Assignment 8 Problem 4

Michael Cai

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4a) Find a function $y(t)$ to model the number of people infected.
$\frac{dy}{dt} = ky(M-y)$ where $M = 5000$, $k = 1.78$, and the initial value is at $t = 0$, $y = 10$
$\frac{dy}{dt} = kyM - ky^2$
$\frac{dy}{dt} = k(My - y^2)$
$\frac{dy}{(My-y^2)} = kdt$
$\int \frac{dy}{My-y^2} = \int kdt$
$\int \frac{1}{y(M-y)} dy = k \int 1dt$
$\int \frac{A}{y} + \frac{B}{M-y} dy = k \int dt$
When $y = 0$, $A = \frac{1}{M}$, and when $y = M$, $B = \frac{1}{M}$
$\int \frac{1}{My} + \frac{1}{M^2-My} dy = k \int dt$
$ln(My) - ln(M^2 - My) = kt + C$
$e^{ln(My)-ln(M^2-My)} = e^{kt+C}$
$\frac{My}{M^2-My} = e^C e^{kt}$
$My = (M^2 - My)e^C e^{kt}$
$My = M^2 e^C e^{kt} - Mye^C e^{kt}$
$My + Mye^C e^{kt} = M^2 e^C e^{kt}$
$y(M + Me^C e^{kt}) = M^2 e^C e^{kt}$
$y = \frac{M^2 e^C e^{kt}}{M+Me^C e^{kt}} = \frac{Me^C e^{kt}}{1+e^C e^{kt}}$
Given the initial values we can solve for the constant, C.
$10 = \frac{Me^C}{1+e^C}$
$10 + 10e^C = Me^C$
$10 = Me^C - 10e^C$
$10 = (M - 10)e^C$
$e^C = \frac{10}{M-10} = \frac{1}{499}$
Therefore the function y(t) is:
$y(t) = \frac{\frac{5000}{499} e^{1.78t}}{1+\frac{1}{499} e^{1.78t}} = \frac{5000e^{1.78t}}{499+e^{1.78t}}$

4b) Sketch a graph of the function $y(t)$ that you found in part (a). Then use your graph to estimate when the rate at which people are becoming infected starts to decrease.
It seems that the rate at which people are becoming infected starts to decrease at time t=4.
(Having issues uploading the graph onto my LaTeX file, so I will include a separate file of just the graph).