Assignment 4: Problem 1

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1. Consider the following integral $\int_0^\pi sin(t)dt$.

(a) Use the Trapezoidal Rule to estimate the integral using $n = 4$ subintervals.

The Trapezoidal Rule states that given a certain number of subintervals, in this case 4, you can estimate the value of an integral using the formula: $T_4 = \frac{\Delta x}{2}(y_o + 2y_1 + 2y_2 + 2y_3 + y_4)$ where y_i is equal to the integrand evaluated at the given x_i value that was calculated from the Δx's.

In this case, $x_0 = 0$ $x_1 = \frac{\pi}{4}$ $x_2 = \frac{\pi}{2}$ $x_3 = \frac{3\pi}{4}$ $x_4 = \pi$ and thus $y_0 = 0$ $y_1 = \frac{\sqrt{2}}{2}$ $y_2 = 1$ $y_3 = \frac{\sqrt{2}}{2}$ $y_4 = 0$.

Therefore $T_4 = \frac{\pi}{8}(y_0 + 2y_1 + 2y_2 + 2y_3 + y_4) = \frac{\pi}{8}(\sqrt{2} + 2 + \sqrt{2}) \approx 1.896119$.

(b) Use Simpson's Rule to estimate the integral using $n = 4$ subintervals.

For Simpson's Rule we use the same x's and y's as in the Trapezoidal rule, but the formula is slightly different. $S_4 = \frac{\Delta x}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4) = \frac{\pi}{12}(2\sqrt{2} + 2 + 2\sqrt{2}) \approx 2.004560$.

(c) Find an upper bound for the Error $|E_T|$ and $|E_S|$ for the Trapezoidal Rule and Simpson's Rule using 4 subintervals, respectively.

To find the upper bound of $|E_T|$ we must use the formula $E_T = \frac{k(b-a)^3}{12n^2}$ where $|f''(x)| \leq k$.

$f'(t) = cos(t)$
$f''(t) = -sin(t)$

The maximum value of $|f''(x)| = 1$ since the vertical bound of cosine and sine is 1, which indicates that $k = 1$.

Therefore $|E_T| = \frac{\pi^3}{12(4^2)} \approx 0.161491$.

The formula for $|E_S| = \frac{L(b-a)^5}{180n^4}$ where $|f^{(4)}(x)| \leq L$.

We have already calculated the first two derivatives of the integrand, so the next two are:

$f'''(t) = -cos(t)$
$f^{(4)}(t) = sin(t)$

And by the same reasoning as in estimating the previous error bound, the maximum of cosine and sine is 1, and thus $k = 1$.

Therefore $|E_S| = \frac{\pi^5}{180(4^4)} \approx 0.006641$.

(d) Evaluate the integral directly and find $|E_T|$ and $|E_S|$.

$\int_0^\pi sin(t)dt = -cos(t)|_0^\pi = -cos(\pi) - (-cos(0)) = 2$
$|E_T| = |2 - 1.896119| = 0.103881$
$|E_S| = |2 - 2.004560| = 0.004560$