Assignment 10 Problem 1

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1. Use the integral test, the comparison test, or the limit comparison test to determine if the following series are convergent or divergent.

(a) $\Sigma_{n=1}^{\infty} \frac{2^n}{3^n - 1}$

Use the Limit Comparison Test, which states:

Suppose $\Sigma a_n$and $\Sigma b_n$are positive series.

If $\lim_{n\to\infty} \frac{a_n}{b_n} = c$ where $0 < C < \infty$ then if Σa_n converges so does Σb_n and vice versa with divergence.

Therefore let $a_n = \frac{2^n}{3^n}$, $b_n = \frac{2^n}{3^n - 1}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\frac{2^n}{3^n}}{\frac{2^n}{3^n-1}} = \frac{2^n}{2^n} \times \frac{3^n - 1}{3^n} = \frac{2^n}{2^n} \times \frac{3^n}{3^n} - \frac{1}{3^n}$

$= 1 \times 1 - 0 = 1$

Since $0 < 1 < \infty$ holds then we know that both series must converge or diverge depending on a_n.

Let's evaluate a_n using the Ratio Test, which states:

$\left|\frac{a_{n+1}}{a_n}\right| = r$, where if $r < 1$ the series converges, and if $r > 1$ the series diverges.

$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\frac{2^{n+1}}{3^{n+1}}}{\frac{2^n}{3^n}}\right| = \frac{3^n}{3^{n+1}} \times \frac{2^{n+1}}{2^n}$

$= \frac{3^n}{3(3^n)} \times \frac{2(2^n)}{2^n} = \frac{2}{3}$

Thus since $r = \frac{2}{3} < 1$ then a_n converges, which means that the original b_n the original series converges by the justification of the Limit Comparison Test and the Ratio Test.

(b) $\Sigma_{n=1}^{\infty} \frac{n sin^2(n)}{n^3+1}$

We know the $sin^2(n)$ function is bounded absolutely by 1.

Thus I use the Comparison Test:

$\frac{n sin^2(n)}{n^3+1} < \frac{n}{n^3+1} < \frac{n}{n^3} = \frac{1}{n^2}$

We know that $\Sigma_{n=1}^{\infty} \frac{1}{n^2}$ converges by the p-test for series. Thus by the Comparison Test the original series must converge.

(c) $\Sigma_{n=1}^{\infty} \frac{1}{n^{\frac{1}{5}}}$

The convergence of this series can easily be checked by using the Integral Test.

$\int_1^{\infty} x^{-\frac{1}{5}} dx = [\frac{5}{4} x^{\frac{4}{5}}]\Big|_1^{\infty} = \infty$

Because the integral diverges then the original series must also diverge.