Modeling Recurrent Safety-critical Events among Commercial Truck Drivers: A Bayesian Hierarchical Jump Power Law Process

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Abstract

JUST COMMENT ON TARGET JOURNALS, NOT REAL ABSTRACT. The target of this manuscript will be statistical journal, so the writing will involve quite a bit of math and simulation. The target journals I have in mind include: a) Journal of the American Statistical Association (JASA, impact factor: 3.412, rank: 5/123), b) Journal of Computational and Graphical Statistics (impact factor: 1.882, rank: 28/123), c) Statistics in Medicine (impact factor: 1.847, rank: 29/123), d) Journal of Applied Statistics (impact factor: 0.767, rank: 84/123). This will be written once the paper is complete.

1. INTRODUCTION

The methods used in trucking safety is usually predictive model, while reliability models that account for multiple events are less used.

Traditional data use retrospective crash reports collected at certain road segments.In studies using these data, there are at most one crash in one shift and recurrent data models do not make sense in these scenarios. However, there can be multiple unsafe driving events in shift generated in NDS, which motivate the application of innovative recurrent event models in transportation safety modeling.

Literature review of recurrent event analyses application in transportation safety science. The most common recurrent event analysis model is probably Poisson regression, which assumes the events in a time interval is generated by a homogeneous Poisson process (Kim et al., 2013). (Chen and Guo, 2016; Li et al., 2017; Liu and Guo, 2019; Liu et al., 2019; Guo et al., 2019; Li et al., 2018).

2. DESCRIPTION OF DATA

[Figure 1 about here.]

Here are the notations for the data:

- Driver $d : 1, 2, \ldots, D$,
- Shift $s, 1, 2, \ldots, S_d$,
- Trip $r : 1, 2, \ldots, R_{d,s}$,
- SCE $i : 1, 2, \ldots, I_{d,s}$.
- $t_{d,s,i}$: time to the i-th SCE for driver d measured from the beginning of the s-shift,
- $n_{d,s,r}$: the number of SCEs for trip r within shift s for driver d,
- $a_{d,s,r}$: the end time of trip r within shift s for driver d.

3. MODELS

3.1 Non-homogeneous Poisson Process (NHPP) and Power Law Process

We assume the time to SCEs t follows a non-homogeneous Poisson process, whose intensity function $\lambda(t)$ is non-constant. The intensity function is assumed to have the following function form

$$\lambda_{\text{PLP}}(t) = \beta\theta^{-\beta}t^{\beta-1}, \tag{1}$$

where the shape parameter β indicates reliability improvement ($\beta < 1$), constant ($\beta = 1$), or deterioration ($\beta > 1$), and the scale parameter θ determines the rate of events. Here we assume the intensity function of a power law process because it has a flexible functional form, relatively simple statistical inference, and is a well-established model (Rigdon and Basu, 1989, 2000).

3.2 Bayesian Hierarchical Power Law Process (PLP)

The Bayesian hierarchical power law process is parameterized as:

$$
\begin{aligned}
t_{d,s,1}, t_{d,s,2}, \cdots, t_{d,s,n_{d,s}} &\sim \text{PLP}(\beta, \theta_{d,s}, \tau_{d,s}) \\
\beta &\sim \text{Gamma}(1, 1) \\
\log \theta_{d,s} &= \gamma_{0d} + \gamma_1 x_{d,s,1} + \gamma_2 x_{d,s,2} + \cdots + \gamma_k x_{d,s,k} \\
\gamma_{01}, \gamma_{02}, \cdots, \gamma_{0D} &\sim \text{i.i.d. } N(\mu_0, \sigma_0^2) \\
\gamma_1, \gamma_2, \cdots, \gamma_k &\sim \text{i.i.d. } N(0, 10^2) \\
\mu_0 &\sim N(0, 5^2) \\
\sigma_0 &\sim \text{Gamma}(1, 1),
\end{aligned}
\tag{2}
$$

where $t_{d,s,i}$ is the time to the i-th event for driver d in shift s, $\tau_{d,s} = a_{d,s,R_{d,s}}$ is the length of time of shift s (truncation time) for driver d, and $n_{d,s} = \sum_{r=1}^{n_{d,s}}$ is the number of SCEs in shift s for driver d. The likelihood function of event times generated from a PLP for driver d in shift s is given in Rigdon and Basu (2000, Section 2.3.2, Page 60):

$$
\begin{aligned}
L_{d,s}(\beta, \gamma_{0d}, \gamma | \mathbf{X}_d, \mathbf{W}_s) &= \Big(\prod_{i=1}^{n_{d,s}} \lambda_{\text{PLP}}(t_{d,s,i})\Big) \exp(-\int_0^{\tau_{d,s}} \lambda(u) du) \\
&= \begin{cases} \exp\Big(-(\tau_{d,s}/\theta_{d,s})^{\beta}\Big), & \text{if } n_{d,s} = 0, \\ \Big(\prod_{i=1}^{n_{d,s}} \beta \theta_{d,s}^{-\beta} t_{d,s,i}^{\beta-1}\Big) \exp\Big(-(\tau_{d,s}/\theta_{d,s})^{\beta}\Big), & \text{if } n_{d,s} > 0, \end{cases}
\end{aligned}
\tag{3}
$$

where $\mathbf{X}_d$ indicates driver specific variables (e.g. driver age and gender), $\mathbf{W}_s$ represents shift specific variables (e.g. precipitation and traffic), and $\theta_{d,s}$ is the function of parameters $\gamma_{0d}, \gamma_1, \gamma_2, \ldots, \gamma_k$ and variables $x_{d,s,1}, x_{d,s,2}, \ldots, x_{d,s,k}$ given in the third line of Equation 2. The full likelihood function for all drivers are:

$$
L = \prod_{d=1}^{D} \prod_{s=1}^{S_d} L_{d,s}(\beta, \gamma_{0d}, \gamma | \mathbf{X}_d, \mathbf{W}_s) \tag{4}
$$

where $L_{d,s}(\beta, \gamma_{0d}, \gamma | \mathbf{X}_d, \mathbf{W}_s)$ is given in Equation 3.

3.3 Bayesian Hierarchical Jump Power Law Process (JPLP)

Since the Bayesian hierarchical PLP in Subsection 3.2 does not account for the rests $(r : 1, 2, \ldots, R_{d,s})$ within shifts and associated potential reliability improvement. In this subsection, we proposes a Bayesian hierarchical JPLP, with the following piecewise intensity function:

$$
\begin{aligned}
\lambda_{\text{JPLP}}(t | d, s, r, \beta, \gamma_{0,d}, \gamma, \mathbf{X}_d, \mathbf{W}_s) &= \begin{cases} \kappa^0 \lambda(t | \beta, \gamma_{0,d}, \gamma, \mathbf{X}_d, \mathbf{W}_s), & 0 < t \leq a_{d,s,1}, \\ \kappa^1 \lambda(t | \beta, \gamma_{0,d}, \gamma, \mathbf{X}_d, \mathbf{W}_s), & a_{d,s,1} < t \leq a_{d,s,2}, \\ \cdots & \cdots \\ \kappa^{R-1} \lambda(t | \beta, \gamma_{0,d}, \gamma, \mathbf{X}_d, \mathbf{W}_s), & a_{d,s,R-1} < t \leq a_{d,s,R}, \end{cases} \\
&= \kappa^{r-1} \lambda(t | d, s, r, \kappa, \beta, \gamma_{0,d}, \gamma, \mathbf{X}_d, \mathbf{W}_s),\ a_{d,s,r-1} < t \leq a_{d,s,r},
\end{aligned}
\tag{5}
$$

where the introduced parameter κ is the percent of intensity function recovery once the driver takes a break, and $a_{d,s,r}$ is the end time of trip r within shift s for driver d. By definition, the end time of the 0-th trip $a_{d,s,0} = 0$, and the end time of the last trip for the d-driver within the s-th shift $a_{d,s,R_{d,s}}$ equals the shift end time $\tau_{d,s}$. We assume that this κ is constant across drivers and shifts.

The Bayesian hierarchical JPLP model is parameterized as

$$
\begin{aligned}
t_{d,s,1}, t_{d,s,2}, \cdots, t_{d,s,n_{d,s}} &\sim \text{JPLP}(\beta, \theta_{d,s}, \tau_{d,s}, \kappa) \\
\beta &\sim \text{Gamma}(1, 1) \\
\log \theta_{d,s} &= \gamma_{0d} + \gamma_1 x_{d,s,1} + \gamma_2 x_{d,s,2} + \cdots + \gamma_k x_{d,s,k} \\
\kappa &\sim \text{Uniform}(0, 1) \\
\gamma_{01}, \gamma_{02}, \cdots, \gamma_{0D} &\sim \text{i.i.d. } N(\mu_0, \sigma_0^2) \\
\gamma_1, \gamma_2, \cdots, \gamma_k &\sim \text{i.i.d. } N(0, 10^2) \\
\mu_0 &\sim N(0, 5^2) \\
\sigma_0 &\sim \text{Gamma}(1, 1),
\end{aligned}
\tag{6}
$$

The notations are identical with those in Equation 2 except for the extra κ parameter. Similarly, the likelihood function of event times generated from a JPLP for driver d on shift s is

$$
L^*_{d,s}(\kappa, \beta, \gamma_{0d}, \gamma | \mathbf{X}_d, \mathbf{W}_s) = \\
\begin{cases}
\exp\Big(-\int_0^{\tau_{d,s}} \lambda_{\text{JPLP}}(u)du\Big), & \text{if } n_{d,s} = 0, \\
\Big(\prod_{i=1}^{n_{d,s}} \lambda_{\text{JPLP}}(t_{d,s,i})\Big) \exp\Big(-\int_0^{\tau_{d,s}} \lambda_{\text{JPLP}}(u)du\Big), & \text{if } n_{d,s} > 0,
\end{cases}
\tag{7}
$$

where the piecewise intensity function $\lambda_{\text{JPLP}}(t_{d,s,i})$ is given in Equation 5.

However, since the intensity function depends on the trip r for the same driver d and shift s, it is hard to write out specific form of Equation 7. Instead, we can rewrite the likelihood function at trip level, where the intensity function λ_{JPLP} is fixed for driver d on shift s and trip r:

$$
L^*_{d,s,r}(\kappa, \beta, \gamma_{0d}, \gamma | \mathbf{X}_d, \mathbf{W}_r) = \\
\begin{cases}
\exp\Big(-\int_{a_{d,s,r-1}}^{a_{d,s,r}} \lambda_{\text{JPLP}}(u)du\Big), & \text{if } n_{d,s,r} = 0, \\
\Big(\prod_{i=1}^{n_{d,s,r}} \lambda_{\text{JPLP}}(t_{d,s,r,i})\Big) \exp\Big(-\int_{a_{d,s,r-1}}^{a_{d,s,r}} \lambda_{\text{JPLP}}(u)du\Big), & \text{if } n_{d,s,r} > 0,
\end{cases}
\tag{8}
$$

where $t_{d,s,r,i}$ is the time to the i-th SCE for driver d on shift s and trip r measured from the beginning of the shift, $n_{d,s,r}$ is the number of SCEs for driver d on shift s and trip r. Compared to the PLP likelihood function given in Equation 4 where $\mathbf{W}_s$ are assumed to be a constant during an entire shift, the rewritten likelihood function for JPLP in Equation 8 assumes external covariates $\mathbf{W}_r$ vary between different trips in a shift. In this way, JPLP can account for the variability between different trips within a shift.

Therefore, the overall likelihood function for drivers $d = 1, 2, \ldots, D$, their corresponding shifts $s = 1, 2, \ldots, S_d$, and trips $r = 1, 2, \ldots, R_{d,s}$ is:

$$
L^* = \prod_{d=1}^{D} \prod_{s=1}^{S_d} \prod_{r=1}^{R_{d,s}} L^*_{d,s,r}, \tag{9}
$$

where $L^*_{d,s,r}$ is a likelihood function given in Equation 8, in which the intensity function λ_{JPLP} has a fixed functional form provided in the last line of Equation 5 for a certain driver d in a given shift s and trip r.

[Figure 2 about here.]

4. SIMULATION STUDY

4.1 Simulation setting

We conducted a simulation study to evaluate the performance of our proposed JPLP. We performed 1,000 simulations to each of following three scenarios with different number of drivers ($D = 10, 25, 50, 75, 100$):

1. Data generated from a PLP and estimated assuming a PLP (PLP),
2. Data generated from a JPLP, but estimated assuming a PLP (JPLP),
3. Data generated from a JPLP and estimated assuming a JPLP (PLP $\leftarrow$ JPLP).

The scenario "data generated from a PLP, but estimated assuming a JPLP" is not considered here since it is not theoretically possible: if the data is generated from a PLP, then there are no breaks within shift and it is impossible to estimate the data assuming a JPLP.

Specifically, for each driver, the number of shifts is simulated from a Poisson distribution with the mean parameter of 10. We assume three predictor variables x_1, x_2, x_3 for θ ($k = 3$) and shift time $\tau_{d,s}$ are generated from the following process:

$$\begin{aligned} x_1 &\sim \text{Normal}(1, 1^2) \\ x_2 &\sim \text{Gamma}(1, 1) \\ x_3 &\sim \text{Poisson}(2) \\ \tau_{d,s} &\sim \text{Normal}(10, 1.3^2) \end{aligned} \tag{10}$$

The parameters and hyperparameters are assigned the following values or generated from the following process:

$$\begin{aligned} \mu_0 = 0.2,&\ \sigma_0 = 0.5, \\ \gamma_{01}, \gamma_{02}, \cdots, \gamma_{0D} &\sim \text{i.i.d. } N(\mu_0, \sigma_0^2) \\ \gamma_1 = 1,&\ \gamma_2 = 0.3,\ \gamma_3 = 0.2 \\ \theta_{d,s} &= \exp(\gamma_{0d} + \gamma_1 x_1 + \gamma_2 x_2 + \gamma_3 x_3) \\ \beta = 1.2,&\ \kappa = 0.8. \end{aligned} \tag{11}$$

After the predictor variables, shift time, and parameters are generated, the time to events $t_{d,s,1}$, $t_{d,s,2}$, $\cdots$, $t_{d,s,n_{d,s}}$ and $t^*_{d,s,1}, t^*_{d,s,2}, \cdots, t^*_{d,s,n_{d,s}}$ are generated from PLP and JPLP:

$$\begin{aligned} t_{d,s,1}, t_{d,s,2}, \cdots, t_{d,s,n_{d,s}} &\sim \text{PLP}(\beta, \theta_{d,s}, \tau_{d,s}) \\ t^*_{d,s,1}, t^*_{d,s,2}, \cdots, t^*_{d,s,n_{d,s}} &\sim \text{JPLP}(\beta, \theta_{d,s}, \tau_{d,s}, \kappa) \end{aligned} \tag{12}$$

The parameters are then inferred using the likelihood functions given in Equation 4 and 9 with probabilistic programming language Stan in R (Carpenter et al., 2017; Stan Development Team, 2018), which uses efficient Hamiltonian Monte Carlo to sample from the posterior distributions. For each simulation, one chain is applied, with 2,000 warmup and 2,000 post-warmup iterations drawn from the posterior distributions.

4.2 Simulation results

The simulation results are shown in the following Table. For the five sets of drivers ($D = 10, 25, 50, 75, 100$) in each of the three scenarios, the mean of posterior mean estimates, mean of estimation bias $\Delta = |\hat{\mu} - \mu|$, and mean of standard error estimates for parameters $\beta, \kappa, \gamma_1, \gamma_2, \gamma_3$ and hyperparameters μ_0 and σ are calculated.

[Table 1 about here.]

5. DATA ANALYSES

The hierarchical Bayesian PLP and JPLP were performance using the probabilistic programming language Stan in R (Carpenter et al., 2017; Stan Development Team, 2018).

6. DISCUSSION

In this article we have proposed a Bayesian hierarchical jump power law process, which accounts for the characteristics of multiple rests within a shift among commercial truck drivers. The simulation results shows XXXX. In the application to a 496 truck driver NDS dataset, the results suggest that XXX.

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List of Figures

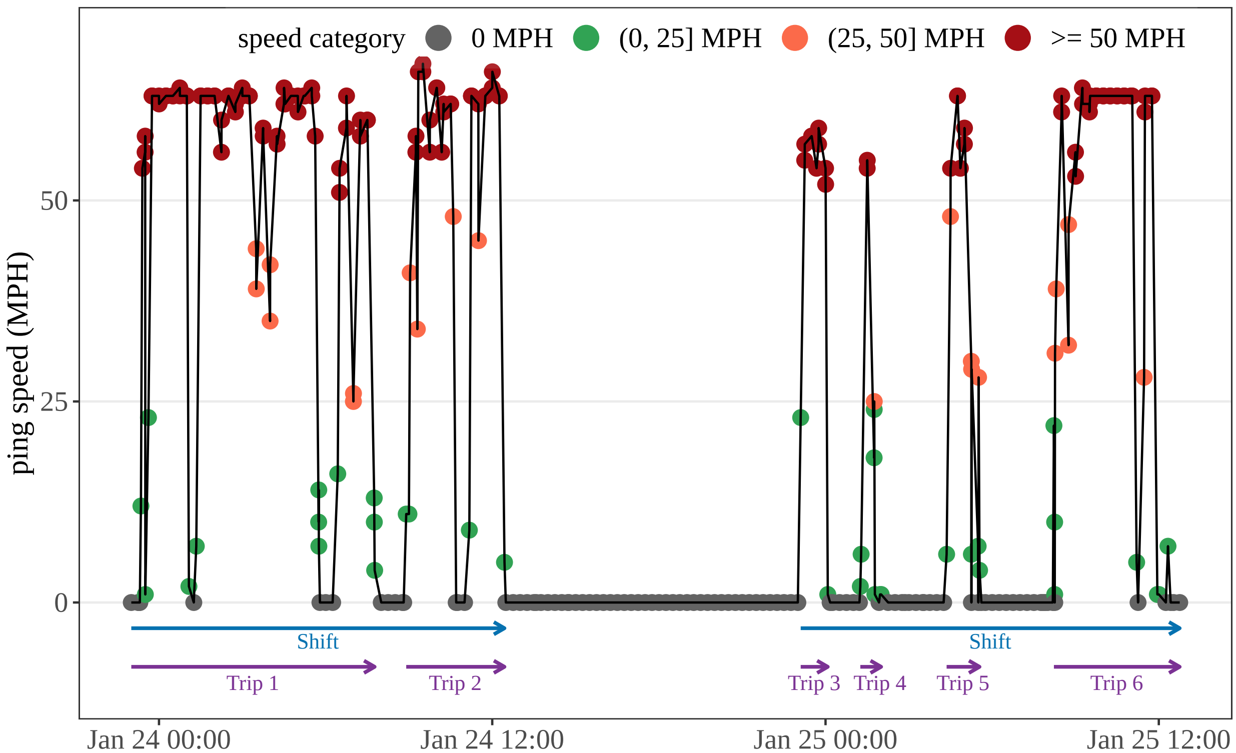

Figure 1: NDS real-time ping data (colored points) and the aggregation process from pings to shifts and trips (colored arrows).

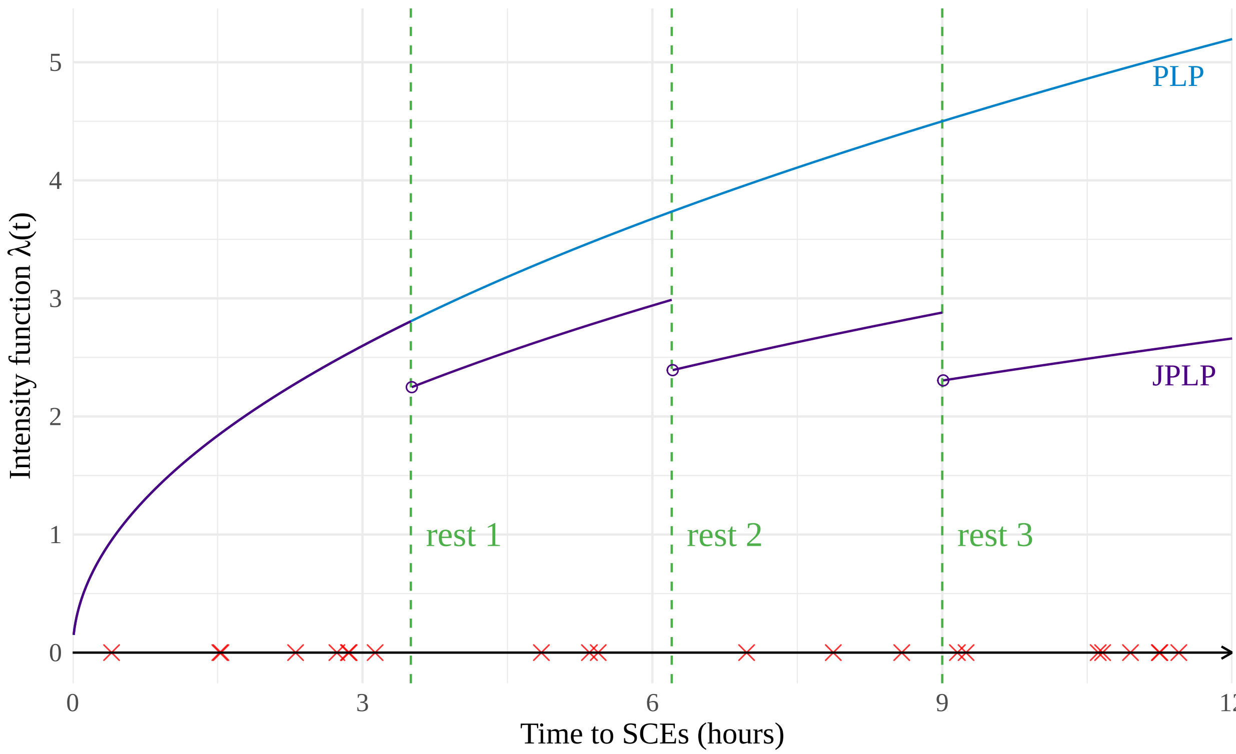

Figure 2: Simulated intensity function of PLP and JPLP. The x-axis shows time in hours since start and y-axis shows the intensity of SCEs. The red crosses mark the time to SCEs and the green vertical lines indicates the time of the rests. Parameter values for simulation: shape parameters $\beta = 1.2$, rate parameter $\theta = 2$, jump parameter $\kappa = 0.8$.

List of Tables

Table 1: Simulation results for PLP, JPLP, and PLP ← JPLP

sim_scenaio	D	estimate	γ_1	γ_2	γ_3	β	κ	μ_0	σ_0
PLP	10	mean $\hat{\mu}$	1.0203	0.3095	0.2067	1.1898		0.1718	0.5527
PLP	25	mean $\hat{\mu}$	1.0066	0.3046	0.2012	1.1955		0.1985	0.5220
PLP	50	mean $\hat{\mu}$	1.0040	0.3033	0.2005	1.1983		0.1932	0.5077
PLP	75	mean $\hat{\mu}$	1.0034	0.3004	0.2007	1.1983		0.1974	0.5091
PLP	100	mean $\hat{\mu}$	1.0009	0.3009	0.2003	1.1994		0.1966	0.5042
PLP	10	bias Δ	0.0203	0.0095	0.0067	0.0102		0.0282	0.0527
PLP	25	bias Δ	0.0066	0.0046	0.0012	0.0045		0.0015	0.0220
PLP	50	bias Δ	0.0040	0.0033	0.0005	0.0017		0.0068	0.0077
PLP	75	bias Δ	0.0034	0.0004	0.0007	0.0017		0.0026	0.0091
PLP	100	bias Δ	0.0009	0.0009	0.0003	0.0006		0.0034	0.0042
PLP	10	s.e.	0.0777	0.0696	0.0413	0.0589		0.2401	0.1722
PLP	25	s.e.	0.0459	0.0414	0.0247	0.0360		0.1392	0.0916
PLP	50	s.e.	0.0316	0.0286	0.0172	0.0254		0.0960	0.0610
PLP	75	s.e.	0.0258	0.0232	0.0139	0.0207		0.0784	0.0497
PLP	100	s.e.	0.0220	0.0198	0.0119	0.0179		0.0667	0.0420
JPLP	10	mean $\hat{\mu}$	1.0331	0.3218	0.2092	1.1774	0.8149	0.1599	0.5696
JPLP	25	mean $\hat{\mu}$	1.0158	0.3081	0.2039	1.1869	0.8084	0.1798	0.5219
JPLP	50	mean $\hat{\mu}$	1.0037	0.3012	0.2039	1.1943	0.8032	0.2014	0.5111
JPLP	75	mean $\hat{\mu}$	1.0060	0.3012	0.2006	1.1942	0.8028	0.2057	0.5097
JPLP	100	mean $\hat{\mu}$	1.0048	0.3003	0.2008	1.1957	0.8023	0.1996	0.5041
JPLP	10	bias Δ	0.0331	0.0218	0.0092	0.0226	0.0149	0.0401	0.0696
JPLP	25	bias Δ	0.0158	0.0081	0.0039	0.0131	0.0084	0.0202	0.0219
JPLP	50	bias Δ	0.0037	0.0012	0.0039	0.0057	0.0032	0.0014	0.0111
JPLP	75	bias Δ	0.0060	0.0012	0.0006	0.0058	0.0028	0.0057	0.0097
JPLP	100	bias Δ	0.0048	0.0003	0.0008	0.0043	0.0023	0.0004	0.0041
JPLP	10	s.e.	0.0992	0.0834	0.0498	0.0828	0.0573	0.2556	0.1854
JPLP	25	s.e.	0.0586	0.0477	0.0288	0.0512	0.0360	0.1453	0.0960
JPLP	50	s.e.	0.0406	0.0334	0.0201	0.0366	0.0256	0.0999	0.0647
JPLP	75	s.e.	0.0331	0.0272	0.0164	0.0298	0.0208	0.0812	0.0519
JPLP	100	s.e.	0.0287	0.0233	0.0141	0.0258	0.0179	0.0699	0.0442
PLP ← JPLP	10	mean $\hat{\mu}$	1.1923	0.3645	0.2434	1.0157		0.0766	0.6599
PLP ← JPLP	25	mean $\hat{\mu}$	1.1769	0.3514	0.2374	1.0260		0.1134	0.6053
PLP ← JPLP	50	mean $\hat{\mu}$	1.1718	0.3531	0.2355	1.0266		0.1146	0.5977
PLP ← JPLP	75	mean $\hat{\mu}$	1.1686	0.3511	0.2346	1.0276		0.1126	0.5960
PLP ← JPLP	100	mean $\hat{\mu}$	1.1674	0.3512	0.2349	1.0287		0.1189	0.5925
PLP ← JPLP	10	bias Δ	0.1923	0.0645	0.0434	0.1843		0.1234	0.1599
PLP ← JPLP	25	bias Δ	0.1769	0.0514	0.0374	0.1740		0.0866	0.1053
PLP ← JPLP	50	bias Δ	0.1718	0.0531	0.0355	0.1734		0.0854	0.0977
PLP ← JPLP	75	bias Δ	0.1686	0.0511	0.0346	0.1724		0.0874	0.0960
PLP ← JPLP	100	bias Δ	0.1674	0.0512	0.0349	0.1713		0.0811	0.0925
PLP ← JPLP	10	s.e.	0.1041	0.0946	0.0559	0.0580		0.2952	0.2078
PLP ← JPLP	25	s.e.	0.0609	0.0546	0.0329	0.0354		0.1671	0.1095
PLP ← JPLP	50	s.e.	0.0423	0.0383	0.0230	0.0250		0.1167	0.0743
PLP ← JPLP	75	s.e.	0.0344	0.0310	0.0186	0.0204		0.0946	0.0601
PLP ← JPLP	100	s.e.	0.0297	0.0266	0.0160	0.0177		0.0810	0.0514