Assignment 7 Problem One

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1. A bead is formed from a sphere of radius 5 by drilling through a diameter of the sphere with a drill bit of radius 3.

(a) Find the volume of the bead.

The volume of the bead will be the volume of a sphere subtracting the volume portion of the removed portion, which amounts to a cylindrical portion extending from (if you were to make the sphere a 2-dimensional circle on an xy-plane) y=-3 to 3, and two circular "caps" that extend on the same plane from y=-5 to -3 and y = 3 to 5.

Thus the equation is:

$V = \frac{4}{3}\pi r^3 - 2$ (Removed portion)

Let's break this into two parts.

The volume of the cylindrical portion is just the integral of cross-sections perpendicular to the y-axis of circles all of equal area πr^2 where $r = 3$ thus the circles are of area 9π.

To find the volume of the "cap", we first must imagine the graph of a circle of radius = 5 on an xy-plane.

The cap is the portion of the function $x = \sqrt{25 - y^2}$ from the bounds $y = 3$ to $y = 5$ revolved about the y-axis.

I calculate the volume of this solid of revolution by using the disk method, which states that the volume, $V = \int_4^5 A(y)dy$ where $A(y)$ is the area of each of the circular cross-sections.

$A(y) = \pi r^2 = \pi(\sqrt{25 - y^2})^2 = \pi(25 - y^2)$

Thus the total volume of the bead is equal to:

$V = \frac{4}{3}\pi(5)^3 - 2(\int_0^4 \pi(3)^2 dy + \int_4^5 \pi(25 - y^2)dy)$

$= \frac{500\pi}{3} - 2(36\pi + \frac{14\pi}{3})$

$= \frac{256\pi}{3}$

(b) Find the volume of the removed portion of the sphere

As calculated previously the volume of the removed portion of the sphere would just be the volume of the sphere subtracting the volume of the bead:

$V = \frac{500\pi}{3} - \frac{256\pi}{3} = \frac{244\pi}{3}$