Hierarchical Jump-point PLP (JPLP) simulation

Miao Cai miao.cai@slu.edu

2020-04-10

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1 Bayesian Hierarchical Jump Power Law Process (JPLP)

1.1 Intensity function of JPLP

Since the Bayesian hierarchical PLP in Subsection **??** does not account for the rests within a shift and associated potential reliability repairment. In this subsection, we proposes a Bayesian hierarchical JPLP, with the following intensity function:

$$\begin{aligned}\lambda_{\text{JPLP}}(t|d,s,r,\beta,\gamma_{0,d},\gamma,\mathbf{X}_d,\mathbf{W}) &= \begin{cases}\kappa^0\lambda(t|\beta,\gamma_{0,d},\gamma,\mathbf{X}_d,\mathbf{W}) & 0 < t \leq a_{d,s,1}\\ \kappa^1\lambda(t|\beta,\gamma_{0,d},\gamma,\mathbf{X}_d,\mathbf{W}) & a_{d,s,1} < t \leq a_{d,s,2}\\ \cdots & \cdots\\ \kappa^{R-1}\lambda(t|\beta,\gamma_{0,d},\gamma,\mathbf{X}_d,\mathbf{W}) & a_{d,s,R-1} < t \leq a_{d,s,R}\end{cases}\\ &= \kappa^{r-1}\lambda(t|d,s,r,\kappa,\beta,\gamma_{0,d},\gamma,\mathbf{X}_d,\mathbf{W}) \quad a_{d,s,r-1} < t \leq a_{d,s,r},\end{aligned} \tag{1}$$

where the introduced parameter κ is the percent of intensity function recovery once the driver takes a break. By definition, $a_{d,s,0} = 0$. We assume that this κ is constant across drivers and shifts.

1.2 Parametrization of Bayesian hierarchical JPLP model

The Bayesian hierarchical JPLP model is parameterized as

$$\begin{aligned} t_{d,s,1}, t_{d,s,2}, \cdots, t_{d,s,n_{d,s}} &\sim \text{JPLP}(\beta, \theta_{d,s}, \kappa)\\ \beta &\sim \text{Gamma}(1,1)\\ \log\theta_{d,s} &= \gamma_{0d} + \gamma_1 x_{d,s,1} + \gamma_2 x_{d,s,2} + \cdots + \gamma_k x_{d,s,k}\\ \kappa &\sim \text{Uniform}(0,1)\\ \gamma_{01}, \gamma_{02}, \cdots, \gamma_{0D} &\sim \text{i.i.d. } N(\mu_0, \sigma_0^2)\\ \gamma_1, \gamma_2, \cdots, \gamma_k &\sim \text{i.i.d. } N(0, 10^2)\\ \mu_0 &\sim N(0, 5^2)\\ \sigma_0 &\sim \text{Gamma}(1,1), \end{aligned} \tag{2}$$

The notations are identical with those in Equation **??** except for the extra κ parameter.

1.3 The likelihood function of JPLP

The likelihood function for driver d on shift s is

$$\begin{aligned} L_{s,d}(\kappa, \beta, \gamma_{0,d}, \gamma|\text{Data}_{d,s}) = &\Big(\prod_{i=1}^{c_{d,s}} \lambda\big(t_{i,d,s}|d, s, r, k, \beta, \gamma_{0,d}, \gamma, \mathbf{X}_d, \mathbf{W})\Big) \\ &\times \exp\Big(-\int_0^{a_{d,s,r}} \lambda\big(u|d,s,r,k,\beta,\gamma_{0,d},\gamma,\mathbf{X}_d,\mathbf{W}\big)du\Big) \end{aligned} \tag{3}$$

The overall likelihood function is

$$L = \prod_d \prod_{s \in d} L_{s,d} \tag{4}$$