Modeling Recurrent Safety-critical Events among Commercial Truck Drivers: A Bayesian Hierarchical Jump Power Law Process

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Abstract

Many transportation safety studies aim to predict crashes based on aggregated road segment data. As an increasing number of naturalistic driving studies are initiated in the recent decade, safety-critical events (SCEs) such as hard brakes have been widely used as a proxy measure of driving risk. Different from real crashes, multiple SCEs can occur in a driving shift and they do not interrupt the state of driving. Motivated by a growing need of analyzing recurrent SCEs and the feature that multiple trips are nested within a shift for commercial truck drivers, we proposed a Bayesian hierarchical non-homogeneous Poisson process with power law process intensity function and an innovative Bayesian hierarchical jump power law process. We specified the parameterization, intensity function, and likelihood function for the two models and demonstrates the estimation results for correctly and wrongly specified models based on simulated data. The two models are then applied to a naturalistic driving data of over 13 million driving records and 8,407 SCEs generated by 496 commercial truck drivers. Supplementary materials including simulated data and parameter estimation for reproducing the work, are available as an online supplement.

Keywords: trucking; safety-critical events; reliability; power law process

1. INTRODUCTION

Traditional trucking safety studies apply classification or count data models to predict the occurrence or number of crashes in certain road segments for a fixed amount of time based on policy reports data (Lord and Mannering, 2010; Savolainen et al., 2011; Mannering and Bhat, 2014). These retrospective crash prediction studies are inherently limited in the sample size of crashes, selection of control groups, and undercount of less severe crashes. Large-scale naturalistic driving studies that continuously record real-world driving data using unobtrusive instruments have recently been proposed as an innovative method for transportation safety research (Guo, 2019; Mehdizadeh et al., 2020). Instead of studying real crashes, naturalistic driving studies use kinematic safety-critical events (SCEs) such as hard brakes as a proxy measure of driving risk. Although SCEs can still be analyzed using classification and count data models (Kim et al., 2013), these events are different from crashes: there can be at most one crash in a working shift and the drivers have to stop once a crash occurs; however, there can be multiple SCEs in a working shift and the drivers can continue driving even if SCEs occur.

Commercial truck drivers are on the road for an extended period of time and commonly face fatigue problems. Investigating the driving reliability using SCEs can contribute to understanding fatigue problems among commercial truck drivers and optimize route and shift scheduling. Previous studies tend to use traditional classification or count data models (Kim et al., 2013; Chen et al., 2016), or change-point models for naturalistic driving data sets (Chen and Guo, 2016; Li et al., 2017, 2018; Liu and Guo, 2019; Liu et al., 2019; Guo et al., 2019). These models are limited in two aspects. First, retrospective reports are collecting crashes at road segments and non-crashes are selected randomly to match the crashes, which resembles a case-control study design. In contrast, naturalistic driving data sets are following the drivers or vehicles for a certain amount of time, so the sampling units are drivers instead of road segments, which resembles a prospective cohort study design (Mehdizadeh et al., 2020). Since the same drivers tend to have similar driving patterns, it is natural to apply hierarchical models that account for driver-level effects. Second, commercial truck drivers, specially long-haul truck drivers, must take at least one break in long-distance transporting required by Federal Motor Carrier Safety Administration (2017). It is reasonable to assume that some level of fatigue alleviation and reliability change occur at these short breaks.

In this article, we first introduced a Bayesian hierarchical non-homogeneous Poisson process with the power law process (PLP) intensity function to model SCEs within shifts. This model accounts for driver-level unobserved heterogeneity by specifying driver-level random intercepts for the rate parameter in PLP. On the other hand, since the Federal Motor Carrier Safety Administration (2017) regulates that drivers who transports property and delivers materials must a) be on duty for no more than 14 hours; b) drive for no more than 11 hours, and c) take a at least 30-minute break by the eight hour of on duty, a property-carrying truck driver must have at least one break if they are on road for more than eight hours. To account for this feature of multiple trips and breaks nested within a shift among commercial truck drivers, we then propose a Bayesian hierarchical jump power law process (JPLP) to take potential reliability changes at the time of rests into consideration.

The structure of this article is as follows. In Section 2, we define our terminology and notation for shifts, trips, and events for naturalistic driving data generated by commercial truck drivers. In Section 3, we specify our proposed PLP and JPLP, their intensity functions and likelihood functions. In Section 4, several simulation studies are conducted to demonstrate the validity of our code and the consequences if the models are not specified correctly. In section 5, we present the results of real data analyses for 496 commercial truck drivers using PLP and JPLP. Strengths, possible limitations, and future research directions are discussed in Section 6. A simulated data set, description on data structure, and Stan and R code for Bayesian PLP and JPLP estimation are provided in the supplementary material.

2. TERMINOLOGY AND NOTATION

Figure 1 presents a time series plot of speed data for a sample truck driver (including two shifts and six trips nested within the shifts) and arrows suggesting shifts and trips. We use $d : 1, 2, \ldots, D$ as the notation for different drivers. A shift $s, 1, 2, \ldots, S_d$ is on-duty periods with no breaks longer than 10 hours for driver d. As the Federal Motor Carrier Safety Administration (2017) requires, a shift must be no more than 14 hours with no more than 11 hours of driving, and this leads to the phenomena that multiple trips $r : 1, 2, \ldots, R_{d,s}$ are separated by breaks longer than 30 minutes but less than 10 hours for each driver d and shift s.

SCEs can occur any time in the trips whenever preset kinematic thresholds are trigger by the driver. We use $i : 1, 2, \ldots, I_{d,s}$ as notations for the i-th SCE for driver d in shift s. For each SCE, $t_{d,s,i}$ is the time to the i-th SCE for driver d measured from the beginning of the s-shift and the rest times between trips are excluded from calculation. $n_{d,s,r}$ is the number of SCEs for trip r within shift s for driver d. $a_{d,s,r}$ is the end time of trip r within shift s for driver d.

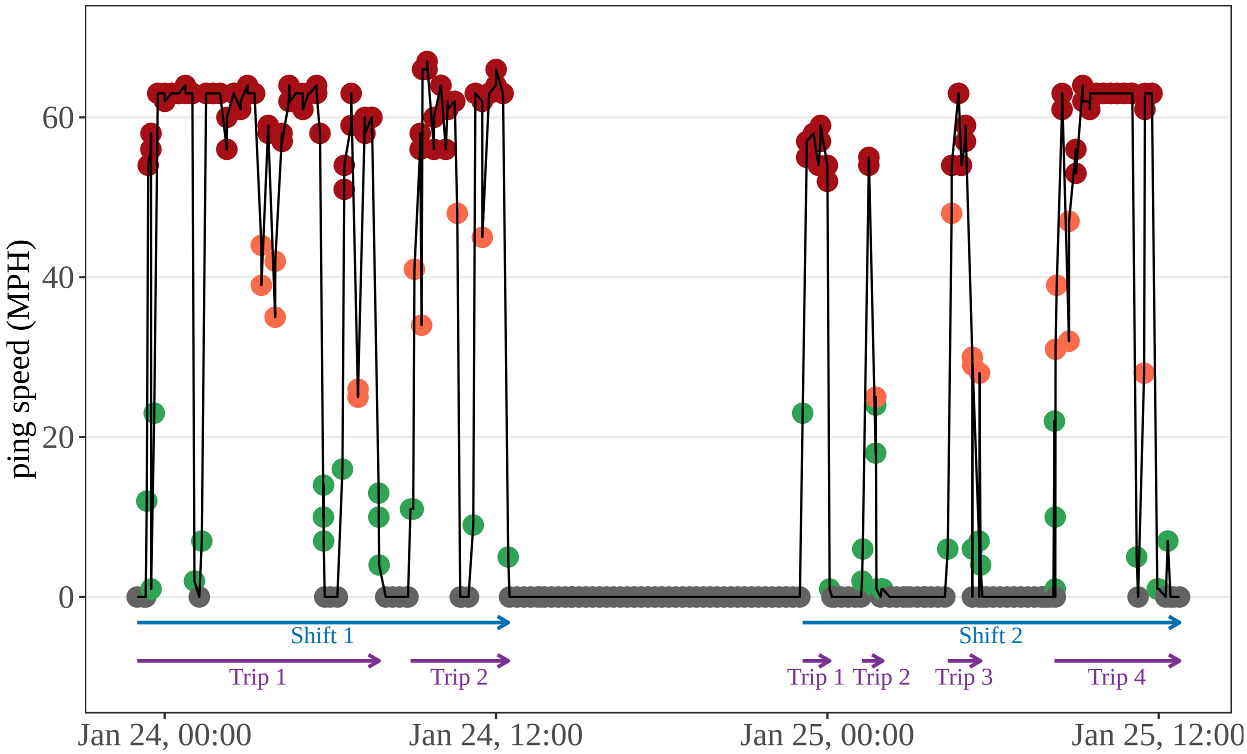

Figure 1: Naturalistic truck driving real-time ping data (points) and the aggregation process from pings to shifts and trips (arrows).

3. MODELS

3.1 Non-homogeneous Poisson Process (NHPP) and Power Law Process

We assume the time to a SCE t follows a non-homogeneous Poisson process, whose intensity function $\lambda(t)$ is non-constant. The intensity function is assumed to have the following function form:

$$\lambda_{\text{PLP}}(t) = \beta\theta^{-\beta}t^{\beta-1}, \tag{1}$$

where the shape parameter β indicates reliability improvement ($\beta < 1$), constant ($\beta = 1$), or deterioration ($\beta > 1$), and the scale parameter θ determines the rate of events. Here we assume the intensity function of a power law process because it has a flexible functional form, relatively simple statistical inference, and is a well-established model (Rigdon and

Basu, 1989, 2000).

3.2 Bayesian Hierarchical Power Law Process (PLP)

The Bayesian hierarchical power law process is parameterized as:

$$
\begin{aligned}
t_{d,s,1}, t_{d,s,2}, \cdots, t_{d,s,n_{d,s}} &\sim \mathrm{PLP}(\beta, \theta_{d,s}, \tau_{d,s}) \\
\beta &\sim \mathrm{Gamma}(1,1) \\
\log\theta_{d,s} &= \gamma_{0d} + \gamma_1 x_{d,s,1} + \gamma_2 x_{d,s,2} + \cdots + \gamma_k x_{d,s,k} \\
\gamma_{01}, \gamma_{02}, \cdots, \gamma_{0D} &\sim \text{i.i.d. } N(\mu_0, \sigma_0^2) \\
\gamma_1, \gamma_2, \cdots, \gamma_k &\sim \text{i.i.d. } N(0, 10^2) \\
\mu_0 &\sim N(0, 5^2) \\
\sigma_0 &\sim \mathrm{Gamma}(1,1),
\end{aligned}
\tag{2}
$$

where $t_{d,s,i}$ is the time to the i-th event for driver d in shift s, $\tau_{d,s} = a_{d,s,R_{d,s}}$ is the length of time of shift s (truncation time) for driver d, and $n_{d,s} = \sum_{r=1}^{n_{d,s}}$ is the number of SCEs in shift s for driver d. The likelihood function of event times generated from a PLP for driver d in shift s is given in Rigdon and Basu (2000, Section 2.3.2, Page 60):

$$
\begin{aligned}
L_{d,s}(\beta, \gamma_{0d}, \gamma | \mathbf{X}_d, \mathbf{W}_s) &= \Big(\prod_{i=1}^{n_{d,s}} \lambda_{\mathrm{PLP}}(t_{d,s,i})\Big) \exp(-\int_0^{\tau_{d,s}} \lambda_{\mathrm{PLP}}(u) du) \\
&= \begin{cases} \exp\Big(-(\tau_{d,s}/\theta_{d,s})^{\beta}\Big), & \text{if } n_{d,s} = 0, \\ \Big(\prod_{i=1}^{n_{d,s}} \beta\theta_{d,s}^{-\beta} t_{d,s,i}^{\beta-1}\Big) \exp\Big(-(\tau_{d,s}/\theta_{d,s})^{\beta}\Big), & \text{if } n_{d,s} > 0, \end{cases}
\end{aligned}
\tag{3}
$$

where $\mathbf{X}_d$ indicates driver specific variables (e.g. driver age and gender), $\mathbf{W}_s$ represents shift specific variables (e.g. precipitation and traffic), and $\theta_{d,s}$ is the function of parameters $\gamma_{0d}, \gamma_1, \gamma_2, \ldots, \gamma_k$ and variables $x_{d,s,1}, x_{d,s,2}, \ldots, x_{d,s,k}$ given in the third line of Equation 2. The full likelihood function for all drivers are:

$$L = \prod_{d=1}^{D} \prod_{s=1}^{S_d} L_{d,s}(\beta, \gamma_{0d}, \gamma | \mathbf{X}_d, \mathbf{W}_s) \tag{4}$$

where $L_{d,s}(\beta, \gamma_{0d}, \gamma | \mathbf{X}_d, \mathbf{W}_s)$ is given in Equation 3.

3.3 Bayesian Hierarchical Jump Power Law Process (JPLP)

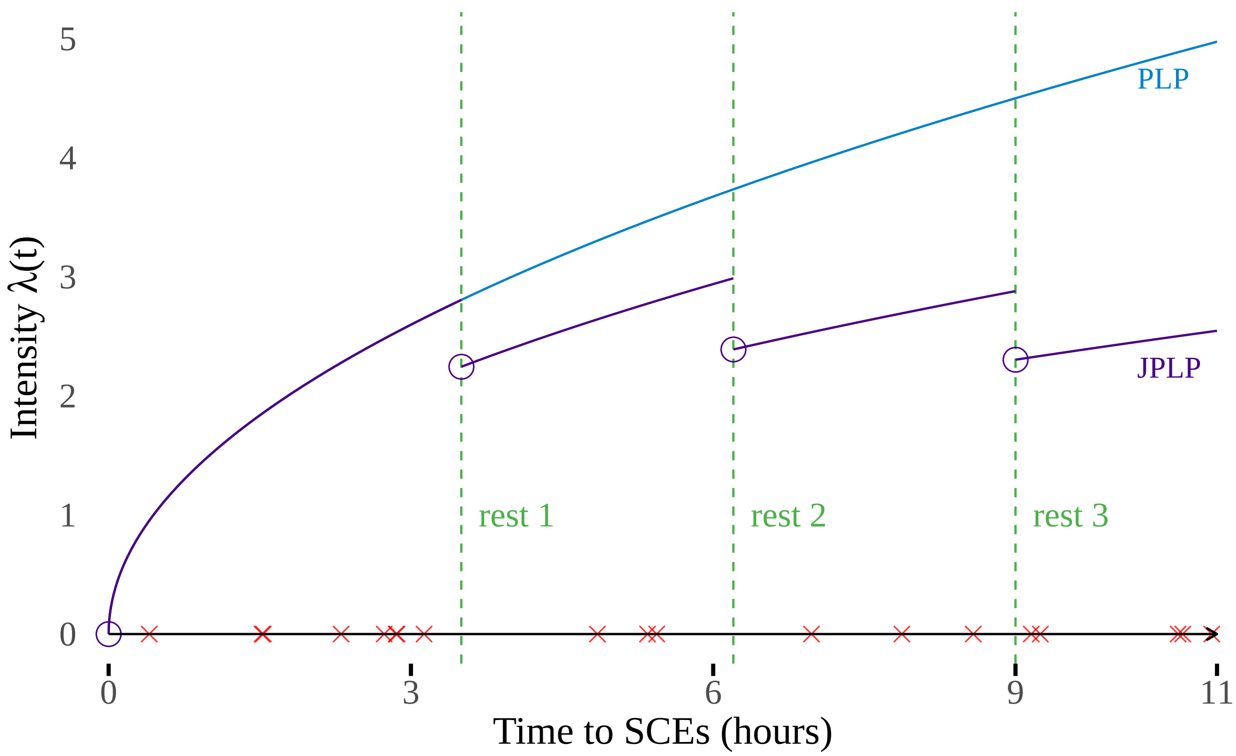

Figure 2: Simulated intensity function of PLP and JPLP. The x-axis shows time in hours since start and y-axis shows the intensity of SCEs. The red crosses mark the time to SCEs and the green vertical lines indicates the time of the rests. Parameter values for simulation: shape parameters $\beta = 1.2$, rate parameter $\theta = 2$, jump parameter $\kappa = 0.8$.

Since the Bayesian hierarchical PLP in Subsection 3.2 does not account for the rests ($r : 1, 2, \ldots, R_{d,s}$) within shifts and associated potential reliability improvement. In this subsection, we proposes a Bayesian hierarchical JPLP with an additional jump parameter κ. Figure 2 presents the intensity functions of PLP and JPLP. The PLP has a smooth curve with concave-down trend (the first segment of the two curves overlaps), while the JPLP has piecewise appearance. Whenever the driver takes a break, the intensity function of a JPLP jump back by a certain percent κ.

Our proposed JPLP has the following piecewise intensity function:

$$\lambda_{\text{JPLP}}(t|d, s, r, \beta, \gamma_{0d}, \gamma, \mathbf{X}_d, \mathbf{W}_s) = \begin{cases} \kappa^0 \lambda(t|\beta, \gamma_{0d}, \gamma, \mathbf{X}_d, \mathbf{W}_s), & 0 < t \leq a_{d,s,1}, \\ \kappa^1 \lambda(t|\beta, \gamma_{0d}, \gamma, \mathbf{X}_d, \mathbf{W}_s), & a_{d,s,1} < t \leq a_{d,s,2}, \\ \cdots & \cdots \\ \kappa^{R-1} \lambda(t|\beta, \gamma_{0d}, \gamma, \mathbf{X}_d, \mathbf{W}_s), & a_{d,s,R-1} < t \leq a_{d,s,R}, \end{cases}$$
$$= \kappa^{r-1} \lambda(t|d, s, r, \kappa, \beta, \gamma_{0d}, \gamma, \mathbf{X}_d, \mathbf{W}_s),\ a_{d,s,r-1} < t \leq a_{d,s,r}, \tag{5}$$

where the introduced parameter κ is the percent of intensity function recovery once the driver takes a break, and $a_{d,s,r}$ is the end time of trip r within shift s for driver d. By definition, the end time of the 0-th trip $a_{d,s,0} = 0$, and the end time of the last trip for the d-driver within the s-th shift $a_{d,s,R_{d,s}}$ equals the shift end time $\tau_{d,s}$. We assume that this κ is constant across drivers and shifts.

The Bayesian hierarchical JPLP model is parameterized as

$$
\begin{aligned}
t_{d,s,1}, t_{d,s,2}, \cdots, t_{d,s,n_{d,s}} &\sim \text{JPLP}(\beta, \theta_{d,s}, \tau_{d,s}, \kappa)\\
\beta &\sim \text{Gamma}(1, 1)\\
\log \theta_{d,s} &= \gamma_{0d} + \gamma_1 x_{d,s,1} + \gamma_2 x_{d,s,2} + \cdots + \gamma_k x_{d,s,k}\\
\kappa &\sim \text{Uniform}(0, 1)\\
\gamma_{01}, \gamma_{02}, \cdots, \gamma_{0D} &\sim \text{i.i.d. } N(\mu_0, \sigma_0^2)\\
\gamma_1, \gamma_2, \cdots, \gamma_k &\sim \text{i.i.d. } N(0, 10^2)\\
\mu_0 &\sim N(0, 5^2)\\
\sigma_0 &\sim \text{Gamma}(1, 1),
\end{aligned}
\tag{6}
$$

The notations are identical with those in Equation 2 except for the extra κ parameter. Similarly, the likelihood function of event times generated from a JPLP for driver d on shift s is

$$
L^*_{d,s}(\kappa, \beta, \gamma_{0d}, \gamma | \mathbf{X}_d, \mathbf{W}_s) = \Big(\prod_{i=1}^{n_{d,s}} \lambda_{\text{JPLP}}(t_{d,s,i})\Big) \exp(-\int_0^{\tau_{d,s}} \lambda_{\text{JPLP}}(u)du)
$$
$$
\begin{cases}
\exp\Big(-\int_0^{\tau_{d,s}} \lambda_{\text{JPLP}}(u)du\Big), & \text{if } n_{d,s} = 0,\\
\Big(\prod_{i=1}^{n_{d,s}} \lambda_{\text{JPLP}}(t_{d,s,i})\Big) \exp\Big(-\int_0^{\tau_{d,s}} \lambda_{\text{JPLP}}(u)du\Big), & \text{if } n_{d,s} > 0,
\end{cases}
\tag{7}
$$

where the piecewise intensity function $\lambda_{\text{JPLP}}(t_{d,s,i})$ is given in Equation 5.

However, since the intensity function depends on the trip r for the same driver d and shift s, it is hard to write out specific form of Equation 7. Instead, we can rewrite the likelihood function at trip level, where the intensity function λ_{JPLP} is fixed for driver d on shift s and trip r:

$$
\begin{aligned}
&L^*_{d,s,r}(\kappa, \beta, \gamma_{0d}, \gamma | \mathbf{X}_d, \mathbf{W}_r) = \\
&\qquad \begin{cases}
\exp\Big(-\displaystyle\int_{a_{d,s,r-1}}^{a_{d,s,r}} \lambda_{\text{JPLP}}(u)du\Big), & \text{if } n_{d,s,r} = 0, \\
\Big(\displaystyle\prod_{i=1}^{n_{d,s,r}} \lambda_{\text{JPLP}}(t_{d,s,r,i})\Big) \exp\Big(-\displaystyle\int_{a_{d,s,r-1}}^{a_{d,s,r}} \lambda_{\text{JPLP}}(u)du\Big), & \text{if } n_{d,s,r} > 0,
\end{cases}
\end{aligned}
\tag{8}
$$

where $t_{d,s,r,i}$ is the time to the i-th SCE for driver d on shift s and trip r measured from the beginning of the shift, $n_{d,s,r}$ is the number of SCEs for driver d on shift s and trip r. Compared to the PLP likelihood function given in Equation 4 where $\mathbf{W}_s$ are assumed to be a constant during an entire shift, the rewritten likelihood function for JPLP in Equation 8 assumes external covariates $\mathbf{W}_r$ vary between different trips in a shift. In this way, JPLP can account for the variability between different trips within a shift.

Therefore, the overall likelihood function for drivers $d = 1, 2, \ldots, D$, their corresponding shifts $s = 1, 2, \ldots, S_d$, and trips $r = 1, 2, \ldots, R_{d,s}$ is:

$$
L^* = \prod_{d=1}^{D} \prod_{s=1}^{S_d} \prod_{r=1}^{R_{d,s}} L^*_{d,s,r}, \tag{9}
$$

where $L^*_{d,s,r}$ is a likelihood function given in Equation 8, in which the intensity function λ_{JPLP} has a fixed functional form provided in the last line of Equation 5 for a certain driver d in a given shift s and trip r.

4. SIMULATION STUDY

4.1 Simulation setting

We conducted a simulation study to evaluate the performance of our proposed JPLP. We performed 1,000 simulations to each of the following three scenarios with different number of drivers ($D = 10, 25, 50, 75, 100$):

1. Data generated from a PLP and estimated assuming a PLP (PLP),
2. Data generated from a JPLP, but estimated assuming a PLP (JPLP),
3. Data generated from a JPLP and estimated assuming a JPLP (PLP $\leftarrow$ JPLP).

The scenario "data generated from a PLP, but estimated assuming a JPLP" is not considered here since it is not theoretically possible: if the data is generated from a PLP, then there are no breaks within shift and it is impossible to estimate the data assuming a JPLP.

Specifically, for each driver, the number of shifts is simulated from a Poisson distribution with the mean parameter of 10. We assume three predictor variables x_1, x_2, x_3 for θ ($k = 3$) and shift time $\tau_{d,s}$ are generated from the following process:

$$\begin{aligned}
x_1 &\sim \text{Normal}(1, 1^2)\\
x_2 &\sim \text{Gamma}(1, 1)\\
x_3 &\sim \text{Poisson}(2)\\
\tau_{d,s} &\sim \text{Normal}(10, 1.3^2)
\end{aligned} \tag{10}$$

The parameters and hyperparameters are assigned the following values or generated from the following process:

$$
\begin{aligned}
\mu_0 = 0.2,\ \sigma_0 = 0.5, \\
\gamma_{01}, \gamma_{02}, \cdots, \gamma_{0D} \sim \text{i.i.d. } N(\mu_0, \sigma_0^2) \\
\gamma_1 = 1,\ \gamma_2 = 0.3,\ \gamma_3 = 0.2 \\
\theta_{d,s} = \exp(\gamma_{0d} + \gamma_1 x_1 + \gamma_2 x_2 + \gamma_3 x_3) \\
\beta = 1.2,\ \kappa = 0.8.
\end{aligned}
\tag{11}
$$

After the predictor variables, shift time, and parameters are generated, the time to events $t_{d,s,1},\ t_{d,s,2},\ \cdots,\ t_{d,s,n_{d,s}}$ and $t^*_{d,s,1}, t^*_{d,s,2}, \cdots, t^*_{d,s,n_{d,s}}$ are generated from PLP and JPLP:

$$
\begin{aligned}
t_{d,s,1}, t_{d,s,2}, \cdots, t_{d,s,n_{d,s}} \sim \text{PLP}(\beta, \theta_{d,s}, \tau_{d,s}) \\
t^*_{d,s,1}, t^*_{d,s,2}, \cdots, t^*_{d,s,n_{d,s}} \sim \text{JPLP}(\beta, \theta_{d,s}, \tau_{d,s}, \kappa)
\end{aligned}
\tag{12}
$$

The parameters are then estimated using the likelihood functions given in Equation 4 and 9 with probabilistic programming language Stan in R (Carpenter et al., 2017; Stan Development Team, 2018), which uses efficient Hamiltonian Monte Carlo to sample from the posterior distributions. For each simulation, one chain is applied, with 2,000 warmup and 2,000 post-warmup iterations drawn from the posterior distributions.

4.2 Simulation results

The simulation results are shown in Table 1. For the five sets of drivers ($D = 10, 25, 50, 75, 100$) in each of the three scenarios, the mean of posterior mean estimates, mean of estimation bias $\Delta = |\hat{\mu} - \mu|$, and mean of standard error estimates for parameters $\beta, \kappa, \gamma_1, \gamma_2, \gamma_3$ and hyperparameters μ_0 and σ are calculated.

Table 1: Simulation results for PLP, JPLP, and PLP $\leftarrow$ JPLP

sim_scenaio	D	estimate	γ_1	γ_2	γ_3	β	κ	μ_0	σ_0
PLP	10	bias Δ	0.0203	0.0095	0.0067	0.0102		0.0282	0.0527
PLP	25	bias Δ	0.0066	0.0046	0.0012	0.0045		0.0015	0.0220
PLP	50	bias Δ	0.0040	0.0033	0.0005	0.0017		0.0068	0.0077
PLP	75	bias Δ	0.0034	0.0004	0.0007	0.0017		0.0026	0.0091
PLP	100	bias Δ	0.0009	0.0009	0.0003	0.0006		0.0034	0.0042
PLP	10	s.e.	0.0777	0.0696	0.0413	0.0589		0.2401	0.1722
PLP	25	s.e.	0.0459	0.0414	0.0247	0.0360		0.1392	0.0916
PLP	50	s.e.	0.0316	0.0286	0.0172	0.0254		0.0960	0.0610
PLP	75	s.e.	0.0258	0.0232	0.0139	0.0207		0.0784	0.0497
PLP	100	s.e.	0.0220	0.0198	0.0119	0.0179		0.0667	0.0420
JPLP	10	bias Δ	0.0331	0.0218	0.0092	0.0226	0.0149	0.0401	0.0696
JPLP	25	bias Δ	0.0158	0.0081	0.0039	0.0131	0.0084	0.0202	0.0219
JPLP	50	bias Δ	0.0037	0.0012	0.0039	0.0057	0.0032	0.0014	0.0111
JPLP	75	bias Δ	0.0060	0.0012	0.0006	0.0058	0.0028	0.0057	0.0097
JPLP	100	bias Δ	0.0048	0.0003	0.0008	0.0043	0.0023	0.0004	0.0041
JPLP	10	s.e.	0.0992	0.0834	0.0498	0.0828	0.0573	0.2556	0.1854
JPLP	25	s.e.	0.0586	0.0477	0.0288	0.0512	0.0360	0.1453	0.0960
JPLP	50	s.e.	0.0406	0.0334	0.0201	0.0366	0.0256	0.0999	0.0647
JPLP	75	s.e.	0.0331	0.0272	0.0164	0.0298	0.0208	0.0812	0.0519
JPLP	100	s.e.	0.0287	0.0233	0.0141	0.0258	0.0179	0.0699	0.0442
PLP $\leftarrow$ JPLP	10	bias Δ	0.1923	0.0645	0.0434	0.1843		0.1234	0.1599
PLP $\leftarrow$ JPLP	25	bias Δ	0.1769	0.0514	0.0374	0.1740		0.0866	0.1053
PLP $\leftarrow$ JPLP	50	bias Δ	0.1718	0.0531	0.0355	0.1734		0.0854	0.0977
PLP $\leftarrow$ JPLP	75	bias Δ	0.1686	0.0511	0.0346	0.1724		0.0874	0.0960
PLP $\leftarrow$ JPLP	100	bias Δ	0.1674	0.0512	0.0349	0.1713		0.0811	0.0925
PLP $\leftarrow$ JPLP	10	s.e.	0.1041	0.0946	0.0559	0.0580		0.2952	0.2078
PLP $\leftarrow$ JPLP	25	s.e.	0.0609	0.0546	0.0329	0.0354		0.1671	0.1095
PLP $\leftarrow$ JPLP	50	s.e.	0.0423	0.0383	0.0230	0.0250		0.1167	0.0743
PLP $\leftarrow$ JPLP	75	s.e.	0.0344	0.0310	0.0186	0.0204		0.0946	0.0601
PLP $\leftarrow$ JPLP	100	s.e.	0.0297	0.0266	0.0160	0.0177		0.0810	0.0514

When the models were specified correctly, the biases converges to 0 as the number of drivers increases; the standard errors converges to 0 proportional to $\sqrt{D}$ (the square root of the number of drivers), which is consistent with the central limit theorem. When the models are not specified correctly, there are still a fair amount of biases when the number of drivers increases and the speed of converging to zero is not consistent with the central limit theorem. These large-scale simulation and estimation results in Table 1 suggest that the Stan codes for PLP and JPLP are valid and can be applied for real data estimation.

5. REAL DATA ANALYSIS

A naturalistic driving data set was provided to the research team by a national commercial trucking company in North America. The data set includes 13,187,289 ping records in 2015 and 2016 generated by 496 regional drivers who move freights in regional routes that may include several surrounding states. Each ping records the date and time (year, month, day, hour, minute, and second), latitude and longitude (specific to five decimal places), driver identification number, and speed at that time point. Historic weather data were queried from the DarkSky Application Programming Interface (API), which allows us to query historic real-time and hour-by-hour nationwide historic weather conditions according to latitude, longitude, date, and time (The Dark Sky Company, LLC, 2020).

The hierarchical Bayesian PLP and JPLP were performance using the probabilistic programming language Stan in R (Carpenter et al., 2017; Stan Development Team, 2018). The convergence of Hamiltonian Monte Carlo was assessed using Gelman-Rubin diagnostic $\hat{R}$ (Gelman et al., 1992), effective sample size (ESS), and trace plots.

6. DISCUSSION

In this article, we proposed a Bayesian hierarchical NHPP with PLP intensity function and an innovative Bayesian hierarchical JPLP to model naturalistic truck driving data. Our motivation comes from more popular use of naturalistic driving data sets in the recent decade and real-life truck driving characteristics of multiple trips nested within shifts. The proposed JPLP accounts for the characteristics of multiple rests within a shift among commercial truck drivers. The intensity functions, parameterization forms, and likelihood functions are specified separately. Simulation studies showed the consistency of the Bayesian hierarchical estimation if the models are specified correctly, as well as the persistent bias when the models are not specified correctly.

The SCEs generated from naturalistic truck driving data are different from previous reliability problems and models in two aspects. Most previous studies either assume minimal repair or perfect repair (also known as renewal process). A minimal repair assumes the unit after the repair is exactly the same as it is before the repair (for example, our first proposed NHPP with a PLP intensity function), while a perfect repair assumes the unit is a completely new unit after repair (Rigdon and Basu, 2000). In the scenario of truck driving, although it is reasonable to assume that the drivers experience a perfect repair when they take a break of longer than 10 hours, researchers would not expect the reliability to be perfectly repaired or minimally repaired during a short break of around half an hour. Instead, a partial repair is a more proper assumption here.

On the other hand, even though some studies proposed partial repair reliability models such as the modulated PLP (Lakey and Rigdon, 1993; Black and Rigdon, 1996), none of these previously proposed models fit for naturalistic truck driving data since the repairs are independent of the SCEs. These previous models are based on high-tech devices such as aircraft manufacturing, which need immediate repair once a failure is detected. However in the case of naturalistic truck driving data sets, SCEs such as hard brakes and headway do not seriously influence the driving and drivers generally will keep driving even if SCEs occur.

The repair (breaks) can be considered as independent of SCEs. Although our case study is based on naturalistic truck driving data sets, the JPLP can applied to any type of drivers who drive for a long distance with at least one break.

Our work can be extended in several aspects. First, the assumption of proportion reliability jump may not hold. Other proper assumptions include reliability jumping for a fixed-amount jump or jumping dependent on the length of the rest. Additionally, in our proposed JPLP, the length of breaks within shifts are ignored to simply the parameterization and likelihood function. In truck transportation practice, longer breaks certainly have larger effects on reliability jump, hence the relationship between reliability jump and the length of breaks can have more complex functional forms, so it would be of interest to test different forms reliability change as a function of length of break.

SUPPLEMENTARY MATERIALS

Since the real truck driving data provided by our industry partner are confidential and cannot be made publicly accessible, we provided a simulated data set that is similar to our real data in data structure but has fewer drivers to be completed within a reasonable amount of time. The online supplementary materials contain the R code to simulate PLP and JPLP data, explanation on the data structure, and Stan and R code for Bayesian hierarchical PLP and JPLP estimation. The supplementary materials can be access at `https://github.com/caimiao0714/JPLP_sim`.

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