Section #10

- problem sets 7–9
- example of renormalization
- big picture again.

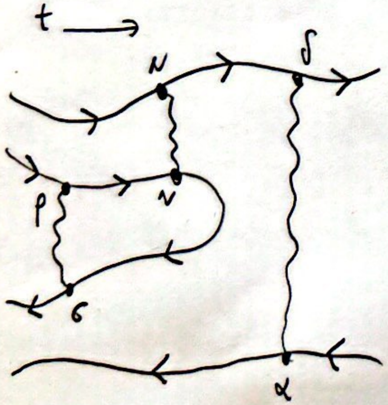
PS 7, (1b): show Dirac eq. has solns. $\begin{pmatrix} \sqrt{p\cdot\sigma}\,\xi_s \\ \sqrt{p\cdot\bar\sigma}\,\xi_s \end{pmatrix} e^{-ip\cdot x}$.

Several students stated that we only have to show this for $p^\mu = (m, \vec{0})$ by "Lorentz invariance". But that's just a special case. The right argument would be ① show for $p^\mu = (m,\vec{0})$, ② note that solns. of Dirac eq. become other solns. of Dirac eq. under LT, by LI, ③ show that $U[\Lambda]$ on the $p^\mu = (m,\vec{0})$ soln. gives the above soln.

PS 8, (3b): lots of algebra errors. Strongly recommend using computer for stuff like this, esp. after traces done.

PS 8, (3c): many used $t = E_{cm}^2 \frac{\cos\theta - 1}{2}$ from PS 5 but that result was for massless particles.

PS 9, (1b): there is some confusion on how to write spinor terms. The order matters because there are implicit matrix multiplications. The general rule is that we build up each expression by following each spinor line. (Not called a Feynman rule, left implicit.)



particle in : $u^s(p)$ out : $\bar{u}^s(p)$ [typo in lecture]

antiparticle in : $\bar{v}^s(p)$ out : $v^s(p)$

top line : $\bar{u}\,\gamma^\delta\,\not{k}\,\gamma^\mu\,u$ — the left to right order is against the arrows on lines.

bottom line : $\bar{v}\,\gamma^\alpha\,v$ — caution: other sources have time going right to left,

middle line : $\bar{v}\,\gamma^\sigma\,\not{k}\,\gamma^\nu\,\not{k}\,\gamma^\rho\,u$ — bottom to top, or top to bottom.

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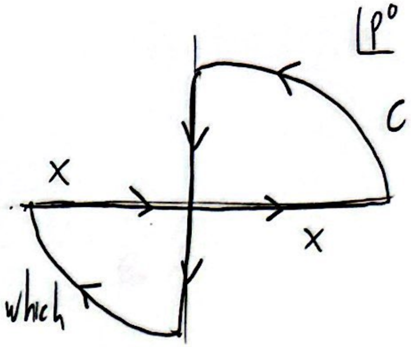
PS 9, (2d): how does Wick rotation work? Loop integral is of the form

$$I = \int d^4p\, f(p^2) = \int_{-\infty}^{\infty} dp^0 \int_{-\infty}^{\infty} dp^1 \int_{-\infty}^{\infty} dp^2 \int_{-\infty}^{\infty} dp^3\, f(p^{0\,2} - p^{1\,2} - p^{2\,2} - p^{3\,2})$$



Consider just the p^0 integral, regard as contour integral. There are singularities in p^0 which are shifted off real axis by $i\epsilon$ in Feynman propagator. Now consider contour C.

$$\left.\begin{aligned} &C \text{ does not contain singularities} \longrightarrow \oint f(p^0)\, dp^0 = 0 \\ &f \text{ falls off at large } |p^0| \longrightarrow \int_{\llcorner} f(p^0)\, dp^0 = \int_{\urcorner} f(p^0)\, dp^0 = 0 \end{aligned}\right\} \rightarrow \int_{\rightarrow} f(p^0)\, dp^0 = \int_{\uparrow} f(p^0)\, dp^0$$

Can replace the integral over real p^0 with one that goes up the imaginary p^0 axis. We define $p^0 = i p_E^0$, so

$$\int_{\uparrow} f(p^0)\, dp^0 = \int_{-\infty}^{\infty} i\, dp_E^0\, f(i p_E^0) \longrightarrow I = \int d^4 p_E\, f(\underbrace{-p_E^{0\,2} - p^{1\,2} - p^{2\,2} - p^{3\,2}}_{\text{"}-p_E^2\text{"}})$$

So integrand now has 4d spherical sym. That's the point.

A baby example of renormalization.

renormalization — rephrase theory so it is easier to compute physical observables. { often requires artfully removing dependence on ill-behaved but unphysical quantities; can introduce new scales into problem.

regularization — remove idealizations to make things finite, math. defined.

Often together, but not synonymous.

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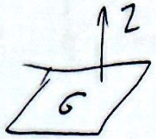
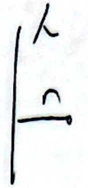
An example from high school E&M. If you know dim. of ϵ_0 then you know

• q $\quad V \sim \frac{q}{\epsilon_0 r}$ $\qquad$ similarly  $V \sim \frac{\sigma z}{\epsilon_0}$ $\qquad$ but  $V \sim \frac{\lambda}{\epsilon_0} f(r)$

but $f(r)$ must be dimensionless which is impossible?! Resolution: if you just do the integral, V is <u>infinite</u>. So how do we describe motion of charge in this potential?

("dimensional transmutation" allows us to write f)

① regularize: finite length L. V finite but now clunky to compute.

or ② renormalize: realize only changes in V observable, so add on a constant to make $V(a) = 0 \rightarrow V(r) = \frac{\lambda}{\epsilon_0} \log \frac{a}{r}$.

Renormalization adds new scale. Practically useful to set $a \sim$ typical r of particle so V isn't huge.

QFT is more complex but has some features in common. in a scalar theory...

actually not that bad because div. are often only logarithmic. $\log M_{pl}/\text{GeV} \sim 50$ only.

- set $\mathcal{L} = \underbrace{\frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2}_{\mathcal{L}_0} \underbrace{- \frac{g}{4!}\phi^4}_{\mathcal{L}_{int}}$

- calculate $\mathcal{M}$ as series in $g \rightarrow$ get infinite terms from loops. $\xrightarrow{\text{regularize}}$ loops finite but very big.

(similarly find physical mass of particle is finite but very far from m)

- renormalization: redefine the split between $\mathcal{L}_0$ and $\mathcal{L}_{int}$ so we get a good perturbation series.

in the renormalized Lagrangian, $\mathcal{L}_0$ is actually a good rough guide to the physics.

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For example, rewrite

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \underset{\substack{\uparrow \\ \text{physical mass}}}{\frac{1}{2} m_p^2 \phi^2} - \underset{\substack{\uparrow \\ \text{physical coupling}}}{\frac{g_p}{4!}\phi^4} + \frac{1}{2}(\delta Z)(\partial_\mu \phi)^2 - \frac{1}{2}\delta m^2 \phi^2 - \frac{\delta g}{4!}\phi^4$$

to make physical predictions, you need to already know m_p and g_p but then you can predict other stuff.

where m_p and g_p determined recursively, fixing counterterms. Get well-behaved series in g_p. In dim. reg. there is a scale μ and terms have powers of $\log p^2/\mu^2$ for typical momentum p $\rightarrow$ corrections small if we let $\mu \sim p$

An example from high school mechanics. What happens in universe w/ regular ρ?

- collapses (but to which point?)
- stays static (but what about gravity?)

Force analysis: get $\infty - \infty$, indefinite. Potential: $\nabla^2 V = \rho$ so we <u>can't</u> have V constant, must collapse. But V is not uniquely defined, so this doesn't tell us <u>how</u> it collapses. Even relative accelerations are not uniquely defined!

So a "pure renormalization" approach fails. If we regularize by making the universe finite then we clearly get collapse, but this introduces a center, breaking translational sym, which is <u>anomalous</u> here. ~~[Or we can specify boundary conditions for V, which also break translational sym. → what happens here depends on that]~~ In general, regulators can break syms. and results can depend on choice of regulator, so we must be careful!

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what QFT is: unification of waves & particles, SR & QM.

particle physics, condensed matter, nuclear physics; quantum optics, cosmology

AMO, chemistry

acoustics, hydro, engineering, E&M

everyday, solar system, galaxies

$$\begin{tikzcd}
 & \text{RQFT} \arrow[rr] \arrow[dl] \arrow[d] & & \text{RQM} \arrow[d] \arrow[dr] & \\
\text{RCFT} \arrow[dr] & \text{NRQFT} \arrow[r] \arrow[d] & \text{NRQM} \arrow[dr] & & \text{RCM} \arrow[dl] \\
 & \text{NRCFT} \arrow[rr, leftrightarrow] & & \text{NRCM} &
\end{tikzcd}$$

how QFT works (at least in this course)

$$\begin{tikzcd}
 & \text{conservation laws} \arrow[r, leftrightarrow, "\text{Noether}"] & \text{symmetries} \arrow[d, "\text{rep.}"] & & & & \text{phase space} \arrow[d] \\
\text{expt.} \arrow[ur] \arrow[r] & \text{particles} \arrow[r, "\text{rep}"] & \text{fields} \arrow[r, "\text{EFT}", "{([\mathcal{O}]\leq 4)}"'] & \mathcal{L} \arrow[r, "\text{c.q.}"] & \text{quantum thy} \arrow[r, "\text{Feynman rules}"] \arrow[d, "\text{Wick}", "\text{Dyson}"'] & \mathcal{M} \arrow[r, "\text{reg.}", "\text{and renorm.}"'] & \text{predictions} \\
 & & & & \text{correlation fncs.} \arrow[ur, "\text{LSZ}"'] & &
\end{tikzcd}$$

$[\phi, \pi] \sim \delta$

$\{\Psi, \Psi^\dagger\} \sim \delta$

- spin 0 $\rightarrow$ scalar ϕ
- spin $\frac{1}{2}$ $\rightarrow$ Dirac Ψ
- spin 1 $\rightarrow$ vector A^μ
- helicity 1 $\rightarrow$ vector A^μ w/ gauge sym.

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