

Assignment 3 Problem 1

Michael Cai

February 12, 2016

1. Find the area of the region bounded by $y = \frac{3x^2+x}{(x^2+1)(x+1)}$, $y = 0$, $x = 0$, and $x = 1$.

For the integral $\int_0^1 \frac{3x^2+x}{(x^2+1)(x+1)}$ we first use partial fraction decomposition because the integrand is a rational function (a ratio of polynomials).

$$\text{Thus } \int_0^1 \frac{3x^2+x}{(x^2+1)(x+1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+1}$$

We then cross multiply and consider the equality of the numerator of the integrand with the cross-multiplied terms of the partial fractions.

$$3x^2 + x = (Ax + B)(x + 1) + C(x^2 + 1)$$

If $x = -1$ then $C = 1$.

We then expand the RHS to solve for the match the coefficients on the LHS and the RHS.

$$3x^2 + x = Ax^2 + Ax + Bx + B + Cx^2 + C$$

$$3x^2 + x = (A + C)x^2 + (A + B)x + B + C$$

Since the constant term on the LHS is 0, then $B + C = 0 \rightarrow B = -1$

Since the coefficient of the linear term, x , on the LHS = 1 then $1 = A + B \rightarrow A = 2$

$$\text{Therefore: } \frac{3x^2+x}{(x^2+1)(x+1)} = \frac{2x-1}{x^2+1} + \frac{1}{x+1}$$

$$\int_0^1 \frac{3x^2+x}{(x^2+1)(x+1)} = \int_0^1 \frac{2x-1}{x^2+1} + \int_0^1 \frac{1}{x+1}$$

$$= \int_0^1 \frac{2x}{x^2+1} - \int_0^1 \frac{1}{x^2+1} + \int_0^1 \frac{1}{x+1}$$

Consider the $\int_0^1 \frac{2x}{x^2+1}$ term:

We use u-substitution and set $u = x^2 + 1$ and thus $du = 2xdx$ and the limits become $u = 2$ and $u = 1$.

Consider the $\int_0^1 \frac{1}{x+1}$ term:

Here we also use u-substitution and set $u = x + 1$ and thus $du = dx$ with the limits becoming $u = 2$ and $u = 1$.

Now if we integrate all 3 of the separate terms, we get two with natural logs and one with arctan.

$$\begin{aligned} &= \ln|u|_1^2 - \tan^{-1}(x)|_0^1 + \ln|u|_1^2 \\ &= 2\ln|2| - \frac{\pi}{4} \end{aligned}$$