Assignment 5 Problem Four

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4. Find the area of the shaded region below *(picture not included). We are given the two outer bounds, -2 and 3. Now we must find the interior intersections.

Let $f(x) = \frac{x^3}{3} - x$ and $g(x) = \frac{x}{3}$.

$f(x) = g(x) \implies \frac{x^3}{3} - x = \frac{x}{3}$

$\frac{x^3}{3} - \frac{4x}{3} = 0$

$\frac{1}{3}x(x^2 - 4) = 0$

Therefore the roots are $x = 0$ and $x \pm 2$.

Thus to calculate the total area bounded within this interval by these two functions, we need to specify, which function is the "larger" function over given intervals.

Thus the total area bounded by these two functions equals:

$\int_{-2}^{0} f(x) - g(x) + \int_{0}^{2} g(x) - f(x) + \int_{2}^{3} f(x) - g(x)$

$= \int_{-2}^{0} \frac{x^3}{3} - x - \frac{x}{3} + \int_{0}^{2} \frac{x}{3} - \frac{x^3}{3} + x + \int_{2}^{3} \frac{x^3}{3} - x - \frac{x}{3}$

Now we can easily solve for this value by splitting all three definite integrals into their component pieces and integrating separately (by applying basic power rule).

Therefore, the total area bounded by the shaded region equals:

$= \frac{4}{3} + \frac{4}{3} + \frac{25}{12}$

$= \frac{19}{4}$