Quantitative Finance

ISSN: (Print) (Online) Journal homepage: www.tandfonline.com/journals/rquf20

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To cite this article: Marie-Hélène Gagnon, Gabriel J. Power & Dominique Toupin (2022) Forecasting market index volatility using Ross-recovered distributions, Quantitative Finance, 22:2, 255-271, DOI: 10.1080/14697688.2021.1939407

To link to this article: https://doi.org/10.1080/14697688.2021.1939407

Published online: 19 Jul 2021.

Quantitative Finance, 2022
Vol. 22, No. 2, 255–271, https://doi.org/10.1080/14697688.2021.1939407

Forecasting market index volatility using Ross-recovered distributions

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(*Received 1 July 2020; accepted 28 May 2021; published online 19 July 2021*)

The Ross recovery theorem shows that option data can reveal the market's true (physical) expectations. We adapt this approach to international index options data (S&P, FTSE, CAC, SMI, and DAX) to improve volatility forecasting. We separate implied volatility into Ross-recovered expected volatility and a risk preference proxy. We investigate the performance of these variables, constructed domestically or globally, to forecast realized volatility as well as index excess returns. The results show evidence of significantly improved forecasts and yield new insights on the international dynamics of risk expectations and preferences. Across indexes, models using Ross-recovered, value-weighted global measures of risk preferences perform best. The findings suggest that the recovery theorem is empirically useful.

Keywords: Ross recovery; Risk-neutral; Volatility; Forecast; Options; International; Physical distribution

1. Introduction

The value of an equity index such as the S&P 500 reflects the market's expectations about the index's future returns as well as the market's risk preferences. According to Ross (2015), it is possible to recover the market's expectations using only data on index option prices, obviating the need to estimate the physical distribution of returns from historical price data. In particular, the recovery theorem allows for model-free tests as it does not make any parametric assumptions on the utility function of the representative agent. More specifically, the theorem shows how to separately identify market expectations and risk preferences that are implied in option prices.

Ross's seminal paper has sparked significant interest and theoretical debate (Borovička *et al.* 2016, Jensen *et al.* 2019, Schneider and Trojani 2019). The recovery theorem shows promise for empirical applications because historical distributions constructed from past realized returns tend to be unreliable. Therefore, it is hard to overstate the importance of learning more about the distribution of expected returns, whether it is for purposes of asset pricing, portfolio allocation, capital budgeting, forecasting, or risk management.

In this paper, we empirically implement the recovery theorem using option data for five major indexes. To assess the theorem's empirical relevance and to document the two components of option-implied volatility (physical volatility expectations and risk preferences), we examine the performance of Ross-recovered volatility expectations and risk preferences to forecast realized volatility. Specifically, we make two contributions.

First, we investigate the empirical relevance of the theorem in international equity markets. We ask whether the Ross-recovered distribution contains new and useful information compared with the risk-neutral distribution. Thus, we do not aim to determine whether Ross-recovered distributions are 'true', but whether they are empirically informative.

We find that both Ross-recovered components help improve volatility forecasts. Therefore, applying Ross's theorem is empirically relevant, as it yields significant predictive improvements. We build on the volatility forecasting literature, which shows that risk-neutral† volatility (RNV) is a better predictor of future volatility than is volatility measured from past returns (e.g. Christensen and Prabhala 1998, Blair *et al.* 2001, Szakmary *et al.* 2003, Poon and Granger 2005, Christiansen *et al.* 2012, or Taylor *et al.* 2010). Thus, we show

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† For the USA, in effect, RNV is the VIX.

that volatility forecasts can be improved when the RND is decomposed using the recovery theorem.†

Our paper is related and complementary to Audrino *et al.* (2019). They implement the recovery theorem for the S&P500 index using a different nonparametric approach and show that the recovered distribution allows for improved market timing. Our paper also builds on to the literature estimating forward-looking pricing kernels, such as Linn *et al.* (2018) and Barone-Adesi *et al.* (2020). These contributions investigate the possible mismatch between physical and risk-neutral distributions and its role in generating the pricing kernel puzzle.

Our second contribution concerns cross-country investor risk preferences. This issue is of fundamental importance to international finance, as it concerns the sources of systemic risk worldwide and how investor preferences are formed at a global level. Given the available decomposition that is provided by applying the recovery theorem, we investigate whether risk preferences and volatility expectations are better measured locally or globally. This question has been examined previously in asset pricing models (e.g. Griffin 2002, Bekaert *et al.* 2005, Carrieri *et al.* 2007, or Beaulieu *et al.* 2016). These contributions aimed at assessing whether known risk factors (such as Fama and French factors) are better proxied using local or global measures. So far, however, there is no comparable in-depth exercise for variables explaining realized volatility. Some recent studies, however, have made significant progress (see e.g. Buncic and Gisler 2016, 2017). For instance, Zhang *et al.* (2020) conclude that volatility across different country indexes has a significant local component that is not driven by the US market.

Furthermore, the literature on risk preferences has documented that global measures of behavior towards risk are appropriate (Beer *et al.* 2011, Baker *et al.* 2012, Bollerslev *et al.* 2014, and Schmeling 2009). This is where our contribution lies, as Ross's theorem allows one to reconcile the local components of risk premia and the global nature of sentiment and preferences by jointly analyzing the two components without imposing specific parametric hypotheses. While this literature is rich, our study is the first to use true, forward-looking risk preferences to investigate their global or domestic origin. Our results suggest there is an important global component to risk preferences, one that is largely unaffected by local culture or bias. We find that global measures of risk preferences lead to more accurate volatility forecasts, so if culture affects investors, it is not through risk preferences. Our results therefore suggest that fluctuations in risk preferences tend to spread globally.

1.1. Related literature on the recovery theorem

Since Breeden and Litzenberger (1978), a large literature has examined what we can learn about the market's expectations and investor risk aversion by first constructing the risk-neutral distribution (RND) implied from option data (e.g. Jackwerth and Rubinstein 1996, Jackwerth 2000, Bliss and Panigirtzoglou 2004). Ross's (2015) recovery theorem provides a novel approach to learn about the market's expectations using only options data. A rich theoretical literature has soon emerged. Carr and Yu (2012) extend Ross recovery to a continuous state-space setting for bounded diffusions. Park (2016) also finds sufficient conditions for Ross recovery in a continuous state space. Walden (2017) extends the result for unbounded diffusions. Qin and Linetsky (2016) establish Ross recovery in the more general state-space of Borel right processes. Borovička *et al.* (2016) question whether Ross's theorem recovers uniquely the true physical distribution. Dubynskiy and Goldstein (2013) are also critical of Ross's bounding restrictions on state vector dynamics. However, Jensen *et al.* (2019) show how to generalize Ross recovery. In particular, their results do not depend on time homogeneity assumptions. Lastly, Schneider and Trojani (2019) develop an alternative, but related, theoretical framework to recover accurate physical distributions.

The empirical literature related to the recovery theorem is smaller but growing. Martin and Ross (2019) use the recovery theorem to study properties of the 'long bond', namely the distant end of the yield curve. Bakshi *et al.* (2018) test the recovery theorem using 30-year Treasury bond futures. Jackwerth and Menner (2020) investigate the performance of the recovered distribution to predict future realized returns for the US S&P 500 index. Using distributional tests, they reject the hypothesis that S&P index realized returns are drawn from the Ross-recovered empirical distribution. Jensen *et al.* (2019) also find that the recovered distribution predicts future returns poorly. However, Audrino *et al.* (2019) find evidence of profitable market timing strategies for the S&P500 using moments of the recovered distribution.

Our paper extends the empirical literature on the recovery theorem in three significant ways. First, our empirical study investigates volatility forecasts as well as returns. Our objective is to show that careful application of the theorem allows for the recovery of useful information that helps to better forecast volatility. Our study is related to, but distinct from, Jackwerth and Menner (2020). They aim to assess whether Ross's recovered distribution is the true distribution of future returns, as predicted in Ross (2015), and whether the recovered distribution is biased relative to the distribution of future realized returns. In contrast, our objective is to show that the recovered distribution, while imperfect, is useful empirically and relevant to finance researchers and practitioners.

Second, our study is not limited to the US S&P 500 index, but rather considers five major international equity index markets including the S&P, adding robustness and depth to our results. Third, this paper proposes a novel empirical design to implement Ross recovery, yielding more economically plausible outputs than what is obtained by directly applying Ross's theorem to the data. In fact, implementing the theorem empirically turns out to be challenging. For example, we show that a straightforward implementation of the recovery theorem leads to distributions that are not economically plausible, a result in line with Jackwerth and Menner (2020).

Our approach yields two main results. First, the empirical implementation of the recovery theorem leads to better forecasts of realized volatility for all indexes in our sample, compared to the traditional benchmark of using risk-neutral volatility. The improvements are statistically and

† Jensen *et al.* (2019) show that Ross-recovered volatility predicts future realized volatility, but their analysis is only for the US market.

economically significant, as R^2 increases by up to 8%. Thus, we conclude that the recovery theorem is empirically successful in our setting and that more accurate market expectations and risk preferences can be recovered since the new information significantly improves volatility forecasts.

Second, we find additional insights stemming from the fact that the recovery theorem allows for a separation of volatility expectations from a risk preference proxy. In doing so, we find that both components are useful to predict market volatility. We further exploit the decomposition made possible by the recovery theorem and separate the variance risk premium (VRP) into two components to conduct predictive regressions of index excess returns. Our analysis suggests that the VRP's explanatory power for excess returns stems mostly from one component, the 'expected change in uncertainty'.

We further find that risk preferences are global, but volatility expectations are generally domestic. This result suggests that the markets in our sample are sufficiently integrated that using global risk preferences is preferable to using domestic measures to forecast volatility, and that international market participants have common, globally shared risk preferences in those markets. In contrast, we do not typically find improvements in forecast performance from models using only global risk-neutral volatility.

2. Recovering expectations and risk preferences from option data

2.1. Ross's recovery theorem

This section summarizes the recovery theorem (Ross 2015), focusing on some key aspects for our empirical implementation. Simply put, Ross's theorem states that option prices can be exploited to infer both the probability that a future state of the world will occur as well as the magnitude of this event. In fact, the theorem states that one can recover the physical distribution of returns using only option prices, without making strong assumptions regarding investor utility and risk preferences. Since academics and practitioners alike would like to know more accurately the probabilities associated with future market conditions and given that the recovery theorem offers a new way forward, the implications of such a claim are potentially immense for finance and economics.

Let the stochastic discount factor (SDF) be a random variable m that relates the price p of an asset today to its final value (random variable x) in each possible future state of nature.

$$p = \mathrm{E}[mx] \tag{1}$$

If the Law of One Price holds the SDF exists, and it is strictly positive in the absence of arbitrage. Thus, knowing the SDF and the probability of getting to each future state, one can price any asset based on the values it attains in each state. Assuming a representative investor, the SDF is proportional to his or her marginal utility, so it takes on higher values in 'bad' states and vice versa.

Consider an Arrow-Debreu security (state price) paying off \$1 in state j and zero otherwise. If the current state is i, the state price from i to j is:

$$p_{i,j} = \mathrm{E}[mx] = \pi_{i,j} m_{i,j} \times 1 \tag{2}$$

where $\pi_{i,j}$ is the probability of going from state i to state j and $m_{i,j}$ is the SDF for this state. The asset's price can be directly observed, but $\pi_{i,j}$ and $m_{i,j}$ cannot be separately identified. Equivalently, we can express this relation using risk-neutral probabilities:

$$p_{i,j} = \mathrm{E}^*[R_f^{-1} x] = \pi^*_{i,j} R_f^{-1} \tag{3}$$

where R_f is the risk-free interest rate and the asterisk (*) denotes risk-neutral probabilities. The latter can be recovered directly from financial data using principles of option pricing theory, as options can be represented as a sum of Arrow-Debreu securities (e.g. Rubinstein 1994). Thus, knowing any two of the following is sufficient to learn the value of the third one: (i) the physical probability distribution of attaining each state of nature, ($\pi_{i,j}$) in eq. (2); (ii) the SDF of each state of nature; (iii) the risk-neutral probability distribution of attaining each state of nature *or* the state price of each state.

State prices can be recovered directly from option prices (Breeden and Litzenberger 1978), leaving two unknown quantities. Therefore, researchers in this literature make assumptions about either the physical probability distribution in order to learn about the SDF, or vice versa. Jackwerth (2000), Aït-Sahalia and Lo (2000), and Rosenberg and Engle (2002) assume that the physical distribution of expected returns is the same as the historical distribution, and use this evidence to study the SDF. However, it is well known that it is difficult to obtain a reliable physical distribution of asset returns from historical data (e.g. Conrad *et al.* 2013).

Others such as Bliss and Panigirtzoglou (2004) assume a specific form for the SDF (e.g. time-separable power utility), allowing them to test the predictive power of option data for realized returns. This approach, however, is based on restrictive assumptions on the representative investor. Thus, the validity of the results from this literature is sensitive to the correctness of the assumptions made about one of the three quantities previously discussed. In contrast, the recovery theorem solves this problem by letting us recover all three quantities directly from option prices, under less restrictive assumptions.

Ross considers state prices obtained from option data and defines a matrix P where each element p_{ij} is the state price of state j when we are currently in state i. These state prices are computed from assets that are contingent claims on the same Markovian state variable X. Note that X could be an equity index, in which case P would be computed from index option prices at a specific date. In order to end up with only one possible matrix P that represents the evolution of X, Ross (2015) makes a first assumption:

> *Assumption 1*: The process for X is time-homogenous on a finite state space.

This means that the matrix P represents state prices from time 0 to t as well as those from time t to $t+1$, $t+1$ to $t+2$, etc. In economic terms, the probabilities of going from one given state to another are constant across time. Loosely speaking, we can interpret this as a 'steady state' assumption where the probability of transitioning from one state to another will not be affected by current market conditions.

Another assumption is needed to separately identify the physical probabilities and the SDF:

Assumption 2: The SDF is transition-independent and is of the following form:

$$m_{i,j} = \delta \frac{d_j}{d_i} \tag{4}$$

where δ is a positive constant representing the market's average discount rate and $d(\cdot)$ is a function of the corresponding state of nature. Thus, the SDF is not dependent on intermediate states that are reached before attaining the final state j. For instance, a time-additive utility function implies a transition-independent SDF. Intuitively, in a time-additive setup, path dependency is ruled out. Essentially, in this setting, one cares only about the starting and ending states of nature and not about how one gets from the starting state of nature to the ending state of nature. With those assumptions, eq. (2) can be reformulated as:

$$p_{i,j} = m_{i,j}\pi_{i,j} = \delta \frac{d_j}{d_i}\pi_{i,j} \tag{5}$$

If the state space that can be reached by the Markovian variable X has a finite number of elements n, the state price matrix P is $n \times n$. The (physical) probability transition matrix F, with its elements $\pi_{i,j}$ is also $n \times n$. Then, eq. (5) can be rewritten as:

$$P = \delta D^{-1} F D \tag{6}$$

where $D_{n\times n}$ is a matrix with elements d_i on the diagonal and zeros elsewhere. Knowing that F is a probability transition matrix, its rows must add up to 1, therefore $F\vec{1} = \vec{1}$ where $\vec{1}$ is a vector of ones. The previous equation can now be written as:

$$PD^{-1}\vec{1} = \delta D^{-1}\vec{1} \tag{7}$$

Finally, we define the vector $D^{-1}\vec{1}$ to obtain the following familiar form:

$$P_{n\times n} z_{n\times 1} = \delta z_{n\times 1} \tag{8}$$

Eq. (8) is a classical characteristic root problem where the eigenvectors (z) and the eigenvalues (δ) correspond to a square matrix (P). Without additional assumptions, the solution will be expressed as complex numbers having a real and imaginary part. Additional steps are needed to insure a solution without an imaginary part, to be consistent with financial market data. The Perron-Frobenius theorem states that for a characteristic root problem such as eq. (8), if P is non-negative and irreducible, there exists only one strictly positive eigenvector and that its corresponding eigenvalue is positive and real. This eigenvalue is the largest absolute in the possible eigenvalues. Thus, if P is positive and irreducible, (8) has a unique positive solution. Since the elements of P are state prices, P is positive under the no-arbitrage condition. To make sure that it is also irreducible, we need one last assumption:

Assumption 3: The Markovian variable X can reach any state j from a state i in a finite number of steps.

In economic terms, this means there is always a strictly positive probability of reaching any state j at any point. The Perron-Frobenius theorem additionally states that the solution will be unique.

Now that (8) has a unique positive solution, we can compute the SDF, the discount rate δ and the physical probability transition matrix F using only the matrix of state prices, P. To this end, we only need to look at option prices for different strikes and maturities on a particular asset, e.g. an equity index.

2.2. *From options to the matrix of state prices P*

State prices at a specific date can be computed from the prices of call options C for a given maturity T and a continuum of strike prices K (Breeden and Litzenberger 1978):

$$s(K,T) = \frac{\partial^2 C(K,T)}{\partial K^2} \tag{9}$$

where $s(K,T)$ is the state price corresponding to a state of nature where the value of the asset is K at time T.† If we consider a finite state space of m maturities and n strike prices, the state prices can be grouped in a matrix $S_{n\times m}$. This matrix is an 'implied state price surface', a transformation of the commonly used implied volatility surface. It is not, however, equal to the state price matrix $P_{n\times n}$ that we need. Indeed, $S_{n\times m}$ consists of prices of contingent claims (valued today) that pay off $1 if the underlying asset equals K at time T, for different values of (K,T). Meanwhile, $P_{n\times n}$ consists of prices of contingent claims (valued in state i) that pay off 1$ if the final asset value is K (state j). In P, the maturity of the contingent claims is fixed. P can be computed for any maturity. This choice is arbitrary since by assumption the underlying process is time-homogenous.

To obtain a plausible solution for P, we use the fact that there is one row of P that is already known. Indeed, if we consider that P represents monthly transitions from state to state, the row of P where ($i =$ today's state) and ($j =$ every strike) is equal to the column of S where ($K =$ every strike) and ($T = 1$ month). The remaining rows of P correspond to state prices we would observe today if we were in a different state than i. Assuming that the underlying process is time-homogenous, P and S are related by:

$$S_{:,t}^T P = S_{:,t+1}^T. \tag{10}$$

and since $m > n$, there are enough equations to find the n^2 unknown state prices.

† A state of nature could be defined by any number of variables. For example, state i could be defined as a state where the S&P500 is at 2200, market volatility is at 20%, and the US economy is in a recession. A change in any of those variables would mean that we are now in a different state of nature. Empirically however, we are limited by the kind of contingent claims that are traded on the market. Thus, we implicitly assume that the possible future states of nature are completely defined by the strike prices of an asset.

2.3. Empirical considerations

2.3.1. The implied state price surface *S*. First, we apply a method presented by Birru and Figlewski (2012) to obtain from option prices a risk-neutral probability density function for any given maturity. From eq. (3), state prices for a given maturity equal risk-neutral probabilities that are discounted at the risk-free rate. By repeating this step for all m maturities needed, we obtain the complete implied state price surface S. If n discrete strike prices are considered, we can infer a $S_{n\times m}$ matrix of implied state prices.

2.3.2. The transition state prices matrix *P*. According to eq. (10), the state price surface S has to be split in two overlapping submatrices in order to recover P. The lag time τ between those submatrices corresponds to the horizon of the resulting state price transition matrix P. Denoting the submatrices by $A^T = S_{:,[1:m-\tau]}$ and $B^T = S_{:,[1+\tau:m]}$, finding the solution to eq. (10) can be written as:

$$\min_{P\geq 0} ||AP - B||^2. \tag{11}$$

This is equivalent to n separate least-squares problems:

$$\min_{p_j\geq 0} ||Ap_j - B||^2, j = 1, 2, \ldots, n \tag{12}$$

where p_j and b_j are the j-th columns of P and B, respectively.

There are known algorithms to solve this problem, but the results are not plausible for a distribution of expected returns. Audrino *et al.* (2019) note that the matrix A appears to be ill-conditioned, as the solution P is highly sensitive to small perturbations of A. This is clear from Figure 1, Panel A, which shows an example of P solved from eq. (12). Graphically, this figure represents an 'unstable' solution for P. Indeed, since the state price transition matrix P is a slight transformation of the risk-neutral probability transition matrix, we would expect this figure to look roughly like a ridge along the diagonal. Such a shape would represent a distribution where, for each possible beginning state of nature i, the most probable state at time τ is a state that is close to i. For example, suppose the beginning state is a stock index level of 2000. Then, the most probable state at time τ would be at a level near 2000. This pattern is visible in some parts of Figure 1, Panel A, but many observations (including spikes) are far from the diagonal, especially on the left side of the panel.

2.3.3. Obtaining a plausible distribution of returns. Our empirical approach to obtain a plausible matrix P relies on three further constraints to the optimization problem in eq. (12). First, we impose unimodality for each vector of state prices for a given final state of nature j, i.e. p_j in eq. (12). This constraint is based on a suggestion in Ross (2015) and is also implemented in Jackwerth and Menner (2020).

Second, we constrain the solution to P to ensure coherence with the row of P that is observable from today's state prices. By definition, the matrix P and the matrix S have one row in common, corresponding to the state of nature (e.g. index level) for the current date (see Ross 2015). This constraint is added to the optimization problem in eq. (12).

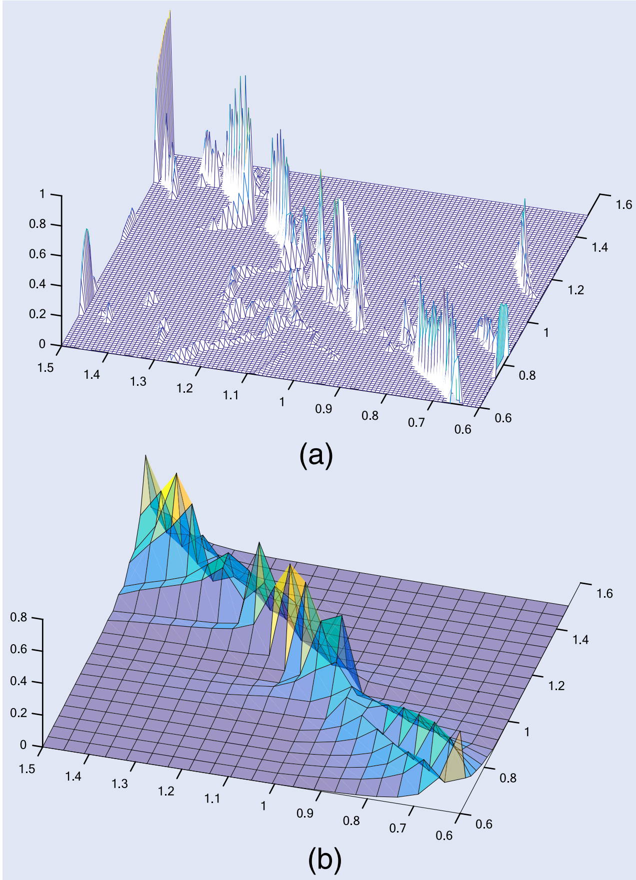

Figure 1. State price transition matrix P obtained from least-squares problem. Example from option data on the S&P500 index. The X and Y axes are moneyness in states i and j (strike prices relative to the index level that day). The z-axis is the corresponding state price p_{ij}. Panel A presents a direct application of eq. (12) while panel B shows the results using our novel approach, necessary to obtain a plausible transition matrix P.

Third, we limit the number of state prices considered. We use a relatively small number of strike prices for the dimensions of P. At first, it is tempting to use a fine grid for option moneyness (strike prices) in order to obtain a finer probability distribution representing the market's true expectations. This approach, however, turns out to be problematic when solving the optimization problem in eq. (12) numerically, as it makes the solution more sensitive to the input data. This difficulty implies a necessary tradeoff. Figure 1, Panel B, shows how our approach leads to a plausible matrix P. This solution for the transition matrix is obtained using a grid of 21 moneyness points† in P.‡

† Moneyness is the strike relative to the level of the index that day (mn = strike / level of index).

‡ Our robustness analysis suggests that using 21 points in moneyness is an appropriate tradeoff given our goal of measuring variance of the recovered distribution. For other purposes, one could obtain a finer distribution of recovered expectations with the help of some additional manipulations. Once the pricing kernel for a coarse grid of moneyness is obtained, one can fill in its gap by fitting it to a curve (e.g., a spline) and then use it to transform a fine risk-neutral distribution into an equally fine probability distribution of recovered expectations.

Finally, we apply the recovery theorem on option data for several international indexes by solving the optimization problem in eq. (12) using the previously stated constraints. The implied state price surface S is discretized in $m = 100$ evenly spaced maturities† from 1 month to two years. On the strike axis, we discretize moneyness from 0.6 to 1.5 ($n = 21$). The implied state price matrix S is then split in two submatrices with a lag of $\tau = 1$ month. Thus, the recovered matrix of physical transition probabilities F represents expectations for 1-month-ahead returns. The relevant physical probabilities are contained in the row of F that corresponds to the current state of nature (i.e. moneyness = 1). The volatility of this physical distribution can be interpreted as the market's volatility expectations for each international index. It is therefore computed from variance and annualized for ease of comparison and interpretation.

Figure 2 shows time series of each variable used in our predictive regressions. Panels A to E show this information for each country. In each panel, the first plot shows the three measures of volatility that we compute, namely realized, risk-neutral, and Ross-recovered. We obtain risk-neutral and recovered volatilities from the one-month horizon risk-neutral distribution computed from option price data, i.e. when we previously obtain state prices for the matrix S. Realized volatility (see e.g. Gatheral *et al.* 2018) is computed from intraday data (see the Data section).

In each country, risk-neutral volatility is typically higher than realized volatility (consistent with the literature) since it embeds risk aversion, but realized volatility spikes during crisis periods. Ross-recovered volatility is generally lower than risk-neutral volatility. This makes economic sense because the recovery theorem aims to separate physical distributions from the risk-aversion component.

The second plot shows global (value-weighted) measures of risk-neutral and recovered volatilities. The third plot shows monthly innovations (differences) of risk-neutral (RNV) and recovered (REV) volatilities, while the fourth plot shows the global equivalents. The fifth plot shows the difference between recovered and risk-neutral volatilities, which can be interpreted as a proxy for risk aversion. These risk preference corrections are time-varying, but fluctuate less than do volatility series.

3. Empirical forecasting methodology

In this section, we present our strategy to investigate the forecasting performance of variables drawn from the Ross-recovered distribution for each of the five international equity index markets in our dataset. Our objectives are twofold: (i) To assess whether incorporating information from the recovered distribution improves the performance of a model that forecasts index realized volatility, compared to a model that uses only the risk-neutral distribution. (ii) To determine whether market risk expectations and risk preferences are globally shared or domestic. Our approach therefore contributes to an active literature on volatility forecasting (e.g. Corsi 2008, Christiansen *et al.* 2012, or Mittnik *et al.* 2015).

We perform a series of eight predictive regressions for each stock market index. The dependent variable is h-period ahead realized volatility for each index. Table 1, panel A (below), describes the variables used in the regressions. The independent variables are (domestic or global) risk-neutral volatility (RNV) and Ross-recovered physical expected volatility (REV), as well as a (domestic or global) risk preference proxy.‡ The global versions of the factors (gRNV and gREV) are value-weighted. As in Bollerslev *et al.* (2014), we use relative market capitalizations of the components of each equity index§ at each date over the sample period.

This risk preference proxy is the correction factor needed to go from risk-neutral to recovered physical volatility. The difference between the two volatilities is therefore a model-free measure of risk preferences. Our approach requires no further assumptions (e.g. on utility) and further aligns with our goal of predicting volatility. Thus, the recovery theorem allows for new insights by studying the variable (REV – RNV).

This variable is related to the theoretical definition of the variance risk premium (VRP) in Bollerslev *et al.* (2009). However, it is defined over volatility rather than variance, given our objective of forecasting future volatility. Therefore, the risk preference proxy is labeled in this paper as a Ross-recovered VoRP. Furthermore, note that the empirical VRP in Bollerslev *et al.* (2009) and in Bollerslev *et al.* (2014) relies on past realized variance to obtain a proxy for the theoretical VRP defined in their model. The recovery theorem contributes to this literature by providing a novel method to directly measure the theoretical VRP, since one can use a forward-looking physical variance instead of a historical realized variance. This quantity has both an economic and financial interpretation. Economically, it proxies for investor risk preferences. Financially, it proxies for a market 'fear' premium.

4. Data

Our data sources are Optionmetrics Ivy DB US and Europe, Datastream, Federal Reserve Bank of St. Louis (FRED), and Oxford-Man Realized Library. Market capitalizations used for the global value-weighted measures are obtained from Datastream. Country risk-free rates index are obtained from FRED. Realized variances computed as the sum of the 5- minute realized variance and squared overnight returns of the past month for each index are obtained from the Oxford-Man Institute's realized library's website (Gerd *et al.* 2009). Index option data are obtained from OptionMetrics Ivy DB U.S. and Europe. The selected indexes are those with the most options available among the markets covered by this database. They are the S&P500 index for the United States, the DAX index for Germany, the SMI index for Switzerland, the CAC index for

† Using 100 maturities over 2 years corresponds to a step of about 1 week. This level of resolution is needed in order to fully capture the information when options are available at weekly maturities.

‡ Note that using VIX instead of RNV for the U.S. S&P makes no difference in the results.

§ As an indication, over the full period the average weights for the S&P500, FTSE, CAC, SMI, and DAX are respectively 69.4%, 14.3%, 6.0%, 4.8%, and 5.5%.

France, and the FTSE index for the United Kingdom. These markets are significant for the Eurozone and have active index options markets. Strike prices, maturities, and implied volatilities are extracted for all available options on the selected indexes.

Descriptive statistics for the raw option data are presented in Table 2. The usual filters used in the literature (see e.g. Carr and Wu 2009) when working with options are applied to the raw data. Options are excluded if they are expiring in five days or less or having implied volatility that is negative or above 100%. Only out-of-the-money options are used to obtain the volatility surface. As a result of applying the filters, options that violate no-arbitrage conditions are eliminated.

We predict next-month ($h = 21$ business days) realized volatility using time-t risk-neutral volatility (RNV) or Ross-recovered physical volatility (REV). Volatility is

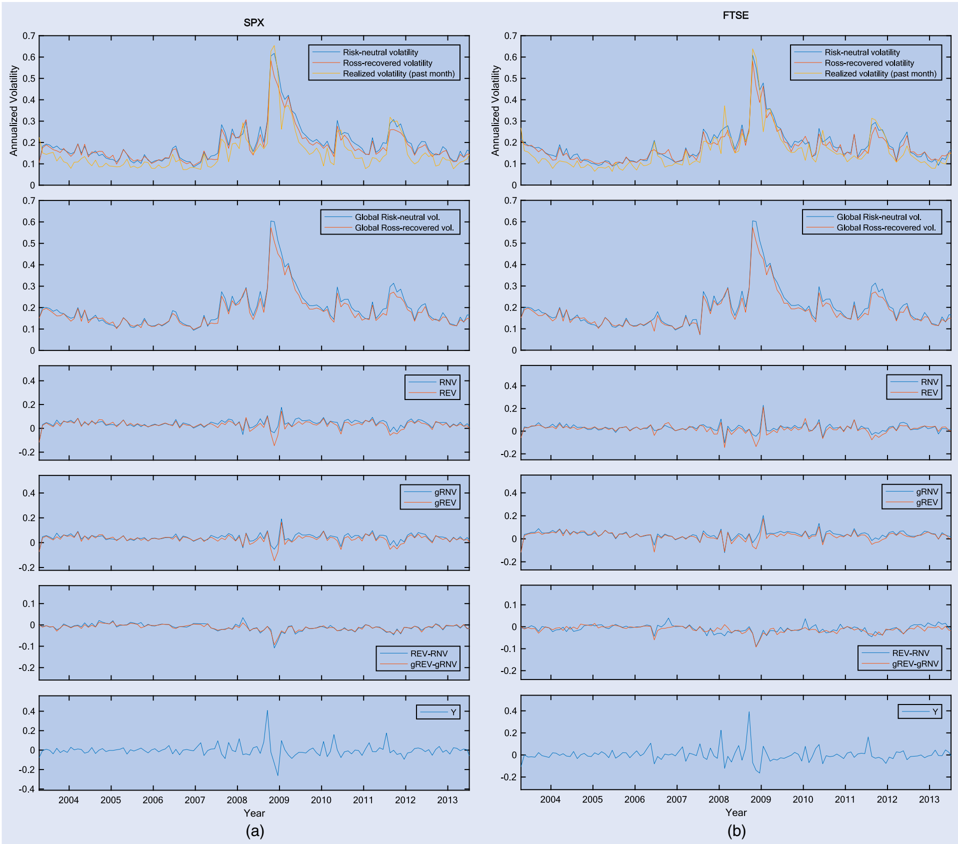

Figure 2. Panel A: Ross-recovered, realized, and risk-neutral volatility (S&P500 index options). This panel shows monthly observations of each explanatory variable used in our volatility forecasts, for S&P index options data. The period covered is 4/2003-8/2013. The first two plots show domestic and global volatility levels. Third and fourth are plots of monthly volatility innovations, domestic (RNV, REV) and global (gRNV, gREV). Fifth is the difference between Ross-recovered and risk-neutral volatilities, a risk aversion proxy. The last plot (Y) is the dependent variable, i.e. realized volatility innovations. We compute Ross-recovered and risk-neutral volatility from options data and realized volatility from intraday data. Panel B: Ross-recovered, realized, and risk-neutral volatility (FTSE index options). This panel shows monthly observations of each explanatory variable used in our volatility forecasts, for FTSE index options data. The period covered is 4/2003-8/2013. The first two plots show domestic and global volatility levels. Third and fourth are plots of monthly volatility innovations, domestic (RNV, REV) and global (gRNV, gREV). Fifth is the difference between Ross-recovered and risk-neutral volatilities, a risk aversion proxy. The last plot (Y) is the dependent variable, i.e. realized volatility innovations. We compute Ross-recovered and risk-neutral volatility from options data and realized volatility from intraday data. Panel C: Ross-recovered, realized, and risk-neutral volatility (CAC index options). This panel shows monthly observations of each explanatory variable used in our volatility forecasts, for CAC index options data. The period covered is 4/2003-8/2013. The first two plots show domestic and global volatility levels. Third and fourth are plots of monthly volatility innovations, domestic (RNV, REV) and global (gRNV, gREV). Fifth is the difference between Ross-recovered and risk-neutral volatilities, a risk aversion proxy. The last plot (Y) is the dependent variable, i.e. realized volatility innovations. We compute Ross-recovered and risk-neutral volatility from options data and realized volatility from intraday data. Panel D: Ross-recovered, realized, and risk-neutral volatility (SMI index options). This panel shows monthly observations of each explanatory variable used in our volatility forecasts, for SMI index options data. The period covered is 4/2003-8/2013. The first two plots show domestic and global volatility levels. Third and fourth are plots of monthly volatility innovations, domestic (RNV, REV) and global (gRNV, gREV). Fifth is the difference between Ross-recovered and risk-neutral volatilities, a risk aversion proxy. The last plot (Y) is the dependent variable, i.e. realized volatility innovations. We compute Ross-recovered and risk-neutral volatility from options data and realized volatility from intraday data. Panel E: Ross-recovered, realized, and risk-neutral volatility (DAX index options). This panel shows monthly observations of each explanatory variable used in our volatility forecasts, for DAX index options data. The period covered is 4/2003-8/2013. The first two plots show domestic and global volatility levels. Third and fourth are plots of monthly volatility innovations, domestic (RNV, REV) and global (gRNV, gREV). Fifth is the difference between Ross-recovered and risk-neutral volatilities, a risk aversion proxy. The last plot (Y) is the dependent variable, i.e. realized volatility innovations. We compute Ross-recovered and risk-neutral volatility from options data and realized volatility from intraday data.

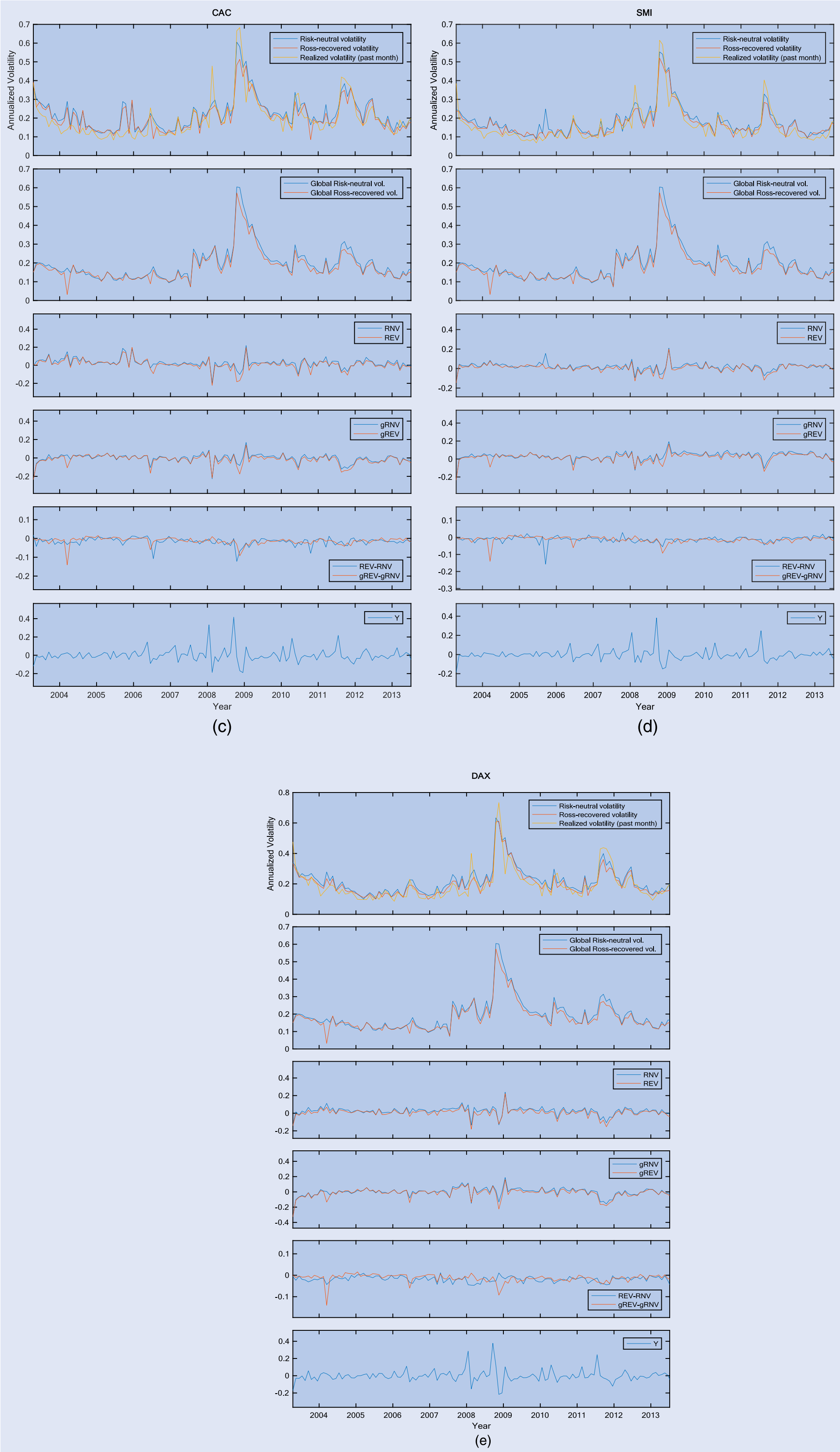

Figure 2. Continued.

a persistent variable, which can cause econometric challenges in predictive regressions. Thus, in our regressions we subtract the past-month realized volatility from the regressand and the regressors. Our regressions therefore predict next-month differences in realized volatility (i.e. 'unexpected' RV). We obtain the balanced predictive eq. (13)†:

$$\mathrm{RealizedVol}_{\mathrm{t,t+h}} - \mathrm{RealizedVol}_{\mathrm{t-h,t}} = Y_{t,t+h} = \alpha + \beta X_t + e_t \tag{13}$$

† E.g., REV = Ross-recovered expected volatility at time t minus past realized volatility from *t-h* to *t*.

Table 1. Definition of the variables used in the predictive regressions Overview of the variables used in the analysis and the predictive regressions performed.

Panel A—Variables used in the regressions	
Dependent variable	
Future 'unpredicted' part of the realized volatility of an index:	Y = RealizedVol$_{t,t+h}$ – RealizedVol$_{t-h,t}$
RealizedVol$_{t-h,t}$ = 5-min intraday index volatility from time *t-h* to *t*	
Explanatory variables	
Domestic risk-neutral volatility:	**RNV**$_t$ = (Volatility of the risk-neutral dist.)$_t$ – RealizedVol$_{t-h,t}$
Domestic Ross volatility:	**REV**$_t$ = (Volatility of the true expected dist.)$_t$ – RealizedVol$_{t-h,t}$
Domestic risk preference proxy (Ross recovered VoRP):	**REV**$_t$ – **RNV**$_t$
Global risk-neutral volatility:	**gRNV**$_t$ = (Global vol. of the 5 RND)$_t$ – RealizedVol$_{t-h,t}$
Global Ross volatility:	**gREV**$_t$ = (Global vol. of the true expected dist.)$_t$ – RealizedVol$_{t-h,t}$
Global risk preference proxy: (Ross recovered VoRP)	**gREV**$_t$ – **gRNV**$_t$
Panel B—Predictive regressions	
Domestic regressions	
1-Benchmark:	$Y = \beta_0 + \beta_1 \mathbf{RNV}_t + e_t$
2-True expected vol. from Ross:	$Y = \beta_0 + \beta_1 \mathbf{REV}_t + e_t$
3-Two components (old + new):	$Y = \beta_0 + \beta_1 \mathbf{RNV}_t + \beta_2 (\mathbf{REV}_t - \mathbf{RNV}_t) + e_t$
Global regressions	
4-Benchmark:	$Y = \beta_0 + \beta_1 \mathbf{gRNV}_t + e_t$
5-True expected vol. from Ross:	$Y = \beta_0 + \beta_1 \mathbf{gREV}_t + e_t$
6-Two components (old + new):	$Y = \beta_0 + \beta_1 \mathbf{gRNV}_t + \beta_2 (\mathbf{gREV}_t - \mathbf{gRNV}_t) + e_t$
Hybrid domestic ± global regressions	
7-Global vol., domestic risk pref.:	$Y = \beta_0 + \beta_1 \mathbf{gRNV}_t + \beta_2 (\mathbf{REV}_t - \mathbf{RNV}_t) + e_t$
8-Domestic vol., global risk pref.:	$Y = \beta_0 + \beta_1 \mathbf{RNV}_t + \beta_2 (\mathbf{gREV}_t - \mathbf{gRNV}_t) + e_t$

Table 2. Descriptive statistics for the raw option data.

	S&P500	FTSE	CAC	SMI	DAX
Number of options per day	642.2	376.7	362.7	475.9	664.2
Number of strikes per day	135.6	78.7	57.7	86.3	107.0
Number of maturities per day	12.0	10.2	13.3	12.2	14.2

This table reports, for each equity index, the mean of each of the variables relating to characteristics of the raw option data. The source of the data is Optionmetrics Ivy DB USA and Europe. The data used in the analysis run from 2003/04 to 2013/08.

where X_t are predictive variables (risk-neutral or Ross-recovered) described in Table 1. Descriptive statistics for the explanatory variables are presented in Table 3, panels A to E. They show that our variables are stationary and that the quantities, as defined, are not persistent, which is expected given our variable definition. Correlations between domestic and global variables vary by country, justifying investigating whether, for each country, risk anticipations and risk preferences are best explained by domestic or global variables.†

5. Results

5.1. *Volatility forecasts*

This section presents the results of our predictive regression analysis on volatilities. The models are estimated using OLS coefficient estimates with Newey-West standard errors (e.g. Christensen and Prabhala 1998). This approach is suitable given that our observations are non-overlapping and that the variables are stationary. Table 4, panels A to E, reports regression results for each country index. We group our regression specifications into three categories. Regressions 1–3 report results using only domestic variables, while regressions 4–6 report results using global variables and regressions 7–8 are for hybrid models.

First, we find that for all indexes domestic recovered volatility (REV) and domestic risk-neutral volatility (RNV) are significant predictors of realized volatility. Furthermore, REV outperforms RNV for a majority of indexes (S&P500, FTSE, SMI) based on adjusted R^2, AIC and BIC (regressions 1–2). Then, in regression 3 we decompose recovered volatility into risk-neutral volatility and a risk preference proxy (REV-RNV). This new information is statistically significant once more for the S&P500, FTSE and SMI.

Second, turning to global variables, we find that the two global risk variables (gREV and gRNV) are significant predictors of realized volatility. Moreover, gREV outperforms gRNV for all indexes (regressions 4–5). Further, if we separate gREV into gRNV and a global risk preference proxy (regression 6), we find that the risk preference variable is significant for a majority of indexes (S&P500, FTSE, SMI).

Third, looking at hybrid models (regressions 7–8), we find that a model with domestic RNV and global risk preferences

† The correlation coefficients between the volatilities and their equally-weighted global counterpart in that case are 0.74 and 0.79 (RNV, REV) and 0.58 for the risk preferences (REV-RNV).

Table 3. Panels A to E: Descriptive statistics for the time series variables.

Panel A: S&P500	Y	RNV	REV	REV-RNV	gRNV	gREV	gREV-gRNV
Nb of obs.	123	123	123	123	123	123	123
Mean	– 0.0029	0.0404	0.0293	– 0.0111	0.0411	0.0290	– 0.0120
Std Dev.	0.06	0.03	0.04	0.02	0.03	0.04	0.01
Autocorr(1)	0.037	0.034	0.131	0.546	– 0.008	0.111	0.567
ADF	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Correlations							
RNV		1.00	0.88	– 0.04	0.97	0.86	0.00
REV			1.00	0.43	0.89	0.97	0.45
REV-RNV				1.00	0.04	0.42	0.96
gRNV					1.00	0.91	0.07
gREV						1.00	0.47
gREV-gRNV							1.00
Panel B: FTSE	Y	RNV	REV	REV-RNV	gRNV	gREV	gREV-gRNV
Nb of obs.	123	123	123	123	123	123	123
Mean	0.0010	0.0278	0.0186	– 0.0092	0.0366	0.0246	– 0.0120
Std Dev.	0.06	0.03	0.04	0.02	0.04	0.04	0.02
Autocorr(1)	– 0.079	– 0.102	0.024	0.485	– 0.012	0.044	0.504
ADF	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Correlations							
RNV		1.00	0.90	0.15	0.84	0.81	0.06
REV			1.00	0.56	0.74	0.83	0.36
REV-RNV				1.00	0.08	0.36	0.72
gRNV					1.00	0.92	– 0.04
gREV						1.00	0.35
gREV-gRNV							1.00
Panel C: CAC	Y	RNV	REV	REV-RNV	gRNV	gREV	gREV-gRNV
Nb of obs.	123	123	123	123	123	123	123
Mean	0.0004	0.0264	0.0103	– 0.0161	– 0.0043	– 0.0176	– 0.0133
Std Dev.	0.08	0.05	0.06	0.02	0.05	0.05	0.02
Autocorr(1)	– 0.076	0.029	0.126	0.169	0.140	0.194	0.284
ADF	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Correlations							
RNV		1.00	0.94	0.15	0.70	0.65	0.02
REV			1.00	0.48	0.65	0.65	0.16
REV-RNV				1.00	0.06	0.20	0.41
gRNV					1.00	0.94	0.06
gREV						1.00	0.41
gREV-gRNV							1.00
Panel D: SMI	Y	RNV	REV	REV-RNV	gRNV	gREV	gREV-gRNV
Nb of obs.	123	123	123	123	123	123	123
Mean	– 0.0006	0.0190	0.0077	– 0.0114	0.0333	0.0199	– 0.0133
Std Dev.	0.06	0.04	0.04	0.02	0.05	0.05	0.02
Autocorr(1)	– 0.027	0.010	0.134	0.053	0.199	0.173	0.301
ADF	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Correlations							
RNV		1.00	0.89	– 0.16	0.73	0.71	0.01
REV			1.00	0.31	0.74	0.76	0.12
REV-RNV				1.00	0.08	0.17	0.24
gRNV					1.00	0.92	– 0.11
gREV						1.00	0.29
gREV-gRNV							1.00
Panel E: DAX	Y	RNV	REV	REV-RNV	gRNV	gREV	gREV-gRNV
Nb of obs.	123	123	123	123	123	123	123
Mean	– 0.0015	0.0245	0.0066	– 0.0179	– 0.0023	– 0.0156	– 0.0133
Std Dev.	0.07	0.05	0.05	0.01	0.06	0.06	0.02
Autocorr(1)	0.043	0.008	0.108	0.266	0.300	0.299	0.287
ADF	0.00	0.00	0.00	0.02	0.00	0.00	0.00

(*Continued*).

Table 3. Continued.

Panel E: DAX	Y	RNV	REV	REV-RNV	gRNV	gREV	gREV-gRNV
Correlations							
RNV		1.00	0.96	– 0.05	0.82	0.78	0.02
REV			1.00	0.24	0.82	0.79	0.10
REV-RNV				1.00	0.05	0.14	0.31
gRNV					1.00	0.95	0.06
gREV						1.00	0.36
gREV-gRNV							1.00

This table reports, for each equity index, the descriptive statistics of the time series variables used in the regression models of this paper. They are monthly time series from 2003/04 to 2013/08. *RNV* is the annualized volatility of the risk-neutral distribution minus the realized volatility of the past month. *REV* is the annualized volatility of the true expected distribution of future returns according to the Recovery Theorem (Ross 2015) minus the realized volatility of the past month. *gRNV* and *gREV* are the global versions constructed by forming value-weighted global variables, minus the domestic realized volatility of the past month. The dependent variable *Y* is defined as the future realized volatility over the next month minus the realized volatility of the past month as measured by intraday returns data. Autocorr(1) is the first order autocorrelation. ADF is the *p*-value of an augmented Dickey-Fuller unit root test.

performs especially well (regression 8). The two variables are significant across indexes and the model has the highest adjusted R^2 for most European indexes. These results suggest that while volatility expectations are often market-specific, risk aversion is better proxied by a global variable. Therefore, we find overall that the best performing model for each index is one that uses at least one predictive variable obtained from applying the recovery theorem.† More specifically, the improvements in adjusted R^2 from using the best model instead of the domestic RNV model (regression 1) range from 1.8% to 8.1% across indexes.

The global REV model (regression 5) generates the best performance for the S&P500 and CAC indexes according to our criteria, while the hybrid model with local RNV and global risk preferences (regression 8) performs best for the other indexes (FTSE, SMI, DAX). This finding suggests that risk preferences have more explanatory power when measured globally. Further support comes from the fact that forecast performance is always improved by replacing the domestic risk preferences proxy (regression 3) with a global one.

5.2. *Forecasting excess returns: recovered VRP vs. standard VRP*

Since we have found that recovered quantities are useful to forecast volatility, we now move on the harder task of forecasting excess returns. To explore the usefulness of Ross recovery to explain market excess returns in different countries, we run forecast regressions of returns over the Variance Risk Premium (VRP). Our regressions include the standard VRP specification as well as specifications using Ross recovery. We further examine domestic vs. global specifications. Note that for the return regressions, unlike the volatility forecasts, we use variances (e.g. VIX squared rather than VIX for the U.S.) to be consistent with the literature on return forecasts. In order to account for this slight difference, we change our terminology accordingly.

VRP is conventionally defined as the difference between risk-neutral variance (RNV) from option data and realized variance (RZV) computed from historical high-frequency returns data. Letting REV stand for (Ross) recovered variance, we divide VRP in two, namely REV-RNV and REV-RZV. The first component of VRP relates to a ‘risk adjustment’, since it equals the physical (recovered) forward-looking variance minus the risk-neutral forward-looking variance. We call this the ‘recovered VRP’ (since we cannot know if it captures the true VRP). The second component, REV-RZV, captures how the physical distribution is updated with new information, since REV is forward-looking while RZV is historical. This difference can be described as the ‘expected change in uncertainty’. According to financial theory, both components are relevant, but they represent different elements of risk to investors.

Table 5 presents results for the monthly excess returns forecasts. Panels A to E report our findings by country. To provide a baseline, we first compare our results using the standard VRP (regression [1] in each panel) with those in Bollerslev *et al.* (2009) and Bollerslev *et al.* (2014). We find that our slope coefficients and other regression results are similar to theirs. For instance, for the SPX the VRP slope coefficient at the 1-month horizon is 0.56 and significant. This can be compared to the 0.5 coefficient found by Bollerslev *et al.* (2014). Likewise, the VRP slope coefficients for the other country indexes are significant and similar in size to those reported by Bollerslev *et al.* (2014).

Looking next at specifications that use Ross recovery, we find that at the 1-month horizon, for all market indexes except the SPX, the best model is the global version of the Ross-recovered VRP. For the SPX, the domestic recovered VRP and traditional VRP perform about equally well. This finding is consistent with the prior literature suggesting that foreign equity markets benefit from incorporating international information, especially from the US market (which constitutes a large part of the global measure). The VRP slope coefficients and the adj. R^2s are consistent with prior literature (e.g. Bollerslev *et al.* 2014). The improvements in adj. R^2 from using recovered VRP can be economically substantial. For instance, for the SMI, adj. R^2 increases from 2.7% to 8.3%.

Interestingly, if we break down VRP into two components using Ross recovery, we find that one component (REV-RZV) is always significant but the other (RNV-REV) is rarely so. By

† The BIC measure for the DAX is the exception.

learning more about the relative importance of the two components of VRP, we can better understand the sources of risk that investors can use to predict higher market returns. Moreover, the finding that recovery is less useful for the SPX is consistent with prior research that has applied Ross recovery to the US market (e.g. Jackwerth and Menner 2020). This result suggests that the SPX may not be representative of all equity indexes in terms of exploiting the recovery

Table 4. Panels A to E: Results of the predictive regressions on future realized volatility.

Panel A: S&P500	1	2	3	4	5	6	7	8
	Domestic models			Global models			Hybrid models	
Intercept	– 0.035***	– 0.027***	– 0.027***	– 0.041***	– 0.029***	– 0.029***	– 0.032***	– 0.023***
	(0.008)	(0.007)	(0.006)	(0.008)	(0.006)	(0.005)	(0.005)	(0.005)
RNV	0.80***		0.81***					0.80***
	(0.21)		(0.19)					(0.19)
REV		0.82***						
		(0.19)						
REV – RNV			0.84**				0.70**	
			(0.37)				(0.33)	
gRNV				0.92***		0.89***	0.90***	
				(0.20)		(0.17)	(0.17)	
gREV					0.89***			
					(0.17)			
gREV – gRNV						0.86**		1.00**
						(0.38)		(0.42)
Adj. R^2	0.169	0.223	0.216	0.210	0.250°	0.243	0.242	0.217
AIC	– 351.9	– 360.2	– 358.2	– 358.2	– 364.5°	– 362.5	– 362.2	– 358.3
BIC	– 346.3	– 354.6	– 349.8	– 352.6	– 358.9°	– 354.1	– 353.8	– 349.9
Panel B: FTSE	1	2	3	4	5	6	7	8
	Domestic models			Global models			Hybrid models	
Intercept	– 0.020***	– 0.012**	– 0.013*	– 0.020***	– 0.015**	– 0.008	– 0.011	– 0.009
	(0.007)	(0.006)	(0.008)	(0.006)	(0.006)	(0.008)	(0.007)	(0.008)
RNV	0.77***		0.71***					0.74***
	(0.17)		(0.16)					(0.16)
REV		0.70***						
		(0.12)						
REV – RNV			0.68**				0.79**	
			(0.32)				(0.36)	
gRNV				0.56***		0.58***	0.53***	
				(0.13)		(0.13)	(0.12)	
gREV					0.64***			
					(0.11)			
gREV – gRNV						1.05***		0.89***
						(0.37)		(0.31)
Adj. R^2	0.181	0.219	0.213	0.106	0.162	0.167	0.153	0.224°
AIC	– 359.7	– 365.6°	– 363.7	– 349.1	– 357.0	– 356.7	– 354.7	– 365.4
BIC	– 354.1	– 360.0°	– 355.2	– 343.4	– 351.3	– 348.3	– 346.2	– 356.9
Panel C: CAC	1	2	3	4	5	6	7	8
	Domestic models			Global models			Hybrid models	
Intercept	– 0.015**	– 0.005	– 0.011	0.004	0.012*	0.011	0.009	– 0.007
	(0.007)	(0.006)	(0.008)	(0.007)	(0.007)	(0.007)	(0.007)	(0.007)
RNV	0.59***		0.58***					0.59***
	(0.13)		(0.12)					(0.13)
REV		0.52***						
		(0.11)						
REV – RNV			0.22				0.34	
			(0.23)				(0.24)	
gRNV				0.71***		0.70***	0.70***	
				(0.14)		(0.14)	(0.13)	
gREV					0.68***			
					(0.13)			
gREV – gRNV						0.55		0.63**
						(0.39)		(0.27)
Adj. R^2	0.163	0.160	0.160	0.215	0.232°	0.227	0.217	0.181
AIC	– 306.6	– 306.1	– 305.2	– 314.4	– 317.2°	– 315.4	– 313.8	– 308.4
BIC	– 301.0	– 300.5	– 296.7	– 308.8	– 311.6°	– 307.0	– 305.4	– 299.9

(*Continued*).

Table 4. Continued.

Panel D: SMI	1	2	3	4	5	6	7	8
	Domestic models			Global models			Hybrid models	
Intercept	– 0.014**	– 0.006	– 0.009	– 0.016***	– 0.011**	– 0.007	– 0.014**	– 0.005
	(0.006)	(0.006)	(0.005)	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)
RNV	0.71***		0.76***					0.71***
	(0.15)		(0.15)					(0.15)
REV		0.73***						
		(0.13)						
REV – RNV			0.56***				0.22	
			(0.14)				(0.27)	
gRNV				0.47***		0.51***	0.46***	
				(0.10)		(0.10)	(0.11)	
gREV					0.54***			
					(0.10)			
gREV – gRNV						0.80*		0.65**
						(0.40)		(0.27)
Adj. R^2	0.186	0.210	0.206	0.111	0.160	0.160	0.108	0.217°
AIC	– 350.0	– 353.7	– 352.1	– 339.2	– 346.2	– 345.2	– 337.8	– 353.8°
BIC	– 344.4	– 348.0°	– 343.7	– 333.6	– 340.6	– 336.8	– 329.4	– 345.4

Panel E: DAX	1	2	3	4	5	6	7	8
	Domestic models			Global models			Hybrid models	
Intercept	– 0.022***	– 0.007	– 0.026***	0.000	0.008	0.007	– 0.009	– 0.015***
	(0.006)	(0.006)	(0.009)	(0.006)	(0.007)	(0.006)	(0.009)	(0.006)
RNV	0.85***		0.85***					0.85***
	(0.15)		(0.15)					(0.15)
REV		0.78***						
		(0.14)						
REV – RNV			– 0.20				– 0.49	
			(0.43)				(0.49)	
gRNV				0.64***		0.63***	0.64***	
				(0.13)		(0.13)	(0.13)	
gREV					0.62***			
					(0.13)			
gREV – gRNV						0.51		0.58**
						(0.42)		(0.26)
Adj. R^2	0.287	0.253	0.282	0.265	0.282	0.277	0.267	0.305°
AIC	– 336.5	– 330.8	– 334.7	– 332.7	– 335.6	– 333.8	– 332.1	– 338.6°
BIC	– 330.8°	– 325.1	– 326.3	– 327.1	– 330.0	– 325.4	– 323.7	– 330.2

Monthly time series of next-month realized volatility minus past-month realized volatility are regressed over predictive variables, with different models. *RNV* is the annualized volatility of the risk-neutral distribution minus the realized volatility of the past month. *REV* is the annualized volatility of the true expected distribution of future returns according to the Recovery Theorem (Ross 2015) minus the realized volatility of the past month. *gRNV* and *gREV* are the global versions constructed by forming value-weighted global variables, minus the domestic realized volatility of the past month. The regression coefficients and their significance level are presented (*, **, *** for 10%, 5% and 1% respectively). Newey-West corrected standard errors for each regression coefficient are presented in parentheses. Adjusted R^2 in decimal form, Akaike information criterion (AIC) and Bayesian information criterion (BIC) are also presented for each model. The best performing model according to each of these criteria is noted by a (°).

theorem, since the theorem is more useful for other markets. We have also examined forecasts at a 4-month horizon, as prior research found a peak in index return predictability at the 3–4 month horizon. These results are broadly similar to the 1-mo horizon results, and the adj. R^2s are higher than for the 1-month horizon, as in the prior VRP literature.† Thus, recovery is useful to improve predictions of excess returns using VRP at various horizons.

† For brevity, 4-month horizon results on excess returns forecasts are omitted from the paper, but available upon request.

5.3. Discussion of the results

There are two main implications to our results. First, for the five markets in our sample, Ross recovery improves realized volatility forecasts compared to the risk-neutral volatility benchmark. These findings support the empirical relevance of the recovery theorem, in addition to being of interest to the volatility forecasting literature. In all cases, the improvements are statistically and economically significant.

Our results can be contrasted with those of Jensen *et al.* (2019), who study the US market. We find, as they do, that recovered volatility is a less biased predictor than is the VIX.

However, their forecasts generate lower R^2s using recovered volatility than using VIX, while we find that recovered volatility generally produces a higher R^2 for the US as well as for European markets. Relatedly, Jackwerth and Menner (2020) argue that the recovered distribution is biased and not equal to the true distribution of returns. However, our results suggest

Table 5. One-month horizon excess returns forecasts.

	1	2	3	4	5	6	7	8
Panel A: SPX	Domestic models			Global models			Hybrid models	
Intercept	.000	.004	.002	.010***	.002	.005	.004	.012***
	(.004)	(.004)	(.004)	(.004)	(.004)	(.004)	(.005)	(.004)
RNVar-RZVar	.56***							
	(.12)							
REVar-RZVar		.49***	.54***					
		(.11)	(.16)					
REVar-RNVar			− .30	.62				
			(.60)	(.54)				
gRNVar-gRZVar					.53***			
					(.11)			
gREVar-gRZVar						.47***	.50***	
						(.09)	(.18)	
gREVar-gRNVar							− .17	.92
							(.86)	(.70)
Adj. R^2	.130°	.129	.126	.014	.117	.120	.113	.024
AIC	− 419.1°	− 419.1	− 417.6	− 403.8	− 417.3	− 417.7	− 415.8	− 405.0
BIC	− 413.5°	− 413.4	− 409.1	− 398.2	− 411.7	− 412.1	− 407.4	− 399.4
	1	2	3	4	7	8	6	5
Panel B: FTSE	Domestic models			Global models			Hybrid models	
Intercept	.004	.005	.008*	.010***	.003	.005	.005	.009**
	(.004)	(.004)	(.004)	(.004)	(.004)	(.004)	(.004)	(.004)
RNVar-RZVar	.26***							
	(.07)							
REVar-RZVar		.26***	.20					
		(.04)	(.13)					
REVar-RNVar			.52	.99				
			(.87)	(.60)				
gRNVar-gRZVar					.27***			
					(.08)			
gREVar-gRZVar						.25***	.24*	
						(.07)	(.13)	
gREVar-gRNVar							.06	.60
							(.72)	(.49)
Adj. R^2	.023	.030	.027	.021	.030	.033°	.025	.007
AIC	− 421.7	− 422.6	− 421.3	− 421.5	− 422.6	− 423.1°	− 421.1	− 419.8
BIC	− 416.1	− 417.0	− 412.9	− 415.8	− 417.0	− 417.4°	− 412.6	− 414.2
Panel C: CAC	Domestic models			Global models			Hybrid models	
Intercept	.004	.006	.011*	.015***	.002	.005	.007	.013***
	(.005)	(.005)	(.006)	(.005)	(.005)	(.005)	(.005)	(.005)
RNVar-RZVar	.28***							
	(.04)							
REVar-RZVar		.28***	.18					
		(.03)	(.13)					
REVar-RNVar			.84	1.47**				
			(.99)	(.68)				
gRNVar-gRZVar					.44***			
					(.08)			
gREVar-gRZVar						.42***	.35**	
						(.07)	(.15)	
gREVar-gRNVar							0.45	1.23**
							(.73)	(.54)
Adj. R^2	.055	.067	.074°	.061	.060	.073	.069	.038
AIC	− 381.3	− 382.9	− 382.8	− 382.1	− 381.9	− 383.6°	− 382.2	− 379.1
BIC	− 375.7	− 377.3	− 374.3	− 376.5	− 376.3	− 378.0°	− 373.8	− 373.5

(*Continued*).

Table 5. Continued.

	1	2	3	4	7	8	6	5
Panel D: SMI		Domestic models			Global models		Hybrid models	
Intercept	.006	.007*	.010*	.012***	.004	.006	.011**	.015***
	(.004)	(.004)	(.006)	(.005)	(.004)	(.004)	(.005)	(.004)
RNVar-RZVar	.25**							
	(.11)							
REVar-RZVar		.24***	.15					
		(.09)	(.18)					
REVar-RNVar			.78	1.41				
			(1.36)	(.88)				
gRNVar-gRZVar					.29***			
					(.08)			
gREVar-gRZVar						.31***	.19*	
						(.04)	(.10)	
gREVar-gRNVar							.88	1.30***
							(.60)	(.44)
Adj. R^2	.027	.032	.030	.030	.042	.065	.083°	.073
AIC	− 435.8	− 436.4	− 435.2	− 436.1	− 437.7	− 440.7	− 442.1°	− 441.7
BIC	− 430.1	− 430.8	− 426.8	− 430.4	− 432.1	− 435.0	− 433.7	− 436.1°
Panel E: DAX		Domestic models			Global models		Hybrid models	
Intercept	.008	.010*	.014*	.019***	.005	.008	.007	.013**
	(.005)	(.005)	(.008)	(.007)	(.005)	(.005)	(.006)	(.005)
RNVar-RZVar	.24***							
	(.08)							
REVar-RZVar		.23***	.19					
		(.06)	(.12)					
REVar-RNVar			.68	1.47				
			(1.28)	(1.06)				
gRNVar-gRZVar					.40***			
					(.09)			
gREVar-gRZVar						.36***	.37**	
						(.08)	(.14)	
gREVar-gRNVar							− .05	.76
							(.82)	(.64)
Adj. R^2	.022	.025	.020	.011	.038	.040°	.032	.006
AIC	− 350.8	− 351.1	− 349.5	− 349.5	− 352.8	− 353.1°	− 351.1	− 348.8
BIC	− 345.2	− 345.5	− 341.1	− 343.9	− 347.2	− 347.5°	− 342.6	− 343.2

For each country market index (panels A through E), time series of next-month excess returns are regressed over one or more predictive variables, according to different model specifications (1)–(8). *RNVar* is the annualized variance of the risk-neutral distribution. *REVar* is the annualized variance according to the Recovery Theorem (Ross 2015). RZVar is the annualized realized variance. *gRNV*ar, *gREVar* and *gRZVar* are the global versions constructed by forming value-weighted global variables using the country market indexes in our sample. The regression coefficients and their significance levels are presented (*, **, *** for 10%, 5% and 1% respectively) in columns (1)–(8) for each model. Newey-West corrected standard errors for each regression coefficient are presented in parentheses. Adjusted R^2s are reported in decimal form. Akaike information criterion (AIC) and Bayesian information criterion (BIC) statistics are also presented for each model. The best performing model according to each of these criteria is noted by a (°).

that recovery yields empirical gains in forecast regressions relative to using traditional predictors. Therefore, while prior empirical research finds weaknesses in the performance of the recovery theorem (e.g. Bakshi *et al.* 2018), our results support its empirical relevance. This is an important distinction, because the theorem need not perfectly recover investor preferences and expectations to yield the most accurate model available.

Second, among the best-performing models, risk preferences are always globally measured. For three out of four European indexes (FTSE, DAX and SMI), the best model uses domestic volatility expectations and global risk preferences. These findings suggest that investor appetite for risk is determined globally, since investors can invest internationally and markets are fairly well integrated. In contrast, volatility expectations can have an important domestic component and thus are often best measured locally. For the S&P500 and the CAC index, the best-performing model uses only global Ross-recovered volatility, implying that risk expectations and risk preferences are both global. For the S&P500, this result may be explained by the heavy US weight in the global value-weighted measures (as argued by Bollerslev *et al.* 2014).† For the CAC index, the global model outperforms models using domestic risk expectations.

Overall, our results echo the evidence on international investor sentiment (Baker *et al.* 2012) and further support Bollerslev *et al.* (2014), who find that VRP in different countries is best measured globally rather than domestically. Our results provide a way to disaggregate the evidence

† Given the heavy weight of the S&P500, we could argue that the global measures are close to the domestic US measures, but with additional international information.

and they show that this finding remains true when we use forward-looking physical distributions rather than historical distributions.

The literature on risk preferences has documented that global measures of behavior toward risk are appropriate (Schmeling 2009, Beer *et al.* 2011, Baker *et al.* 2012, Bollerslev *et al.* 2014). Applying the Recovery theorem allows us to to dig further, as we separate recovered risk preferences from volatility expectations. Our findings on generally local expectations but global risk preferences relate to the fact that there is a worldwide pool of investors following different country equity indexes. Thus, their risk preferences are better measured globally, consistent with Baker *et al.* (2012), Bollerslev *et al.* (2014), Schmeling (2009) and Beer *et al.* (2011).

Our evidence for the empirical relevance of Ross recovery is further strengthened by our analysis focusing on excess return forecasts. Summarizing our results on excess return forecasts using VRP and Ross-recovered VRP, we find that: (1) Ross-recovered VRP performs at least as well as conventional VRP. This is true at the 1-month horizon and, as a robustness check, the 4-month horizon. (2) The reason why Ross-recovery is less useful to forecast returns appears to be that the 'risk adjustment' component of recovered VRP is not always significant. The other component, reflecting change in uncertainty, is always significant. (3) For the SPX, a domestic model is best but for other countries, global measures perform better, a result in line with the literature (e.g. Bollerslev *et al.* 2014).

6. Conclusion

In this paper, we develop an empirical framework to implement the recovery theorem (Ross 2015) and use it to forecast volatility and excess returns in international equity indexes. We assess the extent to which variables obtained from the Ross-recovered physical distribution help explain future realized volatility and excess returns. Overall, the new information recovered using the theorem, combined with risk-neutral volatility, significantly outperforms models containing only risk-neutral volatility. This result holds internationally for several major international stock indexes. Furthermore, models using global measures of risk preferences perform best, indicating a shared risk preference component among investors in these markets, and further documenting the sources of risk in equity markets.

Our results lead us to the following joint conclusion. (i) The recovery theorem has empirical relevance. (ii) The empirical methodology presented in this paper brings us closer to recovering accurate market expectations and risk preferences. (iii) Globally-determined risk preferences play a meaningful role in forecasting realized volatility. (iv) When splitting the VRP in two (using recovery) to forecast excess returns, one (the change in the level of uncertainty) is highly significant while the other (the adjustment for risk aversion) seldom is. The improved volatility forecasting performance obtained using Ross recovery has numerous practical applications in finance, in terms of improving portfolio allocation or through risk management strategies, such as computing Value at risk or Expected shortfall under the recovered 'true' forward-looking distribution rather than the historical distribution.

Acknowledgement

The authors gratefully acknowledge financial support from SSHRC, FRQSC, the Fonds de recherche AMF-GIRIF, the Chaire de recherche Industrielle-Alliance, and the Salles des Marchés FSA Jean-Turmel et Carmand-Normand. We thank conference participants at the Optionmetrics annual conference, French Finance Association (AFFI) meetings, INFINITI International Finance conference, Canadian Economic Science meetings [Société canadienne de science économique], Trondheim Business School Finance & Banking Conference, and in workshops at the Norwegian University of Science & Technology and Aix-en-Provence. Any errors are our own.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Funding

This work was supported by Fonds de Recherche du Québec-Société et Culture [grant number 01]; Chaire de recherche FSA Industrielle-Alliance [grant number 01]; Fonds AMF-GIRIF [grant number 01]; Social Sciences and Humanities Research Council of Canada [grant number 435].

References

Aït-Sahalia, Y. and Lo, A. W., Nonparametric risk management and implied risk aversion. *J. Econom.*, 2000, **94**(1-2), 9–51.

Audrino, F., Huitema, R., and Ludwig, M. An empirical implementation of the Ross Recovery theorem as a prediction device. *J. Financ. Econom.*, 2019, forthcoming.

Baker, M., Wurgler, J. and Yuan, Y., Global, local, and contagious investor sentiment. *J. Financ. Econ.*, 2012, **104**(2), 272–287.

Bakshi, G., Chabi-Yo, F. and Gao, X., A recovery that We Can trust? Deducing and testing the restrictions of the recovery theorem. *Rev. Financ. Stud.*, 2018, **31**(2), 532–555.

Barone-Adesi, G., Fusari, N., Sala, C. and Mira, A., Option market trading activity and the estimation of the pricing kernel: A Bayesian approach. *J. Econom.*, 2020, **216**(2), 430–449.

Beaulieu, M.-C., Gagnon, M.-H. and Khalaf, L., Less is more: Testing financial integration using identification-robust asset pricing models. *J. Int. Financ. Markets, Inst. Money*, 2016, **45**, 171–190.

Beer, F., Nouyrigat, G. and Zouaoui, M., How does investor sentiment affect stock market crises? Evidence from panel data. *Financ. Rev.*, 2011, **46**(4), 723–747.

Bekaert, G., Harvey, C. R. and Ng, A., Market integration and contagion. *J. Bus.*, 2005, **78**(1), 39–69.

Birru, J. and Figlewski, S., Anatomy of a meltdown: The risk-neutral density for the S&P 500 in the fall of 2008. *J. Financ. Markets*, 2012, **15**(2), 151–180.

Blair, B. J., Poon, S. H. and Taylor, S. J., Forecasting S&P 100 volatility: The incremental information content of implied volatilities and high-frequency index returns. *J. Econom.*, 2001, **105**(1), 5–26.

Bliss, R. R. and Panigirtzoglou, N., Option-implied risk aversion estimates. *J. Finance*, 2004, **59**(1), 407–446.

Bollerslev, T., Marrone, J., Xu, L. and Zhou, H., Stock return predictability and variance risk premia: Statistical inference and international evidence. *J. Financ. Quant. Anal.*, 2014, **49**(3), 633–661.

Bollerslev, T., Tauchen, G. and Zhou, H., Expected stock returns and variance risk premia. *Rev. Financ. Stud.*, 2009, **22**(11), 4463–4492.

Borovička, J., Hansen, L. P. and Scheinkman, J. A., Misspecified recovery. *J. Finance*, 2016, **71**(6), 2493–2544.

Breeden, D. T. and Litzenberger, R. H., Prices of state-contingent claims implicit in option prices. *J. Bus.*, 1978, **51**(4), 621–651.

Buncic, D. and Gisler, K. I., Global equity market volatility spillovers: A broader role for the United States. *Int. J. Forecast.*, 2016, **32**(4), 1317–1339.

Buncic, D. and Gisler, K. I., The role of jumps and leverage in forecasting volatility in international equity markets. *J. Int. Money. Finance*, 2017, **79**, 1–19.

Carr, P. and Wu, L., Variance risk premiums. *Rev. Financ. Stud.*, 2009, **22**(3), 1311–1341.

Carr, P. and Yu, J., Risk, return, and Ross recovery. *J. Derivatives*, 2012, **20**(1), 38–59.

Carrieri, F., Errunza, V. and Hogan, K., Characterizing world market integration through time. *J. Financ. Quant. Anal.*, 2007, **42**(4), 915–940.

Christensen, B. and Prabhala, N., The relation between implied and realized volatility. *J. Financ. Econ.*, 1998, **50**(2), 125–150.

Christiansen, C., Schmeling, M. and Schrimpf, A., A comprehensive look at financial volatility prediction by economic variables. *J. Appl. Econom.*, 2012, **27**(6), 956–977.

Conrad, J., Dittmar, R. and Ghysels, E., Ex ante skewness and expected stock returns. *J. Finance*, 2013, **68**(1), 85–124.

Corsi, F., A simple approximate long-memory model of realized volatility. *J. Financ. Econom.*, 2008, **7**, 174–196.

Dubynskiy, S. and Goldstein, R. S. Recovering Drifts and Preference Parameters from Financial Derivatives. 2013. Available at SSRN: https://papers.ssrn.com/sol3/papers.cfm?abstract_id = 2244394.

Gatheral, J., Jaisson, T. and Rosenbaum, M., Volatility is rough. *Quant. Finance*, 2018, **18**(6), 933–949.

Gerd, H., Lunde, A., Shephard, N. and Sheppard, K., *Oxford-Man Institute's Realized Library v. 0.2. Oxford-Man Institute*, 2009 (University of Oxford: Oxford).

Griffin, J. M., Are the Fama and French factors global or country specific? *Rev. Financ. Stud.*, 2002, **15**(3), 783–803.

Jackwerth, J. C., Recovering risk aversion from option prices and realized returns. *Rev. Financ. Stud.*, 2000, **13**(2), 433–451.

Jackwerth, J.C. and Menner, M. Does the Ross recovery theorem work empirically? *J. Financ. Econ.*, 2020, forthcoming.

Jackwerth, J. C. and Rubinstein, M., Recovering probability distributions from option prices. *J. Finance*, 1996, **51**(5), 1611–1631.

Jensen, C. S., Lando, D. and Pedersen, L. H., Generalized recovery. *J. Financ. Econ.*, 2019, **133**(1), 154–174.

Linn, M., Shive, S. and Shumway, T., Pricing kernel monotonicity and conditional information. *Rev. Financ. Stud.*, 2018, **31**(2), 493–531.

Martin, I. W. and Ross, S. A. Notes on the yield curve. *J Financ. Econ.*, 2019, **134**(3), 689–702.

Mittnik, S., Robinzonov, N. and Spindler, M., Stock market volatility: Identifying major drivers and the nature of their impact. *J. Bank. Financ.*, 2015, **58**, 1–14.

Park, H., Ross recovery with recurrent and transient processes. *Quant. Finance*, 2016, **16**(5), 667–676.

Poon, S.-H. and Granger, C. W. J., Practical issues in forecasting volatility. *Financ. Anal. J.*, 2005, **61**(1), 45–56.

Qin, L. and Linetsky, V., Positive eigenfunctions of Markovian pricing operators: Hansen-Scheinkman factorization, Ross recovery, and long-term pricing. *Oper. Res.*, 2016, **64**(1), 99–117.

Rosenberg, J. V. and Engle, R. F., Empirical pricing kernels. *J. Financ. Econ.*, 2002, **64**(3), 341–372.

Ross, S., The recovery theorem. *J. Finance*, 2015, **70**(2), 615–648.

Rubinstein, M., Implied binomial trees. *J. Finance*, 1994, **49**(3), 771–818.

Schmeling, M., Investor sentiment and stock returns: Some international evidence. *J. Empir. Financ.*, 2009, **16**(3), 394–408.

Schneider, P. and Trojani, F., (Almost) model-free recovery. *J. Finance*, 2019, **74**(1), 323–370.

Szakmary, A., Ors, E., Kim, J. K. and Davidson, W. N., The predictive power of implied volatility: Evidence from 35 futures markets. *J. Bank. Financ.*, 2003, **27**, 2151–2175.

Taylor, S. J., Yadav, P. K. and Zhang, Y., The information content of implied volatilities and model-free volatility expectations: Evidence from options written on individual stocks. *J. Bank. Financ.*, 2010, **34**(4), 871–881.

Walden, J., Recovery with unbounded diffusion processes. *Rev. Financ.*, 2017, **21**(4), 1403–1444.

Zhang, Y., Ma, F. and Liao, Y., Forecasting global equity market volatilities. *Int. J. Forecast.*, 2020, **36**(4), 1454–1475.