Assignment 6 Problem One

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1. Find the volume of the following solid. The solid lies between planes perpendicular to the x-axis at $x = -1$ and $x = 1$. The cross sections perpendicular to the x-axis are circular disks whose diameters run from the parabola $y = x^2$to the parabola $y = 2 - x^2$.

The shape that is formed by this description is a lemon-shaped object that is bounded by the two planes perpendicular to the x-axis at $x = -1$ and $x = 1$. Because the cross-sections are circular disks, we have to calculate the area of the disks and to do that we must find the radius. Because the curves $y = x^2$ and $y = 2 - x^2$ intersect at $y = 1$ when $x = -1$ and $x = 1$, we know that the line $y = 1$ slices the shape into two perfect halves. Thus the radius of one of the disks can be calculated by subtracting 1 from the top curve $y = 2 - x^2$, $r = 1 - x^2$. The volume formula we will use is $\int A(x)dx$, where $A(x) = \pi r^2$ because the cross sections are circular disks, which run perpendicular to the x-axis. Thus for calculating the volume of the shape, we have $\int_{-1}^{1} A(x)dx = \int_{-1}^{1} \pi r^2 dx = \int_{-1}^{1} \pi(1 - x^2)dx = \pi \int_{-1}^{1} 1 - 2x^2 + x^4 dx$.

$= \pi[x - \frac{2}{3}x^3 + \frac{1}{5}x^5]\Big|_{-1}^{1} = \pi[1 - \frac{2}{3}(1) + \frac{1}{5}(1) - (-1 - \frac{2}{3}(-1) + \frac{1}{5}(-1)] = \frac{16\pi}{15}$

Therefore the volume of the following solid is equal to $\frac{16\pi}{15}$.