CAIM | CROWN AGENTS INVESTMENT MANAGEMENT

ASSET ALLOCATION AND PORTFOLIO CONSTRUCTION

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ASSET ALLOCATION AND PORTFOLIO CONSTRUCTION

INGREDIENTS FOR STRATEGIC ASSET ALLOCATION

Ingredient	“Modern” Portfolio Theory	Modern Portfolio Construction
Expected return for each asset	Often uses average historical returns	Forward-looking return expectations
Expected risk for each asset	Volatility – variance or standard deviations	Simulated path-dependent scenarios, no need to assume Gaussian, or any parametrised distributions
Description of comovement between assets	Variance-covariance matrix	
Portfolio optimization methodology	Mean-variance “Markowitz” optimisation	Downside risk (CVaR) optimization, e.g. Uryasev etc.
Pros/cons	Simple, easy to interpret	Much better analysis of “ugly” data, focus on more relevant risk measures
Cons	Fragile, very sensitive to input error. Assumption of normal distributions, covariance matrix “dumbs down” comovement information	More complex to set up and execute; senior stakeholders might be unfamiliar with CVaR and its interpretation

INGREDIENTS FOR ASSET ALLOCATION AND PORTFOLIO CONSTRUCTION

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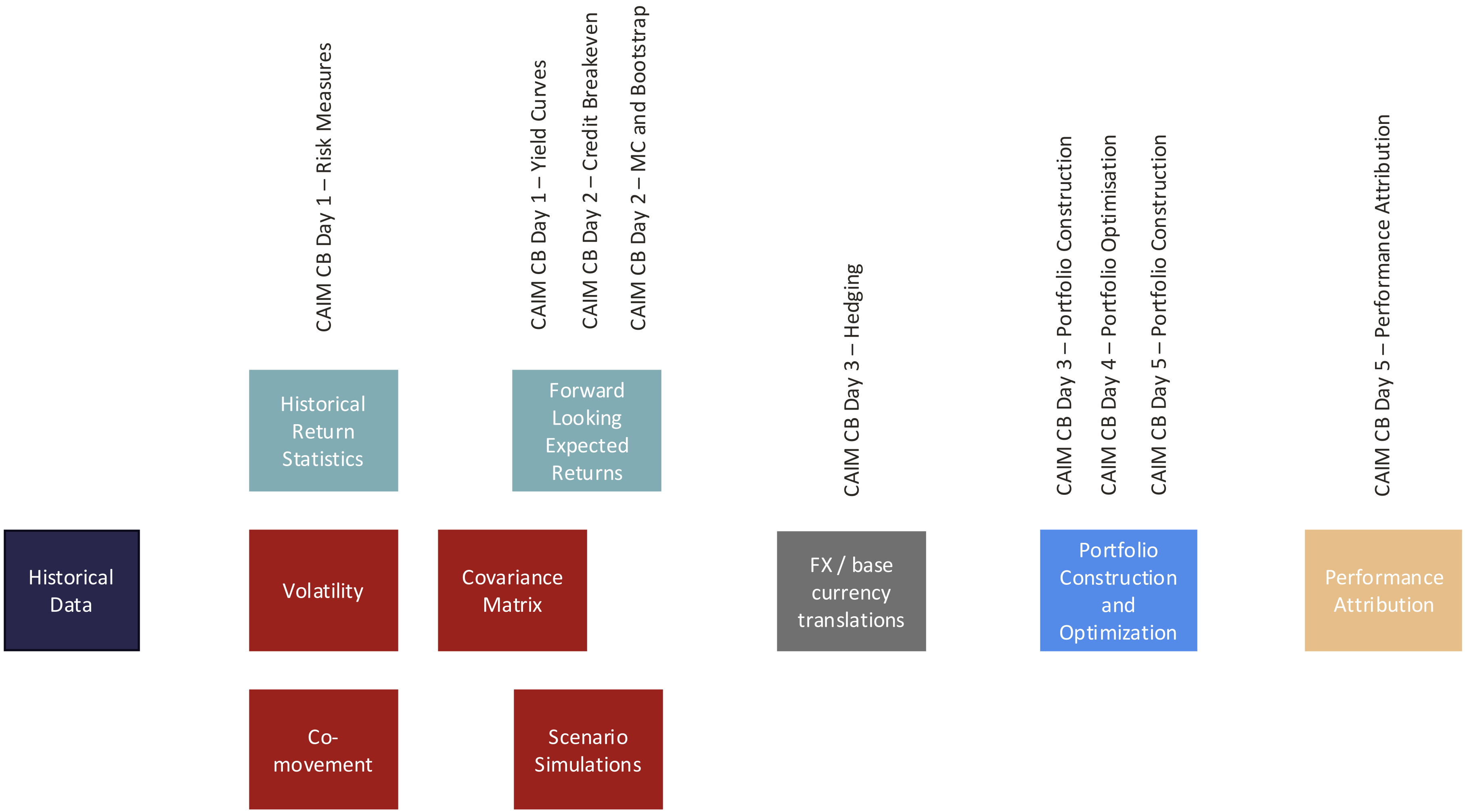
You’ve built and used relevant tools this week



PORTFOLIO OPTIMIZATION ANNEX

"MODERN" PORTFOLIO THEORY

In matrix form, for a given "risk tolerance" $q \in [0, \infty)$, the efficient frontier is found by minimizing the following expression:

$$w^T \Sigma w - q * R^T w$$

where

- w is a vector of portfolio weights and $\sum_i w_i = 1$. (The weights can be negative, which means investors can short a security.);
- Σ is the covariance matrix for the returns on the assets in the portfolio;
- $q \geq 0$ is a "risk tolerance" factor, where 0 results in the portfolio with minimal risk and ∞ results in the portfolio infinitely far out on the frontier with both expected return and risk unbounded; and
- R is a vector of expected returns.
- $w^T \Sigma w$ is the variance of portfolio return.
- $R^T w$ is the expected return on the portfolio.

The above optimization finds the point on the frontier at which the inverse of the slope of the frontier would be *q* if portfolio return variance instead of standard deviation were plotted horizontally. The frontier in its entirety is parametric on *q*.

QUADRATIC PROGRAMMING

solve.QP {quadprog} R Documentation

Solve a Quadratic Programming Problem

Description

This routine implements the dual method of Goldfarb and Idnani (1982, 1983) for solving quadratic programming problems of the form *min(-d^T b + 1/2 b^T D b)* with the constraints *A^T b >= b_0*.

$$-q * R^T w + w^T \Sigma w$$

Usage

```
solve.QP(Dmat, dvec, Amat, bvec, meq=0, factorized=FALSE)
```

Arguments

`Dmat`	matrix appearing in the quadratic function to be minimized.
`dvec`	vector appearing in the quadratic function to be minimized.
`Amat`	matrix defining the constraints under which we want to minimize the quadratic function.
`bvec`	vector holding the values of *b_0* (defaults to zero).
`meq`	the first `meq` constraints are treated as equality constraints, all further as inequality constraints (defaults to 0).
`factorized`	logical flag: if `TRUE`, then we are passing *R^(-1)* (where *D = R^T R*) instead of the matrix *D* in the argument `Dmat`.

Dmat = covariance matrix

Dvec = returns (or zeros)

Amat = constraint groups

bvec = constraint values

Value

a list with the following components:

`solution`	vector containing the solution of the quadratic programming problem.
`value`	scalar, the value of the quadratic function at the solution

CVAR OPTIMISATION IS A LINEAR PROGRAMMING PROBLEM

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- Portfolio weights x, j assets $x_j \geqslant 0 \quad \text{for } j = 1, \ldots, n, \quad \text{with } \sum_{i=1}^{n} x_j = 1.$
- Portfolio returns f(x, y) $f(\boldsymbol{x}, \boldsymbol{y}) = -[x_1 y_1 + \cdots + x_n y_n] = -\boldsymbol{x}^{\mathsf{T}} \boldsymbol{y}.$
- Minimum mean return constraint $\mu(\boldsymbol{x}) \leqslant -R$
- CVaR $$\tilde{F}_{\beta}(\boldsymbol{x}, \alpha) = \alpha + \frac{1}{q(1-\beta)} \sum_{k=1}^{q} [-\boldsymbol{x}^{\mathsf{T}} \boldsymbol{y}_k - \alpha]^{+}.$$
  - Utilize auxiliary variables u $$\alpha + \frac{1}{q(1-\beta)} \sum_{k=1}^{q} u_k$$
  - Subject to $u_k \geqslant 0 \quad \text{and} \quad \boldsymbol{x}^{\mathsf{T}} \boldsymbol{y}_k + \alpha + u_k \geqslant 0 \qquad \text{for } k = 1, \ldots, r.$

Source: T.R. Rockafellar and S.P. Uryasev. Optimization of Conditional Value–at–Risk. Journal of Risk, 2:21–41, 2000.

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