Assignment 5 Problem Three

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February 27, 2016

3. If the birth rate of a population is $b(t) = 2200e^{0.0024t}$ people per year and the death rate is $d(t) = 1460e^{0.018t}$ people per year, find the area between these curves for $0 \le t \le 10$. What does this area represent? $\quad b(10) \approx 2253.44$ and $d(10) \approx 1747.94$

Because these are strictly increasing on this interval and because at $t = 10$, $b(10) > d(10)$, this indicates that throughout the interval $b(t) > d(t)$. Because $d(t)$ has a faster exponential growth rate than $b(t)$ there will be a point when $d(t)$ outpaces $b(t)$, but since we've shown that $b(10) > d(10)$ then we can conclude that the two functions have yet to intersect, and that in this interval $b(t)$ is larger than $d(t)$.

$\int_0^{10} 2200e^{0.0024t} - 1460e^{0.018t}dt = 2200 \int_0^{10} e^{0.0024t}dt - 1460 \int_0^{10} e^{0.0018t}dt$

$= 2200[\frac{1}{1.0024}e^{1.0024t}]\Big|_0^{10} - 1460[\frac{1}{1.0018}e^{1.0018t}]\Big|_0^{10}$

$= 2200[\frac{1}{1.0024}e^{1.0024(10)} - \frac{1}{1.0024}e^{1.0024(0)}] - 1460[\frac{1}{1.0018}e^{1.0018(10)} - \frac{1}{1.0018}e^{1.0018(0)}]$

≈ 6269.61

This area represents a 6269.61 person population increase over the timeframe t $[0,10]$ (since it was the integral of the birth rate minus the death rate, i.e. the cumulative population change).