Assignment 3 Problem 1

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1. Find the area of the region bounded by $y = \frac{3x^2+x}{(x^2+1)(x+1)}$, $y = 0$, $x = 0$, and $x = 1$.

For the integral $\int_0^1 \frac{3x^2+x}{(x^2+1)(x+1)}$ we first use partial fraction decomposition because the integrand is a rational function (a ratio of polynomials).
Thus $\int_0^1 \frac{3x^2+x}{(x^2+1)(x+1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+1}$
We then cross multiply and consider the equality of the numerator of the integrand with the cross-multiplied terms of the partial fractions.
$3x^2 + x = (Ax + B)(x + 1) + C(x^2 + 1)$
If $x = -1$ then $C = 1$.
We then expand the RHS to solve for the match the coefficients on the LHS and the RHS.
$3x^2 + x = Ax^2 + Ax + Bx + B + Cx^2 + C$
$3x^2 + x = (A + C)x^2 + (A + B)x + B + C$
Since the constant term on the LHS is 0, then $B + C = 0 \to B = -1$
Since the coefficient of the linear term, x, on the LHS $= 1$ then $1 = A + B \to A = 2$
Therefore: $\frac{3x^2+x}{(x^2+1)(x+!)} = \frac{2x-1}{x^2+1} + \frac{1}{x+1}$
$\int_0^1 \frac{3x^2+x}{(x^2+1)(x+1)} = \int_0^1 \frac{2x-1}{x^2+1} + \int_0^1 \frac{1}{x+1}$
$= \int_0^1 \frac{2x}{x^2+1} - \int_0^1 \frac{1}{x^2+1} + \int_0^1 \frac{1}{x+1}$

Consider the $\int_0^1 \frac{2x}{x^2+1}$ term:
We use u-substitution and set $u = x^2 + 1$ and thus $du = 2xdx$ and the limits become $u = 2$ and $u = 1$.
Consider the $\int_0^1 \frac{1}{x+1}$ term:
Here we also use u-substitution and set $u = x + 1$ and thus $du = dx$ with the limits becoming $u = 2$ and $u = 1$.
Now if we integrate all 3 of the separate terms, we get get two with natural logs and one with arctan.
$= ln|u||_1^2 - tan^{-1}(x)\big|_0^1 + ln|u||_1^2$
$= 2ln|2| - \frac{\pi}{4}$