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Coupled GARCH(1,1) model

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We introduce a coupled GARCH model for the intraday and overnight volatility, using the implied jump magnitude from option markets and the earnings calendar to model anticipated shocks. We estimate the model on DJIA and report on the accuracy of the forecasts.

Keywords: Volatility forecasting; GARCH model; Implied volatility; Realized volatility

JEL Classification: C12, C13

1. Introduction

Volatility estimation for financial asset returns is of interest to several areas. It is crucial in derivatives pricing and risk management, for example. Volatility in stock prices exceeds what can be attributed to the arrival of new information (Grossman and Shiller 1981). More recently, interest in excess volatility has focused on microstructure activity bursts. Much of the early advances in volatility forecasting extended the GARCH model following the seminal work of Engle (1982). GARCH models fail to capture the long memory in volatility. Baillie *et al.* (1996) introduced the Fractionally Integrated GARCH model where the kernel decays as a power law instead of an exponential. The GARCH-X model was later developed to improve the prediction of squared returns augmented with additional exogenous variables, such as interest rate levels (Brenner *et al.* 1996) and trading volumes (Fleming *et al.* 2008). With the rapid improvement in data processing and acquisition techniques, Hörmann *et al.* (2013) proposed a functional GARCH model to take advantage of the more continuous flow of information from high-frequency financial data. Blanc *et al.* (2014) showed that intraday and overnight volatility both propagate with a power-law kernel, whereas their coupling (the effect of overnight volatility on subsequent intraday volatility) decays much faster. This suggests that the dynamics of both intraday and overnight volatility are self-exciting long-memory processes, whereas the market process for digesting news may be exponential. Gatheral *et al.* (2018) showed that volatility is rough with a sub-Brownian exponent. As explained by Hardiman *et al.* (2013), at the microstructure scale, volatility can be modeled as a self-exciting Hawkes process where the feedback is close to the critical value, which implies that the intraday volatility process is mostly endogenous. Blanc *et al.* (2017) show that the quadratic generalization of the Hawkes model is asymptotically equivalent to a Quadratic ARCH (QARCH) model and captures most stylistic facts about volatility. Gatheral *et al.* (2020) argued that the rough Heston model arises as the limit of natural Hawkes process-based models of price and order flow.

To be most useful to practitioners, volatility estimates should couple volatility dynamics from the above models with the information available from news sources and event calendars. First, amongst anticipated news events, earnings announcements can account for the majority of annualized stock price volatility. Linton and Wu (2020) and Dhaene and Wu (2020) considered a coupled GARCH model where overnight shocks are treated as t-distributed innovations and showed that the role of overnight volatility has increased in the last decades. Their approach is well-suited to earnings announcements which are indeed fat-tailed innovations, but it does not take advantage of the information that is encoded in option prices and is readily available to practitioners. The volatility in the GARCH model is driven by squared close-to-close returns, but this is a weak signal in that it ignores everything that happens in-between. With access to intraday data, one can compute realized variance (RV) (Andersen and Bollerslev 1998) and the realized kernel (RK) (Barndorff–Nielsen and Shephard 2002). Andersen *et al.* (2003) argued that the realized volatility could be an accurate measure of the latent volatility process. Hansen *et al.* (2012) further developed a realized GARCH framework that incorporates realized measures of systematic and idiosyncratic volatility. As Cliff *et al.* (2008) pointed out, the overnight returns behave differently from those of regular trading hours, which inspires us to scrutinize overnight volatility and open-to-close volatility individually.

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We aim to improve volatility forecasts by separating intraday realized volatility from overnight volatility and drawing from the option markets to incorporate information on anticipated news shocks. Specifically, our model adds the implied jump magnitude from stock option prices to the overnight volatility. News shocks are important to practitioners: earnings announcements, corporate actions, drug approvals, and road shows are some noteworthy examples. From the perspective of real trading or trade execution algorithms, the information flow from the market open is clearly beneficial to estimate volatility in the trading session.

In this paper, we introduce a new framework in section 2 that combines a GARCH structure for returns with auxiliary market factors. Models under our framework are called coupled GARCH models since underlying open-to-close returns, close-to-close returns, and their variance are modeled jointly. In section 3, we review loss functions and statistical tests for forecast evaluation. In section 4, we describe the data used for the analysis. In sections 5 and 6, we estimate our coupled GARCH(1,1) model on DJIA and comment on forecast accuracy compared to four benchmark models.

2. Coupled GARCH(p,q) model

In this section, we introduce the coupled GARCH model. The key variable of interest is the conditional variance for close-to-open variance $h_{co,t} = \mathrm{var}(r_{co,t}|\mathcal{F}_{t-1})$ and open-to-close variance $h_{oc,t} = \mathrm{var}(r_{oc,t}|\mathcal{F}_{t-1}, P_t^{open})$, where filtration $\mathcal{F}_t$ is generated by daily close-to-close returns up to time t at market close time. $r_{co,t} = \log(P_{t-1}^{close}/P_t^{open})$ is the log return from last close to today's open and $r_{oc,t} = \log(P_t^{close}/P_t^{open})$ is the log return from today's open to today's close. In the classical GARCH(1,1) model, conditional variance is a function of previous return and variance. In the coupled GARCH model, the open-to-close (close-to-open) conditional variance will also depend on previous overnight (trading hour) returns, volatility, and a vector (M_t or N_t) of adapted factors that represent the market impact of trading and information flow. Since market is closed overnight, we denote N_t as the news and information flow after market close and M_t as the market impact of trading and information flow during trading hours. Given the martingale condition, we will assume $\mathbb{E}(r_{co,t}|\mathcal{F}_{t-1}) = \mathbb{E}(r_{oc,t}|\mathcal{F}_{t-1}, P_t^{open}) = 0$. The general formulation of coupled GARCH(p,q) is as follows

$$\begin{aligned}
r_{co,t} &= \sqrt{h_{co,t}}\,\epsilon_{1,t}\\
r_{oc,t} &= \sqrt{h_{oc,t}}\epsilon_{2,t}\\
h_{co,t} &= f_1\left(N_{t-1}, h_{co,t-1}, r_{co,t-1}, h_{oc,t-1}, r_{oc,t-1}, \ldots,\right.\\
&\qquad \left. h_{co,t-p}, r_{co,t-p}, h_{oc,t-p}, r_{oc,t-p}\right)\\
h_{oc,t} &= f_2\left(M_t, h_{co,t}, r_{co,t}, h_{co,t-1}, r_{co,t-1}, h_{oc,t-1}, r_{oc,t-1}, \ldots,\right.\\
&\qquad \left. h_{co,t-q}, r_{co,t-q}, h_{oc,t-q}, r_{oc,t-q}\right) \qquad (1)
\end{aligned}$$

where $\epsilon_{1,t}, \epsilon_{2,t} \sim \mathcal{N}(0,1)$ are i.i.d standard normal variables. We include $h_{co,t}$ and $r_{co,t}$ in f_2 since they are determinant when the market is open. If we take a diagonal f_1 and f_2, this model can be reduced to GARCH(1,1) for r_{oc} and r_{co} separately. Similarly, the GARCH(1,1) model for close-to-close returns is a special case of our framework, as we will show next.

Example 1 Close-to-close return is defined as $r_t = r_{co,t} + r_{oc,t}$ with its conditional variance $h_t \triangleq \mathrm{var}(r_t|\mathcal{F}_{t-1})$. If we set

$$\begin{aligned}
h_{co,t} &= \frac{\omega}{2} + \alpha r_{co,t-1}^2 + \beta h_{co,t-1} + \alpha r_{oc,t-1} r_{co,t-1}\\
h_{oc,t} &= \frac{\omega}{2} + \alpha r_{oc,t-1}^2 + \beta h_{oc,t-1} + \alpha r_{oc,t-1} r_{co,t-1} \qquad (2)
\end{aligned}$$

The close-to-close conditional variance is calculated as

$$\begin{aligned}
h_t &= \mathrm{var}\left((r_{co,t} + r_{oc,t})|\mathcal{F}_{t-1}\right)\\
&= \mathrm{var}\left(r_{co,t}|\mathcal{F}_{t-1}\right) + \mathrm{var}\left(r_{oc,t}|\mathcal{F}_{t-1}\right)\\
&= h_{co,t} + \mathbb{E}\left(r_{oc,t}^2|\mathcal{F}_{t-1}\right)\\
&= h_{co,t} + \mathbb{E}\left(\mathbb{E}\left(r_{oc,t}^2|\mathcal{F}_{t-1}, P_t^{open}\right)|\mathcal{F}_{t-1}\right)\\
&= h_{co,t} + \mathbb{E}\left(h_{oc,t}|\mathcal{F}_{t-1}\right) \qquad (3)
\end{aligned}$$

Since $h_{oc,t}$ is $\mathcal{F}_{t-1}$ measurable,

$$\begin{aligned}
h_t &= h_{co,t} + h_{oc,t}\\
&= \omega + \alpha(r_{co,t-1} + r_{co,t-1})^2 + \beta\left(h_{co,t-1} + h_{oc,t-1}\right)\\
&= \omega + \alpha r_{t-1}^2 + \beta h_{t-1} \qquad (4)
\end{aligned}$$

Equation (4) matches the standard GARCH(1,1) model

$$\begin{aligned}
r_t &= \sigma_t e_t\\
\sigma_t^2 &= \omega + \alpha r_{t-1}^2 + \beta\sigma_{t-1}^2 \qquad (5)
\end{aligned}$$

2.1. *Coupled GARCH(1,1) model with linear specification*

The coupled GARCH model with a simple linear specification is characterized by the following equations

$$\begin{aligned}
r_{co,t} &= \sqrt{h_{co,t}}\,\epsilon_{1,t}\\
r_{oc,t} &= \sqrt{h_{oc,t}}\,\epsilon_{2,t}\\
h_{co,t} &= N_{t-1} + \gamma_1\, h_{oc,t-1} + \gamma_2\, r_{oc,t-1}^2\\
h_{oc,t} &= M_t + \beta_1\, h_{oc,t-1} + \beta_2\, r_{oc,t-1}^2 + \beta_3\, h_{co,t} + \beta_4\, r_{co,t}^2 \qquad (6)
\end{aligned}$$

The γ_1, γ_2, β_3 and β_4 are coupling parameters, while β_1 and β_2 maintain the GARCH(1,1) structure. We assume the linear dependence of conditional variance rather than a log-linear specification since it is more similar to the original GARCH(1,1) model (Bollerslev 1986) and easier to interpret. The auxiliary positive variables M_t and N_t can be decomposed into systematic volatility and idiosyncratic jump risk. For systematic volatility, we will draw on the CBOE Volatility Index, known as VIX, which is a popular measure of the stock market's expectation of short-term volatility implied by S&P 500 index options. As pointed out by Hao and Zhang (2013), VIX can be regarded as the annualized average of the expected daily variance over the following 30 calendar days under the risk-neutral measure(Q).

$$\left(\frac{\mathrm{VIX}_t}{100}\right)^2 = \mathbb{E}_t^Q\left[\frac{1}{\tau}\int_t^{t+\tau} h_s\, \mathrm{d}s\right]$$

$$\approx \frac{1}{\tau}\sum_{s=1}^{\tau}\mathbb{E}_t^Q\left[h_{t+s}\right] \tag{7}$$

where $\tau = 30$ calendar days and h_s is the instantaneous annualized variance. For the idiosyncratic jump risk, we will draw from information in the option markets, as we will explain below.

2.1.1. Stationarity and ergodicity. The properties of stationarity and ergodicity are important features in the time series analysis. The structure of the coupled GARCH(1,1) model is very similar to that of the generalized autoregressive processes (Bougerol and Picard 1992, Liu 2006). To explore these properties, we first write equation (6) in the matrix form $z_t = A_t z_{t-1} + b_t$ where z_t, A_t, and b_t are defined as follows

$$\begin{aligned}
z_t &= \left(r^2_{co,t}, r^2_{oc,t}, h_{co,t}, h_{oc,t}\right)^\mathsf{T}\\
b_t &= \left(N_{t-1}\epsilon^2_{1,t}, \left(M_t + \left(\beta_3+\beta_4\epsilon^2_{1,t}\right)N_{t-1}\right)\epsilon^2_{2,t}, N_{t-1}, M_t\right.\\
&\quad \left. + \left(\beta_3+\beta_4\epsilon^2_{1,t}\right)N_{t-1}\right)^\mathsf{T}\\
A_t &= \begin{bmatrix}
0 & \gamma_2\epsilon^2_{1,t} & 0 & \gamma_1\epsilon^2_{1,t}\\
0 & \left(\left(\beta_3+\beta_4\epsilon^2_{1,t}\right)\gamma_2+\beta_2\right)\epsilon^2_{2,t} & 0 & \left(\left(\beta_3+\beta_4\epsilon^2_{1,t}\right)\gamma_1+\beta_1\right)\epsilon^2_{2,t}\\
0 & \gamma_2 & 0 & \gamma_1\\
0 & \left(\beta_3+\beta_4\epsilon^2_{1,t}\right)\gamma_2+\beta_2 & 0 & \left(\beta_3+\beta_4\epsilon^2_{1,t}\right)\gamma_1+\beta_1
\end{bmatrix}
\end{aligned} \tag{8}$$

The following assumption defines the relation between the auxiliary processes and the noise terms in the coupled GARCH(1,1) process.

Assumption 2.1 *The innovation processes* $\{\epsilon_{1,t}\}$ *and* $\{\epsilon_{2,t}\}$ *are independent identically distributed variables with mean* 0 *and variance* 1*, the exogenous processes* N_t *and* M_t *are positive, stationary, ergodic and independent of* $\{\epsilon_{1,t}\}$ *and* $\{\epsilon_{2,t}\}$.

Lemma 2.2 (Bougerol and Picard 1992, Straumann and Mikosch 2006) *Let* $\{A_t, t\in\mathbb{Z}\}$ *be a strictly stationary and ergodic sequence of random matrices such that* $\mathbb{E}\log^+\|A_t\|$ *and* $\mathbb{E}\log^+\|b_t\|$ *is finite, we have*

$$\begin{aligned}
\gamma &= \inf_{t\in\mathbb{N}}\frac{1}{t}\mathbb{E}\left(\log\|A_tA_{t-1}\ldots A_1\|\right)\\
&= \lim_{t\to\infty}\frac{1}{t}\mathbb{E}\left(\log\|A_tA_{t-1}\ldots A_1\|\right)
\end{aligned} \tag{9}$$

Thus a necessary and sufficient condition for the existence of a strictly stationary solution to the model $z_t = A_t z_{t-1} + b_t$ *is that*

$$\gamma < 0 \tag{10}$$

Proposition 2.3 *Under Assumption* 2.1*, the coupled GARCH (1,1) model given by* (6) *admits a unique stationary and ergodic solution if and only if the top Lyapunov exponent of the sequence* $\{A_t, t\in\mathbb{Z}\}$ *is negative.*

Proof Considering the norm defined by $\|A\| = \sum|a_{ij}|$, this norm is absolutely multiplicative $\|AB\| \le \|A\|\|B\|$. Since the innovation processes $\{\epsilon_{1,t}\}$ and $\{\epsilon_{2,t}\}$ have a finite variance, the elements of the matrix A_t are integrable and we conclude that $\mathbb{E}\log^+\|A_t\| \le \mathbb{E}\|A_t\| < \infty$ and $\mathbb{E}\log^+\|B_t\| < \infty$ ∎

Proposition 2.4 *The top Lyapunov exponent* γ *of the sequence* A_t *for the coupled GARCH(1,1) model defined in equation* (8) *is*

$$\gamma = \mathbb{E}\left(\log\left[\left(\beta_3+\beta_4\epsilon^2_{1,k}\right)\left(\gamma_1+\gamma_2\epsilon^2_{2,k-1}\right)+\beta_1+\beta_2\epsilon^2_{2,k-1}\right]\right) \tag{11}$$

Proof A_t could be decomposed as $A_t = p_tq_t^\mathsf{T} + u_tv_t^\mathsf{T}$ by four vectors p_t, q_t, u_t and v_t defined as

$$\begin{aligned}
p_t &= \left(\epsilon^2_{1,t}, 0, 1, 0\right)^\mathsf{T}\\
q_t &= (0, \gamma_2, 0, \gamma_1, 0)^\mathsf{T}\\
u_t &= \left(0, \epsilon^2_{2,t}, 0, 1\right)^\mathsf{T}\\
v_t &= \left(0, \left(\beta_3+\beta_4\epsilon^2_{1,t}\right)\gamma_2+\beta_2, 0, \left(\beta_3+\beta_4\epsilon^2_{1,t}\right)\gamma_1+\beta_1\right)^\mathsf{T}
\end{aligned} \tag{12}$$

Since $q_i^\mathsf{T}p_j = 0$ and $v_i^\mathsf{T}p_j = 0$ for any pair (i,j),

$$\begin{aligned}
\|A_tA_{t-1}\ldots A_1\| &= \left\|\prod_{k=1}^{t}\left(p_kq_k^\mathsf{T}+u_kv_k^\mathsf{T}\right)\right\|\\
&= \left\|A_t\prod_{k=1}^{t-1}\left(p_kq_k^\mathsf{T}+u_kv_k^\mathsf{T}\right)\right\|\\
&= \left\|A_t\prod_{k=1}^{t-1}u_kv_k^\mathsf{T}\right\|
\end{aligned} \tag{13}$$

Furthermore,

$$\begin{aligned}
&\mathbb{E}\left(\log\|A_tA_{t-1}\ldots A_1\|\right)\\
&= \mathbb{E}\left(\log\|A_t\prod_{k=1}^{t-1}u_kv_k^\mathsf{T}\|\right)\\
&= \mathbb{E}\left(\log\|A_tu_{t-1}\left(\prod_{k=2}^{t-1}v_k^\mathsf{T}u_{k-1}\right)v_1^\mathsf{T}\|\right)\\
&= \mathbb{E}\left(\log\left|\prod_{k=2}^{t-1}v_k^\mathsf{T}u_{k-1}\right|\|A_tu_{t-1}v_1^\mathsf{T}\|\right)\\
&= \mathbb{E}\left(\log\left(\prod_{k=2}^{t-1}\left[\left(\beta_3+\beta_4\epsilon^2_{1,k}\right)\left(\gamma_1+\gamma_2\epsilon^2_{2,k-1}\right)\right.\right.\right.\\
&\quad \left.\left.\left.+\beta_1+\beta_2\epsilon^2_{2,k-1}\right]\right)\|A_tu_{t-1}v_1^\mathsf{T}\|\right)\\
&= \sum_{k=2}^{t-1}\mathbb{E}\log\left[\left(\beta_3+\beta_4\epsilon^2_{1,k}\right)\left(\gamma_1+\gamma_2\epsilon^2_{2,k-1}\right)\right.\\
&\quad \left.+\beta_1+\beta_2\epsilon^2_{2,k-1}\right]+\mathbb{E}\log\|A_tu_{t-1}v_1^\mathsf{T}\|
\end{aligned} \tag{14}$$

Finally, taking the limit we find that

$$\gamma = \lim_{t\to\infty}\frac{1}{t}\mathbb{E}\left(\log\|A_tA_{t-1}\ldots A_1\|\right)$$

$$= \mathbb{E}\left(\log\left[\left(\beta_3 + \beta_4\epsilon_{1,k}^2\right)\left(\gamma_1 + \gamma_2\epsilon_{2,k-1}^2\right) + \beta_1 + \beta_2\epsilon_{2,k-1}^2\right]\right) \tag{15}$$

■

PROPOSITION 2.5 *The necessary and sufficient condition for the existence of a strictly stationary solution of the coupled GARCH*(1,1) *model is* $(\beta_3 + \beta_4)(\gamma_1 + \gamma_2) + \beta_1 + \beta_2 < 1$.

Proof Necessity: according to Jensen's inequality,

$$\begin{aligned}
\gamma &= \mathbb{E}\left(\log\left[\left(\beta_3 + \beta_4\epsilon_{1,k}^2\right)\left(\gamma_1 + \gamma_2\epsilon_{2,k-1}^2\right) + \beta_1 + \beta_2\epsilon_{2,k-1}^2\right]\right)\\
&\le \log\mathbb{E}\left(\left[\left(\beta_3 + \beta_4\epsilon_{1,k}^2\right)\left(\gamma_1 + \gamma_2\epsilon_{2,k-1}^2\right) + \beta_1 + \beta_2\epsilon_{2,k-1}^2\right]\right)\\
&= \log\left(\left[\left(\beta_3 + \beta_4\right)\left(\gamma_1 + \gamma_2\right) + \beta_1 + \beta_2\right]\right)\\
&\le \log(1) = 0
\end{aligned} \tag{16}$$

So its top Lyapunov exponent is negative, which guarantees a stationary solution. To prove sufficiency, we note that if there is a strictly stationary solution,

$$\begin{aligned}
\mathbb{E}\left(h_{oc,t}\right) &= \frac{\mathbb{E}\left(M_t\right) + (\beta_3 + \beta_4)\mathbb{E}\left(N_{t-1}\right)}{1 - \left[\left(\beta_3 + \beta_4\right)\left(\gamma_1 + \gamma_2\right) + \beta_1 + \beta_2\right]} > 0\\
&\Rightarrow \left(\beta_3 + \beta_4\right)\left(\gamma_1 + \gamma_2\right) + \beta_1 + \beta_2 < 1
\end{aligned} \tag{17}$$

■

2.1.2. One-day jump risk. In the Black-Scholes framework, the volatility is a constant; of course, this is not the case in real option markets. The volatility surface in markets captures the information about anticipated news shocks and stochastic effects. To consider the effect of news shocks, we will use the volatility surface to estimate a model that distinguishes a time-invariant base volatility from a one-time jump volatility. We can estimate the jump volatility $\sigma^2_{\text{jump},t,K}$ by comparing the two options with the same strike K and two nearest expiration τ_1 and τ_2 ($\tau_1 < \tau_2$). The average jump volatility $\sigma^2_{\text{jump},t}$ is defined as the $\sigma^2_{\text{jump},t,K}$ weighted by the inverse of the bid–ask across the option montage.

If a jump is expected to occur before τ_1, a positive jump volatility is embedded in options with expiration τ_1 besides the base implied volatility $\sigma^2_{\text{base},K}$:

$$\begin{aligned}
\sigma^2_{IV,t,\tau_1,K} &= \sigma^2_{\text{jump},t,K} + \sigma^2_{\text{base},K}\cdot\tau_1\\
\sigma^2_{IV,t,\tau_2,K} &= \sigma^2_{\text{jump},t,K} + \sigma^2_{\text{base},K}\cdot\tau_2
\end{aligned} \tag{18}$$

where implied volatility $\sigma^2_{IV,t,\tau_i,K}$ is derived by midquote. we calculate the implied jump volatility by solving these linear equations,

$$\sigma^2_{\text{jump},t,K} = \frac{\tau_2\sigma^2_{IV,t,\tau_1,K} - \tau_1\sigma^2_{IV,t,\tau_2,K}}{\tau_2 - \tau_1} \tag{19}$$

However, if a jump is expected between τ_1 and τ_2, the solution might be negative since our focus is on the on-day volatility prediction, we will truncate the jump variance to zero in this case. We take the inverse spreads as weights (Grover and Thomas 2012) and calculate the weighted jump risk as

$$\sigma^2_{\text{jump},t} = \frac{\sum_i s_i^{-1}\max(\sigma^2_{\text{jump},t,K_i}, 0)}{\sum_{i=1} s_i^{-1}} \quad \text{where } s_i = ask_i - bid_i \tag{20}$$

There are two cases to consider with regard to the timing of a jump. In the case where an event is expected but the precise date is unknown (for example, drug testing results and approvals in the bio-pharmaceutical industry), it is reasonable to distribute this jump risk evenly for the remaining days up to the nearest option expiration date $OD(t) = t + \tau_1$. However, in some cases, the date of the event is known. Examples include earnings announcements, elections, FOMC meetings, etc. In these cases, the jump variance collapses on that day.

In the remainder of this study, we will focus on earning announcements: the adjusted jump risk could be defined as

$$\hat{\sigma}^2_{\text{jump},t} \triangleq \begin{cases}
\sigma^2_{\text{jump},t} & ED(T) = t \text{ and } ET(t) \text{ is after hours}\\
\sigma^2_{\text{jump},t} & ED(T) = t+1 \\
& \text{and } ET(t) \text{ is pre} - \text{market}\\
\dfrac{\sigma^2_{\text{jump},t}}{OD(t) - t} & \text{otherwise}
\end{cases} \tag{21}$$

where $OD(t)$ is the nearest option expiration date τ_1 with next earning announcement date $ED(t)$ and earning release time $ET(t)$. Earnings announcements usually happen pre-market or after hours and provide a large contribution to overnight volatility $h_{co,t}$. This overnight volatility effect in turn can spawn continuing volatility into the next trading day through the coupling coefficients (β_3 and β_4). To obtain a parsimonious model, we only incorporate this jump risk in the N_t. Combining the above points, we have specified a coupled GARCH(1,1) model described by the following four equations.

$$\begin{aligned}
r_{co,t} &= \sqrt{h_{co,t}}\,\epsilon_{1,t}\\
r_{oc,t} &= \sqrt{h_{oc,t}}\,\epsilon_{2,t}\\
h_{co,t} &= \omega_{co,1}\overline{\text{VIX}}_{t-1,close} + \omega_{co,2} + \gamma_1\, h_{oc,t-1} + \gamma_2\, r^2_{oc,t-1}\\
&\quad + \gamma_3\,\hat{\sigma}^2_{\text{jump},t-1,c}\\
h_{oc,t} &= \omega_{oc,1}\overline{\text{VIX}}_{t,open} + \omega_{oc,2} + \beta_1\, h_{oc,t-1} + \beta_2\, r^2_{oc,t-1}\\
&\quad + \beta_3\, h_{co,t} + \beta_4\, r^2_{co,t}
\end{aligned} \tag{22}$$

where adjusted $\overline{\text{VIX}}_t$ is defined as

$$\overline{\text{VIX}}_t = \left(\frac{\text{VIX}_t}{100}\right)^2 \tag{23}$$

2.2. Estimation

In this section, we discuss the maximum likelihood estimator for the coupled GARCH model. For the purpose of estimation, we adopt a Gaussian specification,† so that the likelihood function is given by

$$
\begin{aligned}
& f\left(r_{oc,t}, r_{co,t}, r_{oc,t-1}, r_{co,t-1}, \ldots, r_{oc,1}, r_{co,1}\right) \\
&\quad = f\left(r_{oc,t} | r_{co,t}, r_{oc,t-1}, r_{co,t-1}, \ldots, r_{oc,1}, r_{co,1}\right) \\
&\qquad \times f\left(r_{co,t}, r_{oc,t-1}, r_{co,t-1}, \ldots, r_{oc,1}, r_{co,1}\right) \\
&\quad = f\left(\sqrt{h_{oc,t}}\,\epsilon_{oc,t} | r_{co,t}, r_{oc,t-1}, r_{co,t-1}, \ldots, r_{oc,1}, r_{co,1}\right) \\
&\qquad \times f\left(r_{co,t}, r_{oc,t-1}, r_{co,t-1}, \ldots, r_{oc,1}, r_{co,1}\right) \\
&\quad = \frac{1}{\sqrt{2\pi h_{oc,t}}} \exp\left(-\frac{r_{oc,t}^2}{2h_{oc,t}}\right) \\
&\qquad \times f\left(r_{co,t}, r_{oc,t-1}, r_{co,t-1}, \ldots, r_{oc,1}, r_{co,1}\right) \\
&\quad = \prod_{k=1}^{t} \frac{1}{\sqrt{2\pi h_{oc,k}}} \exp\left(-\frac{r_{oc,k}^2}{2h_{oc,k}}\right) \frac{1}{\sqrt{2\pi h_{co,k}}} \exp\left(-\frac{r_{co,k}^2}{2h_{co,k}}\right)
\end{aligned} \tag{24}
$$

Denote by $\theta = (\theta_1, \ldots, \theta_{11})^\intercal$ the coupled GARCH(1,1) model parameters

$$
\begin{aligned}
\theta &= (\theta_1, \ldots, \theta_{11})^\intercal \\
&= \left(\omega_{oc,1}, \omega_{oc,2}, \omega_{co,1}, \omega_{co,2}, \beta_1, \beta_2, \beta_3, \beta_4, \gamma_1, \gamma_2, \gamma_3\right)^\intercal
\end{aligned} \tag{25}
$$

The log joint likelihood function is

$$
\begin{aligned}
L_t\left(\{r_{oc,k}, r_{co,k}\}_{k=1,\ldots,t}, \theta\right) &= \sum_{k=1}^{t} \left(-\frac{1}{2}\log(h_{oc,k}) - \frac{r_{oc,k}^2}{2h_{oc,k}}\right. \\
&\qquad \left. - \frac{1}{2}\log(h_{co,k}) - \frac{r_{co,k}^2}{2h_{co,k}}\right) \\
&\propto \log f\left(r_{oc,t}, r_{co,t}, r_{oc,t-1},\right. \\
&\qquad \left. \times\ r_{co,t-1}, \ldots, r_{oc,1}, r_{co,1}\right)
\end{aligned} \tag{26}
$$

2.3. QMLE inference

In this section, we discuss the asymptotic properties of the quasi-maximum likelihood estimator for the coupled GARCH(1,1) model, similar to the structure analyzed in Straumann and Mikosch (2006) and Nana *et al.* (2013). The joint likelihood in the coupled GARCH model can be decomposed into the sum of two univariate GARCH-extended models. A QMLE of θ is defined as any measurable solution of $\hat{\theta}_n$

$$
\hat{\theta}_n = \underset{\theta \in \Theta}{\operatorname{argmax}}\, L_n(\theta) \tag{27}
$$

where Θ is the parameter space

$$
\Theta \subset (0,\infty) \times [0,\infty) \times (0,\infty) \times [0,\infty) \times [0,\infty)^7 \tag{28}
$$

† We set the mean to zero for simplicity, this implies a slight violation of the Martingale condition. For $h = 0.01$, the effective drift would be $\frac{1}{2h} = 0.00005$

Maximizing the log likelihood $L_n(\theta)$ is equivalent to minimizing

$$
I_n(\theta) = n^{-1} \sum_{k=1}^{n} l_k(\theta),
$$

$$
\text{where } l_k(\theta) = \log(h_{oc,k}) + \frac{r_{oc,k}^2}{h_{oc,k}} + \log(h_{co,k}) + \frac{r_{co,k}^2}{h_{oc,k}} \tag{29}
$$

Thus QMLE is the solution to the equation

$$
\hat{\theta}_n = \underset{\theta \in \Theta}{\operatorname{argmin}}\, I_n(\theta) \tag{30}
$$

Assuming the true values of θ is θ_0

$$
\theta_0 = \left(\omega_{oc,1}, \omega_{oc,2}, \omega_{co,1}, \omega_{co,2}, \beta_1, \beta_2, \beta_3, \beta_4, \gamma_1, \gamma_2, \gamma_3\right)^\intercal \tag{31}
$$

We conjecture that standard QMLE asymptotic normality of QMLE (Francq and Zakoïan 2012, Straumann and Mikosch 2006) can be applied to this framework, so that

$$
\sqrt{n}\left(\hat{\theta}_n - \theta_0\right) \xrightarrow{\mathcal{L}} \mathcal{N}(0, \mathcal{I}_{\theta_0}^{-1} \mathcal{J}_{\theta_0} \mathcal{I}_{\theta_0}^{-1}) \tag{32}
$$

where

$$
\mathcal{J}_\theta = \operatorname{Var}_\theta\left[\frac{\partial l_k(\theta)}{\partial \theta}\right] \quad \text{and} \quad \mathcal{I}_\theta = \mathbb{E}_\theta\left[\frac{\partial^2 l_k(\theta_0)}{\partial\theta\partial\theta^\intercal}\right] \tag{33}
$$

A study of convergence conditions lies beyond the scope of this work.

PROPOSITION 2.6 *The asymptotic normality could be simplified as*

$$
\sqrt{n}\left(\hat{\theta}_n - \theta_0\right) \xrightarrow{\mathcal{L}} \mathcal{N}(0, (\kappa - 1)\mathcal{I}_{\theta_0}^{-1}) \quad \text{where } \kappa = \mathbb{E}\left(\epsilon_{1,t}^4\right) \tag{34}
$$

Proof The expectation of the first-order derivative is given by

$$
\begin{aligned}
\mathbb{E}_{\theta_0}\left[\frac{\partial l_k(\theta_0)}{\partial\theta}\right] &= \mathbb{E}_{\theta_0}\left[(1 - \epsilon_{1,k}^2)\frac{1}{h_{co,k}}\frac{\partial h_{co,k}}{\partial\theta}\right. \\
&\qquad \left. + (1 - \epsilon_{2,k}^2)\frac{1}{h_{oc,k}}\frac{\partial h_{oc,k}}{\partial\theta}\right] \\
&= \mathbb{E}_{\theta_0}\left[(1 - \epsilon_{1,k}^2)\right]\mathbb{E}_{\theta_0}\left[\frac{1}{h_{co,k}}\frac{\partial h_{co,k}}{\partial\theta}\right] \\
&\qquad + \mathbb{E}_{\theta_0}\left[(1 - \epsilon_{2,k}^2)\right]\mathbb{E}_{\theta_0}\left[\frac{1}{h_{oc,k}}\frac{\partial h_{oc,k}}{\partial\theta}\right] = 0
\end{aligned} \tag{35}
$$

Its variance is given by

$$
\begin{aligned}
\mathcal{J}_{\theta_0} &= \operatorname{Var}_{\theta_0}\left[\frac{\partial l_k(\theta_0)}{\partial\theta}\right] \\
&= \mathbb{E}_{\theta_0}\left[\left(\frac{(1 - \epsilon_{1,k}^2)}{h_{co,k}}\frac{\partial h_{co,k}}{\partial\theta} + \frac{(1 - \epsilon_{2,k}^2)}{h_{oc,k}}\frac{\partial h_{oc,k}}{\partial\theta}\right)\right.
\end{aligned}
$$

$$\times \left(\frac{(1-\epsilon_{1,k}^2)}{h_{co,k}} \frac{\partial h_{co,k}}{\partial \theta} + \frac{(1-\epsilon_{2,k}^2)}{h_{oc,k}} \frac{\partial h_{oc,k}}{\partial \theta} \right)^{\intercal} \Bigg]$$

$$= \mathbb{E}_{\theta_0} \left[\frac{(1-\epsilon_{1,k}^2)^2}{h_{co,k}^2} \frac{\partial h_{co,k}}{\partial \theta} \left(\frac{\partial h_{co,k}}{\partial \theta} \right)^{\intercal} \right]$$

$$+ \mathbb{E}_{\theta_0} \left[\frac{(1-\epsilon_{2,k}^2)^2}{h_{oc,k}^2} \frac{\partial h_{oc,k}}{\partial \theta} \left(\frac{\partial h_{oc,k}}{\partial \theta} \right)^{\intercal} \right]$$

$$+ 2\mathbb{E}_{\theta_0} \left[\frac{(1-\epsilon_{1,k}^2)(1-\epsilon_{2,k}^2)}{h_{co,k}h_{oc,k}} \frac{\partial h_{oc,k}}{\partial \theta} \left(\frac{\partial h_{co,k}}{\partial \theta} \right)^{\intercal} \right]$$

$$= (\kappa - 1)\mathbb{E}_{\theta_0} \left[\frac{1}{h_{co,k}^2} \frac{\partial h_{co,k}}{\partial \theta} \left(\frac{\partial h_{co,k}}{\partial \theta} \right)^{\intercal} + \frac{1}{h_{oc,k}^2} \frac{\partial h_{oc,k}}{\partial \theta} \left(\frac{\partial h_{oc,k}}{\partial \theta} \right)^{\intercal} \right] \tag{36}$$

Furthermore, the Fisher information matrix is given by

$$\mathcal{I}_{\theta_0}^{-1} = -\mathbb{E}_{\theta_0} \left[\frac{\partial^2 l_k(\theta_0)}{\partial \theta \partial \theta^{\intercal}} \right]$$

$$= \mathbb{E}_{\theta_0} \left[(1-\epsilon_{1,k}^2) \left(\frac{1}{h_{co,k}} \frac{\partial^2 h_{co,k}}{\partial \theta \partial \theta^{\intercal}} - \frac{1}{h_{co,k}^2} \frac{\partial h_{co,k}}{\partial \theta} \frac{\partial h_{co,k}}{\partial \theta^{\intercal}} \right) \right]$$

$$+ \mathbb{E}_{\theta_0} \left[\frac{\epsilon_{1,k}^2}{h_{co,k}^2} \frac{\partial h_{co,k}}{\partial \theta} \frac{\partial h_{co,k}}{\partial \theta^{\intercal}} \right]$$

$$+ \mathbb{E}_{\theta_0} \left[(1-\epsilon_{2,k}^2) \left(\frac{1}{h_{oc,k}} \frac{\partial^2 h_{oc,k}}{\partial \theta \partial \theta^{\intercal}} - \frac{1}{h_{oc,k}^2} \frac{\partial h_{oc,k}}{\partial \theta} \frac{\partial h_{oc,k}}{\partial \theta^{\intercal}} \right) \right]$$

$$+ \mathbb{E}_{\theta_0} \left[\frac{\epsilon_{2,k}^2}{h_{oc,k}^2} \frac{\partial h_{oc,k}}{\partial \theta} \frac{\partial h_{oc,k}}{\partial \theta^{\intercal}} \right]$$

$$= \mathbb{E}_{\theta_0} \left[\frac{1}{h_{co,k}^2} \frac{\partial h_{co,k}}{\partial \theta} \left(\frac{\partial h_{co,k}}{\partial \theta^{\intercal}} \right) + \frac{1}{h_{oc,k}^2} \frac{\partial h_{oc,k}}{\partial \theta} \left(\frac{\partial h_{oc,k}}{\partial \theta^{\intercal}} \right) \right] \tag{37}$$

combining these matrices, we find that

$$\mathcal{I}_{\theta_0}^{-1} \mathcal{J}_{\theta_0} \mathcal{I}_{\theta_0}^{-1} = (\kappa - 1)\mathcal{I}_{\theta_0}^{-1} \quad \text{if } \mathcal{I}_{\theta_0} \text{ is invertible.} \tag{38}$$

■

2.4. *Out of sample prediction*

The coupled GARCH model can be used to predict both the conditional open-to-close and close-to-open return variance. One of the main advantages of having coupled terms is that rolling forecasts are feasible, i.e. overnight information that becomes available at the open helps us better understand the trading day variance. We denote $\mathcal{F}_t$ as information filtration up to the market close at time t

$$\mathcal{F}_t = \{h_{co,0}, r_{co,0}, h_{oc,0}, r_{oc,0} \ldots, h_{co,t}, r_{co,t}, h_{oc,t}, r_{oc,t}\} \tag{39}$$

2.4.1. At the close. Similar to GARCH(1,1) model, the variance updating mechanism for $h_{co,t+1}$ is fully $\mathcal{F}_t$ measurable and its forecast is

$$\begin{aligned} \mathbb{E}\left[r_{co,t+1}^2 | \mathcal{F}_t\right] &= \mathbb{E}\left[h_{co,t+1} | \mathcal{F}_t\right] \\ &= \omega_{co,1} \overline{\text{VIX}}_{t,close} + \omega_{co,2} + \gamma_1 h_{oc,t} \\ &\quad + \gamma_2 r_{oc,t}^2 + \gamma_3 \hat{\sigma}_{\text{jump},t}^2 \end{aligned} \tag{40}$$

The expected next trading day variance is

$$\begin{aligned} \mathbb{E}\left[r_{oc,t+1}^2 | \mathcal{F}_t\right] &= \mathbb{E}\left[h_{oc,t+1} | \mathcal{F}_t\right] \\ &= \omega_{oc,1} \mathbb{E}\left[\overline{\text{VIX}}_{t+1,open} | \mathcal{F}_t\right] + \omega_{oc,2} \\ &\quad + \beta_1 h_{oc,t} + \beta_2 r_{oc,t}^2 + (\beta_3 + \beta_4)\mathbb{E}\left[h_{co,t+1} | \mathcal{F}_t\right] \\ &\approxeq \omega_{oc,1} \overline{\text{VIX}}_{t,close} + \omega_{oc,2} + \beta_1 h_{oc,t} \\ &\quad + \beta_2 r_{oc,t}^2 + (\beta_3 + \beta_4)\mathbb{E}\left[h_{co,t+1} | \mathcal{F}_t\right] \\ &= \omega_{oc,2} + (\beta_3 + \beta_4)\omega_{co,2} \\ &\quad + \left(\omega_{oc,1} + (\beta_3 + \beta_4)\omega_{co,1}\right) \overline{\text{VIX}}_{t,close} \\ &\quad + (\beta_1 + (\beta_3 + \beta_4)\gamma_1)\, h_{oc,t} \\ &\quad + (\beta_2 + (\beta_3 + \beta_4)\gamma_2)\, r_{oc,t}^2 \\ &\quad + (\beta_3 + \beta_4)\gamma_3 \hat{\sigma}_{\text{jump},t}^2 \end{aligned} \tag{41}$$

Here the intuition is to approximate $\mathbb{E}[\overline{\text{VIX}}_{t+1,open} | \mathcal{F}_t]$ by $\overline{\text{VIX}}_{t,close}$. To test this intuition, we ran a Mincer-Zarnowitz regression of $\text{VIX}_{t+1,open}$ on $\text{VIX}_{t,close}$ from 01/01/2008 to 12/31/2018.

$$\text{VIX}_{t+1,open} = \underset{(0.002)}{0.997} * \text{VIX}_{t,close} + \underset{(0.043)}{0.1593} \tag{42}$$

The p-value for the null hypothesis $H_0 : \beta = 1$ is 0.149, which isn't statistically significant.

2.4.2. Close-to-close prediction. Given the assumption that $r_{oc,t}$ and $r_{co,t}$ are independent and have zero means, we predict the conditional variance of close-to-close return r_t by combining each component variance

$$\begin{aligned} \mathbb{E}\left[r_{t+1}^2 | \mathcal{F}_t\right] &= \mathbb{E}\left[(r_{co,t+1} + r_{oc,t+1})^2 | \mathcal{F}_t\right] \\ &= \mathbb{E}\left[r_{co,t+1}^2 | \mathcal{F}_t\right] + \mathbb{E}\left[r_{oc,t+1}^2 | \mathcal{F}_t\right] \\ &= \mathbb{E}\left[h_{co,t+1} | \mathcal{F}_t\right] + \mathbb{E}\left[h_{oc,t+1} | \mathcal{F}_t\right] \end{aligned} \tag{43}$$

2.4.3. At the open. When the market opens at 9:30 AM in the morning, the market open prices for VIX and the target symbol are public information. The revised open-to-close variance prediction is

$$\begin{aligned} &\mathbb{E}\left[r_{oc,t+1}^2 | \mathcal{F}_t, open\right] \\ &\quad = \mathbb{E}\left[h_{oc,t+1} | \mathcal{F}_t, open\right] \\ &\quad = \omega_{oc,1} \mathbb{E}\left[\overline{\text{VIX}}_{t+1,open} | \mathcal{F}_t, open\right] + \omega_{oc,2} + \beta_1 h_{oc,t} \\ &\qquad + \beta_2 r_{oc,t}^2 + \beta_3 \mathbb{E}\left[h_{co,t+1} | \mathcal{F}_t, open\right] + \beta_4 r_{co,t+1}^2 \end{aligned}$$

$$= \omega_{oc,2} + \beta_3\omega_{co,2} + \omega_{oc,1}\overline{\mathrm{VIX}}_{t+1,open} + \beta_3\omega_{co,1}\overline{\mathrm{VIX}}_{t,close} \\ + (\beta_1 + \beta_3\gamma_1)\, h_{oc,t} \\ + (\beta_2 + \beta_3\gamma_2)\, r^2_{oc,t} + \beta_3\gamma_3\hat{\sigma}^2_{\mathrm{jump},t,c} + \beta_4 r^2_{co,t+1} \tag{44}$$

3. Forecast evaluation

3.1. MSE and QLIKE

Given a forecast for volatility and a measure of realized volatility, it is important to evaluate the goodness of the forecast directly, in addition to the likelihood. There is no unique criterion for selecting the best model; we will compare the models in terms of the mean squared error and QLIKE (Bollerslev *et al.* 1994, Patton 2011),

$$\mathrm{MSE} = \frac{1}{T}\sum_{t=1}^{T}\left(\hat{h}_t^2 - \sigma_t^2\right)^2$$

$$\text{and} \quad \mathrm{QLIKE} = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{\sigma_t^2}{\hat{h}_t^2} + \log \hat{h}_t^2\right) \tag{45}$$

where $\hat{h}_t$ is the daily (intraday) volatility forecast and σ_t is the daily (intraday) volatility. Since daily volatility σ_t is a latent variable, we substitute the realized variance (Andersen and Bollerslev 1998). The realized variance for a specific day is computed with a 5-minute sampling frequency. The log price $p_t = \log(S_t)$ is given by $p_t = p_0 + \int_0^t \sigma_s \,\mathrm{d}B_s$. The RV converges to IV in probability when $n \to \infty$

$$IV = \int_0^t \sigma_s^2 \,\mathrm{d}s \quad \text{and} \quad RV_{t+1} = \sum_{i=1}^{N}\left(p_{t+\frac{i}{N}} - p_{t+\frac{(i-1)}{N}}\right)^2 \tag{46}$$

Obviously, stocks are not traded 24 hours a day due to the market closing overnight and over weekends, we can only observe intraday returns during trading hours. Since RV_t defined above only captures the volatility when the market is open, we need to extend it using the estimator

$$\hat{\sigma}_t^2 = \hat{C}\cdot RV_t = \frac{\sum_{k=1}^{n}(r_t - \bar{r}_t)^2}{\sum_{k=1}^{n} RV_k}\cdot RV_t \tag{47}$$

This estimator is approximately unbiased for σ_t^2 under fairly reasonable assumptions (Martens 2002, Hansen and Lunde 2005). An illustration in figure 1 shows a demonstration of the scaled RV and the variation in daily close-to-close returns from the period of 2011 to 2018 for Pfizer (PFE).

3.2. Realized utility

Besides statistical analysis, investors also care about the economic value of volatility forecasting. We use the realized utility of Bollerslev *et al.* (2018) to evaluate competing models. The average expected utility per unit of wealth UoW^θ is defined as

$$UoW^\theta = \frac{1}{T}\sum_{t=1}^{T}\left(8\%\frac{\sqrt{RV_{t+1}}}{\sqrt{\mathbb{E}_t^\theta\,(RV_{t+1})}} - 4\%\frac{RV_{t+1}}{\mathbb{E}_t^\theta\,(RV_{t+1})}\right) \tag{48}$$

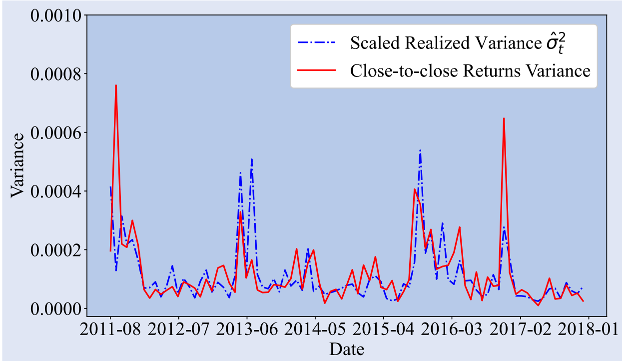

Figure 1. The scaled RV versus real close-to-close returns (PFE) variance from 2011 to 2018. The close-to-close returns variance is the variance of the returns in the past 15 days. Scaled RV is the mean of the past 15 days with scale factor $\hat{C} = 1.08$.

A perfect volatility model would deliver a realized utility of $8\% - 4\% = 4\%$, which indicates that the utility of trading this equity worth 4% of wealth.

3.3. DM t-test

In addition to evaluating and comparing loss functions, we employ Diebold-Mariano (Diebold and Mariano 2002, Diebold 2015) t-statistics to test the significance of the average utility gains and forecast improvements relative to the Coupled GARCH(1,1) model. A positive (negative) DM t-test statistic indicates that the competing model outperforms (underperforms) the Coupled GARCH(1,1) model in terms of both in-sample and out-of-sample realized utility and volatility forecasts.

4. Empirical analysis

In this section, we present empirical results using the VIX index and returns and options data for 29 stocks in DJIA (the ticker DWDP is excluded due to its short trading history). Our sample spans the period from July 1, 2011, to December 31, 2018, which we divide into an in-sample period from July 1, 2011, to December 31, 2017, and an out-of-sample period covering 2018. We adopt 5-min realized variance as the open-to-close variance and scaled realized variance as the close-to-close variance.

4.1. Data description

The following data are required to estimate and update our coupled GARCH model:

- daily option implied one-day-jump $\sigma^2_{\mathrm{jump},t,c}$ with near option expiration date $OD(t)$ for each ticker in DJIA
- next earning announcement date $ED(t)$ and time $ET(t)$ for each ticker in DJIA
- daily VIX open and close priced from 07/01/2011 ––12/31/2018

- 5-min tick data for each DJIA stock from 07/01/2011−−12/31/2018
- daily open and close prices for each DJIA stock from 07/01/2011−−12/31/2018

5. Benchmark models

5.1. Realized GARCH(1,2)

We compare our results to the standard GARCH(1,1) model and to a realized GARCH(1,2) model with a log-linear specification as suggested by Hansen *et al.* (2012). The general realized GARCH(p,q) model takes the form

$$\begin{aligned} r_t &= h_t z_t, z_t \sim i.i.d.\mathcal{N}(0,1) \\ \log h_t^2 &= \omega + \sum_{i=1}^{q} \alpha_i \log x_{t--i} + \sum_{i=1}^{p} \beta_i \log h_{t--i}^2 \\ \log x_t &= \xi + \delta \log h_t^2 + \eta_1 z_t + \eta_2 \left(z_t^2 - 1\right) + u_t, \\ u_t &\sim \mathcal{N}(0,\lambda) \end{aligned} \tag{49}$$

where x_t is set to the realized variance, i.e. $x_t = \mathrm{RV}_t$.

5.2. VIX-Joint GARCH

Hao and Zhang (2013) investigated whether GARCH models jointly estimated with returns and CBOE VIX index could explain the variance risk premium. The authors also suggest that information from risk-neutral measures (such as VIX) can significantly improve the GARCH model's predictivity. Assuming conditional log-normal return, we have

$$\begin{aligned} r_t &= r + \lambda\sqrt{h_t} - \frac{1}{2}h_t + \epsilon_t, \quad \text{where } \epsilon_t|\mathcal{F}_{t-1} \sim i.i.d.\mathcal{N}(0,h_t) \\ h_t &= \omega + \sum_{i=1}^{q} \alpha_i \epsilon_t^2 + \sum_{i=1}^{p} \beta_i h_{t--i} \end{aligned} \tag{50}$$

We choose GARCH(1,1) model jointly estimated with individual VIX as a competing benchmark. VIX is the expected volatility of the S&P 500 index in the following 30 calendar days,

$$\overline{\mathrm{VIX}}_t = \left(\frac{\mathrm{VIX}_t}{100}\right)^2 = \frac{1}{30}\sum_{t=1}^{30} \mathbb{E}_t^Q\left(h_{t+k}\right) \tag{51}$$

Assuming locally risk-neutral valuation (LRNVR) relationship (Duan 1995), the implied VIX ($\overline{\mathrm{VIX}}_t^{Imp}$) is linear in conditional variance of the next period,

$$\overline{\mathrm{VIX}}_t^{Imp} = \frac{\alpha_0}{1-\eta}\left(1 - \frac{1-\eta^n}{n(1-\eta)}\right) + \left(1 - \frac{1-\eta^n}{n(1-\eta)}\right) h_{t+1}$$
$$\text{where } \eta = \alpha_1(1+\lambda^2) + \beta_1 \tag{52}$$

A joint estimation is simply maximizing the following log-likelihood function

$$\begin{aligned} L_{joint} &= -T\ln(2\pi^2\hat{s}^2) - \frac{1}{2}\sum_{t=1}^{T}\left\{\ln(h_t) + \left[r_t - r - \lambda\sqrt{h_t}\right.\right. \\ &\quad \left.\left. + \frac{1}{2}h_t\right]^2 / h_t - \left(\overline{\mathrm{VIX}}_t - \overline{\mathrm{VIX}}_t^{Imp}\right)^2\right\} \end{aligned} \tag{53}$$

where $\hat{s}^2$ is the sample variance $\hat{s}^2 = \mathrm{var}(\overline{\mathrm{VIX}}_t - \overline{\mathrm{VIX}}_t^{Imp})$

5.2.1. Equity VIX. Chicago Board Options Exchange (CBOE) only publishes equity VIX for five companies: Apple, Amazon, Google, Goldman Sachs, and IBM. In order to estimate the VIX-joint GARCH model, we follow the CBOE's white paper (2015) to construct the VIX for each stock (equity VIX) and S&P 500 index (VIX).

$$\begin{aligned} VIX_t &= \left[T_1\sigma_1^2\left(\frac{N_{T_2} - N_{30}}{N_{T_2} - N_{T_1}}\right) + T_2\sigma_2^2\left(\frac{N_{30} - N_{T_1}}{N_{T_2} - N_{T_1}}\right)\right]\frac{365}{30} \\ \sigma_i^2 &= \frac{2}{T_i}\sum_{i=0}^{n}\frac{\Delta K_i}{K_i^2}e^{rT_i}M_i - \frac{1}{T_i}\left(\frac{F}{K_0} - 1\right)^2 \end{aligned} \tag{54}$$

where M_i is the mid-price and K_0 is the first strike below forward price F. We use Constant Maturity Treasury rates as proxies for risk-free rate r. Since Apple, Goldman Sachs, and IBM are included in DJIA, we present the statistical comparison between self-constructed VIX series and those presented by CBOE (table 1).

Table 1. Model fit: VIX levels and statistical comparison.

Ticker	CBOE Ticker	ME	Std Err	MAE	MSE	RMSE	P-value	Correlation	σ Violation
S&P 500	VIX	– 0.17	0.38	0.28	0.18	0.42	0.41	1.00	0.00%
AAPL	VXAPL	– 0.31	1.25	0.97	1.66	1.29	0.23	0.98	0.15%
IBM	VXIBM	– 0.72	1.45	0.86	1.74	1.32	0.12	0.98	0.38%
GS	VXGS	– 0.82	0.94	0.89	1.54	1.24	0.11	0.99	0.05%

This table shows how the implied VIX (equity VIX) fits the CBOE VIX (equity VIX) in levels as well as other statistical properties during the time period from July 1, 2011, to December 31, 2018. The error is calculated as the CBOE's value minus the constructed value. The mean error (ME) calculates the daily average error. The standard error (Std Err) calculates the standard deviation of the error. The mean absolute error (MAE) calculates the daily average absolute errors. The mean squared error (MSE) calculates the daily average squared error. The root mean squared error (RMSE) is simply the squared root of MSE. The P-value is used for testing the null hypothesis that the means of the two series are equal. The correlation coefficient calculates the linear correlation between the calculated VIX and the CBOE's data. One sigma violation represents the probability that our VIX series lie beyond one standard deviation of the CBOE VIX.

5.3. Linton and Wu coupled GARCH

Linton and Wu (2020) proposed a coupled component GARCH model to accommodate distinct properties of intraday and overnight volatility. They utilize a DCS (dynamic conditional score) method with heavy-tailed t-distribution. We implement their model of the form,

$$\begin{bmatrix} 1 & \delta \\ 0 & 1 \end{bmatrix}\begin{bmatrix} r_t^{oc} \\ r_t^{co} \end{bmatrix} = \begin{bmatrix} \mu_t^{D} \\ \mu_t^{N} \end{bmatrix} + \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{21} & \Pi_{22} \end{bmatrix}\begin{bmatrix} r_{t-1}^{oc} \\ r_{t-1}^{co} \end{bmatrix} + \begin{bmatrix} u_t^{oc} \\ u_t^{co} \end{bmatrix} \text{ and } \begin{bmatrix} u_t^{oc} \\ u_t^{co} \end{bmatrix} = \begin{bmatrix} \exp(\lambda_t^{oc})\exp(\sigma^{oc}(t/T))\epsilon_t^{oc} \\ \exp(\lambda_t^{co})\exp(\sigma^{co}(t/T))\epsilon_t^{co} \end{bmatrix} \tag{55}$$

where ϵ_t^j are i.i.d mean zero shocks from t-distributions with v_j degrees of freedom. $\sigma^j(t/T)$ are normalized kernel estimates of of residual u_t^j. The key dynamic lies in the innovation process λ_t^j:

$$\begin{aligned} \lambda_t^{oc} &= \omega_{oc}(1-\beta_{oc}) + \beta_{oc}\lambda_{t-1}^{oc} + \gamma_{oc}m_{t-1}^{oc} + \rho_{oc}m_t^{co} \\ &\quad + \gamma_{oc}^*(m_{t-1}^{oc}+1)\mathrm{sign}(e_{t-1}^{oc}) + \rho_{oc}^*(m_t^{co}+1)\mathrm{sign}(e_t^{co}) \\ \lambda_t^{co} &= \omega_{co}(1-\beta_{co}) + \beta_{co}\lambda_{t-1}^{co} + \gamma_{co}m_{t-1}^{co} + \rho_{co}m_{t-1}^{oc} \\ &\quad + \rho_{co}^*(m_{t-1}^{oc}+1)\mathrm{sign}(e_{t-1}^{oc}) + \gamma_{co}^*(m_{t-1}^{co}+1)\mathrm{sign}(e_{t-1}^{co}) \end{aligned} \tag{56}$$

where $e_t^j = \exp(-\sigma^j(t/T))u_t^j$, $m_t^j = \frac{(1+v_j)(e_t^j)^2}{v_j\exp(2\lambda_t^j)+(e_t^j)^2} - 1$. We follow the estimation procedures by imposing a parabolic Epanechnikov kernel and maximizing the likelihood $\sum_{t=1}^{T}(l_t^{oc} + l_t^{co})$, where

$$\begin{aligned} l_t^j &= -\lambda_t^j - \frac{v_j+1}{2}\log\left(1+\frac{1}{v_j}\frac{(e_t^j)^2}{\exp(2\lambda_t^j)}\right) \\ &\quad + \log\Gamma\left(\frac{v_j+1}{2}\right) - \frac{\log(v_j)}{2} - \log\Gamma\left(\frac{v_j}{2}\right) \end{aligned} \tag{57}$$

6. Empirical results for DJIA stocks

The Dow Jones Industrial Average (DJIA) is a stock market index that indicates the value of 30 large, publicly owned companies based in the United States with a price-weighted methodology. Our sample covers July 1, 2011, to December 31, 2018, and excludes ticker DWDP due to its short trading history. We estimate the model using data from July 1, 2011, to December 31, 2017.

6.1. Summary statistics

Tables 2 and 3 summarize the descriptive statistics for DJIA stocks. Intuitively, all the scale factors $\hat{C}$ are greater than one. We also conduct Augmented Dickey–Fuller test for jump volatility, the column *P-value* suggests strong rejection to null hypothesis that a unit root exists.

Table 2. Summary statistics for equity returns and volatility.

Symbol	$\bar{r}_{oc}$	min r_{oc}	max r_{oc}	$\overline{r^2_{oc}}$	$\bar{r}_{co}$	min r_{co}	max r_{co}	$\overline{r^2_{co}}$
AAPL	– 0.00018	– 0.06863	0.08339	0.00016	0.00078	– 0.11101	0.09422	0.00012
AXP	0.00022	– 0.06325	0.06103	0.00012	0.00017	– 0.07206	0.06464	0.00006
BA	0.00031	– 0.07174	0.06330	0.00015	0.00047	– 0.06545	0.06530	0.00008
CAT	– 0.00031	– 0.10075	0.05571	0.00018	0.00046	– 0.07592	0.06296	0.00011
CSCO	0.00030	– 0.05249	0.04689	0.00012	0.00007	– 0.13642	0.11803	0.00011
CVX	0.00005	– 0.06777	0.05297	0.00012	0.00004	– 0.07125	0.04121	0.00006
DIS	0.00014	– 0.04768	0.05178	0.00010	0.00039	– 0.09348	0.05586	0.00006
GS	– 0.00017	– 0.08042	0.06848	0.00018	0.00015	– 0.06516	0.04376	0.00007
HD	0.00038	– 0.05329	0.06722	0.00012	0.00040	– 0.05403	0.04870	0.00005
IBM	0.00018	– 0.04308	0.04126	0.00009	– 0.00031	– 0.08722	0.06971	0.00006
INTC	0.00066	– 0.05750	0.06421	0.00014	– 0.00025	– 0.09644	0.06578	0.00008
JNJ	0.00029	– 0.08182	0.05275	0.00007	0.00007	– 0.04802	0.03045	0.00002
JPM	0.00006	– 0.06176	0.06873	0.00016	0.00034	– 0.09252	0.06731	0.00009
KO	0.00026	– 0.03846	0.03787	0.00006	– 0.00007	– 0.05924	0.03994	0.00003
MCD	0.00023	– 0.04565	0.04223	0.00007	0.00018	– 0.05884	0.07386	0.00004
MMM	0.00044	– 0.05080	0.03933	0.00009	– 0.00005	– 0.06667	0.03307	0.00004
MRK	0.00007	– 0.07925	0.04625	0.00010	0.00031	– 0.05238	0.07765	0.00006
MSFT	0.00040	– 0.05934	0.05541	0.00013	0.00024	– 0.09032	0.08517	0.00008
NKE	0.00038	– 0.06108	0.06085	0.00014	0.00024	– 0.09806	0.09821	0.00009
PFE	0.00052	– 0.05765	0.04482	0.00009	– 0.00007	– 0.04794	0.09561	0.00005
PG	0.00032	– 0.05350	0.03166	0.00006	– 0.00014	– 0.04659	0.05598	0.00003
TRV	0.00024	– 0.05973	0.04946	0.00009	0.00014	– 0.05281	0.04095	0.00004
UNH	0.00065	– 0.06429	0.05937	0.00014	0.00029	– 0.06574	0.08344	0.00006
UTX	– 0.00014	– 0.06471	0.05176	0.00010	0.00029	– 0.06654	0.03196	0.00005
V	0.00041	– 0.09036	0.13127	0.00015	0.00059	– 0.10397	0.06776	0.00006
VZ	0.00009	– 0.05065	0.03399	0.00008	0.00013	– 0.04199	0.04329	0.00004
WBA	0.00047	– 0.08016	0.06468	0.00017	– 0.00020	– 0.17575	0.09921	0.00008
WMT	0.00046	– 0.10401	0.04622	0.00008	– 0.00019	– 0.07726	0.10402	0.00005
XOM	0.00028	– 0.04369	0.04243	0.00009	– 0.00033	– 0.05911	0.03700	0.00005

6.2. Full linear coupled GARCH (1,1) model

The empirical results for the linear coupled GARCH(1,1) model with all full parameters are summarized and presented in table 4 for DJIA 29 stocks. The fact that $\omega_{oc,1}$ is estimated to be larger than its counterpart $\omega_{co,1}$ simply reflects that the intra-day volatility is generally higher than overnight volatility. Interestingly, we observe that the empirical estimates of the constants $\omega_{co,2}$ are roughly zero across the DJIA, while the estimates of $\omega_{oc,2}$ and the VIX dependent coefficients $\omega_{oc,1}$ and $\omega_{co,1}$ are positive across all tickers. The null values of $\omega_{co,2}$ suggest that overnight volatility goes to zero in absence of any external stimulus. In contrast, the positive constant for intraday volatilities may be related to the reflexive nature of market microstructure, sometimes modeled as Hawkes self-exciting processes (Blanc *et al.* 2017). We notice that estimates of auto-regressive and moving-average coefficients are remarkably similar in DJIA components that span diverse industries and face varying market dynamics. The robust standard errors show that the jump coefficient $\hat{\gamma}_3$ is clearly significant. The estimates of β_3 are closer to one, which suggests that the overnight volatility $h_{co,t}$ and its following intra-day volatility $h_{oc,t}$ are highly collinear.

6.3. Results from the model comparison

Sections 6.4–6.9 document performances of coupled GARCH (1,1) model and four benchmark volatility models. Tables 5– 7 present results in terms of three loss functions: mean squared error (MSE), realized utility, and QLIKE. The best out-of-sample models are highlighted in each row. We observe that coupled GARCH(1,1) and Realized GARCH(1,2) models clearly outperform standard GARCH(1,1), VIX-joint GARCH(1,1) model, and the Linton and Wu coupled GARCH(1,1) model. In most cases the best model is either coupled GARCH or Realized GARCH(1,2). A further analysis shows that our model is better by the MSE measure, however, it slightly underperforms under realized utility and QLIKE loss functions. This is not surprising since the realized GARCH(1,2) model directly utilizes high-frequency realized volatility as an auxiliary variable while our coupled GARCH(1,1) model only extracts information from option markets and market quotes at market open and close.

Tables 8–10 contain results from the model comparison in the form of DM t-test, which corresponds to the null hypothesis that the alternative model is as good as coupled GARCH(1,1) model. A negative DM t-test statistic indicates that the alternative model underperforms the Coupled GARCH(1,1) model, the thresholds for 5% and 1% significance are ± 1.96 and ± 2.58, respectively. The coupled GARCH(1,1) model and realized GARCH(1,2) are still dominant in MSE and QLIKE separately after filtering out the insignificant t-stat, however, the best model for the realized utility measure tilts towards the coupled GARCH(1,1) model. This shift could be explained by the construction of the realized utility: a function of the ratio of true volatility to volatility prediction, which is more sensitive to outliers and erratic predictions in the denominator.

Table 3. Summary statistics for realized variance, implied jump variance and equity VIX.

Symbol	mean RV	min RV	max RV	mean $\sigma^2_{\text{jump},c}$	max $\sigma^2_{\text{jump},c}$	P-value	mean E.VIX	min E.VIX	max E.VIX	scale $\hat{C}$
AAPL	0.00015	0.00001	0.00574	0.11184	2.01248	0.00000	27.69	14.24	60.35	1.72
AXP	0.00012	0.00001	0.00300	0.05623	1.12750	0.00000	23.65	13.43	62.83	1.57
BA	0.00015	0.00001	0.00476	0.06957	1.18969	0.01073	24.96	15.29	58.08	1.58
CAT	0.00019	0.00001	0.00318	0.08879	1.31211	0.01578	27.59	15.56	69.77	1.53
CSCO	0.00014	0.00002	0.00222	0.05966	4.29202	0.00000	25.12	12.73	63.73	1.68
CVX	0.00013	0.00001	0.00269	0.04990	0.80444	0.00013	22.24	12.61	50.45	1.43
DIS	0.00011	0.00001	0.00257	0.04476	1.30548	0.00000	22.99	12.82	95.62	1.54
GS	0.00018	0.00002	0.00417	0.10536	2.25579	0.00770	27.06	15.67	85.90	1.49
HD	0.00012	0.00001	0.00789	0.04671	0.78854	0.00002	21.82	12.54	50.21	1.31
IBM	0.00009	0.00001	0.00200	0.05498	1.32154	0.00000	20.72	12.96	45.76	1.71
INTC	0.00016	0.00002	0.00326	0.06323	2.34420	0.00000	25.34	14.39	57.15	1.37
JNJ	0.00008	0.00001	0.01699	0.02722	0.76086	0.00000	16.29	10.68	36.41	1.03
JPM	0.00016	0.00001	0.00390	0.06189	1.63370	0.00353	25.33	14.18	78.20	1.64
KO	0.00007	0.00001	0.00192	0.02302	0.56305	0.00000	16.57	9.79	37.21	1.25
MCD	0.00007	0.00001	0.00287	0.03752	0.64527	0.00000	17.98	11.69	34.42	1.32
MMM	0.00009	0.00001	0.00253	0.04647	1.06097	0.00000	20.01	12.24	50.88	1.61
MRK	0.00011	0.00001	0.00255	0.03388	0.64085	0.00000	20.70	14.10	41.41	1.43
MSFT	0.00014	0.00001	0.00431	0.06036	2.23901	0.00000	23.58	12.12	47.82	1.57
NKE	0.00014	0.00001	0.00690	0.07253	3.48443	0.00000	24.50	13.11	49.60	1.53
PFE	0.00012	0.00001	0.00237	0.03212	0.64458	0.00011	20.68	12.11	284.70	1.08
PG	0.00007	0.00001	0.00474	0.02825	0.74075	0.00000	16.83	10.60	33.25	1.23
TRV	0.00009	0.00001	0.00196	0.00982	0.44880	0.00000	20.01	11.75	50.93	1.53
UNH	0.00016	0.00002	0.02117	0.06183	2.41348	0.00000	24.70	12.80	323.17	1.17
UTX	0.00011	0.00001	0.00194	0.04684	2.07412	0.00077	21.30	12.47	61.88	1.46
V	0.00014	0.00001	0.00930	0.06946	2.08645	0.00000	23.95	13.33	57.75	1.41
VZ	0.00010	0.00001	0.00315	0.03153	5.67513	0.00000	19.56	12.97	410.62	1.26
WBA	0.00018	0.00002	0.00888	0.05688	1.19081	0.00000	25.93	15.46	45.41	1.40
WMT	0.00008	0.00001	0.00438	0.03456	1.11833	0.00000	18.55	10.87	39.39	1.49
XOM	0.00010	0.00001	0.00233	0.04276	0.77968	0.00004	21.64	10.57	178.67	1.34

The subscripts -oc and -co refer to open-to-close and close-to-close returns. The P-value is for the null hypothesis that a unit root is present in the jump variance $\sigma^2_{\text{jump},c}$ by Augmented Dickey–Fuller test. The E.VIX refers to calculated equity VIX.

Table 4. Summary of parameters with robust standard deviation in brackets.

Symbol	$\hat{\omega}^{\dagger}_{oc,1}$	$\hat{\omega}^{\dagger}_{oc,2}$	$\hat{\omega}^{\dagger}_{co,1}$	$\hat{\omega}^{\dagger}_{co,2}$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3^{\dagger}$
AAPL	13.612	0.103	1.571	0.067	0.145	0.057	0.936	0.081	0.173	0.101	1.568
	(1.254)	(0.045)	(0.543)	(0.010)	(0.021)	(0.010)	(0.134)	(0.032)	(0.048)	(0.036)	(0.238)
AXP	20.191	0.074	6.902	0.000	0.000	0.128	0.763	0.166	0.114	0.007	2.085
	(11.223)	(0.517)	(1.660)	(0.648)	(0.364)	(0.015)	(0.257)	(0.011)	(0.008)	(0.001)	(1.149)
BA	17.884	0.071	0.736	0.000	0.097	0.061	1.000	0.159	0.221	0.057	2.718
	(4.631)	(0.626)	(0.313)	(1.775)	(0.017)	(0.007)	(0.390)	(0.009)	(0.034)	(0.004)	(0.456)
CAT	26.998	0.158	3.510	0.000	0.036	0.048	0.800	0.166	0.179	0.018	4.800
	(2.183)	(1.487)	(3.138)	(1.297)	(0.010)	(0.003)	(0.120)	(0.018)	(0.014)	(0.004)	(2.821)
CSCO	23.840	0.265	9.434	0.000	0.059	0.164	0.233	0.027	0.004	0.000	9.266
	(3.804)	(1.431)	(1.595)	(1.066)	(0.011)	(0.153)	(0.154)	(0.004)	(0.001)	(0.009)	(3.836)
CVX	8.452	0.065	0.000	0.000	0.205	0.083	0.947	0.238	0.278	0.040	2.479
	(1.010)	(2.038)	(3.414)	(1.646)	(0.052)	(0.029)	(0.091)	(0.008)	(0.036)	(0.005)	(1.083)
DIS	16.273	0.147	5.744	0.000	0.015	0.074	0.746	0.133	0.109	0.034	2.881
	(3.980)	(1.291)	(2.664)	(1.367)	(0.002)	(0.009)	(0.094)	(0.013)	(0.028)	(0.008)	(2.559)
GS	23.402	0.001	20.067	0.000	0.053	0.108	1.000	0.227	0.035	0.005	0.595
	(7.080)	(0.123)	(3.331)	(0.586)	(0.008)	(0.007)	(0.192)	(0.042)	(0.004)	(0.001)	(0.528)
HD	11.984	0.108	8.433	0.000	0.016	0.105	0.994	0.615	0.000	0.027	2.645
	(1.333)	(0.066)	(1.881)	(0.840)	(0.001)	(0.015)	(0.091)	(0.036)	(0.006)	(0.005)	(1.226)
IBM	11.921	0.052	5.385	0.009	0.198	0.100	0.681	0.065	0.053	0.013	1.968
	(1.450)	(0.023)	(1.319)	(0.001)	(0.043)	(0.022)	(0.100)	(0.009)	(0.009)	(0.002)	(0.570)
INTC	17.711	0.523	6.916	0.000	0.014	0.109	0.802	0.042	0.049	0.068	4.234
	(3.548)	(0.486)	(0.536)	(0.740)	(0.004)	(0.036)	(0.093)	(0.002)	(0.015)	(0.021)	(2.747)
JNJ	7.195	0.000	1.650	0.004	0.226	0.035	1.000	0.867	0.068	0.000	3.198
	(2.263)	(1.535)	(1.283)	(0.497)	(0.053)	(0.010)	(0.077)	(0.125)	(0.012)	(0.011)	(1.684)
JPM	19.364	0.000	13.472	0.000	0.016	0.052	0.871	0.174	0.160	0.072	1.842
	(1.132)	(0.106)	(4.127)	(0.987)	(0.002)	(0.004)	(0.136)	(0.038)	(0.013)	(0.008)	(1.577)
KO	7.698	0.113	5.477	0.000	0.095	0.093	0.862	0.145	0.000	0.014	3.214
	(2.122)	(3.731)	(0.545)	(0.400)	(0.047)	(0.029)	(0.140)	(0.051)	(0.042)	(0.002)	(0.674)
MCD	8.029	0.132	4.310	0.000	0.074	0.065	0.867	0.203	0.043	0.071	2.276
	(1.177)	(1.627)	(0.084)	(0.322)	(0.008)	(0.003)	(0.120)	(0.021)	(0.003)	(0.008)	(0.859)
MMM	14.248	0.039	6.965	0.000	0.077	0.130	0.778	0.233	0.024	0.007	2.264
	(6.707)	(2.718)	(5.436)	(0.288)	(0.009)	(0.089)	(0.038)	(0.080)	(0.007)	(0.001)	(1.805)
MRK	8.479	0.153	5.626	0.000	0.010	0.049	0.883	0.198	0.187	0.040	5.594
	(1.783)	(0.500)	(2.374)	(0.152)	(0.001)	(0.004)	(0.112)	(0.027)	(0.022)	(0.010)	(0.806)
MSFT	21.526	0.051	6.635	0.000	0.137	0.102	0.780	0.049	0.060	0.090	2.807
	(3.326)	(0.968)	(0.792)	(0.419)	(0.022)	(0.010)	(0.185)	(0.026)	(0.010)	(0.014)	(2.338)
NKE	29.693	0.204	5.437	0.063	0.009	0.099	0.330	0.059	0.027	0.053	3.106
	(7.519)	(0.117)	(0.668)	(0.005)	(0.001)	(0.027)	(0.021)	(0.033)	(0.028)	(0.018)	(0.494)
PFE	13.268	0.160	0.280	0.000	0.000	0.097	0.974	0.197	0.235	0.025	4.779
	(6.533)	(0.014)	(1.341)	(0.366)	(0.085)	(0.034)	(0.257)	(0.019)	(0.037)	(0.003)	(0.965)
PG	7.217	0.135	4.374	0.000	0.144	0.052	0.781	0.187	0.005	0.070	1.571
	(3.477)	(2.082)	(1.319)	(0.232)	(0.019)	(0.005)	(0.039)	(0.012)	(0.001)	(0.013)	(0.263)
TRV	7.937	0.208	8.489	0.000	0.056	0.096	0.877	0.178	0.004	0.055	6.457
	(2.798)	(1.117)	(8.269)	(2.370)	(0.009)	(0.020)	(0.065)	(0.010)	(0.001)	(0.005)	(3.019)
UNH	29.009	0.000	7.314	0.000	0.003	0.051	0.896	0.402	0.087	0.045	1.628
	(8.234)	(6.252)	(0.252)	(5.116)	(0.002)	(0.012)	(0.369)	(0.088)	(0.011)	(0.005)	(0.750)
UTX	13.318	0.120	8.608	0.000	0.209	0.103	0.429	0.187	0.045	0.017	3.044
	(5.401)	(0.540)	(1.621)	(1.947)	(0.025)	(0.012)	(0.105)	(0.009)	(0.004)	(0.001)	(0.727)
V	20.558	0.054	7.903	0.000	0.069	0.042	0.943	0.232	0.035	0.009	2.794
	(1.535)	(0.788)	(2.729)	(0.723)	(0.009)	(0.008)	(0.185)	(0.012)	(0.008)	(0.001)	(0.434)
VZ	10.592	0.266	2.149	0.024	0.148	0.154	0.328	0.290	0.170	0.001	2.740
	(2.898)	(0.035)	(0.091)	(0.009)	(0.029)	(0.051)	(0.091)	(0.090)	(0.126)	(0.001)	(0.599)
WBA	11.143	0.421	9.480	0.000	0.132	0.040	0.998	0.066	0.000	0.077	4.914
	(6.932)	(3.049)	(8.451)	(2.354)	(0.022)	(0.022)	(0.276)	(0.017)	(0.096)	(0.008)	(3.681)
WMT	13.664	0.220	2.285	0.001	0.000	0.003	0.771	0.020	0.049	0.094	5.135
	(2.064)	(0.082)	(1.193)	(0.353)	(0.178)	(0.001)	(0.053)	(0.003)	(0.004)	(0.008)	(2.148)
XOM	10.203	0.033	0.749	0.000	0.139	0.086	0.920	0.183	0.260	0.047	2.056
	(1.156)	(1.146)	(0.646)	(0.451)	(0.052)	(0.008)	(0.049)	(0.012)	(0.027)	(0.011)	(1.992)

† means the value should be multiplied by 10^4, e.g. $\hat{\omega}^{\dagger}_{oc,2} = 10^4 \cdot \hat{\omega}_{oc,2}$, $\hat{\gamma}_3^{\dagger} = 10^4 \cdot \hat{\gamma}_3$.

6.4. Realized Utility

Table 5. Realized Utility of intraday and overnight volatility.

	In-sample realized utility					Out-of-sample realized utility				
Symbol	Standard	Realized	LintonWu	VIX-joint	Coupled	Standard	Realized	LintonWu	VIX-joint	Coupled
Panel A: Realized utility for open-to-close volatility										
AAPL	3.46%	3.57%	3.47%	3.47%	3.58%	3.56%	**3.79%**	3.53%	3.51%	3.69%
AXP	3.53%	3.62%	3.55%	3.53%	3.71%	3.40%	3.26%	3.43%	3.30%	**3.48%**
BA	3.56%	3.56%	3.52%	3.55%	3.64%	3.37%	**3.57%**	3.42%	3.14%	3.45%
CAT	3.60%	3.69%	3.61%	2.92%	3.73%	3.40%	**3.82%**	3.47%	3.39%	3.68%
CSCO	3.51%	3.68%	3.50%	3.59%	3.76%	2.46%	− 5.60%	3.37%	3.56%	**3.71%**
CVX	3.68%	3.75%	3.59%	3.68%	3.76%	3.74%	3.77%	3.69%	3.67%	**3.80%**
DIS	3.57%	3.60%	3.44%	3.57%	3.69%	3.62%	− 1.44%	3.66%	3.57%	**3.68%**
GS	3.62%	3.71%	3.65%	3.64%	3.73%	3.64%	**3.88%**	3.68%	3.60%	3.76%
HD	3.58%	3.61%	3.52%	3.58%	3.71%	3.59%	**3.80%**	3.61%	3.50%	3.70%
IBM	3.54%	3.68%	3.47%	3.55%	3.74%	3.58%	**3.84%**	3.58%	3.51%	3.74%
INTC	3.59%	3.71%	3.59%	3.59%	3.72%	3.44%	**3.79%**	3.57%	3.33%	3.56%
JNJ	3.43%	3.39%	3.46%	3.47%	3.65%	3.62%	**3.64%**	3.48%	2.74%	3.41%
JPM	3.60%	3.67%	3.55%	3.63%	3.74%	3.57%	3.37%	3.57%	3.52%	**3.75%**
KO	3.61%	3.69%	3.57%	3.60%	3.72%	3.64%	**3.90%**	3.66%	3.55%	3.79%
MCD	3.52%	3.64%	3.48%	3.55%	3.72%	3.22%	**3.82%**	3.32%	3.22%	3.61%
MMM	3.60%	3.68%	3.62%	3.60%	3.72%	3.52%	**3.80%**	3.53%	3.07%	3.52%
MRK	3.61%	3.66%	3.44%	3.61%	3.75%	3.54%	**3.85%**	3.64%	3.51%	3.78%
MSFT	3.51%	3.67%	3.57%	3.55%	3.72%	3.21%	3.36%	3.54%	3.37%	**3.68%**
NKE	3.54%	3.66%	3.03%	3.53%	3.72%	3.57%	1.20%	3.10%	3.49%	**3.72%**
PFE	3.61%	3.70%	3.48%	3.60%	3.76%	**3.77%**	3.49%	3.50%	3.64%	**3.77%**
PG	3.54%	3.61%	3.56%	3.56%	3.69%	**3.71%**	3.23%	3.68%	3.60%	**3.71%**
TRV	3.62%	3.67%	3.62%	3.62%	3.74%	3.51%	3.59%	3.56%	3.20%	**3.65%**
UNH	3.39%	3.43%	3.42%	3.36%	3.63%	3.60%	**3.82%**	3.34%	3.38%	3.62%
UTX	3.59%	3.66%	3.53%	3.60%	3.71%	3.45%	**3.82%**	3.42%	3.29%	3.61%
V	3.44%	3.52%	3.49%	3.46%	3.60%	3.68%	**3.76%**	3.60%	3.63%	3.69%
VZ	3.60%	3.67%	3.60%	3.61%	3.69%	3.51%	**3.86%**	3.58%	3.39%	3.67%
WBA	3.43%	3.49%	3.44%	3.43%	3.56%	3.57%	**3.87%**	3.58%	3.42%	3.70%
WMT	3.45%	3.51%	3.52%	3.44%	3.60%	3.46%	3.52%	3.57%	3.25%	**3.71%**
XOM	3.64%	3.73%	3.46%	3.64%	3.76%	3.77%	3.42%	3.29%	3.64%	**3.82%**
Panel B: Realized utility for close-to-close volatility										
AAPL	3.46%	3.57%	3.48%	3.47%	3.54%	3.56%	**3.79%**	3.54%	3.51%	3.60%
AXP	3.53%	3.62%	3.56%	3.53%	3.70%	3.40%	3.26%	3.44%	3.30%	**3.37%**
BA	3.56%	3.56%	3.49%	3.55%	3.64%	3.37%	**3.57%**	3.46%	3.14%	3.37%
CAT	3.60%	3.69%	3.52%	2.92%	3.72%	3.40%	**3.82%**	3.45%	3.39%	3.64%
CSCO	3.51%	3.68%	3.53%	3.59%	3.72%	2.46%	− 5.60%	3.40%	3.56%	**3.61%**
CVX	3.68%	3.75%	3.68%	3.68%	3.76%	3.74%	3.77%	3.72%	3.67%	**3.79%**
DIS	3.57%	3.60%	3.47%	3.57%	3.68%	3.62%	− 1.44%	3.64%	3.57%	**3.63%**
GS	3.62%	3.71%	3.65%	3.64%	3.72%	3.64%	**3.88%**	3.68%	3.60%	3.74%
HD	3.58%	3.61%	3.52%	3.58%	3.71%	3.59%	**3.80%**	3.61%	3.50%	3.73%
IBM	3.54%	3.68%	3.56%	3.55%	3.64%	3.58%	**3.84%**	3.63%	3.51%	3.53%
INTC	3.59%	3.71%	3.62%	3.59%	3.73%	3.44%	**3.79%**	3.54%	3.33%	3.50%
JNJ	3.43%	3.39%	3.36%	3.47%	3.65%	3.62%	**3.64%**	3.42%	2.74%	3.51%
JPM	3.60%	3.67%	3.61%	3.63%	3.74%	3.57%	3.37%	3.57%	3.52%	**3.74%**
KO	3.61%	3.69%	3.61%	3.60%	3.72%	3.64%	**3.90%**	3.66%	3.55%	3.81%
MCD	3.52%	3.64%	3.53%	3.55%	3.71%	3.22%	**3.82%**	3.35%	3.22%	3.62%
MMM	3.60%	3.68%	3.60%	3.60%	3.75%	3.52%	**3.80%**	3.50%	3.07%	3.32%
MRK	3.61%	3.66%	3.51%	3.61%	3.74%	3.54%	**3.85%**	3.65%	3.51%	3.80%
MSFT	3.51%	3.67%	3.58%	3.55%	3.70%	3.21%	3.36%	**3.59%**	3.37%	**3.59%**
NKE	3.54%	3.66%	2.17%	3.53%	3.71%	3.57%	1.20%	2.16%	3.49%	**3.61%**
PFE	3.61%	3.70%	3.62%	3.60%	3.73%	**3.77%**	3.49%	3.59%	3.64%	3.75%
PG	3.54%	3.61%	3.57%	3.56%	3.68%	**3.71%**	3.23%	3.69%	3.60%	**3.71%**
TRV	3.62%	3.67%	3.60%	3.62%	3.74%	3.51%	**3.59%**	3.55%	3.20%	3.58%
UNH	3.39%	3.43%	3.48%	3.36%	3.64%	3.60%	**3.82%**	3.43%	3.38%	3.60%
UTX	3.59%	3.66%	3.59%	3.60%	3.72%	3.45%	**3.82%**	3.44%	3.29%	3.61%
V	3.44%	3.52%	3.48%	3.46%	3.58%	3.68%	**3.76%**	3.59%	3.63%	3.65%
VZ	3.60%	3.67%	3.56%	3.61%	3.67%	3.51%	**3.86%**	3.57%	3.39%	3.73%
WBA	3.43%	3.49%	3.42%	3.43%	3.56%	3.57%	**3.87%**	3.57%	3.42%	3.67%
WMT	3.45%	3.51%	3.52%	3.44%	3.58%	3.46%	3.52%	3.59%	3.25%	**3.60%**
XOM	3.64%	3.73%	3.46%	3.64%	3.76%	3.77%	3.42%	3.38%	3.64%	**3.83%**

6.5. *Volatility forecast*

Table 6. Mean squared error (MSE) of volatility estimation.

	In-sample MSE					Out-of-sample MSE				
Symbol	Standard	Realized	LintonWu	VIX-joint	Coupled	Standard	Realized	LintonWu	VIX-joint	Coupled
Panel A: open-to-close volatility (every component is multiplied by 10^8 compared to its original value)										
AAPL	4.025	3.465	6.198	4.019	3.080	2.680	**0.860**	4.563	2.877	1.866
AXP	2.408	1.859	2.397	2.379	1.571	3.114	3.177	3.278	3.299	**2.605**
BA	2.552	2.499	3.050	2.580	2.144	13.150	**10.397**	28.270	14.120	12.383
CAT	2.928	2.270	4.344	13.963	2.108	7.241	**4.372**	15.838	11.186	4.841
CSCO	1.778	1.653	1.384	2.003	1.035	4.393	5.330	3.538	2.741	**1.597**
CVX	1.728	1.345	1.717	1.736	1.331	1.096	0.951	1.463	1.305	**0.894**
DIS	1.440	1.354	1.505	1.433	1.008	0.934	1.925	1.121	0.993	**0.596**
GS	3.875	3.066	3.757	3.792	2.502	2.789	**1.590**	3.368	2.905	1.772
HD	4.377	4.283	4.501	4.396	3.222	2.542	2.263	3.571	3.001	**2.022**
IBM	0.916	0.682	0.846	0.911	0.612	1.164	**0.502**	1.809	1.310	0.715
INTC	2.438	1.857	1.882	2.493	1.884	5.275	**3.254**	5.742	5.935	3.611
JNJ	17.737	17.759	18.022	17.721	13.961	2.091	**3.841**	5.893	5.117	3.913
JPM	3.225	2.657	3.506	3.128	2.142	2.270	2.128	2.799	2.270	**1.325**
KO	0.559	0.462	0.521	0.556	0.430	0.403	**0.208**	0.464	0.523	0.288
MCD	0.818	0.735	1.310	0.810	0.556	1.471	**0.641**	3.139	1.432	0.846
MMM	0.798	0.692	0.715	0.925	0.651	2.455	1.876	2.805	2.889	**1.674**
MRK	1.179	0.910	2.064	1.212	0.776	1.536	0.936	2.383	1.673	**0.716**
MSFT	2.423	1.877	2.271	2.439	1.761	5.494	4.105	9.958	4.715	**3.011**
NKE	4.604	4.006	16.544	4.566	3.529	2.330	3.566	17.822	2.632	**1.470**
PFE	1.367	1.050	1.206	1.361	0.891	0.928	1.275	1.464	1.232	**0.790**
PG	1.668	1.580	1.660	1.677	1.339	0.580	0.515	0.608	0.822	**0.393**
TRV	0.779	0.675	1.033	0.887	0.497	0.844	0.930	1.437	1.175	**0.696**
UNH	29.135	28.851	27.915	29.381	26.363	6.507	**4.800**	7.213	7.419	4.923
UTX	1.133	0.955	1.305	1.065	0.875	2.503	**1.531**	4.950	2.652	1.987
V	5.578	4.737	5.379	5.765	3.360	2.118	1.857	3.365	2.135	**1.846**
VZ	0.998	0.912	1.219	0.999	0.860	1.204	**0.547**	1.924	1.331	0.847
WBA	11.093	9.647	10.401	11.461	9.390	3.941	**2.176**	5.242	5.768	2.605
WMT	1.824	1.673	1.920	1.792	1.696	1.522	1.220	2.396	1.619	**0.811**
XOM	1.180	0.890	3.131	1.209	0.847	0.563	0.804	8.035	0.778	**0.458**
Panel B: close-to-close volatility (every component is multiplied by 10^8 compared to its original value)										
AAPL	11.849	10.199	17.624	11.829	9.281	7.889	**2.531**	14.449	8.468	6.440
AXP	5.831	4.500	5.696	5.759	3.919	7.538	7.692	8.035	7.988	**6.567**
BA	5.810	5.689	11.165	5.874	4.875	29.939	**23.671**	44.218	32.147	29.088
CAT	6.611	5.125	17.820	31.522	4.857	16.347	**9.869**	37.360	25.253	11.474
CSCO	4.750	4.416	4.056	5.350	4.393	11.736	14.238	7.277	7.322	**5.097**
CVX	3.533	2.748	3.900	3.547	2.801	2.241	1.943	2.599	2.667	**1.875**
DIS	3.298	3.101	4.185	3.282	2.360	2.140	4.411	2.869	2.276	**1.460**
GS	8.254	6.531	12.522	8.078	5.356	5.942	**3.387**	6.896	6.189	3.870
HD	7.269	7.112	7.984	7.301	5.535	4.222	3.759	5.748	4.985	**3.302**
IBM	2.821	2.100	2.813	2.807	1.920	3.586	**1.548**	4.598	4.035	2.910
INTC	4.972	3.787	4.484	5.084	3.951	10.756	**6.634**	17.110	12.101	7.804
JNJ	22.229	22.257	23.270	22.209	18.011	**2.621**	4.813	8.563	6.414	4.789
JPM	8.006	6.595	11.769	7.767	5.490	5.636	5.283	5.928	5.637	**3.387**
KO	0.830	0.685	0.834	0.825	0.698	0.598	**0.309**	0.654	0.776	0.410
MCD	1.493	1.343	2.167	1.479	1.075	2.687	**1.171**	3.698	2.616	1.565
MMM	1.934	1.678	2.703	2.243	1.448	5.952	**4.547**	6.841	7.004	4.733
MRK	2.261	1.745	3.945	2.324	1.706	2.946	1.795	3.404	3.208	**1.462**
MSFT	5.772	4.472	5.386	5.810	4.117	13.088	9.779	14.130	11.233	**7.926**
NKE	11.576	10.072	39.224	11.479	9.149	5.857	8.966	65.912	6.618	**4.415**
PFE	1.624	1.247	1.811	1.617	1.409	**1.103**	1.515	2.026	1.464	1.152
PG	2.824	2.674	3.031	2.838	2.338	0.981	0.872	0.982	1.391	**0.615**
TRV	1.680	1.455	2.977	1.913	1.105	1.820	2.005	2.705	2.534	**1.561**
UNH	40.393	39.999	39.468	40.735	36.881	9.021	**6.655**	10.795	10.285	7.185
UTX	2.246	1.893	3.016	2.110	1.836	4.961	**3.034**	7.231	5.256	3.885
V	11.344	9.634	11.084	11.723	7.427	4.308	**3.777**	6.491	4.343	4.000
VZ	1.499	1.371	2.079	1.501	1.632	1.809	**0.822**	2.944	2.000	1.138
WBA	21.667	18.842	20.090	22.386	18.440	7.697	**4.251**	8.227	11.265	5.299
WMT	4.348	3.986	4.657	4.272	4.114	3.627	2.907	4.947	3.858	**2.129**
XOM	2.232	1.683	5.920	2.286	1.682	1.066	1.521	9.605	1.472	**0.815**

6.6. Likelihood estimate (QLIKE)

Table 7. Quasi likelihood loss for volatility estimation.

	In-sample QLIKE					Out-of-sample QLIKE				
Symbol	Standard	Realized	LintonWu	VIX-joint	Coupled	Standard	Realized	LintonWu	VIX-joint	Coupled
Panel A: QLIKE for open-to-close volatility										
AAPL	−7.907	−7.962	−7.902	−7.908	−7.964	−7.722	**−7.820**	−7.690	−7.699	−7.788
AXP	−8.162	−8.216	−8.177	−8.161	−8.248	−7.886	−7.824	−7.889	−7.845	**−7.930**
BA	−8.068	−8.078	−8.027	−8.069	−8.104	−7.212	−7.227	−7.130	−7.128	**−7.247**
CAT	−7.821	−7.868	−7.800	−7.285	−7.873	−7.115	**−7.280**	−7.054	−7.000	−7.220
CSCO	−7.976	−8.048	−7.980	−7.991	−8.080	−7.248	−4.251	−7.563	−7.651	**−7.719**
CVX	−8.205	−8.240	−8.171	−8.206	−8.233	−7.858	−7.879	−7.824	−7.828	**−7.886**
DIS	−8.245	−8.268	−8.199	−8.249	−8.301	−8.020	−6.063	−8.019	−7.997	**−8.044**
GS	−7.770	−7.811	−7.775	−7.776	−7.812	−7.636	**−7.744**	−7.635	−7.621	−7.682
HD	−8.225	−8.251	−8.200	−8.227	−8.278	−7.888	**−7.980**	−7.871	−7.855	−7.929
IBM	−8.417	−8.484	−8.389	−8.418	−8.504	−8.122	**−8.236**	−8.106	−8.090	−8.195
INTC	−7.890	−7.949	−7.904	−7.889	−7.941	−7.183	**−7.333**	−7.209	−7.135	−7.234
JNJ	−8.688	−8.697	−8.658	−8.697	−8.750	−8.136	**−8.160**	−8.043	−7.845	−8.069
JPM	−7.939	−7.978	−7.918	−7.948	−8.001	−7.841	−7.760	−7.809	−7.819	**−7.911**
KO	−8.684	−8.732	−8.681	−8.685	−8.735	−8.476	**−8.592**	−8.475	−8.439	−8.540
MCD	−8.642	−8.691	−8.594	−8.648	−8.712	−8.037	**−8.271**	−8.014	−8.041	−8.198
MMM	−8.636	−8.678	−8.649	−8.632	−8.673	−7.841	**−7.961**	−7.824	−7.679	−7.852
MRK	−8.258	−8.295	−8.183	−8.254	−8.326	−7.982	**−8.117**	−7.987	−7.964	−8.081
MSFT	−8.005	−8.085	−8.030	−8.014	−8.099	−7.484	−7.542	−7.551	−7.548	**−7.676**
NKE	−7.978	−8.043	−7.563	−7.976	−8.062	−7.706	−6.759	−7.311	−7.666	**−7.775**
PFE	−8.181	−8.228	−8.140	−8.180	−8.249	−8.137	−8.028	−8.024	−8.083	**−8.136**
PG	−8.709	−8.749	−8.709	−8.711	−8.765	−8.353	−8.153	−8.315	−8.305	**−8.359**
TRV	−8.530	−8.565	−8.522	−8.527	−8.574	−7.940	−7.971	−7.937	−7.810	**−7.998**
UNH	−7.873	−7.909	−7.891	−7.873	−7.971	−7.923	**−8.037**	−7.837	−7.843	−7.953
UTX	−8.345	−8.381	−8.325	−8.350	−8.392	−7.840	**−7.996**	−7.792	−7.779	−7.903
V	−8.069	−8.119	−8.098	−8.074	−8.150	−7.943	**−7.998**	−7.902	−7.920	−7.961
VZ	−8.397	−8.433	−8.367	−8.397	−8.421	−7.828	**−7.972**	−7.805	−7.779	−7.895
WBA	−7.746	−7.788	−7.743	−7.746	−7.809	−7.491	**−7.621**	−7.457	−7.428	−7.540
WMT	−8.474	−8.516	−8.471	−8.476	−8.527	−7.949	−7.989	−7.952	−7.884	**−8.065**
XOM	−8.399	−8.444	−8.241	−8.398	−8.450	−8.145	−8.010	−7.817	−8.092	**−8.180**
Panel B: QLIKE for close-to-close volatility										
AAPL	−7.367	−7.422	−7.375	−7.369	−7.419	−7.182	**−7.280**	−7.160	−7.159	−7.215
AXP	−7.720	−7.773	−7.736	−7.719	−7.806	−7.444	−7.382	−7.448	−7.403	**−7.456**
BA	−7.657	−7.666	−7.591	−7.658	−7.696	−6.800	**−6.816**	−6.768	−6.717	−6.805
CAT	−7.414	−7.460	−7.333	−6.878	−7.468	−6.708	**−6.873**	−6.643	−6.593	−6.796
CSCO	−7.485	−7.556	−7.499	−7.500	−7.577	−6.757	−3.759	−7.090	−7.160	**−7.189**
CVX	−7.848	−7.882	−7.842	−7.849	−7.876	−7.501	−7.522	−7.482	−7.470	**−7.524**
DIS	−7.831	−7.854	−7.793	−7.835	−7.888	−7.606	−5.649	−7.595	−7.583	**−7.614**
GS	−7.392	−7.433	−7.379	−7.398	−7.433	−7.258	**−7.366**	−7.256	−7.242	−7.299
HD	−7.972	−7.997	−7.942	−7.974	−8.023	−7.634	**−7.726**	−7.617	−7.601	−7.686
IBM	−7.855	−7.921	−7.861	−7.856	−7.910	−7.559	**−7.674**	−7.562	−7.528	−7.554
INTC	−7.534	−7.593	−7.544	−7.532	−7.594	−6.826	**−6.976**	−6.815	−6.779	−6.857
JNJ	−8.575	−8.584	−8.456	−8.584	−8.632	−8.023	**−8.047**	−7.875	−7.732	−7.994
JPM	−7.485	−7.523	−7.474	−7.493	−7.547	−7.386	−7.305	−7.368	−7.364	**−7.454**
KO	−8.486	−8.534	−8.488	−8.488	−8.529	−8.279	**−8.395**	−8.278	−8.242	−8.348
MCD	−8.341	−8.389	−8.314	−8.347	−8.406	−7.736	**−7.970**	−7.741	−7.740	−7.896
MMM	−8.194	−8.235	−8.179	−8.189	−8.252	−7.398	**−7.518**	−7.371	−7.236	−7.334
MRK	−7.932	−7.970	−7.881	−7.928	−7.989	−7.657	**−7.792**	−7.681	−7.638	−7.757
MSFT	−7.571	−7.651	−7.601	−7.580	−7.664	−7.050	−7.107	−7.167	−7.114	**−7.208**
NKE	−7.517	−7.582	−6.331	−7.515	−7.603	−7.245	−6.298	−6.016	−7.205	**−7.273**
PFE	−8.094	−8.141	−8.095	−8.094	−8.130	**−8.051**	−7.942	−7.966	−7.997	−8.025
PG	−8.446	−8.486	−8.440	−8.448	−8.500	−8.090	−7.890	−8.054	−8.042	**−8.096**
TRV	−8.146	−8.181	−8.119	−8.143	−8.198	−7.556	**−7.587**	−7.552	−7.426	−7.585
UNH	−7.710	−7.745	−7.738	−7.710	−7.803	−7.759	**−7.873**	−7.692	−7.680	−7.767
UTX	−8.003	−8.039	−7.996	−8.008	−8.053	−7.498	**−7.654**	−7.467	−7.437	−7.565
V	−7.714	−7.764	−7.736	−7.719	−7.793	−7.588	**−7.643**	−7.545	−7.565	−7.591
VZ	−8.193	−8.230	−8.133	−8.193	−8.203	−7.625	**−7.769**	−7.600	−7.576	−7.713
WBA	−7.411	−7.453	−7.409	−7.412	−7.473	−7.156	**−7.286**	−7.136	−7.094	−7.193
WMT	−8.040	−8.082	−8.034	−8.041	−8.093	−7.514	−7.555	−7.533	−7.450	**−7.587**
XOM	−8.080	−8.125	−7.925	−8.079	−8.131	−7.827	−7.691	−7.563	−7.773	**−7.861**

6.7. DM t-tests for realized utility

Table 8. Diebold-Mariano t-statistics for testing the significance of realized utility gain.

Symbol	In-sample DM t-tests				Out-of-sample DM t-tests			
	Standard	Realized	LintonWu	VIX-joint	Standard	Realized	LintonWu	VIX-joint
Panel A:DM t-test for realized utility of open-to-close volatility relative to coupled GARCH(1,1) model								
AAPL	−0.332	−0.273	−0.761	−0.190	−1.933	3.079	−2.684	−2.179
AXP	−2.166	−1.461	−1.552	−2.004	0.926	0.948	0.919	0.503
BA	−0.900	−1.470	0.655	−1.024	0.832	1.051	1.011	−2.973
CAT	−1.781	−1.304	−0.500	−3.903	−2.274	0.824	0.053	−1.222
CSCO	−1.789	−3.323	−5.173	−2.422	−2.791	−10.132	−1.226	−2.844
CVX	−2.640	−1.211	−2.092	−2.631	−1.971	−0.212	−2.071	−3.103
DIS	−0.522	−1.902	−1.501	−0.381	−0.097	−11.313	1.493	−1.439
GS	−1.825	−0.996	−1.875	−1.850	−1.701	1.238	−1.058	−1.583
HD	−1.053	−1.058	−1.830	−1.044	−1.315	0.032	−1.010	−1.480
IBM	−1.747	−1.225	−2.396	−1.639	−1.468	2.366	−1.221	−2.712
INTC	−2.179	−1.139	−2.102	−2.137	0.262	2.144	1.774	−2.151
JNJ	−1.015	−1.006	−1.028	−1.023	1.223	1.113	0.715	−0.972
JPM	−1.289	−1.086	−2.303	−1.360	−1.648	−5.067	−1.440	−1.571
KO	−1.590	−1.140	−3.021	−1.174	−1.653	1.821	−2.144	−2.700
MCD	−1.109	−1.169	−2.420	−1.154	−1.645	1.860	−0.853	−1.604
MMM	−1.208	−1.271	−1.888	−1.368	1.127	1.387	1.282	−1.248
MRK	−2.691	−1.519	−4.042	−2.771	−1.200	0.776	−0.694	−1.413
MSFT	−1.352	−1.227	−2.969	−2.207	−1.781	−1.026	1.071	−2.574
NKE	−1.164	−0.929	−0.154	−1.062	−1.406	−10.015	−2.884	−2.897
PFE	−2.847	−2.018	−3.127	−2.720	0.604	−1.429	−1.638	−1.116
PG	−0.717	−0.846	−1.102	−0.575	0.592	−3.016	1.393	−0.995
TRV	−2.599	−1.417	−2.182	−2.791	−0.633	1.122	−0.312	−2.557
UNH	−0.964	−0.982	−1.740	−0.980	0.887	0.981	0.345	0.127
UTX	−2.141	−1.508	−2.524	−2.178	−0.574	1.735	−0.392	−1.991
V	−0.280	−1.365	0.348	0.582	1.210	1.596	−0.587	0.232
VZ	−0.977	−0.862	−0.586	−0.950	−1.591	2.483	0.747	−2.394
WBA	−0.334	−0.544	0.609	−0.096	−0.991	1.286	0.356	−2.162
WMT	−1.441	−1.037	0.902	−1.127	−1.223	0.360	0.925	−2.936
XOM	−2.427	−1.035	−1.025	−2.377	0.266	−2.607	−3.018	−1.215
Panel B: DM t-test for realized utility of close-to-close volatility relative to coupled GARCH(1,1) model								
AAPL	0.543	0.844	−0.620	0.645	−0.425	3.290	−0.188	−0.960
AXP	−1.887	−1.436	−1.845	−1.666	1.022	0.982	0.978	0.917
BA	0.028	−1.009	0.905	−0.102	0.905	1.058	1.042	0.183
CAT	−1.682	−1.175	−0.768	−3.397	−2.203	1.011	−0.058	−0.517
CSCO	−1.717	−1.943	−3.718	−1.886	−2.705	−10.351	−1.050	0.741
CVX	−2.608	−1.197	−1.371	−2.598	−1.702	0.378	−1.251	−2.929
DIS	0.095	−1.677	−1.468	0.208	0.638	−11.375	1.120	−0.666
GS	−1.772	−0.919	−0.140	−1.775	−1.546	1.542	−0.883	−1.487
HD	−1.067	−1.064	−1.825	−1.056	−1.341	−0.331	−1.181	−1.537
IBM	−1.353	−0.719	−0.920	−1.243	0.581	2.837	1.743	0.239
INTC	−2.022	−1.131	−1.338	−1.955	0.775	1.988	1.909	−0.408
JNJ	−1.018	−1.007	−1.021	−1.028	1.140	1.014	0.740	−0.977
JPM	−1.289	−1.084	−1.454	−1.361	−1.501	−3.575	−1.454	−1.584
KO	−1.678	−1.180	−2.700	−1.201	−1.829	1.426	−2.535	−2.719
MCD	−1.106	−1.160	−2.580	−1.147	−1.650	1.824	−0.987	−1.537
MMM	−1.208	−1.272	−1.643	−1.371	1.240	1.391	1.361	−0.704
MRK	−2.798	−1.537	−3.483	−2.894	−1.243	0.028	−2.401	−1.491
MSFT	−1.295	−1.091	−2.257	−1.695	−1.631	0.650	2.027	−2.148
NKE	−1.279	−0.962	−3.010	−1.099	−0.001	−10.353	−5.787	−0.869
PFE	−3.058	−2.251	−2.536	−2.917	−0.179	−2.841	−1.560	−1.831
PG	−0.489	−0.746	−0.495	−0.310	0.439	−3.034	1.380	−1.095
TRV	−2.542	−1.402	−1.839	−2.718	−0.171	1.365	0.202	−2.304
UNH	−0.994	−1.001	−1.698	−1.001	0.863	0.980	0.449	−0.287
UTX	−2.301	−1.686	−2.271	−2.235	−0.528	1.605	−0.343	−2.104
V	0.126	−1.393	0.437	0.741	1.482	1.602	0.779	0.953
VZ	−1.004	−0.902	−0.701	−0.986	−2.151	2.163	−0.507	−2.617
WBA	−0.312	−0.547	0.531	−0.072	−0.787	1.339	−0.016	−2.101
WMT	−1.524	−1.030	1.006	−1.122	−0.229	0.866	1.187	−2.830
XOM	−2.526	−1.056	−1.010	−2.463	0.060	−3.099	−2.907	−1.270

6.8. DM t-test for volatility forecast

Table 9. Diebold-Mariano t-statistics for testing the accuracy of volatility estimation.

	In-sample DM t-tests				Out-of-sample DM t-tests			
Symbol	Standard	Realized	LintonWu	VIX-joint	Standard	Realized	LintonWu	VIX-joint
Panel A:DM t-test for volatility forecast of open-to-close volatility relative to coupled GARCH(1,1) model								
AAPL	– 1.422	– 0.601	– 3.056	– 1.433	– 2.817	3.324	– 4.376	– 3.360
AXP	– 3.206	– 1.423	– 4.687	– 3.295	– 2.241	– 2.038	– 3.357	– 3.062
BA	– 2.193	– 1.561	– 5.413	– 2.348	– 0.782	0.875	– 5.203	– 2.864
CAT	– 2.545	– 0.739	– 6.675	– 36.097	– 2.499	0.866	– 8.262	– 8.931
CSCO	– 3.673	– 3.135	– 4.760	– 5.460	– 4.064	– 5.994	– 4.473	– 2.799
CVX	– 1.405	– 0.043	– 2.358	– 1.429	– 1.532	– 0.525	– 3.253	– 2.496
DIS	– 2.741	– 2.658	– 5.946	– 2.772	– 2.114	– 6.014	– 4.431	– 2.721
GS	– 2.667	– 1.705	– 4.085	– 2.620	– 1.864	0.722	– 4.385	– 2.056
HD	– 0.752	– 0.664	– 0.922	– 0.752	– 1.715	– 0.727	– 3.660	– 2.122
IBM	– 3.677	– 1.144	– 5.060	– 3.658	– 3.070	2.731	– 4.137	– 3.964
INTC	– 3.673	0.203	0.018	– 3.734	– 2.099	0.778	– 3.857	– 2.761
JNJ	– 0.975	– 0.949	– 1.122	– 0.977	1.627	0.128	– 2.033	– 1.193
JPM	– 2.212	– 1.204	– 3.329	– 2.189	– 1.921	– 2.515	– 4.597	– 2.153
KO	– 2.255	– 0.507	– 2.355	– 1.877	– 1.845	1.703	– 3.692	– 3.236
MCD	– 1.046	– 0.757	– 3.888	– 1.042	– 2.130	1.463	– 4.515	– 2.295
MMM	– 1.177	– 0.361	– 0.658	– 2.241	– 1.912	– 0.522	– 3.130	– 2.317
MRK	– 2.792	– 0.930	– 3.253	– 2.855	– 1.546	– 0.647	– 4.350	– 1.935
MSFT	– 1.790	– 0.369	– 2.645	– 2.441	– 3.232	– 2.699	– 6.197	– 3.125
NKE	– 3.035	– 1.204	– 19.793	– 2.532	– 2.404	– 5.322	– 11.079	– 3.295
PFE	– 3.754	– 1.264	– 3.056	– 3.624	– 0.797	– 2.349	– 3.547	– 2.687
PG	– 1.431	– 0.953	– 1.597	– 1.490	– 1.245	– 1.664	– 1.890	– 1.219
TRV	– 2.621	– 1.361	– 5.188	– 3.377	– 0.894	– 2.170	– 5.706	– 4.433
UNH	– 1.094	– 0.939	– 1.093	– 1.079	– 0.942	0.121	– 2.199	– 1.384
UTX	– 2.146	– 0.714	– 3.207	– 1.924	– 1.694	2.570	– 3.642	– 2.541
V	– 1.491	– 0.969	– 1.463	– 1.689	– 1.980	– 0.074	– 4.112	– 1.493
VZ	– 0.785	– 0.311	– 2.405	– 0.806	– 2.327	2.629	– 5.535	– 4.063
WBA	– 2.187	– 0.840	– 3.169	– 1.993	– 1.888	1.236	– 3.995	– 2.157
WMT	– 2.554	0.535	– 5.149	– 1.840	– 3.193	– 2.889	– 5.770	– 3.490
XOM	– 2.135	– 0.234	– 8.706	– 2.207	– 0.974	– 2.657	– 9.573	– 2.325
Panel B: DM t-test for volatility forecast of close-to-close volatility relative to coupled GARCH(1,1) model								
AAPL	– 2.111	– 0.809	– 2.838	– 2.147	– 2.046	3.410	– 3.612	– 2.837
AXP	– 4.195	– 1.570	– 5.839	– 4.320	– 2.013	– 1.777	– 2.955	– 3.168
BA	– 3.015	– 2.048	– 6.750	– 3.203	– 0.366	0.996	– 3.994	– 2.394
CAT	– 2.741	– 0.632	– 9.605	– 39.593	– 2.150	1.174	– 8.368	– 8.137
CSCO	– 0.367	– 0.024	0.357	– 0.968	– 3.875	– 5.963	– 2.747	– 2.273
CVX	– 1.604	0.096	– 3.794	– 1.626	– 1.343	– 0.316	– 3.331	– 2.447
DIS	– 3.465	– 2.715	– 5.651	– 3.523	– 1.980	– 6.142	– 4.367	– 2.637
GS	– 2.909	– 1.993	– 7.825	– 2.873	– 1.954	1.118	– 4.142	– 2.183
HD	– 0.839	– 0.727	– 1.334	– 0.839	– 1.727	– 0.901	– 3.853	– 2.424
IBM	– 5.081	– 1.550	– 6.895	– 5.091	– 2.109	4.100	– 2.238	– 3.521
INTC	– 3.312	0.646	– 2.479	– 3.372	– 1.842	1.156	– 5.440	– 2.461
JNJ	– 0.956	– 0.928	– 1.329	– 0.958	1.514	– 0.036	– 3.006	– 1.334
JPM	– 2.669	– 1.406	– 6.958	– 2.713	– 1.943	– 2.571	– 3.388	– 2.199
KO	– 1.623	0.149	– 2.648	– 1.318	– 1.831	1.538	– 3.084	– 2.963
MCD	– 1.123	– 0.770	– 4.142	– 1.120	– 2.159	1.670	– 3.580	– 2.328
MMM	– 2.385	– 1.295	– 6.947	– 3.677	– 1.359	0.222	– 2.538	– 1.971
MRK	– 2.205	– 0.157	– 3.284	– 2.293	– 1.469	– 0.514	– 2.915	– 1.845
MSFT	– 2.183	– 0.559	– 3.207	– 3.103	– 3.032	– 1.980	– 4.162	– 2.817
NKE	– 4.086	– 1.247	– 21.752	– 3.266	– 1.632	– 5.247	– 12.041	– 2.867
PFE	– 1.200	0.954	– 2.791	– 1.116	0.188	– 1.153	– 3.261	– 1.131
PG	– 1.494	– 0.926	– 2.993	– 1.567	– 1.273	– 2.566	– 2.453	– 1.316
TRV	– 3.077	– 1.500	– 8.328	– 3.765	– 0.698	– 1.881	– 4.981	– 4.072
UNH	– 1.100	– 0.931	– 1.616	– 1.084	– 0.779	0.373	– 2.454	– 1.221
UTX	– 1.905	– 0.292	– 4.151	– 1.589	– 1.706	2.579	– 3.340	– 2.596
V	– 1.684	– 1.010	– 1.710	– 1.941	– 0.965	0.607	– 3.430	– 0.726
VZ	0.377	0.754	– 1.362	0.379	– 2.879	2.151	– 6.284	– 4.405
WBA	– 2.127	– 0.700	– 3.279	– 1.946	– 1.512	1.151	– 2.205	– 2.030
WMT	– 1.441	0.839	– 3.269	– 0.893	– 2.708	– 2.165	– 5.413	– 3.042
XOM	– 2.292	– 0.003	– 10.201	– 2.340	– 1.487	– 3.179	– 9.323	– 2.781

6.9. DM t-test for QLIKE

Table 10. Diebold-Mariano t-statistics for testing the QLIKE.

	In-sample DM t-tests				Out-of-sample DM t-tests			
Symbol	Standard	Realized	LintonWu	VIX-joint	Standard	Realized	LintonWu	VIX-joint
Panel A:DM t-test for QLIKE of open-to-close volatility relative to coupled GARCH(1,1) model								
AAPL	− 1.337	− 0.294	− 0.979	− 1.219	− 3.044	2.513	− 4.785	− 3.768
AXP	− 3.048	− 1.582	− 2.805	− 2.887	0.563	0.754	0.763	− 0.273
BA	− 1.866	− 1.705	− 0.472	− 1.937	0.604	0.879	0.526	− 3.905
CAT	− 2.791	− 1.196	− 4.122	− 29.826	− 2.539	2.592	− 5.403	− 6.898
CSCO	− 2.596	− 3.974	− 6.927	− 5.598	− 3.581	− 15.820	− 1.905	− 3.963
CVX	− 3.314	− 0.782	− 3.560	− 3.311	− 2.402	− 0.566	− 3.548	− 3.952
DIS	− 2.425	− 2.409	− 1.829	− 2.239	− 0.875	− 19.491	− 0.283	− 2.304
GS	− 2.242	− 0.852	− 4.184	− 2.296	− 2.202	2.836	− 2.683	− 2.184
HD	− 1.211	− 1.070	− 2.532	− 1.185	− 1.491	2.075	− 1.830	− 1.696
IBM	− 2.907	− 1.442	− 3.769	− 2.774	− 2.490	2.967	− 2.927	− 4.240
INTC	− 3.464	− 0.748	− 2.597	− 3.663	− 0.467	2.975	0.462	− 3.240
JNJ	− 1.023	− 1.002	− 1.136	− 1.039	1.276	1.352	0.469	− 0.977
JPM	− 1.670	− 1.190	− 4.064	− 1.944	− 2.337	− 8.385	− 3.596	− 2.322
KO	− 2.635	− 1.058	− 4.096	− 1.547	− 2.385	4.048	− 3.750	− 3.789
MCD	− 1.251	− 1.220	− 5.664	− 1.400	− 2.047	2.158	− 2.333	− 2.274
MMM	− 1.331	− 1.129	− 1.966	− 1.691	0.890	1.571	0.954	− 1.433
MRK	− 4.480	− 1.677	− 7.217	− 4.779	− 1.586	2.202	− 4.660	− 2.345
MSFT	− 1.941	− 1.284	− 5.542	− 5.059	− 2.416	− 5.043	− 3.678	− 3.559
NKE	− 2.156	− 0.994	− 13.408	− 1.556	− 2.183	− 18.802	− 12.600	− 4.342
PFE	− 4.375	− 2.250	− 4.790	− 4.089	0.366	− 3.558	− 2.315	− 1.781
PG	− 0.982	− 0.849	− 1.606	− 0.892	0.079	− 8.142	− 1.316	− 2.223
TRV	− 3.283	− 1.402	− 3.670	− 3.931	− 0.949	0.380	− 1.894	− 3.421
UNH	− 0.998	− 0.984	− 2.053	− 1.003	0.702	1.043	− 0.153	− 0.426
UTX	− 3.095	− 1.513	− 3.646	− 3.289	− 1.226	2.613	− 1.388	− 2.748
V	− 0.585	− 1.474	− 0.087	0.129	− 0.263	2.544	− 2.953	− 1.528
VZ	− 1.007	− 0.705	− 1.269	− 1.005	− 2.537	3.865	− 2.627	− 3.313
WBA	− 1.350	− 0.666	0.116	− 1.213	− 1.158	2.356	− 1.276	− 2.663
WMT	− 1.743	− 1.039	0.012	− 1.226	− 2.292	− 0.981	− 1.091	− 3.735
XOM	− 4.080	− 1.005	− 16.070	− 3.902	− 1.712	− 7.693	− 12.786	− 2.200
Panel B: DM t-test for QLIKE of close-to-close volatility relative to coupled GARCH(1,1) model								
AAPL	− 0.233	0.808	− 0.732	− 0.118	− 1.147	3.201	− 1.908	− 2.125
AXP	− 2.859	− 1.575	− 3.879	− 2.616	0.956	0.858	0.911	0.731
BA	− 0.768	− 1.273	− 0.914	− 0.879	0.789	0.968	0.843	− 0.809
CAT	− 2.772	− 1.140	− 6.365	− 28.318	− 2.332	2.787	− 4.192	− 5.619
CSCO	− 2.451	− 2.441	− 5.472	− 4.815	− 3.441	− 16.251	− 1.760	− 0.347
CVX	− 3.339	− 0.785	− 2.448	− 3.331	− 1.973	0.003	− 3.130	− 3.634
DIS	− 1.665	− 2.229	− 1.826	− 1.486	0.054	− 19.699	− 0.052	− 1.344
GS	− 2.185	− 0.757	− 5.230	− 2.219	− 1.948	3.541	− 1.727	− 1.997
HD	− 1.228	− 1.078	− 2.566	− 1.201	− 1.592	1.840	− 2.009	− 1.827
IBM	− 2.177	− 0.405	− 1.631	− 2.041	0.419	3.922	0.943	− 0.550
INTC	− 3.538	− 0.997	− 2.813	− 3.716	0.252	2.725	0.800	− 1.611
JNJ	− 1.025	− 1.003	− 1.374	− 1.042	1.066	1.111	0.123	− 0.993
JPM	− 1.686	− 1.202	− 4.410	− 1.980	− 2.195	− 7.708	− 2.953	− 2.357
KO	− 2.474	− 0.972	− 3.485	− 1.489	− 2.500	3.974	− 3.637	− 3.632
MCD	− 1.237	− 1.185	− 6.393	− 1.374	− 2.014	2.238	− 1.716	− 2.101
MMM	− 1.441	− 1.338	− 2.489	− 1.920	1.233	1.648	1.340	− 0.823
MRK	− 4.112	− 1.576	− 6.136	− 4.383	− 1.630	2.187	− 4.574	− 2.400
MSFT	− 1.886	− 1.153	− 4.310	− 4.641	− 2.131	− 2.022	− 0.293	− 2.872
NKE	− 2.474	− 1.049	− 36.773	− 1.645	− 0.477	− 19.618	− 21.437	− 2.127
PFE	− 3.386	− 1.307	− 3.195	− 3.225	1.158	− 3.319	− 1.840	− 1.567
PG	− 0.754	− 0.750	− 1.683	− 0.621	0.004	− 7.936	− 1.306	− 2.217
TRV	− 3.513	− 1.460	− 4.395	− 4.246	− 0.352	1.088	− 0.467	− 2.977
UNH	− 1.031	− 1.006	− 2.044	− 1.027	0.723	1.134	0.082	− 0.763
UTX	− 3.208	− 1.708	− 3.483	− 3.257	− 1.189	2.406	− 1.232	− 2.890
V	− 0.146	− 1.463	0.103	0.438	0.860	2.396	− 1.448	− 0.045
VZ	− 0.972	− 0.656	− 1.648	− 0.959	− 3.231	3.688	− 4.367	− 3.618
WBA	− 1.274	− 0.660	− 0.006	− 1.117	− 0.881	2.392	− 0.852	− 2.488
WMT	− 1.915	− 1.030	0.604	− 1.230	− 0.946	0.346	0.297	− 3.204
XOM	− 4.158	− 1.010	− 16.912	− 3.971	− 2.010	− 8.073	− 11.872	− 2.249

7. Conclusion

In this paper we have proposed a coupled GARCH model for intraday and overnight volatility that incorporates information from the VIX index and the implied jump variance from the stock options markets. For each component, we further specify autoregressive and cross terms with VIX-dependent coefficients and implied volatility terms. The stationarity conditions and QMLE asymptotics of the estimators are provided. This model is straightforward to estimate and helps improve forecast accuracy over benchmark models. The auxiliary variables (VIX and implied jump) are valuable for improving the forecast of both open-to-close and close-to-close volatility.

Disclosure statement

No potential conflict of interest was reported by the author(s).

References

Andersen, T.G. and Bollerslev, T., Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. *Int. Econ. Rev. (Philadelphia)*, 1998, **39**(4), 885–905.

Andersen, T.G., Bollerslev, T., Diebold, F.X. and Labys, P., Modeling and forecasting realized volatility. *Econometrica*, 2003, **71**(2), 579–625.

Baillie, R.T., Bollerslev, T. and Mikkelsen, H.O., Fractionally integrated generalized autoregressive conditional heteroskedasticity. *J. Econom.*, 1996, **74**(1), 3–30.

Barndorff–Nielsen, O.E. and Shephard, N., Econometric analysis of realized volatility and its use in estimating stochastic volatility models. *J. R. Stat. Soc. Ser. B (Stat. Methodol.)*, 2002, **64**(2), 253–280.

Blanc, P., Chicheportiche, R. and Bouchaud, J.P., The fine structure of volatility feedback II: Overnight and intra-day effects. *Phys. A: Stat. Mech. Appl.*, 2014, **402**, 58–75.

Blanc, P., Donier, J. and Bouchaud, J.P., Quadratic Hawkes processes for financial prices. *Quant. Finance*, 2017, **17**(2), 171–188.

Bollerslev, T., Generalized autoregressive conditional heteroskedasticity. *J. Econom.*, 1986, **31**(3), 307–327.

Bollerslev, T., Engle, R.F. and Nelson, D.B., ARCH models. *Handbook Econom.*, 1994, **4**, 2959–3038.

Bollerslev, T., Hood, B., Huss, J. and Pedersen, L.H., Risk everywhere: Modeling and managing volatility. *Rev. Financ. Stud.*, 2018, **31**(7), 2729–2773.

Bougerol, P. and Picard, N., Strict stationarity of generalized autoregressive processes. *Ann. Probab.*, 1992, **20**(4), 1714–1730.

Brenner, R.J., Harjes, R.H. and Kroner, K.F., Another look at models of the short-term interest rate. *J. Financ. Quant. Anal.*, 1996, **31**(1), 85–107.

Cliff, M., Cooper, M.J. and Gulen, H., Return differences between trading and non-trading hours: Like night and day. *SSRN eLibrary*, 32, 2008.

Dhaene, G. and Wu, J., Incorporating overnight and intraday returns into multivariate GARCH volatility models. *J. Econom.*, 2020, **217**(2), 471–495.

Diebold, F.X., Comparing predictive accuracy, twenty years later: A personal perspective on the use and abuse of Diebold–Mariano tests. *J. Bus. Econ. Stat.*, 2015, **33**(1), 1–1.

Diebold, F.X. and Mariano, R.S., Comparing predictive accuracy. *J. Bus. Econ. Stat.*, 2002, **20**(1), 134–144.

Duan, J.C., The GARCH option pricing model. *Math. Finance*, 1995, **5**(1), 13–32.

Engle, R.F., Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econom. J. Econom. Soc.*, 1982, **50**(4), 987–1007.

Fleming, J., Kirby, C. and Ostdiek, B., The specification of GARCH models with stochastic covariates. *J. Futures Mark. Futures Options Derivative Products*, 2008, **28**(10), 911–934.

Francq, C. and Zakoïan, J.M., Strict stationarity testing and estimation of explosive and stationary generalized autoregressive conditional heteroscedasticity models. *Econometrica*, 2012, **80**(2), 821–861.

Gatheral, J., Jaisson, T. and Rosenbaum, M., Volatility is rough. *Quant. Finance*, 2018, **18**(6), 933–949.

Gatheral, J., Jusselin, P. and Rosenbaum, M., The quadratic rough Heston model and the joint S&P 500/VIX smile calibration problem. arXiv:2001.01789, 2020.

Grossman, S.J. and Shiller, R.J., The determinants of the variability of stock market prices. *Am. Econ. Rev.*, 1981, **71**(2), 222–227.

Grover, R. and Thomas, S., Liquidity considerations in estimating implied volatility. *J. Fut. Mark.*, 2012, **32**(8), 714–741.

Hansen, P.R. and Lunde, A., A forecast comparison of volatility models: Does anything beat a GARCH (1, 1)?. *J. Appl. Econom.*, 2005, **20**(7), 873–889.

Hansen, P.R., Huang, Z. and Shek, H.H., Realized GARCH: A joint model for returns and realized measures of volatility. *J. Appl. Econom.*, 2012, **27**(6), 877–906.

Hao, J. and Zhang, J.E., GARCH option pricing models, the CBOE VIX, and variance risk premium. *J. Financ. Econom.*, 2013, **11**(3), 556–580.

Hardiman, S.J., Bercot, N. and Bouchaud, J.P., Critical reflexivity in financial markets: A Hawkes process analysis. *Euro. Phys. J. B*, 2013, **6**(10), 442.

Hörmann, S., Horváth, L. and Reeder, R., A functional version of the ARCH model. *Econ. Theory.*, 2013, **29**(2), 267–288.

Linton, O. and Wu, J., A coupled component DCS-EGARCH model for intraday and overnight volatility. *J. Econom.*, 2020, **217**(1), 176–201.

Liu, J.C., Stationarity of a Markov-switching GARCH model. *J. Financ. Econom.*, 2006, **4**(4), 573–593.

Martens, M., Measuring and forecasting S&P 500 index–futures volatility using high–frequency data. *J. Futures Mark. Futures Options Derivative Products*, 2002, **22**(6), 497–518.

Nana, G.A.N., Korn, R. and Erlwein-Sayer, C., GARCH-extended models: Theoretical properties and applications. *arXiv*, arXiv:1307.6685, 2013.

Patton, A.J., Volatility forecast comparison using imperfect volatility proxies. *J. Econom.*, 2011, **160**(1), 246–256.

Straumann, D. and Mikosch, T., Quasi-maximum-likelihood estimation in conditionally heteroscedastic time series: A stochastic recurrence equations approach. *Ann. Stat.*, 2006, **34**(5), 2449–2495.