1 Bipartite Representation and Network Architecture

We fully detail the Bipartite representation and GAT architecture described in Section 4. First, a feature extractor converts the MILP instance P to bipartite graph representation $P' = (G, V, C, E)$. The extracted features are shown in Table 1. Then, our policy takes the bipartite graph $P' = (G, V, C, E)$ as input and outputs a score vector with one score per decision variable. We use 2-layer MLPs with 64 hidden units per layer and ReLU as the activation function to map each node feature and edge feature to $\mathbb{R}^L$ where $L = 64$.

Let $\boldsymbol{v}_j, \boldsymbol{c}_I, \boldsymbol{e}_{i,j} \in \mathbb{R}^L$ be the embeddings of the i-th variable, j-th constraint, and the edge connecting their output by the embedding layers V^1, C^1, E^1. We perform two rounds of message passing through the GAT. In the first round, each constraint node $\boldsymbol{c}_i$ attends to its neighbors $\mathcal{N}_i$ using an attention structure with $H = 8$ attention heads:

$$\boldsymbol{c}'_i = \frac{1}{H} \sum_{i=1}^{H} \left(\alpha_{ii,1}^{(h)} \boldsymbol{\theta}_{c,1}^{(h)} \boldsymbol{c}_i + \sum_{j \in \mathcal{N}_i} \alpha_{ij,1}^{(h)} \boldsymbol{\theta}_{v,1}^{(h)} \boldsymbol{v}_j \right)$$

where $\boldsymbol{\theta}_{c,1}^{(h)} \in \mathbb{R}^{L \times L}$ and $\boldsymbol{\theta}_{v,1}^{(h)} \in \mathbb{R}^{L \times L}$ are learnable weights. The updated constraints embeddings $\boldsymbol{c}'_i$ in updated embedding layer C^2 are averaged across H attention heads using attention weights

$$\alpha_{ij,1}^{(h)} = \frac{\exp(\boldsymbol{w}_1^T \rho([\boldsymbol{\theta}_{c,1}^{(h)} \boldsymbol{c}_i, \boldsymbol{\theta}_{v,1}^{(h)} \boldsymbol{v}_j, \boldsymbol{\theta}_{e,1}^{(h)} \boldsymbol{e}_{i,j}]))}{\sum_{k \in \mathcal{N}_i} \exp(\boldsymbol{w}_1^T \rho([\boldsymbol{\theta}_{c,1}^{(h)} \boldsymbol{c}_i, \boldsymbol{\theta}_{v,1}^{(h)} \boldsymbol{v}_k, \boldsymbol{\theta}_{e,1}^{(h)} \boldsymbol{e}_{i,k}]))}$$

where the attention coefficients $\boldsymbol{w}_1 \in \mathbb{R}^{3L}$ and $\boldsymbol{\theta}_{e,1}^{(h)} \in \mathbb{R}^{L \times L}$ are both learnable weights and $\rho(\cdot)$ refers to the LeakyReLU activation function with negative slope 0.2. In the second round, similarly, each variable node attends to its neighbors to get updated variable node embeddings

$$\boldsymbol{v}'_j = \frac{1}{H} \sum_{i=1}^{H} \left(\alpha_{jj,2}^{(h)} \boldsymbol{\theta}_{c,2}^{(h)} \boldsymbol{c}'_i + \sum_{j \in \mathcal{N}_i} \alpha_{ji,2}^{(h)} \boldsymbol{\theta}_{v,2}^{(h)} \boldsymbol{v}_j \right)$$

with attention weights

$$\alpha_{ji,2}^{(h)} = \frac{\exp(\boldsymbol{w}_2^T \rho([\boldsymbol{\theta}_{c,2}^{(h)} \boldsymbol{c}'_i, \boldsymbol{\theta}_{v,2}^{(h)} \boldsymbol{v}_j, \boldsymbol{\theta}_{e,2}^{(h)} \boldsymbol{e}_{i,j}]))}{\sum_{k \in \mathcal{N}_i} \exp(\boldsymbol{w}_2^T \rho([\boldsymbol{\theta}_{c,2}^{(h)} \boldsymbol{c}'_i, \boldsymbol{\theta}_{v,2}^{(h)} \boldsymbol{v}_j, \boldsymbol{\theta}_{e,2}^{(h)} \boldsymbol{e}_{i,k}]))}$$

where $\boldsymbol{w}_2 \in \mathbb{R}^{3L}$ and $\boldsymbol{\theta}_{c,2}^{(h)}, \boldsymbol{\theta}_{v,2}^{(h)}, \boldsymbol{\theta}_{e,2}^{(h)} \in \mathbb{R}^{L \times L}$ are learnable weights. After the two rounds of message passing, we learned MILP embedding consisting of (V_2, C_2), passed to task-specific layers.

2 Benchmark Problem Descriptions and MILP Formulations

We present the MILP formulations for Combinatorial Auction (CA), Maximum Independent Set (MIS), and Minimum Vertex Cover (MVC).

Table 1: Description of the constraint, edge, and variable features in our bipartite state representation $P' = (G, C, E, V)$.

Tensor	Feature	Description
C	obj_cos_sim	Cosine similarity with objective.
	bias	Bias value, normalized.
	is_tight	Tightness indicator in LP solution.
	dualsol_val	Dual solution value, normalized.
E	coef	Constraint coefficient, normalized.
V	type_CONTINUOUS	Indicator of continuous variable
	type_BINARY	Indicator of binary variable
	type_INTEGER	Indicator of integer variable
	coef	Objective coefficient, normalized.
	has_lb	Lower bound indicator.
	has_ub	Upper bound indicator.
	sol_is_at_lb	Solution value equals lower bound.
	sol_is_at_ub	Solution value equals upper bound.
	sol_frac	Solution value fractionality.
	basis_status_BASIC	Indicator of basic simplex basis status.
	basis_status_NONBASIC_LOWER	Indicator of lower simplex basis status.
	basis_status_NONBASIC_UPPER	Indicator of upper simplex basis status.
	basis_status_SUPERBASIC	Indicator of zero simplex basis status.
	reduced_cost	Reduced cost, normalized.
	sol_val	Solution value.

2.1 CA

In a CA instance, we are given n bids $\{(B_i, p_i) : i \in [n]\}$ for m items, where B_i is a subset of items and p_i is its associated bidding price. The objective is to allocate items to bids such that the total revenue is maximized:

$$\max \sum_{i \in [n]} p_i x_i$$
$$\text{s.t.} \sum_{i: j \in B_i} x_i \leq 1, \forall j \in [m],$$
$$x_i \in \{0, 1\}, \forall i \in [n].$$

2.2 MIS

We are given an undirected graph $G = (V, E)$. The goal is to select the largest subset of nodes such that no two nodes in the subsets are connected by an edge in G:

$$\max \sum_{v \in V} x_v$$
$$\text{s.t.} x_u + x_v \leq 1, \forall (u, v) \in E,$$
$$x_v \in \{0, 1\}, \forall v \in V.$$

2.3 MVC

In the MVC problem, we are given an undirected graph $G = (V, E)$ with a weight w_v associated with each node $v \in V$. The objective is to select a subset of nodes $V' \subseteq V$ with the minimum sum of weights such that for every edge in E, at least one of its endpoints is selected in V':

$$\begin{aligned} &\min \sum_{v \in V} w_v x_v \\ \text{s.t.} \quad &x_u + x_v \geq 1, \forall (u, v) \in E, \\ &x_v \in \{0, 1\}, \forall v \in V. \end{aligned}$$

3 Hyperparameter Settings

We present the hyperparameter settings during data collection and evaluation for our experiments. During data collection, for Backdoor, we collect 50 candidate backdoors for every instance and select 5 as positive samples and 5 as negative samples. For PaS, we collect 50 best-found solutions for each training instance with an hour runtime. Then, we collect ten negative samples for each solution. For Configuration, we collect 50 configurations through *SMAC* for every instance and select 5 as positive and 5 as negative samples.

The configuration we selected for SCIP is shown below:

- **branching/clamp**: Minimal relative distance of branching point to bounds when branching on a continuous variable.
- **branching/lpgainnormalize**: Strategy for normalization of LP gain when updating pseudocosts of continuous variables (divide by movement of *l*p value, reduction in *d*omain width, or reduction in domain width of *s*ibling).
- **branching/midpull**: Fraction by which to move branching point of a continuous variable towards the middle of the domain; a value of 1.0 leads to branching always in the middle of the domain.
- **branching/midpullreldomtrig**: Multiply *midpull* by relative domain width if the latter is below this value.
- **branching/preferbinary**: Should branching on binary variables be preferred?
- **branching/scorefac**: Branching score factor to weigh downward and upward gain prediction in sum score function.
- **branching/scorefunc**: Branching score function (*s*um, *p*roduct, *q*uotient).
- **cutselection/hybrid/minortho**: Minimal orthogonality for a cut to enter the LP.
- **cutselection/hybrid/minorthoroot**: Minimal orthogonality for a cut to enter the LP in the root node.
- **lp/colagelimit**: Maximum age a dynamic column can reach before it is deleted from the LP.
- **lp/pricing**: LP pricing strategy (*l*pi default, *a*uto, *f*ull pricing, *p*artial, *s*teepest edge pricing, *q*uickstart steepest edge pricing, *d*evex pricing).

- **lp/rowagelimit**: Maximum age a dynamic row can reach before it is deleted from the LP.
- **nodeselection/childsel**: Child selection rule (*d*own, *u*p, *p*seudo costs, *i*nference, *l*p value, *r*oot LP value difference, *h*ybrid inference/root LP value difference).
- **separating/cutagelimit**: Maximum age a cut can reach before it is deleted from the global cut pool.
- **separating/maxcutsgenfactor**: Factor with respect to *maxcuts* for maximal number of cuts generated per separation round.
- **separating/maxcutsrootgenfactor**: Factor with respect to *maxcutsroot* for maximal number of generated cuts per separation round at the root node.
- **separating/poolfreq**: Separation frequency for the global cut pool.

During the evaluation, for BACKDOOR, backdoor size K is set to 8 for all benchmarks. For PAS, (k_0, k_1, Δ) is set to $(1500, 0, 0)$ for CA, $(500, 500, 10)$ for MIS and $(500, 100, 15)$ for MVC.

4 Additional Tables and Figures

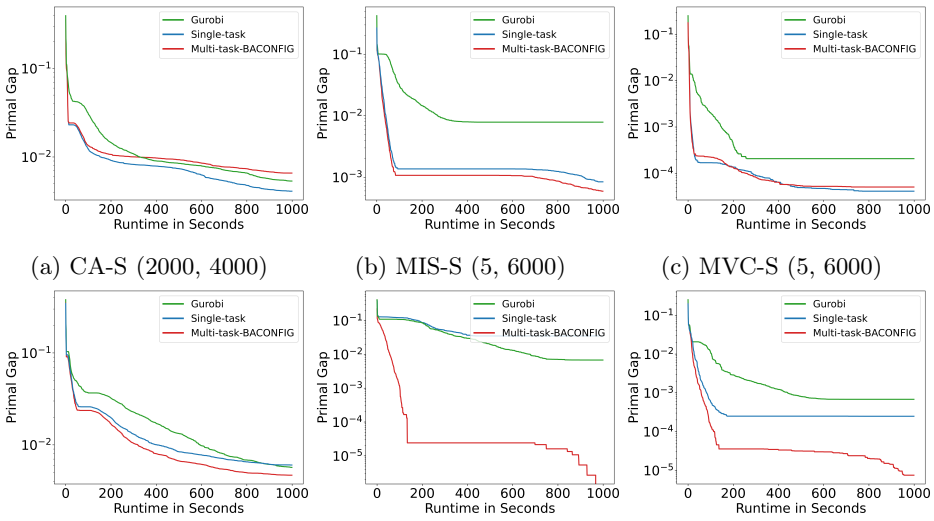
We provide supplementary results through tables and figures to illustrate the performance and comparison of multitask models. Table 2 presents the full primal integral results, highlighting the new task generalization capabilities of our multitask model. Figure 1 visualizes the primal gap results obtained from cross-validation on PAS.

Additionally, Tables 3 and 4, along with Figure 2, compare multitask models on the same tasks, BACKDOOR and PAS. Unlike the consistent performance advantage observed when comparing multitask models to singletask models, the results between multitask models vary. On BACKDOOR, *Multi-task-PASCONFIG* outperforms *Multi-task-BAPAS* on CA and MVC benchmarks, while their performance is similar on MIS. Conversely, on PAS, *Multi-task-BACONFIG* underperforms compared to *Multi-task-BAPAS* on CA and MVC but performs better on MIS.

These differences may stem from variations in data quality across problem domains and the number of instances used to train each model. This highlights the importance of task-specific data characteristics and opens avenues for further exploration in multitask model optimization and data utilization strategies.

Table 2: Primal Integral for CONFIGURATION averaged over 100 test instances for each benchmark. We compare the performance of *SCIP*, *SMAC*, *Single-task* (trained on CONFIGURATION), and *Multi-task-BAPAS*. Results include the mean, standard deviation, and the number of instances each approach wins. The best-performing entries are highlighted in bold for clarity.

		CONFIGURATION Primal Integral		
Benchmarks	Approaches	Mean	Std Dev	Wins
CA-S (2000, 4000)	*SCIP*	597.36	206.55	13
	SMAC	598.30	207.27	9
	Single-task	489.10	**155.86**	34
	Multi-task-BAPAS	**485.84**	194.10	**44**
MIS-S (4, 3000)	*SCIP*	439.27	310.01	**37**
	SMAC	411.70	227.07	24
	Single-task	422.58	190.61	16
	Multi-task-BAPAS	**351.60**	**167.71**	23
MVC-S (4, 3000)	*SCIP*	576.36	213.6	0
	SMAC	264.36	185.60	4
	Single-task	87.35	60.55	37
	Multi-task-BAPAS	**72.94**	**58.54**	**59**
CA-L (3000, 6000)	*SCIP*	247.97	72.19	6
	SMAC	269.15	116.17	13
	Single-task	306.77	74.60	4
	Multi-task-BAPAS	**162.21**	**55.02**	**77**
MIS-L (5, 6000)	*SCIP*	782.65	**166.61**	1
	SMAC	732.51	176.52	5
	Single-task	768.61	172.79	4
	Multi-task-BAPAS	**234.45**	222.67	**90**
MVC-L (5, 6000)	*SCIP*	714.79	**176.49**	0
	SMAC	576.32	204.4	8
	Single-task	398.38	322.22	21
	Multi-task-BAPAS	**231.92**	282.19	**71**



(a) CA-S (2000, 4000) (b) MIS-S (5, 6000) (c) MVC-S (5, 6000)

(d) CA-L (3000, 6000) (e) MIS-L (5, 9000) (f) MVC-L (5, 9000)

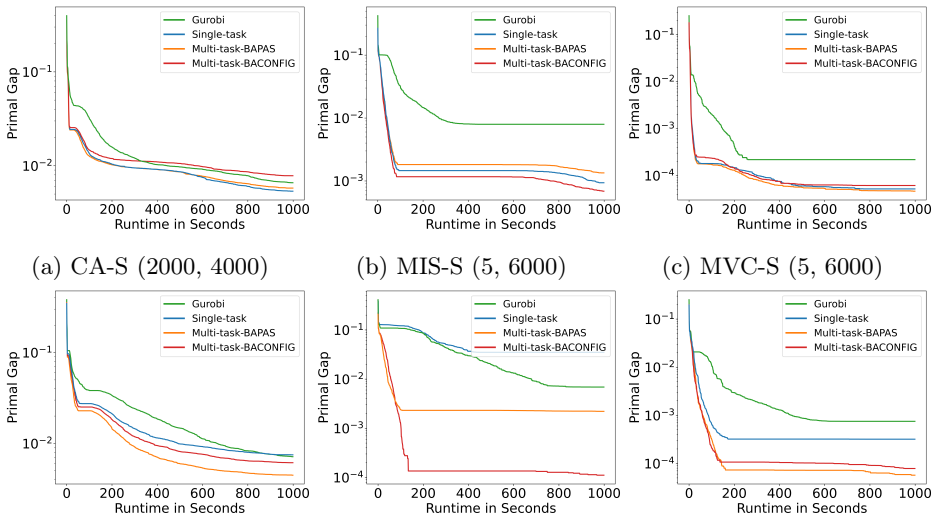
Fig. 1: The Primal Gap (the lower, the better) as a runtime function averaged over 100 test instances on PAS and each benchmark. We compare the performance of *Gurobi* (green line), *Single-task* on PAS (blue line), and *Multi-task-BACONFIG* (red line).

Table 3: Solve Time (seconds) for BACKDOOR averaged over 100 test instances for each benchmark. We compare the performance of *SCIP*, *Single-task* (trained on BACKDOOR), same task multitask model *Multi-task-BAPAS*, and new task multitask model *Multi-task-PASCONFIG*. Results include the mean, standard deviation, and the number of instances each approach wins. The best-performing entries are highlighted in bold for clarity.

		BACKDOOR Solve Time		
Benchmarks	Approaches	Mean	Std Dev	Wins
CA-S (175, 850)	*Gurobi*	253.08	3	132.57
	Single-task	231.23	24	124.91
	Multi-task-BAPAS	218.75	27	121.10
	Multi-task-PASCONFIG	**209.06**	**46**	**117.74**
MIS-S (4, 1250)	*Gurobi*	183.51	24	221.31
	Single-task	172.50	**31**	216.93
	Multi-task-BAPAS	165.02	30	213.47
	Multi-task-PASCONFIG	**164.73**	15	**202.90**
MVC-S (5, 1500)	*Gurobi*	77.47	4	93.26
	Single-task	72.19	30	96.69
	Multi-task-BAPAS	71.43	31	91.52
	Multi-task-PASCONFIG	**67.70**	**35**	**78.47**
CA-L (200, 1000)	*Gurobi*	793.10	13	450.03
	Single-task	749.08	18	479.99
	Multi-task-BAPAS	708.07	31	**413.69**
	Multi-task-PASCONFIG	**696.39**	**38**	419.18
MIS-L (4, 1500)	*Gurobi*	530.41	17	935.79
	Single-task	470.73	27	837.01
	Multi-task-BAPAS	448.30	**33**	792.42
	Multi-task-PASCONFIG	**446.61**	23	**791.89**
MVC-L (5, 2000)	*Gurobi*	427.86	17	697.71
	Single-task	390.04	17	623.76
	Multi-task-BAPAS	379.41	20	653.14
	Multi-task-PASCONFIG	**354.35**	**46**	**607.36**

Table 4: Primal Integral for PAS averaged over 100 test instances for each benchmark. We compare the performance of *SCIP*, *Single-task* (trained on PAS), same task multitask model *Multi-task-BAPAS*, and new task multitask model *Multi-task-BACONFIG*. Results include the mean, standard deviation, and the number of instances each approach wins. The best-performing entries are highlighted in bold for clarity.

		PAS Primal Integral		
Benchmarks	Approaches	Mean	Std Dev	Wins
CA-S (2000, 4000)	*Gurobi*	15.09	**5.19**	3
	Single-task	10.26	5.34	36
	Multi-task-BAPAS	**10.23**	5.57	**39**
	Multi-task-BACONFIG	12.25	5.99	22
MIS-S (5, 6000)	*Gurobi*	17.03	3.78	0
	Single-task	3.58	1.44	31
	Multi-task-BAPAS	3.81	1.28	16
	Multi-task-PASCONFIG	**3.03**	**1.04**	**53**
MVC-S (5, 6000)	*Gurobi*	1.45	0.65	0
	Single-task	0.55	**0.10**	27
	Multi-task-BAPAS	**0.53**	0.13	31
	Multi-task-PASCONFIG	0.54	0.14	**42**
CA-L (3000, 6000)	*Gurobi*	21.46	5.98	3
	Single-task	15.58	5.90	11
	Multi-task-BAPAS	**11.10**	**5.14**	**55**
	Multi-task-PASCONFIG	13.60	5.80	31
MIS-L (5, 9000)	*Gurobi*	38.06	10.75	0
	Single-task	56.23	8.76	0
	Multi-task-BAPAS	4.51	1.82	29
	Multi-task-PASCONFIG	**3.24**	**0.95**	**71**
MVC-L (5, 9000)	*Gurobi*	3.91	2.28	0
	Single-task	1.56	0.57	5
	Multi-task-BAPAS	**1.11**	**0.44**	47
	Multi-task-PASCONFIG	**1.11**	0.46	**48**



(a) CA-S (2000, 4000) (b) MIS-S (5, 6000) (c) MVC-S (5, 6000)

(d) CA-L (3000, 6000) (e) MIS-L (5, 9000) (f) MVC-L (5, 9000)

Fig. 2: The Primal Gap (the lower, the better) as a runtime function averaged over 100 test instances on PAS and each benchmark. We compare the performance of *Gurobi* (green line), *Single-task* on PAS (blue line), *Multi-task-BAPAS* (orange line), and *Multi-task-BACONFIG* (red line).