Assignment 13 Problem Four

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4. Find the length of the curve $x = cos(t)$,$y = t + sin(t)$**,** $0 \leq t \leq \pi$

The arc-length formula for a parametric curve is given by the following:

$\int_{\alpha}^{\beta} \sqrt{\frac{dx}{dt}^2 + \frac{dy}{dt}^2} dt$

$x = cos(t)$

$\frac{dx}{dt} = -sin(t)$

$y = t + sin(t)$

$\frac{dy}{dt} = 1 + cos(t)$

Thus with $\alpha = 0$, and $\beta = \pi$

We have $\int_0^{\pi} \sqrt{(-sin(t))^2 + (1 + cos(t))^2 dt}$

$\int_0^{\pi} \sqrt{sin^2 t + 1 + 2cos(t) + cos^2 t dt}$

Because of the Pythagorean Identity, $cos^2 t + sin^2 t = 1$

$\int_0^{\pi} \sqrt{2 + 2cos(t)} dt$

$\int_0^{\pi} \sqrt{2(1 + cos(t)) dt}$

Recall the double angle formula:

$cos(2\alpha) = cos^2\alpha - sin^2\alpha = 2cos^2\alpha - 1$

Therefore, we consider $cos(t) = cos(2(\frac{t}{2}))$

And thus we get:

$\int_0^{\pi} \sqrt{2(1 + 2cos^2 \frac{t}{2} - 1)} dt$

$\int_0^{\pi} \sqrt{4cos^2 \frac{t}{2}} dt = \int_0^{\pi} 2cos(\frac{t}{2}) dt$

U substitute $u = \frac{t}{2}, du = \frac{1}{2} dt$, $ulim = \frac{\pi}{2}, llim = 0$

$\int_0^{\frac{\pi}{2}} 4cos(u) du$

$4[sin(u)]_0^{\frac{\pi}{2}} = 4$