Distributed algorithms

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Contents

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General

- The distributed system is made of a finite set of **processes** : each process models a **sequencial** program
- Every pair of processes is connected by a **link** through which the processes exchange **messages**
- **Safety** is a property which states that nothing bad should happen
- **Liveness** is a property which states that something good should happen
- Twos kinds of failures are mainly considered
  - **Omissions** : The process omits to send messages it is supposed to send
  - **Arbitrary** : The process sends messages it is not supposed to send
- A **correct** process is a process that does not fail (that does not crash)
- A **Failure detector** is a distributed oracle that provides processes with suspicions about crashed processes
  - It is implemented using *timing assumptions*
  - **Perfect** :
    - *Strong Completness* : Eventually, every process that crashes is permanantly suspected by every other correct process
    - *String Accuracy* : No process is suspected before it crashes
  - **Eventually Perfect** :
    - *Strong Completness*
    - *Eventually Strong Accuracy* : Eventually, no correct process is ever suspected

Fair-loss links

- **FL1. Fai-loss** : If a message is sent infinitely often by p_i to p_jn and neither p_i or p_j crashes then m is delivered infinitely often by p_j
- **FL2. Finite duplication** : If a message m is sent a finite number if times by p_i to p_j, m is delivered a finite number of times by p_j
- **FL3. No creation** : No message is delivered unless it was sent

Stubborn links

- **SL1. Stubborn delivery** : If a process p_i sends a message m to a correct process p_j, and p_i does not crash, then p_j delivers m an infinite number of times
- **SL2. No creation** : No message is delivered unless it was sent

```
Implements: StubbornLinks (sp2p)
Uses : FairLossLinks (flp2p)
upon event <sp2pSend, dest, m> do
  while (true) do
    trigger <flp2pSend, dest, m>
upon event <flp2pDeliver, src, m> do
  trigger <sp2pDeliver, src, m>
```

Reliable (Perfect) links

- **PL1. Validity** : If p_i and p_j are correct
- **PL2. No duplication** : No message is delivered (to a process) more than once
- **PL3. No creation** : No message is delivered unless it was sent
- Roughly speaking, reliable links ensure that messages exchanged between correct processes are *not lost*

```
Implements: PerfectLinks (pp2p)
Uses: StubbornLinks (sp2p)
upon event <Init> do delivered := emptySet
upon event <pp2pSend, dest, m> do
  trigger <sp2Send, dest, m>
upon event <sp2pDeliver, src, m> do
```

```
  if m not in delivered then
    trigger <pp2pDeliver, src, m>
    add m to delivered
```

Reliable Broadcast

Best-effort Broadcast (beb)

- **BEB1. Validity** : If p_i and p_j are correct then every message broadcast by p_i is eventually delivered by p_y
- **BEB2. No duplication** : No message is delivered more than once
- **BEB3. No creation** : No messages is delivered unless it was broadcast

```
Implements: BestEffortBroadcast (beb)
Uses: PerfectLinks (pp2p)
upon event <bebBroadcast, m> do
  forall pi in S do
    trigger <pp2pSend, pi, m>
upon event <pp2pDeliver, pi, m> do
  trigger <dedDeliver, pi, m>
```

Reliable Broadcast (rb)

- **RB1** = BEB1
- **RB2** = BEB2
- **RB3** = RB3
- **RB4. Agreement** : For any message m, if any correct process delivers m, then every correct process delivers m

```
Implements: ReliableBroadcast (rb)
Uses:
  BestEffortBroadcast (beb)
  PerfectFailureDetector (P)
upon event <Init> do
  delivered := emptySet
  correct := S
  forall pi in S do from[pi] := emptySet
upon event <rbBroadcast, m> do
  delivered := delivered U {m}
  trigger <rbDeliver, self, m>
  trigger <bebBroadcast, [data, self, m]>
upon event <crash, pi> do
  correct := correct \{pi}
  forall [pj, m] in from[pi] do
    trigger <bebBroadcast, [data, pj, m]>
upon event <bebDeliver, pi, [data, pj, m]> do
  if m not in delivered then
    delivered := delivered U {m}
    trigger <rbDeliver, pj, m>
    if pi not in correct then
      trigger <bebBroadcast, [data, pj, m]>
    else
      from[pi] := from[pi] U {[pj, m]}
```

Uniform Reliable Broadcast (urb)

- **URB1** = BEB1
- **URB2** = BEB2
- **URB3** = BEB3
- **URB4. Uniform Agreement** : For any message m, if any process delivers m, then every process delivers m

```
Implements: UniformBroadcast (urb)
Uses:
  BestEffortBroadcast (beb)
  PerfectFailureDetector (P)
upon event <Init> do
  correct := S
  delivered := forward := emptySet
  ack[Message] := emptySet
upon event <urbBroadcast, m> do
  forward := forward U {[self, m]}
  trigger <bebBroadcast, [data, self, m]>
upon event <bebDeliver, pi, [data, pj, m]> do
  ack[m] := ack[m] U {pi}
  if [pi, m] not in forward then
    forward := forward U {[pj, m]}
    trigger <bebBroadcast, [data, pj, m]>
upon event (for any [pj, m] in forward) <correct in ack[m]> and <m not in delivered> do
  delivered := delivered U {m}
  trigger <urbDeliver, pj, m>
```

Causal Broadcast

- A **non-blocking** algorithm using the past
- A **blocking** algorithm using **vector clocks**

Causality

- Let m_1 and m_2 be any two messages : $m_1 \rightarrow m_2$ (m_1 causally precedes m_2) iff
  - **C1. Fifo order** : Some process p_i broadcast m_1 before broadcasting m_2
  - **C2. Local order** : Some process p_i delivers m_1 and then broadcast m_2
  - **C3. Transitivity** : There is a message m_3 such that $m_1 \rightarrow m_3$ and $m_3 \rightarrow m_2$

Causal broadcast

- **CO** : If any process p_i delivers a message m_2, then p_i must have delivered every message m_1 such that $m_1 \rightarrow m_2$

Reliable Causal Broadcast (rcb)

- **RB1**, **RB2**, **RB3**, **RB4**
- **CO**

Uniform Causal Broadcast (ucb)

- **URB1**, **URB2**, **URB3**, **URB4**
- **CO**

Reliable Causal Order Broadcast (rco)

```
Implements: ReliableCausalOrderBroadcast (rco)
Uses : ReliableBroadcast (rb)
upon event <Init> do
  delivered := past := emptySet
upon event <rcoBroadcast, m> do
  trigger <rbBroadcast, [data, past, m]>
  past := past U {[self, m]}
upon event <rbDeliver, pi [data, pastm, m]> do
  if m not in delivered then
    forall [sn, n] in pastm do
      if n not in delivered then
        trigger <rcoDeliver, sn, n>
        delivered := delivered U {n}
        past := past U {[self, n]}
    trigger <rcoDeliver, pi, m>
    delivered := delivered U {m}
    past := past U {[pi, m]}
```

```
Implements ReliableCausalOrderBroadcast (rco)
Uses: ReliableBroadcast (rb)
upon event <Init> do
  forall pi in S: VC[pi] := 0
  pending := emptySet
upon event <rcoBroadcast, m> do
  trigger <rcoDeliver, self, m>
  trigger <rbBroadcast, [data, VC, m]>
  VC[self] := VC[self] + 1
upon event <rbDeliver, pj, [data, VCm, m]> do
  if pj not self then
    pending := pending U (pj, [data, VCm, m])
    deliver-pending
procedure deliver-pending is
  while (s, [data, VCm, m]) in pending do
    if forall pk: (VC[pk] >= VCm[pk]) do
      pending := pending - (s, [data, VCm, m])
      trigger <rcoDeliver, self, m>
      VC[s] := VC[s] + 1
```

- These algo ensure causal reliable broadcast
- If we replace reliabe broadcast with uniform reliabe broadcast, these algo would ensure uniform causal broadcast

Total Order Broadcast (tob)

- In **reliable** broadcast, the processes are free to deliver messages in any order they wish
- In **causal** broadcast, the processes need to deliver messages according to some order (causal order)
  - The order imposed by causal broadcast is however partial : some messages might be delivered in different order by the processes
- In **total order** broadcast, the processes must deliver all messages according to the same order(i.e. the order is now total)
  - This order does not need to respect causality (or event FIFO ordering)
- **RB1. Validity** : If p_i and p_j are correct, then every message broadcast by p_i is eventually delivered by p_j

- **RB2. No duplication** : No message is delivered more than once
- **RB3. No creation** : No message os delivered unless it was broadcast
- **RB4. (Uniform) Agreement** : For any message m. If a correct (any) process delivers m, then every correct process delivers m
- **(Uniform) Total order** : Let m and m' be any two messages. Let p_i be any any (correct) process that delivers m without having delivered m'. Then no (correct) process delivers m' before m

(Uniform) Consensus

- In the (uniform) consensus problem the processes propose values and need to agree on one among these values
- **C1. Validity** : Any value decided is a value proposed
- **C2. (Unifrom) Agreement** : No two correct (any) processes decide differently
- **C3. Termination** : Every correct process eventually decides
- **C4. Integrity** : Every process decides at most once

Total Order (to)

```
Implements: TotalOrder (to)
Uses:
  ReliableBroadcast (rb)
  Consensus (cons)
upon event <Init> do
  unordered := delivered := emptySet
  wait := false;
  sn := 1
upon event <toBroadcast, m> do
  trigger <rbBroadcast, m>
upon event <rbDeliver, sm, m> and (m not in delivered) do
  unordered := unordered U {(sm, m)}
upon event (unordered not emptySet) and not wait do
  wait := true
  trigger <Propose, unordered>sn
upon event <Decide, decided>sn do
  unordered := unordered \ decided
  ordered := deterministicSort(decided)
  forall (sm, m) in ordered do
    trigger <toDeliver, sm, m>
    delivered := delivered U {m}
  sn := sn + 1
  wait = false
```

Shared Memory

Regular register

- Assumes only one writer
- Provides *strong* guarantees when there is no concurrent operations
- When some operations are concurrent, the register provides *minimal* guarantees
- `Read()` returns :
  - *The last value* written if there is no concurrent or failed operations

    - Otherwise the last value written on *any* value concurrently written i.e. the input parameter of some `Write()`
- We assume **fail-stop** model
    - Process can fail by crashing (no recovery)
    - Channels are reliable
    - Failure detection is perfect
- We implement a **regular** register
    - Every process p_i has a local copy of the register value v_i
    - Every process reads **locally**
    - The writer writes **globally**

```
Write(v) at pi
  send [W, w] to all
  forall pj, wait until either
    receive [ack] or
    detect [pj]
  return ok

Read() at pi
  return vi

At pi
  when receive [W, w] from pj
    vi := v
    send [ack] to pj
```

- We assume while failure detection is not perfect
    - P_1 is the writer and any process can be reader
    - A mojority of the process is correct
    - Channels are reliable
- We implement a **regular** register
    - Every process p_i maintains a local copy of the register v_i, as well as a sequence number sn_i and a read timestamp rs_i
    - Process p_1 maintains in addition a timestamp ts_1

```
Write(v) at p1
  ts1 ++
  send [W, ts1, v] to all
  when receive [W, ts1, ack] from majority
    return ok

Read() at pi
  rsi ++
  send [R, rsi] to all
  when receive [R, rsi, snj, vj] from majority
    v := vj with the largest snj
    return v

At pi
  when receive [W, ts1, v] from p1
    if ts1 > sni then
      vi = v
      sni := ts1
      send[W, ts1, ack] to p1
  when receive [R, rsj] from pj
    send [R, rsj, sni, vi] to pj
```

Atomic Register

- An **Atomic Register** provides strong guarantees event when there is concurrency and failures : the execution is equivalent to a sequencial and failure-free execution
- Every failed (write) operation appears to be either complete or not to have been invoked at all
- Every complete operation appears to be executed at some instant between its invocation and reply time events
- We implement a **fail-stop** 1-N **atomic register**
  - Every process maintais a local value of the register as well as a sequence number
  - The writer, p_1, maintains, in addition a timestamp ts_1
  - Any process can read in the register

```
Write(v) at p1
  ts1++
  send [W, ts1, v] to all
  forall pi wait until either
    receive [ack] or
    detect [pi]
  return ok
```

```
Read() at pi
  send [W, sni, vi] to all
  forall pi wait until either
    receive [ack] or
    suspect [pj]
  return vi
```

```
At pi
  When pi receive [W, ts, v] from pj
    if ts > sni then
      vi := v
      sni := ts
    send [ack] to pj
```

- We implement a **fail-stop** N-N **atomic register**

```
Write(v) at pi
  send [W] to all
  forall pj wait until either
    receive [W, snj] or
    suspect [pj]
  (sn, id) := (highest snj + 1, i)
  send [W, (sni, id), v] to all
  forall pj wait until either
    receive [W, (sn, id), ack] or
    detect [pj]
  return ok
```

```
Read() at pi
  send [R] to all
  forall pj wait until either
    recieve [R, (snj, idj), vj] or
    suspect pj
  v = vj with the highest (snj, idj)
  (sn, id) = highest (snj, idj)
  send [W, (sn, id), v] to all
```

```
  forall pj wait until either
    receive [W, (sn, id), ack] or
    detect [pj]
  return v

At pi
T1 :
  when receive [W] from pj
    send [W, sn] to pj
  when receive [R] from pj
    send [R, (sn, id), vi] to pj

T2 :
  when receive [W, (snj, idj), v] from pj
    if (snj, idj) > (sn, id) then
      vi := v
      (sn, id) = (snj, idj)
    send [W, (sn, id), ack] to pj
  when receive [W, (snj, idj), v] from pj
    if (snj, idj) > (sn, id) then
      vi := v
      (sn, id) := (snj, idj)
    send [W, (sn, id), ack] to pj
```

- From fail-stop to **fail-silent**
  - We assume a mojority of correct processes
  - In the 1-N algorithm, the writer writes in a majority using a timestamp determined locally and the reader selects a value from a majority and then imposes this value on a majority
  - In the N-N algorithm, the writers determines first the timestamp using a majority

Terminating Reliable Broadcast (TRB)

- Like reliable broadcast, terminating reliable broadcast (TRB) is a communication primitive used to disseminate a message among a set of processes in a reliable way
- TRB is however strictly stronger than (uniform) reliable broadcast
- Like with reliable broadcast, correct processes in TRB agree on the set of messages they deliver
- Like with (uniform) reliable broadcast, every correct process in TRB delivers every message delivered by any correct process
- Unlike with reliable broadcast, every correct process delivers a message, event if the broadcaster crashes
- The problem is defined for a specific broadcaster process $p_i = src$ (know by all processes)
  - Process src is supposed to broadcast a message m (distinct from φ)
  - The other processes need to deliver m if src is correct but may deliver φ is src crashes
- **TRB1. Integrity** : If a prcess delivers a message m, then either m is φ or m was broadcasted by src
- **TRB2. Validity** : If the sender src is correct and broadcasts a message m, then src eventually delivers m
- **TRB3. (Unifrom) Agreement** : For any message m, if a correct (any) process delivers m, then every correct process delivers m
- **TRB4. Termination** : Every correct process eventually delivers exactly one message

```
Implements: trbBroadcast (trb)
Uses:
  BestEffortBroadcast (beb)
  PerfectFailureDetector (P)
  Consensus (cons)
upon event <Init> do
```

```
  prop := 0
  correct := S
upon event <trbBroadcast, m> do
  trigger <bebBroadcast, m>
upon event <crash, src> and (prop = 0) do
  prop := phi
upon event <bebDeliver, src, m> and (prop = 0) do
  prop := m
upon event (prop != 0) do
  trigger <Propose, prop>
upon event <Decide, decision> do
  trigger <trbDeliver, src, decision>
```

- We give an algorithm that implements P unsign TRB. More precisely, we assume that every process p_i can use an infinite number of instances of TRB where p_i is the sender *src*
  1. Every process p_i keeps on trbBroadcasting messages m_{i1}, m_{mi2} etc
  2. If a process p_k delivers φ_i, p_k suspects p_i