Numerical Problem 3

CG is script that takes {j1,j2,J,M} as inputs and yields the ClebschGordan Coefficients attached to corresponding states.

```
Clear[j1, j2, J, M]
CG[j1_, j2_, J_, M_] :=
 Do[

  Do[

   If[m1 + m2 == M, Print[ClebschGordan[{j1, m1}, {j2, m2}, {J, M}],
     "|", j1, ",", m1, "〉", "|", j2, ",", m2, "〉"], 0]

   , {m2, -j2, j2}]

  , {m1, -j1, j1}]
```

- **(1) Use your script to do problem 4.20.**

Problem 4.20 states j1 = 1 and j2 = 1/2. Starting from the J = 3/2 state we will step down to J = 1/2

```
CG[1, 1 / 2, 3 / 2, 3 / 2]
```

$1|1,1\rangle|\frac{1}{2},\frac{1}{2}\rangle$

```
CG[1, 1 / 2, 3 / 2, 1 / 2]
```

$\sqrt{\frac{2}{3}}\,|1,0\rangle|\frac{1}{2},\frac{1}{2}\rangle$

$\frac{1}{\sqrt{3}}|1,1\rangle|\frac{1}{2},-\frac{1}{2}\rangle$

```
CG[1, 1 / 2, 3 / 2, -1 / 2]
```

$\frac{1}{\sqrt{3}}|1,-1\rangle|\frac{1}{2},\frac{1}{2}\rangle$

$\sqrt{\frac{2}{3}}\,|1,0\rangle|\frac{1}{2},-\frac{1}{2}\rangle$

```
CG[1, 1 / 2, 3 / 2, -3 / 2]
```

$1|1,-1\rangle|\frac{1}{2},-\frac{1}{2}\rangle$

```
CG[1, 1 / 2, 1 / 2, 1 / 2]
```

$-\frac{1}{\sqrt{3}}|1,0\rangle|\frac{1}{2},\frac{1}{2}\rangle$

$\sqrt{\frac{2}{3}}\,|1,1\rangle|\frac{1}{2},-\frac{1}{2}\rangle$

```
CG[1, 1 / 2, 1 / 2, -1 / 2]
```

$-\sqrt{\frac{2}{3}}\ |1,-1\rangle\,|\frac{1}{2},\frac{1}{2}\rangle$

$\frac{1}{\sqrt{3}}\ |1,0\rangle\,|\frac{1}{2},-\frac{1}{2}\rangle$

- **(2) Use your script to find all 9 angular momentum eigenstates with L=4, and M = -4 ... +4 that arise from ccoupling two d electrons (l = 2)**

```
CG[2, 2, 4, -4]
```

$1\,|2,-2\rangle\,|2,-2\rangle$

```
CG[2, 2, 4, -3]
```

$\frac{1}{\sqrt{2}}\ |2,-2\rangle\,|2,-1\rangle$

$\frac{1}{\sqrt{2}}\ |2,-1\rangle\,|2,-2\rangle$

```
CG[2, 2, 4, -2]
```

$\sqrt{\frac{3}{14}}\ |2,-2\rangle\,|2,0\rangle$

$\frac{2}{\sqrt{7}}\ |2,-1\rangle\,|2,-1\rangle$

$\sqrt{\frac{3}{14}}\ |2,0\rangle\,|2,-2\rangle$

```
CG[2, 2, 4, -1]
```

$\frac{1}{\sqrt{14}}\ |2,-2\rangle\,|2,1\rangle$

$\sqrt{\frac{3}{7}}\ |2,-1\rangle\,|2,0\rangle$

$\sqrt{\frac{3}{7}}\ |2,0\rangle\,|2,-1\rangle$

$\frac{1}{\sqrt{14}}\ |2,1\rangle\,|2,-2\rangle$

```
CG[2, 2, 4, 0]
```

$\frac{1}{\sqrt{70}} |2,-2\rangle |2,2\rangle$

$2\sqrt{\frac{2}{35}} |2,-1\rangle |2,1\rangle$

$3\sqrt{\frac{2}{35}} |2,0\rangle |2,0\rangle$

$2\sqrt{\frac{2}{35}} |2,1\rangle |2,-1\rangle$

$\frac{1}{\sqrt{70}} |2,2\rangle |2,-2\rangle$

```
CG[2, 2, 4, 1]
```

$\frac{1}{\sqrt{14}} |2,-1\rangle |2,2\rangle$

$\sqrt{\frac{3}{7}} |2,0\rangle |2,1\rangle$

$\sqrt{\frac{3}{7}} |2,1\rangle |2,0\rangle$

$\frac{1}{\sqrt{14}} |2,2\rangle |2,-1\rangle$

```
CG[2, 2, 4, 2]
```

$\sqrt{\frac{3}{14}} |2,0\rangle |2,2\rangle$

$\frac{2}{\sqrt{7}} |2,1\rangle |2,1\rangle$

$\sqrt{\frac{3}{14}} |2,2\rangle |2,0\rangle$

```
CG[2, 2, 4, 3]
```

$\frac{1}{\sqrt{2}} |2,1\rangle |2,2\rangle$

$\frac{1}{\sqrt{2}} |2,2\rangle |2,1\rangle$

```
CG[2, 2, 4, 4]
```

$1 |2,2\rangle |2,2\rangle$

- **Use your script to find the eigenstate with L = 0 that arises from coupling two d electrons (l = 2). Show that the state with L=4, M=0 is orthogonal to the state with L=0.**

```
CG[2, 2, 0, 0]
```

$\frac{1}{\sqrt{5}} |2,-2\rangle |2,2\rangle$

$-\frac{1}{\sqrt{5}} |2,-1\rangle |2,1\rangle$

$\frac{1}{\sqrt{5}} |2,0\rangle |2,0\rangle$

$-\frac{1}{\sqrt{5}} |2,1\rangle |2,-1\rangle$

$\frac{1}{\sqrt{5}} |2,2\rangle |2,-2\rangle$

```
CG[2, 2, 4, 0]
```

$\frac{1}{\sqrt{70}} |2,-2\rangle |2,2\rangle$

$2\sqrt{\frac{2}{35}} |2,-1\rangle |2,1\rangle$

$3\sqrt{\frac{2}{35}} |2,0\rangle |2,0\rangle$

$2\sqrt{\frac{2}{35}} |2,1\rangle |2,-1\rangle$

$\frac{1}{\sqrt{70}} |2,2\rangle |2,-2\rangle$

To prove the states L=0, M=0 and L=4, M=0 are orthogonal we will take their dot product.

<2,2,0,0 | 2,2,4,0> =

$$\frac{1}{\sqrt{5}}\frac{1}{\sqrt{70}} - \frac{1}{\sqrt{5}}2\sqrt{\frac{2}{35}} + \frac{1}{\sqrt{5}}3\sqrt{\frac{2}{35}} - \frac{1}{\sqrt{5}}2\sqrt{\frac{2}{35}} + \frac{1}{\sqrt{5}}\frac{1}{\sqrt{70}}$$

0

Thus, the two states are orthonormal.