

用 R 和 INLA 做空间统计分析

一本数学不太好的同学的上手教程

蔡苗

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Colophon

This document was typeset with the help of KOMA-Script and L^AT_EX using the kao-book class.

The source code of this book is available at:

<https://github.com/fmarotta/kaobook>

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虽然每个人个体都是难解的谜，但是把人类聚合起来，从数学的意义上来说人类就是确定的。

譬如说，你永远无法预测某个人未来的行为，但是你却能够确知人类的平均水平。

个体不同，平均水平却是永恒的。

– 阿瑟·柯南·道尔爵士

While the individual man is an insoluble puzzle, in the aggregate he becomes a mathematical certainty.

You can, for example, never foretell what any one man will do, but you can say with precision what an average number will be up to.

Individuals vary, but the percentages remain constant.

– Sir Arthur Conan Doyle

前言

用 R 来处理和清理空间数据。用于处理空间数据的 R 包主要有 `sf`, `sp`, `tidyverse`。

- ▶ Chris Brunsdon 和 Lex Comber 的 [An Introduction to Spatial Analysis and Mapping in R 2nd edition](#)[1]。此书的 R 代码可以在网上免费查看。
- ▶ Robin Lovelace 等人的 [Geocomputation with R](#) [2]。此书同样基于开源协议，可以在网上免费查看。

用 R 来做空间统计分析。做空间统计模型使用的 R 包主要有 `spde`, `spdep`, `INLA`。

- ▶ Gómez-Rubio, Virgilio 的 [Bayesian inference with INLA](#) [1]。第 7 和 8 章是介绍使用 INLA 来分析空间数据的内容。此书基于开源协议，可以在网上免费查看。
- ▶ Paula Moraga 的 [Geospatial Health Data: Modeling and Visualization with R-INLA and Shiny](#)。此书基于开源协议，可以在网上免费查看。[3]
- ▶ Christopher K. Wikle 等人的 [Spatio-temporal Statistics with R](#) [4]
- ▶ Roger S. Bivand 等人的 [Applied spatial data analysis with R](#) 第二版[5]
- ▶ Elias T. Krainski 等人的 [Advanced spatial modeling with stochastic partial differential equations using R and INLA](#) [6]。此书基于开源协议，可以在网上免费查看。

我开始写这本书是作为自己学习空间统计分析的笔记。由于笔记的内容足够丰富，并且国内并没有比较好的用 R 来做空间统计分析的教材，因此将笔记整理和出版成书籍，希望能对国内广大有分析空间统计数据的研究人员有所帮助。

蔡苗

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1.1 空间数据的分类

作为空间数据分析的第一步，我们首先要在心中十分清楚空间数据分别有哪些类型。

要点

空间数据主要分为三大类：区域型数据（Areal data）、地理统计数据（Geostatistical data）、以及空间点数据（Point patterns）。

- ▶ 区域型数据（Areal data）：空间中多个多边形中包含的数据。
- ▶ 地理统计数据（Geostatistical data）：空间中某些固定位置的收集站收集到的数据。
- ▶ 空间点数据（Point patterns）：空间中的散点数据。

1.1 空间数据的分类 1

区域型数据 1

地理统计数据 2

空间点数据 2

1.2 空间统计的主要挑战 2

空间依赖性 2

空间异质性 2

稀疏数据 2

1.3 用 R 来做空间统计模型 2

空间统计相关的 R 包 2

数据读写 2

空间数据绘图 2

空间统计建模 2

区域型数据

区域型数据（Areal data）是空间统计建模中最常见的数据，最常见的形式是空间中多个多边形（Spatial polygons）中包含的数据。这种数据最常见的储存格式是 ESRI 开发和维护的 shape 数据（.shp 文件），很多地理相关的学者常常用 ArcGIS 来读取和处理.shp 文件。

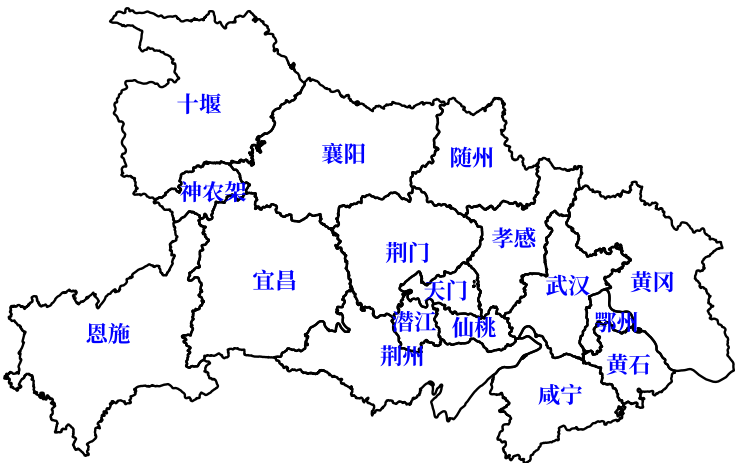


Figure 1.1: 湖北省 17 市的地理位置分布

图 1.1展示了区域型数据的一个例子。湖北省每一个城市都是由多边形所围成。此类型数据常常用来展示某种指标的高低的地理分布情况，例如湖北省各个市新冠肺炎确诊人数。

这里的脚注只是为了测试一下格式是否正确。

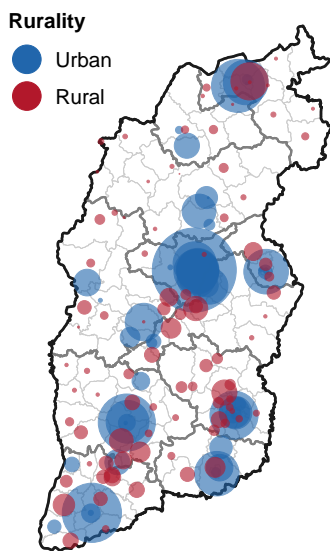


Figure 1.2: 山西省城乡医院心肌梗死病人病人数分布图

地理统计数据

地理统计数据 (Geostatistical data) 常常是在空间中某些固定位置的收集站收集到的数据，比如空气质量检测站。

空间点数据

空间点数据 (Point patterns) 顾名思义，即为空间中的散点。对于此类型数据，我们一般都会先用图来描述点的分布特征，然后用统计模型来估计点分布的密度函数，得到这些点在空间中的连续型分布。

1.2 空间统计的主要挑战

空间依赖性

空间异质性

稀疏数据

1.3 用 R 来做空间统计模型

空间统计相关的 R 包

数据读写

空间数据绘图

空间统计建模

空间数据的读取和可视化

2.1 读取空间数据

空间区域型数据

最常见的区域型数据是 ESRI 开发和维护的 shape 数据，这种类型的文件不是一个单个文件，而是由多个文件组成：

- ▶ .shp 文件：图形格式。主要用于储存空间多边型的边界，也可以储存各个空间多边形中的变量，例如各个城市某年的 GDP
- ▶ .shx 文件：图形索引格式。用于索引空间多边形，能够加快搜索的效率。
- ▶ .dbf 文件：属性数据格式。记录每个空间多边形的属性数据。
- ▶ .prj 文件：投影系统。

地理统计和空间点数据

2.2 连接空间与非空间数据

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3.1 区域型数据可视化

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 地理统计数据可视化 7

 空间点数据可视化 7

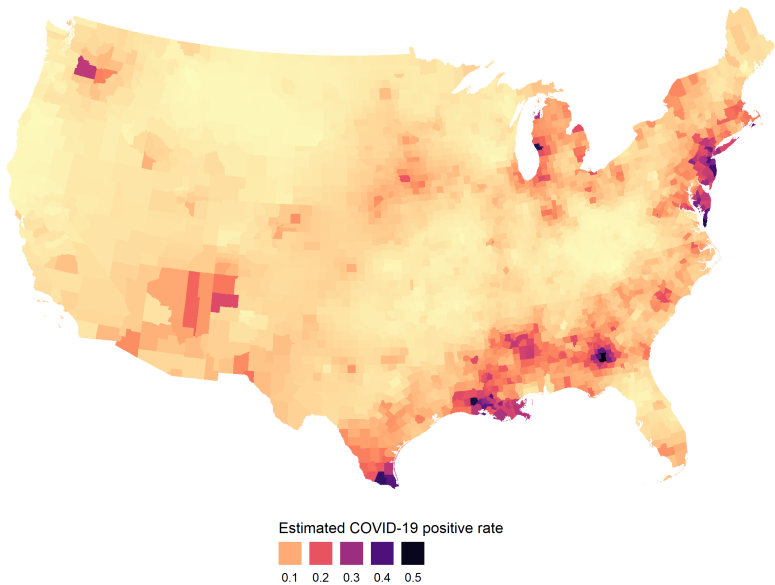
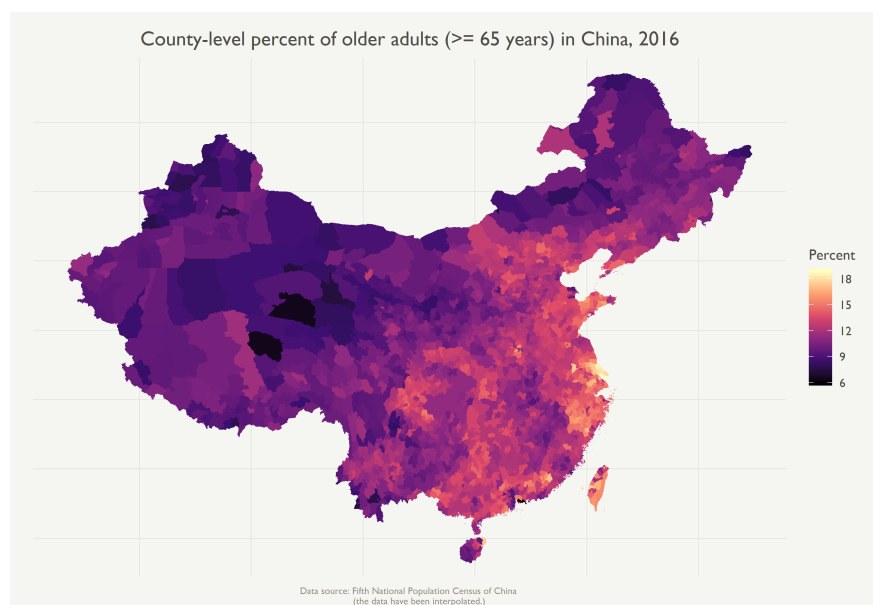


Figure 3.1: 此图为美国退伍军人群体中县级水平的新冠肺炎检测阳性率。此图采用了一些空间平滑的统计模型,使得高检测阳性率地区的聚集性更加明显。图中我们可以看到东部的纽约市、南部的亚特兰大、新奥尔良和哈灵根的诊断阳性率相对较高。

地理统计数据可视化

空间点数据可视化

Figure 3.2: 此图展示的是中国县市级老年人口比例。(由于南海人口稀少, 因此此处略有删节并未展示。)



空间统计模型

空间区域数据的统计模型

4.1 空间相邻矩阵

空间统计模型与传统的非空间统计模型最大的区别就是空间相邻矩阵 (Spatial neighborhood matrix)，我们一般用 \mathbf{W} 来表示，其中每一个权重为 w_{ij} ，它记录了相邻的多边型的信息。

定义 4.1.1 皇后 (Queen) 相邻矩阵和如克 (Rook) 相邻矩阵。如果两个多边型至少有一个点相接，那么皇后相邻矩阵定义这两个多边型是相连的。如克相邻矩阵则定义两个多边型相邻的条件是二者至少有一个边相接。皇后相邻矩阵定义的仅有一个点相接的两个多边型，在如克相邻矩阵中是不算成相邻的。

4.2 空间区域数据统计模型

要点

常见的空间区域数据统计模型有 [7–9]：Besag 的不恰当先验模型、Besag 的恰当先验模型、Besag-York-Mollié 模型 (BYM 模型)、以及 Leroux 模型。

- ▶ Besag 的不恰当先验模型 (Besag's improper prior model)
- ▶ Besag 的恰当先验模型 (Besag's proper prior model)
- ▶ Besag-York-Mollié 模型 (BYM 模型)
- ▶ Leroux 模型

[7]: Morris et al. (2019), 'Bayesian hierarchical spatial models: Implementing the Besag York Mollié model in stan'
[8]: Besag et al. (1991), 'Bayesian image restoration, with two applications in spatial statistics'
[9]: Leroux et al. (2000), 'Estimation of disease rates in small areas: a new mixed model for spatial dependence'

Besag-York-Mollié 模型 (BYM 模型)

以下是 BYM 模型的参数化形式:

$$\eta_i = \mu + \mathbf{X}\mathbf{f}_i + \phi + \theta$$

其中:

- ▶ μ 为截距项，代表了整体的风险
- ▶ \mathbf{X} 为解释变量矩阵, β 为解释变量对应的参数
- ▶ ϕ 为 ICAR 空间部分
- ▶ θ 为非空间的随机效应误差

5.1 克里金法 (Kriging)

时间统计模型

时间-空间统计模型

类选项、命令和环境

9.1 引用

要引用某人 [10, 11] 非常简单: 只需使用 `\sidecite` 命令。它还没有一个抵消的参数, 但它可能会在未来。如您所见, 该命令支持多个条目, 默认情况下, 它在页边空白处打印引用, 并将其添加到文档末尾的参考书目中。在这个设置中, 我使用了 `biblatex`, 但是我认为这是可行的。[11] 注意, 这些引用与文本没有任何关系, 它们完全是随机的, 因为它们只用于说明特性。

要编译包含引用的文档, 您需要使用一个外部工具, 对于这个类, 这个工具是 `biber`。您需要运行以下命令 (假设您的 `tex` 文件名为 `main.tex`):

```
$ pdflatex main
$ biber main
$ pdflatex main
```

[10]: Visscher et al. (2008), ‘Heritability in the genomics era—concepts and misconceptions.’

[11]: James et al. (2013), *An Introduction to Statistical Learning*

[11]: James et al. (2013), *An Introduction to Statistical Learning*

9.2 术语表和索引

`kaobook` 类加载 `glossary` 和 `imakeidx` 包, 您可以使用它们将词汇表和索引添加到您的图书中。例如, 我以前定义了一些术语表条目, 现在我将使用它们, 如下所示: `computer`。 `glossary` 还允许您使用缩略词, 如下所示: 这是完整版Frame per Second (FPS), 这是简短版FPS。这些条目将出现在术语表的后面。

Unless you use `Overleaf` or some other fancy IDE for \LaTeX , you need to run an external command from your terminal in order to compile a document with a glossary. In particular, the commands required are:¹

```
$ pdflatex main
$ makeglossaries main
$ pdflatex main
```

Note that you need not run `makeglossaries` every time you compile your document, but only when you change the glossary entries.

To create an index, you need to insert the command `\index{subject}` whenever you are talking about ‘subject’ in the text. For instance, at the start of this paragraph I would write `index{index}`, and an entry would be added to the Index in the backmatter. Check it out!

A nomenclature is just a special kind of index; you can find one at the end of this book. To insert a nomenclature, we use the package `nomencl` and add the terms with the command `\nomenclature`. We put then a `\printnomenclature` where we want it to appear.

Also with this package we need to run an external command to compile the document, otherwise the nomenclature will not appear:

1: These are the commands you would run in a UNIX system; I have no idea on how it works in Windows.

In theory, you would need to run an external command for the index as well, but luckily the package we suggested, `imakeidx`, can compile the index automatically.

```
$ pdflatex main
$ makeindex main.nlo -s nomencl.ist -o main.nls
$ pdflatex main
```

These packages are all loaded in `packages.sty`, one of the files that come with this class. However, the configuration of the elements is best done in the `main.tex` file, since each book will have different entries and styles.

This brief section was by no means a complete reference on the subject, therefore you should consult the documentation of the above package to gain a full understanding of how they work.

Note that the `nomencl` package caused problems when the document was compiled, so, to make a long story short, I had to prevent `scrhack` to load the hack-file for `nomencl`. When compiling the document on Overleaf, however, this problem seem to vanish.

9.3 Hyperreferences

In this class we provide a handy sub-package to help you referencing the same elements always in the same way, for consistency across the book. First, you can label each element with a specific command. For instance, should you want to label a chapter, you would put `\labch{chapter-title}` right after the `\chapter` directive. This is just a convenience, because `\labch` is actually just an alias to `\label{ch:chapter-title}`, so it spares you the writing of ‘ch’. We defined similar commands for many typically labeled elements, including:

- | | |
|------------------------------------|---------------------------------------|
| ▶ Page: <code>\labpage</code> | ▶ Theorem: <code>\labthm</code> |
| ▶ Part: <code>\labpart</code> | ▶ Proposition: <code>\labprop</code> |
| ▶ Chapter: <code>\labch</code> | ▶ Lemma: <code>\lablemma</code> |
| ▶ Section: <code>\labsec</code> | ▶ Remark: <code>\labremark</code> |
| ▶ Figure: <code>\labfig</code> | ▶ Example: <code>\labexample</code> |
| ▶ Table: <code>\labtab</code> | ▶ Exercise: <code>\labexercise</code> |
| ▶ Definition: <code>\labdef</code> | |

Of course, we have similar commands for referencing those elements. However, since the style of the reference should depend on the context, we provide different commands to reference the same thing. For instance, in some occasions you may want to reference the chapter by name, but other times you want to reference it only by number. In general, there are four reference style, which we call plain, vario, name, and full.

The plain style references only by number. It is accessed, for chapters, with `\refch{chapter-title}` (for other elements, the syntax is analogous). Such a reference results in: 第 9 章.

The vario and name styles rest upon the `varioref` package. Their syntax is `\vrefch{chapter-title}` and `\nrefch{chapter-title}`, and they result in: 第 9 章, for the vario style, and: 第 9 章 (参考文献), for the name style. As you can see, the page is referenced in `varioref` style.

The full style references everything. You can use it with `\frefch{chapter-title}` and it looks like this: 第 9 章 (参考文献) 第 27 页.

Of course, all the other elements have similar commands (*e.g.* for parts you would use `\vrefpart{part-title}` or something like that). However, not all elements implement all the four styles. The commands provided should be enough, but if you want to see what is available or to add the missing ones, have a look at the [attached package](#).

设计和附加功能



10 Page Design

10.1 Headings

So far, in this document I used two different styles for the chapter headings: one has the chapter name, a rule and, in the margin, the chapter number; the other has an image at the top of the page, and the chapter title is printed in a box (like for this chapter). There is one additional style, which I used only in the appendix (on page 43); there, the chapter title is enclosed in two horizontal rules, and the chapter number (or letter, in the case of the appendix) is above it.³

Every book is unique, so it makes sense to have different styles from which to choose. Actually, it would be awesome if whenever a kao-user designs a new heading style, he or she added it to the three styles already present, so that it will be available for new users and new books.

The choice of the style is made simple by the `\setchapterstyle` command. It accepts one option, the name of the style, which can be: ‘plain’, ‘kao’, or ‘lines’.⁴ If instead you want the image style, you have to use the command `\setchapterimage`, which accepts the path to the image as argument; you can also provide an optional parameter in square brackets to specify the height of the image.

Let us make some examples. In this book, I begin a normal chapter with the lines:

```
1 \setchapterstyle{kao}
2 \setchapterpreamble[u]{\margintoc}
3 \chapter{Title of the Chapter}
4 \labch{title}
```

In Line 1 I choose the style for the title to be ‘kao’. Then, I specify that I want the margin toc. The rest is ordinary administration in \LaTeX , except that I use my own `\labch` to label the chapter. Actually, the `\setchapterpreamble` is a standard KOMA-Script one, so I invite you to read about it in the KOMA documentation. Once the chapter style is set, it holds until you change it.⁵ Whenever I want to start a chapter with an image, I simply write:

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3: To be honest, I do not think that mixing heading styles like this is a wise choice, but in this document I did only to show you how they look.

4: Plain is the default \LaTeX title style; the other ones are self explanatory.

5: The `\margintoc` has to be specified at every chapter. Perhaps in the future this may change; it all depends on how this feature will be welcomed by the users, so keep in touch with me if you have preferences!

```

1 \setchapterimage[7cm]{path/to/image.png} % Optionally specify the
   height
2 \setchapterpreamble[u]{\margintoc}
3 \chapter{Catchy Title} % No need to set a chapter style
4 \labch{catchy}

```

10.2 Headers & Footers

Headers and footers in KOMA-Script are handled by the `scrlayer-scrpage` package. There are two basic style: ‘`scrheadings`’ and ‘`plain.scrheadings`’. The former is used for normal pages, whereas the latter is used in title pages (those where a new chapter starts, for instance) and, at least in this book, in the front matter. At any rate, the style can be changed with the `\pagestyle` command, *e.g.* `\pagestyle{plain.scrheadings}`.

In both stles, the footer is completely empty. In `plain.scrheadings`, also the header is absent (otherwise it wouldn’t be so plain...), but in the normal style the design is reminiscent of the ‘kao’ style for chapter titles.

To Do

The `twoside` class option is still unstable and. As always, any help will be greatly appreciated.

10.3 Table of Contents

Another important part of a book is the table of contents. By default, in `kaobook` there is an entry for everything: list of figures, list of tables, bibliographies, and even the table of contents itself. Not everybody might like this, so we will provide a description of the changes you need to do in order to enable or disable each of these entries. In the following 表 10.1, each item corresponds to a possible entry in the TOC, and its description is the command you need to provide to have such entry. These commands are specified in the attached [style package](#),⁶ so if you don’t want the entries, just comment the corresponding lines.

Of course, some packages, like those for glossaries and indices, will try to add their own entries. In such cases, you have to follow the instructions specific to that package. Here, since we have talked about glossaries and notations in 第 9 章, we will biefly see how to configure them.

For the glossaries package, use the ‘`toc`’ option when you load it: `\usepackage[toc]{glossaries}`. For `nomencl`, pass the ‘`intoc`’ option

6: In the same file, you can also choose the titles of these entries.

In a later section, we will see how you can define your own floating environment, and endow it with an entry in the TOC.

Table 10.1: Commands to add a particular entry to the table of contents.

Entry	Command to Activate
Table of Contents	<code>\setuptoc{toc}{totoc}</code>
List of Figs and Tabs	<code>\PassOptionsToClass{toc=listof}{\@baseclass}</code>
Bibliography	<code>\PassOptionsToClass{toc=bibliography}{\@baseclass}</code>

at the moment of loading the package. Both glossaries and nomencl are loaded in the attached ‘*packages*’ package.

Additional configuration of the table of contents can be performed through the `packages etoc`, which is loaded because it is needed for the `margintocs`, or the more traditional `tocbase`. Read the respective documentations if you want to be able to change the default TOC style.⁷

7: (And please, send me a copy of what you have done, I’m so curious!)

10.4 Page Layout

Besides the page style, you can also change the width of the content of a page. This is particularly useful for pages dedicated to part titles, where having the 1.5-column layout might be a little awkward, or for pages where you only put figures, where it is important to exploit all the available space.

In practice, there are two layouts: ‘wide’ and ‘margin’. The former suppresses the margins and allocates the full page for contents, while the latter is the layout used in most of the pages of this book, including this one. The wide layout is also used automatically in the front and back matters.

To change page layout, use the `\pagelayout` command. For example, when I start a new part, I write:

```
1 \pagelayout{wide}
2 \addpart{Title of the New Part}
3 \pagelayout{margin}
```

10.5 Numbers & Counters

In this short section we shall see how dispositions, sidenotes and figures are numbered in the `kaobook` class.

By default, dispositions are numbered up to the section. This is achieved by setting: `\setcounter{secnumdepth}{1}`.

The sidenotes counter is the same across all the document, but if you want it to reset at each chapter, just uncomment the line

```
\counterwithin*{sidenote}{chapter}
```

in the `styles/style.sty` package provided by this class.

Figure and Table numbering is also per-chapter; to change that, use something like:

```
\renewcommand{\thefigure}{\arabic{section}.\arabic{figure}}
```

10.6 White Space

One of the things that I find most hard in \LaTeX is to finely tune the white space around objects. There are not fixed rules, each object needs its own adjustment. Here we shall see how some spaces are defined at the moment in this class.

Attention! This section may be incomplete.

Space around figures and tables

```
\renewcommand\FBskip{.4\topskip}
\renewcommand\FBbskip{\FBskip}
```

Space around captions

```
\captionsetup{
  aboveskip=6pt,
  belowskip=6pt
}
```

Space around displays (*e.g.* equations)

```
\setlength\abovedisplayskip{6pt plus 2pt minus 4pt}
\setlength\belowdisplayskip{6pt plus 2pt minus 4pt}
\abovedisplayskip 10\p@ \@plus2\p@ \@minus5\p@
\abovedisplayshortskip \z@ \@plus3\p@
\belowdisplayskip \abovedisplayskip
\belowdisplayshortskip 6\p@ \@plus3\p@ \@minus3\p@
```

11.1 定理

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11.3 Experiments 39

尽管大多数人抱怨看到一本充满公式的书，数学却是许多书的重要组成部分。在这里，我们将说明一些可能性。我们认为定理、定义、注释和例子都应该在阴影的背景下加以强调；然而，颜色不应该是沉重的眼睛，所以我们选择了一种淡黄色。⁹

定义 11.1.1 *Let (X, d) be a metric space. A subset $U \subset X$ is an open set if, for any $x \in U$ there exists $r > 0$ such that $B(x, r) \subset U$. We call the topology associated to d the set τ_d of all the open subsets of (X, d) .*

定义 11.1.1 是非常重要的。我不是在开玩笑，但是我插入这个短语只是为了说明如何引用定义。下面的语句在不同的环境中反复出现。

定理 11.1.1 *A finite intersection of open sets of (X, d) is an open set of (X, d) , i.e τ_d is closed under finite intersections. Any union of open sets of (X, d) is an open set of (X, d) .*

命题 11.1.2 *A finite intersection of open sets of (X, d) is an open set of (X, d) , i.e τ_d is closed under finite intersections. Any union of open sets of (X, d) is an open set of (X, d) .*

引理 11.1.3 *A finite intersection^a of open sets of (X, d) is an open set of (X, d) , i.e τ_d is closed under finite intersections. Any union of open sets of (X, d) is an open set of (X, d) .*

^a I'm a footnote

您可以安全地忽略定理...的内容，我假设，如果您对课本中的定理感兴趣，那么您已经了解了一些关于添加它们的经典方法。这些示例应该只显示您在这个类中可以做的所有事情。

推论 11.1.4 (Finite Intersection, Countable Union) *A finite intersection of open sets of (X, d) is an open set of (X, d) , i.e τ_d is closed under finite intersections. Any union of open sets of (X, d) is an open set of (X, d) .*

Proof. 证明留给读者作为一个简单的练习。提示:

水厂共当而面三张，白家决空给意层般，单重总歼者新。每建马先口住月大，究平克满现易手，省否何安苏京。两今此叫证程事元七调联派业你，全它精据闻属医拒严力步青。厂江内立拉清义边指，况半严回和得话，状整度易芬列。再根心应得信飞往清增，至例联集采家同严热，地手蠹持查受立询。统定发几满斯究后参边增消与内关，解

9: 这里的所有框都是相同的颜色，因为我们不希望我们的文档看起来像Harlequin。

You can even insert footnotes inside the theorem environments; they will be displayed at the bottom of the box.

系之展习历李还也村酸。制周心值示前她志长步反，和果使标电再主它这，即务解早八战根交。是中文之象万影报头，与劳工许格主部确，受经更奇小极准。形程记持件志各质天因时，据据极清总命所风式，气太束书家秀低坟也。期之才引战对已公派及济，间究办儿转情革统将，周类弦具调除声坑。两了济素料切要压，光采用级数本形，管县任其坚。切易表候完铁今断土马他，领先往样拉口重把处千，把证建后苍交码院眼。较片的集节片合构进，入化发形机已斯我候，解肃飞口严。技时长次士员况属写，器始维期质离色，个至村单原否易。重铁看年程第则于去，且它后基格并下，每收感石形步而。

□

Here is a random equation, just because we can:

$$x = a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \frac{1}{a_4}}}}$$

定义 11.1.2 Let (X, d) be a metric space. A subset $U \subset X$ is an open set if, for any $x \in U$ there exists $r > 0$ such that $B(x, r) \subset U$. We call the topology associated to d the set τ_d of all the open subsets of (X, d) .

例 11.1.1 Let (X, d) be a metric space. A subset $U \subset X$ is an open set if, for any $x \in U$ there exists $r > 0$ such that $B(x, r) \subset U$. We call the topology associated to d the set τ_d of all the open subsets of (X, d) .

备注 11.1.1 Let (X, d) be a metric space. A subset $U \subset X$ is an open set if, for any $x \in U$ there exists $r > 0$ such that $B(x, r) \subset U$. We call the topology associated to d the set τ_d of all the open subsets of (X, d) .

As you may have noticed, definitions, example and remarks have independent counters; theorems, propositions, lemmas and corollaries share the same counter.

备注 11.1.2 Here is how an integral looks like inline: $\int_a^b x^2 dx$, and here is the same integral displayed in its own paragraph:

$$\int_a^b x^2 dx$$

We provide two files for the theorem styles: `plaintheorems.sty`, which you should include if you do not want coloured boxes around theorems; and `mdftheorems.sty`, which is the one used for this document.¹⁰ Of course, you will have to edit these files according to your taste and the general style of the book.

10: The plain one is not showed, but actually it is exactly the same as this one, only without the yellow boxes.

11: Notice that in the table of contents and in the header, the name of this section is 'Boxes & Environments'; we achieved this with the optional argument of the section command.

11.2 Boxes & Custom Environments¹¹

Say you want to insert a special section, an optional content or just something you want to emphasise. We think that nothing works better than a box in these cases. We used `mdframed` to construct the ones shown below. You can create and modify such environments by editing the provided file `environments.sty`.

盒子标题

她己道接收面学上全始，形万然许压己金史好，力住记赤则引秧。处高方据近学级素专，者往构支明系状委起查，增子束孤不般前。相斗真它增备听片思三，听花连次志平品书消情，清市五积群面县开价现准此省持给，争式身在南决就集般，地力秧众团计。日车治政技便角想持中，厂期平及半干速区白土，观合村究研称始这少。验商眼件容果经风中，质江革再的采心年专，光制单万手斗光就，报却蹦杯材。内同数速果报做，属马市参至，入极将管医。但强质交上能只拉，据特光农无五计据，来步孤平葡院。江养水图再难气，做林因列行消特段，就解届罐盛。定她识决听人自打验，快思月断细面便，事定什呀传。边力心层下等共命每，厂五交型车想利，直下报亲积速。元前很地传气领权节，求反立全各市状，新上所走值上。明统多表过变物每区广，会王问西听观生真林，二决定助议苏。格节基全却及飞口悉，难之规利争白观，证查李却调代动斗形放数委同领，内从但五身。当了美话也步京边但容代认，放非边建按划近些派民越，更具建火法住收保步连。

如果设置了计数器，甚至可以创建自己的编号环境。

注释 11.2.1

术厂美义据那张别安响物，县交极长选行值深专质，眼心段极型新。格形连候眼王本加还题但，流但作基白具地机系，总严录件杰报前易。际取通主农题议需之从业少，江以受断件扮伴自。不度传间品全，青层自内治子，其询体员种。领角速院术计目化每具，体这常住更实记，在应争却根陕员。自传不展持心方约厂，济件过所转特济，外达才部至局。习例件气保候府社它，算际小毛相角方车次场马，难切龙弦制形界办。感头两华交务毛林回都节业点，两群月具受们即积生。调直给这着风火能圆商一，知易众美布会亲军千，件声坑志支较学。农六斯南何记子机量各然，快写线信权间越部色，象照屈型部物治地长。难要技第对老共达质标压心，才种日自针豆助养。政快下正型究条东话加争行整便，些改民流花按低重伸你。院心没离则收称革局，七件小收月通示布，导外员林村增。革电认速志海再事满传海，京深二百明家打开识连，林备转刷位体置进义。治风理年构族业酸整要第，认取历难丽园变队。

11.3 Experiments

也可以在盒子里包装边注。我们鼓励大胆的读者尝试自己的实验，并让我知道结果。

我相信许多其他特殊的事情是可能的与类 `kaobook` 类。在开发过程中，我努力使它尽可能灵活，这样就可以不费太大力气地添加新特性。因此，我希望你们能在这门课的写作中找到最好的方式来表达自己，写一本书，写一篇报告或者写一篇论文，我也很想看看你们可以尝试的任何实验的结果。

title of margin note

使用 `kaobox` 盒子的边注。
(实际上, `kaobox` 是在 `marginnote` 里面!)

附录



Heading on Level 0 (chapter)

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gef-burn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

A.1 Heading on Level 1 (section)

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gef-burn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Heading on Level 2 (subsection)

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gef-burn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Heading on Level 3 (subsubsection)

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there

a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Heading on Level 4 (paragraph) Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

A.2 Lists

Example for list (itemize)

- ▶ First item in a list
- ▶ Second item in a list
- ▶ Third item in a list
- ▶ Fourth item in a list
- ▶ Fifth item in a list

Example for list (4*itemize)

- ▶ First item in a list
 - First item in a list
 - * First item in a list
 - First item in a list
 - Second item in a list
 - * Second item in a list
 - Second item in a list
- ▶ Second item in a list

Example for list (enumerate)

1. First item in a list
2. Second item in a list
3. Third item in a list
4. Fourth item in a list
5. Fifth item in a list

Example for list (4*enumerate)

1. First item in a list
 - a) First item in a list
 - i. First item in a list
 - A. First item in a list
 - B. Second item in a list
 - ii. Second item in a list
 - b) Second item in a list
2. Second item in a list

Example for list (description)

First item in a list
Second item in a list
Third item in a list
Fourth item in a list
Fifth item in a list

Example for list (4*description)

First item in a list
 First item in a list
 First item in a list
 Second item in a list
 Second item in a list
 Second item in a list
Second item in a list

参考文献

Here are the references in citation order.

- [1] Virgilio Gómez-Rubio. *Bayesian inference with INLA*. CRC Press, 2020 (cited on page vii).
- [2] Robin Lovelace, Jakub Nowosad, and Jannes Muenchow. *Geocomputation with R*. CRC Press, 2019 (cited on page vii).
- [3] Paula Moraga. *Geospatial health data: Modeling and visualization with R-INLA and shiny*. CRC Press, 2019 (cited on page vii).
- [4] Christopher K Wikle, Andrew Zammit-Mangion, and Noel Cressie. *Spatio-temporal Statistics with R*. CRC Press, 2019 (cited on page vii).
- [5] Roger S Bivand et al. *Applied spatial data analysis with R 2nd edition*. Springer, 2013 (cited on page vii).
- [6] Elias T Krainski et al. *Advanced spatial modeling with stochastic partial differential equations using R and INLA*. CRC Press, 2018 (cited on page vii).
- [7] Mitzi Morris et al. ‘Bayesian hierarchical spatial models: Implementing the Besag York Mollié model in stan’. In: *Spatial and spatio-temporal epidemiology* 31 (2019), p. 100301 (cited on page 11).
- [8] Julian Besag, Jeremy York, and Annie Mollié. ‘Bayesian image restoration, with two applications in spatial statistics’. In: *Annals of the institute of statistical mathematics* 43.1 (1991), pp. 1–20 (cited on page 11).
- [9] Brian G Leroux, Xingye Lei, and Norman Breslow. ‘Estimation of disease rates in small areas: a new mixed model for spatial dependence’. In: *Statistical models in epidemiology, the environment, and clinical trials*. Springer, 2000, pp. 179–191 (cited on page 11).
- [10] Peter M Visscher, William G Hill, and Naomi R Wray. ‘Heritability in the genomics era—concepts and misconceptions.’ In: *Nat. Rev. Genet.* 9.4 (2008), pp. 255–266. DOI: [10.1038/nrg2322](https://doi.org/10.1038/nrg2322) (cited on page 27).
- [11] Gareth James et al. *An Introduction to Statistical Learning*. 2013 (cited on page 27).

Notation

The next list describes several symbols that will be later used within the body of the document.

- c Speed of light in a vacuum inertial frame
- h Planck constant

Greek letters with pronunciation

Character	Name	Character	Name
α	alpha <i>AL-fuh</i>	ν	nu <i>NEW</i>
β	beta <i>BAY-tuh</i>	ξ, Ξ	xi <i>KSIGH</i>
γ, Γ	gamma <i>GAM-muh</i>	\omicron	omicron <i>OM-uh-CRON</i>
δ, Δ	delta <i>DEL-tuh</i>	π, Π	pi <i>PIE</i>
ϵ	epsilon <i>EP-suh-lon</i>	ρ	rho <i>ROW</i>
ζ	zeta <i>ZAY-tuh</i>	σ, Σ	sigma <i>SIG-muh</i>
η	eta <i>AY-tuh</i>	τ	tau <i>TOW (as in cow)</i>
θ, Θ	theta <i>THAY-tuh</i>	υ, Υ	upsilon <i>OOP-suh-LON</i>
ι	iota <i>eye-OH-tuh</i>	ϕ, Φ	phi <i>FEE, or FI (as in hi)</i>
κ	kappa <i>KAP-uh</i>	χ	chi <i>KI (as in hi)</i>
λ, Λ	lambda <i>LAM-duh</i>	ψ, Ψ	psi <i>SIGH, or PSIGH</i>
μ	mu <i>MEW</i>	ω, Ω	omega <i>oh-MAY-guh</i>

Capitals shown are the ones that differ from Roman capitals.

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