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When order execution meets informed trading

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Various participants trade simultaneously in modern financial markets. We study the interaction between the institutional investor driven by a position target and multiple informed traders possessing long-lived information. The problem is formulated as a Stackelberg-Nash game in the linear price impact model. We solve the problem explicitly and obtain the unique equilibrium. Informed traders first engage in predatory trading and then quickly provide much liquidity to the institutional investor. The advantage in transaction costs encourages informed traders to intensify predation but fierce competition forces them to focus on liquidity provision. The institutional investor always employs a well-known U-shape trading strategy regardless of the magnitude of transaction costs and the number of informed traders, and tends to trade more uniformly with high transaction costs and more informed traders. Orders posted by the institutional investor distort the originally efficient price while informed traders trade in the opposite direction to keep the price from deviating too far from the value. We propose a conjecture that transient price impact may be caused by informed trading. An approximately exponential decay is derived with linear permanent and temporary price impacts, and the convergence speed depends on informed traders' transaction costs.

Keywords: Order execution; Informed trading; Price impact; Stochastic differential game; Stackelberg equilibrium

1. Introduction

In modern financial markets, a large number of market participants trade an asset simultaneously for different purposes. They employ suitable trading strategies according to their goals, resulting in various types of trading patterns. More importantly, different trading activities are usually intertwined and affect each other. In the extensive previous literature, both order execution and informed trading have been studied quite thoroughly because both types of trading activities have a direct link to the wealth and have a significant impact on price dynamics. In this paper, we focus on the interaction of these two types of trading activities. By considering a Stackelberg-Nash game, we explore how an institutional investor optimally liquidate a prescribed position in the presence of informed traders who have long-lived information.

In reality, many transactions are driven by position targets. For example, fund managers need to open or unwind a prescribed position within a fixed time horizon when constructing or rebalancing their portfolios. Therefore, they require an order execution strategy to minimize transaction costs. The order execution problem was earliest studied by Bertsimas and Lo (1998) and Almgren and Chriss (2000). Considering linear price impact, the TWAP strategy was found to be optimal. Subsequently, in order to deal with various issues encountered in practice, risk aversion (Almgren and Chriss 2000, Forsyth *et al.* 2012, Cartea *et al.* 2015), nonlinear price impact (Almgren 2003, Alfonsi *et al.* 2010), resilience and transient impact (Gatheral 2010, Alfonsi *et al.* 2012, Gatheral *et al.* 2012, Obizhaeva and Wang 2013, Graewe and Horst 2017, Chen *et al.* 2019), execution uncertainty (Cheng *et al.* 2017, Carmona and Leal 2023), targeting VWAP (Frei and Westray 2015, Cartea and Jaimungal 2016), trading signals (Bechler and Ludkovski 2015, Cartea and Jaimungal 2016, Lehalle and Neuman 2019, Forde *et al.* 2022, Neuman and Voß 2022), and other topics (see Donnelly 2022) have been taken into account extensively. Various strategies are thus developed to cope with different scenarios under different price dynamics, and the trader can use suitable order execution strategies to effectively reduce transaction costs.

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Another important type of market participants are informed traders, who possess long-lived information. They know the value of an asset and can exploit this information asymmetry to earn the spread between price and value. Such information-motivated trading activity was first studied by Kyle (1985) and extended by Back (1992), Back and Pedersen (1998), Baruch (2002), Foucault *et al.* (2016) and Barger and Donnelly (2021). Roughly speaking, although informed traders hide in the market, their orders cause an imbalance in the order flow, and other market participants can adjust the market price based on the order flow to avoid adverse selection. As a result, informed traders disclose long-lived information to the market while making profits through trading. This allows the price to better reflect the value, leading to good price efficiency.

Further, since financial markets involve a large number of participants, besides studying the trading behavior of a single trader, it is also of vital importance to consider the competition between multiple traders. It is worth noting that competition may cause the original result to become very different. For example, Kyle (1985) and Back (1992) showed that a single informed trader will disclose long-lived information uniformly, whereas competition causes the information to be revealed much more rapidly (see Holden and Subrahmanyam 1992, Back *et al.* 2000, Xu and Shi 2024). For order execution, competition triggers predatory trading (Carlin *et al.* 2007), and liquidity provision may also occur in the risk-averse case (Schied and Zhang 2017). In addition, many of the original single-trader models can be extended by considering competition (see Huang *et al.* 2019, Voß 2022, Micheli *et al.* 2023, Neuman and Voß 2023, Dong *et al.* 2024). These models regarding competition are sometimes difficult to solve, but the final results can lead to a better understanding of complex trading patterns in reality. However, current studies mainly focus on competition between the same type of market participants, while competition between different types of traders remains to be further explored. Earlier works are done by Moallemi *et al.* (2012), Choi *et al.* (2019) and Yang and Zhu (2020). Recently, Cont *et al.* (2023) explored the interaction between a liquidity trader and a high-frequency trader, and Bergault and Sánchez-Betancourt (2024) and Cartea *et al.* (2024) considered the broker who provides liquidity to informed traders in over-the-counter markets. Since different types of traders may have different observations and purposes, these models are extremely complex and are hardly tractable. However, such researches deserve further attention because in reality, complex trading activities may stem from competition among different types of market participants and price dynamics are exactly the result of the interaction of different types of market participants.

In this paper, we focus on the interaction of liquidity traders and informed traders. The risk-neutral institutional investor needs to choose an order execution strategy to fulfill the position target within a specified time horizon, and risk-neutral informed traders can trade with long-lived information to maximize their profits. This setup is similar to the one in Choi *et al.* (2019). However, instead of considering the case of one liquidity trader and one informed trader, we study the interaction of one liquidity trader and multiple informed traders. And we choose the price impact model to better consider the impact of transaction costs on trading patterns. In addition, we also take into account the setup in Cont *et al.* (2023) by considering a Stackelberg-Nash game. At the follower level, informed traders know both the institutional investor's trading intention and long-lived information, so they can either engage in predatory trading as described in Carlin *et al.* (2007) and Dong *et al.* (2024), or they can engage in informed trading with long-lived information. We explore how informed traders should balance these two types of trading activities and how they trade in a Nash competition. At the leader level, the institutional investor is not aware of long-lived information, but is aware of the presence of informed traders in the market. We explore how the institutional investor can optimally execute orders in the presence of informed traders.

Our main result is an explicit expression for the unique Stackelberg-Nash equilibrium. For informed traders, given any admissible strategy of the institutional investor, we employ a convex analytic approach to derive a system of coupled linear forward-backward stochastic differential equations (FBSDEs) and further obtain the unique open-loop Nash equilibrium in feedback form. This method was initially proposed by Bank *et al.* (2017) and also used in Neuman and Voß (2022), Forde *et al.* (2022), Voß (2022) and Neuman and Voß (2023). For the institutional investor, given informed traders' overall reactions, we derive the optimal order execution strategy via a calculus of variation argument. The uniqueness can also be guaranteed. As a result, by solving the problem with two steps, we obtain the unique Stackelberg-Nash equilibrium. Despite the complexity of the problem, especially the optimization problem for the institutional investor, trading rates of both informed traders and the institutional investor are solved explicitly.

The price is generally efficient due to the presence of informed traders, but trading activities of the institutional investor disrupt the initial efficiency and activate informed traders. Trading activities of the institutional investor cause the price to deviate from the value, and informed traders can trade in the opposite direction to earn the spread as well as engage in predatory trading to gain additional revenues. From our explicit solution, we find that on the whole informed traders provide liquidity to the institutional investor, but prefer to compete for liquidity at the beginning. Predatory behaviors of informed traders depend on transaction costs and the intensity of competition among informed traders. When transaction costs are high for the institutional investor or low for informed traders, informed traders will engage more in predatory trading. When competition among informed traders is quite fierce, they will focus more on providing liquidity and maintaining price efficiency. Furthermore, the institutional investor always employs the well-known U-shape trading strategy, which means that the institutional investor trades more rapidly at the beginning and at the end. This trading pattern has been found in several papers on transient price impact (Gatheral *et al.* 2012, Obizhaeva and Wang 2013). From our explicit solution, we find that when there are a large number of informed traders in the market, intense competition makes them unconcerned about the trading intention of the institutional investor and fully focus on long-lived information. Permanent price impact caused by order execution are gradually offset by informed trading, and in the linear price impact model we find that this decay is exponential and the convergence speed depends on informed traders' transaction costs. This property is very similar to transient price impact, and our limit equilibrium strategy for the institutional investor matches the result of the order execution problem with temporary and transient price impacts in Chen *et al.* (2019). Therefore, we propose a reasonable conjecture that transient price impact may be caused by informed trading.

This paper is organized as follows. Section 2 formulates the Stackelberg-Nash game between the institutional investor and informed traders in the linear price impact model. Section 3 solves the problem explicitly and provides the unique Stackelberg-Nash equilibrium. Section 4 presents some numerical results and analyzes trading patterns of the institutional investor and informed traders. Section 5 concludes.

2. Model setup

We divide traders in the market into three types: the risk-neutral institutional investor, risk-neutral informed traders, and other market participants. We consider the Stackelberg-Nash game between the institutional investor and $N(\geq 2)$ informed traders. The institutional investor, regarded as the leader, knows the presence of informed traders and needs to fulfill a prescribed position target within a fixed time horizon. Informed traders, regarded as followers, know the institutional investor's position target and have long-lived information, so they can engage in both predatory and informed trading. Other market participants don't know the institutional investor's position target or long-lived information, but they are aware of the presence of informed traders. Therefore, they adjust the market price based on the average information content of each order to avoid adverse selection.

The trading horizon is defined as $[0, T]$. Both the institutional investor and the ith informed trader trade continuously in the market with trading rates v_t and ξ_t^i $(i = 1, 2, \ldots, N)$. We use Y_t to denote the total trading rate of informed traders $\sum_{i=1}^{N} \xi_t^i$. We consider a linear response of other market participants to order flow, which corresponds to linear permanent price impact in Almgren and Chriss (2000) and is also consistent with the linear pricing rule in Kyle (1985). The price process S_t is hence given by

$$dS_t = \gamma(v_t + Y_t)\, dt + \sigma_1\, dB_t^1, \tag{1}$$

where γ and σ_1 are positive constants and B_t^1 is a Brownian motion. The first term is linear permanent price impact and the second term is stochastic fluctuations inherent in price dynamics. Informed traders are privy to long-lived information, which enables them to know the value of the asset V_t. We follow Foucault *et al.* (2016) and Xu and Shi (2024) to simply assume that the value process V_t is driven by

$$dV_t = \sigma_2\, dB_t^2, \tag{2}$$

where σ_2 is a positive constant and B_t^2 is a Brownian motion. B_t^1 and B_t^2 are assumed to be independent from each other. As risk-neutral informed traders only respond to the spread between price and value, we define the spread $\Lambda_t = V_t - S_t$, which is driven by

$$d\Lambda_t = -\gamma(v_t + Y_t)\, dt + \sigma\, dW_t, \tag{3}$$

where the positive constant σ and the Brownian motion W_t satisfy that $\sigma W_t = \sigma_2 B_t^2 - \sigma_1 B_t^1$. Due to the presence of informed traders, the price is generally efficient before the institutional investor arrives, which means that the price does not deviate too much from the value at time 0 ($|\Lambda_0|$ is small). When the institutional investor starts trading, large orders make the price deviate significantly from the value, and Λ_0 is trivial compared to the institutional investor's impact. Therefore, we simply suppose $\Lambda_0 = 0$ so that the inefficiency of the price during the trading horizon results from order execution and other fluctuations.

The institutional investor knows the price is efficient at time 0 ($\Lambda_0 = 0$) due to the presence of informed traders. The institutional investor has an initial position Q_0 and needs to unwind the position. The institutional investor's position process Q_t follows

$$dQ_t = v_t\, dt. \tag{4}$$

We consider deterministic trading strategies for the institutional investor, and the class of admissible trading trajectories is defined as

$$\mathcal{B} = \left\{ Q :\ Q_t = Q_0 + \int_0^t v_t\, dt,\ v \text{ is deterministic}, \right.$$
$$\left. \int_0^T |v_t|^2\, dt < +\infty,\ Q_T = 0 \right\}. \tag{5}$$

For risk-neutral informed traders, their positions don't affect the development of trading strategies. For simplicity we assume the initial position $q_0^i = 0$ $(i = 1, 2, \ldots, N)$. The ith informed trader's position process q_t^i follows

$$dq_t^i = \xi_t^i\, dt. \tag{6}$$

We consider adaptive trading strategies for informed traders. $\{\mathcal{F}_t\}_{0 \leq t \leq T}$ is the filtration generated by Λ_t, and the class of admissible trading rates is defined as

$$\mathcal{A} = \left\{ \xi \text{ is } \mathcal{F}_t\text{-adapted} :\ \mathbb{E} \int_0^T |\xi_t|^2\, dt < +\infty \right\}. \tag{7}$$

Note that a positive trading rate v_t or ξ_t^i means buying and negative means selling.

Both the institutional investor and informed traders suffer transaction costs due to linear temporary price impact. As these two types of market participants are likely to trade at different frequencies, we separately consider transaction costs of the two levels as in Cont *et al.* (2023) to describe different transaction costs for different types of market participants. The transaction price for the institutional investor is given by

$$\widetilde{S}_t^I = S_t + \eta_1 v_t, \tag{8}$$

where η_1 is a positive constant. We define $m_1 = \frac{\gamma}{\eta_1}$ to measure the magnitude of transaction costs. Informed traders form a symmetric Nash competition and their temporary price impact is assumed to be interwined (Carlin *et al.* 2007, Schied and Zhang 2017, Micheli *et al.* 2023) because they trade on the same time scale. The transaction price for informed traders is given by

$$\widetilde{S}_t^f = S_t + \eta_2 Y_t, \tag{9}$$

where η_2 is a positive constant† . We also define $m_2 = \frac{\gamma}{\eta_2}$. A bigger η or a smaller m means higher transaction costs. Given that informed traders in the market may be played by high-frequency traders, they have more sophisticated order placement strategies as well as lower latency, thus having lower transaction costs. As a result, η_2 should be smaller than η_1, and m_2 is larger than m_1.

For risk-neutral informed traders, at every time t they actually earn the spread between the value and the transaction price. Given the institutional investor's trading trajectory Q and other informed traders' trading strategies ξ^{-i}, the trading revenue of the ith informed trader is hence given by

$$\begin{aligned} J^i(\xi^i;\xi^{-i},Q) &= \mathbb{E}\int_0^T (V_t - \widetilde{S}_t^f)\,\mathrm{d}q_t^i \\ &= \mathbb{E}\int_0^T (-\eta_2 Y_t \xi_t^i + \Lambda_t \xi_t^i)\,\mathrm{d}t. \end{aligned} \tag{10}$$

For the risk-neutral institutional investor, given informed traders' overall reaction $Y(Q)$, the cost of trading is $S_0Q_0 + \int_0^T \widetilde{S}_t^l\,\mathrm{d}Q_t$. Note that an admissible trading rate v leads to $Q_T = 0$, so with (1), (4) and (8) we have the cost of trading

$$\begin{aligned} S_0Q_0 + \int_0^T \widetilde{S}_t^l\,\mathrm{d}Q_t &= S_0Q_0 + \int_0^T S_t\,\mathrm{d}Q_t + \eta_1\int_0^T (v_t)^2\,\mathrm{d}t \\ &= S_TQ_T - \int_0^T Q_t\,\mathrm{d}S_t + \eta_1\int_0^T (v_t)^2\,\mathrm{d}t \\ &= -\int_0^T \gamma Q_t(v_t + Y_t)\,\mathrm{d}t \\ &\quad + \eta_1\int_0^T (v_t)^2\,\mathrm{d}t - \int_0^T \sigma_1 Q_t\,\mathrm{d}B_t^1 \\ &= \frac{\gamma}{2}(Q_0)^2 - \eta_1\int_0^T \left[-(v_t)^2 + m_1Q_tY_t\right]\mathrm{d}t \\ &\quad - \int_0^T \sigma_1 Q_t\,\mathrm{d}B_t^1. \end{aligned}$$

The risk-neutral institutional investor aims to minimize the expected cost of trading, so we define the performance functional of the institutional investor‡

$$J^0(Q;Y(Q)) = \mathbb{E}\int_0^T \left[-(v_t)^2 + m_1Q_tY_t\right]\mathrm{d}t. \tag{11}$$

Definition 1 The institutional investor's trading trajectory $Q^* \in \mathcal{B}$ and informed traders' trading strategies $(\xi^{i,*}(Q^*), \xi^{2,*}(Q^*), \ldots, \xi^{N,*}(Q^*)) \in \mathcal{A}^N$ form a Stackelberg-Nash equilibrium if

- given an institutional investor's trading trajectory $Q \in \mathcal{B}$, informed traders' trading strategies $(\xi^{i,*}(Q), \xi^{2,*}(Q), \ldots, \xi^{N,*}(Q)) \in \mathcal{A}^N$ form an open-loop Nash equilibrium, which means it holds that

$$J^i(\xi^i(Q);\xi^{-i,*}(Q),Q) \le J^i(\xi^{i,*}(Q);\xi^{-i,*}(Q),Q)$$

  for any $i = 1, 2, \ldots, N$ and any $\xi^i(Q) \in \mathcal{A}$.
- given informed traders' overall equilibrium reactions $Y^*(Q)$ to the institutional investor's trading trajectory Q, it holds that

$$J^0(Q;Y^*(Q)) \le J^0(Q^*;Y^*(Q^*))$$

  for any $Q \in \mathcal{B}$.

3. Main results

In this section we derive the explicit expression of the unique Stackelberg-Nash equilibrium between the institutional investor and informed traders. According to the definition, we solve the problem with two steps. We first find the unique open-loop Nash equilibrium that solves

$$\max_{\xi^i \in \mathcal{A}} J^i(\xi^i;\xi^{-i},Q), \quad i = 1, 2, \ldots, N$$

for any institutional investor's trading trajectory $Q \in \mathcal{B}$. After deriving informed traders' overall equilibrium reactions $Y^*(Q)$, We then find the unique optimal trading trajectory Q^* that solves the problem

$$\max_{Q\in\mathcal{B}} J^0(Q;Y^*(Q)).$$

† We can add an term $\eta_0(v_t + Y_t)$ into $\widetilde{S}_t^l$ and $\widetilde{S}_t^f$ to additionally consider interwined transaction prices for the institutional investor and informed traders. However, this will make an already complex model more difficult to tackle.

‡ We can add an additional term $-\phi\int_0^T Q_t^2\,\mathrm{d}t$ into $J^0(Q;Y(Q))$ to consider the institutional investor's risk aversion, and similarly, quadratic running and terminal penalties can be added into $J^i(\xi^i;\xi^{-i},Q)$ to describe informed traders' risk aversion. What's more, an additional term $-\eta_3\int_0^T(\xi_t^i)^2\,\mathrm{d}t$ can be added into $J^i(\xi^i;\xi^{-i},Q)$ to take into account other transaction costs like market frictions or a transaction tax (see Strehle 2017, Casgrain and Jaimungal 2020). However, the complicated model, especially the problem for the leader level, is computationally challenging and can't be solved explicitly with these extensions. For simplicity we opt to only consider the risk-neutral case and transaction costs arising from temporary price impact.

3.1. Solution to informed traders

Inspired by Bank *et al.* (2017), Neuman and Voß (2022), Forde *et al.* (2022), Voß (2022) and Neuman and Voß (2023), we employ a convex analytic approach to derive a system of coupled linear forward-backward stochastic differential equations (FBSDEs) and further obtain the unique open-loop Nash equilibrium for informed traders.

LEMMA 3.1 *For any $i = 1, 2, \dots, N$, given the institutional investor's trading trajectory $Q \in \mathcal{B}$ and other informed traders' trading rates $\xi^{-i} \in \mathcal{A}^{N-1}$, the functional $J^i(\xi^i; \xi^{-i}, Q)$ defined in (10) is strictly concave for $\xi^i \in \mathcal{A}$. As a result, there exists at most one open-loop Nash equilibrium for informed traders.*

Proof Let $Q \in \mathcal{B}$ and $\xi^{-i} \in \mathcal{A}^{N-1}$ be fixed. Consider two different trading rates $\xi^i, \widetilde{\xi}^i \in \mathcal{A}$ with the corresponding $\Lambda, \widetilde{\Lambda}$ and $q, \widetilde{q}$. For any $\varepsilon \in (0, 1)$ it holds that $\varepsilon\xi^i + (1-\varepsilon)\widetilde{\xi}^i \in \mathcal{A}$ with the corresponding Λ^ε and q^ε. With (3) and (6) we have

$$\begin{aligned} &\Lambda_t - \Lambda_t^\varepsilon = -\gamma(1-\varepsilon)(q_t^i - \widetilde{q}_t^i) \quad \text{and} \\ &\widetilde{\Lambda}_t - \Lambda_t^\varepsilon = \gamma\varepsilon(q_t^i - \widetilde{q}_t^i). \end{aligned}$$

We then obtain

$$\begin{aligned} &\varepsilon J^i(\xi^i; \xi^{-i}, Q) + (1-\varepsilon) J^i(\widetilde{\xi}^i; \xi^{-i}, Q) \\ &\quad - J^i(\varepsilon\xi^i + (1-\varepsilon)\widetilde{\xi}^i; \xi^{-i}, Q) \\ &= \mathbb{E}\int_0^T -\eta_2(\varepsilon(\xi_t^i)^2 + (1-\varepsilon)(\widetilde{\xi}_t^i)^2 \\ &\quad - (\varepsilon\xi_t^i + (1-\varepsilon)\widetilde{\xi}_t^i)^2)\mathrm{d}t \\ &\quad + \mathbb{E}\int_0^T [\varepsilon(\Lambda_t - \Lambda_t^\varepsilon)\xi_t^i + (1-\varepsilon)(\widetilde{\Lambda}_t - \Lambda_t^\varepsilon)\widetilde{\xi}_t^i]\mathrm{d}t \\ &= -\eta_2\varepsilon(1-\varepsilon)\mathbb{E}\int_0^T (\xi_t^i - \widetilde{\xi}_t^i)^2\,\mathrm{d}t \\ &\quad - \frac{\gamma}{2}\varepsilon(1-\varepsilon)\mathbb{E}(q_T^i - \widetilde{q}_T^i)^2 < 0. \end{aligned}$$

As a result, we know $J^i(\xi^i; \xi^{-i}, Q)$ is strictly concave for $\xi^i \in \mathcal{A}$. Next, with the same argumentation via contradiction from Schied and Zhang (2017), Voß (2022) and Neuman and Voß (2023), we know there exists at most one open-loop Nash equilibrium. ■

LEMMA 3.2 *Let $Q \in \mathcal{B}$ and $\xi^{-i} \in \mathcal{A}^{N-1}$ be fixed. For any trading rate $\xi^i \in \mathcal{A}$ and any direction $w^i \in \mathcal{A}$, we have*

$$\begin{aligned} &\langle \nabla J^i(\xi^i; \xi^{-i}, Q), w^i \rangle \\ &= \lim_{\varepsilon \to 0} \frac{J^i(\xi^i + \varepsilon w^i; \xi^{-i}, Q) - J^i(\xi^i; \xi^{-i}, Q)}{\varepsilon} \\ &= \mathbb{E}\int_0^T w_t^i \left(-\eta_2 Y_t - \eta_2\xi_t^i + \Lambda_t - \gamma\int_t^T \xi_u^i\,\mathrm{d}u\right)\mathrm{d}t. \quad (12) \end{aligned}$$

Proof With straightforward computations, we have

$$\begin{aligned} &J^i(\xi^i + \varepsilon w^i; \xi^{-i}, Q) - J^i(\xi^i; \xi^{-i}, Q) \\ &= \mathbb{E}\int_0^T \Big(-\eta_2(Y_t + \varepsilon w_t^i)(\xi_t^i + \varepsilon w_t^i) \\ &\quad + \left(\Lambda_t - \gamma\int_0^t \varepsilon w_u^i\,\mathrm{d}u\right)(\xi_t^i + \varepsilon w_t^i)\Big)\mathrm{d}t \\ &\quad - \mathbb{E}\int_0^T (-\eta_2 Y_t\xi_t^i + \Lambda_t\xi_t^i)\,\mathrm{d}t \\ &= \varepsilon\mathbb{E}\int_0^T \left(-\eta_2 Y_t w_t^i - \eta_2\xi_t^i w_t^i + \Lambda_t w_t^i - \gamma\xi_t^i\int_0^t w_u^i\,\mathrm{d}u\right)\mathrm{d}t \\ &\quad + \varepsilon^2\mathbb{E}\int_0^T \left(-\eta_2(w_t^i)^2 - \gamma w_t^i\int_0^t w_u^i\,\mathrm{d}u\right)\mathrm{d}t. \end{aligned}$$

We can obtain (12) after applying Fubini's theorem. ■

LEMMA 3.3 *Let $Q \in \mathcal{B}$ be fixed. If $(\xi^{1,*}, \xi^{2,*}, \dots, \xi^{N,*}) \in \mathcal{A}^N$ satisfies the following system of coupled FBSDEs*

$$\begin{cases} \mathrm{d}\Lambda_t = -\gamma(v_t + Y_t)\,\mathrm{d}t + \sigma\,\mathrm{d}W_t, \\ \Lambda_0 = 0, \\ -\eta_2\,\mathrm{d}Y_t - \eta_2\,\mathrm{d}\xi_t^i + \mathrm{d}\Lambda_t + \gamma\xi_t^i\,\mathrm{d}t = \mathrm{d}M_t^i, \quad i = 1, 2, \dots, N \\ -\eta_2 Y_T - \eta_2\xi_T^i + \Lambda_T = 0, \quad i = 1, 2, \dots, N \end{cases} \tag{13}$$

for suitable square integrable martingales $(M_t^i)_{0\le t\le T}$, $i = 1, 2, \dots, N$, then $(\xi^{1,*}, \xi^{2,*}, \dots, \xi^{N,*})$ forms an open-loop Nash equilibrium.

Proof If $(\xi^{1,*}, \xi^{2,*}, \dots, \xi^{N,*}) \in \mathcal{A}^N$ satisfies (13), we know

$$\begin{aligned} M_t^i - M_T^i &= -\eta_2(Y_t^* - Y_T^*) - \eta_2(\xi_t^{i,*} - \xi_T^{i,*}) \\ &\quad + \Lambda_t^* - \Lambda_T^* - \int_t^T \gamma\xi_u^{i,*}\,\mathrm{d}u \\ &= -\eta_2 Y_t^* - \eta_2\xi_t^{i,*} + \Lambda_t^* - \gamma\int_t^T \xi_u^{i,*}\,\mathrm{d}u. \end{aligned}$$

We then obtain

$$\begin{aligned} \langle \nabla J^i(\xi^{i,*}; \xi^{-i,*}, Q), w^i \rangle &= \mathbb{E}\left[\int_0^T w_t^i\left(M_t^i - M_T^i\right)\mathrm{d}t\right] \\ &= \mathbb{E}\left[\int_0^T \mathbb{E}\left[w_t^i\left(M_t^i - M_T^i\right) \mid \mathcal{F}_t\right]\mathrm{d}t\right] \\ &= \mathbb{E}\left[\int_0^T w_t^i\left(M_t^i - \mathbb{E}\left[M_T^i \mid \mathcal{F}_t\right]\right)\mathrm{d}t\right] \\ &= 0 \end{aligned}$$

for any $w^i \in \mathcal{A}$ and any $i = 1, 2, \dots, N$, which is a sufficient condition for the optimality of $\xi^{i,*}$ due to the strict concavity of $J^i(\xi^i; \xi^{-i}, Q)$ shown in Lemma 3.1. In other words, no informed trader has an incentive to deviate from $(\xi^{1,*}, \xi^{2,*}, \dots, \xi^{N,*})$ because $\xi^{i,*}$ has maximized $J^i(\xi^i; \xi^{-i,*}, Q)$ for any $i = 1, 2, \dots, N$. As a result, an open-loop Nash equilibrium is formed by $(\xi^{1,*}, \xi^{2,*}, \dots, \xi^{N,*})$. ■

LEMMA 3.4 *Let $Q \in \mathcal{B}$ be fixed. If $Y^* \in \mathcal{A}$ satisfies FBSDE*

$$\begin{cases} \mathrm{d}\Lambda_t = -\gamma(v_t + Y_t)\,\mathrm{d}t + \sigma\,\mathrm{d}W_t, \\ \Lambda_0 = 0, \\ -\eta_2(N+1)\,\mathrm{d}Y_t + N\,\mathrm{d}\Lambda_t + \gamma Y_t\,\mathrm{d}t = \mathrm{d}M_t, \\ Y_T = \dfrac{N}{\eta_2(N+1)}\Lambda_T \end{cases} \tag{14}$$

for some suitable square integrable martingale $(M_t)_{0\le t\le T}$, then $\xi^{i,*} = \frac{1}{N}Y^* \in \mathcal{A}$ $(i = 1, 2, \ldots, N)$ satisfy (13).

Proof If $Y^* \in \mathcal{A}$ satisfies (14), let $\xi^{i,*} = \frac{1}{N}Y^*$ $(i = 1, 2, \ldots, N)$ and we have

$$\begin{aligned} &-\eta_2\,\mathrm{d}Y_t^* - \eta_2\,\mathrm{d}\xi_t^{i,*} + \mathrm{d}\Lambda_t^* + \gamma\xi_t^{i,*}\,\mathrm{d}t \\ &= \frac{1}{N}\left[-\eta_2(N+1)\,\mathrm{d}Y_t^* + N\,\mathrm{d}\Lambda_t^* + \gamma Y_t^*\,\mathrm{d}t\right] = \frac{1}{N}\mathrm{d}M_t, \\ &-\eta_2 Y_T^* - \eta_2\xi_T^{i,*} + \Lambda_T^* = -\eta_2\left(1 + \frac{1}{N}\right)Y_T^* + \Lambda_T^* = 0. \end{aligned}$$

With $M_t^i = \frac{1}{N}M_t$ $(i = 1, 2, \ldots, N)$, we know $\xi^{i,*}$s solve (13). ■

We are now ready to provide the solution to informed traders. We first introduce two constants and one function

$$\rho_1 = \frac{N-1}{N+1}, \quad \rho_2 = \frac{N}{N+1}, \quad \varphi(t) = \mathrm{e}^{-m_2\rho_1(T-t)}. \tag{15}$$

THEOREM 3.5 *Let the institutional investor's trading trajectory $Q \in \mathcal{B}$ be fixed. There exists a unique open-loop Nash equilibrium for informed traders. The total reaction to the institutional investor's trading trajectory in feedback form is given by*

$$Y_t^* = \frac{1}{\eta_2}f(t)\Lambda_t + g(t) \tag{16}$$

where

$$\begin{aligned} f(t) &= \frac{\rho_1 N}{N - \varphi(t)}, \\ g(t) &= -\frac{m_2\rho_2}{N - \varphi(t)}\left(Q_t + \int_t^T \varphi(s)v_s\,\mathrm{d}s\right). \end{aligned} \tag{17}$$

The equilibrium strategies is given by $\xi^{i,*} = \frac{1}{N}Y^*$ $(i = 1, 2, \ldots, N)$.

Proof We first have to find a solution to (14). Substituting the ansatz (16) into (14) we have

$$\begin{cases} -\eta_2(N+1)\,\mathrm{d}\left(\dfrac{1}{\eta_2}f(t)\Lambda_t + g(t)\right) \\ -\gamma(N-1)\left(\dfrac{1}{\eta_2}f(t)\Lambda_t + g(t)\right)\mathrm{d}t \\ -\gamma N v_t\,\mathrm{d}t = \mathrm{d}M_t - \sigma N\,\mathrm{d}W_t, \\ \dfrac{1}{\eta_2}f(T)\Lambda_T + g(T) = \dfrac{\rho_2}{\eta_2}\Lambda_T. \end{cases}$$

We can rewrite the above differential equation as

$$\begin{aligned} &\left[-(N+1)f'(t) + m_2(N+1)f^2(t) - m_2(N-1)f(t)\right]\Lambda_t\,\mathrm{d}t \\ &\quad - \gamma(N-1)g(t)\,\mathrm{d}t - \gamma N v_t\,\mathrm{d}t - \eta_2(N+1) \\ &\quad \times\left[g'(t) - m_2 f(t)v_t - m_2 f(t)g(t)\right]\mathrm{d}t \\ &= \sigma(N+1)f(t)\,\mathrm{d}W_t + \mathrm{d}M_t - \sigma N\,\mathrm{d}W_t, \end{aligned}$$

which yields the system

$$\begin{cases} f'(t) = m_2 f^2(t) - m_2\rho_1 f(t) \\ g'(t) - m_2\left(f(t) - \rho_1\right)g(t) = m_2\left(f(t) - \rho_2\right)v_t \\ M_t = \sigma\left[N - (N+1)f(t)\right]W_t \end{cases} \tag{18}$$

with the terminal condition $f(T) = \rho_2,\ g(T) = 0$. The first equation can be uniquely solved by $f(t)$ in (17), and then the second equation can be uniquely solved by $g(t)$ in (17). The suitable square integrable martingale $(M_t)_{0\le t\le T}$ is given by the third equation. As a result, Y_t^* in (16) with $f(t)$ and $g(t)$ in (17) can solve FBSDE (14). With Lemma 3.4, $\xi^{i,*} = \frac{1}{N}Y^*$ $(i = 1, 2, \ldots, N)$ is hence a solution to the system of coupled FBSDEs (13). According to Lemma 3.3, we know $\xi^{i,*}$s form an open-loop Nash equilibrium. The uniqueness can also be guaranteed due to Lemma 3.1. ■

COROLLARY 3.6 *Informed traders' expected total equilibrium reaction to the institutional investor's trading trajectory $Q \in \mathcal{B}$ can be written by*

$$\mathbb{E}Y_t^* = -\frac{m_2\rho_2}{\varphi_t}\left(\frac{\varphi_0 Q_0}{N - \varphi_0} + \int_0^t \varphi_s v_s\,\mathrm{d}s + \frac{\varphi_0}{N - \varphi_0}\int_0^T \varphi_t v_t\,\mathrm{d}t\right) \tag{19}$$

Proof With (3) and (16) we have

$$\mathrm{d}\Lambda_t + m_2 f_t\Lambda_t\,\mathrm{d}t = -\gamma(v_t + g_t)\,\mathrm{d}t + \sigma\,\mathrm{d}W_t.$$

With the fact $(f_t\varphi_t)' = m_2 f_t^2\varphi_t$, we have

$$\mathrm{d}f_t\varphi_t\Lambda_t = -\gamma f_t\varphi_t(v_t + g_t)\,\mathrm{d}t + \sigma f_t\varphi_t\,\mathrm{d}W_t.$$

With $\Lambda_0 = 0$ we obtain

$$\mathbb{E}\left[f_t\Lambda_t\right] = -\frac{\gamma}{\varphi_t}\int_0^t f_u\varphi_u(v_u + g_u)\,\mathrm{d}u$$

In addition, $f_t - \rho_1 = f_t\varphi_t/N$, and with (16)–(18) we have

$$\begin{aligned} \mathbb{E}Y_t^* &= g_t - \frac{m_2}{\varphi_t}\int_0^t f_u\varphi_u v_u\,\mathrm{d}u - \frac{m_2}{\varphi_t}\int_0^t f_u\varphi_u g_u\,\mathrm{d}u \\ &= g_t - \frac{m_2 N}{\varphi_t}\int_0^t \left(f_t - \rho_1\right)v_u\,\mathrm{d}u \\ &\quad - \frac{N}{\varphi_t}\int_0^t \left[g_t' - m_2\left(f(t) - \rho_2\right)v_t\right]\mathrm{d}u \\ &= -\frac{N - \varphi_t}{\varphi_t}g_t + \frac{N}{\varphi_t}g_0 - \frac{m_2\rho_2}{\varphi_t}\left(Q_t - Q_0\right) \\ &= \frac{m_2\rho_2}{\varphi_t}\left[\left(Q_0 + \int_t^T \varphi_s v_s\,\mathrm{d}s\right)\right. \\ &\quad \left. - \frac{N}{N - \varphi_0}\left(Q_0 + \int_0^T \varphi_t v_t\,\mathrm{d}t\right)\right] \\ &= -\frac{m_2\rho_2}{\varphi_t}\left(\frac{\varphi_0 Q_0}{N - \varphi_0} + \int_0^t \varphi_s v_s\,\mathrm{d}s + \frac{\varphi_0}{N - \varphi_0}\int_0^T \varphi_t v_t\,\mathrm{d}t\right) \end{aligned}$$

■

3.2. Solution to the institutional investor

As we consider multiple informed traders at the follower level, the optimization problem at the leader level is rather complicated. Given informed traders' expected total equilibrium reaction $\mathbb{E}Y_t^*(Q)$ shown in Corollary 3.6, we now solve the optimal order execution strategy for the institutional investor via a calculus of variation argument. With (19), the trading revenue of the institutional investor can be rewritten as

$$\begin{aligned} J^0(Q;Y^*(Q)) &= \int_0^T \left(-(v_t)^2 + m_1 Q_t \mathbb{E}\left[Y_t^*\right]\right) \mathrm{d}t. \\ &= -\int_0^T (v_t)^2\,\mathrm{d}t - m_1 m_2 \rho_2 \\ &\quad\times\left[\frac{\varphi_0 Q_0}{N-\varphi_0}\int_0^T \frac{Q_t}{\varphi_t}\mathrm{d}t\right. \\ &\quad+\int_0^T \frac{Q_t}{\varphi_t}\left(\int_0^t \varphi_s v_s\,\mathrm{d}s\right)\mathrm{d}t \\ &\quad\left.+\frac{\varphi_0}{N-\varphi_0}\int_0^T \varphi_t v_t\,\mathrm{d}t \int_0^T \frac{Q_t}{\varphi_t}\mathrm{d}t\right]. \end{aligned} \tag{20}$$

Lemma 3.7 *The functional $J^0(Q;Y^*(Q))$ defined in* (20) *is strictly concave for $Q \in \mathcal{B}$.*

Proof Consider two different trading trajectories $Q, \widetilde{Q} \in \mathcal{B}$ with corresponding trading rates $v, \widetilde{v}$. We have $\widetilde{Q}_0 = Q_0$ and $Q_T = \widetilde{Q}_T = 0$. For any $\varepsilon \in (0,1)$ it holds that $\varepsilon Q + (1-\varepsilon)\widetilde{Q} \in \mathcal{B}$ with corresponding $v^\varepsilon = \varepsilon v + (1-\varepsilon)\widetilde{v}$. With the fact

$$\int_0^t \varphi_s (v_s - \widetilde{v}_s)\mathrm{d}s = \varphi_t\left(Q_t - \widetilde{Q}_t\right) - m_2\rho_1 \int_0^t \varphi_s\left(Q_s - \widetilde{Q}_s\right)\mathrm{d}s,$$

we obtain

$$\begin{aligned} &\varepsilon J^0(Q;Y^*(Q)) + (1-\varepsilon)J^0(\widetilde{Q};Y^*(\widetilde{Q})) \\ &\quad - J^0(\varepsilon Q + (1-\varepsilon)\widetilde{Q}; Y^*(\varepsilon Q + (1-\varepsilon)\widetilde{Q})) \\ &= -\varepsilon(1-\varepsilon)\int_0^T (v_t - \widetilde{v}_t)^2\,\mathrm{d}t - m_1 m_2 \rho_2 \varepsilon(1-\varepsilon) \\ &\quad\times\left[\int_0^T \frac{Q_t - \widetilde{Q}_t}{\varphi_t}\left(\int_0^t \varphi_s\,(v_s - \widetilde{v}_s)\,\mathrm{d}s\right)\mathrm{d}t\right. \\ &\quad\left.+\frac{\varphi_0}{N-\varphi_0}\int_0^T \varphi_t\,(v_t - \widetilde{v}_t)\,\mathrm{d}t \int_0^T \frac{Q_t - \widetilde{Q}_t}{\varphi_t}\,\mathrm{d}t\right] \\ &= -\varepsilon(1-\varepsilon)\int_0^T (v_t - \widetilde{v}_t)^2\,\mathrm{d}t + m_1 m_2 \rho_2 \varepsilon(1-\varepsilon) \\ &\quad\times\left[-\int_0^T \left(Q_t - \widetilde{Q}_t\right)^2\mathrm{d}t\right. \\ &\quad+\frac{N}{N-\varphi_0} m_2\rho_1 \int_0^T\int_0^t \frac{\varphi_s}{\varphi_t}\left(Q_t - \widetilde{Q}_t\right)\left(Q_s - \widetilde{Q}_s\right)\mathrm{d}s\,\mathrm{d}t \\ &\quad\left.+\frac{\varphi_0}{N-\varphi_0} m_2\rho_1 \int_0^T\int_t^T \frac{\varphi_s}{\varphi_t}\left(Q_t - \widetilde{Q}_t\right)\left(Q_s - \widetilde{Q}_s\right)\mathrm{d}s\,\mathrm{d}t\right] \end{aligned}$$

We also have

$$\begin{aligned} &m_2\rho_1 \int_0^T\int_0^t \frac{\varphi_s}{\varphi_t}\left(Q_t - \widetilde{Q}_t\right)\left(Q_s - \widetilde{Q}_s\right)\mathrm{d}s\,\mathrm{d}t \\ &\le m_2\rho_1 \int_0^T\int_0^t \frac{\varphi_s}{\varphi_t}\frac{\left(Q_t - \widetilde{Q}_t\right)^2 + \left(Q_s - \widetilde{Q}_s\right)^2}{2}\mathrm{d}s\,\mathrm{d}t \\ &= \frac{1}{2}\int_0^T \frac{\left(Q_t - \widetilde{Q}_t\right)^2}{\varphi_t}\int_0^t m_2\rho_1\varphi_s\,\mathrm{d}s\,\mathrm{d}t \\ &\quad+\frac{1}{2}\int_0^T \frac{m_2\rho_1}{\varphi_t}\int_0^t \varphi_s\left(Q_s - \widetilde{Q}_s\right)^2\mathrm{d}s\,\mathrm{d}t \\ &= \frac{1}{2}\int_0^T \frac{\left(Q_t - \widetilde{Q}_t\right)^2}{\varphi_t}\left(\int_0^t \mathrm{d}\varphi_s\right)\mathrm{d}t \\ &\quad+\frac{1}{2}\int_0^T \varphi_t\left(Q_t - \widetilde{Q}_t\right)^2\int_t^T \frac{m_2\rho_1}{\varphi_s}\,\mathrm{d}s\,\mathrm{d}t \\ &= \frac{1}{2}\int_0^T \frac{\left(Q_t - \widetilde{Q}_t\right)^2}{\varphi_t}(\varphi_t - \varphi_0)\,\mathrm{d}t \\ &\quad+\frac{1}{2}\int_0^T \varphi_t\left(Q_t - \widetilde{Q}_t\right)^2\left(\frac{1}{\varphi_t} - \frac{1}{\varphi_T}\right)\mathrm{d}t \\ &= \int_0^T \left(Q_t - \widetilde{Q}_t\right)^2\mathrm{d}t - \frac{1}{2}\int_0^T\left(\varphi_t + \frac{\varphi_0}{\varphi_t}\right)\left(Q_t - \widetilde{Q}_t\right)^2\mathrm{d}t, \end{aligned}$$

and similarly,

$$\begin{aligned} &m_2\rho_1 \int_0^T\int_t^T \frac{\varphi_s}{\varphi_t}\left(Q_t - \widetilde{Q}_t\right)\left(Q_s - \widetilde{Q}_s\right)\mathrm{d}s\,\mathrm{d}t \\ &\quad\le -\int_0^T \left(Q_t - \widetilde{Q}_t\right)^2\mathrm{d}t \\ &\qquad+\frac{1}{2\varphi_0}\int_0^T\left(\varphi_t + \frac{\varphi_0}{\varphi_t}\right)\left(Q_t - \widetilde{Q}_t\right)^2\mathrm{d}t. \end{aligned}$$

We then obtain

$$\begin{aligned} &\varepsilon J^0(Q;Y^*(Q)) + (1-\varepsilon)J^0(\widetilde{Q};Y^*(\widetilde{Q})) \\ &\quad - J^0(\varepsilon Q + (1-\varepsilon)\widetilde{Q}; Y^*(\varepsilon Q + (1-\varepsilon)\widetilde{Q})) \\ &\le -\varepsilon(1-\varepsilon)\int_0^T (v_t - \widetilde{v}_t)^2\,\mathrm{d}t + m_1 m_2\rho_2\varepsilon(1-\varepsilon) \\ &\quad\times\left[-\int_0^T \left(Q_t - \widetilde{Q}_t\right)^2\mathrm{d}t\right. \\ &\quad+\frac{N}{N-\varphi_0}\left(\int_0^T \left(Q_t - \widetilde{Q}_t\right)^2\mathrm{d}t - \frac{1}{2}\int_0^T\left(\varphi_t + \frac{\varphi_0}{\varphi_t}\right)\right. \\ &\quad\left.\times\left(Q_t - \widetilde{Q}_t\right)^2\mathrm{d}t\right) \\ &\quad+\frac{\varphi_0}{N-\varphi_0}\left(-\int_0^T \left(Q_t - \widetilde{Q}_t\right)^2\mathrm{d}t\right. \\ &\quad\left.\left.+\frac{1}{2\varphi_0}\int_0^T\left(\varphi_t + \frac{\varphi_0}{\varphi_t}\right)\left(Q_t - \widetilde{Q}_t\right)^2\mathrm{d}t\right)\right] \\ &= -\varepsilon(1-\varepsilon)\int_0^T (v_t - \widetilde{v}_t)^2\,\mathrm{d}t - \frac{1}{2}m_1 m_2\rho_2\varepsilon(1-\varepsilon) \\ &\quad\times\frac{N-1}{N-\varphi_0}\int_0^T\left(\varphi_t + \frac{\varphi_0}{\varphi_t}\right)\left(Q_t - \widetilde{Q}_t\right)^2\mathrm{d}t \end{aligned}$$

$$< 0$$

As a result, we know $J^0(Q; Y^*(Q))$ is strictly concave for $Q \in \mathcal{B}$. ■

LEMMA 3.8 *Let $\mathcal{B}^0$ denote $\{h \in \mathcal{B} : h_0 = h_T = 0\}$. For any trading trajectory $Q \in \mathcal{B}$ and any direction $h \in \mathcal{B}^0$, we have*

$$\begin{aligned}
&\left\langle \nabla J^0(Q; Y^*(Q)), h \right\rangle \\
&= \lim_{\varepsilon \to 0} \frac{J^0(Q + \varepsilon h; Y^*(Q + \varepsilon h)) - J^0(Q; Y^*(Q))}{\varepsilon} \\
&= \int_0^T h_t \left(2v_t' + m_1 m_2 \rho_2 \left[\frac{Q_0}{N - \varphi_0} \left(\varphi_t - \frac{\varphi_0}{\varphi_t} \right) \right.\right. \\
&\quad + \frac{\varphi_0}{N - \varphi_0} \left(\varphi_t \int_0^T \frac{v_s}{\varphi_s} \mathrm{d}t - \frac{1}{\varphi_t} \int_0^T \varphi_s v_s \, \mathrm{d}s \right) \\
&\quad \left.\left. + \left(\varphi_t \int_t^T \frac{v_s}{\varphi_s} \mathrm{d}s - \frac{1}{\varphi_t} \int_0^t \varphi_s v_s \, \mathrm{d}s \right) \right] \right) \mathrm{d}t. \qquad (21)
\end{aligned}$$

Proof With straightward computations, we have

$$\begin{aligned}
&J^0(Q + \varepsilon h; Y^*(Q + \varepsilon h)) - J^0(Q; Y^*(Q)) \\
&= \varepsilon \int_0^T \left(-2h_t' v_t - m_1 m_2 \rho_2 \right. \\
&\quad \times \left[\frac{\varphi_0 Q_0}{N - \varphi_0} \frac{h_t}{\varphi_t} + \frac{h_t}{\varphi_t} \left(\int_0^t \varphi_s v_s \, \mathrm{d}s \right) + \frac{Q_t}{\varphi_t} \left(\int_0^t \varphi_s h_s' \, \mathrm{d}s \right) \right. \\
&\quad \left.\left. + \frac{\varphi_0}{N - \varphi_0} \frac{h_t}{\varphi_t} \left(\int_0^T \varphi_s v_s \, \mathrm{d}s \right) \right] \right) \mathrm{d}t \\
&\quad - \varepsilon m_1 m_2 \rho_2 \frac{\varphi_0}{N - \varphi_0} \int_0^T \varphi_t h_t' \, \mathrm{d}t \int_0^T \frac{Q_t}{\varphi_t} \mathrm{d}t \\
&\quad - \varepsilon^2 \left[\int_0^T \left((h_t')^2 + m_1 m_2 \rho_2 \frac{h_t}{\varphi_t} \left(\int_0^t \varphi_s h_s' \, \mathrm{d}s \right) \right) \mathrm{d}t \right. \\
&\quad \left. + m_1 m_2 \rho_2 \frac{\varphi_0}{N - \varphi_0} \int_0^T \varphi_t h_t' \, \mathrm{d}t \int_0^T \frac{h_t}{\varphi_t} \mathrm{d}t \right].
\end{aligned}$$

With the fact

$$\begin{aligned}
&\int_0^T h_t' v_t \, \mathrm{d}t = - \int_0^T h_t v_t' \, \mathrm{d}t, \\
&\int_0^t \varphi_s h_s' \, \mathrm{d}s = \varphi_t h_t - m_2 \rho_1 \int_0^t \varphi_s h_s \, \mathrm{d}s, \\
&m_2 \rho_1 \int_t^T \frac{Q_s}{\varphi_s} \mathrm{d}s = \frac{Q_t}{\varphi_t} + \int_t^T \frac{v_s}{\varphi_s} \mathrm{d}s
\end{aligned}$$

and Fubini's theorem, we have

$$\begin{aligned}
&\left\langle \nabla J^0(Q; Y^*(Q)), h \right\rangle \\
&= \int_0^T \left(2h_t v_t' - m_1 m_2 \rho_2 \left[\frac{\varphi_0 Q_0}{N - \varphi_0} \frac{h_t}{\varphi_t} + \frac{h_t}{\varphi_t} \left(\int_0^t \varphi_s v_s \, \mathrm{d}s \right) + Q_t h_t - m_2 \rho_1 \frac{Q_t}{\varphi_t} \left(\int_0^t \varphi_s h_s \, \mathrm{d}s \right) \right.\right. \\
&\quad \left.\left. + \frac{\varphi_0}{N - \varphi_0} \frac{h_t}{\varphi_t} \left(\int_0^T \varphi_s v_s \, \mathrm{d}s \right) \right] \right) \mathrm{d}t + m_1 m_2 \rho_2 \frac{m_2 \rho_1 \varphi_0}{N - \varphi_0} \int_0^T \varphi_t h_t \, \mathrm{d}t \int_0^T \frac{Q_t}{\varphi_t} \mathrm{d}t. \\
&= \int_0^T \left(2h_t v_t' - m_1 m_2 \rho_2 \left[\frac{\varphi_0 Q_0}{N - \varphi_0} \frac{h_t}{\varphi_t} + \frac{h_t}{\varphi_t} \left(\int_0^t \varphi_s v_s \, \mathrm{d}s \right) + Q_t h_t - m_2 \rho_1 \varphi_t h_t \left(\int_t^T \frac{Q_s}{\varphi_s} \mathrm{d}s \right) \right.\right. \\
&\quad \left.\left. + \frac{\varphi_0}{N - \varphi_0} \frac{h_t}{\varphi_t} \left(\int_0^T \varphi_s v_s \, \mathrm{d}s \right) \right] \right) \mathrm{d}t + m_1 m_2 \rho_2 \frac{m_2 \rho_1 \varphi_0}{N - \varphi_0} \int_0^T \varphi_t h_t \, \mathrm{d}t \int_0^T \frac{Q_t}{\varphi_t} \mathrm{d}t. \\
&= \int_0^T \left(2h_t v_t' - m_1 m_2 \rho_2 \left[\frac{\varphi_0 Q_0}{N - \varphi_0} \frac{h_t}{\varphi_t} + \frac{h_t}{\varphi_t} \left(\int_0^t \varphi_s v_s \, \mathrm{d}s \right) - \varphi_t h_t \left(\int_t^T \frac{v_s}{\varphi_s} \mathrm{d}s \right) \right.\right. \\
&\quad \left.\left. + \frac{\varphi_0}{N - \varphi_0} \frac{h_t}{\varphi_t} \left(\int_0^T \varphi_s v_s \, \mathrm{d}s \right) \right] \right) \mathrm{d}t + m_1 m_2 \rho_2 \frac{\varphi_0}{N - \varphi_0} \int_0^T \varphi_t h_t \, \mathrm{d}t \left(\frac{Q_0}{\varphi_0} + \int_0^T \frac{v_t}{\varphi_t} \mathrm{d}t \right).
\end{aligned}$$

We then obtain (21). ■

Before providing the solution to the institutional investor, we first introduce a constant

$$\lambda = \sqrt{m_1 m_2 \rho_2 + m_2^2 \rho_1^2}. \qquad (22)$$

THEOREM 3.9 *For the institutional investor, the optimal trading trajectory $Q^* \in \mathcal{B}$ is uniquely given by*

$$Q_t^* = \frac{Q_0}{2} + \frac{b}{\lambda} \left[\mathrm{e}^{\lambda t} - \mathrm{e}^{\lambda(T-t)} \right] - \frac{c}{2} (T - 2t) \qquad (23)$$

with the optimal trading rate

$$v_t^* = b \left[\mathrm{e}^{\lambda(T-t)} + \mathrm{e}^{\lambda t} \right] + c \qquad (24)$$

where

$$\begin{aligned}
b &= - \frac{m_1 \lambda Q_0 [N + m_2 \rho_2 T]}{2m_1 N \left(\mathrm{e}^{\lambda T} - 1 \right) + \lambda T [\lambda (N+1) \left(\mathrm{e}^{\lambda T} - 1 \right) + m_2 \rho_1 (N-1) \left(\mathrm{e}^{\lambda T} + 1 \right)]}, \\
c &= - \frac{Q_0}{T} - \frac{2b \left(\mathrm{e}^{\lambda T} - 1 \right)}{\lambda T}.
\end{aligned} \qquad (25)$$

Proof Due to the strict concavity of $J^0(Q; Y^*(Q))$ shown in Lemma 3.7, we only have to find a trading trajectory $Q \in \mathcal{B}$ such that $\langle \nabla J^0(Q; Y^*(Q)), h\rangle = 0$ for any direction $h \in \mathcal{B}^0$. A sufficient condition is

$$\begin{aligned}2v_t' + m_1 m_2 \rho_2 \Bigg[&\frac{Q_0}{N-\varphi_0}\left(\varphi_t - \frac{\varphi_0}{\varphi_t}\right) \\ &+ \frac{\varphi_0}{N-\varphi_0}\left(\varphi_t \int_0^T \frac{v_s}{\varphi_s}\mathrm{d}t - \frac{1}{\varphi_t}\int_0^T \varphi_s v_s\,\mathrm{d}s\right) \\ &+ \left(\varphi_t \int_t^T \frac{v_s}{\varphi_s}\mathrm{d}s - \frac{1}{\varphi_t}\int_0^t \varphi_s v_s\,\mathrm{d}s\right)\Bigg] = 0. \qquad (26)\end{aligned}$$

By differentiating we have

$$\begin{aligned}2v_t'' + m_1 m_2^2 \rho_1 \rho_2 \Bigg[&\frac{Q_0}{N-\varphi_0}\left(\varphi_t + \frac{\varphi_0}{\varphi_t}\right) \\ &+ \frac{\varphi_0}{N-\varphi_0}\left(\varphi_t \int_0^T \frac{v_s}{\varphi_s}dt + \frac{1}{\varphi_t}\int_0^T \varphi_s v_s\,\mathrm{d}s\right) \\ &+ \left(\varphi_t \int_t^T \frac{v_s}{\varphi_s}\mathrm{d}s + \frac{1}{\varphi_t}\int_0^t \varphi_s v_s\,\mathrm{d}s\right)\Bigg] - 2m_1 m_2 \rho_2 v_t = 0. \qquad (27)\end{aligned}$$

Multiply the left-hand side of (26) by $m_2\rho_1$ and add the left-hand side of (27), and then divide by $2\varphi_t$ (consider $(m_2\rho_1(26) + (27))/(2\varphi_t)$) to obtain

$$\begin{aligned}&\frac{-m_1 m_2 \rho_2 v_t + m_2 \rho_1 v_t' + v_t''}{\varphi_t} + m_1 m_2^2 \rho_1 \rho_2 \\ &\times \left[\frac{Q_0}{N-\varphi_0} + \frac{\varphi_0}{N-\varphi_0}\int_0^T \frac{v_s}{\varphi_s}\mathrm{d}t + \int_t^T \frac{v_s}{\varphi_s}\mathrm{d}s\right] = 0.\end{aligned}$$

By differentiating we have

$$\frac{\begin{array}{l}-m_1 m_2 \rho_2 v_t' + m_2 \rho_1 v_t'' + v_t''' \\ -m_2\rho_1\left(-m_1 m_2 \rho_2 v_t + m_2 \rho_1 v_t' + v_t''\right)\end{array}}{\varphi_t} - m_1 m_2^2 \rho_1 \rho_2 \frac{v_t}{\varphi_t} = 0,$$

which yields

$$v_t''' - \left(m_1 m_2 \rho_2 + m_2^2 \rho_1^2\right) v_t' = 0.$$

This ODE has the general solution

$$v_t = a\,\mathrm{e}^{-\lambda t} + b\,\mathrm{e}^{\lambda t} + c.$$

Substituting it into (26) and considering the coefficients of φ_t and $\frac{\varphi_0}{\varphi_t}$, we have

$$\begin{aligned}&Q_0 + \frac{a}{\lambda + m_2\rho_1}\left(1 - N\,\mathrm{e}^{-\lambda T}\right) \\ &\quad + \frac{b}{\lambda - m_2\rho_1}\left(N\,\mathrm{e}^{\lambda T} - 1\right) - \frac{N-1}{m_2\rho_1}c = 0, \qquad (28)\\ &Q_0 + \frac{a}{\lambda - m_2\rho_1}\left(N - \mathrm{e}^{-\lambda T}\right) \\ &\quad + \frac{b}{\lambda + m_2\rho_1}\left(\mathrm{e}^{\lambda T} - N\right) - \frac{N-1}{m_2\rho_1}c = 0. \qquad (29)\end{aligned}$$

Subtract the left-hand side of (29) from the left-hand side of (28), and we then have

$$\left(a\,\mathrm{e}^{-\lambda T} - b\right)\left(\frac{\mathrm{e}^{\lambda T} - N}{\lambda + m_2\rho_1} - \frac{N\,\mathrm{e}^{\lambda T} - 1}{\lambda - m_2\rho_1}\right) = 0,$$

which yields $a = b\,\mathrm{e}^{\lambda T}$. Therefore, v_t has the expression in (24). In addition, with the terminal condition $Q_T = 0$, c has the expression in (25). Substituting a and c into (28) we obtain the expression of b in (25). Finally, with (24) and (25) we have

$$\begin{aligned}Q_t &= Q_0 + \int_0^t v_t\,\mathrm{d}t = Q_0 + \frac{b}{\lambda}\left[\mathrm{e}^{\lambda t} - \mathrm{e}^{\lambda(T-t)}\right] \\ &\quad + \frac{b}{\lambda}\left[\mathrm{e}^{\lambda T} - 1\right] + ct \\ &= Q_0 + \frac{b}{\lambda}\left[\mathrm{e}^{\lambda t} - \mathrm{e}^{\lambda(T-t)}\right] - \frac{1}{2}(Q_0 + cT) + ct,\end{aligned}$$

which yields (23). As a result, we find a trading trajectory Q^* in (23) which satisfies $\langle \nabla J^0(Q^*; Y^*(Q^*)), h\rangle = 0$ for any direction $h \in \mathcal{B}^0$. With Lemma 3.7 we can conclude Q^* in (23) is the unique optimal trading trajectory for the institutional investor. ■

3.3. Stackelberg-Nash equilibrium

With two steps, we have first solved the problem for informed traders at the follower level and then solved the problem for the institutional investor at the leader level. With Theorems 3.5 and 3.9, we are now ready to state our main result.

THEOREM 3.10 *There exists a unique Stackelberg-Nash equilibrium between the risk-neutral institutional investor and $N (\geq 2)$ risk-neutral informed traders. For the institutional investor, the equilibrium trading trajectory Q^* is given by* (23) *with the equilibrium trading rate v^* in* (24)*. For informed traders, equilibrium strategies are symmetric and the total trading rate is given by* (16).

After obtaining the main result, it's natural to consider the case when $N = 1$ and N is sufficient large. The results are shown in the next two corollaries.

COROLLARY 3.11 *Consider one institutional investor and one informed trader. There exists a unique Stackelberg equilibrium between them. For the institutional investor, the equilibrium trading trajectory Q^* is given by*

$$Q_t^* = \frac{Q_0}{2}\left[1 + \frac{\mathrm{e}^{\lambda(T-t)} - \mathrm{e}^{\lambda t}}{\mathrm{e}^{\lambda T} - 1}\right] \qquad (30)$$

with the equilibrium trading rate

$$v_t^* = -\frac{\lambda Q_0}{2}\frac{\mathrm{e}^{\lambda(T-t)} + \mathrm{e}^{\lambda t}}{\mathrm{e}^{\lambda T} - 1} \qquad (31)$$

where $\lambda = \sqrt{\frac{m_1 m_2}{2}}$. For the informed trader, the equilibrium trading rate ξ^* in feedback form is given by

$$\xi_t^* = \frac{\frac{1}{\eta_2}\Lambda_t + \frac{m_2^2}{2}\int_t^T (T-s)v_s^*\,\mathrm{d}s}{m_2(T-t) + 2}. \qquad (32)$$

In addition, the expected trading rate $\mathbb{E}\xi^*$ *can be written as*

$$\mathbb{E}\xi_t^* = \frac{m_2 Q_0}{2}\left[\frac{1}{m_2 T + 2} - \frac{e^{\lambda(T-t)} - e^{\lambda t}}{2\left(e^{\lambda T} - 1\right)}\right] \tag{33}$$

Proof We sketch the proof as procedures are highly similar. By solving (18) with $N = 1$ we have

$$f(t) = \frac{1}{m_2(T-t)+2}, \quad g(t) = \frac{\frac{m_2^2}{2}\int_t^T (T-s)v_s\,\mathrm{d}s}{m_2(T-t)+2}.$$

We also have

$$\mathbb{E}\xi_t = \frac{m_2 Q_0}{m_2(T-t)+2} - \frac{m_2}{2}Q_t + \frac{m_2^2}{2}\frac{\int_0^T Q_s\,\mathrm{d}s}{m_2(T-t)+2}.$$

When it comes to (26), only by differentiating we can get an ODE which can be easily solved. We then get (31) and (30). Finally by substituting we obtain (33). ■

COROLLARY 3.12 *When N is large,* λ *and b in* (22) *and* (25) *is approximately given by*

$$\lambda = \sqrt{m_1 m_2 + m_2^2},$$

$$b = -\frac{m_1 \lambda Q_0}{2m_1\left(e^{\lambda T} - 1\right) + \lambda T\left[\lambda\left(e^{\lambda T} - 1\right) + m_2\left(e^{\lambda T} + 1\right)\right]}.$$

Informed traders' total trading rate is approximately given by

$$Y_t^* = \frac{1}{\eta_2}\Lambda_t.$$

Proof Let $N \to \infty$. We have $\rho_1, \rho_2 \to 1$. $f(t) \to 1$ and $g(t) \to 0$. ■

Furthermore, we have to show that our result doesn't admit price manipulation for the institutional investor (see Huberman and Stanzl 2004, Gatheral 2010, Gatheral *et al.* 2012). If we let $Q_0 = 0$, our result shows that the optimal strategy for the institutional investor is $Q_t^* \equiv 0$ with $v_t^* \equiv 0$ regardless of the number of informed traders, and the corresponding performance functional $J^0(Q^*; Y^*(Q^*)) = 0$. Therefore, $J^0(Q; Y^*(Q)) \leq J^0(Q^*; Y^*(Q^*)) = 0$ for any $Q \in \mathcal{B}$. The performance functional is non-positive, and the cost of trading is non-negative on average. As a result, no price manipulation can be carried out by the institutional investor.

4. Illustrations and analysis

From the equilibrium derived in the above section, we notice that trading activities of the institutional investor and informed traders are affected by some factors like the magnitude of transaction costs and the number of informed traders. In this section, we present some numerical results to visualize these impacts. We let $Q_0 = 1$ and $T = 1$.

4.1. *Impact of transaction costs*

In order to better illustrate the impact of transaction costs, let us consider the equilibrium between one institutional investor and one informed trader shown in Corollary 3.11. We separately study the impact of m_1 and m_2 in figures 1 and 2. Firstly, the institutional investor always chooses the U-shape trading strategy, which means that the institutional investor trades rapidly at the beginning and at the end. The informed trader engages in both predatory trading and liquidity provision. The informed trader competes for liquidity at the beginning and then provides much more liquidity. Such trading pattern enables the informed trader to earn the spread between price and value by providing liquidity and gain additional revenues through predatory trading. More specifically, on the one hand the institutional investor's order execution strategy disturbs the originally efficient price so that the informed trader trades in the opposite direction to turn long-lived information into profits. On the other hand, the informed trader knows the trading intention of the institutional investor so that predatory trading can be carried out.

Specific equilibrium strategies depend heavily on the magnitude of transaction costs. In general, when transaction costs are high for the institutional investor (smaller m_1) or low for the informed trader (bigger m_2), the informed trader will engage more in predatory trading as shown in figures 1 and 2, but the reasons are different. When m_2 is fixed, high transaction costs restricts the flexibility of order execution and prevent the institutional investor from trading rapidly within a small time interval, so the institutional investor tend to trade relatively uniformly. This makes the order execution strategy more similar to TWAP strategy and gives the informed trader a better chance for predatory trading. Conversely, if the institutional investor can trade with low transaction costs, it's advisable to trade rapidly at the beginning because intense trading can significantly disturb the price and hence drive the informed trader to trade in the opposite direction to earn the spread. When m_1 is fixed, low transaction costs enable the informed trader to adjust the position more flexibly, so the informed trader can compete for more liquidity at the beginning and easily give it back at the end. However, the institutional investor can strategically trade more rapidly at the beginning to inspire the informed trader to provide liquidity earlier. As a result, high transaction costs for the institutional investor (smaller m_1) and low transaction costs for the informed trader (bigger m_2) lead to similar trading patterns of the informed trader but different trading patterns of the institutional investor. The institutional investor trades more uniformly with smaller m_1, and U-shape is more pronounced with bigger m_2.

4.2. *Multiple informed traders and transient price impact*

We fix m_1 and m_2 to focus on the impact of the number of informed traders. It can be seen in figure 3 that predatory trading is mitigated and liquidity provision is carried out earlier and in greater quantities when competition between informed traders is intense. Intense competition forces informed traders to concentrate more on liquidity provision and prevents them from earning additional revenues

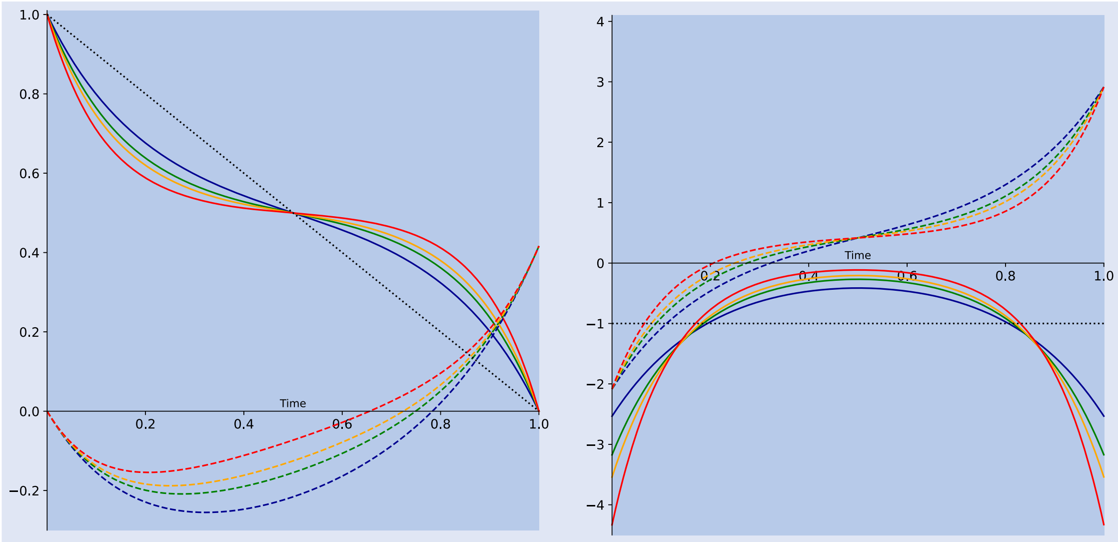

Figure 1. Expected positions (left) and expected trading rates (right) with varing m_1. Solid lines represents the institutional investor and dashed lines represents the informed trader. The black dotted line corresponds to TWAP strategy. Blue, green, orange and red correspond to $m_1 = 5, 8, 10, 15$. $m_2 = 10$.

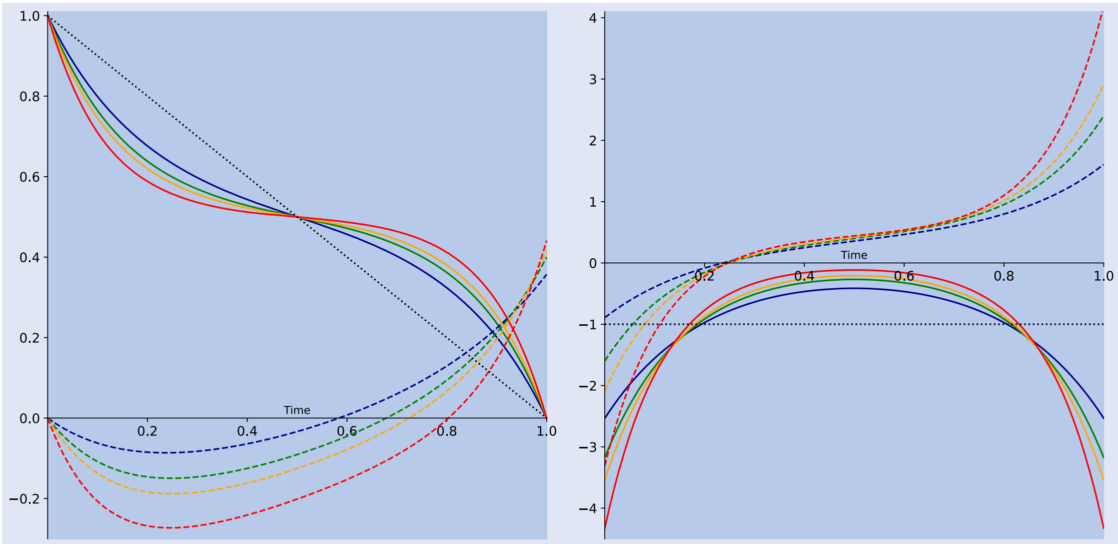

Figure 2. Expected positions (left) and expected trading rates (right) with varing m_2. Solid lines represents the institutional investor and dashed lines represents the informed trader. The black dotted line corresponds to TWAP strategy. Blue, green, orange and red correspond to $m_2 = 5, 8, 10, 15$. $m_1 = 10$.

by predatory trading. Meanwhile, as the number of informed traders increases, the institutional investor tends to trade more uniformly, but the limit order execution strategy is still U-shaped according to Corollary 3.12. Furthermore, as shown in figure 4, informed traders' trading activities prevent the price from being further distorted by the institutional investor, keeping the price and the transaction price for the institutional investor from deviating too far from the value of the asset. And with more informed traders, price distortion is smaller and the transaction price is better for the institutional investor.

From this result, we find that the cases of single informed trader and multiple informed traders are significantly different. While single informd trader is choosing when to engage in predatory trading and when to provide liquidity, multiple informed traders focus more on long-lived information because of competition. When competition is intense, informed traders seem to be unconcerned about the trading intention of the institutional investor and trade exclusively based on the spread between price and value. Also, by Corollary 3.12, informed traders in reality needn't know the trading intention of the institutional investor. The interaction between these two types of market participants can depend entirely on price dynamics. The institutional investor executes large orders and distorts the price, and informed traders observe the price deviation and trade in the opposite direction to earn the spread.

Moreover, the optimal order execution strategy for the institutional investor is U-shaped, which is consistent with the results obtained in some models regarding transient price impact (Obizhaeva and Wang 2013, Gatheral *et al.* 2012). Since informed traders are a fairly important group of market participants, we can't help wondering whether informed traders are related to price resilience. We further examine the

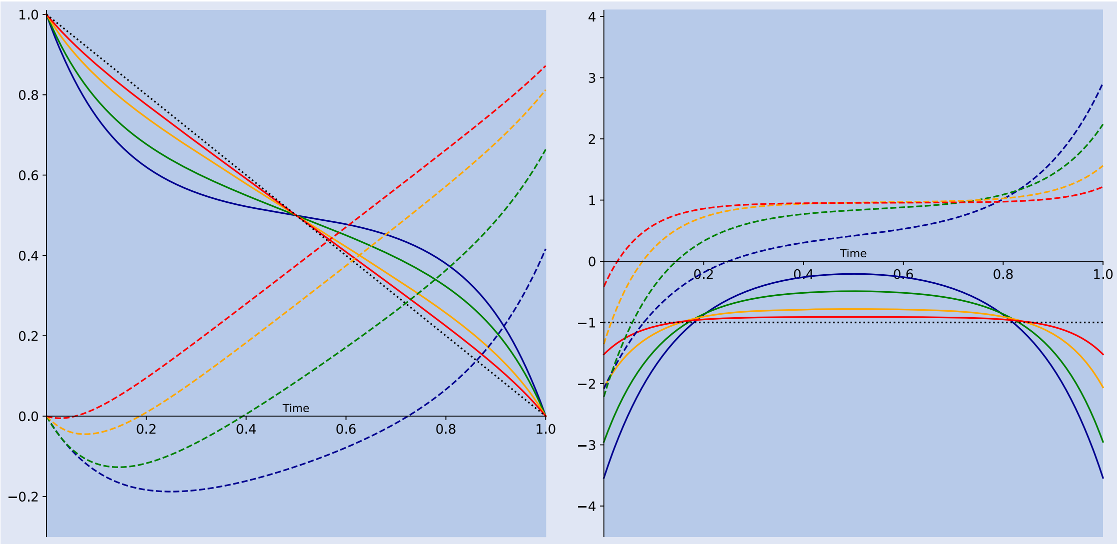

Figure 3. Expected positions (left) and expected trading rates (right) with varing N. Solid lines represents the institutional investor and dashed lines represents overall behaviors of informed traders. The black dotted line corresponds to TWAP strategy. Blue, green, orange and red correspond to $N = 1, 2, 5, 20$. $m_1 = m_2 = 10$.

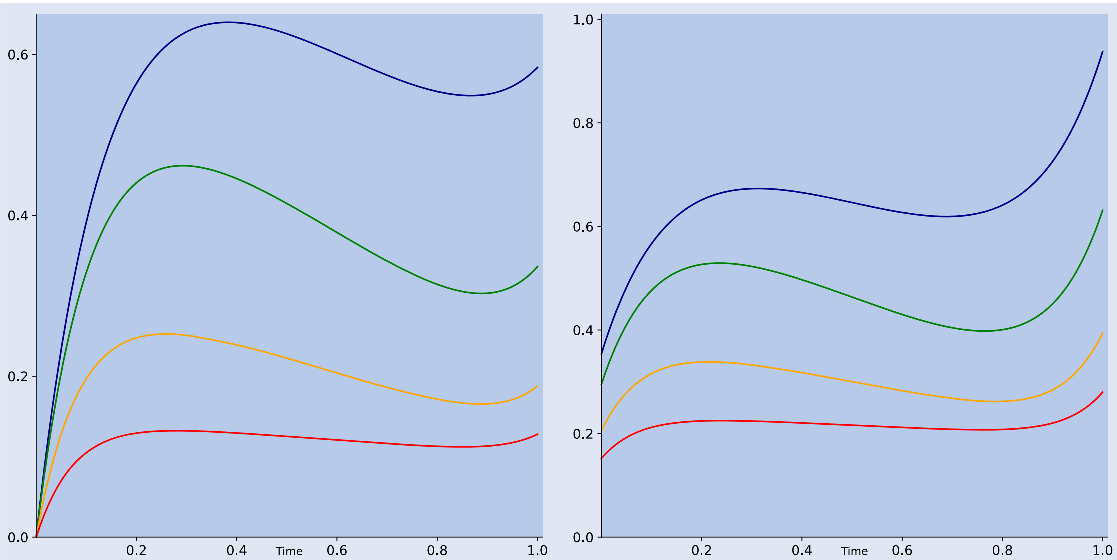

Figure 4. $\mathbb{E}[\Lambda_t]$ (left) and $\mathbb{E}[V_t - \widetilde{S}_t^l]$ (right) with varing N. Blue, green, orange and red correspond to $N = 1, 2, 5, 20$. $m_1 = m_2 = 10$. $\gamma = 1$.

impact of informed traders on price dynamics in our results. According to Corollary 3.12, when many informed traders compete in the market, the spread Λ_t approximately follows

$$\mathrm{d}\Lambda_t = -\gamma v_t \,\mathrm{d}t - m_2 \Lambda_t \,\mathrm{d}t + \sigma \,\mathrm{d}W_t,$$

which yields

$$\Lambda_t = -\gamma \int_0^t \mathrm{e}^{-m_2(t-s)} v_s \,\mathrm{d}s + \int_0^t \mathrm{e}^{-m_2(t-s)} \sigma \,\mathrm{d}W_t.$$

The affected price process hence follows

$$S_t = V_t - \Lambda_t = S_0 + \gamma \int_0^t \mathrm{e}^{-m_2(t-s)} v_s \,\mathrm{d}s - \int_0^t \mathrm{e}^{-m_2(t-s)} \sigma \,\mathrm{d}W_t + \sigma_2 B_t^2.$$

With no order execution ($v = 0$), the original price only affected by informed traders is given by

$$S_t^0 = S_0 - \int_0^t \mathrm{e}^{-m_2(t-s)} \sigma \,\mathrm{d}W_t + \sigma_2 B_t^2,$$

which doesn't have to be a martingale. S_t^0 can have a short-term trend due to order flow imbalance, which have been discovered and studied in Cartea and Jaimungal (2016). When the institutional investor executes orders in the market, the transaction price for the institutional investor is given by

$$\widetilde{S}_t^l = S_t^0 + \gamma \int_0^t \mathrm{e}^{-m_2(t-s)} v_s \,\mathrm{d}s + \eta_1 v_t,$$

which is exactly the model setup in Neuman and Voß (2022, 2023). And the second term is well-known transient price impact introduced in Obizhaeva and Wang (2013) and Gatheral *et al.* (2012). More importantly, our Corollary 3.12 derived from the stackelberg-Nash game between the institutional investor and informed traders is exactly the result of the order execution problem with temporary and transient price impacts in Chen *et al.* (2019). As a result, we propose a reasonable conjecture that transient price impact may be caused by informed traders and their competition. While other market participants adjust the market price based on the average information content of each order, informed traders possess long-lived information and know that current order flow contains no true information. Informed traders gradually offset price distortion caused by the institutional investor to earn the spread and maintain price efficiency. The decay is exponential and the convergence speed m_2 highly depends on informed traders' transaction costs. In addition, it's worth noting that such exponential decay is derived in the linear price impact model and in the risk-neutral case, the decay kernel might be different as assumed in Gatheral *et al.* (2012) because of nonlinear price impact and risk aversion of informed traders. Finally, Nash competition between institutional investors at the leader level can also be considered in our Stackelberg-Nash game, and based on our conjecture we think the result shoud be consistent with that derived from the liquidation game with temporary and transient price impacts.

5. Conclusion

In this paper, we study the interaction of order execution and informed trading. We consider a Stackelberg-Nash game and solve the unique equilibrium explicitly. Informed traders provide liquidity to the institutional investor on the whole to earn the spread between price and value, but prefer to first compete for liquidity to gain additional revenues by predatory trading. The institutional investor employs a well-known U-shape execution strategy to discourage informed traders from predation and inspire them to provide liquidity. Informed traders and their competition offset price distortion caused by the institutional investor, and the decay is found to be approxiamtely exponential in the linear price impact model. These results give a plausible explanation for our conjecture that price resilience and transient price impact may be caused by informed trading.

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