UNIVERSITATEA TEHNICĂ
DIN CLUJ-NAPOCA

PROIECT TEORIA SISTEMELOR I

Student

Caia-Hoanas Cristina-Elena

grupa 30126

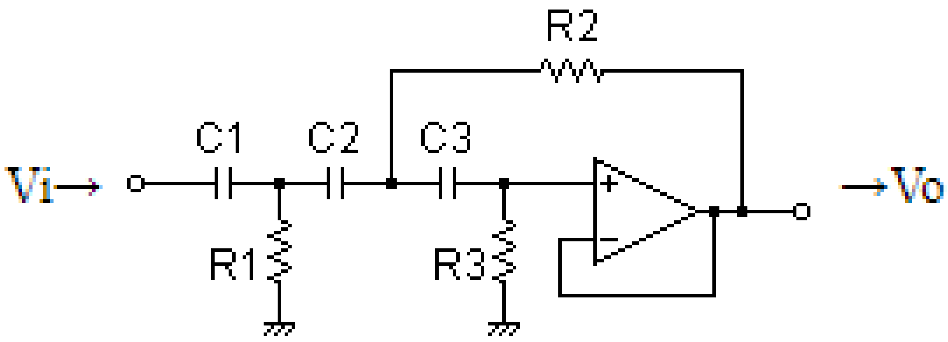
Facultate: Automatica si Calculatoare

Profesor indrumator

Profesor As. Drd. Ing Morar Dora

SISTEMUL - Sallen Key filtru trece sus, ordin 3

1a.



- componentele

active - 3 condensatoare pasive – 3 rezistente amplificator operational

- semnale

de intrare: u – tensiunea de intrare

de iesire: y – tensiunea de iesire

- valorile:

R1 = 54kΩ = 54000Ω   R2 = 27kΩ = 27000Ω   R3 = 108kΩ = 108000Ω

C1 = 2* 10^{-5} F   C2 = 10^{-5} F   C3 = 10^{-5} F

- marimile implicate – Ω (ohm)   si   F (Farad)

1b.

MODELUL MATEMATIC

$$I : u = u_{C1} + u_{R1}$$

$$II : u_{R1} = u_{C2} + u_{C3} + u_{R3}$$

$$III : u_{C3} = u_2$$

N1: $i_{C1} = i_{C2} + i_{R1}$

N2: $i_{C2} = i_{R2} + i_{C3}$

N3: $i_{C3} = i_{R3}$

$$x_1 = u_{C1}$$
$$x_2 = u_{C2}$$
$$x_3 = u_{C3}$$

$$V_+ = V_-$$
$$V_- = y$$
$$u_{R3} = V_+ - 0$$
$$=> y = u_{R3}$$

u/x/y:

din I: $u = x_1 + x_2 + x_3 + u_{R3} = x_1 + x_2 + x_3 + R3C3\frac{dx_3}{dt} => \frac{dx_3}{dt} = \frac{1}{R3C3}(u - x_1 - x_2 - x_3)$

din N2: $C2\frac{dx_2}{dt} = \frac{u_{R2}}{R2} + C3\frac{dx_3}{dt} => C2\frac{dx_2}{dt} = \frac{x_3}{R2} + \frac{1}{R_3}(u - x_1 - x_2 - x_3) => \frac{dx_2}{dt} = \frac{1}{C2}(\frac{x_3}{R2} + \frac{1}{R_3}(u - x_1 - x_2 - x_3))$

din N1:
$$C1\frac{dx_1}{dt} = C2\frac{dx_2}{dt} + \frac{u_{R1}}{R1} => C1\frac{dx_1}{dt} = C2\frac{dx_2}{dt} + \frac{u - x_1}{R1} => C1\frac{dx_1}{dt} = \frac{x_3}{R2} + \frac{1}{R_3}(u - x_1 - x_2 - x_3) + \frac{u - x_1}{R1}$$
$$=> \frac{dx_1}{dt} = \frac{1}{C1}(\frac{x_3}{R2} + \frac{1}{R_3}(u - x_1 - x_2 - x_3) + \frac{u - x_1}{R1})$$

$$\dot{x} = Ax + Bu$$

$$y = Cx + D$$

=>

$$\dot{x} = \begin{pmatrix} -1.38 & -0.46 & 1.38 \\ -0.92 & -0.92 & 2.77 \\ -0.92 & -0.92 & -0.92 \end{pmatrix} x + \begin{pmatrix} 2.77 \\ 0.92 \\ 0.92 \end{pmatrix} u$$

$$y = \begin{pmatrix} -1 & -1 & -1 \end{pmatrix} x + [1]$$

2.

u/y:

$$u = u_{C1} + u_{R1} = u_{C1} + u_{C2} + u_{C3} + y$$

$$u = \frac{1}{C1}\int i_{C1}dt + \frac{1}{C2}\int i_{C2}dt + \frac{1}{C3}\int i_3 dt + y$$

$$u = \frac{1}{C1}\int (\frac{u_{R1}}{R1} + \frac{u_{R2}}{R2} + \frac{y}{R3})dt + \frac{1}{C2}\int (\frac{u_{R2}}{R2} + \frac{y}{R3})dt + \frac{1}{C3}\int i_3 dt + y$$

$$u = \frac{1}{C1}\int \frac{y}{R3}dt + \frac{1}{C1R1}\int u_{R1}dt + \frac{1}{C1R2}\int u_{R2}dt + \frac{1}{C2R2}\int u_{R2}dt + \frac{1}{C2R3}\int ydt + \frac{1}{C3R3}\int ydt + y$$

$$=> u = (\frac{1}{C1R3} + \frac{1}{C2R3} + \frac{1}{C3R3})\int ydt + \frac{1}{C1R1}\int u_{R1}dt + (\frac{1}{C1R2} + \frac{1}{C2R2})\int u_{R2}dt + y$$

$$=> u = (\frac{1}{C1R3} + \frac{1}{C2R3} + \frac{1}{C3R3})\int ydt + \frac{1}{C1R1}(\frac{1}{C2}\iint (\frac{\frac{1}{C3}\int \frac{y}{R3}}{R2} + \frac{y}{R3})dt + \frac{1}{C3}\iint \frac{y}{R3}dt + \int ydt) + (\frac{1}{C1R2} + \frac{1}{C2R2})\frac{1}{C3}\iint \frac{y}{R3}dt + y$$

$$\int u_{R1}dt = \int (u_{C2} + u_{C3} + u_{R3})dt = \int (\frac{1}{C2}\int i_{C2})dt + \int (\frac{1}{C3}\int i_{C3})dt + \int ydt$$

$$= \frac{1}{C2}\iint (\frac{u_{R2}}{R2} + \frac{y}{R3})dt + \frac{1}{C3}\iint (\frac{y}{R3})dt + \int ydt$$

- $$\int u_{R2}dt = \int u_{C3}dt = \int (\frac{1}{C3}\int i_{C3})dt = \frac{1}{C3}\iint i_{C3}dt = \frac{1}{C3}\iint i_{R3}dt = \frac{1}{C3}\iint \frac{y}{R3}dt$$

$$=> u = (\frac{1}{C1R3} + \frac{1}{C2R3} + \frac{1}{C3R3})\int ydt + \frac{1}{C1R1C2R2C3R3}\iiint ydt + \frac{1}{C1R1C2R3}\iint ydt + \frac{1}{C1R1C3R3}\iint ydt + \frac{1}{C1R1}\int ydt + (\frac{1}{C1R2C3R3} + \frac{1}{C2R2C3R3})\iint ydt + y$$

$$=> u = (\frac{1}{C1R3} + \frac{1}{C2R3} + \frac{1}{C3R3} + \frac{1}{C1R1})\int ydt + (\frac{1}{C1R1C2R3} + \frac{1}{C1R1C3R3} + \frac{1}{C2R2C3R3} + \frac{1}{C1R2C3R3})\iint ydt + \frac{1}{C1R1C2R2C3R3}\iiint ydt + y$$

-voi deriva cu ordinul 3 toata relatia:

$$\frac{\partial^3 u}{\partial t^3} = (\frac{1}{C1R3} + \frac{1}{C2R3} + \frac{1}{C3R3} + \frac{1}{C1R1})\frac{\partial^2 y}{\partial t^2} + (\frac{1}{C1R1C2R3} + \frac{1}{C1R1C3R3} + \frac{1}{C2R2C3R3} + \frac{1}{C1R2C3R3})\frac{dy}{dt} + \frac{1}{C1R1C2R2C3R3}y + \frac{\partial^3 y}{\partial t^3}$$

-aplic Laplace:

$$s^3U(s) = s^3Y(s) + s^2Y(s)(\frac{1}{C1R3} + \frac{1}{C2R3} + \frac{1}{C3R3} + \frac{1}{C1R1}) + sY(s)(\frac{1}{C1R1C2R3} + \frac{1}{C1R1C3R3} + \frac{1}{C2R2C3R3} + \frac{1}{C1R2C3R3}) + Y(s)\frac{1}{C1R1C2R2C3R3}$$

FUNCTIA DE TRANSFER:

$$H(s)=\frac{Y(s)}{U(s)}=\frac{s^3}{s^3+s^2(\frac{1}{C1R3}+\frac{1}{C2R3}+\frac{1}{C3R3}+\frac{1}{C1R1})+s(\frac{1}{C1R1C2R3}+\frac{1}{C1R1C3R3}+\frac{1}{C2R2C3R3}+\frac{1}{C1R2C3R3})+\frac{1}{C1C2C3R1R2R3}}$$

RELATIA DINTRE SPATIUL STARILOR SI FUNCTIA DE TRANSFER – verificare

$$A=\begin{pmatrix} \frac{-1}{C1}(\frac{1}{R3}+\frac{1}{R1}) & -\frac{1}{R3C1} & \frac{1}{C1}(\frac{1}{R2}-\frac{1}{R3}) \\ -\frac{1}{C2R3} & -\frac{1}{C2R3} & \frac{1}{C2}(\frac{1}{R2}-\frac{1}{R3}) \\ -\frac{1}{R3C3} & -\frac{1}{R3C3} & -\frac{1}{R3C3} \end{pmatrix} \quad B=\begin{pmatrix} \frac{1}{C1}(\frac{1}{R3}+\frac{1}{R1}) \\ \frac{1}{C2R3} \\ \frac{1}{C3R3} \end{pmatrix} \quad C=\begin{pmatrix} -1 & -1 & -1 \end{pmatrix} \quad D=[1]$$

$$
\left(sI-A\right)^{-1}=\begin{pmatrix} s+\frac{1}{C1}(\frac{1}{R3}+\frac{1}{R1}) & \frac{1}{R3C1} & -\frac{1}{C1}(\frac{1}{R2}-\frac{1}{R3}) \\ \frac{1}{C2R3} & s+\frac{1}{C2R3} & -\frac{1}{C2}(\frac{1}{R2}-\frac{1}{R3}) \\ \frac{1}{R3C3} & \frac{1}{R3C3} & s+\frac{1}{R3C3} \end{pmatrix}^{-1}
$$

$$
H\left(s\right)=C(sI-A)^{-1}B+D
$$

$$
\det(sI-A)=\begin{pmatrix} s+\frac{1}{C1}(\frac{1}{R3}+\frac{1}{R1}) & \frac{1}{R3C1} & -\frac{1}{C1}(\frac{1}{R2}-\frac{1}{R3}) \\ \frac{1}{C2R3} & s+\frac{1}{C2R3} & -\frac{1}{C2}(\frac{1}{R2}-\frac{1}{R3}) \\ \frac{1}{R3C3} & \frac{1}{R3C3} & s+\frac{1}{R3C3} \end{pmatrix}=
$$

$$
=\frac{1}{C1C2C3R1R2R3}+\frac{s}{C1C2R1R3}+\frac{s}{C1C3R1R3}+\frac{s}{C1C3R2R3}+\frac{s^2}{C1R1}+\frac{s^2}{C1R3}+\frac{s}{C2C3R2R3}+\frac{s^2}{C2R3}+\frac{s^2}{C3R3}+s^3=
$$

$$
\frac{C1R1s(C2R2s(C3R3s+1)+C3R2s+1)+C2s(C3R1R2s+C3R2R3s+R1+R2)+C3R2s+1}{C1C2C3R1R2R3}
$$

$$\left(sI-A\right)^{t}=\begin{pmatrix} s+\frac{1}{C1}(\frac{1}{R3}+\frac{1}{R1}) & \frac{1}{C2R3} & \frac{1}{R3C3} \\ \frac{1}{R3C1} & s+\frac{1}{C2R3} & \frac{1}{R3C3} \\ -\frac{1}{C1}(\frac{1}{R2}-\frac{1}{R3}) & -\frac{1}{C2}(\frac{1}{R2}-\frac{1}{R3}) & s+\frac{1}{R3C3} \end{pmatrix}$$

$$a_{11}=(-1)^{1+1}\begin{pmatrix} s+\frac{1}{C2R3} & \frac{1}{R3C3} \\ -\frac{1}{C2}(\frac{1}{R2}-\frac{1}{R3}) & s+\frac{1}{R3C3} \end{pmatrix}=(-1)^{2}(\frac{1}{C2C3R2R3}+\frac{s}{C2R3}+\frac{s}{C3R3}+s^{2})=\frac{sC2R2(sC3R3+1)+sC3R2+1}{C2C3R2R3}$$

$$a_{12}=(-1)^{1+2}\begin{pmatrix} \frac{1}{R3C1} & \frac{1}{R3C3} \\ -\frac{1}{C1}(\frac{1}{R2}-\frac{1}{R3}) & s+\frac{1}{R3C3} \end{pmatrix}=(-1)^{3}(\frac{1}{C1C3R2R3}+\frac{s}{C1R3})=-\frac{sC3R2+1}{C1C3R2R3}$$

$$a_{13}=(-1)^{1+3}\begin{pmatrix} \frac{1}{R3C1} & s+\frac{1}{C2R3} \\ -\frac{1}{C1}(\frac{1}{R2}-\frac{1}{R3}) & -\frac{1}{C2}(\frac{1}{R2}-\frac{1}{R3}) \end{pmatrix}=\frac{s}{C1R2}-\frac{s}{C1R3}=\frac{s(R3-R2)}{C1R2R3}$$

$$a_{21}=(-1)^{2+1}\begin{pmatrix} \dfrac{1}{C2R3} & \dfrac{1}{R3C3} \\ -\dfrac{1}{C2}(\dfrac{1}{R2}-\dfrac{1}{R3}) & s+\dfrac{1}{R3C3} \end{pmatrix}=(-1)^{3}(\frac{1}{C2C3R2R3}+\frac{s}{C2R3})=-\frac{sC3R2+1}{C2C3R2R3}$$

$$a_{22}=(-1)^{2+2}\begin{pmatrix} s+\dfrac{1}{C1}(\dfrac{1}{R3}+\dfrac{1}{R1}) & \dfrac{1}{R3C3} \\ -\dfrac{1}{C1}(\dfrac{1}{R2}-\dfrac{1}{R3}) & s+\dfrac{1}{R3C3} \end{pmatrix}=\frac{1}{C1C3R1R3}+\frac{1}{C1C3R2R3}+\frac{s}{C1R1}+\frac{s}{C1R1}+\frac{s}{C3R3}+s^{3}=$$

$$=\frac{R1(sC1R2(sC3R3+1)+sC3R2+1)+sC3R2R3+R2}{C1C3R1R2RR3}$$

$$a_{23}=(-1)^{2+3}\begin{pmatrix} s+\dfrac{1}{C1}(\dfrac{1}{R3}+\dfrac{1}{R1}) & \dfrac{1}{C2R3} \\ -\dfrac{1}{C1}(\dfrac{1}{R2}-\dfrac{1}{R3}) & -\dfrac{1}{C2}(\dfrac{1}{R2}-\dfrac{1}{R3}) \end{pmatrix}=(-1)^{5}(-\frac{1}{C1C2R1R2}+\frac{1}{C1C2R1R3}-\frac{s}{C2R2}+\frac{s}{C2R3})$$

$$=-\frac{(R2-R3)(sC1R1+1)}{C1C2R1R2R3}$$

$$a_{31}=(-1)^{3+1}\begin{pmatrix} \dfrac{1}{C2R3} & \dfrac{1}{R3C3} \\ s+\dfrac{1}{C2R3} & \dfrac{1}{R3C3} \end{pmatrix}=-\frac{s}{R3C3}$$

$$a_{32}=(-1)^{3+2}\begin{pmatrix} s+\dfrac{1}{C1}(\dfrac{1}{R3}+\dfrac{1}{R1}) & \dfrac{1}{R3C3} \\ \dfrac{1}{R3C1} & \dfrac{1}{R3C3} \end{pmatrix}=-(\frac{1}{C1C3R1R3}+\frac{s}{C3R3})=-\frac{1+C1R1s}{C1C3R1R3}$$

$$a_{33}=(-1)^{3+3}\begin{pmatrix} s+\dfrac{1}{C1}(\dfrac{1}{R3}+\dfrac{1}{R1}) & \dfrac{1}{C2R3} \\ \dfrac{1}{R3C1} & s+\dfrac{1}{C2R3} \end{pmatrix}=\frac{C1R1s(C2R3s+1)+C2s(R1+R3)+1}{C1C2R1R3}$$

$$(sI-A)^{*}=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$H(s) = C\frac{1}{\det(sI - A)}(sI - A)^{*}B + D$$

$(sI - A)^{-1}$ este:

$$\frac{C1R1 + C1C2C3R1R2R3s^2 + C1C2R1R2s + C1C3R1R2s}{C1R1s + C2R1s + C1C2C3R1R2R3s^3 + C1C2R1R2s^2 + C1C3R1R2s^2 + C2C3R1R2s^2 + C2C3R2R3s^2 + C2R2s + C3R2s + 1} \quad \frac{-C2R1 - C2C3R1R2s}{C1R1s + C2R1s + C1C2C3R1R2R3s^3 + C1C2R1R2s^2 + C1C3R1R2s^2 + C2C3R1R2s^2 + C2C3R2R3s^2 + C2R2s + C3R2s + 1} \quad \frac{C2C3R1R3s - C2C3R1R2s}{C1R1s + C2R1s + C1C2C3R1R2R3s^3 + C1C2R1R2s^3 + C1C3R1R2s^2 + C2C3R1R2s^2 + C2C3R2R3s^2 + C2R2s + C3R2s + 1}$$

$$\frac{-C1R1 - C1C3R1R2s}{C1R1s + C2R1s + C1C2C3R1R2R3s^3 + C1C2R1R2s^2 + C1C3R1R2s^2 + C2C3R1R2s^2 + C2C3R2R3s^2 + C2R2s + C3R2s + 1} \quad \frac{C2R1 + C2R2 + C1C2C3R1R2R3s^2 + C1C2R1R2s + C2C3R1R2s + C2C3R2R3s}{C1R1s + C2R1s + C1C2C3R1R2R3s^3 + C1C2R1R2s^2 + C1C3R1R2s^2 + C2C3R1R2s^2 + C2C3R2R3s^2 + C2R2s + C3R2s + 1} \quad \frac{C3R3 + C1C3R1R3s - C3R2 - C1C3R1R2s}{C1R1s + C2R1s + C1C2C3R1R2R3s^3 + C1C2R1R2s^2 + C1C3R1R2s^2 + C2C3R1R2s^2 + C2C3R2R3s^2 + C2R2s + C3R2s + 1}$$

$$\frac{-C1C2R1R2s}{C1R1s + C2R1s + C1C2C3R1R2R3s^3 + C1C2R1R2s^2 + C1C3R1R2s^2 + C2C3R1R2s^2 + C2C3R2R3s^2 + C2R2s + C3R2s + 1} \quad \frac{-C2R2 - C1C2R1R2s}{C1R1s + C2R1s + C1C2C3R1R2R3s^3 + C1C2R1R2s^2 + C1C3R1R2s^2 + C2C3R1R2s^2 + C2C3R2R3s^2 + C2R2s + C3R2s + 1} \quad \frac{C3R2 + C1C2C3R1R2R3s^2 + C1C3R1R2s + C2C3R1R2s + C2C3R2R3s}{C1R1s + C2R1s + C1C2C3R1R2R3s^3 + C1C2R1R2s^2 + C1C3R1R2s^2 + C2C3R1R2s^2 + C2C3R2R3s^2 + C2R2s + C3R2s + 1}$$

$$C(sI - A)^{-1}B = \frac{-C1R1s - C2R1s - C1C2R1R2s^2 - C1C3R1R2s^2 - C2C3R1R2s^2 - C2C3R2R3s^2 - C2R2s - C3R2s - 1}{C1R1s + C2R1s + C1C2C3R1R2R3s^3 + C1C2R1R2s^2 + C1C3R1R2s^2 + C2C3R1R2s^2 + C2C3R2R3s^2 + C2R2s + C3R2s + 1}$$

$$H(s) = C(sI - A)^{-1}B + D = \frac{-C1R1s - C2R1s - C1C2R1R2s^2 - C1C3R1R2s^2 - C2C3R1R2s^2 - C2C3R2R3s^2 - C2R2s - C3R2s - 1}{C1R1s + C2R1s + C1C2C3R1R2R3s^3 + C1C2R1R2s^2 + C1C3R1R2s^2 + C2C3R1R2s^2 + C2C3R2R3s^2 + C2R2s + C3R2s + 1} + [1] =$$

$$\frac{C1C2C3R1R2R3s^3}{C1R1s + C2R1s + C1C2C3R1R2R3s^3 + C1C2R1R2s^2 + C1C3R1R2s^2 + C2C3R1R2s^2 + C2C3R2R3s^2 + C2R2s + C3R2s + 1}$$

-impart atat numaratorul, cat si numitorul cu C1C2C3R1R2R3 => FT rezultat simbolic

$$H(s) = \frac{Y(s)}{U(s)} = \frac{s^3}{s^3 + s^2\left(\frac{1}{C1R3} + \frac{1}{C2R3} + \frac{1}{C3R3} + \frac{1}{C1R1}\right) + s\left(\frac{1}{C1R1C2R3} + \frac{1}{C1R1C3R3} + \frac{1}{C2R2C3R3} + \frac{1}{C1R2C3R3}\right) + \frac{1}{C1C2C3R1R2R3}}$$

CONCLUZIE : spatiul starilor si FT sunt egale.

FT rezultat numeric

- valorile:

R1 = 54kΩ R2 = 27kΩ R3 = 108kΩ

C1 = 2* 10^{-5} F C2 = 10^{-5} F C3 = 10^{-5}

$$H(s)=\frac{s^3}{s^3+s^2(\frac{1}{2*10^{-5}*108*10^3}+\frac{1}{10^{-5}*108*10^3}+\frac{1}{10^{-5}*108*10^3}+\frac{1}{2*10^{-5}*108*10^3})}$$

$$\frac{s^3}{+s(\frac{1}{2*10^{-5}*54*10^3*10^{-5}*108*10^3}+\frac{1}{2*10^{-5}*54*10^3*10^{-5}*108*10^3}+\frac{1}{2*10^{-5}*27*10^3*10^{-5}*108*10^3}+\frac{1}{2*10^{-5}*27*10^3*10^{-5}*108*10^3})}$$

$$\frac{s^3}{+\frac{1}{2*10^{-5}*10^{-10}*54*27*108*10^9}} \Rightarrow H(s)=\frac{s^3}{s^3+3.23s^2+6.84s+3.17}$$

3.SINGULARITATILE : zerourile (radacinile numaratorului) si polii (radacinile numitorului)

- valorile:

R1 = 54kΩ R2 = 27kΩ R3 = 108kΩ

C1 = 2* 10^{-5} F C2 = 10^{-5} F C3 = 10^{-5} F

-zerouri: s1 = s2 = s3 = 0;

-poli:

$$s^3 + 3.23s^2 + 6.84s + 3.17 = 0$$

$$s(s^2 + 3.23s + 6.84) + 3.17 = 0$$

$$s(s(s + 3.23) + 6.84) + 3.17 = 0$$

$$(s + 1.07)^3 + 3.36(s + 1.07) - 1.69 = 0$$

$$(s + 0.60)(s^2 + 2.62s + 5.25) = 0$$

$$s + 0.60 = 0 => s_1 = 0.60$$

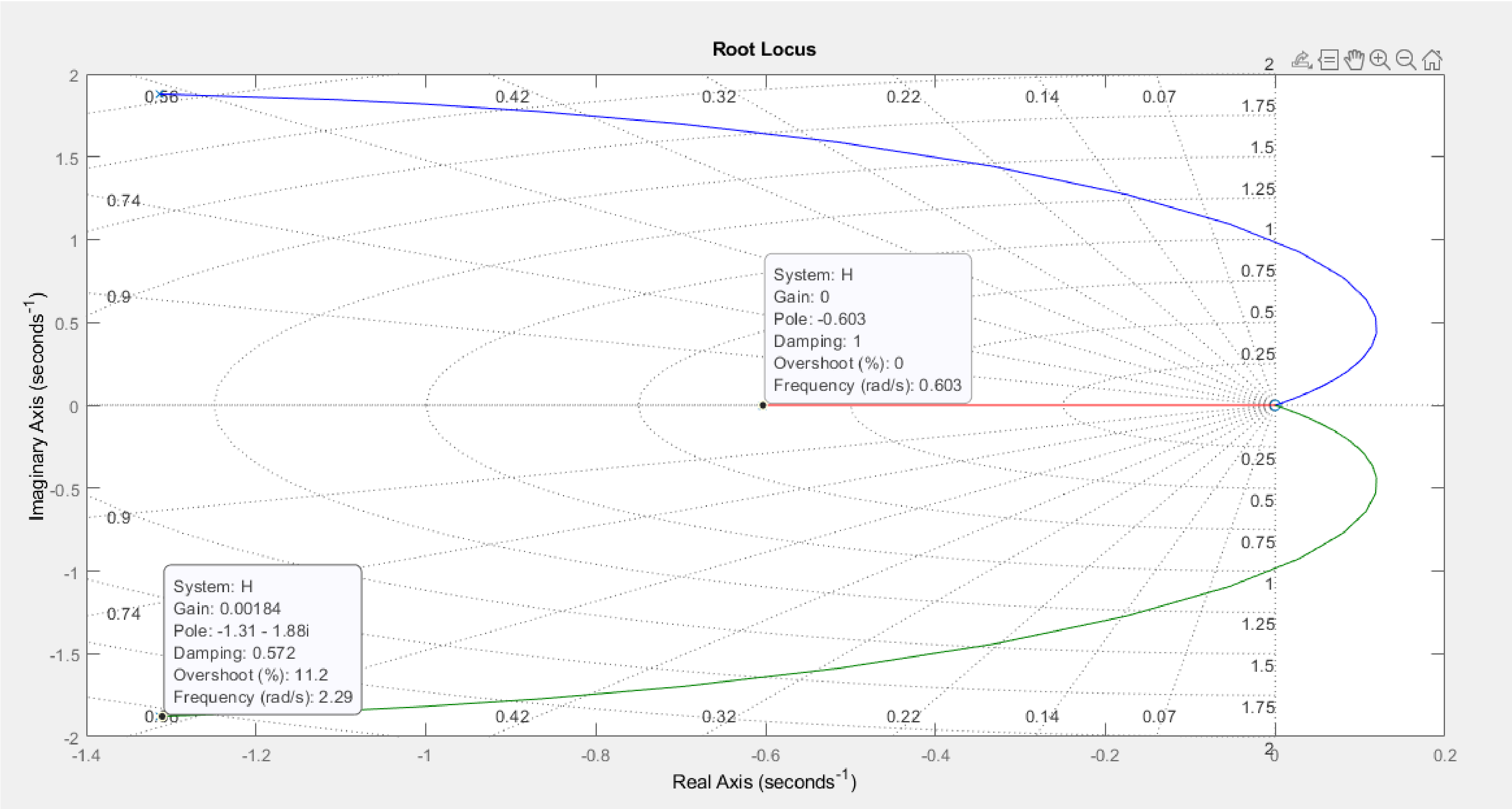
$$s^2 + 2.62s + 5.25 = 0$$

$$=> s_2 = -1.31 - 1.87i \qquad => s_3 = -1.31 + 1.87i$$

```
P=[1 3.23 6.84 3.17];
roots(P)
s1 = -0.60;        s2 = -1.31 + 1.87i;              s3 = -1.31 – 1.87i;
```

-in plan complex:

```
H = tf([1 0 0 0] , [1 3.23 6.84 3.17])
rlocus(H);
grid;
```


Root Locus
System: H
Gain: 0
Pole: -0.603
Damping: 1
Overshoot (%): 0
Frequency (rad/s): 0.603
System: H
Gain: 0.00184
Pole: -1.31 - 1.88i
Damping: 0.572
Overshoot (%): 11.2
Frequency (rad/s): 2.29
Imaginary Axis (seconds^-1)
Real Axis (seconds^-1)

5.FORMA MINIMALA FT – determinare

$$H(s) = \frac{s^3}{s^3 + 3.23s^2 + 6.84s + 3.17}$$

pas 1: determinarea parametrilor Markov - $\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5$ (2*n-1 = 5)

- impart polinomul de la numitor la polinomul de la numarator

$$
\begin{array}{l|l}
s^3 & s^3 + 3.23s^2 + 6.84s + 3.17 \\
-s^3 - 3.23s^2 - 6.84s - 3.17 & 1 - 3.23\frac{1}{s} + 3.59\frac{1}{s^2} + 7.33\frac{1}{s^3} - 37.99\frac{1}{s^4} + 61.19\frac{1}{s^5} \\
\hline
3.23s^2 + 10.43s + 22.09 + 10.23\frac{1}{s} & \\
\ldots &
\end{array}
$$

=>
$$
\begin{array}{ll}
\gamma_0 = 1 & \gamma_3 = 7.33 \\
\gamma_1 = -3.23 & \gamma_4 = -37.99 \\
\gamma_2 = 3.59 & \gamma_5 = 61.19
\end{array}
$$

```
%% parametrii Markov
num = [1 0 0 0];
den = [1 3.23 6.84 3.17];
markov = deconv([num zeros(1,6)],den)      % ca sa returneze primii 6 parametrii
```

pas 2: Matricea Hankel

$$H_{3x3} = \begin{pmatrix} \gamma_1 & \gamma_2 & \gamma_3 \\ \gamma_2 & \gamma_3 & \gamma_4 \\ \gamma_3 & \gamma_4 & \gamma_{15} \end{pmatrix} = \begin{pmatrix} -3.23 & 3.59 & 7.33 \\ 3.59 & 7.33 & -37.99 \\ 7.33 & -37.99 & 61.19 \end{pmatrix}$$

pas 3: verificarea rangului H_{3x3}

$$\det(H) = \begin{pmatrix} -3.23 & 3.59 & 7.33 \\ 3.59 & 7.33 & -37.99 \\ 7.33 & -37.99 & 61.19 \end{pmatrix} = -3.23*61.19*7.33 - 2*37.99*3.59*7.33 - 7.33^3 + 37.99^2*3.23 - 61.19*3.59^{2=}$$

=31.0906

$\det(H_{3x3}) \neq 0$ => rang (H_{3x3}) =3

⇨ H(s) este in forma minimala

```
%% FT minim
num = [1 0 0 0];
den = [1 3.23 6.78 3.17];
```

```
H =

               s^3
  ------------------------------
  s^3 + 3.23 s^2 + 6.78 s + 3.17

Continuous-time transfer function.


Hm =

               s^3
  ------------------------------
  s^3 + 3.23 s^2 + 6.78 s + 3.17

Continuous-time transfer function.
```

4.REALIZARILE DE STARE CORESPUNZATOARE FORMELOR CANONICE

OBSERVATIE: pentru a putea determina formele FCC si FCO primul pas este aducerea functiei de transfer in stare minimala (pasul 5)

- forme canonice: trecerea din FT in modelul in spatiul starilor

$$H(s) = d + \frac{b_{n-1}s^{n-1} + \dots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \dots + a_1 s + a_0}$$

- unde $d = \lim_{s \to 0} H(s)$

FORMA CANONICA DE CONTROL (FCC) : este necesara rescrierea functiei de transfer deoarece gradul numaratorului trebuie sa fie mai mic decat cel al numitorului

$$\left(\begin{array}{c|c} A_{FCC} & B_{FCC} \\ \hline C_{FCC} & D \end{array}\right) = \left(\begin{array}{ccccc|c} -a_{n-1} & -a_{n-2} & \dots & -a_1 & -a_0 & 1 \\ 1 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 & 0 \\ \hline b_{n-1} & b_{n-2} & \dots & b_1 & b_0 & d \end{array}\right).$$

- **simbolic**

H(s)=

$$=1+\frac{-s^2\left(\frac{1}{C1R3}+\frac{1}{C2R3}+\frac{1}{C3R3}+\frac{1}{C1R1}\right)-s\left(\frac{1}{C1R1C2R3}+\frac{1}{C1R1C3R3}+\frac{1}{C2R2C3R3}+\frac{1}{C1R2C3R3}\right)-\frac{1}{C1C2C3R1R2R3}}{s^3+s^2\left(\frac{1}{C1R3}+\frac{1}{C2R3}+\frac{1}{C3R3}+\frac{1}{C1R1}\right)+s\left(\frac{1}{C1R1C2R3}+\frac{1}{C1R1C3R3}+\frac{1}{C2R2C3R3}+\frac{1}{C1R2C3R3}\right)+\frac{1}{C1C2C3R1R2R3}}$$

$\Rightarrow$ $FCC=\begin{pmatrix} A_{FCC} & B_{FCC} \\ C_{CC} & D \end{pmatrix}$ =

$$\left(\begin{array}{ccc|c}
-\frac{1}{C1R3}-\frac{1}{C2R3}-\frac{1}{C3R3}-\frac{1}{C1R1} & -\frac{1}{C1R1C2R3}-\frac{1}{C1R1C3R3}-\frac{1}{C2R2C3R3}-\frac{1}{C1R2C3R3} & -\frac{1}{C1C2C3R1R2R3} & 1 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
\hline
-\frac{1}{C1R3}-\frac{1}{C2R3}-\frac{1}{C3R3}-\frac{1}{C1R1} & -\frac{1}{C1R1C2R3}-\frac{1}{C1R1C3R3}-\frac{1}{C2R2C3R3}-\frac{1}{C1R2C3R3} & -\frac{1}{C1C2C3R1R2R3} & 1
\end{array}\right)$$

- **numeric**

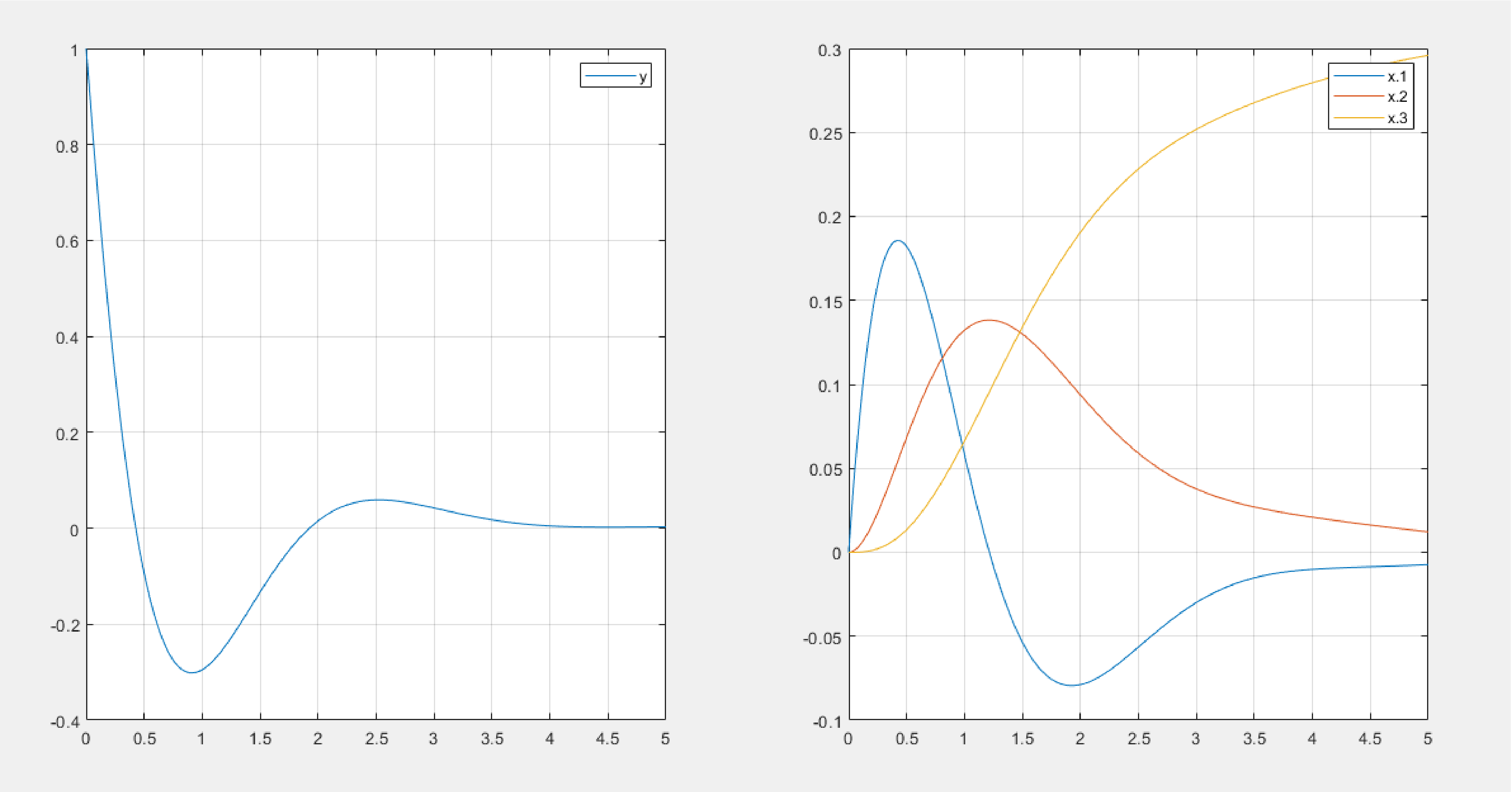
$$H(s) = 1 + \frac{-3.23s^2 - 6.84s - 3.17}{s^3 + 3.23s^2 + 6.84s + 3.17}$$

$$FCC = \begin{pmatrix} A_{FCC} & B_{FCC} \\ C_{CC} & D \end{pmatrix} = \left(\begin{array}{ccc|c} -3.23 & -6.84 & -3.17 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline -3.23 & -6.84 & -3.17 & 1 \end{array}\right)$$

```
%% FCC

num = [1 0 0 0];
den = [1 3.23 6.84 3.17];
b2=-3.23; b1=-6.48; b0=-3.17;
a2=3.23; a1=6.48; a0=3.17;
[A,b,c,d]=tf2ss(num,den) %obtinerea FCC (FT to ss)

sistem = ss(A,b,c,d);
```

```
x0=[0,0,0] % conditii initiale
t=0:0.01:5;
u=ones(1,length(t)); % semnal treapta
[y,t,x] = lsim(sistem,u,t,x0); %raspunsul sistemului afisat grafic

subplot(121); plot(t,y); legend('y'); grid;
subplot(122); plot(t,x); legend('x.1', 'x.2', 'x.3'); grid;
```

```
A =

   -3.2300   -6.7800   -3.1700
    1.0000         0         0
         0    1.0000         0


b =

     1
     0
     0


c =

   -3.2300   -6.7800   -3.1700


d =

     1
```



y
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
x.1
x.2
x.3
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

- **ecuatile de stare/iesire:**

$$\begin{cases}\dot{x}_1 = -(\frac{1}{C1R3}+\frac{1}{C2R3}+\frac{1}{C3R3}+\frac{1}{C1R1})\cdot x_1 - (\frac{1}{C1R1C2R3}+\frac{1}{C1R1C3R3}+\frac{1}{C2R2C3R3}+\frac{1}{C1R2C3R3})\cdot x_2 - \frac{1}{R_1R_2R_3C_1C_2C_3}\cdot x_3 + u \\ \dot{x}_2 = x_1 \\ \dot{x}_3 = x_2 \\ y = -(\frac{1}{C1R3}+\frac{1}{C2R3}+\frac{1}{C3R3}+\frac{1}{C1R1})\cdot x_1 - (\frac{1}{C1R1C2R3}+\frac{1}{C1R1C3R3}+\frac{1}{C2R2C3R3}+\frac{1}{C1R2C3R3})\cdot x_2 - \frac{1}{R_1R_2R_3C_1C_2C_3}\cdot x_3 + u\end{cases}$$

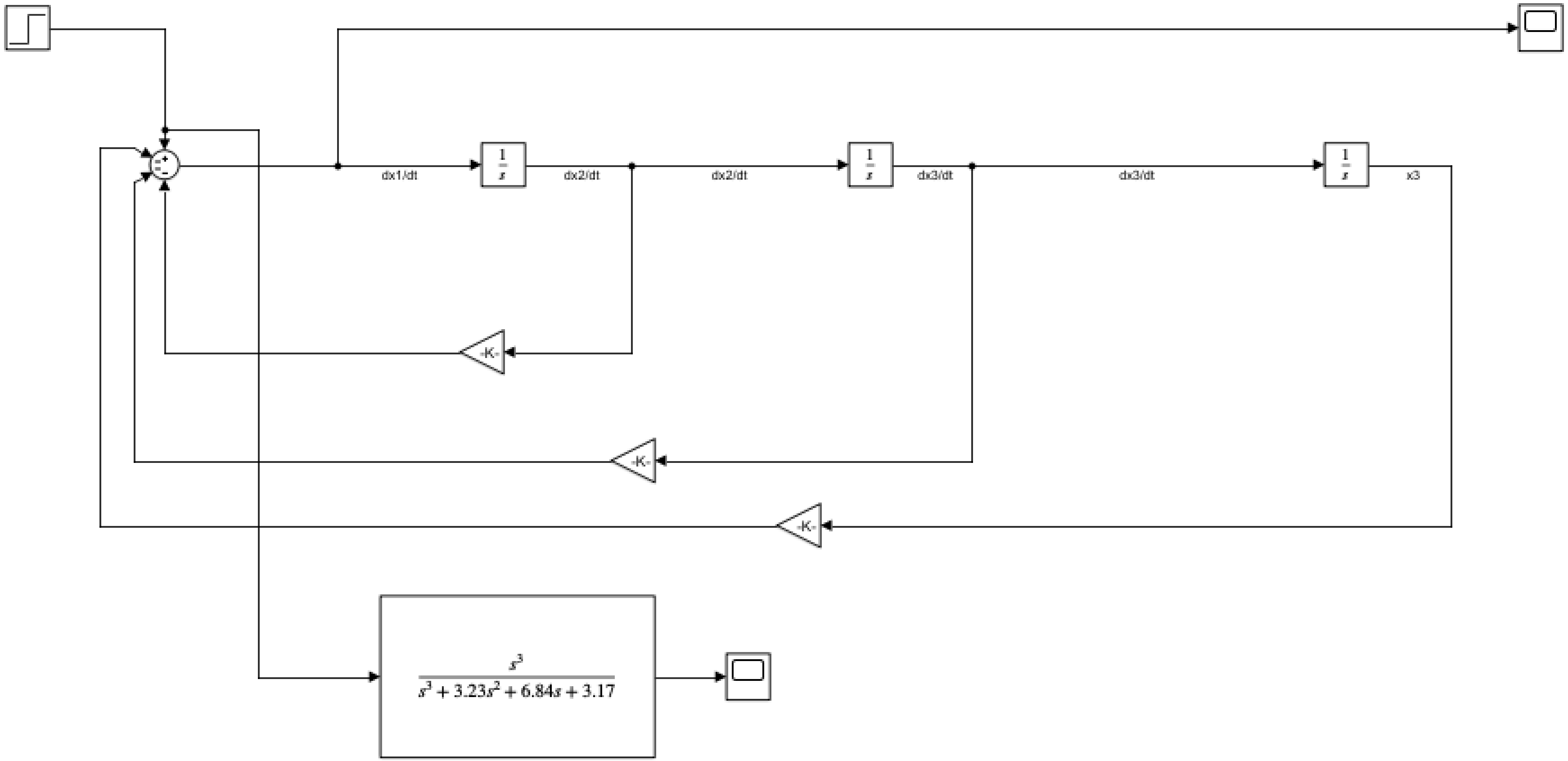
$$\frac{dx_1}{dt} = -3.23x_1 - 6.84x_2 - 3.17x_3 + u$$

$$\frac{dx_2}{dt} = x_1$$

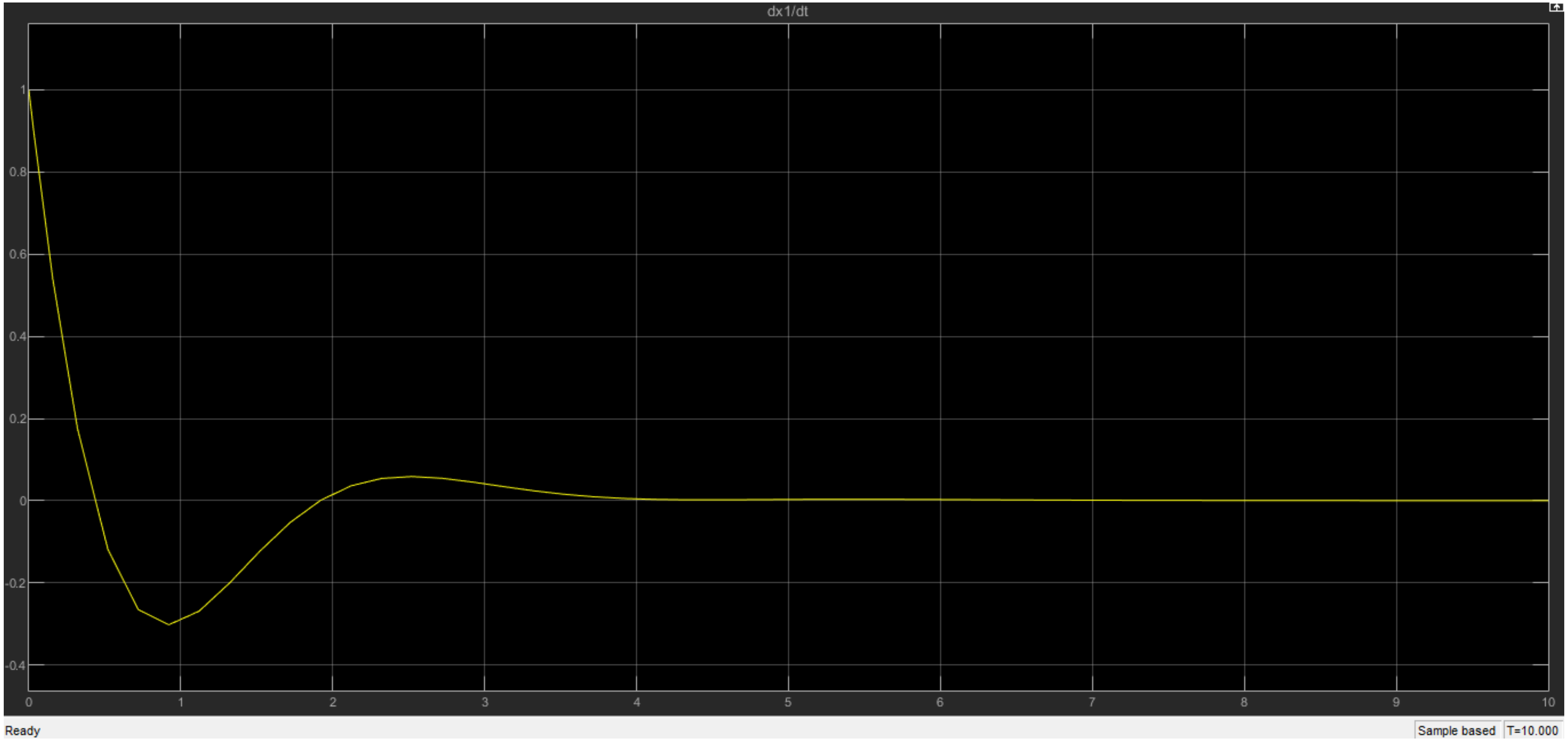
$$\frac{dx_3}{dt} = x_2$$

$$y = -3.23x_1 - 6.84x_2 - 3.17x_3 + u$$

FCC schema bloc:



FCC grafic:


dx1/dt
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
1
2
3
4
5
6
7
8
9
10
Ready
Sample based
T=10.000

FORMA CANONICA DE OBSERVARE (FCO) :

$$\left(\begin{array}{c|c} A_{FCO} & B_{FCO} \\ \hline C_{FCO} & D \end{array}\right) = \left(\begin{array}{ccccc|c} -a_{n-1} & 1 & \dots & 0 & 0 & b_{n-1} \\ -a_{n-2} & 0 & \dots & 0 & 0 & b_{n-2} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ -a_1 & 0 & \dots & 0 & 1 & b_1 \\ -a_0 & 0 & \dots & 0 & 0 & b_0 \\ \hline 1 & 0 & \dots & 0 & 0 & d \end{array}\right).$$

OBSERVATIE:

$A_{FCO} = A_{FCC}^T$

$B_{FCO} = C_{FCC}^T$

$C_{FCO} = B_{FCC}^T$

acelasi D

- **simbolic**

H(s)=

$$=1+\frac{-s^2\left(\frac{1}{C1R3}+\frac{1}{C2R3}+\frac{1}{C3R3}+\frac{1}{C1R1}\right)-s\left(\frac{1}{C1R1C2R3}+\frac{1}{C1R1C3R3}+\frac{1}{C2R2C3R3}+\frac{1}{C1R2C3R3}\right)-\frac{1}{C1C2C3R1R2R3}}{s^3+s^2\left(\frac{1}{C1R3}+\frac{1}{C2R3}+\frac{1}{C3R3}+\frac{1}{C1R1}\right)+s\left(\frac{1}{C1R1C2R3}+\frac{1}{C1R1C3R3}+\frac{1}{C2R2C3R3}+\frac{1}{C1R2C3R3}\right)+\frac{1}{C1C2C3R1R2R3}}$$

$$FCO = \begin{pmatrix} A_{FCO} & B_{FCO} \\ C_{CO} & D \end{pmatrix} =$$

$$\left[\begin{array}{ccc|c}
-\frac{1}{C1R3}-\frac{1}{C2R3}-\frac{1}{C3R3}-\frac{1}{C1R1} & 1 & 0 & \frac{1}{C1R3}+\frac{1}{C2R3}+\frac{1}{C3R3}+\frac{1}{C1R1} \\
-\frac{1}{C1R1C2R3}-\frac{1}{C1R1C3R3}-\frac{1}{C2R2C3R3}-\frac{1}{C1R2C3R3} & 0 & 1 & \frac{1}{C1R1C2R3}+\frac{1}{C1R1C3R3}+\frac{1}{C2R2C3R3}+\frac{1}{C1R2C3R3} \\
-\frac{1}{C1C2C3R1R2R3} & 0 & 0 & \frac{1}{C1C2C3R1R2R3} \\
\hline
1 & 0 & 0 & 1
\end{array}\right]$$

- **numeric**

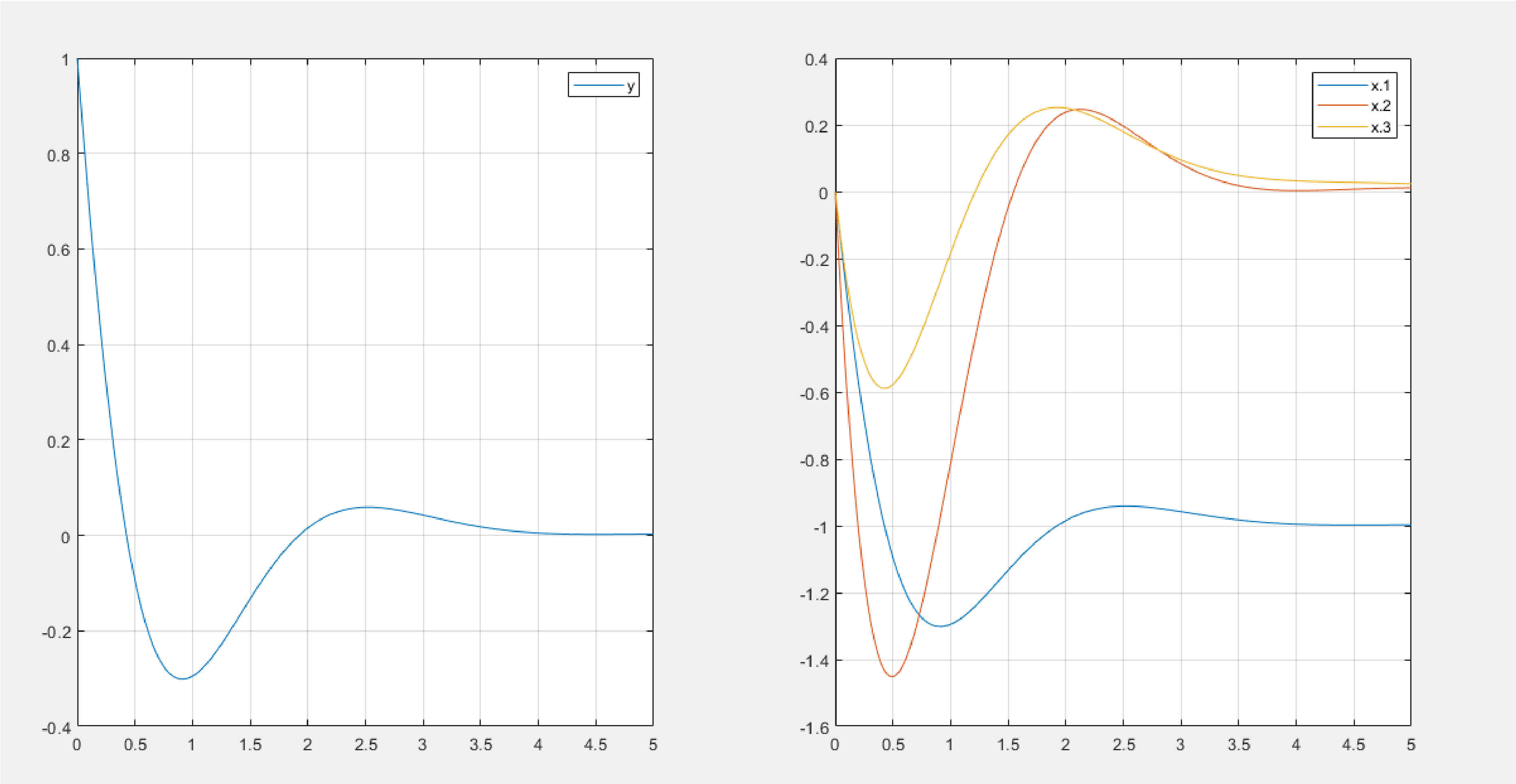
$$H(s) = 1 + \frac{-3.23s^2 - 6.84s - 3.17}{s^3 + 3.23s^2 + 6.84s + 3.17}$$

$$FCO = \begin{pmatrix} A_{FCO} & B_{FCO} \\ C_{CO} & D \end{pmatrix} = \begin{array}{ccc|c} -3.23 & 1 & 0 & -3.23 \\ -6.48 & 0 & 1 & -6.48 \\ -3.17 & 0 & 0 & -3.17 \\ \hline \mathbf{1} & \mathbf{O} & \mathbf{O} & 1 \end{array}$$

```
%% FCO

A=[-3.23 -6.84 -3.17;
   1 0 0;
   0 1 0];
b=[1; 0; 0];
c=[-3.23 -6.84 -3.17];
d=[1];
num = [1 0 0 0];
den = [1 3.23 6.48 3.17];
b2=-3.23; b1=-6.48; b0=-3.17;
a2=3.23; a1=6.48; a0=3.17;
[A,b,c,d]=tf2ss(num,den); %obtinerea FCC (FT to ss)
Afco = A'
```

```
Bfco=c'
Cfco=b'
Dfco=d
sistem = ss(Afco,Bfco,Cfco,Dfco);
x0=[0,0,0] % conditii initiale
t=0:0.01:5;
u=ones(1,length(t)); % semnal treapta
[y,t,x] = lsim(sistem,u,t,x0); %raspunsul sistemului afisat grafic
subplot(121); plot(t,y); legend('y'); grid;
subplot(122); plot(t,x); legend('x.1', 'x.2', 'x.3'); grid;
```

```
Afco =

   -3.2300    1.0000         0
   -6.7800         0    1.0000
   -3.1700         0         0


Bfco =

   -3.2300
   -6.7800
   -3.1700


Cfco =

     1     0     0


Dfco =

     1
```



y
x.1
x.2
x.3

- **ecuatiile de stare/iesire:**

$$\begin{cases} \dot{x}_1 = -(\frac{1}{C1R3} + \frac{1}{C2R3} + \frac{1}{C3R3} + \frac{1}{C1R1}) \cdot x_1 - (\frac{1}{C1R1C2R3} + \cdot x_2 + (\frac{1}{C1R3} + \frac{1}{C2R3} + \frac{1}{C3R3} + \frac{1}{C1R1})u \\ \dot{x}_2 = -(\frac{1}{C1R1C2R3} + \frac{1}{C1R1C3R3} + \frac{1}{C2R2C3R3} + \frac{1}{C1R2C3R3})x_1 + x_3 + (\frac{1}{C1R1C2R3} + \frac{1}{C1R1C3R3} + \frac{1}{C2R2C3R3} + \frac{1}{C1R2C3R3})u \\ \dot{x}_3 = \frac{1}{R_1R_2R_3C_1C_2C_3} x_1 + \frac{1}{R_1R_2R_3C_1C_2C_3} u \\ y = \cdot x_1 + u \end{cases}$$

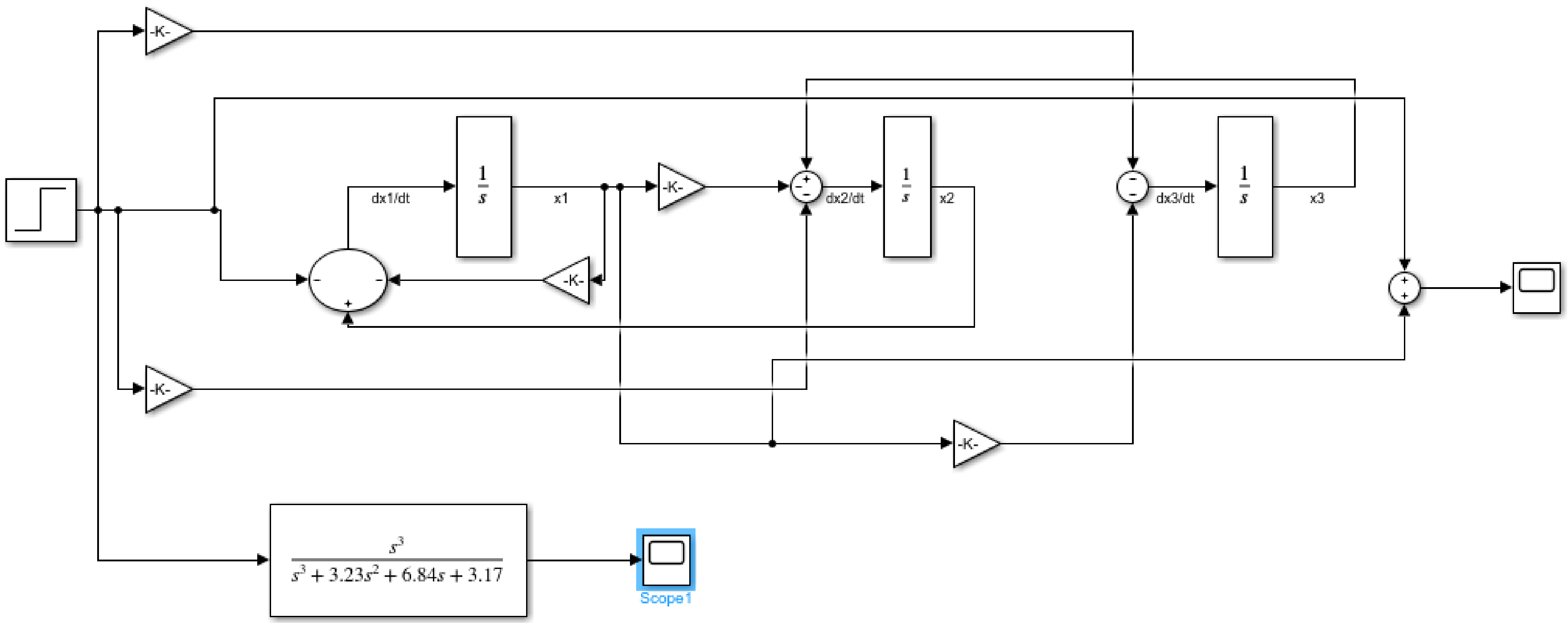
$$\frac{dx_1}{dt} = -3.23x_1 + x_2 - 3.23u$$

$$\frac{dx_2}{dt} = -6.84x_1 + x_3 - 6.84u$$

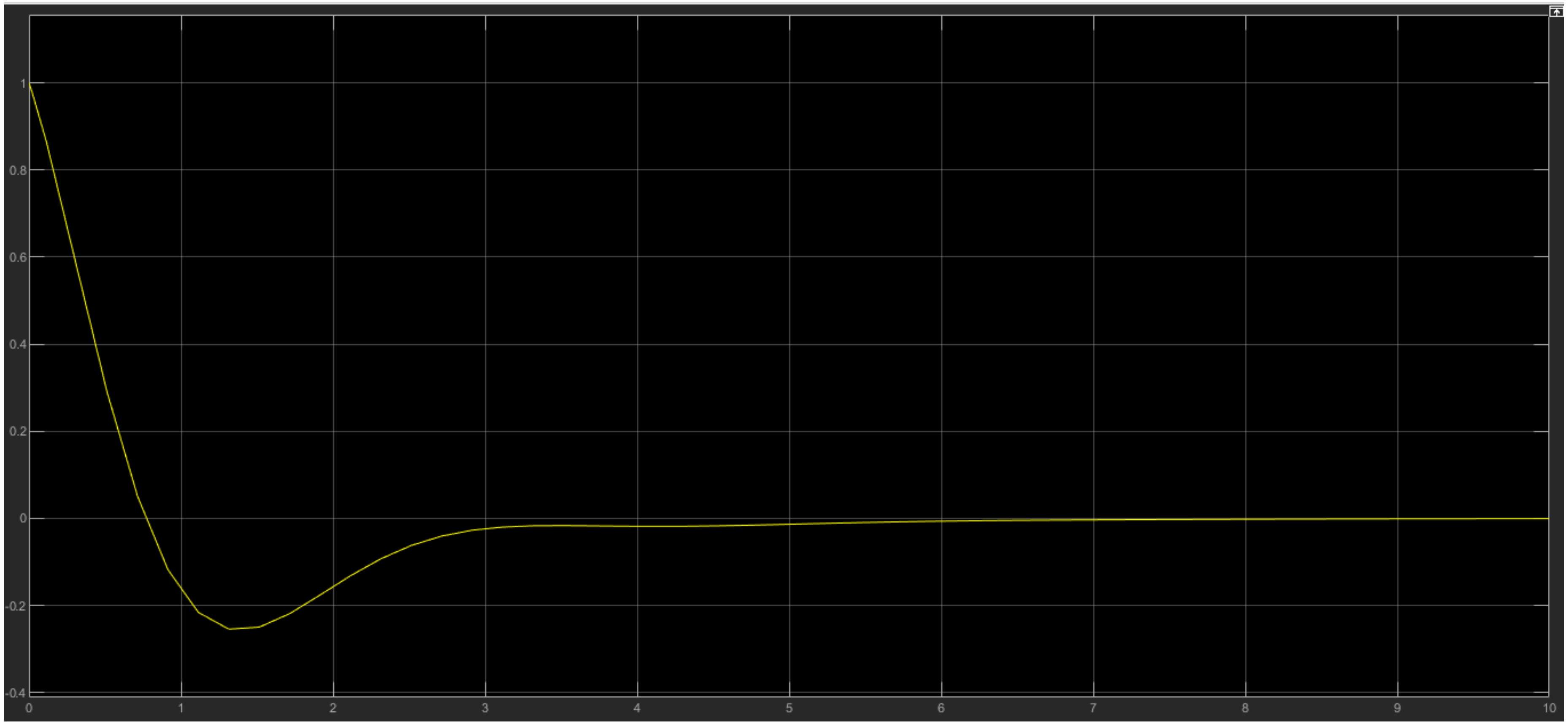
$$\frac{dx_3}{dt} = -3.17x_1 - 3.17u$$

$$y = x_1 + u$$

FCO schema bloc:



FCO grafic:



6. STABILITATE

INTERNA – utilizand valorile proprii ale matricei de stare

$$A = \begin{pmatrix} -1.38 & -0.46 & 1.38 \\ -0.92 & -0.92 & 2.77 \\ -0.92 & -0.92 & -0.92 \end{pmatrix} \qquad B = \begin{pmatrix} 2.77 \\ 0.92 \\ 0.92 \end{pmatrix} \qquad C = \begin{pmatrix} -1 & -1 & -1 \end{pmatrix} \qquad D = [1]$$

-pentru ca sistemul sa fie **intern stabil**, trebiue ca toate valorile proprii sa fie in semiplanul stang

$$\mathrm{Re}\{\hat{\lambda}_i\} < 0, \forall \hat{\lambda}_i \in \wedge(A)$$

$$\det(\lambda I - A) = 0 => \begin{vmatrix} \lambda + 1.38 & 0.46 & -1.38 \\ -0.92 & -0.92 & 2.77 \\ -0.92 & -0.92 & -0.92 \end{vmatrix} =$$

$$(\lambda + 0.92)^3 - 1.26\lambda - 2.54\lambda + \lambda * 0.46 * (\lambda + 0.92)$$

$$= \lambda^3 + 3.23\lambda^2 + 6.78\lambda + 3.12 = 0$$

```
P=[1 3.23 6.78 3.12];
roots(P)
```

```
ans =

  -1.3152 + 1.8636i
  -1.3152 - 1.8636i
  -0.5997 + 0.0000i
```

$$\lambda_1 = -1.31 + 1.86i$$

=> $$\lambda_2 = -1.31 - 1.86i$$

$$\lambda_3 = 0.59$$

- **tabelul Routh-Hurwitz**

λ^3	1	6.78
λ^2	3.22	3.12
λ^1	5.81	0
λ^0	-3.12	0

$$b_1 = -\frac{\begin{vmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{vmatrix}}{a_{n-1}}, \quad b_2 = -\frac{\begin{vmatrix} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{vmatrix}}{a_{n-1}}, \ldots$$

$$c_1 = -\frac{\begin{vmatrix} a_{n-1} & a_{n-3} \\ b_1 & b_2 \end{vmatrix}}{b_1}, \quad c_2 = -\frac{\begin{vmatrix} a_{n-1} & a_{n-5} \\ b_1 & b_3 \end{vmatrix}}{b_1}, \ldots$$

$$b_1 = \frac{-\begin{vmatrix} 1 & 6.87 \\ 3.22 & 3.12 \end{vmatrix}}{3.22} = \frac{-(3.12 - 21.83)}{3.22} = 5.81$$

$$b_2 = \frac{-\begin{vmatrix} 1 & 0 \\ 3.22 & 0 \end{vmatrix}}{2} = 0$$

$$c_1 = \frac{-\begin{vmatrix} 3.22 & 3.12 \\ 5.81 & 0 \end{vmatrix}}{2} = 3.12$$

OBSERVATIE: polinomul caracteristic P(s) are toate radacinile in semiplanul stang ⇔ pe prima coloana a tabelului nu exista nicio schimbare de semn. (adevarat in acest caz)

⇨ SISTEM STABIL INTERN

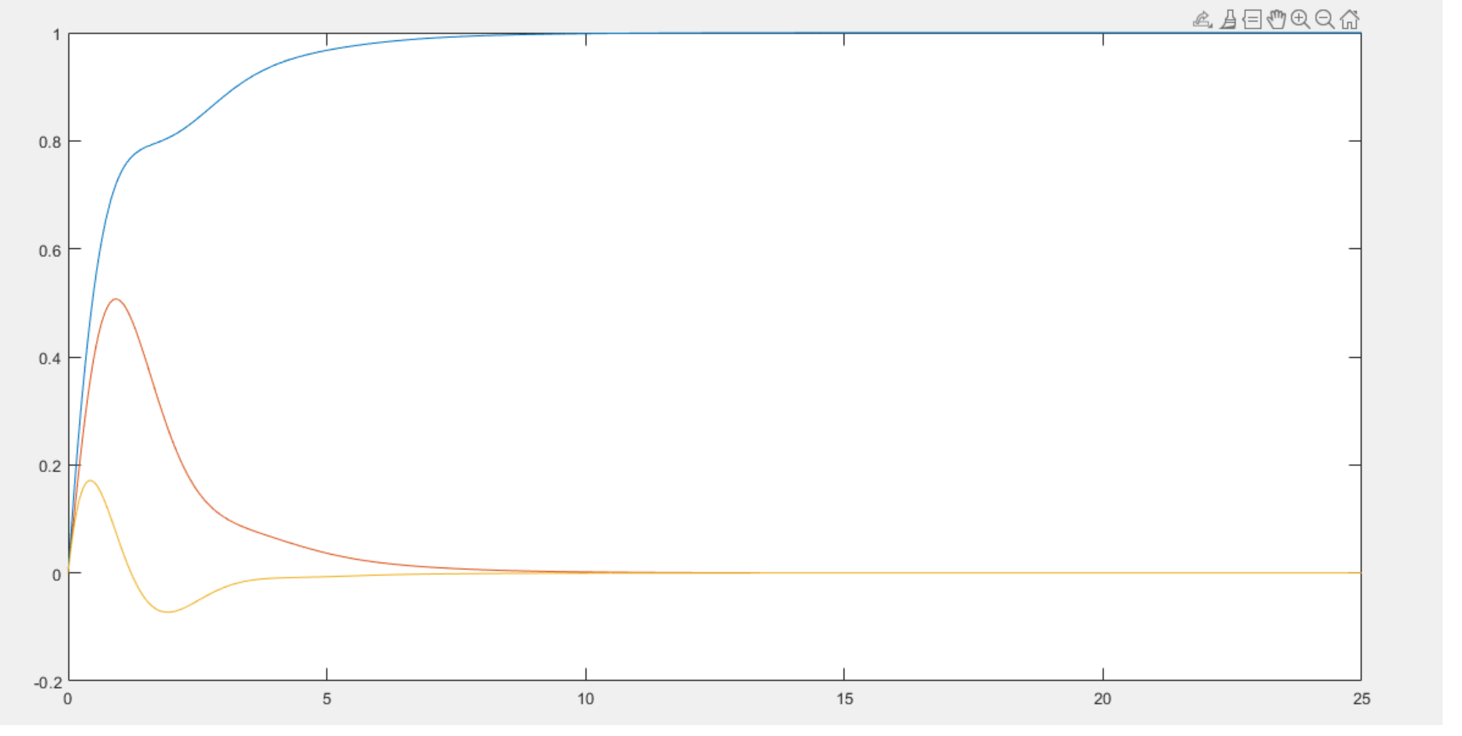
EXTERNA – ulilizand polii sistemului

$$H\left(s\right)=C(sI-A)^{-1}B+D=\frac{\beta(s)}{\alpha(s)}$$

-pentru ca sistemul sa fie **extern stabil**, trebiue ca toati polii functiei de transfer in forma mainimala sa fie in semiplanul stang

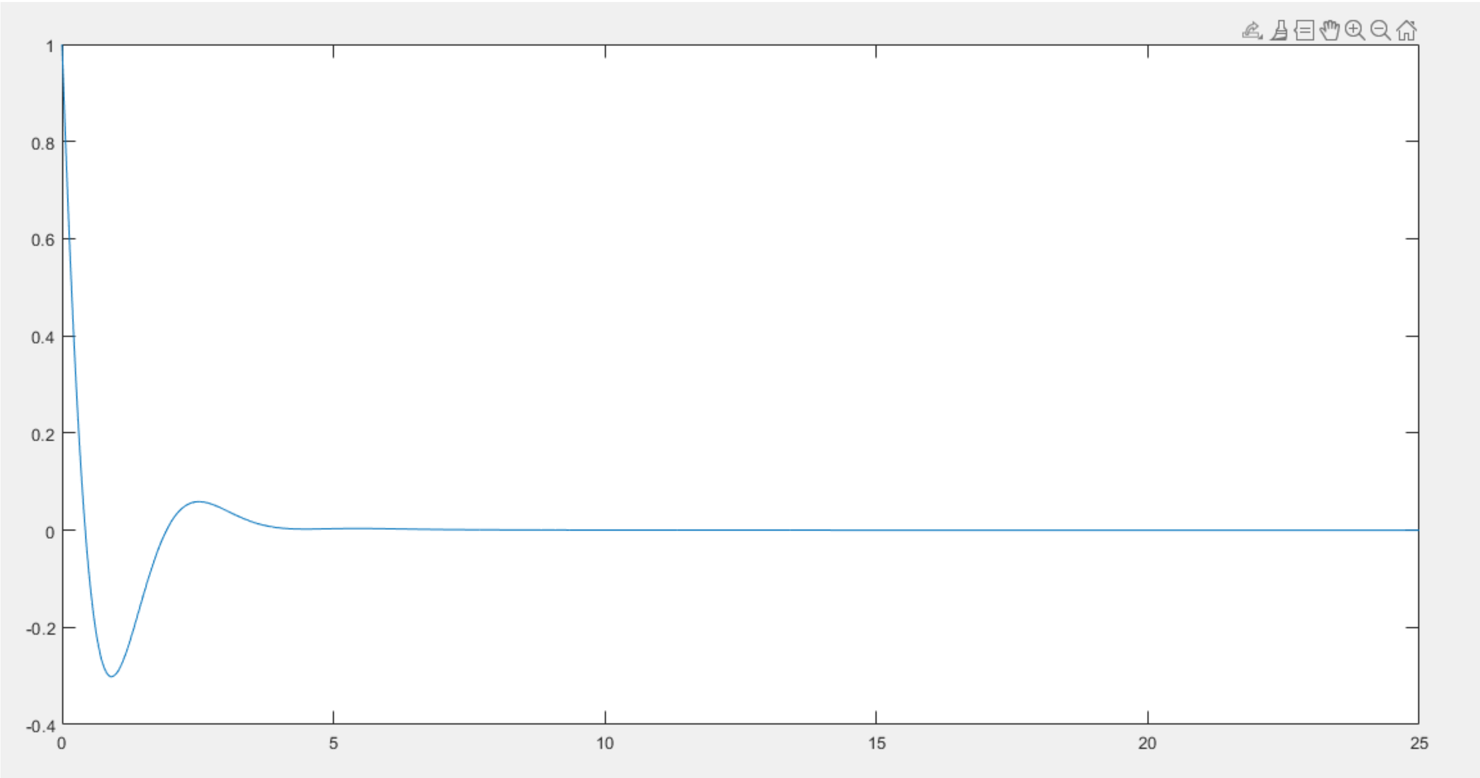
$$\mathrm{Re}\{\hat{s}_i\}<0,\forall\,\hat{s}_i\in\{s\mid\alpha(s)=0\}$$

```
%% stabilitate INTERNA
A=[-1.38 -0.46 1.38;
   -0.92 -0.92 2.77;
   -0.92 -0.92 -0.92]
B=[1.38; 0.92; 0.92];
C=[-1 -1 -1];
D=[1];
sys=ss(A,B,C,D);
t=0:0.01:25 ;
u=(t>=0);
[y,t,x]=lsim(sys,u,t);
plot(t,x);
```




```
%% stabilitate EXTERNA
A=[-1.38 -0.46 1.38;
   -0.92 -0.92 2.77;
   -0.92 -0.92 -0.92]
B=[1.38; 0.92; 0.92];
C=[-1 -1 -1];
D=[1];
```

```
sys=ss(A,B,C,D);
t=0:0.01:25 ;
u=(t>=0);
[y,t,x]=lsim(sys,u,t);
plot(t,y);
```



OBSERVATIE: pe baza figurii => sistemul se stabilizeaza.

- Stabilitatea interna implica stabilitatea externa => SISTEM EXTERN STABIL

7.

DET. STABILITATE INTERNA PRIN ECUATIA ALGEBRICA LYAPUNOV + EXTRAGEREA FUNCTIEI CANDIDAT

-conform teoriei lui Lyapunov, se caut o functie scalar de tip "energie" pozitiv definita , având derivata nagativa , ceea ce semnifica un caracter disipativ al sistemului stabil:

$$V(x) = x^T P x, \quad P = P^T > 0$$

-sistemul este stabil daca si numai daca:

$$A^T P + PA < 0$$

$$A^T P + PA = -Q, \quad Q = Q^T > 0$$

Observație: În funcția `lyap` din MATLAB, ecuația matricială este de forma $AX + XA^T + Q = 0$. De aceea, este necesar apelul funcției `lyap` cu matricea A transpusă.

Metoda prezintă condițiile necesară și suficientă de determinare a stabilității interne (în sens Lyapunov).

-folosind Matlab, am calculat P. Apoi am aflat valorile proprii pentru P:

```
%% Lyapunov
A=[-1.38 -0.46 1.38;
    -0.92 -0.92 2.77;
    -0.92 -0.92 -0.92]
B=[1.38; 0.92; 0.92];
C=[-1 -1 -1];
D=[1];

Q=eye(3);
P=lyap(A',Q)
Val_pr_P=eig(P)

%%simulare pt conditii initiale
t=0:1:25;
    %u=(t>=0)
u=zeros(1,length(t));
x0=[11 8 1];
[y,t,x]=lsim(ss(A,B,C,D),u,t,x0);
figure;
V=zeros(1,length(t));
for i=1:length(t)
    V(i)=x(i,:)*P*x(i,:)';
end
plot(t,V);grid
eig(A);
```

```
Q=2*eye(3);
P =lyap(A',Q)
eig(P)
```

OBSERVATIE: valorile proprii ale matricei P sunt toate > 0 => sistemul este INTERN ASIMPTOTIC STABIL

$V(x) = x^T P x$ => V(x) = $\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \begin{pmatrix} 1.54 & -0.63 & -0.59 \\ -0.63 & 0.98 & 0.42 \\ -0.59 & 0.42 & 1.46 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ =

= $1.54x_1^2 + 0.98x_2^2 + 1.46x_3^2 - 1.26x_1x_2 - 1.18x_1x_3 + 0.84x_2x_3$

8.

FUNCTIA PONDERE / RASPUNSUL LA IMPULS

$$h(t) = L^{-1}\left\{H(s)\right\}$$

$$h(t) = L^{-1}\{\frac{s^3}{s^3 + 3,23s^2 + 6,84s + 3,17}\} =$$

$$= L^{-1}\{1 + \frac{-3,23s^2 - 6,84s - 3,17}{s^3 + 3,23s^2 + 6,84s + 3,17}\} =$$

$$= L^{-1}\{1 + \frac{-3,23s^2 - 6,84s - 3,17}{(s + 0,6)(s^2 + 2,62s + 5.25)}\}$$

$$\frac{-3,23s^2 - 6,84s - 3,17}{(s + 0,6)(s^2 + 2,62s + 5.25)} = \frac{A}{(s + 0,6)} + \frac{Bs + C}{(s^2 + 2,62s + 5.25)}$$

$$-3,23s^2 - 6,84s - 3,17 = A(s^2 + 2,62s + 5.25) + (Bs + C)(s + 0,6)$$

$$-3,23s^2 - 6,84s - 3,17 = s^2(A + B) + s(2,32A + 0,6B + C) + 5,25A + 0,6C$$

$$A + B = -3,23$$
$$2,32A + 0,6B + C = -6,84$$
$$5,25A + 0,6C = -3,17$$
$$B = -3,23 - A; C = -5,28 - 8,75A;$$
$$2,62A - 1,93 - 0,6A - 5,28 - 8,75A = -6,84 => -6,73A = 0,37$$
$$A = -0.05; B = -3,18; C = -4,84$$

$$s^2 + 2,62s + 5.25 = s^2 + 2s * 1,31 + 1,31^2 - 1,31^2 + 5,25 = (s + 1,31)^2 + 3,54$$
$$L^1\{\frac{s + 1,52}{s^2 + 2,62s + 5.25}\} = L^1\{\frac{s + 1,52}{(s + 1,31)^2 + 3,54}\} = L^1\{\frac{s + 1,31 - 0,21}{(s + 1,31)^2 + 3,54}\} =$$
$$= L^1\{\frac{s + 1,31}{(s + 1,31)^2 + 3,54}\} + L^1\{\frac{-0,21}{(s + 1,31)^2 + 3,54}\} =$$
$$= L^1\{\frac{s + 1,31}{(s + 1,31)^2 + 3,54}\} - 0,21L^1\{\frac{1}{(s + 1,31)^2 + 3,54}\} =$$
$$= \cos(\sqrt{3,54t})e^{-1,31t} - 0,21(\frac{1}{\sqrt{3.54}}\sin(\sqrt{3,54t})e^{-1,31t}) =$$
$$= e^{-1,31t}(\cos(1.87t) - 0.1\sin(1.87t))$$
$$h(t) = \delta(t) - 0,05e^{-0,6t} - 3,18e^{-1,31t}(\cos(1.87t) - 0.1\sin(1.87t))$$

$$y(t) = L^{-1}\{H(s) * U(s)\} = y_l(t) + y_p(t)$$

$y_l =$ componenta tranzitorie(libera)

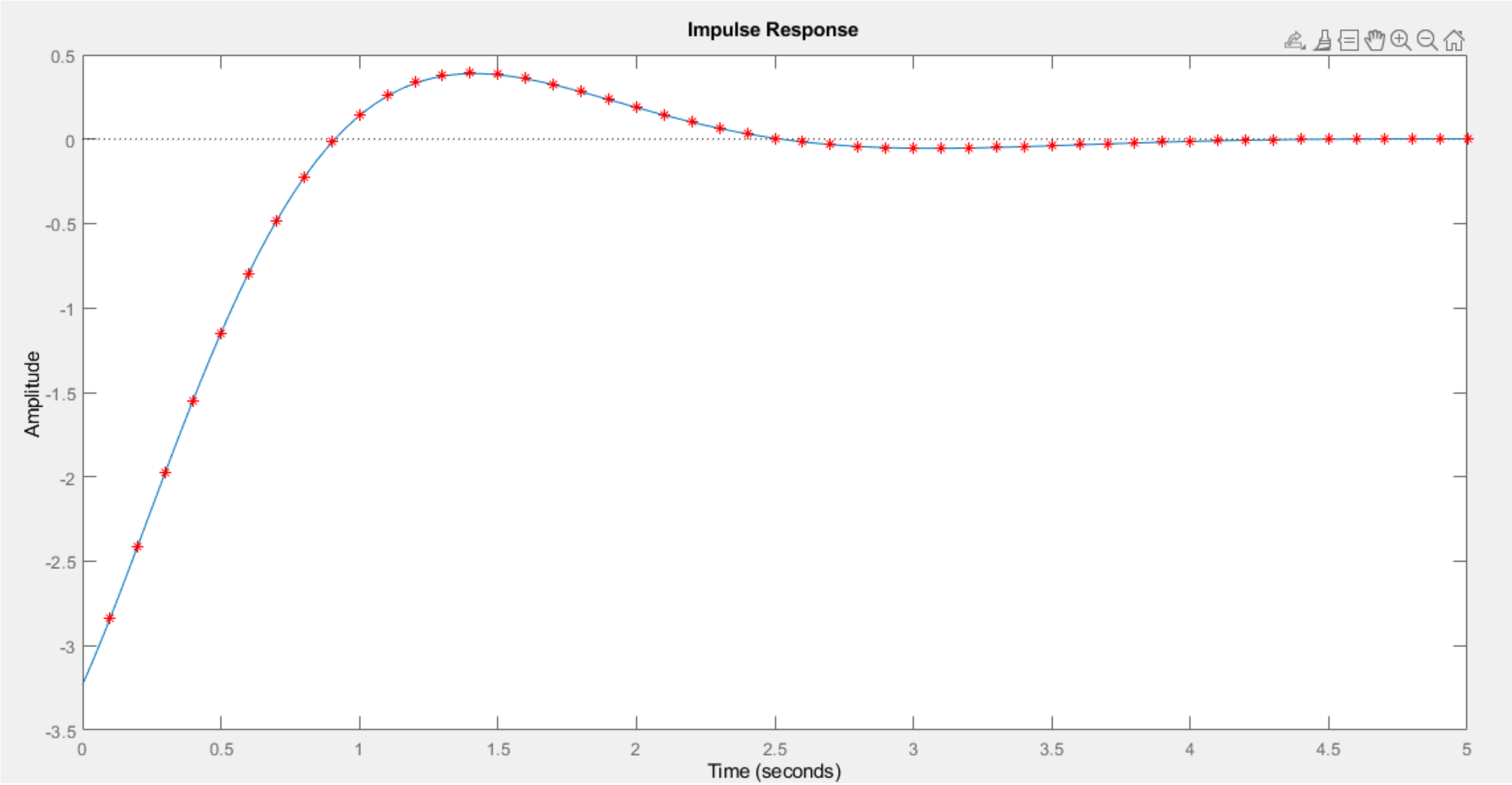
$y_p(t)$ =componenta permanenta(raspuns fortat)->depinde de u

$y_l = h(t) = L^{-1}\{H(s)\}$ =y(t)

$y_p(t)$ =0

-moduri $e^{\hat{s}_1 t} = e^{-0.6t}$ $e^{\mathrm{Re}\{s\hat{2,}3\}t} \sin(\mathrm{Im}\{s2\hat{,}3\}t) = e^{-1.31t} \sin 1.87t$

```
%% raspunsul la impuls
H=tf([1 0 0 0],[1 3.23 6.84 3.17])
impulse(H)
hold on
t=0:0.1:10
h=dirac(t)-0.05*exp(-0.6*t)-3.18*exp(-1.31*t).*(cos(1.87*t)+0.1*sin(1.87*t))
plot(t,h,'r*')
```



OBSERVATIE:

albastru-raspunsul la impuls calculat de Matlab

rosu – pct din pasul 0.1 in 0.1 la rasp la impuls calculat

putem observa din plot ca linia cu albastru(raspunsul la impuls calculat de Matlab cu functia impluse) si linia rosie(functia pondere calculata analitic) se suprapun

RASPUNSUL INDICIAL / RASPUNSUL LA TREAPTA

$$y(t)=L^{-1}\left\{H(s)\frac{1}{s}\right\}$$

$$y(t)=L^{-1}\{\frac{s^3}{s^3+3,23s^2+6,84s+3,17}\frac{1}{s}\}=$$

$$y(t)=L^{-1}\{\frac{s^2}{s^3+3,23s^2+6,84s+3,17}\}=$$

$$=L^{-1}\{\frac{s^2}{(s+0,6)(s^2+2,62s+5.25)}\}$$

$$\frac{s^2}{(s+0,6)(s^2+2,62s+5.25)}=\frac{A}{(s+0,6)}+\frac{Bs+C}{(s^2+2,62s+5.25)}$$

$$-3,23s^2-6,84s-3,17=A(s^2+2,62s+5.25)+(Bs+C)(s+0,6)$$

$$-3,23s^2-6,84s-3,17=s^2(A+B)+s(2,32A+0,6B+C)+5,25A+0,6C$$

$$A+B=1$$

$$2,32A+0,6B+C=0$$

$$5,25A+0,6C=0$$

$$B=1-A;C=-8,75A;$$

$$2,62A-0,6A+0,6-8,75A=0=>-6,73A=-0,6$$

$$A=0.08;B=0,92;C=-0,7$$

$$y(t)=L^{1}\{\frac{0,08}{s+0,6}+\frac{0,92s-0,7}{s^2+2,62s+5.25}\}=$$

$$= -0,05L^{1}\{\frac{1}{s+0,6}\} + 0,92L^{1}\{\frac{s-0.76}{s^2+2,62s+5.25}\} =$$

$$s^2+2,62s+5.25 = s^2+2s*1,31+1,31^2-1,31^2+5,25 = (s+1,31)^2+3,54$$

$$L^{1}\{\frac{s-0,76}{s^2+2,62s+5.25}\} = L^{1}\{\frac{s-0,76}{(s+1,31)^2+3,54}\} = L^{1}\{\frac{s+1,31-2,07}{(s+1,31)^2+3,54}\} =$$

$$= L^{1}\{\frac{s+1,31}{(s+1,31)^2+3,54}\} + L^{1}\{\frac{-2.07}{(s+1,31)^2+3,54}\} =$$

$$= L^{1}\{\frac{s+1,31}{(s+1,31)^2+3,54}\} - 2.07L^{1}\{\frac{1}{(s+1,31)^2+3,54}\} =$$

$$= \cos(\sqrt{3,54}t)e^{-1,31t} - 2.07(\frac{1}{\sqrt{3.54}}\sin(\sqrt{3,54}t)e^{-1,31t}) =$$

$$= e^{-1,31t}(\cos(1.87t) - 1,1\sin(1.87t))$$

$$y(t) = 0,08e^{-0,6t} + 0,92e^{-1,31t}(\cos(1.87t) - 1.1\sin(1.87t))$$

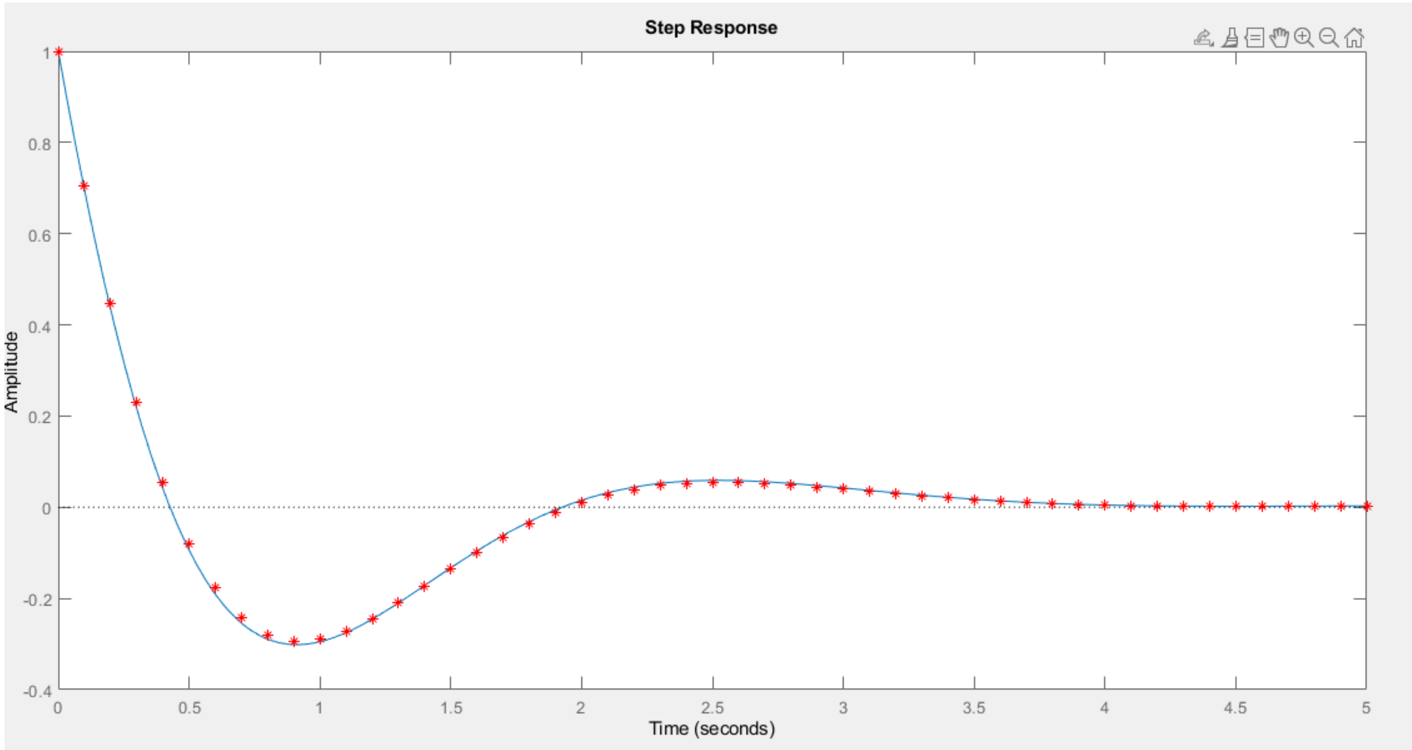
$$y(t) = L^{-1}\{H(s)*U(s)\} = y_l(t) + y_p(t)$$

$y_l =$ componenta tranzitorie(libera) $y_p(t)$ =componenta permanenta(raspuns fortat)->depinde de u

$y_l = h(t) = L^{-1}\{H(s)\}$ =y(t) $y_p(t)$ =0

-moduri: $e^{\hat{s}_1 t} = e^{-0.6t}$ $e^{\mathrm{Re}\{s\hat{2,3}\}t}\sin(\mathrm{Im}\{s\hat{2,3}\}t) = e^{-1.31t}\sin 1.87t$

```
%% raspunsul la treapta

H=tf([1 0 0 0],[1 3.23 6.78 3.17])
step(H)
hold on
t=0:0.1:10
y=0.08*exp(-0.6*t)+exp(-1.31*t).*(0.92*cos(1.87*t)-1.01*sin(1.87*t))
plot(t,y,'r*')
```



RASPUNSUL LA RAMPA

$$y(t) = L^{-1}\left\{H(s)\frac{1}{s^2}\right\}$$

$$y(t) = L^{-1}\{\frac{s^3}{s^3+3,23s^2+6,84s+3,17}\frac{1}{s^2}\} =$$

$$y(t) = L^{-1}\{\frac{s}{s^3+3,23s^2+6,84s+3,17}\} =$$

$$= L^{-1}\{\frac{s}{(s+0,6)(s^2+2,62s+5.25)}\}$$

$$\frac{s}{(s+0,6)(s^2+2,62s+5.25)} = \frac{A}{(s+0,6)} + \frac{Bs+C}{(s^2+2,62s+5.25)}$$

$$-3,23s^2-6,84s-3,17 = A(s^2+2,62s+5.25)+(Bs+C)(s+0,6)$$

$$-3,23s^2-6,84s-3,17 = s^2(A+B)+s(2,32A+0,6B+C)+5,25A+0,6C$$

$$A + B = 0$$
$$2,32A + 0,6B + C = 1$$
$$5,25A + 0,6C = 0$$
$$B = -A; C = -8,75A;$$
$$2,62A - 0,6A - 8,75A = 1 => -6,73A = 1$$
$$A = -0,14; B = 0,14; C = 1,22$$
$$y(t) = L^1\{\frac{-0,14}{s+0,6} + \frac{0,14s+1,22}{s^2+2,62s+5.25}\} =$$
$$= 1,22L^1\{\frac{1}{s+0,6}\} + 0,14L^1\{\frac{s+8,71}{s^2+2,62s+5.25}\} =$$
$$s^2 + 2,62s + 5.25 = s^2 + 2s*1,31 + 1,31^2 - 1,31^2 + 5,25 = (s+1,31)^2 + 3,54$$
$$L^1\{\frac{s-8,71}{s^2+2,62s+5.25}\} = L^1\{\frac{s+8,71}{(s+1,31)^2+3,54}\} = L^1\{\frac{s+1,31+7,4}{(s+1,31)^2+3,54}\} =$$
$$= L^1\{\frac{s+1,31}{(s+1,31)^2+3,54}\} + L^1\{\frac{7,4}{(s+1,31)^2+3,54}\} =$$
$$= L^1\{\frac{s+1,31}{(s+1,31)^2+3,54}\} + 7,4L^1\{\frac{1}{(s+1,31)^2+3,54}\} =$$
$$= \cos(\sqrt{3,54}t)e^{-1,31t} + 7,4(\frac{1}{\sqrt{3.54}}\sin(\sqrt{3,54}t)e^{-1,31t}) =$$
$$= e^{-1,31t}(\cos(1.87t) + 3.95\sin(1.87t))$$
$$y(t) = -0.14e^{-0,6t} + 0,14e^{-1,31t}(\cos(1.87t) + 3,95\sin(1.87t))$$

$$y(t) = L^{-1}\{H(s)*U(s)\} = y_l(t) + y_p(t)$$

$y_l =$ componenta tranzitorie(libera)

$y_p(t)$ =componenta permanenta(raspuns fortat)->depinde de u

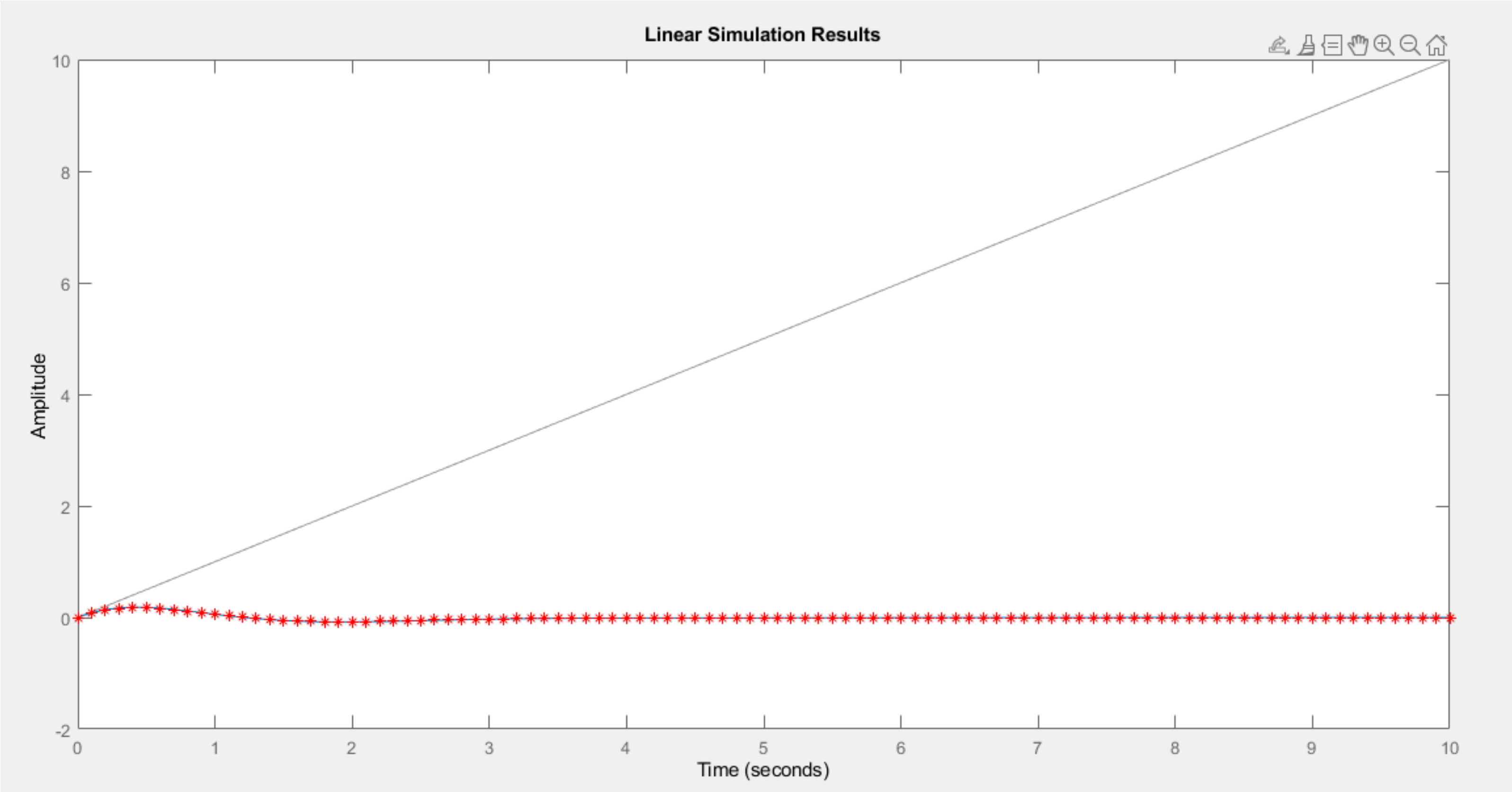
$y_l = h(t) = L^{-1}\{H(s)\}$ =y(t)

$y_p(t)$ =0

-moduri:

$$e^{\hat{s}_1 t} = e^{-0.6t} \qquad e^{\mathrm{Re}\{\hat{s}2,3\}t}\sin(\mathrm{Im}\{\hat{s}2,3\}t) = e^{-1.31t}\sin 1.87t$$

```
%% Raspunsul la rampa
H=tf([1 0 0 0],[1 3.23 6.78 3.17])
t=0:0.1:10
u=t %rampa
lsim(H,u,t);
hold on
y=-0.14*exp(-0.6*t)+0.14*exp(-1.31*t).*(cos(1.87*t)+3.95*sin(1.87*t))
plot(t,y,'r*')
```



9.

T – constanta de timp

$$\hat{T} = \left|\frac{1}{\hat{s}}\right|; \quad \mathring{T} = \left|\frac{1}{\mathring{s}}\right|$$

Poli:

$$\left.\begin{array}{l}\hat{s}_1 = \text{-0.60} \\ \hat{s}_2 = \text{-1.31+1.87i} \\ \hat{s}_3 = \text{-1.31-1.87i}\end{array}\right| \Rightarrow \hat{T}_1 = \left|\frac{1}{\hat{s}_1}\right| = 1.66\text{sec}$$

$\hat{s}_1 = \text{-0.60}$ este cel mai din dreapta pol care $\in R_-$ => $\hat{s}_1 = \text{-0.60}$ este polul dominant => facem referire la $T = 1.66\text{sec}$

Zerouri:

$$\left.\begin{array}{l}\overset{o}{s}_1 = 0 \\ \overset{o}{s}_2 = 0 \\ \overset{o}{s}_3 = 0\end{array}\right|$$

K – factor de proportionalitate

$K = H(0) = \frac{1}{3.17} = 0.31$

ξ **- factor de amortizare**

ω_n - **pulsatia naturala**

$$\omega_n = \sqrt{5.25} = 2.29$$

$$s^2 + 2\xi\omega_n s + \omega_n{}^2 = s^2 + 2\xi 2.29s + 5.25 => \xi = 0.57$$

-sistem de ordin I: $H(s) = \frac{k}{Ts+1}$

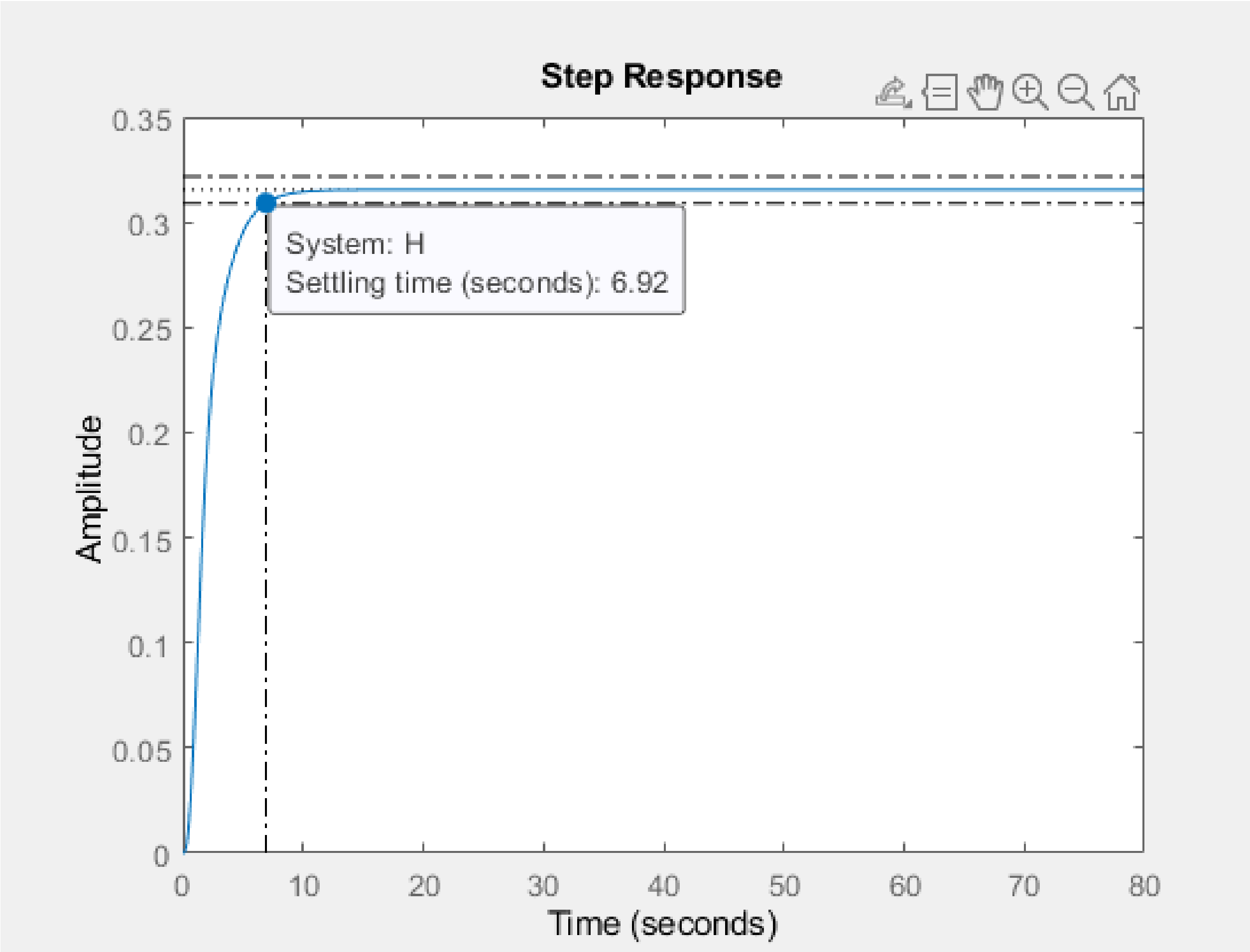
-sistem de ordin II: $H(s) = \frac{k\omega_n}{s^2 + 2\xi\omega_n s + \omega_n{}^2}$

Sistemul este stabil extern=> **PERFORMANTELE SISTEMULUI**

I. IN REGIM TRANZITORIU

- **timpul de raspuns (settling time)** – timpul in care sistemul se stabilizeaza $t_r = 4T$; (ordin I)

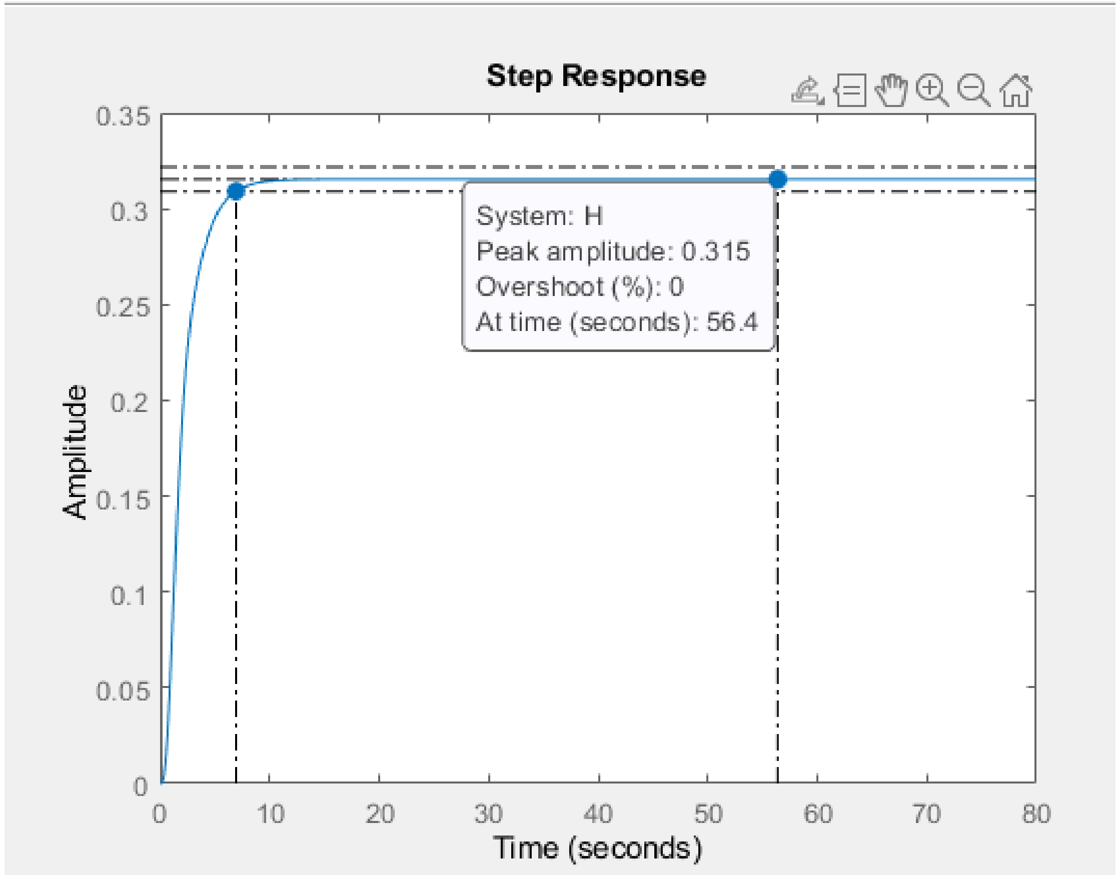
$t_r = 4T = 4\frac{1}{0.6} = 4*1.66 = 6.66$ sec



- **suprareglajul (peak response - overshoot)**

$$\sigma = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \in [0,1]$$

Deoarece polul dominant este -0.6 (sistem de ordin I) = nu avem suprareglaj => $\sigma = 0\%$



- **pulsatia naturala/ de oscilatie** $(=\mathrm{Im}\{\hat{s}_{1,2}\})$

Deoarece polul dominant este -0.6 (sistem de ordin I) = nu avem pulsatie a oscilatiilor

II. IN REGIM STATIONAR

- **erorile stationare (1- steady state)**

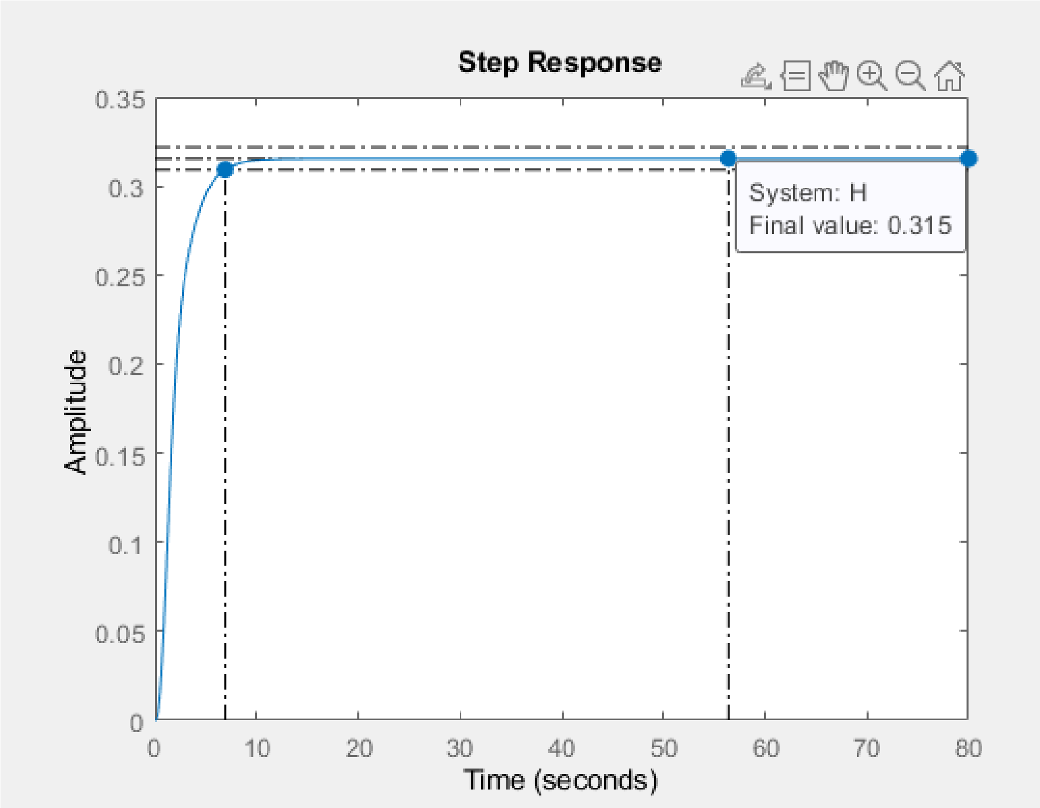
$$\varepsilon_{ss}=\lim_{t\to\infty}u(t)-y(t)=\lim_{s\to 0}s(U(s)-Y(s))=\lim_{s\to 0}s(U(s)-H(s)*U(s))$$

$$=>\varepsilon_{ss}=\lim_{s\to 0}sU(s)(1-H(s))$$

- **la pozitie:** semnifica abilitatea sistemului de a urmari o intrare de tip treapta unitara, deci u(t)=1(t)

intrare treapta : U(s) = $\frac{1}{s}$

$$\varepsilon_{ssp}=1-H(0)=1-\frac{0}{s^3+3.23s^2+6.84s+3.17}=1$$



- **la viteza (intrare de tip rampa):**

$$E_{ssv} = \lim_{s \to 0} \frac{1 - H(s)}{s}$$

descrie abilitatea sistemului de a urmari o intrare de tip rampa unitare, deci u(t)=t

- valoarea va fi ∞ deoarece eroarea stationara la pozitie este $\neq 0$.

10.

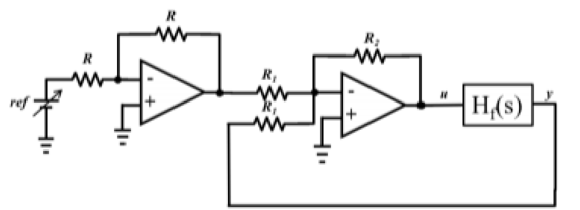
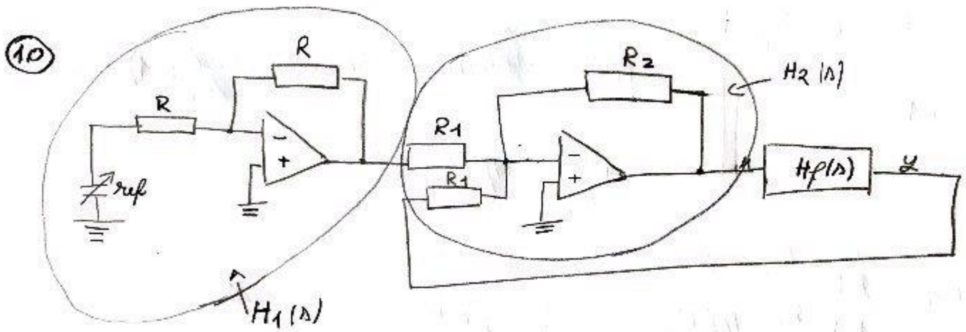


Figure 1: Structura unui sistem de reglare cu regulator proporţional

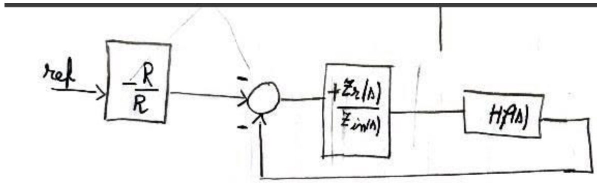
a.

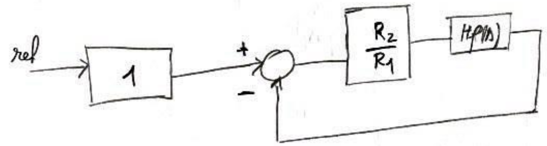


$$\cdot\ H_1(s) = \frac{-Z_r(s)}{Z_{in}(s)}$$

$$H_1(s) = \frac{-R}{R} = -1 \quad ;$$

Hd





$$Hd = \frac{R_2}{R_1} \cdot H_f(s)$$

$$H_0 = \frac{\frac{R_2}{R_1} H_f(s)}{1 + \frac{R_2}{R_1} H_f(s)}$$

$$H(s)=\frac{-Zr(s)}{Zin(s)}$$

$$H_1(s)=\frac{-R}{R}$$

$$Y(s)=\frac{-Zr(s)}{Zin_1(s)}U_1(s)-\frac{Zr(s)}{Zin_2(s)}U_2(s)=\frac{-Zr(s)}{Zin_1(s)}(U_1(s)+U_2(s))$$

$$Y(s)=H*U(s)\Rightarrow H=\frac{-Zr(s)}{Zin_1(s)}=\frac{-R2}{R1}$$

$$Hd=\frac{R2}{R1}Hf(s)=\frac{R2}{R1}\frac{s^3}{s^3+3.23s^2+6.84s+3.17}; \quad Hr=1;\ H_{des}=Hd$$

$$H_o(s)=\frac{Hdes(s)}{1+Hdes(s)}=\frac{\frac{R2}{R1}Hf(s)}{1+\frac{R2}{R1}Hf(s)}=\frac{\frac{R2}{R1}\frac{s^3}{s^3+3.23s^2+6.84s+3.17}}{1+\frac{R2}{R1}\frac{s^3}{s^3+3.23s^2+6.84s+3.17}}=\frac{R2*s^3}{(R1+R2)s^3+3.23R1s^2+6.84R1s+3.17R1}$$

b.

$$\frac{R2}{R1}\in(0,\infty)\Rightarrow k=\frac{R2}{R1}\in(0,\infty)\Rightarrow k>0$$

$n=3$

$m=3$ => $n_a=0$ (nr de asimptote)

polii

$\hat{s}_1 = \text{-}0.60$

$\hat{s}_2 = \text{-}1.31\text{+}1.87\text{i}$

$\hat{s}_3 = \text{-}1.31\text{-}1.87\text{i}$

zerouri:

$\overset{o}{s}_1 = 0$

$\overset{o}{s}_2 = 0$

$\overset{o}{s}_3 = 0$

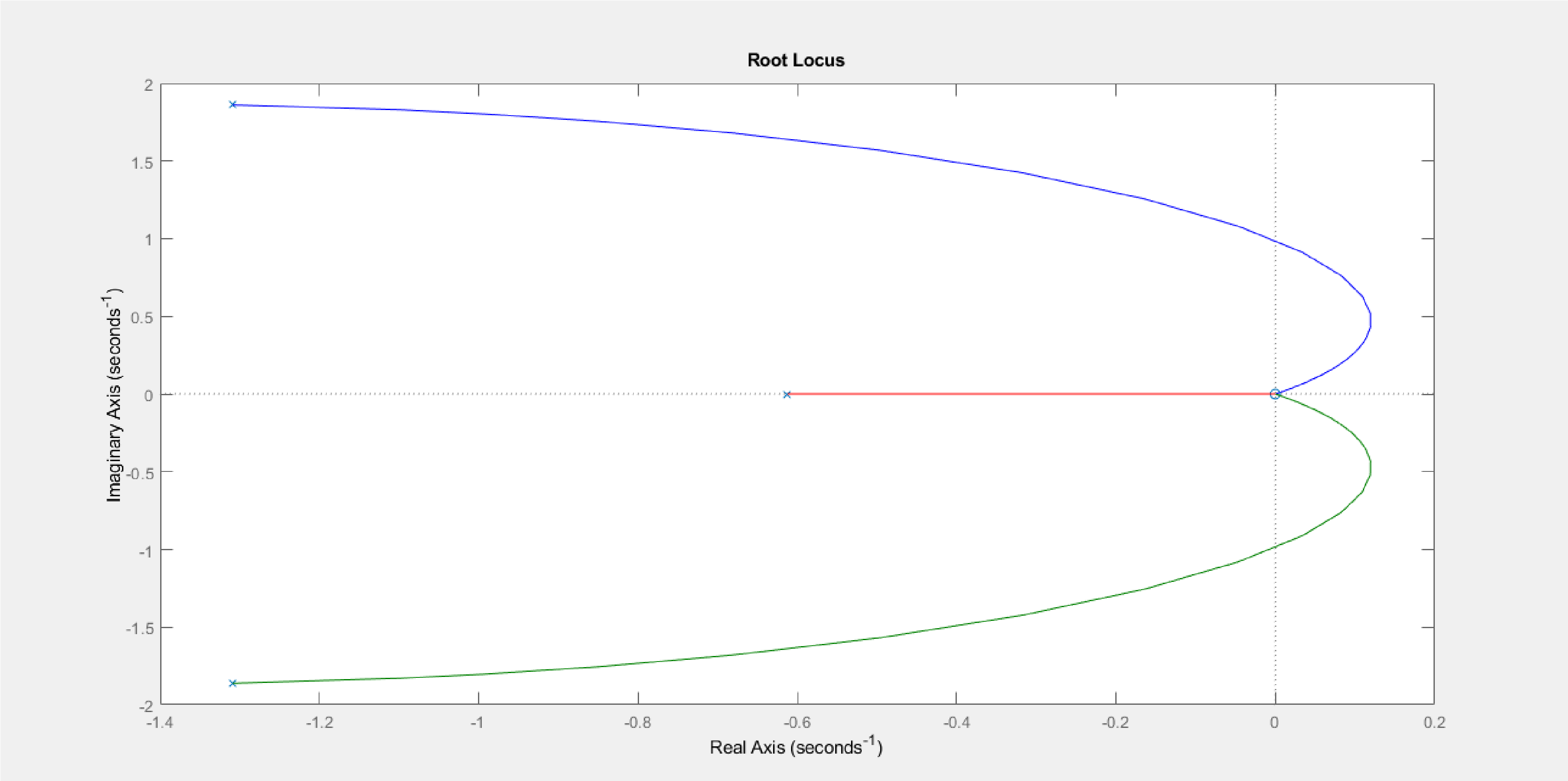
$s_x \in R, s_x \in LR <=>$

$$1 + k * H'des(s) = 0 <=> \left\{ \begin{aligned} &\angle \frac{a}{b} = \angle a - \angle b \\ &\angle a * b = \angle a + \angle b \end{aligned} \right\}$$

$$\sum_{i=1}^{m} \angle s_x - \overset{0}{s}_i - \sum_{i=1}^{n} \angle s_x - \hat{s}_i = \angle \frac{-1}{k * k'}$$

$k' > 0 : s_x \in R \cap LR <=>$ are in partea dreapta un nr impar de singularitati

$\hat{s}_1 = \text{-}0.60$: 3 zerouri=>zona permisa



Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1.4
-1.2
-1
-0.8
-0.6
-0.4
-0.2
0
0.2

- **unghiurile de plecare din poli:**

$$\varnothing_{\hat{s}_1} = \sum \angle(\hat{s}_1 - \overset{0}{s}_i) - \sum \angle(\hat{s}_1 - \hat{s}_i) - (2l+1)\Pi = 3\angle -0.6 - (\angle(-0.6+1.31-1.87i) + \angle(-0.6+1.31+1.87i)) - (2l+1)\Pi = 540 - (2l+1)\Pi \rightarrow l = 1 => 0^0$$

$$\varnothing_{\hat{s}_2} = \sum \angle(\hat{s}_2 - \overset{0}{s}_i) - \sum \angle(\hat{s}_2 - \hat{s}_i) - (2l+1)\Pi = 174.24 - (2l+1)\Pi \rightarrow l = -1 => 354.24^0$$

$$\varnothing_{\hat{s}_3} = \sum \angle(\hat{s}_3 - \overset{0}{s}_i) - \sum \angle(\hat{s}_3 - \hat{s}_i) - (2l+1)\Pi = -5.76$$

- **unghiurile de sosire in zerouri:**

$$\varnothing_{\overset{0}{s}_1} = -\sum \angle(\overset{0}{s}_1 - \overset{0}{s}_i) + \sum \angle(\overset{0}{s}_1 - \hat{s}_i) + (2l+1)\Pi = -0 + \angle 0.6 + \angle(1.31-1.87i) + \angle(1.31+1.87i) + (2l+1)\Pi \rightarrow l = 0 => \Pi$$

$$\varnothing_{\overset{0}{s}_2} = -\sum \angle(\overset{0}{s}_2 - \overset{0}{s}_i) + \sum \angle(\overset{0}{s}_2 - \hat{s}_i) + (2l+1)\Pi = -0 + 0 + (2l+1)\Pi \rightarrow l = 0 => \Pi$$

$$\varnothing_{\overset{0}{s}_3} = -\sum \angle(\overset{0}{s}_3 - \overset{0}{s}_i) + \sum \angle(\overset{0}{s}_3 - \hat{s}_i) + (2l+1)\Pi = -0 + 0 + (2l+1)\Pi \rightarrow l = 0 => \Pi$$

```
%% calcul pt faze
x=3*atan2d(0,-0.6)-atan2d(-1.87,0.71)-atan2d(1.87,0.71);
x=3*atan2d(1.87,-1.31)-atan2d(1.87,-0.71)-atan2d(2*1.87,0);
y=atan2d(1.87,1.31)+atan2d(-1.87,1.31)+atan2d(0,0.6)
```

- **puncte de intalnire: de desprindere/ apropiere (cu axa reala):**

$$\frac{dH'des(s)}{ds}=0 \Rightarrow \left(\frac{R2}{R1}\frac{s^3}{s^3+3.23s^2+6.84s+3.17}\right)'=0 \Rightarrow \frac{R2}{R1}(3s^2(s^3+3.23s^2+6.84s+3.17)-s^3(3s^2+6.46s+6.84)=0$$

$$\Rightarrow \frac{R2}{R1}s^2(3.23s^2+13.68s+9.51)=0$$

$$\Rightarrow s_1=s_2=0\in LR-punct\_de\_apropiere$$

$$\Delta=187.14-122.86=64.24$$

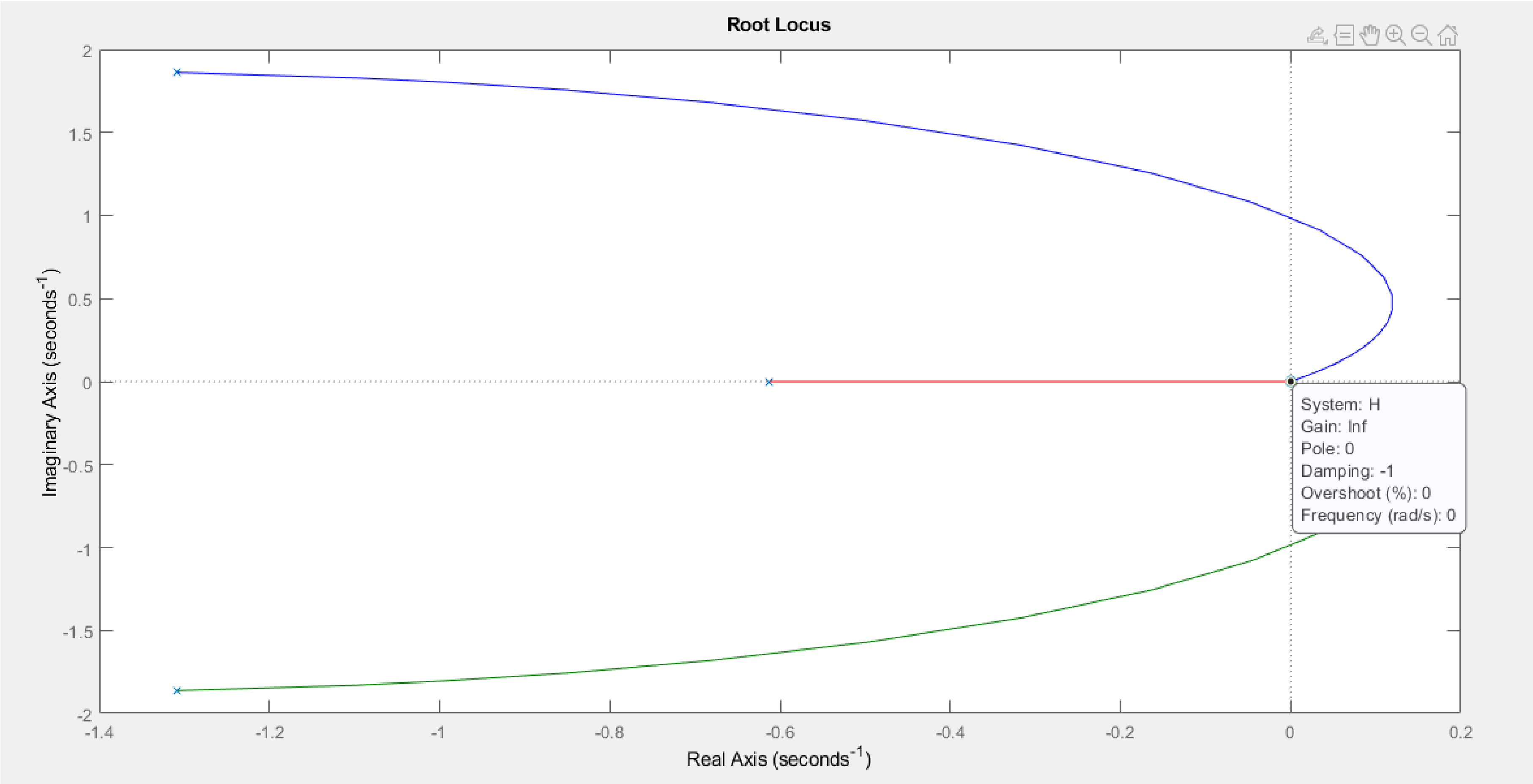
$$s_{3,4}=\frac{-13.68\pm 8.01}{2*3.23}$$

$$s_3=-0.87\notin LR$$

$$s_4=-3.35\notin LR$$

- k apropiere:

$$k_{apr}=\frac{-1}{H'des(s)}\xrightarrow{s=0}\infty$$



-k critic (intersectia cu axa imaginara)

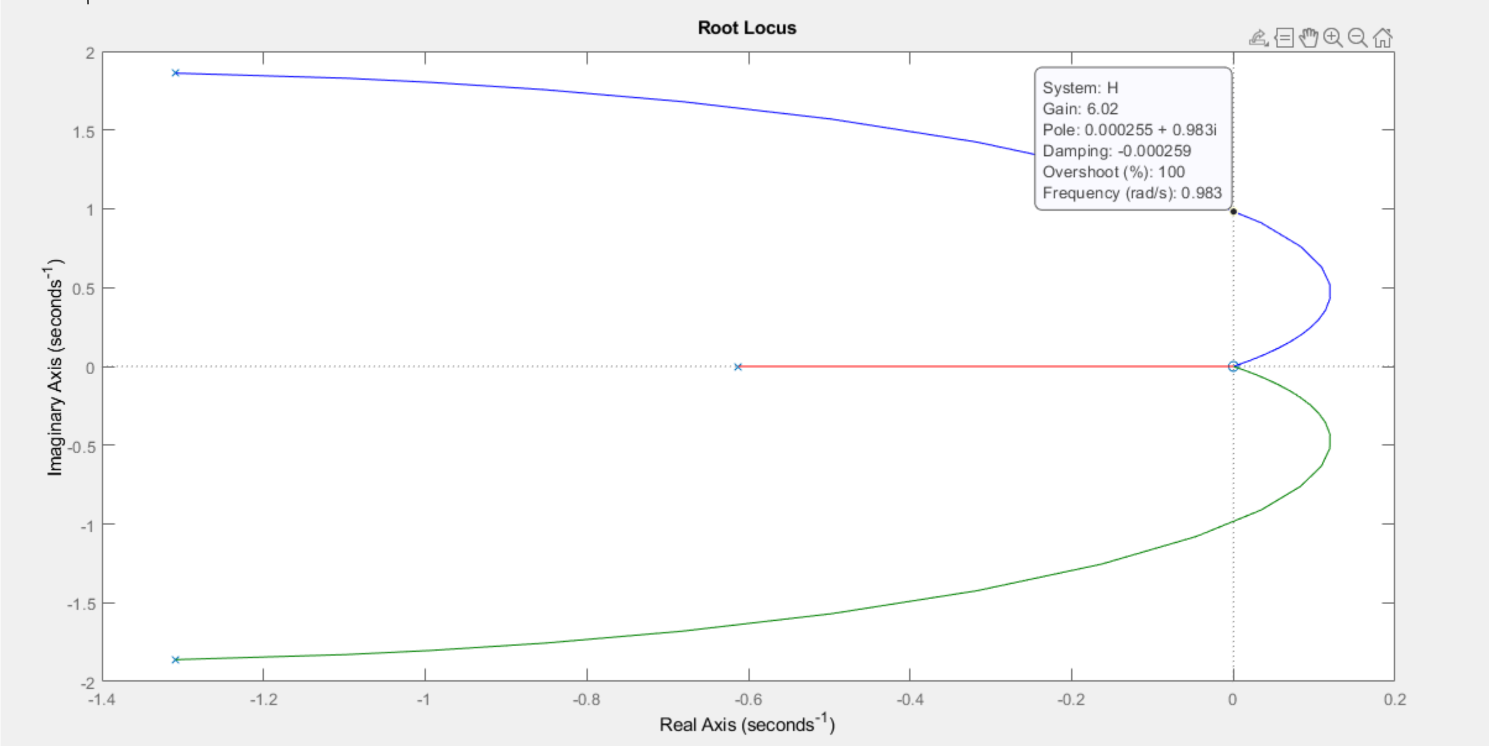
$$1 + k * H'des(s) = 0 => 1 + k * \frac{s^3}{s^3 + 3.23s^2 + 6.84s + 3.17} = 0 => (1+k)s^3 + 3.23s^2 + 6.84s + 3.17 = 0$$

$$Ruth - Hurwitz:$$

s^3	$1+k$	6.84
s^2	3.23	3.17
s^1	$\dfrac{3.17k-18.92}{3.23}$	0
s^0	3.17	

$$b_1 = \frac{-\begin{vmatrix} 1+k & 6.87 \\ 3.22 & 3.17 \end{vmatrix}}{3.22} = \frac{3.17k-18.92}{3.23}$$

$$b_1 > 0 \Rightarrow \frac{3.17k-18.92}{3.23} \Rightarrow 3.17k > 18.92 \Rightarrow k > 5.96 \Rightarrow k_{cr} = 5.96$$



- Ho extern stabil pentru k $\in$ (0,5.96)

- k $\in$ (0,5.96):

$\hat{s}_{o1,2} \in C_-$ $\quad$ moduri: $e^{\hat{s}_{o3}t}$ $\quad$ regim: oscilant amortizat

$\hat{s}_{o3} \in R_-$ $\quad$ $e^{\mathrm{Re}\{\hat{s1,2}\}t}\sin(\mathrm{Im}\{\hat{s}_{o1,2}\}t)$

-**k=5.96:**

$\hat{s}_{o1,2} \in C, \mathrm{Re} = 0$ $\quad$ moduri: $e^{\hat{s}_{o3}t}$ $\quad$ regim: oscilant intretinut

$\hat{s}_{o3} \in R_-$ $\quad$ $\sin(\mathrm{Im}\{\hat{s}_{o1,2}\}t)$

- k $\in$ (5.96,inf):

$\hat{s}_{o1,2} \in C_+$ $\quad$ moduri: $e^{\hat{s}_{o3}t}$ $\quad$ regim: oscilant neamortizat

$\hat{s}_{o3} \in R_-$ $\quad$ $e^{\mathrm{Re}\{\hat{s1,2}\}t}\sin(\mathrm{Im}\{\hat{s}_{o1,2}\}t)$

- **Sensibilitate mare, deoarece daca modific valoarea lui k, se va modifica regimul.**

c.

c3) $\dfrac{R2}{R1}$ =? a.i. tr_min = tr – ¼*tr = 4.99

Tr=4.99 => 4T=4.99 => T=1.24 =>

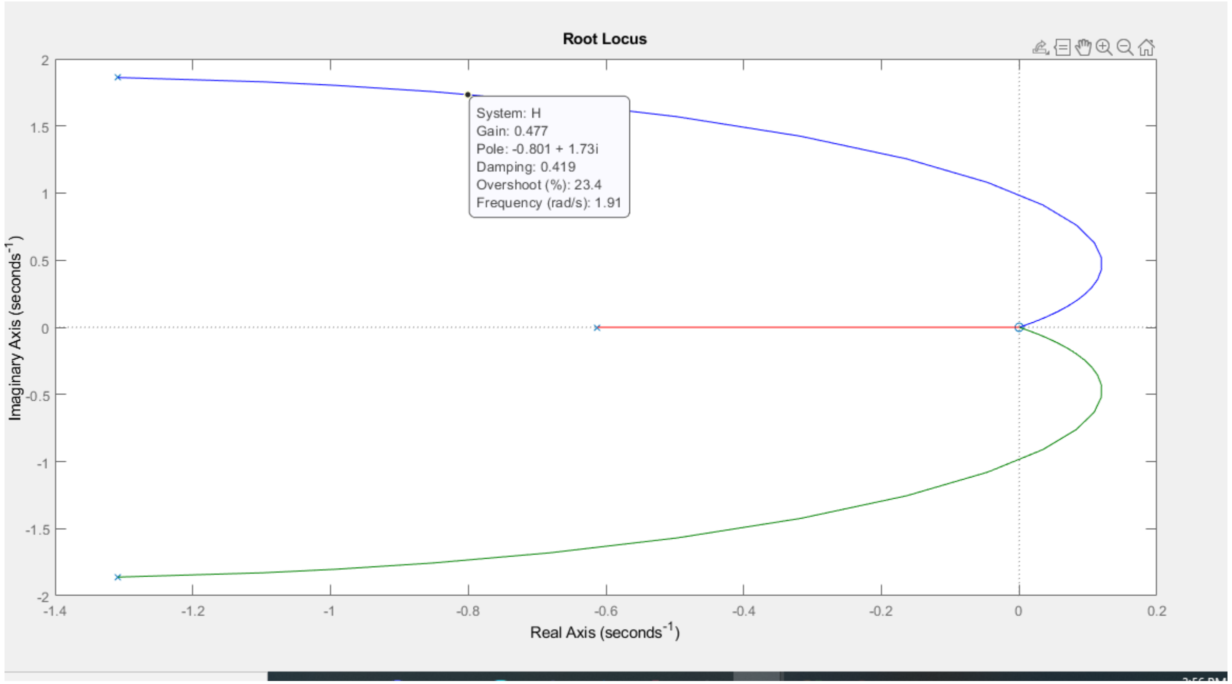
-pt ca polul dominant = -0.6 este real => Ts+1=0 **=>** $\hat{s}_i$ =-1/T = -0.8 $\notin$ LR

Obs: polii sunt complecsi:

$$s^2 + 2\xi\omega_n + {\omega_n}^2$$

$$tr = \frac{4}{\xi\omega_n} => 4.99 = \frac{4}{\xi\omega_n} => \xi\omega_n = 0.8 => \mathrm{Re}\{\hat{s}\} = -0.8$$

-pe grafic: unde polul=0.8 => k=0.47

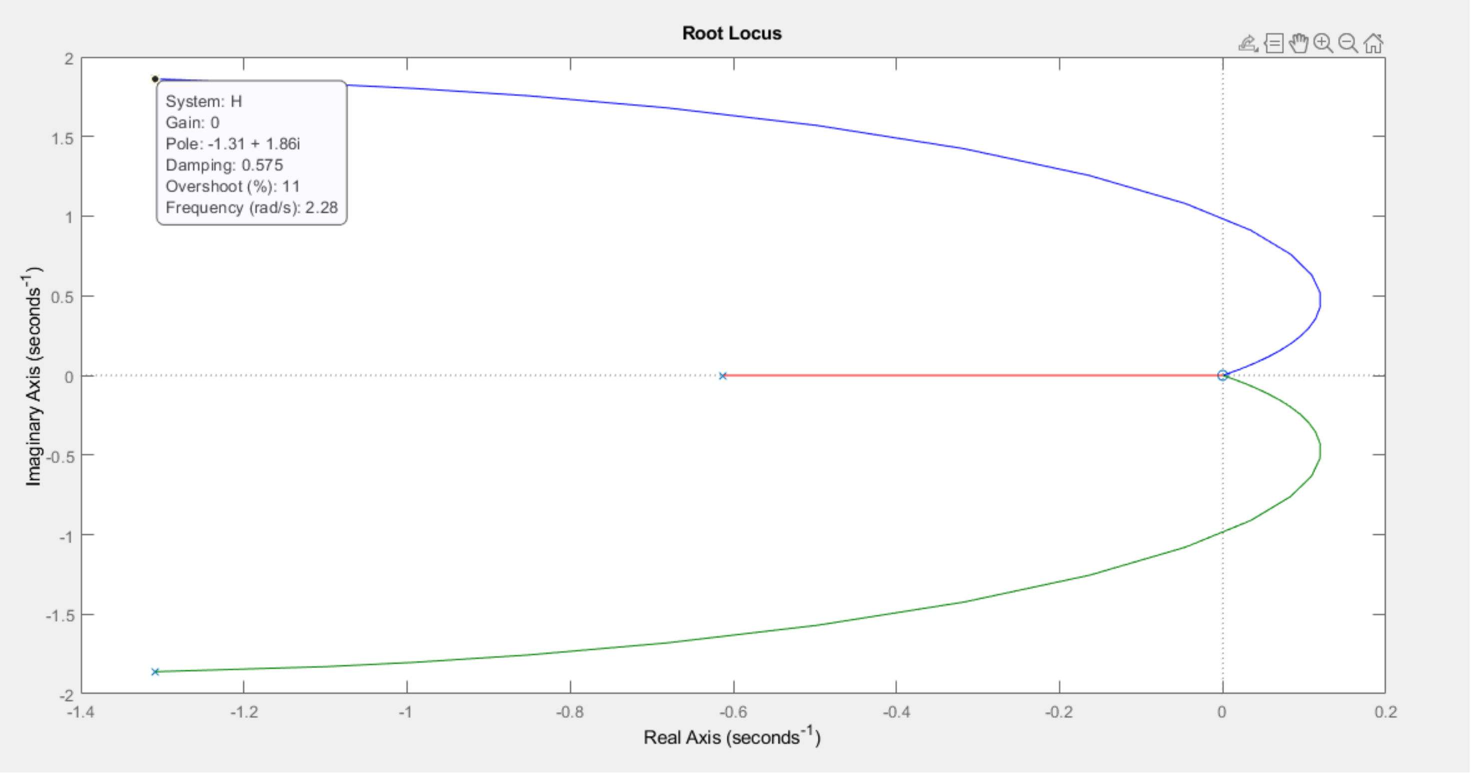


$\frac{R2}{R1}$ =? a.i $\sigma = 0$ -deoarece $\sigma = 0$ => $\frac{R2}{R1} \in$ R

c2) $\frac{R2}{R1}$ a.i. tr minim

-poli complecsi: $s^2 + 2\xi\omega_n + \omega_n^2 = s^2 + 2.62 + 5.25 => \xi\omega_n = 1.31$ $tr = \frac{4}{\xi\omega_n}$ = $\frac{4}{1.31}$ =3.05

-pe grafic: unde polul=1.31 => k=0 (in polul imaginar)

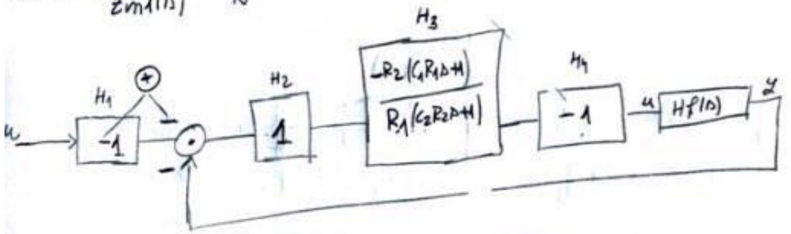


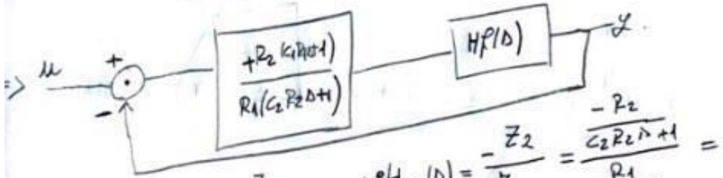
11.

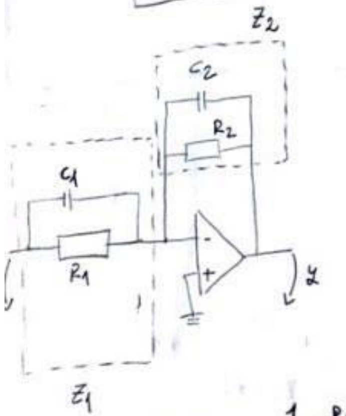
• $H_1(s) = \frac{-R}{R} = -1$

$$Y(s) = \frac{-Z_r(s)}{Z_{in1}(s)} U_1(s) - \frac{Z_r(s)}{Z_{in2}(s)} U_2(s) = \frac{-Z_r(s)}{Z_{in1}(s)}\left(U_1(s) + U_2(s)\right) \Rightarrow$$

$$H_2(s) = \frac{-Z_r(s)}{Z_{in1}(s)} = \frac{-R}{R} = -1$$







• $H_3(s) = -\frac{Z_2}{Z_1} = \frac{\frac{-R_2}{C_2R_2s+1}}{\frac{R_1}{C_1R_1s+1}} =$

$$= \frac{-R_2(C_1R_1s+1)}{R_1(C_2R_2s+1)}$$

• $H_4 = -\frac{R}{R}$

$\frac{1}{R_1} + \frac{1}{R_2} = R_1R_2$

$$C_1 \| R_1 \Rightarrow Z_1 = \frac{Z_{C1}\cdot Z_{R1}}{Z_{C1}+Z_{R1}} = \frac{\frac{1}{sC_1}\cdot R_1}{\frac{1}{sC_1}+R_1} = \frac{\frac{R_1}{sC_1}}{\frac{1+sC_1R_1}{sC_1}} = \frac{R_1}{C_1R_1s+1}$$

a.

$$H(s) = -\frac{Zr(s)}{Zin(s)}$$

$$H_1(s) = \frac{-R}{R} = -1$$

$$H_2(s) = \frac{-R}{R} = -1$$

$$H_4(s) = \frac{-R}{R} = -1$$

$$C1 \parallel R1 \Rightarrow Z_1 = \frac{Z_{C1}Z_{R1}}{Z_{C1}+Z_{R1}} = \frac{\frac{1}{sC1}R1}{\frac{1}{sC2}+R1} = \frac{R1}{C1R1s+1}$$

$$C2 \parallel R2 \Rightarrow Z_2 = \frac{R2}{C2R2s+1}$$

$$H_3(s) = H_R(s) = \frac{-Z_2}{Z_1} = \frac{\frac{-R2}{C2R2s+1}}{\frac{R1}{C1R1s+1}} = \frac{R2}{R1}\frac{C2R2s+1}{C1R1s+1}$$

b.

$Hr = 1;$

$$Hd = Hdes = \frac{R2}{R1}\frac{C2R2s+1}{C1R1s+1}\frac{s^3}{s^3+3.23s^2+6.84s+3.17}$$

$$H_o(s) = \frac{Hdes(s)}{1+Hdes(s)} = \frac{C1R1R2s^4+R2s^3}{(R1C2R2+C1R1R2)s^4+(3.23R1C2R2+R2+R1)s^3+(6.84R1C2R2+3.23R1)s^2+3.17R1C2R2+3.17R1}$$

c.

$$H_{/o}(s) = \frac{Hd(s)}{1+Hdes(s)} = \frac{HR(s)*Hf(s)}{1+HR(s)*Hf(s)}$$

$$= \frac{\frac{R2}{R1}\left(\frac{T1s+1}{T2s+1}\right)\frac{s^3}{s^3+3.23s^2+6.84s+3.17}}{1+HR(s)*Hf(s)} =$$

$$= \frac{R2(T1s+1)s^3}{R1(T2s+1)(s^3+3.23s^2+6.84s+3.17)+R2T1s^4+R2s^3} =$$

$$= \frac{R2(T1s+1)s^3}{(R1T2s+R1)(s^3+3.23s^2+6.84s+3.17)+R2T1s^4+R2s^3} =$$

$$= \frac{R2T1s+R2s}{R1T2s^4+R1T2s^3*3.23+6.84R1T2s^2+R1T2*3.17s+R1s^3+3.23R1s^2+6.84R1s+3.17R1+R2T1s^4+R2s^3}$$

$numitorul : R1T2s^4+R1T2s^3*3.23+6.84R1T2s^2+R1T2*3.17s+R1s^3+3.23R1s^2+6.84R1s+3.17R1+R2T1s^4+R2s^3=0$

$R1 = 3k\Omega$

$R2 = 5k\Omega$

$C1 = 10^{-3}F$

$C2 = 2*10^{-3}F$

$$T1 = R1C1 = \frac{1}{10^3}*3*10^3 = 3$$

$$(3*10^3T2+5*10^3T1)s^4+(3.23*3*10^3T2+8*10^3)s^3+(6.84*3*10^3T2+3.23*3*10^3)s^2+(3.17*3*10^3T2+6.84*3*10^3)s+3.17*3*10^3=0$$

$$3000T2s^4+5000s^4+9.69*10^3T2s^3+8000s^3+20.52*10^3T2s^2+9.69*10^3s^2+9.51*10^3T2s+20.52*10^3s+9.51*10^3=0$$

$$T2[3000s^4+9690s^3+20520s^2+9510s]+5000s^4+8000s^3+9690s^2+20520s+9510=0$$

$$T2*A+B=0\,|:A=>T2+\frac{B}{A}=0\,|:T2=>1+\frac{1}{T2}\frac{B}{A}=0=>$$

$$1+\frac{1}{T2}\frac{5000s^4+8000s^3+9690s^2+20520s+9510}{3000s^4+9690s^3+20520s^2+9510s}=0$$

$$H=\frac{5000s^4+8000s^3+9690s^2+20520s+9510}{3000s^4+9690s^3+20520s^2+9510s}$$

$$zerouri:5000s^4+8000s^3+9690s^2+20520s+9510=0$$

$$\overset{\circ}{s}_1=0.25+1.44i;$$

$$\overset{\circ}{s}_2=0.25-1.44i;$$

$$\overset{\circ}{s}_3=-1.54;$$

$$\overset{\circ}{s}_4=-0.57;$$

$$polii: 3000s^4 + 9690s^3 + 20520s^2 + 9510s = 0$$

$$\hat{s}_1 = 0;$$

$$\hat{s}_2 = -1.31 + 1.87i;$$

$$\hat{s}_3 = -1.31 - 1.87i;$$

$$\hat{s}_4 = -0.6;$$

$$n = 4; m = 4; \Rightarrow n_a = 0;$$

$$\frac{1}{T2} > 0 \Rightarrow k > 0$$

-unghiurile de plecare din poli

$$\varnothing_{\hat{s}_1} = \sum \angle(\hat{s}_1 - \overset{0}{s}_i) - \sum \angle(\hat{s}_1 - \hat{s}_i) - (2l+1)\Pi = \angle(-0.25 - 1.44i) + \angle(-0.25 + 1.44i) + \angle 1.54 + \angle 0.57 - \angle(-1.31 - 1.87i) - \angle(-1.31 + 1.87i) - \angle 0.6 - (2l+1)\Pi = 0 - (2l+1)\Pi \rightarrow l = 0 \Rightarrow -180^0$$

$$\varnothing_{\hat{s}_2} = \sum \angle(\hat{s}_2 - \overset{0}{s}_i) - \sum \angle(\hat{s}_2 - \hat{s}_i) - (2l+1)\Pi = 148.86 - (2l+1)\Pi \rightarrow l = -1 \Rightarrow 328.86^0$$

$$\varnothing_{\hat{s}_3} = \sum \angle(\hat{s}_3 - \overset{0}{s}_i) - \sum \angle(\hat{s}_3 - \hat{s}_i) - (2l+1)\Pi = -31.14^0$$

$$\varnothing_{\hat{s}_4} = \sum \angle(\hat{s}_4 - \overset{0}{s}_i) - \sum \angle(\hat{s}_4 - \hat{s}_i) - (2l+1)\Pi = -180^0$$

-unghiurile de sosire

$$\varnothing_{\overset{0}{s_1}} = -\sum \angle(\overset{0}{s_1} - \overset{0}{s_i}) + \sum \angle(\overset{0}{s_1} - \hat{s_i}) + (2l+1)\Pi = -\angle(0.25+1.44i-0.25+1.44i) - \angle(0.25+1.44i+1.54) - \angle(0.25+1.44i+0.37) + \angle(0.25+1.44i) + \angle(0.25+1.44i+1.31-1.87i) +$$
$$+\angle(0.25+1.44i+1.31+1.87i) + \angle(0.25+1.44i+0.6) + (2l+1)\Pi =$$
$$= -6.56 + (2l+1) \rightarrow l = 0 => 172.8^0$$
$$\varnothing_{\overset{0}{s_2}} = -\sum \angle(\overset{0}{s_2} - \overset{0}{s_i}) + \sum \angle(\overset{0}{s_2} - \hat{s_i}) + (2l+1)\Pi = -0 + 0 + (2l+1)\Pi = 360 - 172.8 = -187.2^0$$
$$\varnothing_{\overset{0}{s_3}} = -\sum \angle(\overset{0}{s_3} - \overset{0}{s_i}) + \sum \angle(\overset{0}{s_3} - \hat{s_i}) + (2l+1)\Pi = -\angle(-1.54-0.25-1.44i) - \angle(-1.54-0.25+1.44i) - \angle(-1.54+0.57) + \angle -1.54 + \angle(-1.54+1.31-1.87i) + \angle(1.54+1.31+1.87i) +$$
$$+\angle(0.6-1.54) + (2l+1)\Pi \rightarrow l = 0 => 180^0$$
$$\varnothing_{\overset{0}{s_3}} = -\sum \angle(\overset{0}{s_4} - \overset{0}{s_i}) + \sum \angle(\overset{0}{s_4} - \hat{s_i}) + (2l+1)\Pi = 180^0$$

-punctele de intalnire

$$\frac{dH^{'}des(s)}{ds} = 0 => (4*5*10^3*s^3 + 2*8*10^3*s^2 + 2*9690s + 2052)(3*10^3*s^4 + 9690s^3 + 20520s^2 + 9510s) -$$
$$-(5*10^3*s^4 + 8*10^3*s^3 + 9690s^2 + 20520s + 9510)(12*10^3*s^3 + 3*9690*s^2 + 2*20520s + 9510) = 0$$

```
%% puncte de intalnire

p1=[0 20e3 16e2 19380 2052]

p2=[3e3 9690 20520 9510 0]

A1=conv(p1,p2)


p3=[5e3 8e3 9690 20520 9510]
```

p4=[0 12e3 3*9690 2*20520 9510]

A2=conv(p3,p4)

P=A1-A2

roots(P)

```
Command Window

P =

           0           0   -42750000   -69996000  -486818100  -751616520  -984337560  -565921080   -90440100


ans =

   0.0831 + 3.0283i
   0.0831 - 3.0283i
  -0.4819 + 1.1489i
  -0.4819 - 1.1489i
  -0.5867 + 0.0000i
```

$$s_1 = 0.083 + 3.02i \notin LR$$

$$s_2 = 0.083 - 3.02i \notin LR$$

$$s_3 = -0.48 + 1.14i \notin LR$$

$$s_4 = -0.48 - 1.14i \notin LR$$

$$s_5 = -0.58 \notin LR$$

-k critic

$$1+k*H(s)=0$$

$$3000s^4+9690s^3+20520s^2+9510s+5000ks^4+8000ks^3+9690ks^2+20520ks+9510k=0$$

$$(3000+5000k)s^4+(9690+8000k)s^3+(20520+9690k)s^2+(9510+20520k)s+9510k=0$$

$$\begin{array}{llll}
s^4 & 3000+5000k & 20520+9690k & 9510k \\
s^3 & 9690+8000k & 9510+20520 & 0 \\
s^2 & \dfrac{48426*10^3k}{9690+8000k} & 9510k & 0 \\
s^1 & \dfrac{-6086*10^6k^4-480728880k^3-4324206510k^2}{484260} & 0 & 0 \\
s^0 & 9510k & 0 & 0
\end{array}$$

$$b1=\frac{-\det\begin{pmatrix}3000 & 5000k \\ 9690 & 8000k\end{pmatrix}}{9690+8000k}=\frac{48426*10^3k}{9690+8000k}$$

$$c1=\frac{-\det\begin{pmatrix}9690+8000k & 9510+20520k \\ \dfrac{48426*10^3k}{9690+8000k} & 9510k\end{pmatrix}}{\dfrac{48426*10^3k}{9690+8000k}}=\frac{-6086*10^6k^4-480728880k^3-4324206510k^2}{484260}$$

$3000 + 5000k > 0$

$9690 + 8000k > 0$

$\frac{48426 * 10^3 k}{9690 + 8000k} > 0$ => $k > 2.7 => k_{cr} = 2.7$

$\frac{-6086 * 10^6 k^4 - 480728880k^3 - 4324206510k^2}{484260} > 0$

$9510k > 0$

- Ho extern stabil pentru k $\in$ (0,2.7)

- k $\in$ (0,2.7):

$\hat{s}_{o1,2} \in C_-$

$\hat{s}_{o3}, \hat{s}_{o4} \in R_-$ moduri: $e^{\hat{s}_{o3}t}, e^{\hat{s}_{o4}t}$

$e^{\mathrm{Re}\{\hat{s}1,2\}t} \sin(\mathrm{Im}\{\hat{s}_{o1,2}\}t)$

regim: oscilant amortizat

-**k=2.7:**

$\hat{s}_{o1,2} \in C, \mathrm{Re} = 0$

$\hat{s}_{o3}, \hat{s}_{o4} \in R_-$ moduri: $e^{\hat{s}_{o3}t}, e^{\hat{s}_{o4}t}$

$\sin(\mathrm{Im}\{\hat{s}_{o1,2}\}t)$

regim: oscilant intretinut

- k $\in$ (2.7,inf):

$\hat{s}_{o1,2} \in C_+$

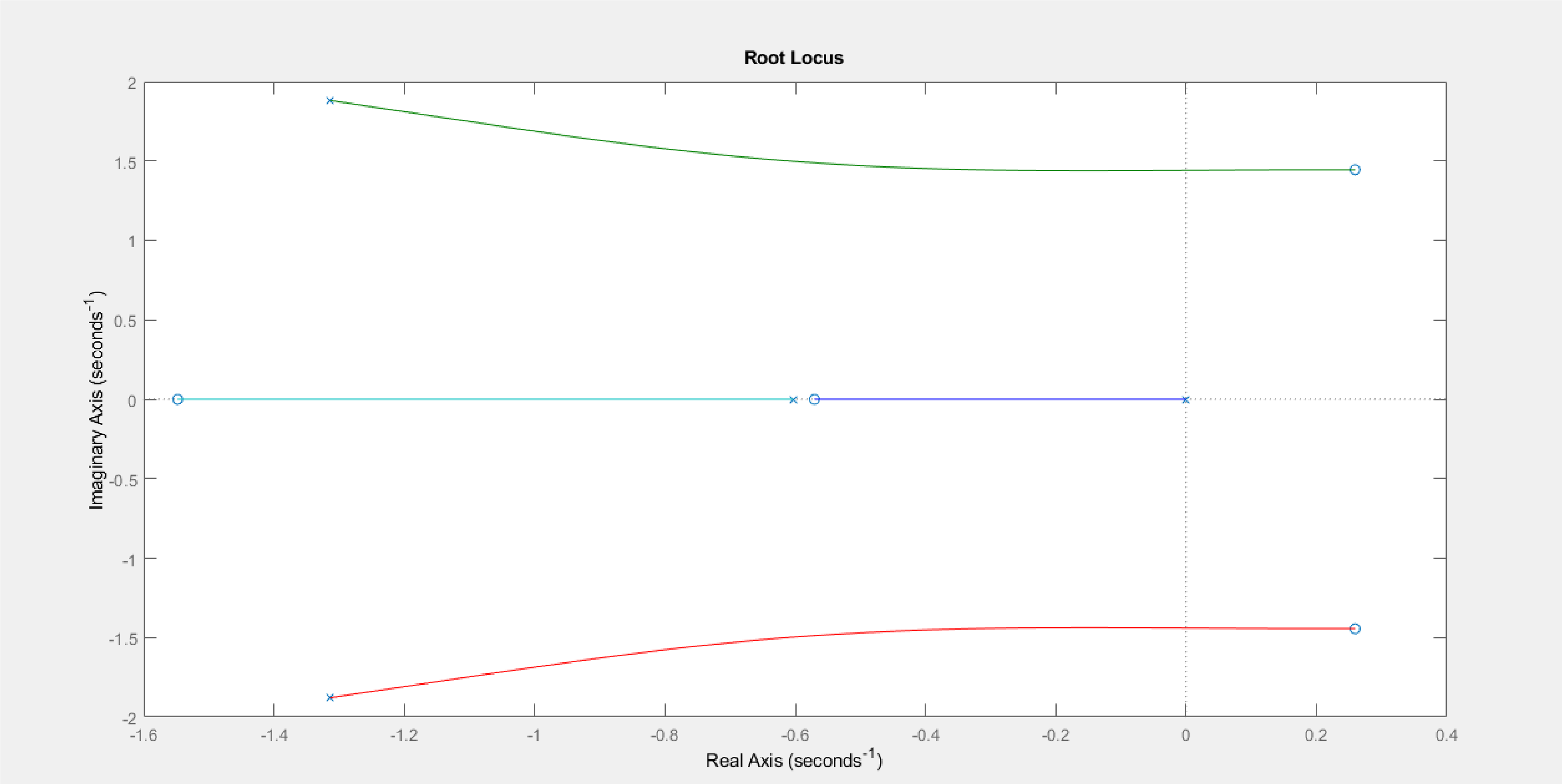
moduri: $e^{\hat{s}_{o3}t}, e^{\hat{s}4t}$

$\hat{s}_{o3}, \hat{s}_{o4} \in R_-$

$e^{\mathrm{Re}\{\hat{s}1,2\}t} \sin(\mathrm{Im}\{\hat{s}_{o1,2}\}t)$

regim: oscilant neamortizat

Sensibilitate mare, deoarece daca modific valoarea lui k, se va modifica regimul.



d.

polul dominant este s=0 => sistem de ordin I: Ts+1;

-vor trebui date valori pentru Ho de la supbct b., apoi:
step(H) – pt performante.