

Simulate NHPP

Miao Cai*

2019-05-20

Contents

1	A few concepts	1
2	NHPP and PLP	1
3	Inverse algorithm to simulate NHPP	2
4	Simulation using R	2
4.1	The most straightforward way of simulation - a loop	2
4.2	A more efficient way of simulation - vectorized function	3
	References	3

1 A few concepts

Mean function of a point process:

$$\Lambda(t) = E(N(t))$$

$\Lambda(t)$ is the expected number of failures through time t .

Rate of Occurrence of Failures (ROCOF): When Λ is differentiable, the ROCOF is:

$$\mu(t) = \frac{d}{dt}\Lambda(t)$$

The ROCOF can be interpreted as the instantaneous rate of change in the expected number of failures.

Intensity function: The intensity function of a point process is

$$\lambda(t) = \lim_{\Delta t \rightarrow 0} \frac{P(N(t, t + \Delta t] \geq 1)}{\Delta t}$$

When there is no simultaneous events, ROCOF is the same as intensity function.

2 NHPP and PLP

Nonhomogeneous Poisson Process (NHPP): The NHPP is a Poisson process whose intensity function is non-constant.

Power law process (PLP): When the intensity function of a NHPP is:

$$\lambda(t) = \frac{\beta}{\theta} \left(\frac{t}{\theta} \right)^{\beta-1}$$

*Department of Epidemiology and Biostatistics, College for Public Health and Social Justice, Saint Louis University. Email: miao.cai@slu.edu

Where $\beta > 0$ and $\theta > 0$, the process is called the power law process (PLP).

Therefore, the mean function $\Lambda(t)$ is the integral of the intensity function:

$$\Lambda(t) = \int_0^t \lambda(t)dt = \int_0^t \frac{\beta}{\theta} \left(\frac{t}{\theta}\right)^{\beta-1} = \left(\frac{t}{\theta}\right)^{\beta}$$

3 Inverse algorithm to simulate NHPP

The algorithm used in this tutorial is the review provided by Pasupathy (2010). He mentioned that the earliest inversion technique was devised by Cinlar (2013), which was based on a property of NHPP with a continuous expectation function $\Lambda(t)$.

Theorem 3.1 (Cinlar's inversion algorithm). *Let $\Lambda(t)$ be a positive-valued, continuous, nondecreasing function. Then the random variables T_1, T_2, \dots are event times corresponding to a NHPP with expectation function $\Lambda(t)$ if and only if $\Lambda(T_1), \Lambda(T_2), \dots$ are the event times corresponding to a HPP with rate one.*

Theorem 3.1 can be used to generate failures from a NHPP: First generate event times from a HPP with rate one, then invert $\Lambda(\cdot)$ to get the event times. Here are the steps/algorithms provided by Pasupathy (2010) to generate NHPP failure times.

- (0) Initialize $s = 0$
- (1) Generate $u \sim U(0, 1)$
- (2) Set $s \leftarrow s - \log(u)$
- (3) Set $t \leftarrow \inf\{\Lambda(v) \geq s\}$
- (4) Deliver t
- (5) Go to Step (1)

Here the Step (3) is essentially getting the inverse of $\Lambda(v)$ if $\Lambda(v)$ is a continuous function.

$$s = \Lambda(t) = \left(\frac{t}{\theta}\right)^{\beta}$$

$$s^{1/\beta} = \frac{t}{\theta}$$

$$\theta \cdot s^{1/\beta} = t$$

Therefore, the inverse of $s = \Lambda(t)$ is $t = \theta \cdot s^{1/\beta}$

4 Simulation using R

We don't have to use a loop to iteratively sample as this algorithm does. Instead, we can use a vectorized form in R to simulate event times using this algorithm.

4.1 The most straightforward way of simulation - a loop

In this simulation, we randomly set parameters $\beta = 2, \theta = 10$.

```
set.seed(123)
s = 0; N = 10; t = rep(NA_real_, N) #initialization
beta = 2; theta = 10 # random parameters

for (i in 1:N) {
  u = runif(1)
  s = s - log(u)
  t[i] = theta*s^(1/beta)
```

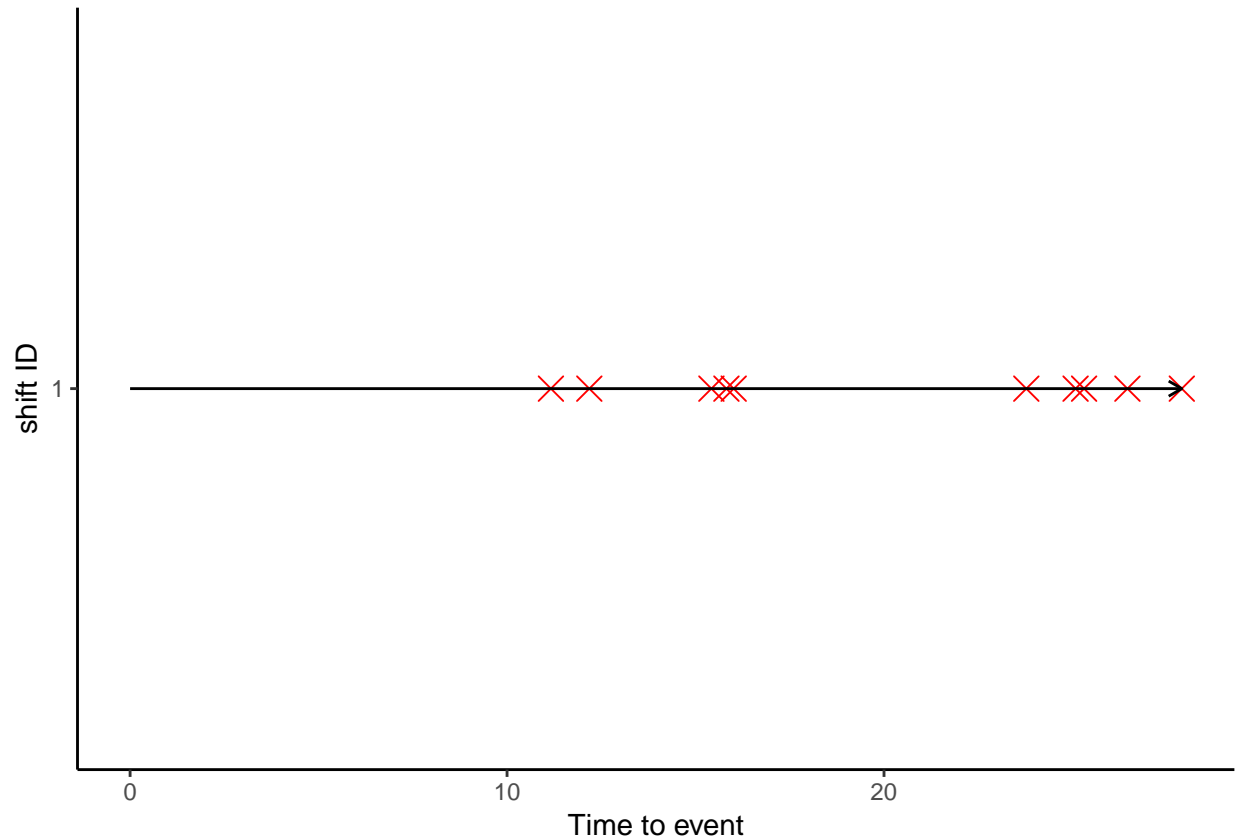
```
}
```

```
t
```

```
## [1] 11.16361 12.18250 15.42151 15.81973 16.01255 23.77566 25.08242
```

```
## [8] 25.30830 26.45814 27.90026
```

An arrow plot of these failures would be:



Since $\beta = 2 > 1$, the reliability of this system is deteriorating. The failures become more and more intense/frequent at the right side of the plot.

4.2 A more efficient way of simulation - vectorized function

References

Cinlar, Erhan. 2013. *Introduction to Stochastic Processes*. Courier Corporation.

Pasupathy, Raghu. 2010. "Generating Homogeneous Poisson Processes." *Wiley Encyclopedia of Operations Research and Management Science*.