

# PWA for $\psi(3686) \rightarrow \phi\pi^+\pi^-(\phi K^+K^-)$

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# Outline

- Motivation
- Data and MC sample
- Event selection
- PWA
- Summary

# Motivation

- Search for strangeness partner of  $Z_c(3900)$  in  $\phi K$  channel, as well as other possible resonances.
- Measure the branching fractions of various  $\psi(3686)$  decay processes with high precision.

# Data and MC sample

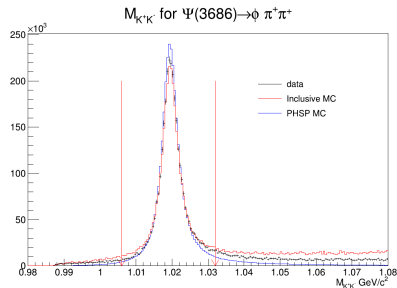
- BOSS version 7.0.9.p01
- Data sample
  - 2021 2.26B  $\psi(3686)$  sample
- Official inclusive MC sample including
  - $\psi(3686) \rightarrow K^+K^-\pi^+\pi^-$  ( $0.08 \times 10^6$ )
  - $\psi(3686) \rightarrow K^+K^-K^+K^-$  ( $1.72 \times 10^6$ )
- Exclusive MC sample
  - PHSP generate  $\psi(3686) \rightarrow \phi\pi^+\pi^-$ ,  $\phi \rightarrow K^+K^-$  ( $6 \times 10^8$ )
  - PHSP generate  $\psi(3686) \rightarrow \phi K^+K^-$ ,  $\phi \rightarrow K^+K^-$  ( $5 \times 10^8$ )

# Event selection

- Good charged Track:
  - $|V_r| < 1.0 \text{ cm}$  and  $|V_z| < 10.0 \text{ cm}$
  - $|\cos \theta| < 0.93$
- 4 good charged tracks & net charge is 0
- PID (TOF and dE/dx)
  - Kaon:  $\text{Prob}(K) > \text{Prob}(\pi)$  and  $\text{Prob}(K) > 0.001$
  - Pion:  $\text{Prob}(\pi) > \text{Prob}(K)$  and  $\text{Prob}(\pi) > 0.001$

# Event selection for $\psi(3686) \rightarrow \phi\pi^+\pi^-$

- PID and charge:  $K^+K^-\pi^+\pi^-$
- 4C fit:  $\chi_{4c}^2 < 200$
- $\phi$  Signal region:  
 $1.006 < M_{K^+K^-} < 1.032 \text{ GeV}/c^2$   
with signal purity at 80% level

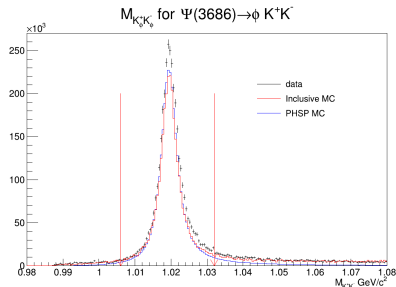


# Event selection for $\psi(3686) \rightarrow \phi K^+ K^-$

- PID and charge:  $K^+ K^- K^+ K^-$
- 4C fit:  $\chi^2_{4c} < 200$
- Two kaons are labeled with subscript  $\phi$  as the mass of them is closest to that of  $\phi$ , that is  $K^+_{\phi} K^-_{\phi} K^+ K^-$
- $\phi$  Signal region:

$$1.006 < M_{K^+_{\phi} K^-_{\phi}} < 1.032 \text{ GeV}/c^2$$

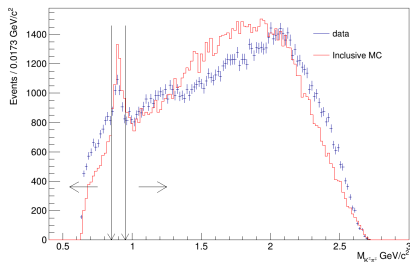
with signal purity at 80% level



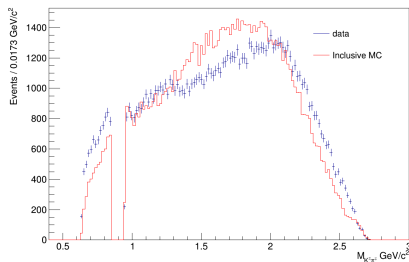
# Veto $K^*(892) \rightarrow K^\pm \pi^\mp$ for $\psi(3686) \rightarrow \phi \pi^+ \pi^-$

- $M_{K^\pm \pi^\mp} < 0.85$  or  $M_{K^\pm \pi^\mp} > 0.95$  is required

$M_{K^\pm \pi^\mp}$  without veto  $K^*(892)$



$M_{K^\pm \pi^\mp}$  with veto of  $K^*(892)$





Inclusive MC sample is used to study potential backgrounds. After applying above cuts, topology analysis is performed based on the inclusive MC. The main decay channels of backgrounds shows in Appendix(II)

- Peaking backgrounds in  $\psi(3686) \rightarrow \phi\pi^+\pi^-$ :
  - mainly from  $k^*(892) \rightarrow (K\pi)^{+-}$  : 2%
- Other small backgrounds levels
  - in  $\psi(3686) \rightarrow \phi\pi^+\pi^-$  : 3%
  - in  $\psi(3686) \rightarrow \phi K^+K^-$  : 0.3%

# Continuum background

Non- $\phi$  background levels in topology analysis:

- $\psi(3686)$  directly to  $K^+K^-\pi^+\pi^-$  : 5.4%
- $\psi(3686)$  directly to  $K^+K^-K^+K^-$  : 5.3%

Backgrounds above can be further suppressed in  $\phi$  reweight, and then will fix in the fit of PWA as backgrounds.

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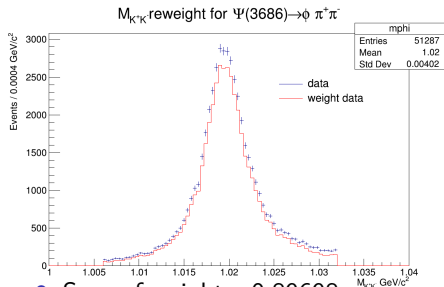
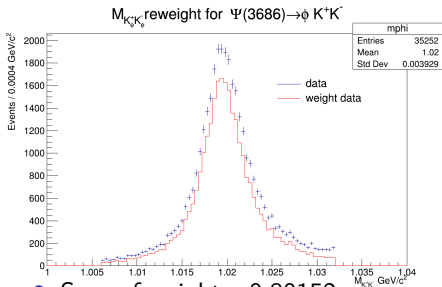
Fit the remaining data with a double-Gaussian function for signal and second-order polynomial for background. The probability density function(PDF) is described as

$$Q_i = \frac{F_s(\xi_r^i, \hat{\alpha}_i)}{F_s(\xi_r^i, \hat{\alpha}_i) + F_b(\xi_r^i, \hat{\alpha}_i)}$$

- Reference :M.Williams,M.Bellis&C.A.Meyer,Journal of Instrumentation,4(10), P10003(2009)

# $\phi(1020)$ reweight

## reweight results

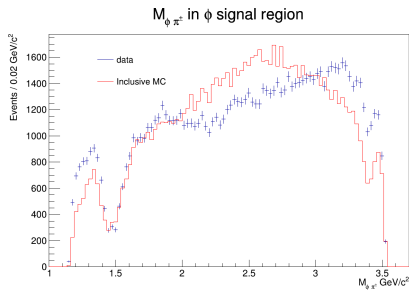
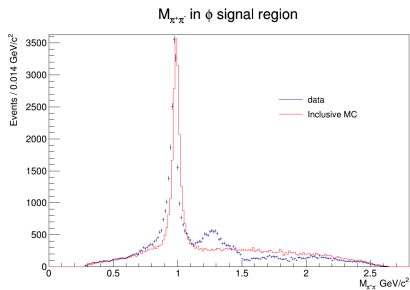


- Sum of weight : 0.80152

- Sum of weight : 0.89698

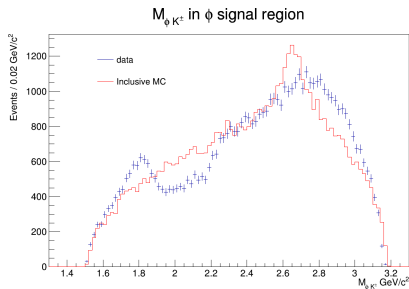
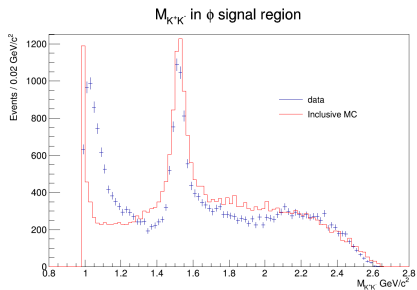
The summed weights is consistent with the number of signal events in the inclusive MC.

# $M_{f_{\pi^+\pi^-}}$ and $M_{b_{\phi\pi^\pm}}$ for $\psi(3686) \rightarrow \phi\pi^+\pi^-$



- Some difference between data and MC is observed

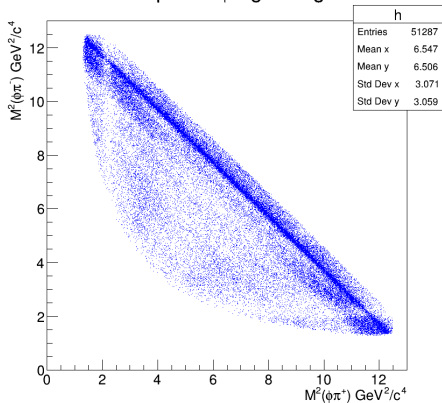
# $M_{f_{K^+K^-}}$ and $M_{b_{\phi K^\pm}}$ for $\psi(3686) \rightarrow \phi K^+ K^-$



- Some difference between data and MC is observed.

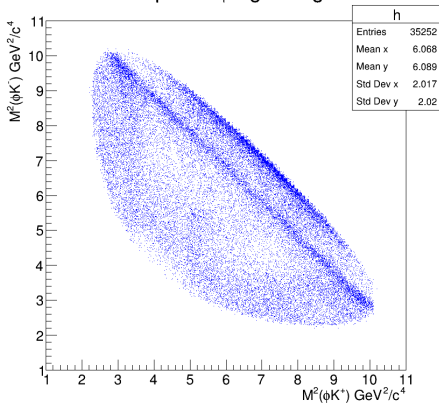
# Dalitz plots for $\psi(3686) \rightarrow \phi\pi^+\pi^-$

Dalitz plot in  $\phi$  signal region



*Dalitz plots for  $\psi(3686) \rightarrow \phi\pi^+\pi^-$*

Dalitz plot in  $\phi$  signal region



*Dalitz plots for  $\psi(3686) \rightarrow \phi K^+ K^-$*

## PWA: resonances describe

- The amplitudes are composed with covariant tensors

$$\frac{d\sigma}{d\Phi} = \frac{1}{2} \sum_{\mu=1}^2 A^\mu A^{*\mu} = \frac{1}{2} \sum_{i,j} \Lambda_i \Lambda_j^* \sum_{\mu=1}^2 U_i^\mu U_j^{*\mu}$$

- reference: B.S.Zou and D.V.Bugg, Eur.Phys.J.A 16,537-547(2003)
- Most propagators are modeled using relativistic Breit-Wigner function.

$$f = \frac{1}{m_0^2 - m^2 - im_0\Gamma_0}$$

- Propagator for  $f_0(980)$  is modeled using the Flatté formula

$$f = \frac{1}{m_0^2 - m^2 - im_0(\Gamma_{\pi\pi} + \Gamma_{K\bar{K}})}$$



## PWA: likelihood functions

$$\ln L = \sum_{i=1}^N \ln N_{\text{gen}} \sum_{\mu=1}^2 A^{\mu}(\xi_i, \alpha) A^{*\mu}(\xi_i, \alpha) - N \ln \sum_{j=1}^{N_{\text{acc}}} \sum_{\mu=1}^2 A^{\mu}(\xi_j, \alpha) A^{*\mu}(\xi_j, \alpha)$$

$A^{\mu}$ —amplitude under given parameters  $\alpha$  and phase space point  $\xi$

$N$ —number of observed events

$N_{\text{gen}}$ —number of events generated by simulation

$N_{\text{acc}}$ —number of accepted events after detector acceptance simulation.

Non- $\phi$  backgrounds are described by add resonance with infinite width into the fit, which is expressed in the likelihood function as:

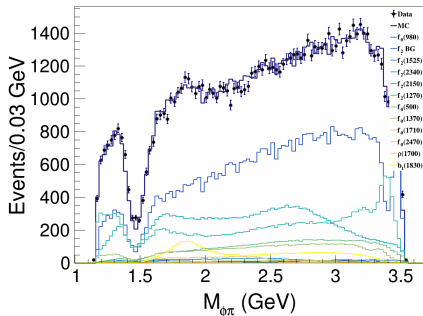
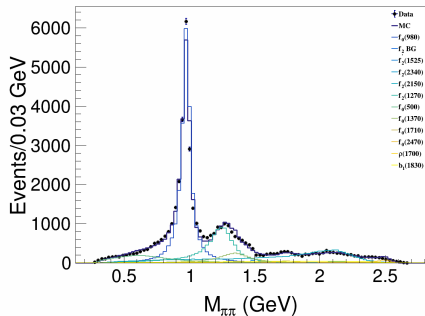
$$\ln L(\text{signal}) = \ln L(\text{data}) - \ln L(\text{background})$$

# PWA :fit strategy

- Strategy to add resonances
  - First a baseline solution from knowledge and eyes
  - Add resonance considering significance
- Simultaneous fit on  $K^+K^-$  and  $\pi^+\pi^-$  modes
  - Constraint on: mass & width of resonances, relative amplitude ratios ,total fraction of all resonances
- PWACG - Partial Wave Analysis Code Generator
  - GitHub[<https://github.com/caihao/PWACG>]

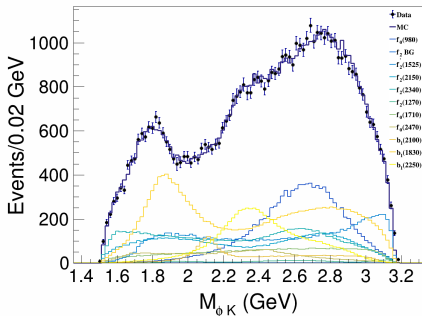
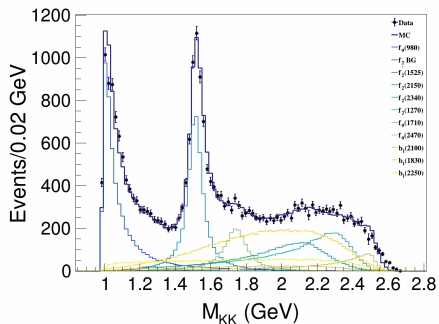
# PWA baseline for $\psi(3686) \rightarrow \phi\pi^+\pi^-$

- Error bar is data; blue line is the result of PWA



# PWA baseline for $\psi(3686) \rightarrow \phi K^+ K^-$ (I)

- Error bar is data; blue line is the result of PWA



# PWA baseline $\psi(3686) \rightarrow \phi\pi^+\pi^-$

mode name	fraction(%)	mass(MeV)	PDG mass	width(MeV)	PDG width	significance
$f_0(980)$	49.08	$972.42 \pm 0.49$	$990.0 \pm 20$	-	-	>10
$f_2(1950)$	23.60	$2149.5 \pm 5.6$	$1936 \pm 12$	$436.1 \pm 1.0$	$464 \pm 24$	>10
$f_2(1270)$	19.81	$1275.6 \pm 2.8$	$1275 \pm 0.8$	$197.6 \pm 5.1$	$186.6^{+2.8}_{-2.2}$	>10
$f_0(500)$	7.569	$535.4 \pm 9.0$	400 - 800	$643 \pm 21$	100 - 800	>10
$f_0(1370)$	6.404	$1350 \pm 12$	1200 - 1500	$265.4 \pm 9.0$	200 - 500	>10
$b_1(1830)$	5.251	$1833.9 \pm 5.8$	-	$320.7 \pm 6.2$	-	>10
$f_0(2470)$	3.158	$2521.4 \pm 4.8$	$2346^{+6}_{-7}$	$114.0 \pm 1.4$	$75^{+14}_{-12}$	>10
$f_2(2340)$	2.004	$2358 \pm 16$	$2346^{+21}_{-10}$	$322 \pm 23$	$331^{+27}_{-18}$	>10
$f_0(1710)$	0.9106	$1742.9 \pm 3.4$	$1733^{+8}_{-7}$	$140.1 \pm 7.5$	$150^{+12}_{-10}$	>10
$f'_2(1525)$	0.8256	$1512.0 \pm 1.3$	$1517 \pm 2.4$	$86.2 \pm 2.6$	$86.0 \pm 4$	9.74
$\rho(1900)$	0.1089	$1851.4 \pm 7.8$	$1880 \pm 10$	$131 \pm 24$	$69 \pm 15$	7.10
BG	1.137	-	-	-	-	-
sum fraction	119.9	-	-	-	-	-

# PWA baseline $\psi(3686) \rightarrow \phi K^+ K^-$

mode name	fraction(%)	mass(MeV)	PDG mass	width(MeV)	PDG width	significance
$b_1(1830)$	30.35	$1833.9 \pm 5.8$	-	$320.7 \pm 6.2$	-	>10
$f_2(2340)$	15.19	$2358 \pm 16$	$2346^{+21}_{-10}$	$322 \pm 23$	$331^{+27}_{-18}$	>10
$f_2'(1525)$	14.04	$1512.0 \pm 1.3$	$1517 \pm 2.4$	$86.2 \pm 2.6$	$86.0 \pm 4$	>10
$f_0(980)$	11.93	$972.42 \pm 0.49$	$990.0 \pm 20$	-	-	>10
$b_1(2250)$	11.62	$2309 \pm 39$	-	$380 \pm 18$	-	>10
$f_2(1950)$	10.82	$2149.5 \pm 5.6$	$2095^{+17}_{-19}$	$436.1 \pm 1.0$	$287^{+32}_{-24}$	>10
$f_0(2470)$	9.970	$2521.4 \pm 4.8$	$2346^{+6}_{-7}$	$114.0 \pm 1.4$	$75^{+14}_{-12}$	>10
$f_0(1710)$	4.829	$1742.9 \pm 3.4$	$1733^{+8}_{-7}$	$140.1 \pm 7.5$	$150^{+12}_{-10}$	>10
$b_1(2100)$	3.543	$2109 \pm 22$	-	$135 \pm 23$	-	7.56
$f_2(1270)$	0.6639	$1275.6 \pm 2.8$	$1275 \pm 0.8$	$197.6 \pm 5.1$	$186.6^{+2.8}_{-2.2}$	9.67
BG	$<10^{-8}$	-	-	-	-	-
sum fraction	113.0	-	-	-	-	-

- $f_0(980)$  Faltte parameters fit result  $g_1 = 63.8 \pm 1.2 \text{ MeV}$ ,  $g_2 = 1.31 \pm 0.13$

## Discussion about $b_1$ states

According to B.S.Zou's article, there are speculations about  $(s\bar{s}q\bar{q})$  four-quark states which could decay to  $\phi K$  or  $\phi\pi$ .

Partial wave amplitudes for  $\psi(2s) \rightarrow X\pi(K), X \rightarrow \phi\pi(K)$ :

- For X being a  $\rho'(1^{--})$  state, there is only one independent amplitude since both  $\psi(2s) \rightarrow \rho'\pi(K)$  and  $\rho' \rightarrow \phi\pi(K)$  are limited to a P wave
- For X being a  $b_1(1^{+-})$  state, there are four independent amplitudes since  $\psi(2s) \rightarrow b_1\pi(K)$  and  $b_1 \rightarrow \phi\pi(K)$  both can have both S and D waves

## $b_1$ possible $J^P$ assignments

Possible  $J^P$  assignments  $1^-$  and  $1^+$ , for all  $b_1$  resonance are tested statistical significance

mode	1+ over 1-			1- over 1+		
	$\Delta \ln L$	$\Delta ndf$	sig.	$\Delta \ln L$	$\Delta ndf$	sig.
$b_1(1830)$	857.9	20	>30	49.35	8	8.61
$b_1(2100)$	42.75	10	7.56	26.53	4	6.50
$b_1(2250)$	277.9	10	22.6	45.79	4	8.89

The significance tests favor all  $b_1$  resonances are  $1^+$  states.



## Signal efficiency

$0.7 \times 10^6 \psi(3683) \rightarrow \phi K^+ K^-$  and  $1.17 \times 10^6 \psi(3683) \rightarrow \phi \pi^+ \pi^-$  MC samples with amplitude information were generated to estimate the detector response and event selection efficiency.

Cut	before	after	efficiency(%)
detector response	1170823	410618	35.07
$M_{K^+ K^-} \in (1.006, 1.032)\text{MeV}/c^2$	410618	376488	91.68
$M_{K^\pm \pi^\mp} \notin (0.85, 0.95)\text{MeV}/c^2$	376488	346238	91.94
total	1170823	346238	29.57

Cut	before	after	efficiency(%)
detector response	705882	199484	28.26
$M_{K^+ K^-} \in (1.006, 1.032)\text{MeV}/c^2$	199484	183602	92.03
total	705882	183602	26.01

# Branch Fraction Measurement(preliminary)

Calculate the branching ratio

$$\frac{\Gamma(f \rightarrow K^+ K^-)}{\Gamma(f \rightarrow \pi^+ \pi^-)} = \frac{B(f \rightarrow K^+ K^-)}{B(f \rightarrow \pi^+ \pi^-)} \times \frac{\varepsilon_2 N(\psi' \rightarrow \phi K^+ K^-)}{\varepsilon_1 N(\psi' \rightarrow \phi \pi^+ \pi^-)}$$

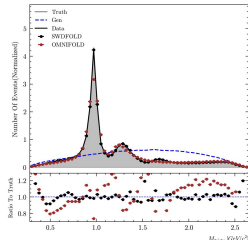
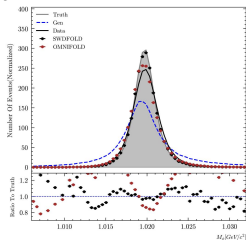
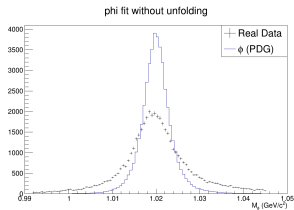
Where  $\varepsilon_1$  represents the signal efficiency of  $\psi' \rightarrow \phi K^+ K^-$ ,  $\varepsilon_2$  represents the signal efficiency of  $\psi' \rightarrow \phi \pi^+ \pi^-$ , N is the total number of events after selection for the corresponding process, and B represents the branch ratio for a certain resonance in all  $\psi' \rightarrow \phi K^+ K^-$  or  $\psi' \rightarrow \phi \pi^+ \pi^-$  process.

# Branch Fraction Measurement(preliminary)

mode	$B(f \rightarrow K^+ K^-)/\%$	$B(f \rightarrow \pi^+ \pi^-)/\%$	$\frac{\Gamma(f \rightarrow K^+ K^-)}{\Gamma(f \rightarrow \pi^+ \pi^-)}$	pdg value
$f_0(980)$	11.93	49.08	0.19	0.52
$f_2(1270)$	0.6639	19.81	0.026	0.054
$f_2'(1525)$	14.04	0.8256	13	13
$f_0(1710)$	4.829	0.9106	4.2	4.3
$f_2(1950)$	10.82	23.60	0.36	-
$f_2(2340)$	15.19	2.004	5.9	-
$f_0(2470)$	9.970	3.158	2.5	-
$b_1(1830)$	30.35	5.251	4.5	-

# Data Unfolding

Due to the significant impact of detector resolution on the real data of  $\phi$ , current fitting method for  $\phi$  involves fixing values to the PDG values. The next step will be to apply unfolding to the real  $\phi$  data in order to achieve a more accurate fit.



- arXiv[<https://doi.org/10.48550/arXiv.2406.01635>]

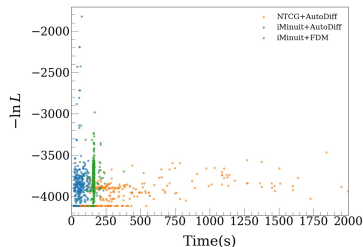
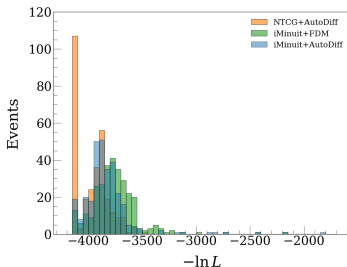
## Summary and to-do list

- The baseline is determined and the fit is stable.
- The branching ratio is calculated.
- Three  $b_1$  states have been observed significantly in the  $\phi\pi$  and  $\phi K$  system.
- Estimate systematic errors and calculate statistical errors.
- Calculate the branching ratio error.
- Using data unfolding in our fit.

# Appendix(I)

## PWACG - Partial Wave Analysis Code Generator

- PWA program based on JAX tensor calculation
- Multi-threaded GPU computation and low memory use
- jinja2 code generator: building programs flexibly



- arXiv[<https://doi.org/10.48550/arXiv.2403.09225>]

# Appendix(II): topoanalysis result for $\psi(3686) \rightarrow \phi\pi^+\pi^-$

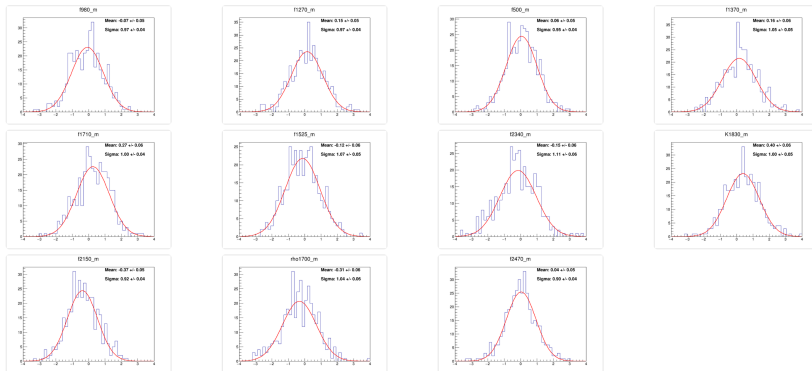
No.	iDcyTr	nEtr	Decay Chain
1	1	16334	$\psi(2S) \rightarrow \pi^+\pi^-\phi, \phi \rightarrow K^+K^-$
2	2	13949	$\psi(2S) \rightarrow \phi f_0(980), \phi \rightarrow K^+K^-, f_0(980) \rightarrow \pi^+\pi^-$
3	0	1885	$\psi(2S) \rightarrow \pi^+\pi^- K^+K^-$
4	6	734	$\psi(2S) \rightarrow \rho^0 K^+K^-, \rho^0 \rightarrow \pi^+\pi^-$
5	8	483	$\psi(2S) \rightarrow \phi f_0(1710), \phi \rightarrow K^+K^-, f_0(1710) \rightarrow \pi^+\pi^-$
6	3	251	$\psi(2S) \rightarrow \phi f_0(980), \phi \rightarrow K^+K^-, f_0(980) \rightarrow \pi^+\pi^-\gamma_f$
7	4	234	$\psi(2S) \rightarrow \pi^-\bar{K}^{*0} K^+, \bar{K}^{*0} \rightarrow \pi^+K^-$
8	5	220	$\psi(2S) \rightarrow \pi^+ K^{*0} K^-, K^{*0} \rightarrow \pi^- K^+$
9	11	135	$\psi(2S) \rightarrow \pi^+ K_2^{*0} K^-, K_2^{*0} \rightarrow \pi^- K^+$
10	7	132	$\psi(2S) \rightarrow \pi^+\pi^-\phi\gamma_f, \phi \rightarrow K^+K^-$
11	12	123	$\psi(2S) \rightarrow \pi^-\bar{K}_2^{*0} K^+, \bar{K}_2^{*0} \rightarrow \pi^+K^-$
12	18	83	$\psi(2S) \rightarrow \phi f'_2, \phi \rightarrow K^+K^-, f'_2 \rightarrow \pi^+\pi^-$
13	13	24	$\psi(2S) \rightarrow \pi^+\pi^- a_0, a_0 \rightarrow K^+K^-$
14	-	197	modes less than 20 entries

## Appendix(II): topoanalysis result for $\psi(3686) \rightarrow \phi K^+ K^-$

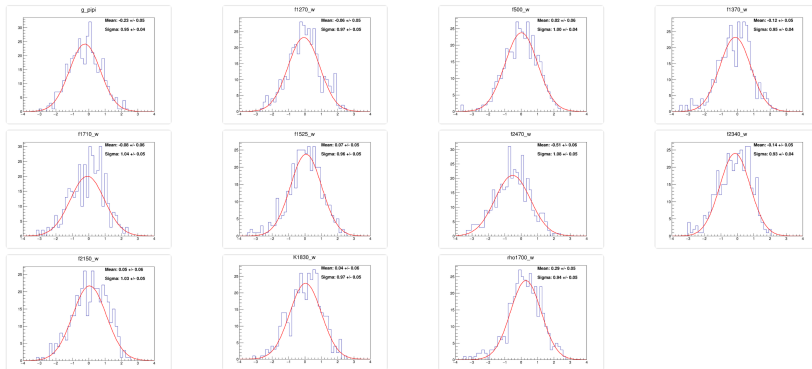
No.	iDcyTr	nEtr	Decay Chain
1	0	24240	$\psi(2S) \rightarrow K^+ K^- \phi, \phi \rightarrow K^+ K^-$
2	1	7320	$\psi(2S) \rightarrow \phi f_2', \phi \rightarrow K^+ K^-, f_2' \rightarrow K^+ K^-$
3	2	3342	$\psi(2S) \rightarrow \phi f_0(980), \phi \rightarrow K^+ K^-, f_0(980) \rightarrow K^+ K^-$
4	3	2007	$\psi(2S) \rightarrow K^+ K^+ K^- K^-$
5	4	477	$\psi(2S) \rightarrow \phi f_0(1710), \phi \rightarrow K^+ K^-, f_0(1710) \rightarrow K^+ K^-$
6	5	47	$\psi(2S) \rightarrow K^+ K^- \phi \gamma_f, \phi \rightarrow K^+ K^-$
7	12	39	$\psi(2S) \rightarrow \phi f_2', \phi \rightarrow K^+ K^-, f_2' \rightarrow K^+ K^- \gamma_f$
8	7	18	$\psi(2S) \rightarrow K^+ K^- f_2', f_2' \rightarrow K^+ K^-$
9	-	30	modes less than 20 entries



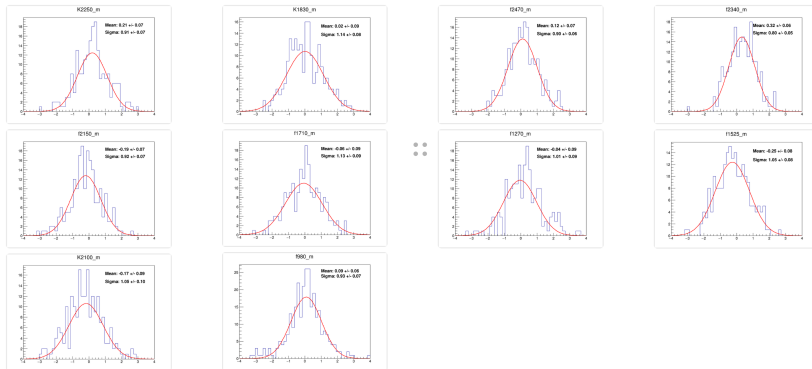
# Appendix(III):pull distribution of fit parameters in $\phi\pi^+\pi^-$



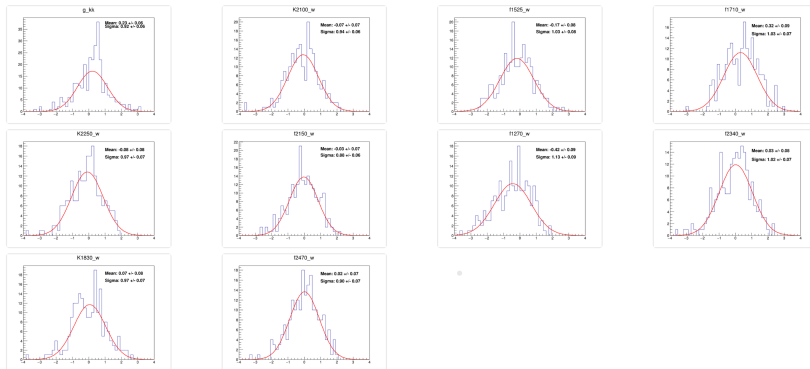
# Appendix(III):pull distribution of fit parameters in $\phi\pi^+\pi^-$



# Appendix(III):pull distribution of fit parameters in $\phi K^+ K^-$



# Appendix(III):pull distribution of fit parameters in $\phi K^+ K^-$



*Thanks!*