Assignment 8 Problem 1

Michael Cai

March 27, 2016

1a) Find the total volume of the flask.

$\int_{-1}^{0} \pi(1-y^2)dy + \int_{0}^{2} \pi(\frac{3}{8}cos(\frac{\pi y}{2}) + \frac{5}{8})^2 dy + \int_{2}^{4} \pi(\frac{1}{2})^2 dy$

Let's consider each portion of this equation as portions 1, 2, and 3 respectively. We will sum them all up in the end.

Portion 1:

$\pi \int_{-1}^{0} 1 - y^2 dy$

$= \pi[y - \frac{1}{3}y^3]\Big|_{-1}^{0}$

$= \pi[-1 - \frac{1}{3}(-1)^3] = \frac{2\pi}{3}$

Portion 2:

$\pi \int_{0}^{2} (\frac{3}{8}cos(\frac{\pi y}{2}) + \frac{5}{8})^2 dy$

$= \pi \int_{0}^{2} \frac{9}{64}cos^2(\frac{\pi y}{2}) + 2(\frac{5}{8})(\frac{3}{8}cos(\frac{\pi y}{2})) + \frac{25}{64}dy$

$= \frac{9\pi}{64} \int_{0}^{2} cos^2(\frac{\pi y}{2})dy + \frac{15\pi}{32} \int_{0}^{2} cos(\frac{\pi y}{2})dy + \pi \int_{0}^{2} \frac{25}{64}dy$

The u-substitution that we will use on the first two components will be the same:

Let $u = \frac{\pi y}{2}$ and thus $du = \frac{\pi}{2}dy$, and the limits will (upper and lower) will equal π and 0 respectively.

Thus:

$= \frac{18}{64} \int_{0}^{\pi} cos^2(u)du + \frac{30}{32} \int_{0}^{\pi} cos(u)du + \pi \int_{0}^{2} \frac{25}{64}dy$

Through the use of the half-angle formula to reduce the first term and all terms evaluated we get:

$= \frac{18}{64}[\frac{\pi}{2}] + 0 + \frac{25}{32}[\pi] = \frac{59\pi}{64}$

Portion 3:

$\pi \int_{2}^{4} (\frac{1}{2})^2 dy = \frac{\pi}{2}$

Thus the total volume of the flask equals:

$= \frac{2\pi}{3} + \frac{59\pi}{64} + \frac{\pi}{2} = \frac{401\pi}{192}$

1b) Write the integral that represents work necessary to empty the flask.

$W = \int_{-1}^{4} \rho\pi x^2(4-y)dy$

Where x represents:

$x =$

$\{\sqrt{1-y^2}$ from $-1 \leq y < 0$

$\{\frac{3}{8}cos(\frac{\pi y}{2}) + \frac{5}{8}$ from $0 \leq y < 2$

$\{\frac{1}{4}$ from $2 \leq y \leq 4$

1c) Use Simpson's rule with n=10 to approximate the value of the integral in part b.

$\Delta y = \frac{4-(-1)}{10} = \frac{1}{2}$

$y_0 = -1, y_1 = -\frac{1}{2}$ ... $y_9 = \frac{7}{2}, y_{10} = 4$

$S_{10} = \frac{1}{6}(f(y_0) + 4f(y_1) + 2f(y_2)...4f(y_9) + f(y_{10}))$ where $f(y) = \rho\pi x^2(4-y)$ where x represents different $f(y)$ depending on the interval.

$S_{10} \approx 19.429219\rho$