Assignment 12 Problem Two

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2. Use a power series to approximate the definite integral $\int_0^{0.4} ln(1+x^4)$ to six decimal places.

$f(x) = ln(1 + x^4)$

$f'(x) = \frac{4x^3}{1+x^4}$

First we try $\frac{1}{1+x^4}$

$\frac{1}{1+x^4} = \frac{1}{1-(-x^4)}$

$= \Sigma_{n=0}^{\infty}(-x^4)^n = \Sigma(-1)^n x^{4n}$

Thus, $f'(x) = 4x^3\Sigma(-1)^n x^{4n} = \Sigma 4(-1)^n x^{4n+3}$

Integrating to solve for $f(x)$ we get:

$f(x) = ln(1 + x^4) = \int \Sigma_{n=0}^{\infty} 4(-1)^n x^{4n+3} = \Sigma_{n=0}^{\infty} 4(-1)^n \frac{x^{4n+4}}{4n+4}$

We integrate one final time to get the original problem:

$\int_0^{0.4} ln(1 + x^4) = \int_0^{0.4} \Sigma_{n=0}^{\infty} 4(-1)^n \frac{x^{4n+4}}{4n+4} = \Sigma_{n=0}^{\infty} 4(-1)^n \frac{x^{4n+5}}{(4n+4)(4n+5)}\Bigg|_0^{0.4} = \Sigma_{n=0}^{\infty} 4(-1)^n \frac{0.4^{4n+5}}{(4n+4)(4n+5)}$

To make the estimation we use the Alternating Series Estimation technique, which demonstrates intuitively that the partial sums of terms in an alternating series can provide arbitrarily good estimations of the true sum (or in this case the true value of the integral).

For this particular estimation because of the rapid growth in the denominator due to the exponential term multiplied by the polynomial term, the estimation converges quickly at $n = 2$ the terms being added are already within our error bound.

At $n = 2$

$\Sigma_{n=0}^{2} 4(-1)^n \frac{0.4^{4n+5}}{(4n+4)(4n+5)} = 0.00203361$, which is within 0.000001 of the true value of the integral.