Computational Methods for Macroeconomics

Hanbaek Lee
University of Cambridge

Michaelmas 2024

Assignment 1

A household consumes and supplies labor in the following way:

$$\begin{aligned} R(a,z) &= \max\{W(a,z), N(a,z)\} \\ W(a,z) &= \max_{c,n,a'} \quad log(c) - \eta \frac{1}{1+\frac{1}{\chi}} n^{1+\frac{1}{\chi}} + \beta v(a') \\ &\quad \text{s.t.} \quad c + a' = zw(1-\tau)n + a(1 + r(1-\tau)) + T \\ N(a,z) &= \max_{c,a'} \quad log(c) + \beta v(a') \\ &\quad \text{s.t.} \quad c + a' = b + a(1 + r(1-\tau)) + T \end{aligned}$$

The idiosyncratic productivity z satisfies $log(z) \sim_{iid} N(0, \sigma_z)$. The production sector operates as follows:

$$Y = \max_{L} AK^{\alpha}L^{1-\alpha} - wL - rK$$

The government budget balance condition is as follows:

$$\underbrace{\int (wzh + ar)\tau d\Phi}_{\text{Revenue}} = \underbrace{\int_{\{\mathcal{NW}\}} b d\Phi + T}_{\text{Spending}}$$

Wage is determined in the following competitive effective labor market:

$$L^* = \int z n^*(z) d\Phi$$

1. Label and interpret the model ingredients properly.
2. Characterize the individual labor supply curve.
3. Characterize the aggregate labor supply curve.
4. Characterize the aggregate labor demand curve.
5. Suppose the following parameter levels:

$$\alpha = 0.3, \quad \tau = 0.15, \quad \bar{z} = 1, \quad A = 1, \quad r = 0.04, \quad \beta = 0.96$$

   Define and characterize the stationary recursive competitive equilibrium.

6. Visualize the aggregate supply and demand curves in the labor market.

7. Visualize the comparative statics of the wage with respect to the change in A (horizontal axis: A, vertical: w).

8. (Method of simulated moments) Estimate the parameters $(\eta, b, \chi, \sigma_z)$ to match the following hypothetical moments in general equilibrium:

   - Total working hours: 0.33.
   - Unemployment rate: 0.06 (Assume unemployed = non-working).
   - Unemployment benefit: 25% of average wage income.
   - STD/Mean ratio of income: 0.70.

9. Counterfactual exercise: How does an increase in tax rate from 0.15 to 0.16 affects the total income inequality? (including non-working households). Check the various inequality measures.

10. Counterfactual exercise: Suppose η increases instantaneously by 20% (Say, Covid 19). How does it affects the total income inequality? (including non-working households). Check the various inequality measures.