

Assignment 11 Problem Three

Michael Cai

April 17, 2016

3. Suppose that the power series $\sum c_n(x-a)^n$ satisfies the given conditions. Show that the radius of convergence of the power series is R . Condition 1:

$c_n \neq 0$ for all n , and

Condition 2:

$\lim_{n \rightarrow \infty} \left| \frac{c_n}{c_{n+1}} \right| = R$, where R is some positive number.

The Ratio Test gives the radius of convergence.

$$\left| \frac{c_{n+1}(x-a)^{n+1}}{c_n(x-a)^n} \right| = \left| \frac{c_{n+1}}{c_n} \right| \times |x-a| < 1$$

$$\implies |x-a| < \left| \frac{c_n}{c_{n+1}} \right|$$

Thus the radius of convergence equals $\left| \frac{c_n}{c_{n+1}} \right|$, which equals R as you take the limit. Thus the radius of convergence of the power series is R .