Note

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1 General Structure

The code mainly consists of three parts. Class `formFun` calculates the form factor of the deuteron nucleon vertex and the nucleon electromagnetic form factor; Code in the namespace `leptonTensor` gives results for the lepton tensor; Code in the namespace `nuclearTensor` calculates the nuclear tensor. We use the numerical solutions of deuteron wave functions in Ref. [1].

Denote the nuclear electromagnetic current matrix element in the momentum space as

$$\mathcal{M}^{\mu}_{p\,(n),P\,(F)} = \left\langle \boldsymbol{p}_1; \boldsymbol{p}_2 \left| \hat{J}^{\mu}(0) \right| \boldsymbol{P} \right\rangle_{p\,(n),P\,(F)} ,$$

where the first lower index $p\,(n)$ represents that proton (neutron) absorbs the exchanged photon and the second lower index $P\,(F)$ represents the Plane wave contribution (with the Final state interactions) to the Feynman amplitudes. More details can be found in the manuscript arXiv:2210.10560. When the momentum of the proton are much larger than the momentum of the neutron in the final states, the dominant contribution to the nuclear tensor is

$$\mathcal{M}^{\mu}_{p,P}\mathcal{M}^{\nu\dagger}_{p,P} + \mathcal{M}^{\mu}_{p,P}\mathcal{M}^{\nu\dagger}_{p,F} + \mathcal{M}^{\mu}_{p,F}\mathcal{M}^{\nu\dagger}_{p,P} + \mathcal{M}^{\mu}_{p,F}\mathcal{M}^{\nu\dagger}_{p,F} ,$$

and the numerical code is given in the namespace `nuclearTensor`. Function `pwdcsfun` calculates the contribution related to $\mathcal{M}^{\mu}_{p,P}\mathcal{M}^{\nu\dagger}_{p,P}$.

After average over initial states, sum over final states, and taking Dirac trace, $\bar{\sum}\mathcal{M}^{\mu}\mathcal{M}^{\nu\dagger}$ is a rank 2 tensor constructed by $g^{\mu\nu}$, $P^{\mu}P^{\nu}$, $P^{\mu}p_1^{\nu} + p_1^{\mu}P^{\nu}$, etc. The coefficients of $g^{\mu\nu}$ and other rank 2 momentum tensors are polynomials of the Lorentz scalar production of the four-momentum. Before calculating the differential cross section, arrays storing the exponent vector should be imported using function `importpwcoeMat` for plane wave amplitudes.

In the namespace `nuclearTensor`, our code evaluates the differential cross section of the quasi-free scattering from the proton in the deuteron

$$\mathrm{d}\sigma = \frac{\alpha^3}{8(4\pi)^4}\frac{|\boldsymbol{p}_1|^2\beta}{m_d^2E_\gamma^2t^2}\frac{L^{\mu\nu}W_{\mu\nu}}{||\boldsymbol{p}_1|(m_d+\nu) - E_1|\boldsymbol{q}|\cos\theta_1|}\mathrm{d}\Omega_p\mathrm{d}\Omega_{ll}\mathrm{d}s_{ll}\mathrm{d}s_{pn}\mathrm{d}t \; . \tag{1}$$

The neutron momentum is integrated out. $s_{pn} = (p_1 + p_2)^2$ is the squared invariant mass of the proton and the neutron, and $s_{ll} = (p_3 + p_4)^2$ is the squared invariant mass of the lepton pair. Ω_{ll} is the solid angle of lepton pair in the l^+l^- center of mass frame, and Ω_p is the solid angle of proton in the laboratory frame (with $\boldsymbol{q}$ as z axis). $t = q^{\mu}q_{\mu}$ is the squared transfer momentum and $\beta = \sqrt{1 - 4m_e^2/s_{ll}}$. $\boldsymbol{p}_1$ and θ_1 are the three-momentum and the polar angle of the final proton, respectively. For the arguments in the function prototypes like `pwdcsfuntot(double Egama, double thetaea, double phiea, double slla, double tqsqa, double spna, double p1a, double phi1a, double res[])`, `Egama` is the incident photon energy E_γ, `thetaea` is the lepton polar angle θ_e, `phiea` is the lepton azimuthal angle ϕ_e, `slla` is the squared invariant mass of the lepton pair s_{ll}, `tqsqa` is is the squared transfer momentum t, `spna` is the squared invariant mass of the proton and the neutron s_{pn}, `p1a` is the three-momentum $|\boldsymbol{p}_1|$ of the final proton and `phi1a` is the azimuthal angle ϕ_1 of the final proton with reference to $\boldsymbol{q}$.

References

[1] Franz Gross and Alfred Stadler. Covariant spectator theory of *np* scattering: Effective range expansions and relativistic deuteron wave functions. Phys. Rev. C, 82:034004, 2010. doi: 10.1103/PhysRevC.82.034004.

[2] Ron L. Workman, William J. Briscoe, and Igor I. Strakovsky. Partial-Wave Analysis of Nucleon-Nucleon Elastic Scattering Data. Phys. Rev. C, 94(6):065203, 2016. doi: 10.1103/PhysRevC.94.065203.