Assignment 9 Problem Two

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2. Find the limit of the sequence $\left\{\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}...\right\}$

I define the sequence with the recursive definition of $a_n = \sqrt{2a_{n-1}}$ for $n > 1$, and where $a_1 = \sqrt{2}$

$\lim_{n\to\infty} a_n \to L$

Therefore, both a_n and a_{n-1} will approach the same limit L since one step should not make any difference in value when taking the limit to infinity.

Thus we have $a_n = \sqrt{2a_{n-1}}$

$L = \sqrt{2L}$

$L^2 = 2L$

$L^2 - 2L = 0$

$L(L-2) = 0$

$L = 0, 2$

Thus the limit as $n \to \infty$ of $a_n = 2$ since obviously the infinite sum cannot equal 0.