Assignment 9 Problem One

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1. Find the limits of each of the following sequences:

(a) $a_n = (4^n n)^{\frac{1}{n}} = 4(n^{\frac{1}{n}})$
$n = 1, 2, 3, 4, 5, ...$
$a_n = 4, 4(2)^{\frac{1}{2}}, 4(3)^{\frac{1}{3}}, 4(4)^{\frac{1}{4}}, 4(5)^{\frac{1}{5}}...$
Suppose $\lim_{n\to\infty} n^{\frac{1}{n}} \to L$
$a_n = n^{\frac{1}{n}} \to L$
$ln(n^{\frac{1}{n}}) \to ln(L)$
$\frac{1}{n}ln(n) \to ln(L)$
$\frac{ln(n)}{n} \to ln(L)$
Using L'Hospitals to evaluate the RHS, which is currently in indeterminate form.
$\frac{ln(n)}{n} = \frac{\frac{1}{n}}{1} = \frac{1}{n} \to ln(L)$
$e^n = L$
Taking the limit as $n \to \infty$, $L \to 1$.
Thus $\lim_{n\to\infty} a_n = 4$

(b) $a_n = n - \sqrt{n^2 - n}$
$n - \sqrt{n^2 - n} = \frac{n+\sqrt{n^2-n}}{n+\sqrt{n^2-n}}(n - \sqrt{n^2 - n})$
$= \frac{n^2-(n^2-n)}{n+\sqrt{n^2-n}}$
$= \frac{n}{n+n\sqrt{1-\frac{1}{n}}}$
$= \frac{1}{1+\sqrt{1-\frac{1}{n}}}$
If you take the limit as $n \to \infty, a_n \to \frac{1}{2}$