Assignment 5 Problem One

Michael Cai

February 27, 2016

1. Find all the values of $p$for which the integral $\int_0^1 x^p ln(x)dx$ converges.

$\int_0^1 x^p ln(x)dx = \lim_{b\to 0^+} \int_b^1 x^p ln(x)dx$

First we use IBP, and let $u = ln(x)$ and $du = \frac{1}{x}dx$. $dv = x^p dx$ and $v = \frac{1}{p+1}x^{p+1}$.

$= \lim_{b\to 0^+} \frac{1}{p+1}x^{p+1}ln(x)|_b^1 - \frac{1}{p+1}\int_b^1 x^p dx$

$= \lim_{b\to 0^+} \frac{1}{p+1}[1^{p+1}ln(1) - b^{p+1}ln(b)] - \frac{1}{p+1}(\frac{1}{p+1}x^{p+1})|_b^1$

$= \lim_{b\to 0^+} \frac{1}{p+1}[1^{p+1}ln(1) - b^{p+1}ln(b)] - \frac{1}{(p+1)^2}[1^{p+1} - b^{p+1}]$

$= \frac{1}{p+1}[-b^{p+1}ln(b)] - \frac{1}{(p+1)^2} - \frac{b^{p+1}}{(p+1)^2}$

Because the $\lim_{b\to 0^+} -b^{p+1}$ grows toward 0 and $ln(b)$ grows toward $-\infty$. Because we have an indeterminate form, we must use L'Hospitals Rule.

$-b^{p+1}ln(b) = -\frac{ln(b)}{b^{-p-1}}$

$= -\frac{\frac{1}{b}}{(-p-1)b^{-p-2}} = -\frac{1}{(-p-1)b^{-p-1}} = \frac{b^{p+1}}{p+1}$

Now to plug this back into the original equation:

$\lim_{b\to 0^+} \frac{1}{p+1}[-b^{p+1}ln(b)] - \frac{1}{(p+1)^2} = \lim_{b\to 0^+} \frac{b^{p+1}}{p+1} - \frac{1}{(p+1)^2}$

Therefore for this integral to converge, $p > -1$ so that there will be no 0's in the denominators of the two separate terms, and so that b will have a positive exponent (thus again ensuring no 0's will be in the denominators).