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Market making with inventory control and order book information

E. DONATONI, S. PATERLINI and F. BAZZANA*

Department of Economics and Management, University of Trento, Trento, Italy

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We introduce a new market making algorithm, the Signal-adaptive (SA) strategy, that takes into account the empirical recurrences of the historical intra-day price series of IPOs and the current risk properties addressed by the market maker. Empirical analysis on IPO high-frequency data allows to compare the performance of the SA strategy with two state-of-art competing strategies, suggesting that the SA strategy could be suitable for the price support activity of the IPO in the first days of its issuance.

Keywords: Algorithmic trading; High-frequency trading; Initial public offerings; Underwriter; Fintech

JEL classifications: G14, G18

1. Introduction

The use of high-frequency trading (HFT) by investment banks, hedge funds and institutional investors has become increasingly relevant in recent times (Kauffman *et al.* 2015). This subset of algorithmic trading techniques is characterized by high operating speed, automation without human intervention and a technological competitive advantage in terms of low latency and co-location (Gomber *et al.* 2011).

The presence of high-frequency traders (HFt) in the financial markets has effects in terms of systemic risk (Jain *et al.* 2016, Klein 2020), market quality† and integrity (Frino and Lepone 2012). A particular ramification of the set of techniques that define this trading environment is called 'liquidity provision' or 'market-making'.

The 'market making' strategy is characterized by the simplicity of its implementation structure: the trader must continuously place on the market both buy (BID quote) and sell (ASK quote) orders in order to provide liquidity to the market. Its profitability depends on the combination between the difference in the prices of the BID-ASK orders (called 'spread') and econometric characteristics of the price of the traded financial instrument (Stoikov and Saglam 2008).

Taking into account the main costs that the market maker has to face, that is the so-called inventory costs and adverse selection costs, the literature divides into two research streams to cover the most relevant risks; first, the so-called inventory-based approach (Garman 1976, Ho and Stoll 1981), that specifically refers to the treatment of issues related to inventory costs of the market maker, due to the possession of assets in the portfolio; second, the information-based approach (Glosten and Milgrom 1985, Kyle 1985), which expressly concerns the treatment of issues related to adverse selection problems between the market maker and informed traders. Such approaches recognize inventory risk and information risk as the key-risks. The market making strategy must then be able to adequately consider the management of such risks according to the specific case of use.

Historically, the market making strategy is known to be one of the most important operations performed by the underwriter of an Initial Public Offering (IPO) in order to support the price of newly listed security (Ellis *et al.* 1993). In particular, the market making activity performed by the underwriter seems to be a standalone profit center conditioned by the econometric characteristics of the price performance of an IPO.

Therefore, taking into account the multitude of theories (Boudriga *et al.* 2009, Jamaani and Alidarous 2019) that attempt to explain what governs IPO price returns, which have on average positive performances in the first days of stock release (Loughran and Ritter 2004), we assume here not to consider the risks of adverse selection and to focus mainly on inventory risk management. Anyway, it is important to remind that, in the context of the definition of the market making

*Corresponding author. Email: flavio.bazzana@unitn.it

† In terms of information efficiency (Zhang 2010), volatility (Hagströmer and Nordén 2013, Brogaard *et al.* 2014, Caivano 2015) and liquidity (Menkveld 2013, Kirilenko *et al.* 2017)

algorithms for practitioners, it is necessary to evaluate correctly the quote prices in order to avoid the problem of adverse selection by more informed subjects.

In order to understand the profit margins deriving from an operation of this type, a new market making algorithm, the so-called Signal-Adaptive (SA) market making strategy, is introduced with the aim to exploit the empirical recurrences characterizing the historical series of the prices of IPOs and managing the main risks associated with this activity. The algorithm is called Signal-Adaptive because it considers econometric signals from the historical price series of the IPO under observation.

The paper is thus organized as follows. Section 2 sets out the logic behind the implementation of the market making algorithm used to test the profitability of the algorithmic trading strategy for the first days of issuance of IPOs. Section 3 describes the ideas behind the development of the high-frequency interpretation system of an order book used for backtesting the signal-adaptive strategy in comparison with other two market-making strategies found in literature. Section 4 discusses the empirical results of the market making strategies on a high-frequency dataset of 20 IPOs listed on the French Euronext market.

2. The signal-adaptive market making strategy

The new market making strategy presented in this section aims to provide a market making algorithmic structure that can be easily modified according to the ambitions of the market maker, both in terms of liquidity and return-risk profile.

Therefore, the idea behind the creation of the SA strategy is to exploit the generation of signals to alter subjectively the status of the quotas that are placed by the market maker. An example is given by the arrival of a positive signal indicating a forecast of an increase in the price of traded security in the next period. In this case, the strategy should intercept this signal, making the BID share more aggressive (by raising it) and making the ASK share more passive (by raising it). The mirror logic is used in the case of a negative signal. Instead, if a neutral signal arrives, indicating the static nature of the price in the next period, the strategy should not alter the state characterizing the quotes.

2.1. *Main objective*

With reference to the IPO context (Ellis *et al.* 1993), the SA strategy aims at exploiting the recurring short-term empirical features in order to give importance to the profitability of the strategy, as well as to partially mitigating inventory risk to plain hedging.

In this respect, information-risk coverage would not seem necessary because of the multitude of theories (Boudriga *et al.* 2009, Jamaani and Alidarous 2019) that try to explain the recurring patterns that commonly occur when analyzing the short-term performance (for instance, regarding the first day) of an IPO. Therefore, when a market maker decides to operate in the first days of a company's listing on the stock exchange, he must give importance mainly to the inventory risk to a degree appropriate to the risk-return profile that he wants to preserve.

For example, in the case of a risk-averse underwriter that wants to support the price of an IPO by providing maximum liquidity without altering the parameters of best BID and best ASK of the market, a market making strategy which simply copies the best values in the market seems to be appropriate. On the contrary, if the Market Maker wants to strongly consider the inventory-risk, it could be obliged to renounce to give maximum support in terms of liquidity to the price of the stock.

Therefore, the SA strategy is a quotes management strategy that allows the underwriter to actively support the price of the stock with market making strategies while considering the inventory-risk and it aims to be the basis for implementing more complex strategies and becoming a state-of-art benchmark for comparative performance evaluation.

2.2. *Strategy implementation*

The SA market making strategy requires the definition of the relevant signals and the starting point for the calculation of spread.

In order to choose the signals necessary to define the evolution of the market making spread, it is necessary for the first instance to understand what may be the factors that affect the spread to a greater extent (Bollen *et al.* 2004, Bouchaud *et al.* 2018), that is:

(i) Standard deviation of best prices: measures the risk of the stock mid-price's temporal evolution. The higher this risk, the larger the spread must be in order to adequately remunerate the market maker for the risk taken;

(ii) Quantity of stocks in inventory: measures the inventory risk associated with the market making activity. The quotes of bidMM and askMM (market maker quotes) that define the spread of market making should decrease if the amount of stock in inventory is positive and tends to increase, so as to 'force' the sale of the stock and restore an inventory balance (quantity of stock in inventory = 0). On the contrary, bidMM and askMM should increase if the quantity of stock in inventory is negative (the market maker has gone short) to bring more pressure to buy the stock;

(iii) Difference between market ASK and BID total size: if the difference between the total size of ASK and BID orders grows, a decline in mid price is expected due to higher sales volume. In this case, the market maker should avoid buying the stock that is about to lose value and therefore must lower the BID and ASK shares in order to sell (or shorten) the stock. The mirror logic is applied in the case of the higher volume of BID orders in comparison to the volume of ASK orders.

Table 1 summarizes the effects of the above-mentioned signals on the market making spread. The calculation of the market making spread starts from the mid-price value, defined as the average between the best BID and best ASK market

Table 1. Effect of signals on market making quotes (BID MM, ASK MM) and spread (Spread MM).

Signal	Direction	BID MM	ASK MM	Spread MM
Standard deviation of best prices	↑	↓	↑	↑
	↓	↑	↓	↓
Quantity of stocks in inventory	↑	↓	↓	=
	↓	↑	↑	=
Difference between market ASK and BID total size	↑	↓	↓	=
	↓	↑	↑	=

Note: The direction of the signal symbolizes an increase (up arrow) or decrease (down arrow) in the reference parameter. The effects of such a change in the value of the signal parameter are represented with the same logic on market making quotes and spread.

values, as it can be considered a good estimate of the value of the transactions involving the stock.

$$\text{Mid Price} = \frac{\text{Best BID} + \text{Best ASK}}{2} \tag{1}$$

For the calculation of the ASK quote (askMM) placed by the market maker, the standard deviation of the historical values of the best ASK on the market is considered as the basis for the calculation of the distance from the mid-price. For the BID quote (bidMM) placed by the market maker, the standard deviation of the historical values of the best BID on the market is taken into account.

Thus, the distance from the mid-price for the ASK side can range from 0 to the standard deviation of Best Ask, SD(Best ASK) while the distance from the mid-price for the BID side can range from 0 to SD(best BID).

$$\text{askMM} - \text{Mid Price} \rightarrow \; [0, \text{SD(Best ASK)}] \tag{2a}$$

$$\text{Mid Price} - \text{bidMM} \rightarrow \; [0, \text{SD(Best BID)}] \tag{2b}$$

The standard deviation fraction used to calculate the distance to the mid-price is the result of a weighted average of the effects of Inventory and Total Size signals:

$$\begin{aligned}\text{askMM} = \text{MidPrice} + \text{SD(BestASK)} \\ \times [\alpha \text{Signal1ASK} + \beta \text{Signal2ASK}]\end{aligned} \tag{3a}$$

$$\begin{aligned}\text{bidMM} = \text{MidPrice} - \text{SD(BestBID)} \\ \times [\alpha \text{Signal1BID} + \beta \text{Signal2BID}]\end{aligned} \tag{3b}$$

where Signal1 refers to the signal given by the 'difference between market ASK and BID total sizes' and Signal 2 refers to the signal given by the 'quantity of stocks in inventory'.

In the calculation system devised, it is needed a formula that, given as input the difference in absolute value between ASK total size and BID total size, returns as output a value close to 1 the larger the difference in absolute value. In other words, the formula is defined by a concave function whose output is within the set of values $[0, 1]$. Therefore, this function can be used to calculate the parameters needed to estimate the market making spread.

This formula is also used for the quantification of the intensity of Signal2, the one related to the inventory-risk:

$$\text{Signal Value Generator}(\mathit{Input}) = 1 - \left(\frac{1}{\text{abs}(\mathit{Input}) + 1}\right) \tag{4}$$

To fully understand how the Signal Value Generator (SVG) is used, the SA market making's pseudocode strategy is shown in Algorithm 1. Note that the algorithm requires as Input the signals of 'quantity of stocks in inventory' (qInventory) and 'difference between total ASK and total BID sizes' (sizeDiff). After that, the signal values are defined according to the values of the above-mentioned signals through a ramified decision logic. Finally, the market making quotas are calculated by equations (3a) and (3b).

Algorithm 1 - Pseudocode of Signal-Adaptive market making strategy

Require: *MidPrice*, *stdBestBID*, *stdBestASK*, *qInventory*, *sizeDiff*;

Define SizeDiff-Signal (Signal1) Quote → α;
Define Inventory-Signal (Signal2) Quote → β;
Define Vector 2x1 of SignalAskValues → *SignalAskValues*[];
Define Vector 2x1 of SignalBidValues → *SignalBidValues*[];
if *sizeDiff* > 0 **then**
  Define Spread portion for BID → $\mathit{SignalBidValues}[1] = \mathit{SVG}(\mathit{sizeDiff})$;
  Define Spread portion for ASK → $\mathit{SignalAskValues}[1] = 0$;
else if *sizeDiff* < 0 **then**
  Define Spread portion for BID → $\mathit{SignalBidValues}[1] = 0$;
  Define Spread portion for ASK → $\mathit{SignalAskValues}[1] = \mathit{SVG}(\mathit{sizeDiff})$;
else
  Define Spread portion for BID → $\mathit{SignalBidValues}[1] = 0$;
  Define Spread portion for ASK → $\mathit{SignalAskValues}[1] = 0$;
end if
if *qInventory* > 0 **then**
  Define Spread portion for BID → $\mathit{SignalBidValues}[2] = \mathit{SVG}(\mathit{qInventory}) \times \mathit{qInventory}^2$;
  Define Spread portion for ASK → $\mathit{SignalAskValues}[2] = 0$;
else if *qInventory* < 0 **then**
  Define Spread portion for BID → $\mathit{SignalBidValues}[2] = 0$;
  Define Spread portion for ASK → $\mathit{SignalAskValues}[2] = \mathit{SVG}(\mathit{qInventory}) \times \mathit{qInventory}^2$;
else
  Define Spread portion for BID → $\mathit{SignalBidValues}[2] = 0$;
  Define Spread portion for ASK → $\mathit{SignalAskValues}[2] = 0$;
end if
Define Market Maker BID Order → **bidMM** $= \mathit{MidPrice} - \mathit{stdBestBid} \times (\alpha \times \mathit{SignalBidValues}[1] + \beta \times \mathit{SignalBidValues}[2])$;
Define Market Maker ASK Order → **askMM** $= \mathit{MidPrice} + \mathit{stdBestBid} \times (\alpha \times \mathit{SignalBidValues}[1] + \beta \times \mathit{SignalBidValues}[2])$;
return bidMM, askMM;

By changing the parameters α and β it is possible to give more influence on one signal than to another. Here, we set α

Table 2. List of IPOs composing the dataset for backtesting (ordered by date).

Date MM/DD/YY	Company name	ISIN code	Location	Market
1/17/17	TECHNIP F.M.C. P.L.C.	GB00BDSFG982	Paris	Euronext
2/8/17	LYSOGENE	FR0013233475	Paris	Euronext
2/8/17	OSMOZIS	FR0013231180	Paris	Euronext
2/15/17	INVENTIVA S.A.	FR0013233012	Paris	Euronext
3/7/17	TIKEHAU CAPITAL S.C.A.	FR0013230612	Paris	Euronext
3/20/17	MECELEC COMPOSITES	FR0000061244	Paris	Euronext
4/6/17	X-FAB SILICON FOUNDRIES S.E.	BE0974310428	Paris	Euronext
5/12/17	PRODWAYS GROUP	FR0012613610	Paris	Euronext
6/7/17	VALBIOTIS S.A.	FR0013254851	Paris	Euronext
6/9/17	BALYO S.A.	FR0013258399	Paris	Euronext
6/12/17	ANTALIS INTERNATIONAL	FR0013258589	Paris	Euronext
6/16/17	ALD S.A.	FR0013258662	Paris	Euronext
7/11/17	IMPLANET S.A.	FR0010458729	Paris	Euronext
7/21/17	UV GERMI S.A.	FR0011898584	Paris	Euronext
9/14/17	PHARMASIMPLE S.A.	BE0974302342	Paris	Euronext
9/18/17	M2I CORP	FR0013270626	Paris	Euronext
10/13/17	BIOM'UP S.A.	FR0013284080	Paris	Euronext
10/16/17	ADEUNIS S.A.C.A.	FR0013284627	Paris	Euronext
10/20/17	SMCP S.A.	FR0013214145	Paris	Euronext
12/6/17	ADVICENNE PHARMA S.A.	FR0013296746	Paris	Euronext

and β equal to 0.5, assuming the same degree of importance of the two signals.† In fact, it can be noticed that the performance results regarding the inventory risk management for α and β equal to 0.5 are within the range outlined by the backtesting performed with the other two sets of values considered.

Finally, we use the value $qInventory^2$ in the calculation of the SVG for the Signal2 in order to increase the impact of the inventory-risk hedging strategy component. Without the inclusion of this parameter, the strategy seems to hedge the inventory risk too weakly and the square of *qInventory* appears to be a sufficient measure to deal with this issue.

3. Data and methodological set up

This section reports the logic of interpretation of the high-frequency dataset that is used for backtesting the Market Making algorithms described in Section 2 against state-of-art approaches.

3.1. *The dataset*

High-frequency data concerning IPOs listed in AMF Euronext Paris in 2017 are used. The database used for the backtesting and empirical validation of the proposed strategy is the one provided by BEDOFIH ('Base Européenne de Données Financières à Haute Fréquence').‡ In particular, BEDOFIH offers high-frequency data categorized by type and order time arrival. Therefore, HFT data are divided into 'Current Orders', that is orders arrived in the order book during the analysis period, 'Historic Orders', which are valid orders already present in the order book before the analysis period, and 'Trades', which correspond to details of trades executed in the order book. Such partition provides an organizational structure to manage high-frequency data by sorting them by millisecond.§ Additional information regarding the parameters contained in the above mentioned HFT dataset could be found in Appendix 1.

Table 2 reports all the companies identified as IPO in Euronext Paris, during the year 2017, that compose the dataset. Descriptive statistics regarding the IPOs mentioned in table 2 can be found in Appendix 5. The time interval under observation is three trading days: the day of the IPO market entry and the two days immediately following the IPO market entry. To this end, the time interval in which the market making strategies are executed must take into account the opening and closing times of AMF Euronext Paris, 9:00 a.m. and 5:30 p.m., respectively.

Summing up, the data analyzed are all the orders of each type arrived in the Euronext Paris market for the securities reported in table 2 in the time interval defined above. The arrival of these orders, together with the calculation of the parameters that define the characteristics of the order book, is of fundamental importance in outlining the behavior of the market making strategies.

3.2. *Understanding high-frequency data*

In order to be able to estimate the profit and the strategy effectively performed by a high-frequency trading algorithm, it is necessary to be able to calculate in every moment (or to the arrival of every order in the order book) the parameters characterizing the high-frequency strategy. Pseudocode (Algorithm 2) reports the approach we propose to follow to

† For completeness, robustness test for values of $\alpha = [0.4; 0.6]$ and $\beta = [0.6; 0.4]$ result in qualitatively similar results and are available in Appendix 4. Research is high on the agenda for the development of data-driven methods to select the optimal values of α and β.

‡ We use AMF Euronext Paris data obtained via BEDOFIH. This database includes trades and orders with the highest frequency contained in the most important European Stock Markets such as London Stock Exchange, BATS, CHI-X, Deutsche Boerse Xetra, AMF Euronext Paris and Eurex (Derivatives Markets).

§ We assume that the sequence of orders in the provided database follows the temporary arrival of the orders in the market.

evaluate the performance of each market making algorithm on past price data of each financial instrument. With this aim, the High-Frequency Interpretation System (HFIS) calculates all the defining parameters of a high-frequency algotrading strategy within a defined time interval (e.g. a trading day). In our implementation, only the parameters relevant to the execution of the market making algorithms under observation are computed, but the proposed approach can be easily modified to allow the calculation of additional parameters.

The HFIS operation reported in Algorithm 2 needs three different types of datasets as inputs, which are able to define the total workload of the orders arrived in the market during the analyzed time interval. The mentioned datasets, defined in table 3, are obtained from the BEDOFIH database described in Section 3.1.

In particular, after having defined the parameters for the time interval considered, the 'rolling times' necessary for the execution of the while loop are initialized. The while cycle repeats itself every time a new order arrives on the market in order to take into consideration automatically every order that enters in the order book at high frequency.

Since HFIS is used on past data, it is necessary to update the historical dataset every time an order is taken into consideration. The 'OrderBook' data structure collects all valid orders on the market in the time interval identified by the current while loop. Then, each time a while loop repeats, the new order to be considered is removed from the historical dataset† and added to the 'OrderBook' data structure.

The newly created 'OrderBook' is then analyzed at the end of the loop to define the market making quotas for the current strategy. This analysis produces the input parameters for the market making strategy‡ in order to make the algorithmic strategy responsive for each new order arrival in the market.

The 'TradesOB' data structure is involved to assess whether there have been any trades during the time interval defined by the previous cycle and the current cycle and to assess a potential interaction of the market making quotes with the market. If any trades are executed, 'OrderBook' is updated and then a consistency check (Roşu 2009, Law and Viens 2020) is done to ensure the goodness of the calculation of the input parameters for the market making strategy.

Once the inputs are provided, Algorithm 2 returns the algotrading orders and profit (i.e. Output of the AlgoTradingFunction() function), the best ASK and best BID prices (i.e. Output of the bCalc() function) and the sizes of BID and ASK queues (i.e. Output of the sizesOB() function).

The steps of the bCalc() procedure for the calculation of best values in the 'OrderBook' are illustrated in the pseudocode in Algorithm 3. Basically, the logic behind this function is to initially separate BID orders from ASK orders. After that, the vectors containing BID and ASK orders must be ordered in ascending and descending order, respectively. This allows to easily pick the best price at the end of them.

The same order separation logic is adopted by the sizesOB() function, with the difference that afterwards the total size values for each order side are calculated (as shown in Algorithm 4).

The usefulness of HFIS represented in Algorithm 2 is that it allows the market maker to get the outputs whenever a new order arrives in the order book. Consequently, by collecting this data and ordering them with a time logic for the time interval analyzed it is easy to represent the trend of these values during the whole time period considered.

Finally, the HFIS can be exploited by changing the 'AlgoTradingFunction()' according to the trading strategy that needs to be analyzed, i.e. by using the SA strategy illustrated in Algorithm 1.

† I.e. CurrentOB input parameter of Algorithm 2.

‡ In the Signal-Adaptive case, the input parameters are inventory and total queue sizes.

Algorithm 2 - High frequency order book analysis for algorithmic trading.

Require: $HistoryOB, CurrentOB, TradesOB$
  Initialize order book $\rightarrow OrderBook = HistoryOB$;
  Define trading times $\rightarrow StartTime,\ EndTime$;
  Initialize rolling times $\rightarrow OldTime = NewTime =$ *First time contained in CurrentOB*;
  Define initial algotrading orders $\rightarrow AlgoTradingFunction() \rightarrow AlgoTradingOrders$;
  while $OldTime \leq EndTime$ **do**
    NewTime = First time contained in the CurrentOB;
    Define time range $\rightarrow RangeT = NewTime - OldTime$;
    Update order book $\rightarrow OrderBook = OrderBook + CurrentOB(RangeT)$;
    Update CurrentOB $\rightarrow CurrentOB = CurrentOB - CurrentOB(RangeT)$;
    if $TradesOB(RangeT)$ *is not empty* **then**
      if $Priority(AlgoTradingOrders) > Priority(OrderBookTradingOrders)$ **then**
        Execute algotrading orders;
        Calculate algotrading profit;
      end if
      Execute remaining trading orders contained in the order book;
    end if
    Scan order book for consistency;
    Calculation of best prices $\rightarrow bCalc() \rightarrow BestASK, BestBID, MIDprice$;
    Calculation of BID and ASK queue sizes $\rightarrow sizesOB() \rightarrow totalASKsize, totalBIDsize$;
    Define new algotrading orders $\rightarrow AlgoTradingFunction() \rightarrow AlgoTradingOrders$;
    Update rolling times $\rightarrow OldTime = NewTime$;
  end while
  return Algotrading orders and profit, best prices and sizes of BID and ASK queues

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Algorithm 3 - bCalc(): calculation of best BID and best ASK prices.

Require: $OrderBook$
  Remove non-Limit Orders from the OrderBook $\rightarrow OrderBook$;
  Create Vector of BID Limit Orders $\rightarrow BIDorders$;
  Create Vector of ASK Limit Orders $\rightarrow ASKorders$;
  Sort BIDorders in ascending order $\rightarrow BIDorders$;
  Sort ASKorders in descending order $\rightarrow ASKorders$;
  Calculate Best BID $\rightarrow bestBID = BIDorders(end)$;
  Calculate Best ASK $\rightarrow bestASK = ASKorders(end)$;
  return bestBID, bestASK

Table 3. Definition of input parameters for Algorithm 2.

Input parameter	Definition
HistoryOB	All the still-valid orders arrived in the period before the analyzed time interval.
CurrentOB	All the orders arriving in the analyzed time interval.
TradesOB	All the trades performed during the analyzed time interval.

Algorithm 4 - sizesOB(): calculation of sizes of BID and ASK queues.

```
Require: OrderBook
  Remove non-Limit Orders from the OrderBook → OrderBook;
  Create Vector of BID Limit Orders → BIDorders;
  Create Vector of ASK Limit Orders → ASKorders;
  Initialize → ASKsize = 0, BIDsize = 0;
  for i = 1:length(BIDorders) do
    BIDsize = BIDsize + size(BIDorders(i));
  end for
  for i = 1:length(BIDorders) do
    ASKsize = ASKsize + size(ASKorders(i));
  end for
  return BIDsize, ASKsize
```

4. Backtesting of market making strategies

In this section, we report the results of the backtesting operation for the performance of the strategy illustrated in Section 2. For performance comparison, two other state-of-art market making strategies are considered: Best-Values (BV) strategy and Inventory-Hedging (IH) strategy. The BV strategy consists simply in listing by following the levels of best values in the market (best ASK and best BID), while the IH strategy was proposed by Avellaneda and Stoikov (2008) with the aim of hedging the inventory risk by assuming that the market maker's decisions are based on a risk-adverse utility function. A detailed description of the two strategies is reported in Appendix 3.

4.1. *Simulation set-up*

We briefly describe the assumptions made for the performance simulation of market making strategies. Similar assumptions have also been used in previous research work (Guéant *et al.* 2013, Huang *et al.* 2015, Fushimi and Rojas 2018, Lu and Abergel 2018, Baldacci *et al.* 2020, Law and Viens 2020). The purpose of those assumptions is to make market maker as simple as possible, while at the same time allowing to easily add details that could make simulations more consistent with a specific case of use. The use case we focus on is that of the use of market making algorithms in the context of IPO, and therefore a reference figure can be the underwriter of the IPO. To this end, the following assumptions were made in the implementation of the order book analysis algorithm, described in Section 3:

(i) Unitary order size: the market maker is able to place orders to buy and sell up to 1 stock. This assumption has been made to simplify the execution of the strategy and to increase the probability of execution of market maker orders. In fact, the increase in size in the orders can lead to an incomplete execution of the orders due to the constraint of the priority that governs the queues of the order book †;

(ii) Zero starting inventory: it is assumed that the market maker begins to implement its strategy with a starting inventory that does not contain stock. Then, at the end of the trading day, if the market maker has stocks in inventory, it is obliged to sell them (or to buy them in case of negative inventory). This assumption simplifies the analysis of the effect of each market making strategy on the inventory‡;

(iii) Unlimited cash availability: the market maker is potentially able to shorten stock (making the inventory negative) indefinitely. This avoids the complication of analyzing the case of market maker bankruptcy, which can interrupt the process of strategy analysis;

(iv) No transaction costs: the simulation does not consider the presence of transaction costs in the placement of market making orders. This choice is due to the better transparency of the profit obtained with the strategy and the inaccurate quantification of transaction costs currently in the market §;

(v) Update of market making quotes every order time arrival: the process of calculating the quotes of orders placed by the market maker is updated every time an order (or a set of orders if they arrive at the same time) arrives in the market. More precisely, the market making strategy updates its output each time after the order book consistency check due to the arrival of new orders. Therefore, in this case, the possibility for the market maker to modify its orders in periods of static order book does not exist.¶

Furthermore, in order to fully understand the empirical results, we briefly describe below how the market maker's wealth is calculated during the execution of the strategy. The idea behind this is that the market maker can at any time buy a stock at the best ASK price of the market and sell a stock at the best BID price of the market. Consequently, if its inventory is

† The assumption of constant order size is in line with previous research works such as Law and Viens (2020), Huang *et al.* (2015), Lu and Abergel (2018), and Baldacci *et al.* (2020).

‡ The backtesting state in which the market makers start with an inventory containing zero stocks is in line with previous research work such as Guéant *et al.* (2013).

§ The transaction costs were not considered during the backtesting in previous research such as Lu and Abergel (2018).

¶ The assumption of zero latency in the positioning of market maker's orders in the market is in line qith previous research work such as Fushimi and Rojas (2018).

Table 4. IPOs' aggregate performance during the first three days.

IPOs aggregate performance	1st day	2nd day	3rd day	Average
Daily return	– 0.001	0.005	– 0.005	0.000
Daily standard deviation	0.141	0.098	0.080	0.106
Mean (Market spread)	0.096	0.131	0.236	0.118
Std (Market spread)	0.758	0.113	0.509	0.460
Mean (ASK-BID size difference)	– 151 048.620	– 88 168.392	– 96 983.719	– 112 066.910
Std (ASK-BID size difference)	69 982.464	37 224.207	37 417.988	48 208.220
N. temporary order arrivals	2965.850	1564.700	1362.700	1964.417

Note: The parameters shown are the result of averaging the individual parameters of the 20 analyzed IPOs composing the backtesting dataset.

Table 5. Aggregate backtest performance (mean and standard deviation) of BestValues (BV), InventoryHedging (IH) and SignalAdaptive (SA) market making strategies for the first three days of issuance of IPOs.

Performance parameter	Day	BV	IH	SA
Final wealth	First	2.085 (4.124)	0.498 (0.884)	1.031 (1.538)
	Second	0.855 (1.554)	0.703 (1.685)	0.640 (1.044)
	Third	3.135 (7.457)	0.251 (0.676)	0.507 (0.991)
Mean (Wealth)	First	– 0.162 (2.753)	0.256 (0.485)	0.534 (0.869)
	Second	0.382 (0.675)	0.217 (0.685)	0.155 (0.428)
	Third	0.385 (1.169)	0.114 (0.275)	0.205 (0.301)
Std (Wealth)	First	1.555 (3.260)	0.154 (0.145)	0.632 (1.190)
	Second	0.453 (0.984)	0.454 (0.772)	0.359 (0.634)
	Third	0.612 (1.586)	0.104 (0.205)	0.186 (0.325)
Final inventory	First	– 8.400 (48.161)	– 0.200 (1.576)	– 0.450 (1.432)
	Second	3.150 (8.349)	1.900 (4.712)	0.400 (2.113)
	Third	– 10.250 (28.364)	2.050 (4.957)	0.200 (1.963)
Mean (Inventory)	First	– 6.033 (25.036)	0.005 (1.005)	– 0.080 (0.951)
	Second	0.604 (4.564)	0.635 (1.845)	0.272 (1.225)
	Third	– 4.675 (12.457)	0.448 (0.915)	0.154 (0.920)
Std (Inventory)	First	9.218 (16.717)	0.735 (0.660)	1.115 (1.012)
	Second	2.570 (3.762)	1.058 (1.522)	0.975 (0.441)
	Third	4.984 (10.600)	0.788 (1.618)	0.821 (0.666)
Mean (SpreadMM)	First	0.027 (0.578)	0.090 (0.276)	0.337 (1.005)
	Second	0.222 (0.529)	0.023 (0.007)	0.128 (0.181)
	Third	0.317 (0.785)	0.061 (0.176)	0.189 (0.544)
Std (SpreadMM)	First	0.841 (3.023)	0.060 (0.204)	0.244 (0.537)
	Second	0.209 (0.412)	0.006 (0.013)	0.096 (0.129)
	Third	0.589 (2.054)	0.060 (0.262)	0.133 (0.397)
N. inventory changes/Order arrivals	First	0.021 (0.018)	0.011 (0.013)	0.026 (0.019)
	Second	0.019 (0.021)	0.014 (0.020)	0.030 (0.019)
	Third	0.018 (0.017)	0.005 (0.011)	0.025 (0.023)

Note: The parameters shown are the result of averaging the individual parameters of the 20 analyzed IPOs composing the backtesting dataset.

negative the wealth is calculated as:

$$\text{Wealth MM}_t = \text{Quantity of stocks in inventory}_t \times \text{BestBID}_t + \text{Cash}_t \tag{5}$$

Symmetrically, if the inventory is positive:

$$\text{Wealth MM}_t = \text{Quantity of stocks in inventory}_t \times \text{BestASK}_t + \text{Cash}_t \tag{6}$$

where 'Cash' means the liquidity earned (or spent) due to market making operations.† For example, if the market maker has just shortened a stock, its cash amount rises by the selling price of the stock. In turn, if the market maker has just bought a stock the amount of cash decreases by the purchase price of the stock. In this case, the total amount of cash may be negative at certain times during the execution of the strategy.

4.2. Backtesting results

This section illustrates the results of the backtesting of the market making strategies underlying the constraints mentioned in the previous paragraph.

Table 4 shows the aggregate performance data of the IPOs listed in table 2. All the showed data are the result of an average between all the single values obtained for each IPO. Those quantities are explained in table A3.

As far as the backtesting of the performance of market making strategies is concerned, the relative aggregate results are presented in table 5; these results are calculated as the average

† Notice that (5) and (6) assume unlimited liquidity on both ASK and BID (as the quantity at time t can be infinite, as per the unlimited cash availability, which is an assumption we rely on.

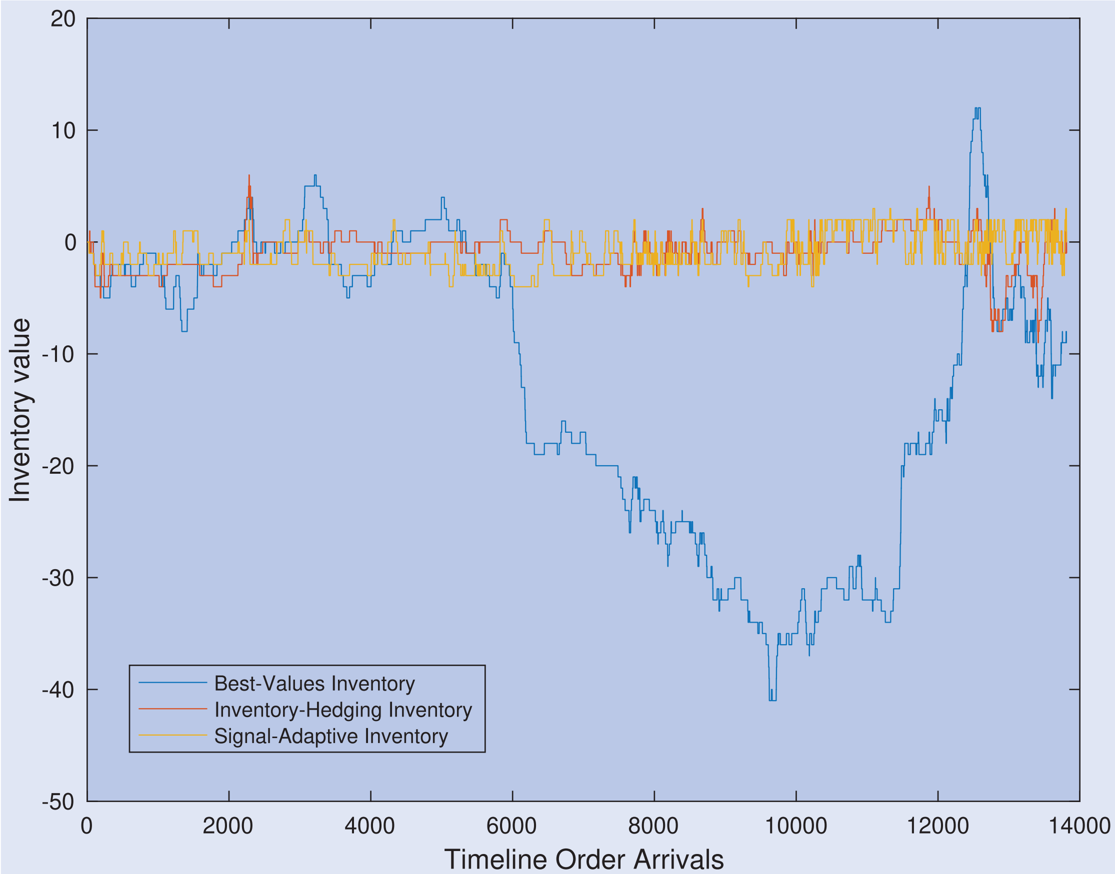

Figure 1. Inventory evolution process of BestValues, InventoryHedging and SignalAdaptive Market Making strategies. The graph is the result of running the Algorithm 2 on the high-frequency data of Technip FMC's first day of listing (17/01/2017).

of single data obtained from each IPO. Table A4 explains the parameters used to compare the performances of market making strategies.

Table 5 reports backtesting data of Market Making strategies. The 'Final Wealth' provides information on the degree of profitability of the strategy. Analyzing backtesting values, the BV strategy seems to be on average the most profitable. On the contrary, the IH strategy is on average the least profitable, even less than the SA strategy for the first and third day.

Still, when interpreting the performance results of a strategy, the strategy's risk profile must also be considered. To this end, the 'Std (Final Wealth)' points out the higher riskiness of the BV strategy compared to others. The least risky strategy is the IH strategy for the first and the third day, being consistent with the consensus that a higher return is linked to a higher risk.

If the inventory risk needs to be analyzed separately, the aggregate parameter 'Mean (Inventory)' can be useful. It is clear that the SA strategy maintains a balance of inventory (understood as the market maker's propensity to hold on average zero inventory during the execution of the strategy) more efficient than competing strategies. On the contrary, the inventory balance value performed by the BV strategy seems to indicate a great inventory risk assumed by the market maker who decides to perform this strategy.

To better contextualize the inventory risk, it is necessary to analyze also the 'Std (Inventory)' parameter, which is a proxy of the volatility of the inventory value held by the market maker. In this sense, the SA strategy seems to best hedge the inventory risk. Figure 1 displays the inventory values relating to market making strategies applied on the first day (17/01/2017) of trading for Technip F.M.C. P.L.C., supporting the findings described in table 5.

To assess the 'aggressiveness' of the strategy we compute the 'Mean (Spread MM)', 'Std (Spread MM)' and 'N Inventory Changes/Order Arrivals'. Aggressiveness is defined as the propensity of the strategy to buy and sell securities in the market, and therefore the ability to support the price of the security. Looking at the values of 'Mean (Spread MM)' and 'Std (Spread MM)', the IH strategy seems to be more static in the difference of price between the quotations of ASK and BID introduced in the market. The BV strategy, given its nature, introduces the same values of spread aggregates of the market spread illustrated in table 2.

By taking into account the average of the first three days of issue of IPOs, the SA strategy places on average the orders with the highest spread which turns out to be more volatile than the IH strategy. For our purposes, this information is incomplete if not compared with the 'N. Inventory Changes/Order Arrivals' parameter, which is able to give an indication of how many times the market maker has bought or sold the securities compared to how many times it has evaluated order placement. The most active strategy in this sense is the SA, which seems to widely modify the spread according to the signals received in order to trade the securities effectively.

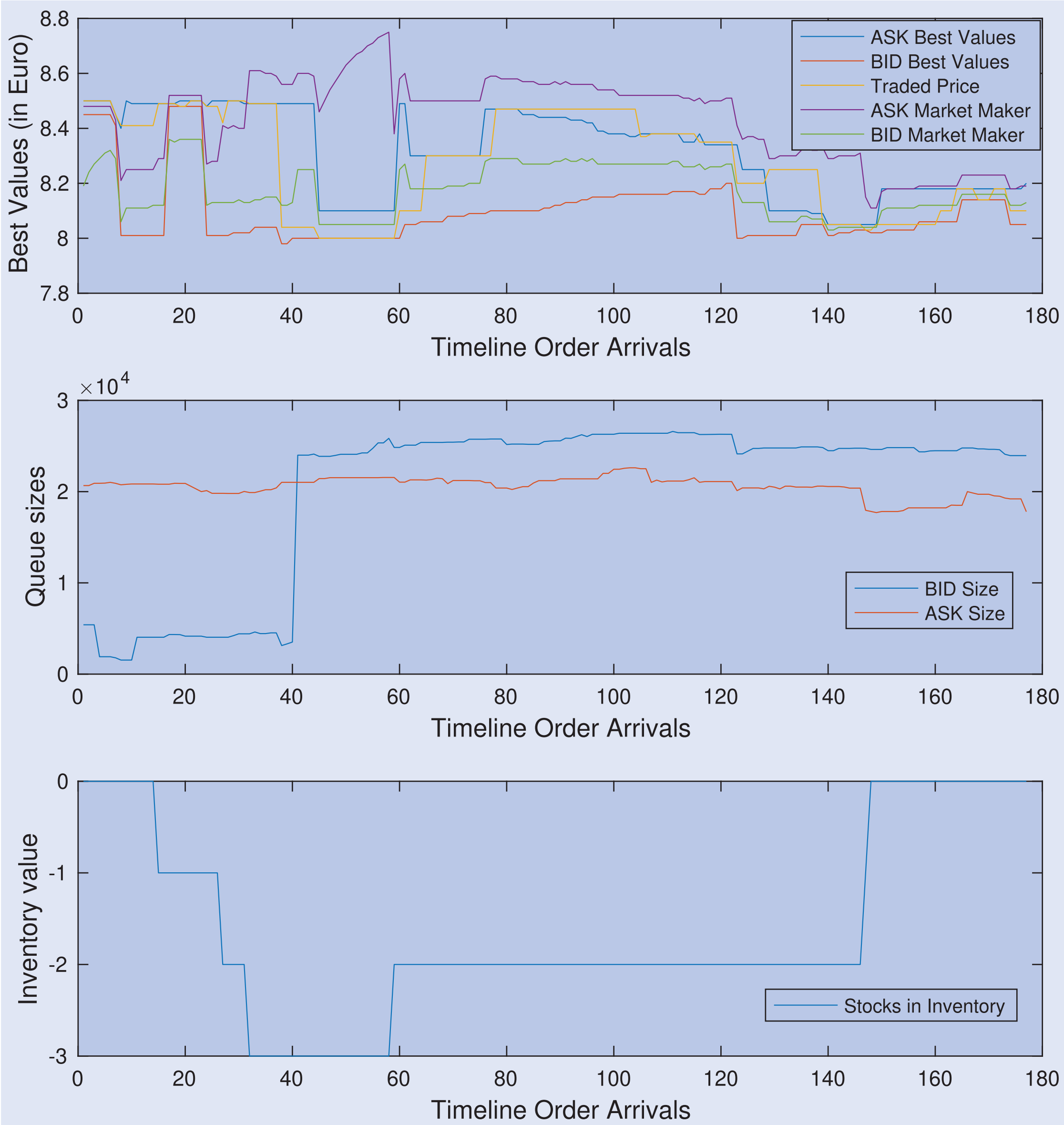

Figure 2. Signal-adaptive (SA) market making strategy quotes reacting to inventory and total queue sizes signals. The graph is the result of running the Algorithm 2 on the high-frequency data of Inventiva's third day of listing (17/02/2017). The first graph represents the change in market making and best values during the analyzed trading day. The second one shows, for the same trading day, the variation of the values of Queue Sizes for ASK and BID orders that define the signal 'total queue sizes'. The third chart shows, for the same trading day, the inventory situation of the market maker who is executing the SA market making strategy.

An illustration of this logic of spread adaptation by the SA strategy is shown in figure 2. In fact, it is possible to notice that when the market maker's inventory reaches high values (in terms of absolute value), the spread moves to avoid that the imbalance persists and aims to return to the equilibrium value. In such case, when the inventory reaches the value of -3 the market maker quote a very high ASK price in order to avoid selling further stocks. Moreover, figure 2 illustrates the adaptation of the market making quotes to the signal deriving from the difference in the dimensions of the queues from the BID and ASK part. It is possible to note that the lower aggressiveness in selling the stock corresponds to an inversion in the size of the queues (when the size of the BID queue becomes larger than the size of the ASK queue).

Summing up, the SA strategy is capable of reaching a good level of profitability when the historical series of the stock price has persistent phases of growth (or decline). This is in line with the idea behind the strategy, which aims to interpret signals that may suggest a future price movement. In other words, foreseeing a growth of the price in the next period, the market maker modified the spread in order to try to buy the stock because it will increase its value in the following period.

5. Conclusions

This paper introduces the SA strategy, that is a new market making algorithm able to exploit the empirical recurrences of the historical intra-day price series of IPOs. The SA strategy has been tested in a high-frequency order book interpretation environment that simulates the algorithmic trading activity performed by a market maker. By comparing it with two market making strategies representing the antipodes in the inventory risk management profile, we can conclude that the SA strategy maintains a high return profile related to strict inventory control and it is the most effective in terms of trades made with the market. Therefore, the SA strategy could be suitable for an underwriter who must actively support the price of the IPO in the first days of its issuance.

The best performance of SA is due to the logic of interpretation of econometric signals deriving from the historical price series and the inventory trend. In a few words, the SA strategy exploits expectations of growth/decrease in the price of the stock to buy/sell it at the right time. The price to which this exchange is executed is stated by the spread of market making, which reflects the situation of the inventory of the market maker in order to allow an efficient management of the risk of inventory. On the negative side, the SA strategy might suffer moments of market turbulence because sudden changes of the prices can exploit the not adequate speed of modification of the quotas outlining the spread of market making. However, the SA is capable of solving such issues either by inserting new signals or by increasing the weight of the inventory risk management component (at the expense of lower returns).

Backtesting results, carried out on historical data from 20 French IPOs in 2017, show that on average this type of high-frequency activity is profitable for all the strategies considered. This may suggest that a component of the profitability obtained from the market making activity carried out by an underwriter operating to sustain the price of an IPO could be due to the defining logic of this algorithmic trading strategy. Further analysis on a larger dataset should be performed to provide further empirical evidence.

Beyond the academic contribution, the HFIS introduced with this paper could be useful for researchers and practitioners working in the world of HFT to simulate the execution of high-frequency algorithmic strategies. In the particular case of simulating market making activity, the SA strategy can represent a basis for implementing more complex strategies or a state-of-art benchmark for comparative performance evaluation. However, further research should be devoted to relax the assumptions behind the considered framework, as they can make its application to a real-world context still not realistic enough.

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ORCID

S. Paterlini http://orcid.org/0000-0003-4269-4496
F. Bazzana http://orcid.org/0000-0003-1467-6776

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Appendices

Appendix 1. BEDOFIH dataset

Appendix 1 illustrates the dataset parameters defining the input data necessary for the Algorithm 2 execution. For the datasets 'Current Orders' and 'Historic Orders', the parameters are the same and are set out in the table A1.

Differently, the parameters of the BEDOFIH 'Trades' dataset taken into account are contained in table A2.

Appendix 2. Backtesting parameters definition

Appendix 2 illustrates the parameters defining the backtesting results. In particular, the aggregate performance data of the IPOs during the first three days of issuance is defined by the parameters contained in table A3.

Differently, the aggregate performance data of the market making strategies during the first three days of issuance of IPOs is defined by the parameters contained in table A4.

Appendix 3. Conventional market making strategies

Appendix 3 describes the strategies that are used as a benchmark for the proposed SA strategy. The first 'naive' strategy to be explained is the most basic and able to provide more liquidity to the market. The second strategy, proposed by Avellaneda and Stoikov (2008), is created with the primary objective of managing inventory risk given the risk aversion of the market maker. The decisive steps in its derivation are explained on the basis of the assumptions reported in Avellaneda and Stoikov (2008).

A.1. Best-values (BV) strategy

The first market making strategy to be analyzed aims to provide as much liquidity to the market as possible without giving importance to the presence of inventory risk that the market maker can assume.

Algorithm 5 - BestValues(): Best-Values Market Making Strategy

Require: *MidPrice*, *MarketSpread*
Define market maker BID order $\rightarrow$ **bidMM** $= MidPrice - MarketSpread/2$
Define market maker ASK order $\rightarrow$ **askMM** $= MidPrice + MarketSpread/2$
return bidMM, askMM

As shown in Algorithm 5, this strategy of placing orders in the market by the market maker is simply based on the detection of the best BID and best ASK in the market at the time of evaluation. The inputs of the algorithm are, for this purpose, the mid-price and the market spread defined by the best ASK and the best BID in the market. In fact, the market maker enters BID and ASK orders of price exactly equal to the value of the best BID and best ASK, respectively. In doing so, it does not change the values of best BID and best ASK that characterize the market, by supporting the maintenance of the market spread. Consequently, this strategy is characterized by high inventory risk.

In the event of a substantial change of value in the price of the stock, the market maker adopting the BV strategy is forced to buy or sell (or, in case, to short) a large quantity of stock that could make its overall wealth vary considerably (based on market movements).

A.2. Inventory-hedging (IH) strategy

This strategy is based on the work by Avellaneda and Stoikov (2008). They argue that the market maker faces an inventory-risk given the diffusive nature of the mid-price stock and a transaction risk due to

Table A1. Definition of 'Current Orders' and 'Historic Orders' input parameters for Algorithm 2.

Input parameter	Definition
Validation time	Date (YYYYMMDD) and Time (hh:mm:ss, milliseconds) of order validity.
Release time	Date (YYYYMMDD) and Time (hh:mm:ss, milliseconds) of when a order is released.
Modification time	Date (YYYYMMDD) and Time (hh:mm:ss, milliseconds) of when a order is modified.
Order ID	Fundamental identifier of the order.
Order side	Buy or Sell order specification.
Order price	Order price. For market orders it is zero.
Order size	Initial order size.
Order type	Order Type (Market, Limit, Stop, Pegged).
Order state	State of the order. (i.e. Partially filled).

Table A2. Definition of 'Trades' input parameters for Algorithm 2.

Input parameter	Definition
Validation time	Date (YYYYMMDD) and Time (hh:mm:ss, milliseconds) of trade validity.
Buy order ID	Fundamental identifier of the buy-side order.
Sell order ID	Fundamental identifier of the sell-side order.
Trade price	Transaction price.
Trade quantity	Transaction quantity.

Table A3. Definition of backtest parameters for IPO aggregate performance.

Backtest parameter	Definition
Daily return	Calculated as (FinalPrice − StartingPrice)/StartingPrice.
Daily standard deviation	Volatility of the vector of the prices treated for the security during the day of analysis.
Mean (Market spread)	Average of the market spread (best ASK - best BID) values for the stock on the day of analysis.
Std (Market spread)	Volatility of the market Spread values for the stock on the day of analysis.
Mean (ASK-BID size difference)	Average of the size differences between ASK size and BID size at every moment during the analyzed day.
Std (ASK-BID size difference)	Volatility of the size differences between ASK size and BID size at every order arrival during the analyzed day.
N. temporary order arrivals	Number of order arrivals for each trading day.

Table A4. Definition of the backtest parameters for the aggregate performance of market-making strategies.

Backtest parameter	Definition
Final wealth MM	Final profit obtained from the market making activity.
Mean (Wealth MM)	Average of all the states of wealth experienced by the market maker during the execution of the strategy.
Std (Wealth MM)	Volatility of all the states of wealth experienced by the market maker during the execution of the strategy.
Final inventory	Number of stocks hold (or sold) by the market maker at the end of the trading day.
Mean (Inventory)	Average of all the inventory states experienced by the market maker during the execution of the strategy.
Std (Inventory)	Volatility of all the inventory states experienced by the market maker during the execution of the strategy.
Mean (Spread MM)	Average of the price differences between the ASK orders and the BID orders submitted by the market maker.
Std (Spread MM)	Volatility of the price differences between the ASK orders and the BID orders submitted by the market maker.
N. inventory changes/Order arrivals	Ratio between the number of times the inventory has changed since the arrival of the previous order and the total number of times the orders have arrived.

lower priority in case of a queue of orders for a certain price. In order to limit the inventory-risk, they propose an HFT order book simulation model to test their 'hedging-risk' strategy.

The strategy proposed by Avellaneda and Stoikov (2008), which in this context will be called IH strategy, is based on the assumption that the agent operating as market maker aims to maximize its utility defined by a 'value function'.

In fact, the agent's goal is to maximize the expected exponential utility of his profit and loss profile at a specific terminal time T. In this case, the choice of a convex risk measure given by the exponential nature of the utility function is particularly convenient, since ≪it will allow us to define reservation (or indifference) prices which are independent of the agent's wealth≫ (Avellaneda and Stoikov 2008).

In this model, the most important component to limit inventory risk is to define the spread starting from two distinct values called 'reservation prices'. In particular, the reservation BID price is the price that would make the agent indifferent between his current inventory and his current inventory plus one stock. Instead, the reservation ASK price is the price that would make the agent indifferent between his current inventory and his current inventory minus one stock.

In order to define formulas characterizing the reservation prices, Avellaneda and Stoikov (2008) model in the first instance an inactive market maker who does not place any limit order in the market and simply holds an inventory of q stocks at the time until the terminal time (of execution of the strategy) T. The passivity of the market maker becomes then useful later when considering the possibility of placing limit orders.

The agent's value function is defined as follows:

$$v(x, s, q, t) = -\exp(-\gamma x) \times \exp(-\gamma qs) \times \exp\left(\frac{\gamma^2 q^2 \sigma^2 (T-t)}{2}\right) \tag{A1}$$

where x is the market maker's initial wealth in dollars, s is the mid-price of the stock at time T, q is the quantity of stocks in inventory, t is the current time $\leq$ terminal time T, S_T is the stock final mid-price, γ is the risk adversion parameter, and σ is the volatility of the stock.

The value function (A1) reported above can be written as:

$$v(x, s, q, t) = -\exp(-\gamma x) \times \exp(-\gamma qs) \times \exp\left(\frac{\gamma^2 q^2 \sigma^2 (T-t)}{2}\right) \tag{A2}$$

Consequently, the reservation BID price is defined as:

$$v(x - r^b(s, q, t), s, q+1, t) = v(x, s, q, t) \tag{A3}$$

While the reservation ASK price is defined as:

$$v(x + r^a(s, q, t), s, q-1, t) = v(x, s, q, t) \tag{A4}$$

A simple calculation involving the three formulas above allows us to define the reservation price formulas as:

$$r^b(s, q, t) = s + (-1 - 2q) \times \frac{\gamma \sigma^2 (T-t)}{2} \tag{A5a}$$

$$r^a(s, q, t) = s + (1 - 2q) \times \frac{\gamma \sigma^2 (T-t)}{2} \tag{A5b}$$

The paper shows the steps to calculate the following distances from reservation prices, derived from the optimization problem defined by Ho and Stoll (1981):

$$\delta^b = \gamma q \sigma^2 (T-t) + \frac{1}{\gamma} \ln\left(1 + \frac{\gamma}{k}\right) \tag{A6a}$$

$$\delta^a = -\gamma q \sigma^2 (T-t) + \frac{1}{\gamma} \ln\left(1 + \frac{\gamma}{k}\right) \tag{A6b}$$

where σ^2 is the volatility of the stock, and k is the calibration parameter.This market making strategy therefore quotes a fixed spread

Table A5. Robustness test performance.

Performance parameter	Day	SA ($\alpha = 0.4$)	SA ($\alpha = 0.5$)	SA ($\alpha = 0.6$)
Final wealth	First	1.074 (1.493)	1.031 (1.538)	0.914 (1.402)
	Second	0.683 (1.046)	0.640 (1.044)	0.750 (1.088)
	Third	0.451 (0.928)	0.507 (0.991)	0.435 (0.904)
Mean (Wealth)	First	0.533 (0.839)	0.534 (0.869)	0.477 (0.826)
	Second	0.164 (0.454)	0.155 (0.428)	0.244 (0.448)
	Third	0.184 (0.276)	0.205 (0.301)	0.179 (0.266)
Std (Wealth)	First	0.644 (1.186)	0.632 (1.190)	0.616 (1.183)
	Second	0.363 (0.637)	0.359 (0.634)	0.383 (0.639)
	Third	0.180 (0.295)	0.186 (0.325)	0.172 (0.299)
Final inventory	First	– 0.450 (1.638)	– 0.450 (1.432)	– 0.350 (1.387)
	Second	0.350 (1.899)	0.400 (2.113)	0.300 (2.227)
	Third	0.400 (1.984)	0.200 (1.963)	0.250 (2.023)
Mean (Inventory)	First	– 0.210 (1.354)	– 0.080 (0.951)	0.046 (1.022)
	Second	0.201 (1.143)	0.272 (1.225)	0.357 (1.283)
	Third	0.175 (1.151)	0.154 (0.920)	0.171 (0.960)
Std (Inventory)	First	1.107 (1.006)	1.115 (1.012)	1.169 (1.046)
	Second	0.908 (0.461)	0.975 (0.441)	1.028 (0.492)
	Third	0.950 (0.744)	0.821 (0.666)	0.850 (0.693)
Mean (SpreadMM)	First	0.302 (0.913)	0.337 (1.005)	0.366 (1.097)
	Second	0.114 (0.161)	0.128 (0.181)	0.140 (0.204)
	Third	0.162 (0.435)	0.189 (0.544)	0.213 (0.653)
Std (SpreadMM)	First	0.227 (0.495)	0.244 (0.537)	0.265 (0.598)
	Second	0.084 (0.109)	0.096 (0.129)	0.103 (0.154)
	Third	0.116 (0.317)	0.133 (0.397)	0.150 (0.477)
N. inventory changes/Order arrivals	First	0.028 (0.020)	0.026 (0.019)	0.024 (0.018)
	Second	0.032 (0.019)	0.030 (0.019)	0.030 (0.019)
	Third	0.025 (0.023)	0.025 (0.023)	0.022 (0.020)

Note: Aggregate robustness test performance of Signal-Adaptive (SA) market making strategy for the first three days of issuance of IPOs with α parameter of 0.4, 0.5 and 0.6. The parameters shown are the result of averaging the individual parameters of the 20 analyzed IPOs composing the backtesting dataset.

equal to:

$$\delta^a + \delta^b = \frac{2}{\gamma} \ln\left(1 + \frac{\gamma}{k}\right) \tag{A7}$$

Centered around the reservation prices in the following way:

$$\text{bidMM} = r^b - \delta^b \tag{A8a}$$

$$\text{askMM} = r^a + \delta^a \tag{A8b}$$

The previously explained market making strategy is shown in Algorithm 6 and this code is used for the backtesting reported in Section 4.

Algorithm 6 - InventoryHedging(): Inventory-Hedging Market Making Strategy

Require: *MidPrice*, *stdMidPrice*, *riskAdversion*, *qInventory*, *tTot*, *tNow*, *k*

Normalization of Time Values → $tNow = tNow/tTot$; $tTot = 1$;

Calculation of BID reservation price → $rb = MidPrice + (-1 - 2 \times qInventory) \times (riskAdversion \times (stdMidPrice^2) \times (tTot - tNow))/2$;

Calculation of ASK reservation price → $ra = MidPrice + (1 - 2 * qInventory) \times (riskAdversion \times (stdMidPrice^2) \times (tTot - tNow))/2$;

Calculation of BID distance from BID reservation price → $db = (riskAdversion \times qInventory \times (stdMidPrice^2) \times (tTot - tNow)) + (1/riskAdversion) \times log(1 + riskAdversion/k)$;

Calculation of ASK distance from ASK reservation price → $da = -1 \times (riskAdversion \times qInventory \times (stdMidPrice^2) \times (tTot - tNow)) + (1/riskAdversion) \times log(1 + riskAdversion/k)$;

Define market maker BID order → $bidMM = rb - db$;

Define market maker ASK order → $askMM = ra + da$;

return bidMM, askMM

Appendix 4. Signal-Adaptive strategy: robustness test

Appendix 4 illustrates the results of the robustness test regarding the Signal-Adaptive (SA) strategy. The results, reported in table A5, represent the backtest performance of the SA strategy with parameters α of 0.4, 0.5 and 0.6. The beta parameter is simply calculated $\beta = 1 - \alpha$.

Appendix 5. Euronext 2017 IPOs: descriptive statistics

Appendix 5 shows the descriptive statistics of the 20 IPOs quoted on Euronext Paris in 2017. Details regarding starting trade price, returns and standard deviation after one day, one week and one month can be found in table A6.

Table A6. Descriptive statistics of the 20 IPOs quoted in 2017 on Euronext Paris stock market.

Company	Starting price (EUR)	Parameter	1st day	1st week*	1st month*
TECHNIP F.M.C. P.L.C.	35.350	Return	– 0.023	– 0.030	– 0.089
		Std. Deviation	0.166	0.632	1.356
LYSOGENE	6.850	Return	– 0.006	– 0.085	– 0.180
		Std. Deviation	0.210	0.226	0.451
INVENTIVA S.A.	8.540	Return	– 0.004	– 0.028	– 0.021
		Std. Deviation	0.104	0.125	0.120
TIKEHAU CAPITAL S.C.A	21.000	Return	0.050	0.153	0.186
		Std. Deviation	0.388	1.301	0.966
X-FAB SILICON FOUNDRIES S.E.	7.800	Return	0.042	0.051	0.026
		Std. Deviation	0.228	0.239	0.152
PRODWAYS GROUP	5.100	Return	0.059	0.180	0.331
		Std. Deviation	0.111	0.327	0.630
BALYO S.A.	5.710	Return	– 0.107	– 0.158	– 0.107
		Std. Deviation	0.160	0.326	0.277
ANTALIS INTERNATIONAL	2.980	Return	0.013	– 0.131	– 0.332
		Std. Deviation	0.056	0.147	0.294
ALD S.A.	14.330	Return	0.000	– 0.043	0.082
		Std. Deviation	0.052	0.285	0.757
BIOM'UP S.A.	11.000	Return	– 0.046	– 0.087	– 0.170
		Std. Deviation	0.202	0.404	0.501
SMCP S.A.	22.000	Return	– 0.045	– 0.012	– 0.077
		Std. Deviation	0.183	0.622	0.622
ADVICENNE PHARMA S.A.	14.050	Return	– 0.004	– 0.009	– 0.025
		Std. Deviation	0.072	0.051	0.093
OSMOZIS	10.900	Return	– 0.026	0.135	0.116
		Std. Deviation	0.101	0.643	0.526
MECELEC COMPOSITES	1.380	Return	0.072	– 0.047	– 0.061
		Std. Deviation	0.024	0.049	0.047
VALBIOTIS S.A.	10.440	Return	– 0.027	0.037	– 0.035
		Std. Deviation	0.221	0.269	0.370
IMPLANET S.A.	0.740	Return	– 0.014	– 0.055	– 0.219
		Std. Deviation	0.007	1.131	3.331
UV GERMI S.A.	5.670	Return	– 0.039	– 0.007	– 0.029
		Std. Deviation	0.086	0.028	0.083
PHARMASIMPLE S.A.	32.480	Return	– 0.002	– 0.107	– 0.198
		Std. Deviation	0.014	2.447	3.208
M2I CORP	4.050	Return	0.101	0.233	0.276
		Std. Deviation	0.174	0.735	0.662
ADEUNIS S.A.C.A.	10.190	Return	– 0.009	– 0.033	– 0.095
		Std. Deviation	0.271	0.233	0.488

*The returns and standard deviations for the 1st week and 1st month have been computed on daily prices for the considered time interval, while the statistic relative to 1st day are computed with a intra-day (high-frequency) logic by using the HFIS provided in Section 3.2.