Gamma-Poisson distribution

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1 Gamma-Poisson distribution introduction

Let us assume that:

$$\lambda \sim \text{Gamma}(\alpha, \beta)$$
$$X|\lambda \sim \text{Poisson}(\lambda)$$

Then we have:

$$X \sim \text{Gamma-Poisson}(\alpha, \beta)$$

The Gamma-Poisson distribution is a α-parameter distribution, with the probability mass function of:

$$f(x) = \frac{\Gamma(x+\beta)\alpha^x}{\Gamma(\beta)(1+\alpha)^{\beta+x}x!}, \quad x = 0, 1, 2, \ldots$$

The mean and variance are:

$$E(X) = \alpha\beta$$
$$V(X) = \alpha\beta + \alpha^2\beta$$
$$= \alpha\beta(1+\alpha)$$

where α measures the overdispersion.

We might assume that:

$$\log \beta = \mathbf{X}\gamma, \text{ and } \alpha \text{ is fixed}$$

and α is a fixed value.

2 Simulation

3 Estimation