Assignment 2 Problem 3

Michael Cai

February 7, 2016

3.
(a) *Use the formulas for* $sin(A+B)$ *and* $sin(A-B)$ *to show that:*
$sinAcosB = \frac{1}{2}[sin(A-B) + sin(A+B)]$

The Angle Addition Identity for sin states:
$sin(A+B) = sinAcosB + cosAsinB$
$sin(A-B) = sinAcosB - cosAsinB$

Prove: $sinAcosB = \frac{1}{2}[sin(A-B) + sin(A+B)$

$sin(A+B) = sinAcosB + cosAsinB$
$sinAcosB = sin(A+B) - cosAsinB$
Now add $sin(A-B)$ to both sides
$sinAcosB + sin(A-B) = sin(A+B) - cosAsinB + sin(A-B)$
Expand on the left hand side
$sinAcosB + sinAcosB - cosAsinB = sin(A = B) + sin(A+B) - cosAsinB$
$2sinAcosB - cosAsinB + cosAsinB = sin(A-B) + sin(A+B)$

Therefore $sinAcosB = \frac{1}{2}(sin(A-B) + sin(A+B))$

(b) *Use part (a) to evaluate* $\int sin(3x)cos(x)dx$
Recall from part (a) that $sinAcosB = \frac{1}{2}(sin(A-B) + sin(A+B))$
Plug it in:
$\int sin(3x)cos(x)dx = \frac{1}{2}(sin(3x-x) + sin(3x+x))$
$= \frac{1}{2}\int sin(2x) + sin(4x)dx$
$= \frac{1}{2}\int sin(2x)dx + \frac{1}{2}\int sin(4x)dx$

First consider $\int sin(2x)dx$:
By the double angle formula, $\int sin(2x)dx = \int 2sinxcosxdx$
Therefore, we I use u-substitution on $\in 2sinxcosxdx$ and set $u = sinx$ making $du = cosxdx$.
Thus $\int 2sinxcosx = 2\int udu = 2(\frac{1}{2}u^2 + C) = u^2 + C = sin^2(x) + C$

Now we consider $\int sin(4x)dx$:
Also apply the double angle formula.
$\int sin(4x)dx = \int sin(2(2x))dx = \int 2sin(2x)cos(2x)dx$
If you expand both the sin(2x) and cos(2x) components of this new integral using the double angle formula you get:
$2\int(2sin(x)cos(x))(cos^2(x) - sin^2(x))dx$
$= 2\int 2sinxcos^3(x) - 2sin^3(x)cos(x)dx$
$= 4\int cos^3(x)sin(x)dx - 4\int sin^3(x)cos(x)dx$

Now we use u-substitution for both integrals (u for the first and u' for the second)
Let $u = cos(x)$ and thus $du = -sin(x)dx$.
Let $u' = sin(x)$ and thus $du' = cos(x)dx$.

Therefore, $4\int cos^3(x)sin(x)dx - 4\int sin^3(x)cos(x)dx = 4\int u^3du - 4\int u'^3du'$
$= -4(\frac{1}{4}u^4 + C) - 4(\frac{1}{4}u'^4 + C')$
$= -cos^4(x) - sin^4(x) + C''$

Plugging it all back together we get:
$\int sin(3x)cos(x)dx = \frac{1}{2}(sin(2x) + sin(4x))$
$= \frac{1}{2}(-cos^4(x) - sin^4(x) + C'') + \frac{1}{2}(sin^2(x) + C)$
$= -\frac{1}{2}cos^4(x) - \frac{1}{2}sin^4(x) + \frac{1}{2}sin^2(x) + C$