THE EXTANT *SIDDHĀNTA ŚEKHARA*: AN ERROR IN ONE OF ITS SINE VALUES

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The *Siddhānta Śekhara*, an astronomical work of Śrīpati (eleventh century A.D.) in its *Sphuṭādhyāya*, has given 24 sine values, as in the other Siddhāntic works. The sine value for 78° 45′, as given in the extant text, is observed to be erroneous by 2′. A study of the relevant textual portion reveals that this must be due to the defective copying of the portion concerned. The paper attempts to throw light on this aspect.

Hindu astronomers in their sine tables have given the values of $R \sin \theta$ and not of $\sin \theta$. The value of R adopted is 60′ (*Siddhānta-tattva-viveka*[1]), 120′ (*Pañcasiddhāntikā*[2] and *Siddhāntaśiromaṇi*[3]), 150′ (*Khaṇḍakhādyaka*[4]), 3,270′ (*Brāhmasphuṭasiddhānta*[5]), 3,415′ (*Siddhānta Śekhara*[6]), 3,438′ (*Āryabhaṭīyam*[7], *Mahāsiddhānta*[8], *Siddhāntaśiromaṇi*[9] and *Sūryasiddhānta*[10]), 3,437′ 44″ (*Vaṭeśvara-Siddhānta*[11]) and 3,437′ 44″ 48‴ (Mādhava[12]). In the last six works the value of R adopted is the circumference of a circle in minutes divided by 2π. In the *Siddhānta Śekhara* of Śrīpati, the value of π used is $\sqrt{10}$, which in turn gives the value of R as 3,415′. If we take $\pi = 3 \cdot 1416$, as given by Āryabhaṭa I, R comes out to be nearly 3,437′ 44″. This is the value adopted in *Vaṭeśvara-Siddhānta*. But in the *Āryabhaṭīyam*, the *Mahāsiddhānta*, the *Siddhāntaśiromaṇi* and the *Sūryasiddhānta*, the approximate value 3,438′, has been followed. Mādhava, a fourteenth-century astronomer of Kerala, was well aware of the value of π up to 11 decimal places[13] and hence was able to adopt the most accurate value of R. It would appear that the other four values for R must have been adopted arbitrarily.

Āryabhaṭa II (the *Mahāsiddhānta*) and Bhāskara II (the *Siddhānta-śiromaṇi*) have given the same table of *jyā* which would be derived from the table of differences of Āryabhaṭa I with the exception that the value of *jyā* for 60° given by them is 2,977′, while that given by Āryabhaṭa I is 2,978′. In the *Sūryasiddhānta* the values given are exactly the same as those given by Āryabhaṭa I.

A study of the values given by Āryabhaṭa I, Brahmagupta and Śrīpati shows that, except in one case, the errors in the values of R sines stated by them are nearly of the same order. Since the ācāryas were unfamiliar with the decimal fractional notation and stated the values of R sines only up to

integral numbers, their values of sin θ are not quite exact. If we denote by θ' the angle calculated from their values of $R \sin \theta$, then $\triangle \theta = \theta' - \theta$ has a small value except for $\theta = 30°$ and $\theta = 90°$. This is shown in Table I.

TABLE I

		(Āryabhaṭa)	(Brahmagupta)	(Śrīpati)
3°	45′	+ 9″	+ 8″	— 21″
7°	30′	+ 15″	+ 11″	— 46″
11°	15′	+ 17″	+ 4″	— 14″
15°	0′	+ 11″	— 22″	+ 8″
18°	45′	— 7″	— 7″	+ 19″
22°	30′	— 42″	— 26″	+ 10″
26°	15′	— 39″	— 20″	— 28″
30°	0′	0″	0″	+ 35″
33°	45′	— 4″	+ 22″	+ 53″
37°	30′	+ 7″	+ 28″	+ 16″
41°	15′	+ 29″	— 5″	+ 27″
45°	0′	— 4″	— 32″	+ 19″
48°	45′	+ 16″	+ 47″	+ 42″
52°	30′	+ 45″	— 27″	— 30″
56°	15′	+ 42″	+ 11″	— 52″
60°	0′	+ 72″	+ 12″	+ 64″
63°	45′	+ 75″	+ 31″	+ 25″
67°	30′	+111″	— 14″	— 9″
71°	15′	+ 85″	— 93″	+ 46″
75°	0′	+ 33″	+104″	+ 84″
78°	45′	+ 16″	— 54″	—732″
82°	30′	+180″	— 12″	+102″
86°	15′	+342″	+ 14″	+300″
90°	0′	0″	0″	0″

A study of this Table shows that the values stated by Brahmagupta are more nearly correct, especially for the large angles, than those stated by Āryabhaṭa I and Śrīpati. But a more important point emerges that Śrīpati's value of $R \sin \theta$ for $\theta = 78° 45'$ is exceptionally low. The value stated by him for this angle as found in the extant *Siddhānta Śekhara* is *nagasāgarāmarāḥ*, i.e. 3,347′. Since, conceivably Śrīpati's value for this particular angle cannot be so much in error, it was thought at first that the value stated by Śrīpati was *nāgasāgarāmarāḥ*, i.e. 3,348′ and due to the defective copying of the word

nāga had been changed into *naga*. But an examination of the metre showed that it cannot be *nāga*. It was then conjectured that originally it must have been *nava* which would give $R \sin (78° \ 45') = 3{,}349'$. This would make $\Delta\theta = -117''$ which would be comparable to other values of $\Delta\theta$ in the neighbourhood.

This conjecture is supported by the *Utkramajyā* value of $11° \ 15'$. It is easy to prove that

$R \sin\theta + R$ versin $\left(\dfrac{\pi}{2}-\theta\right) = R.$ The *Utkramajyā* value for $11° \ 15'$ as given by Śrīpati is $66'$ (*ṣaḍrasā*). Therefore $R \sin (78° \ 45')$ must be $3{,}415'-66' = 3{,}349'$ and not $3{,}347'$.

Another method to get the same result would be to apply the method given by Āryabhaṭa I in the *Āryabhaṭīyam*[14]. This gives the relationship between successive *khaṇḍajyās*. According to this

$$R \sin (\theta+\alpha) - R \sin \theta = R \sin \theta - R \sin (\theta-\alpha)$$

$$- \frac{R \sin \theta}{R \sin \alpha},$$

where $\alpha = 3° \ 45'$. Hence,

$$R \sin (78° \ 45') - R \sin 75° = R \sin 75° - R \sin (71° \ 15')$$

$$- \frac{R \sin 75°}{223}.$$

According to Śrīpati, $R \sin (71° \ 15')$ is given by *vedarāmadaśanā*, i.e. $3{,}234'$ and $R \sin 75°$ is given by *navāṅkadṛagvahnayo*, i.e. $3{,}299'$. Inserting these values in the above relation, we have

$$R \sin (78° \ 45') = 2 \times 3{,}299' - 3{,}234' - \frac{3{,}299'}{223}$$

$$= 3{,}364' - 14{\cdot}8'$$

$$= 3{,}349{\cdot}2'.$$

Since Śrīpati states only the integral values, $R \sin (78° \ 45') = 3{,}349'$.

The *jyā* values have been stated in the *Katapayādi* system by the commentator, and here the value given is '*savāṅgagaḥ*,' i.e. $3{,}347$. Babuāji Misra, who has edited and partly commented on the *Siddhānta Śekhara*, after stanzas 75 of the fourth chapter, does not include the five stanzas giving the *jyā* and *Utkramajyā* values as part of the main text, because the values of revolutions, etc., are stated in the *Katapayādi* system only by Makkibhaṭṭa, the commentator of the earlier portion. It seems therefore that the error in the *jyā* value of $78° \ 45'$ in the *Siddhānta Śekhara* had crept in even before Makkibhaṭṭa wrote his commentary.

REFERENCES

1 *Siddhānta-tattva-viveka* of Kamalākara Bhaṭṭa, edited with notes by Sudhākara Dvivedi and Muralidhara Jha, Banaras, 1925, p. 168.

2 *Pañchasiddhāntikā* of Varāha Mihira, edited by G. Thibault and Sudhākara Dvivedi, 4, 6–11.

3 *Siddhāntaśiromaṇi, Spaṣṭādhikāra*, 13.

4 *Khaṇḍakhādyaka* of Brahmagupta, 3, 6.

5 *Brāhmasphuṭasiddhānta* of Brahmagupta, *Spaṣṭādhikāra*, 2–5.

6 *Siddhānta Śekhara* of Śrīpati, edited by Babuāji Misra. Pt. I, 1932.   *Sphuṭādhyāya*, 7–10.

7 *Āryabhaṭīyam* of Āryabhaṭa I, *Daśagītikā, Āryā*, 12.

8 *Mahāsiddhānta* of Āryabhaṭa II, edited with his own commentary by Sudhākara Dvivedi 1910.   *Spaṣṭādhikāra*, 4–5½.

9 *Siddhāntaśiromaṇi, Spaṣṭādhikāra*, 3–6.

10 *Sūrya Siddhānta*, 2, 17–22.

11 *Vaṭeśvara-Siddhānta*, Vol. 1, edited by R. S. Sharma and M. Misra. Published by Institute of Astronomical and Sanskrit Research, New Delhi, 1962, pp. 212–309.

12 Mādhava as quoted by the commentator Śankara Vāriar in *Tantrasaṃgraha* of Nīlkanṭha, Somasutvan, edited by Sūranād Kunjan Pillai, T.S.S. No. 188, p. 19.

13 Nīlakaṇṭha's commentary on *Āryabhaṭīyam*, edited by K. S. Sastri. T.S.S. No. 101, p. 42. Nīlakaṇṭha says: *Saṅgamagrāmajo mādhavaḥ punaratyāsannaṃ paridhisankhyāmuktavān. Bibudhanetragajāhihutaśanatriguṇavedabhavāraṇabāhavaḥ navanikharvamite vṛtivistare paridhimānamidaṃ jagadurbudhāḥ.*

14 *Āryabhaṭīyam, Gaṇitapāda*, 12.