



ICPC Templates

我们需要更深入浅出一些

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Chapter 1

字符串

1.1 最小表示法

```
1 // == Main ==
2 int i = 0, j = 1, k = 0;
3 while (k < n && i < n && j < n)
4     if (a[(i + k) % n] == a[(j + k) % n]) k++;
5     else {
6         if (a[(i + k) % n] > a[(j + k) % n]) i = i + k + 1;
7         else j = j + k + 1;
8         if (i == j) i++;
9         k = 0;
10    }
11 ans = min(i, j);
```

1.2 Border 理论

1.2.1 关键结论

定理 1: 对于一个字符串 s , 若用 t 表示其最长的 Border, 则有 $\mathcal{B}(s) = \mathcal{B}(t) \cup \{t\}$ 。

定理 2: 一个字符串的 Border 与 Period 一一对应。具体地, $\text{pre}(s, i) \in \mathcal{B}(s) \iff |s| - i \in \mathcal{P}(s)$ 。

弱周期引理:

$$\forall p, q \in \mathcal{P}(s), p + q \leq |s| \implies \gcd(p, q) \in \mathcal{P}(s)$$

定理 3: 若字符串 t 是字符串 s 的前缀, 且 $a \in \mathcal{P}(s), b \in \mathcal{P}(t), b \mid a, |t| \geq a$, 且 b 是 t 的整周期, 则有 $b \in \mathcal{P}(s)$ 。

周期引理:

$$\forall p, q \in \mathcal{P}(s), p + q - \gcd(p, q) \leq |s| \implies \gcd(p, q) \in \mathcal{P}(s)$$

定理 4: 对于文本串 s 和模式串 t , 若 $|t| \geq \frac{|s|}{2}$, 且 t 在 s 中至少成功匹配了 3 次, 则每次匹配的位置形成一个等差数列, 且公差为 t 的最小周期。

定理 5: 一个字符串 s 的所有长度不小于 $\frac{|s|}{2}$ 的 Border 的长度构成一个等差数列。

定理 6: 一个字符串的所有 Border 的长度排序后可以划分成 $\lceil \log_2 |s| \rceil$ 个连续段, 使得每段都是一个等差数列。

定理 7: 回文串的回文前/后缀即为该串的 Border。

定理 8: 若回文串 s 有周期 p , 则可以把 $\text{pre}(s, p)$ 划分成长度为 $|s| \bmod p$ 的前缀和长度为 $p - |s| \bmod p$ 的后缀, 使得它们都是回文串。

定理 9: 若 t 是回文串 s 的最长 Border 且 $|t| \geq \frac{|s|}{2}$, 则 t 在 s 中只能匹配 2 次。

定理 10: 对于任意一个字符串以及 $u, v \in \text{Ssuf}(s), |u| < |v|$, 一定有 u 是 v 的 Border。

定理 11: 对于任意一个字符串 s 以及 $u, v \in \text{Ssuf}(s), |u| < |v|$, 一定有 $2|u| \leq |v|$ 。

定理 12: $|\text{Ssuf}(s)| \leq \log_2 |s|$ 。

1.2.2 KMP

```

1 // == Main ==
2 // n is |s|, m is |t|.
3 for (int i = 1, j = 0; i <= n; i++) {
4     while (j && t[j + 1] != s[i]) j = pi[j];
5     if (t[j + 1] == s[i])
6         if (++j == m) {

```

```

7         // ...
8         j = nxt[j];
9     }
10 }

```

1.3 Z 函数

```

1 // == Main ==
2 // n is |s|.
3 for (int i = 2, j = 0; i <= n; i++) {
4     if (i < j + z[j]) z[i] = min(z[i - j + 1], j + z[j] - i);
5     while (i + z[i] <= n && s[i + z[i]] == s[1 + z[i]]) z[i]++;
6     if (i + z[i] > j + z[j]) j = i;
7 }

```

1.4 Manacher

```

1 // == Main ==
2 // t is the original string, n is |t|.
3 string s = "^#";
4 for (char i : t) s.push_back(i), s.push_back('#');
5 s.push_back('@');
6 for (int i = 1, j = 0; i <= 2 * n + 1; i++) {
7     if (i <= j + p[j]) p[i] = min(p[2 * j - i], j + p[j] - i);
8     while (s[i - p[i] - 1] == s[i + p[i] + 1]) p[i]++;
9     if (i + p[i] > j + p[j]) j = i;
10 }

```

1.5 AC 自动机

```

1 // == Preparations ==
2 #include <queue>
3 // == Main ==
4 struct ACAM {
5     int tot, fail[200005], delta[200005][26];
6
7     void insert(string s, int id) {
8         int now = 0;
9         for (char c : s) {
10             int v = c - 'a';
11             if (!delta[now][v]) delta[now][v] = ++tot;

```

```

12         now = delta[now][v];
13     }
14     return;
15 }
16 void build() {
17     queue<int> q;
18     for (int c = 0; c < 26; c++)
19         if (delta[0][c]) q.push(delta[0][c]);
20     while (!q.empty()) {
21         int now = q.front();
22         q.pop();
23         for (int c = 0; c < 26; c++)
24             if (delta[now][c]) fail[delta[now][c]] = delta[fail[now]][c],
25                 ↪ q.push(delta[now][c]);
26             else delta[now][c] = delta[fail[now]][c];
27     }
28     return;
29 } ac;

```

1.6 回文自动机 PAM

```

1 // == Main ==
2 struct PAM {
3     int tot, delta[500005][26], len[500005], fail[500005], ans[500005];
4     string s;
5     int lst;
6
7     PAM() {tot = 1; len[0] = 0; len[1] = -1; fail[0] = fail[1] = 1;}
8     int getfail(int now, int i) {
9         while (s[i - len[now] - 1] != s[i]) now = fail[now];
10        return now;
11    }
12    void insert(int i) {
13        int now = getfail(lst, i);
14        if (!delta[now][s[i] - 'a']) {
15            len[++tot] = len[now] + 2;
16            fail[tot] = delta[getfail(fail[now], i)][s[i] - 'a'];
17            delta[now][s[i] - 'a'] = tot;
18            ans[tot] = ans[fail[tot]] + 1;
19        }
20        lst = delta[now][s[i] - 'a'];
21        return;
22    }
23 } p;

```

1.7 后缀自动机

1.7.1 普通 SAM

```

1 // == Main ==
2 struct SAM {
3     int tot, lst;
4     int len[2000005], siz[2000005], link[2000005];
5     int delta[2000005][26];
6
7     SAM() {link[0] = -1;}
8     void insert(char ch) {
9         int c = ch - 'a', now = ++tot;
10        len[now] = len[lst] + 1;
11        siz[now] = 1;
12        for (int p = lst; p != -1; p = link[p])
13            if (!delta[p][c]) delta[p][c] = tot;
14            else if (len[delta[p][c]] == len[p] + 1) {link[now] = delta[p][c];
15                ↪ break;}
16            else {
17                int q = delta[p][c], v = ++tot;
18                len[v] = len[p] + 1;
19                memcpy(delta[v], delta[q], sizeof(delta[v]));
20                link[v] = link[q], link[q] = v, link[now] = v;
21                for (int i = p; delta[i][c] == q; i = link[i]) delta[i][c] = v;
22                break;
23            }
24        lst = now;
25        return ;
26    }
27 } sam;

```

1.7.2 广义 SAM

注意自动机空间要开 Trie 的两倍。

```

1 // == Main ==
2 struct GSAM {
3     int tot;
4     int delta[2000005][26], link[2000005], len[2000005];
5     struct Trie {
6         int tot, trie[1000005][26], st[1000005];
7
8         void insert(string s) {
9             int now = 0;
10            for (char c : s) {

```



```

11         int id = c - 'a';
12         if (!trie[now][id]) trie[now][id] = ++tot;
13         now = trie[now][id];
14     }
15     return;
16 }
17 } tr;
18
19 GSAM() {link[0] = -1;}
20 int insert(int c, int lst) {
21     int now = ++tot;
22     len[now] = len[lst] + 1;
23     for (int p = lst; p != -1; p = link[p])
24         if (!delta[p][c]) delta[p][c] = now;
25         else if (len[delta[p][c]] == len[p] + 1) {link[now] = delta[p][c];
26             ↪ break;}
27         else {
28             int q = delta[p][c], v = ++tot;
29             len[v] = len[p] + 1;
30             memcpy(delta[v], delta[q], sizeof(delta[v]));
31             link[v] = link[q], link[q] = v, link[now] = v;
32             for (int i = p; i != -1 && delta[i][c] == q; i = link[i]) delta[i][c]
33                 ↪ = v;
34             break;
35         }
36     return now;
37 }
38 void build() {
39     queue<int> q;
40     tr.st[0] = 0;
41     q.push(0);
42     while (!q.empty()) {
43         int now = q.front();
44         q.pop();
45         for (int i = 0; i < 26; i++)
46             if (tr.trie[now][i])
47                 tr.st[tr.trie[now][i]] = insert(i, tr.st[now]),
48                 ↪ q.push(tr.trie[now][i]);
49     }
50     return;
51 }
52 } gsam;

```

1.8 后缀排序

```

1 // == Preparations ==
2 int sa[2000005], rk[2000005], b[1000005], cp[2000005];
3 // == Main ==
4 for (int i = 1; i <= n; i++) b[rk[i] = s[i]]++;
5 for (int i = 1; i < 128; i++) b[i] += b[i - 1];
6 for (int i = n; i >= 1; i--) sa[b[rk[i]]--] = i;
7 memcpy(cp, rk, sizeof(cp));
8 for (int i = 1, j = 0; i <= n; i++)
9     if (cp[sa[i]] == cp[sa[i - 1]]) rk[sa[i]] = j;
10    else rk[sa[i]] = ++j;
11 for (int w = 1; w < n; w <= 1) {
12     memcpy(cp, sa, sizeof(cp));
13     memset(b, 0, sizeof(b));
14     for (int i = 1; i <= n; i++) b[rk[cp[i] + w]]++;
15     for (int i = 1; i <= n; i++) b[i] += b[i - 1];
16     for (int i = n; i >= 1; i--) sa[b[rk[cp[i] + w]]--] = cp[i];
17     memcpy(cp, sa, sizeof(cp));
18     memset(b, 0, sizeof(b));
19     for (int i = 1; i <= n; i++) b[rk[cp[i]]]++;
20     for (int i = 1; i <= n; i++) b[i] += b[i - 1];
21     for (int i = n; i >= 1; i--) sa[b[rk[cp[i]]]--] = cp[i];
22     memcpy(cp, rk, sizeof(cp));
23     for (int i = 1, j = 0; i <= n; i++)
24         if (cp[sa[i]] == cp[sa[i - 1]] && cp[sa[i] + w] == cp[sa[i - 1] + w])
25             ↪ rk[sa[i]] = j;
26         else rk[sa[i]] = ++j;
27 }

```

Chapter 2

数论

2.1 Miller Rabin 和 Pollard Rho

2.1.1 Miller Rabin

```
1 // == Preparations ==
2 const int prime[] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};
3
4 long long power(long long a, long long b, long long mod) {
5     long long ans = 1;
6     while (b) {
7         if (b & 1) ans = (__int128)ans * a % mod;
8         a = (__int128)a * a % mod;
9         b >>= 1;
10    }
11    return ans % mod;
12 }
13 // == Main ==
14 inline int Miller_Rabin(long long n) {
15     if (n == 1) return 0;
16     if (n == 2) return 1;
17     if (n % 2 == 0) return 0;
18     long long u = n - 1, t = 0;
19     while (u % 2 == 0) u /= 2, t++;
20     for (int i = 0; i < 12; i++) {
21         if (prime[i] % n == 0) continue;
22         long long x = power(prime[i] % n, u, n);
23         if (x == 1) continue;
24         int flag = 0;
25         for (int j = 1; j <= t; j++) {
26             if (x == n - 1) {flag = 1; break;}
27             x = (__int128)x * x % n;
28         }
29         if (!flag) return 0;
```

```

30     }
31     return 1;
32 }

```

2.1.2 Pollard Rho

```

1 // == Preparations ==
2 #include <chrono>
3 #include <random>
4
5 mt19937_64 gen(chrono::system_clock::now().time_since_epoch().count());
6 /*
7 Miller Rabin
8 */
9 // == Main ==
10 long long Pollard_Rho(long long n) {
11     long long s = 0, t = 0, c = gen() % (n - 1) + 1;
12     for (int goal = 1; ; goal <= 1, s = t) {
13         long long val = 1;
14         for (int step = 1; step <= goal; step++) {
15             t = ((__int128)t * t + c) % n;
16             val = ((__int128)val * abs(t - s) % n;
17             if (!val) return n;
18             if (step % 127 == 0) {
19                 long long d = __gcd(val, n);
20                 if (d > 1) return d;
21             }
22         }
23         long long d = __gcd(val, n);
24         if (d > 1) return d;
25     }
26 }
27 void factor(long long n) {
28     if (n < ans) return ;
29     if (n == 1 || Miller_Rabin(n)) {
30         // n = 1 或 n 是一个质因子。
31         // ...
32     }
33     long long p;
34     do p = Pollard_Rho(n);
35     while (p == n);
36     while (n % p == 0) n /= p;
37     factor(n), factor(p);
38     return ;
39 }

```

2.2 常用数论算法

2.2.1 exgcd

求出来的解满足 $|x| \leq b, |y| \leq a$ 。

```

1 // == Main ==
2 int exgcd(int a, int b, int &x, int &y) {
3     if (b == 0) {x = 1; y = 0; return a;}
4     int m = exgcd(b, a % b, y, x);
5     y -= a / b * x;
6     return m;
7 }

```

2.2.2 CRT

没用。

```

1 // == Preparations ==
2 #include <vector>
3 // == Main ==
4 int CRT(vector<pair<int, int>> &a) {
5     int M = 1;
6     for (auto i : a) M *= i.second;
7     int res = 0;
8     for (auto i : a) {
9         int t = inv(M / i.second, i.second);
10        res = (res + (long long)i.first * (M / i.second) % M * t % M) % M;
11    }
12    return res;
13 }

```

2.2.3 exCRT

需保证 lcm 在 long long 范围内。

```

1 // == Preparations ==
2 long long exgcd(long long a, long long b, long long &x, long long &y);
3 // == Main ==
4 long long exCRT(vector<pair<long long, long long>> vec) {
5     long long ans = vec[0].first, mod = vec[0].second;
6     for (int i = 1; i < (int)vec.size(); i++) {
7         long long a = mod, b = vec[i].second, c = vec[i].first - ans % b;
8         long long x, y;

```

```

9      long long g = exgcd(a, b, x, y);
10     if (c % g != 0) return -1;
11     b /= g;
12     x = (__int128)x * (c / g) % b;
13     ans += x * mod;
14     mod *= b;
15     ans = (ans % mod + mod) % mod;
16 }
17 return ans;
18 }

```

2.2.4 exLucas

```

1 // == Preparations ==
2 int power(int a, long long b, int mod);
3 int exgcd(int a, int b, int &x, int &y);
4 int CRT(vector<pair<int, int>> &a);
5 // == Main ==
6 int inv(int n, int p) {
7     int x, y;
8     exgcd(n, p, x, y);
9     return (x % p + p) % p;
10 }
11 int fac(long long n, int p, int pk) {
12     if (n == 0) return 1;
13     int res = 1;
14     for (int i = 1; i < pk; i++)
15         if (i % p != 0) res = (long long)res * i % pk;
16     res = power(res, n / pk, pk);
17     for (int i = 1; i <= n % pk; i++)
18         if (i % p != 0) res = (long long)res * i % pk;
19     return (long long)res * fac(n / p, p, pk) % pk;
20 }
21 int C(long long n, long long m, int p, int pk) {
22     long long x = n, y = m, z = n - m;
23     int res = (long long)fac(x, p, pk) * inv(fac(y, p, pk), pk) % pk * inv(fac(z, p,
24         ↪ pk), pk) % pk;
25     long long e = 0;
26     while (x) e += x / p, x /= p;
27     while (y) e -= y / p, y /= p;
28     while (z) e -= z / p, z /= p;
29     return (long long)res * power(p, e, pk) % pk;
30 }
31 int exLucas(long long n, long long m, int p) {
32     vector<pair<int, int>> a;
33     for (int i = 2; i * i <= p; i++)
34         if (p % i == 0) {

```

```

34         int pk = 1;
35         while (p % i == 0) pk *= i, p /= i;
36         a.emplace_back(C(n, m, i, pk), pk);
37     }
38     if (p != 1) a.emplace_back(C(n, m, p, p), p);
39     return CRT(a);
40 }

```

2.3 万能欧几里得算法

问题描述:

给出一个幺半群 (S, \cdot) 和元素 $u, r \in S$, 以及一条直线 $y = \frac{ax+b}{c}$ 。

画出平面中所有坐标为正整数的横线和竖线, 维护一个 f , 初值为单位元 e 。

从原点出发, 先向 y 轴正方向走直到到达直线与 y 的交点, 然后沿直线走一直走到与 $x = n$ 的交点为止。

每当经过一条横线时, 执行 $f \leftarrow fu$, 经过一条竖线时执行 $f \leftarrow fr$ 。特别地, 在 y 轴上行走时不考虑竖线, 同时经过横线和竖线时先执行前者。

求最终的 f 。记为 $\text{euclid}(a, b, c, n, u, r)$ 。

其中 $a, b \geq 0, n, c > 0$ 。

做法:

$$\text{euclid}(a, b, c, n, u, r) = \begin{cases} r^n & m = 0 \\ u^{\lfloor \frac{b}{c} \rfloor} \cdot \text{euclid}(a \bmod c, b \bmod c, c, n, u, u^{\lfloor \frac{a}{c} \rfloor} r) & a \geq c \vee b \geq c \\ r^{\lfloor \frac{c-b-1}{a} \rfloor} u \cdot \text{euclid}(c, (c-b-1) \bmod a, a, m-1, r, u) \cdot r^{n - \lfloor \frac{cm-b-1}{a} \rfloor} & \text{otherwise} \end{cases}$$

设一次乘法的复杂度为 $O(T)$, 则复杂度为 $O(T \log(a+c) \log(a+n+c))$ 。

```

1 // == Preparations ==
2 struct Node {
3     // ...
4
5     Node operator*(const Node &x) const {
6         // ...
7     }
8 };
9 // == Main ==
10 Node power(Node a, long long b) {
11     Node ans = Node(/* 幺元 */);
12     while (b) {
13         if (b & 1) ans = ans * a;
14         a = a * a;

```

```

15         b >>= 1;
16     }
17     return ans;
18 }
19 Node Euclid(int a, int b, int c, long long n, Node r, Node u) {
20     long long m = (a * n + b) / c;
21     if (!m) return power(r, n);
22     if (a >= c || b >= c)
23         return power(u, b / c) * Euclid(a % c, b % c, c, n, power(u, a / c) * r, u);
24     return power(r, (c - b - 1) / a) * u *
25         Euclid(c, (c - b - 1) % a, a, m - 1, u, r) * power(r, n - (c * m - b - 1) /
26             ↪ a);

```

Chapter 3

数据结构

3.1 K-D Tree

```
1 // == Main ==
2 template<const int Dim = 2>
3 struct KDTree {
4     using point = array<int, Dim>;
5     struct node {
6         point p, l, r;
7         int val, siz, sum;
8         node *ls, *rs;
9
10        node() = default;
11        node(point _p, int _val = 0):
12            p(_p), l(_p), r(_p), val(_val), siz(1), sum(_val), ls(nullptr),
13            ↪ rs(nullptr) {}
14        void pushup() {
15            l = r = p, siz = 1, sum = val;
16            for (int i = 0; i < Dim; i++) {
17                if (ls) l[i] = min(l[i], ls->l[i]), r[i] = max(r[i], ls->r[i]);
18                if (rs) l[i] = min(l[i], rs->l[i]), r[i] = max(r[i], rs->r[i]);
19            }
20            if (ls) siz += ls->siz, sum += ls->sum;
21            if (rs) siz += rs->siz, sum += rs->sum;
22            return ;
23        }
24    };
25    vector<node*> root;
26    using itor = typename vector<node*>::iterator;
27
28    node *build(itor l, itor r, int dim = 0) {
29        if (l == r) return nullptr;
30        int mid = (r - l) / 2;
31        nth_element(l, l + mid, r, [&dim](const node &x, const node &y) {return
32            ↪ x.p[dim] < y.p[dim];});
```

```

31     node *now = new node(*(l + mid));
32     now->ls = build(l, l + mid, (dim + 1) % Dim);
33     now->rs = build(l + mid + 1, r, (dim + 1) % Dim);
34     now->pushup();
35     return now;
36 }
37 void getnode(node *now, vector<node> &vec) {
38     if (!now) return ;
39     vec.push_back(*now);
40     getnode(now->ls, vec), getnode(now->rs, vec);
41     delete now;
42     return ;
43 }
44 void insert(point p, int val) {
45     vector<node> tmp({node(p, val)});
46     while (!root.empty() && root.back()->siz == (int)tmp.size())
47         getnode(root.back(), tmp), root.pop_back();
48     sort(tmp.begin(), tmp.end(), [](const node &x, const node &y) {return x.p <
49         ⇨ y.p;});
49     vector<node> vec;
50     for (node i : tmp)
51         if (!vec.empty() && vec.back().p == i.p) vec.back().val += i.val;
52         else vec.push_back(i);
53     root.push_back(build(vec.begin(), vec.end()));
54     return ;
55 }
56 int query(point ll, point rr, node *now) {
57     if (!now) return 0;
58     int flag = 1;
59     for (int i = 0; i < Dim; i++)
60         if (now->r[i] < ll[i] || now->l[i] > rr[i]) return 0;
61         else flag &= ll[i] <= now->l[i] && now->r[i] <= rr[i];
62     if (flag) return now->sum;
63     flag = 1;
64     for (int i = 0; i < Dim; i++) flag &= ll[i] <= now->p[i] && now->p[i] <=
65         ⇨ rr[i];
65     return flag * now->val + query(ll, rr, now->ls) + query(ll, rr, now->rs);
66 }
67 int query(point ll, point rr) {
68     int ans = 0;
69     for (node *rt : root) ans += query(ll, rr, rt);
70     return ans;
71 }
72 ~KDTree() {
73     vector<node> tmp;
74     for (node *rt : root) getnode(rt, tmp);
75 }
76 };

```

3.2 Link Cut Tree

代码维护的是点权异或和。

```

1 // == Main ==
2 struct LinkCutTree {
3     int fa[100005], son[100005][2], siz[100005], swp[100005];
4     int val[100005], Xor[100005];
5
6     void pushup(int now) {
7         siz[now] = siz[son[now][0]] + siz[son[now][1]] + 1;
8         Xor[now] = Xor[son[now][0]] ^ val[now] ^ Xor[son[now][1]]; // 此处更新信息。
9         return;
10    }
11    void pushdown(int now) {
12        if(!swp[now]) return ;
13        swap(son[now][0], son[now][1]);
14        swp[son[now][0]] ^= 1, swp[son[now][1]] ^= 1;
15        swp[now] = 0;
16        // 此处将信息 pushdown
17        return;
18    }
19    int isRoot(int now) {return now != son[fa[now]][0] && now != son[fa[now]][1];}
20    int get(int now) {return now == son[fa[now]][1];}
21    void rotate(int x) {
22        int y = fa[x], z = fa[fa[x]], chk = get(x);
23        if (!isRoot(y)) son[z][get(y)] = x;
24        son[y][chk] = son[x][chk ^ 1], fa[son[x][chk ^ 1]] = y;
25        son[x][chk ^ 1] = y, fa[y] = x;
26        fa[x] = z;
27        pushup(y), pushup(x);
28        return;
29    }
30    void splay(int now) {
31        vector<int> stk;
32        stk.push_back(now);
33        for (int i = now; !isRoot(i); i = fa[i]) stk.push_back(fa[i]);
34        while (!stk.empty()) pushdown(stk.back()), stk.pop_back();
35        for (int f; f = fa[now], !isRoot(now); rotate(now))
36            if (!isRoot(f)) rotate(get(f) == get(now) ? f : now);
37        return;
38    }
39    void access(int now) { // 打通到根的链
40        for (int lst = 0; now; lst = now, now = fa[now]) splay(now), son[now][1] =
41            ↪ lst, pushup(now);
42        return;
43    }
44 }
```

```

43 void makeRoot(int now) {access(now); splay(now); swp[now] ^= 1; return;} // 设置
    ↳ 根
44 void link(int u, int v) {makeRoot(u); fa[u] = v; return;} // 连接
45 void cut(int u, int v) {makeRoot(u); access(v); splay(v); son[v][0] = fa[u] = 0;
    ↳ return;} // 切割
46 int find(int now) { // 找根
47     access(now), splay(now);
48     pushdown(now);
49     while (son[now][0]) now = son[now][0], pushdown(now);
50     splay(now);
51     return now;
52 }
53 void split(int u, int v) {makeRoot(u); access(v); splay(u); return;} // 剖出 u ~
    ↳ v 的链
54 void update(int u, int _val) {split(u, u); val[u] = Xor[u] = _val; return ;} //
    ↳ 修改操作, split 后做就行了, 此处为单点修改。
55 int query(int u, int v) {split(u, v); return Xor[u];} // 查询操作, split 后做就行
    ↳ 了。
56 int isConnected(int u, int v) {return find(u) == find(v);} // 查询两个点是否连通。
57 };

```

Chapter 4

图论

4.1 Tarjan

4.1.1 强连通分量

```
1 // == Main ==
2 void Tarjan(int now) {
3     dfn[now] = low[now] = ++Index;
4     s.push(now);
5     for (int i = g.hd[now]; i; i = g.nxt[i])
6         if (!dfn[g.to[i]]) {
7             Tarjan(g.to[i]);
8             low[now] = min(low[now], low[g.to[i]]);
9         } else if (!scc[g.to[i]]) low[now] = min(low[now], dfn[g.to[i]]);
10    if (low[now] == dfn[now]) {
11        scc_cnt++;
12        for (int x = 0; x != now; s.pop()) {
13            x = s.top();
14            scc[x] = scc_cnt;
15        }
16    }
17    return ;
18 }
```

4.1.2 割边与边双

割边:

```
1 // == Main ==
2 void Tarjan(int now, int fa) {
3     dfn[now] = low[now] = ++Index;
4     for (int i = g.hd[now]; i; i = g.nxt[i])
5         if (!dfn[g.to[i]]) {
```

```

6         Tarjan(g.to[i], now);
7         low[now] = min(low[now], low[g.to[i]]);
8         if (low[g.to[i]] > dfn[now])
9             printf("A Bridge of the Input Garph is (%d, %d)\n", now, g.to[i]);
10    } else if (g.to[i] != fa) low[now] = min(low[now], dfn[g.to[i]]);
11    return ;
12 }

```

边双:

```

1 // == Main ==
2 void Tarjan(int now, int fa) {
3     dfn[now] = low[now] = ++Index;
4     s.push(now);
5     for (int i = g.hd[now]; i; i = g.nxt[i])
6         if (!dfn[g.to[i]]) {
7             Tarjan(g.to[i], now);
8             low[now] = min(low[now], low[g.to[i]]);
9         } else if (g.to[i] != fa) low[now] = min(low[now], dfn[g.to[i]]);
10    if (low[now] == dfn[now]) {
11        bcc_cnt++;
12        for (int x = 0; x != now; s.pop()) {
13            x = s.top();
14            bcc[x] = bcc_cnt;
15        }
16    }
17    return ;
18 }

```

4.1.3 割点与点双

割点:

```

1 // == Main ==
2 void Tarjan(int now, int root) {
3     dfn[now] = low[now] = ++Index;
4     int sons=0, flag=0;
5     for (int i=g.hd[now]; i; i = g.nxt[i], sons++)
6         if (!dfn[g.to[i]]) {
7             Tarjan(g.to[i], now);
8             low[now] = min(low[now], low[g.to[i]]);
9             if (now!=root && low[g.to[i]] == dfn[now] && !flag)
10                printf("A Cut Vertex of the Input Graph is %d.", now), flag=1;
11        } else low[now] = min(low[now], dfn[g.to[i]]);
12    if (now == root && sons >= 2)
13        printf("A Cut Vertex of the Input Graph is %d.", now);

```

```

14     return ;
15 }

```

点双:

```

1 // == Main ==
2 void Tarjan(int now) {
3     dfn[now] = low[now] = ++Index;
4     s.push(now);
5     for (int i = g.hd[now]; i; i = g.nxt[i], sons++)
6         if (!dfn[g.to[i]]) {
7             Tarjan(g.to[i]);
8             low[now] = min(low[now], low[g.to[i]]);
9             if (low[g.to[i]] == dfn[now]) {
10                 printf("BCC #%d:\n", ++bcc_cnt);
11                 for (int x = 0; x != g.to[i]; s.pop())
12                     printf("%d ", x = s.top());
13                 printf("%d\n", now);
14             }
15             } else low[now] = min(low[now], dfn[g.to[i]]);
16     return ;
17 }

```

4.2 欧拉路径

```

1 // == Preparations ==
2 #include <vector>
3
4 vector<int> g[100005], ans;
5 // == Main ==
6 void dfs(int now) {
7     while (!g[now].empty()) {
8         int to = g[now].back();
9         g[now].pop_back();
10        dfs(to);
11    }
12    ans.push_back(now);
13    return ;
14 }

```

4.3 二分图匹配

4.3.1 最大匹配

```

1 // == Preparations ==
2 int chos[100005], vis[100005];
3 struct graph { /* ... */ } g;
4 // == Main ==
5 int dfs(int now) {
6     for (int i = g.hd[now]; i; i = g.nxt[i]) {
7         if (vis[g.to[i]]) continue;
8         vis[g.to[i]] = true;
9         if (!chos[g.to[i]] || dfs(chos[g.to[i]])) {
10             chos[g.to[i]] = now;
11             return 1;
12         }
13     }
14     return 0;
15 }
16
17 for (int i = 1; i <= n; i++) {
18     memset(vis, 0, sizeof(vis));
19     ans += dfs(i);
20 }

```

4.3.2 最大权匹配

```

1 // == Preparations ==
2 int vis[1005], mat[1005], pre[1005];
3 long long g[505][1005];
4 long long w[1005], slack[1005];
5 // edge: g[u][n + v] = w;
6 // == Main ==
7 for (int i = 1; i <= n; i++) {
8     w[i] = ~0x3f3f3f3f3f3f3f3f;
9     for (int j = n + 1; j <= n + n; j++) w[i] = max(w[i], (long long)g[i][j]);
10 }
11 for (int i = 1; i <= n; i++) {
12     memset(vis, 0, sizeof(vis));
13     memset(slack, 0x3f, sizeof(slack));
14     memset(pre, 0, sizeof(pre));
15     int now = i, ri = 0;
16     while (1) {
17         int id = 0;
18         long long delta = 0x3f3f3f3f3f3f3f3f;
19         for (int j = n + 1; j <= n + n; j++)

```

```

20         if (!vis[j]) {
21             long long val = w[now] + w[j] - g[now][j];
22             if (val < slack[j]) slack[j] = val, pre[j] = ri;
23             if (slack[j] < delta) delta = slack[j], id = j;
24         }
25     w[i] -= delta;
26     for (int j = n + 1; j <= n + n; j++)
27         if (vis[j]) w[j] += delta, w[mat[j]] -= delta;
28         else slack[j] -= delta;
29     vis[ri = id] = 1;
30     if (mat[ri]) now = mat[ri];
31     else break;
32 }
33 while (ri) {
34     mat[ri] = mat[pre[ri]];
35     if (!pre[ri]) {mat[ri] = i; break;}
36     ri = pre[ri];
37 }
38 }
39 long long ans = 0;
40 for (int i = 1; i <= n + n; i++) ans += w[i];
41 printf("%lld\n", ans);
42 for (int i = n + 1; i <= n + n; i++) printf("%d ", mat[i]);
43 puts("");

```

4.4 网络流

4.4.1 最大流

```

1 // == Preparations ==
2 #include <queue>
3 // == Main ==
4 struct Dinic {
5     int s, t;
6     struct graph {
7         int tot, hd[205];
8         int nxt[10005], to[10005], dt[10005];
9         graph() {tot = 1;}
10        void add(int u, int v, int w) {
11            nxt[++tot] = hd[u];
12            hd[u] = tot;
13            to[tot] = v;
14            dt[tot] = w;
15            return ;
16        }
17    } g;

```

```

18     int cur[205], dis[205];
19
20     void add_edge(int u, int v, int f) {g.add(u, v, f), g.add(v, u, 0); return;}
21     int bfs() {
22         memset(dis, 0, sizeof(dis));
23         queue<int>q;
24         q.push(s);
25         dis[s] = 1;
26         while (!q.empty()) {
27             int now = q.front();
28             q.pop();
29             cur[now] = g.hd[now];
30             for (int i = g.hd[now]; i; i = g.nxt[i])
31                 if (g.dt[i] && !dis[g.to[i]]) dis[g.to[i]] = dis[now] + 1,
32                     ↪ q.push(g.to[i]);
33         }
34         return dis[t];
35     }
36     long long dinic(int now, long long flow) {
37         if (now == t) return flow;
38         long long used = 0;
39         for (int i = cur[now]; i && used < flow; i = g.nxt[i])
40             if (g.dt[i] && dis[g.to[i]] == dis[now] + 1) {
41                 long long k = dinic(g.to[i], min(flow - used, (long long)g.dt[i]));
42                 g.dt[i] -= k, g.dt[i ^ 1] += k;
43                 used += k;
44                 cur[now] = i;
45             }
46         if (used == 0) dis[now] = 0;
47         return used;
48     }
49     long long solve() {
50         long long ans = 0;
51         while (bfs()) ans += dinic(s, 0x3f3f3f3f3f3f3f3f);
52         return ans;
53 } F;

```

4.4.2 费用流

原始对偶:

```

1 // == Preparations ==
2 #include <queue>
3 // == Main ==
4 struct PrimalDual {
5     int n, s, t;

```

```

6  struct graph {
7      int tot, hd[805];
8      int nxt[30005], to[30005], flw[30005], cst[30005];
9
10     graph() {tot = 1;}
11     void add(int u, int v, int f, int c) {
12         nxt[++tot] = hd[u];
13         hd[u] = tot;
14         to[tot] = v;
15         flw[tot] = f;
16         cst[tot] = c;
17         return ;
18     }
19 } g;
20 int h[805], dis[805], f[805], pre[805];
21 struct node {
22     int id, val;
23
24     node() = default;
25     node(int _id, int _val): id(_id), val(_val) {}
26     bool operator<(const node &x) const {return val > x.val;}
27 };
28
29 void add_edge(int u, int v, int f, int c) {g.add(u, v, f, c), g.add(v, u, 0, -c);
    ↪ return;}
30 void spfa() {
31     queue<int> q;
32     memset(h, 0x3f, sizeof(h));
33     h[s] = 0;
34     q.push(s);
35     while (!q.empty()) {
36         int now = q.front();
37         q.pop();
38         f[now] = 0;
39         for (int i = g.hd[now]; i; i = g.nxt[i])
40             if (g.flw[i] && h[g.to[i]] > h[now] + g.cst[i]) {
41                 h[g.to[i]] = h[now] + g.cst[i];
42                 if (!f[g.to[i]]) q.push(g.to[i]), f[g.to[i]] = 1;
43             }
44     }
45     return ;
46 }
47 int dijkstra() {
48     priority_queue<node> q;
49     memset(dis, 0x3f, sizeof(dis));
50     memset(pre, 0, sizeof(pre));
51     q.emplace(s, dis[s] = 0);
52     while (!q.empty()) {
53         int now = q.top().id, tmp = q.top().val;

```

```

54     q.pop();
55     if (dis[now] != tmp) continue;
56     for (int i = g.hd[now]; i; i = g.nxt[i])
57         if (g.flw[i] && dis[g.to[i]] > dis[now] + g.cst[i] + h[now] -
58             ↪ h[g.to[i]]) {
59             q.emplace(g.to[i], dis[g.to[i]] = dis[now] + g.cst[i] + h[now] -
60                 ↪ h[g.to[i]]);
61             pre[g.to[i]] = i ^ 1;
62         }
63     }
64     return pre[t];
65 }
66 pair<int, int> solve() {
67     int flow = 0, cost = 0;
68     spfa();
69     while (dijkstra()) {
70         for (int i = 1; i <= n; i++)
71             if (dis[i] < 0x3f3f3f3f) h[i] += dis[i];
72         int mnflow = 0x3f3f3f3f;
73         for (int i = t; i != s; i = g.to[pre[i]]) mnflow = min(mnflow,
74             ↪ g.flw[pre[i] ^ 1]);
75         for (int i = t; i != s; i = g.to[pre[i]]) g.flw[pre[i] ^ 1] -= mnflow,
76             ↪ g.flw[pre[i]] += mnflow;
77         flow += mnflow;
78         cost += mnflow * h[t];
79     }
80     return {flow, cost};
81 }
82 } F;

```

4.4.3 上下界

$f(u, v)$ 表示边 (u, v) 的流量, $f(u)$ 表示 u 的出流减入流, $c(u, v)$ 表示边 (u, v) 的容量。
 对于每条边给定一个流量下界 $b(u, v)$, 需要额外满足 $\forall(u, v), b(u, v) \leq f(u, v) \leq c(u, v)$ 。

无源汇上下界可行流

没有源点和汇点, 对于所有点满足 $f(u) = 0$, 求一个可行的流。

先强制每条边流到流量下界, 建立虚拟源汇点 s, t , 对于每个点 u 考虑此时的净流量:

- $f(u) = 0$: 满足条件, 不用管。
- $f(u) > 0$: 出流大于入流, 从 u 向 t 连容量为 $f(u)$ 的边。
- $f(u) < 0$: 入流大于出流, 从 s 向 u 连容量为 $-f(u)$ 的边。

将原图中每条边的容量设为 $c(u, v) - b(u, v)$ ，则从 s 到 t 的流相当于增加调整流量的过程。
若 s 的出边流满（等同于 t 的入边流满），则找到了一条可行流。

有源汇上下界可行流

连一条 t 到 s 容量正无穷下界为 0 的边，然后跑无源汇上下界可行流即可，流量为新增边的流量。

有源汇上下界最大流

求出可行流后删掉 t 到 s 的边，在残量网络上跑 s 到 t 的最大流，该最大流加上原本的可行流即为答案。

有源汇上下界最小流

同理，改成求 t 到 s 的最大流，原可行流减去该最大流即为答案。

有源汇上下界最小费用流

做法是一样的，所有新增边费用为 0。

需要注意求最小流时需要改成费用最大。

4.4.4 有负圈的最小费用最大流

先钦定所有负圈边流满，即上下界均为流量。然后对于负边建反向、容量相同、费用为相反数的边用于退流原边。

这样就转化成了有源汇上下界最小费用最大流。

4.5 k 短路

复杂度为 $O((n + m) \log n + k \log k)$ 。

```

1 // == Preparations ==
2 int ontree[200005];
3 struct graph {
4     int tot, hd[5005];
5     int nxt[200005], to[200005];
6     long long dt[200005];
7
8     void add(int u, int v, long long w) {
9         nxt[++tot] = hd[u];
10        hd[u] = tot;
11        to[tot] = v;
12        dt[tot] = w;
13        return ;
14    }

```

```

15 } g;
16 long long dis[5005];
17 struct node {
18     int id;
19     long long val;
20
21     node() = default;
22     node(int _id, long long _val): id(_id), val(_val) {}
23     bool operator<(const node &x) const {return val > x.val;}
24 };
25 priority_queue<node> q;
26 int vis[5005];
27
28 // 以下左偏树
29 struct HeapNode {
30     long long val;
31     int to, dist;
32     HeapNode *ls, *rs;
33
34     HeapNode() = default;
35     HeapNode(long long _val, int _to): val(_val), to(_to), dist(1), ls(nullptr),
        ↪ rs(nullptr) {}
36 };
37 struct Heap {
38     HeapNode *root[5005];
39
40     HeapNode *merge(HeapNode *u, HeapNode *v) {
41         if (!u) return v;
42         if (!v) return u;
43         if (u->val > v->val) swap(u, v);
44         HeapNode *p = new HeapNode(*u);
45         p->rs = merge(u->rs, v);
46         if (!p->ls || p->ls->dist < p->rs->dist) swap(p->ls, p->rs);
47         if (p->rs) p->dist = p->rs->dist + 1;
48         else p->dist = 1;
49         return p;
50     }
51 } h;
52
53 struct Node {
54     HeapNode *id;
55     long long val;
56
57     Node() = default;
58     Node(HeapNode *_id, long long _val): id(_id), val(_val) {}
59     bool operator<(const Node &x) const {return val > x.val;}
60 };
61 priority_queue<Node> Q;
62 // == Main ==

```

```

63 void dfs(int now) {
64     vis[now] = 1;
65     for (int i = g.hd[now]; i; i = g.nxt[i])
66         if (!vis[g.to[i]] && dis[g.to[i]] == dis[now] + g.dt[i]) ontree[i] = 1,
            ↪ dfs(g.to[i]);
67     return ;
68 }
69 void dfs2(int now) {
70     for (int i = g.hd[now]; i; i = g.nxt[i])
71         if (ontree[i]) h.root[g.to[i]] = h.merge(h.root[g.to[i]], h.root[now]),
            ↪ dfs2(g.to[i]);
72     return;
73 }
74
75 memset(dis, 0x3f, sizeof(dis));
76 q.emplace(n, dis[n] = 0);
77 while (!q.empty()) {
78     int now = q.top().id;
79     long long tmp = q.top().val;
80     q.pop();
81     if (tmp != dis[now]) continue;
82     for (int i = g.hd[now]; i; i = g.nxt[i])
83         if (dis[g.to[i]] > dis[now] + g.dt[i]) q.emplace(g.to[i], dis[g.to[i]] =
            ↪ dis[now] + g.dt[i]);
84 }
85 dfs(n);
86 for (int i = 1; i <= n; i++)
87     for (int j = g.hd[i]; j; j = g.nxt[j])
88         if (!ontree[j] && g.to[j] != n)
89             h.root[g.to[j]] = h.merge(h.root[g.to[j]], new HeapNode(dis[i] + g.dt[j]
            ↪ - dis[g.to[j]], i));
90 dfs2(n);
91 if (h.root[1]) Q.emplace(h.root[1], dis[1] + h.root[1]->val);
92 while (!Q.empty()) { // 每次取出来一条路径
93     HeapNode *now = Q.top().id;
94     long long d = Q.top().val;
95     Q.pop();
96     if (now->ls) Q.emplace(now->ls, d - now->val + now->ls->val);
97     if (now->rs) Q.emplace(now->rs, d - now->val + now->rs->val);
98     HeapNode *tmp = h.root[now->to];
99     if (tmp) Q.emplace(tmp, d + tmp->val);
100 }

```

4.6 全局最小割

时间复杂度为 $O(|V|^3)$ 。

```

1 // == Preparations ==
2 int g[605][605], vis1[605], vis2[605];
3 long long w[605];
4 // == Main ==
5 long long Stoer_Wagner() {
6     long long ans = 0x3f3f3f3f3f3f3f3f;
7     for (int i = 1; i < n; i++) {
8         int s = 0, t = 0;
9         memset(vis2, 0, sizeof(vis2));
10        memset(w, 0, sizeof(w));
11        for (int j = 1; j <= n - i + 1; j++) {
12            int now = 0;
13            for (int k = 1; k <= n; k++)
14                if (!vis1[k] && !vis2[k] && w[k] >= w[now]) now = k;
15            s = t, t = now;
16            vis2[now] = 1;
17            for (int k = 1; k <= n; k++) w[k] += g[k][now];
18        }
19        ans = min(ans, w[t]);
20        vis1[t] = 1;
21        for (int j = 1; j <= n; j++)
22            if (j != s) g[s][j] += g[t][j], g[j][s] += g[j][t];
23    }
24    return ans;
25 }

```

4.7 支配树

$idom_u$ 为 u 在支配树上的父亲。

最后 id 形成 dfs 序。

```

1 // == Preparations ==
2 #include <vector>
3
4 struct graph {
5     int tot, hd[200005];
6     int nxt[300005], to[300005];
7
8     void add(int u, int v) {
9         nxt[++tot] = hd[u];
10        hd[u] = tot;
11        to[tot] = v;
12        return ;
13    }
14 } g, fg;

```



```

15 int timer, fa[200005], dfn[200005], id[200005];
16 int sdom[200005], idom[200005];
17 struct dsu {
18     int fa[200005], mn[200005];
19
20     dsu() {for (int i = 1; i < 200005; i++) fa[i] = mn[i] = i;}
21     int find(int x) {
22         if (x == fa[x]) return x;
23         int tmp = find(fa[x]);
24         if (dfn[sdom[mn[fa[x]]]] < dfn[sdom[mn[x]]]) mn[x] = mn[fa[x]];
25         return fa[x] = tmp;
26     }
27 } d;
28 vector<int> vec[200005];
29 int siz[200005];
30 // == Main ==
31 void dfs(int now) {
32     id[dfn[now] = ++timer] = now;
33     for (int i = g.hd[now]; i; i = g.nxt[i])
34         if (!dfn[g.to[i]]) fa[g.to[i]] = now, dfs(g.to[i]);
35     return ;
36 }
37 void solve() {
38     dfs(1);
39     for (int i = 1; i <= n; i++) sdom[i] = i;
40     for (int i = timer; i >= 1; i--) {
41         int u = id[i];
42         for (int v : vec[u]) {
43             d.find(v);
44             if (sdom[d.mn[v]] == u) idom[v] = u;
45             else idom[v] = d.mn[v];
46         }
47         if (i == 1) continue;
48         for (int j = fg.hd[u]; j; j = fg.nxt[j]) {
49             if (!dfn[fg.to[j]]) continue;
50             if (dfn[fg.to[j]] < dfn[sdom[u]]) sdom[u] = fg.to[j];
51             else if (dfn[fg.to[j]] > dfn[u]) {
52                 d.find(fg.to[j]);
53                 if (dfn[sdom[d.mn[fg.to[j]]]] < dfn[sdom[u]]) sdom[u] =
54                     ↪ sdom[d.mn[fg.to[j]]];
55             }
56             vec[sdom[u]].push_back(u);
57             d.fa[u] = fa[u];
58         }
59     }
60     for (int i = 2; i <= timer; i++)
61         if (idom[id[i]] != sdom[id[i]]) idom[id[i]] = idom[idom[id[i]]];
62     return ;

```

4.8 弦图

4.8.1 MCS 最大势算法。

```

1 // == Preparations ==
2 #include <vector>
3
4 int pos[/* ... */], p[/* ... */];
5 vector<int> vec[/* ... */];
6 // == Main ==
7 for (int i = 1; i <= n; i++) pos[i] = vec[0].size(), vec[0].push_back(i);
8 for (int i = 1, j = 0; i <= n; i++, j++) {
9     while (vec[j].empty()) j--;
10    int u = p[i] = vec[j].back();
11    vec[j].pop_back();
12    pos[u] = -1;
13    for (int k = g.hd[u]; k; k = g.nxt[k])
14        if (pos[g.to[k]] != -1) {
15            int v = g.to[k];
16            pos[vec[l[v]].back()] = pos[v];
17            swap(vec[l[v]][pos[v]], vec[l[v]].back());
18            vec[l[v]].pop_back();
19            pos[v] = vec[++l[v]].size();
20            vec[l[v]].push_back(v);
21        }
22 }
23 reverse(p + 1, p + n + 1);

```

4.8.2 弦图判定

跑 MCS，然后判断是否为完美消除序列。

具体地，对于每个 p_i ，找到与之相连且在它之后出现的点，按出现顺序记为 c_1, c_2, \dots, c_k ，我们只需要判断 c_1 与 c_j 之间是否有边即可。因为这个团中其他边会在 $p_{c_2}, p_{c_3}, \dots, p_{c_k}$ 中被判断。

4.8.3 求弦图的团数与色数

求团数：

设 $N(x)$ 为完美消除序列中在 x 之后且与 x 相连的点的集合，则弦图的最大团一定可以被表示为 $\{x\} + N(x)$ ，则 $|\{x\} + N(x)|$ 的最大值就是弦图的团数。

求色数：

考虑按完美消除序列从后往前考虑，贪心染 mex，这样需要的颜色数量等于团数。由于团数小于等于

色数，这样取到等号，一定最小。

4.8.4 求弦图的最大独立集和最小团覆盖

最大独立集：

按完美消除序列从前往后贪心。正确性证明：每次考虑最靠前的极大团，选最前面的点不劣于选其他点，且优于不选点。

最小团覆盖：

取最大独立集中的每个点 x 对应的团 $\{x\} + N(x)$ ，这样需要的团的数量等于最大独立集的大小。由于最大独立集小于等于最小团覆盖，这样取到等号，一定最小。

4.8.5 tricks

区间图是弦图，完美消除序列为按区间右端点从小到大排序。

树上距离不超过 k 的点连边是弦图，完美消除序列为 bfs 序的逆序。

Chapter 5

多项式

5.1 牛顿迭代

用于解决下列问题：

已知函数 G 且 $G(F(x)) = 0$ ，求多项式 $F \pmod{x^n}$ 。

结论：

$$F(x) = F_*(x) - \frac{G(F_*(x))}{G'(F_*(x))} \pmod{x^n}$$

其中 $F_*(x)$ 为做到 $x^{n/2}$ 时的答案。

5.2 FFT

```
1 // == Preparations ==
2 struct complex {
3     double a, b;
4
5     complex() = default;
6     complex(double _a, double _b): a(_a), b(_b) {}
7     complex operator+(const complex &x) const {return complex(a + x.a, b + x.b);}
8     complex operator-(const complex &x) const {return complex(a - x.a, b - x.b);}
9     complex operator*(const complex &x) const {return complex(a * x.a - b * x.b, a *
10         ↪ x.b + b * x.a);}
11     complex operator/(const complex &x) const {
12         double t = b * b + x.b * x.b;
13         return complex((a * x.a + b * x.b) / t, (b * x.a - a * x.b) / t);
14     }
15     complex &operator+=(const complex &x) {return *this = *this + x;}
16     complex &operator-=(const complex &x) {return *this = *this - x;}
17     complex &operator*=(const complex &x) {return *this = *this * x;}
```

```

17     complex &operator/=(const complex &x) {return *this = *this / x;}
18 };
19 // == Main ==
20 void FFT(vector<complex> &f, int flag) const {
21     int n = f.size();
22     vector<int> swp(n);
23     for (int i = 0; i < n; i++) {
24         swp[i] = swp[i >> 1] >> 1 | ((i & 1) * (n >> 1));
25         if (i < swp[i]) std::swap(f[i], f[swp[i]]);
26     }
27     for (int mid = 1; mid < n; mid <= 1) {
28         complex w1(cos(pi / mid), flag * sin(pi / mid));
29         for (int i = 0; i < n; i += mid << 1) {
30             complex w(1, 0);
31             for (int j = 0; j < mid; j++, w *= w1) {
32                 complex x = f[i + j], y = w * f[i + mid + j];
33                 f[i + j] = x + y, f[i + mid + j] = x - y;
34             }
35         }
36     }
37     return;
38 }

```

5.3 常用 NTT 模数及其原根

模数	原根	分解
167772161	3	$5 \times 2^{25} + 1$
469762049	3	$7 \times 2^{26} + 1$
998244353	3	$119 \times 2^{23} + 1$
1004535809	3	$479 \times 2^{21} + 1$
2013265921	31	$15 \times 2^{27} + 1$
2281701377	3	$17 \times 2^{27} + 1$

5.4 多项式模板

```

1 // == Preparations ==
2 #include <vector>
3 // == Main ==
4 namespace Poly {
5     const int mod = 998244353, G = 3, invG = 332748118;
6
7     inline int power(int a, int b) {
8         int ans = 1;
9         while (b) {

```

```

10         if (b & 1) ans = (long long)ans * a % mod;
11         a = (long long)a * a % mod;
12         b >>= 1;
13     }
14     return ans % mod;
15 }
16
17 struct poly: vector<int> {
18     poly(initializer_list<int> &&arg): vector<int>(arg) {}
19     template<typename... argT>
20     poly(argT &&...args): vector<int>(forward<argT>(args)...) {}
21
22     poly operator+(const poly &b) const {
23         const poly &a = *this;
24         poly ans(max(a.size(), b.size()));
25         for (int i = 0; i < (int)ans.size(); i++)
26             ans[i] = ((i < (int)a.size() ? a[i] : 0) + (i < (int)b.size() ? b[i]
27                 ↪ : 0)) % mod;
28         return ans;
29     }
30     poly operator+=(const poly &b) {return *this = *this + b;}
31     poly operator-(const poly &b) const {
32         const poly &a = *this;
33         poly ans(max(a.size(), b.size()));
34         for (int i = 0; i < (int)ans.size(); i++)
35             ans[i] = ((i < (int)a.size() ? a[i] : 0) - (i < (int)b.size() ? b[i]
36                 ↪ : 0) + mod) % mod;
37         return ans;
38     }
39     poly operator-=(const poly &b) {return *this = *this - b;}
40     void NTT(poly &g, int flag) const {
41         int n = g.size();
42         vector<unsigned long long> f(g.begin(), g.end());
43         vector<int> swp(n);
44         for (int i = 0; i < n; i++) {
45             swp[i] = swp[i >> 1] >> 1 | ((i & 1) * (n >> 1));
46             if (i < swp[i]) std::swap(f[i], f[swp[i]]);
47         }
48         for (int mid = 1; mid < n; mid <= 1) {
49             int w1 = power(flag ? G : invG, (mod - 1) / mid / 2);
50             vector<int> w(mid);
51             w[0] = 1;
52             for (int i = 1; i < mid; i++) w[i] = (long long)w[i - 1] * w1 % mod;
53             for (int i = 0; i < n; i += mid << 1)
54                 for (int j = 0; j < mid; j++) {
55                     int t = (long long)w[j] * f[i + mid + j] % mod;
56                     f[i + mid + j] = f[i + j] - t + mod;
57                     f[i + j] += t;
58                 }
59         }
60     }

```

```

57         if (mid == 1 << 10)
58             for (int i = 0; i < n; i++) f[i] %= mod;
59     }
60     int inv = flag ? 1 : power(n, mod - 2);
61     for (int i = 0; i < n; i++) g[i] = f[i] % mod * inv % mod;
62     return;
63 }
64
65 // 下面是基于转置原理的 NTT, 相对朴素版本效率更高。
66 void NTT(poly &g, int flag) const {
67     int n = g.size();
68     vector<int> f(g.begin(), g.end());
69     if (flag) {
70         for (int mid = n >> 1; mid >= 1; mid >>= 1) {
71             int w1 = power(G, (mod - 1) / mid / 2);
72             vector<int> w(mid);
73             w[0] = 1;
74             for (int i = 1; i < mid; i++) w[i] = (long long)w[i - 1] * w1 %
75                 ↪ mod;
76             for (int i = 0; i < n; i += mid << 1)
77                 for (int j = 0; j < mid; j++) {
78                     int t = (long long)(f[i + j] - f[i + mid + j] + mod) *
79                         ↪ w[j] % mod;
80                     f[i + j] = f[i + j] + f[i + mid + j] >= mod ?
81                         f[i + j] + f[i + mid + j] - mod : f[i + j] + f[i +
82                         ↪ mid + j];
83                     f[i + mid + j] = t;
84                 }
85             }
86         for (int i = 0; i < n; i++) g[i] = f[i];
87     } else {
88         for (int mid = 1; mid < n; mid <= 1) {
89             int w1 = power(invG, (mod - 1) / mid / 2);
90             vector<int> w(mid);
91             w[0] = 1;
92             for (int i = 1; i < mid; i++) w[i] = (long long)w[i - 1] * w1 %
93                 ↪ mod;
94             for (int i = 0; i < n; i += mid << 1)
95                 for (int j = 0; j < mid; j++) {
96                     int t = (long long)w[j] * f[i + mid + j] % mod;
97                     f[i + mid + j] = f[i + j] - t < 0 ? f[i + j] - t + mod :
98                         ↪ f[i + j] - t;
99                     f[i + j] = f[i + j] + t >= mod ? f[i + j] + t - mod : f[i
100                     ↪ + j] + t;
101                 }
102             }
103         int inv = power(n, mod - 2);
104         for (int i = 0; i < n; i++) g[i] = (long long)f[i] * inv % mod;
105     }
106 }

```

```

100     return;
101 }
102
103 poly operator*(poly b) const {
104     poly a(*this);
105     int n = 1, len = (int)(a.size() + b.size()) - 1;
106     while (n < len) n <= 1;
107     a.resize(n), b.resize(n);
108     NTT(a, 1), NTT(b, 1);
109     poly c(n);
110     for (int i = 0; i < n; i++) c[i] = (long long)a[i] * b[i] % mod;
111     NTT(c, 0);
112     c.resize(len);
113     return c;
114 }
115
116 poly operator*=(const poly &b) {return *this = *this * b;}
117
118 poly inv() const {
119     poly f = *this, g;
120     g.push_back(power(f[0], mod - 2));
121     int n = 1;
122     while (n < (int)f.size()) n <= 1;
123     f.resize(n << 1);
124     for (int len = 2; len <= n; len <= 1) {
125         poly tmp(len), ff(len << 1);
126         for (int i = 0; i < len >> 1; i++) tmp[i] = g[i] * 2 % mod;
127         for (int i = 0; i < len; i++) ff[i] = f[i];
128         g.resize(len << 1);
129         NTT(g, 1), NTT(ff, 1);
130         for (int i = 0; i < len << 1; i++) g[i] = (long long)g[i] * g[i] %
            ↪ mod * ff[i] % mod;
131         NTT(g, 0);
132         g.resize(len);
133         for (int i = 0; i < len; i++) g[i] = (tmp[i] - g[i] + mod) % mod;
134     }
135     g.resize(size());
136     return g;
137 }
138
139 poly sqrt() const { // need F[0] = 1.
140     poly f = *this, g;
141     g.push_back(1);
142     int n = 1;
143     while (n < (int)f.size()) n <= 1;
144     f.resize(n << 1);
145     for (int len = 2; len <= n; len <= 1) {
146         poly tmp(len), ff(len << 1);
147         for (int i = 0; i < len >> 1; i++) tmp[i] = g[i] * 2 % mod;
148         for (int i = 0; i < len; i++) ff[i] = f[i];
149         g.resize(len << 1);
150         NTT(g, 1);

```



```

148         for (int i = 0; i < len << 1; i++) g[i] = (long long)g[i] * g[i] %
            ↪ mod;
149         NTT(g, 0);
150         g += ff;
151         g *= tmp.inv();
152         g.resize(len);
153     }
154     g.resize(size());
155     return g;
156 }
157 poly derivative() const {
158     poly f(*this);
159     for (int i = 1; i < (int)f.size(); i++) f[i - 1] = (long long)f[i] * i %
            ↪ mod;
160     f.pop_back();
161     return f;
162 }
163 poly integral() const {
164     poly f(*this);
165     f.push_back(0);
166     for (int i = f.size() - 1; i >= 1; i--) f[i] = (long long)f[i - 1] *
            ↪ power(i, mod - 2) % mod;
167     f[0] = 0;
168     return f;
169 }
170 poly ln() const {
171     poly f((derivative() * inv()).integral());
172     f.resize(size());
173     return f;
174 }
175 poly exp() const { // 需要满足 F[0] = 0
176     poly f(*this), g;
177     g.push_back(1);
178     int n = 1;
179     while (n < (int)size()) n <<= 1;
180     f.resize(n);
181     for (int len = 2; len <= n; len <<= 1) {
182         poly tmp(g);
183         g.resize(len);
184         g = g.ln();
185         for (int i = 0; i < len; i++) g[i] = (f[i] - g[i] + mod) % mod;
186         g[0] = (g[0] + 1) % mod;
187         g *= tmp;
188         g.resize(len);
189     }
190     g.resize(size());
191     return g;
192 }
193 };

```

```

194
195 inline poly power(poly f, int b) { // 需要满足 F[0] = 1
196     f = f.ln();
197     for (int i = 0; i < (int)f.size(); i++) f[i] = (long long)f[i] * b % mod;
198     f = f.exp();
199     return f;
200 }
201 // 不要求 F[0] = 1 的多项式快速幂，但是我忘记怎么用了，记得去回顾一下！
202 poly power(poly f, int b1, int b2 = -1) {
203     if (b2 == -1) b2 = b1;
204     int n = f.size(), p = 0;
205     reverse(f.begin(), f.end());
206     while (!f.empty() && !f.back()) f.pop_back(), p++;
207     if (f.empty() || (long long)p * b1 >= n) return poly(n);
208     int v = f.back();
209     int inv = power(v, mod - 2);
210     for (int &i : f) i = (long long)i * inv % mod;
211     reverse(f.begin(), f.end());
212     f = f.ln();
213     for (int &i : f) i = (long long)i * b1 % mod;
214     f = f.exp();
215     reverse(f.begin(), f.end());
216     for (int i = 1; i <= p * b1; i++) f.push_back(0);
217     reverse(f.begin(), f.end());
218     f.resize(n);
219     v = power(v, b2);
220     for (int &i : f) i = (long long)i * v % mod;
221     return f;
222 }
223 }

```

Chapter 6

杂项

6.1 取模类

```
1 // == Main ==
2 struct mint {
3     static const int mod = 998244353;
4     int v;
5
6     mint() = default;
7     mint(int _v): v((_v % mod + mod) % mod) {}
8     explicit operator int() const {return v;}
9     mint operator+(const mint &x) const {return v + x.v - (v + x.v < mod ? 0 : mod);}
10    mint &operator+=(const mint &x) {return *this = *this + x;}
11    mint operator-(const mint &x) const {return v - x.v + (v - x.v >= 0 ? 0 : mod);}
12    mint &operator-=(const mint &x) {return *this = *this - x;}
13    mint operator*(const mint &x) const {return (long long)v * x.v % mod;}
14    mint &operator*=(const mint &x) {return *this = *this * x;}
15    mint inv() const {
16        mint a(*this), ans(1);
17        int b(mod - 2);
18        while (b) {
19            if (b & 1) ans *= a;
20            a *= a;
21            b >>= 1;
22        }
23        return ans;
24    }
25    mint operator/(const mint &x) const {return *this * x.inv();}
26    mint &operator/=(const mint &x) {return *this = *this / x;}
27    mint operator-() {return mint(-v);}
28 };
```

6.1.1 Barrett 约减

当模数不固定时可以加速。

用法：在构造函数中传模数，使用方法为 `F.reduce(x)`，其中 x 是需要取模的数。

```

1 // == Main ==
2 struct Barrett {
3     unsigned long long b, m;
4     Barrett(unsigned long long b = 2): b(b), m((__uint128_t(1) << 64) / b) {}
5     unsigned long long reduce(long long x) {
6         unsigned long long r = (__uint128_t(x + b) * m) >> 64;
7         unsigned long long q = (x + b) - b * r;
8         return q >= b ? q - b : q;
9     }
10 } F;

```

6.2 对拍脚本

```

1  #!/usr/bin/bash
2
3  declare -i num=0
4
5  while [ true ]; do
6      ./mkdata > in.txt
7      time ./mine < in.txt > out.txt
8      ./correct < in.txt > ans.txt
9      diff out.txt ans.txt
10     if [ $? -ne 0 ]; then
11         echo "WA"
12         break
13     fi
14     num=num+1
15     echo "Passed $num tests."
16 done

```

6.3 VS Code 配置

6.3.1 User Tasks

```

1  {
2      // See https://go.microsoft.com/fwlink/?LinkId=733558
3      // for the documentation about the tasks.json format

```

```

4  "version": "2.0.0",
5  "tasks": [
6    {
7      "type": "shell",
8      "label": "My C++ Runner",
9      "detail": "Build and Run Current C++ Program",
10     "command": [ // 三个编译方式保留一个即可。
11       "clear",
12       "&&",
13       "g++ ${file} -o ${fileDirname}/${fileBasenameNoExtension}
14       ↪ -std=c++14 -Wall -Wextra && echo '== Normal ==',
15       "g++ ${file} -o ${fileDirname}/${fileBasenameNoExtension}
16       ↪ -std=c++14 -Wall -Wextra -O2 && echo '== O2 ==',
17       "g++ ${file} -o ${fileDirname}/${fileBasenameNoExtension}
18       ↪ -std=c++14 -Wall -Wextra -fsanitize=undefined,address && echo
19       ↪ '== UB Check ==',
20       "&&",
21       "gnome-terminal -- bash -c \"ulimit -s 524288; time
22       ↪ ${fileDirname}/${fileBasenameNoExtension}; read -p 'Press ENTER
23       ↪ to continue...'; exit\"
24     ],
25     "problemMatcher": [ // 非必要
26       "$gcc"
27     ],
28     "group": { // 非必要
29       "kind": "build",
30       "isDefault": true
31     },
32     "presentation": { // 非必要
33       "showReuseMessage": false
34     }
35   ]
36 ]
37 }

```

6.3.2 设置

- 字体大小: 16。 ("editor.fontSize": 16)
- 添加多个光标的方式: ctrl。 ("editor.multiCursorModifier": "ctrlCmd")
- 不适用空格代替 Tab。 ("editor.insertSpaces": false)
- 不允许 Enter 进行代码补全。 ("editor.acceptSuggestionOnEnter": "off")
- 标尺: 110。 ("editor.rulers": [110])
- 平滑。 ("editor.cursorSmoothCaretAnimation": "on")

- 标题栏外观。 ("window.titleBarStyle": "custom")

totally:

```
1 {  
2   "editor.fontSize": 16,  
3   "editor.multiCursorModifier": "ctrlCmd",  
4   "editor.insertSpaces": false,  
5   "editor.acceptSuggestionOnEnter": "off",  
6   "editor.rulers": [110],  
7   "editor.cursorSmoothCaretAnimation": "on",  
8   "window.commandCenter": false  
9 }
```

6.3.3 快捷键

- 切换块注释: Ctrl+Shift+A -> Ctrl+Shift+/
• 运行任务: Ctrl+Shift+B -> F11
• 向上移动行: Alt+up -> Ctrl+Shift+up
• 向下移动行: Alt+down -> Ctrl+Shift+down