Assignment 2 Problem 2

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2. *Let K_m denote the following indefinite integral:*
$K_m(x) = \int_x^m sin(x)dx$ for $m = 0, 1, 2...$

(a) *Evaluate* $K_1(x)$.
$K_1(x) = \int x^1 sin(x)dx$
Integration by parts would be the strategy to solve the integral because the integrand is a product. Therefore, we let $u = x$ and thus $du = dx$ and $dv = sinxdx$ and thus $v = -cosx$.
$\int x^1 sin(x) = -xcosx + \int cosxdx$
$= -xcosx + sinx + C$

(b) *Show* $K_m(x) = -x^m cos(x) + mx^{m-1} sin(x) - m(m-1)K_{m-2}x$
$K_m(x) = \int x^m sin(x)dx$
Because the integrand is a product, I am going to use integration by parts and select $u = x^m$ and thus $du = mx^{m-1}dx$. I choose $dv = sin(x)dx$ and thus $v = -cos(x)$.
Therefore $\int x^m sin(x)dx = -x^m cos(x) + \int mx^{m-1} cos(x)dx$

Now consider $\int mx^{m-1} cos(x)dx$ which equals $m \int x^{m-1} cos(x)dx$
Again we use integration by parts because the integrand is a product.
I choose $u = x^{m-1}$ and thus $du = (m-1)x^{m-2}dx$. I choose $dv = cos(x)dx$ and thus $v = sin(x)$.
Therefore $m \int x^{m-1} cos(x)dx = m(x^{m-1} sin(x) - (m-1) \int x^{m-2} sin(x)dx$
$= mx^{m-1} sin(x) - m(m-1) \int x^{m-2} sin(x)dx$
This last integral $\int x^{m-2} sin(x)dx$ is equal to $K_{m-2}(x)$.

Thus, plugging this integral back into the original equation, where we left off:
$\int x^m sin(x)dx = -x^m cos(x) + \int mx^{m-1} cos(x)dx$
$= \int x^m sin(x)dx = -x^m cos(x) + mx^{m-1} sin(x) - m(m-1)K_{m-2}(x)$

(c) *Calculate $K_0(x)$, $K_2(x)$, and $K_4(x)$*
$K_0(x) = \int x^0 sin(x)dx = \int sin(x)dx = -cos(x) + C$
$K_2(x) = \int x^2 sin(x)dx$
Because the integrand is a product, we again apply integration by parts.
I choose $u = x^2$ and thus $du = 2xdx$. I choose $dv = sin(x)dx$ and thus $v = -cos(x)$.
Therefore $\int x^2 sin(x)dx = -x^2 cos(x) + 2\int xcos(x)dx$.
Consider the integral $\int xcos(x)dx$
Again we apply IBP. Let $u = x$ and thus $du = dx$. Let $dv = cos(x)dx$ and thus $v = sin(x)$.
$\int xcos(x)dx = xsin(x) - \int sin(x)dx = xsin(x) - cos(x) + C$

Plugging this back into the original equations we get:
$\int x^2 sin(x)dx = -x^2 cos(x) + 2\int xcos(x)dx = -x^2 cos(x) + 2(xsin(x) - cos(x) + C)$
$= -x^2 cos(x) + 2xsin(x) + 2cos(x) + C' : C' = 2C$

$K_4(x) = \int x^4 sin(x)dx$
Recall the formula for $K_m(x)$:
$\int x^m sin(x)dx = -x^m cos(x) + mx^{m-1} sin(x) - m(m-1)K_{m-2}(x)$
Thus we choose m = 4 and we get:
$= \int x^4 sin(x)dx = -x^4 cos(x) + 4x^3 sin(x) - 4(3)K_2(x)$
Then we plug in $K_2(x)$ and we get:
$= -x^4 cos(x) + 4x^3 sin(x) + 12x^2 cos(x) + 24xsin(x) + 24cos(x) + C$

(d) *Solve the definite integral $\int_0^\pi x^4 sin(x)dx$*
To get the answer, we just need to evaluate the indefinite integral that we just calculated with the bounds 0 and π.
$\int_0^\pi x^4 sin(x)dx$ equals
$-\pi^4 cos(\pi) + 4\pi^3 sin(\pi) + 12\pi^2 cos(\pi) + 24\pi sin(\pi) + 24cos(\pi)$
minus $(0^4 cos(0) + 4(0)^3 sin(0) + 12(0)^2 cos(0) + 24(0)sin(0) + 24cos(0))$
$= \pi^4 - 12\pi^2 - 24 - 24 = \pi^4 - 12\pi^2 - 48.$