Data Structures and Algorithms

Discussion 4

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- Stacks and Queues
- Recursion
- Priority Queues
- Function Objects
- Programming Problems

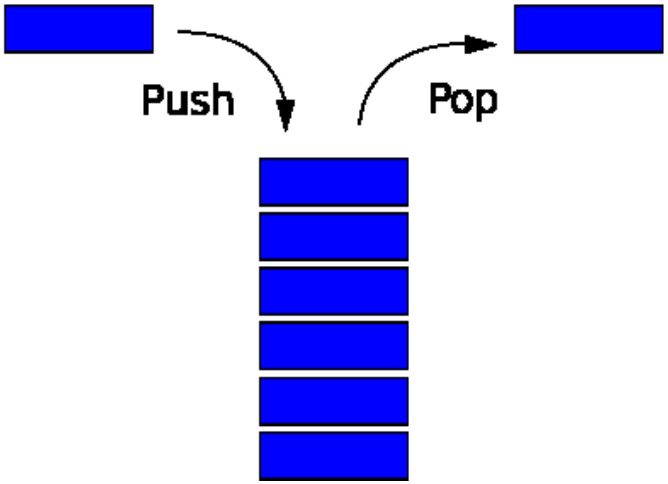
Stacks and Queues

Stack and Queue

- Containers for “inserting” and “removing”
  - Insert();
  - Remove();
  - Top(); or Front();
  - Size()
  - Empty();
- Included in STL
  - #include <stack>
  - #include <queue>

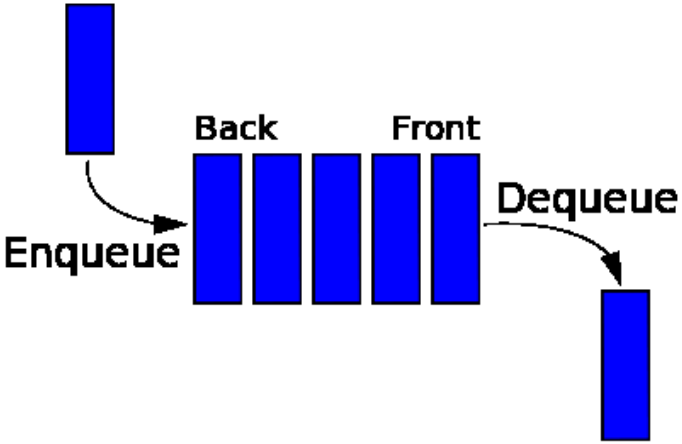
Stack

- Last-in First-out (LIFO)
- Member functions
  - push();
  - pop();
  - top();
  - size()
  - empty();



Queue

- First-in First-out (FIFO)
- Member functions
  - push();
  - pop();
  - front();
  - size()
  - empty();



POP QUIZ!

If you have an unsorted stack, how do you sort it using only one other stack (O(n) extra space)?

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```
//s1 is the original stack, s2 is a stack

while s1 is not empty:
  temp = s1.top(); s1.pop();
  while(s2 is not empty) AND (s2.top() > temp):
    s1.push(s2.top()); s2.pop();
  s2.push(temp)
```

Recursion

Recursion

- **Recursion** = solves a problem by solving smaller instances of the same problem
- All recursive functions can be expressed iteratively (and vice versa)

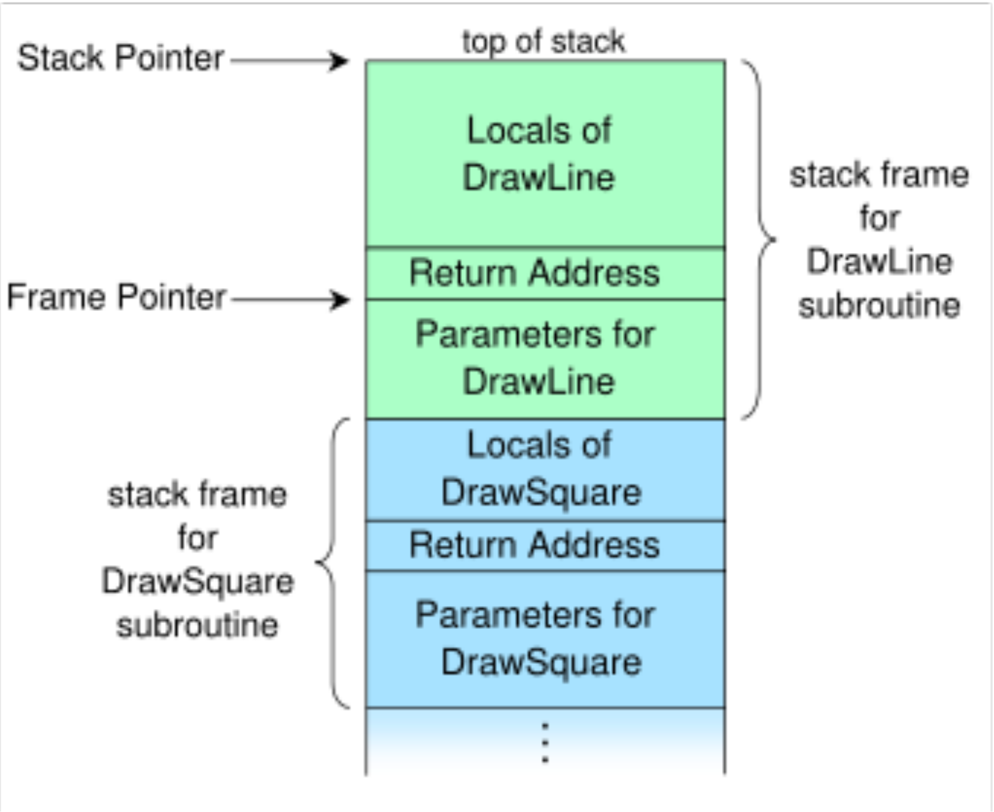
- Recursion advantages:
  - Can be a more intuitive way to solve the problem
  - Some data structures (like trees) are easier to traverse using recursion

Recursion - The Call Stack

- Pseudocode / Python:

```
def drawSquare(args):
    drawLine(args)

def drawLine(args):
    foo = 5
    bar = 3.14
    # do more stuff
```



Recursion

The function call stack is called that because it is **literally** a stack. Functions are logically evaluated in “LIFO” order.

Let's compare a recursive implementation of factorial to its equivalent with an explicit stack rather than the call stack...

Recursion - Factorial Function

```
int fact(int n){
  if (n >= 2){
    // recursive call pushes
    // another stack frame
    // which must store n
    return n * fact(n-1);
  }
  else {
    // base case
    return 1;
  }
}
```

```
int fact(int n){
  stack<int> s;
  // push all recursive calls
  while (n >= 2){
    // push n onto stack
    s.push(n--);
  }

  int f = 1; // base case
  while (!s.empty()){
    f *= s.top();
    s.pop();
  }
  return f;
}
```

Recursion - Factorial Function

Wait a minute…

- That implementation sucks.

---------->

- It takes O(n) space. Do we need that?

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```

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Recursion - Factorial Function

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Better:

```
int fact(int n) {
   int f = 1; //base case
   while(n >= 2) {
      //accumulate n
      f *= n--;
   }
   return f; //done
```

```
int fact(int n){
   stack<int> s;
   // push all recursive calls
   while (n >= 2){
      // push n onto stack
      s.push(n--);
   }

   int f = 1; // base case
   while (!s.empty()){
      f *= s.top();
      s.pop();
   }
   return f;
}
```

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Recursion - Factorial Function

Why is this one better?

- Doesn't store `n` for each frame.
- It only needs O(1) space (no stack!)

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int fact(int n) {
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```

Connect to tail recursion…

- Compiler optimizes by turning recursion into a loop
- Can "reuse" the old stack frame.

```
int fact(int n){
  return fact_help(n,1);
}
int fact_help(int n, int f){
  if (n >= 2){
    // "accumulate" n
    return fact_help(n-1,
f*n);
  }
  else { return f; } // done
}
```

Recursion

Recall that some problems are hard (i.e. unnatural) to implement with tail recursion.

What's an example of such a problem?

What can we say about problems that need > O(1) space?

Recursion

Recall that some problems are hard (i.e. unnatural) to implement with tail recursion.

What's an example of such a problem?

- **Tree algorithms.**
- **How to combine results from multiple subtrees?**

What can we say about problems that need > O(1) space?

- **"Reusing" stack frames isn't a great trick anymore.**
- **Number of them we need will depend on input.**

Recursion

Speaking of trees…

- Review
  How do you (deep) copy a tree?
  (Will be helpful in P2!)

- Challenge (Interview question!)
  It turns out you can do tree traversal in O(1) **extra** space.
  How? (Hint: put some wasted space already in the tree to good use!)

Priority Queues

Priority Queue

- An abstract container type that supports two operations:

(1) Insert a new item
(2) Remove the item with the **largest key**

- Can have different implementations (binary heap, pairing heap, etc.)

STL Priority Queue

Implemented with a binary heap (You will implement this in project 2!)

`push`: Inserts an item into the priority queue
`pop`: Removes (without returning) the highest priority item
`top`: Returns (without removing) the highest priority item
`empty`: Returns true if the priority queue is empty
`size`: Returns the number of items in the priority queue

STL Priority Queue

These complexities are specific to the STL priority queue - not inherent to the priority queue itself, which is abstract

push	O(log N)
pop	O(log N)
top	O(1)
empty	O(1)
size	O(1)

STL Priority Queue

```
template <
     class T,
     class Container = vector<T>,
     class Compare = less<typename Container::value_type>
   > class priority_queue
```

Template arguments:

- The type of object stored in the priority queue
- The underlying container used (the default is usually fine)
- A function object (functor) type used to determine the priority between two objects
  - Default: `less<T>`

STL Priority Queue

```
template <
     class T,
     class Container = vector<T>,
     class Compare = less<typename Container::value_type>
   > class priority_queue
```

If you want to override the comparison functor, you must also set the container type used (can still use default vector<TYPE>).

GOOD: `priority_queue<int, vector<int>, greater<int>>`

BAD: `priority_queue<int, greater<int>>`

Function Objects (Functors)

Functors Review

- **Functors**: Objects that can be called as if they were ordinary functions
- Often used with STL containers and algorithms
- Made possible by overloading operator ()

```
class MyClass { … };
MyClass classObject;
classObject(); //looks like a function, but
               //really an object
```

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Functor Example

```
class People {
   int age;
public:
   int get_age() {return age;}
};

class PeopleCompare {
public:
   bool operator()(People& p1, People& p2) {
      return p1.get_age() < p2.get_age();
   }
};
```

Functor Example (2)

```
//assuming we already have two People
//objects to work with
PeopleCompare my_functor;
if(my_functor(person1, person2))
   cout << "Person 1 is the youngest\n";
else
   cout << "Person 2 is the youngest\n";
```

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Functor + Priority_queue

Priority queue of type People using the functor PeopleCompare:

```
std::priority_queue <People, std::
  vector<People>, PeopleCompare>
  people_queue;
```

Programming Problems

Problem 1

Using two stacks, how can you implement a queue?

What is the worst case complexity for each operation?

What is the amortized complexity for each operation?

Problem 2

Given two strings, how do you check if one is an anagram (permutation) of the other?

Problem 3

You have 20 bottles of pills

Bottle 1 has one pill, Bottle 2 has 2 pills, etc.

19 bottles have 1.0 gram pills

1 bottle has 1.1 gram pills

You are also given a scale that provides an exact measurement (can only use once)

How do you find the bottle that has the 1.1 gram pills?

Problem 3 - Solution

Since 1+2+3+...+19+20 = 210

Answer: (weight – 210) / 0.1

Example: if weight is 211.3, then bottle #13 is heavy bottle

Problem 4

How could you implement a double-ended priority queue (a queue where it is efficient to get **both** the minimum and the maximum values)?