

Assignment 9 Problem Three

Michael Cai

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3. For each of the following series, write out the first few terms of each series to show how the series starts. Then, find the sum of the series.

(a) $\sum_{n=2}^{\infty} \frac{1}{4^n}$

$$a_n = \left\{ \frac{1}{16}, \frac{1}{64}, \frac{1}{256}, \dots \right\}$$

$$s_n = \sum ar^k = \sum_{n=0}^{\infty} \left(\frac{1}{16}\right) \left(\frac{1}{4}\right)^n = \frac{\frac{1}{16}}{1-\frac{1}{4}} = \frac{1}{12}$$

(b) $\sum_{n=0}^{\infty} \left(\frac{1}{2^n} + \frac{(-1)^n}{5^n}\right)$

$$a_n = \left\{ 2, \frac{3}{10}, \frac{29}{100}, \frac{117}{1000}, \dots \right\}$$

First we will split the series up into two separate series and sum the separate parts.

$$= \sum_{n=0}^{\infty} \frac{1}{2^n} + \sum_{n=0}^{\infty} \frac{(-1)^n}{5^n}$$

Note that the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{5^n}$ is of the form $\left\{ 1, -\frac{1}{5}, \frac{1}{25}, -\frac{1}{125}, \dots \right\}$

Which could also be written as $\sum_{n=0}^{\infty} \left(\frac{1}{25}\right)^n - \sum_{n=0}^{\infty} \left(-\frac{1}{5}\right) \left(\frac{1}{25}\right)^n$

Thus evaluating all 3 geometric series together we have:

$$\frac{1}{1-\frac{1}{2}} + \frac{1}{1-\frac{1}{25}} - \frac{\frac{1}{5}}{1-\frac{1}{25}} = 2 + \frac{25}{24} - \frac{5}{24} = \frac{17}{6}$$

*Realized I did this in an overly complicated way initially. Can be simplified to just:

$$= \frac{1}{1-\frac{1}{2}} + \frac{1}{1-(-\frac{1}{5})} = 2 + \frac{5}{6} = \frac{17}{6}$$