters of the *chādaka* and the *chādya*.

Then, a second circle with the centre is drawn whose radius is equal to half the *bimbam* (*diameter*) of the *chādya* (eclipsed) body.

Through the centre draw the two (perpendicular) lines along the north - south and the east - west directions.

The first circle is called the *mānaikyārdha vṛtta*. On this (outer circle) mark the points representing the *spārśika valanam* from the east and the *mokṣa valanam* from the west.

In the case of the solar eclipse, consider the *spārśika* and *mokṣa valanams* respectively from the west and the east.

The (positive and negative) signs i.e., north and south directions of the valanam are the same as those of the śara at the beginning and the end of the eclipse.

Śloka 20: From the centre of the circle the latitude (śara) at the middle (madhya of the eclipse) in its direction (and the other two for sparśa and mokṣa) are marked. Circles are drawn with half the diameter of the eclipsing body and the points of the commencement (sparśa) and

the end (*mokṣa*) latitude (*śara*) as the centres. [The point of contact of this circle and the circle with half of the diameter of the eclipsing body (*chādraka*) as the radius is the point at which the eclipse commences.]

From the point of the middle of the eclipse, measuring the śara (of the middle of eclipse) in its direction, three points representing the śara values (respectively at the sparśa, madhya and moksa) are marked.

With the point of *sparśa śara* as the centre draw a circle with radius equal to that of the *chādaka*. The point where this circle touches the circle representing the eclipsed body (i.e. the inner circle) is the point of *sparśa* (commencement of the eclipse).

Similarly, the point of contact of the the circle having the point of *mokṣa* śara as the centre (and the radius of the *chādaka*) with the circle of the *chādya* (i.e. the second circle is the point of *moksa*.

A similar circle with madhya śara is also drawn.

- (i) If this circle covers the *chādya circle* (inner circle) and goes beyond, then the eclipse is *total*.
- (ii) If it covers a portion of the inner circle then the eclipse is partial.
- (iii) If the circle does not cut the inner circle, there will be no eclipse.

Śloka 21, 22 and 23: Find the obscuration (grāsamāna) for a given time.

The arc joining the three śara points (of the beginning, the middle and the end) will be in the shape of a bow (dhanur ākāra). The path from the point of madhya śara to that of the sparśa śara is called the "path of getting eclipsed" (grahana mārga). The path from the point of the middle śara to that of the mokṣa śara is called the "path of release" (mokṣa mārga).

Draw another circle with the same centre (as that of the *chādya* circle) and radius equal to half the *difference* of the diameters of the *chādya* 

and chādaka. Take the points of intersection of this circle with the grahaṇa mārga and the mokṣa marga. With these as centres and the radius equal to that of the chādaka (eclipser) draw circles. The points of contact of these two circles with the chādya circle respectively give the points of the beginning and the end of totality (i.e., sammīlanam and the unmīlanam).

The given time (*iṣṭakāla* from the commencement of the eclipse) multiplied by the measure (in *aṅgulas*) of the path (of eclipse or release, as the case is) and divided by the *sthiti* is the obscuration (*grāsa*) at the given time.

Finding the amount of obscuration (grāsa) at a given time (iṣṭakāla):

(a) If the given time is between the *sparśa* and the middle of the eclipse (i.e., if the time interval from *sparśa* is less than the *sparśa sthiti*) then

(b) If the given time is between the middle of the eclipse and the mokṣa, then

# CHAPTER 5 SŪRYAGRAHAŅĀDHIKĀRAḤ

# (Computation of Solar Eclipse)

Śloka 1: For the instant of newmoon (darśānta) the tropical ascendant (sāyana lagna) reduced by three signs (tribhoṇa) is worked out. Its declination (krānti) in palas reduced from or added to (the latitude of the place, akṣa), for the different or the same directions, is the natāṃśa; the same reduced from ninety (degrees) is the unnatāṃśa.

Obtaining of nata, unnata, vitribhalagna is explained.

(i) Find the true Sun, true Moon and *lagna* for the instant of the *darśānta* (i.e. new moon).

Add ayanāṃśa to the (nirayaṇa) lagna to get the sāyana lagna. Subtract 3 rāśis from it to get the (sāyana) vitribha (or tribhoṇa) lagna.

- (ii) Find the krānti (declination) of the (sāyana) vitribha lagna.
- (iii) Then,  $n\bar{a}t\bar{a}m\acute{s}a=kr\bar{a}nti-ak\dot{s}\bar{a}m\acute{s}a=\delta-\phi$  where  $\delta$  = declination of the *tribhonalagna* and  $\phi$  = latitude of the place.

Conventionally, for a place in the northern hemisphere its latitude is taken as southern. However, the above definition, with the modern convention, is clear.

(iv) Unnatāṃśa = 90° – natāṃśa.

Example: Samvat 1657, Śā.Śa. 1522, Aṣāḍha kṛṣṇa amāvāsyā. The darśānta (end of amāvāsyā) = 29|24 gh., Ayanāṃśa =  $17^{\circ} 57' 20''$ .

At the instant of the darśānta, we have

True Sun =  $3^R$  0° 35′ 08″ (nirayana)

 $\therefore S\bar{a}yana \text{ Sun} = 3^R 18^\circ 32' 28''$ 

 $S\bar{a}yana\ lagna = 8^R\ 25^{\circ}\ 14'\ 58''$ 

 $\therefore$  Vitribha lagna =  $5^R$  25° 14′ 58″

Akṣāṃśa (of the place),  $\phi = 24^{\circ} 35' 09'' N$ .

Krānti of Vitribha lagna,  $\delta = 1^{\circ} 54' 58'' N$ .

 $Natāmśa = \delta - \phi = 1^{\circ} 54' 38'' - 24^{\circ} 35' 09'' = 22^{\circ} 40' 31''$  South.

 $Unnatāmśa = 90^{\circ} - 22^{\circ} 40' 31'' = 67^{\circ} 19' 29''$ .

Ślokas 2 and 3: The Rsine (jyā) of the difference of the tribhonalagna and the (longitude of the) Sun divided by 30 is the (mean, madhya) elongation (lambana) in ghaṭīs. That multiplied by Rsine of unnatāmśa and divided by 120 is the corrected (sphuṭa) elongation; this is added to or subtracted from the instant of new moon (darśānta) according as the tribhonalagna is greater or lesser than the Sun.

The Rsine (jyā) of natāmśa added with 1/12th of itself and (the sum) divided by 8 is nati in angulas etc. in the direction of the natāmśa.

Obtaining of lambana and nati is explained.

#### (i) Lambana:

Let  $SR = S\bar{a}y$ ana Ravi,  $SVL = S\bar{a}y$ ana vitribha lagna at the darś $\bar{a}$ nta. Find the bhuja of (SR - SVL), denoted by Bhuja.

Then madhya lambana (mean elongation) is given by

$$Madhya \, lambana = \frac{Jy\bar{a} \, (Bhuja)}{30} \, ghat\bar{l}s$$

Then the corrected (sphuṭa) madhya lambana is given by

$$Spuța\ lambana = \frac{(Madhya\ lambana) \times (Jyā\ of\ Unnatāmśa)}{120}$$

ghațīs.

i.e. sphuṭa lambana =  $4\sin(SR - SVL)\cos(\delta - \phi)ghaṭ\bar{i}s$ .

If SVL > SR and the  $dar \dot{s} \bar{a} nta$  is in the western hemisphere (i.e., between the noon and the sunset), then the  $sphu\dot{t}a$  lambana is added to the  $dar \dot{s} \bar{a} nta$   $(ghat \bar{l}s)$ .

On the otherhand, if SR > SVL and the  $dar \dot{s} \bar{a} nta$  is in the eastern hemisphere (i.e., between the sunrise and the noon) then the  $sphu\dot{t}a$  lambana is subtracted from the  $dar \dot{s} \bar{a} nta$  to get the lambana corrected  $dar \dot{s} \bar{a} nta$ .

#### (ii) Nati:

$$Nati = \frac{1}{8} \left[ Jy\bar{a} \left( Nat\bar{a}m\acute{s}a \right) + \frac{Jy\bar{a} \left( Nat\bar{a}m\acute{s}a \right)}{12} \right] ang$$

$$=\frac{1}{8} \times \frac{13}{12} \times Jy\bar{a}$$
 (Natāṃśa) aṅg.

The direction of the nati is the same as that of the natāmśa.

**Example**: In the example considered, we have  $Nat\bar{a}m\acute{s}a = 22^{\circ} 40' 31'' S$ 

 $S\bar{a}yana$  Ravi,  $SR=\,3^R\,18^\circ\,32^\prime\,28^\prime\prime$ ,  $Unnat\bar{a}m\acute{s}a=\,67^\circ\,19^\prime\,29^\prime\prime$ 

Sāyana Vitribha Lagna, SVL =  $5^R$  25° 14′ 58″, Bhuja of (SR – SVL) =  $66^\circ$  42′ 30″

: Madhya lambana = 
$$\frac{Jy\bar{a} (66^{\circ} 42' 30^{\circ})}{30} = 3|40 \text{ gh.}$$

Note: At this corrected darśānta true Sun, tribhona lagna, etc., are found out and a further lambana corrected darśānta is obtained. This process is iterated till we obtain a convergent value of the darśānta.

∴ Spaṣṭa lambana = 
$$\frac{(3|40) \ Jy\bar{a} \ (67^{\circ} \ 19' \ 28'')}{120} \approx 3|22 \ gh.$$

The spaṣṭa lambana is additive since SVL > SR and the darśānta is in the western hemisphere (paścima kapāla).

Adding this to the darśānta, we get

lambana corrected darsānta = (29|24+3|22) gh. = 32|46 gh.

Repeating the same process for the above corrected darśānta viz.  $32|46 \, gh.$ , we get the second lambana corrected darśānta as  $32|49 \, gh.$ 

We drop further iterations since the difference between the two successive darśānta timings is only 3 vig. At 32|49 gh., we shall find the nati.

We have  $natāmśa = 30^{\circ} 36' 05''$ ,  $unnatāmśa = 59^{\circ} 23' 55''$ .

$$Nati = \frac{1}{8} \times \frac{13}{12} Jy\bar{a} (30^{\circ} 36' 05'') = 8|15 \text{ ang.}$$

Nati is always taken negative.

Ślokas 4 and 5: The (nine) blocks (pindas) of elongation (lambana) are 77, 141, 188, 219, 235, 240, 236, 224 and 200. The (bhuja) of the difference between the tribhonalagna and the Sun divided by 11 (gives) the elapsed block (gata pinda).

The difference between the to-be-covered (gamya) and the elapsed (gata) blocks is multiplied by the remainder (bhuja minus elapsed block) and divided by 11. (The result) divided by 60 and subtracted from or added to (the elapsed block according as) the to-be-covered (bhogya, gamya) block is lesser or greater (than the elapsed block) is the (mean, madhya) elongation (lambana). The corrected one (sphuṭa lambana), by successive approximation (asakṛt), and hence nati are obtained as (explained) earlier.

Obtaining lambana by another method is explained.

The nine lambana piṇḍas are 77, 141, 188, 219, 235, 240, 236, 224, 200. Find the bhuja of the difference between the sāyana vitribha lagna and the sāyana Ravi. Divide by 11 and let the quotient be q and the remainder be r. The quotient q represents the number of piṇḍas completed.

Multiply the remainder r by the difference  $(q+1)^{th}$  piṇḍa minus the  $q^{th}$  piṇḍa and divide by 11. The result is combined (added or subtacted as the case may be) with the  $q^{th}$  piṇḍa. Dividing the resultant piṇḍa by 60, we get the madhyama (mean) lambana in ghaṭ̄s. This is corrected to get the sphuṭa lambana as explained earlier (in Ślokas 2 and 3).

**Example**: At the darśānta instant, we have

Sāyana Ravi  $SR = 3^R 18^{\circ} 32' 28''$  and sāyana vitribha lagna

$$SVL = 5^{R} 25^{\circ} 14' 58''$$
,  $SVL - SR = 2^{R} 6^{\circ} 42' 30''$ ,  $Bhuja = 66^{\circ} 42' 30''$ 

Dividing  $Bhuja = 66^{\circ}42'30''$  by 11, we have

$$q = 6$$
 and  $r = 0^{\circ} 42' 30''$ .

Now, the 6th pinda is 240 and the 7th pinda is 236.

∴ 
$$7^{th}$$
 piṇḍa -  $6^{th}$  piṇḍa =  $236 - 240 = -4$ 

Now, 
$$\frac{r \times (-4)}{11} = \frac{(0^{\circ} 42' 30'') (-4)}{11} = 0|15|27 \text{ vig.}$$

The required  $pinda = 240 \text{ vig.} - 01527 \text{ vig.} \approx 23944 \text{ vig.}$ 

Dividing by 60, we get

(Madhya) Lambana = 
$$3|59|44$$
 gh.

 $Jy\bar{a}$  (unnatāṃśa) =  $Jy\bar{a}$  (67° 19′ 29″) = 110′ 35″ (see eg., under Ślokas 2, 3)

Sphuṭa lambana = 
$$\frac{(3|59|44)(110|35)}{120}$$
 gh. =  $3|40$  gh.

:. Lambana corrected darśānta = 29|24 + 3|40 = 33|04 gh.

(The earlier obtained value is 32|49 gh.)

Ślokas 6 and 7: The latitude is corrected (spaṣṭa śara of the Moon) with the nati and also (obtained) are the obscuration (channam) and the half-duration (sthiti) as earlier (explained in Chapter 4).

The *lambanam*, kept separately (in two places), is added to and subtracted from the instant of the end of the *tithi* (newmoon) reduced

by and added with the half-duration (*sthiti*). The positive or negative corrected latitude (of the Moon, *spaṣṭa śara*) as also the half-duration (are obtained) by subtracting and adding repeatedly these from and to the computed values.

Finding the śara and sthiti is explained.

(i) Find the true longitudes of the Moon (M) and its  $P\bar{a}ta$  (node P). Obtain the  $jy\bar{a}$  of  $sap\bar{a}ta$  Candra (M+P) i.e.,  $120\sin{(M+P)}$   $kal\bar{a}s$ . Then

The nati corrected sara is given by

(Śara and nati may be positive or negative. Their algebraic sum is taken).

(ii) Let  $d_1$  and  $d_2$  be the bimbams (diameters) of the Sun and the Moon.

$$\therefore Gr\bar{a}sa = \begin{bmatrix} \frac{1}{2} (d_1 + d_2) & - Spaṣṭa śara \\ ang. \end{bmatrix} ang.$$

Then the (mean) half-duration (sthiti) given by

$$Sthiti = \frac{\sqrt{(2 \text{ \'sara} + \text{gr\bar{a}sa}) (\text{gr\bar{a}sa})}}{(MDM - SDM)} \times 180 \text{ gh.}$$

where śara is the corrected (spaṣṭa) śara, MDM and SDM are respectively the true daily motions of the Moon and the Sun.

Khagrāsa = Grāsa - Sun's bimbam.

In the case of *total* solar eclipse, the half-duration of totality *marda* is obtained by replacing *grāsa* by *khagrāsa*.

**Example**: We have the corrected  $dar \dot{s} ant a = 32 | 49 \, gh$ . At this instant,

True Moon, 
$$M = 3^R 1^{\circ} 2' 46''$$
,  $P\bar{a}ta$ ,  $P = 2^R 26^{\circ} 21' 38''$ 

$$M + P = 5^R 27^{\circ} 24' 24'', Bhuja = 2^{\circ} 35' 36''$$

However, in the printed commentary, the bhuja is given as  $2^{\circ}$  16' 36''.

$$\therefore$$
 Sara =  $\frac{3}{4} \times Jy\bar{a} (2^{\circ} 16' 36'') = 3|34 N ang.$ 

(since  $M + P < 6^R$ , śara is northern).

We have obtained nati = 8|15 ang. (S). Therefore nati corrected sara,

Spasta śara = 
$$3|34 - 8|15 = 4|41 S$$
 ang.

(Note: Nati is taken always as South).

Sun's bimbam 
$$d_1 = \frac{2 \times (56'58'')}{11} = 10|21$$
 ang.

Moon's bimbam 
$$d_2 = \frac{819' \, 04''}{74} = 11 | 04 \text{ ang.}$$

$$Gr\bar{a}sa = \frac{1}{2}(d_1 + d_2) - spaṣṭa śara$$

$$= 10|42 - 4|41 = 6|01$$
 ang.

 $Sthiti = 2 \mid 16 \text{ gh.}$ 

where MDM = 819'04'' and SDM = 56'58'' are the true daily motions of the Moon and the Sun.

Since grāsa < Sun's bimbam, the solar eclipse is not total.

Śloka 8: The improved instants of the commencement of the eclipse and its release are obtained by applying the *lambana* (correction). The half-intervals between the middle (of the eclipse) and the beginning and the end (of the eclipse) are corrected and the rest are explained in (the context of) lunar eclipse.

Obtaining the sparśa and mokṣa of the solar eclipse.

Subtracting the sthiti from and adding the stithi to the darśānta respectively we get the tentative instants of sparśakāla and mokṣakāla. Considering these instants as iṣṭakālas, determine sāyana Ravi, sāyana lagna, nati and śara. From the corrected sthiti, obtain the above parameters again. Thus, by successive iterations the final convergent values for sthiti, darśānta, sparśa kāla and mokṣa kāla etc. are obtained.

We shall consider a modern example.

Example: August 11, 1999, Wednesday, at Bangalore [Longitude, 77° E' 35'; Latitude, 13°N].

At 5-30 a.m. (IST) we have

True (*nirayaṇa*) Sun =  $3^R 24^{\circ} 03' 31''$ 

True (nirayaṇa) Moon =  $3^R$  17° 59′

Sun's true daily motion, SDM = 57'35''

Moon's true daily motion, MDM = 837'.

The instant of conjuction of the Sun and the Moon,

 $Darśanta = 16^h 43^m 27^s IST.$ 

At the  $dar \hat{s} ant a$  i.e., at  $16^h 43^m 27^s IST$ , we have

True Sun =  $114^{\circ} 30' 27''$ , True moon =  $114^{\circ} 30' 27''$ 

Ayanāṃśa = 23° 51', sāyana lagna = 288° 20'

∴ Sāyana Vitribha Lagna = 198° 20′ ≡ SVL

Declination of  $SVL = -7^{\circ} 21' 01'' = \delta$ 

$$Nat\bar{a}m\acute{s}a = \delta - \phi = -7^{\circ} 21' 01'' - 13^{\circ} = 20^{\circ} 21' 01'' S$$

where  $\phi = 13^{\circ} N$ , the akṣāṃśa (latitude) of Bangalore. Considering the numerical value of the natāṃśa, we have

$$Unnat\bar{a}m\acute{s}a = 90^{\circ} - 20^{\circ} 21' 01'' = 69^{\circ} 38' 59''$$

:.  $Jy\bar{a} (unnat\bar{a}m\acute{s}a) = 120' \sin(69^{\circ} 38' 59'') = 112' 30''$ 

#### Finding lambana and nati:

Sāyana Ravi, SR = 138° 21′ 27″

Madhya Lambana = 
$$\frac{Jy\bar{a} (SVL - SR)}{30} = 3|27 \text{ gh.}$$

Spaṣṭa Lambana = 
$$\frac{(3|27) \times (112|30)}{120}$$
 =  $3|14$  gh.

i.e., 
$$1^h 17^m 36^s$$

Lambana corrected darśānta =  $16^{h} 43^{m} 27^{s} + 1^{h} 17^{m} 36^{s} = 18^{h} 01^{m} 03^{s}$ 

i.e.,  $12^h 31^m 03^s$  from 5-30 a.m. IST

#### At the first corrected darśānta:

At  $18^h 01^m 03^s$  (IST), we have

Sāyana Ravi SR = 138° 24′ 32″, Sāyana Lagna = 307° 57′

∴ Sāyana Vitribha Lagna, SVL = 217° 57′

Spaṣṭa Lambana = 3|29|22 gh. =  $1^h 23^m 45^s$ 

: Second corrected darśānta =  $16^h 43^m 27^s + 1^h 23^m 45^s = 18^h 07^m 12^s$ 

#### At the second corrected darśānta:

At  $18^h 07^m 12^s$  (IST), we have

Sāyana Ravi = 138° 24' 47", Sāyana Lagna = 310° 26'

∴ Sāyana Vitribha Lagna SVL = 220° 26′

Spaṣṭa Lambana =  $3^{gh} 29^{vig} 16^{pvg} = 1^h 23^m 43^s$ 

.. Third corrected  $dar \dot{s} ant a = 16^h 43^m 27^s + 1^h 23^m 43^s = 18^h 07^m 10^s$ .

Since the difference between the second and third corrected instants of

darśanta is only 2 seconds, we take  $18^h$   $07^m$   $10^s$  as the finally corrected darśanta.

#### Nati at the corrected darśanta:

At the corrected  $dar \hat{s} ant a = 18^h \ 07^m \ 10^s$  we have

 $Natāmśa = 28^{\circ} 17' 43''$ 

:. 
$$Nati = [Jy\vec{a} (28^{\circ} 17' 43'')] \times \frac{13}{12 \times 8} = 7|42 \text{ ang. } (S)$$

True Moon  $M = 115^{\circ} 19' 06''$ , Rāhu  $R = 109^{\circ} 10' 56''$ 

 $Jy\bar{a} (Moon - R\bar{a}hu) = 120' \sin(M - R) = 12'50''$ 

$$\therefore \acute{S}ara = \frac{3}{4} Jy\bar{a} (M - R) = 9 | 37 ang. (N)$$

:. Spasta śara = Nati + Śara (algebraic sum) = -7|42 + 9|37 = 1|55 ang.

Bimba (diameters of the Sun and the Moon):

Moon's diameter 
$$d_1 = \frac{\text{Moon's true daily motion}}{74}$$

$$=\frac{837'}{74}=11|19 \text{ angulas}|$$

Suns diameter  $d_2 = \frac{2}{11} \times \text{Sun's true daily motion}$ 

$$=\frac{2}{11}\times 57'35''=10|28 \text{ angulas}$$

i.e.,  $d_1 = 11|19$  ang. and  $d_2 = 10|28$  ang.

Bimbardha yoga = 
$$\frac{1}{2}(d_1 + d_2) = 10|53$$
 ang.

Grāsa (Channam) = 
$$\frac{1}{2} (d_1 + d_2) - \acute{s}ara = 10 |53 - 1|55 = 8 |58|$$
 ang.

$$Sthiti = \frac{\sqrt{(2 \times \acute{s}ara + gr\bar{a}sa) \times gr\bar{a}sa}}{(MDM-SDM)} \times 180 \text{ gh.} =$$

$$2|28|26 \ gh. \equiv 0^h \ 59^m \ 23^s$$

#### Sparśa and Makṣa:

Sparśa kāla = Darśānta - sthiti

$$= 18^{h} 07^{m} 10^{s} - 0^{h} 59^{m} 23^{s} = 17^{h} 7^{m} 47^{s}$$
 (IST)

#### Spārśika lambana and nati:

Sparśa kāla =  $17^h 7^m 47^s$ 

At that instant, we have

 $S\bar{a}$ yana Ravi  $SR = 138^{\circ} 22' 25''$ ,  $S\bar{a}$ yana Lagna = 294° 27'

Sāyana Vitribha Lagna, SVL = 204° 27′

Declination of  $SVL = 9^{\circ} 41' 30'' S$ 

 $Natāmśa = 22^{\circ} 41' 30'' S$ ,  $Unnatāmśa = 67^{\circ} 18' 30''$ 

Madhya Lambana = 3gh 39vig

## S104 KARAŅAKUTŪHALAM OF BHĀSKARĀCĀRYA II

Spasta Lambana =  $3^{gh} 22^{vig} 23^{pvg} = 1^{h} 20^{m} 57^{s}$ 

Lambana corrected sparśa  $k\bar{a}la = 17^h 7^m 47^s + 1^h 20^m 57^s = 18^h 28^m 44^s$ .

At the lambana corrected sparśa kāla:

At  $18^h \ 28^m \ 44^s$  (IST), we have

True Sun = 114° 34′ 39″, True Moon = 115° 31′ 38″ (both *nirayaṇa*)

Sāyana Lagna = 315° 56'

Sāyana Vitribha Lagna, SVL = 225° 56′

Declination of  $SVL = -16^{\circ} 59' 34'' \equiv \delta$ 

 $Nat\bar{a}m\acute{s}a=\delta-\phi=29^{\circ}\,59'\,34''\,S\,,\;\;Unnat\bar{a}m\acute{s}a=60^{\circ}\,0'\,26''$ 

Nati = 8|07 ang. S,  $R\bar{a}hu = 109^{\circ} 10' 56''$ , S' ara = 9|57 ang. N

Spaṣṭa śara = -8|07+9|57 = 1|50 aṅg. N

 $Gr\bar{a}sa\ (Channam) = 10|53 - 1|50 = 9|03 \text{ ang}$ 

 $Sthiti = 2|28|39 gh. = 0^h 59^m 28^s$ 

First corrected sparśa kāla:

Sparśa kāla = Darśānta – sthiti

 $= 18^{h} 07^{m} 10^{s} - 0^{h} 59^{m} 28^{s} = 17^{h} 07^{m} 42^{s} IST$ 

#### At the corrected sparśa kāla:

At  $17^h 07^m 42^s$  (IST), proceeding as before, we have

Spasta Lambana =  $3^{gh} 22^{vig} = 1^h 20^m 57^s$ 

Lambana corrected sparśa kāla

$$= 17^{h} 07^{m} 42^{s} + 1^{h} 20^{m} 57^{s} = 18^{h} 28^{m} 39^{s}$$
 (IST)

Natāmśa =  $30^{\circ} 0' 27'' S$ , Unnatāmśa =  $59^{\circ} 59' 33''$ 

Nati = 8|07|36 ang. S, Śara = 9|56|42 ang. N

Spaṣṭa śara =  $-8|07|36 + 9|56|42 \approx 1|49$  aṅg.

 $Gr\bar{a}sa = 9|04$  ang.

$$Sthiti = 2|28|41 gh. \equiv 0^h 59^m 28^s$$

Second corrected sparśa kāla

$$= 18^{h} 07^{m} 10^{s} - 0^{h} 59^{m} 28^{s} = 17^{h} 07^{m} 42^{s}$$
 (IST)

Since the first and second corrected instants of sparśa  $k\bar{a}la$  are the same, we take the finally corrected sparśa  $k\bar{a}la$  as  $17^h~07^m~42^s$  (IST)

#### Mokṣa kāla :

We have

Mokṣa kāla = Darśānta + sthiti

$$= 18^h \ 07^m \ 10^s + 0^h \ 59^m \ 23^s \ = 19^h \ 06^m \ 33^s \ (\mathrm{IST})$$

Mokṣa kāla lambana, sthiti etc. :

At the mokṣa kāla  $19^h 06^m 33^s$  (IST), we have

 $S\bar{a}yana \text{ Ravi}, SR = 138^{\circ} 27' 10''$ 

S106

Sāyana Lagna = 326°, Sāyana Vitribha Lagna, SVL = 236°

Declination of  $SVL = 19^{\circ} 42' 22'' S$ 

Natāniśa = 32° 42′ 22″ S, Unnatāmśa = 57° 17′ 38″

Madhya lambana =  $3|57|55 \text{ gh.} \equiv 1^h 35^m 10^s$ 

Spasta lambana =  $3|20|11 \text{ gh.} \equiv 1^h 20^m 04^s$ 

: Lambana corrected mokṣa kāla

$$= 19^{h} 06^{m} 33^{s} + 1^{h} 20^{m} 04^{s} = 20^{h} 26^{m} 37^{s} IST$$

### At the lambana corrected mokṣa kāla:

 $Natamśa = 36^{\circ} 33' 35'' S$ ,  $Unnatamśa = 53^{\circ} 26' 25''$ 

Nati = 9|40|46 ang., Sara = 11|43|37 ang.

Spaṣṭa śara = 2|02 ang.

 $Gr\bar{a}sa = 8|51$  ang.

$$Sthiti = 2|28|08 gh. \equiv 0^h 59^m 15^s$$

First corrected moksa kāla = Darśānta + sthiti

= 
$$18^h 07^m 10^s + 0^h 59^m 15^s = 19^h 06^m 25^m$$
 (IST)

#### Second corrected mokṣa kāla:

At  $19^h 06^m 25^s$  IST, we have

 $S\bar{a}yana \text{ Ravi} = 138^{\circ} 31' 10'' \equiv SR$ ,  $S\bar{a}yana \text{ Lagna} = 325^{\circ} 59'$ 

Sāyana Vitribha Lagna = 235° 59′ ≡ SVL

Declination of  $SVL = 19^{\circ} 42' 08'' S$ 

 $Natamśa = 32^{\circ} 42' 08'' S$ ,  $Unnatamśa = 57^{\circ} 17' 52''$ 

Madhya lambana = 3|57|57 gh. =  $1^h 35^m 11^s$ 

Spaṣṭa lambana =  $3|20|14 \text{ gh.} = 1^h 20^m 06^s$ 

Lambana corrected moksa kāla

$$= 19^h \ 06^m \ 25^s + 1^h \ 20^m \ 06^s = 20^h \ 26^m \ 31^s$$
 (IST)

At this instant i.e., at  $20^h 26^m 31^s$ :

We have

 $Natam\acute{s}a = 36^{\circ} 34' 05'' S$ ,  $Unnatam\acute{s}a = 53^{\circ} 25' 55''$ 

Nati = 9|40|52 ang., Sara = 11|43|32 ang.

Spaṣṭa śara = 2|2|39 aṅg.

 $Gr\bar{a}sa = 8|51$  ang.

 $Sthiti = 2|28|07 gh. \equiv 0^h 59^m 15^s$ 

Second corrected  $mok ilde{s}a$   $k ilde{a} la = 18^h 07^m 10^s + 0^h 59^m 15^s = 19^h 06^m 25^s$  (IST).

Since the instance of the first and second corrected  $mokṣa k\bar{a}la$  are the same, we take the finally corrected  $mokṣa k\bar{a}la$  as  $19^h~06^m~25^s$  (IST).

Further, since grāsa < Sun's bimba, the solar eclipse is partial.

Summary of the solar eclipse which occurred on August 11, 1999:

Sparśa kāla (beginning) :  $17^h 07^m 42^s$  IST

Darśānta (middle) :  $18^h 07^m 10^s IST$ 

Moksa kāla (end) :  $19^h 06^m 25^s$  IST

#### at Bangalore

Note: According to the *Indian Astronomical Ephemeris*, at Bangalore,  $Spar\acute{s}a$   $k\bar{a}la$  is  $17^h$   $12^m$   $04^s$  IST and the middle at  $18^m$   $12^m$   $03^s$  IST. Since the *mokṣa*  $k\bar{a}la$  of the eclipse at Bangalore is after the sunset  $(18^h$   $42^m$   $05^s$  IST), it is not visible.

Śloka 9: One-twelfth of the Sun and one-sixteenth of the Moon, even if eclipsed, are not to the declared. The Moon is smoky (in colour) in less than half eclipse, dark in half-eclipse and reddish brown in total eclipse. The Sun (in a solar eclipse) is black.

The colour of the eclipsed Sun and Moon is mentioned.

In a solar eclipse, if the  $gr\bar{a}sa$  is less than or equal to  $\frac{1}{12}^{th}$  of the Sun's diameter (bimbam), then the eclipse is not predicted.

Similarly, in a lunar eclipse, if the  $gr\bar{a}sa$  is less than or equal to  $\frac{th}{16}$  of the Moon's diameter (bimbam), the eclipse is not predicted.

In a lunar eclipse, if the magnitude is less than half then the Moon's colour is one of *smoke*; if the magnitude is  $\frac{1}{2}$  then the colour is black. In a total lunar eclipse the Moon's colour is reddish brown (or tawny, piśanga). In a solar eclipse, the Sun is always black.