Jump-point PLP (JPLP)

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Contents

1 NHPP and PLP

Intensity function: The intensity function of a point process is

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{P(N(t, t + \Delta t] \geq 1)}{\Delta t}$$

When there is no simultaneous events, ROCOF is the same as intensity function.

Nonhomogeneous Poisson Process (NHPP): The NHPP is a Poisson process whose intensity function is non-constant.

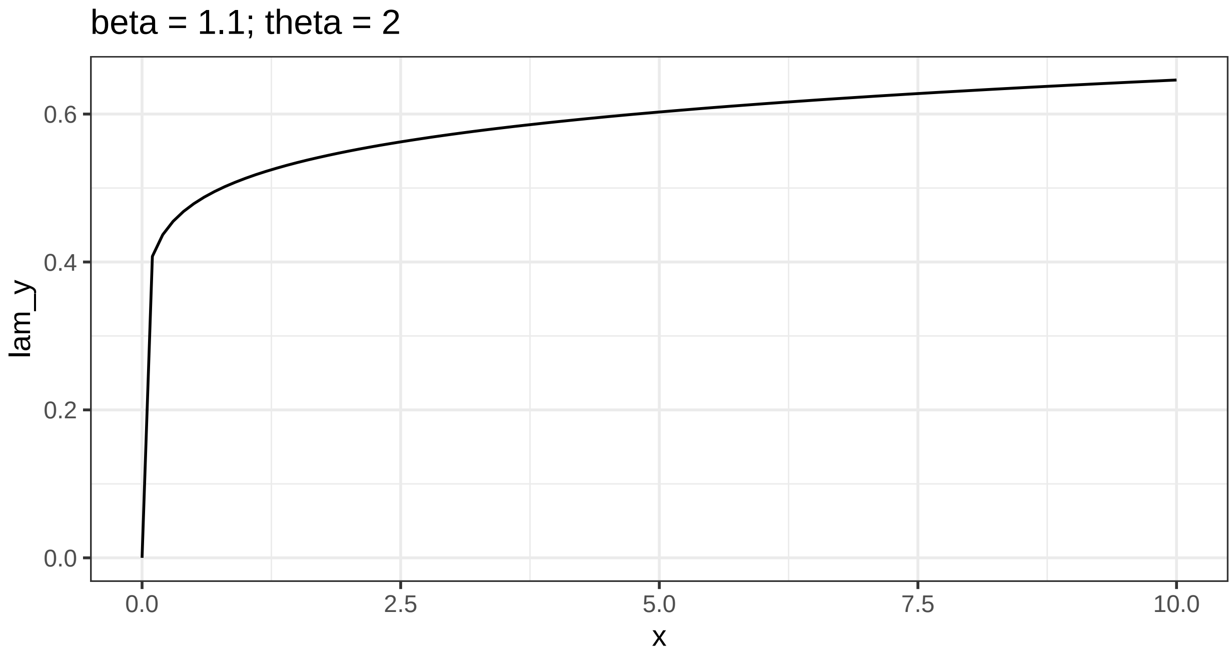
Power law process (PLP): When the intensity function of a NHPP is:

$$\lambda(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta - 1}$$

Where $\beta > 0$ and $\theta > 0$, the process is called the power law process (PLP).

Therefore, the mean function $\Lambda(t)$ is the integral of the intensity function:

$$\Lambda(t) = \int_0^t \lambda(t)dt = \int_0^t \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta - 1} = \left(\frac{t}{\theta}\right)^{\beta}$$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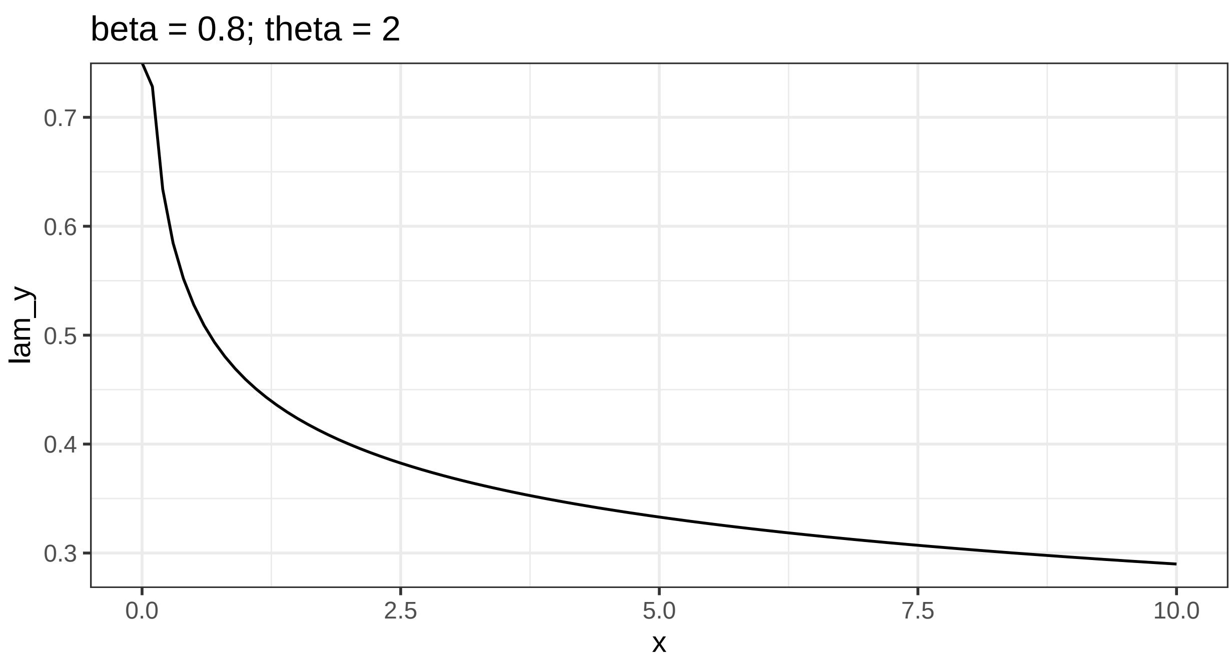
0.4
0.2
0.0
lam_y
0.0
2.5
5.0
7.5
10.0
x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0.3
lam_y
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2.5
5.0
7.5
10.0
x

2 Jump-point PLP (JPLP)

At the time of the failure or rest, the intensity will bounce back at a certain percent κ, and $0 < \kappa < 1$.

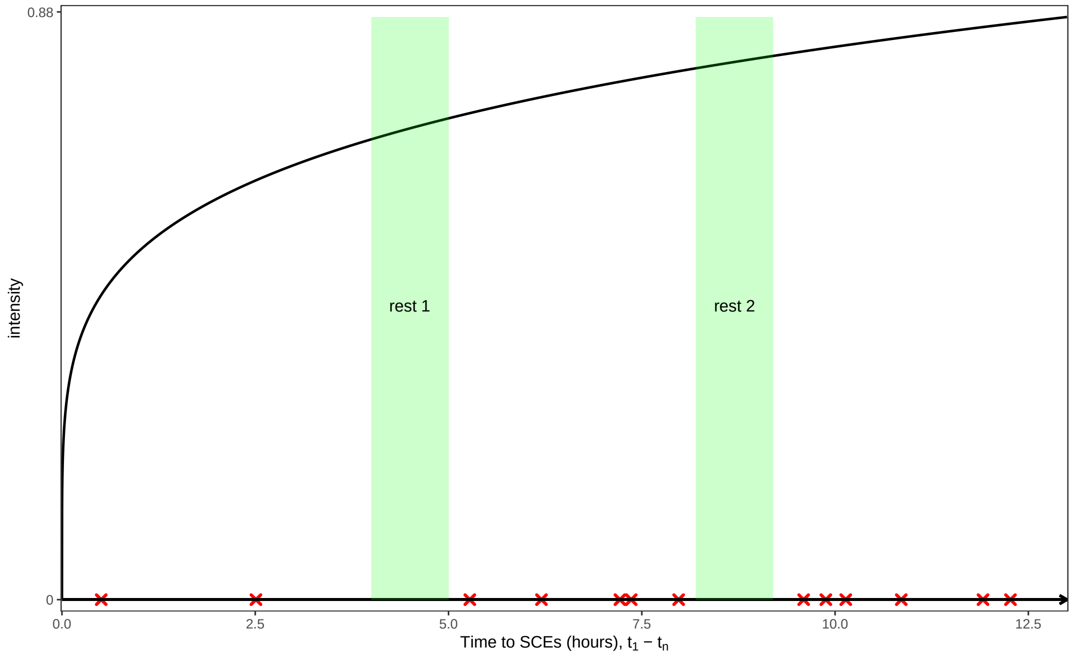

Figure 1: Intensity function, time to SCEs, and rest time within a shift generated from a NHPP with a PLP intensity function, $\beta = 1.2$, $\theta = 2$

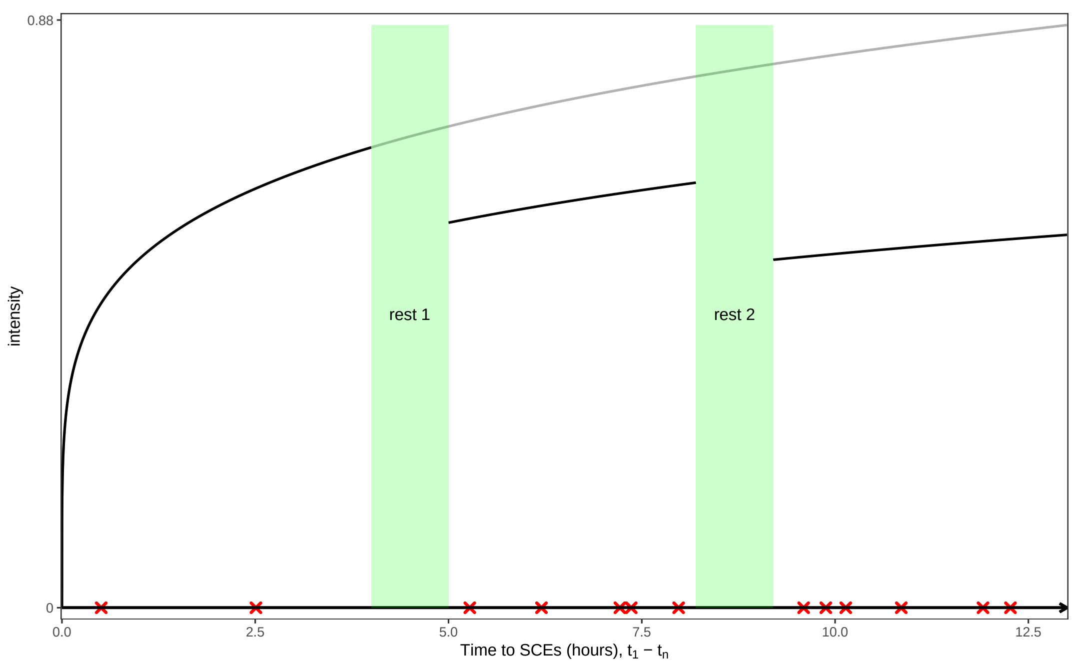

Figure 2: Intensity function, time to SCEs, and rest time within a shift with a jump-point PLP intensity function, $\beta = 1.2$, $\theta = 2$, $\kappa = 0.8$

3 Complete intensity function

The complete intensity function of a NHPP with a PLP intensity function is:

$$\lambda^\star(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1}$$

The complete intensity function of a JPLP is then:

$$\lambda_{JPLP}(t) = \begin{cases} \lambda^\star(t) & 0 \leq t \leq t_1 \\ \kappa\lambda^\star(t) & t_1 < t \leq t_2 \\ \kappa^2\lambda^\star(t) & t_2 \leq t \leq t_3 \\ \cdots & \cdots \\ \kappa^{n-1}\lambda^\star(t) & t_{n-1} \leq t \leq t_n \\ \kappa^n\lambda^\star(t) & t_n \leq t < \tau \end{cases} \tag{1}$$

where $t_1, t_2, \cdots, t_n$ are the time to the 1st, 2nd, $\cdots$, and n-th event. τ is the truncation time.

4 Joint likelihood function

4.1 The first event

The survival function for the first event is

$$\begin{aligned} S_1(t_1) &= P(T_1 > t_1) \\ &= \exp\left(-\int_0^{t_1}\lambda(x)dx\right) \\ &= \exp\left(-\int_0^{t_1}\lambda^\star(x)dx\right) \end{aligned}$$

The likelihood function is then

$$\begin{aligned} f_1(t_1) &= -S_1'(t_1) \\ &= \lambda^\star(t_1)\exp\left(-\int_0^{t_1}\lambda^\star(x)dx\right) \end{aligned}$$

4.2 The second event

The survival function for the second event is

$$\begin{aligned} S_2(t_2) &= P(T_2 > t_2|t_1) \\ &= P(N(t_1, t_2]) = 0) \\ &= \exp\left(-\int_{t_1}^{t_2}\kappa\lambda^\star(x)dx\right) \end{aligned}$$

The associated likelihood function is:

$$
\begin{aligned}
f_2(t_2) &= -S_2'(t_2) \\
&= \kappa\lambda^\star(t_2)\exp\left(-\int_{t_1}^{t_2}\kappa\lambda^\star(x)dx\right)
\end{aligned}
$$

4.3 The n-th event

Likewise, the likelihood function of the n-th event is:

$$
\begin{aligned}
f_n(t_n|t_1, t_2, \cdots, t_n) &= -S_n'(t_n) \\
&= \kappa^{n-1}\lambda^\star(t_n)\exp\left(-\int_{t_{n-1}}^{t_n}\kappa^{n-1}\lambda^\star(x)dx\right)
\end{aligned}
$$

4.4 All events

The likelihood function of all events is:

$$
\begin{aligned}
&f(t_1, t_2, \cdots, t_n) \\
&= f_1(t_1)f_2(t_2|t_1)\cdots f_n(t_n|t_1, t_2, \cdots, t_{n-1}) \\
&= \kappa^0\lambda^\star(t_1)\exp\left(-\int_0^{t_1}\lambda^\star(x)dx\right)\times\kappa^1\lambda^\star(t_2)\exp\left(-\int_{t_1}^{t_2}\kappa\lambda^\star(x)dx\right)\times\cdots\times\quad\kappa^{n-1}\lambda^\star(t_n)\exp\left(-\int_{t_{n-1}}^{t_n}\kappa^{n-1}\lambda^\star(x)dx\right) \\
&= \kappa^{n(n-1)/2}\left(\prod_{i=1}^{n}\lambda^\star(t_i)\right)\times\exp\left(-\left[\left(\frac{t_1}{\theta}\right)^\beta-\left(\frac{0}{\theta}\right)^\beta+\kappa\left(\frac{t_2}{\theta}\right)^\beta-\kappa\left(\frac{t_1}{\theta}\right)^\beta+\cdots\kappa^{n-1}\left(\frac{t_n}{\theta}\right)^\beta-\kappa^{n-1}\left(\frac{t_{n-1}}{\theta}\right)^\beta\right]\right) \\
&= \kappa^{n(n-1)/2}\left(\prod_{i=1}^{n}\lambda^\star(t_i)\right)\times\exp\left(-\left[(1-\kappa)\left(\frac{t_1}{\theta}\right)^\beta+\kappa(1-\kappa)\left(\frac{t_2}{\theta}\right)^\beta+\cdots+\kappa^{n-2}(1-\kappa)\left(\frac{t_{n-1}}{\theta}\right)^\beta+\kappa^{n-1}\left(\frac{t_n}{\theta}\right)^\beta\right]\right) \\
&= \kappa^{n(n-1)/2}\left(\prod_{i=1}^{n}\lambda^\star(t_i)\right)\exp\left(\left[\sum_{j=1}^{n-1}\kappa^{j-1}(1-\kappa)\left(\frac{t_j}{\theta}\right)^\beta\right]+\kappa^{n-1}\left(\frac{t_n}{\theta}\right)^\beta\right)
\end{aligned}
$$