Assignment 6 Problem Two

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2. Find the volume of the following solid. The base of the solid is the disk $x^2 + y^2 \leq 1$. The cross-section by planes perpendicular to the y-axis between $y = -1$ and $y = 1$ are isosceles right triangles with one leg in the disk.

Because the cross-sections by planes are perpendicular to the y-axis, we use the following formula to calculate the volume $\int A(y)dy$, where $A(y)$ is the area of the shapes that form the cross-sections. In this case the area formula for an isosceles right triangle is $A(y) = \frac{b^2}{2}$. To find the length of the base of the triangle, which lies along the circular base of the solid, we must calculate the distance between one edge of the circle to the other in terms of y. From $x^2 + y^2 \leq 1$, we get $x = \sqrt{1 - y^2}$, which implies that the triangle base is twice the distance from the y-axis to the curve $x = \sqrt{1 - y^2}$, or $2\sqrt{1 - y^2}$. Therefore, the volume is $\int_{-1}^{1} A(y)dy = \int_{-1}^{1} \frac{(2\sqrt{1-y^2})^2}{2} dy$

$= 2\int_{-1}^{1}(\sqrt{1 - y^2})^2 dy = 2\int_{-1}^{1} 1 - y^2 dy = 2[y - \frac{1}{3}y^3]\Big|_{-1}^{1} = 2[(1 - \frac{1}{3}(1)^3) - (-1 - \frac{1}{3}(-1))] = \frac{8}{3}$

Therefore, the volume of the solid is $\frac{8}{3}$.