Assignment 2 Problem 1

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1. *Let f be twice differentiable with $f(0) = 6$, $f(1) = 5$, and $f'(1) = 2$. Use integration by parts together with the given information to evaluate the integral $\int_0^1 x f''(x) dx$*

For the integral $\int_0^1 x f''(x) dx$ it is obvious that we must use integration by parts since the integrand is a product. Therefore, I am going to choose $u = x$ and thus $du = dx$ and $dv = f''(x) dx$ and thus $v = f'(x)$ because I want the u term to become simpler upon using IBP.

Therefore, $\int_0^1 x f''(x) dx = x f'(x) - \int_0^1 f'(x) dx$
$= x f'(x)\big|_1^0 - \int_0^1 f'(x) dx$
$= x f'(x)\big|_1^0 - f(x)\big|_1^0$
$= 1 f'(1) - 0 f'(0) - (f(1) - f(0))$
$= 2 - 0 - 5 + 6$

$\int_0^1 x f''(x) dx = 3$

The process to find the answer is a pretty straightforward integration by parts problem. After choosing the u's and the v's via Integration by Parts, I just evaluated the integral and then plugged in the values given by the problem to find the answer.