

71117408_梅洛瑜

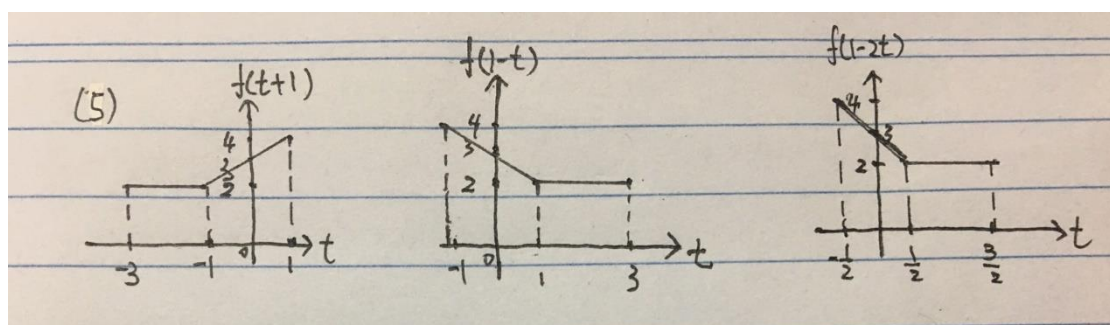
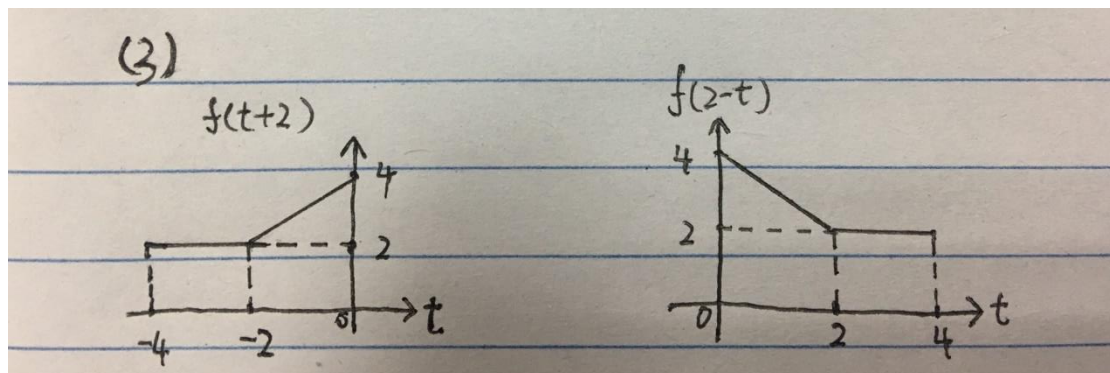
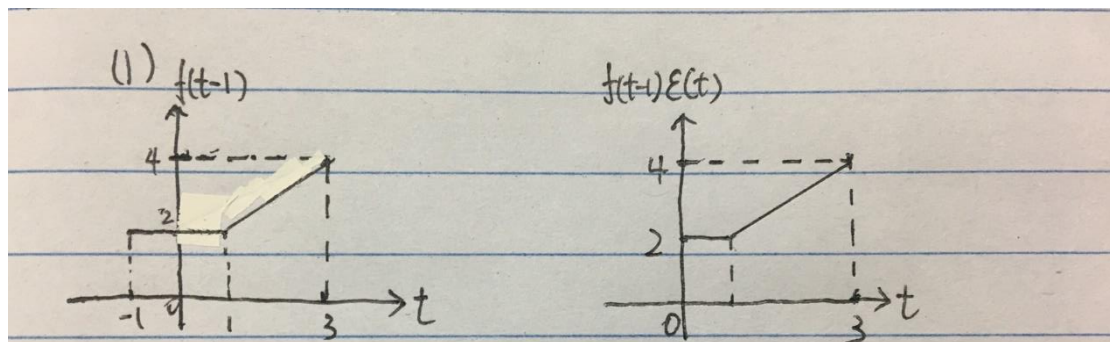
1.4:

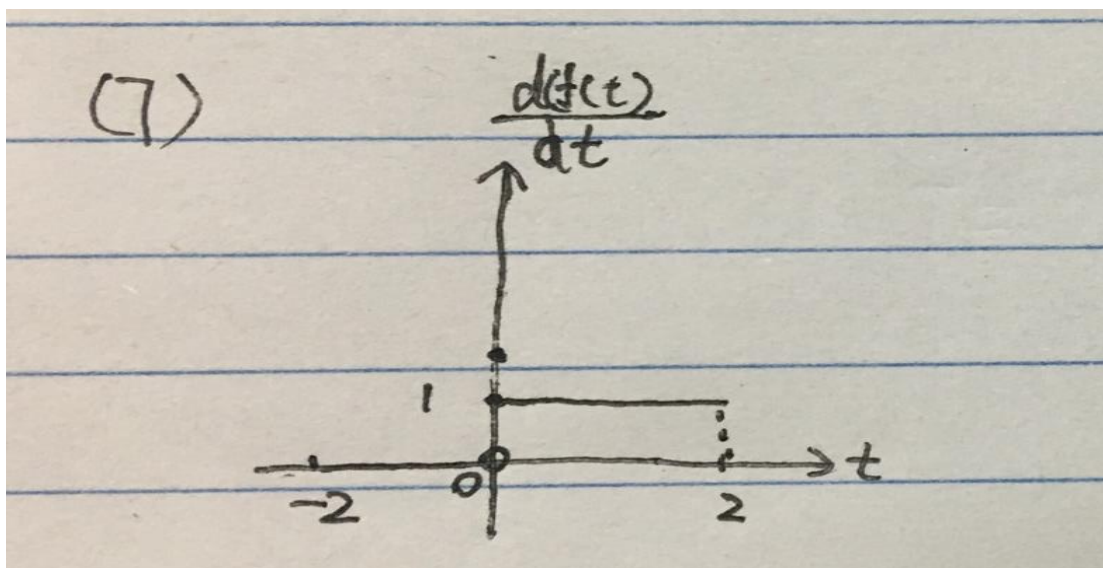
- a) $f(k) = \varepsilon(k+2)$
- b) $f(k) = \varepsilon(k-3)$
- c) $f(k) = \varepsilon(-k+2)$
- d) $f(k) = (-1)^k \varepsilon(k)$

1.5:

- (1) 序列为周期序列, $T=0.3$
- (2) 对于 $k=1$, 不存在周期 T 使 $f_3(1) = f_3(1+mT)$ ($m=0, \pm 1, \pm 2 \dots$)
- (3) $\cos t$ 周期 $T_1 = 2\pi$, $\sin(\pi t)$ 周期 $T_2 = 2$, $\frac{T_1}{T_2} = \pi$ 不为有理数, 故 f_5 不具有周期性

1.6:





1.10:

(1)

$$\begin{aligned}
 \frac{d^2}{dt^2} [\cos t + \sin 2t] \varepsilon(t) &= \frac{d}{dt} \{ [-\sin t + 2 \cos(2t)] \varepsilon(t) + [\cos t + \sin 2t] \sigma(t) \} \\
 &= \frac{d}{dt} \{ [-\sin t + 2 \cos(2t)] \varepsilon(t) + \sigma(t) \} \\
 &= [-\cos t - 4 \sin(2t)] \varepsilon(t) + [-\sin t + 2 \cos(2t)] \sigma(t) + \sigma^2(t) \\
 &= [-\cos t - 4 \sin(2t)] \varepsilon(t) + 2 \sigma(t) + \sigma^2(t)
 \end{aligned}$$

(2)

$$\begin{aligned}
 (1+t) \frac{d}{dt} [e^{-t} \delta(t)] &= (1+t) * [-e^{-t} \delta(t) + e^{-t} \delta'(t)] \\
 &= (1+t) * [-\delta(t) + \delta(t) + \delta'(t)] \\
 &= (1+t) * \delta'(t) = \delta'(t) + \delta(t)
 \end{aligned}$$

$$\text{Tips: } \begin{cases} e^{-at} \delta'(t) = \delta'(t) + a \delta(t) \\ t \delta'(t) = -\delta(t) \end{cases}$$

(3)

$$\text{令 } f(t) = \frac{\sin(\pi t)}{t}, \text{ 则 } \int_{-\infty}^{\infty} \frac{\sin(\pi t)}{t} \delta(t) dt = \int_{-\infty}^{\infty} f(t) \delta(t) dt = f(0) = 1$$

(4)

$$\int_{-\infty}^{\infty} e^{-2t} [\delta(t) + \delta'(t)] dt = \int_{-\infty}^{\infty} e^{-2t} \delta'(t) dt + \int_{-\infty}^{\infty} e^{-2t} \delta(t) dt = 2 + 1 = 3$$

(5)

$$\int_{-\infty}^{\infty} [t^2 + \sin\left(\frac{\pi}{4}t\right) \delta(t+2)] dt = [t^2 + \sin\left(\frac{\pi}{4}t\right)] \Big|_{t=-2} = 3$$

(7)

$$\int_{-\infty}^{\infty} (t^3 + 2t^2 - 2t + 1) \delta'(t-1) dt = [t^3 + 2t^2 - 2t + 1]' \Big|_{t=1} = 5$$