Assignment 13 Problem Three

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3. Find the area enclosed by the y-axis and the curve: $x = t - t^2$**,** $y = 1 + e^{-t}$

$y = 1 + e^{-t}$
$y - 1 = e^{-t}$
$ln(y-1) = -t$
$t = -ln(y-1)$
$x = -ln(y-1) - (-ln(y-1))^2$

To find the area enclosed by the $y-$axis and the curve, I set $x = 0$ to find the intersections with the $y-$axis.

$0 = -ln(y-1) - (-ln(y-1))^2$
$0 = ln(y-1) + ln(y-1)^2$
$0 = ln(y-1)(1 + ln(y-1))$

In this factored form, each factor is an intersection point with the $y-$axis.

$ln(y-1) = 0$
$y - 1 = e^0$
$y = 2$
$1 + ln(y-1) = 0$
$ln(y-1) = -1$
$y - 1 = e^{-1}$
$y = \frac{1}{e} + 1$

Recall:

$t = -ln(y-1)$
$t = -ln(2-1) = -ln(1) = 0$

Thus $\alpha = 0$

$t = -ln(\frac{1}{e} + 1 - 1) = -ln(\frac{1}{e}) = -ln(1) - (-ln(e)) = ln(e) = 1$

Thus $\beta = 1$

Now finally to set up our equation to find the area enclosed in the curve we use the formula:

$\int_\alpha^\beta g(t)f'(t)dt$, where $g(t) = 1 + e^{-t}$ and $f'(t) = 1 - 2t$

$= \int_0^1 (1 + e^{-t})(1 - 2t)dt$
$= \int_0^1 1 - 2t + e^{-t} - 2te^{-t}dt$
$= \int_0^1 1dt - 2\int_0^1 tdt + \int_0^1 e^{-t} - 2\int_0^1 te^{-t}dt$

For the integral on the end we must use IBP:

Let $u = t$, $du = dt$, $dv = e^{-t}dt$, $v = -e^{-t}$

And we get for the total equation:

$=\int_0^1 1dt - 2\int_0^1 tdt + \int_0^1 e^{-t} - 2(-te^{-t} + \int_0^1 e^{-t}dt)$
$= [t]_0^1 - 2[\frac{1}{2}t^2]_0^1 + [-e^{-t}]_0^1 + 2te^{-t}\Big|_0^1 - 2[-e^{-t}]_0^1$
$= \frac{3}{e} - 1$