Assignment 4 Problem 2

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2. Consider the error function, $erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$

(a) Use Simpson's Rule with n = 10 to estimate erf(1)

To calculate S_{10} for the $erf(1)$ we must first find the Δx's and the corresponding y's.
$\Delta x = \frac{1-0}{10} = \frac{1}{10}$
Therefore, $x_0 = 0$ $x_1 = \frac{1}{10}$ $x_2 = \frac{2}{10}$ $x_3 = \frac{3}{10}$ ... $x_{10} = \frac{10}{10}$
And thus, we calculate y_0 through y_{10} in the following way:
Let m represent $\frac{2}{\sqrt{\pi}}$
Then $y_i = me^{-x_i^2}$
Thus $S_{10} = \frac{1}{30}(y_0 + 4y_1 + 2y_2 + ... + 2y_8 + 4y_9 + y_{10}) \approx 0.842702$

(b) In the interval [0,1] $|\frac{d^4}{dt^4}(e^{-t^2})| \leq 12$. **Use this information to give an upper bound for the magnitude of error of the estimate in part (a).**

The information given indicates that in the equation $|E_S| = \frac{L(b-a)^5}{180n^4}$ that $L = 12 * \frac{2}{\sqrt{\pi}} = \frac{24}{\sqrt{\pi}}$ then $|E_S| = \frac{L(1)^5}{180(10^4)} \approx 7.522528 * 10^{-6}$