

# Bayesian Hierarchical Censored Poisson Regression

JQT paper with 200 drivers

*Miao Cai\**

2018-10-25

## 1 Poisson distribution

## 2 Censored Poisson regression

Logistic regression ignores the intensity of the critical events with any number greater than 0 categorized into 1. So we adopt a Bayesian hierarchical Poisson regression to model the effect of cumulative driving time on the occurrence of critical events. Each driver has a random intercept and a random slope on cumulative driving time.

$$\begin{aligned} N &\sim \text{POIS}(t \cdot \lambda) \\ \lambda_{d(i)} &= \exp(\beta_{0,d(i)} + \beta_{1,d(i)} \cdot \text{CT} + \xi \cdot \mathbf{W} + \nu \cdot \mathbf{D}) \end{aligned} \tag{1}$$

Where  $N$  is the number of critical events for driver  $d(i)$  in time interval  $j$ , and it has a Poisson distribution with parameter  $\lambda$ . The other variables are identical as those described in Equation ??.

```
load("t.Rdata")

library(rstan)
library(rstanarm)
library(shinystan)
options(mc.cores=parallel::detectCores())

t1 = t[1:20000,]

poistan = '
data {
  int<lower=0> n; //total # of obs
  int<lower=0> k;

  int<lower=0> driver_num[n]; //driver id
  int<lower=0> cnum[n]; //count outcome
  real<lower=0> drivetime_cum[n]; //cumulative time of driving
  real<lower=0> travelTime[n];
  int<lower=0> precip_bi[n]; //precipitation
}
parameters{
  real beta0[k];
  real beta1[k];
  real beta2;
  real mu0;
  real mu1;
  real<lower=0> sigma0;
```

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\*Department of Epidemiology and Biostatistics, Saint Louis University. Email address [miao.cai@slu.edu](mailto:miao.cai@slu.edu)

```

    real<lower=0> sigma1;
  }
model{
  for(i in 1:n){
    cnum[i] ~ poisson( travelTime[i] * exp(beta0[driver_num[i]] + beta1[driver_num[i]]*drivetime_cum[i])
  }
  //PRIORS
  beta2 ~ normal(0, 10);
  for(j in 1:k){
    beta0[j] ~ normal(mu0, sigma0);
    beta1[j] ~ normal(mu1, sigma1);
  }
  //HYPERPRIORS
  mu0 ~ normal(0, 10);
  mu1 ~ normal(0, 10);
  sigma0 ~ gamma(1, 1);
  sigma1 ~ gamma(1, 1);
}
'

datstan = list(n = nrow(t1),
               k = max(t1$driver_num),
               travelTime = t1$travelTime,
               driver_num = t1$driver_num,
               cnum = t1$cnum,
               drivetime_cum = t1$drivetime_cum,
               precip_bi = t1$precip_bi)

hfitpois20000 <- stan(model_code=poistan, model_name="Poisstan", data=datstan, iter=5000, warmup = 1000,
save(hfitpois20000, file = "hfitpois20000.Rdata")

launch_shinystan(hfitpois20000)

```