

Cryptography and security

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Markdown version on *github*

General

- $b \in \mathbb{Z}_p^*$ has a square root if and only if $b^{\frac{p-1}{2}} \mod p = 1$

Diffie Helman

- We check that X and Y are in $\langle g \rangle$
- Use a KDF to fix bad distribution of g^{xy}
- We check the lower order $X \neq 1, X^2 \neq 1$
- If $n = pq$ then \mathbb{Z}_n ring is isomorphic to $\mathbb{Z}_p \times \mathbb{Z}_q$ and \mathbb{Z}_n^* ring is isomorphic to $\mathbb{Z}_p^* \times \mathbb{Z}_q^*$

RSA

- Square and multiply algorithm to compute x^e or x^d
- Primality test : Verify that a number is prime
- To check if a number is coprime to another one use euclid algorithm
- To compute the inverse of an elem use extended euclid algorithm
- $\varphi(p^\alpha) = (p-1)p^{\alpha-1}$
- We can compute square root of n in $\mathcal{O}(\log n)^3$

Elliptic Curve

- All finite fields have a cardinality of form p^k where p is a prime number. This prime number p is called the **characteristic** of the field.
- A **binary** field is a field with characteristic equal to 2
- Over a field \mathbb{R} , an elliptic curve with parameters a and b consists of a special point \mathcal{O} called the *point at infinity* and the points (x, y) which are the solutions of the equation $y^2 = x^3 + ax + by$
- Elliptic Curve over a **Prime Field**
 - The **discriminant** is $\Delta = -16(4a^3 + 27b^2)$
 - The curve is **non-singular** iff $\Delta \neq 0$
 - We define the **j-invariant** $j = 1728 \frac{4a^3}{4a^3 + 27b^2}$, two isomorphic curves have the same j-invariant
- Elliptic Curve over a **Binary Field**

- **Ordinary** curves are defined by two fields elements denoted a_2 and a_6

$$E_{a_2, a_6}(\mathbb{K}) = \{\mathcal{O}\} \cup \{(x, y) \in \mathbb{K}^2; y^2 + xy = x^3 + a_2x^2 + a_6\}$$

- We define the **j-invariant** $j = \frac{1}{\Delta}$
- Simple factoring method : Pollard's (also called $p - 1$ algorithm)
- **Elliptic Curve Method** (ECM) is the best method to find p when it is small
- **ECDH** key exchange protocol is the variant of Diffie-Hellman protocol working over an elliptic curve group
 - We have two participants U and V using the same subgroup of order n generated by some point G over an elliptic curve.
 - They both select their secret key $d_U, d_V \in \mathbb{Z}_n^*$
 - They compute their public key $Q_U = d_U.G$ and $Q_V = d_V.G$ which are points and exchange them.
 - Then, they both check that the received public key is actually a point of the curve which is generated by G , different from the point at infinity, and that its order is a factor of n .
 - They both compute the point P , either by $P = d_U.Q_V$ or by $P = d_V.Q_U$
 - They take the first coordinate x_p of P and convert it into a byte string Z
 - Finally they compute $K = KDF(Z)$

Symmetric Encryption