Assignment 11 Problem Two

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2. Consider the series $\Sigma_{n=1}^{\infty}\frac{(-1)^{n+1}(x+2)^n}{n2^n}$
(a) Determine the radius and interval of convergence
Using the Ratio Test:
$\left|\frac{\frac{(x+2)^{n+1}}{(n+1)(2^{n+1})}}{\frac{(x+2)^n}{n2^n}}\right| = |\frac{(x+2)^{n+1}}{(x+2)^n}| \times \frac{n}{n+1} \times \frac{2^n}{2^{n+1}} \implies \frac{1}{2}|x+2| < 1 \implies |x+2| < 2 \implies -2 < x+2 < 2 = -4 < x < 0$
Thus the radius of convergence is 2.

Now we check the endpoints.
When $x = -4$ the original series equals:
$\Sigma\frac{(-1)^{n+1}(-2)^n}{n2^n} = \Sigma\frac{(-1)^{n+1}(-1)^n 2^n}{n2^n} = \Sigma\frac{(-1)^{n+1}(-1)^n}{n}$
By this form we know that when $n$is even then the whole term will be negative because of $(-1)^{n+1}$; however, the whole term will be negative also when $n$is odd because of $(-1)^n$.
Therefore this series is the equivalent of the negative Harmonic Series, which we know to be divergent.
When $x = 0$ the original series equals:
$\Sigma\frac{(-1)^{n+1}2^n}{n2^n}$, which is the Alternative Harmonic series, which we know to be convergent.
Therefore the interval of convergence is $(-4, 0]$.
(b) For what values of x does the series converge absolutely?
The series converges absolutely for the interval $(-4, 0)$ because within that interval the term $|(x+2)^n| < 2^n$, and thus 2^n becomes the dominant term in the fraction, which because it is in the denominator, causes the entire series to converge.
(c) For what values of x does the series converge conditionally?
The series converges conditionally only for the value $x = 0$, as was tested when we were determining the interval of convergence. Conditional convergence basically states that the series (at a particular x value) converges only if there is an alternating term $(-1)^n$ or $(-1)^{n+1}$.