Section 5

Diagrams in 2(c) and their meanings.

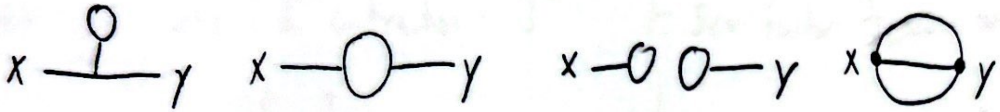
x <u>means</u> $(t, \vec{x})$!

<u>don't</u> draw diagrams like

Makes it very hard to avoid duplicates and see sym. factor!

λ^0: 

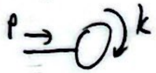
all diagrams with loops are ∞, need renorm.!

λ^2: 

"tadpole", will be cancelled

prop. renorm.

does not contribute to prop by mom. cons.



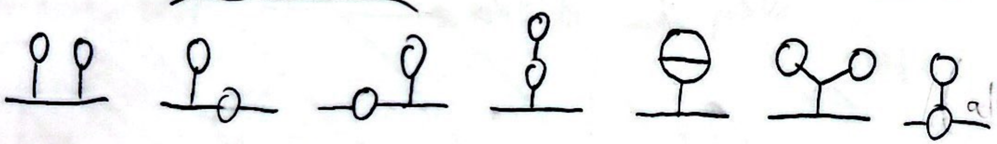
$p + k - k = 0$

not a valid diagram.

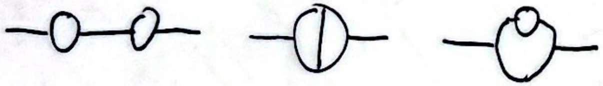
$\langle 0 | T(\phi(x)\phi(y) \int dz_1 \phi(z_1)^3 \int dz_2 \phi(z_2)^3) | 0 \rangle$

the <u>vertices</u> are z_1 and z_2, not x and y.

not amputated.

λ^4: tadpoles: 

all (vanish after renormalization)

prop. renorm.: 

essentially square of previous

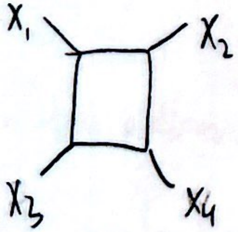
fundamentally new

has prev. as subdiagram.

note:  is not distinct.

draw carefully to avoid double counting. work methodically to avoid missing.

To compute sym. factors, consider a diagram with no sym factor.



from: $\frac{1}{4!}\left(\frac{1}{3!}\right)^4 \langle 0 | T(\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) \int dz_1 \phi(z_1)^3 \int dz_2 \phi(z_2)^3 \int dz_3 \phi(z_3)^3 \int dz_4 \phi(z_4)^3) | 0 \rangle$

$4!$ ways to match z_i with x_i. 3^4 ways to choose which copy of $\phi(z_i)$ contracts with $\phi(x_i)$. 2^4 ways to contract the rest.

Symmetry factors arise when there are fewer contractions that yield the same diagram.

$$\langle 0 | T\left(\phi(x)\phi(y)\int dz_1\,\phi(z_1)^3 \int dz_2\,\phi(z_2)^3\right)|0\rangle \times \frac{1}{2!}\,\frac{1}{3!^2}$$

choose whether z_1 or z_2 contracts with x : 2

choose which $\phi(z_i)$ do contraction : 3^2

remaining choice: only 2

Sym. factor $\frac{1}{2}$.

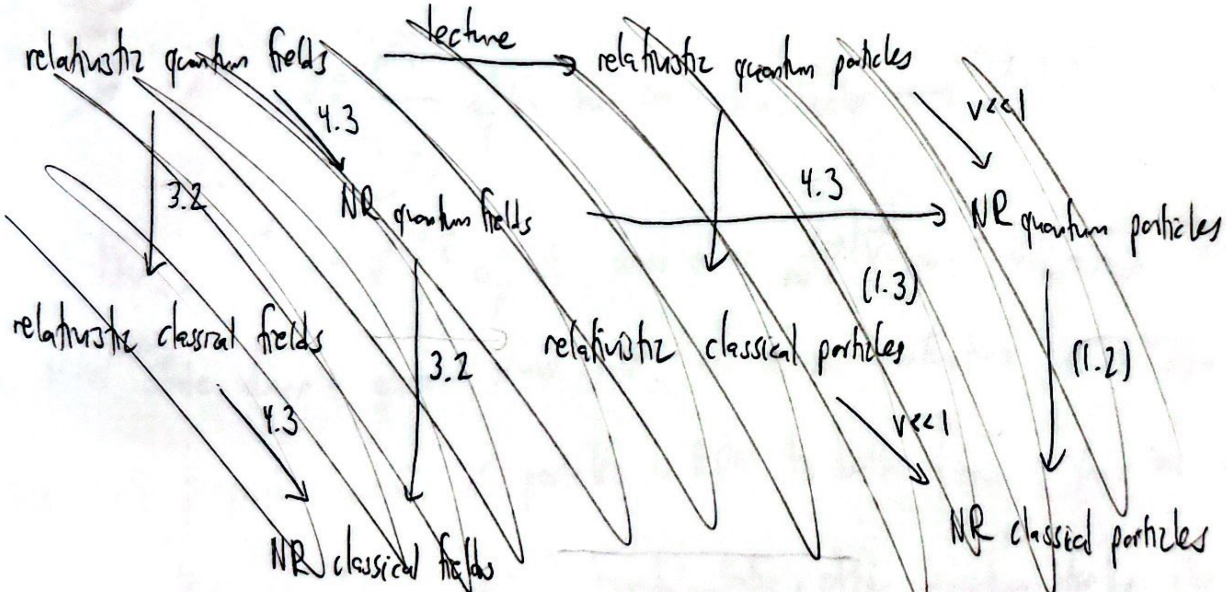
how does this differ from previous case? naively 4 choices (choose which $\phi(z_1)$ to contract with which $\phi(z_2)$ but

$\phi(z_1)\phi(z_1)\phi(z_2)\phi(z_2)$ same as $\phi(z_1)\phi(z_1)\phi(z_2)\phi(z_2)$

the two edges are symmetric.

$\boxed{\text{don't write } \partial_i \chi \chi^\dagger \ldots}$

Remarks on problem 3: we're completing the big picture!



a lot of people wrote

$$\partial_\mu \phi = \partial_0 \phi + \partial_i \phi$$

$$(\partial_\mu \phi)(\partial^\mu \phi) = (\partial_0 \phi + \partial_i \phi)(\partial^0 \phi + \partial^i \phi)$$

Correct is

$$(\partial_\mu \phi)(\partial^\mu \phi) = \sum_{\mu=0123} \partial_\mu \phi\, \partial^\mu \phi$$

$$= \partial_0 \phi \partial^0 \phi + \partial_i \phi \partial^i \phi.$$

In this problem you get a NR QFT where χ annihilates NR particles $\longrightarrow$ recover single particle states $|\psi\rangle = \int d\vec{x}\; \psi(\vec{x},t)|\vec{x}\rangle$ with wavefunctions satisfying SE, $i\dot{\psi} = -\nabla^2\psi/2m$. also multiparticle states $\int d\vec{x}\, d\vec{y}\; \psi(\vec{x},\vec{y})\, \chi^\dagger(\vec{x})\chi^\dagger(\vec{y})|0\rangle$.

The big picture:

$$\begin{tikzcd}
 & \text{rel. QFT} \arrow[dl, "3.2"'] \arrow[dr, "4.3"] \arrow[rrr, leftrightarrow, "\text{lecture}"] & & & \text{rel. quantum particles} \arrow[dl, "v \ll 1"'] \arrow[dr, "(1.3)"] & \\
\text{rel. CFT} \arrow[dr, "4.3"'] & & \text{nonrel. QFT} \arrow[r, "4.3"] \arrow[dl, "3.2"] & \text{nonrel. quantum particles} \arrow[dr, "1.2"'] & & \text{rel. classical particles} \arrow[dl, "v \ll 1"] \\
 & \text{nonrel. CFT} & & & \text{nonrel. classical particle} &
\end{tikzcd}$$

in EM, we start with relativistic photon and electron fields. electrons are massive and have conserved charge → their # stays the same in low energy expts. photons are massless & produced in great quantity. to describe everyday physics, reduce

photon Q field → classical EM field

electron Q field → NR classical electrons

A nagging Q: why no position-space wavefunctions in relativistic QFT? The issue is

$|\vec{x}\rangle = \psi^\dagger(\vec{x})|0\rangle = \int d\vec{p}\, \frac{e^{-i\vec{p}\cdot\vec{x}}}{\sqrt{2E_p}}\, a^\dagger_{\vec{p}}|0\rangle$ has rel. norm. factor → $\langle\vec{y}|\vec{x}\rangle$ not a delta function! falls on scale $1/m$.

N-W fixed: $|\vec{x}\rangle_{NW} = \int d\vec{p}\, e^{-i\vec{p}\cdot\vec{x}}\, a^\dagger_{\vec{p}}|0\rangle$ does obey ${}_{NW}\langle\vec{y}|\vec{x}\rangle_{NW} = \delta(\vec{y}-\vec{x})$. but does not play well with Lorentz transformations: boosting N-W state doesn't give a N-W state, and N-W wavefunctions spread superluminally (though falls on scale $1/m$).

Bottom line: we can't pinpoint locations of particles in RQM to better than $\sim 1/m$. We express locality in terms of fields.

Problem 4: what is the source "made of"? Fundamentally, other quantum fields, like $J(x) = \chi^\dagger(x)\chi(x)$. So why can we treat it as classical & localized? For single particles, $\langle\psi|J(x)|\psi\rangle = |\psi(x)|^2$, the norm of the wave function!

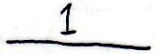
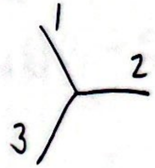
So this problem describes, e.g. two χ particles localized at $\vec{0}$ and $\vec{R}$.

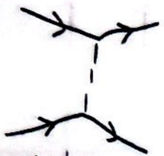
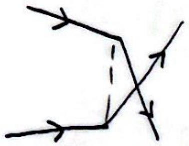
More Feynman diagrams.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}M^2\phi^2 + |\partial_\mu\psi|^2 - m^2|\psi|^2 - g\psi^*\psi\phi$$

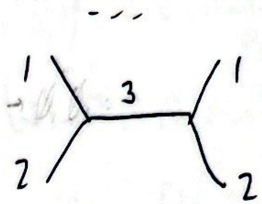
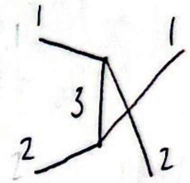
 $= \dfrac{i}{p^2-M^2+i\epsilon}$  $= \dfrac{i}{p^2-m^2+i\epsilon}$  $= -ig$

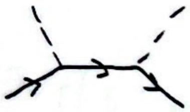
$$\mathcal{L} = \sum_{i=1}^{3}\left(\frac{1}{2}(\partial_\mu\phi_i)^2 - \frac{1}{2}m_i^2\phi_i^2\right) - g\phi_1\phi_2\phi_3.$$

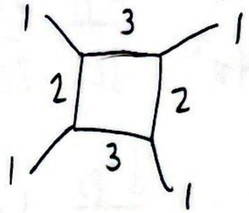
 $= \dfrac{i}{p^2-m_i^2+i\epsilon}$  $= -ig.$

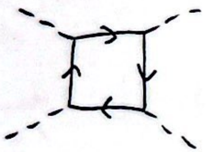
$\psi\psi \to \psi\psi$:  

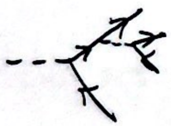
t channel u channel.

$\phi_1\phi_2 \to \phi_1\phi_2$:  

$\phi\psi \to \phi\psi$:  gives  

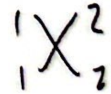
$\phi_1\phi_1 \to \phi_1\phi_1$:  + more.

$\phi\phi \to \phi\phi$:  + many more

$\phi \to \psi\psi^*\psi\psi^*$  + many more.

What about $\mathcal{L}_{int} \supset -g\phi_1^2\phi_2^2$? on a diagram with no symmetry factors, we would have

$$-ig\int dx\ \underbrace{\phi_1(x)\phi_1(x)}_{2\text{ choices}}\underbrace{\phi_2(x)\phi_2(x)}_{2\text{ choices}} \longrightarrow \text{vertex } \textrm{B}$$

 $= -4ig.$

Phase space integrals. keep calm and do the integral. Example: $d\sigma/d\Omega$ in CM frame for $p_A, p_B \to p_1, p_2$ all massless.

$$d\sigma = \frac{1}{2E_A \cdot 2E_B \cdot |v_A - v_B|} \prod_f \frac{d^3 p_f}{(2\pi)^3} \frac{1}{2E_f} |M|^2 (2\pi)^4 \delta^4 (p_A + p_B - \Sigma p_f)$$

$p_A = (E_{cm}/2, 0, 0, E_{cm}/2)$

$p_B = (E_{cm}/2, 0, 0, -E_{cm}/2)$

$p_1 = (E_1, \vec{p}_1)$

$p_2 = (E_2, \vec{p}_2)$

$E_i = |\vec{p}_i|$

$$= \frac{1}{2E_{cm}^2} \frac{1}{(2\pi)^2} \frac{d^3 p_1}{2E_1} \frac{d^3 p_2}{2E_2} |M|^2 \delta(E_{cm} - E_1 - E_2) \delta^{(3)}(\vec{p}_1 + \vec{p}_2)$$

$$= \frac{1}{32\pi^2 E_{cm}^2} \frac{d^3 p_1}{|p_1|^2} |M|^2 \delta(E_{cm} - E_1 - E_2)$$

$d^3 p_1 = |\vec{p}_1|^2 d|\vec{p}_1| \, d\Omega$

$$= \frac{1}{32\pi^2 E_{cm}^2} d|\vec{p}_1| \, d\Omega \, |M|^2 \delta(E_{cm} - 2|\vec{p}_1|)$$

$$= \frac{1}{64\pi^2 E_{cm}^2} d\Omega \, |M|^2. \quad \checkmark$$

if the final particles are identical, factor of $\frac{1}{2}$ to avoid overcounting.