

Example Candidate Responses

Cambridge
O Level

Cambridge O Level
Additional Mathematics

4037

Paper 2

Cambridge Secondary 2

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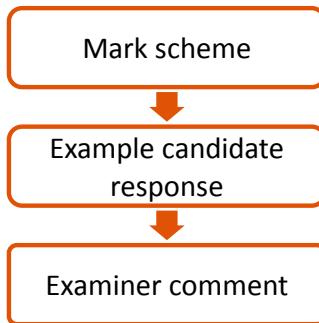
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Introduction

The main aim of this booklet is to exemplify standards for those teaching Cambridge O Level Additional Mathematics (4037), and to show how different levels of candidates' performance relate to the subject's curriculum and assessment objectives.

In this booklet candidate responses have been chosen to exemplify a range of answers. Each response is accompanied by a brief commentary explaining the strengths and weaknesses of the answers.

For ease of reference the following format for each component has been adopted:



For each question an extract from the mark scheme, as used by examiners, is followed by examples of marked candidate responses, each with an examiner comment on performance. Comments are given to indicate where and why marks were awarded, and how additional marks could have been obtained. In this way, it is possible to understand what candidates have done to gain their marks and what they still have to do to improve them.

This document provides illustrative examples of candidate work. These help teachers to assess the standard required to achieve marks beyond the guidance of the mark scheme.

Past papers, Examiner Reports and other teacher support materials are available on Teacher Support at <http://teachers.cie.org.uk>

Assessment at a glance

All candidates will take two written papers.

The syllabus content will be assessed by Paper 1 and Paper 2.

Paper 1	Duration	Marks
10–12 questions of various lengths There will be no choice of question.	2 hours	80
Paper 2	Duration	Marks
10–12 questions of various lengths There will be no choice of question.	2 hours	80

Teachers are reminded that the latest syllabus is available on our public website at www.cie.org.uk and Teacher Support at <http://teachers.cie.org.uk>

Paper 2**Question 1****Mark scheme**

1	(i)		B3,2,1,0	2 correctly placed in Venn diagram; 1, 3, 4, 6 correctly placed; 12, 8, 0, 7, 9, 10 correctly placed; 11, 5 correctly placed
	(ii)	3	B1ft	correct or correct ft <i>their</i> (i), provided non-zero
	(iii)	{4, 6}	B1ft	correct or correct ft <i>their</i> (i), provided not the empty set

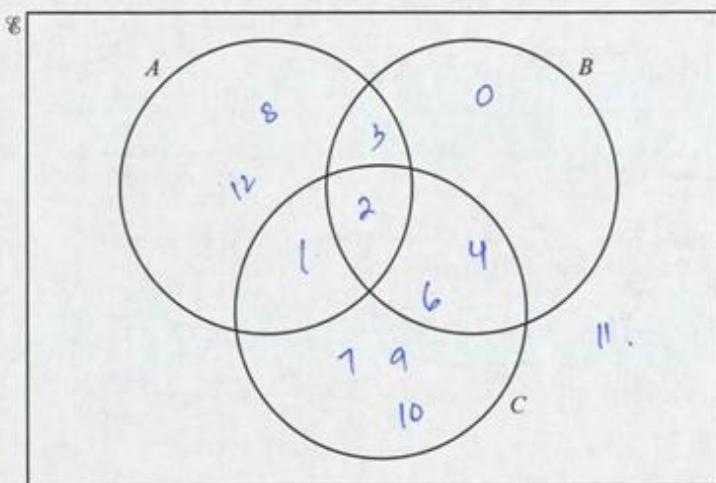
Example candidate response – high

- 1 The universal set contains all the integers from 0 to 12 inclusive. Given that

$$A = \{1, 2, 3, 8, 12\}, \quad B = \{0, 2, 3, 4, 6\} \quad \text{and} \quad C = \{1, 2, 4, 6, 7, 9, 10\},$$

- (i) complete the Venn diagram.

[3]



- (ii) state the value of $n(A' \cap B' \cap C)$.

[1]

$$n(A' \cap B' \cap C) = 3$$

- (iii) write down the elements of the set $A' \cap B \cap C$.

[1]

$$\text{Answer: } \{6, 9\}$$

Marks awarded = (i) 2/3, (ii) 1/1, (iii) 1/1

Total marks awarded = 4 out of 5

Examiner comment – high

The candidate has clearly understood all the set notation being assessed. There is one minor error in the first part of the question as the integer 5 has been omitted. The candidate may have avoided this error if they had listed the integers from 1 to 12 and deleted them as they placed the values in the Venn diagram. This candidate has given a well-presented and well thought out solution to this question.

Candidates providing answers at grade A usually earned 4 or 5 marks for this question.

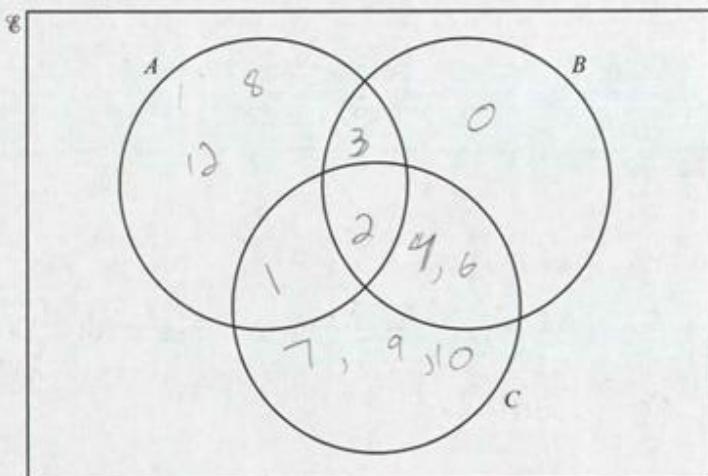
Example candidate response – middle

- 1 The universal set contains all the integers from 0 to 12 inclusive. Given that

$$A = \{1, 2, 3, 8, 12\}, \quad B = \{0, 2, 3, 4, 6\} \quad \text{and} \quad C = \{1, 2, 4, 6, 7, 9, 10\},$$

- (i) complete the Venn diagram.

[3]



- (ii) state the value of $n(A' \cap B' \cap C)$.

[1]

$$n(A' \cap B' \cap C) = 3.$$

- (iii) write down the elements of the set $A' \cap B \cap C$.

[1]

$$A' \cap B \cap C = \{4, 6\} \cup \{2, 4, 6\}.$$

Marks awarded = (i) 2/3, (ii) 1/1, (iii) 0/1

Total marks awarded = 3 out of 5

Examiner comment – middle

This candidate has shown some understanding of the set notation being assessed. They have indicated some method – underlining and circling elements in the given sets. This was a sensible approach. They have omitted two elements from the Venn diagram. This could have been avoided if the candidate had also listed the elements in the Universal set. The candidate understands the notation for the number of elements and has chosen the correct set in part (ii). In part (iii), the candidate has misinterpreted the set as $B \cap C$.

Candidates giving middle grade answers generally scored 2 marks for part (i). The usual reason for this was the candidate omitting the 5 and 11. Occasionally a candidate working at this grade used notation inappropriately. To improve their mark, some candidates needed to take better care with their notation. For example, in part (ii) the answer was sometimes given as {3} rather than simply 3.

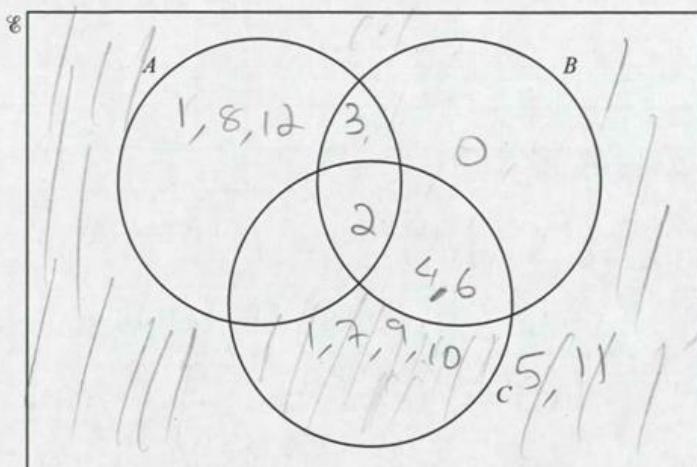
Example candidate response – low

- 1 The universal set contains all the integers from 0 to 12 inclusive. Given that

$$A = \{1, 2, 3, 8, 12\}, \quad B = \{0, 2, 3, 4, 6\} \quad \text{and} \quad C = \{1, 2, 4, 6, 7, 9, 10\},$$

- (i) complete the Venn diagram,

[3]



- (ii) state the value of $n(A' \cap B \cap C)$,

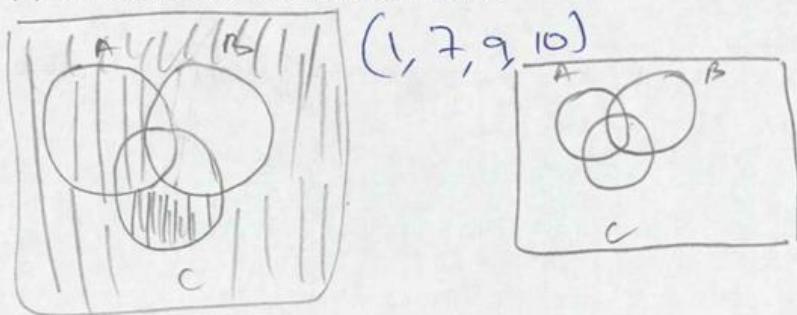
[1]

$$n((\cancel{1, 7, 9, 10}) - 12) = 12$$

$$n = 8$$

- (iii) write down the elements of the set $A' \cap B \cap C$.

[1]



Marks awarded = (i) 2/3, (ii) 0/1, (iii) 0/1

Total marks awarded = 2 out of 5

Examiner comment – low

This candidate has shown understanding of Venn diagrams. In part (i) they have used a method with partial success. This candidate needs to recall or interpret set notation more accurately in order to improve. In part (ii), the candidate has interpreted the notation for the number of elements as the sum of the elements. Other candidates at grade E misinterpreted similarly. In part (iii), the candidate has redrawn the Venn diagram and shaded out some relevant sections. This was a good approach, although they would have benefitted from re-reading the question as they have misread B as B' in the set they were attempting to identify.

Question 2

Mark scheme

2 (i)	$[P =] \begin{pmatrix} 60 & 70 & 58 \\ 50 & 52 & 34 \end{pmatrix}$ and $[Q =] (120 \quad 300)$	B2	or $[P =] \begin{pmatrix} 50 & 52 & 34 \\ 60 & 70 & 58 \end{pmatrix}$ and $[Q =] (300 \quad 120)$ or B1 if one error may be written as an unevaluated product; B0 if choice of P and Q offered
(ii)	$(22\,200 \quad 24\,000 \quad 17\,160)$	B2	must have brackets and must not have commas; must be a 1 by 3 matrix; must be from correct product; working may be seen in (i) or B1 for any two elements correct
(iii)	The total (amount of revenue) from all (three) flights. oe	B1	do not accept, e.g. The total amount from each flight; must be a comment not just a figure; must not contain a contradiction

Example candidate response – high

- 2 The table shows the number of passengers in Economy class and in Business class on 3 flights from London to Paris. The table also shows the departure times for the 3 flights and the cost of a single ticket in each class.

Departure time	Number of passengers in Economy class	Number of passengers in Business-class
0930	60	50
1330	70	52
1545	58	34
Single ticket price (£)	120	300

- (i) Write down a matrix, P, for the numbers of passengers and a matrix, Q, of single ticket prices, such that the matrix product QP can be found. [2]

$$P = \begin{bmatrix} 60 & 50 \\ 70 & 52 \\ 58 & 34 \end{bmatrix} \quad Q = \begin{bmatrix} 120 & 300 \\ 300 & \end{bmatrix} \quad Q = \begin{bmatrix} 120 & 300 \\ 300 & \end{bmatrix}$$

$$P = \begin{bmatrix} 60 & 70 & 58 \\ 50 & 52 & 34 \end{bmatrix}$$

- (ii) Find the matrix product QP . [2]

$$\begin{bmatrix} 120 & 300 \end{bmatrix} \begin{bmatrix} 60 & 70 & 58 \\ 50 & 52 & 34 \end{bmatrix}$$

$$\begin{bmatrix} 7200 + 15000 & 8400 + 15600 & 6960 + 10200 \end{bmatrix}$$

- (iii) Given that $R = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, explain what information is found by evaluating the matrix product QPR . [1]

$$QPR = \begin{bmatrix} 22200 & 24000 & 17160 \end{bmatrix}$$

$$QPR = QR$$

Therefore matrix R is the multiplicative identity matrix because it does not have any effect on the product.

Marks awarded = (i) 2/2, (ii) 2/2, (iii) 0/1

Total marks awarded = 4 out of 5

Examiner comment – high

This candidate has shown they are able to interpret information and display it as a matrix of the correct order. Initially, this candidate selected two matrices of the incorrect order. At some point they have realised that they have chosen unsuccessfully and corrected their error. This is good practice – candidates would do well to check and evaluate answers they have given as they work through a question such as this. This helps to ensure that they have indeed given the answer they were asked for. The candidate then goes on in part (ii) to correctly evaluate the matrix product and gives evidence of their method. Their solution is clear, neat and well set out. This candidate may have improved in part (iii) if they had attempted to find the matrix product $\mathbf{Q}\mathbf{P}\mathbf{R}$ as they have misinterpreted \mathbf{R} as the multiplicative identity. The candidate may also have done better if they had realised that they were being assessed on their ability to interpret the data given in the matrix. This means interpreting the data in the context of the question rather than considering the structure and nature of the form of the answer.

Example candidate response – middle

- 2 The table shows the number of passengers in Economy class and in Business class on 3 flights from London to Paris. The table also shows the departure times for the 3 flights and the cost of a single ticket in each class.

Departure time	Number of passengers in Economy class	Number of passengers in Business class
0930	60	50
1330	70	52
1545	58	34
Single ticket price (£)	120	300

- (i) Write down a matrix, P, for the numbers of passengers and a matrix, Q, of single ticket prices, such that the matrix product QP can be found. [2]

$$P = \begin{pmatrix} 60 & 50 \\ 70 & 52 \\ 58 & 34 \end{pmatrix} \quad Q = \begin{pmatrix} 120 \\ 300 \end{pmatrix}$$

- (ii) Find the matrix product QP .

$$QP = PQ = \begin{pmatrix} 60 & 50 \\ 70 & 52 \\ 58 & 34 \end{pmatrix} \begin{pmatrix} 120 \\ 300 \end{pmatrix} = \begin{pmatrix} 60 \times 120 + 50 \times 300 \\ 70 \times 120 + 52 \times 300 \\ 58 \times 120 + 34 \times 300 \end{pmatrix} = \begin{pmatrix} 22200 \\ 24000 \\ 17160 \end{pmatrix} \quad [2]$$

- (iii) Given that $R = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, explain what information is found by evaluating the matrix product QPR . [1]

$$QPR = \begin{pmatrix} 22200 \\ 24000 \\ 17160 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

It is the total amount of money collected for all three departure times.

Marks awarded = (i) 0/2, (ii) 1/2, (iii) 1/1

Total marks awarded = 2 out of 5

Examiner comment – middle

This candidate has shown some understanding of how to apply matrices to solve a problem. In part (i), they have chosen matrices of the wrong order in part (i). Rather than realising that they had made an error in part (i) they have stated that $QP = PQ$. This candidate would have improved if they had recalled that matrix multiplication is not commutative. If they had realised this, they may have gone back to their answer to part (i) and corrected their error in that part. They have shown some skill in multiplying matrices in part (ii) and therefore earn a special case 1 mark for finding the correct 3 values.

Example candidate response – low

- 2 The table shows the number of passengers in Economy class and in Business class on 3 flights from London to Paris. The table also shows the departure times for the 3 flights and the cost of a single ticket in each class.

Departure time	Number of passengers in Economy class	Number of passengers in Business class
0930	60	50
1330	70	52
1545	58	34
Single ticket price (£)	120	300

- (i) Write down a matrix, P, for the numbers of passengers and a matrix, Q, of single ticket prices, such that the matrix product QP can be found. [2]

$$QP = \begin{bmatrix} 60 & 50 \\ 70 & 52 \\ 58 & 34 \end{bmatrix} \begin{bmatrix} 120 \\ 300 \end{bmatrix}$$

- (ii) Find the matrix product QP . [2]

$$QP = \begin{bmatrix} 60 \times 120 + 50 \times 300 \\ 70 \times 120 + 52 \times 300 \\ 58 \times 120 + 34 \times 300 \end{bmatrix} = \begin{bmatrix} 22200 \\ 24000 \\ 17160 \end{bmatrix}$$

- (iii) Given that $R = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, explain what information is found by evaluating the matrix product QPR . [1]

It shows the total amount of money earned by each of the 3 flights for their one departure.

Marks awarded = (i) 0/2, (ii) 1/2, (iii) 0/1

Total marks awarded = 1 out of 5

Examiner comment – low

The candidate has shown some skill in interpreting the question. More care needed to be taken in the reading of which matrix was which as this was important and the candidate has reversed the labels **P** and **Q**. They have indicated that they are able to apply matrix multiplication in part **(ii)** and have earned a special case 1 mark for finding the three correct values. In part **(iii)** they have attempted to make a comment in context, which was the correct approach. The comment made, however, refers to **each** of the 3 flights, rather than **all** the flights. To earn the mark, candidates needed to indicate the matrix product represented the **total amount for all the flights**. This needed to be clearly and unambiguously stated.

Some candidates working at this grade indicated that the matrix product **QPR** could not be found as the matrices were not of the correct order. This should have been a clue that the matrices they had formed at the start were not correct. Some candidates, working at a slightly higher level, realised this and successfully corrected their earlier error.

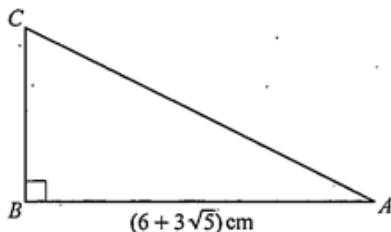
Question 3

Mark scheme

3 (i) $\frac{(36+15\sqrt{5})}{(6+3\sqrt{5})} \times \frac{(6-3\sqrt{5})}{(6-3\sqrt{5})} \text{ oe}$ $\frac{216+90\sqrt{5}-108\sqrt{5}-225}{-9}$ $1+2\sqrt{5} \text{ cao}$ <p>Alternative method:</p> $36+15\sqrt{5} = (6a+15b) + (3a+6b)\sqrt{5}$ $6a+15b=36$ $3a+6b=15$ $a=1 \text{ and } b=2$	M1 DM1 A1	or $\frac{(12+5\sqrt{5})}{(2+\sqrt{5})} \times \frac{(2-\sqrt{5})}{2-\sqrt{5}} \text{ oe}$ or $\frac{24+10\sqrt{5}-12\sqrt{5}-25}{-1}$ or $-(24+10\sqrt{5})-12\sqrt{5}-25$ allow $a=1$ and $b=2$
(ii) $\left[AC^2 = (6+3\sqrt{5})^2 + \text{their} (1+2\sqrt{5})^2 \right]$ $= 36 + 36\sqrt{5} + 45 + \text{their} (1+4\sqrt{5}+20)$ $102 + 40\sqrt{5} \text{ cao}$	M1 A1	correct or correct ft expansions, using Pythagoras with $(6+3\sqrt{5})$ and <i>their BC</i> ignore attempts to square root after correct answer seen

Example candidate response – high

3 Do not use a calculator in this question.



The diagram shows the right-angled triangle ABC , where $AB = (6 + 3\sqrt{5})\text{cm}$ and angle $B = 90^\circ$. The area of this triangle is $\frac{(36 + 15\sqrt{5})}{2}\text{cm}^2$.

- (i) Find the length of the side BC in the form $(a + b\sqrt{5})\text{cm}$, where a and b are integers.

[3]

$$\begin{aligned} \text{Area} &= \frac{1}{2} ab \\ 36+15\sqrt{5} &= \frac{1}{2} (6+3\sqrt{5})BC \\ \frac{36+15\sqrt{5}}{2} &= \frac{1}{2} (6+3\sqrt{5})BC \\ BC(6+3\sqrt{5}) &= 36+15\sqrt{5} \\ BC(6+3\sqrt{5}) &= 36+15\sqrt{5} \\ BC &= \frac{36+15\sqrt{5}}{6+3\sqrt{5}} \end{aligned} \quad \begin{aligned} BC &= \frac{36+15\sqrt{5}}{6+3\sqrt{5}} \times \frac{6-3\sqrt{5}}{6-3\sqrt{5}} \\ &= \frac{6-3\sqrt{5}(36+15\sqrt{5})}{(6)^2 - (3\sqrt{5})^2} \\ &= \frac{216 + 90\sqrt{5} - 108\sqrt{5} - (45 \times 5)}{36 - (9 \times 5)} \\ &= \frac{216 - 18\sqrt{5}}{-9} \\ &= \frac{-9(1+2\sqrt{5})}{-9} = 1+2\sqrt{5} \\ \therefore a &= 1 \text{ and } b = 2 \quad (\text{Ans}) \end{aligned}$$

- (ii) Find $(AC)^2$ in the form $(c + d\sqrt{5})\text{cm}^2$, where c and d are integers.

[2]

$$\begin{aligned} AC^2 &= (1+2\sqrt{5})^2 + (6+3\sqrt{5})^2 \\ &= \{(1)^2 + 2 \cdot 1 \cdot 2\sqrt{5} + (2\sqrt{5})^2\} + \{(6)^2 + 2 \cdot 6 \cdot 3\sqrt{5} + (3\sqrt{5})^2\} \\ &= \{1 + 4\sqrt{5} + (4 \times 5)\} + \{36 + 36\sqrt{5} + (9 \times 5)\} \\ &= (1 + 4\sqrt{5} + 20) + (36 + 36\sqrt{5} + 45) \\ &= (21 + 4\sqrt{5}) + (81 + 36\sqrt{5}) \\ &= 21 + 4\sqrt{5} + 81 + 36\sqrt{5} \\ &= 102 + 40\sqrt{5} \end{aligned} \quad \begin{aligned} \therefore c &= 102 \\ d &= 40 \quad (\text{Ans}) \end{aligned}$$

Marks awarded = (i) 3/3, (ii) 2/2

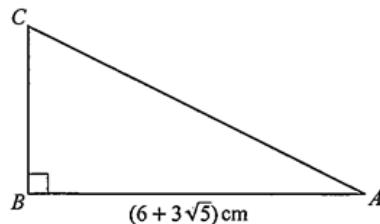
Total marks awarded = 5 out of 5

Examiner comment – high

In this question, candidates were not allowed to use a calculator. This instruction indicated that candidates needed to show full and clear method. This candidate has done exactly what was needed. They have omitted some brackets at one point in part (i), but have recovered in the next line of working and it is very clear that they fully understand what is required and the method to achieve the correct answer. Part (ii) was also fully correct, with every line of method shown. The candidate has identified that Pythagoras' theorem is appropriate and has applied it correctly. Each step is shown and the candidate has been careful and accurately found the answer. This is an exemplary answer.

Example candidate response – middle

3 Do not use a calculator in this question.



The diagram shows the right-angled triangle ABC , where $AB = (6 + 3\sqrt{5})\text{cm}$ and angle $B = 90^\circ$. The area of this triangle is $\frac{(36 + 15\sqrt{5})}{2}\text{cm}^2$.

- (i) Find the length of the side BC in the form $(a + b\sqrt{5})\text{cm}$, where a and b are integers.

[3]

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\frac{36 + 15\sqrt{5}}{2} = \frac{1}{2} \times (6 + 3\sqrt{5}) \times BC$$

$$BC = \frac{36 + 15\sqrt{5}}{6 + 3\sqrt{5}}$$

$$BC = \frac{36 + 15\sqrt{5}}{6 + 3\sqrt{5}} \times \frac{6 - 3\sqrt{5}}{6 - 3\sqrt{5}}$$

$$BC = \frac{216 - 108\sqrt{5} + 90\sqrt{5} - 225}{36 - 45}$$

$$BC = \frac{-9 - 18\sqrt{5}}{-9}$$

$$BC = (2 + 18\sqrt{5})\text{cm}$$

- (ii) Find $(AC)^2$ in the form $(c + d\sqrt{5})\text{cm}^2$, where c and d are integers.

[2]

$$(AC)^2 = (6 + 3\sqrt{5})^2 + (2 + 18\sqrt{5})^2$$

$$(AC)^2 = 36 + 36\sqrt{5} + 45 + 4 + 72\sqrt{5} + 1620$$

$$(AC)^2 = 1705 + 108\sqrt{5}$$

Marks awarded = (i) 2/3, (ii) 1/2

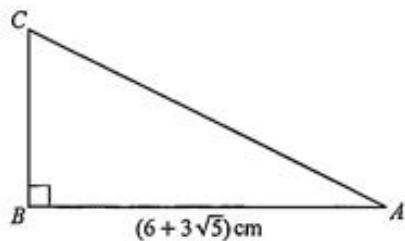
Total marks awarded = 3 out of 5

Examiner comment – middle

This candidate has understood the method that is required to solve the problem in part (i). They have applied the method correctly and have shown enough correct method to indicate that they have not used a calculator. They have made a slip in the very last step and misevaluated the answer. This candidate would have benefitted from double checking their answer. This could have been done by multiplying their answer by $6 + 3\sqrt{5}$ and checking that the result was $36 + 15\sqrt{5}$. In part (ii) the candidate has used the correct method applying Pythagoras correctly with their answer to part (i). They have, again, shown enough clear method to be awarded the method mark.

Example candidate response – low

3 Do not use a calculator in this question.



The diagram shows the right-angled triangle ABC , where $AB = (6 + 3\sqrt{5})\text{cm}$ and angle $B = 90^\circ$. The area of this triangle is $\frac{36 + 15\sqrt{5}}{2}\text{cm}^2$.

- (i) Find the length of the side BC in the form $(a + b\sqrt{5})\text{cm}$, where a and b are integers. [3]

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times AB \times BC \\ \frac{36 + 15\sqrt{5}}{2} &= \frac{1}{2} \times (6 + 3\sqrt{5}) \times BC \\ BC &= \frac{36 + 15\sqrt{5}}{6 + 3\sqrt{5}} \\ &= \boxed{1 + 2\sqrt{5}} \text{ cm} \end{aligned}$$

- (ii) Find $(AC)^2$ in the form $(c + d\sqrt{5})\text{cm}^2$, where c and d are integers. [2]

$$\begin{aligned} (6 + 3\sqrt{5})^2 + (1 + 2\sqrt{5})^2 &= AC^2 \\ [36 + 36\sqrt{5} + 45] + [1 + 4\sqrt{5} + 20] &= AC^2 \\ 81 + 36\sqrt{5} + 21 + 4\sqrt{5} &= AC^2 \\ AC^2 &= 102 + 40\sqrt{5} \text{ cm}^2. \end{aligned}$$

Marks awarded = (i) 0/3, (ii) 2/2

Total marks awarded = 2 out of 5

Paper 2

Examiner comment – low

This candidate has shown, in part (ii), that they are able to perform simple operations with surds. In part (i), even though the correct answer has been given, the candidate has not been awarded any marks. Candidates were instructed not to use a calculator. In order to be awarded marks, candidates needed to give full and clear method to show that they fully understood what was required.

Question 4

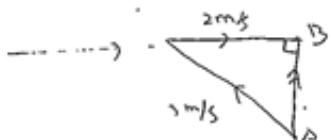
Mark scheme

4 (i) $\cos(x) = \frac{2}{3}$ oe soi 48.189...° or 131.810...° or 0.8410... rad or 2.3(00...) rad oe isw with reference axis indicated by comment, e.g. “to the bank” or “upstream”, etc. or clearly marked on a diagram	M1 A1	Alternatively $\sin(y) = \frac{2}{3}$ oe soi 41.810...° or 0.7297 ... or 0.73(0) rad oe isw with reference axis indicated by comment, e.g. “to the perpendicular with the bank”, etc. or clearly marked on a diagram If M0 then SC1 for an unsupported answer of 138.189...° or 2.4118... rad or 318.189...° or 5.5534... rad with reference axis indicated by comment, e.g. “on a bearing of” or “from North” or clearly marked on a diagram
(ii) $\text{Speed} = \sqrt{9 - 4} (= \sqrt{5})$ or $3 \sin 48.2$ or $2 \tan 48.2$ or $3 \cos 41.8$ or $\frac{2}{\tan 41.8}$ or $\sqrt{2^2 + 3^2 - 2 \times 2 \times 3 \cos 48.2}$ oe or 2.236(0...) rot to 4 or more figs or 2.24 [m/s] soi $\text{time} = \frac{80}{\text{their } \sqrt{5}}$ oe 35.66 to 35.8 (seconds) oe	B1 M1 A1	Or Distance = $\frac{80}{\sin 48.2} = 107.(33\dots)$ oe soi $\text{time} = \frac{\text{their } 107.33\dots}{3}$ ignore subsequent rounding or attempted conversion to, e.g. minutes but A0 if answer spoiled by continuation of method if no working, so B0 M0, then allow B3 for an answer 35.66 to 35.8 oe

Example candidate response – high

- 4 A river, which is 80 m wide, flows at 2 ms^{-1} between parallel, straight banks. A man wants to row his boat straight across the river and land on the other bank directly opposite his starting point. He is able to row his boat in still water at 3 ms^{-1} . Find

(i) the direction in which he must row his boat, [2]



$$\frac{\sin 90}{3} = \frac{\sin \theta}{2}$$

$$\frac{2}{3} = \sin \theta$$

$$\theta = 41.8^\circ$$

$$360 - 41.8$$

at a bearing of 318.2°

(ii) the time it takes him to cross the river. [3]

$$3^2 = 2^2 + x^2$$

$$9 - 4 = x^2$$

$$\sqrt{5} = x$$

$$\text{speed} = \sqrt{5} \text{ m/s.}$$

$$t = \frac{80 \text{ m}}{\sqrt{5} \text{ m/s}}$$

$t = 35.8 \text{ sec}$ Ans.

Marks awarded = (i) 1/2, (ii) 3/3

Total marks awarded = 4 out of 5

Examiner comment – high

This candidate has drawn a clear and correct diagram in part (i). This is very good practice in questions of this type and should be encouraged. The candidate has correctly applied trigonometry and found a correct angle. They would have earned both marks if they had labelled the angle they were finding on the diagram or given a correct explanation in words. The comment *at a bearing of 318.2°* is not valid as north is not indicated. The candidate would have been awarded the mark if they had said, for example, *perpendicular to the bank* or similar, which would have been valid for the angle found. In part (ii), the candidate has used the most reliable method as they have used the figures given in the question to find the resultant speed and completed the method correctly and accurately. As they have used the surd form $\sqrt{5}$, rather than decimalising at this point, they have given a final answer which is sufficiently accurate. Using working values of sufficient accuracy to produce a final answer which is of acceptable form is very good practice.

Example candidate response – middle

- 4 A river, which is 80m wide, flows at 2 ms^{-1} between parallel, straight banks. A man wants to row his boat straight across the river and land on the other bank directly opposite his starting point. He is able to row his boat in still water at 3 ms^{-1} . Find

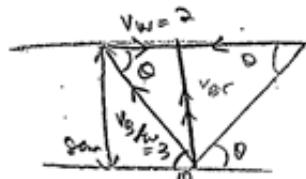
- (i) the direction in which he must row his boat,

[2]

$$V_w = 2\text{ ms}^{-1}$$

$$V_{B/w} = 3\text{ ms}^{-1}$$

$$V_{B/w} = V_B - V_w$$



$$V_B = \sqrt{2^2 + 3^2} \\ = 5$$

$$\cos \theta = \frac{2}{5}$$

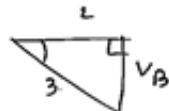
~~48.2° upstream.~~

- (ii) the time it takes him to cross the river.

[3]

$$V_B^2 = 2^2 + 3^2 \\ V_B = \sqrt{2^2 + 3^2} \\ V_B = \sqrt{13}$$

$$V_{B/w} + V_w = V_B \\ 3 + 2 = 5$$



$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{Time} = \frac{\text{distance}}{\text{speed}} \\ = \frac{80}{5}$$

$$= 16 \text{ secs}$$

Marks awarded = (i) 2/2, (ii) 1/3

Total marks awarded = 3 out of 5

Examiner comment – middle

The candidate starts the question well. Part (i) is fully correct. A labelled diagram has been drawn and the angle is identified. The candidate also comments “upstream” which is correct for this angle. In part (ii), the candidate has given two calculations for the required speed and chooses the incorrect value to work with. The candidate has drawn a second, correct diagram in this part and considering this diagram may have prevented the error made. The speed required cannot be more than 3 as this is the hypotenuse of the triangle in the diagram and hence the longest side of the forces triangle. The candidate may be confused by how to calculate simple relative velocity when the velocity is the magnitude of the vector given, rather than when the velocity is given in vector component form.

Example candidate response – low

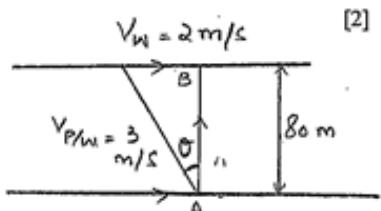
- 4 A river, which is 80 m wide, flows at 2 ms^{-1} between parallel, straight banks. A man wants to row his boat straight across the river and land on the other bank directly opposite his starting point. He is able to row his boat in still water at 3 ms^{-1} . Find

- (i) the direction in which he must row his boat,

$$\sin \theta = \frac{2}{3}$$

$$\theta = \sin^{-1}(0.7)$$

$$= 44.4^\circ \approx 44^\circ$$



The man must row his boat at a bearing of
or 044° .

- (ii) the time it takes him to cross the river.

[3]

V_p = Velocity of boat

V_w = Velocity of river water

$V_{p/w}$ = Velocity of boat relative to water

$$V_p = V_{p/w} + V_w = 3 + 2 = 5 \text{ m/s}$$

$$\text{Time taken} = \frac{\text{Distance}}{\text{Speed}} = \frac{80}{5}$$

$$= 16 \text{ seconds}$$

Marks awarded = (i) 1/2, (ii) 1/3

Total marks awarded = 2 out of 5

Examiner comment – low

The candidate starts the part (i) well. A labelled diagram has been drawn and the angle is identified. The candidate has made a premature approximation error in rounding $\frac{2}{3}$ to 0.7. The angle found from this is therefore incorrect. Candidates should be encouraged to keep all working values as accurate as reasonably possible to ensure the accuracy of their final answer. This candidate would have done better if they had used the fractions key on their calculator rather than decimalising and introducing the error. If they had maintained their accuracy, the labelled diagram would have been sufficient for the second mark. The comment *the man must row his boat at a bearing of 044°* is not correct as north has not been indicated in the question, but this would not have been taken into account as the angle had been marked.

The candidate may improve by revising how to calculate simple relative velocity when the velocity is the magnitude of the vector given. In part (ii), the candidate incorrectly uses the speed 5. They have calculated relative velocity for the case where the objects are moving in the same straight line, rather than when at right angles. They may have avoided this error if they had considered their diagram in part (i). The speed required cannot be more than 3 as this is the hypotenuse of the triangle in the diagram and hence the longest side of the forces triangle.

Question 5**Mark scheme**

5	<p>Substitution of either $4 - x$ or $4 - y$ into equation of curve and brackets expanded</p> <p>$12x^2 - 52x + 48 [= 0]$ or $12y^2 - 44y + 32 [= 0]$ oe</p> <p>Solve their 3-term quadratic $x = \frac{4}{3}$ and 3 isw</p> <p>$y = \frac{8}{3}$ and 1 isw</p>	M1	<p>condone one sign error or slip in either equation of curve or expansion of brackets; condone omission of $= 0$, BUT $4 - x$ or $4 - y$ must be correct</p>
		A1	
		M1 A1	<p>dep on a valid substitution attempt or $x = \frac{4}{3}$ $y = \frac{8}{3}$ not from wrong working</p>
		A1	<p>or $x = 3$ $y = 1$ not from wrong working</p> <p>if no working, allow full marks for fully correct answer only.</p>

Example candidate response – high

5 Solve the simultaneous equations

$$\begin{aligned} & 2x^2 + 3y^2 = 7xy, \quad (1) \\ \text{From (2)} & \quad x + y = 4. \quad (2) \end{aligned} \quad [5]$$

$x = 4 - y$, substitute into (1). Either .

$2(4-y)^2 + 3y^2 = 7(4-y)y$. ✓ $y = 2.7 \text{ or } y = 1$

$2[16 - 8y + y^2] + 3y^2 = 7y(4-y)$ When,

$32 - 16y + 2y^2 + 3y^2 - 28y + 7y^2 = 0$ $y = 2.7, x = 1.3 \quad \text{A0}$

$12y^2 - 44y + 32 = 0$ ✓ $y = 1, x = 3 \quad \text{A1}$

Possible solutions:

$(1.3, 2.7)$
 $(3, 1)$

By quadratic formula,

$$y = \frac{-(-44) \pm \sqrt{(-44)^2 - 4(12)(32)}}{2(12)} \quad \text{M1}$$

✓

Total marks awarded = 4 out of 5

Examiner comment – high

This candidate clearly understands what is required to arrive at the correct solution. They have labelled and referenced the equations. This shows a logical thought process, even though it did not gain any credit in itself. They have indicated what their method involves and have detailed it fully and correctly. The only way this candidate could have improved their overall solution would have been not rounding the decimal values to 2 significant figures. The rubric suggests that inexact answers should be given to 3 significant figures. In this case, the candidate would have done better to have kept their solutions as fractions, rather than decimals.

Example candidate response – middle

5 Solve the simultaneous equations

$$\begin{aligned} 2x^2 + 3y^2 &= 7xy, \\ x + y &= 4. \end{aligned} \quad [5]$$

$$\begin{array}{l}
 x = 4 - y \\
 2(4-y)^2 + 3y^2 = 7y(4-y) \\
 2(16 - 8y + y^2) + 3y^2 = 28y - 7y^2 \\
 \text{M1} \quad 32 - 16 + 2y^2 + 3y^2 - 28y + 7y^2 = 0 \\
 16 + 12y^2 - 28y = 0. \\
 \text{A0} \quad 12y^2 - 28y + 16 = 0. \\
 4(3y^2 - 7y + 4) = 0 \\
 3y^2 - 7y + 4 = 0
 \end{array}
 \left| \begin{array}{l}
 3y^2 - 3y - 4y + 4 = 0 \\
 3y(y-1) - 4(y-1) = 0 \\
 (3y-4)(y-1) = 0 \\
 y = 1 \quad y = 4/3
 \end{array} \right. \quad \begin{array}{l}
 \text{M1} \\
 \text{AO}
 \end{array}$$

$$\begin{array}{l}
 \text{when } y = 1, \\
 x = 4 - 1 \\
 x = 3 \quad \rightarrow (3, 1)
 \end{array}$$

$$\begin{array}{l}
 \text{when } y = 4/3, \\
 x = 4 - \frac{4}{3} \\
 x = \frac{8}{3} \quad \rightarrow \left(\frac{8}{3}, \frac{4}{3}\right) \quad \text{AO}
 \end{array}$$

Total marks awarded = 2 out of 5

Examiner comment – middle

This candidate has understood the method required to solve the problem. They have made a slip when expanding the first bracket. They have not recovered from this and therefore have earned the two method marks only. The first mark has been awarded for a correct substitution and correct expansion of brackets with no more than one error. The second mark has been awarded for the correct factorisation of their quadratic equation, following their error. This candidate needed to give a little more care and attention to detail. If they had done so, they would most likely have earned a much higher mark. This question was very well answered by candidates on the whole and generally marks were lost through inaccurate, rather than incorrect, methods.

Example candidate response – low

5 Solve the simultaneous equations

Substitute ① into ②

$$2x^2 + 3y^2 = 7xy, \quad (2)$$

[5]

$$\begin{matrix} x+y=4 \\ \hookrightarrow y=x-4 \end{matrix} \quad \text{①} \times \quad \text{M0}$$

$$2x^2 + 3(x-4)^2 = 7x(x-4)$$

$$2x^2 + 3x^2 - 24x + 48 = 7x^2 - 28x$$

$$5x^2 - 7x^2 - 24x + 28x + 48 = 0$$

$$\text{So } y = -2-4 \text{ or } y = 12-4$$

$$y = -6 \text{ or } y = 8$$

$$-2x^2 + 4x + 48 = 0$$

$$x^2 - 2x - 24 = 0$$

$$x^2 - 12x + 2x - 24 = 0$$

$$x(x-12) + 2(x-12) = 0 \quad \text{M1}$$

$$x = -2 \text{ or } x = 12$$

Total marks awarded = 1 out of 5

Examiner comment – low

Again, this candidate has shown that they have an understanding of a valid approach. The substitution chosen is incorrect in this case, however. This has resulted in a significant loss of marks, as the correct substitution is fundamental to the method. One mark has been awarded for the correct solution of their quadratic equation which has resulted from an attempt at substitution. The candidate has shown sufficient correct method, following their earlier error, to earn the method mark awarded for this step. This candidate may have improved their result by substituting the values found for x and y into the equation they have labelled 2, to check that the values satisfied both equations. This is good practice when solving simultaneous equations.

Question 6

Mark scheme

6 (a) $(x-2) \log 6 = \log\left(\frac{1}{4}\right)$ oe or $\log_6\left(\frac{1}{4}\right) = x-2$ oe 1.23 or 1.226(29...) rot to 4 or more figures isw	M1 A1	or $x \log 6 = \log\left(\frac{36}{4}\right)$ oe or $x \log 6 - \log 36 = \log 1 - \log 4$ oe correct answer or 1.22 implies M1
(b) Method 1 $\log\left(\frac{8 \times 2y^2 \times 16y}{64y}\right) = \log 4^2$ oe $y = 2$ Method 2 $\log 2 + 2 \log y + 3 \log 2 + 4 \log 2 + \log y - 6 \log 2 - \log y = 4 \log 2$ $y = 2$	B3 B1 B3,2,1,0 B1	or B2 if at most one error or omitted step or B1 if at most two errors or omitted steps not from wrong working <u>LHS terms</u> $\log 2y^2 = \log 2 + 2 \log y;$ $\log 8 = 3 \log 2;$ $\log 16y = 4 \log 2 + \log y;$ $-\log 64y = -6 \log 2 - \log y;$ <u>RHS term</u> $2 \log 4 = 4 \log 2$ not from wrong working

Example candidate response – high

6 (a) Solve $6^{x-2} = \frac{1}{4}$. [2]

$$\begin{aligned} 6^x &= 9 \\ \frac{6^x}{6^2} &= \frac{1}{4} \\ 6^x &= \log_{\sqrt{6}} 9 \\ 6^x &= 36 \\ 6^x &= \frac{36}{4} \end{aligned}$$

(b) Solve $\log_a 2y^2 + \log_a 8 + \log_a 16y - \log_a 64y = 2 \log_a 4$. [4]

$$\begin{aligned} \log_a \left(\frac{2y^2 \times 8 \times 16y}{64y} \right) &= \log_a 4^2 & 4y^2 - 16 &= 0 \\ \log_a \frac{256y^3}{64y} &= \log_a 16 & (2y)^2 - 4^2 &= 0 \\ \frac{256y^3}{64y} &= 16 & (2y - 4)(2y + 4) &= 0 \\ 4y^2 &= 16 & y = \frac{4}{2} \text{ or } y = -\frac{-4}{2} \\ 4y^2 &= 16 & y = 2 \text{ or } y = -2 \end{aligned}$$

Marks awarded = (a) 2/2, (b) 3/4

Total marks awarded = 5 out of 6

Examiner comment – high

The candidate has applied a useful method in part (a). They have manipulated the terms correctly and taken logarithms at a valid point. The final answer has been given to an acceptable accuracy and the method is detailed and clear. In part (b), they have chosen to combine the logarithms. This was the simplest and most direct approach. The candidate has anti-logged correctly. The final mark has not been awarded as the candidate needed to have discarded the negative solution. In questions such as this, involving solving equations where the unknown is the argument of a logarithm, candidates should check that all the solutions given are valid and whether any need to be disregarded.

Example candidate response – middle

6 (a) Solve $6^{x-2} = \frac{1}{4}$. [2]

$$\begin{aligned} \lg 6^{x-2} &= \lg \left(\frac{1}{4}\right) \\ x-2 &= \frac{\lg 0.25}{\lg 6} \\ x-2 &= -0.774 \\ x &= 1.226 \end{aligned}$$

(b) Solve $\log_a 2y^2 + \log_a 8 + \log_a 16y - \log_a 64y = 2 \log_a 4$. [4]

$$\begin{aligned} 2\log_a 2y + 3\log_a 2 &\neq 4\log_a 2 \\ 2\log_a 2y + 3\log_a 2 + 4\log_a 2 - \log_a 64y &= 2 + 4\log_a 2 \\ 2\log_a 2y + 3\log_a 2 + 4\log_a 2 - \log_a y - 6\log_a 2 &= 4\log_a 2 \\ \log_a 2y + \log_a 2(3+4-6-4) &= 0 \\ \log_a 2y + \log_a 2(-3) &= \log_a 2y - 3\log_a 2 \\ \log_a 2y - \log_a 2 &= 0 \\ \boxed{y=2} \end{aligned}$$

Marks awarded = (a) 2/2, (b) 1/4

Total marks awarded = 3 out of 6

Examiner comment – middle

This candidate has used a successful method in part (a). They have taken logarithms as their first step and done so correctly. They have been careful with the accuracy of their method and have also given their answer to an acceptable accuracy. In part (b), the candidate has chosen the more involved approach of separating the logarithms. The second and fourth terms on the left hand side and the term on the right hand side have been dealt with correctly and so the candidate has been awarded B1. The answer, although it appears to be correct, has been obtained from incorrect working and so no credit has been given. This candidate has interpreted the argument of the first logarithm i.e. $2y^2$ as $(2y)^2$. The y has been omitted from the argument of the third term. A common error made by candidates giving solutions at grade C was to treat $\log_a 16y$ as $\log_a (2y)^4$ and hence rewrite it as $4\log_a 2y$.

Example candidate response – low

6 (a) Solve $6^{x-2} = \frac{1}{4}$.

$$3(2)^{x-2} = \frac{1}{2^2}$$

$$3(2)^{x-2} = 2^{-2}$$

$$3 \log_2 2^{x-2} = \log_2 2^{-2}$$

$$3x - 6 = -2$$

$$3x = 4$$

$$\boxed{x = \frac{4}{3}}$$

(b) Solve $\log_a 2y^2 + \log_a 8 + \log_a 16y - \log_a 64y = 2 \log_a 4$.

$$(\log_a 2y^2 + \log_a 8) + \log_a \left(\frac{16y}{64y}\right) = \log_a 8.$$

$$\log_a 4y + \cancel{\log_a} + \log_a 1 - \log_a 4 = \log_a 8 - \log_a 8.$$

$$\log_a 4y - \log_a 4 = 0$$

$$\log_a \left(\frac{14y}{4}\right) = 0$$

$$\log_a y = \log_a 1.$$

$$\therefore \boxed{y = 1}$$

Marks awarded = (a) 0/2, (b) 1/4

Total marks awarded = 1 out of 6

Examiner comment – low

The candidate has attempted to manipulate the left-hand side and right-hand side of the given equation to take logarithms to the same base in part (a). In doing so, they have made an error when separating the left-hand term into factors. This was unnecessary work. They clearly realised that taking logarithms was the correct method. If they had done this, using a consistent base, in the first line, they may well have been more successful. When they do take logarithms to base 2 at the start of their second column of working, they make a second error, which enables them to arrive at a solution. The method is incorrect and no marks are awarded, even though there is evidence that the candidate had some appreciation of what was appropriate. The candidate attempts to combine the arguments of the logarithms in part (b). They have some success in doing this. Two of the left hand terms are correctly combined using the division rule. The first term on the left-hand side has been incorrectly manipulated. The term on the right-hand side has also been incorrectly manipulated – the candidate may have thought that $4^2 = 8$ or thought that $2\log_a 4 = \log_a(2 \times 4)$. All the remaining steps follow through correctly from these errors and so the candidate earns B1. It is possible to check the answer by substituting the value of y into the left-hand side of the original equation to ensure it did indeed match the right. This could have been done with any base, for example, base 10. Although this would NOT have been acceptable as a solution, it would have been a perfectly sensible check.

Question 7

Mark scheme

7	$\frac{n(n-1)(n-2)(n-3)(2^4)}{4 \times 3 \times 2 \times 1} = 10 \frac{n(n-1)(2^2)}{2 \times 1}$ <p style="text-align: center;">or better</p>	M3	<p>condone omitting the factor of n and/or $n - 1$; must have dealt with factorials</p> <p>M2 if one slip/omission or M1 if two slips/omissions</p> <p>or</p> <p>B1 for $\frac{n(n-1)}{2}(2)^2[x^2]$ seen</p> <p>and</p> <p>B1 for $\frac{n(n-1)(n-2)(n-3)}{24}(2)^4[x^4]$ seen</p>
	$n^2 - 5n - 24 [= 0]$ oe $(n + 3)(n - 8) [= 0]$ $n = 8$ only	A1	<p>equivalent must be 3-terms, e.g. $n^2 - 5n = 24$</p>
		M1	<p>or any valid method of solution for their 3-term quadratic</p>
		A1	<p>A0 if -3 also given as a final solution, i.e. not discarded If zero scored, allow SC1 for $n = 8$ unsupported or without correct method</p>

Example candidate response – high

- 7 In the expansion of $(1+2x)^n$, the coefficient of x^4 is ten times the coefficient of x^2 . Find the value of the positive integer, n . [6]

$$T_{r+1} = \frac{n}{r} C_2 (2x)^3$$

$$(1+2x)^n = 1 + \sum_{r=1}^n n C_r 2^r x^r + n C_2 4x^2 + n C_3 8x^3 + n C_4 16x^4.$$

$$n C_4 16x^4 = 10 \times n C_2 4x^2.$$

$$16 n C_4 = 40 n C_2.$$

$$\frac{n!}{(n-r)! r!}$$

$$16 \times \frac{60}{(n-4)! 4!} \times 1 = 40 \times \frac{60}{(n-2)! 2!} \times \frac{1}{1!}$$

$$(2 \cdot 5 \times 24) \cdot (n-4)! = 2 \cdot (n-2)!$$

$$\frac{60}{2} = \frac{(n-2)!}{(n-4)!}$$

$$30 = \frac{(n-2)(n-3)(n-4)(n-5)}{(n-4)(n-5)}$$

$$30 = (n-2)(n-3)$$

$$n^2 - 3n - 2n + 6 = 30$$

$$n^2 - 5n - 24 = 0$$

$$\frac{5 \pm \sqrt{25 - 6 \cdot 100}}{2}$$

$$\frac{5 \pm \sqrt{25 - (-96)}}{2}$$

$$\frac{5+11}{2}, \frac{5-11}{2}$$

$$\boxed{n=8 \text{ or } -3 \text{ answer}}$$

Total marks awarded = 5 out of 6

Examiner comment – high

This candidate clearly understood what was required to answer this question. The method used was well-presented and the logical thought progression was clear. The candidate starts by forming the binomial expansion of terms, including the general binomial coefficients nC_1 and so on. The method is sound and the candidate forms the correct equation in terms of nC_4 and nC_2 . At that point, the candidate deals with the binomial coefficients. The candidate forms the correct equation in terms of n – they have omitted some brackets when doing this, but have recovered from that slip in the next line, so this is condoned. They solve the correct equation and find a correct pair of solutions. This candidate would have improved if they had reread the question more carefully and realised that n needed to be a positive integer. In fact, for this syllabus, candidates are only expected to work with values of n that are positive integers.

Example candidate response – middle

- 7 In the expansion of $(1+2x)^n$, the coefficient of x^4 is ten times the coefficient of x^2 . Find the value of the positive integer, n . [6]

$$1^n + nC_1(2x) + nC_2(2x)^2 + nC_3(2x)^3 + nC_4(2x)^4$$

$$10(nC_2)x^2 = (nC_4)16x^4$$

$$\frac{40}{2} \left[\frac{n(n-1)}{2} \right] = 16 \left[\frac{n(n-1)(n-2)(n-3)}{4} \right]$$

M2

$$20(n^2 - n) = 4[(n^2 - n)(n^2 - 3n - 2n + 6)]$$

$$20n^2 - 20n = 4[(n^2 - n)(n^2 - 5n + 6)]$$

$$20n^2 - 20n = 4[n^4 - 5n^3 + 6n^2 - n^3 + 5n^2 - 6n]$$

$$20n^2 - 20n = 4[n^4 - 6n^3 + 11n^2 - 6n]$$

$$20n^2 - 20n = 4n^4 - 24n^3 + 44n^2 - 24n$$

$$4n^4 - 24n^3 + 44n^2 - 20n^2 - 24n + 20n = 0$$

$$4n^4 - 24n^3 + 24n^2 - 4n = 0$$

$$4n(n^3 - 6n^2 + 6n - 4) = 0$$

$$n^3 - 6n^2 + 6n - 4 = 0$$

AO

$$\frac{20}{4} \left(\frac{n^2 - n}{n^2 - n} \right) = n^2 - 5n + 6$$

$$n^2 - 5n + 6 - 5 = 0$$

$$n^2 - 5n + 1 = 0$$

AO

$$n = \frac{5 \pm \sqrt{25 - 4}}{2}$$

M1

$$n = 2.4, n = 0.2$$

AO

Total marks awarded = 3 out of 6

Examiner comment – middle

This candidate clearly understood what was required to answer this question. The method used was well-presented and the logical thought progression was clear. The candidate starts by forming the binomial expansion of terms, including the general binomial coefficients nC_1 and so on. The method is sound and the candidate forms the correct equation in terms of nC_4 and nC_2 . At that point, the candidate deals with the binomial coefficients and forms an equation in terms of n . However, they have made a slip in writing $4!$ as 4 and therefore are awarded M2 for this part of the method. Their resulting quadratic is not correct and so they are not awarded the accuracy mark. They do show correct method, applying the quadratic formula correctly, in order to solve their equation and so earn the method mark for that. This candidate would have done better if they had taken a little more care with the application of their method. Checking the formula sheet at the

start of the examination paper would have indicated that $\binom{n}{r} = \frac{n!}{(n-r)! r!}$. Perhaps the candidate did not

make the connection between $\binom{n}{r}$ and nC_r and therefore the useful nature of given information in the formula sheet was not appreciated. It is important that the information given to candidates in the formula sheet on page 2 of each examination is appreciated and fully understood by the candidates taking the examination. The work that the candidate deleted indicates that they first attempted to multiply out the brackets, but then realised that, since the value of n was a positive integer and must be greater than 1 for the expansion to exist in the first place, division by n and by $n - 1$ was a reasonable method step.

Example candidate response – low

- 7 In the expansion of $(1 + 2x)^n$, the coefficient of x^4 is ten times the coefficient of x^2 . Find the value of the positive integer, n . [6]

$$\left(\frac{1}{1} + n\right) \binom{n}{1} (2x) + \binom{n}{2} (1)(2x)^2 + \binom{n}{3} (1)(2x)^3 + \binom{n}{4} ((1)(2x)^4)$$

$$\begin{aligned} & 1 + 2xn + \frac{n^2 - n}{2} (4x^2) + \frac{n^3 - 3n^2 + 2n}{6} \\ & (8x^3) + \frac{n^4 - 6n^3 + 11n^2 - 6n}{24} (16x^4) \end{aligned}$$

$$10 \left(\frac{n^2 - n}{2} \right) (16x^2) = \left(\frac{n^4 - 6n^3 + 11n^2 - 6n}{24} \right) (16x^4) \quad \text{M2}$$

$$10(2n^2 - 2n) = 2(n^4 - 6n^3 + 11n^2 - 6n) \quad \text{M2}$$

$$2(20n^2 - 20n) = 2n^4 - 12n^3 + 22n^2 + 12n.$$

$$60n^2 - 60n = 2n^4 - 12n^3 + 22n^2 + 12n$$

$$2n^4 - 12n^3 + 22n^2 - 60n^2 + 12n + 60n = 0$$

$$2n^4 - 12n^3 - 38n^2 + 72n = 0$$

$$2n(n^3 - 6n^2 - 19n + 36) = 0 \quad \text{M0}$$

$$n^3 - 6n^2 - 19n + 36 = 0.$$

$$\frac{(n^3 - 3n^2 + 2n)(n-3)}{1 \times 2 \times 3 \times 4} = \frac{-2}{24}$$

$$\begin{aligned} & n^4 - 3n^3 + 2n^2 - 3n^3 + 9n^2 - 6n \\ & = n^4 - 6n^3 + 11n^2 - 6n \end{aligned}$$

$$\begin{aligned} & \binom{n}{r} = \frac{n(n-1)\dots(n-r+1)}{r \times r-1 \times 2} \\ & = \frac{n^2 - n}{2} \end{aligned}$$

$$\frac{n(n-1)(n-2)}{1 \times 2 \times 3} = \frac{(n^2 - n)(n-2)}{6}$$

$$n^3 - 2n^2 - n^2 + 2n =$$

$$n^3 - 3n^2 + 2n =$$

Total marks awarded = 2 out of 6

Examiner comment – low

The candidate starts the question correctly, expanding using the binomial theorem and finding the second and fourth terms correctly. The candidate has included the x^2 and x^4 elements when forming their equation. This was a slip in method. If they had removed them with no errors in the next line, this would have not counted as an error. However, in the next line, there is a sign error and so they are awarded M2 only (for either line). The candidate chooses to multiply out the factors, rather than divide, and does not solve their cubic equation. In general, division rather than factoring can result in a loss of marks. For example, solutions to equations may be omitted should candidates divide common factors. In a question such as this, where it is clear that the value must be an integer and also be greater than 1, candidates who divided rather than multiplied out were more successful. Candidates may improve should they be clear when it is appropriate to do this.

Comment [CT1]: I am not sure what this means? Either take it out or clarify please.

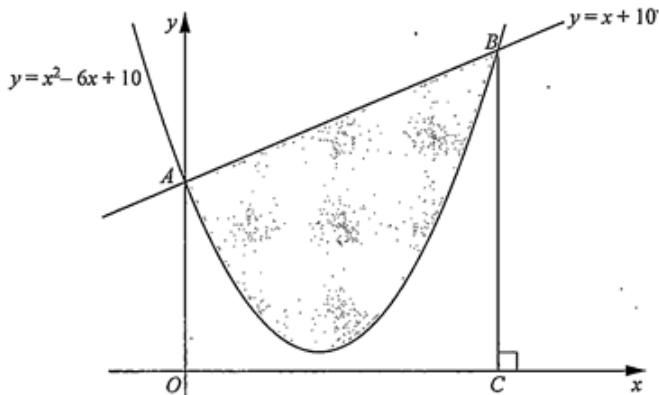
Question 8

Mark scheme

8	<p>Method 1 (Separate areas subtracted)</p> <p>$[x_B = x_C =] 7$ soi</p> $\left[\int(x^2 - 6x + 10)dx \right] = \frac{x^3}{3} - \frac{6x^2}{2} + 10x$ <p>Correct or correct ft substitution of limits 0 and <i>their</i> 7 into <i>their</i> $\left[\frac{x^3}{3} - \frac{6x^2}{2} + 10x \right]$</p> $\frac{1}{2}(10+17) \times 7 \text{ oe or}$ $\int_0^7 (x+10)dx = \left[\frac{x^2}{2} + 10x \right]_0^7 = \frac{(7)^2}{2} + 10(7) \text{ oe}$ <p><i>their</i> $\left(\frac{189}{2} - \frac{112}{3} \right)$</p> <p>$\frac{343}{6}$ or $57\frac{1}{6}$ or 57.2 to 3 sf or 57.16(6...) rot to 4 figs isw</p>	<p>B1</p> <p>M2</p> <p>DM1</p> <p>B2</p> <p>M1</p> <p>A1</p>	<p>or M1 for at least one term correct</p> <p>dep on at least M1 being earned; evidence of substitution must be seen in <i>their</i> integral which must be at least two terms; condone omission of lower limit;</p> <p>or M1 for</p> $\frac{1}{2}(\text{their } 10 + \text{their } 17) \times \text{their } 7 \text{ oe}$ <p>or B1 for</p> $\int(x+10)dx = \frac{x^2}{2} + 10x$ <p>dep on a genuine attempt to integrate the equation of the curve; must be <i>their</i> area trapezium/under the line – <i>their</i> attempt at area under curve</p> <p>from full and correct working with no omitted steps</p>
	<p>Method 2 (Subtracting and using integration once)</p> <p>$[x_B = x_C =] 7$ soi</p> $\int(-x^2 + 7x)dx$ $\left[-\frac{x^3}{3} + \frac{7x^2}{2} \right] \text{ oe or } \left[\frac{x^3}{3} - \frac{7x^2}{2} \right] \text{ oe}$ <p>Correct or correct ft substitution of limits 0 and <i>their</i> 7</p> <p>into <i>their</i> $\left[-\frac{x^3}{3} + \frac{7x^2}{2} \right]$</p> <p>$\frac{343}{6}$ or $57\frac{1}{6}$ or 57.2 to 3 sf or 57.16(6...) rot to 4 figs isw</p>	<p>B1</p> <p>B1</p> <p>M3</p> <p>M2</p> <p>A1</p>	<p>condone omission of dx</p> <p>or M2 for</p> $\int(px^2 + qx)dx = \frac{px^3}{3} + \frac{qx^2}{2} \text{ oe either with } p = \pm 1 \text{ or } q = \pm 7$ <p>or M1 for $\int(px^2 + qx)dx = \frac{px^3}{3} + \frac{qx^2}{2}$ with non-zero constants p and q, with $p \neq \pm 1$ and $q \neq \pm 7$</p> <p>dep on a valid integration attempt; evidence of substitution must be seen; condone omission of lower limit;</p> <p>from full and correct working with no omitted steps</p>

Example candidate response – high

8



The graph of $y = x^2 - 6x + 10$ cuts the y -axis at A . The graphs of $y = x^2 - 6x + 10$ and $y = x + 10$ cut one another at A and B . The line BC is perpendicular to the x -axis. Calculate the area of the shaded region enclosed by the curve and the line AB , showing all your working. [8]

$$\begin{aligned}
 & \text{At } A \quad x=0 \\
 & \therefore y = 0^2 - 6(0) + 10 \\
 & \quad = 10 \\
 & \therefore A(0, 10) \\
 \\
 & \text{Area of } ABCO = \frac{1}{2}(AO + BO) \times OC \quad \text{Area enclosed} \\
 & = \frac{1}{2} \times (10 + 17) \times 7 \\
 & = \frac{1}{2} \times 17 \times 7 \\
 & = \frac{1}{2} \times 119 \\
 & = 59.5 \text{ unit}^2 \\
 \\
 & \text{Area of shaded region} \\
 & = \int_0^7 (x^2 - 6x + 10) dx \\
 & = \left[\frac{x^3}{3} - 3x^2 + 10x \right]_0^7 \\
 & = \left[\frac{7^3}{3} - 3(7)^2 + 10(7) \right] - 0 \\
 & = \frac{343}{3} - 98 + 70 \\
 & = 86.33 \\
 & \cancel{\text{86.33 units}} - \cancel{16.5}
 \end{aligned}$$

$\therefore 0 = x^2 - 7x$
 $x(x-7) = 0$
 $x = 0 \quad x = 7$
 $y = 0 + 10 \quad y = 7 + 10$
 $= 10 \quad = 17$
 $\therefore B = (7, 17)$

Total marks awarded = 6 out of 8

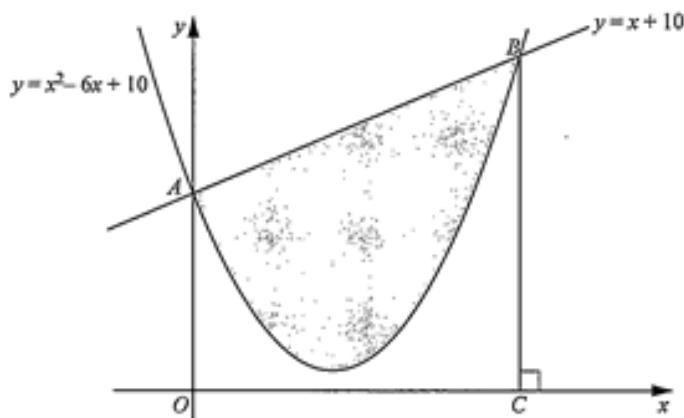
Examiner comment – high

This candidate understands the key steps in the method required to solve the problem. There is one error and the final step has been omitted resulting in two marks not being awarded. The candidate starts correctly, realising that finding the coordinates of the point B is the first valid step in the method. They then correctly state the calculation for finding the area of the trapezium $OABC$ and earn B2 for that. Integrating was essential and this candidate integrates the equation of the curve between the correct limits. The substitution of the limits has been shown. This was essential as candidates were directed to show all their method in the question itself. The candidate omits to make the connection between the two areas they have found and therefore are not awarded the final method mark. Their answer would not have been accurate as they have made some arithmetic slips in the process.

It is good practice to show full method in all questions as this is expected as outlined in the assessment objectives for this syllabus. In a question such as this, where it is specifically demanded, candidates are not awarded a significant proportion of the marks if the full method is not shown.

Example candidate response – middle

8



The graph of $y = x^2 - 6x + 10$ cuts the y -axis at A . The graphs of $y = x^2 - 6x + 10$ and $y = x + 10$ cut one another at A and B . The line BC is perpendicular to the x -axis. Calculate the area of the shaded region enclosed by the curve and the line AB , showing all your working. [8]

$$\text{At } A, x = 0$$

$$y = 10$$

$$A(0, 10)$$

For B

$$x^2 - 6x + 10 = x + 10$$

$$x^2 - 7x = 0$$

$$x = 0, 7$$

$$y = 7 + 10 = 17$$

B1

$$B = (7, 17)$$

Apply definite integral
for area under graph,

$$= \int_0^7 (x^2 - 6x + 10) dx$$

$$= \int_0^7 \frac{x^3}{3} - \frac{6x^2}{2} + 10x$$

M2

$$= \int_0^7 \frac{x^3}{3} - 3x^2 + 10x$$

$$= \frac{(7)^3}{3} - 3(7)^2 + 70$$

M1

$$= 37.3 \text{ cm}^2$$

A

Total marks awarded = 4 out of 8

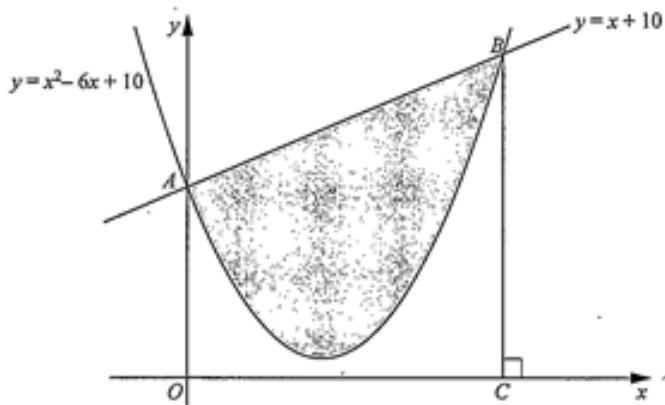
Examiner comment – middle

This candidate has some understanding of the method required here. The candidate has correctly found the coordinates of B – the valid first step in solving the problem. To evaluate a plane area, integration is required at some point. The candidate has correctly integrated the equation of the curve between the correct limits and the full method for that has been shown. The notation has not been used correctly as the integral sign is still present, even though the candidate has integrated. This is condoned. The candidate has not dealt with the area of the trapezium $OABC$ and no further marks can be awarded as the method is incomplete. This candidate may have improved their mark if they had realised that they had found the area between the curve and the x -axis rather than the shaded area. It may be that this candidate thought that they had found the area required with the calculation shown indicating a misinterpretation of the area given by the integral.

Other candidates working at this level omitted key method steps. The most significant method steps that were omitted were the substitution of the limits into the integral (or integrals, depending on their approach). Also, it was not uncommon for the coordinates of B to be given as $(5, 15)$ with a slip having been made in solving the equations of the line and curve – this resulted in accuracy being lost.

Example candidate response – low

8



The graph of $y = x^2 - 6x + 10$ cuts the y -axis at A . The graphs of $y = x^2 - 6x + 10$ and $y = x + 10$ cut one another at A and B . The line BC is perpendicular to the x -axis. Calculate the area of the shaded region enclosed by the curve and the line AB , showing all your working. [8]

$$\begin{aligned}
 & y = x^2 - 6x + 10 \\
 & A \text{ at } A \quad x = 0 \\
 & y = 0^2 - 6(0) + 10 \\
 & (0, 10) \text{ A} \\
 & (x+10) = x^2 - 6x + 10 \\
 & x^2 - 7x \\
 & x(x-7) = 0 \\
 & x = 7 \\
 & B(7, 17) \quad \checkmark \boxed{B1} \\
 & \int_{0}^{7} (x+10) - (x^2 - 6x + 10) \, dx \quad \boxed{B1} \\
 & \boxed{\Delta} = 94.5 \text{ A under tangent}
 \end{aligned}$$

Total marks awarded = 2 out of 8

Examiner comment – low

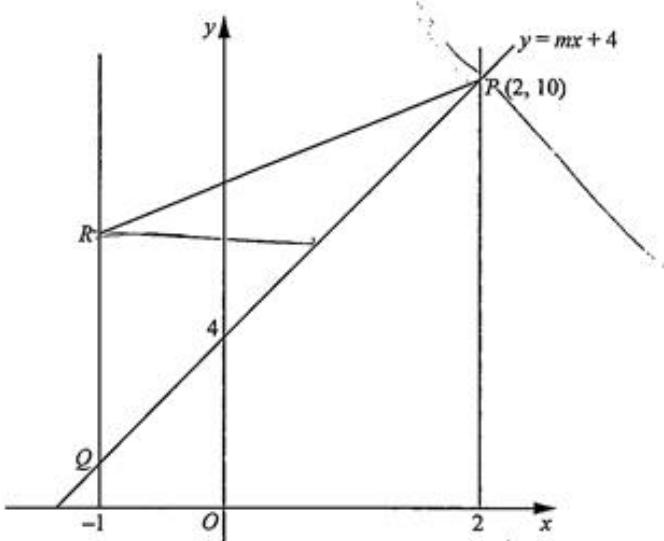
The candidate has some appreciation of the method required. They have correctly found the coordinates of B . They have integrated the equation of the line to find the area of the trapezium $OABC$. They have not shown the substitution of the limits to find the answer. It is possible that the candidate understood what was required and was not awarded marks as the method was asked for and not shown. It is also possible that the candidate calculated the definite integral using the numerical integration function on their calculator. Candidates would improve their mark if they realise that this function is useful to check their solution, but they should not be used as a replacement for method and that method must be shown. One of the syllabus aims is to *integrate information technology (IT) to enhance the mathematical experience*. This is not an assessment objective, however, and the usefulness and limitations of using a calculator to demonstrate method must be clearly understood by candidates who wish to improve their results.

Question 9**Mark scheme**

9	(i)	$10 = 2m + 4$ soi	M1	or $[m =] \frac{10 - 4}{2 - 0}$ oe soi
		$m = 3$	A1	
	(ii)	1	B1	
	(iii)	$\frac{10 - y_R}{2 - 1} = 1$ oe soi $(-1, 7)$ or $x = -1$ and $y = 7$	M1 A1	or $y = x + 8$ oe if $y = 7$ only stated, provided that $x = -1$ is soi in working allow both marks if M0 then B1 for $y = 7$ only with no working
	(iv)	Use of $m_1 m_2 = -1$ with <i>their m</i> from (i) $y - 10 = \left(\text{their } -\frac{1}{3}\right)(x - 2)$	M1 A1	may be implied by perpendicular gradient seen in equation or $\left(\text{their } -\frac{1}{3}\right)x + c$ and $10 = \left(\text{their } -\frac{1}{3}\right)2 + c$
		$3y + x = 32$ isw	A1	allow for correct equation with integer coefficients in any simplified form
	(v)	$\left(\frac{1}{2}, \text{their } \frac{11}{2}\right)$ oe isw	B1,B1ft	ft <i>their y_Q</i> or M1 for $\left(\frac{2-1}{2}, \frac{10+1}{2}\right)$ seen
	(vi)	4.5 oe cao	B2	not from wrong working or M1 for any correct method with correct coordinates

Example candidate response – high

9 Solutions by accurate drawing will not be accepted.



The line $y = mx + 4$ meets the lines $x = 2$ and $x = -1$ at the points P and Q respectively. The point R is such that QR is parallel to the y -axis and the gradient of RP is 1. The point P has coordinates $(2, 10)$.

- (i) Find the value of m . (2)

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{6}{2}$$

$$= \frac{10 - 4}{2 - 0} = 3.$$

- (ii) Find the y -coordinate of Q . (1)

$$\begin{aligned} y &= 3x + 4 \\ y &= 3(-1) + 4 \\ &= -3 + 4 \\ &\boxed{y = 1} \end{aligned}$$

- (iii) Find the coordinates of R . (2)

$$\begin{aligned} \text{line } RP: \\ y &= 1x + c. \\ 10 &= 2 + c \\ 10 - 2 &= c \\ c &= 8. \\ y &= x + 8 \end{aligned}$$

$$\begin{aligned} \text{line } RQ: \\ R(-1, y) \\ y &= -1 + 8 \\ &= 7 \\ &\boxed{(-1, 7)} \end{aligned}$$

Marks awarded = (i) 2/2, (ii) 1/1, (iii) 2/2

Example candidate response – high, continued

- (iv) Find the equation of the line through P , perpendicular to PQ , giving your answer in the form $ax + by = c$, where a , b and c are integers. [3]

$$\begin{array}{l} \text{gradient} = -\frac{1}{3} \\ y - y_1 = -\frac{1}{3}(n - n_1) \\ y - 10 = -\frac{1}{3}(n - 2) \\ y - 10 = -\frac{1}{3}n + \frac{2}{3} \end{array} \quad \left| \begin{array}{l} y = -\frac{1}{3}n + \frac{32}{3} \\ 3y + n = 32 \\ n + 3y = 32 \end{array} \right.$$

- (v) Find the coordinates of the midpoint, M , of the line PQ . $P = (2, 10)$ $Q = (-1, 1)$ [2]

$$\begin{aligned} \text{Midpoint} &= \left(\frac{n_1 + n_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-1 + 2}{2}, \frac{10 + 1}{2} \right) \\ &= (0.5, 5.5) \end{aligned}$$

- (vi) Find the area of triangle QRM . [2]

area of triangle:

$$\begin{aligned} &\left| \begin{array}{ccc|c} 0.5 & -1 & -1 & 6.5 \\ 5.5 & +1 & 7 & 5.5 \end{array} \right| \\ &= (6.5 + 1 + 5.5) - (3.5 + 1 + 5.5) \\ &= -12 - (-3) \\ &= -12 + 3 \\ &= -9 \quad \therefore 9 \text{ square units} \end{aligned}$$

Marks awarded = (iv) 3/3, (ii) 2/2, (vi) 0/2

Total marks awarded = 10 out of 12

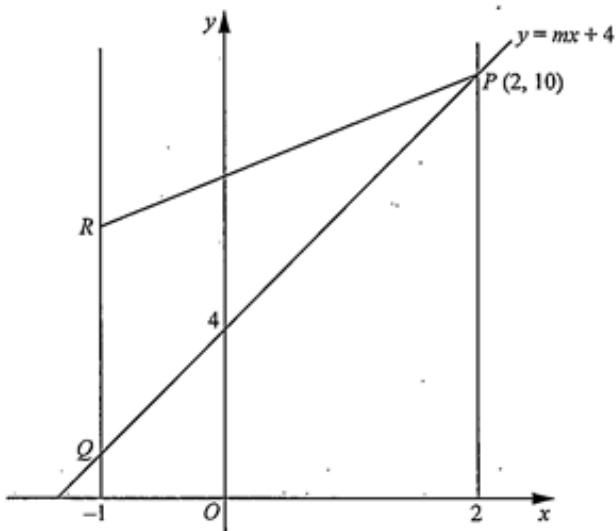
Examiner comment – high

This question was well-answered by the majority of candidates. Candidates who gave well-presented and clear methods, taking care to be accurate, performed best overall in this question.

In part (i), this candidate writes down and uses the general formula for the gradient of a straight line and then uses it appropriately. This is good practice and reduces the possibility of making an error by, for example, reversing the coordinates. Using their correct value from part (i), the candidate forms the correct equation for the line through P and Q and uses that to answer part (ii) correctly. This was also a valid and reasonable approach to use to solve the problem. In part (iii), the candidate forms the equation of the line RP using the given gradient. The method has been detailed clearly and logically with no errors seen. In part (iv) the candidate again correctly uses the value found in part (i) and shows the general form of the equation of a straight line. The correct coordinates have been used to form the equation and the candidate has taken care to ensure all the requirements of the question have been met. In part (v) the candidate again shows full and clear method. The stating of the general form for finding midpoints was useful and is likely to have ensured that this candidate made no errors with signs in this part. In part (vi), the candidate has chosen the popular shoelace method of finding the area of the triangle. This was not the simplest method in this case. The candidate may have benefitted from a moment of reflection at this point. In this particular question, the easiest method of solution was to use half base times height, as both the base and height were very easy to state from the coordinates found. The candidate has omitted the factor of a half from their shoelace formula attempt and therefore the method is incorrect and no marks are awarded.

Example candidate response – middle

9 Solutions by accurate drawing will not be accepted.



The line $y = mx + 4$ meets the lines $x = 2$ and $x = -1$ at the points P and Q respectively. The point R is such that QR is parallel to the y -axis and the gradient of RP is 1. The point P has coordinates $(2, 10)$.

- (i) Find the value of m .

[2]

$$\begin{aligned} y &= 10 - 2m + 4 \\ m &= \frac{6}{2} \end{aligned}$$

$$\therefore m = 3$$

- (ii) Find the y -coordinate of Q .

[1]

$$\begin{aligned} y &= -3 + 4 \\ y &= 1 \end{aligned}$$

- (iii) Find the coordinates of R .

[2]

$$\begin{aligned} y &= u + c \\ 10 &= 2 + c \\ c &= 8 \end{aligned}$$

$$\begin{aligned} y &= m + b \\ y &= 5 \\ \therefore R &(-1, 5) \end{aligned}$$

Marks awarded = (i) 2/2, (ii) 1/1, (iii) 1/2

Example candidate response – middle, continued

- (iv) Find the equation of the line through P , perpendicular to PQ , giving your answer in the form $ax + by = c$, where a , b and c are integers. [3]

$$\begin{aligned} m_{PQ} &= -\frac{1}{3} \\ y - 1 &= -\frac{1}{3}(x - 2) \\ y - 1 &= -\frac{1}{3}x + \frac{2}{3} \\ y &= -\frac{1}{3}x + \frac{5}{3} \\ 3y &= -x + 5 \\ x + 3y &= 5 \end{aligned}$$

- (v) Find the coordinates of the midpoint, M , of the line PQ . [2]

$$\left(\frac{2+(-1)}{2}, \frac{10+1}{2} \right)$$

$$M \left(\frac{3}{2}, \frac{11}{2} \right)$$

- (vi) Find the area of triangle QRM . [2]

$$\frac{1}{2} \left| 1 \times 5 \times \frac{3}{2} \times 1 \right|$$

$$\frac{1}{2} \left| -9 - 1 \right|$$

$$\therefore 5 \text{ units}^2$$

Marks awarded = (iv) 3/3, (v) 1/2, (vi) 0/2

Total marks awarded = 8 out of 12

Examiner comment – middle

In part (i), this candidate uses the equation of the line given in the question and substitutes the coordinates of P into it. This was the best approach as only information explicitly given in the question needed to be used. This reduced the possibility of introducing an unnecessary error. Part (ii) was also answered using the equation given in the question, along with the correct answer from part (i). The candidate starts part (iii) using a correct approach. They would have done better if they had paid more attention to the presentation of their work, however. After correctly stating $y = x + c$ and finding $c = 8$, and so earning the first mark, they miscopy their equation as $y = x + 6$ and therefore have the incorrect coordinates for R . This impacts the overall mark awarded as all points needed to be correct for marks to be awarded in part (vi). In part (iv), the candidate correctly uses their value from part (i). They use the $y = mx + c$ form of the equation of a straight line and find c correctly. Some candidates working at this level needed to take more care with their arithmetic

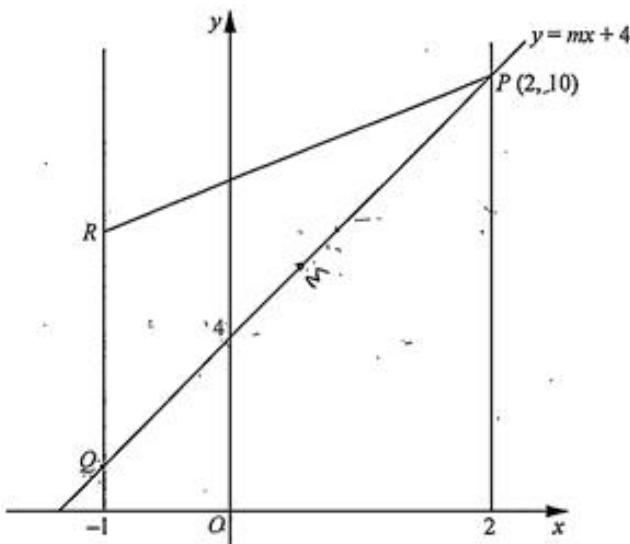
in order to improve as it was not uncommon for c to be given as $\frac{28}{3}$ following $10 = -\frac{2}{3} + c$. This sign slip

may have been avoided if these candidates had included the step $10 + \frac{2}{3} = c$ in their working, rather than calculating this mentally. In part (v) this candidate has detailed a correct method for finding the midpoint, so have clearly understood what is required. An arithmetic slip results in only 1 of the 2 possible marks being awarded. Many candidates working at grade C made errors when dealing with the negative coordinates.

Commonly, candidates at this grade would give $\left(\frac{2-1}{2}, \frac{10-1}{2}\right)$ as their method. These candidates may have improved if they had stated $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ initially. This may have reduced the likelihood of such a sign error being made. In part (vi), candidates needed to be using the correct points with a correct method. This candidate did not have the correct points and therefore no marks were awarded.

Example candidate response – low

- 9 Solutions by accurate drawing will not be accepted.



The line $y = mx + 4$ meets the lines $x = 2$ and $x = -1$ at the points P and Q respectively. The point R is such that QR is parallel to the y -axis and the gradient of RP is 1. The point P has coordinates $(2, 10)$.

- (i) Find the value of m .

$$\begin{aligned} \cancel{y = mx + 4} &= 10 \\ \cancel{x = 2} & \\ \cancel{m} &= 3 \end{aligned}$$

$$\begin{aligned} \cancel{y = mx + 4} &= 9 \\ \cancel{x = 0} & \\ \cancel{m} &= 3 \end{aligned}$$

$$\begin{aligned} \frac{10-4}{2-0} &= 3 \\ 6 &= 3x \\ x &= 2 \end{aligned}$$

[2]

- (ii) Find the y -coordinate of Q .

[1]

$$\begin{aligned} \cancel{\frac{2+y}{2} = 1} & \\ \cancel{2+y} &= 2 \\ y &= -2+2 \\ y &= 0 \end{aligned}$$

$$\begin{aligned} (1, 10) & \\ (-1, 0) & \end{aligned}$$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 10 &= 3(-1 - 2) \\ y &= -6 + 10 \\ y &= 4 \end{aligned}$$

- (iii) Find the coordinates of R .

[2]

$$R = \cancel{(-1, 0)}$$

Marks awarded = (i) 2/2, (ii) 0/1, (iii) 2/2

Example candidate response – low, continued

- (iv) Find the equation of the line through P , perpendicular to PQ , giving your answer in the form $ax + by = c$, where a , b and c are integers. [3]

$$\begin{aligned} & y = mx + c \\ & y - y_1 = m(x - x_1) \\ & y - 10 = 3(x - 2) \quad \rightarrow 6. 3x + 4 = y \\ & y = 3x - 6 + 10 \quad \therefore 3x - y = -4 \\ & y = 3x + 4 \end{aligned}$$

- (v) Find the coordinates of the midpoint, M , of the line PQ . [2]

$$\begin{aligned} M_P &= \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \\ &= \frac{2 + (-1)}{2}, \frac{10 + 4}{2} \\ &= \frac{1}{2}, \frac{14}{2} \\ M_P &= (0.5, 7) \end{aligned}$$

- (vi) Find the area of triangle QRM . [2]

$$\text{Area}_{\triangle QRM} = \frac{1}{2} \left(\begin{array}{cccc} Q & R & M & Q \\ x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{array} \right)$$

Marks awarded = (iv) 0/3, (v) 2/2, (vi) 0/2

Total marks awarded = 6 out of 12

Examiner comment – low

Candidates working at this grade tended to make more arithmetic slips than most. They often did not seem to read the question as carefully as was required and often omitted key elements of the method. For example, in part (iv), it was common for the coordinates of a point other than P to be used in forming the equation or for the equation not to be simplified to the required form.

This candidate starts the question well in part (i). After initial confusion, which they have deleted, they apply a correct method and have earned both marks. In part (ii), they have returned to using the unsimplified form of the equation of the line. This was unnecessary and may have been responsible for the arithmetic slip the candidate makes in simplification. This candidate has already found the y -intercept of the line PQ . If they had considered how reasonable it was, therefore, for Q to have the same y -coordinate as the intercept, they may have realised they had made an error and checked through their method. Checking how reasonable an answer is against given or known information is good practice in questions that have many, connected parts, such as this one. The candidate has the coordinates of R stated correctly. No method has been shown and, if their coordinates had been incorrect, no marks would have been awarded. Candidates should be encouraged to show full and clear method to reduce the possibility of errors being made and to increase the likelihood of method marks or full marks being awarded. In part (iv), the candidate again shows that they have not made connections between relevant parts of this question. They may have benefited from reading the question more carefully and highlighting key words and phrases such as *perpendicular*. They use the gradient of PQ they have found in part (i), rather than finding the perpendicular gradient. If they had thought about the information given in the question and their answer to part (i), they should have realised that they had simply found the equation of the line PQ . This may have alerted them to their error and they may have improved their mark in this question. In part (v), this candidate states the general form of the midpoint. Even though they are not using the correct values, they do apply the correct method, following through their points. Clear and full method is shown and full credit can be given. In part (vi), the candidate considers applying the shoelace method for finding the area. They do not make progress with their method and marks are not awarded for a general statement such as this alone.

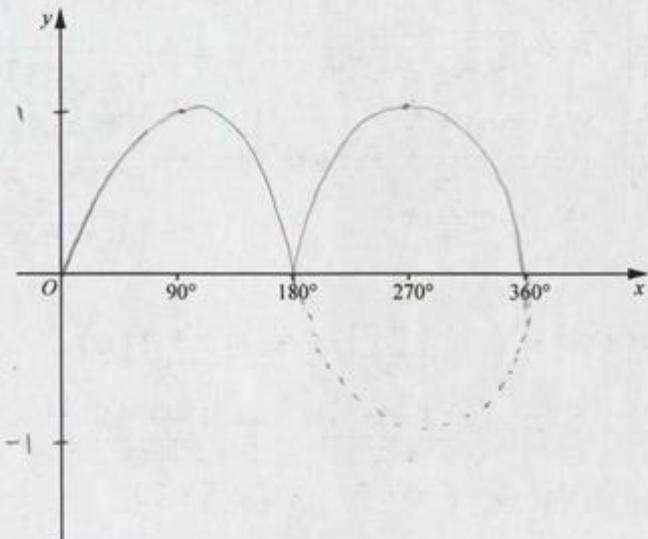
Question 10

Mark scheme

10 (a) 	B2,1,0 correct sinusoidal/reflected sinusoidal shape, all above x -axis with intent to have all maximum points of equal height; 2 maximum points of intended equal height only over 0 to 360; all max points clearly at $y = 1$; cusp at 180
(b)(i) $[hg(x) =] \frac{e^{\ln(4x-3)} + 3}{4}$ fully correct and completion to $[hg(x) =] x$	M1 Alternative method $y = \ln(4x - 3)$ and change of subject to x $\text{oe},$ A1 fully correct and comment that $h(x) = g^{-1}(x) \text{ oe}$
(ii) 	B2,1,0 correct shape; 1 marked on the y -axis or $(0, 1)$ stated close by; curve with positive gradient in first quadrant only
(iii) $x \geq 0 \text{ or } [0, \infty)$	B1 not domain ≥ 0
(iv) $y \geq 1 \text{ or } [1, \infty)$	B1 or $h(x) \geq 1, h \geq 1 \text{ etc.}$

Example candidate response – high

- 10 (a) The function f is defined by $f:x \mapsto |\sin x|$ for $0^\circ \leq x \leq 360^\circ$. On the axes below, sketch the graph of $y = f(x)$. [2]



- (b) The functions g and hg are defined, for $x \geq 1$, by

$$\begin{aligned} g(x) &= \ln(4x - 3), \\ hg(x) &= x. \end{aligned}$$

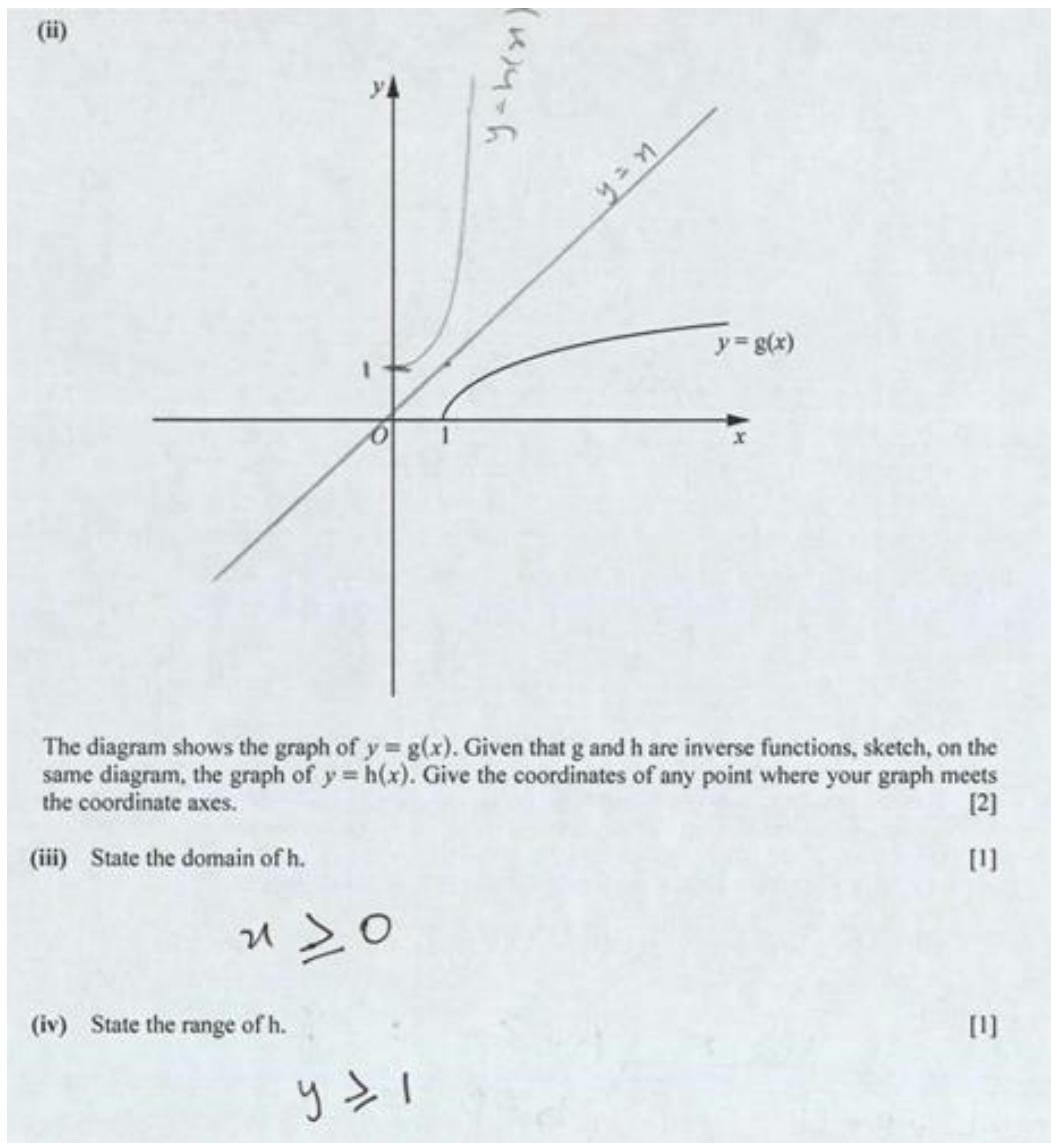
- (i) Show that $h(x) = \frac{e^x + 3}{4}$. [2]

$$\begin{aligned} h(\ln(4x-3)) &= x \\ \ln(\cancel{4x-3}) &\cancel{+ 3} = \ln(4x-3) \\ h^{-1}(x) &= \ln(4x-3) \end{aligned}$$

Marks awarded = (a) 2/2, (b) (i) 0/2

Total marks awarded = 2 out of 4

Example candidate response – high, continued



Marks awarded = (b) (ii) 2/2, (b) (iii) 1/1, (b) (iv) 1/1

Total marks awarded = 6 out of 8

Examiner comment – high

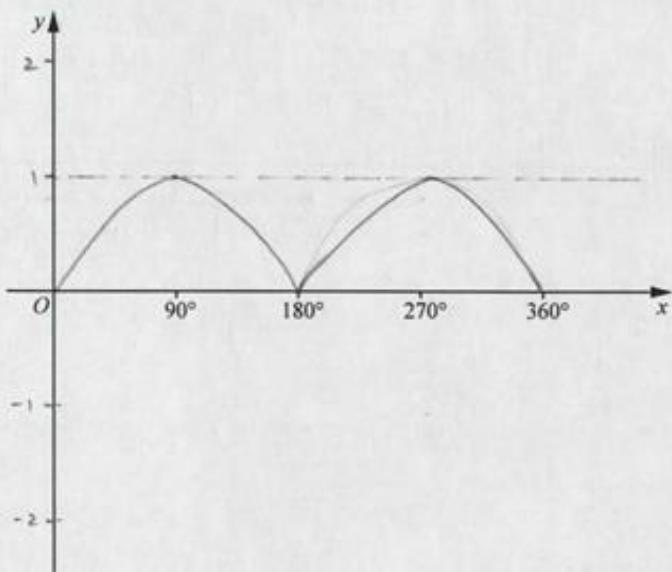
Candidates giving answers at grade A displayed a good and broad working knowledge of functions.

In part (a) this candidate has drawn a good graph, indicating all the key features. The value 1 has been marked on the y -axis, the cusp is positioned correctly and the shape is reasonably good. The candidate has taken care and has two maxima of roughly equal height. As is often apparent in graphs of the modulus of a function, the candidate has drawn the part of the original sine curve that fell below the axis on the graph using dotted lines. This is good practice and is perfectly acceptable.

The candidate starts part (b) (i) using a correct approach. They do not seem to realise what the next step in their method actually is, however. If they had recalled that the inverse of h^{-1} is h and therefore gone on to find the inverse function of what they were given, they would most likely have been successful here too. Many candidates working at this level stated that, as $hg(x) = x$, then h was g^{-1} and correctly found this. In part (b) (ii), the candidate draws a neat and accurate graph, again paying careful attention to detail and key features such as symmetry, shape and the y -intercept. The scale chosen for the vertical axis is the most appropriate as the candidate has attempted to use a square scale and therefore avoids skewing the graph in one direction. They have also drawn the line $y = x$, clearly indicating their method and that they are aware of the symmetrical relationship between the graphs. In part (b) (iii) and (iv), the candidate has chosen the correct notation and the correct values and has shown good knowledge of the domain and range of a function.

Example candidate response – middle

- 10 (a) The function f is defined by $f:x \rightarrow |\sin x|$ for $0^\circ \leq x \leq 360^\circ$. On the axes below, sketch the graph of $y = f(x)$. [2]



- (b) The functions g and hg are defined, for $x \geq 1$, by

$$\begin{aligned} g(x) &= \ln(4x - 3), \\ hg(x) &= x. \end{aligned}$$

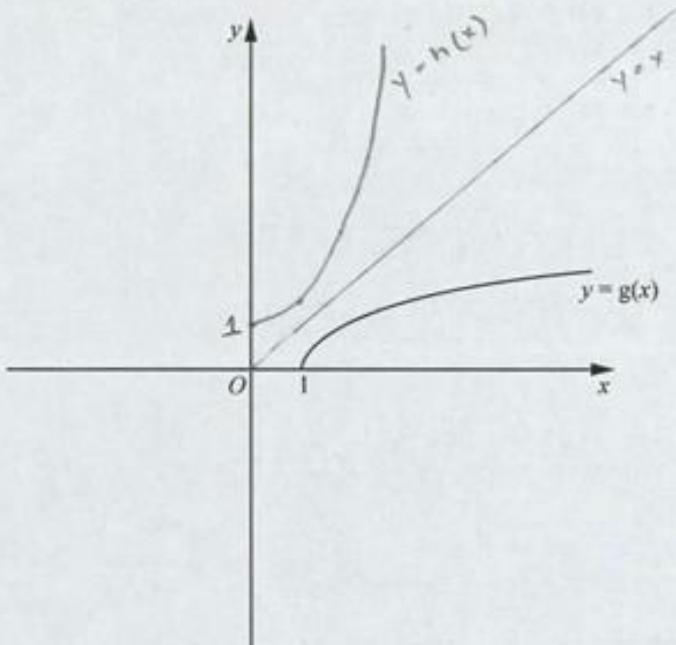
- (i) Show that $h(x) = \frac{e^x + 3}{4}$. [2]

$$\begin{aligned} g(x) &= y \\ y &= \ln(4x - 3) \\ e^y &= \ln e(4x) - \ln e(3) \\ u_n &= e^y + 3 \\ u &= \frac{e^y + 3}{4} \\ g^{-1}(x) &= \frac{e^x + 3}{4} \\ \boxed{h(x) = \frac{e^x + 3}{4}} \quad (g^{-1}(x) = h(x)) \end{aligned}$$

Marks awarded = (a) 2/2, (b) (i) 0/2

Example candidate response – middle, continued

(ii)



The diagram shows the graph of $y = g(x)$. Given that g and h are inverse functions, sketch, on the same diagram, the graph of $y = h(x)$. Give the coordinates of any point where your graph meets the coordinate axes. [2]

(iii) State the domain of h .

[1]

$$h(x) \geq 0$$

(iv) State the range of h .

[1]

$$h(x) \geq 1$$

Marks awarded = (b) (ii) 2/2, (b) (iii) 0/1, (b) (iv) 1/1

Total marks awarded = 5 out of 8

Examiner comment – middle

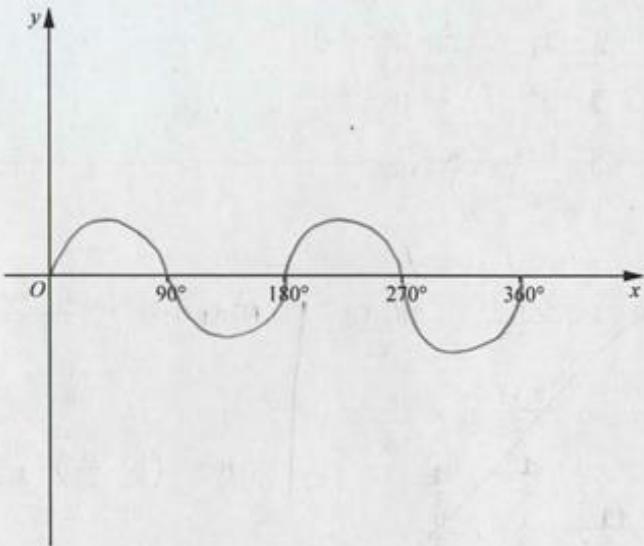
Candidates working at this level generally showed a reasonable working knowledge of functions. Graphs were usually reasonably well drawn and all sections of the question attempted, but notation was not always correct and sometimes there was evidence of exploratory rather than logical method.

This candidate has drawn a reasonable graph in part **(a)**. The graph is good enough to earn both marks – having a good enough shape, and all the key features required in the mark scheme being present. The candidate has drawn a helpful dotted line at $y = 1$ to help them get the level correct. This is good practice.

In part **(b) (i)**, the candidate has realised the $h = g^{-1}$ and attempted to find the inverse of g . They have made fundamental errors in manipulating the logarithms in their second and third lines, however, and therefore no marks have been awarded as the method was flawed. This candidate perhaps would not have made this error if they had had a better understanding of logarithmic and exponential functions. The graph the candidate has drawn in part **(b) (ii)** is neat and clear with all key features indicated and scores full marks. The scale chosen for the vertical axis is the most appropriate as the candidate has attempted to use a square scale and therefore avoids skewing the graph in one direction. They have also drawn the line $y = x$, clearly indicating their method and that they are aware of the symmetrical relationship between the graphs. Some candidates working at this level ignored the information given regarding the inverse nature of the functions h and g . These candidates used the function given in part **(b) (i)** for their sketch here. Often, these candidates omitted to restrict their graph to the first quadrant or they marked the y -intercept at $\frac{3}{4}$ rather than 1. In parts **(b) (iii)** and **(iv)**, this candidate has a clear understanding of which set of values is the domain and which is the range. The notation used for the domain in part **(b) (iii)** is not correct, however, as the candidate has used $h(x)$ rather than just x . Candidates would do better if they paid careful attention to detail such as this, as marks are only awarded if the notation used for domains and ranges is correct.

Example candidate response – low

- 10 (a) The function f is defined by $f: x \mapsto |\sin x|$ for $0^\circ \leq x \leq 360^\circ$. On the axes below, sketch the graph of $y = f(x)$. [2]



- (b) The functions g and hg are defined, for $x \geq 1$, by

$$\begin{aligned} g(x) &= \ln(4x - 3), \\ hg(x) &= x. \end{aligned}$$

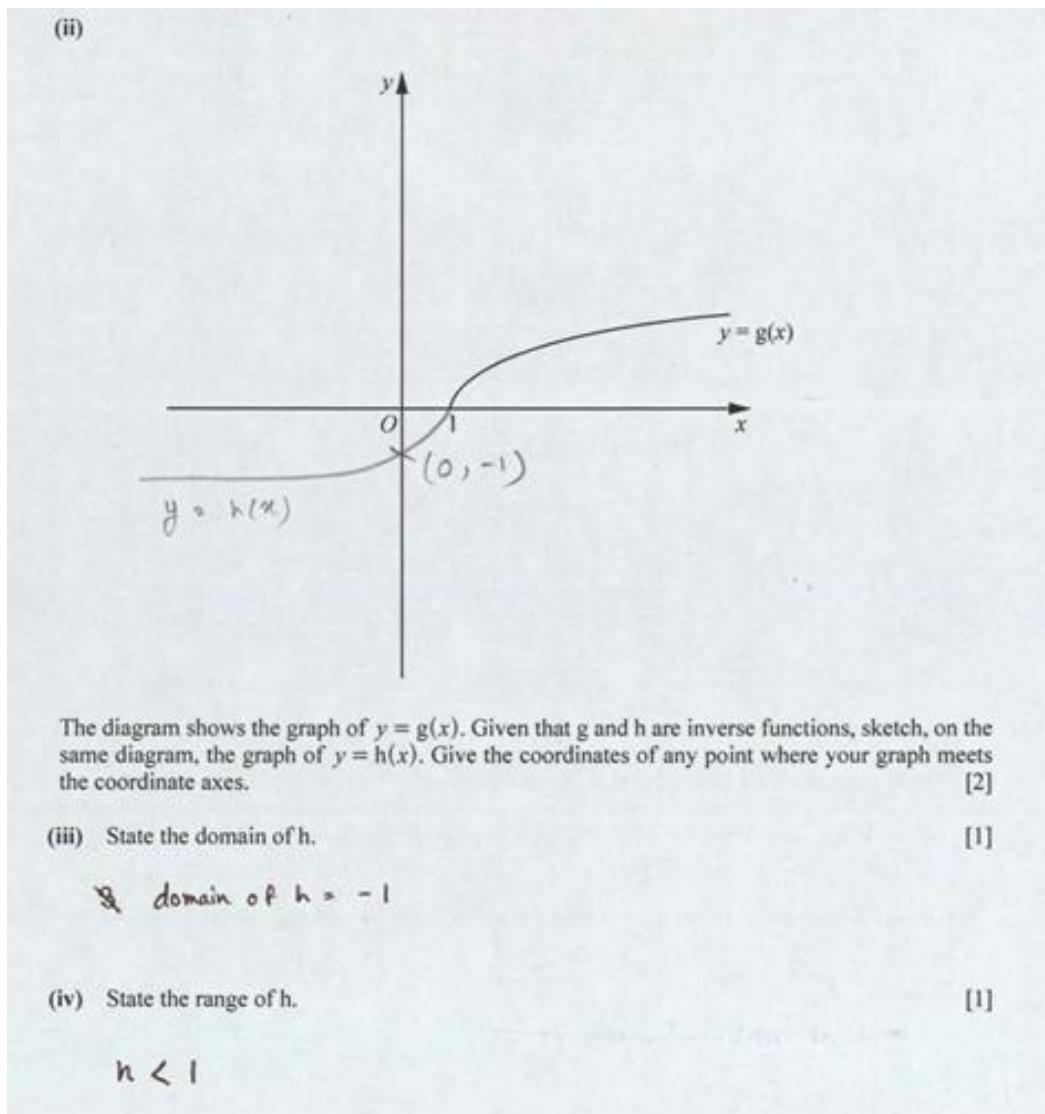
- (i) Show that $h(x) = \frac{e^x + 3}{4}$. [2]

Let,

$$\begin{aligned} g(x) &= y \\ \therefore y &= \ln(4x - 3) \\ 4x - 3 &= e^y \\ x &= \frac{e^y + 3}{4} \\ \therefore g(x) &= \frac{e^x + 3}{4} \quad \text{Ans} \end{aligned}$$

Marks awarded = (a) 0/2, (b) (i) 1/2

Example candidate response – low, continued



Marks awarded = (b) (ii) 0/2, (b) (iii) 0/1, (b) (iv) 0/1

Total marks awarded = 1 out of 8

Examiner comment – low

Candidates working at this grade often showed limited knowledge of functions. The graphs produced at this level often had key features omitted or the shapes were not drawn accurately enough to be awarded credit.

In part (a), it is possible this candidate has simply not made the connection between the graph of a function and the graph of the modulus of that function. The graph given is sinusoidal, but not a basic sine curve, as the period is different. The candidate has omitted to mark at least 1 on the vertical scale. This candidate may have improved if they had started by drawing a basic sine curve. Once that had been done, any section of the curve under the x -axis could have been reflected above and the section under the axis erased.

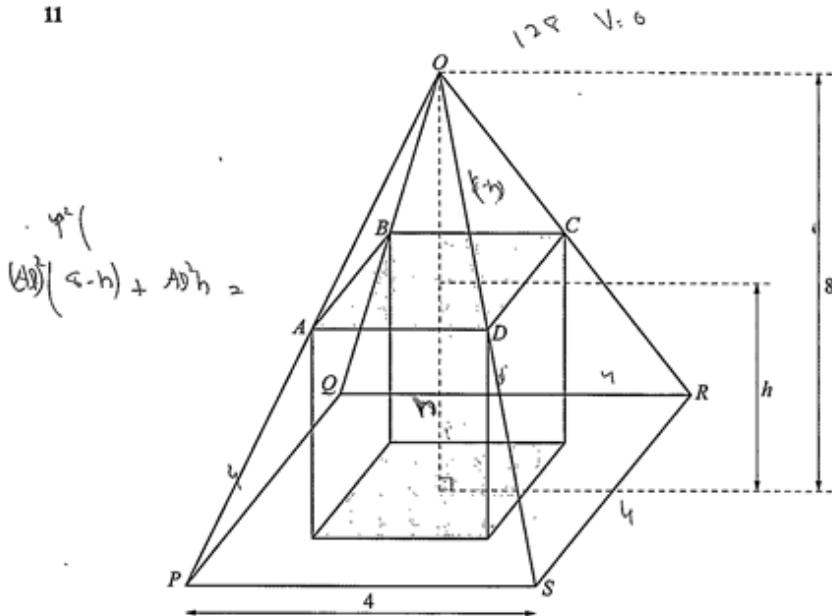
In part (b) (i), the candidate shows some skill in being able to find the inverse of a function. This question required candidates to show that a given result was true. This being the case, candidates using this approach were required to justify why it was valid to do so. This candidate has not quite concluded the argument, labelling their inverse function as $g(x)$ rather than $g^{-1}(x)$ and omitting the statement $h(x) = g^{-1}(x)$. This candidate may have improved in part (b) (ii) if they had recalled that the graph of a function and its inverse are reflections of each other in the line $y = x$. This candidate has not drawn the line on their diagram which would have been helpful. It was not uncommon for candidates working at this grade to draw graphs such as this or to reflect the given graph in one of the coordinate axes. In part (b) (iii) and (iv), the candidate shows some confusion between domain and range. In part (b) (iv) they give the domain for their graph as the range, although they have at least used appropriate notation for a range. In part (b) (iii) they state the point as a single value rather than a set of values. This candidate may have improved if they had understood that the domain of the inverse is the range of the function and vice versa. This would have enabled them to have quoted the correct answers for the last two parts of the question regardless of whether or not their graph was correctly drawn in part (b) (ii). Candidates should be encouraged, where possible, to use information given to them in the question, rather than using their own values or answers. This ensures that accuracy is maintained and marks can be awarded.

Question 11**Mark scheme**

11 (i) $\frac{8-h}{8}$ or $8 : 8 - h$ soi $\frac{8-h}{8} \times 4$ oe $h\left(\frac{8-h}{8} \times 4\right)^2$ oe expand and simplify to $\frac{h^3}{4} - 4h^2 + 16h$ AG	M1 A1 M1 A1	or $\frac{8}{8-h}$ or $8 - h : 8$ soi or $4 \div \frac{8}{8-h}$ oe <i>h</i> must be in the numerator of the expression for this mark;;
(ii) $\frac{3}{4}h^2 - 8h + 16$ oe <i>their</i> $\left(\frac{3}{4}h^2 - 8h + 16\right) = 0$ and attempt to solve $\frac{8}{3}$ oe only	B1 M1 A2	must be a 3-term quadratic; must be an attempt at a derivative or A1 for $h = \frac{8}{3}$ and 8 allow 2.67 or 2.66(6...) rot to 4 or more figs for $\frac{8}{3}$

Example candidate response – high

11



The diagram shows a cuboid of height h units inside a right pyramid $OPQRS$ of height 8 units and with square base of side 4 units. The base of the cuboid sits on the square base $PQRS$ of the pyramid. The points A, B, C and D are corners of the cuboid and lie on the edges OP, OQ, OR and OS , respectively, of the pyramid $OPQRS$. The pyramids $OPQRS$ and $OABCD$ are similar.

- (i) Find an expression for AD in terms of h and hence show that the volume V of the cuboid is given by $V = \frac{h^3}{4} - 4h^2 + 16h$ units 3 . [4]

similar pyramids

$$\frac{8}{8-h} = \frac{4}{AD}$$

$$\frac{AD}{8-h} = \frac{32-h}{8}$$

$$AD = \frac{1}{2}(8-h)$$

$$V = (AD^2) \times h$$

~~$$(32-h)^2$$~~

$$V = \left(\frac{8-h}{2}\right)^2 \times h$$

~~$$64h^3 - 16h^2 \times h^2 \times h$$~~
~~$$32h - 16h^2 + h^3$$~~

$$V = \frac{(64 - 16h + h^2)h}{4}$$

$$= \frac{64h - 16h^2 + h^3}{4}$$

$$\frac{h^3 - 4h^2 + 16h}{4}$$

shown

Marks awarded = (i) 4/4

Example candidate response – high, continued

- (ii) Given that h can vary, find the value of h for which V is a maximum. [4]

$$\frac{dV}{dh} = \frac{3h^2}{4} - 8h + 16 \quad \checkmark \quad \text{B1}$$

$$3h^2 - 32h + 64 = 0 \quad \checkmark$$

$$(h-8)(h-\frac{8}{3}) \quad \text{M1}$$

$$h=8 \quad h=\frac{8}{3} \quad \checkmark \quad \text{reject}$$

$h=8$ gives maximum values so $\boxed{h=8}$ ans.

$$\begin{array}{r} 2162 \\ 2 | 96 \\ 2 | 48 \\ 2 | 24 \\ 3 | 12 \\ 3 | 4 \\ 1 \end{array}$$

Marks awarded = (ii) 3/4

Total marks awarded = 7 out of 8

Examiner comment – high

Candidates working at this grade usually chose the simplest methods and gave clear and detailed evidence of method to support their answer. This was important in part (i) where the answer was given.

In part (i) the candidate forms a correct proportional relationship from the information given. They then use this to find the expression asked for and to derive the formula given.

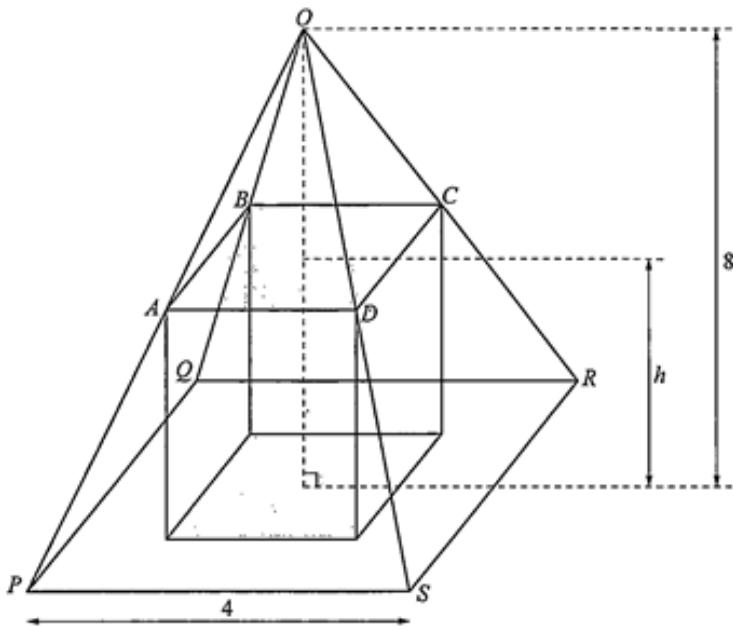
In part (ii), the candidate uses the correct approach. They have differentiated the given formula for the volume and equated the result to zero. They have given a pair of factors in the solution of the quadratic they have found and then stated the roots of the equation. The candidate would have earned all the marks if they had perhaps re-read the question at this point. If they had done so, they may have realised, as many

candidates did, that 8 was not a possible value and so the answer was $\frac{8}{3}$. Other candidates applied the second derivative test and also arrived at the correct final answer.

It is likely that this candidate has found their factors using the roots of the equation given by their calculator, as the form of the factors does not match the form of the equation. Candidates should try to avoid this. Method marks are only awarded if the solutions to the equation are correct in the case when a candidate has omitted to show their method. Candidates should be encouraged to use their calculator to check their solutions rather than as a replacement for method being shown.

Example candidate response – middle

11



- (i) Find an expression for AD in terms of h and hence show that the volume V of the cuboid is given by $V = \frac{h^3}{4} - 4h^2 + 16h$ units 3 .

$$AD = \left(4 - h\right) \left(\frac{8-h}{2}\right) \checkmark$$

$$\left(\frac{8}{3}\right)\left(\frac{8-h}{2}\right) \cdot \sqrt{2} \frac{h^3}{4} - 4h^2 + 16.$$

Marks awarded = (i) 2/4

Example candidate response – middle, continued

- (ii) Given that
- h
- can vary, find the value of
- h
- for which
- V
- is a maximum.

[4]

$$\frac{dV}{dh} = \frac{3h^2 - 8h + 16}{4}$$

✓

$$\frac{3h^2 - 8h + 16}{4} = 0$$

$$\frac{3h^2 - 32h + 64}{4} = 0$$

$$3h^2 - 32h + 64 = 0$$

mark

$$h=8$$

$$h=2.667$$

$$h=\frac{8}{3}$$

When V is a maximum.

Marks awarded = (ii) 3/4

Total marks awarded = 5 out of 8

Examiner comment – middle

Candidates at this grade show some understanding of the method required to solve the problem. Occasionally, methods were chosen that caused a significant increase in the complexity of the work required. For example, candidates compared areas or volumes rather than lengths. Also, some candidates at this level chose to compare lengths in similar triangles within the pyramids, rather than simply using the information given in the question. These approaches increased the possibility of an error being made. Candidates would do better to realise that using the dimensions given in the question resulted in the quickest and simplest method of solution.

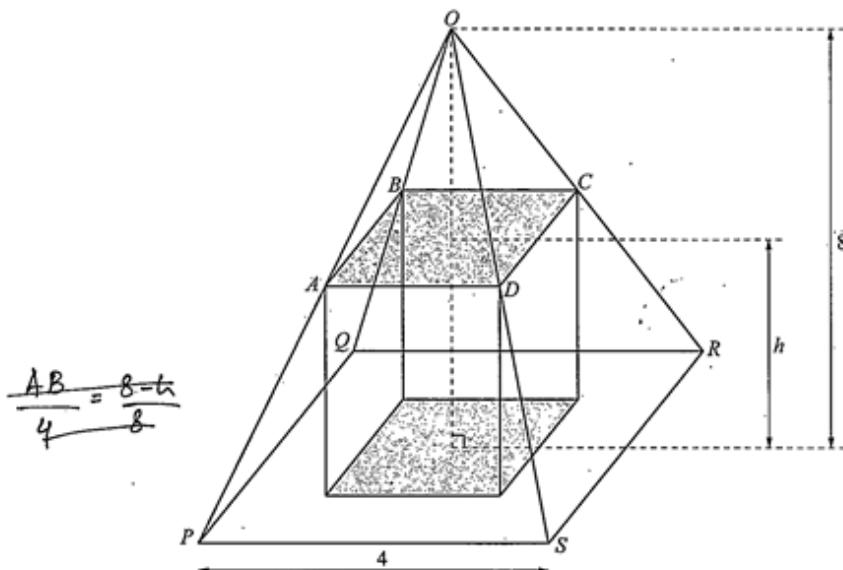
In part (i), this candidate states the correct expression for AD without any evidence of method. The form of the expression given is sufficient for the candidate to be awarded 2 marks. This comes directly from the dimensions given. It would have been useful if the candidate showed the first step in their thinking which is likely to have been something along the lines of $PS = \frac{1}{2} \times 8$ therefore $AD = \frac{1}{2} \times (8 - h)$.

This candidate omits the method required to find the given formula for the volume. They have attempted to use the maximum value they have found in part (ii) in some way here, but this is not valid. This candidate may have improved if they had reread the question and realised that as the pyramids were square-based, they had all the dimensions required to find the volume of the cuboid. This question directly assessed problem solving skills mentioned in the Assessment Objectives in this syllabus i.e. *formulate problems into mathematical terms and select and apply appropriate techniques of solution*.

In part (ii) the candidate successfully differentiates the given formula and equates to zero. No method of solution is shown and candidates should be aware that method marks can only be awarded if an answer is incorrect if the method is given. In this case the candidate's answers are correct. It is useful to check answers using a calculator. It is much better to detail the full method of solution first. This candidate rejects the correct answer in favour of the incorrect one. If they had reread the question or applied a relevant test, this would have been less likely to happen. It is important to encourage students at this level to make connections between parts of questions that are linked to check how reasonable their answers are.

Example candidate response – low

11



The diagram shows a cuboid of height h units inside a right pyramid $OPQRS$ of height 8 units and with square base of side 4 units. The base of the cuboid sits on the square base $PQRS$ of the pyramid. The points A , B , C and D are corners of the cuboid and lie on the edges OP , OQ , OR and OS , respectively, of the pyramid $OPQRS$. The pyramids $OPQRS$ and $OABCD$ are similar.

- (i) Find an expression for AD in terms of h and hence show that the volume V of the cuboid is given

$$\text{by } V = \frac{h^3}{4} - 4h^2 + 16h \text{ units}^3.$$

$$\text{Volume of pyramid} = (4)^2 \times 8 \quad [4]$$

$$\frac{AD}{4} = \frac{8-h}{8} \quad \checkmark \rightarrow AD = \frac{32-4h}{8} = 4 \left(\frac{8-h}{8} \right) = \frac{8-h}{2}$$

$$\text{Volume of cuboid} = BL^2 \times b \times h \quad \checkmark$$

$$= \cancel{\left(\frac{32-4h}{8} \right)} \times \left(\frac{8-h}{2} \right) \times (h) \times (\text{breadth})$$

Marks awarded = (i) 2/4

Example candidate response – low, continued

- (ii) Given that h can vary, find the value of h for which V is a maximum. [4]

$$\frac{dV}{dh} = \frac{h^3}{4} - 4h^2 + 16h = \frac{3h^2}{4} - 2(4h) + 16$$

✓

$$\frac{3h^2}{4} - \frac{8h}{1} + \frac{16}{1} = 0 \approx$$

$$\frac{3h^2 - 32h + 64}{4} = 0 \rightarrow 3h^2 - 32h + 64 = 0$$

Either $\cancel{h = -2.7}$ (fd不符)
OR
MO $h = -8$ (N.A) \cancel{x}

$$\frac{d^2V}{dh^2} = 6h - 32, \text{ for } h = -2.7 = -48.2 (< 0) \text{ max.}$$

$$\therefore V = \frac{(2.7)^3}{4} + 4(2.7)^2 + 16(2.7) = 77.3 \text{ cm units}^3$$

Marks awarded = (ii) 1/4

Total marks awarded = 3 out of 8

Examiner comment – low

Candidates working at this level showed some understanding of the methods required. They were able to start both parts of the question, but most often, were unsure of how to complete the methods successfully.

This candidate starts part (i) correctly – forming a correct proportional relationship and correctly stating an expression for AD . They do not realise that the dimension they are missing, the *breadth*, is given by the fact that the pyramid is square based. Re-reading the question may have helped this candidate score more highly in this part.

In part (ii), the candidate again starts the method successfully. The expression given in part (i) has been correctly differentiated. The candidate then manipulates the equation into a more useful form and does so correctly. They then state two solutions without giving any method. Both solutions are incorrect and the candidate cannot be awarded the methods mark or the accuracy marks as the method had been omitted. This candidate would have done better if the method had been stated. This candidate has not made the connection between the practical context of the question and the mathematics they are using. If they had made that connection, they may have realised that the values of h they had found were highly unlikely to be correct as they were negative. Candidates need to make sense of their answers in the context of the question, where a context is given. This candidate has done more work than was necessary in this part. They have found what they believe to be the maximum volume. This was not penalised in any way, but did waste some time.

Question 12**Mark scheme**

12	(i) $-120 + 104 + 22 - 6 = 0$ or correct unsimplified form, e.g. $15(-2)^3 + 26(-2)^2 - 11(-2) - 6 = 0$ or $15(-8) + 26(4) - 11(-2) - 6 = 0$	B1	or correct synthetic division $\begin{array}{r} -2 \\ \hline 15 & 26 & -11 & -6 \\ & -30 & 8 & 6 \\ \hline 15 & -4 & -3 & 0 \end{array}$
(ii)	Substituting $x = 3$ into $15x^3 + 26x^2 - 11x - 6$	M1	or correct synthetic division $\begin{array}{r} 3 \\ \hline 15 & 26 & -11 & -6 \\ & 45 & 213 & 606 \\ \hline 15 & 71 & 202 & 600 \end{array}$
	600	A1	correct answer implies M1; must be explicitly identified as answer if using synthetic/long division methods by e.g. circling
(iii)	$(x - 1)(15x^3 + 26x^2 - 11x - 6)$ soi	B1	by inspection or division; may be implied by e.g. $(ax + b)(15x^3 + 26x^2 - 11x - 6)$ and $a = 1, b = -1$ seen in later work comparing coefficients
	Multiply out $(x \pm 1)(15x^3 + 26x^2 - 11x - 6)$ and compare coefficients of x^3 or x to quartic	M1	or multiply out, e.g. $(ax + b)(15x^3 + 26x^2 - 11x - 6)$ and compare coefficients of x^3 or x to quartic
	$p = 11$	A1	correct p or q implies M1; correct p and q www implies B1 M1
	$q = 5$	A1	

Example candidate response – high

- 12 (i) Show that $x = -2$ is a root of the polynomial equation $15x^3 + 26x^2 - 11x - 6 = 0$. [1]

$$f(-2) = 15(-2)^3 + 26(-2)^2 - 11(-2) - 6 \quad \cancel{\text{and}}$$

$$R = 0$$

$(x+2)$ is a factor.

- (ii) Find the remainder when $15x^3 + 26x^2 - 11x - 6$ is divided by $x - 3$. [2]

$$f(3) = 15(3)^3 + 26(3)^2 - 11(3) - 6$$

$$R = 600$$

- (iii) Find the value of p and of q such that $15x^3 + 26x^2 - 11x - 6$ is a factor of $15x^4 + px^3 - 37x^2 + qx + 6$. [4]

$$(x-a)(15x^3 + 26x^2 - 11x - 6)$$

$$15x^4 + 26x^3 - 11x^2 + 6x - 15ax^3 - 26ax^2 + 11ax - 6a$$

$$\cancel{15x^4 + 26x^3 - 15ax^3} - 11x^2 - 26ax^2 - 6x + 11ax - 6a$$

$$-11 - 26a = -37$$

$$-26a = -37$$

$$a = \frac{-26}{-26}$$

$$a = 1$$

$$26 - 15(1) = p \quad \cancel{\text{and}} \quad -6 + 11(1) = q$$

$$p = 11$$

$$q = 5$$

$$p = 11$$

$$q = 5$$

Marks awarded = (i) 1/1, (ii) 2/1, (iii) 4/4

Total marks awarded = 7 out of 7

Examiner comment – high

Candidates often answered part (i) and part (ii) of this question very successfully. The simplest approaches – using the factor theorem and the remainder theorem respectively – were usually made by better candidates. Part (iii) was more challenging, with only the best candidates using the neatest method of solution which was to find the linear factor and expand. The linear factor in this case was easily found by inspection and it was acceptable to simply state what it was.

This candidate shows full and correct method for parts (i) and (ii). In part (iii), they realise that the linear factor can be found and construct a simple general factor then expand. They are accurate and clear in their working. They quickly arrive at the correct linear factor and the correct values for p and q .

Example candidate response – middle

- 12 (i) Show that $x = -2$ is a root of the polynomial equation $15x^3 + 26x^2 - 11x - 6 = 0$. [1]

$$\begin{aligned} 15(-2)^3 + 26(-2)^2 - 11(-2) - 6 &= 0 \\ -120 + 104 + 22 - 6 &= 0 \\ -16 + 16 &= 0 \\ 0 &= 0 \end{aligned}$$

shown.

- (ii) Find the remainder when $15x^3 + 26x^2 - 11x - 6$ is divided by $x - 3$. [2]

$$\begin{aligned}
 n &= 3 \\
 R &= 15(3)^3 + 26(3)^2 - 11(3) - 6 \\
 &= 15(27) + 26(9) - 11(3) - 6 \\
 &= 405 + 234 - 33 - 6 \\
 R &= 600
 \end{aligned}$$

- (iii) Find the value of p and of q such that $15x^3 + 26x^2 - 11x - 6$ is a factor of $15x^4 + px^3 - 37x^2 + qx + 6$. [4]

$$\begin{array}{l}
 15n^3 + 26n^2 - 11n - 6 = 0 \\
 \hline
 -2 \mid \begin{array}{r} 15 & 26 & -11 & -6 \\ \times & -30 & 8 & 6 \\ \hline 15 & -4 & -3 & 0 \end{array} \\
 (n+2)(15n^2 - 4n - 3) = 0 \\
 n = -2, \quad 15n^2 - 4n - 3 = 0 \\
 3n(5n + 1) + 1(5n + 1) = 0 \\
 n = -\frac{1}{5}, \quad n = -\frac{1}{3} \\
 \\
 15(-2)^4 + p(-2)^3 - 37(-2)^2 + q(-2) + 1 \\
 98 - 8p - 2q = 0 \\
 98 = 8p + 2q \\
 \text{(Cross)} \\
 \frac{56}{27} + \frac{p}{27} = \frac{q}{3} \\
 \frac{56}{27} = \frac{p + 9q}{27} \\
 2p + 2q = 112 \\
 8p + 2q = 98 \\
 \hline
 -6p = 14 \\
 p = -2.33, q = 58.3
 \end{array}$$

Marks awarded = (i) 1/1, (ii) 2/2, (iii) 2/4

Total marks awarded = 5 out of 7

Examiner comment – middle

Candidates working at this level generally answered part (i) and part (ii) correctly. Many candidates chose to divide out or use synthetic division to answer these parts. Occasionally this caused an error. Some candidates who used a division method omitted to identify the remainder in part (ii) which was a requirement for both marks to be awarded. Commonly 1 or 2 marks only were awarded for part (iii), which these candidates found challenging.

This particular candidate has used the factor theorem and remainder theorem in parts (i) and (ii) and done so very well. In part (iii) they have found the roots of the cubic equation and used two of them to form a correct pairs of equations in p and q . They have been awarded 2 marks for arriving at this point in their valid method correctly. They have made an error in solving their equations and have therefore not earned the final two marks. The candidate may have improved if they had checked, for example, that -2 was a root of the quartic they had found with their p and q . They should have realised it was not and this should have resulted in the candidate checking their method for a possible error.

Example candidate response – low

- 12 (i) Show that $x = -2$ is a root of the polynomial equation $15x^3 + 26x^2 - 11x - 6 = 0$. [1]

$$15(-2)^3 + 26(-2)^2 - 11(-2) - 6 = 0$$

$$15(-8) + 26(4) + 22 - 6 = 0$$

$$-120 + 104 + 22 - 6 = 0$$

$$0 = 0 \quad \underline{\text{shown}}.$$

- (ii) Find the remainder when $15x^3 + 26x^2 - 11x - 6$ is divided by $x - 3$. [2]

$$x - 3 = 0$$

$$x = 3$$

$$15(3)^3 + 26(3)^2 - 11(3) - 6$$

$$15(27) + 26(9) - 33 - 6$$

$$405 + 234 - 27$$

$$612 \quad \underline{\text{Ans.}}$$

- (iii) Find the value of p and of q such that $15x^3 + 26x^2 - 11x - 6$ is a factor of

$$15x^4 + px^3 - 37x^2 + qx + 6$$

[4]

$$15x^4 + px^3 - 37x^2 + qx + 6 = 15x^3 + 26x^2 - 11x - 6$$

$$x^3 = x^3$$

$$px^3 = 15x^3$$

$$p = 15$$

$$x = x$$

$$qx = -11x$$

$$q = -11$$

$$p = 15, q = -11 \quad \underline{\text{Ans.}}$$

Marks awarded = (i) 1/1, (ii) 1/2, (iii) 0/4

Total marks awarded = 2 out of 7

Examiner comment – low

Candidates working at grade E usually demonstrated some knowledge of factors and remainders. Occasionally they made arithmetic slips in part (i) and part (ii). In part (iii) many candidates substituted $x = -2$ into the quartic and equated to 0. This was valid as -2 was a root of the cubic and therefore also a root of the quartic. They then, however, tended to substitute $x = 3$ into the quartic and equate to 600. This was not a valid approach. The candidate had misinterpreted the information from part (ii) and they made no further progress.

In this case, the candidate successfully completes part (i). In part (ii), their method is correct. They make an arithmetic slip in calculating $-33 - 6$ as -27 and therefore have not been awarded the accuracy mark. Part (iii) required a good understanding of factors of polynomials. This candidate offers the other, most common solution at this grade. They have equated the quartic and the cubic and written down the values of p and q that are given by comparing coefficients. This candidate may have realised that the correct method was not quite as "simple" as this if they had looked at the total mark awarded for the question.

This part of this question proved to be a good discriminator across all grades.

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