Assignment 7 Problem Three

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3. The graph of the equation $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$ **is one of a family of curves called "astroids" because of their starlike appearance. Find the length of this particular astroid by finding the length of half of the first quadrant portion, namely,** $y = (1 - x^{\frac{2}{3}})^{\frac{3}{2}}, \frac{\sqrt{2}}{4} \leq x \leq 1$, **and multiplying by** 8.

The arc length formula is given by $L = \int \sqrt{1-(y')^2}dx$

$y = (1 - x^{\frac{2}{3}})^{\frac{3}{2}}$

$y' = \frac{3}{2}(1 - x^{\frac{2}{3}})^{\frac{1}{2}} * (\frac{-2}{3}x^{\frac{-1}{3}}) = \frac{-(1-x^{\frac{2}{3}})^{\frac{1}{2}}}{x^{\frac{1}{3}}}$

$(y')^2 = \frac{1-x^{\frac{2}{3}}}{x^{\frac{2}{3}}} == \frac{1}{x^{\frac{2}{3}}} - 1$

$L = \int_{\frac{\sqrt{2}}{4}}^{1} \sqrt{1 + \frac{1}{x^{\frac{2}{3}}} - 1}dx = \int_{\frac{\sqrt{2}}{4}}^{1} \sqrt{x^{-\frac{2}{3}}}dx = \int_{\frac{\sqrt{2}}{4}}^{1} x^{-\frac{1}{3}}dx$

$L = [\frac{3}{2}x^{\frac{2}{3}}]\Big|_{\frac{\sqrt{2}}{4}}^{1} = \frac{3}{2} - \frac{3}{2}(\frac{\sqrt{2}}{4})^{\frac{2}{3}} = \frac{3}{2} - \frac{3}{2}(\frac{(3\sqrt{2})^4}{(3\sqrt{2})^8}) = \frac{3}{2} - \frac{3}{2}(\frac{1}{2}) = \frac{3}{4}$

Then multiplying by 8 to get the length of the whole astroid we get $L_a = 8 * \frac{3}{4} = 6$