Assignment 10 Problem 3

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3. Determine whether the following alternating series is absolutely convergent, conditionally convergent, or divergent.

$\Sigma_{n=0}^{\infty} \frac{(-1)^n}{2n+3}$

First I am going to check whether or not the series is absolutely convergent using the Limit Comparison Test.

$\Sigma_{n=0}^{\infty} \frac{1}{2n+3}$

$a_n = \frac{1}{n}$

$b_n = \frac{1}{2n+3}$

$\frac{a_n}{b_n} = \frac{\frac{1}{n}}{\frac{1}{2n+3}} = \frac{2n+3}{n} = 2 + \frac{3}{n}$

As $\lim_{n\to\infty} 2 + \frac{3}{n} = 2$

Because 2 is a constant between 0 and infinity, if a_n converges/diverges so does b_n.

We know that a_n diverges because it is the Harmonic Series.

Therefore, b_n diverges, and thus the original series is not absolutely convergent.

Let us check whether it is conditionally convergent using the Alternating Series Test.

To conduct this test we must check 3 conditions for $b_n = \frac{1}{2n+3}$

1. $b_n > 0$ (positive)

2.$\{b_n\}$ is decreasing

3. $\lim_{n\to\infty} b_n = 0$

The series is positive, decreasing, and as we take the limit it approaches 0. Thus all 3 conditions are met, and therefore the alternating series $\Sigma_{n=1}^{\infty}(-1)^{n-1}b_n$ converges.

Thus, the original series is conditionally convergent.