

Assignment 9

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1. Consider the IV regression model below and given certain conditions assess whether the IV estimator is consistent.

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 W_i + u_i$$

(a) $Cov(Z_i, W_i) = 0$

$$\beta^{IV} \rightarrow \frac{Cov(Z, Y)}{Cov(Z, X)}$$

$$Cov(Z, Y) = Cov(Z, \beta_0 + \beta_1 X_i + \beta_2 W_i + u_i) = \beta_1 Cov(Z, X_i)$$

Thus: $\beta^{IV} \rightarrow \beta_1$

Yes the IV estimator is consistent.

(b) $Cov(Z_i, W_i) \neq 0$

Then following the same procedure we instead get:

$$Cov(Z, Y) = \beta_1 Cov(Z, X_i) + Cov(Z, W_i)$$

And thus the estimator converges in probability to $\beta_1 + \frac{Cov(Z, W_i)}{Cov(Z, X)}$

Which is thus biased and inconsistent.

2. How does fertility affect labor supply?

(a) Yes, on average women with more than two children work about 5 hours less than women with less than two children.

(b) It is inappropriate for estimating the causal effect of fertility (*morekids*) on labor supply (*weeksm1*) because both of those decisions are made by the mother's preference (and thus this model suffers from endogeneity).

(c) Yes, the effect is statistically significant but not very large. Couples, whose first two children are of the same sex, have a 6% higher rate of having a third child when compared to couples, who do not meet this criteria.

(d) *samesex* is a valid instrument because $Cov(samesex, morekids) \neq 0$ because of the statistically significant relationship that we just identified in part (c). Also because $Cov(samesex, u_i) = 0$ since the parents do not have the ability to choose whether or not their first two children are of the same gender, thus avoiding the endogeneity encountered in the initial regression.

(e) The fertility effect now causes mothers to work on average 6 hours less as opposed to the 5 hours less identified earlier using the instrument.

(f) The Wu-Hausman test for endogeneity gave a p-value of 0.2, and thus we cannot reject the null hypothesis. Therefore, we cannot conclude that the regressor is endogenous.

(g) The F-stat for *morekids* is large (the p-value for the F-stat is very small), thus *samesex* is not a weak instrument for *morekids*.

(h) No, my estimates do not change. The instrument is still significant and the Wu-Hausman test still does not have a significant p-value.

(i) The Sargan test yields a 4.183 test statistic and a p-value of 0.124 so therefore we are unable to reject the null hypothesis, and thus we cannot conclude that the instruments are endogenous.

(j) My results differ with all of the tests now being significant (Weak instruments, Wu-Hausman, and Sargan). This might happen because after we control for these other demographic effects then the observation of data for women having more or less children or working more or less will demonstrate more clearly the endogenous relationship/the choice being made to have more or fewer kids and to work more or less.