Assignment 7 Problem 4

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4. Two people walk along different paths starting at the origin, such that they have the same positive x-coordinate at each time. One follows the positive x-axis, while the other follows the curve $y = \frac{2}{3}x^{\frac{3}{2}}$. Find the point at which one person has walked twice as far as the other.

Again, we use the arc length formula to measure the "distance traveled" by the person walking along the arc.

$y = \frac{2}{3}x^{\frac{3}{2}}$

$y' = x^{\frac{1}{2}}$

$(y')^2 = x$

$L = \int_0^b \sqrt{1+x}dx$, where b indicates the x-coordinate that the person walking along the arc is currently on.

Let $u = 1+x$, thus $du = dx$, and the limits equal $ulim = b+1$ and $llim = 1$.

$= \int_1^{b+1} u^{\frac{1}{2}} du$

$= [\frac{2}{3}u^{\frac{3}{2}}]\Big|_1^{b+1} = \frac{2}{3}(b+1)^{\frac{3}{2}} - \frac{2}{3}$

We know that the distance of the man walking along the x-axis is just going to be x, whereas the distance for the man walking along the arc is going to be given by the formula I just derived.

Because an arc will always be longer than an x-value at a given point x (we know this if we approximate an arc to be a straight line, then the hypotenuse would represent that straightened arc and the bottom edge of the triangle would represent the x-axis), we know that the man walking along the arc will always be ahead of the man walking along the x-axis at points $x > 0$.

Thus we must calculate the point in time when the arc man is twice as far as the x-axis man.

$\frac{2}{3}(b+1)^{\frac{3}{2}} - \frac{2}{3} = 2b$

$\frac{1}{3}(b+1)^{\frac{3}{2}} - \frac{1}{3} = b$

$\frac{1}{3}(b+1)^{\frac{3}{2}} = b + \frac{1}{3}$

$(b+1)^{\frac{3}{2}} = 3b+1$

$(b+1)^3 = (3b+1)^2$

If you expand the factorization and set the polynomial equal to zero you get:

$x^3 - 6x^2 - 3x = 0$

$x(x^2 - 6x - 3) = 0$

Using the quadratic formula to solve for the roots of the quadratic term:

$x = \frac{6 \pm \sqrt{6^2 - 4(1)(-3)}}{2}$

$x = \frac{6 \pm \sqrt{48}}{2} = \frac{6 \pm 4\sqrt{3}}{2} = 3 \pm 2\sqrt{3}$, but since $3 - 2\sqrt{3}$ is negative, which would not make sense in the context of the problem, the two roots that we get are $x = 0$ and $x = 3 + 2\sqrt{3}$.

Thus, the point at which the person walking along the arc has walked double the distance as the one walking along the axis will be $x = 3 + 2\sqrt{3}$.