Assignment 6 Problem Four

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4. Let D be the region graphed below

Let E_1 be the solid obtained by revolving D around the x-axis. Let E_2 be the solid obtained by revolving D around the y-axis. Use Simpson's rule with eight subdivisions to approximate the volumes of E_1 and E_2.

$S_8 : \Delta x = \frac{10-2}{8} = 1$

$x_0 = 2$, $x_1 = 3$, $x_2 = 4$, $x_3 = 5$, $x_4 = 6$, $x_5 = 7$, $x_6 = 8$, $x_7 = 9$, $x_8 = 10$

$y_0 = 0$, $y_1 = 1.5$, $y_2 = 2$, $y_3 = 2.2$, $y_4 = 3$, $y_5 = 3.8$, $y_6 = 4$, $y_7 = 3$, $y_8 = 0$

When calculating the volume of this shape revolved around the x-axis, the cross sections will be disks, and thus the volume will be $\int A(x)dx = \int \pi r^2 dx$. The r's are the distance from the x-axis to the y-value on the curve. Because Simpson's rule uses parabolas to approximate curves, the volume approximation must be split into the separate parabolic curves used for estimation.

$E_1 = \int_2^{10} \pi f(x)^2 dx = \pi \int_2^{10} f(x)^2 dx$. We can think of $f(x)^2$ as a new function $g(x)$. Since we are just given the shape of the curve and not the actual function, we can approximate the volume of the solid by instead calculating the area under the new curve $g(x)$, which is formed by $f(x)^2$ on the interval $[2, 10]$, and this we can do using the Simpson's rule approximation.

$E_1 = \pi * S_{8,g(x)} = \pi * \frac{1}{3}[0^2 + 4*1.5^2 + 2*2^2 + 4*2.2^2 + 2*3^2 + 4*3.8^2 + 2*4^2 + 4*3^2 + 0^2] \approx \frac{180.12\pi}{3} \approx 188.621.$

The volume of $E_1 \approx 188.621$ cubic units.

The formula to calculate the volume of E_2 is $E_2 = \int A(y)dy$, but because of the shape of the curve you cannot use the washer method. The washer method would not work because the function is not strictly increasing with x, and so if you were to create washers with the outer edge on $x = 10$, then you would have extra empty space to consider between $x = [8, 10]$ (assuming $x = 8$ is the inflection point of the curve).

Therefore, you must employ the shell method (cross-sections cutting parallel to the y-axis).

$E_2 = \int_2^{10} 2\pi r h \Delta r = 2\pi \int_2^{10} r h \Delta r$. The $r$here is just the x-coordinate, and the h here is the y-coordinate.

Thus, $E_2 = 2\pi \int_2^{10} x f(x) dx$. Let's think of $xf(x)$ as a new function $g(x)$. We can again approximate the volume of the solid by instead calculating the area under the new curve $g(x)$.

$E_2 = 2\pi * S_{8,g(x)} = 2\pi * \frac{1}{3}[2*0 + 3*4(1.5) + 4*2(2) + 5*4(2.2) + 6*2(3) + 7*4(3.8) + 8*2(4) + 9*4(3) + 10*0]$

$E_2 \approx 821.841$

The volume of $E_2 \approx 821.841$ cubic units.