A visualization of random effects model and fixed effects model

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1 Random intercept model

1.1 Statistical model

In this section, we will set up the statistical model for a random-intercept model:

$$
\begin{aligned}
&y_{ij} = \beta_{0j} + \beta_1 * x_{1i} + \beta_2 * x_{2i} + \beta_3 * x_{3i} + \epsilon_i \\
&\beta_{0j} \sim N(\mu_0, \sigma_0^2)
\end{aligned}
$$

Where $i = 1, 2, \cdots, N$ is the index for individual observations, so the total sample size is N in this case. $j = 1, 2, \cdots, J$ is the index for groups, so there are J groups/clusters in this case. $\beta_{01}, \beta_{02}, \cdots, \beta_{0J}$ are the random intercepts in this model. $\beta_1, \beta_2, \cdots, \beta_K$ are the fixed-effects parameters, and in this case, $K = 3$. The number of predictor variables can vary, and this data generating process can be generalized using a more succinct matrix form style as:

$$
\begin{aligned}
&y_{ij} = \beta_{0j} + \mathbf{X}\beta + \epsilon_i \\
&\beta_{0j} \sim N(\mu_0, \sigma_0^2)
\end{aligned}
$$

where $\mathbf{X}$ is a $N \times K$ data matrix, and β is a $K \times 1$ column vector.

We assume that the random intercepts β_{0j} has a normal distribution with two hyperparameters: mean μ_0 and standard deviation σ_0.

1.2 Simulation parameters

```r
set.seed(666)

Total_obs = 500
group = 10
Group_Obs = Total_obs/group

b1 = 1
mu = 1
sigma = 1
epsilon = 2

# y = b0 + b1 + e
j = rep(1:group, Group_Obs)
b0 = rnorm(group, 1, 5)
x = rnorm(Total_obs, 5, 5)

y = rep(b0, Group_Obs) + b1*x + rnorm(Total_obs)

dat = data.frame(y, j, x, b0, b1)
```

```r
plot(x, y)
for (i in 1:group) {
  abline(b0[i], b1)
}

reg1 <- lm(y ~ x,data = dat)
abline(reg1, col = "grey", lwd = 3)

reg2 <- lm(y ~ factor(j) + x,data = dat)
reg2_b0 = reg2$coefficients[labels(reg2$coefficients) == "(Intercept)"]
reg2_b1 = reg2$coefficients[labels(reg2$coefficients) == "x"]
abline(reg2_b0, reg2_b1, col = "blue", lwd = 3)


require(lme4)
reg3 <- lmer(y ~ x + (1|j), dat)
summary(reg3)
```

1.3 Including Plots

You can also embed plots, for example:

```
## Loading required package: ggplot2
```