Assignment 12 Problem One

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1. Find a power series representation for $f(x) = ln(x^2+4)$ and graph f and the first three partial sums $s_n(x)$ i.e. when $n = 1, 2, 3$ on the same plot. What happens as n increases?

$f(x) = ln(x^2+4)$

To solve for this PSR, let's take the derivative and then integrate the resulting power series.

$f'(x) = \frac{2x}{x^2+4}$

Let's first find the PSR for a simpler rational function: $\frac{1}{x^2+4}$

$\frac{1}{x^2+4} = \frac{1}{4-(-x^2)} = \frac{1}{4}\left(\frac{1}{1-\frac{-x^2}{4}}\right)$

Thus the power series representation for the function inside the parentheses is just a simple geometric series with $r = \frac{x^2}{4}$

$\frac{1}{x^2+4} = \frac{1}{4}\Sigma_{n=0}^{\infty}(\frac{-x^2}{4})^n = \frac{1}{4}\Sigma_{n=0}^{\infty}(-1)^n(\frac{1}{4})^n(-x)^{2n} = \Sigma_{n=0}^{\infty}(-1)^n(\frac{1}{4})^{n+1}x^{2n}$ (since $(-x)^{2n} = x^{2n}$)

$\frac{2x}{x^2+4} = \Sigma_{n=0}^{\infty}2(-1)^n(\frac{1}{4})^{n+1}x^{2n+1}$

$\int\frac{2x}{x^2+4} = \int\Sigma_{n=0}^{\infty}2(-1)^n(\frac{1}{4})^{n+1}x^{2n+1}$

$ln(x^2+4) = C + \Sigma_{n=0}^{\infty}2(-1)^n(\frac{1}{4})^{n+1}\frac{x^{2n+2}}{2n+2}$

Solve for C when $x = 0$

$ln(4) = C$

Thus the PSR for $ln(x^2+4)$ is:

$ln(x^2+4) = ln(4) + \Sigma_{n=0}^{\infty}(-1)^n(\frac{1}{4})^{n+1}\frac{x^{2n+2}}{n+1}$

The first three partial sums $s_n(x)$ indexing from 0 are as follows:

$s_0(x) = ln(4) + \frac{x^2}{4}$

$s_1(x) = ln(4) + \frac{x^2}{4} - \frac{x^4}{32}$

$s_2(x) = ln(4) + \frac{x^2}{4} - \frac{x^4}{32} + \frac{x^6}{192}$

As $n$increases the plots approximate the actual function more and more accurately.

(Image included in a separate document. Could not figure out the error that LyX kept giving me preventing me from attaching the image)