

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS General Certificate of Education Ordinary Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

8 1 8 7 0 8 9 1 0 6

ADDITIONAL MATHEMATICS

4037/21

Paper 2 May/June 2012

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 80.

For Examiner's Use			
1			
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3			
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10			
11			
12			
Total			

This document consists of 16 printed pages.



Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where *n* is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\csc^2 A = 1 + \cot^2 A$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2}bc \sin A$$

1	(i)	Given that $\mathbf{A} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$	$\begin{bmatrix} 1 & -3 \\ 2 & 5 \end{bmatrix}$, find the inverse matrix \mathbf{A}^{-1} .
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(ii) Use your answer to part (i) to solve the simultaneous equations

$$4x - 3y = -10,$$

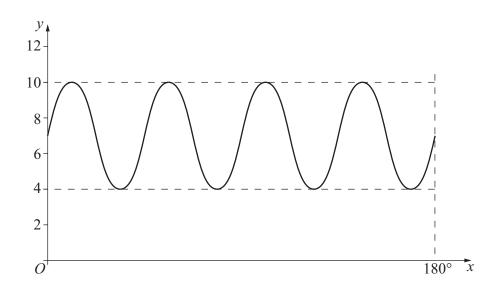
2x + 5y = 21. [2]

A cuboid has a square base of side $(2 + \sqrt{3})$ cm and a volume of $(16 + 9\sqrt{3})$ cm³. Without using a calculator, find the height of the cuboid in the form $(a + b\sqrt{3})$ cm, where a and b are integers. [4]

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3 (a)

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The diagram shows a sketch of the curve $y = a\sin(bx) + c$ for $0^{\circ} \le x \le 180^{\circ}$. Find the values of a, b and c.

(b) Given that $f(x) = 5\cos 3x + 1$, for all x, state

(i) the period of f,

[1]

(ii) the amplitude of f.

[1]

4 (i) Find $\frac{d}{dx}(x^2 \ln x)$.

[2] For Examiner's Use

(ii) Hence, or otherwise, find $\int x \ln x \, dx$.

[3]

5 (a) Solve the equation $3^{2x} = 1000$, giving your answer to 2 decimal places.

For Examiner's Use

[2]

(b) Solve the equation
$$\frac{36^{2y-5}}{6^{3y}} = \frac{6^{2y-1}}{216^{y+6}}$$
.

[4]

6 By shading the Venn diagrams below, investigate whether each of the following statements is true or false. State your conclusions clearly.

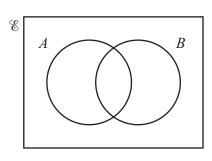
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[2]

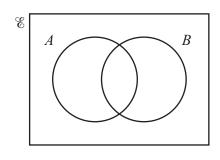
[2]

[2]

(i) $A \cap B' = (A' \cap B)'$

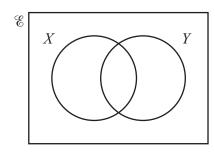


 $A \cap B'$

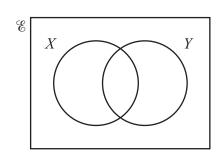


 $(A' \cap B)'$

(ii) $X \cap Y = X' \cup Y'$

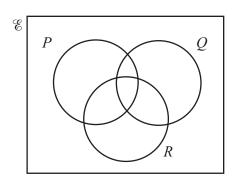


 $X \cap Y$

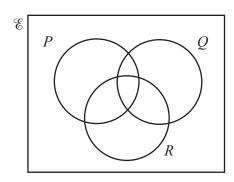


 $X' \cup Y'$

(iii) $(P \cap Q) \cup (Q \cap R) = Q \cap (P \cup R)$



 $(P \cap Q) \cup (Q \cap R)$



 $Q \cap (P \cup R)$

7 Given that $f(x) = x^2 - \frac{648}{\sqrt{x}}$, find the value of x for which f''(x) = 0.

[6] For Examiner's Use

8	Relative to an origin O, the position vectors of the points A and B are $2\mathbf{i} - 3\mathbf{j}$ and $11\mathbf{i} + 42\mathbf{j}$
	respectively.

For Examiner's Use

(i) Write down an expression for \overrightarrow{AB} .

[2]

The point *C* lies on *AB* such that $\overrightarrow{AC} = \frac{1}{3} \overrightarrow{AB}$.

(ii) Find the length of \overrightarrow{OC} .

[4]

The point D lies on \overrightarrow{OA} such that \overrightarrow{DC} is parallel to \overrightarrow{OB} .

(iii) Find the position vector of D.

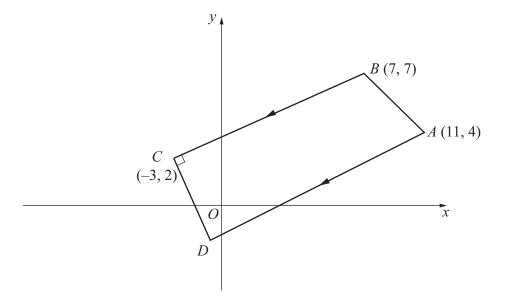
[2]

A particle moves in a straight line so that, ts after passing through a fixed point O, its velocity, $v \,\text{ms}^{-1}$, is given by $v = 2t - 11 + \frac{6}{t+1}$. Find the acceleration of the particle when it is at instantaneous rest.

For Examiner's Use

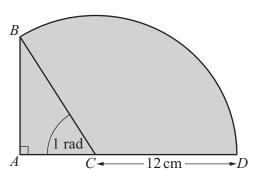
10 Solutions to this question by accurate drawing will not be accepted.





The diagram shows a trapezium ABCD with vertices A(11, 4), B(7, 7), C(-3, 2) and D. The side AD is parallel to BC and the side CD is perpendicular to BC. Find the area of the trapezium ABCD.

11



For Examiner's Use

The diagram shows a right-angled triangle ABC and a sector CBDC of a circle with centre C and radius $12 \, \text{cm}$. Angle ACB = 1 radian and ACD is a straight line.

(i) Show that the length of AB is approximately 10.1 cm.

[1]

(ii) Find the perimeter of the shaded region.

[5]

(iii) Find the area of the shaded region.

[4]

For Examiner's Use

12 Answer only one of the following two alternatives.						
EITHER						
	The equation of a curve is $y = 2x^2 - 20x + 37$.					
	(i)	Express y in the form $a(x + b)^2 + c$, wh	here a, b and c	are integers. [3]		
((ii)	Write down the coordinates of the stati	ionary point on	n the curve. [1]		
	A fu	function f is defined by $f: x \mapsto 2x^2 - 20$	0x + 37 for x >	> k . Given that the function $f^{-1}(x)$ exists,		
(i	iii)	write down the least possible value of	k,	[1]		
(iv)	sketch the graphs of $y = f(x)$ and $y = f^{-1}$	1(x) on the axe	es provided, [2]		
((v)	obtain an expression for f^{-1} .		[3]		
	OR					
	A fu be v	function g is defined by $g: x \mapsto 5x^2 + y$ written as $g: x \mapsto 5(x-4)^2 + q$.	px + 72, where	re p is a constant. The function can also		
	(i)	Find the value of p and of q .		[3]		
((ii)	Find the range of the function g.		[1]		
(i	iii)	Sketch the graph of the function on the	e axes provided	d. [2]		
(iv)	Given that the function h is defined by $gh(x) = 12$.	$h: x \mapsto \ln x, v$	where $x > 0$, solve the equation [4]		
Start	you	r answer to Question 12 here.				
In dia		which associan was an anasymin a	EITHER			
maic	cate	which question you are answering.	OR			
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