① $f(x) = e^{2x}$

$u = 2x$

$du = 2dx$

$f(x) = e^u$

$f'(x) = e^u \cdot du$

$= e^{2x} \cdot 2dx$

$f'(x) = e^{2x} \cdot 2$

$= 2e^{2x}$

$u = 2x$

$du = 2dx$

$f'(x) = 2e^u$

$f''(x) = 2e^u \cdot du$

$= 2e^{2x} \cdot 2 = \boxed{4e^{2x}}$

For $f'(x)$

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{e^{2x+2h} - e^{2x}}{h} = \frac{e^{2x}(e^{2h} - 1)}{h}$

$\to e^{2h} = 1 + 2h$

$= \frac{e^{2x}(2\cancel{h})}{\cancel{h}} = 2e^{2x}$

For $f''(x)$

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{2e^{2x} \cdot e^{2h} - 2e^{x}}{h} = \frac{2e^{2x}(e^{2h} - 1)}{h} = \frac{2e^{2x} \cdot 2\cancel{h}}{\cancel{h}}$

$= 4e^{2x}$

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② $f(x) = \cos(x)$

$f'(x) = \boxed{-\sin(x)}$

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{\cos(x+h) - \cos(x)}{h} = \frac{(\cos(x) \cdot \cos(h) - \sin(x)\sin(h)) - \cos(x)}{h}$

$\frac{\cos(x)[\cos(h) - 1] - \sin(x)\sin(h)}{h} = \cos x \cdot \frac{\cos(h) - 1}{0} - \frac{\sin x \sin h}{\cancel{h}}$

$= \cos(x) \cdot 0 - \sin(x) \cdot 1$

$= -\sin(x)$

$\lim_{h \to 0} \frac{\cos(h) - 1}{h} = 0$

$\lim_{h \to 0} \frac{\sin(h)}{h} = 1$

③ $f(x,y) = x^2 + y^2 = 25 \longrightarrow x^2 + y^2 = 25$

$$x^2 + y^2 = 25$$
$$-x^2 \qquad -x^2$$
$$\sqrt{y^2} = \sqrt{25 - x^2}$$
$$y = \pm\sqrt{25 - x^2}$$

Find $\frac{dy}{dx} =$

$$y = (25 - x^2)^{1/2}$$
$$y' = \frac{1}{2}(25 - x^2)^{-1/2} \cdot (-2x)$$
$$y' = \frac{-\cancel{2}x}{\cancel{2}} \cdot (25 - x^2)^{-1/2}$$
$$\frac{dy}{dx} = y' = \boxed{\frac{-x}{\sqrt{25 - x^2}}} \rightarrow \text{explicit differentiation}$$

$$2x + 2y \cdot \frac{dy}{dx} = 0$$
$$-2x \qquad\qquad -2x$$
$$\frac{\cancel{2}y \cdot \frac{dy}{dx}}{y} = \frac{-\cancel{2}x}{y}$$
$$y' = \frac{dy}{dx} = \boxed{\frac{-x}{y}} \rightarrow \text{implicit}$$

since we want to see how y change as a function of x, this is the answer

when you solve explicitly for y, you get $\frac{-x}{\sqrt{25-x^2}}$.
But when you consider relationship between x & y on the constraint $x^2+y^2=25$, you get $\frac{-x}{y}$.
The explicit option depends on posiv̶e̶g you choose whereas implicit option is general

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④ $\int_0^1 x^3 dx - \int_0^1 2x\,dx + \int_0^1 1\,dx$

power rule $\int x^n = \frac{x^{n+1}}{n+1}$

$$= \left[\frac{x^{3+1}}{3+1}\right]_0^1 - \left[\frac{2x^{1+1}}{1+1}\right]_0^1 + \left[\frac{x^{0+1}}{0+1}\right]_0^1$$

$$= \left[\frac{x^4}{4}\right]_0^1 - \left[\frac{\cancel{2}x^2}{\cancel{2}}\right]_0^1 + \left[\frac{x}{1}\right]_0^1$$

$$= \left(\frac{1}{4} - 0\right) - (1 - 0) + (1 - 0) = \frac{1}{4} - \cancel{\frac{4}{4}} + \cancel{\frac{4}{4}} = \boxed{\frac{1}{4}}$$

⑤ $\int x e^x \, dx$ $\quad u = x \quad u'(x) = 1$
$\quad v = e^x \quad v'(x) = e^x$

$$F(x) = \int u'(x) v(x)\, dx + \int u(x) \cdot v'(x)\, dx$$

$$x \cdot e^x = \int e^x \, dx + \int x \cdot e^x \, dx$$

$$x \cdot e^x = e^x + \int x \cdot e^x \, dx$$
$$-e^x \qquad -e^x$$

$$\int x \cdot e^x \, dx = x \cdot e^x - e^x$$
$$\boxed{= e^x (x-1)}$$

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⑥ $\int 2x \cdot \sqrt{1+x^2} \, dx$

$u = 1+x^2$
$du = 2x \, dx$
$dx = \dfrac{du}{2x}$

$$= \int \cancel{2x} \cdot \sqrt{u} \, \frac{du}{\cancel{2x}}$$

$$= \int u^{1/2} \, du \longrightarrow \frac{u^{1/2+1}}{\frac{1}{2}+1}$$

$$= \frac{u^{3/2}}{\frac{3}{2}} = \frac{2}{3} \cdot u^{3/2} = \frac{2}{3} u^{3/2}$$

$$\boxed{= \frac{2}{3} (1+x^2)^{3/2}}$$