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A model of dynamic information production for initial public offerings

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We develop a multi-period information-theoretic model of initial public offering (IPO) in the presence of an adverse selection problem that addresses both underpricing in an IPO and subsequent underperformance in the long run. In this model, information asymmetry exists among the owner of a firm going IPO, underwriter(s), informed analysts and uninformed investors. Information asymmetry between the owner and the investors is reduced through both the initial information production by some investors and the evaluations by informed analysts in the subsequent periods as new information arrives on the market. By incorporating future uncertainty, subsequent information revelation, certain firm-specific constraints and the actions of the agents, the optimal or sub-optimal actions of the agents are identified. The model explains why firms going public are underpriced at the IPO and, on average, underperform in the long run. The results are also compatible with social comparison explanations from a behavioral finance perspective.

Keywords: Initial public offering; Information asymmetry; Bayesian equilibrium; Social comparison

JEL Classifications: G32, C11, C61, D81, D82

1. Introduction

Initial Public Offering (IPO) is a process that makes a young firm become a publicly traded corporation which facilitates access to equity capital (common shares) and other sources. On the one hand, it provides the firm with some funding for initial investment, expansion, and other types of operating activities. On the other hand, it can help the owner-manager (the owner, henceforth) sell the firm in a strategic way to maximize her final wealth. From an owner's point of view, the IPO process is the first of several steps in diversifying her wealth by selling a part of the firm through common shares and then strategically selling the remaining shares of the firm to outside investors over multiple periods. However, asymmetric information about the future prospects of a firm would be a major hindrance in attaining the owner's final objective. If outside investors are uncertain about the true quality of the firm and hence face an adverse selection problem, they may not make accurate investment decisions. Due to the increased idiosyncratic risk of their investment, a higher risk premium may be requested to justify such an investment [Megginson and Weiss 1991, Ljungqvist and Wilhelm 2003, and others]. In a financial market with different quality types of firms, the absence of full information about a high-quality firm poses a threat to the success of selling the firm at its fair value due to the existence of moral hazard problem among firms (see Ferris *et al.* 2013) where lower quality firms may mimic strategies of high-quality firms. So, both the owner's moral hazard problem and the investors' adverse selection problem are tied to asymmetric information in the market. In this paper, we introduce a multi-period information-theoretic model of IPO in a Bayesian setup and consider the optimization problems of both the owner and informed institutional investors (underwriters). Our model is also consistent with signaling and behavioral finance approaches via suitable assumptions to address both IPO underpricing of a new issue and its potential long-term underperformance.

If the owner of a firm does not reveal any information, outside investors are, on average, likely to value the firm's security inaccurately, where the financial security of a high-quality

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firm is likely to be underpriced and that of a low-quality firm is likely to be overpriced. An obvious incentive-compatible strategy for the owner of a higher-quality firm to maximize her wealth is to reveal information sequentially to the public at different stages of the firm's life. Since it may not be possible to produce enough information to completely reveal the true value of the firm at any point in time, one effective strategy for the owner would be to invite, at a cost, a certain group of informed investors (say, analysts and underwriters), who are endowed with technological resources and capacity to engage in information production, to process and evaluate information, and to reveal about the quality of the firm. This conjecture is supported by both theoretical and empirical literature, for evidence, see Sherman and Titman (2002), Lowry *et al.* (2017), and Hu *et al.* (2021), among others. In addition, since the firm has a longer life, the owner's strategic compensation package to the informed investors should entice them to further engage in information production without additional compensation (for various reasons, such as continuing to invest in the firm or investing more in the firm, support the firm for future business, etc.). It would also entice other outside analysts (who could not undertake the business at the IPO process but were interested in investing in such a firm for portfolio management) to engage in information production to sequentially reveal the true quality of the firm. The empirical literature is quite rich in the subsequent information production that addresses the likeliness of following up with recommendations after the quiet period of the IPO by the underwriting firms and others (including Bradley *et al.* 2003, O'Brien *et al.* 2005, James and Korceski 2006). The new information revealed to the public at the time of IPO by underwriters/analysts should help reduce the information asymmetry about the firm and increase investors' interest in engaging into further information production as they invest in the firm's security at the time of IPO or become interested in investing in the firm's security after IPO. This initial information production and investors' subsequent interest in information production would help the owner maximize her wealth by strategically choosing a fraction of the share to sell at the time of IPO and the remaining shares at its fair value in the future. Since these informed investors are helping the owner in the IPO process by engaging in information production, the owner, in return, compensates them by offering cash (flat fee) and a certain number of securities at a discounted price to recoup information production cost. The owner, then, would be able to strategically choose an equilibrium (IPO) price for its share and a fraction of the firm's share to complete the IPO. This discounted price would be considered 'money on the table' for the investors as part of their compensation in the IPO. It is known as 'underpricing' in the IPO literature and is one of the most extensively documented puzzles in financial economics.

The first description of the underpricing phenomenon that accompanies initial public offering (IPO) is found in McDonald and Fisher (1972), Logue (1973) and Ibbotson (1975). Later, Ritter (1991) and Loughran and Ritter (1995), in their extensive studies, also document this anomaly as a puzzling phenomenon. During 2001–2021, mean first-day return was 18.20% and money left on the table was \$130.22 billion.† In explaining the IPO underpricing phenomenon, a body of literature identifies it as a response to the asymmetric information between insiders (the owner(s)) and outsiders (investors) regarding the value of the firm or its future growth potential (see Boulton *et al.* 2010, 2011, Chang *et al.* 2017). Ibbotson *et al.* (1988) report an average initial return of 16.4% for IPOs made from 1960 to 1987, computed from the offer price to the closing price on the first day of trading in the secondary market. Benveniste and Spindt (1989) consider an information-gathering model where information asymmetry among investors is also highlighted. In this model, underwriters get demand and price feedback from informed investors. Moreover, IPO underpricing compensates for information production and the cost of the potential purchase of overpriced issues. Lee *et al.* (1999) also consider differential information among investors and provide evidence from Singapore IPOs that large investors who are better informed about IPO value can take advantage of this information advantage in their IPO application strategies. Another explanation of the underpricing puzzle popular with investment bankers and other practitioners is that underpricing generates publicity about the firm making the IPO and induces investors to learn more about that firm. They contend that this 'publicity effect' or 'desire for publicity effect' leads to a run-up in the secondary market share price and, consequently, is in the best interests of firms going public (Chemmanur 1993). Rajan and Servaes (1997) examine analysts/informed investors following IPOs and find that higher underpricing leads to increased analysts following the issue after the IPO. In their paper on internet firms, Demers and Lewellen (2003) explains IPO underpricing as a substitute for conventional marketing and publicity techniques. From a behavioral finance perspective, Chang (2011) finds evidence for a 'social comparison' explanation of IPO underpricing based on a sample of Taiwan IPOs whose offer prices are influenced by peers in the same industry considerably (see also the references there for other behavioral finance approaches). Similar empirical evidence of social/peer comparison for both IPO issuing firms and investors in India (studying IPOs from April 2004 to February 2013) is reported by Jain and Madhukar (2015). Considering stocks listed on CRSP US Stock Database from 01/04/2000 to 03/30/2022, we also detect such a peer or social comparison effect to explain abnormal IPO returns, price-to-earnings ratios, and market capitalization-to-total assets ratios. See section 5 and table 1 for details.

While the stock price run-up in the secondary market justifies the 'underpricing' phenomenon, financial economists are puzzled by the long-run 'underperformance' findings in the empirical research. Firms that are underpriced in the IPOs, on average, underperform in the long-run (see Ooi et al. (2019)). Ritter (1991) and Loughran and Ritter (1995) also detect this phenomenon and propose that the severe underperformance of IPOs implies that investors may systematically be too optimistic about the prospects of the firms that are issuing

† Ritter, Jay. 'IPOs 2021 Underpricing '. Retrieved from https://site.warrington.ufl.edu/ritter/ipo-data/.

Table 1. Regression results for IPO returns, P/E ratios and market capitalization from CRSP US Stock Database from 01/04/2000 to 03/30/2022.

Independent Variable	Dependent Variable $\mathbf{AR}_j$	$\mathbf{PE}_i$	$\mathbf{MC/TA}_i$
AR_j	0.044***		
	(0.015)		
AR_k	0.027*		
	(0.016)		
AR_l	– 0.003		
	(0.015)		
PE_j		0.064***	
		(0.022)	
PE_k		0.007	
		(0.022)	
PE_l		– 0.021	
		(0.022)	
MC/TA_j			0.062***
			(0.022)
MC/TA_k			0.022
			(0.022)
MC/TA_l			0.039*
			(0.022)
Constant	0.012***	15.513	2.836***
	(0.002)	(9.527)	(0.186)
Observations	4,178	1,807	2,001
R-squared	0.003	0.005	0.006
Adj. R-squared	0.002	0.004	0.005
F-Stat	3.767***	3.212**	4.114***

Note: Following Jay Ritter,† we exclude IPOs with an offer price of less than $5.00. We also exclude ADRs, unit offers, closed-end funds, REITs, natural resource limited partnerships, small best efforts offers, banks and S&Ls. AR stands for abnormal returns which are calculated by subtracting index return from the IPO return on the first day of trading. 'i' indicates stock i and $j/k/l$ means stocks that went public before stock i. Stock j went public before stock k, and stock k went public before stock l, such that $i < j < k < l$. PE stands for price to earnings ratio. MC/TA means market capitalization to total assets ratio. Standard error is reported in the parenthesis under each coefficient. *, **, *** refer to 10%, 5%, and 1% statistical significance respectively.

equity for the first time. Loughran (1997), Rajan and Servaes (1997), and Chou *et al.* (2009) also report over-optimistic post-issue earnings forecasts, subsequent deteriorating performance after the IPO, and continuous investment of the issuers on low-quality projects. One interpretation of the findings in Loughran (1997) is that firms deliberately and successfully manage to improve earnings before going public to mislead investors. Another implication is the overoptimism of the issuers/ managers on the net present value of their projects. Teoh et al. (1998) and Rangan (1998) confirm that issuers with high levels of discretionary accruals (which boost earnings relative to cash flows) have the worst subsequent stock returns. Brav and Gompers (1997) report that long-run underperformance usually occurs in small firms (with high market-to-book stocks) that are not backed by venture capitalists. The conflicts of interest between issuers and the intermediaries are also mentioned in the literature, see, for example, Jenkinson and Ljungqvist (2001) and the references there. Lyandres *et al.* (2008) find evidence that when an investor factor is added to the three-factor Fama-French model, the long-run underperformance is reduced substantially. Ritter (2011) points out that firms that expand more rapidly than their internally generated cash permits wind up disappointing investors on average. Moreover, Brau *et al.* (2012) document that firms that make an acquisition in the first year after going public subsequently underperform. The literature about the long-run performance of IPOs is still mostly empirical. Moreover, there is no consensus on whether the IPOs really underperform or whether such underperformance of IPOs is significant compared to similar (e.g. small and high-growth) firms. See, for example, Ljungqvist *et al.* (2006), Ritter (2011), and the references there. However, this long-run underperformance phenomenon strikingly implicates the importance of re-evaluation of issuing firm's future prospects (Ritter and Welch 2002).

Earlier theoretical literature focused on modeling IPO decisions as a single shot or dissipative signaling to investors. See, for example, Allen and Faulhaber (1989), Welch (1989), Grinblatt and Hwang (1989), and Chemmanur (1993). In the presence of asymmetric information, Chemmanur (1993) proposes strategic pricing and allocation of shares among a group of investors for producing information at IPO with a reduced IPO price. In this model, underpricing serves as publicity to entice further information production after IPO so that the secondary market price is more reflective of the true value of the firm. However, this spirit of desire to learn more by other investors after the IPO has not been addressed completely, and the long-run underperformance is left as an open problem. Aggarwal *et al.* (2002), Benninga *et al.* (2005), and Ljungqvist *et al.* (2006) model the IPO decision of the firm in a dynamic framework. Aggarwal *et al.* (2002) develop a model where managers strategically underprice IPOs to generate information momentum and hence to maximize personal wealth from selling shares at lockup expiration. However, it is unclear from their model about the explicit role of the underwriter and the conflicts of interest between the owner and the underwriter. Benninga *et al.* (2005) study the timing dimension of the decision to go public by examining the possibility of privatizing publicly traded firms. Ljungqvist *et al.* (2006) consider a 'hot' market with optimistic investors for the optimization problem of the issuers by focusing on the demand side of the IPOs. Their model covers underpricing, hot issue markets, and the long-run underperformance, which are all linked to the presence of a class of irrational investors.

We attempt to extend these theoretical models by focusing on the roles of further information production in the post-IPO market, the firm-specific constraints, and the preferences of the underwriter(s). We develop a theoretical multi-period dynamic model to capture the IPO underpricing and to explain the long-run performance of IPOs. In this three-period model (with actually six stages), the insider (owner) is assumed to know the firm's quality type with certainty. She goes public at the time '0' by inviting a group of investors to produce information and allocating a certain number of shares at a discounted price. When the issue is traded in the secondary market, all investors (including those who participate in the IPO) trade on this security. The security price in the secondary market is essentially a function of all investors' expectations based on the current information set in a Bayesian setup. If the owner finds that the market's perceived value is lower than the actual stock price (based on the actual quality type), she waits to sell residual shares in the secondary market and vice versa. This feature of the model addresses why an overvalued firm issues equity in the secondary market and why its stock price performs poorly in the long run.

This paper focuses on an information-production model in a principal-agent framework though it also has signaling and behavioral finance components. We consider three quality types of firms: high, medium, and low qualities. One can argue that a medium-quality firm is more likely to represent a typical firm in a given industry with a better chance of mimicking a high-quality firm (compared to a low-quality firm), but also with a higher risk of being identified as a low-quality firm. In this model, the strategies of the owner would depend on certain firm-specific characteristics and constraints which are not perfectly known by the outsiders. The informed investors would only know a set of plausible strategies of a high-quality firm based on their prior beliefs and experience in a certain industry. In the presence of uncertainty/information asymmetry and a relatively rich set of strategies (of a high-quality firm), a lower-quality firm would have the ability to act like a high-quality firm. This reasoning is also compatible with the social comparison explanation of IPO underpricing. See, for example, Chang (2011), Jain and Madhukar (2015) as well as our US-market empirical findings in section 5. So, the owner's aim is to find the best decision in a certain set of (admissible) strategies to maximize her expected total gains from the sale of the shares by staying in the game. Following the empirical evidence on the initial optimistic beliefs of the investors, we assume the post-IPO evaluations of the analysts are more accurate than the initial evaluations. We find that the initial amount lost during the IPO underpricing is often compensated by the future gains from the increased secondary market values as long as the owner of a high-quality firm chooses to sell a minimal amount of the shares before the terminal time. Moreover, on average, a high-quality firm has a greater equilibrium IPO underpricing and smaller IPO share allocation than the other firms. even with similar firm-specific prospects and constraints. However, a fully separating equilibrium is not expected since the actions of the owners don't necessarily reveal the actual quality types. There is still a certain range of strategies of high-quality firms, which allows the other quality types to partially mimic a feasible strategy, supporting a partial pooling equilibrium. For example, if the initial price reduction range for a high-quality firm in a specific industry is believed to be between 10% and 40%, then the owner (regardless of the quality type) should choose the IPO price in this range to stay in the game. The initial allocations and the IPO price chosen by the owners become a part of the subsequent information revelation, improving the quality of the analysts' evaluations at date 1.

We also specify the posterior net gains of the underwriters depending on the fee and contract structures. We then discuss the implications of the model for the optimization problems of both the owner and the underwriters. In particular, we discuss how the warrant rate affects the optimal choices of the agents. For the empirical results concerning warrant rates, we refer the reader to Barry *et al.* (1991), Dunbar (1995), and Torstila (2001). An interesting finding of the model is that the underwriter's profit is usually maximized when the underpricing is minimal for a high-quality firm. However, the underwriter may not fully take advantage of this scenario since he wouldn't know the actual quality type and the firm-specific information with certainty in advance. Another implication of the model is that the long run underperformance of many issues may also be explained by 'initial overperformance' due to several factors that include overoptimism of the market, information asymmetry, and potential conflicts between the owners and the underwriters.

The rest of the paper is organized as follows: In section 2, we describe the model. The specification of the information production process and the payoff structure is shown in section 3, and we characterize the equilibrium for our model in section 4. In section 5, we describe the empirical implications of our model on numerical examples, relating it to the existing evidence. We conclude the paper in section 6. A mathematical appendix and the references are given at the end.

2. The model and problem formulation

The problem is designed as a three-period dynamic game-theoretic model between the owner of the firm and a syndicate of underwriters/ investment banks. For simplicity, we assume there is one leading underwriter who manages both the IPO process and the subsequent sales of the shares (also acting as a broker for the firm). The other underwriters and analysts are considered informed investors at date 0. We do not explicitly model the problem of outside (uninformed) investors or informed analysts who are considered price-takers or small investors. Moreover, the optimization problem of each underwriter and their agency conflicts in the syndicate are ignored. We only consider the overall gains of the underwriters who are involved in the process of both the IPO and post-IPO sales of the firm's shares.

All the players in the game, including the owner, are assumed to be risk-neutral. The owner has perfect information about her firm's prospects, quality type and projects while the investors have some noisy, imperfect information about the firm. When investors investigate the firm based on the information available to them at that particular time, they receive some (noisy) information that is still assumed to be incomplete. The analysts re-evaluate the firm in the subsequent periods and investors incorporate that into their incomplete information set.

We assume that the owner has access to a project which is to be implemented at time 0 and be completed at date 2, offering cash flows at the end of period 2. The quality type of the firm takes values in the set $\{H, M, L\}$, also indicating the quality type of the cash flow from the project which can be high (H), medium (M) or low (L). The owner knows the actual type of the project and hence the type of the firm. However, investors (outsiders) have asymmetric information about the quality type of the project and hence the type of the firm. They only know that it can be high, medium or low with certain probabilities at times $t = 0$, 1 and at the beginning of date 2. They can observe the owner's actions in each period (although not always perfectly) when the owner decides to sell some fraction of the remaining ownership. At the end of period 2 ($t = 2+$), the quality type of the firm is known by all investors. We denote the value of a firm with quality type S at time t by V_t^S, for $t = 0, 1, 2$ and $S \in \{H, M, L\}$. We sometimes drop S when the quality type is given or clear from context. Initially, the investors value the firm by assigning prior probabilities $P(V = V^S) = p_S$, for $S = H$, M and L, which form the initial information set Ω_0 with $p_L + p_M + p_H = 1$ and $0 < V^L < V^M < V^H$. Then, the uninformed investors assign the expected value of the firm as the (initial) fair market price V_0:

$$V_0 \triangleq E[V|\Omega_0] = \sum_{S=L,M,H} p_S V^S. \tag{1}$$

To address information asymmetry, some analysts and informed investors are invited to produce information as part of the IPO process in period 0. These investors are assumed to bid for (the entire block of available) shares if they assign a good evaluation on the firm. The information production process is organized by the (leading) underwriter who also manages the distribution of the shares among the bidders. He collects fees from the owner for his services. For simplicity, we consider a firm-commitment contract in the sense that the unsold shares would be taken over by the syndicate of the underwriters, but the distribution of the fees among the underwriters is ignored in this work. The reader can refer to Torstila (2001) for an empirical study regarding the fee distribution in a syndicate. By offering equity at a lower price, the owner compensates the investors who produce information and bid for shares. Some amount of underpricing would also be beneficial for the underwriter to reduce the costs and risks regarding the actual quality type (including the reputation costs for incorrect evaluations or recommendations). On the other hand, the underwriter might not impose a very low IPO price since this may conflict with the owner's preferences and can also reduce the amount of fees collected, which are proportional to the IPO share price.

Now, we describe the fee rate, f_t, for $t = 0$, 1 and 2. The rates include the IPO fees, transaction costs and other brokerage fees. They depend on the information available at time t for each quality type S. The initial rate f_0 may be larger due to the IPO costs (advertising, compensation for the analysts, official procedures, etc.), and the extent of the initial uncertainty involved. This rate, which usually clusters around 7% for US IPOs and is a bit smaller in European IPOs (Chen and Ritter 2000), may also depend on the initial demand for the shares of the firm, the size of the IPO and the amount of IPO underpricing (e.g. a decreasing function of demand, IPO underpricing and IPO size). See also Jenkinson and Ljungqvist (2001) and Torstila (2001) for IPO spreads and management fees. The rates f_1 and f_2 depend on the evaluation results (as a proxy for liquidity of the shares). We assume that the rate f_i is a decreasing function of the implied market price V_i at date i. Since our model assumes the risk neutrality, the optimization problem would involve some linear programming with constraints. For tractability purposes in numerical examples, we consider a fee structure which is linear or quadratic in V_{IPO}. The initial fee rate, f_0, is given by

$$f_0 = (V_{IPO}/V_0)^2 (f_{base} + f_{IPO}), \tag{2}$$

where f_{base} is the base fee for each period, the ratio V_{IPO}/V_0 represents an underpricing incentive by the underwriter, and f_{IPO} accounts for the extra costs of the IPO process or risks of the initial uncertainty. The fee rate at time 1 is proportional to f_{base}: $f_1 = (V_{IPO}/V_1) f_{base}$ which reflects the impact of the IPO underpricing on the post-IPO market price. Finally, the uncertainty risk is minimal at date 2, and so f_2 is simply taken as a constant to account for fixed transaction/brokerage fees, e.g. $f_2 = f_{base}$. Some portion of the fees is paid by warrants (shares of the firm, for simplicity) and the remaining part by cash. Let l_0 and l_1 denote the proportion of the fees paid as warrants at times 0 and 1, respectively. They are determined by the underwriter, and their values depend on the IPO price, the market price V_i and a choice of base level, l: $l_0 = l(V_0/V_{IPO})^2$ and $l_1 = l(V_1/V_{IPO})$, where for simplicity, we assume that there are two levels of l at each date 0 and 1, namely $l_{\min}$ and $l_{\max}$ where $0 < l_{\min} \le l_{\max}$. The underwriter decides a (sub)optimal value of l_0 at date 0 and that of l_1 at date 1, making a choice of $l \in \{l_{\min}, l_{\max}\}$.†

The number N_0 of investors available for information production depends on the information cost for each investor and on V_{IPO} (or the amount of price reduction, $V_0 - V_{IPO}$). We can consider, $N_0 = N_0(V_{IPO})$, as a non-increasing function of V_{IPO} on an interval $0 < V_{IPO} \le V_{IPO}^{up}$, where V_{IPO}^{up} is an upper bound for the IPO price in a particular market determined by the underwriters at date 0. When the IPO is completed at $t = 0$ and the post-IPO market price of the shares is observed, the firm's projects are evaluated by some analysts whose number N_1 depends on the post IPO returns. The owner decides the IPO price as well as the share proportions (Δ_0, Δ_1) to sell in a dynamic setting at periods 0 and 1, respectively, by considering all the constraints and the future actions of the underwriter and the investors. Some of these constraints are imposed by the underwriter, for example the initial share proportion Δ_0 can only be between two specified values $\epsilon_{\min}$ and $\epsilon_{\max}$. On the other hand, there are some firm-specific constraints (ϵ_0 and ϵ_1) for (Δ_0, Δ_1) related to the operating costs, fund-raising, project completion, etc. Such constraints may not be directly observable by outsiders including the underwriters until date 2. However we assume that the owner observes the actions of the underwriter.

We now specify the extensive form of the game: There are two stages at each period. In period 0, at the first stage, the owner is the sole owner of her firm. Based on the actual quality type, the constraints and the contract that specifies the fee and warrant rates, she decides to sell an optimal proportion Δ_0 of shares at price V_{IPO} at date 0. Then the underwriter invites all of the potential N_0 investors to produce information about the firm. Compensation for the information production is assured through discounted price. Investors evaluate the firm and receive additional (noisy) information that reduces their informational disadvantage with respect to the owner. The outcome of the evaluation is either 'good' or 'bad'. At the

† The initial fee, warrant rate and cost terms are modeled with quadratic (rather than linear) functions to emphasize the significant role of the IPO price.

second stage, investors who receive good evaluation bid for entire shares which are allocated among the bidders (e.g. proportionately). When IPO is completed in period 0, the related information (e.g. the number of good evaluations) is made public and is reflected in the secondary market price during the first stage of period 1. Then the owner decides a proportion Δ_1 of the shares to sell at this price V_1 by also taking the future expected price into account. Investors observe the price V_1 and some analysts engage in a post-IPO evaluation process in the second stage of period 1. The number of analysts at date 1, N_1, depends on V_1, V_{IPO} and V_0. In particular, a significant amount of IPO underpricing would attract more analysts to review the firm. The underwriter collects all the evaluation results, updates the probability of quality types and hence the firm value V_2 which sets the long-run price for the remaining shares of the firm. The game for the owner ends when she sells all the shares and pays the relevant fees to the underwriter who, then, has access to the firm-specific information and observes the actual quality type. The underwriter's total wealth consists of the total fees collected plus the market value of the warrants and shares owned at date 2 minus the total costs (including the IPO expenditures, payments to the affiliated analysts, future reputation implications of handling lower quality IPO firms, etc.).

2.1. The owner's optimization problem

The objective of the owner is to complete the IPO in period 0 and sell the rest of the ownership in the subsequent periods to maximize the expected total net gains from the sales. For simplicity, we assume the interest rate is equal to zero (so no discounting is needed). When the owner decides V_{IPO} and the initial share allocations (Δ_0, Δ_1) to be sold at times 0 and 1, the aggregate wealth of the owner from the sale of the shares is

$$\begin{aligned} W = (1 - f_0(1 - l_0))V_{IPO}\Delta_0 + (1 - f_1(1 - l_1))V_1\Delta_1 \\ + (1 - \Delta_0 - \Delta_1 - \Delta^u)(1 - f_2)V_2 \end{aligned} \tag{3}$$

where $\Delta^u = l_0 f_0 \Delta_0 + l_1 f_1 \Delta_1$ represents the proportion of the total shares to be paid as fees to the underwriter and $1 - \Delta_0 - \Delta_1 - \Delta^u$ represents the remaining shares (if any) that are sold at date 2. The share proportions, Δ_0 and Δ_1, have the following bounds† :

$$0 < \max(\epsilon_0, \epsilon_{\min}) \le \Delta_0 \le \epsilon_{\max} \le \epsilon_1 \le \Delta_0 + \Delta_1 \le 1 - \Delta^u, \tag{4}$$

where ϵ_0 is the minimum proportion of the shares the owner needs to sell during the IPO and ϵ_1 is the minimum proportion of the shares to be sold by time $t = 1$ (especially if the completion of the projects and investments require it). Moreover, $\epsilon_{\min}$ and $\epsilon_{\max}$ are the (lower and upper) bounds for the proportion of shares that a typical high-quality firm sells during IPO.

REMARK 1 *The constraint, $\Delta_0 \in [\epsilon_{\min}, \epsilon_{\max}]$, is an assumption from the 'social comparison' perspective. We assume the firm-specific threshold, ϵ_0, is less than $\epsilon_{\max}$. Even if a firm (e.g. a low-quality one) needs extra cash for operating expenses such that $\epsilon_0 > \epsilon_{\max}$ may hold, the remaining cash need of $\epsilon_0 - \epsilon_{\max}$ can also be borrowed, perhaps with a small additional cost. So* (4) *is a reasonable constraint for feasible allocations.*

The actual firm-specific values of ϵ_0 and ϵ_1 may not be fully observed by the underwriter until date 2. However, they are known by the owner who has an informational advantage. By taking the expected future values of the share prices into account, the owner aims to maximize her expected wealth W over all feasible V_{IPO}, Δ_0 and Δ_1 values subject to (4). We denote the optimal value of W by W^*: $W^* = \max_{\Delta_0,\Delta_1,V_{IPO}} W$. Without any information production, the firm's market value is given by (1). So this is the amount the owner is expected to receive when there is sufficient demand for the shares. It is almost always a better strategy for high-quality firms and most medium quality firms to reduce information asymmetry via a costly information production process.

In this model, let N_0 denote the number of investors who are invited to evaluate the firm at a cost and reduce informational disadvantages of outsiders with respect to the owner. The outcome of an evaluation is either 'good' or 'bad', independently of other evaluations. The conditional probability (which is unknown by the outsiders) that an informed investor's evaluation is good given the true type of the firm's quality is denoted by $P(e_0 = G|S) = \alpha_S$, for $S = L, M$ and H, respectively, where it is reasonable to assume that $0 < \alpha_L < \alpha_M < \alpha_H < 1$. The posterior probabilities of the investors with a good evaluation are derived from the Bayes' rule:

$$P(S|e_0 = G) = \frac{P(e_0 = G|S)P(S)}{P(e_0 = G)} = \frac{\alpha_S p_S}{\sum_{S' \in \{L,M,H\}} \alpha_{S'} p_{S'}}. \tag{5}$$

Now, let X_0 denote the number of informed investors who assign a good evaluation. Moreover for each quality type $S = L, M$ and H, introduce the posterior probability weight

$$\gamma_S(r_0) = P(S|X_0 = r_0). \tag{6}$$

Then the secondary market value V_1 of the firm satisfies

$$V_1(r_0) = E[V|X_0 = r_0] = \sum_S \gamma_S(r_0)V^S, \tag{7}$$

as a function of r_0. The expression (7) dictates the new market value of the firm after the observed value of X_0 is made public.

2.2. The underwriter's problem

The underwriter doesn't know the actual quality type of the firm until the end of time 2, denoted 2+. The initial price, V_0, reflects all the information available at date 0 about the quality type of the firm, and therefore $E[V_1] = E[V_2] = E[V] = V_0$, from his perspective. The aggregate revenue (gains) of the underwriters consists of the fees collected and the profit/loss from the shares owned:

$$Gains = f_0 V_{IPO}\Delta_0(1 - l_0) + f_1 V_1 \Delta_1(1 - l_1)$$

† When the fee rates are not too large and ϵ_1 is not very close to 1, the feasibility condition $\epsilon_1 \le 1 - \Delta^u$ in (4) holds. A sufficient condition for this is $\epsilon_1 \le \frac{1}{1+l_{\max}f_0}$, which can be verified using linear programming arguments.

$$+ (1 - \Delta^u - \Delta_0 - \Delta_1)V_2 f_2$$
$$+ (1 - \Delta^u - \Delta_0 - \Delta_1)(V - V_2) + V\Delta^u. \quad (8)$$

We consider the following expression for the total costs of the underwriter(s):

$$Costs = cost_0 f_0 V_{IPO}\Delta_0(1 - l_0) + cost_1 f_1 V_1 \Delta_1(1 - l_1) + cost_2(1 - \Delta^u - \Delta_0 - \Delta_1)V_2 f_2 + cost_{2+}\Delta^u V, \text{with}$$
$$cost_0 = c_0\left(\frac{V_{IPO}}{V_0}\right)^2, \quad cost_1 = c_1\frac{V_{IPO}}{V_1} + c_1',$$
$$cost_2 = c_2\frac{V_{IPO}}{V_2} \quad \text{and} \quad cost_{2+} = c_2\frac{V_{IPO}}{V} \quad (9)$$

where the expressions $cost_0$, $cost_1$ and $cost_2$ represent the proportions of the cash fees, which are held by the underwriter and go to the relevant costs at dates 0, 1 and 2, respectively. All of these cost terms are increasing functions of the IPO price to account for the advertising costs as well as costs related to potential complaints/legal issues from the (institutional) customers. On the other hand, they are decreasing functions of the market price of the security at each date to account for liquidity issues/future reputation costs. The quadratic form in $cost_0$ is mainly for technical and tractability reasons. The terminal cost $cost_{2+}$ applies only to the warrants and is inversely proportional to the actual firm value V (accounting for the long term risks in keeping the warrants). The net terminal wealth or profit of the underwriter is then given by $W^u = Gains - Costs$ using (8) and (9). The underwriter's aim is to select an optimal pair (l_0, l_1) at dates 0 and 1 from a given set to maximize $E[W^u]$ sequentially.

3. Information production and the firm's value

We ignore the optimization problem of marginal (uninformed) investors and consider the aggregate wealth of the underwriters/investment banks that manage the IPO process. Depending on the quality type of the firm, the public information set and other constraints, the owner decides the optimal value of V_{IPO} which also determines N_0, the number of investors to be invited. Before going into details of the second stage, we set the following standing assumptions for the rest of the paper:

Assumption 1 The evaluations by N_0 investors at date 0 are statistically independent, and the posterior probabilities given in (6) serve as the prior probabilities at date 1: $P(S|\Omega_1) = \gamma_S$, which also determine the secondary market price V_1^S, for $S = L, M$ and H.

Assumption 2 The firm-specific proportions, ϵ_0 and ϵ_1, are unknown by the outsiders until date 2, but certain bounds for ϵ_0 and V_{IPO} exist: $\epsilon_{\min} \le \epsilon_0 \le \epsilon_{\max}$ and $V_{IPO} \le V_{IPO}^{up}$ (reflecting expectations of investors for a H-quality firm's IPO allocation and price from a *social comparison* perspective).

Assumption 3 It is not optimal for the owner of a lower quality firm to reveal the actual quality type (due to higher fees, liquidity costs, demand issues etc.).† Moreover, there is sufficient demand for the shares as long as the owner stays in the game.

Assumption 4 The underwriter's information set is updated after an evaluation result at each date. His decision to determine the (sub)optimal warrant rate at date t is sequential in nature: He decides l_t^* based on the information set, Ω_t, for each $t = 0, 1$.

Assumption 5 The (discounted) market price process $\{V_0, V_1, V_2\}$ satisfies a *martingale* property from the outsiders' point of view: $E[V_i|\Omega_j] = V_j$, for $0 \le j \le i \le 2$. This means that the expected (discounted) market value of the firm at future periods depends only on the most recent known market value based on the updated information set at that time (disregarding the previous information sets).

Assumption 6 The initial evaluation probabilities α_S and total proportion Δ_1 of shares sold at date 1 may not be observed by the investors/underwriters perfectly until date 2.

Assumption 7 The evaluation of the firm by each analyst at date 1 is independent of the others. The outcome is again either 'good' ($e_1 = G$) or 'bad' with conditional probabilities $\beta_S \triangleq P(e_1 = G|S)$, for $S = L, M$ and H, satisfying $0 < \beta_L < \beta_M < \beta_H < 1$. Moreover, $\alpha_H < \beta_H$ and $\beta_L < \alpha_L$, meaning that the analysts have a higher (lower, respectively) probability of assigning a good evaluation to a H (L, respectively) quality firm at time 1.

Assumption 8 After the evaluations by analysts in period 1 are made public, the long-run market price V_2 is determined and shared by all investors. However the actual value V^S is only known after the firm's remaining shares are sold.

Now, using the notation of (7) and (6) as well as these assumptions, we have the following result for the secondary market price of the firm:

Lemma 1 *Assume that n investors produce information at date* 0. *Let X_0 denote the number of good evaluations, V_1 be the firm's secondary market price, T be the true quality type of the firm (H, M or L), and $E^T[.]$ denote the expected value operator from the owner's perspective. Then V_1 satisfies the following:*

(a) $E^T[V_1] = \sum_{r=0}^{n}\sum_S \gamma_S(r)V^S\binom{n}{r}\alpha_T^r(1-\alpha_T)^{n-r}$, *where* $\gamma_S(r) = \frac{\alpha_S^r(1-\alpha_s)^{n-r}p_S}{\sum_{S'}\alpha_{S'}^r(1-\alpha S')^{n-r}p_{S'}}$.
(b) $E[V_1|X_0 = r+1] > E[V_1|X_0 = r]$, *for each* $r = 0, \ldots, n-1$.
(c) $E^H[V_1] \ge E^M[V_1] \ge E^L[V_1]$.

Remark 2 *Part* (b) *of Lemma* 1 *indicates that the secondary market price is a strictly increasing function of the 'good' evaluations for a fixed n. Since the owner of a higher quality firm expects a larger number of 'good' evaluations than the owner of a lower quality one, we get the monotonicity result of part* (c): *The expected price increases with the quality type.*

† Otherwise, a lower quality firm may find it less costly to go with a reservation expected wealth by taking an action that reveals the actual quality type with probability one.

Investors adapt the market value in (7) at date 1 until additional information about the firm is revealed. The owner decides Δ_1, the proportion of shares to sell at this price, before the evaluations by analysts are conducted at the second part of date 1. If $X_1 = r$ of N_1 investors assigned a good evaluation at this time, then the new value of the firm from the perspective of uninformed investors is

$$V_2(r) = E[V|X_1 = r, \Omega_1] = \sum_S \gamma'_S(r) V^S, \qquad (10)$$

where $\gamma'_S(r) = P(S|X_1 = r, \Omega_1)$ is the new posterior probability weight.

Lemma 2 *Assume that n analysts evaluated the firm's projects after the IPO and X_1 is not observed yet. Let T be the true type of the firm (H, M or L). Then the expected price of the firm (from the owner's perspective) in the secondary market at time 2 satisfies:*

$$E^T[V_2|\Omega_1] = \sum_{r=0}^{n} \sum_S \gamma'_S V^S \binom{n}{r} \beta_T^r (1-\beta_T)^{n-r}, \qquad (11)$$

where $\gamma'_S = \frac{\beta_S^r (1-\beta_S)^{n-r} \gamma_S}{\sum_{\hat{S}} \beta_{\hat{S}}^r (1-\beta_{\hat{S}})^{n-r} \gamma_{\hat{S}}}$.

Note that the expected value from the owner's point of view is different from that of the investors' valuation since the owner knows the actual quality type of the firm. She decides how much equity to sell in every period by considering the expected future prices.

4. Equilibrium characterization

A Bayesian equilibrium approach for both the owner and the underwriter is addressed in a *principal-agent* type game theoretic setting. At each point in time, the owner makes a decision to maximize the expected value of her total net payoffs by taking the corresponding strategies of the underwriter(s) into account.† At every stage of the game, underwriter updates his beliefs using the updated information set (and Bayes' rule) and makes a decision. The underwriter's incentives to achieve a separation among the quality types may only work partially in this model. The strategies of different owners would also differ but they are not *fully revealing* in most cases resulting in an *imperfectly separating equilibrium*. We define the equilibrium as a five-tuple $(l_0^*, l_1^*; V_{IPO}^*, \Delta_0^*, \Delta_1^*)$ such that (l_0^*, l_1^*) optimize the expected profit of the underwriter given the triple $(V_{IPO}^*, \Delta_0^*, \Delta_1^*)$ which maximize the expected wealth of the owner. Below, the details are given backwards in time.

4.1. Period 2

Assume that $X_1 = r_1$ of N_1 analysts assign good evaluations at time 1, and this information is made available to all the parties at the first stage of period 2. Then the resulting price $V_2(r_1)$, as in (10), is the market value of the firm whose shares can be traded at this value until the end of period 2. There is no optimization problem for the owner who simply sells the remaining $1 - \Delta_0 - \Delta_1 - \Delta^u$ proportion of shares (if any left) at price V_2 and hence her payoff is given by the equation (3). The fees are collected by the underwriter, and the game ends for the owner.

At the second stage of date 2 (stage 2+), the underwriter and then all the investors observe the actual value V^S (hence the quality type) of the firm and the final cash flows are realized. The realized net wealth of the underwriter is then given by $W^u = Gains - Costs$, where *Gains* and *Costs* are as in (8) and (9) of subsection 2.2.

4.2. Period 1

All the agents observe the market price V_1 at time 1, following the result of the initial information production. The owner's (optimal) IPO price V_{IPO}^* and the initial allocation Δ_0^* are already known by the investors. The probability of a good evaluation of the firm (or its remaining projects) is now given by β_s for each quality-type S. The owner should decide which fraction of ownership to sell at the prevailing market value V_1 by considering the current evaluation probabilities β_s, the future uncertainty and the underwriter's warrant rate decision l_1. The owner knows that a certain number, N_1, of analysts evaluate the firm, and the long-run price V_2 will be a direct result of the evaluations. The probability of obtaining a particular realization, say $X_1 = r$ good evaluations out of N_1, conditional on the quality types are computed using the binomial distribution: $P(X_1 = r|S) = \binom{N_1}{r}\beta_S^r(1-\beta_s)^{N_1-r}$ for $S \in \{L, M, H\}$. The owner can compute the corresponding posterior probabilities $\gamma'_S(r)$ for each quality type S and for each $r \in \{0, 1, \ldots, N_1\}$. So, she would know the expected value of the firm as given in Lemma 2. The realized value of X_1 will be known at the end of the period. For the problems of both the owner and the underwriter, we first introduce the following expression: Given $V_1(.)$ as a function of V_{IPO}, let

$$h(v) = (c'_1 - f_2)V_1(v) + (c_1 - c_2(1 - f_2))v, \qquad (12)$$

for $v > 0$, where c_1, c'_1 and c_2 are as in cost functions $cost_1$ and $cost_2$ of subsection 2.2. Then, the optimal strategies of both the owner and the underwriter are given in Lemma 3 below. It basically tells us that the underwriter switches between the warrant rates, depending on the sign of $h(V_{IPO})$. Moreover, the choice of the optimal proportion of the shares Δ_1^* that the owner sells at date 1 is determined by comparing the current value V_1 with a scaled version of the expected future value V_2.

Lemma 3 *If the number of good evaluations at date 1, X_1, is made public, then:*

(a) ***The underwriter's problem at date 1***. *The optimal choice of l_1 is given by*

$$l_1^* = \begin{cases} (V_1/V_{IPO})l_{\max}, & \text{if } h(V_{IPO}) > 0 \\ (V_1/V_{IPO})l_{\min}, & \text{if } h(V_{IPO}) < 0 \end{cases}$$

where h is as in (12) *above.*

† This can be considered a Stackelberg game in which the owner is the leader of the game while the underwriter is a follower.

(b) ***The owner's problem at date 1***. *Let the pair* $(l_0, l_1) = (l_0^*, l_1^*)$ *be the underwriter's sub-optimal warrant selection and the proportion* Δ_1^* *be the owner's response. Then,*
- *Case 1: If* $V_1 < \frac{1+l_1 f_1}{1-f_1(1-l_1)}(1-f_2)E^T[V_2|\Omega_1]$, *then* $\Delta_1^* = \epsilon_1 - \Delta_0^*$.
- *Case 2: If* $V_1 > \frac{1+l_1 f_1}{1-f_1(1-l_1)}(1-f_2)E^T[V_2|\Omega_1]$, *then* $\Delta_1^* = \frac{1-(1+l_0 f_0)\Delta_0^*}{1+l_1 f_1}$.
- *Case 3: If* $V_1 = \frac{1+l_1 f_1}{1-f_1(1-l_1)}(1-f_2)E^T[V_2|\Omega_1]$, *then* Δ_1^* *can be any value between* $\epsilon_1 - \Delta_0^*$ *and* $\frac{1-(1+l_0 f_0)\Delta_0^*}{1+l_1 f_1}$, *e.g. their average.*

4.3. Period 0

Following the timeline through a backward procedure, the owner's strategic decision to sell some proportion of the firm is at hand in period zero. The owner has to decide an optimal pair (Δ_0, V_{IPO}) to maximize the expected payoff from the combined proceeds of all sales.

The secondary market price V_1 of the equity is guided by the investors' evaluated price which depends on the number X_0 of investors who assign good evaluations. The probability of obtaining a particular realization r_0 of X_0 for a given N_0, conditional on the firm type S is given by $P(X_0 = \delta_0|S) = \binom{N_0}{r_0}\alpha_S^{r_0}(1-\alpha_s)^{N_0-r_0}$ using (conditional) Binomial distribution. The actual post-IPO market price V_1 can be higher or lower than the owner's expectation and the owner can exercise her option to sell some or all of her remaining shares at this market price. So the optimization problem to be solved at time 0 involves finding the optimal values $\Delta_0^*, \Delta_1^{*,0}$ and V_{IPO}^* (and hence the amount of information cost, in other words, how many investors N_0 to invite) that maximize the expected terminal wealth from the sale of all shares of the firm.

The initial optimal allocations $(\Delta_0^*, \Delta_1^{*,0})$ and the optimal IPO price V_{IPO}^* are obtained simultaneously by considering the actual quality type, all the constraints, the reactions of the underwriter and the investors, and future random variables regarding the values of the firm and the number of good evaluations at times 0 and 1. More specifically, the optimal wealth of the owner satisfies

$$\begin{aligned} E[W^*] = \max_{\Delta_0,\Delta_1,V_{IPO}} &\{V_{IPO}\Delta_0(1-f_0(1-l_0)) \\ &+ E^T[V_1]\Delta_1(1-f_1(1-l_1)) \\ &+ (1-\Delta_0-\Delta_1-\Delta^u)E^T[(1-f_2)V_2]\} \end{aligned} \tag{13}$$

where $\Delta^u = l_0 f_0 \Delta_0 + l_1 f_1 \Delta_1$ represents the proportion of shares to be paid as fees to the underwriter, the superscript T indicates the true quality type and $(\Delta_0, \Delta_1, \Delta^u)$ satisfy the inequality (4). The optimization problem is linear in (Δ_0, Δ_1) but nonlinear and implicit in V_{IPO} since the future expected prices and the optimal values of the pair (l_0, l_1) which are determined by the underwriter also depend on V_{IPO}. In practice, the problem can only be solved numerically by using discretized values of V_{IPO}. Then $E^T[V_1]$ and $E^T[V_2]$ from the perspective of the owner are computed for each fixed V_{IPO}. The equilibrium values are obtained sequentially: At time 0, we obtain the optimal values Δ_0^* and V_{IPO}^*. The actual optimal value Δ_1^* is obtained at time 1. We denote the estimate of Δ_1^* that is obtained at time 0 by $\Delta_1^{*,0}$ to distinguish between these two allocations.

For each fixed value of $V_{IPO} = v$, the expectations $E^T[V_1]$ and $E^T[V_2]$ depend on v implicitly. Moreover, the underwriter's decision (l_0, l_1) and the fee rates also depend on v. Hence, given the best response $(l_0, l_1) = (l_0^*, l_1^*)$ of the underwriter, the expected value W can be written as a function of (v, Δ_0, Δ_1) as follows:

$$\begin{aligned} E^T[W] = E^T[(1-f_2)V_2] &+ \Delta_0\{v(1-f_0(1-l_0)) \\ &- E^T[V_2(1-f_2)](1+l_0 f_0)\} \\ &+ \Delta_1\{E^T[V_1](1-f_1(1-l_1)) \\ &- E^T[V_2(1-f_2)](1+l_1 f_1)\} \end{aligned} \tag{14}$$

where the dependence of each term on v is implicit in the equation.

NOTATION 1 (a) *For each fixed v, define*

$$\hat{W}(v) \triangleq \max_{(\Delta_0,\Delta_1)} f(v,\Delta_0,\Delta_1) = f(v,\Delta_0^*(v),\Delta_1^{*,0}(v)).$$

(b) *Let* l_* *(*l^**, respectively) denote the optimal choice of* l *at time 0 (time 1, respectively)*

Then the optimal initial allocations $\Delta_0^*(v)$ and $\Delta_1^*(v)$ can be obtained by a linear programming approach for each v. Finally, W is maximized numerically by solving $W^* = \max_{v \le V_0} \hat{W}(v)$ over a set of discretized values of v. In general, the equilibrium results depend on the constraints and the assumptions on the parameters. The following lemma summarizes all possible cases:

LEMMA 4 *Let* $V_{IPO} = v$ *be a feasible IPO price and consider the cubic function*

$$g(v) = c_0 v^3/V_0^2 - [1 + c_2(1-f_2)]v + V_0(1-f_2).$$

(a) ***The underwriter's problem at date 0***. *The optimal choice of* l_0 *is given by*
$$l_0^* = \begin{cases} (V_0/v)^2 l_{\max}, & \text{if } g(v) > 0 \\ (V_0/v)^2 l_{\min}, & \text{if } g(v) < 0. \end{cases}$$

(b) ***The owner's problem at date 0***. *Given the optimal strategies of the underwriter,* $l_0^* = l_*(V_0/v)^2$ *and* $l_1^* = l^* V_1/v$*, the initial optimal proportions* $\Delta_0^*(v)$ *and* $\Delta_1^{*,0}(v)$ *satisfy the followings:*

(i) *If* $\max\left(\frac{1-f_0(1-l_0)}{1+l_0 f_0}v, \frac{1-f_1(1-l_1)}{1+l_1 f_1}E^T[V_1(v)]\right) < (1-f_2)E^T V_2$, *then* $\Delta_0^* = \epsilon_0$ *and* $\Delta_1^{*,0} = \epsilon_1 - \epsilon_0$.

(ii) *If* $\frac{1-f_1(1-l_1)}{1+l_1 f_1}E^T[V_1(v)] < (1-f_2)E^T V_2 < \frac{1-f_0(1-l_0)}{1+l_0 f_0}v$, *then* $\Delta_0^* = \epsilon_{\max}$ *and* $\Delta_1^{*,0} = \epsilon_1 - \epsilon_{\max}$.

(iii) *If* $\frac{1-f_0(1-l_0)}{1+l_0 f_0}v < (1-f_2)E^T V_2 < \frac{1-f_1(1-l_1)}{1+l_1 f_1}E^T[V_1(v)]$, *then* $\Delta_0^* = \epsilon_0$ *and* $\Delta_1^{*,0} = \frac{1-(1+f_0 l_0)\epsilon_0}{1+l_1 f_1}$.

(iv) *If* $\min\left(\frac{1-f_0(1-l_0)}{1+l_0 f_0}v, \frac{1-f_1(1-l_1)}{1+l_1 f_1}E^T[V_1(v)]\right) > (1-f_2)E^T V_2$, *then* $\Delta_0^* = \epsilon_{\max}$ *and* $\Delta_1^{*,0} = \frac{1-(1+f_0 l_0)\epsilon_{\max}}{1+l_1 f_1}$.

Table 2. The warrant rate decision at dates 0 and 1.

Condition	l_*	l^*
$g(V_{IPO}) > 0, h(V_{IPO}) > 0$	$l_{\max}$	$l_{\max}$
$g(V_{IPO}) > 0, h(V_{IPO}) < 0$	$l_{\max}$	$l_{\min}$
$g(V_{IPO}) < 0, h(V_{IPO}) > 0$	$l_{\min}$	$l_{\max}$
$g(V_{IPO}) < 0, h(V_{IPO}) < 0$	$l_{\min}$	$l_{\min}$

REMARK 3 (a) *When the inequalities in part* (b) *of Lemma* 4 *are not strict, there may be multiple solutions to optimal allocations. The underwriter is indifferent among such solutions: He may choose one of the solutions or their average, or leave the decision to the owner.*

(b) *The owner observes the choice l_0^* of Lemma* 4 *and can estimate l_1^* of Lemma* 3 *by computing the expected value of the underwriter's problem as a function of V_{IPO} at date* 0. *Table* 2 *includes four possible scenarios of the pair (l_*, l^*) based on the functions h and g of Lemmas* 3 *and* 4.

PROPOSITION 1 *Consider a finite set of potential values of V_{IPO} in a closed interval. Then a* ***partially (or imperfectly) separating*** *equilibrium $(l_0^*, l_1^*; V_{IPO}^*, \Delta_0^*, \Delta_1^*)$ always exists in the sense that the owners have different strategies which are not fully revealing.*

REMARK 4 (a) *If the model allows the underwriter to observe the firm-specific constraints, then this would result in a pure pooling equilibrium (mimicking the strategies of the owner of a high-quality firm) or fully separating equilibrium (when the owner of a lower quality firm cannot afford mimicry). However we are not going to study these cases in more detail here.*

(b) *The owner optimizes her realized total wealth, W^*, using the optimal choices $(V_{IPO}^*, \Delta_0^*, \Delta_1^*)$. However, the underwriter's problem can only be optimized partially and sequentially (in a noisy information set). So his expected total realized profit is not guaranteed to be the best possible.*

A simpler case, where an analytic solution for V_{IPO}^* is possible on a closed interval, involves fixed numbers N_0 and N_1 of investors and analysts available at dates 0 and 1, respectively, to evaluate the firm's projects:

PROPOSITION 2 *Assume that N_0 and N_1 are constant. If $\hat{V}_{IPO}$ is a local maximum IPO price (satisfying one set of the conditions in table* 2*), then it is given by*

$$\hat{V}_{IPO} = V_0 \cdot \sqrt{\frac{1}{3}\left(l_* + \frac{1 - f_{base}(\Delta_1^{*,0}/\Delta_0^*)}{f_{base} + f_{IPO}}\right)}, \qquad (15)$$

where l_* equals $l_{\min}$ or $l_{\max}$ depending on part (a) *of Lemma* 4, *and Δ_0^* and $\Delta_1^{*,0}$ are as in part* (b) *of Lemma* 4. *The absolute maximum occurs either at this local maximum or at a boundary point of the feasible set (see Remark* 3(a) *for the boundary values).*

5. Empirical considerations and model implications

Certain firm-specific characteristics, initial beliefs/experience /expectations of underwriter(s) about the quality types of the firms as well as the owner's ability to partially match such expectations and imitate a better quality type play a key role in this model. We assume that the owners, informed investors and underwriters have access to information for similar IPO offerings in a particular industry or country, and expect a high-quality firm to have certain firm-specific characteristics and IPO decision strategies. This observation allows the owner to select her optimal strategies in a set of feasible choices in a framework of peer imitation/comparison. The resulting initial optimal allocation, Δ_0^*, doesn't reveal the actual quality type *per se* but it may provide a signal to the underwriter for his warrant decision and potentially improve accuracy of analysts' evaluations.

Therefore a partial imitation of the strategy of a high-quality company by the owners of lower quality firms without revealing the quality type is one of the main contributions of our paper.

5.1. Numerical examples

We now provide some examples and describe their model implications. For each IPO price V_{IPO}, the underwriter's best response l_0^* and the estimated value of l_1^* at time 0 are computed based on the model parameters. Then the owner's optimal allocation Δ_0^* and the estimated value $\Delta_1^{*,0}$ are obtained from the owner's perspective. Finally, the expected wealth of the owner is computed on a grid of feasible IPO prices to determine the best IPO price V_{IPO}^* numerically.

EXAMPLE 1 *Consider the IPO of a firm in a certain industry with potential market values $V^H = 25$, $V^M = 20$, and $V^L = 16$ (in million dollars for a total of one million shares) depending on the quality type.*† *The initial probabilities of the firm's quality are $p_H = 0.35, p_M = 0.40$ and $p_L = 0.25$, respectively, implying $V_0 = 20.75$. The initial 'good' evaluation probabilities are $\alpha_H = 0.80, \alpha_M = 0.70$ and $\alpha_L = 0.40$. The probabilities of a 'good' evaluation at date 1 are $\beta_H = 0.85, \beta_M = 0.55$ and $\beta_L = 0.3$. Moreover, $\epsilon_{\min} = 0.18$, $\epsilon_{\max} = 0.30$, and $V_{IPO}^{up} = 20.80$.*

The number of investors available at each IPO price v is given by a function, $N_0(v)$ as follows: If $v = V_{IPO}^{up} = 20.80$, $N_0(v) = 33$. Then, $N_0(v)$ increases by 3 investors for each price reduction of 5 cents if $v \geq 20.05$ dollars. Below 20.05, when $18.85 \leq v < 20.05$, $N_0(v)$ increases by 2 investors for each reduction of 5 cents, and so on. The number of analysts at date 1 is fixed: $N_1 = 25$, and the fee schedule is given by $f_0 = (0.09)(V_{IPO}/V_0)^2$, $f_1 = (0.05)(V_{IPO}/V_1)$ and $f_2 = 0.05$. Moreover, the warrant rate parameters are $l_{\max} = 0.3$ and $l_{\min} = 0.15$.

The firm-specific constraints satisfy $0.18 \leq \epsilon_0 \leq 0.30 < \epsilon_1$. The underwriter's cost parameters are $c_0 = 0.33$, $c_1 = 0.17$, $c_1' = 0.16$ and $c_2 = 0.3$. His warrant rate choice at date 0 is determined by the expression $g(V_{IPO})$ of

† So the share price and the firm value will be represented by the same quantity, by an abuse of notation.

Lemma 4: If $V_{IPO} < 20.45$, then $g(V_{IPO}) > 0$ and hence $l_0^* = (V_0/V_{IPO})^2 l_{\max}$. Otherwise, $l_0^* = (V_0/V_{IPO})^2 l_{\min}$. The total profit of the underwriter(s) is determined by the fees collected, the terminal value of the warrants and the total costs which depend on the IPO price, the actual quality type, and the allocations of the owner. We now discuss the optimization problem for each quality type separately.

EXAMPLE 2 ***High-quality firm***: *The optimal expected gains of the owner of a high-quality firm, $E^H(W^*)$, depends on ϵ_0, ϵ_1 and the warrant allocations of the underwriter. Table 2 shows that $E^H(W^*)$ is maximized for smaller values of the firm-specific proportions, ϵ_0 and ϵ_1. For $\epsilon_0 = 0.20$ and $\epsilon_1 = 0.55$ fixed, we describe the optimal actions of both the owner and underwriter below.*

The owner's problem. *The expected future market prices of the firm are expected to increase for smaller IPO prices. These values and the expected wealth of the owner versus the IPO price (with increments of 0.05) are shown in figure 1. The optimal expected wealth $E^H(W^*)$ is attained at $V_{IPO} = 19.45$ for which $E^H(W^*) = 22.2688$ and $N_0 = 102$.*

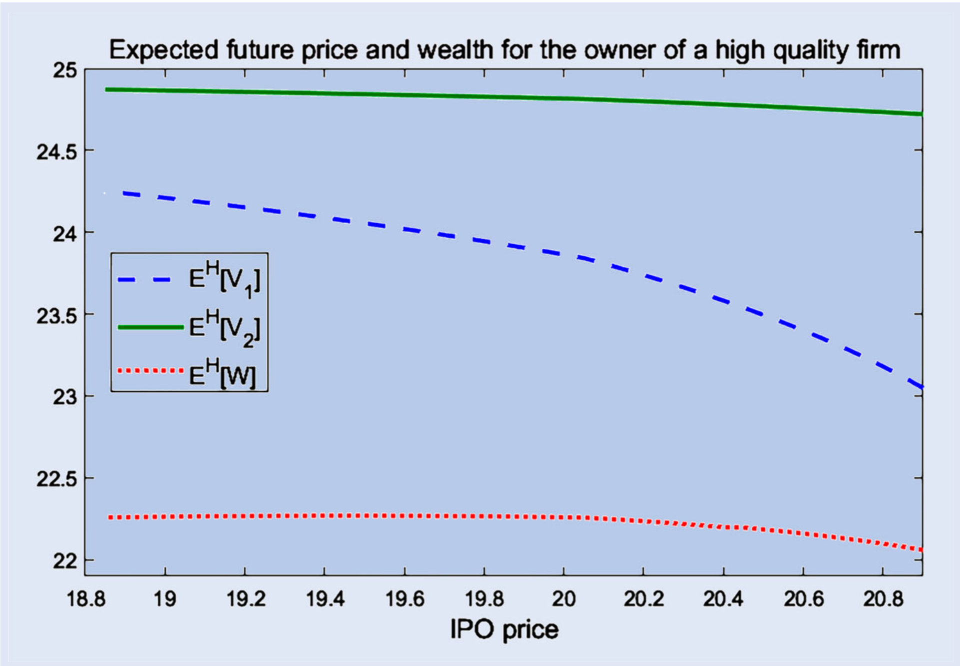

Figure 1. The secondary market prices, $E^H(V_1)$ and $E^H(V_2)$, tend to increase as the IPO price decreases. Expected wealth is maximized at $V_{IPO} = 19.45$.

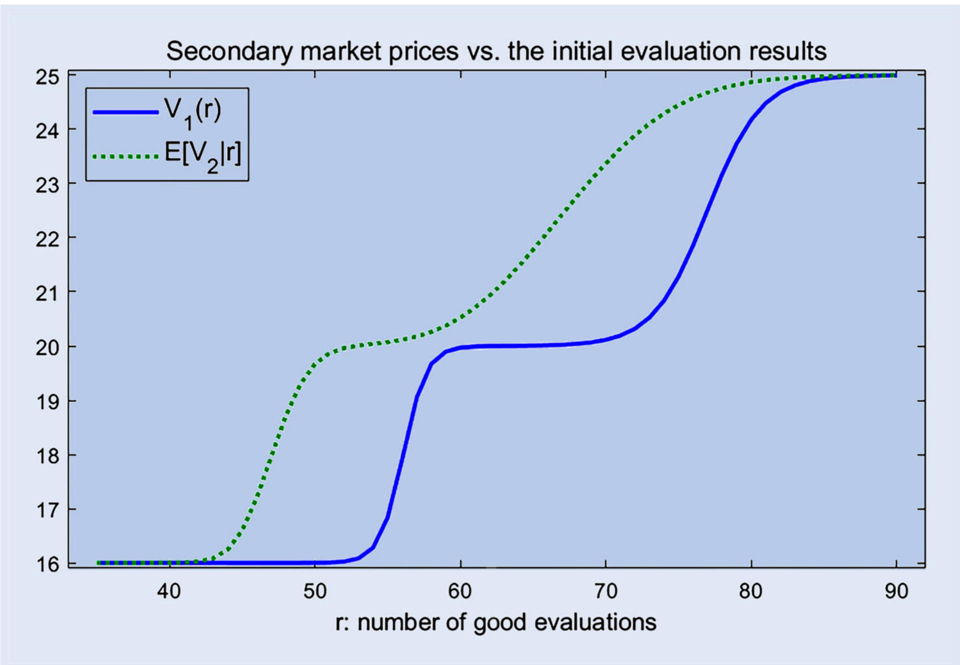

Figure 2. The secondary market prices are increasing with r.

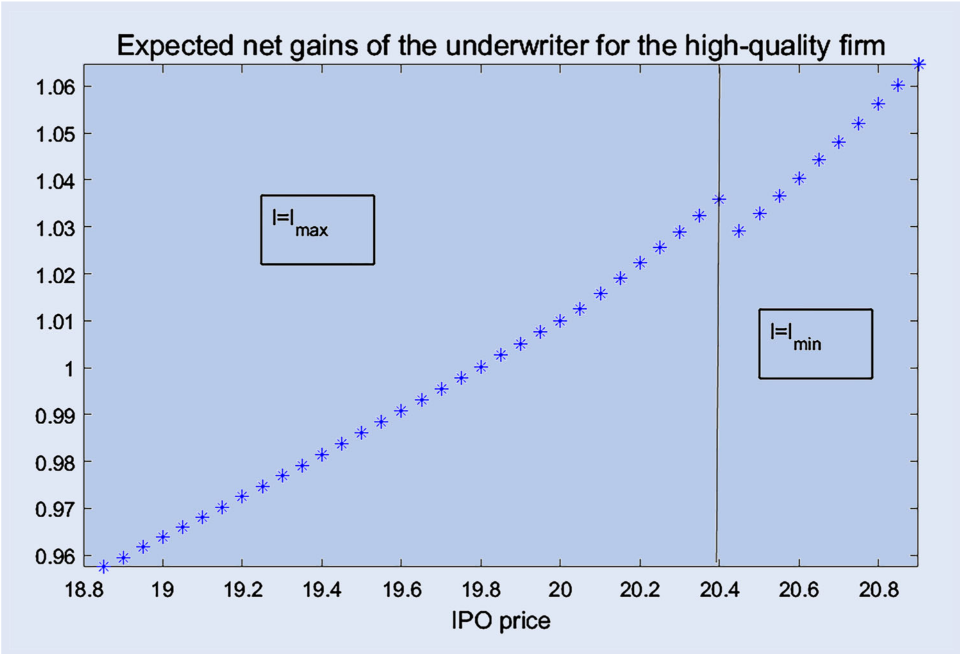

Figure 3. Expected net gains and optimal action of the underwriter.

The owner sells $\Delta_0^* = \epsilon_0 = 0.20$ *proportion of the shares at this price. Figure 2 is a truncated plot of the secondary market price* $V_1(r)$ *and the conditional expectation* $E^H[V_2|r]$ *as a function of the number of 'good' evaluations, r. For a high-quality firm, r is expected to be large. So the market value at date 1,* $V_1(r)$*, is typically close to* $V^H = 25$*. The optimal allocation* Δ_1 *is guessed to be* $\epsilon_1 - \epsilon_0 = 0.35$ *at date 0 but its actual value is determined at date 1. For example, if 77 of 102 investors assigned a 'good' evaluation, then the market price at date 1 is* $V_1(77) = 22.5147$*, and the owner's expected market price at date 2 is* $E^H[V_2|r = 77] = 24.6776$*, satisfying Case 1 of Lemma 3(b). So,* $\Delta_1 = \epsilon_1 - \epsilon_0 = 0.35$ *at date 1. The remaining shares are worth* V_2 *at date 2. The true value,* $V^H = 25$*, applies only after the projects are completed.*

The underwriter's problem. *The values* $V_{IPO} = 19.45$ *and* $\Delta_0 = 0.20$ *are determined by the owner at date 0, and* $l_0^* = (V_0/V_{IPO})^2 l_{\max} = (\frac{20.75}{19.45})^2(0.3) = 0.34144$ *is determined by the underwriter. Now, if* $r = 77$ *investors assigned a 'good' evaluation at date 1, the updated price is* $V_1 = 22.5147$*, and the revised probabilities for the quality types are* $\gamma_H = 0.5029, \gamma_M = 0.4971$ *and* $\gamma_L = 0$*. So, it is almost equally likely for the quality of the firm to be high or medium from the perspective of underwriter who may not solve his optimization problem perfectly (the first best solution may not be attained). However, his expected (realized) profit can be computed from the owner's perspective. Figure 3 shows that the best IPO price would be around the upper bound, 20.8. Using other potential values of* (ϵ_0, ϵ_1) *for a high-quality firm, we find that the best IPO price for the underwriter is usually at (or near) the upper bound, and sometimes at the cut-off point of the warrant rate shift where* $g(V_{IPO}) = 0$ *in Lemma 4. The comparative statics of the equilibrium IPO price, owner's expected wealth and the underwriters' expected total profit are given in tables 3 and 4 using several values of* ϵ_0, ϵ_1 *and* β_H*. Panel A in table 3 indicates that (for fixed* ϵ_1*) as* ϵ_0 *increases, the owner predicts a smaller expected wealth though the optimal IPO price tends to be larger. However, the underwriter's expected profit slightly increases with* ϵ_0*. It implies the gains from a larger allocation of IPO shares to his favorite customers together with higher fees collected from the owner would dominate the costs of a higher IPO price, on average. Panel B of table 3 shows the optimal IPO price turns out to be increasing with* ϵ_1*. The owner's expected wealth still decreases with* ϵ_1 *due to the decreased amount of shares sold at the higher price in date 2. The underwriter's expected net gains also decreases with* ϵ_1*. So, both the owner and underwriter would benefit from a smaller value of* ϵ_1*, e.g. if there is a less costly borrowing opportunity for the owner at date 1. Furthermore, table 4 indicates that underpricing is positively correlated with the evaluation uncertainty for a high-quality firm (using* β_H *as a proxy of evaluation certainty), supporting empirical findings of relationship between (ex-ante or ex-post) uncertainty and underpricing.*

EXAMPLE 3 ***Low-quality firm:*** *For a low-quality firm, the owner's IPO allocation,* Δ_0*, is at or around* $\epsilon_{\max} = 0.30$ *with an IPO price of 20.80. Moreover,* Δ_1 *is either* $1 - \epsilon_{\max} - \Delta^u$ *or* $\epsilon_1 - \epsilon_{\max}$*. The latter case occurs especially for relatively large values of fees at date 1 and smaller values of* V_{IPO}*, indicating that the underwriter's incentives may cause the owner to keep some shares until date 2. The kink at* $V_{IPO} = 20.45$ *in figure 4 is due to a transition from* $l_{\max}$ *(when* $V_{IPO} < 20.45$*) to* $l_{\min}$*. The owner's expected final wealth,* $E^L[W]$*, is increasing with both* V_{IPO} *and l but (slightly) decreasing with* ϵ_1 *(see table 5). On the other hand, the expected wealth* $E^L[W^u]$ *of the underwriter is not monotonic with* V_{IPO} *but increasing with* ϵ_1*. The absolute maximum of* $E^L[W^u]$ *usually occurs at or after the cut-off point, 20.45, where the warrant rate is minimal. A local maximum point is attained at smaller* V_{IPO} *values, and it is occasionally the absolute maximum for small* ϵ_1*. See figure 4 and table 5 for the best IPO price,* V_{IPO}^u*, which would maximize the underwriter's net gains. The owner can*

Table 3. The equilibrium quantities for the high-value firm depending on ϵ_0 and ϵ_1.

ϵ_0	ϵ_1	$\mathbf{V}^*_{IPO}$	$\mathbf{EW}^*$	$\mathbf{EW}^u(\mathbf{V}^*_{IPO})$
Panel A. The equilibrium values for $\epsilon_1 = 0.5$				
0.18	0.55	19.05	22.3792	0.9565
0.2	0.55	19.45	22.2688	0.9837
0.22	0.55	19.85	22.1674	1.0134
0.24	0.55	20.05	22.0741	1.0348
0.26	0.55	20.05	21.9827	1.0459
0.28	0.55	20.05	21.8912	1.0570
0.3	0.55	20.45	21.8066	1.0829
Panel B. The equilibrium for $\epsilon_0 = 0.20$				
0.2	0.40	20.05	22.3665	1.0739
0.2	0.45	20.05	22.3300	1.0535
0.2	0.5	19.75	22.2965	1.0181
0.2	0.55	19.45	22.2688	0.9837
0.2	0.60	19.20	22.2460	0.9523
0.2	0.65	18.95	22.2273	0.9212
0.2	0.70	18.85	22.2116	0.8967

Table 4. The equilibrium quantities versus β_H when $\epsilon_0 = 0.2$ and $\epsilon_1 = 0.55$.

β_H	V^*_{IPO}	$E[W(V^*_{IPO})]$	$\mathbf{E}[\mathbf{W}^u(\mathbf{V}^*_{IPO})]$
0.80	19.20	22.2163	1.0230
0.85	19.45	22.2688	0.9837
0.90	19.65	22.3076	0.9531
0.95	19.75	22.3295	0.9345

take full advantage of her informational superiority and over-optimism of the market's view of the firm at date 0. Therefore, the final expected wealth, $E[W^L]$, would be larger than the firm's market values at both dates 1 and 2 for reasonably large IPO price levels (figure 5). The following relationship

Table 5. The equilibrium quantities for the low-value firm when $\epsilon_0 = 0.3$ and ϵ_1 varies.

ϵ_1	$\mathbf{V}^*_{IPO}$	$E[W^*]$	$E[W^u(V^*_{IPO})]$	$\mathbf{E}[\mathbf{W}^u_*]$	$\mathbf{V}^u_{IPO}$
0.45	20.80	16.4027	0.6684	0.6995	20.45
0.50	20.80	16.4009	0.6811	0.7040	20.45
0.55	20.80	16.3990	0.6939	0.7125	20.45
0.60	20.80	16.3972	0.7066	0.7211	20.45
0.65	20.80	16.3954	0.7193	0.7296	20.50
0.70	20.80	16.3936	0.7320	0.7385	20.55
0.75	20.80	16.3918	0.7447	0.7480	20.60

always holds: $V^L = V_{2+} \le E^L[V_2] \le E^L[V_1] \le V_0$. So information production and subsequent evaluations contribute to gradual corrections of the market price.

EXAMPLE 4 ***Medium-quality firm.*** *For all admissible IPO prices, $\Delta_0^* = \epsilon_0$ (Δ_0^* would be $\epsilon_{\max}$ only when $V_{IPO} > 20.80$ is allowed). Moreover, the initial optimal allocation estimate is $\Delta_1^{*,0} = 1 - \Delta_0^* - \Delta^u$ which depends on l and ϵ_0 (getting larger when $l = l_{\min}$ and ϵ_0 is smaller) but not so much on ϵ_1 in the range of parameters considered. The expected wealth of the owner, $E^M[W]$, decreases with ϵ_0 (table 6 and figure 6(a)) while the underwriter's wealth increases (figure 6(b)). The extent of the optimal underpricing depends on the over-optimism and evaluation quality by analysts (captured by probabilities α and β) and is usually very small. In the range of IPO prices considered, the expected secondary market price, $E^M[V_1]$, has been above the actual value V^M and mostly above V_{IPO}, thanks to the over-optimism of the market. However the expected long-run price $E^M[V_2]$ gets much closer to V^M (figure 7). With the largest IPO allocation of $V^*_{IPO} = 20.80$, both the owner's wealth and the underwriter's wealth would be optimized although the underwriter wouldn't know it until date 2.*

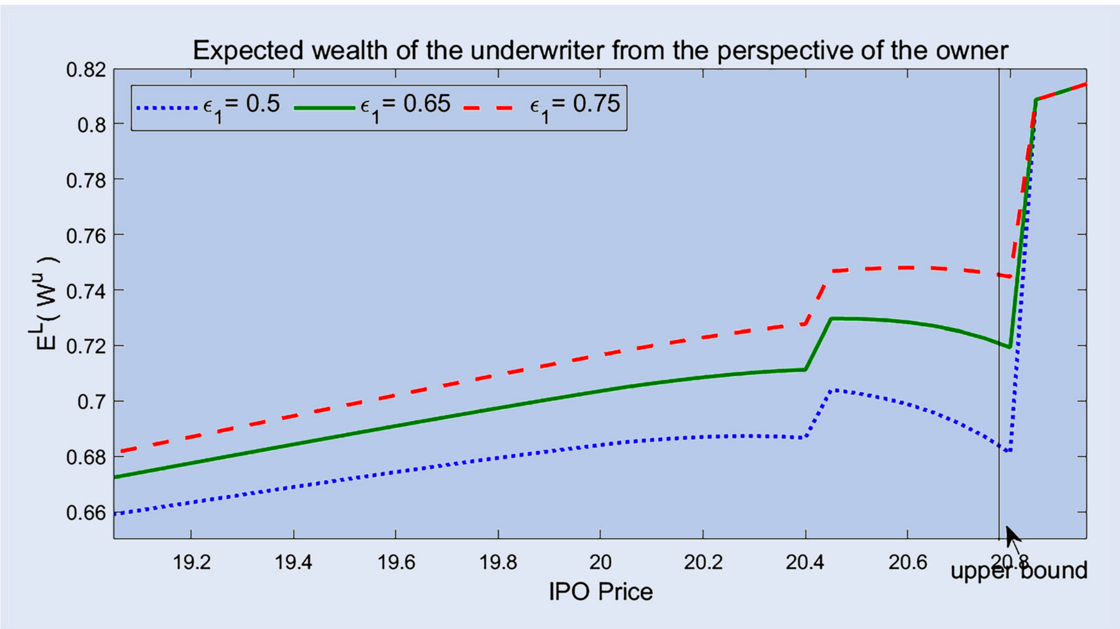

Figure 4. Expected net gains of the underwriter versus the IPO price for the low-quality firm.

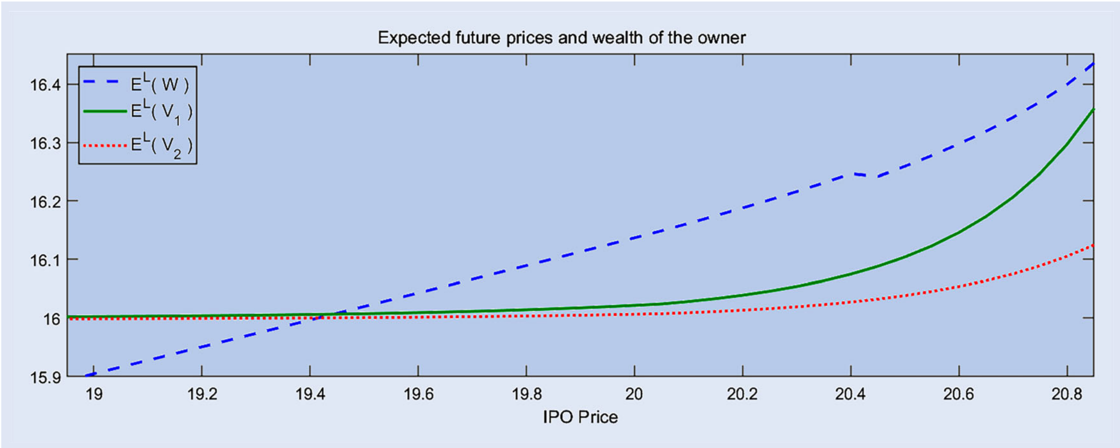

Figure 5. Expected wealth of the owner and underwriter for the medium-quality firm versus the IPO price.

Table 6. The equilibrium quantities for the medium-value firm depending on ϵ_0.

ϵ_0	$\mathbf{V}^*_{IPO}$	$E[W^*]$	$\mathbf{E}[\mathbf{W}^u_*]$	$\mathbf{V}^u_{IPO}$
0.18	20.80	20.1135	0.79	20.80
0.20	20.80	20.0845	0.8024	20.80
0.22	20.80	20.0555	0.8135	20.80
0.24	20.80	20.0265	0.8246	20.80
0.26	20.80	19.9975	0.8357	20.80
0.28	20.80	19.9685	0.8468	20.80
0.30	20.80	19.9395	0.8578	20.80

5.2. *Model implications*

For most of the reasonable model parameters, the following results hold:

High-quality firm. The owner takes full advantage of the information production to reduce information asymmetry. The secondary market price V_1^H is close to the true value V^H. Moreover, the part (a) of Lemma 4 usually applies: $\Delta_0^* = \epsilon_0^H$ and $\Delta_0^* + \Delta_1^{*,0} = \epsilon_1^H$. The owner sells the minimum possible shares at times 0 and 1. The level of the underpricing is positively correlated with the uncertainty of the evaluations at date 1. The underwriter's expected net gains are usually larger when the IPO price is higher. This suggests that the IPOs of (presumably) high-quality firms backed by large investment banks would offer a relatively lower underpricing compared to a firm which is more likely to be of lower quality. The positions of the owner and the underwriter contradict regarding the warrant rate and the initial allocation ϵ_0. The underwriter benefits from a higher warrant rate and ϵ_0 while the owner prefers a minimal warrant rate and ϵ_0.

Low-quality firm. The owner prefers a minimal amount of information revelation/production at date 0. Depending on the parameter values, the extra fees that are paid to the underwriter may force the owner to follow an imitation strategy of

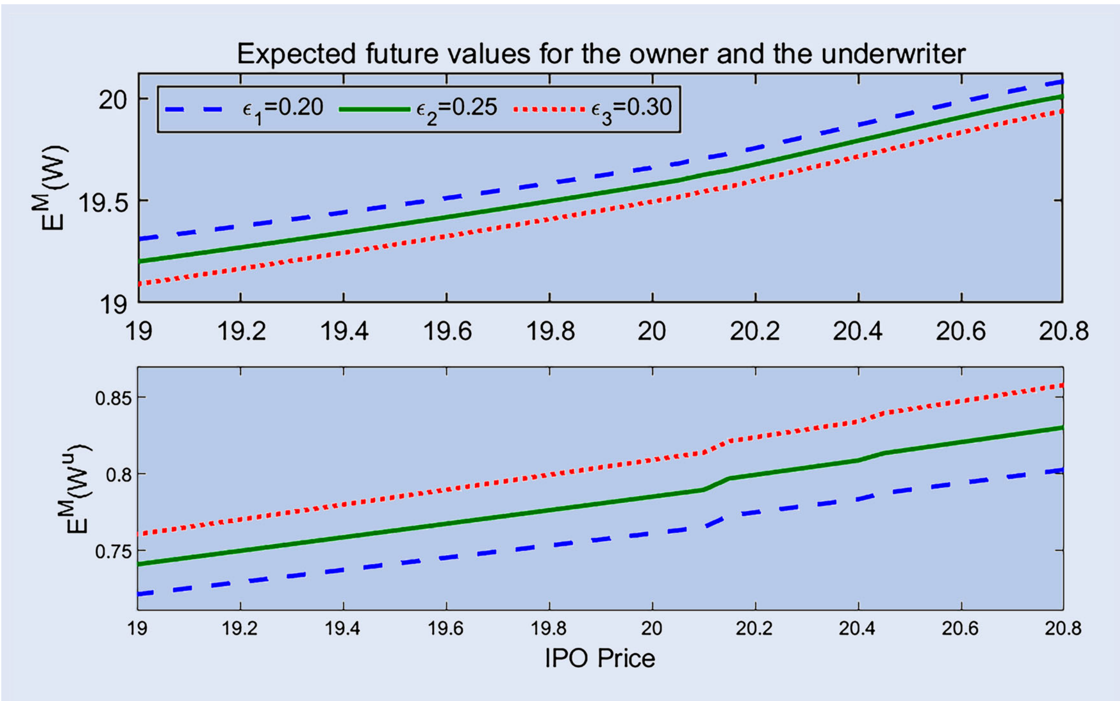

Figure 6. Expected wealth of the owner and underwriter for the medium-quality firm versus the IPO price.

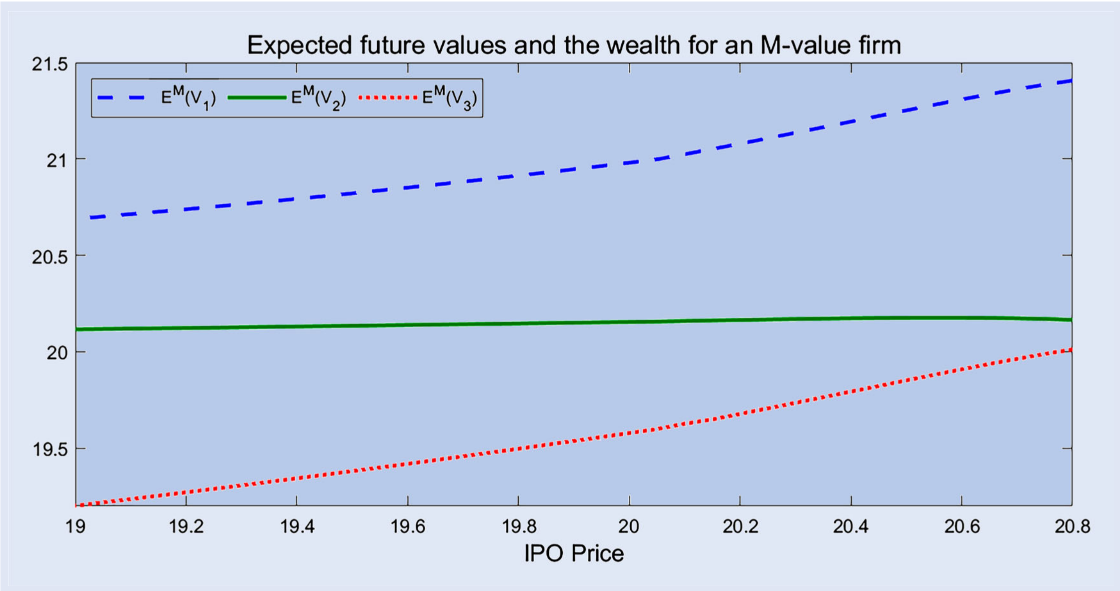

Figure 7. Expected future values of the medium-quality firm vs. IPO price.

a high-quality firm in order to disguise its quality-type. The owner is conservative about reducing the IPO price unless it helps to increase the warrant rate thanks to a relatively higher initial value of the warrants (as proportions of higher share prices). The long-run market price, V_2, gets closer to the true value V^L compared with V_1 on average. Moreover, the initial allocation Δ_0^* is usually $\epsilon_{\max}$ whileΔ_1 depends on whether the net (of the fees) payoff at time 1 is smaller than the expected payoff at date 2, resulting in either $\Delta_1 = \epsilon_1 - \epsilon_{\max}$ or $1 - \Delta_0^* - \Delta^u$ (the part (b) or (d) of Lemma 4 applies).

Medium-quality firm. The owner has a rich set of strategies, in general. The initial allocation Δ_0^* can range from ϵ_0 to $\epsilon_{\max}$. Likewise, Δ_1 ranges between $\epsilon_1 - \epsilon_{\max}$ and $1 - \epsilon_0 - \Delta^u$. A potential price run-up in the secondary market may still indicate a false signal about the quality type (and some investors might expect the price to increase further in the long run). However, when the analysts at date 1 provide more accurate evaluations, the market value at date 2 gets very close to the actual value V^M. A likely scenario is $V_1^M > V_2^M \geq V^M$. If a majority of the firms going IPO are of medium-quality type, which can initially follow a mimicking (or peer comparison) strategy of a high-quality firm, then this observation would be a reasonable explanation of the long-run underperformance observations in empirical research.

Underwriter. The first-best solution to the underwriter's optimization problem is not obtained in this model due to hidden information by the owner whose actions may not signal the actual quality type with certainty. The underwriter would still have some (control) tools to keep the owner's IPO price and allocation values close to his (sub-) optimal values. This reduces the impact of imitations or actions of the owner from a 'social comparison' perspective. One such a control strategy is to adjust the fees to cover the possible costs arising from non-optimal IPO price selections (in our model it is implicit as a function of the IPO price). A more direct control mechanism is to adjust the warrant rates for the fees as needed. We show that these tools provide only sub-optimal choices for the underwriter who takes the IPO price and the initial market price as a proxy for the initial warrant and fee rates. In general, the cut-off point for a higher warrant rate occurs at a lower equilibrium IPO price, and the underwriter benefits from that if the firm is of a high-quality one. Since it is less likely for a low-quality firm to reduce the price to that level (except that the owner may sometimes reduce it further to increase the warrant rate), mostly the minimum warrant rate applies in that case. Therefore, our model captures the empirical observations that the warrants would increase the underwriters' total compensation (Torstila 2001, Barry *et al.* 1991) or decrease the IPO costs (Dunbar 1995) on average.

6. Conclusion

We model the IPO price and the subsequent values of a firm in a dynamic game between the owner and the underwriter(s) in a multi-period asymmetric information-theoretic setting. We characterize the equilibrium allocations (Δ_0^*, Δ_1^*) and optimal IPO price V_{IPO} of the owner, as well as the (sub)-optimal warrant rates (l_0^*, l_1^*) of the underwriter(s). We find that the owner of a lower quality firm can imitate a potential strategy of a higher quality firm without fully revealing the actual quality-type (resulting in a *partially separating equilibrium*). The firm-specific constraints, potential (or expected) strategies of higher quality firms and the actual quality type of the firm are key factors for such an imitation strategy in the presence of *social comparison* actions of the agents. However, the owner of a high-quality firm can usually afford and benefit from a more significant underpricing/initial price reduction than the other firms. The level of underpricing increases with the uncertainty in the analysts' evaluation accuracy.

The underwriter's profit is not always maximized at the optimal choice of the owner. It is usually larger for managing the IPO of a high-quality firm. The optimal profit depends on some trade-off among the fees collected, the costs involved (both are proportional to IPO price) and the actual value of the warrants.

The model predicts that the market price of a firm would get closer to the actual value after evaluation by the investors/analysts. Moreover, the 'underperformance phenomenon' reported in empirical studies may actually be some 'initial overperformance' arising from a combination of overoptimism of the market, information asymmetry, and potential conflicts between the owner and the underwriter.

Some Potential Extensions for Future Work.

- One extension of this work would be considering random/uncertain quality types of projects. For example, the owner may not herself know the actual quality type of the firm with certainty, either. She may believe a project of the firm is of high quality (with some degree of over-optimism) but the result of the evaluations at date 1 may not agree with her beliefs.
- Another extension would be to allow the accuracy of the evaluations (measured by the probability vectors α and β) to depend on the IPO price or other parameters of choice rather than given exogenously.
- Using a continuous random variable (e.g. a beta family of distributions), we can actually consider an interval of (infinitely many) quality types, e.g. in a beta-binomial model.
- One can also consider risk-averse agents with expected utility or mean-variance preferences.

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Appendix

This part includes the proofs of lemmas/propositions as well as the tables and figures stated in the text.

Proof of Lemma 1. (a) From the owner's point of view, the expected price will be

$$E^T[V_1] = \sum_{r=0}^{n} E^T[V_1|X=r]P^T(X=r),$$

where $P^T(.)$ is the probability measure with respect to the owner's information set where the actual quality type is T and $P^T(e_0 = G|T) = \alpha_T$. Hence $P^T(X_0 = r) = \binom{n}{r}\alpha_T^r(1-\alpha_T)^{n-r}$, and $E^T[V_1|X_0 = r] = E[V_1|X_0 = r] = E[V|X_0 = r]$ which is independent of the owner's firm-specific information and is given by (7). Moreover, by (6) and Bayes' rule, we obtain

$$\begin{aligned}\gamma_S(r) &= P(S|X_0 = r)\\ &= \frac{P(X_0 = r|S)p_s}{P(X_0 = r)}\\ &= \frac{\binom{n}{r}\alpha_S^r(1-\alpha_s)^{n-r}p_S}{\sum_{\acute{S}}\binom{n}{r}\alpha_{\acute{S}}^r(1-\alpha_{\acute{S}})^{n-r}p_{\acute{S}}}\\ &= \frac{\alpha_S^r(1-\alpha_s)^{n-r}p_S}{\sum_{\acute{S}}\alpha_{\acute{S}}^r(1-\alpha_{\acute{S}})^{n-r}p_{\acute{S}}}.\end{aligned}$$

(b) The identity $E[V_1|X = r+1] > E[V_1|X = r]$ holds if and only if

$$\frac{\sum_S \alpha_S^{r+1}(1-\alpha_s)^{n-(r+1)}p_S V^S}{\sum_S \alpha_S^{r+1}(1-\alpha_S)^{n-(r+1)}p_S} > \frac{\sum_S \alpha_S^{r}(1-\alpha_s)^{n-r}p_S V^S}{\sum_S \alpha_S^{r}(1-\alpha_S)^{n-r}p_S}$$

which is equivalent to expression

$$\left(\sum_S \frac{\alpha_s}{1-\alpha_s}u_S V^S\right)\left(\sum_S u_S\right) > \left(\sum_S \frac{\alpha_s}{1-\alpha_s}u_S\right)\left(\sum_S u_S V^S\right) \tag{A1}$$

where $u_S = \alpha_S^r(1-\alpha_S)^{n-r}p_S > 0$. After writing the expressions in both sides of (A1) explicitly and simplifying the resulting inequality, we get the equivalent identity

$$u_L u_M \alpha_{M,L}(V^M - V^L) + u_L u_H \alpha_{H,L}(V^H - V^L) + u_H u_M \alpha_{H,M}(V^H - V^M) > 0 \tag{A2}$$

where $\alpha_{S_1,S_2} = \frac{\alpha_{S_1}}{1-\alpha_{S_1}} - \frac{\alpha_{S_2}}{1-\alpha_{S_2}}$, for each ordered pair of states (S_1, S_2) where $S_1 \neq S_2$. Since the function $\frac{x}{1-x}$ is strictly increasing on $(0,1)$ and $0 < \alpha_L < \alpha_M < \alpha_H$, each α_{S_1,S_2} pair in (A2) is positive. The other terms on the left-hand-side of inequality (A2) are also positive. Therefore, the result holds.

(c) Heuristically, this is a result of the monotonicity identity $E^H[X_0] > E^M[X_0] > E^L[X_0]$ (the higher the quality type, the larger is the number of good evaluations, on average) and part (b). A rigorous proof is quite lengthy and is omitted here. ■

Proof of Lemma 2. Given S or $V_1 = V_1^S$, X_1 has a (conditional) binomial distribution with parameters $N_1 = n$ and β_S (with respect to the probability measure P). Again by Bayes' rule,

$$\begin{aligned}\gamma_S'(r) = P(S|X_1 = r, \Omega_1) &= \frac{P(X_1 = r|S)\gamma_S}{P(X_1 = r)}\\ &= \frac{\binom{n}{r}\beta_S^r(1-\beta_s)^{n-r}\gamma_S}{\sum_{\acute{S}\in\{L,M,H\}}\binom{n}{r}\beta_{\acute{S}}^r(1-\beta_{\acute{S}})^{n-r}\gamma_{\acute{S}}}\\ &= \frac{\beta_S^r(1-\beta_S)^{n-r}\gamma_S}{\sum_{\acute{S}\in\{L,M,H\}}^{r}(1-\beta_{\acute{S}})_{\acute{S}}^{n-r}\gamma_{\acute{S}}}.\end{aligned}$$

The rest is similar to the proof of Lemma 1(a) above. ■

Proof of Lemma 3. (a) We want to maximize the (conditional) expected value of the underwriters' total profit which is a linear function of l_1. Using (8)–(9) and $E[V_2|\Omega_1] = V_1$, the expected net wealth can be written as

$$\begin{aligned}E[W^u|\Omega_1] &= E[Gains|\Omega_1] - E[Costs|\Omega_1]\\ &= l_1 f_1 \Delta_1\{(c_1' - f_2)V_1 + (c_1 - c_2(1-f_2))V_{IPO}\}\\ &\quad + \textit{other terms}.\end{aligned}$$

So, applying the function $h(\cdot)$ in (12) and noting that V_1 depends on V_{IPO}, we maximize $l_1 f_1 \Delta_1 h(V_{IPO}) = l \cdot f_{base}\Delta_1 h(V_{IPO})$ as a linear function of $l \in \{l_{\min}, l_{\max}\}$. When $h(V_{IPO}) > 0$, $l = l_{\max}$ is selected and so $l_1 = l\frac{V_1}{V_{IPO}} = \frac{l_{\max}V_1}{V_{IPO}}$. Similarly, $l = l_{\min}$ is selected for $h(V_{IPO}) < 0$.

(b) With $\Delta^u = l_0 f_0 \Delta_0 + f_1\Delta_1 l_1$, $\Delta_0 = \Delta_0^*$ and $(l_0, l_1) = (l_0^*, l_1^*)$ which are known, the owner decides the optimal value, Δ_1^*, based on the expected price levels, (3) and the constraint $\epsilon_1 \le \Delta_0^* + \Delta_1 \le 1 - l_0 f_0 \Delta_0^* - f_1\Delta_1 l_1$ from (4). So in the continuation game, the optimal fraction of shares the owner is willing to sell at date 1 maximizes the expression

$$\Delta_1\{V_1(1 - f_1(1-l_1)) - (1 + l_1 f_1)(1-f_2)E[V_2|\Omega_1]\} \tag{A3}$$

subject to the condition

$$\epsilon_1 - \Delta_0^* \le \Delta_1 \le \frac{1 - (1 + l_0 f_0)\Delta_0^*}{1 + l_1 f_1}.$$

Since (A3) is linear in Δ_1, the result easily follows. ■

Proof of Lemma 4. (a) The proof is similar to that of Lemma 3. After writing the expected net wealth of the underwriter at time 0, we focus on the part that involves l_0:

$$E[W^u] = l_0 f_0 \Delta_0\{V_0(1-f_2) - V_{IPO}(1 - cost_0 + c_2(1-f_2))\} + \textit{other terms}, \tag{A4}$$

where $cost_0 = c_0(\frac{V_{IPO}}{V_0})^2$. Since "*other terms*" that include Δ_1 are unknown by the underwriter at date 0, he can only maximize (A4) in this sequential optimization problem. Clearly, this (sub)optimal value of l_0 depends on the sign of$V_0(1-f_2) - V_{IPO}(1 - cost_0 + c_2(1-f_2))$.

(b) Now, we have $E^T[V_2(1-f_2)|\Omega_0] = (1-f_2)E^T[V_2]$. The equation (14) is linear in the initial allocations Δ_0 and Δ_1. Moreover, the constraints are linear in Δ_0 and Δ_1. So, the results follow from linear programming arguments. ■

Proof of Proposition 1. If v is a feasible IPO price, then underwriter's corresponding sub-optimal choice l_0^* as well as the owner's optimal allocations and expected wealth can be determined as in Lemma 4. Moreover, an optimal IPO price V_{IPO}^* from the owner's perspective always exists in a closed and bounded (compact) feasible set. Given that the owner observes l_0^* and can estimate l_1^*, she has access to complete information to solve her optimal expected wealth problem. Depending on the constraints given in part (b) of Lemma 4, the corresponding optimal allocations of the owner may not reveal the actual quality type. ■

Proof of Proposition 2. When N_0 and N_1 are constant, the expected values of V_1 and V_2 don't depend on V_{IPO}. Therefore, the expected wealth of the owner is given by

$$E[W] = V_{IPO}\{\Delta_0^*(1 + l_*(f_{base} + f_{IPO})) - \Delta_1^{*,0} f_b\} - \frac{V_{IPO}^3}{V_0^2}(f_{base} + f_{IPO})\Delta_0^* + \textit{other terms},$$

which is a (differentiable) cubic function of V_{IPO}. Taking its derivative with respect to V_{IPO}, the first order condition specifies the local maximum at (15) if it is in the feasible set which is a closed and bounded (compact) interval. So, an absolute maximum is guaranteed on this interval (either at the local maximum or at a boundary point). ■