

Assignment 7 Problem Two

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March 19, 2016

2. A Bundt cake, well known for having a ringed shape, is formed by revolving around the y-axis the region bounded by the graph of $y = \sin(x^2 - 1)$ and around the x-axis over the interval $1 \leq x \leq \sqrt{1 + \pi}$. Find the volume of the cake.

As shown by the graph provided on the problem sheet, the most practical way to find the volume of a bundt cake would be to use the shell method for the solid of revolution created by the function $y = \sin(x^2 - 1)$ on the interval $1 \leq x \leq \sqrt{1 + \pi}$.

The formula for shell method is given by the following: $V = \int 2\pi rh dx$, where r is the radius and h is the height of the cylindrical shells that are formed.

The r in this case is the distance measured in x from the y-axis, where $x = 0$ to the function, so thus the $r = x$.

The h in this case is the distance between the x-axis and the height of the function, which is just $y = \sin(x^2 - 1) = h$

Thus the volume is:

$$V = \int_1^{\sqrt{1+\pi}} 2\pi x \sin(x^2 - 1) dx = 2\pi \int_1^{\sqrt{1+\pi}} x \sin(x^2 - 1) dx$$

Let $u = x^2 - 1$, thus $du = 2x dx$, and the limits are $u_{lim} = \pi$ and $l_{lim} = 0$.

$$V = \pi \int_0^\pi \sin(u) du$$

$$V = \pi [-\cos(u)] \Big|_0^\pi = \pi [\cos(u)]_\pi^0 = \pi [\cos(0) - \cos(\pi)] = 2\pi$$