PORTFOLIO OPTIMISATION

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“MODERN” PORTFOLIO THEORY

In matrix form, for a given "risk tolerance" $q \in [0, \infty)$, the efficient frontier is found by minimizing the following expression:

$$w^T \Sigma w - q * R^T w$$

where

- w is a vector of portfolio weights and $\sum_i w_i = 1.$ (The weights can be negative, which means investors can short a security.);
- Σ is the covariance matrix for the returns on the assets in the portfolio;
- $q \geq 0$ is a "risk tolerance" factor, where 0 results in the portfolio with minimal risk and ∞ results in the portfolio infinitely far out on the frontier with both expected return and risk unbounded; and
- R is a vector of expected returns.
- $w^T \Sigma w$ is the variance of portfolio return.
- $R^T w$ is the expected return on the portfolio.

The above optimization finds the point on the frontier at which the inverse of the slope of the frontier would be q if portfolio return variance instead of standard deviation were plotted horizontally. The frontier in its entirety is parametric on q.

QUADRATIC PROGRAMMING

solve.QP {quadprog} R Documentation

Solve a Quadratic Programming Problem

Description

This routine implements the dual method of Goldfarb and Idnani (1982, 1983) for solving quadratic programming problems of the form *min(-d^T b + 1/2 b^T D b)* with the constraints *A^T b >= b_0*.

$-q * R^T w + w^T \Sigma w$

Usage

```
solve.QP(Dmat, dvec, Amat, bvec, meq=0, factorized=FALSE)
```

Arguments

`Dmat`	matrix appearing in the quadratic function to be minimized.
`dvec`	vector appearing in the quadratic function to be minimized.
`Amat`	matrix defining the constraints under which we want to minimize the quadratic function.
`bvec`	vector holding the values of *b_0* (defaults to zero).
`meq`	the first `meq` constraints are treated as equality constraints, all further as inequality constraints (defaults to 0).
`factorized`	logical flag: if `TRUE`, then we are passing *R^(-1)* (where *D = R^T R*) instead of the matrix *D* in the argument `Dmat`.

Dmat = covariance matrix
Dvec = returns (or zeros)
Amat = constraint groups
bvec = constraint values

Value

a list with the following components:

`solution`	vector containing the solution of the quadratic programming problem.
`value`	scalar, the value of the quadratic function at the solution

CVAR OPTIMISATION IS A LINEAR PROGRAMMING PROBLEM

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- Portfolio weights x, j assets $$x_j \geqslant 0 \quad \text{for } j = 1, \ldots, n, \quad \text{with } \sum_{j=1}^{n} x_j = 1.$$
- Portfolio returns f(x, y) $$f(\boldsymbol{x}, \boldsymbol{y}) = -[x_1 y_1 + \cdots + x_n y_n] = -\boldsymbol{x}^{\mathsf{T}}\boldsymbol{y}.$$
- Minimum mean return constraint $$\mu(\boldsymbol{x}) \leqslant -R$$
- CVaR $$\tilde{F}_\beta(\boldsymbol{x}, \alpha) = \alpha + \frac{1}{q(1-\beta)} \sum_{k=1}^{q} [-\boldsymbol{x}^{\mathsf{T}}\boldsymbol{y}_k - \alpha]^+.$$
  - Utilize auxiliary variables u $$\alpha + \frac{1}{q(1-\beta)} \sum_{k=1}^{q} u_k$$
  - Subject to $$u_k \geqslant 0 \quad \text{and} \quad \boldsymbol{x}^{\mathsf{T}}\boldsymbol{y}_k + \alpha + u_k \geqslant 0 \qquad \text{for } k = 1, \ldots, r.$$

Source: T.R. Rockafellar and S.P. Uryasev. Optimization of Conditional Value–at–Risk. Journal of Risk, 2:21–41, 2000.

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