TeX work for Assignment 4

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1

Let β = fraction of workers going from New York to Chicago.

Let γ = fraction of workers going from Chicago to New York.

Then the laws of motion for N_{t+1}, C_{t+1}, and P_{t+1} are:

$N_{t+1} = (1-\beta)N_t + \gamma C_t$

$C_{t+1} = (1-\gamma)C_t + \beta N_t$

$P_{t+1} = P_t$ since there are no births or deaths in this particular Lake Model.

2

The A matrix is as follows:

$$\begin{bmatrix} 1-\beta & \gamma \\ \beta & 1-\gamma \end{bmatrix}$$

3

Because there are no births nor are there deaths, then the growth rate, $g = 0$. Thus normally in this case, $\hat{A} = \frac{1}{1+g} * A$ but since $g = 0$ then $\hat{A} = A$

4

To find the steady state, we must find the eigenvector [n,c] for A given the $\lambda = 1$ to satisfy the equation:

$\hat{A} \begin{bmatrix} n \\ c \end{bmatrix} = \lambda \begin{bmatrix} n \\ c \end{bmatrix}$ where $\lambda = 1$.

When $\lambda = 1$ we have the following equation for the nullspace of $\hat{A}$.

$$\begin{bmatrix} -\beta & \gamma \\ \beta & -\gamma \end{bmatrix} \times \begin{bmatrix} n \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -\beta & \gamma \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} n \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{-\gamma}{\beta} \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} n \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Therefore, $n - \frac{\gamma}{\beta}c = 0$, which implies that the corresponding eigenvector to $\lambda = 1$ is c$\begin{bmatrix} \frac{\gamma}{\beta} \\ 1 \end{bmatrix}$ where $c \in R$ (reals). Thus given a certain γ and β the eigenvector should be adjusted accordingly.

5

The probability that a worker is in Chicago at time t and ends up in New York at time $t + 3$ is determined by the transition matrix:

P $= \begin{bmatrix} 1-\beta & \gamma \\ \beta & 1-\gamma \end{bmatrix}$, where β represents the transition probability from New York to Chicago, and γ represents the transition probability from Chicago to New York.

The four paths by which a worker can start in Chicago and end up in New York from time t to time $t + 3$ are as follows:

(1) $C \to C \to C \to N$, which corresponds to the probability $(1-\gamma)^2\gamma$

(2) $C \to C \to N \to N$, which corresponds to the probability $(1-\gamma)(\gamma)(1-\beta)$

(3) $C \to N \to C \to N$, which corresponds to the probability $(\gamma)(\beta)(\gamma)$

(4) $C \to N \to N \to N$, which corresponds to the probability $(\gamma)(1-\beta)^2$