Assignment 8 Problem 2

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2. Let D be the region between $y = sin(x)$ and the x-axis on the interval $0 \leq x \leq \pi$. Find the centroid of D.

The centroid formula for a two-dimensional object as given in class is the following:

$\bar{x} = \frac{\int_a^b xf(x)dx}{\int_a^b f(x)dx}$

$\bar{y} = \frac{\frac{1}{2}\int_a^b f(x)^2 dx}{\int_a^b f(x)dx}$

Thus the specific formula for the problem that we are given is the following:

$\bar{x} = \frac{\int_0^\pi xsin(x)dx}{\int_0^\pi sin(x)dx}$

$\bar{y} = \frac{\frac{1}{2}\int_0^\pi sin^2(x)dx}{\int_0^\pi sin(x)dx}$

The numerator of $\bar{x}$ can be calculated with a simple IBP:

$\int_0^\pi xsin(x)dx$

Let $u = x$ and $dv = sin(x)dx$

Then $du = dx$ and $v = -cos(x)$

$\int_0^\pi xsin(x)dx = -xcos(x)\Big|_0^\pi + \int_0^\pi cos(x)dx$

$= \pi$

Because $\int_0^\pi sin(x)dx = [-cos(x)]\Big|_0^\pi = 2$

$\bar{x} = \frac{\pi}{2}$

The numerator of $\bar{y}$ can be calculated using the half-angle identity:

$\frac{1}{2}\int_0^\pi sin^2(x)dx = \frac{1}{2}\int_0^\pi (\frac{1-cos(2x)}{2})dx$

$= \frac{1}{2}[\frac{1}{2}x - \frac{1}{4}sin(2x)]\Big|_0^\pi = \frac{\pi}{4}$

Therefore:

$\bar{y} = \frac{\pi}{8}$

The centroid $(\bar{x}, \bar{y})$ is located at the coordinate $(\frac{\pi}{2}, \frac{\pi}{8})$.