Hierarchical Jump-point PLP (JPLP) Estimation

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1 Theory

1.1 Intensity function of JPLP

We proposes a Bayesian hierarchical JPLP, with the following piecewise intensity function:

$$
\lambda_{\mathrm{JPLP}}(t|d,s,r,\beta,\gamma_{0,d},\gamma,\mathbf{X}_d,\mathbf{W}_s) =
\begin{cases}
\kappa^0\lambda(t|\beta,\gamma_{0,d},\gamma,\mathbf{X}_d,\mathbf{W}_s) & 0 < t \le a_{d,s,1}, \\
\kappa^1\lambda(t|\beta,\gamma_{0,d},\gamma,\mathbf{X}_d,\mathbf{W}_s) & a_{d,s,1} < t \le a_{d,s,2}, \\
\dots & \dots \\
\kappa^{R-1}\lambda(t|\beta,\gamma_{0,d},\gamma,\mathbf{X}_d,\mathbf{W}_s) & a_{d,s,R-1} < t \le a_{d,s,R}, \\
\end{cases}
$$
$$
= \kappa^{r-1}\lambda(t|d,s,r,\kappa,\beta,\gamma_{0,d},\gamma,\mathbf{X}_d,\mathbf{W}_s) \quad a_{d,s,r-1} < t \le a_{d,s,r},
\tag{1}
$$

where the introduced parameter κ is the percent of intensity function recovery once the driver takes a break, and $a_{d,s,r}$ is the end time of trip r within shift s for driver d. By definition, the end time of the 0-th trip $a_{d,s,0} = 0$, and the end time of the last trip for the d-driver within s-shift $a_{d,s,R}$ equals the shift end time $\tau_{d,s}$. We assume that this κ is constant across drivers and shifts.

1.2 Parameterization of JPLP

The Bayesian hierarchical JPLP model is parameterized as

$$
\begin{aligned}
t_{d,s,1}, t_{d,s,2}, \cdots, t_{d,s,n_{d,s}}, \tau_{d,s} &\sim \mathrm{JPLP}(\beta, \theta_{d,s}, \kappa) \\
\beta &\sim \mathrm{Gamma}(1,1) \\
\log \theta_{d,s} &= \gamma_{0d} + \gamma_1 x_{d,s,1} + \gamma_2 x_{d,s,2} + \cdots + \gamma_k x_{d,s,k} \\
\kappa &\sim \mathrm{Uniform}(0,1) \\
\gamma_{01}, \gamma_{02}, \cdots, \gamma_{0D} &\sim \text{i.i.d. } N(\mu_0, \sigma_0^2) \\
\gamma_1, \gamma_2, \cdots, \gamma_k &\sim \text{i.i.d. } N(0, 10^2) \\
\mu_0 &\sim N(0, 5^2) \\
\sigma_0 &\sim \mathrm{Gamma}(1,1),
\end{aligned}
\tag{2}
$$

1.3 Likelihood function of JPLP

The likelihood function of event times generated from a JPLP for driver d on shift s is

$$
\begin{aligned}
&L_{s,d}^*(\kappa, \beta, \gamma_{0d}, \gamma | \mathbf{X}_d, \mathbf{W}_s) = \\
&\begin{cases}
\exp\left(-\int_0^{a_{d,s,R}} \lambda_{\mathrm{JPLP}}\big(u|d,s,r,k,\beta,\gamma_{0d},\gamma,\mathbf{X}_d,\mathbf{W}_s\big)du\right), & \text{if } n_{d,s} = 0, \\[2mm]
\left(\prod_{i=1}^{n_{d,s}} \lambda_{\mathrm{JPLP}}\big(t_{i,d,s}|d,s,r,k,\beta,\gamma_{0d},\gamma,\mathbf{X}_d,\mathbf{W}_s\big)\right) \\
\times \exp\left(-\int_0^{a_{d,s,R}} \lambda_{\mathrm{JPLP}}\big(u|d,s,r,k,\beta,\gamma_{0d},\gamma,\mathbf{X}_d,\mathbf{W}_s\big)du\right), & \text{if } n_{d,s} > 0,
\end{cases}
\end{aligned}
\tag{3}
$$

where $t_{i,d,s}$ is the time to the i-th SCE for driver d on shift s, $n_{d,s}$ is the number of SCEs for driver d on shift s. Therefore, the overall likelihood function for drivers $d \in 1, 2, \ldots, D$ and their corresponding shifts $s \in d$ is:

$$
L^* = \prod_d \prod_{s \in d} L_{s,d}^*.
\tag{4}
$$

Since λ_{JPLP} is a piecewise likelihood function that depends on event time and trip time, we will not spell out the details of the full likelihood or log likelihood.

2 One driver scenario

2.1 Simulating data for multiple shifts from one driver

```
pacman::p_load(rstan, tidyverse, data.table)
source("functions/JPLP_functions.R")


dt = sim_mul_jplp()
str(dt)

## List of 3
##  $ event_time:'data.frame':  76 obs. of  2 variables:
##   ..$ shift_id  : int [1:76] 1 1 1 1 1 1 1 1 2 2 ...
##   ..$ event_time: num [1:76] 1 2.7 2.96 4.47 5.79 ...
##  $ trip_time :'data.frame':  15 obs. of  2 variables:
```

```
##    ..$ shift_id : int [1:15] 1 2 3 3 4 5 5 6 7 7 ...
##    ..$ trip_time: num [1:15] 7.44 5.68 2.99 6.65 5.95 3.31 7.58 3.6 2.64 5.34 ...
##  $ shift_time:'data.frame':  10 obs. of  3 variables:
##    ..$ shift_id  : int [1:10] 1 2 3 4 5 6 7 8 9 10
##    ..$ start_time: num [1:10] 0 0 0 0 0 0 0 0 0 0
##    ..$ end_time  : num [1:10] 9.38 9.73 10.4 10.87 11.36 ...
```