Lecture tomorrow

Monte Carlo so far

Goal: Analyse the properties of some estimator or statistic $T(x) = \theta$

Q 2.5

$$T(x) = \hat{\sigma}^2 \quad (\text{or } S^2) \quad \Rightarrow S^2 = \frac{1}{n-1} \quad \& \quad \hat{\sigma}^2 = \frac{1}{n}$$

Given some data $X \sim N(0, \sigma^2)$, we approximate/est $\text{bias}(T(x)) = E(T(x)) - \theta_{true}$ numerically using $\underbrace{M \text{ MC repititions}}$ of this experiment, by:

$T_m(x) = \hat{\sigma}^2$ from m^{th} repitition

$$\text{Bias} = \underbrace{\frac{1}{M}\sum_{m=1}^{M} T_m(x)} - \underbrace{\theta_{true}}$$

Set by the user

Here $\theta_{true} = \sigma^2$

$$\hat{E}(T(x)) = \frac{1}{M}\sum_{m=1}^{M} T_m(x) \xrightarrow[M \to \infty]{LLN} E(T(x))$$

Q 2.6

Numerical experiment of the theoretic property that given $X \sim \chi^2_\nu$, then

$$E(X) = \nu \quad (df)$$

$\Rightarrow E(X) \simeq \bar{X}$ given a sample and

$$E(\bar{X}) = E(X) \ (\text{unbiased}) \simeq \frac{1}{M}\sum_{m=1}^{M} \bar{X}_m \xrightarrow[m \to \infty]{LLN} E(\bar{X})$$

(i.e)

$\hat{E}(x)$ is a MC estimate of $E[\bar{x}]$

2.4)

LS vs (Robust) M-estimation

Lm vs rLm( ) → in R

in $y = \beta_0 + \beta_1 x + \varepsilon$

How do we compare the 2 estimators of $\beta = (\beta_0, \beta_1)$

1) $\widehat{Bias}(\hat{\beta}) = \frac{1}{M} \sum_{m=1}^{M} \hat{\beta}_m - \beta_{true} = \bar{\hat{\beta}} - \beta_{true}$

2) $var(\hat{\beta}) = \frac{1}{M-1} \sum_{m=1}^{M} (\hat{\beta}_m - \bar{\hat{\beta}})^2 = \widehat{SE}(\hat{\beta})^2$

3) $MSE(\hat{\beta}) = E\left[(\hat{\beta} - \beta_{true})^2\right]$

$$\simeq \frac{1}{M} \sum_{m=1}^{M} (\hat{\beta}_m - \beta_{true})^2 = \widehat{MSE}(\hat{\beta})$$

All 3 = Indicators of Performance.

↳ Can also use $RMSE = \sqrt{MSE}$

Q 2.3)

$\bar{x}$ vs "truncated mean"

→ 1, 2 & 3

Another common indicator of performance is the Confidence Interval.

Given a sample of MC estimations $\{\hat{\theta}_1, \dots, \hat{\theta}_m\}$ C.I could be

a) based on the Normal approximation

$$\bar{\hat{\theta}} \pm 1.96\ \hat{SE}(\hat{\theta})$$

$$\left(\bar{x} \pm 1.96 \frac{s}{\sqrt{n}}\right)$$

$$\text{(i.e)} \left(\frac{1}{m} \sum_{i=1}^{m} \hat{\theta}_m \pm 1.96 \left(\frac{1}{m-1} \sum (\hat{\theta}_m - \bar{\hat{\theta}})^2\right)^{1/2}\right)$$

b) Quantile based "naive"

$$\left[\{\hat{\theta}\}_{.025}\ ,\ \{\hat{\theta}\}_{.975}\right]$$