

Mathematical and Statistical Foundations for Data
Science(CMPINF 2105)
“Pen and Paper” Homework 1: Linear Algebra & Linear Systems
(Modules 1 & 2)

1. Given vectors $\vec{a} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$, calculate $\vec{a} + \vec{b}$, $\vec{a} - \vec{b}$, and $2\vec{a} - 3\vec{b}$.
2. Find the dot product of the vectors $\vec{c} = \begin{bmatrix} 6 \\ 0 \\ -2 \end{bmatrix}$ and $\vec{d} = \begin{bmatrix} -1 \\ 4 \\ -1 \end{bmatrix}$.
3. If matrix $M = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$ and matrix $N = \begin{bmatrix} 1 & 0 \\ -3 & 4 \end{bmatrix}$, what is $M + N$?
4. Calculate 3 times matrix $P = \begin{bmatrix} 7 & -2 \\ -6 & 3 \end{bmatrix}$.
5. Let matrix $A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$ and vector $\vec{b} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$. Find the product $A \cdot \vec{b}$.
6. Take the matrix dot product of matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and matrix $B = \begin{bmatrix} -1 & 3 \\ 4 & -2 \end{bmatrix}$.
7. (a) Find the eigenvalues of matrix $M = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.
(b) Find an eigenvector \vec{v} corresponding to the largest eigenvalue of matrix M from the prior question.
(c) Verify that your eigenvector from the previous question is indeed an eigenvector of matrix M .
8. Solve the following system of linear equations for the unknown variables x_1 and x_2 . If the solution is unknown, then explain why the problem cannot be solved.

$$3x_1 + 5x_2 = 59$$

$$7x_1 + 2x_2 = 99$$

9. Solve the following system of linear equations for the unknown variables x_1 , x_2 , and x_3 . If the solution is unknown, then explain why the problem cannot be solved.

$$9x_1 + 9x_2 - 7x_3 = 6$$

$$3x_1 - 2x_2 - 9x_3 = 3$$

$$6x_1 + 0x_2 + 1x_3 = -10$$

10. Solve the following system of linear equations for the unknown variables x_1 , x_2 , and x_3 . If the solution is unknown, then explain why the problem cannot be solved.

$$11x_1 + 3x_2 + 9x_3 = 237$$

$$5x_1 + 4x_2 + 2x_3 = 101$$

$$37x_1 + 18x_2 + 24x_3 = -777$$