

Modeling Recurrent Safety-critical Events among Commercial Truck Drivers: A Bayesian Hierarchical Jump Power Law Process

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Abstract

Many transportation safety studies aim to predict crashes based on aggregated road segment data. As an increasing number of naturalistic driving studies have been initiated in the recent decade, safety-critical events (SCEs) such as hard brakes are widely used as a proxy measure of driving risk. Different from real crashes, multiple SCEs can occur in a driving shift and they do not interrupt the state of driving. Motivated by a growing need of analyzing recurrent SCEs and the feature that multiple trips are nested within a shift for commercial truck drivers, we proposed  a Bayesian hierarchical non-homogeneous Poisson process with power law process intensity function and an innovative Bayesian hierarchical jump power law process. We specified the parameterization, intensity functions, and likelihood functions for the two models and presented the estimation results for correctly and wrongly specified models based on simulated data. The two models are then applied to a naturalistic driving data of over 13 million driving records and 8,407 SCEs generated by 496 commercial truck drivers. Supplementary materials including simulated data and parameter estimation for reproducing the work, are available as an online supplement.

Keywords: trucking; safety-critical events; reliability; power law process

State what conclusions we draw from the study.

1. INTRODUCTION

19 Traditional trucking safety studies apply classification or count data models to predict the
20 occurrence or number of crashes in certain road segments for a fixed amount of time based on
21 policy reports data (Lord and Mannering, 2010; Savolainen et al., 2011; Mannering and Bhat,
22 2014). These retrospective crash prediction studies are inherently limited in the sample size
23 of crashes, selection of control groups, and undercount of less severe crashes. Large-scale
24 naturalistic driving studies that continuously record real-world driving data using unobtru-
25 sive instruments have recently been proposed as an innovative method for transportation
26 safety research (Guo, 2019). Instead of studying real crashes, naturalistic driving studies use
27 kinematic safety-critical events (SCEs) such as hard brakes as a proxy measure of driving
28 risk. Although SCEs can still be analyzed using classification and count data models (Kim
29 et al., 2013), these events are inherently different from crashes: there can be at most one
30 crash in a working shift and the drivers have to stop once a crash occurs; in contrast, there
31 can be multiple SCEs in a working shift and the drivers can continue driving even if SCEs
32 occur.

33 Commercial truck drivers are on the road for an extended period of time and commonly
34 face fatigue problems. Investigating the driving reliability using SCEs can contribute to
35 understanding fatigue problems among commercial truck drivers and optimize route and
36 shift scheduling. Previous studies tend to use traditional classification or count data models
37 (Kim et al., 2013; Chen et al., 2016), or change-point models for naturalistic driving data
38 sets (Chen and Guo, 2016; Li et al., 2017, 2018; Liu and Guo, 2019; Liu et al., 2019; Guo
39 et al., 2019). These models are limited in two aspects. First, retrospective reports are



Say clearly and concisely what the main objective of this paper is.

40 collecting crashes at road segments and non-crashes are selected randomly to match the
41 crashes, which resembles a case-control study design. By comparison, naturalistic driving
42 data sets follow the drivers or vehicles for a certain amount of time. Therefore, the sampling
43 units in naturalistic driving data sets are drivers instead of road segments, which resembles
44 a prospective cohort study design (Mehdizadeh et al., 2020). Since the same drivers tend to
45 have similar driving patterns, it is logical to apply hierarchical models that account for driver-
46 level effects to naturalistic driving data sets. Second, commercial truck drivers, especially
47 long-haul truck drivers, must take at least one break in long-distance transporting as required
48 by Federal Motor Carrier Safety Administration (2017). Researcher would assume that some
safety
49 level of fatigue alleviation and ~~reliability~~ change occur at these short breaks.

50 In consideration of these gaps, we first introduced ~~a~~ a Bayesian hierarchical non-homogeneous
51 Poisson process with the power law process (PLP) intensity function to model SCEs within
52 shifts. This model accounts for driver-level unobserved heterogeneity by specifying driver-
53 level random intercepts for the rate parameter in PLP. On the other hand, since the Federal
54 Motor Carrier Safety Administration (2017) regulates that drivers who transports ~~property~~
55 and delivers ~~materials~~ materials must a) be on duty for no more than 14 hours; b) drive for no more
56 than 11 hours, and c) take ~~a~~ at least 30-minute break by the eighth hour of ~~on~~ duty, a property-
57 carrying truck driver must have at least one break if they are on road for more than eight
58 hours. To account for this feature of multiple trips and breaks nested within a shift among
59 commercial truck drivers, we then propose a Bayesian hierarchical jump power law process
60 (JPLP) to take potential reliability changes at the time of rests into consideration.

61 The structure of this article is as follows. In Section 2, we define our terminology and
62 notation for shifts, trips, and events for naturalistic driving data generated by commercial

*How does this differ
from (b) above?*

63 truck drivers. In Section 3, we specify our proposed PLP and JPLP models, their inten-
 64 sity functions and likelihood functions. In Section 4, simulation studies are conducted to
 65 demonstrate the validity of our code and the consequences if the models are not specified
 66 correctly. In section 5, we present the results of real data analyses for 496 commercial truck
 67 drivers using PLP and JPLP. Strengths, possible limitations, and future research directions
 68 are discussed in Section 6. A simulated data set, description on data structure, and Stan and
 69 R code for Bayesian PLP and JPLP estimation are provided in the supplementary material.

70

2. TERMINOLOGY AND NOTATION

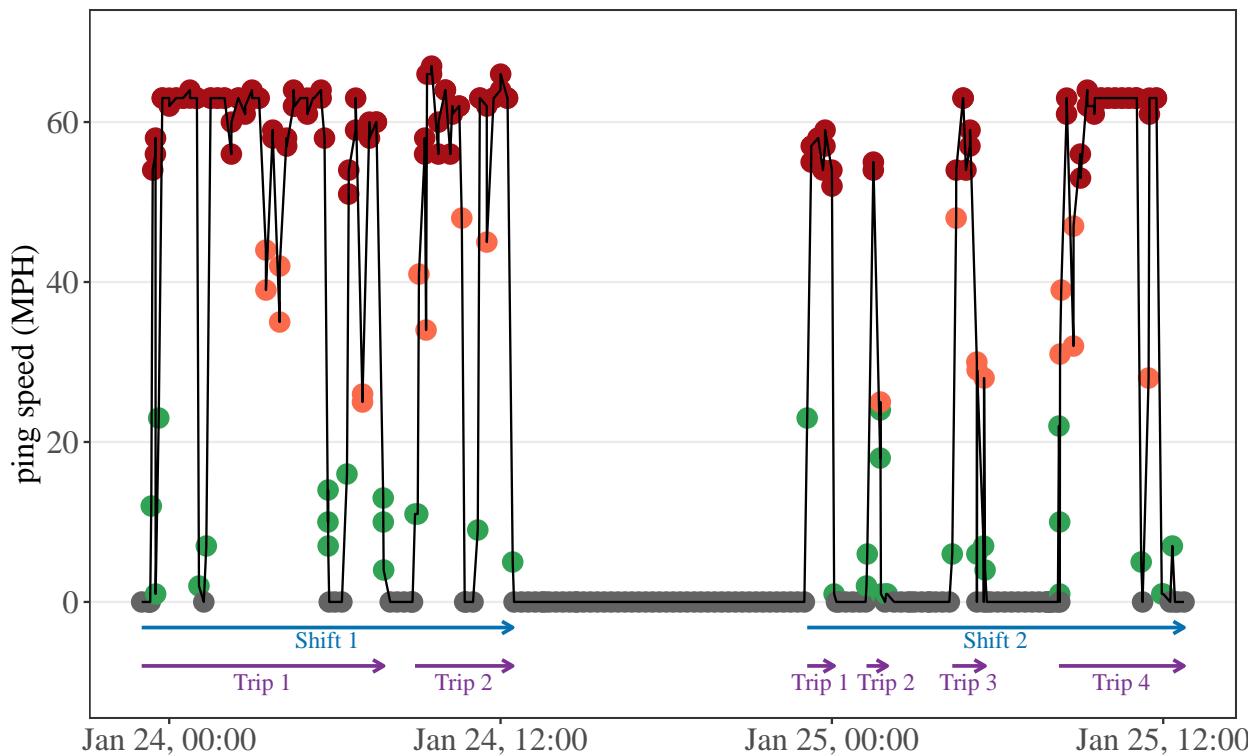


Figure 1: Naturalistic truck driving real-time ping data (points) and the aggregation process from pings to shifts and trips (arrows).

71 Figure 1 presents a time series plot of speed data for a sample truck driver (including two
 72 shifts and six trips nested within the shifts) and arrows suggesting shifts and trips. We use

Say either $d=1, 2, \dots, D$ or $d \in \{1, 2, \dots, D\}$

Similarly, here

index

74 $d = \{1, 2, \dots, D\}$ as the notation for different drivers. A shift $s = \{1, 2, \dots, S_d\}$ is on-duty

74 periods with no breaks longer than 10 hours for driver d . As the Federal Motor Carrier Safety

75 Administration (2017) requires, a shift must be no more than 14 hours with no more than

76 11 hours of driving, and this leads to the phenomena that multiple trips $r = \{1, 2, \dots, R_{d,s}\}$

77 are separated by breaks longer than 30 minutes but less than 10 hours for each driver d and

78 shift s .

79 SCEs can occur any time in the trips whenever preset kinematic thresholds are triggered

80 by the driver. We use $i = \{1, 2, \dots, I_{d,s}\}$ as notations for the i -th SCE for driver d in shift

81 s . For each SCE, $t_{d,s,i}$ is the time to the i -th SCE for driver d measured from the beginning

82 of the s -shift and the rest times between trips are excluded from calculation. $n_{d,s,r}$ is the

83 number of SCEs for trip r within shift s for driver d . $a_{d,s,r}$ is the end time of trip r within

84 shift s for driver d .

85

3. MODELS

3.1 Non-homogeneous Poisson Process (NHPP) and Power Law Process

We assume the time to a SCE follows a non-homogeneous Poisson process, whose intensity function $\lambda(t)$ is non-constant. The intensity function is assumed to have the following function form

$$\lambda_{PLP}(t) = \beta \theta^{-\beta} t^{\beta}, \quad \frac{\beta}{\theta} \left(\frac{t}{\theta}\right)^{\beta-1} \quad (1)$$

87 where the shape parameter β indicates reliability improvement ($\beta < 1$), constant ($\beta = 1$),

88 or deterioration ($\beta > 1$), and the scale parameter θ determines the rate of events. Here

89 we assume the intensity function of a power law process because it has a flexible functional

This makes it easier to see
that θ is a scale parameter.

90 form, relatively simple statistical inference, and is a well-established model (Rigdon and
91 Basu, 1989, 2000).

92 3.2 Bayesian Hierarchical Power Law Process (PLP)

The Bayesian hierarchical power law process is parameterized as:

$$t_{d,s,1}, t_{d,s,2}, \dots, t_{d,s,n_{d,s}} \sim \text{PLP}(\beta, \theta_{d,s}, \tau_{d,s})$$

$$\beta \sim \text{Gamma}(1, 1)$$

$$\log \theta_{d,s} = \gamma_{0d} + \gamma_1 x_{d,s,1} + \gamma_2 x_{d,s,2} + \dots + \gamma_k x_{d,s,k}$$

$$\gamma_{01}, \gamma_{02}, \dots, \gamma_{0D} \sim \text{i.i.d. } N(\mu_0, \sigma_0^2) \quad (2)$$

$$\gamma_1, \gamma_2, \dots, \gamma_k \sim \text{i.i.d. } N(0, 10^2)$$

$$\mu_0 \sim N(0, 5^2)$$

$$\sigma_0 \sim \text{Gamma}(1, 1),$$

where $t_{d,s,i}$ is the time to the i -th event for driver d in shift s , $\tau_{d,s} = a_{d,s,R_{d,s}}$ is the length of time of shift s (truncation time) for driver d , and $n_{d,s} = \sum_{r=1}^{n_{d,s}}$ is the number of SCEs in

shift s for driver d . The likelihood function of event times generated from a PLP for driver

Rigdon and Basu (2000, Section 2.3.2, Page 60) as

$$\begin{aligned} L_{d,s}(\beta, \gamma_{0d}, \gamma | \mathbf{X}_d, \mathbf{W}_s) &= \left(\prod_{i=1}^{n_{d,s}} \lambda_{\text{PLP}}(t_{d,s,i}) \right) \exp\left(- \int_0^{\tau_{d,s}} \lambda_{\text{PLP}}(u) du\right) \\ &= \begin{cases} \exp\left(-(\tau_{d,s}/\theta_{d,s})^\beta\right), & \text{if } n_{d,s} = 0, \\ \left(\prod_{i=1}^{n_{d,s}} \beta \theta_{d,s}^{-\beta} t_{d,s,i}^{\beta-1} \right) \exp\left(-(\tau_{d,s}/\theta_{d,s})^\beta\right), & \text{if } n_{d,s} > 0, \end{cases} \quad (3) \end{aligned}$$

where \mathbf{X}_d indicates driver specific variables (e.g. driver age and gender), \mathbf{W}_s represents shift specific variables (e.g. precipitation and traffic), and $\theta_{d,s}$ is the function of parameters $\gamma_{0d}, \gamma_1, \gamma_2, \dots, \gamma_k$ and variables $x_{d,s,1}, x_{d,s,2}, \dots, x_{d,s,k}$ given in the third line of Equation 2.

The full likelihood function for all drivers are:

$$L = \prod_{d=1}^D \prod_{s=1}^{S_d} L_{d,s}(\beta, \gamma_{0d}, \gamma | \mathbf{X}_d, \mathbf{W}_s)$$

93 where $L_{d,s}(\beta, \gamma_{0d}, \gamma | \mathbf{X}_d, \mathbf{W}_s)$ is given in Equation(1).

Models like this are often used in the literature for repairable systems reliability. Here, a "failure" can be thought of as the occurrence of a SCE.

94 3.3 Bayesian Hierarchical Jump Power Law Process (JPLP)

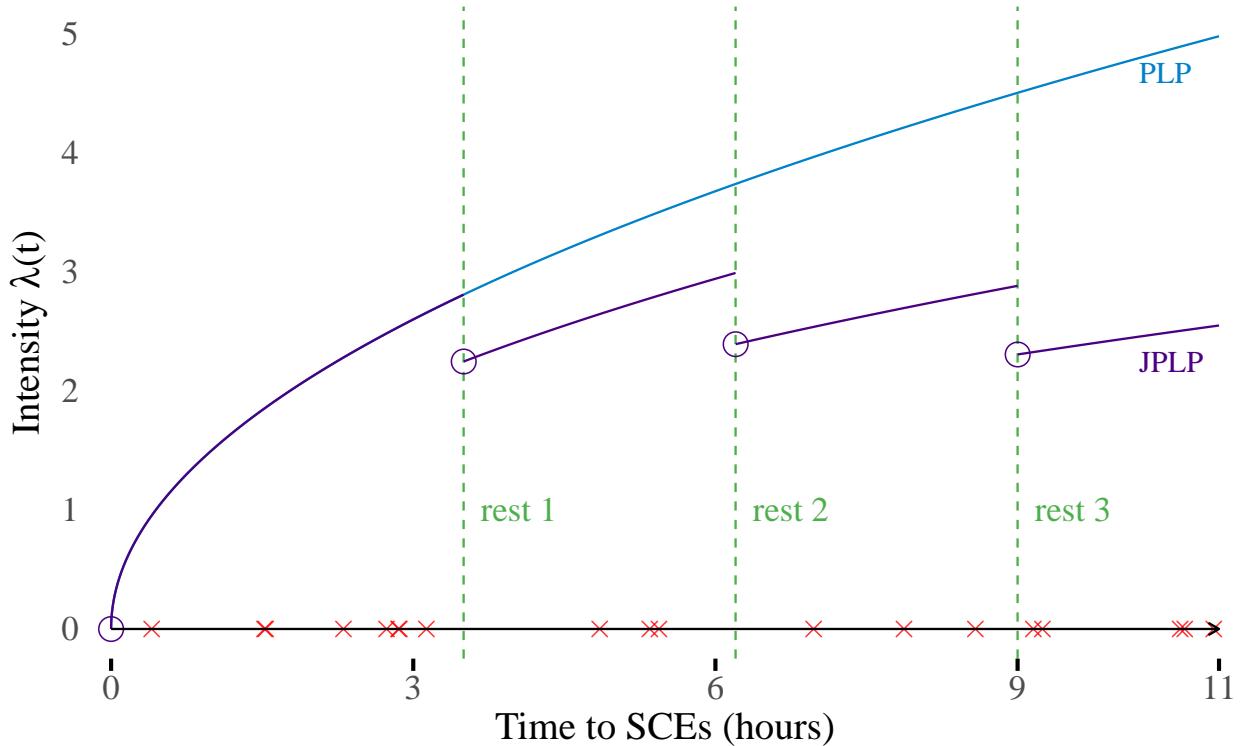


Figure 2: Simulated intensity function of PLP and JPLP. The x -axis shows time in hours since start and y -axis shows the intensity of SCEs. The crosses mark the time to SCEs and the vertical dotted lines indicates the time of the rests. Parameters for this simulated data set: shape parameters $\beta = 1.2$, rate parameter $\theta = 2$, jump parameter $\kappa = 0.8$.

95 Since the Bayesian hierarchical PLP in Subsection 3.2 does not account for the rests
 96 ($r = \{1, 2, \dots, R_{d,s}\}$) within shifts and associated potential ~~reliability~~ improvement. In this
 97 subsection, we propose a Bayesian hierarchical JPLP with an additional jump parameter κ .
 98 Figure 2 presents the intensity functions of PLP and JPLP. The intensity function of PLP
 99 is a smooth curve with concave-down trend when $\beta > 1$ (the first segment of the two curves
 100 overlaps), while the JPLP has a piecewise appearance. Whenever the driver takes a break,
 101 the intensity function of a JPLP jump by a certain amount κ .

Our proposed JPLP has the following piecewise intensity function:

$$\begin{aligned}
 \lambda_{\text{JPLP}}(t|d, s, r, \beta, \gamma_{0d}, \gamma, \mathbf{X}_d, \mathbf{W}_s) = \\
 \left\{ \begin{array}{ll} \kappa^0 \lambda(t|\beta, \gamma_{0d}, \gamma, \mathbf{X}_d, \mathbf{W}_s), & 0 < t \leq a_{d,s,1}, \\ \vdots & \text{...} \\ \kappa^1 \lambda(t|\beta, \gamma_{0d}, \gamma, \mathbf{X}_d, \mathbf{W}_s), & a_{d,s,1} < t \leq a_{d,s,2}, \\ \text{...} & \text{...} \\ \kappa^{R-1} \lambda(t|\beta, \gamma_{0d}, \gamma, \mathbf{X}_d, \mathbf{W}_s), & a_{d,s,R-1} < t \leq a_{d,s,R}, \\ = \kappa^{r-1} \lambda(t|d, s, r, \kappa, \beta, \gamma_{0d}, \gamma, \mathbf{X}_d, \mathbf{W}_s), & a_{d,s,r-1} < t \leq a_{d,s,r}, \end{array} \right. \tag{5}
 \end{aligned}$$

\vdots

102 where the introduced parameter κ is the amount of intensity function change once the driver
 103 takes a break, and $a_{d,s,r}$ is the end time of trip r within shift s for driver d . By definition,
 104 the end time of the ~~0-th~~ trip $a_{d,s,0} = 0$, and the end time of the last trip for the d -driver
 105 within the s -th shift $a_{d,s,R_{d,s}}$ equals the shift end time $\tau_{d,s}$. We assume that this κ is constant
 106 across drivers and shifts.

be changed by a factor that ranges from 0 to 2.

The Bayesian hierarchical JPLP model is parameterized as:

$$t_{d,s,1}, t_{d,s,2}, \dots, t_{d,s,n_{d,s}} \sim \text{JPLP}(\beta, \theta_{d,s}, \tau_{d,s}, \kappa)$$

$$\beta \sim \text{Gamma}(1, 1)$$

$$\log \theta_{d,s} = \gamma_{0d} + \gamma_1 x_{d,s,1} + \gamma_2 x_{d,s,2} + \dots + \gamma_k x_{d,s,k}$$

$$\kappa \sim \text{Uniform}(0, 2)$$

$$\gamma_{01}, \gamma_{02}, \dots, \gamma_{0D} \sim \text{i.i.d. } N(\mu_0, \sigma_0^2)$$

$$\gamma_1, \gamma_2, \dots, \gamma_k \sim \text{i.i.d. } N(0, 10^2)$$

$$\mu_0 \sim N(0, 5^2)$$

$$\sigma_0 \sim \text{Gamma}(1, 1),$$

The notations are identical with those in Equation 2 except for the extra κ parameter.

Here we set the prior distribution for κ as uniform(0, 2), which is assuming that the intensity

function can jump down to 0 or increase by up to 100% at the time of tests. Similarly, the

likelihood function of event times generated from a JPLP for driver d on shift s is:

$$L_{d,s}^*(\kappa, \beta, \gamma_{0d}, \gamma | \mathbf{X}_d, \mathbf{W}_s) = \left(\prod_{i=1}^{n_{d,s}} \lambda_{\text{JPLP}}(t_{d,s,i}) \right) \exp\left(- \int_0^{\tau_{d,s}} \lambda_{\text{JPLP}}(u) du\right)$$

$$\begin{cases} \exp\left(- \int_0^{\tau_{d,s}} \lambda_{\text{JPLP}}(u) du\right), & \text{if } n_{d,s} = 0, \\ \left(\prod_{i=1}^{n_{d,s}} \lambda_{\text{JPLP}}(t_{d,s,i}) \right) \exp\left(- \int_0^{\tau_{d,s}} \lambda_{\text{JPLP}}(u) du\right), & \text{if } n_{d,s} > 0, \end{cases} \quad (7)$$

¹⁰⁷ where the piecewise intensity function $\lambda_{\text{JPLP}}(t_{d,s,i})$ is given in Equation (5)

parentheses

However, since the intensity function depends on the trip r for the same driver d on

shift s , it is hard to write out the specific form of Equation (7). Instead, we can rewrite the likelihood function at trip level, where the intensity function λ_{JPLP} is fixed for driver d on shift s and trip r :

$$L_{d,s,r}^*(\kappa, \beta, \gamma_0 d, \gamma | \mathbf{X}_d, \mathbf{W}_r) = \begin{cases} \exp \left(- \int_{a_{d,s,r-1}}^{a_{d,s,r}} \lambda_{JPLP}(u) du \right), & \text{if } n_{d,s,r} = 0, \\ \left(\prod_{i=1}^{n_{d,s,r}} \lambda_{JPLP}(t_{d,s,r,i}) \right) \exp \left(- \int_{a_{d,s,r-1}}^{a_{d,s,r}} \lambda_{JPLP}(u) du \right), & \text{if } n_{d,s,r} > 0, \end{cases} \quad (8)$$

Equation numbers
always go inside
parentheses.

108 where $t_{d,s,r,i}$ is the time to the i -th SCE for driver d on shift s and trip r measured from
109 the beginning of the shift, $n_{d,s,r}$ is the number of SCEs for driver d on shift s and trip r .

110 Compared to the PLP likelihood function given in Equation (4) where \mathbf{W}_s are assumed to be
111 fixed numbers during an entire shift, the rewritten likelihood function for JPLP in Equation

112 (8) assumes external covariates \mathbf{W}_r vary between different trips in a shift. In this way, JPLP
113 can account for the variability between different trips within a shift.

Therefore, the overall likelihood function for drivers $d = 1, 2, \dots, D$, their corresponding shifts $s = \{1, 2, \dots, S_d\}$, and trips $r = \{1, 2, \dots, R_{d,s}\}$ is

$$L^* = \prod_{d=1}^D \prod_{s=1}^{S_d} \prod_{r=1}^{R_{d,s}} L_{d,s,r}^*, \quad (9)$$

114 where $L_{d,s,r}^*$ is a likelihood function given in Equation (8) in which the intensity function λ_{JPLP}
115 has a fixed functional form provided in the last line of Equation (5) for a certain driver d in a
116 given shift s and trip r .

Interchange sections
4 and 5.

118 4.1 Simulation setting

119 We conducted a simulation study to evaluate the performance of our proposed NHPP and
 120 JPLP under different ~~simulation~~^{for} scenarios. We performed 1,000 simulations ~~to~~^{for} each of the
 121 following three scenarios with different number of drivers $D = \{10, 25, 50, 75, 100\}$:

- 122 1. Data generated from a PLP and estimated assuming a PLP (PLP),
- 123 2. Data generated from a JPLP, but estimated assuming a PLP (JPLP),
- 124 3. Data generated from a JPLP and estimated assuming a JPLP (PLP \leftarrow JPLP).

125 The scenario “data generated from a PLP, but estimated assuming a JPLP” is not con-
 126 sidered here since it is not theoretically possible: if the data ~~is~~^{one} generated from a NHPP with

127 PLP intensity function, then there are no breaks within shifts and it is pointless to estimate
 128 the data assuming a JPLP. *I think this is possible. Think of the PLP as a JPLP with $K=1$. There might be breaks, but the breaks have no effect.*

Specifically, for each driver, the number of shifts is simulated from a Poisson distribution
 with the mean parameter of 10. We assume there are three predictor variables x_1, x_2, x_3 for
 θ ($k = 3$). x_1, x_2, x_3 and the shift time $\tau_{d,s}$ are generated from the following process:

Think of a simpler example, a regression model.

True model is $y = \beta_0 + \beta_1 x + \epsilon$.

What if we assume

$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \epsilon$?

IOW, what if we try to overfit the model?

$$\begin{aligned} x_1 &\sim \text{Normal}(1, 1^2) \\ x_2 &\sim \text{Gamma}(1, 1) \\ x_3 &\sim \text{Poisson}(2) \\ \tau_{d,s} &\sim \text{Normal}(10, 1.3^2) \end{aligned} \tag{10}$$

The parameters and hyperparameters are assigned the following values or generated from

the following process:

$$\mu_0 = 0.2, \sigma_0 = 0.5,$$

$$\gamma_{01}, \gamma_{02}, \dots, \gamma_{0D} \sim \text{i.i.d. } N(\mu_0, \sigma_0^2)$$

$$\gamma_1 = 1, \gamma_2 = 0.3, \gamma_3 = 0.2 \quad (11)$$

$$\theta_{d,s} = \exp(\gamma_{0d} + \gamma_1 x_1 + \gamma_2 x_2 + \gamma_3 x_3)$$

$$\beta = 1.2, \kappa = 0.8.$$

either the

After the predictor variables, shift time, and parameters are generated, the time to events

~~$t_{a,s,1}, t_{a,s,2}, \dots, t_{a,s,n_{d,s}}$ and $t_{d,s,1}^*, t_{d,s,2}^*, \dots, t_{d,s,n_{d,s}}^*$~~ are generated from PLP and JPLP:

$$\begin{aligned} & t_{a,s,1}, t_{a,s,2}, \dots, t_{a,s,n_{d,s}} \sim \text{PLP}(\beta, \theta_{a,s}, \tau_{a,s}) \\ & t_{d,s,1}^*, t_{d,s,2}^*, \dots, t_{d,s,n_{d,s}}^* \sim \text{JPLP}(\beta, \theta_{d,s}, \tau_{d,s}, \kappa) \end{aligned} \quad (12)$$

129 The parameters are then estimated using the likelihood functions given in Equation 10

130 and 9 using the probabilistic programming language Stan in R (Carpenter et al., 2017; Stan

131 Development Team, 2018), which uses efficient Hamiltonian Monte Carlo to sample from the

132 posterior distributions. For each simulation, one chain is applied, with 2,000 warmup and

133 2,000 post-warmup iterations drawn from the posterior distributions.

134 4.2 Simulation results

135 The simulation results are shown in Table 1. For the five sets of drivers $D = \{10, 25, 50, 75, 100\}$

136 in each of the three scenarios, the mean of posterior mean estimates, mean of estimation

137 bias $\Delta = |\hat{\mu} - \mu|$, and mean of standard error estimates for parameters $\beta, \kappa, \gamma_1, \gamma_2, \gamma_3$ and

*why did you take the absolute value?
It would be interesting to know the
direction (pos. or neg.) of bias. Can
you recover this info without having to
redo all the simulations?*

¹³⁸ hyperparameters μ_0 and σ are calculated.

Table 1: Simulation results for PLP, JPLP, and $\text{PLP} \leftarrow \text{JPLP}$

sim_scenao	D	estimate	γ_1	γ_2	γ_3	β	κ	μ_0	σ_0
PLP	10	bias Δ	0.0203	0.0095	0.0067	0.0102		0.0282	0.0527
PLP	25	bias Δ	0.0066	0.0046	0.0012	0.0045		0.0015	0.0220
PLP	50	bias Δ	0.0040	0.0033	0.0005	0.0017		0.0068	0.0077
PLP	75	bias Δ	0.0034	0.0004	0.0007	0.0017		0.0026	0.0091
PLP	100	bias Δ	0.0009	0.0009	0.0003	0.0006		0.0034	0.0042
PLP	10	s.e.	0.0777	0.0696	0.0413	0.0589		0.2401	0.1722
PLP	25	s.e.	0.0459	0.0414	0.0247	0.0360		0.1392	0.0916
PLP	50	s.e.	0.0316	0.0286	0.0172	0.0254		0.0960	0.0610
PLP	75	s.e.	0.0258	0.0232	0.0139	0.0207		0.0784	0.0497
PLP	100	s.e.	0.0220	0.0198	0.0119	0.0179		0.0667	0.0420
JPLP	10	bias Δ	0.0331	0.0218	0.0092	0.0226	0.0149	0.0401	0.0696
JPLP	25	bias Δ	0.0158	0.0081	0.0039	0.0131	0.0084	0.0202	0.0219
JPLP	50	bias Δ	0.0037	0.0012	0.0039	0.0057	0.0032	0.0014	0.0111
JPLP	75	bias Δ	0.0060	0.0012	0.0006	0.0058	0.0028	0.0057	0.0097
JPLP	100	bias Δ	0.0048	0.0003	0.0008	0.0043	0.0023	0.0004	0.0041
JPLP	10	s.e.	0.0992	0.0834	0.0498	0.0828	0.0573	0.2556	0.1854
JPLP	25	s.e.	0.0586	0.0477	0.0288	0.0512	0.0360	0.1453	0.0960
JPLP	50	s.e.	0.0406	0.0334	0.0201	0.0366	0.0256	0.0999	0.0647
JPLP	75	s.e.	0.0331	0.0272	0.0164	0.0298	0.0208	0.0812	0.0519
JPLP	100	s.e.	0.0287	0.0233	0.0141	0.0258	0.0179	0.0699	0.0442
$\text{PLP} \leftarrow \text{JPLP}$	10	bias Δ	0.1923	0.0645	0.0434	0.1843		0.1234	0.1599
$\text{PLP} \leftarrow \text{JPLP}$	25	bias Δ	0.1769	0.0514	0.0374	0.1740		0.0866	0.1053
$\text{PLP} \leftarrow \text{JPLP}$	50	bias Δ	0.1718	0.0531	0.0355	0.1734		0.0854	0.0977
$\text{PLP} \leftarrow \text{JPLP}$	75	bias Δ	0.1686	0.0511	0.0346	0.1724		0.0874	0.0960
$\text{PLP} \leftarrow \text{JPLP}$	100	bias Δ	0.1674	0.0512	0.0349	0.1713		0.0811	0.0925
$\text{PLP} \leftarrow \text{JPLP}$	10	s.e.	0.1041	0.0946	0.0559	0.0580		0.2952	0.2078
$\text{PLP} \leftarrow \text{JPLP}$	25	s.e.	0.0609	0.0546	0.0329	0.0354		0.1671	0.1095
$\text{PLP} \leftarrow \text{JPLP}$	50	s.e.	0.0423	0.0383	0.0230	0.0250		0.1167	0.0743
$\text{PLP} \leftarrow \text{JPLP}$	75	s.e.	0.0344	0.0310	0.0186	0.0204		0.0946	0.0601
$\text{PLP} \leftarrow \text{JPLP}$	100	s.e.	0.0297	0.0266	0.0160	0.0177		0.0810	0.0514

seems to

¹³⁹ When the models were specified correctly, the biases converge to 0 as the number of

¹⁴⁰ drivers increases; the standard errors converge to 0 roughly proportional to the square root

¹⁴¹ of the number of drivers (\sqrt{D}), which is consistent with the central limit theorem. When the

¹⁴² models are not specified correctly, there are still a fair amount of biases when the number

143 of drivers increases and the speed of converging to zero is not consistent with either the
144 other two correctly specified simulation scenarios or the central limit theorem. The Gelman-
145 Rubin diagnostic \hat{R} were all lower than 1.1 and no low effective sample size (ESS) issues
146 were reported in Stan, suggesting that steady posterior distributions were reached while
147 estimating ~~for~~ the simulated data sets.
the parameters of
^

148 5. REAL DATA ANALYSIS

149 5.1 Data description

150 A naturalistic truck driving data set was provided ~~to the research team by a national com-~~
151 mercial trucking company in North America. The data set includes 496 regional drivers
152 who move freights in regional routes that may include several surrounding states. A total
153 of 13,187,289 ping records were generated between April 2015 and March 2016, with a total
154 traveled distance of 20,042,519 miles in 465,641 hours (average speed 43 miles per hour).
155 Each ping records the date and time (year, month, day, hour, minute, and second), latitude
156 and longitude (specific to five decimal places), driver identification number, and speed at
157 that time point. These pings were then aggregated into 64,860 shifts and 180,408 trips.

158 *In addition*
159 ~~On the other hand~~, 8,407 kinematic SCEs were recorded independent of the pings, includ-

160 ing 3941 (46.9%) headway, 3576 (42.5%) hard brakes, 869 (10.3%) collision mitigation, and
161 21 (0.2%) rolling stability. Historic weather data, including precipitation probability, precip-
162 itation intensity, and wind speed, were queried from the DarkSky Application Programming
163 Interface, which provides historic real-time and hour-by-hour nationwide historic weather
164 conditions for specific latitude-longitude-date-time combinations (The Dark Sky Company,
LLC, 2020). The weather data were then merged back to pings data and aggregated to shift-

*Describe these critical events in
greater detail, including thresholds,
if possible.*

the T set

New paragraph

165 and trip-level by taking the mean

Maybe just do
collision mitigation.

166 5.2 Real data analysis results

167 We applied the hierarchical Bayesian PLP and JPLP models to this data as specified in

168 Equations (2) and (6). Since we have four types of SCEs, we then applied the JPLP to the four

169 different types of SCEs separately. Collision mitigation and rolling stability were combined

170 as one type because the later J is very scare and will yield very unstable estimates if

171 modeled alone. Samples of the posterior distributions were drawn using the probabilistic

172 programming language Stan in R (Carpenter et al., 2017; Stan Development Team, 2018).

173 The convergence of Hamiltonian Monte Carlo was checked using Gelman-Rubin diagnostic

174 statistics \hat{R} (Gelman et al., 1992), effective sample size (ESS), and trace plots.

175 Table 2 presents the posterior mean, 95% credible interval (CI), Gelman-Rubin diagnostic

176 statistics \hat{R} , and ESS for the sample 496 regional drivers using PLP and JPLP. In both the

177 PLP and JPLP models, the posterior means of the shape parameters β are less than one

178 and the 95% credible intervals exclude one, indicating SCEs occur in the early stages of the

179 shifts. In JPLP, the reliability jump parameter κ was close to 1, suggesting that within

180 shift, rests have very minor effects on the reliability of SCEs. Figure 3 shows the random

181 intercepts γ_{0d} in both the PLP and JPLP present a fair amount of variability across different

182 drivers. Besides, the random intercepts γ_{0d} are on average larger in JPLP than those in PLP

183 models, while variability of random intercepts is similar in the two models. These patterns

184 are consistent with the parameter estimates of μ_0 and σ_0 in Table 2. All the Gelman-

185 Rubin diagnostic statistics \hat{R} are less than 1.1 and the ESSs are greater than 1,000. The

given Supplementary Material

186 trace plots of important variables in the Appendix figure 4 indicate that all four chains for

Figure 3, which gives histograms for estimates of the estimates of the random intercepts, indicates that there is considerable variability across drivers

- 187 the parameters are well mixed. All these evidence suggests that a steady state posterior
 188 distribution have been reached for the two models.

Table 2: Posterior mean, 95% credible interval, Rhat, and effective sample size (ESS) of PLP and JPLP models for 496 commercial truck drivers

Parameters	Power law process				Jump power law process			
	mean	95% CI	\hat{R}	ESS	mean	95% CI	\hat{R}	ESS
β	0.968	(0.948, 0.988)	1.000	6,500	0.962	(0.940, 0.985)	1.001	3,798
κ					1.020	(0.995, 1.045)	1.000	5,400
μ_0	3.038	(2.397, 3.688)	1.001	2,979	3.490	(2.899, 4.091)	1.001	3,079
σ_0	0.974	(0.897, 1.058)	1.000	9,581	0.982	(0.905, 1.066)	1.000	9,050
Age	0.003	(-0.005, 0.012)	1.001	2,250	0.004	(-0.005, 0.012)	1.001	2,566
Race: black	-0.113	(-0.329, 0.103)	1.002	1,951	-0.130	(-0.342, 0.087)	1.001	2,277
Race: other	-0.343	(-0.707, 0.021)	1.001	2,833	-0.361	(-0.729, 0.010)	1.001	3,334
Gender: female	-0.071	(-0.441, 0.300)	1.001	3,069	-0.071	(-0.435, 0.296)	1.001	4,162
Mean speed	0.019	(0.016, 0.023)	1.000	20,229	0.015	(0.013, 0.018)	1.000	19,827
Speed variation	0.026	(0.017, 0.034)	1.000	24,825	0.017	(0.013, 0.022)	1.000	13,127
Preci. intensity	-3.608	(-6.181, -0.935)	1.000	22,025	-2.136	(-3.785, -0.368)	1.000	24,397
Preci. prob.	0.397	(0.168, 0.628)	1.000	21,416	0.121	(-0.050, 0.296)	1.000	25,329
Wind speed	0.018	(0.008, 0.029)	1.000	32,980	0.010	(0.001, 0.018)	1.000	33,093

95% CI: 95% credible interval; ESS: effective sample size;

PLP: power law process; JPLP: jump power law process;

Preci. intensity: precipitation intensity; Precip. prob.: precipitation probability.

Maybe we should ignore rolling stability.

- 189 We further estimated the JPLP models for different types of SCEs (headway, hard brakes,
 190 and collision mitigation and rolling stability), and the results are presented in Table 3.
 191 Headway and hard brake are similar: they have very close posterior mean and 95% credible
 intervals for parameters β and κ , although the hyperparameters for random intercepts are
 192 quite different. The results that $\beta < 1$ and $\kappa > 1$ suggest that headway and hard brake tend
 193 to occur in the early stages of driving shifts, and taking short breaks will slightly increase
 the intensity of these two events. In contrast, collision mitigation and rolling stability show a
 194 different pattern: they tend to occur in later stages of driving shifts, and taking short breaks
 195 although the credible intervals contain 1.
 196 different pattern: they tend to occur in later stages of driving shifts, and taking short breaks

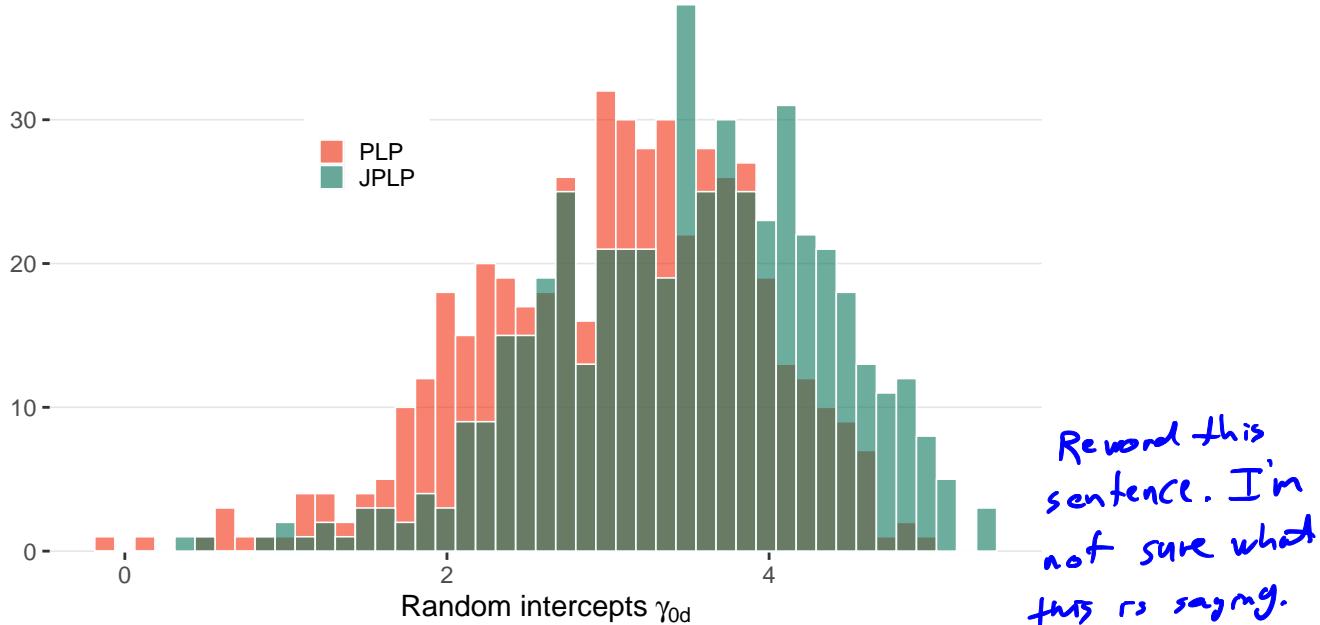


Figure 3: Histogram of random intercepts γ_{0d} across the 496 drivers.

¹⁹⁷ will reduce the intensity of the event. The variability across drivers σ_0 is more evidence for
¹⁹⁸ headway than hard brake, collision mitigation and rolling stability.

¹⁹⁹

6. DISCUSSION

²⁰⁰ In this article, we proposed a Bayesian hierarchical NHPP with PLP intensity function
²⁰¹ and an innovative Bayesian hierarchical JPLP to model naturalistic truck driving data. Our
²⁰² motivation comes from more popular use of naturalistic driving data sets in the recent decade
²⁰³ and real-life truck driving characteristics of multiple trips nested within shifts. The proposed
²⁰⁴ JPLP accounts for the characteristics of multiple rests within a shift among commercial
²⁰⁵ truck drivers. The intensity functions, parameterization forms, and likelihood functions are
²⁰⁶ specified separately. Simulation studies showed the consistency of the Bayesian hierarchical
²⁰⁷ estimation if the models are specified correctly, as well as the persistent bias when the models
²⁰⁸ are not specified correctly. A case study of 496 commercial truck drivers demonstrates a fair
considerable

Table 3: Parameter estimates and 95% credible intervals for jump power law process on 496 truck drivers, stratified by different types of safety-critical events

Parameters	Headway	Hard brake	Collision mitigation & Rolling stability
β	0.989 (0.956, 1.023)	0.922 (0.889, 0.955)	1.020 (0.950, 1.096)
κ	1.034 (0.998, 1.071)	1.034 (0.996, 1.072)	0.890 (0.821, 0.964)
μ_0	7.096 (6.083, 8.139)	3.470 (2.770, 4.199)	4.729 (3.836, 5.666)
σ_0	1.564 (1.411, 1.730)	1.073 (0.973, 1.182)	0.922 (0.786, 1.074)
Age	-0.006 (-0.020, 0.009)	0.011 (0.001, 0.021)	0.002 (-0.009, 0.012)
Race: black	0.184 (-0.170, 0.546)	-0.312 (-0.565, -0.064)	0.113 (-0.153, 0.386)
Race: other	0.306 (-0.340, 0.967)	-0.539 (-0.968, -0.106)	0.100 (-0.373, 0.605)
Gender: female	0.266 (-0.343, 0.870)	-0.217 (-0.654, 0.230)	-0.181 (-0.675, 0.309)
Mean speed	-0.026 (-0.031, -0.021)	0.043 (0.039, 0.047)	0.039 (0.032, 0.046)
Speed variation	-0.009 (-0.017, -0.002)	0.017 (0.010, 0.024)	0.013 (-0.002, 0.027)
Preci. intensity	-0.771 (-4.306, 3.188)	-1.912 (-3.924, 0.269)	-0.676 (-6.329, 6.297)
Preci. prob.	0.694 (0.376, 1.015)	-0.495 (-0.724, -0.263)	0.808 (0.206, 1.423)
Wind speed	0.003 (-0.009, 0.015)	0.019 (0.005, 0.034)	0.000 (-0.025, 0.026)

Preci. intensity: precipitation intensity; Precip. prob.: precipitation probability.

amount of variability exist across drivers. Headway and hard brake tend to occur in early stages while collision mitigation and rolling stability tend to occur in later stages.

The SCEs generated from naturalistic truck driving data are different from previous reliability problems and models in two aspects. Most previous studies either assume minimal repair or perfect repair (also known as renewal process). A minimal repair assumes the unit after the repair is exactly the same as it is before the repair (for example, our first proposed NHPP with a PLP intensity function), while a perfect repair assumes the unit is a completely new unit after repair (Rigdon and Basu, 2000). In the scenario of truck driving, although it is reasonable to assume that the drivers experience a perfect repair when they take a break of longer than 10 hours, researchers would not expect the reliability to be either perfectly

Insert my 2 paragraphs in email.

219 repaired or minimally repaired during a short break of around half an hour. Instead, a partial
220 repair is a more proper assumption here.

221 On the other hand, even though some studies proposed partial repair reliability models
222 such as the modulated PLP (Lakey and Rigdon, 1993; Black and Rigdon, 1996), none of these
223 previously proposed models fit for the data here since the repairs are independent of the SCEs
224 in naturalistic driving data. These previous models are based on high-tech devices such as
225 aircraft manufacturing, which need immediate repair once a failure is detected. However in
226 the case of naturalistic truck driving data sets, SCEs such as hard brakes and headway do
227 not seriously influence the driving and drivers generally will keep driving even if SCEs occur.
228 The repair (breaks) can be considered as independent of SCEs in our study. Although our
229 case study is based on naturalistic truck driving data sets, the JPLP can applied to any type
230 of drivers who drive for a long distance with at least one break.

231 Our work can be extended in several aspects in the future. First, the assumption of pro-
232 portion reliability jump may not hold. Other proper assumptions include reliability jumping
233 for a fixed-amount jump or jumping dependent on the length of the rest. Additionally, in our
234 proposed JPLP, the length of breaks within shifts are ignored to simply the parameterization
235 and likelihood function. In truck transportation practice, longer breaks certainly have larger
236 effects on reliability jump, hence the relationship between reliability jump and the length of
237 breaks can have more complex functional forms, so it would be of interest to test different
238 forms of reliability change as a function of the length of break.

**Do other JASA papers talk about
future work? We may want to
delete this pt.**

Move this to the supplementary material.

239

Appendix A TRACE PLOTS

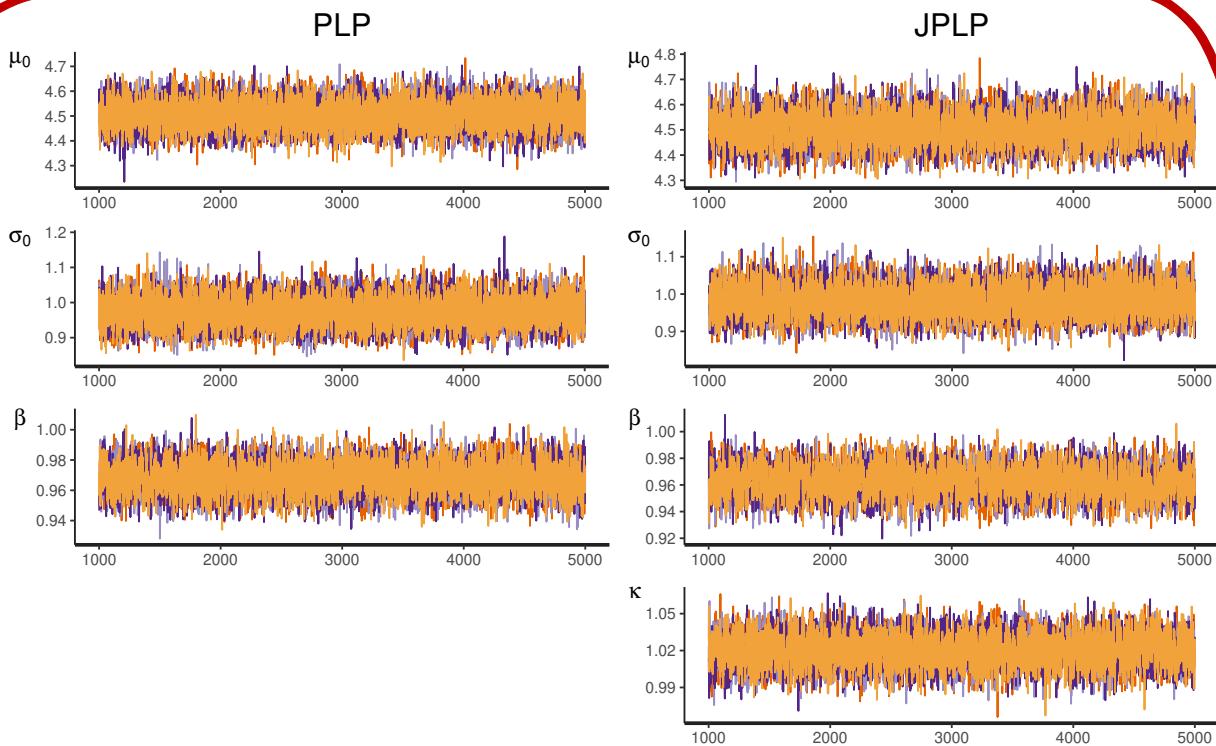


Figure 4: Trace plots of select parameters in PLP (left column) and JPLP (right column) for all types of SCEs

240

SUPPLEMENTARY MATERIALS

241 Since the real truck driving data provided by our industry partner are confidential and
242 cannot be made publicly accessible, we provided a simulated data set that is similar to
243 our real data in data structure but has fewer drivers to be completed within a reasonable
244 amount of time. The online supplementary materials contain the R code to simulate PLP
245 and JPLP data, explanation on the data structure, and Stan and R code for Bayesian
246 hierarchical PLP and JPLP estimation. The supplementary materials can be access at
247 https://github.com/caimiao0714/JPLP_sim.

, which means that the computations can

248

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251

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254 DarkSky company for providing us five million free calls to their historic weather API.

255

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