**Computational Methods for Macroeconomics**
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Michaelmas 2024

# Assignment 3

(Aiyagari, 1994) A continuum of measure one of households is considered, and they live forever in discrete time. Each household consumes, and saves. The labor endowment $z$ is exogenously determined by an $AR(1)$ process. The recursive formulation of the household's problem is as follows:

$$
\begin{aligned}
v(a, z; \Phi) = \max_{c, a'} \quad & log(c) + \beta \mathbb{E}_z v(a', z'; \Phi') \\
\text{s.t.} \quad & c + a' = w(\Phi)z + a(1 + r(\Phi)) \\
& a' \geq \underline{a} = 0 \\
& log(z') = \rho log(z) + \sigma\sqrt{1-\rho^2}\epsilon, \quad \epsilon \sim N(0,1) \\
& \Phi' = G(\Phi)
\end{aligned}
$$

where apostrophe indicates future allocations. $\Phi$ is the joint distribution of the individual states $(a, z)$. The borrowing limit $\underline{a}$ is given as 0. The prices $w(\Phi)$ and $r(\Phi)$ are determined at the competitive labor and capital input markets. Now we consider a production sector that operates using the CRS Cobb-Douglas production function:

$$\max_{K,L} AK^{\alpha}L^{1-\alpha} - (r(\Phi) + \delta)K - w(\Phi)L$$

Suppose the aggregate TFP $A = 1$ fixed. Then we close the economy by introducing the following market clearing conditions at which the prices $(w, r)$ are determined:

$$
\begin{array}{lcc}
 & \underline{\text{Supply}} & \underline{\text{Demand}} \\
\text{[Capital market]} & \int a d\Phi = & K \\
\text{[Labor market]} & \int z d\Phi = & L
\end{array}
$$

The parameters levels are set as in Aiyagari (1994), as follows:

$$\rho = 0.9, \quad \sigma = 0.2, \quad \alpha = 0.36, \quad \beta = 0.96.$$

The idiosyncratic labor endowment process is discretized by Tauchen method using 7 grid points covering $\pm 3$ standard-deviation range.

(a) Why does a household's need to understand the aggregate state $\Phi$?

(b) In the long run, will the economy reach a stationary state or not? Why?

(c) Given the problem, define the stationary recursive competitive equilibrium (SRCE).

(d) How large is the labor supply in each period?

(e) Solve the general equilibrium model using

- the interpolated search
- the histogram (non-stochastic) method for the simulation.

(f) Replicate Figures IIa and IIb in Aiyagari (1994).

(g) Based on the solution, simulate 10,000 households for 100 periods, and plot the aggregate wealth time-series. How much does it fluctuate? What does this mean?

(h) In the simulated sample above (stochastic simulation), collect the bottom 10% of the households in terms of wealth at $t = 50$, and plot the average wealth dynamics over the next 50 years, with the 95% interval. Is it realistic?

(i) In the simulated sample above (stochastic simulation), collect the top 10% of the households in terms of wealth at $t = 50$, and plot the average wealth dynamics over the next 50 years, with the 95% interval. Is it realistic?

(j) Pool the whole household samples from the simulation except for the initial 50 periods. Plot the Lorenz curve and calculate the Gini coefficient.

(k) Consider a permanent jump in $A = 1 \rightarrow A = 1.2$. Solve the general equilibrium again as in (e), and plot the Lorenz curve and calculate the Gini coefficient. How different is it from (j)?