Assignment 6 Problem Three

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3. Let D be the region bounded by $y = \sqrt{x}$, $y = 2 - x$, and the x-axis. Let E be the solid formed by revolving D about the line $y = 4$. What is the volume of E?

The plot (was unable to load the graphic) shows the two functions plotted as well as the line $y = 4$. The way to calculate the volume of the solid that is revolved about the line $y = 4$ is to split the problem into two smaller problems. The whole solid is comprised of two separate portions from the intervals $[0, 1]$ and $[1, 2]$ since the two functions intersect at the point $x = 1$. The cross-sections of this solid will look like washers, which are a larger disk with a smaller disk cut out of the middle.

For the interval $[0, 1]$ the radius of the larger disk - let's call it R_1 - is formed by the distance between $y = 4$ and $y = 0$, thus $R_1 = 4$. The radius of the smaller disk - let's call it r_1 - is formed by the distance between $y = 4$ and $y = \sqrt{x}$, thus $r_1 = 4 - \sqrt{x}$. For the interval $[1, 2]$ the radius of the larger disk, R_2, is formed by the distance between $y = 4$ and $y = 0$, thus $R_2 = 4$, and the radius of the smaller disk, r_2, is formed by the distance between $y = 4$ and $y = 2 - x$, thus $r_2 = 4 - (2 - x) = 2 + x$.

Therefore the volume of the entire solid can be calculated as:

$\int_0^1 \pi(4)^2 - \pi(4-\sqrt{x})^2 dx + \int_1^2 \pi(4)^2 - \pi(2+x)^2 dx = \pi \int_0^1 16 - (16 - 8\sqrt{x} + x) dx + \pi \int_1^2 16 - (4 + 4x + x^2) dx = \pi[\frac{16}{3}x^{\frac{3}{2}} - \frac{1}{2}x^2]\Big|_0^1 + \pi[12x - 2x^2 - \frac{1}{3}x^3]\Big|_1^2$

$= \frac{17\pi}{2}$