ICPC Templates

我们需要更深入浅出一些

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Contents

Chapter 1

字符串

1.1 最小表示法

```
// == Main ==
int i = 0, j = 1, k = 0;
while (k < n && i < n && j < n)
    if (a[(i + k) % n] == a[(j + k) % n]) k++;
    else {
        if (a[(i + k) % n] > a[(j + k) % n]) i = i + k + 1;
        else j = j + k + 1;
        if (i == j) i++;
        k = 0;
    }
ans = min(i, j);
```

1.2 Border 理论

1.2.1 关键结论

定理 1：对于一个字符串 s，若用 t 表示其最长的 Border，则有 $\mathcal{B}(s) = \mathcal{B}(t) \cup \{t\}$。

定理 2：一个字符串的 Border 与 Period 一一对应。具体地，$\text{pre}(s, i) \in \mathcal{B}(s) \iff |s| - i \in \mathcal{P}(s)$。

弱周期引理：

$$\forall p, q \in \mathcal{P}(s), p + q \leq |s| \implies \gcd(p, q) \in \mathcal{P}(s)$$

定理 3：若字符串 t 是字符串 s 的前缀，且 $a \in \mathcal{P}(s), b \in \mathcal{P}(t), b \mid a, |t| \geq a$，且 b 是 t 的整周期，则有 $b \in \mathcal{P}(s)$。

周期引理：

$$\forall p, q \in \mathcal{P}(s), p + q - \gcd(p, q) \leq |s| \implies \gcd(p, q) \in \mathcal{P}(s)$$

定理 4：对于文本串 s 和模式串 t，若 $|t| \geq \frac{|s|}{2}$，且 t 在 s 中至少成功匹配了 3 次，则每次匹配的位置形成一个等差数列，且公差为 t 的最小周期。

定理 5：一个字符串 s 的所有长度不小于 $\frac{|s|}{2}$ 的 Border 的长度构成一个等差数列。

定理 6：一个字符串的所有 Border 的长度排序后可以划分成 $\lceil \log_2 |s| \rceil$ 个连续段，使得每段都是一个等差数列。

定理 7：回文串的回文前/后缀即为该串的 Border。

定理 8：若回文串 s 有周期 p，则可以把 $\mathrm{pre}(s, p)$ 划分成长度为 $|s| \bmod p$ 的前缀和长度为 $p - |s| \bmod p$ 的后缀，使得它们都是回文串。

定理 9：若 t 是回文串 s 的最长 Border 且 $|t| \geq \frac{|s|}{2}$，则 t 在 s 中只能匹配 2 次。

定理 10：对于任意一个字符串以及 $u, v \in \mathrm{Ssuf}(s), |u| < |v|$，一定有 u 是 v 的 Border。

定理 11：对于任意一个字符串 s 以及 $u, v \in \mathrm{Ssuf}(s), |u| < |v|$，一定有 $2|u| \leq |v|$。

定理 12：$|\mathrm{Ssuf}(s)| \leq \log_2 |s|$。

1.2.2 KMP

```
// == Main ==
// n is |s|, m is |t|.
for (int i = 1, j = 0; i <= n; i++) {
    while (j && t[j + 1] != s[i]) j = pi[j];
    if (t[j + 1] == s[i])
        if (++j == m) {
```

```
            // ...
            j = nxt[j];
        }
}
```

1.3 Z 函数

```
// == Main ==
// n is |s|.
for (int i = 2, j = 0; i <= n; i++) {
    if (i < j + z[j]) z[i] = min(z[i - j + 1], j + z[j] - i);
    while (i + z[i] <= n && s[i + z[i]] == s[1 + z[i]]) z[i]++;
    if (i + z[i] > j + z[j]) j = i;
}
```

1.4 Manacher

```
// == Main ==
// t is the original string, n is |t|.
string s = "^#";
for (char i : t) s.push_back(i), s.push_back('#');
s.push_back ('@');
for (int i = 1, j = 0; i <= 2 * n + 1; i++) {
    if (i <= j + p[j]) p[i] = min (p[2 * j - i], j + p[j] - i);
    while (s[i - p[i] - 1] == s[i + p[i] + 1]) p[i]++;
    if (i + p[i] > j + p[j]) j = i;
}
```

1.5 AC 自动机

```
// == Preparations ==
#include <queue>
// == Main ==
struct ACAM {
    int tot, fail[200005], delta[200005][26];

    void insert(string s, int id) {
        int now = 0;
        for (char c : s) {
            int v = c - 'a';
            if (!delta[now][v]) delta[now][v] = ++tot;
```

```
            now = delta[now][v];
        }
        return;
    }
    void build() {
        queue<int> q;
        for (int c = 0; c < 26; c++)
            if (delta[0][c]) q.push(delta[0][c]);
        while (!q.empty()) {
            int now = q.front();
            q.pop();
            for (int c = 0; c < 26; c++)
                if (delta[now][c]) fail[delta[now][c]] = delta[fail[now]][c],
                ↪  q.push(delta[now][c]);
                else delta[now][c] = delta[fail[now]][c];
        }
        return;
    }
} ac;
```

1.6 回文自动机 PAM

```
// == Main ==
struct PAM {
    int tot, delta[500005][26], len[500005], fail[500005], ans[500005];
    string s;
    int lst;

    PAM() {tot = 1; len[0] = 0; len[1] = -1; fail[0] = fail[1] = 1;}
    int getfail(int now, int i) {
        while (s[i - len[now] - 1] != s[i]) now = fail[now];
        return now;
    }
    void insert(int i) {
        int now = getfail(lst, i);
        if (!delta[now][s[i] - 'a']) {
            len[++tot] = len[now] + 2;
            fail[tot] = delta[getfail(fail[now], i)][s[i] - 'a'];
            delta[now][s[i] - 'a'] = tot;
            ans[tot] = ans[fail[tot]] + 1;
        }
        lst = delta[now][s[i] - 'a'];
        return;
    }
} p;
```

1.7 后缀自动机

1.7.1 普通 SAM

```
// == Main ==
struct SAM {
    int tot, lst;
    int len[2000005], siz[2000005], link[2000005];
    int delta[2000005][26];

    SAM() {link[0] = -1;}
    void insert(char ch) {
        int c = ch - 'a', now = ++tot;
        len[now] = len[lst] + 1;
        siz[now] = 1;
        for (int p = lst; p != -1; p = link[p])
            if (!delta[p][c]) delta[p][c] = tot;
            else if (len[delta[p][c]] == len[p] + 1) {link[now] = delta[p][c];
            ↪  break;}
            else {
                int q = delta[p][c], v = ++tot;
                len[v] = len[p] + 1;
                memcpy(delta[v], delta[q], sizeof(delta[v]));
                link[v] = link[q], link[q] = v, link[now] = v;
                for (int i = p; delta[i][c] == q; i = link[i]) delta[i][c] = v;
                break;
            }
        lst = now;
        return ;
    }
} sam;
```

1.7.2 广义 SAM

注意自动机空间要开 Trie 的两倍。

```
// == Main ==
struct GSAM {
    int tot;
    int delta[2000005][26], link[2000005], len[2000005];
    struct Trie {
        int tot, trie[1000005][26], st[1000005];

        void insert(string s) {
            int now = 0;
            for (char c : s) {
```

```
                int id = c - 'a';
                if (!trie[now][id]) trie[now][id] = ++tot;
                now = trie[now][id];
            }
            return;
        }
    } tr;

    GSAM() {link[0] = -1;}
    int insert(int c, int lst) {
        int now = ++tot;
        len[now] = len[lst] + 1;
        for (int p = lst; p != -1; p = link[p])
            if (!delta[p][c]) delta[p][c] = now;
            else if (len[delta[p][c]] == len[p] + 1) {link[now] = delta[p][c]; break;}
            else {
                int q = delta[p][c], v = ++tot;
                len[v] = len[p] + 1;
                memcpy(delta[v], delta[q], sizeof(delta[v]));
                link[v] = link[q], link[q] = v, link[now] = v;
                for (int i = p; i != -1 && delta[i][c] == q; i = link[i]) delta[i][c] = v;
                break;
            }
        return now;
    }
    void build() {
        queue<int> q;
        tr.st[0] = 0;
        q.push(0);
        while (!q.empty()) {
            int now = q.front();
            q.pop();
            for (int i = 0; i < 26; i++)
                if (tr.trie[now][i])
                    tr.st[tr.trie[now][i]] = insert(i, tr.st[now]), q.push(tr.trie[now][i]);
        }
        return;
    }
} gsam;
```

1.8 后缀排序

```
// == Preparations ==
int sa[2000005], rk[2000005], b[1000005], cp[2000005];
// == Main ==
for (int i = 1; i <= n; i++) b[rk[i] = s[i]]++;
for (int i = 1; i < 128; i++) b[i] += b[i - 1];
for (int i = n; i >= 1; i--) sa[b[rk[i]]--] = i;
memcpy(cp, rk, sizeof(cp));
for (int i = 1, j = 0; i <= n; i++)
    if (cp[sa[i]] == cp[sa[i - 1]]) rk[sa[i]] = j;
    else rk[sa[i]] = ++j;
for (int w = 1; w < n; w <<= 1) {
    memcpy(cp, sa, sizeof(cp));
    memset(b, 0, sizeof(b));
    for (int i = 1; i <= n; i++) b[rk[cp[i] + w]]++;
    for (int i = 1; i <= n; i++) b[i] += b[i - 1];
    for (int i = n; i >= 1; i--) sa[b[rk[cp[i] + w]]--] = cp[i];
    memcpy(cp, sa, sizeof(cp));
    memset(b, 0, sizeof(b));
    for (int i = 1; i <= n; i++) b[rk[cp[i]]]++;
    for (int i = 1; i <= n; i++) b[i] += b[i - 1];
    for (int i = n; i >= 1; i--) sa[b[rk[cp[i]]]--] = cp[i];
    memcpy(cp, rk, sizeof(cp));
    for (int i = 1, j = 0; i <= n; i++)
        if (cp[sa[i]] == cp[sa[i - 1]] && cp[sa[i] + w] == cp[sa[i - 1] + w]) rk[sa[i]] = j;
        else rk[sa[i]] = ++j;
}
```

Chapter 2

数论

2.1 Miller Rabin 和 Pollard Rho

2.1.1 Miller Rabin

```
// == Preparations ==
const int prime[] = {2, 3, 5, 7, 9, 11, 13, 17, 19, 23, 29, 31, 37};

long long power(long long a, long long b, long long mod) {
    long long ans = 1;
    while (b) {
        if (b & 1) ans = (__int128)ans * a % mod;
        a = (__int128)a * a % mod;
        b >>= 1;
    }
    return ans % mod;
}
// == Main ==
inline int Miller_Rabin(long long n) {
    if (n == 1) return 0;
    if (n == 2) return 1;
    if (n % 2 == 0) return 0;
    long long u = n - 1, t = 0;
    while (u % 2 == 0) u /= 2, t++;
    for (int i = 0; i < 12; i++) {
        if (prime[i] % n == 0) continue;
        long long x = power(prime[i] % n, u, n);
        if (x == 1) continue;
        int flag = 0;
        for (int j = 1; j <= t; j++) {
            if (x == n - 1) {flag = 1; break;}
            x = (__int128)x * x % n;
        }
        if (!flag) return 0;
```

```
    }
    return 1;
}
```

2.1.2 Pollard Rho

```
// == Preparations ==
#include <chrono>
#include <random>

mt19937_64 gen(chrono::system_clock::now().time_since_epoch().count());
/*
Miller Rabin
*/
// == Main ==
long long Pollard_Rho(long long n) {
    long long s = 0, t = 0, c = gen() % (n - 1) + 1;
    for (int goal = 1; ; goal <<= 1, s = t) {
        long long val = 1;
        for (int step = 1; step <= goal; step++) {
            t = ((__int128)t * t + c) % n;
            val = (__int128)val * abs(t - s) % n;
            if (!val) return n;
            if (step % 127 == 0) {
                long long d = __gcd(val, n);
                if (d > 1) return d;
            }
        }
        long long d = __gcd(val, n);
        if (d > 1) return d;
    }
}
void factor(long long n) {
    if (n < ans) return ;
    if (n == 1 || Miller_Rabin(n)) {
        // n = 1 或 n 是一个质因子。
        // ...
    }
    long long p;
    do p = Pollard_Rho(n);
    while (p == n);
    while (n % p == 0) n /= p;
    factor(n), factor(p);
    return ;
}
```

2.2 常用数论算法

2.2.1 exgcd

求出来的解满足 $|x| \leq b, |y| \leq a$。

```
// == Main ==
int exgcd(int a, int b, int &x, int &y) {
    if (b == 0) {x = 1; y = 0; return a;}
    int m = exgcd(b, a % b, y, x);
    y -= a / b * x;
    return m;
}
```

2.2.2 CRT

没用。

```
// == Preparations ==
#include <vector>
// == Main ==
int CRT(vector<pair<int, int>> &a) {
    int M = 1;
    for (auto i : a) M *= i.second;
    int res = 0;
    for (auto i : a) {
        int t = inv(M / i.second, i.second);
        res = (res + (long long)i.first * (M / i.second) % M * t % M) % M;
    }
    return res;
}
```

2.2.3 exCRT

需保证 lcm 在 `long long` 范围内。

```
// == Preparations ==
long long exgcd(long long a, long long b, long long &x, long long &y);
// == Main ==
long long exCRT(vector<pair<long long, long long>> vec) {
    long long ans = vec[0].first, mod = vec[0].second;
    for (int i = 1; i < (int)vec.size(); i++) {
        long long a = mod, b = vec[i].second, c = vec[i].first - ans % b;
        long long x, y;
```

```
        long long g = exgcd(a, b, x, y);
        if (c % g != 0) return -1;
        b /= g;
        x = (__int128)x * (c / g) % b;
        ans += x * mod;
        mod *= b;
        ans = (ans % mod + mod) % mod;
    }
    return ans;
}
```

2.2.4 exLucas

```
// == Preparations ==
int power(int a, long long b, int mod);
int exgcd(int a, int b, int &x, int &y);
int CRT(vector<pair<int, int>> &a);
// == Main ==
int inv(int n, int p) {
    int x, y;
    exgcd(n, p, x, y);
    return (x % p + p) % p;
}
int fac(long long n, int p, int pk) {
    if (n == 0) return 1;
    int res = 1;
    for (int i = 1; i < pk; i++)
        if (i % p != 0) res = (long long)res * i % pk;
    res = power(res, n / pk, pk);
    for (int i = 1; i <= n % pk; i++)
        if (i % p != 0) res = (long long)res * i % pk;
    return (long long)res * fac(n / p, p, pk) % pk;
}
int C(long long n, long long m, int p, int pk) {
    long long x = n, y = m, z = n - m;
    int res = (long long)fac(x, p, pk) * inv(fac(y, p, pk), pk) % pk * inv(fac(z, p, pk), pk) % pk;
    long long e = 0;
    while (x) e += x / p, x /= p;
    while (y) e -= y / p, y /= p;
    while (z) e -= z / p, z /= p;
    return (long long)res * power(p, e, pk) % pk;
}
int exLucas(long long n, long long m, int p) {
    vector<pair<int, int>> a;
    for (int i = 2; i * i <= p; i++)
        if (p % i == 0) {
```

```cpp
            int pk = 1;
            while (p % i == 0) pk *= i, p /= i;
            a.emplace_back(C(n, m, i, pk), pk);
        }
    if (p != 1) a.emplace_back(C(n, m, p, p), p);
    return CRT(a);
}
```

2.3 万能欧几里得算法

问题描述：
给出一个幺半群 $(S,\cdot)$ 和元素 $u,r\in S$，以及一条直线 $y=\frac{ax+b}{c}$。
画出平面中所有坐标为正整数的横线和竖线，维护一个 f，初值为单位元 e。
从原点出发，先向 y 轴正方向走直到到达直线与 y 的交点，然后沿直线走一直走到与 $x=n$ 的交点为止。
每当经过一条横线时，执行 $f\leftarrow fu$，经过一条竖线时执行 $f\leftarrow fr$。特别地，在 y 轴上行走时不考虑竖线，同时经过横线和竖线时先执行前者。
求最终的 f。记为 $\mathrm{euclid}(a,b,c,n,u,r)$。
其中 $a,b\geq 0,n,c>0$。

做法：

$$\mathrm{euclid}(a,b,c,n,u,r)=\begin{cases} r^n & m=0\\ u^{\lfloor\frac{b}{c}\rfloor}\cdot\mathrm{euclid}(a \bmod c, b \bmod c, c, n, u, u^{\lfloor\frac{a}{c}\rfloor}r) & a\geq c\vee b\geq c\\ r^{\lfloor\frac{c-b-1}{a}\rfloor}u\cdot\mathrm{euclid}(c,(c-b-1) \bmod a, a, m-1, r, u)\cdot r^{n-\lfloor\frac{cm-b-1}{a}\rfloor} & \text{otherwise}\end{cases}$$

设一次乘法的复杂度为 $O(T)$，则复杂度为 $O(T\log(a+c)\log(a+n+c))$。

```cpp
// == Preparations ==
struct Node {
    // ...

    Node operator*(const Node &x) const {
        // ...
    }
};
// == Main ==
Node power(Node a, long long b) {
    Node ans = Node(/* 幺元 */);
    while (b) {
        if (b & 1) ans = ans * a;
        a = a * a;
```

```
        b >>= 1;
    }
    return ans;
}
Node Euclid(int a, int b, int c, long long n, Node r, Node u) {
    long long m = (a * n + b) / c;
    if (!m) return power(r, n);
    if (a >= c || b >= c)
        return power(u, b / c) * Euclid(a % c, b % c, c, n, power(u, a / c) * r, u);
    return power(r, (c - b - 1) / a) * u *
        Euclid(c, (c - b - 1) % a, a, m - 1, u, r) * power(r, n - (c * m - b - 1) /
         ↪  a);
}
```

Chapter 3

数据结构

3.1 K-D Tree

```cpp
// == Main ==
template<const int Dim = 2>
struct KDTree {
    using point = array<int, Dim>;
    struct node {
        point p, l, r;
        int val, siz, sum;
        node *ls, *rs;

        node() = default;
        node(point _p, int _val = 0):
            p(_p), l(_p), r(_p), val(_val), siz(1), sum(_val), ls(nullptr),
             ↪ rs(nullptr) {}
        void pushup() {
            l = r = p, siz = 1, sum = val;
            for (int i = 0; i < Dim; i++) {
                if (ls) l[i] = min(l[i], ls->l[i]), r[i] = max(r[i], ls->r[i]);
                if (rs) l[i] = min(l[i], rs->l[i]), r[i] = max(r[i], rs->r[i]);
            }
            if (ls) siz += ls->siz, sum += ls->sum;
            if (rs) siz += rs->siz, sum += rs->sum;
            return ;
        }
    };
    vector<node *> root;
    using itor = typename vector<node>::iterator;

    node *build(itor l, itor r, int dim = 0) {
        if (l == r) return nullptr;
        int mid = (r - l) / 2;
        nth_element(l, l + mid, r, [&dim](const node &x, const node &y) {return
         ↪ x.p[dim] < y.p[dim];});
```

```
        node *now = new node(*(l + mid));
        now->ls = build(l, l + mid, (dim + 1) % Dim);
        now->rs = build(l + mid + 1, r, (dim + 1) % Dim);
        now->pushup();
        return now;
    }
    void getnode(node *now, vector<node> &vec) {
        if (!now) return ;
        vec.push_back(*now);
        getnode(now->ls, vec), getnode(now->rs, vec);
        delete now;
        return ;
    }
    void insert(point p, int val) {
        vector<node> tmp({node(p, val)});
        while (!root.empty() && root.back()->siz == (int)tmp.size())
            getnode(root.back(), tmp), root.pop_back();
        sort(tmp.begin(), tmp.end(), [](const node &x, const node &y) {return x.p < y.p;});
        vector<node> vec;
        for (node i : tmp)
            if (!vec.empty() && vec.back().p == i.p) vec.back().val += i.val;
            else vec.push_back(i);
        root.push_back(build(vec.begin(), vec.end()));
        return ;
    }
    int query(point ll, point rr, node *now) {
        if (!now) return 0;
        int flag = 1;
        for (int i = 0; i < Dim; i++)
            if (now->r[i] < ll[i] || now->l[i] > rr[i]) return 0;
            else flag &= ll[i] <= now->l[i] && now->r[i] <= rr[i];
        if (flag) return now->sum;
        flag = 1;
        for (int i = 0; i < Dim; i++) flag &= ll[i] <= now->p[i] && now->p[i] <= rr[i];
        return flag * now->val + query(ll, rr, now->ls) + query(ll, rr, now->rs);
    }
    int query(point ll, point rr) {
        int ans = 0;
        for (node *rt : root) ans += query(ll, rr, rt);
        return ans;
    }
    ~KDTree() {
        vector<node> tmp;
        for (node *rt : root) getnode(rt, tmp);
    }
};
```

3.2 Link Cut Tree

代码维护的是点权异或和。

```
// == Main ==
struct LinkCutTree {
    int fa[100005], son[100005][2], siz[100005], swp[100005];
    int val[100005], Xor[100005];

    void pushup(int now) {
        siz[now] = siz[son[now][0]] + siz[son[now][1]] + 1;
        Xor[now] = Xor[son[now][0]] ^ val[now] ^ Xor[son[now][1]]; // 此处更新信息。
        return;
    }
    void pushdown(int now) {
        if(!swp[now]) return ;
        swap(son[now][0], son[now][1]);
        swp[son[now][0]] ^= 1, swp[son[now][1]] ^= 1;
        swp[now] = 0;
        // 此处将信息 pushdown
        return;
    }
    int isRoot(int now) {return now != son[fa[now]][0] && now != son[fa[now]][1];}
    int get(int now) {return now == son[fa[now]][1];}
    void rotate(int x) {
        int y = fa[x], z = fa[fa[x]], chk = get(x);
        if (!isRoot(y)) son[z][get(y)] = x;
        son[y][chk] = son[x][chk ^ 1], fa[son[x][chk ^ 1]] = y;
        son[x][chk ^ 1] = y, fa[y] = x;
        fa[x] = z;
        pushup(y), pushup(x);
        return;
    }
    void splay(int now) {
        vector<int> stk;
        stk.push_back(now);
        for (int i = now; !isRoot(i); i = fa[i]) stk.push_back(fa[i]);
        while (!stk.empty()) pushdown(stk.back()), stk.pop_back();
        for (int f; f = fa[now], !isRoot(now); rotate(now))
            if (!isRoot(f)) rotate(get(f) == get(now) ? f : now);
        return;
    }
    void access(int now) { // 打通到根的链
        for (int lst = 0; now; lst = now, now = fa[now]) splay(now), son[now][1] =
        ↪ lst, pushup(now);
        return;
    }
```

```
    void makeRoot(int now) {access(now); splay(now); swp[now] ^= 1; return;} // 设置
    ↪ 根
    void link(int u, int v) {makeRoot(u); fa[u] = v; return;} // 连接
    void cut(int u, int v) {makeRoot(u); access(v); splay(v); son[v][0] = fa[u] = 0;
    ↪ return;} // 切割
    int find(int now) { // 找根
        access(now), splay(now);
        pushdown(now);
        while (son[now][0]) now = son[now][0], pushdown(now);
        splay(now);
        return now;
    }
    void split(int u, int v) {makeRoot(u); access(v); splay(u); return;} // 剖出 u ~
    ↪ v 的链
    void update(int u, int _val) {split(u, u); val[u] = Xor[u] = _val; return ;} //
    ↪ 修改操作，split 后做就行了，此处为单点修改。
    int query(int u, int v) {split(u, v); return Xor[u];} // 查询操作，split 后做就行
    ↪ 了。
    int isConnected(int u, int v) {return find(u) == find(v);} // 查询两个点是否连通。
};
```

Chapter 4

图论

4.1 Tarjan

4.1.1 强连通分量

```
// == Main ==
void Tarjan(int now) {
    dfn[now] = low[now] = ++Index;
    s.push(now);
    for (int i = g.hd[now]; i; i = g.nxt[i])
        if (!dfn[g.to[i]]) {
            Tarjan(g.to[i]);
            low[now] = min(low[now], low[g.to[i]]);
        } else if (!scc[g.to[i]]) low[now] = min(low[now], dfn[g.to[i]]);
    if (low[now] == dfn[now]) {
        scc_cnt++;
        for (int x = 0; x != now; s.pop()) {
            x = s.top();
            scc[x] = scc_cnt;
        }
    }
    return ;
}
```

4.1.2 割边与边双

割边：

```
// == Main ==
void Tarjan(int now, int fa) {
    dfn[now] = low[now] = ++Index;
    for (int i = g.hd[now]; i; i = g.nxt[i])
        if (!dfn[g.to[i]]) {
```

```
            Tarjan(g.to[i], now);
            low[now] = min(low[now], low[g.to[i]]);
            if (low[g.to[i]] > dfn[now])
                printf("A Bridge of the Input Garph is (%d, %d)\n", now, g.to[i]);
        } else if (g.to[i] != fa) low[now] = min(low[now], dfn[g.to[i]]);
    return ;
}
```

边双：

```
// == Main ==
void Tarjan(int now, int fa) {
    dfn[now] = low[now] = ++Index;
    s.push(now);
    for (int i = g.hd[now]; i; i = g.nxt[i])
        if (!dfn[g.to[i]]) {
            Tarjan(g.to[i], now);
            low[now] = min(low[now], low[g.to[i]]);
        } else if (g.to[i] != fa) low[now] = min(low[now], dfn[g.to[i]]);
    if (low[now] == dfn[now]) {
        bcc_cnt++;
        for (int x = 0; x != now; s.pop()) {
            x = s.top();
            bcc[x] = bcc_cnt;
        }
    }
    return ;
}
```

4.1.3 割点与点双

割点：

```
// == Main ==
void Tarjan(int now, int root) {
    dfn[now] = low[now] = ++Index;
    int sons=0, flag=0;
    for (int i=g.hd[now]; i; i = g.nxt[i], sons++)
        if (!dfn[g.to[i]]) {
            Tarjan(g.to[i], now);
            low[now] = min(low[now], low[g.to[i]]);
            if (now!=root && low[g.to[i]] == dfn[now] && !flag)
                printf("A Cut Vertex of the Input Graph is %d.", now), flag=1;
        } else low[now] = min(low[now], dfn[g.to[i]]);
    if (now == root && sons >= 2)
        printf("A Cut Vertex of the Input Graph is %d.", now);
```

```
    return ;
}
```

点双：

```
// == Main ==
void Tarjan(int now) {
    dfn[now] = low[now] = ++Index;
    s.push(now);
    for (int i = g.hd[now]; i; i = g.nxt[i], sons++)
        if (!dfn[g.to[i]]) {
            Tarjan(g.to[i]);
            low[now] = min(low[now], low[g.to[i]]);
            if (low[g.to[i]] == dfn[now]) {
                printf("BCC #%d:\n", ++bcc_cnt);
                for (int x = 0; x != g.to[i]; s.pop())
                    printf("%d ", x = s.top());
                printf("%d\n", now);
            }
        } else low[now] = min(low[now], dfn[g.to[i]]);
    return ;
}
```

4.2 欧拉路径

```
// == Preparations ==
#include <vector>

vector<int> g[100005], ans;
// == Main ==
void dfs(int now) {
    while (!g[now].empty()) {
        int to = g[now].back();
        g[now].pop_back();
        dfs(to);
    }
    ans.push_back(now);
    return ;
}
```

4.3 二分图匹配

4.3.1 最大匹配

```
// == Preparations ==
int chos[100005], vis[100005];
struct graph {/* ... */} g;
// == Main ==
int dfs(int now) {
    for (int i = g.hd[now]; i; i = g.nxt[i]) {
        if (vis[g.to[i]]) continue;
        vis[g.to[i]] = true;
        if (!chos[g.to[i]] || dfs(chos[g.to[i]])) {
            chos[g.to[i]] = now;
            return 1;
        }
    }
    return 0;
}

for (int i = 1; i <= n; i++) {
    memset(vis, 0, sizeof(vis));
    ans += dfs(i);
}
```

4.3.2 最大权匹配

```
// == Preparations ==
int vis[1005], mat[1005], pre[1005];
long long g[505][1005];
long long w[1005], slack[1005];
// edge: g[u][n + v] = w;
// == Main ==
for (int i = 1; i <= n; i++) {
    w[i] = ~0x3f3f3f3f3f3f3f3f;
    for (int j = n + 1; j <= n + n; j++) w[i] = max(w[i], (long long)g[i][j]);
}
for (int i = 1; i <= n; i++) {
    memset(vis, 0, sizeof(vis));
    memset(slack, 0x3f, sizeof(slack));
    memset(pre, 0, sizeof(pre));
    int now = i, ri = 0;
    while (1) {
        int id = 0;
        long long delta = 0x3f3f3f3f3f3f3f3f;
        for (int j = n + 1; j <= n + n; j++)
```

```cpp
                if (!vis[j]) {
                    long long val = w[now] + w[j] - g[now][j];
                    if (val < slack[j]) slack[j] = val, pre[j] = ri;
                    if (slack[j] < delta) delta = slack[j], id = j;
                }
            w[i] -= delta;
            for (int j = n + 1; j <= n + n; j++)
                if (vis[j]) w[j] += delta, w[mat[j]] -= delta;
                else slack[j] -= delta;
            vis[ri = id] = 1;
            if (mat[ri]) now = mat[ri];
            else break;
        }
        while (ri) {
            mat[ri] = mat[pre[ri]];
            if (!pre[ri]) {mat[ri] = i; break;}
            ri = pre[ri];
        }
}
long long ans = 0;
for (int i = 1; i <= n + n; i++) ans += w[i];
printf("%lld\n", ans);
for (int i = n + 1; i <= n + n; i++) printf("%d ", mat[i]);
puts("");
```

4.4 网络流

4.4.1 最大流

```cpp
// == Preparations ==
#include <queue>
// == Main ==
struct Dinic {
    int s, t;
    struct graph {
        int tot, hd[205];
        int nxt[10005], to[10005], dt[10005];
        graph() {tot = 1;}
        void add(int u, int v, int w) {
            nxt[++tot] = hd[u];
            hd[u] = tot;
            to[tot] = v;
            dt[tot] = w;
            return ;
        }
    } g;
```

```
    int cur[205], dis[205];

    void add_edge(int u, int v, int f) {g.add(u, v, f), g.add(v, u, 0); return;}
    int bfs() {
        memset(dis, 0, sizeof(dis));
        queue<int>q;
        q.push(s);
        dis[s] = 1;
        while (!q.empty()) {
            int now = q.front();
            q.pop();
            cur[now] = g.hd[now];
            for (int i = g.hd[now]; i; i = g.nxt[i])
                if (g.dt[i] && !dis[g.to[i]]) dis[g.to[i]] = dis[now] + 1,
                 ↪ q.push(g.to[i]);
        }
        return dis[t];
    }
    long long dinic(int now, long long flow) {
        if (now == t) return flow;
        long long used = 0;
        for (int i = cur[now]; i && used < flow; i = g.nxt[i])
            if (g.dt[i] && dis[g.to[i]] == dis[now] + 1) {
                long long k = dinic(g.to[i], min(flow - used, (long long)g.dt[i]));
                g.dt[i] -= k, g.dt[i ^ 1] += k;
                used += k;
                cur[now] = i;
            }
        if (used == 0) dis[now] = 0;
        return used;
    }
    long long solve() {
        long long ans = 0;
        while (bfs()) ans += dinic(s, 0x3f3f3f3f3f3f3f3f);
        return ans;
    }
} F;
```

4.4.2 费用流

原始对偶:

```
// == Preparations ==
#include <queue>
// == Main ==
struct PrimalDual {
    int n, s, t;
```

```
struct graph {
    int tot, hd[805];
    int nxt[30005], to[30005], flw[30005], cst[30005];

    graph() {tot = 1;}
    void add(int u, int v, int f, int c) {
        nxt[++tot] = hd[u];
        hd[u] = tot;
        to[tot] = v;
        flw[tot] = f;
        cst[tot] = c;
        return ;
    }
} g;
int h[805], dis[805], f[805], pre[805];
struct node {
    int id, val;

    node() = default;
    node(int _id, int _val): id(_id), val(_val) {}
    bool operator<(const node &x) const {return val > x.val;}
};

void add_edge(int u, int v, int f, int c) {g.add(u, v, f, c), g.add(v, u, 0, -c); return;}
void spfa() {
    queue<int> q;
    memset(h, 0x3f, sizeof(h));
    h[s] = 0;
    q.push(s);
    while (!q.empty()) {
        int now = q.front();
        q.pop();
        f[now] = 0;
        for (int i = g.hd[now]; i; i = g.nxt[i])
            if (g.flw[i] && h[g.to[i]] > h[now] + g.cst[i]) {
                h[g.to[i]] = h[now] + g.cst[i];
                if (!f[g.to[i]]) q.push(g.to[i]), f[g.to[i]] = 1;
            }
    }
    return ;
}
int dijkstra() {
    priority_queue<node> q;
    memset(dis, 0x3f, sizeof(dis));
    memset(pre, 0, sizeof(pre));
    q.emplace(s, dis[s] = 0);
    while (!q.empty()) {
        int now = q.top().id, tmp = q.top().val;
```

```
            q.pop();
            if (dis[now] != tmp) continue;
            for (int i = g.hd[now]; i; i = g.nxt[i])
                if (g.flw[i] && dis[g.to[i]] > dis[now] + g.cst[i] + h[now] - h[g.to[i]]) {
                    q.emplace(g.to[i], dis[g.to[i]] = dis[now] + g.cst[i] + h[now] - h[g.to[i]]);
                    pre[g.to[i]] = i ^ 1;
                }
        }
        return pre[t];
    }
    pair<int, int> solve() {
        int flow = 0, cost = 0;
        spfa();
        while (dijkstra()) {
            for (int i = 1; i <= n; i++)
                if (dis[i] < 0x3f3f3f3f) h[i] += dis[i];
            int mnflow = 0x3f3f3f3f;
            for (int i = t; i != s; i = g.to[pre[i]]) mnflow = min(mnflow, g.flw[pre[i] ^ 1]);
            for (int i = t; i != s; i = g.to[pre[i]]) g.flw[pre[i] ^ 1] -= mnflow, g.flw[pre[i]] += mnflow;
            flow += mnflow;
            cost += mnflow * h[t];
        }
        return {flow, cost};
    }
} F;
```

4.4.3 上下界

$f(u,v)$ 表示边 (u,v) 的流量，$f(u)$ 表示 u 的出流减入流，$c(u,v)$ 表示边 (u,v) 的容量。
对于每条边给定一个流量下界 $b(u,v)$，需要额外满足 $\forall(u,v), b(u,v) \le f(u,v) \le c(u,v)$。

无源汇上下界可行流

没有源点和汇点，对于所有点满足 $f(u)=0$，求一个可行的流。
先强制每条边流到流量下界，建立虚拟源汇点 s,t，对于每个点 u 考虑此时的净流量：

- $f(u)=0$：满足条件，不用管。
- $f(u)>0$：出流大于入流，从 u 向 t 连容量为 $f(u)$ 的边。
- $f(u)<0$：入流大于出流，从 s 向 u 连容量为 $-f(u)$ 的边。

将原图中每条边的容量设为 $c(u,v)-b(u,v)$，则从 s 到 t 的流相当于增加调整流量的过程。

若 s 的出边流满（等同于 t 的入边流满），则找到了一条可行流。

有源汇上下界可行流

连一条 t 到 s 容量正无穷下界为 0 的边，然后跑无源汇上下界可行流即可，流量为新增边的流量。

有源汇上下界最大流

求出可行流后删掉 t 到 s 的边，在残量网络上跑 s 到 t 的最大流，该最大流加上原本的可行流即为答案。

有源汇上下界最小流

同理，改成求 t 到 s 的最大流，原可行流减去该最大流即为答案。

有源汇上下界最小费用流

做法是一样的，所有新增边费用为 0。

需要注意求最小流时需要改成费用最大。

4.4.4 有负圈的最小费用最大流

先钦定所有负圈边流满，即上下界均为流量。然后对于负边建反向、容量相同、费用为相反数的边用于退流原边。

这样就转化成了有源汇上下界最小费用最大流。

4.5 k 短路

复杂度为 $O((n+m)\log n + k\log k)$。

```cpp
// == Preparations ==
int ontree[200005];
struct graph {
    int tot, hd[5005];
    int nxt[200005], to[200005];
    long long dt[200005];

    void add(int u, int v, long long w) {
        nxt[++tot] = hd[u];
        hd[u] = tot;
        to[tot] = v;
        dt[tot] = w;
        return ;
    }
```

```
} g;
long long dis[5005];
struct node {
    int id;
    long long val;

    node() = default;
    node(int _id, long long _val): id(_id), val(_val) {}
    bool operator<(const node &x) const {return val > x.val;}
};
priority_queue<node> q;
int vis[5005];

// 以下左偏树
struct HeapNode {
    long long val;
    int to, dist;
    HeapNode *ls, *rs;

    HeapNode() = default;
    HeapNode(long long _val, int _to): val(_val), to(_to), dist(1), ls(nullptr),
     ↪ rs(nullptr) {}
};
struct Heap {
    HeapNode *root[5005];

    HeapNode *merge(HeapNode *u, HeapNode *v) {
        if (!u) return v;
        if (!v) return u;
        if (u->val > v->val) swap(u, v);
        HeapNode *p = new HeapNode(*u);
        p->rs = merge(u->rs, v);
        if (!p->ls || p->ls->dist < p->rs->dist) swap(p->ls, p->rs);
        if (p->rs) p->dist = p->rs->dist + 1;
        else p->dist = 1;
        return p;
    }
} h;

struct Node {
    HeapNode *id;
    long long val;

    Node() = default;
    Node(HeapNode *_id, long long _val): id(_id), val(_val) {}
    bool operator<(const Node &x) const {return val > x.val;}
};
priority_queue<Node> Q;
// == Main ==
```

```cpp
void dfs(int now) {
    vis[now] = 1;
    for (int i = g.hd[now]; i; i = g.nxt[i])
        if (!vis[g.to[i]] && dis[g.to[i]] == dis[now] + g.dt[i]) ontree[i] = 1,
         ↪  dfs(g.to[i]);
    return ;
}
void dfs2(int now) {
    for (int i = g.hd[now]; i; i = g.nxt[i])
        if (ontree[i]) h.root[g.to[i]] = h.merge(h.root[g.to[i]], h.root[now]),
         ↪  dfs2(g.to[i]);
    return;
}

memset(dis, 0x3f, sizeof(dis));
q.emplace(n, dis[n] = 0);
while (!q.empty()) {
    int now = q.top().id;
    long long tmp = q.top().val;
    q.pop();
    if (tmp != dis[now]) continue;
    for (int i = g.hd[now]; i; i = g.nxt[i])
        if (dis[g.to[i]] > dis[now] + g.dt[i]) q.emplace(g.to[i], dis[g.to[i]] =
         ↪  dis[now] + g.dt[i]);
}
dfs(n);
for (int i = 1; i <= n; i++)
    for (int j = g.hd[i]; j; j = g.nxt[j])
        if (!ontree[j] && g.to[j] != n)
            h.root[g.to[j]] = h.merge(h.root[g.to[j]], new HeapNode(dis[i] + g.dt[j]
             ↪  - dis[g.to[j]], i));
dfs2(n);
if (h.root[1]) Q.emplace(h.root[1], dis[1] + h.root[1]->val);
while (!Q.empty()) { // 每次取出来一条路径
    HeapNode *now = Q.top().id;
    long long d = Q.top().val;
    Q.pop();
    if (now->ls) Q.emplace(now->ls, d - now->val + now->ls->val);
    if (now->rs) Q.emplace(now->rs, d - now->val + now->rs->val);
    HeapNode *tmp = h.root[now->to];
    if (tmp) Q.emplace(tmp, d + tmp->val);
}
```

4.6 全局最小割

时间复杂度为 $O(|V|^3)$。

```cpp
// == Preparations ==
int g[605][605], vis1[605], vis2[605];
long long w[605];
// == Main ==
long long Stoer_Wagner() {
    long long ans = 0x3f3f3f3f3f3f3f3f;
    for (int i = 1; i < n; i++) {
        int s = 0, t = 0;
        memset(vis2, 0, sizeof(vis2));
        memset(w, 0, sizeof(w));
        for (int j = 1; j <= n - i + 1; j++) {
            int now = 0;
            for (int k = 1; k <= n; k++)
                if (!vis1[k] && !vis2[k] && w[k] >= w[now]) now = k;
            s = t, t = now;
            vis2[now] = 1;
            for (int k = 1; k <= n; k++) w[k] += g[k][now];
        }
        ans = min(ans, w[t]);
        vis1[t] = 1;
        for (int j = 1; j <= n; j++)
            if (j != s) g[s][j] += g[t][j], g[j][s] += g[j][t];
    }
    return ans;
}
```

4.7 支配树

$idom_u$ 为 u 在支配树上的父亲。

最后 id 形成 dfs 序。

```cpp
// == Preparations ==
#include <vector>

struct graph {
    int tot, hd[200005];
    int nxt[300005], to[300005];

    void add(int u, int v) {
        nxt[++tot] = hd[u];
        hd[u] = tot;
        to[tot] = v;
        return ;
    }
} g, fg;
```

```
int timer, fa[200005], dfn[200005], id[200005];
int sdom[200005], idom[200005];
struct dsu {
    int fa[200005], mn[200005];

    dsu() {for (int i = 1; i < 200005; i++) fa[i] = mn[i] = i;}
    int find(int x) {
        if (x == fa[x]) return x;
        int tmp = find(fa[x]);
        if (dfn[sdom[mn[fa[x]]]] < dfn[sdom[mn[x]]]) mn[x] = mn[fa[x]];
        return fa[x] = tmp;
    }
} d;
vector<int> vec[200005];
int siz[200005];
// == Main ==
void dfs(int now) {
    id[dfn[now] = ++timer] = now;
    for (int i = g.hd[now]; i; i = g.nxt[i])
        if (!dfn[g.to[i]]) fa[g.to[i]] = now, dfs(g.to[i]);
    return ;
}
void solve() {
    dfs(1);
    for (int i = 1; i <= n; i++) sdom[i] = i;
    for (int i = timer; i >= 1; i--) {
        int u = id[i];
        for (int v : vec[u]) {
            d.find(v);
            if (sdom[d.mn[v]] == u) idom[v] = u;
            else idom[v] = d.mn[v];
        }
        if (i == 1) continue;
        for (int j = fg.hd[u]; j; j = fg.nxt[j]) {
            if (!dfn[fg.to[j]]) continue;
            if (dfn[fg.to[j]] < dfn[sdom[u]]) sdom[u] = fg.to[j];
            else if (dfn[fg.to[j]] > dfn[u]) {
                d.find(fg.to[j]);
                if (dfn[sdom[d.mn[fg.to[j]]]] < dfn[sdom[u]]) sdom[u] =
                 ↪ sdom[d.mn[fg.to[j]]];
            }
        }
        vec[sdom[u]].push_back(u);
        d.fa[u] = fa[u];
    }
    for (int i = 2; i <= timer; i++)
        if (idom[id[i]] != sdom[id[i]]) idom[id[i]] = idom[idom[id[i]]];
    return ;
```

```
}
```

4.8 弦图

4.8.1 MCS 最大势算法。

```
// == Preparations ==
#include <vector>

int pos[/* ... */], p[/* ... */];
vector<int> vec[/* ... */];
// == Main ==
for (int i = 1; i <= n; i++) pos[i] = vec[0].size(), vec[0].push_back(i);
for (int i = 1, j = 0; i <= n; i++, j++) {
    while (vec[j].empty()) j--;
    int u = p[i] = vec[j].back();
    vec[j].pop_back();
    pos[u] = -1;
    for (int k = g.hd[u]; k; k = g.nxt[k])
        if (pos[g.to[k]] != -1) {
            int v = g.to[k];
            pos[vec[l[v]].back()] = pos[v];
            swap(vec[l[v]][pos[v]], vec[l[v]].back());
            vec[l[v]].pop_back();
            pos[v] = vec[++l[v]].size();
            vec[l[v]].push_back(v);
        }
}
reverse(p + 1, p + n + 1);
```

4.8.2 弦图判定

跑 MCS，然后判断是否为完美消除序列。

具体地，对于每个 p_i，找到与之相连且在它之后出现的点，按出现顺序记为 $c_1, c_2, \ldots, c_k$，我们只需要判断 c_1 与 c_j 之间是否有边即可。因为这个团中其他边会在 $p_{c_2}, p_{c_3}, \ldots, p_{c_k}$ 中被判断。

4.8.3 求弦图的团数与色数

求团数：

设 $N(x)$ 为完美消除序列中在 x 之后且与 x 相连的点的集合，则弦图的最大团一定可以被表示为 $\{x\} + N(x)$，则 $|\{x\} + N(x)|$ 的最大值就是弦图的团数。

求色数：

考虑按完美消除序列从后往前考虑，贪心染 mex，这样需要的颜色数量等于团数。由于团数小于等于

色数，这样取到等号，一定最小。

4.8.4 求弦图的最大独立集和最小团覆盖

最大独立集：
按完美消除序列从前往后贪心。正确性证明：每次考虑最靠前的极大团，选最前面的点不劣于选其他点，且优于不选点。

最小团覆盖：
取最大独立集中的每个点 x 对应的团 $\{x\} + N(x)$，这样需要的团的数量等于最大独立集的大小。由于最大独立集小于等于最小团覆盖，这样取到等号，一定最小。

4.8.5 tricks

区间图是弦图，完美消除序列为按区间右端点从小到大排序。

树上距离不超过 k 的点连边是弦图，完美消除序列为 bfs 序的逆序。

Chapter 5

多项式

5.1 牛顿迭代

用于解决下列问题：

> 已知函数 G 且 $G(F(x)) = 0$，求多项式 $F \pmod{x^n}$。

结论：

$$F(x) = F_*(x) - \frac{G(F_*(x))}{G'(F_*(x))} \pmod{x^n}$$

其中 $F_*(x)$ 为做到 $x^{n/2}$ 时的答案。

5.2 FFT

```
// == Preparations ==
struct complex {
    double a, b;

    complex() = default;
    complex(double _a, double _b): a(_a), b(_b) {}
    complex operator+(const complex &x) const {return complex(a + x.a, b + x.b);}
    complex operator-(const complex &x) const {return complex(a - x.a, b - x.b);}
    complex operator*(const complex &x) const {return complex(a * x.a - b * x.b, a *
    ↪  x.b + b * x.a);}
    complex operator/(const complex &x) const {
        double t = b * b + x.b * x.b;
        return complex((a * x.a + b * x.b) / t, (b * x.a - a * x.b) / t);
    }
    complex &operator+=(const complex &x) {return *this = *this + x;}
    complex &operator-=(const complex &x) {return *this = *this - x;}
    complex &operator*=(const complex &x) {return *this = *this * x;}
```

```cpp
    complex &operator/=(const complex &x) {return *this = *this / x;}
};
// == Main ==
void FFT(vector<complex> &f, int flag) const {
    int n = f.size();
    vector<int> swp(n);
    for (int i = 0; i < n; i++) {
        swp[i] = swp[i >> 1] >> 1 | ((i & 1) * (n >> 1));
        if (i < swp[i]) std::swap(f[i], f[swp[i]]);
    }
    for (int mid = 1; mid < n; mid <<= 1) {
        complex w1(cos(pi / mid), flag * sin(pi / mid));
        for (int i = 0; i < n; i += mid << 1) {
            complex w(1, 0);
            for (int j = 0; j < mid; j++, w *= w1) {
                complex x = f[i + j], y = w * f[i + mid + j];
                f[i + j] = x + y, f[i + mid + j] = x - y;
            }
        }
    }
    return;
}
```

5.3 常用 NTT 模数及其原根

模数	原根	分解
167772161	3	$5 \times 2^{25} + 1$
469762049	3	$7 \times 2^{26} + 1$
998244353	3	$119 \times 2^{23} + 1$
1004535809	3	$479 \times 2^{21} + 1$
2013265921	31	$15 \times 2^{27} + 1$
2281701377	3	$17 \times 2^{27} + 1$

5.4 多项式模板

```cpp
// == Preparations ==
#include <vector>
// == Main ==
namespace Poly {
    const int mod = 998244353, G = 3, invG = 332748118;

    inline int power(int a, int b) {
        int ans = 1;
        while (b) {
```

```
        if (b & 1) ans = (long long)ans * a % mod;
        a = (long long)a * a % mod;
        b >>= 1;
    }
    return ans % mod;
}

struct poly: vector<int> {
    poly(initializer_list<int> &&arg): vector<int>(arg) {}
    template<typename... argT>
    poly(argT &&...args): vector<int>(forward<argT>(args)...) {}

    poly operator+(const poly &b) const {
        const poly &a = *this;
        poly ans(max(a.size(), b.size()));
        for (int i = 0; i < (int)ans.size(); i++)
            ans[i] = ((i < (int)a.size() ? a[i] : 0) + (i < (int)b.size() ? b[i] : 0)) % mod;
        return ans;
    }
    poly operator+=(const poly &b) {return *this = *this + b;}
    poly operator-(const poly &b) const {
        const poly &a = *this;
        poly ans(max(a.size(), b.size()));
        for (int i = 0; i < (int)ans.size(); i++)
            ans[i] = ((i < (int)a.size() ? a[i] : 0) - (i < (int)b.size() ? b[i] : 0) + mod) % mod;
        return ans;
    }
    poly operator-=(const poly &b) {return *this = *this - b;}
    void NTT(poly &g, int flag) const {
        int n = g.size();
        vector<unsigned long long> f(g.begin(), g.end());
        vector<int> swp(n);
        for (int i = 0; i < n; i++) {
            swp[i] = swp[i >> 1] >> 1 | ((i & 1) * (n >> 1));
            if (i < swp[i]) std::swap(f[i], f[swp[i]]);
        }
        for (int mid = 1; mid < n; mid <<= 1) {
            int w1 = power(flag ? G : invG, (mod - 1) / mid / 2);
            vector<int> w(mid);
            w[0] = 1;
            for (int i = 1; i < mid; i++) w[i] = (long long)w[i - 1] * w1 % mod;
            for (int i = 0; i < n; i += mid << 1)
                for (int j = 0; j < mid; j++) {
                    int t = (long long)w[j] * f[i + mid + j] % mod;
                    f[i + mid + j] = f[i + j] - t + mod;
                    f[i + j] += t;
                }
```

```cpp
            if (mid == 1 << 10)
                for (int i = 0; i < n; i++) f[i] %= mod;
        }
        int inv = flag ? 1 : power(n, mod - 2);
        for (int i = 0; i < n; i++) g[i] = f[i] % mod * inv % mod;
        return;
    }

    // 下面是基于转置原理的 NTT，相对朴素版本效率更高。
    void NTT(poly &g, int flag) const {
        int n = g.size();
        vector<int> f(g.begin(), g.end());
        if (flag) {
            for (int mid = n >> 1; mid >= 1; mid >>= 1) {
                int w1 = power(G, (mod - 1) / mid / 2);
                vector<int> w(mid);
                w[0] = 1;
                for (int i = 1; i < mid; i++) w[i] = (long long)w[i - 1] * w1 % mod;
                for (int i = 0; i < n; i += mid << 1)
                    for (int j = 0; j < mid; j++) {
                        int t = (long long)(f[i + j] - f[i + mid + j] + mod) * w[j] % mod;
                        f[i + j] = f[i + j] + f[i + mid + j] >= mod ?
                            f[i + j] + f[i + mid + j] - mod : f[i + j] + f[i + mid + j];
                        f[i + mid + j] = t;
                    }
            }
            for (int i = 0; i < n; i++) g[i] = f[i];
        } else {
            for (int mid = 1; mid < n; mid <<= 1) {
                int w1 = power(invG, (mod - 1) / mid / 2);
                vector<int> w(mid);
                w[0] = 1;
                for (int i = 1; i < mid; i++) w[i] = (long long)w[i - 1] * w1 % mod;
                for (int i = 0; i < n; i += mid << 1)
                    for (int j = 0; j < mid; j++) {
                        int t = (long long)w[j] * f[i + mid + j] % mod;
                        f[i + mid + j] = f[i + j] - t < 0 ? f[i + j] - t + mod : f[i + j] - t;
                        f[i + j] = f[i + j] + t >= mod ? f[i + j] + t - mod : f[i + j] + t;
                    }
            }
            int inv = power(n, mod - 2);
            for (int i = 0; i < n; i++) g[i] = (long long)f[i] * inv % mod;
        }
```

```
        return;
    }

    poly operator*(poly b) const {
        poly a(*this);
        int n = 1, len = (int)(a.size() + b.size()) - 1;
        while (n < len) n <<= 1;
        a.resize(n), b.resize(n);
        NTT(a, 1), NTT(b, 1);
        poly c(n);
        for (int i = 0; i < n; i++) c[i] = (long long)a[i] * b[i] % mod;
        NTT(c, 0);
        c.resize(len);
        return c;
    }
    poly operator*=(const poly &b) {return *this = *this * b;}
    poly inv() const {
        poly f = *this, g;
        g.push_back(power(f[0], mod - 2));
        int n = 1;
        while (n < (int)f.size()) n <<= 1;
        f.resize(n << 1);
        for (int len = 2; len <= n; len <<= 1) {
            poly tmp(len), ff(len << 1);
            for (int i = 0; i < len >> 1; i++) tmp[i] = g[i] * 2 % mod;
            for (int i = 0; i < len; i++) ff[i] = f[i];
            g.resize(len << 1);
            NTT(g, 1), NTT(ff, 1);
            for (int i = 0; i < len << 1; i++) g[i] = (long long)g[i] * g[i] % mod * ff[i] % mod;
            NTT(g, 0);
            g.resize(len);
            for (int i = 0; i < len; i++) g[i] = (tmp[i] - g[i] + mod) % mod;
        }
        g.resize(size());
        return g;
    }
    poly sqrt() const { // need F[0] = 1.
        poly f = *this, g;
        g.push_back(1);
        int n = 1;
        while (n < (int)f.size()) n <<= 1;
        f.resize(n << 1);
        for (int len = 2; len <= n; len <<= 1) {
            poly tmp(len), ff(len << 1);
            for (int i = 0; i < len >> 1; i++) tmp[i] = g[i] * 2 % mod;
            for (int i = 0; i < len; i++) ff[i] = f[i];
            g.resize(len << 1);
            NTT(g, 1);
```

```
            for (int i = 0; i < len << 1; i++) g[i] = (long long)g[i] * g[i] %
             ↪  mod;
            NTT(g, 0);
            g += ff;
            g *= tmp.inv();
            g.resize(len);
        }
        g.resize(size());
        return g;
    }
    poly derivative() const {
        poly f(*this);
        for (int i = 1; i < (int)f.size(); i++) f[i - 1] = (long long)f[i] * i %
         ↪  mod;
        f.pop_back();
        return f;
    }
    poly integral() const {
        poly f(*this);
        f.push_back(0);
        for (int i = f.size() - 1; i >= 1; i--) f[i] = (long long)f[i - 1] *
         ↪  power(i, mod - 2) % mod;
        f[0] = 0;
        return f;
    }
    poly ln() const {
        poly f((derivative() * inv()).integral());
        f.resize(size());
        return f;
    }
    poly exp() const { // 需要满足 F[0] = 0
        poly f(*this), g;
        g.push_back(1);
        int n = 1;
        while (n < (int)size()) n <<= 1;
        f.resize(n);
        for (int len = 2; len <= n; len <<= 1) {
            poly tmp(g);
            g.resize(len);
            g = g.ln();
            for (int i = 0; i < len; i++) g[i] = (f[i] - g[i] + mod) % mod;
            g[0] = (g[0] + 1) % mod;
            g *= tmp;
            g.resize(len);
        }
        g.resize(size());
        return g;
    }
};
```

```

    inline poly power(poly f, int b) { // 需要满足 F[0] = 1
        f = f.ln();
        for (int i = 0; i < (int)f.size(); i++) f[i] = (long long)f[i] * b % mod;
        f = f.exp();
        return f;
    }
    // 不要求 F[0] = 1 的多项式快速幂，但是我忘记怎么用了，记得去回顾一下！
    poly power(poly f, int b1, int b2 = -1) {
        if (b2 == -1) b2 = b1;
        int n = f.size(), p = 0;
        reverse(f.begin(), f.end());
        while (!f.empty() && !f.back()) f.pop_back(), p++;
        if (f.empty() || (long long)p * b1 >= n) return poly(n);
        int v = f.back();
        int inv = power(v, mod - 2);
        for (int &i : f) i = (long long)i * inv % mod;
        reverse(f.begin(), f.end());
        f = f.ln();
        for (int &i : f) i = (long long)i * b1 % mod;
        f = f.exp();
        reverse(f.begin(), f.end());
        for (int i = 1; i <= p * b1; i++) f.push_back(0);
        reverse(f.begin(), f.end());
        f.resize(n);
        v = power(v, b2);
        for (int &i : f) i = (long long)i * v % mod;
        return f;
    }
}
```

Chapter 6

杂项

6.1 取模类

```cpp
// == Main ==
struct mint {
    static const int mod = 998244353;
    int v;

    mint() = default;
    mint(int _v): v((_v % mod + mod) % mod) {}
    explicit operator int() const {return v;}
    mint operator+(const mint &x) const {return v + x.v - (v + x.v < mod ? 0 : mod);}
    mint &operator+=(const mint &x) {return *this = *this + x;}
    mint operator-(const mint &x) const {return v - x.v + (v - x.v >= 0 ? 0 : mod);}
    mint &operator-=(const mint &x) {return *this = *this - x;}
    mint operator*(const mint &x) const {return (long long)v * x.v % mod;}
    mint &operator*=(const mint &x) {return *this = *this * x;}
    mint inv() const {
        mint a(*this), ans(1);
        int b(mod - 2);
        while (b) {
            if (b & 1) ans *= a;
            a *= a;
            b >>= 1;
        }
        return ans;
    }
    mint operator/(const mint &x) const {return *this * x.inv();}
    mint &operator/=(const mint &x) {return *this = *this / x;}
    mint operator-() {return mint(-v);}
};
```

6.1.1 Barrett 约减

当模数不固定时可以加速。

用法：在构造函数中传模数，使用方法为 `F.reduce(x)`，其中 x 是需要取模的数。

```cpp
// == Main ==
struct Barrett {
    unsigned long long b, m;
    Barrett(unsigned long long b = 2): b(b), m((__uint128_t(1) << 64) / b) {}
    unsigned long long reduce(long long x) {
        unsigned long long r = (__uint128_t(x + b) * m) >> 64;
        unsigned long long q = (x + b) - b * r;
        return q >= b ? q - b : q;
    }
} F;
```

6.2 对拍脚本

```bash
#!/usr/bin/bash

declare -i num=0

while [ true ]; do
    ./mkdata > in.txt
    time ./mine < in.txt > out.txt
    ./correct < in.txt > ans.txt
    diff out.txt ans.txt
    if [ $? -ne 0 ]; then
        echo "WA"
        break
    fi
    num=num+1
    echo "Passed $num tests."
done
```

6.3 VS Code 配置

6.3.1 User Tasks

```json
{
    // See https://go.microsoft.com/fwlink/?LinkId=733558
    // for the documentation about the tasks.json format
```

```
    "version": "2.0.0",
    "tasks": [
        {
            "type": "shell",
            "label": "My C++ Runner",
            "detail": "Build and Run Current C++ Program",
            "command": [ // 三个编译方式保留一个即可。
                "clear",
                "&&",
                "g++ ${file} -o ${fileDirname}/${fileBasenameNoExtension} -std=c++14 -Wall -Wextra && echo '== Normal =='",
                "g++ ${file} -o ${fileDirname}/${fileBasenameNoExtension} -std=c++14 -Wall -Wextra -O2 && echo '== O2 =='",
                "g++ ${file} -o ${fileDirname}/${fileBasenameNoExtension} -std=c++14 -Wall -Wextra -fsanitize=undefined,address && echo '== UB Check =='",
                "&&",
                "gnome-terminal -- bash -c \"ulimit -s 524288; time ${fileDirname}/${fileBasenameNoExtension}; read -p 'Press ENTER to continue...'; exit\""
            ],
            "problemMatcher": [ // 非必要
                "$gcc"
            ],
            "group": { // 非必要
                "kind": "build",
                "isDefault": true
            },
            "presentation": { // 非必要
                "showReuseMessage": false
            }
        }
    ]
}
```

6.3.2 设置

- 字体大小：16。（`"editor.fontSize": 16`）
- 添加多个光标的方式：ctrl。（`"editor.multiCursorModifier": "ctrlCmd"`）
- 不适用空格代替 Tab。（`"editor.insertSpaces": false`）
- 不允许 Enter 进行代码补全。（`"editor.acceptSuggestionOnEnter": "off"`）
- 标尺：110。（`"editor.rulers": [110]`）
- 平滑。（`"editor.cursorSmoothCaretAnimation": "on"`）

- 标题栏外观。（"window.titleBarStyle": "custom"）

totally：

```json
{
    "editor.fontSize": 16,
    "editor.multiCursorModifier": "ctrlCmd",
    "editor.insertSpaces": false,
    "editor.acceptSuggestionOnEnter": "off",
    "editor.rulers": [110],
    "editor.cursorSmoothCaretAnimation": "on",
    "window.commandCenter": false
}
```

6.3.3 快捷键

- 切换块注释：Ctrl+Shift+A -> Ctrl+Shift+/
- 运行任务：Ctrl+Shift+B -> F11
- 向上移动行：Alt+up -> Ctrl+Shift+up
- 向下移动行：Alt+down -> Ctrl+Shift+down