

## Eigenfunction

Here we define the Hermite eigenfunction for the harmonic oscillator in one dimension.

$$\text{In[15]:= } \psi[n_, x_, \alpha_] := \left( \frac{\alpha}{2^n n! * \sqrt{\pi}} \right)^{\frac{1}{2}} \text{HermiteH}[n, \alpha * x] e^{-\alpha^2 * x^2 / 2}$$

Let's define some constants :

$$\text{In[5]:= } w_1 = \sqrt{\frac{0.1463}{15840}};$$

$$w_2 = \sqrt{\frac{0.5356}{14520}};$$

$$\alpha_1 = (w_1 * 15840)^{\frac{1}{4}};$$

$$\alpha_2 = (w_2 * 14520)^{\frac{1}{4}};$$

This the wavefunction in two dimensions.

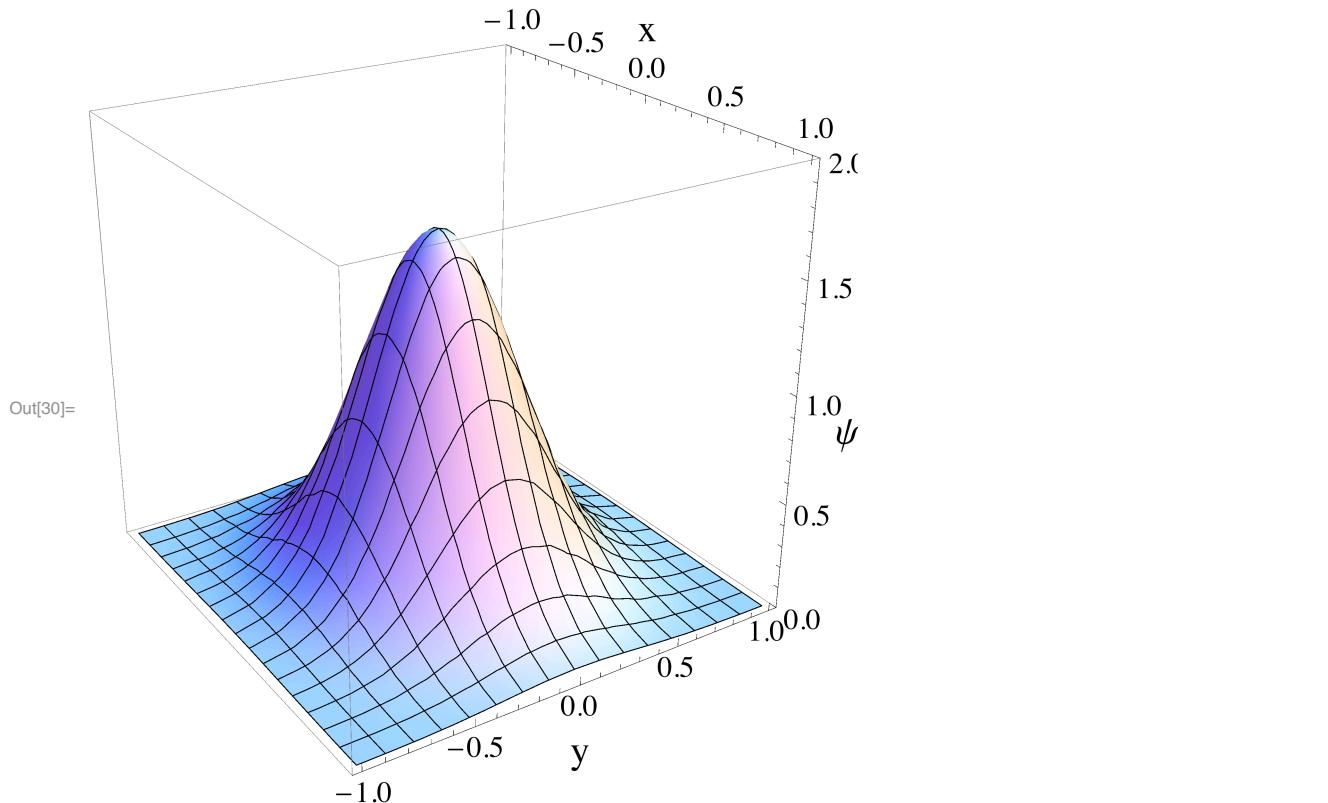
$$\text{In[9]:= } \Psi[n_, m_] := \psi[n, x, \alpha_1] * \psi[m, y, \alpha_2]$$

(a) Plot the wave functions for

- (1) the ground state of this 2D harmonic oscillator as a function of x and y.

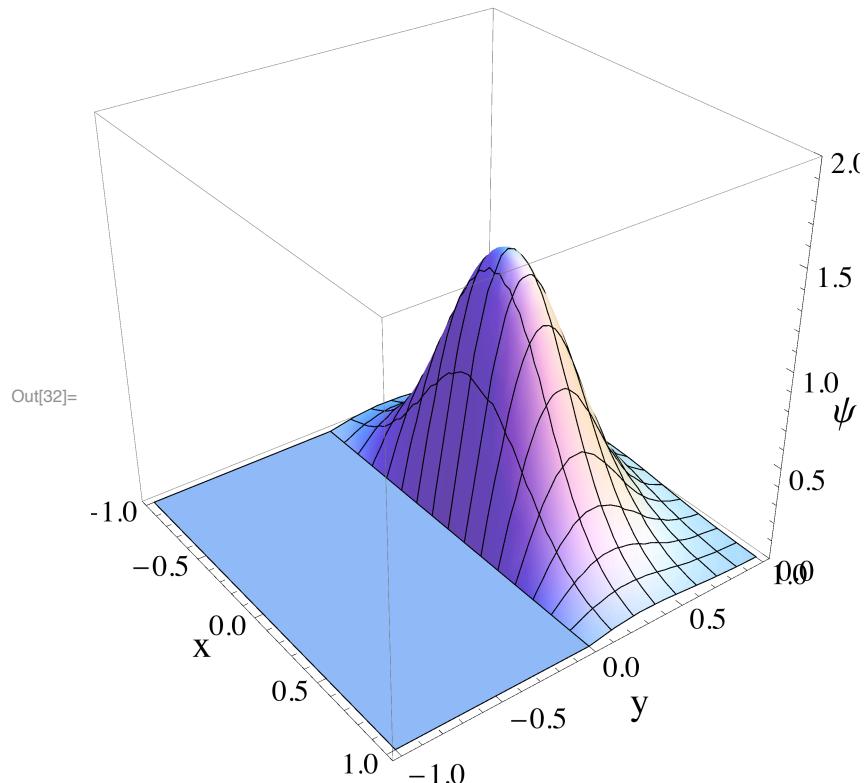
```
In[26]:= a = 1;
b = 1;
min = 0;
max = 2;

Plot3D[\Psi[0, 0], {x, -a, a}, {y, -b, b}, PlotRange -> {min, max}, BoxRatios -> Automatic,
AxesLabel -> {Text[Style["x", FontSize -> 18]], Text[Style["y", FontSize -> 18]],
Text[Style["\u03c8", FontSize -> 18]]}, AxesStyle -> {Directive[14], Directive[14], Directive[14]}]
```



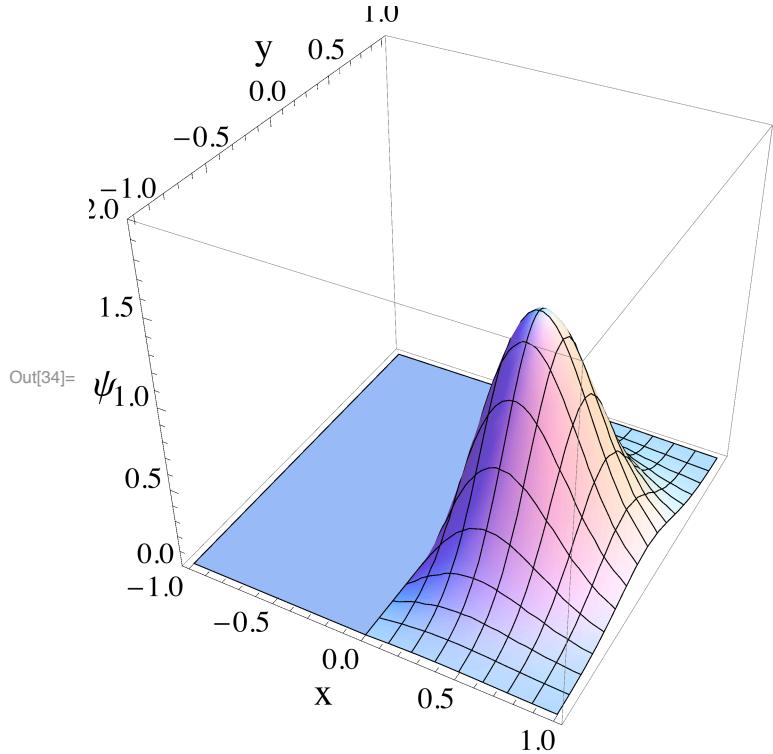
■ (2) the  $n_1=0, n_2=1$  state.

```
In[32]:= Plot3D[\Psi[0, 1], {x, -a, a}, {y, -b, b}, PlotRange -> {min, max}, BoxRatios -> Automatic,
AxesLabel -> {Text[Style["x", FontSize -> 18]], Text[Style["y", FontSize -> 18]],
Text[Style["\u03c8", FontSize -> 18]]}, AxesStyle -> {Directive[14], Directive[14], Directive[14]}]
```



■ (3) the  $n_1 = 1, n_2 = 0$  state.

```
In[34]:= Plot3D[\Psi[1, 0], {x, -a, a}, {y, -b, b}, PlotRange -> {min, max}, BoxRatios -> Automatic,
AxesLabel -> {Text[Style["x", FontSize -> 18]], Text[Style["y", FontSize -> 18]],
Text[Style["\psi", FontSize -> 18]]}, AxesStyle -> {Directive[14], Directive[14], Directive[14]}]
```

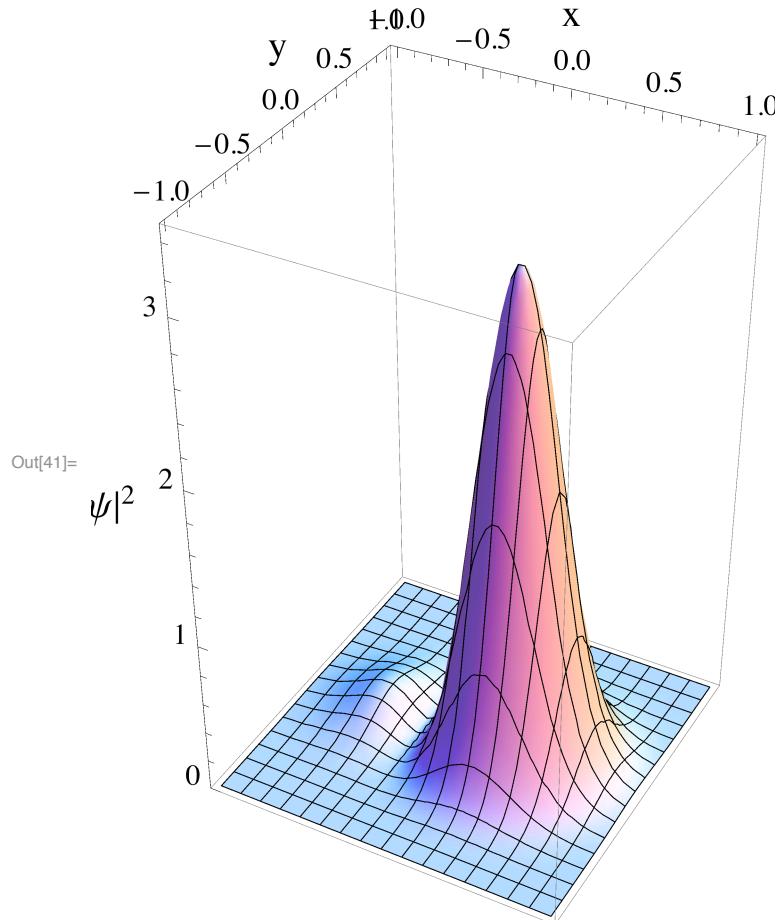


■ (c) Plot "(x,y,t) at t= 0, \tau/4, \tau/2, 3\tau/4, \tau to show a full period of its motion.

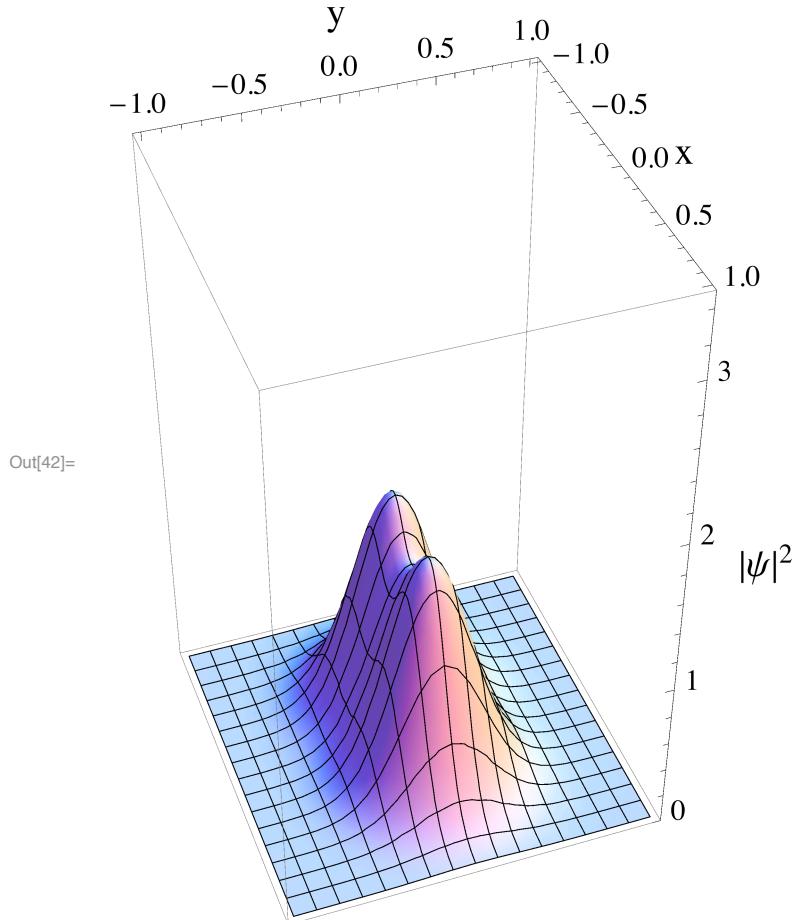
```
In[35]:= P[t_] := \frac{1}{2} (\Psi[0, 0]^2 + \Psi[1, 0]^2) + \Psi[0, 0] \Psi[1, 0] \cos[t (\epsilon_2 - \epsilon_1)]
```

```
In[36]:= \tau := \frac{2 \pi}{(\epsilon_2 - \epsilon_1)}
```

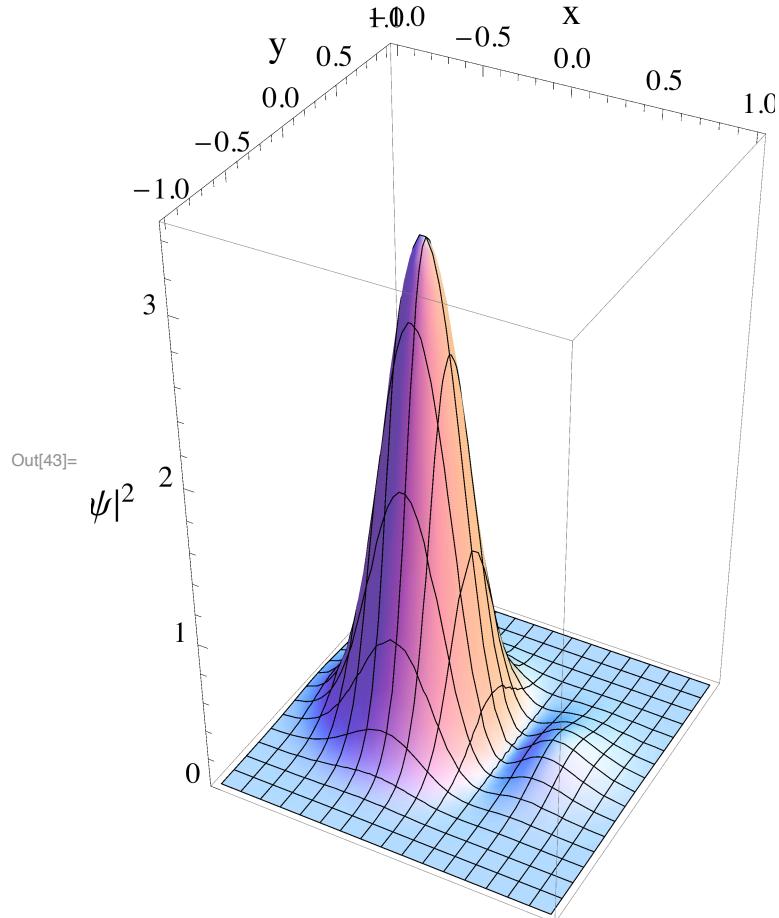
```
In[41]:= Plot3D[P[0], {x, -a, a}, {y, -b, b}, PlotRange -> {min, 3.5},  
BoxRatios -> Automatic, AxesLabel -> {Text[Style["x", FontSize -> 18]],  
Text[Style["y", FontSize -> 18]], Text[Style["|ψ|^2", FontSize -> 18]]},  
AxesStyle -> {Directive[14], Directive[14], Directive[14]}]
```



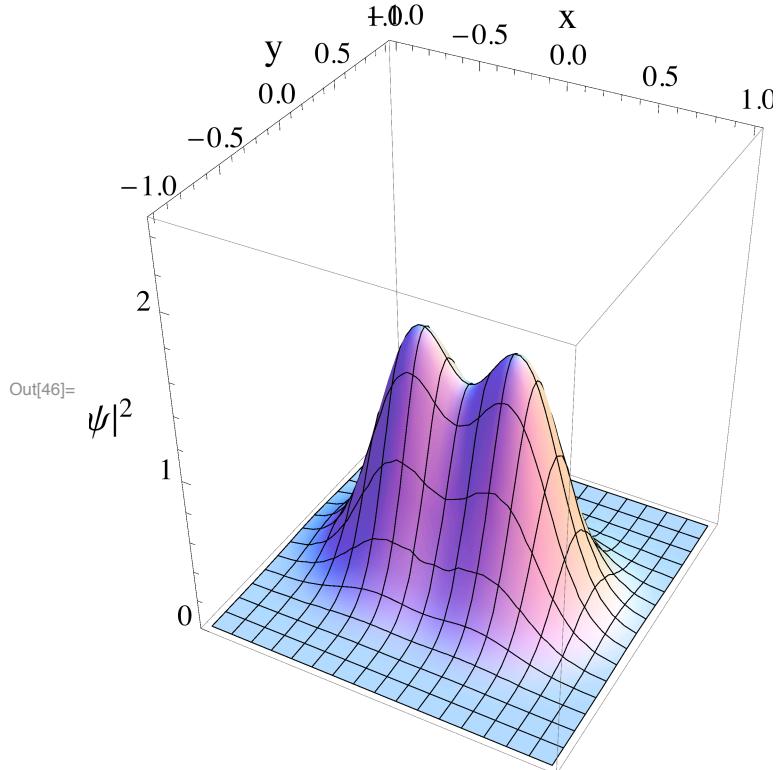
```
In[42]:= Plot3D[P[\tau / 4], {x, -a, a}, {y, -b, b}, PlotRange -> {min, 3.5},  
BoxRatios -> Automatic, AxesLabel -> {Text[Style["x", FontSize -> 18]],  
Text[Style["y", FontSize -> 18]], Text[Style["|\psi|^2", FontSize -> 18]]},  
AxesStyle -> {Directive[14], Directive[14], Directive[14]}]
```



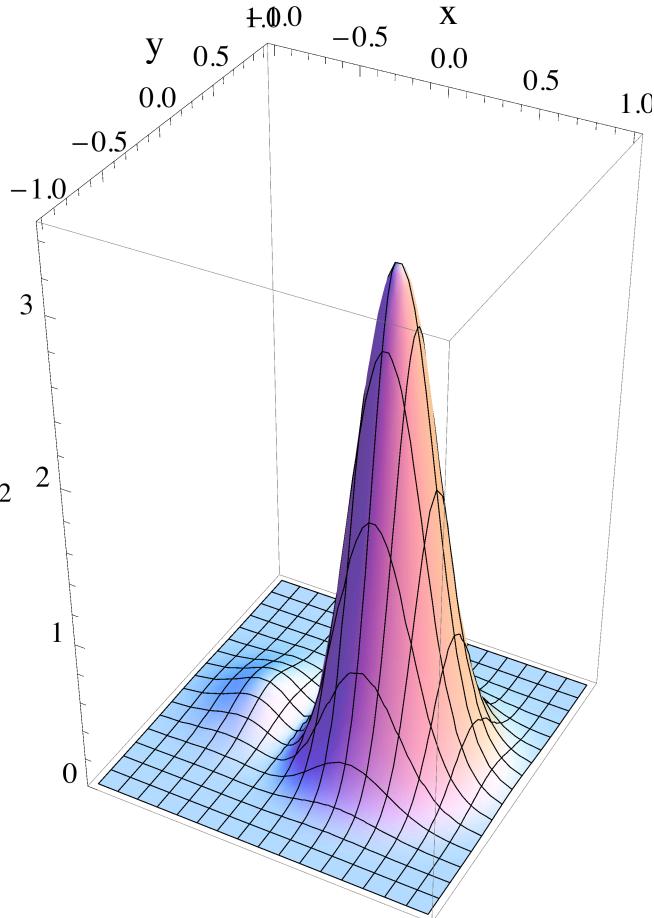
```
In[43]:= Plot3D[P[\tau/2], {x, -a, a}, {y, -b, b}, PlotRange -> {min, 3.5},  
BoxRatios -> Automatic, AxesLabel -> {Text[Style["x", FontSize -> 18]],  
Text[Style["y", FontSize -> 18]], Text[Style["|\psi|^2", FontSize -> 18]]},  
AxesStyle -> {Directive[14], Directive[14], Directive[14]}]
```



```
In[46]:= Plot3D[P[3 \[tau]/4], {x, -a, a}, {y, -b, b}, PlotRange \[Rule] {min, 2.5},  
BoxRatios \[Rule] Automatic, AxesLabel \[Rule] {Text[Style["x", FontSize \[Rule] 18]],  
Text[Style["y", FontSize \[Rule] 18]], Text[Style["|\[psi]|^2", FontSize \[Rule] 18]]},  
AxesStyle \[Rule] {Directive[14], Directive[14], Directive[14]}]
```



```
In[47]:= Plot3D[P[τ], {x, -a, a}, {y, -b, b}, PlotRange → {min, 3.5},
BoxRatios → Automatic, AxesLabel → {Text[Style["x", FontSize → 18]],
Text[Style["y", FontSize → 18]], Text[Style["|ψ|^2", FontSize → 18]]},
AxesStyle → {Directive[14], Directive[14], Directive[14]}]
```



Out[47]=

 $|\psi|^2$ 

- Let's try a different mixed state.

```
In[53]:= P2[t_] :=  $\frac{1}{2} (\Psi[2, 0]^2 + \Psi[0, 1]^2) - \Psi[2, 0] \Psi[0, 1] \cos[t (E_2 - E_1)]$ 
```

```
In[58]:= Plot3D[P2[t / 2], {x, -a, a}, {y, -b, b}, PlotRange -> {min, 2.5},  
BoxRatios -> Automatic, AxesLabel -> {Text[Style["x", FontSize -> 18]],  
Text[Style["y", FontSize -> 18]], Text[Style["|ψ|^2", FontSize -> 18]]},  
AxesStyle -> {Directive[14], Directive[14], Directive[14]}]
```

