Section 4

- review of PS 3
- practice with diagrams
  - complex scalars
  - scalar Yukawa theory.

$$\mathcal{L} = (\partial_\mu \phi^*)(\partial_\mu \phi) - m^2 \phi^* \phi$$

what does Wick's theorem look like? recall $\phi \sim b + c^\dagger$.

$\langle 0 | T \underset{b+c^\dagger}{\phi(x)} \underset{b+c^\dagger}{\phi(y)} | 0 \rangle$ has to be zero.

more formal: act with the generator of $U(1)$ sym. Q,

$$\langle 0 | T \phi(x) \phi(y) | 0 \rangle = \underbrace{\langle 0 | e^{-i\alpha Q}}_{\langle 0 |} \underbrace{e^{i\alpha Q}\, T(\phi(x) e^{-i\alpha Q}\, e^{i\alpha Q} \phi(y)) e^{-i\alpha Q}}_{e^{i\alpha}\, T(\phi(x)\phi(y))\, e^{i\alpha}} \underbrace{e^{i\alpha Q} | 0 \rangle}_{| 0 \rangle}$$

so this correlator is equal to $e^{2i\alpha}$ times itself $\rightarrow$ must be zero. Similarly $\langle 0 | T \phi^*(x) \phi^*(y) | 0 \rangle = 0$.

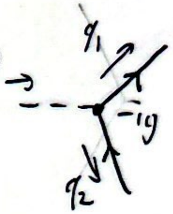
only nonzero one is $\langle 0 | T \phi^*(x) \phi(y) | 0 \rangle = :\phi^*(x) \phi(y): + D_F(x-y)$.

Scalar Yukawa theory: $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 + |\partial_\mu \psi|^2 - M^2 |\psi|^2 - g \psi^* \psi \phi$.

Simple interacting field theory. $\quad H_{int} = g \int d^3x\, \psi^*(x) \psi(x) \phi(x)$

Q: what is amp. for $\phi \to \psi\bar{\psi}$ decay? let $\phi \sim a + a^\dagger$, $\psi \sim b + c^\dagger$.

time left to right.



$|i\rangle = \sqrt{2E_p}\, a_p^\dagger |0\rangle$

$|f\rangle = \sqrt{4E_{q_1}E_{q_2}}\, b_{q_1}^\dagger c_{q_2}^\dagger |0\rangle$

$\mathcal{M} = \langle f | U_I(\infty, -\infty) | i \rangle$.

$U_I(\infty,-\infty) = T\exp\left(-i\int_{-\infty}^{\infty} H_{int,I}(t)\,dt\right)$

leading order result:

$$\mathcal{M} = -ig \langle f | \int d^4x\; \underbrace{\psi^\dagger(x)}_{\substack{b^\dagger + c \\ \downarrow}} \underbrace{\psi(x)}_{\substack{b + c^\dagger \\ \downarrow \\ \text{must hit } \langle f|}} \underbrace{\phi(x)}_{\substack{a + a^\dagger \\ \downarrow \\ \text{must hit } |i\rangle}} | i \rangle$$

$= -ig\,(2\pi)^4\,\delta(q_1 + q_2 - p)$. energy factors cancel

integral is $\int dx\, e^{ip\cdot x} \times e^{-iq_1\cdot x} e^{-iq_2\cdot x}$ gives delta func.

Q: what is amp. for $\psi\psi$ scattering?

$|i\rangle = \sqrt{4E_{p_1}E_{p_2}}\, b_{p_1}^\dagger b_{p_2}^\dagger |0\rangle$

$|f\rangle = \sqrt{4E_{q_1}E_{q_2}}\, b_{q_1}^\dagger b_{q_2}^\dagger |0\rangle$

$\mathcal{M} = \langle f | U_I(\infty,-\infty) | i \rangle$

no contrib at $O(g^0)$, $O(g^1)$

$$\mathcal{M} = \frac{(-ig)^2}{2} \langle f | \int d^4x_1\, d^4x_2\; T\left(\psi^\dagger(x_1)\psi(x_1)\phi(x_1)\psi^\dagger(x_2)\psi(x_2)\phi(x_2)\right) | i \rangle$$

the rule for Wick: you can contract any number of pairs to get D_F's, and the rest is a normal ordered product. we must contract the ϕ's.

$$\mathcal{M} = \frac{(-ig)^2}{2} \langle f | \int\!\!\int d^4x_1\, d^4x_2 :\psi^\dagger(x_1)\psi(x_1)\psi^\dagger(x_2)\psi(x_2): D_F(x_1 - x_2) | i \rangle.$$

$$= \frac{(-ig)^2}{2} \int d^4x_1\, d^4x_2\; \underbrace{\langle f | \psi^\dagger(x_1)\psi^\dagger(x_2) | 0 \rangle}_{e^{i(q_1\cdot x_1 + q_2\cdot x_2)} + e^{i(q_1\cdot x_2 + q_2\cdot x_1)}} \underbrace{\langle 0 | \psi(x_1)\psi(x_2) | i \rangle}_{e^{-i(p_1\cdot x_1 + p_2\cdot x_2)} + e^{-i(p_1\cdot x_2 + p_2\cdot x_1)}} D_F(x_1 - x_2)$$

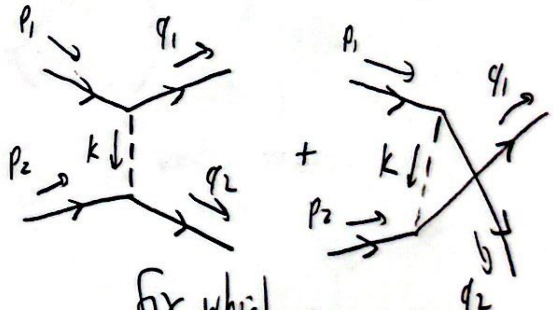
four terms, of which there are pairs of duplicates. yet...

$$\mathcal{M} = (-ig)^2 \int d^4x_1\, d^4x_2 \int \frac{d^4k}{(2\pi)^4} \frac{i e^{ik\cdot(x_1-x_2)}}{k^2-m^2+i\epsilon} \times (\text{phase factors})$$

$$= (-ig)^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2-m^2+i\epsilon} \int d^4x_1 d^4x_2 \Big(e^{i(q_1-p_1+k)\cdot x_1} e^{i(q_2-p_2-k)\cdot x_2}$$
$$+ e^{i(q_2-p_1+k)\cdot x_1} e^{i(q_1+p_2-k)\cdot x_2} \Big)$$

$$= (-ig)^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2-m^2+i\epsilon} \Big((2\pi)^8 \delta(q_1-p_1+k)\,\delta(q_2-p_2-k) + (2\pi)^8 \delta(q_2-p_1+k)(q_1+p_2-k) \Big)$$

$$= (-ig)^2 (2\pi)^4 \delta(p_1+p_2-q_1-q_2) \left(\frac{i}{(p_1-q_1)^2-m^2+i\epsilon} + \frac{i}{(p_1-q_2)^2-m^2+i\epsilon} \right).$$



fix which curve is which momentum.

the scalar Yukawa Feynman rules:

$\xrightarrow{k}$ $= \dfrac{i}{k^2-m^2+i\epsilon}$

$\overset{\to k}{- - - -}$ $= \dfrac{i}{k^2-m^2+i\epsilon}$

 $= -ig$

momentum conserved at vertices

Sum over diagrams. divide by sym. factor.

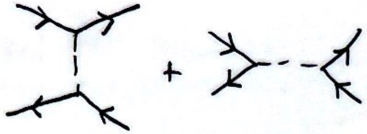
Connect only --- to --- and $\to$ to $\to$.

incoming arrow on incoming line $\Big\}$ is <u>particle</u>
outgoing arrow on outgoing line

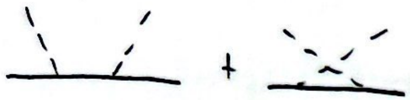
only connected, amputated diagrams (no legit proof of this now)

Fun exercises! Find the leading diagrams for...

$\psi\psi^* \to \psi\psi^*$



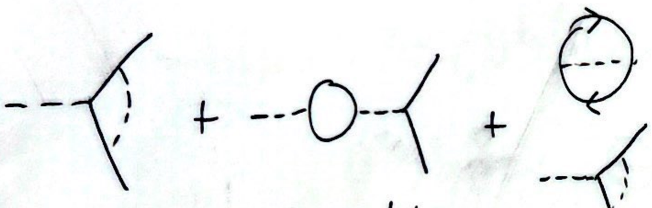
$\psi\phi \to \psi\phi$



$\phi \to \psi\psi^*$ to $O(g^2)$

0.

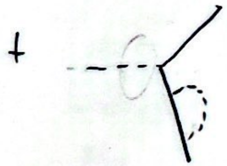
to $O(g^3)$



not amputated

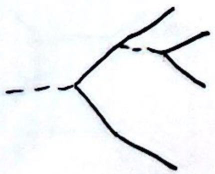
not connected

only one diagram counts for us.



not amputated

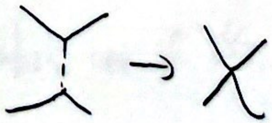
$\phi \to \psi\psi^*\psi\psi^*$



+ perms. (12 total)

effective field theory: if ϕ very heavy, its prop $\sim 1/m^2$. at low energies.

equiv. to a theory without ϕ:



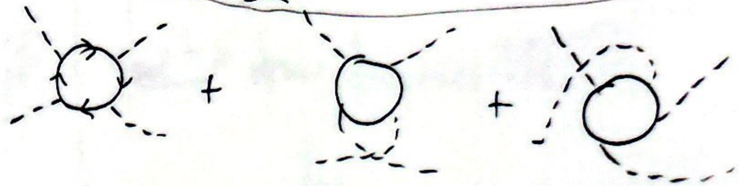
$\sim \frac{g^2}{m^2}\psi^*\psi\psi^*\psi$.

$\psi\psi \to \psi\psi$ has:

"penguin diagram"

$\phi\phi \to \phi\phi$

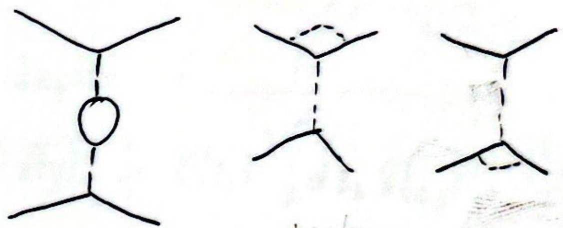
$O(g^4)$



total of 6 distinct diagrams.

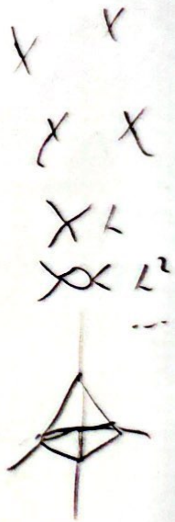
$\psi\psi^* \to \psi\psi^*$

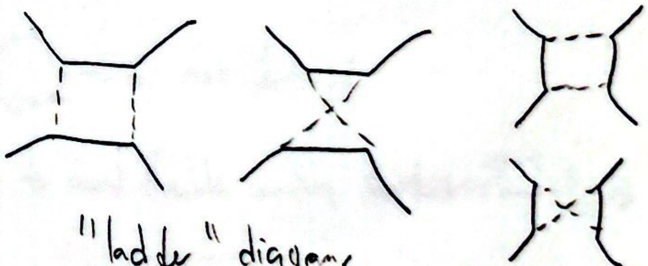
$O(g^4)$



prop. renorm.

vertex renorm.





"ladder" diagrams.

10 diagrams total?

challenge: in ϕ^4 theory

$\phi\phi \to \phi\phi$

at $O(\lambda^4)$ $D=?$