



Cambridge Assessment International Education

Cambridge International Advanced Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	
FURTHER MATHEMA	ATICS		9231/21
Paper 2		Oc	tober/November 2019
			3 hours
Candidates answer or	n the Question Paper.		
Additional Materials:	List of Formulae (MF10)		

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Where a numerical value is necessary, take the acceleration due to gravity to be 10 m s^{-2} .

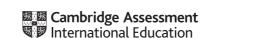
The use of a calculator is expected, where appropriate.

Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.

You are reminded of the need for clear presentation in your answers.

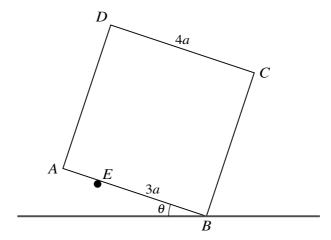
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.



BLANK PAGE

<i>P</i> 18 3	8 m s ⁻² . F	ind the 1	nagnitud	de of the	e transv	erse con	nponent	of the ac	eceleratio	on of P	at this in
•••••	•••••		•••••	•••••	•••••	•••••	•••••	•••••	•••••	•••••	
•••••									•••••		
•••••			•••••	•••••	•••••			••••••	•••••	•••••	
•••••			•••••								
•••••			•••••		•••••						
•••••											
•••••			•••••		•••••	••••••		•••••	••••••	•••••	
•••••											
•••••											
			•••••		•••••	•••••				•••••	
					•••••				• • • • • • • • • • • • • • • • • • • •		
						•••••			•••••		
•••••					•••••	•••••			•••••	•••••	
	•••••							•••••	• • • • • • • • • • • • • • • • • • • •		
•••••			•••••								
•••••											
•••••			•••••		•••••	•••••			•••••	•••••	
					•••••			•••••	•••••	•••••	
					•••••			•••••	•••••	•••••	
•••••	•••••										



A uniform square lamina ABCD of side 4a and weight W rests in a vertical plane with the edge AB inclined at an angle θ to the horizontal, where $\tan \theta = \frac{1}{3}$. The vertex B is in contact with a rough horizontal surface for which the coefficient of friction is μ . The lamina is supported by a smooth peg at the point E on AB, where BE = 3a (see diagram).

Find expressions in terms of W for the normal reaction forces at E and B .	[5]

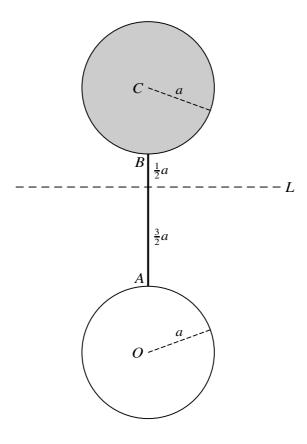
		•••
		•••
		•••
		•••
		•••
		•••
		•••
(;;)	Given that the lamine is about to aline find the value of u	21
(11)	Given that the lamina is about to slip, find the value of μ .	3]
		•••
		•••
		•••
		•••
		•••
		•••
		•••
		•••
		•••
		•••

W1	coefficient of restitution between each pair of spheres is e . Sphere A is projected directly toward th speed u .
i)	Show that the speed of A after its collision with B is $\frac{1}{2}u(1-e)$ and find the speed of B.
he	ere B now collides with sphere C . Subsequently there are no further collisions between any of the sphere C .
he	res.
he	res.
he	res.
he	
he	res.
he	res.
he	res.

A particle P of mass m is attached to one end of a light inextensible string of length a. The other end

projected vertically downwards with speed $\sqrt{(2ag)}$ so that it begins to move along a circular path string becomes slack when OP makes an angle θ with the upward vertical through O .
Show that $\cos \theta = \frac{2}{3}$.

(ii)	Find the greatest height, above the horizontal through O , reached by P in its subsequent motion. [4]



A thin uniform rod AB has mass λM and length 2a. The end A of the rod is rigidly attached to the surface of a uniform hollow sphere (spherical shell) with centre O, mass 3M and radius a. The end B of the rod is rigidly attached to the surface of a uniform solid sphere with centre C, mass 5M and radius a. The rod lies along the line joining the centres of the spheres, so that CBAO is a straight line. The horizontal axis L is perpendicular to the rod and passes through the point of the rod that is a distance $\frac{1}{2}a$ from B (see diagram). The object consisting of the rod and the two spheres can rotate freely about L.

(i)	Show that the moment of inertia of the object about L is $\left(\frac{408 + 7\lambda}{12}\right)Ma^2$.	[6]
		•••••
		•••••
		•••••
		•••••

		•••
		.
		•••
		•••
		•••
		· • • •
The	period of small oscillations of the object about L is $5\pi\sqrt{\left(\frac{2a}{g}\right)}$.	
1110	$\bigvee \left(\begin{array}{c} g \end{array} \right)$	
(ii)	Find the value of λ .	[6]
		· • • •
		•••
		•••
		• • • •
		.
		•••
		•••
		· • • •
		· • • •
		•••
		•••

6

he c	ibuted. A 95% confidence interval for the population mean height, μ metres, is calculated fr lata as $1.65 \le \mu \le 1.85$.	all or
(i)	Find an unbiased estimate for the population variance.	[3
		•••
		•••
		•••
		•••
		•••
		•••
		•••
		•••
(11)	Denoting the height of a member of the club by x metres, find Σx^2 for this sample of 9 members	er [4

7

The time, T days, before an electrical component develops a fault has distribution function F given by

		$F(t) = \begin{cases} 1 - e^{-at} \\ 0 \end{cases}$	otherwise,	
whe	re a is a positive constant	nt. The mean value of	T is 200.	
(i)	Write down the value of	of a.		I
(ii)	Find the probability tha	t an electrical compone	ent of this type develo	os a fault in less than 150 da
A pi		n of these compo		
othe	iece of equipment conta		nents, which develop	faults independently of e
othe is gr	iece of equipment conta	after 150 days, at least	nents, which develop	faults independently of e
othe is gr	iece of equipment conta er. The probability that, a reater than 0.99. Find the smallest possi	after 150 days, at least ble value of n .	nents, which develop tone of the n compon	faults independently of e ents has not developed a fa
othe is gr	iece of equipment conta er. The probability that, a reater than 0.99. Find the smallest possi	after 150 days, at least ble value of n .	nents, which develop tone of the n compon	faults independently of e ents has not developed a fa
othe is gr	iece of equipment conta er. The probability that, a reater than 0.99. Find the smallest possi	after 150 days, at least ble value of n .	nents, which develop tone of the n compon	faults independently of e ents has not developed a fa
othe is gr	iece of equipment conta er. The probability that, a reater than 0.99. Find the smallest possi	after 150 days, at least ble value of n .	nents, which develop tone of the n compon	faults independently of e ents has not developed a fa
othe is gr	iece of equipment conta er. The probability that, a reater than 0.99. Find the smallest possi	after 150 days, at least ble value of n .	nents, which develop tone of the n compon	faults independently of e ents has not developed a fa
othe is gr	iece of equipment conta er. The probability that, a reater than 0.99. Find the smallest possi	after 150 days, at least ble value of n .	nents, which develop tone of the n compon	faults independently of elents has not developed a fa
othe is gr	iece of equipment conta er. The probability that, a reater than 0.99. Find the smallest possi	after 150 days, at least ble value of n .	nents, which develop tone of the n compon	faults independently of elents has not developed a fa
othe is gr	iece of equipment conta er. The probability that, a reater than 0.99. Find the smallest possi	after 150 days, at least ble value of n .	nents, which develop tone of the n compon	faults independently of elents has not developed a fa
othe is gr	iece of equipment conta er. The probability that, a reater than 0.99. Find the smallest possi	after 150 days, at least ble value of n .	nents, which develop tone of the n compon	faults independently of ea

A random sample of 8 elephants from region A is taken and their weights, x tonnes, are recorded (1 tonne = $1000 \mathrm{kg}$.) The results are summarised as follows.
$\Sigma x = 32.4 \qquad \Sigma x^2 = 131.82$
A random sample of 10 elephants from region B is taken. Their weights give a sample mean of 3.78 tonnes and an unbiased variance estimate of 0.1555 tonnes ² . The distributions of the weights of elephants in regions A and B are both assumed to be normal with the same population variance. Test at the 10% significance level whether the mean weight of elephants in region A is the same as the mean weight of elephants in region B .

9 A random sample of five pairs of values of x and y is taken from a bivariate distribution. The values are shown in the following table, where p and q are constants.

х	1	2	3	4	5
y	4	p	q	2	1

The equation of the regression line of y on x is y = -0.5x + 3.5.

(i)	Find the values of p and q .	[7]

		•••
		•••
		•••
		•••
		•••
		•••
		•••
		•••
		•••
(ii)	Find the value of the product moment correlation coefficient.	3]
		•••
		•••
		•••
		•••
		•••
		•••
		•••
		•••
		•••
		•••
		•••
		•••
		•••

	10	The random	variable <i>X</i> has	probability	density	function	f given	ı by
--	-----------	------------	-----------------------	-------------	---------	----------	---------	------

has probability density function f given by
$$f(x) = \begin{cases} \frac{1}{30} \left(\frac{8}{x^2} + 3x^2 - 14 \right) & 2 \le x \le 4, \\ 0 & \text{otherwise.} \end{cases}$$

(i)	Find the distribution function of X .	[3]
	random variable Y is defined by $Y = X^2$.	
(ii)	Find the probability density function of Y .	[4]

(iii)	Find the value of y such that $P(Y < y) = 0.8$. [3]

11 Answer only **one** of the following two alternatives.

EITHER

The points A and B are a distance 1.2 m apart on a smooth horizontal surface. A particle P of mass $\frac{2}{3}$ kg is attached to one end of a light spring of natural length 0.6 m and modulus of elasticity 10 N. The other end of the spring is attached to the point A. A second light spring, of natural length 0.4 m and modulus of elasticity 20 N, has one end attached to P and the other end attached to B.

Show that when P is in equilibrium $AP = 0.75$ m.	
particle P is displaced by 0.05 m from the equilibrium position to Show that P performs simple harmonic motion and state the per-	

		•••••
		•••••
(iii)	Find the speed of <i>P</i> when it passes through the equilibrium position.	[2]
		•••••
		••••••
(iv)	Find the speed of P when its acceleration is equal to half of its maximum value.	[3]
		•••••
		•••••

OR

The number of puncture repairs carried out each week by a small repair shop is recorded over a period of 40 weeks. The results are shown in the following table.

Number of repairs in a week	0	1	2	3	4	5	≥ 6
Number of weeks	6	15	9	6	3	1	0

 (i) Calculate the mean and vari suitability of a Poisson dist 							[3
	•••••	•••••		•••••			
				•••••		•••••	
		•••••		•••••			
llowing table shows some of t	he expect	ed freque	ncies, cor				
llowing table shows some of to weeks using a Poisson distrib	he expect oution wit	ed freque h mean 1.	ncies, cor 6.	rect to 3 c			a period o
ecords over a longer period of llowing table shows some of to weeks using a Poisson distribution. Number of repairs in a week Expected frequency	he expect	ed freque	ncies, cor		lecimal pl	aces, for	
lowing table shows some of t weeks using a Poisson distrib Number of repairs in a week Expected frequency	he expect oution wit 0 8.076	ed freque h mean 1. 1 12.921	ncies, cor 6. 2 10.337	3 5.513	lecimal pl	aces, for	a period o
lowing table shows some of t weeks using a Poisson distrib Number of repairs in a week Expected frequency	he expect oution wit 0 8.076	ed freque h mean 1. 1 12.921	ncies, cor 6. 2 10.337	3 5.513	lecimal pl	aces, for	a period of $\geqslant 6$
lowing table shows some of t weeks using a Poisson distrib Number of repairs in a week Expected frequency	he expect oution wit 0 8.076	ed freque h mean 1. 1 12.921	ncies, cor 6. 2 10.337	3 5.513	lecimal pl	aces, for	a period of $\geqslant 6$
lowing table shows some of t weeks using a Poisson distrib Number of repairs in a week Expected frequency	he expect oution wit 0 8.076	ed freque h mean 1. 1 12.921	ncies, cor 6. 2 10.337	3 5.513	lecimal pl	aces, for	a period of $\geqslant 6$
lowing table shows some of t weeks using a Poisson distrib Number of repairs in a week Expected frequency	he expect oution wit 0 8.076	ed freque h mean 1. 1 12.921	ncies, cor 6. 2 10.337	3 5.513	lecimal pl	aces, for	a period of $\geqslant 6$
llowing table shows some of to weeks using a Poisson distribution. Number of repairs in a week	he expect oution wit 0 8.076	ed freque h mean 1. 1 12.921	ncies, cor 6. 2 10.337	3 5.513	lecimal pl	aces, for	a period of $\geqslant 6$

(iii)	Carry out a goodness of fit test of a Poisson distribution with mean 1.6, using a 10% significance level. [8]

Additional Page

If you use the following fined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.	

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.