

71117408_梅洛瑜

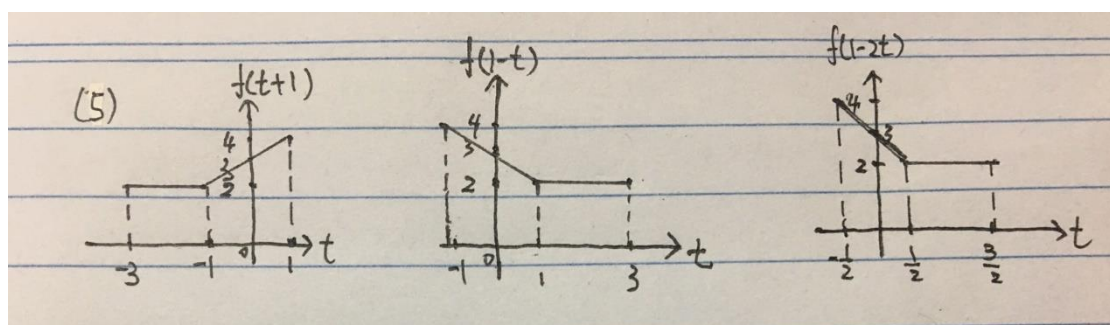
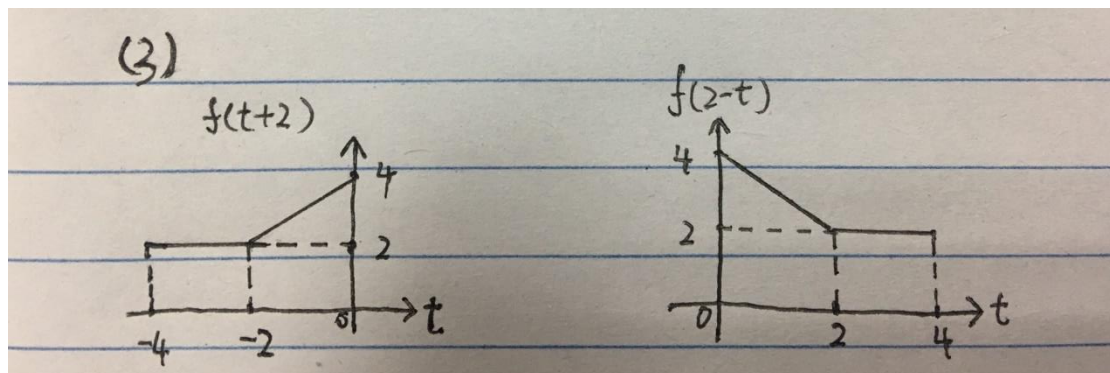
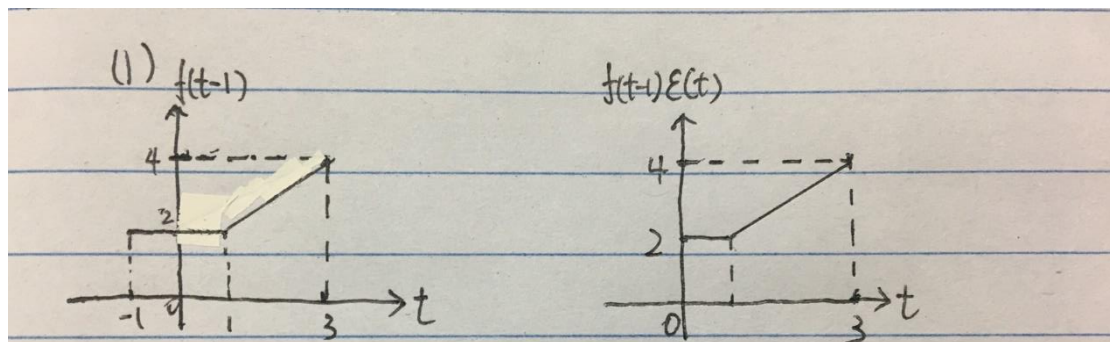
1.4:

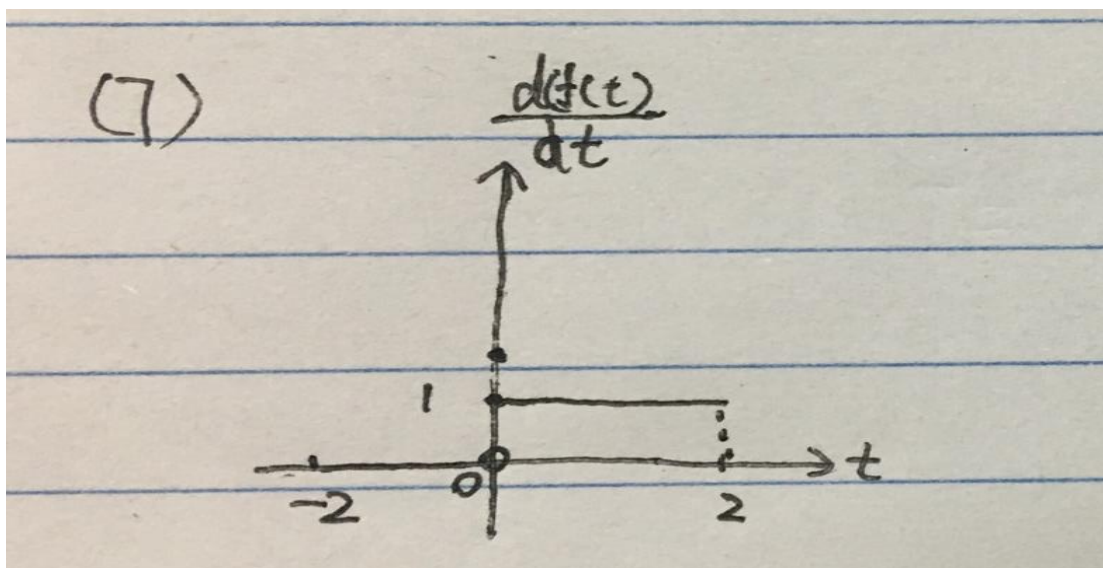
- a) $f(k) = \varepsilon(k+2)$
- b) $f(k) = \varepsilon(k-3)$
- c) $f(k) = \varepsilon(-k+2)$
- d) $f(k) = (-1)^k \varepsilon(k)$

1.5:

- (1) 序列为周期序列, $T=0.3$
- (2) 对于 $k=1$, 不存在周期 T 使 $f_3(1) = f_3(1+mT) (m=0, \pm 1, \pm 2 \dots)$
- (3) $\cos t$ 周期 $T_1 = 2\pi$, $\sin(\pi t)$ 周期 $T_2 = 2$, $\frac{T_1}{T_2} = \pi$ 不为有理数, 故 f_5 不具有周期性

1.6:





1.10:

(1)

$$\begin{aligned}\frac{d^2}{dt^2}[\cos t + \sin 2t]\varepsilon(t) &= \frac{d}{dt}\{[-\sin t + 2\cos(2t)]\varepsilon(t) + [\cos t + \sin 2t]\sigma(t)\} \\ &= \frac{d}{dt}\{[-\sin t + 2\cos(2t)]\varepsilon(t) + \sigma(t)\} \\ &= [-\cos t - 4\sin(2t)]\varepsilon(t) + [-\sin t + 2\cos(2t)]\sigma(t) + \sigma^2(t) \\ &= [-\cos t - 4\sin(2t)]\varepsilon(t) + 2\sigma(t) + \sigma^2(t)\end{aligned}$$

(2)

$$\begin{aligned}(1+t)\frac{d}{dt}[e^{-t}\delta(t)] &= (1+t) * [-e^{-t}\delta(t) + e^{-t}\delta'(t)] \\ &= (1+t) * [-\delta(t) + \delta(t) + \delta'(t)] \\ &= (1+t) * \delta'(t) = \delta'(t) + \delta(t)\end{aligned}$$

$$\text{Tips: } \begin{cases} e^{-at}\delta'(t) = \delta'(t) + a\delta(t) \\ t\delta'(t) = -\delta(t) \end{cases}$$

(3)

$$\text{令 } f(t) = \frac{\sin(\pi t)}{t}, \text{ 则 } \int_{-\infty}^{\infty} \frac{\sin(\pi t)}{t} \delta(t) dt = \int_{-\infty}^{\infty} f(t) \delta(t) dt = f(0) = 1$$

(4)

$$\int_{-\infty}^{\infty} e^{-2t}[\delta(t) + \delta'(t)] dt = \int_{-\infty}^{\infty} e^{-2t}\delta'(t) dt + \int_{-\infty}^{\infty} e^{-2t}\delta(t) dt = 2 + 1 = 3$$

(5)

$$\int_{-\infty}^{\infty} [t^2 + \sin(\frac{\pi}{4}t)]\delta(t+2) dt = [t^2 + \sin(\frac{\pi}{4}t)] \Big|_{t=-2} = 3$$

(7)

$$\int_{-\infty}^{\infty} (t^3 + 2t^2 - 2t + 1)\delta'(t-1) dt = [t^3 + 2t^2 - 2t + 1]' \Big|_{t=1} = 5$$

1.23:

(1)

$$\begin{cases} y_{zs}(t) = \int_0^t \sin x f(x) dx \\ y_{zi}(t) = e^{-t} x(0) \end{cases} \quad \text{故 } y(t) = y_{zi}(t) + y_{zs}(t) \text{ 满足可分解性}$$

同时,

$$e^{-t}[x_1(0) + x_2(0)] = e^{-t}x_1(0) + e^{-t}x_2(0), \text{ 满足零输入线性}$$

$$\int_0^t \sin x [f_1(x) + f_2(x)] dx = \int_0^t \sin x f_1(x) dx + \int_0^t \sin x f_2(x) dx, \text{ 满足零状态线性}$$

故系统为线性的

(3)

$$\begin{cases} y_{zs}(t) = \sin[x(0) * t] \\ y_{zi}(t) = \int_0^t f(x) dx \end{cases} \quad \text{故 } y(t) = y_{zi}(t) + y_{zs}(t) \text{ 满足可分解性}$$

但, $\sin[(x_1(0) + x_2(0)) * t] \neq \sin[x_1(0) * t] + \sin[x_2(0) * t]$ 不满足零输入特性, 故为非线性系统

(5)

$$\begin{cases} y_{zs}(k) = kx(0) \\ y_{zi}(k) = \sum_{j=0}^k f(j) \end{cases} \quad \text{故 } y(t) = y_{zi}(t) + y_{zs}(t) \text{ 满足可分解性}$$

且,

$$k[x_1(0) + x_2(0)] = kx_1(0) + kx_2(0) \text{ 满足零输入线性}$$

$$\sum_{j=0}^k [f_1(j) + f_2(j)] = \sum_{j=0}^k f_1(j) + \sum_{j=0}^k f_2(j) \text{ 满足零状态线性}$$

故为线性系统

1.25:

(3)

$$af_1(t) \cos(2\pi t) + bf_2(t) \cos(2\pi t) = af_1(t) \cos(2\pi t) + bf_2(t) \cos(2\pi t) \text{ 系统满足齐次性与可加性}$$

$$y_{zs}(t - t_d) \neq f(t - t_1) \cos(2\pi t) \text{ 时变}$$

$$t < 0, f(t) = 0, y_{zs}(t) = f(t) \cos(2\pi t) = 0 \quad \text{因果}$$

$$|f(t)| < \infty, |y_{zs}(t)| = |f(t) \cos(2\pi t)| < \infty \quad \text{稳定}$$

故为线性时变因果稳定系统

1.32

由 LT1 系统为线性时不变因果稳定系统

$$\text{故 } y(t) = \varepsilon(t) - \varepsilon(t-1) - \varepsilon(t-2) + \varepsilon(t-3) = f(t) - f(t-1) - f(t-2) + f(t-3)$$

2.14

根据 $h(t)$ 定义有:

$$h'(t) + 2h(t) = \sigma'(t) - \sigma(t)$$

$$h'(0_-) = h(0_-) = 0$$

先求出 $h'(0_+)$, 令:

$$h'(t) = a\sigma'(t) + b\sigma(t) + r_1(t)$$

$$h(t) = a\sigma(t) + r_2(t)$$

故 $\begin{cases} a = 1 \\ b = -2 \end{cases}$, 对 $h'(t)$ 从 0^- 到 0^+ 求积分

$$h(0_+) - h(0_-) = b$$

$$\text{故 } h(0_+) = b = -2$$

当 $t > 0$ 时有 $h'(t) + 2h(t) = 0$, 微分方程的特征根为 -2

故系统的冲激响应为: $h(t) = C_1 e^{-2t}$

带入初始条件 $h(0_+) = -2$, 得出 $C_1 = -2$

故冲激响应为: $h(t) = \sigma(t) - 2e^{-2t}$

阶跃响应为冲击响应的积分: $g(t) = \int_{-\infty}^t h(\tau) d\tau = \varepsilon(t) - 2e^{-2t}$

2.17

(1)

$$y_{zs}(t) = \int_{-\infty}^{\infty} f_1(\tau) f_2(t - \tau) d\tau = \int_{-\infty}^{\infty} \tau \varepsilon(t) \varepsilon(t - \tau) d\tau = \int_0^t \tau d\tau = \frac{1}{2} \tau^2$$

(3)

$$y_{zs}(t) = \int_{-\infty}^{\infty} f_1(\tau) f_2(t - \tau) d\tau = \int_{-\infty}^{\infty} e^{-2\tau} \varepsilon(\tau) e^{-2(t-\tau)} \varepsilon(t - \tau) d\tau$$

$$= \int_{-\infty}^{\infty} e^{-2t} \varepsilon(\tau) \varepsilon(t - \tau) d\tau = \int_0^t e^{-2t} d\tau = t e^{-2t}$$

(5)

$$\begin{aligned} y_{zs}(t) &= \int_{-\infty}^{\infty} f_1(t - \tau) f_2(\tau) d\tau = \int_{-\infty}^{\infty} (t - \tau) \varepsilon(t - \tau) e^{-2\tau} \varepsilon(\tau) d\tau \\ &= \int_0^t t e^{-2\tau} d\tau - \int_0^t \tau e^{-2\tau} d\tau = -\frac{1}{2} t e^{-2t} + -\frac{1}{2} t e^{-2t} + \frac{1}{4} e^{-2t} = \frac{1}{4} e^{-2t} \end{aligned}$$

(7)

$$\begin{aligned} y_{zs}(t) &= \int_{-\infty}^{\infty} f_1(t - \tau) f_2(\tau) d\tau = \int_{-\infty}^{\infty} \sin(\pi\tau) \varepsilon(\tau) [\varepsilon(t - \tau) - \varepsilon(\tau)] d\tau \\ &= \int_0^t \sin(\pi\tau) d\tau - \int_0^{t-4} \sin(\pi\tau) d\tau = \frac{1}{\pi} [\cos(\pi t) - \cos[\pi(t - 4)]] \end{aligned}$$

(9)

$$\begin{aligned} y_{zs}(t) &= \int_{-\infty}^{\infty} f_1(\tau) f_2(t - \tau) d\tau = \int_{-\infty}^{\infty} \tau \varepsilon(\tau - 1) \varepsilon(t - \tau + 3) d\tau \\ &= \int_1^{t+3} \tau d\tau = \frac{1}{2} t^2 + 3t + 4 \end{aligned}$$