Example Candidate Responses
Paper 2

Cambridge O Level
Mathematics (Syllabus D) 4024

For examination from 2018

In order to help us develop the highest quality resources, we are undertaking a continuous programme of review; not only to measure the success of our resources but also to highlight areas for improvement and to identify new development needs.

We invite you to complete our survey by visiting the website below. Your comments on the quality and relevance of our resources are very important to us.

www.surveymonkey.co.uk/r/GL6ZNJB

Would you like to become a Cambridge International consultant and help us develop support materials?

Please follow the link below to register your interest.

www.cambridgeinternational.org/cambridge-for/teachers/teacherconsultants/

Contents

Introduction

The main aim of this booklet is to exemplify standards for those teaching Cambridge O Level Mathematics (Syllabus D) 4024, and to show how candidates' performance (high, middle, low) relate to the subject's curriculum and assessment objectives.

In this booklet candidate responses have been chosen from June 2018 scripts to exemplify a range of answers.

For each question, the response is annotated with a clear explanation of where and why marks were awarded or omitted. This is followed by examiner comments on how the answer could have been improved. In this way, it is possible for you to understand what candidates have done to gain their marks and what they could do to improve their answers. There is also a list of common mistakes candidates made in their answers for each question.

This document provides illustrative examples of candidate work with examiner commentary. These help teachers to assess the standard required to achieve marks beyond the guidance of the mark scheme. Therefore, in some circumstances, such as where exact answers are required, there will not be much comment.

The questions and mark schemes and pre-release material used here are available to download from the School Support Hub. These files are:

June 2018 Question Paper 21

June 2018 Paper 21 Mark Scheme

Past exam resources and other teacher support materials are available on the School Support Hub:

www.cambridgeinternational.org/support

How to use this booklet

This booklet goes through the paper one question at a time, showing you the high level response for each question. The candidate answers are set in a table. In the left-hand column are the candidate answers, and in the right-hand column are the examiner comments.

Example Candidate Response – Question 1, high	Examiner comments
1 (a) Evaluate $\frac{4}{11} - \frac{2}{7}$. $\frac{4 \times 7}{11 \times 7} - \frac{2 \times 11}{7 \times 11}$ ① $\frac{28 - 22}{77} = \frac{6}{77}$ ② (b) Evaluate 0.9×0.011. 0.011 $\times \; 0.9$ ③ $7 \times 11 = 77$ Answer .... $\frac{6}{77}$ .... [1]	① The candidate shows a clear method to write both fractions with a common denominator. ② They subtract the numerators to reach the correct answer. Mark for (a) = 1 out of 1 ③ They perform a long multiplication to reach the correct

Answers are by real candidates in exam conditions. These show you the types of answers for each level. Discuss and analyse the answers with your learners in the classroom to improve their skills.

Examiner comments are alongside the answers. These explain where and why marks were awarded. This helps you to interpret the standard of Cambridge exams so you can help your learners to refine their exam technique.

How the candidate could have improved their answer

The candidate could have written the two equivalent fractions as the first step. It was not necessary to show the multiplications leading to these fractions.

This section explains how the candidate could have improved each answer. This helps you to interpret the standard of Cambridge exams and helps your learners to refine their exam technique.

Common mistakes candidates made in this question

They did not write the fractions with a common denominator and simply subtracted the numerators and denominators separately leading to an answer of 2/4.

Often candidates were not awarded marks because they misread or misinterpreted the questions.
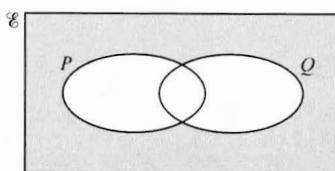
Lists the common mistakes candidates made in answering each question. This will help your learners to avoid these mistakes and give them the best chance of achieving the available marks.

Question 1

Example Candidate Response – high	Examiner comments

1 (a) Use set notation to describe the shaded region in the Venn diagram.

$\mathscr{E}$

P   Q

Answer $(P \cup Q)'$ [1]

(b) $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
     $A = \{x : x \text{ is a factor of 12}\}$ 1,2,3,4,6,12
     $B = \{x : x \text{ is a multiple of 2}\}$ 2,4,6,8,10,12
     $C = \{x : x \text{ is a square number}\}$ 1,4,9

(i) Show this information on the Venn diagram below.

$\mathscr{E}$

A   B

3   2,6,12   8,10
        4
    1
        9
C
    5,7,8,11
U=1,2,3,4,5,6,7,8,9,10,11,12

[2]

(ii) Find $n(A \cap B)$.

Answer   4 [1]

(iii) Find $n(A \cap (B \cup C)')$.

Answer   1 [1]

(iv) One subset in the Venn diagram in **part (b)(i)** has no elements.

Use set notation to describe this subset.

$n(B \cap C) = 0$

Answer $n(B \cap C) = 0$ [1]

(1) The candidate confuses giving the number of elements in a set with describing the set using set notation.
Mark for (a) = 1 out of 1
Mark for (b)(i) = 2 out of 2
Mark for (b)(ii) = 1 out of 1
Mark for (b)(iii) = 1 out of 1
Mark for (b)(iv) = 0 out of 1

Example Candidate Response – high, continued	Examiner comments
(c) (i) Write 540 as the product of its prime factors. 540 $\begin{array}{r|r} 2 & 540 \\ 2 & 270 \\ 3 & 135 \\ 3 & 45 \\ 3 & 15 \\ & 5 \end{array} = 2^2 \times 3^3 \times 5$ Answer $2^2 \times 3^3 \times 5$ .... [2] (ii) p is the smallest possible integer such that $540p$ is a square number. Find $\sqrt{540p}$, giving your answer as the product of its prime factors. $2^2 \times 3^3$ Answer ②$2^2 \times 3^3$ .... [2]	Mark for (c)(i) = 2 out of 2 ② The candidate doesn't connect the previous answer with this part and they do not show that it is necessary to have all even numbers for the indices of a square number. Mark for (c)(ii) = 0 out of 2 **Total mark awarded = 7 out of 10**
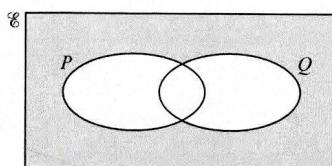
How the candidate could have improved their answer

- **(b)(iv)** The candidate should have described the required subset, instead of giving the number of elements in a set.
- **(c)(ii)** The answer would have been improved by connecting the previous answer with this part question and the candidate should have known that it was necessary to have all even powers for a square number, written as a product of its prime factors.

Example Candidate Response – middle	Examiner comments

1 (a) Use set notation to describe the shaded region in the Venn diagram.
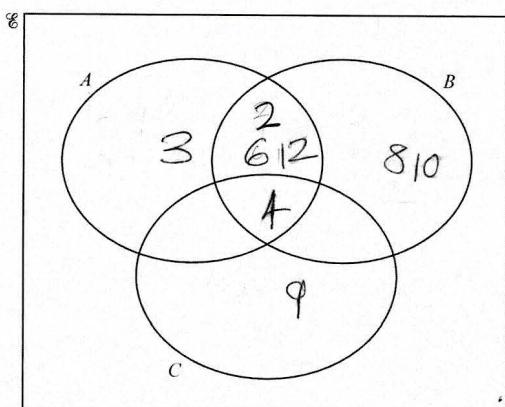
Answer ......$(P \cup Q)'$...... [1]

Mark for (a) = 1 out of 1

**(b)** $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
$A = \{x : x \text{ is a factor of } 12\}$
$B = \{x : x \text{ is a multiple of } 2\}$
$C = \{x : x \text{ is a square number}\}$

(i) Show this information on the Venn diagram below.

[2]

Mark for (b)(i) = 1 out of 2

(ii) Find $n(A \cap B)$.

Answer ......①3...... [1]

(iii) Find $n(A \cap (B \cup C)')$.

Answer ......②0...... [1]

(iv) One subset in the Venn diagram in **part (b)(i)** has no elements.

Use set notation to describe this subset.

Answer ......③$(A \cap C)$...... [1]

① The candidate shows all the correct elements in the required subset, but only gives the answer 3 instead of 4.
Mark for (b)(ii) = 0 out of 1

② The candidate shows the correct element, 3, in the correct subset on the diagram, but gives the incorrect answer of 0 instead of 1.
Mark for (b)(iii) = 0 out of 1

③ The answer is incorrect. It is possible however, to describe an acceptable Follow Through alternative subset e.g. $A \cap C \cap B'$.
Mark for (b)(iv) = 0 out of 1

Example Candidate Response – middle, continued	Examiner comments
(c) (i) Write 540 as the product of its prime factors. $540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$ $= 2^2 \times 3^3 \times 5^1$ Answer $2^2 \times 3^3 \times 5$ [2]	Mark for (c)(i) = 2 out of 2
(ii) p is the smallest possible integer such that $540p$ is a square number. Find $\sqrt{540p}$, giving your answer as the product of its prime factors. $540 = 2^2 \times 3^3 \times 5$ $540p = 2^2 \times 3^2 \times 3^2 \times 5^2$ $\sqrt{540p} = (2^2 \times 3^8 \times 3^2 \times 5)^{2 \times \frac{1}{2}}$ $= 2 \times 3 \times 3 \times 5$ $= 9$ Answer $2 \times 3 \times 3 \times 5$ [2]	Mark for (c)(ii) = 2 out of 2 **Total mark awarded = 6 out of 10**
(a) Sami invests $2000 in an account paying compound interest at a rate of 1.8% per year. Calculate the **total interest** paid to Sami after 3 years. Compound Interest $= 2000 \times \left(1 + \frac{1.8}{100}\right)^3$ $= 2000 \times (1 + 0.018)^3$ $= 2000 \times (1.018)^3$ $= 2000 \times 1.054977832$ $= 2109.955664$ $= \$2109.96$ Answer $ 2109.96 [3]	
(b) Theresa takes out a loan. She repays the loan over one year at a rate of $54 per month. The total she repays is 8% greater than the value of the original loan. Work out the value of the original loan. $100\% = \$54 \times 12$ months $= \$648$ $108\% = ?$ more $\frac{108 \times 648}{100} = 699.84$ Answer $ 699.84 [3]	
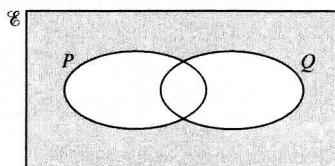
How the candidate could have improved their answer

- **(b)(ii)** The candidate had all the correct elements in the subset, but gave answer as 3 not 4. They might have only counted the elements 2, 6 and 12 as being the correct required elements and disregarded the '4'.
- **(b)(iii)** The candidate should have given 1 as the answer. It is possible that they mistakenly thought, that the given set notation, they had to identify either of the two empty subsets on his diagram.
- **(b)(iv)** The correct subset was not identified. The candidate could have given an alternative answer of $A \cap C \cap B'$, which would have identified an empty subset, for his diagram.

Example Candidate Response – low | Examiner comments
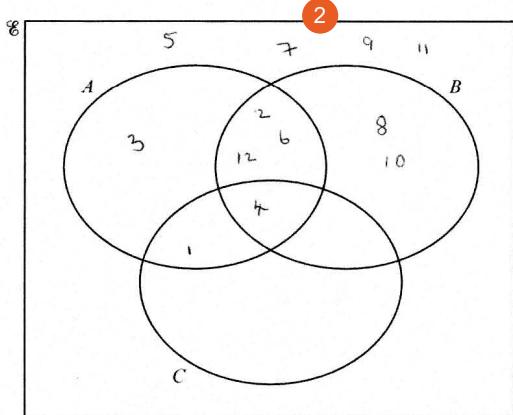
1 (a) Use set notation to describe the shaded region in the Venn diagram.

𝓔

P   Q

Answer ......$P \cap Q$............... [1] ①

① The candidate does not give the correct answer of $(P \cup Q)'$ or $P' \cap Q'$.
Mark for (a) = 0 out of 1

(b) 𝓔 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
$A = \{x : x$ is a factor of 12$\}$ 1, 2, 3, 4, 6, 12
$B = \{x : x$ is a multiple of 2$\}$ 2, 4, 6, 8, 10, 12
$C = \{x : x$ is a square number$\}$ 4, 1

(i) Show this information on the Venn diagram below.

②

𝓔
5   7   9   11
A   B
    2
  3   6   8
    12   10
        4
    1
C

[2]

② Nine has been put in the incorrect subset.
Mark for (b)(i) = 1 out of 2
Mark for (b)(ii) = 1 out of 1

(ii) Find n(A ∩ B).

Answer ....4.......... [1]

(iii) Find n(A ∩ (B ∪ C)').

Answer ....4..........③ [1]

③ The element, 3, has been put in the correct position on the Venn diagram, but the wrong answer of 4 instead of 1 has been given.
Mark for (b)(iii) = 0 out of 1

(iv) One subset in the Venn diagram in **part (b)(i)** has no elements.

Use set notation to describe this subset.

Answer ....∅..........④ [1]

④ The candidate gives the symbol for the empty set, instead of describing the required subset, using set notation.
Mark for (b)(iv) = 0 out of 1

Example Candidate Response – low, continued	Examiner comments

(c) (i) Write 540 as the product of its prime factors.

```
5 8 | 5 4 0
4   | 1 0 8
3   | 2 7
3   | 9
3   | 3
    | 1
```

⑤

Answer   5 × 4 × 3 × 3 × 3 ... [2]

(ii) *p* is the smallest possible integer such that 540*p* is a square number.

Find $\sqrt{540p}$, giving your answer as the product of its prime factors.

$$\sqrt{540p} = \frac{1}{(540p)^2}$$

⑥ $=$

Answer .... [2]

! (a) Sami invests $2000 in an account paying compound interest at a rate of 1.8% per year.

Calculate the **total interest** paid to Sami after 3 years.

$1.8\% \times 2000 \times 3 y s$

$= \frac{18}{100} \times \frac{2000}{1} \times \frac{3}{1}$

$= \frac{10\,899}{100} = \$108$

Answer $ ....1.0.8............ [3]

(b) Theresa takes out a loan.
She repays the loan over one year at a rate of $54 per month.
The total she repays is 8% greater than the value of the original loan.

Work out the value of the original loan.

$\$54 \times 12 = \$648$

$8\% \text{ of } 648$

$= \frac{8}{100} \times \frac{648}{1}$

$= 51,84$

$\therefore \quad 648 - 51,840$

$= \$596,16$

Answer $ ......5.9.6.1.6.............. [3]

⑤ The candidate needs to show the factors of 4, as 2 × 2.
Mark for (c)(i) = 1 out of 2

⑥ The candidate does not recognise that this part question follows on from the previous part and that the indices of a square number must all be even numbers.
Mark for (c)(ii) = 0 out of 2

Total mark awarded = 3 out of 10

How the candidate could have improved their answer

- **(a)** The candidate needed to give correct answer of (*P* ∪ *Q*)′ or *P*′ ∩ *Q*′.
- **(b)(i)** The candidate should have put the element '9', in the correct position on the Venn diagram.
  (iii) The correct answer was 1, as the single element, '3', had been put in the correct subset on the Venn diagram.
  (iv) The candidate puts the empty set, ∅ , instead of *A*′ ∩ *B* ∩ *C*.
- **(c)(i)** The correct answer was 2 × 2 instead of 4.
  (ii) The candidate did not see the connection with previous part, which was needed here.

Common mistakes candidates made in this question

Candidates confused finding the number of elements in a subset with listing all the elements in that particular subset.

Question 2

Example Candidate Response – high	Examiner comments
2 **(a)** Sami invests $2000 in an account paying compound interest at a rate of 1.8% per year. Calculate the **total interest** paid to Sami after 3 years. $$= 2000\left(1 + \frac{1.8}{100}\right)^3$$ $$= 2000 * 1.055$$ $$= 2110 \quad \textbf{①}$$ Answer $ ....2110.............. [3]	**①** The candidate correctly finds the amount after the 3 years, but now needs to subtract the original sum invested in order to obtain the total interest paid. Mark for (a) = 2 out of 3
(b) Theresa takes out a loan. She repays the loan over one year at a rate of $54 per month. The total she repays is 8% greater than the value of the original loan. Work out the value of the original loan. $$= 54 \times 12$$ $$= 648$$ $$648 = 108$$ $$x = 100$$ $$x = \frac{648 \times 100}{108}$$ $$= 600$$ Answer $ .....600.............. [3]	Mark for (b) = 3 out of 3 **Total mark awarded = 5 out of 6**

How the candidate could have improved their answer

(a) The candidate correctly found the amount $2110, but did not subtract the original investment of $2000 to obtain the total interest paid.

Example Candidate Response – middle	Examiner comments
2 (a) Sami invests $2000 in an account paying compound interest at a rate of 1.8% per year. Calculate the **total interest** paid to Sami after 3 years. $A\left(1+\frac{R}{100}\right)^{years}$ $2000\left(1+\frac{1\cdot8}{100}\right)^3 = 2000\left(1\cdot018\right)^3 = 2000\times1\cdot05498$ $= 2109\cdot96$ 2109.96 − 2 35·F$2110 Answer $ ...2H....................... [3]	**1** The candidate correctly finds the amount after the 3 years, but now needs to subtract the original sum invested, in order to obtain the total interest paid. Mark for (a) = 2 out of 3
(b) Theresa takes out a loan. She repays the loan over one year at a rate of $54 per month. The total she repays is 8% greater than the value of the original loan. Work out the value of the original loan. $54\times12 = \$648$ $x+\frac{8}{100}x = 648+x$ $x+\frac{8x}{100} = 648+x \quad 8x = 64800+100x$ $x+\frac{8x}{100} = 648+x \quad \frac{8x}{100}=648 \quad 8x=64800$ $x+8x-x=648 \quad \frac{8x}{100}=648 \quad x=\frac{64800}{8}=8100$ Answer $ .....8100.................... [3]	**2** The candidate correctly finds the total amount repaid, $648. **3** An incorrect equation has been given here. It should be: $x + \frac{8x}{100} = 648$. Mark for (b) = 1 out of 3 **Total mark awarded = 3 out of 6**

How the candidate could have improved their answer

- **(a)** The candidate did not subtract the original investment from the amount, to obtain the total interest paid.

- **(b)** The candidate correctly obtained $648, but did not know that the original loan was $x = \frac{648}{1.08}$ or equivalent, which was the next step that was needed.

Example Candidate Response – low	**Examiner comments**
2 (a) Sami invests $2000 in an account paying compound interest at a rate of 1.8% per year. Calculate the **total interest** paid to Sami after 3 years. $$\frac{PRT}{100} = \frac{2000 \times 1.8 \times 3}{100}$$ ① $$= 108$$ Answer $ ...108........ [3]	① The candidate is incorrectly using the Simple Interest formula and not the Compound Interest formula. Mark for (a) = 0 out of 3
(b) Theresa takes out a loan. She repays the loan over one year at a rate of $54 per month. The total she repays is 8% greater than the value of the original loan. Work out the value of the original loan. 54×12 $= 648$ ② $8\% \times 648 = 51.84$ 596.16 $648 - 51.84 = 59.16$ ③ Answer $ .... [3]	② The candidate correctly finds the total amount repaid, $648. ③ The candidate should now use the calculation: original loan = $\left(\dfrac{648}{108}\right) \times 100$. Mark for (b) = 1 out of 3 **Total mark awarded = 1 out of 6**

How the candidate could have improved their answer

- **(a)** The Simple Interest formula was incorrectly used, instead of using Compound Interest.
- **(b)** The candidate was able to obtain $648, but did not know that $x = \dfrac{648}{1.08}$ was the next step that was required.

Common mistakes candidates made in this question

- **(a)** Some candidates did not subtract the original investment of $2000 from the new amount of $2110, in order to obtain the total interest paid.
- **(b)** Some candidates made the error of finding 8% of $648 and then subtracting this amount from $648 to obtain their answer.

Question 3

Example Candidate Response – high	Examiner comments

3 (a) Solve $4(p-3) = 2p+7$.

$4p - 12 = 2p + 7$

$4p - 2p = 12 + 7$

$2p = 19$

$p = {}^{19}\!/\!_2 = 9.5$

Answer $p =$ ....9.5........ [2]

Mark for (a) = 2 out of 2

(b) Solve these simultaneous equations.

$2x - y = 5$
$7x + 2y = 1$

Show your working.

$2x - y = 5$ ---①
$7x + 2y = 1$ ---②

from ①
$2x - y = 5$
$y = 2x - 5$ ---③

put ③ into ②

$7x + 2y = 1$

$7x + 2(2x - 5) = 1$

$7x + 4x - 10 = 1$

$\sim 11x = 10 + 1$

$11x = 11$

$x = 1$

$2x - y = 5$

$2(1) - y = 5$

$2 - y = 5$

$2 - 5 = y$

$-3 = y$

$y = -3$

Answer $x =$ ....1........

$y =$ ....-3........ [3]

Mark for (b) = 3 out of 3

Example Candidate Response – high, continued	Examiner comments

(c) Simplify $\dfrac{m^2 + 3m}{2m^2 + 5m - 3}$.

$$\frac{m^2 + 3m}{2m^2 + 5m - 3}$$

$$\frac{m(m+3)}{m(2m+5) - 3} \quad \textbf{①}$$

$$\frac{m(m+3)}{m\left(2m+5 - \frac{3}{m}\right)}$$

$$\frac{m+3}{2m+5 - \frac{3}{m}}$$

Answer $\dfrac{m+3}{2m+5 - 3/m}$ [3]

① The candidate factorises the numerator correctly.

② The candidate needs to factorise the denominator correctly as (2m – 1)(m + 3) in order to be able to progress to the final answer.
Mark for (c) = 1 out of 3

(d) b is directly proportional to the cube of a.

Given that $b = 4$ when $a = 2$, find b when $a = 5$.

$$b \propto a^3$$
$$b = Ka^3$$
$$4 = K(2^3)$$
$$4 = 8K$$
$$\frac{4}{8} = K$$
$$K = \frac{4}{8} = \frac{1}{2}$$

$$b = Ka^3$$
$$b = \frac{1}{2} \times 5^3$$
$$b = \frac{5^3}{2}$$
$$b = \frac{125}{2} = 62.5$$

Answer $b = $ 62.5 [3]

Mark for (d) = 3 out of 3

Total mark awarded = 9 out of 11

How the candidate could have improved their answer

(a) The candidate was able to factorise the numerator, but was unable to factorise the quadratic expression and hence could not proceed to cancel the required terms, in order to obtain the final answer.

Example Candidate Response – middle	Examiner comments
3 **(a)** Solve $4(p-3) = 2p + 7$. $4(p-3) = 2p+7$ $4p - 12 = 2p+7$ $4p - 2p = 7 + 12$ $\dfrac{2p}{2} = \dfrac{19}{2}$ Answer $p = $ .....9,5................ [2]	Mark for (a) = 2 out of 2
(b) Solve these simultaneous equations. $2x - y = 5$ $7x + 2y = 1$ Show your working. $2x - y = 5$ (i) $7x + 2y = 1$ (ii) $y = 2x - 5$ in ① $7x + 2(2x-5) = 1$ $7x + 4x - 10 = 1$ $\dfrac{11x}{11} = \dfrac{11}{11}$ $x = 1$ in (i) $2(1) - y = 5$ $2 - y = 5$ $-3 = y$ Answer $x = $ ...1................................. $y = $ ...-3................ [3]	Mark for (b) = 3 out of 3

Example Candidate Response – middle, continued	Examiner comments

(c) Simplify $\dfrac{m^2 + 3m}{2m^2 + 5m - 3}$.

1

$$\frac{m \times m + 3 \times m}{2 \times m \times m + 5 \times m - 3}$$

2

$$\frac{1 + 3}{2 + 5 - 3}$$

$$\frac{4}{4}$$

Answer ....1............................... [3]

1 The candidate needs to factorise the numerator as $m(m + 3)$.

2 The candidate needs to factorise the denominator as $(2m - 1)(m + 3)$.
Mark for (c) = 0 out of 3

(d) b is directly proportional to the cube of a.

Given that $b = 4$ when $a = 2$, find b when $a = 5$.

$$b \propto a^3$$
$$b = ka^3$$
3
$$4 = k2^3$$
$$\frac{4}{8} = \frac{8k}{8}$$
$$k = 2 \qquad b \propto 2a^3$$
$$b = 2(5)^3$$
$$b = 250$$

4

Answer $b =$ ~~125~~ 250 [3]

3 The correct equation of direct proportionality has been used.

4 The candidate incorrectly evaluates the constant of proportionality as 2, instead of $\frac{1}{2}$.
Mark for (d) = 1 out of 3

Total mark awarded = 6 out of 11

How the candidate could have improved their answer

- **(c)** The candidate did not show any understanding of what was required in factorising, either the numerator or denominator.
- **(a)** The candidate correctly used $b = ka^3$, but obtained $k = 2$ instead of $k = \frac{1}{2}$, from incorrect working. The candidate should have reached the equation of proportionality, $b = \frac{1}{2}(5)^3$ next and from there obtained the correct final answer.

Example Candidate Response – low	**Examiner comments**

3 (a) Solve $4(p-3) = 2p+7$.

$$4p - 12 = 2p + 7$$

$$\frac{2p}{2} = \frac{19^{9 \cdot 5}}{2}$$

$$p = 9 \cdot 5$$

Answer $p =$ .... $9 \cdot 5$ .... [2]

Mark for (a) = 2 out of 2

(b) Solve these simultaneous equations.

$$2x - y = 5$$
$$7x + 2y = 1$$

Show your working.

$$7 \times 2x - y = 5 \times 7$$
$$2 \times 7x + 2y = 1 \times 2$$
$$14x - 7y = 35$$
$$\underline{14x + 4y = 2}$$

① $\quad = 11y = 33$

$$\frac{}{11} \quad \frac{}{11}$$

$$y = 3$$
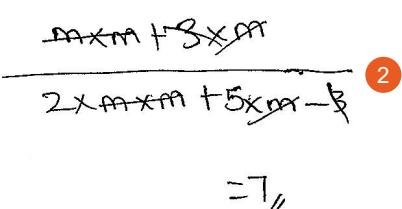
$$4 \times 2x - y = 5 \times 4$$
$$2 \times 7x + 2y = 1 \times 2$$
$$8x - 4y = 20$$
$$\underline{14x + 4y = 2}$$
$$-6x + 0 = 18$$
$$\frac{-6x}{-6} = \frac{18}{-6}$$
$$x = 3$$

① The correct method of elimination has been used, but makes arithmetic error by obtaining
$y = 3$ and not $y = -3$.
Mark for (b) = 1 out of 3

Answer $x =$ .... 3 ....

$y =$ .... 3 .... [3]

Example Candidate Response – low, continued	Examiner comments

(c) Simplify $\dfrac{m^2 + 3m}{2m^2 + 5m - 3}$.

$$\frac{m \times m + 3 \times m}{2 \times m \times m + 5 \times m - 3}$$

②

$$= 7$$

② The candidate is not showing any correct factorisation. It needs to show numerator is $m(m + 3)$ and denominator is $(2m - 1)(m + 3)$.
Mark for (c) = 0 out of 3

Answer 7 [3]

(d) b is directly proportional to the cube of a.

Given that $b = 4$ when $a = 2$, find b when $a = 5$.

③ $b \propto a^3$

$4 \propto 2^3$

$b \propto 5^3$

$b \propto 125$

③ This is not sufficient. The candidate needs to write the equation of proportionality here, $b = ka^3$.
Mark for (d) = 0 out of 3

Total mark awarded = 3 out of 11

Answer $b =$ 7 [3]

How the candidate could have improved their answer

- **(b)** The candidate used the correct method of elimination in reaching $-11y = 33$, but then lost accuracy and gave $y = 3$, so was unable to obtain either of the two correct values for x or y, thus no further marks were earned.
- **(a)** The candidate did not show any correct factorising of either the numerator of the denominator, so no marks could be awarded.
- **(b)** The equation of proportionality should have been written, $b = ka^3$, not just $b \propto a^3$, in order to be awarded marks.

Common mistakes candidates made in this question

(a) Some candidates did not always read the question carefully enough. Some gave answers that referred to $b = \dfrac{1}{a^3}$ or $b = \sqrt[3]{a}$

Question 4

Example Candidate Response – high	Examiner comments

4

| T | R | I | G | O | N | O | M | E | T | R | Y |

Twelve lettered tiles spelling the word TRIGONOMETRY are placed inside a bag.

(a) A tile is taken at random from the bag.

Find the probability that the tile shows a letter R.
Give your answer as a fraction in its simplest form.

No. of Rs = 2   Total tiles = 12

Probability = $\frac{2}{12}^{1}_{6}$ = $\frac{1}{6}$

Answer .... 1/6 .... [1]

Mark for (a) = 1 out of 1

(b) All the tiles are placed back in the bag, a tile is then taken at random and placed on the table.
A second tile is taken at random and placed to the right of the first tile.
A third tile is taken at random and placed to the right of the second tile.

1st   2nd   3rd

☐   ☐   ☐

Find the probability that, in the order the tiles were placed on the table, they spell GET.

Probability is without replacement

G = $\frac{1}{12}$ E = $\frac{1}{11}$ T = $\frac{2}{10}$ = $\frac{1}{5}$

Probability of GET = $\frac{1}{12} \times \frac{1}{11} \times \frac{1}{5}$ = $\frac{1}{660}$

Answer .... 1/660 .... [2]

Mark for (b) = 2 out of 2
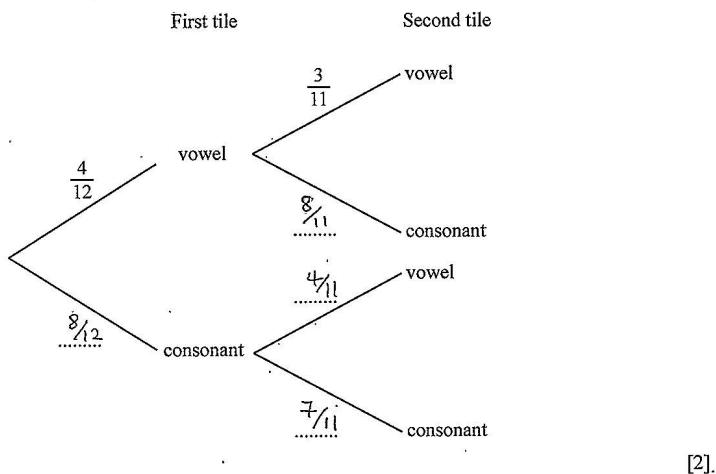
Example Candidate Response – high, continued	Examiner comments
(c) Vowels are the letters A, E, I, O and U. All other letters are consonants. All the twelve tiles are placed back in the bag and two tiles are taken at random, without replacement. **(i)** Complete the tree diagram. First tile Second tile $\frac{4}{12}$ vowel $\frac{3}{11}$ vowel $\frac{8}{11}$ consonant $\frac{8}{12}$ consonant $\frac{4}{11}$ vowel $\frac{7}{11}$ consonant [2]	Mark for (c)(i) = 2 out of 2
(ii) Find the probability that the tiles both show vowels. $\frac{4}{12} \times \frac{3}{11} = \frac{1}{11}$ Answer $\frac{1}{11}$ [1]	Mark for (c)(ii) =1 out of 1
(iii) Find the probability that one tile shows a vowel and one tile shows a consonant. $\frac{4}{12} \times \frac{8}{11} = \frac{8}{33}$ ① Answer $8/33$ [2]	① The candidate is only using one out of the two possible routes. Mark for (c)(iii) = 1 out of 2 **Total mark awarded = 7 out of 8**

How the candidate could have improved their answer

(c)(iii) The candidate only used the one route on the tree diagram and needed to use the other route as well. So the calculation $\left(\frac{4}{12} \times \frac{8}{11}\right) + \left(\frac{8}{12} \times \frac{4}{11}\right)$ was needed.

Example Candidate Response – middle	Examiner comments
4	

T R I G O N O M E T R Y

Twelve lettered tiles spelling the word TRIGONOMETRY are placed inside a bag.

(a) A tile is taken at random from the bag.

Find the probability that the tile shows a letter R.
Give your answer as a fraction in its simplest form.

$$P(R) = \frac{2}{12} \quad \frac{1}{6}$$

$$= \frac{1}{6}$$

Answer .... $\frac{1}{6}$ .... [1]

(b) All the tiles are placed back in the bag, a tile is then taken at random and placed on the table.
A second tile is taken at random and placed to the right of the first tile.
A third tile is taken at random and placed to the right of the second tile.

1st   2nd   3rd

☐ ☐ ☐

Find the probability that, in the order the tiles were placed on the table, they spell GET.

$$P(GET) = \frac{1}{12} \times \frac{1}{11} \times \frac{1}{10} = $$

$$= \frac{1}{1320}$$

Answer .... $\frac{1}{1320}$ .... [2]

**Examiner comments:**

Mark for (a) = 1 out of 1

1 The candidate needs to use the fraction $\frac{2}{10}$ here instead, since there are 2 letter Ts left in the bag.
Mark for (b) = 0 out of 2

Example Candidate Response – middle, continued	Examiner comments

(c) Vowels are the letters A, E, I, O and U.
All other letters are consonants.
All the twelve tiles are placed back in the bag and two tiles are taken at random, without replacement.

(i) Complete the tree diagram.

First tile   Second tile

$\frac{3}{11}$ vowel

vowel

$\frac{4}{12}$

$\frac{8}{11}$ consonant

$\frac{4}{11}$ vowel

$\frac{8}{12}$

consonant

$\frac{7}{11}$ consonant

[2]

Mark for (c)(i) = 2 out of 2

(ii) Find the probability that the tiles both show vowels.

$P(VV) = \frac{4}{12} \times \frac{3}{11}$

$= \frac{4}{44} = \frac{1}{11}$

Answer .... $\frac{1}{11}$ .... [1]

Mark for (c)(ii) = 1 out of 1

(iii) Find the probability that one tile shows a vowel and one tile shows a consonant.

$P(V \& C) = \left(\frac{4}{12} \times \frac{8}{11}\right) + \left(\frac{8}{12} + \frac{7}{11}\right)$

$= \frac{8}{33} + \frac{32}{132} + \frac{56}{132}$

Answer .... $\frac{2}{3}$ .... [2]

$= \frac{32 + 56}{132}$

$= \frac{88}{...}$

2 The candidate correctly shows the product of the two probabilities.

3 The two probabilities have been added here, instead of multiplying them as required and is also using the incorrect fraction of $\frac{7}{11}$.

Mark for (c)(iii) = 1 out of 2

Total mark awarded = 5 out of 8

How the candidate could have improved their answer

- **(b)** The candidate had the correct probabilities for the first two tiles drawn, but should have had $\frac{2}{10}$ for the third.
- **(c)(iii)** The candidate incorrectly used $+ \frac{7}{11}$ in the second pair of probabilities, instead of $\times \frac{4}{11}$.

Example Candidate Response – low	Examiner comments
4	

T R I G O N O M E T R Y

Twelve lettered tiles spelling the word TRIGONOMETRY are placed inside a bag.

(a) A tile is taken at random from the bag.

Find the probability that the tile shows a letter R.
Give your answer as a fraction in its simplest form.

$$\frac{\cancel{2}^{1}}{\cancel{12}_{6}}$$

$$= \frac{1}{6}$$

Answer ....$\frac{1}{6}$........................ [1]

Mark for (a) = 1 out of 1

(b) All the tiles are placed back in the bag, a tile is then taken at random and placed on the table.
A second tile is taken at random and placed to the right of the first tile.
A third tile is taken at random and placed to the right of the second tile.

1st   2nd   3rd

G   E   T

Find the probability that, in the order the tiles were placed on the table, they spell GET.

$$1 - 0,08$$
$$= 0,92$$ ①

① The candidate needs to use the correct product of $\frac{1}{12} \times \frac{1}{11} \times \frac{2}{10}$ here.
Mark for (b) = 0 out of 2

Answer ......$0,92$....................... [2]

Example Candidate Response – low, continued	Examiner comments
(c) Vowels are the letters A, E, I, O and U. All other letters are consonants. All the twelve tiles are placed back in the bag and two tiles are taken at random, without replacement.	

(i) Complete the tree diagram.

First tile   Second tile

$\frac{4}{12}$ — vowel

vowel — $\frac{3}{11}$ — vowel

vowel — $\frac{5}{11}$ — consonant

$\frac{6}{12}$ — consonant

consonant — $\frac{3}{11}$ — vowel

consonant — $\frac{5}{11}$ — consonant

[2]

2 The correct denominators are given for all of the fractions, but all of the numerators are incorrect. Candidates need to remember that the probabilities on a pair of branches, must total 1.
Mark for (c)(i) = 0 out of 2

(ii) Find the probability that the tiles both show vowels.

$$\frac{4^1}{12 \, 4} \times \frac{3^1}{11}$$

$$= \frac{1}{11}$$

Answer ....$\frac{1}{11}$...... [1]

Mark for (c)(ii) = 1 out of 1

(iii) Find the probability that one tile shows a vowel and one tile shows a consonant.

$$= 2\left(\frac{6}{12} \times \frac{3}{11}\right) \times \left(\frac{4}{12} \times \frac{5}{11}\right)$$

$$= \frac{3^1}{22} \times \frac{5}{33}$$

$$= \frac{5}{242}$$

Answer ....$\frac{5}{242}$...... [2]

3 The candidate correctly multiplies the Follow Through probabilities on the tree diagram, which is sufficient for the method here.
Mark for (c)(iii) = 1 out of 2

Total mark awarded = 3 out of 8

How the candidate could have improved their answer

- **(b)** The candidate should have shown the multiplication of the 3 probabilities, $\frac{1}{12} \times \frac{1}{11} \times \frac{2}{10}$.
- **(c)(i)** The tree diagram was completed inaccurately and did not show an understanding that the probabilities on the pairs of branches must total 1.
- **(c)(iii)** The candidate needed to use the pair of probabilities on the other route as well.

Common mistakes candidates made in this question

- **(c)(i)** Candidates need to ensure that a pair of probabilities, on the branches of a probability tree diagram, when added together, must total 1.
- **(c)(iii)** Some candidates did not always identify that there were two possible routes that satisfied the condition for one tile showing a vowel and the other showing a constant.
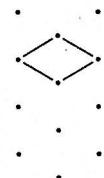
Question 5

| Example Candidate Response – high	Examiner comments
5 (a) 1, 7, 13, 19, 25, … **(i)** Find an expression, in terms of *n*, for the *n*th term of this sequence. Difference= 7−1=6 6n − 5 a−d = 1−6 = −5 *Answer* ....6n−5................ [2] **(ii)** Explain why 251 is not a term in this sequence. 6(43) − 5 = 253 6(42) − 5 = 247 *Answer* 251 is not a term in this sequence because if 6 continues to be added on to the term it will never give 251 [1] **(b)** Here is another sequence. 5, 8, 13, 20, 29, … The *p*th term of this sequence is $p^2 + 4$. Write down an expression, in terms of *p*, for the *p*th term of these sequences. **(i)** −2, 1, 6, 13, 22, … *Answer* ....$p^2 − 3$............ [1] **(ii)** 7, 12, 19, 28, 39, … **1** 3r + 4 n³ + 2n *Answer* .... [1]	Mark for (a)(i) = 2 out of 2 Mark for (a)(ii) =1 out of 1 Mark for (b)(i) =1 out of 1 **1** This is not the answer required, i.e. $p^2 + 2p + 4$. Mark for (b)(ii) =0 out of 1
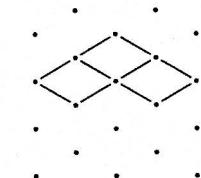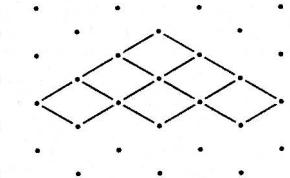
Example Candidate Response – high, continued	Examiner Comments

(c) The diagrams below show the first three patterns in a sequence. The patterns are made from short diagonal lines.

Pattern 1   Pattern 2   Pattern 3

(i) Draw Pattern 4 on the dotty grid below.

[1]

Mark for (c)(i) = 1 out of 1

(ii) Complete the table below for the number of short lines in Patterns 4 and 5.

Pattern	1	2	3	4	5
Number of short lines	4	10	18	28	40

[2]
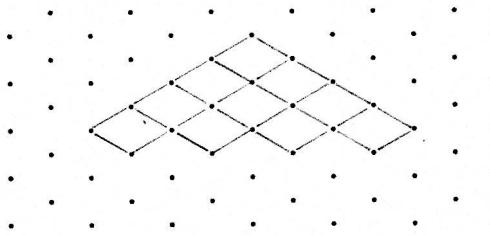
Mark for (c)(ii) = 2 out of 2

(iii) Find an expression, in terms of t, for the number of short lines in Pattern t.

Mark for (c)(iii) = 2 out of 2

Total mark awarded = 9 out of 10

Answer ...... $n^2 + 3n$ ...... [2]

How the candidate could have improved their answer

(b)(ii) The correct answer of $p^2 + 2p + 4$ should have been given.

Example Candidate Response – middle	Examiner comments

5 (a)   1, 7, 13, 19, 25, …

(i) Find an expression, in terms of n, for the nth term of this sequence.

$$n = a + (n-1)6$$

①

Answer $n = a + (n-1)6$ [2]

① The answer is partly correct as it is of the form $6n + k$.
Mark for (a)(i) = 1 out of 2

(ii) Explain why 251 is not a term in this sequence.

Answer It does not follow the order of the sequence. **②** [1]

② The answer is too vague.
Acceptable answers would be:
$6n – 5 = 251$
$6n = 256$
so $n = 42.666$
so n is not a whole number
OR
247 is in the sequence and the next term is 253.
Mark for (a)(ii) = 0 out of 1
Mark for (b)(i) = 1 out of 1
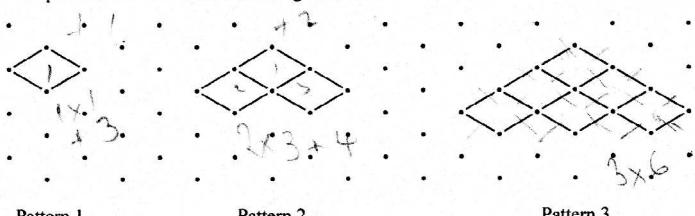
(b) Here is another sequence.

5, 8, 13, 20, 29, …

The pth term of this sequence is $p^2 + 4$.

Write down an expression, in terms of p, for the pth term of these sequences.

(i) –2, 1, 6, 13, 22, …

The pth term of this sequence
$d = $ difference of 3
$= p^2 - 3$

Answer $p^2 - 3$ [1]

(ii) 7, 12, 19, 28, 39, …

$a + (n-1)d$

③

Answer p^2 [1]

③ Although the candidate shows an appreciation that the expression contains p^2, the full answer of $p^2 + 2p + 4$ is required.
Mark for (b)(ii) = 0 out of 1

Example Candidate Response – middle, continued	Examiner comments

(c) The diagrams below show the first three patterns in a sequence. The patterns are made from short diagonal lines.

Pattern 1   Pattern 2   Pattern 3

(i) Draw Pattern 4 on the dotty grid below.

[1]

Mark for (c)(i) = 1 out of 1

(ii) Complete the table below for the number of short lines in Patterns 4 and 5.

Pattern	1	2	3	4	5
Number of short lines	4	10	18	28	40

[2]

Mark for (c)(ii) = 2 out of 2
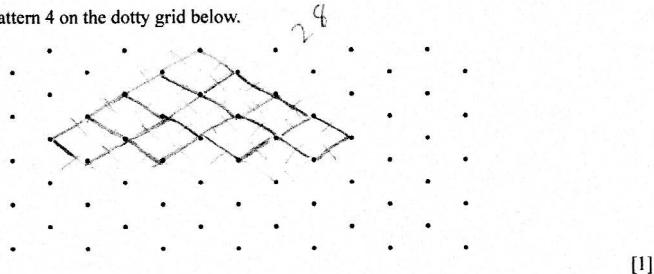
(iii) Find an expression, in terms of t, for the number of short lines in Pattern t.

Pattern $t = (t \times$ number of boxes + even numbers $)$
$= t \times$ the (boxes) + even number

④

Answer .... [2]

④ The candidate is not giving an answer which contains $t^2 + ...$ so does not score any mark.
Mark for (c)(iii) = 0 out of 2

Total mark awarded = 5 out of 10

How the candidate could have improved their answer

- **(a)(i)** The answer contained '6n' so was partly correct, but needed to be 6n – 5 for a fully correct answer.
- **(a)(ii)** An acceptable reason was needed, such as: '256 is not exactly divisible by 6' or '247 is in the sequence and the next one is 253'.
- **(b)(ii)** The candidate did appreciate that the expression included p^2, but needed to give the correct answer of $p^2 + 2p + 4$.
- **(c)(iii)** The candidate did not arrive at an expression containing t^2, so was unable to gain any mark here.

Example Candidate Response – low	Examiner comments

5. **(a)**   1, 7, 13, 19, 25, …

   (i) Find an expression, in terms of n, for the nth term of this sequence.

   ①

   Answer .... $n + 6$ .... [2]

   ① The candidate's answer is not of the form $6n + k$, so does not score a mark.
   Mark for (a)(i) = 0 out of 2

   (ii) Explain why 251 is not a term in this sequence.

   ②
   $n + 6 = 251$
   $n = 251 - 6$

   Answer It does not follow the pattern. $n+6 = 251$; $n = 251-6 = 245$.
   .... [1]

   ② The candidate's equation is incorrect, so further progress cannot be made.
   It should be $6n - 5 = 251$.
   Mark for (a)(ii) = 0 out of 1

   **(b)** Here is another sequence.

                    5, 8, 13, 20, 29, …

   The pth term of this sequence is $p^2 + 4$.

   Write down an expression, in terms of p, for the pth term of these sequences.

   (i) −2, 1, 6, 13, 22, …

   $(p + 2) \times 2$

   $2(p + 2)$

   ③

   Answer ..... $2(p + 2)$ .... [1]
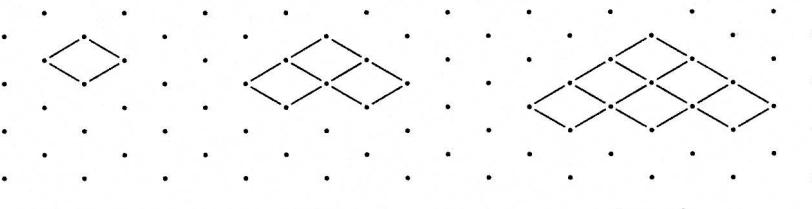
   ③ This is not the correct answer of $p^2 \square 3$.
   Mark for (b)(i) = 0 out of 1

   (ii) 7, 12, 19, 28, 39, …

   ④

   Answer ..... $2p - 2$ .... [1]

   ④ This is not the correct answer of $p^2 + 2p + 4$.
   Mark for (b)(ii) = 0 out of 1

| Example Candidate Response – low, continued	Examiner comments

(c) The diagrams below show the first three patterns in a sequence. The patterns are made from short diagonal lines.

Pattern 1   Pattern 2   Pattern 3

(i) Draw Pattern 4 on the dotty grid below.

[1]
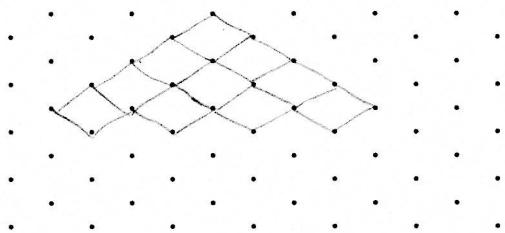
Mark for (c)(i) = 1 out of 1

(ii) Complete the table below for the number of short lines in Patterns 4 and 5.

Pattern	1	2	3	4	5
Number of short lines	4	10	18	28	36

⑤

[2]

⑤ The candidate gives incorrect value of 36, instead of the correct value of 40.
Mark for (c)(ii) = 1 out of 2

(iii) Find an expression, in terms of t, for the number of short lines in Pattern t.

$t = t \times \text{add number}$

$(t + 3) \times 2$

Answer $t + t + \text{++} \ 2$ .... [2]

⑥

⑥ The answer is not of the form $t^2 + ...$ so it is not awarded.
Mark for (c)(iii) = 0 out of 2

Total mark awarded = 2 out of 10

How the candidate could have improved their answer

- **(a)(i)** The sequence was based on the 6 times table, so the answer should have included the term 6n.
- **(a)(ii)** The candidate needed to give an acceptable reason such as: '256 is not exactly divisible by 6' or '247 is in the sequence and the next one is 253'.
- **(b)(i)** The correct answer of $p^2 - 3$ should have been given.
- **(b)(ii)** The correct answer was $p^2 + 2p + 4$.
- **(c)(ii)** The candidate needed to give the answer 40 and not 36, in their table.
- **(c)(iii)** The candidate did not arrive at an expression containing t^2, so was unable to gain any mark here.
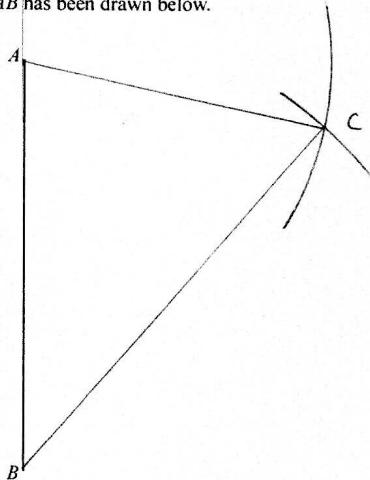
Common mistakes candidates made in this question

Candidates should recognise that they may need to work out the differences between the terms in a sequence twice, before the difference becomes constant and that this then shows that the sequence is based on a quadratic expression.

Question 6

Example Candidate Response – high	Examiner comments

6 (a) *ABC* is a triangle with *AC* = 6 cm and *BC* = 9 cm.
   AB has been drawn below.

(i)   Using a ruler and a pair of compasses only, construct triangle *ABC*.   [2]

(ii) Measure *BÂC* .

1 Answer ....82°.......................... [1]

(b) A rectangular field has dimensions 220 m by 350 m, each correct to the nearest 10 metres.

Calculate the upper bound for the area of the field.

225 × 355

= 79875   Answer ....79875............ m² [2]

Mark for (a)(i) = 2 out of 2

1 The answer is inaccurate and is outside the accepted tolerance for measuring the angle with a protractor.
Mark for (a)(ii) = 0 out of 1
Mark for (b) = 2 out of 2
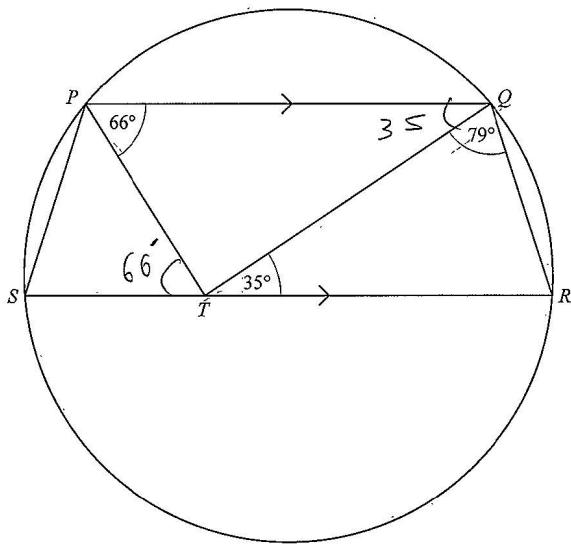
Example Candidate Response – high, continued	Examiner comments

(c)

The points P, Q, R and S lie on the circumference of a circle.
$PQRS$ is a trapezium with PQ parallel to SR.
T is the point on SR such that $Q\hat{P}T = 66°$, $Q\hat{T}R = 35°$ and $T\hat{Q}R = 79°$.

(i) Find $P\hat{T}S$, giving a reason for your answer.

Answer $P\hat{T}S = $ ....66°.... because ...Alternate angles in a parallelogram

......are equal.. [2]

Mark for (c)(i) = 2 out of 2

(ii) Find $P\hat{T}Q$.

180 −101

Answer ....79............ [1]

Mark for (c)(ii) = 1 out of 1

(iii) Complete the statements below to show that triangle $P\overline{Q}T$ is congruent to triangle RTQ.

1. Angle PTQ = Angle ....$R\hat{Q}T$..........

2. Angle PQT = Angle ....$R\hat{T}Q$..........

3. Side QT = ...~~PQ~~ Side TQ..........

Triangle PQT is congruent to triangle RTQ.

Congruency condition ......Angle, Angle, Side
AAS [3]

Mark for (c)(iii) = 3 out of 3

Total mark awarded = 10 out of 11
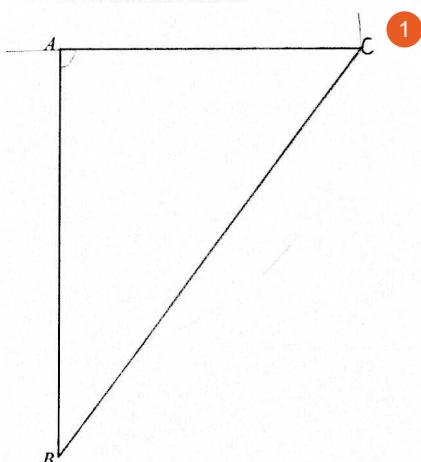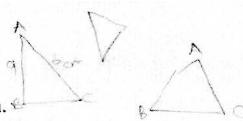
How the candidate could have improved their answer

(a)(ii) The candidate needed to measure the angle more accurately.

Example Candidate Response – middle	Examiner comments

6 **(a)** *ABC* is a triangle with *AC* = 6 cm and *BC* = 9 cm. *AB* has been drawn below. ①

(i) Using a ruler and a pair of compasses only, construct triangle *ABC*. [2]

(ii) Measure $B\hat{A}C$.

Answer ....69°.......................... [1]

(b) A rectangular field has dimensions 220 m by 350 m, each correct to the nearest 10 metres.

Calculate the upper bound for the area of the field.

L × W
225 × 355 m ②

Answer ...73 125.............. m² [2]

60

65   55
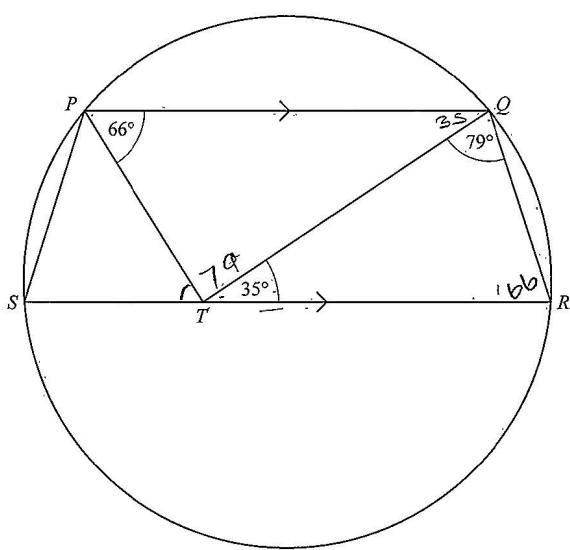
① The candidate's construction is only showing one arc used from *A*.
Mark for (a)(i) = 1 out of 2
Mark for (a)(ii) = 1 out of 1

② The correct upper bounds have been multiplied, for both the width and the length of the field, but an incorrect answer has been given.
Mark for (b) = 1 out of 2

Example Candidate Response – middle, continued	Examiner comments

(c)

The points *P*, *Q*, *R* and *S* lie on the circumference of a circle.
PQRS is a trapezium with *PQ* parallel to *SR*.
T is the point on *SR* such that $Q\hat{P}T = 66°$, $Q\hat{T}R = 35°$ and $T\hat{Q}R = 79°$.

(i) Find $P\hat{T}S$, giving a reason for your answer.

Answer $P\hat{T}S$ = ...66.. because ....Angles on a straight line add.... up to 180°

[2]

(ii) Find $P\hat{T}Q$.

Answer ....79°........ [1]

(iii) Complete the statements below to show that triangle *PQT* is congruent to triangle *RTQ*.

1. Angle *PTQ* = Angle ...79° $R\hat{Q}T$......

2. Angle *PQT* = Angle ...$R\hat{T}Q$...........

3. Angle $T\hat{P}Q$ = Angle $Q\hat{R}T$...........

Triangle *PQT* is congruent to triangle *RTQ*.

Congruency condition ....AAA.......... [3]

③ The candidate is not giving the correct reason here, for alternate angles.
Mark for (c)(i) = 1 out of 2
Mark for (c)(ii) = 1 out of 1

④ The candidate gives the correct pairs of angles in 1 and 2, but needs to mention here in 3, that side QT is a common side.

⑤ AAA is not a condition of congruency. It needs to be AAS here.
Mark for (c)(iii) = 1 out of 3

Total mark awarded = 6 out of 11
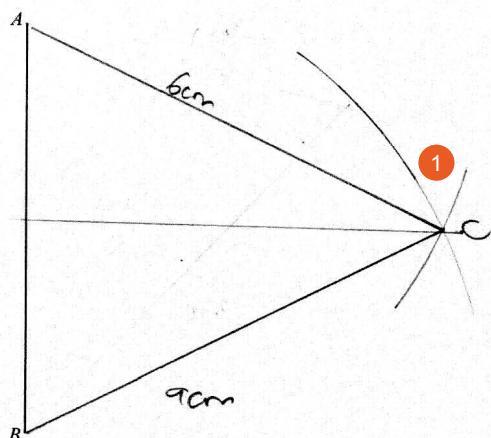
How the candidate could have improved their answer

- **(a)(i)** The candidate needed to draw the other arc from *B* as well and join the points to form the triangle.
- **(b)** The correct upper bounds were used for both the width and the length, but the candidate did not multiply them correctly to get the correct answer.
- **(c)(i)** The correct reason should have been given here for 'alternate angles'.
- **(c)(iii)** The candidate needed to mention that side *QT* was a common side.

Example Candidate Response – low

Examiner comments

6 **(a)** *ABC* is a triangle with *AC* = 6 cm and *BC* = 9 cm.
AB has been drawn below.

6cm

9cm

(i) Using a ruler and a pair of compasses only, construct triangle *ABC*. [2]

(ii) Measure *BÂC* .

Answer ....65°................ [1]
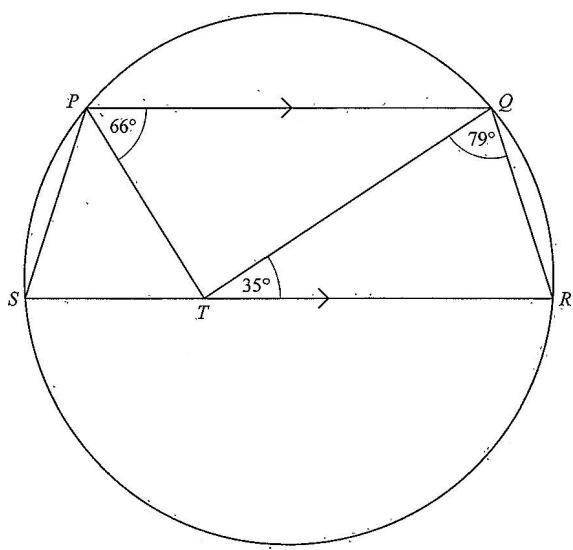
1 The candidate only draws one correct arc from *B*.
Mark for (a)(i) = 1 out of 2
Mark for (a)(ii) = 1 out of 1

(b) A rectangular field has dimensions 220 m by 350 m, each correct to the nearest 10 metres.

Calculate the upper bound for the area of the field.

Area = L × B

= 220 × 350   = 77600

7700

Answer ....77000............ m² [2]

2 The upper bounds are incorrect. They should be 225 m and 355 m.
Mark for (b) = 0 out of 2

Example Candidate Response – low, continued	Examiner comments

(c)

The points P, Q, R and S lie on the circumference of a circle.
$PQRS$ is a trapezium with PQ parallel to SR.
T is the point on SR such that $Q\hat{P}T = 66°$, $Q\hat{T}R = 35°$ and $T\hat{Q}R = 79°$.

(i) Find $P\hat{T}S$, giving a reason for your answer.

Answer $P\hat{T}S = $ **145°** because Angle on a straight line
$= 180° - 35° = 145°$ [2]

(ii) Find $P\hat{T}Q$.

180 - 145

Answer ....35°........ [1]

(iii) Complete the statements below to show that triangle PQT is congruent to triangle RTQ.

1. Angle $PTQ = $ Angle ...TQR...

2. Angle $PQT = $ Angle ...PST RTQ...

3. Angle QTR - Angle PST

Triangle PQT is congruent to triangle RTQ.

Congruency condition .... [3]

Examiner comments:

3 The answer is incorrect. It should be 66 degrees.

4 The answer is incorrect. It should be alternate angles.
Mark for (c)(i) = 0 out of 2

5 Follow Through answer of 35 degrees is incorrect.
Mark for (c)(ii) = 0 out of 1

6 The candidate correctly completes statements 1 and 2.

7 This statement is incorrect and should be identifying the fact that, QT is a common side.

8 The candidate needs to give the case of congruency, which is AAS.
Mark for (c)(iii) = 1 out of 3

Total mark awarded = 3 out of 11

How the candidate could have improved their answer

• **(a)(i)** The candidate needed to draw a second arc and form the triangle.
• **(b)** The candidate did not give either of the correct bounds needed.
• **(c)(i)** The correct angle was not given, 66 degrees, nor stated the correct reason for 'alternate angles'.
• **(c)(ii)** The candidate did not give a correct Follow Through angle from his '66 degrees'.
• **(c)(iii)** The answer needed to mention that side QT was a common side and that the case of congruency was AAS.

## Common mistakes candidates made in this question
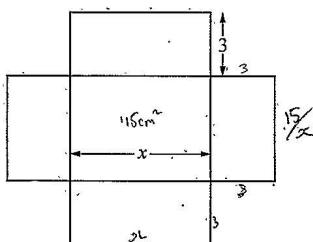
• **(a)(ii)** Measuring the obtuse angle at *A*, instead of the angle *BAC*.
• **(b)** Some candidates did not use the correct upper bounds for either the width or the length.
• **(c)(iii)** The majority of candidates did not show an appreciation of the difference, between congruent triangles and similar triangles. Many candidates thought that showing that the triangles had 3 pairs of equal angles, was sufficient for congruency to be proven.

Question 7

Example Candidate Response – high	**Examiner comments**

7

The diagram shows the net of an open box of height 3 cm.
The area of the base of the box is 15 cm².
The length of the rectangular base is x cm.
The total area of the net is A cm².

(a) Show that $A = 15 + 6x + \dfrac{90}{x}$.

breadth of the box = $\dfrac{15}{x}$ cm

$\therefore A = 15 + 2(3x) + 2\left(\dfrac{15}{x} \times 3\right)$

$A = 15 + 6x + 2\left(\dfrac{45}{x}\right)$

$A = 15 + 6x + \dfrac{90}{x}$

[2]

Mark for (a) = 2 out of 2

(b) Graham has one of these open boxes.
The total area of the net of his box is 65 cm².

Write down an equation in x and solve it to find the length of the base of Graham's box.
Give your answer correct to 2 decimal places.

$65 \text{cm}^2 = 15 + 6x + \dfrac{90}{x}$

$50 = 6x + \dfrac{90}{x}$

$50 = \dfrac{6x^2 + 90}{x}$

$50x = 6x^2 + 90$

$6x^2 - 50x + 90 = 0$

$x = 5.70$

Mark for (b) = 4 out of 4

Answer .... 5.70 .... cm [4]

Example Candidate Response – high, continued	Examiner comments
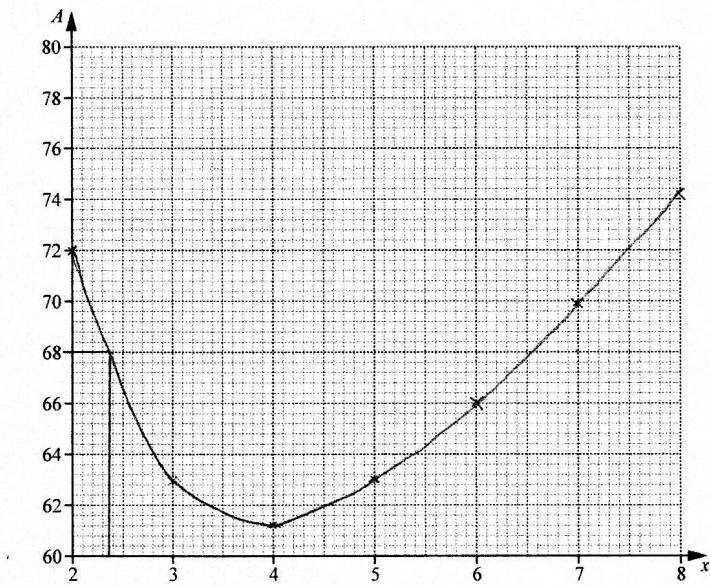
(c) (i) Complete the table below for $A = 15 + 6x + \dfrac{90}{x}$.

x	2	3	4	5	6	7	8
A	72	63	61.5	63	66	69.9	74.33

[1]

Mark for (c)(i) = 1 out of 1

(ii) Draw the graph of $A = 15 + 6x + \dfrac{90}{x}$ for $2 \leqslant x \leqslant 8$.

[2]

Mark for (c)(ii) = 2 out of 2

(iii) Delilah has one of these open boxes.
The area of the net of her box is $68\,\text{cm}^2$.

Use your graph to find the length and width of Delilah's box.

lenght = x = 2·36
width = $\dfrac{15}{x}$ = $\dfrac{15}{2·36}$ = 6·36

Answer length .... 2·36 .... cm
        width .... 6·36 .... cm [2]

1 The candidate correctly reads the value $x = 2.36$ from the graph, but then misunderstands the question, and instead of taking the other reading from the graph, chooses to do an incorrect calculation, to get the value for the width.
Mark for (c)(iii) = 1 out of 2

**Total mark awarded = 10 out of 11**
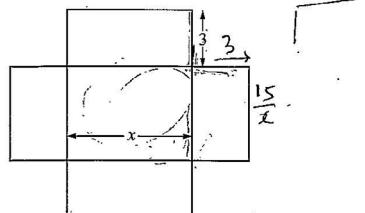
How the candidate could have improved their answer

(c)(iii) The candidate read the width, 2.36, from the graph correctly. However, they did not obtain the value for the length from the graph, but from the calculation $\dfrac{15}{2.36}$, which gave a value of 6.36, which was outside the range of the correct answer. If the candidate had given the value of the length, read from the graph, which was correct, then full marks would have been obtained.

Example Candidate Response – middle	**Examiner comments**

7

The diagram shows the net of an open box of height 3 cm.
The area of the base of the box is 15 cm^2.
The length of the rectangular base is x cm.
The total area of the net is A cm^2.

$15\,cm^2 = x \times$

(a) Show that $A = 15 + 6x + \dfrac{90}{x}$.

$15 + 2\left(3x \dfrac{2}{2} + \dfrac{15}{x} 6\right) 3$

$A = 15 + 6x + \left(\dfrac{15}{x} \times 3\right)$

$150 + 6x + 2\left(\dfrac{15}{x} \times 3\right)$

$A = 15 + 6x + 2\left(\dfrac{45}{x}\right)$

$A = 15 + 6x + \dfrac{90}{x}$

[2]

Mark for (a) = 2 out of 2

(b) Graham has one of these open boxes.
The total area of the net of his box is 65 cm^2.

Write down an equation in x and solve it to find the length of the base of Graham's box.
Give your answer correct to 2 decimal places.

$\dfrac{15}{1} + \dfrac{6x}{1} + \dfrac{90}{x} = 65$

$15 + 6x + \dfrac{90}{x} = 65\,cm^2$

$15 + 6x + \dfrac{90}{x} = 65$

$\dfrac{15x + 6x + 90}{x} = 65$

$15 - 65 + 6x + \dfrac{90}{x} = 0$

$90 + 15 = 65x - 6x$

$21x + 90 = 65x$

$50 + 6x + 90 =$

$\dfrac{105}{59} \quad \dfrac{59x}{59}$

$90 = 65x - 21x$

$6x = 50 - 90$

$= 1.7797$

$90 = 44x$

$6x = -40$

≈ 1.78

$\dfrac{90}{44} = x$

$x = \dfrac{-40}{6}$

$x = 2.05$

$x = \pm 6.$

Answer .... 2.05 cm [4]

2.05

① The candidate makes the error here of not obtaining $6x^2$, when putting the LHS of the equation over the common denominator, x.

② The candidate is not achieving a quadratic equation, so cannot score further marks for method.

③ The answer is either 5.70 or 2.63, so the answer is not awarded the SC marks available. Mark for (b) = 0 out of 4

© UCLES 2018

4024/21/M/J/18

Example Candidate Response – middle, continued	Examiner comments
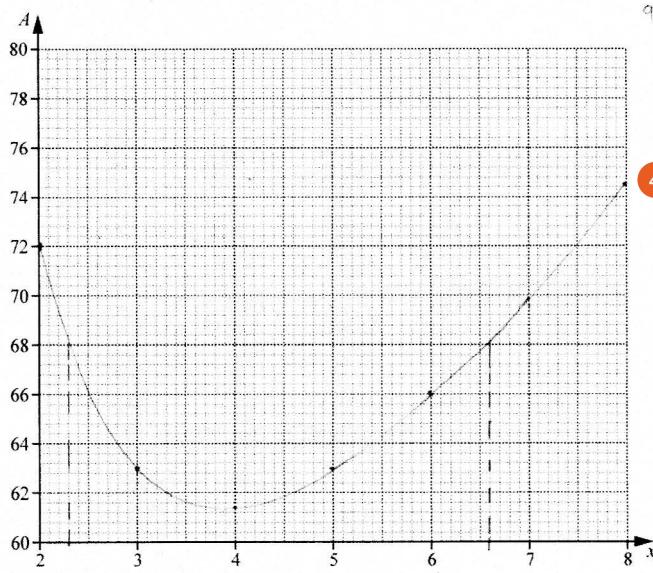
13

$15 + 6x + \dfrac{90}{x} \leqslant 72$

(c) (i) Complete the table below for $A = 15 + 6x + \dfrac{90}{x}$.

x	2	3	4	5	6	7	8
A	72	63	61.5	63	66	69.9	74.3

$15 + 6x + 90 = 72x$

$15 + 90 = 72x - 6_2$

$105 = 66x$

$\dfrac{105}{66} = \dfrac{66x}{66}$

[1]

Mark for (c)(i) = 1 out of 1

(ii) Draw the graph of $A = 15 + 6x + \dfrac{90}{x}$ for $2 \leqslant x \leqslant 8$.

$21x + 90 = 72x$

$90 = 72x - 21$

$\dfrac{90}{51} = 51x$

[2]

4 The plot at (8, 74.25) is inaccurate and is outside the tolerance range.
Mark for (c)(ii) = 1 out of 2

(iii) Delilah has one of these open boxes.
The area of the net of her box is $68\,cm^2$.

Use your graph to find the length and width of Delilah's box.

Answer   length .2.:.3..er...6.6. cm

width .6.:.5.2.er..2.27.cm [2]

$\dfrac{15}{2.3}$ or $\dfrac{15}{6.6}$

Mark for (c)(iii) = 2 out of 2

Total mark awarded = 6 out of 11
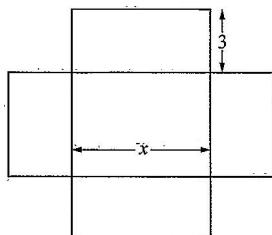
How the candidate could have improved their answer

- **(b)** The candidate needed to form the quadratic equation which was required.
- **(c)(ii)** The answer needed to be more accurate with the plot of the point at (8, 74.25), which was outside of the allowed tolerance.

Example Candidate Response – low	Examiner comments

7

The diagram shows the net of an open box of height 3 cm.
The area of the base of the box is 15 cm².
The length of the rectangular base is x cm.
The total area of the net is A cm².

(a) Show that $A = 15 + 6x + \dfrac{90}{x}$.

length = 6x
Area of base = 15

Area = Length × Length
= 15 + 6x + $\dfrac{90}{x}$

[2]

1 The candidate is giving insufficient evidence, of how the three components that make up the total area of the net, are obtained. The acceptable answer is:
$15 + (2)(3x) + (2)\left(\dfrac{15}{x}\right)(3)$.

Mark for (a) = 0 out of 2

(b) Graham has one of these open boxes.
The total area of the net of his box is 65 cm².

Write down an equation in x and solve it to find the length of the base of Graham's box.
Give your answer correct to 2 decimal places.

$15 + 6x + \dfrac{90}{x} = 65$

$65x = 15 + 6x + 90$

$65x - 6x = 15 + 90$

$59x = 105$

$x = \dfrac{105}{59}$

$= 1.78$

2 The equation in the first line of working is correct, but the second line has two errors in it and is not now the required quadratic equation, needed, in order to make further progress.

Answer ....1.78....... cm [4]

3 The answer is not 2.63 or 5.70, so does not earn the SC marks available.
Mark for (b) = 0 out of 4
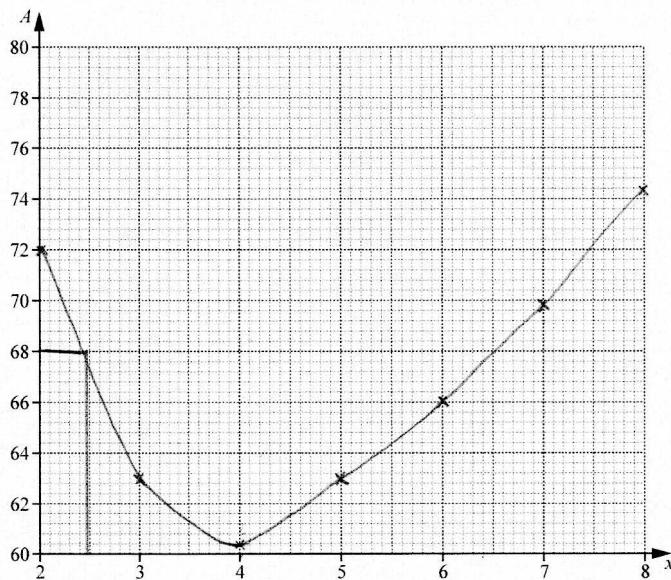
Example Candidate Response – low, continued	Examiner comments

(c) (i) Complete the table below for $A = 15 + 6x + \dfrac{90}{x}$.

x	2	3	4	5	6	7	8
A	72	63	61.5	63	66	69.9	74.25

[1]

Mark for (c)(i) = 1 out of 1

(ii) Draw the graph of $A = 15 + 6x + \dfrac{90}{x}$ for $2 \leqslant x \leqslant 8$.

[2]

4 The first reading from the graph is correct, but there is no evidence of a second reading being taken from the graph.

5 The plot at (4, 61.5) is inaccurate and is outside the tolerance range.
Mark for (c)(ii) = 1 out of 2

(iii) Delilah has one of these open boxes.
The area of the net of her box is 68 cm².

Use your graph to find the length and width of Delilah's box.

Mark for (c)(iii) = 1 out of 2

Answer length ......2.4........... cm

width ......2.4........... cm [2]

Total mark awarded = 3 out of 11

How the candidate could have improved their answer

- **(a)** There should have been more explanation of how the constituent areas of the net were arrived at. A good answer would have been:

$$A = 15 + 2(3x) + 2\left(3\left(\frac{15}{x}\right)\right)$$

- **(b)** The candidate needed to arrive at the quadratic equation required at this stage.
- **(c)(ii)** The plot at point (4, 61.5) was inaccurate.
- **(c)(iii)** The reading from the graph of 2.4 was accurate, but the other reading for the length was not.

Common mistakes candidates made in this question
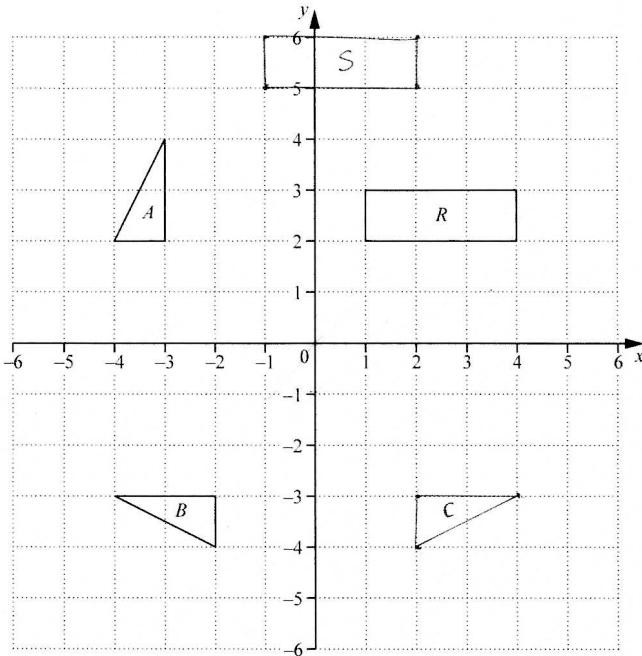
- **(a)** Weaker responses from candidates did not give sufficient explanation of how the constituent areas of the net, were arrived at. For example, it was not sufficient to state that $2 \times \dfrac{45}{x} = \dfrac{90}{x}$ without showing that $\dfrac{45}{x}$ comes from $3 \times \dfrac{15}{x}$.
- **(c)(iii)** Candidates need to ensure that when they are asked to give readings from their graph, then they should do so. Some candidates correctly gave one reading, but then used this value in a calculation to obtain the other. This resulting value did not always fall within the allowed accuracy range for a reading taken from the graph.

Question 8

Example Candidate Response – high	Examiner comments

8   The grid shows triangles *A* and *B* and rectangle *R*.

(a) Triangle *A* is mapped onto triangle *B* by the **single** transformation K.

Find the matrix representing transformation K.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} -3 & -3 & -4 \\ 4 & 2 & 2 \end{pmatrix} = \begin{pmatrix} -4 & -2 & -2 \\ -3 & -3 & -4 \end{pmatrix}$$

$$\begin{pmatrix} -3a+4b & -3+2b & -4a+2b \\ -8c+4d & -3c+2d & -4d+2d \end{pmatrix} \quad \begin{matrix} -3c+4b=-8 \\ -4c+2d=-4 \end{matrix}$$

$$-3a+4b=-4 \qquad a=0 \quad b=-1$$
$$-3a+2b=-2 \qquad c=1 \quad d=0 \qquad Answer \quad \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad [2]$$

(b) Triangle *B* is mapped onto triangle *C* by a reflection in the *y*-axis.

On the diagram, draw triangle *C*. [1]

(c) Triangle *A* is mapped onto triangle *C* by the **single** transformation L.

Describe fully the **single** transformation L.

Answer   reflection on the line −*x* ① [2]

(d) Rectangle *R* is mapped onto rectangle *S* by a translation by the vector $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$.

On the diagram, draw rectangle *S*. [2]

Mark for (a) = 2 out of 2

Mark for (b) = 1 out of 1

① The candidate correctly identifies that the transformation is a reflection, but gives the incorrect equation of the line. It should be *y* = *x*.
Mark for (c) = 1 out of 2
Mark for (d) = 2 out of 2

Total mark awarded = 6 out of 7

How the candidate could have improved their answer

(c) The answer correctly stated that transformation was a reflection, but needed to give the line as *y* = *x* and not *y* = −*x*.

Example Candidate Response – middle   **Examiner comments**

8 The grid shows triangles *A* and *B* and rectangle *R*.

(a) Triangle *A* is mapped onto triangle *B* by the **single** transformation K.

Find the matrix representing transformation K.

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \times \begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$$

① Answer $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ [2]

① The candidate gives a 2 × 2 matrix, but only the top row is correct.
Mark for (a) = 1 out of 2
Mark for (b) = 1 out of 1

(b) Triangle *B* is mapped onto triangle *C* by a reflection in the *y*-axis.

On the diagram, draw triangle *C*.   [1]

(c) Triangle *A* is mapped onto triangle *C* by the **single** transformation L.

Describe fully the **single** transformation L.

Answer *Reflection in the line y = -x axis* [2] ②

② The candidate identifies that the required transformation is a reflection, but gives the incorrect equation of the line. It should be *y = x*.
Mark for (c) = 1 out of 2
Mark for (d) = 2 out of 2

(d) Rectangle *R* is mapped onto rectangle *S* by a translation by the vector $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$.

On the diagram, draw rectangle *S*.   [2]

Total mark awarded = 5 out of 7
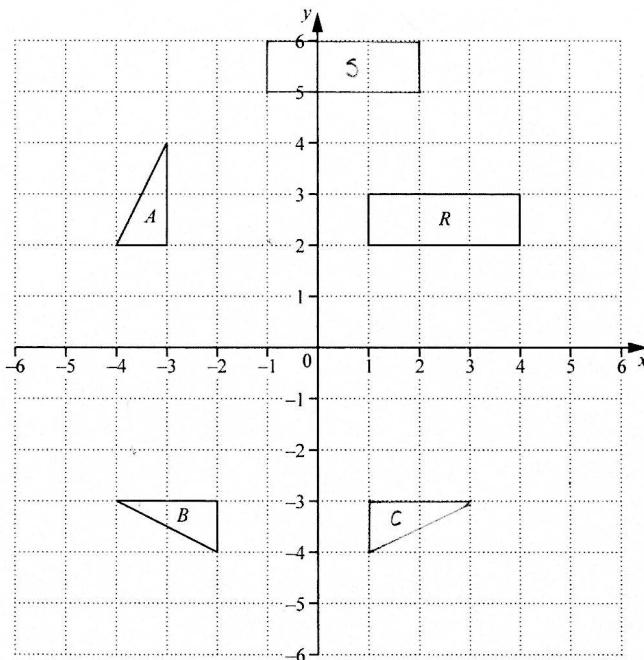
How the candidate could have improved their answer

- **(a)** Only the top row of the matrix correct.
- **(c)** The answer correctly stated that it was a reflection transformation, but needed to give the line as *y = x* and not *y = −x*.

Example Candidate Response – low | Examiner comments

8 The grid shows triangles A and B and rectangle R.

(a) Triangle A is mapped onto triangle B by the **single** transformation K.

Find the matrix representing transformation K.

Answer $\begin{pmatrix} 0 & 1 \\ -5 & -5 \end{pmatrix}$ [2]

(b) Triangle B is mapped onto triangle C by a reflection in the y-axis.

On the diagram, draw triangle C. [1]

1 The candidate correctly identifies that the required matrix is of order 2 × 2, but is not giving either a correct row or a correct column here.
Mark for (a) = 0 out of 2

2 The triangle is of the correct size and orientation, but it is not in the correct position for the required reflection.
Mark for (b) = 0 out of 1

Example Candidate Response – low, continued	Examiner comments
(c) Triangle A is mapped onto triangle C by the **single** transformation L. Describe fully the **single** transformation L. Answer ....180° clockwise Rotation.................................... [2] (d) Rectangle R is mapped onto rectangle S by a translation by the vector $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$. On the diagram, draw rectangle S. [2]	**3** The correct single transformation has not been identified here. It should be a reflection in the line $y = x$. Mark for (c) = 0 out of 2 Mark for (d) = 2 out of 2 **Total mark awarded = 2 out of 7**

How the candidate could have improved their answer

- **(a)** The candidate gave a 2 × 2 matrix for the answer, but neither a row nor column was correct.
- **(b)** The candidate drew a triangle of the same size and correct orientation, but it was a reflection in the line $x = -0.5$ and not the y axis.
- **(c)** The correct answer was single transformation and the reflection in the line $y = x$.

## Common mistakes candidates made in this question
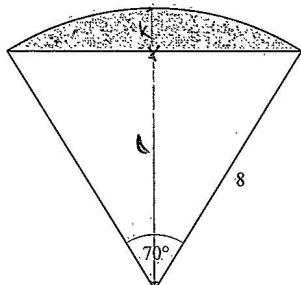
(c) The most common mistake was for candidates to give the incorrect line $y = -x$, for the equation of the line of reflection.

Question 9

Example Candidate Response – high	Examiner comments

9

The diagram shows a sector of a circle of radius 8 cm and angle 70°.

(a) Calculate the shaded area.

Area of Shaded area = Area of Sector − Area of triangle

$$= \left(\frac{70}{360} \times \pi r^2\right) - \left(\frac{1}{2} r^2 \sin\theta\right)$$

$$= \left(\frac{70}{360} \times 64 \times \pi\right) - \left(\frac{1}{2} \times 64 \times 0.9397\right)$$

$$= 39.10 - 30.07$$

$$= 9.03 \text{ cm}$$
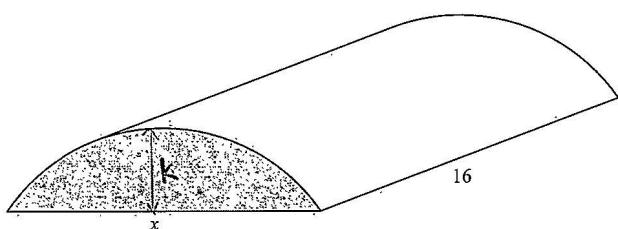
Answer ....9.03.......... cm² [4]

Mark for (a) = 4 out of 4

Example Candidate Response – high, continued	Examiner comments

(b)

A piece of chocolate is in the shape of a prism with the shaded area from **part (a)** being its cross section.
The rectangular base of the chocolate is 16 cm by x cm.
The piece of chocolate is to be placed in a box which is a cuboid of size 16 cm by x cm by 1.5 cm.

(i) Show that the chocolate will fit inside the box.

To prove this we prove that $1.5 \geq$ the height of the chocolate
(We There is no need to bother about side x as both the breadth of the chocolate and the box are the same i-e x)

(Make reference to diagram in 9a)
($+k = 8$ (radius of circle) where k is the height of the chocolate
($= 8\cos(\frac{10}{2}) = 8\cos 35$
$= 6.55$cm
$k = 8 - 6.55$
$= 1.45$cm
Thus the dimension of the chocolate is 16 cm by 2cm × 1.45cm
Thus ∴ It can fit into the cuboid box. [3]

(ii) These boxes are to be packed in cartons in the shape of a cuboid.
The size of each carton is 48 cm by $4x$ cm by 24 cm.

Find the maximum number of boxes that can be packed inside one carton.

No of boxes = $\frac{\text{Volume of 1 carton}}{\text{Volume of 1 box}}$

$\frac{\cancel{3}48 \times \cancel{4x} \times \cancel{24}}{16 \times x \times 1.5} = 2 \times 4 \times 24 = 192$

Answer ....192............ [2]

1 The candidate should more accurately, use the 'greater than' inequality sign here, not the 'greater than or equal to' sign. This does not however, detract from an otherwise, excellently laid out answer.
Mark for (b)(i) = 3 out of 3

Mark for (b)(ii) = 2 out of 2

Total mark awarded = 9 out of 9
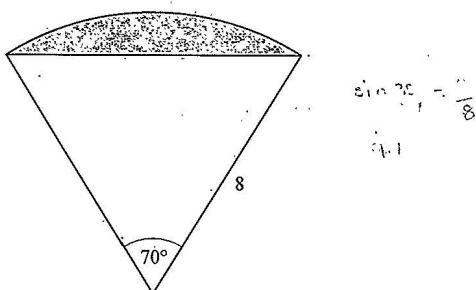
How the candidate could have improved their answer

The candidate achieved full marks.

Example Candidate Response – middle	Examiner comments

9

The diagram shows a sector of a circle of radius 8 cm and angle 70°.

(a) Calculate the shaded area.

Area of segment = Area of sector − Area of triangle

$$= \left(\frac{70}{360} \times 2\pi \frac{r^2}{\theta}\right) - \left(\frac{1}{2} \times 8 \times 8 \times \sin 70\right)$$ ①

$$= \left(\frac{7}{36} \times \frac{64\pi}{16\pi}\right) - \frac{1}{2} 32 \times \sin 70$$

$$= \frac{448}{36} \frac{\cancel{H2}\pi}{36} - \frac{300-070}{30.1}$$

$$= 39.1 - 30.1$$

$$= 9$$

Answer ....9.......................... cm² [4]

① The two correct formulae needed have been used in order to calculate the shaded area.

The candidate is approximating the values of the answers, to the areas of the sector and the triangle, too early.

This leads to an inaccurate final answer.
Mark for (a) = 3 out of 4
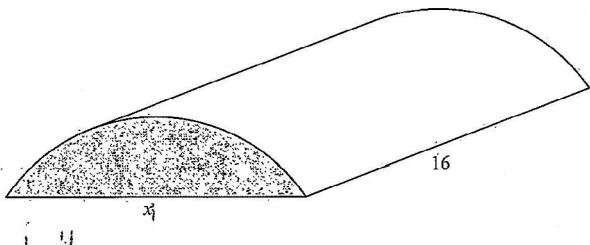
Example Candidate Response – middle, continued	Examiner comments

(b)

A piece of chocolate is in the shape of a prism with the shaded area from **part (a)** being its cross section.
The rectangular base of the chocolate is 16 cm by x cm.
The piece of chocolate is to be placed in a box which is a cuboid of size 16 cm by x cm by 1.5 cm.

(i) Show that the chocolate will fit inside the box.

x = length of the chord from (a)
= $2h\sin\left(\frac{\theta}{2}\right)$
= $2 \times 8 \times \sin(70/2)$
= $16 \times \sin 35 = 9.18$ cm

Volume of prism = $9 \times 16 = 144$ cm³

Volume of box = $16 \times 1.5 \times 9.18 = 220.32$ cm³

∴ There will still be space in the box if the chocolate is kept inside.

2

[3]

2 The candidate does not show an understanding, that it is necessary to show that the height of the chocolate prism must be less than 1.5 cm.
Mark for (b)(i) = 0 out of 3

(ii) These boxes are to be packed in cartons in the shape of a cuboid.
The size of each carton is 48 cm by $4x$ cm by 24 cm.

Find the maximum number of boxes that can be packed inside one carton.

Volume of carton = $48 \times 4x \times 24$
= $48 \times 4(9.18) \times 24$
= 42301.44

Volume of a box = 220.32 $\frac{42301.44}{220.32} = 192$

∴ 192 boxes can be packed in carton.

Answer ....192 boxes.......... [2]

Mark for (b)(ii) = 2 out of 2

Total mark awarded = 5 out of 9
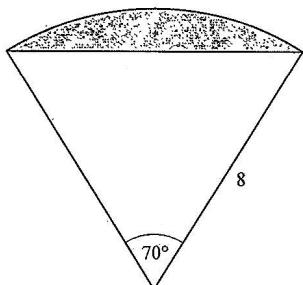
## How the candidate could have improved their answer

- **(a)** The formulae for the area of a sector and the area of a triangle was correct, but in the working out stage, there was early approximation, before the final answer was reached. This led to the latter being inaccurate.
- **(b)** The correct answer was 1.45 or 1446 to 1.447 so the height of the chocolate prism is less than 1.5 cm.

Example Candidate Response – low	Examiner comments

9

The diagram shows a sector of a circle of radius 8 cm and angle 70°.

(a) Calculate the shaded area.

$$\frac{\theta}{360} \times \pi r^2 = \frac{70}{360} \times \pi \times 8^2 = 489.39.$$

area of circle $= \frac{\pi r^2}{2} = \frac{\pi \times 8^2}{2} = 100.5$

$469 - 100.5 = 368 \ cm^2$

$100.5 - 39 = 61.4$

Answer .... 61.4 ... cm² [4]

Examiner comments
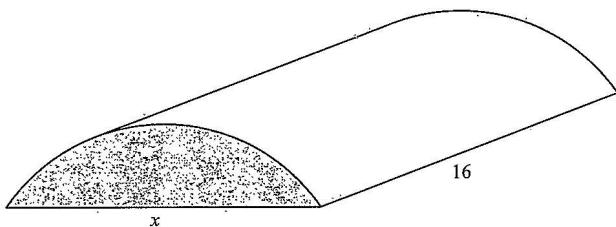
1 The correct formula has been shown for the calculation of the area of the sector.

2 The candidate has used the incorrect method for calculating the area of the triangle.
Mark for (a) = 1 out of 4

Example Candidate Response – low, continued	Examiner comments

(b)

A piece of chocolate is in the shape of a prism with the shaded area from **part (a)** being its cross section.
The rectangular base of the chocolate is 16 cm by x cm.
The piece of chocolate is to be placed in a box which is a cuboid of size 16 cm by x cm by 1.5 cm.

(i) Show that the chocolate will fit inside the box.

$$Box = 16 \times x \times 1.5 = 24x.$$

$$Chocolate = 16 \times x = 16x.$$

③

$$24 \times 61.4 \approx 1473.6 \; box$$

$$Chocolate = 16 \times 61.4 = 982.4 \; chocolate.$$

[3]

③ The candidate does not show an understanding of what is required here. Namely, that it is necessary to show that the height of the shaded cross section must be less than 1.5 cm.
Mark for (b)(i) = 0 out of 3

(ii) These boxes are to be packed in cartons in the shape of a cuboid.
The size of each carton is 48 cm by $4x$ cm by 24 cm.

Find the maximum number of boxes that can be packed inside one carton.

$$48 \times 4x \times 24 = 4608x \; cm^3 . \times 61.4 = 282931.2$$

④

$$\frac{}{982.4}$$

$$\approx 288$$

Answer .... 288 boxes [2]

④ The candidate correctly evaluates the size of a carton, $4608x$ cm^3, but does not then divide this by $16 \times 1.5x$ cm^3.
Mark for (b)(ii) = 0 out of 2

Total mark awarded = 1 out of 9

How the candidate could have improved their answer

- **(a)** The candidate should have used the correct method for finding the area of the triangle.
- **(b)(i)** The candidate should have shown that the height of the chocolate bar was less than 1.5 cm.
- **(b)(ii)** The candidate did not see the connection between the volume of the box and the volume of the carton was $3 \times 4 \times 16$.

Common mistakes candidates made in this question

- **(a)** Some candidates lost the accuracy in their answers because of too early approximation in the method.
- **(b)(i)** The majority of candidates misunderstood that the requirement of this question was to find the height, h cm, of the piece of chocolate and show that it was less than 1.5 cm. Very few candidates used the correct trigonometry, $h = 8 \square 8 \cos 35°$, which showed $h = 1.45$ cm to 3 significant figures. The vast majority of candidates tried to compare the volume of the box to the volume of the carton.

Question 10

Example Candidate Response – high	Examiner comments

10 A boat leaves A and travels 12 km to B.

**(a)** The boat leaves A at 10 25 and travels at an average speed of 15 km/h.
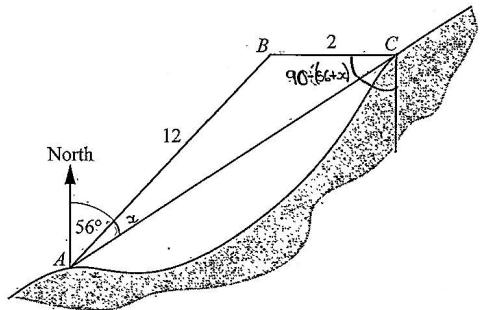
At what time does the boat arrive at B?

time = $\dfrac{\text{Distance}}{\text{speed}}$ = $\dfrac{12}{15}$ = 0.8 hours $\times 60$ = 48 minutes

$$\begin{array}{r} 10\ \ 25 \\ +\ \ \ \ 48 \\ \hline 11\ \ \phi 3 \end{array}$$

Answer ......**11 13**.................. [2]

Mark for (a) = 2 out of 2

(b)

The bearing of B from A is 056°.
B is 2 km due west of C.

Calculate AC.

$A\hat{B}C$ $180 = 90 - (56+x) + x + A\hat{B}C$

$180 = 90 - 56 - x + x + A\hat{B}C$

$180 = 34 + A\hat{B}C$

$A\hat{B}C = 180 - 34$

$A\hat{B}C = 146°$

$AC^2 = AB^2 + BC^2 - 2(AB \times BC)\cos\tilde{B}$

$AC^2 = 12^2 + 2^2 - 2(12 \times 2)\cos 146$

$AC^2 = 144 + 4 - 48(-0.829)$

$AC^2 = 148 + 39.794$

$AC^2 = 187.794$

$AC = \sqrt{187.794}$

$AC = 13.7$

Answer ....**13.7**............ km [4]
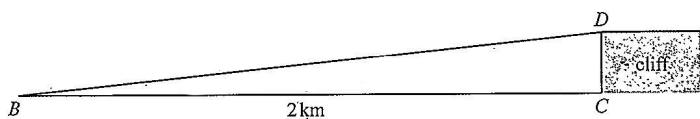
Mark for (b) = 4 out of 4

Example Candidate Response – high, continued	Examiner comments

(c)

C is the base of a cliff.
The top of the cliff, D, is vertically above C.
DC is perpendicular to BC and $DC = 105$ m.

Calculate the angle of elevation of D from B.

$$\text{Tan } \theta = \frac{105}{2}$$

$$\text{Tan } \theta = 52.5$$

$$\theta = \text{Tan}^{-1}(52.5)$$

$$\theta = 88.9°$$

Answer ....88.9°.................. [2]

1 The candidate does not convert the metres and kilometres, to a common unit, before doing the $\tan^{-1}$ calculation.
Mark for (c) = 1 out of 2

Total mark awarded = 7 out of 8

How the candidate could have improved their answer

(c) It was necessary to convert metres to kilometres, or kilometres into metres first of all, before proceeding with the calculation.

Example Candidate Response – middle	Examiner comments

10 A boat leaves A and travels 12 km to B.

**(a)** The boat leaves A at 10 25 and travels at an average speed of 15 km/h.

At what time does the boat arrive at B?

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$= \frac{12 \text{ km}}{15 \text{ km/h}}$$

$$= 0.8 = 48 \text{ minutes}$$

10 25
48
10 73

10 25
48

Answer ......11h 13 mins...... [2]

Mark for (a) = 2 out of 2

(b)

The bearing of B from A is 056°.
B is 2 km due west of C.

Calculate AC.

$a^2 = b^2 + c^2 - 2bc \cos A$

$2bc \cos A = b^2$

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\tan \theta = \frac{12}{2}$

$\theta = 80.5$

$AC^2 = 12^2 + 2^2 - 2(12)(2) \cos(80.5)$

$\tan \theta = \frac{12}{2}$

$\theta = \tan^{-1}\left(\frac{12}{2}\right)$

$\theta = 80.5$
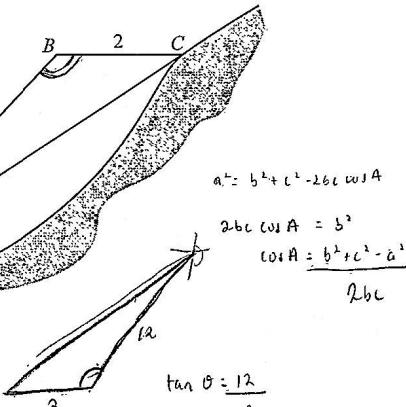
$AC^2 = 12^2 + 2^2 - 2(12)(2) \cos(80.5)$ ①

$AC^2 = \overline{140}$

$AC = \sqrt{140}$

$AC = 11.8$

Answer ......11.8...... km [4]

① The candidate identifies that the solution to find side AC, requires the use of the Cosine rule. However, the value of the angle 80.5 degrees used, is incorrect.
Mark for (b) = 2 out of 4
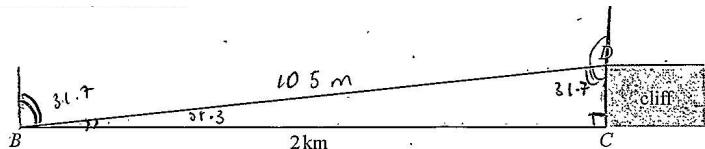
Example Candidate Response – middle, continued	Examiner comments

(c)

C is the base of a cliff.
The top of the cliff, D, is vertically above C.
DC is perpendicular to BC and DC = 105 m.

Calculate the angle of elevation of D from B.

~~100 = 200 m =~~

100 m = 1km
2 km

200 cm

$\cos \theta = \dfrac{200}{105}$

$\theta = \cos^{-1}\left(\dfrac{200}{105}\right)$

$\theta = 58.3$

58.3 + 90 = 148.3

180 - 148.3

31.7

Answer .... 4 148.3 .... [2]

2 The candidate has not converted metres and kilometres correctly to a common unit, and is not using the $\tan^{-1}$ function. Mark for (c) = 0 out of 2

Total mark awarded = 4 out of 8

How the candidate could have improved their answer

- **(a)** The candidate correctly identified that it was necessary to use the Cosine rule to calculate side *AC*, but lost the accuracy required, in order to be able to obtain the correct answer.
- **(c)** It was necessary to use $\tan^{-1}$ *DBC*.

Example Candidate Response – low

10 A boat leaves A and travels 12 km to B.

**(a)** The boat leaves A at 10 25 and travels at an average speed of 15 km/h.
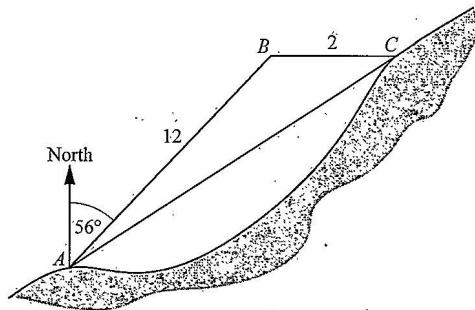
At what time does the boat arrive at B?

$$t = \frac{d}{s} = \frac{12}{15} = 0.8 \text{ minutes}.$$

$$0.8 + 10:25$$

Answer .... 11:05. .... [2]

(b)

The bearing of B from A is 056°.
B is 2 km due west of C.

Calculate AC.

SoH CAH ToA.

$$90 - 56 = 34°$$

$$\frac{\sin 34°}{\cancel{X} \; AC} = \frac{2}{\cancel{X} \; AC}$$

$$\frac{AC \times \sin 34}{\sin 34} = \frac{2}{\sin 34}$$

$$\cos = \frac{Adj}{hyp}$$

$$\cos 34 = \frac{12}{X \; AC}$$

$$= 3.5$$

$$AC = 14.5 \text{ km}$$

Answer .... 14.5 .... km [4]

Examiner comments

1 The candidate knows that to find the time, requires $\frac{\text{distance}}{\text{speed}}$.
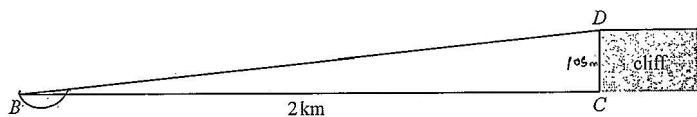
2 They should now convert 0.8 hr to minutes, and add to the 10:25. Mark for (a) = 1 out of 2

3 Incorrect right-angled triangle trigonometry has been used, instead of using the Cosine rule. Mark for (b) = 0 out of 4
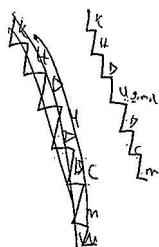
Example Candidate Response – low, continued	Examiner comments
(c) *C is the base of a cliff.* *The top of the cliff, D, is vertically above C.* *DC is perpendicular to BC and DC = 105 m.* Calculate the angle of elevation of D from B. $105 \div 1000 = 0.105$ km $Tan = \dfrac{0.105}{2} = 0.05 = 2.9°$ $180 - 2.9 = 177.1$ *Answer* .... 177.1° .... [2]	④ The candidate realises that it is necessary to change 105m and 2km, to a common unit first of all, before using the tan⁻¹ function. Mark for (c) = 1 out of 2 **Total mark awarded = 2 out of 8**

How the candidate could have improved their answer

- **(a)** The candidate correctly evaluated the time taken for the journey as 0.8 hr, but did not correctly convert this to minutes and add it onto the start time of the journey, in order to find the arrival time.
- **(b)** The candidate needed to use the Cosine rule here to calculate the length of *AC*, but tried to use right-angled triangle trigonometry instead.
- **(c)** The candidate correctly used the inverse tan *DBC*, but then subtracted this value from 180 degrees to obtain the final answer, which was unnecessary.

Common mistakes candidates made in this question
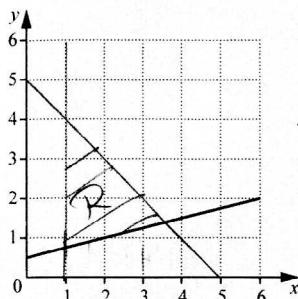
- **(a)** Some candidates incorrectly gave the time taken for the journey as $\dfrac{\text{distance}}{\text{speed}}$.
- **(b)** Candidates often did not convert the 105m and 2km to a common unit, before proceeding with the calculation.

Question 11

Example Candidate Response – high	Examiner comments

11 (a)

The grid shows the line $4y = x + 2$.

By drawing appropriate lines, indicate the region R defined by all these inequalities.

$$x \geqslant 1 \qquad x + y \leqslant 5 \qquad 4y \geqslant x + 2 \qquad [3]$$

(b) A is the point $(-1, 3)$ and B is the point $(5, 5)$.

(i) Calculate the length AB.

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(5 - (-1))^2 + (5 - 3)^2}$$
$$= \sqrt{6^2 + 2^2}$$
$$= \sqrt{36 + 4}$$
$$= \sqrt{40} = 6.3245$$

Answer _____6·32_____ [2]

(ii) Find the equation of the line **perpendicular** to AB that passes through the midpoint of AB.

$(-1, 3) \; (5, 5)$ M/P of AB = $\left(\dfrac{x_2 + x_1}{2}, \dfrac{y_2 + y_1}{2}\right)$

Gradient = $\dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{5 - 3}{3 - (-1)}$ $= \left(\dfrac{5-1}{2}\right), \dfrac{5+3}{2}$

$= \dfrac{2}{+4}$ $= \dfrac{4}{2}, \dfrac{8}{2}$

$mm' = -1$ MIP = $(2, 4)$

$\dfrac{1}{2} m' = -1$ $= \dfrac{1}{2}$ $y = mx + c$

$m' = -2$

$y = mx + c$
$y = -2x + c$
$3 = -2(-1) + c$
$3 = 2 + c$
$3 - 2 = c$
$c = 1$
$y = -2x + 1$

Answer _____$y = -2x + 1$_____ [4]

Mark for (a) = 3 out of 3

Mark for (b)(i) = 2 out of 2

1 The candidate finds the correct coordinates of the midpoint of AB.

2 This shows knowledge that the product of the gradients for a line and a perpendicular line, must be –1.

3 The candidate needs to obtain the correct equation, which is $y = -3x + 10$.
Mark for (b)(ii) = 2 out of 4

Total mark awarded = 7 out of 9
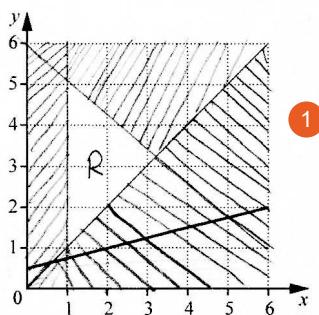
How the candidate could have improved their answer

(b)(ii) The candidate correctly found the coordinates of the midpoint of *AB* and knew that the product of the gradients of the two perpendicular lines was –1. The candidate needed to obtain the correct gradient of □3 for the perpendicular line, in order to progress further.

Example Candidate Response – middle	Examiner comments

11 (a)

The grid shows the line $4y = x + 2$.

By drawing appropriate lines, indicate the region R defined by all these inequalities.

$$x \geqslant 1 \qquad x + y \leqslant 5 \qquad 4y \geqslant x + 2 \qquad [3]$$

(b) A is the point $(-1, 3)$ and B is the point $(5, 5)$.

(i) Calculate the length AB.

$$5 - 3 = 2$$
$$5 + 1 = 6$$
$$\sqrt{6^2 + 2^2}$$

Answer .... 6.32 .... [2]

(ii) Find the equation of the line **perpendicular** to AB that passes through the midpoint of AB.

$$5 - 1 = 4 \qquad 5 + 3 = 8$$
$$\frac{4}{2} \qquad \frac{8}{2} = (2, 4)$$
$$\frac{\Delta y}{\Delta x} = \frac{1}{3}$$
$$y - y_1 = m(x - x_1)$$
$$y - y_1 = \frac{1}{3}(x - x_1)$$
$$3 \times y - 4^{\times 3} = \frac{1}{8}(x - 2) \times 8$$
$$3y - 12 = x - 2$$
$$\frac{3y}{3} = \frac{1}{3}x + 10$$
$$3y = \frac{1}{3}x + 10$$

Answer .... [4]

Examiner comments:

1 The candidate draws the correct line for $x = 1$, but the other required line $x + y = 5$, is incorrectly drawn. Also, the candidate draws the unnecessary line $y = x$, so the region, R, is incorrectly shaded.
Mark for (a) = 1 out of 3

Mark for (b)(i) = 2 out of 2

2 The correct coordinates of the midpoint of AB are obtained.

3 The candidate obtains the correct gradient of AB.

4 The candidate is not using the correct gradient of the line which is perpendicular to AB.
Mark for (b)(ii) = 1 out of 4

Total mark awarded = 4 out of 9
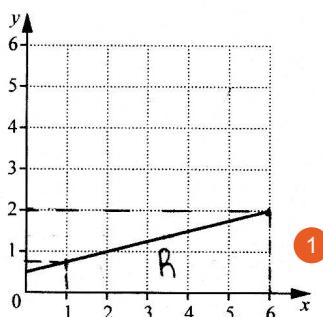
How the candidate could have improved their answer

- **(a)** The candidate correctly drew the line $x = 1$, but could not draw the other required line correctly, so could not identify the correct region required.
- **(b)(ii)** The candidate correctly found the midpoint of AB, but was unable to evaluate the gradient of the perpendicular line and consequently find the equation of the perpendicular line, as required.

Example Candidate Response – low	Examiner comments

11 (a)

The grid shows the line $4y = x + 2$.

By drawing appropriate lines, indicate the region R defined by all these inequalities.

$$x \geqslant 1 \qquad x + y \leqslant 5 \qquad 4y \geqslant x + 2 \qquad [3]$$

(b) A is the point (−1, 3) and B is the point (5, 5).

(i) Calculate the length AB.

$$y = 5 - 3 = 2$$
$$x = 5 - (-1) = 5 + 1 = 6$$

$$AB^2 = 2^2 + 6^2$$
$$AB^2 = 4 + 36$$
$$AB^2 = 40$$
$$AB = \sqrt{40} = 6.324$$

Answer .... 6.32 .... [2]

(ii) Find the equation of the line **perpendicular** to AB that passes through the midpoint of AB.

$$\tfrac{1}{2}(x_2 - x_1) , \tfrac{1}{2}(y_2 - y_1)$$
$$\tfrac{1}{2}(5 - (-1)) , \tfrac{1}{2}(5 - 3)$$
$$\tfrac{1}{2}(6) , \tfrac{1}{2}(2)$$
$$3 , 1$$

Answer .... [4]

Examiner comments:

1. The region labelled R is incorrect. The candidate needs to draw a ruled line at x = 1 and the ruled line for the equation x + y = 5, then label, R, the region bounded by these two lines and the given line.
Mark for (a) = 0 out of 3

Mark for (b)(i) = 2 out of 2

2. The candidate works out the correct displacement of the midpoint, relative to point A (−1, 3). But now needs to do (−1, 3) + (3, 1) which gives the correct midpoint (2, 4).
Mark for (b)(ii) = 0 out of 4

Total mark awarded = 2 out of 9

How the candidate could have improved their answer

- **(a)** The candidate needed to draw a longer line at x = 1 in order to score the mark for this line.
- **(b)(ii)** The candidate did not add on the movement of x = 3 units and y = 1 unit, to the point A (−1, 3), in order to obtain the correct midpoint (2, 4).

Common mistakes candidates made in this question

(b)(ii) Candidates should remember that the product of the gradients of two perpendicular lines is −1.