Generalized additive modeling and dialectology
Lecture 3 of advanced regression for linguists

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This lecture

- Introduction
  - Some words about logistic regression
  - Generalized additive mixed-effects regression modeling
  - Standard Italian and Tuscan dialects

- Material: Standard Italian and Tuscan dialects

- Methods: `R` code

- Results

- Discussion

# A linear regression model

- *linear model*: linear relationship between predictors and dependent variable: $y = a_1 x_1 + ... + a_n x_n$
  - Non-linearities via explicit parametrization: $y = a_1 x_1^2 + a_2 x_1 + ...$
  - Interactions not very flexible
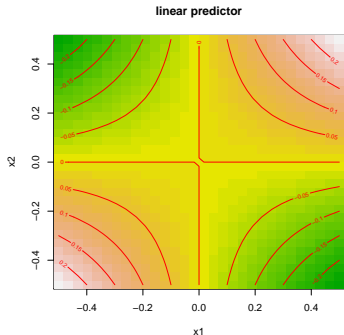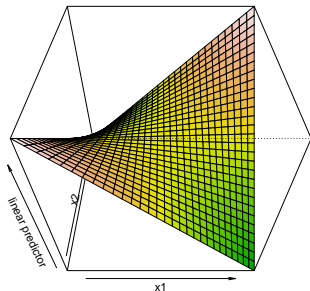
linear predictor

# A generalized linear regression model

- *generalized linear model*: linear relationship between predictors and dependent variable via link function: $g(y) = a_1 x_1 + ... + a_n x_n$
- Examples of link functions:
  - $y^2 = x \Rightarrow y = \sqrt{x}$
  - $\log(y) = x \Rightarrow y = e^x$
  - $\text{logit}(p) = \log(\frac{p}{1-p}) = x \Rightarrow p = \frac{e^x}{e^x+1}$
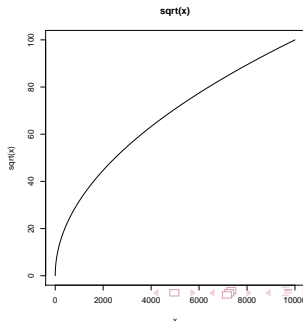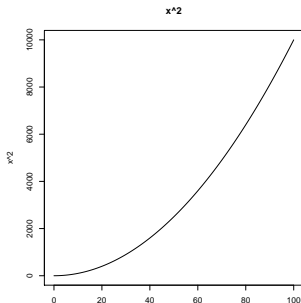
Logistic regression

- Dependent variable is binary (1: success, 0: failure), not continuous
- Transform to continuous variable via log odds: $\log(\frac{p}{1-p}) = \text{logit}(p)$
- Done automatically in regression by setting `family="binomial"`
- interpret coefficients w.r.t. success as logits: in R: `plogis(x)`
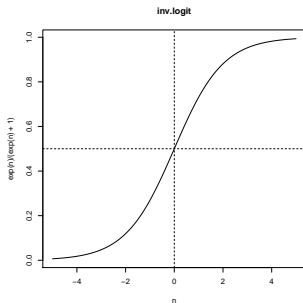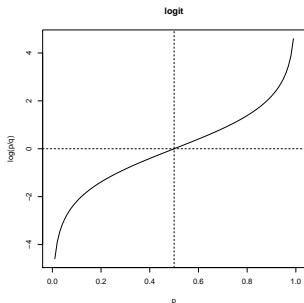
A generalized additive model (1)

▶ *generalized additive model (GAM)*: relationship between individual predictors and (possibly transformed) dependent variable is estimated by a non-linear smooth function: $g(y) = s(x_1) + s(x_2, x_3) + a_4 x_4 + ...$
  ▶ multiple predictors can be combined in a (hyper)surface smooth

Contour plot
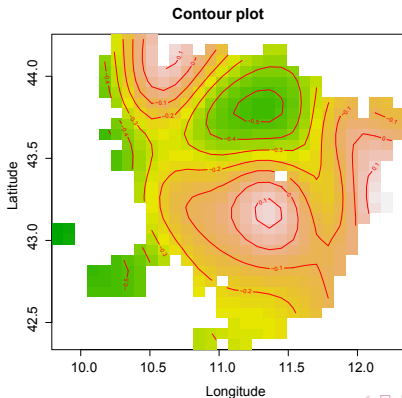
A generalized additive model (2)

- ▶ Advantage of GAM over manual specification of non-linearities: the optimal shape of the non-linearity is determined automatically
  - ▶ appropriate degree of smoothness is automatically determined on the basis of cross validation to prevent overfitting

- ▶ Choosing a smoothing basis
  - ▶ Single predictor or isotropic predictors: thin plate regression spline
    - ▶ Efficient approximation of the optimal (thin plate) spline
  - ▶ Combining non-isotropic predictors: tensor product spline

- ▶ Generalized Additive Mixed Modeling:
  - ▶ Random effects can be treated as smooths as well (Wood, 2008)
  - ▶ R: gam and bam (package mgcv)

- ▶ For more (mathematical) details, see Wood (2006)

Standard Italian and Tuscan dialects

- Standard Italian originated in the 14th century as a written language
- It originated from the prestigious Florentine variety
- The *spoken* standard Italian language was adopted in the 20th century
  - People used to speak in their local dialect

- In this study, we investigate the relationship between standard Italian and Tuscan dialects
  - We focus on lexical variation
  - We attempt to identify which social, geographical and lexical variables influence this relationship

Material: lexical data

- We used lexical data from the *Atlante Lessicale Toscano* (ALT)
  - We focus on 2060 speakers from 213 locations and 170 concepts
  - Total number of cases: 384,454
  - For every case, we identified if the lexical form was different from standard Italian (1) or the same (0)

Geographic distribution of locations

Material: additional data

- In addition, we obtained the following information:
  - Speaker age
  - Speaker gender
  - Speaker education level
  - Speaker employment history
  - Number of inhabitants in each location
  - Average income in each location
  - Average age in each location
  - Frequency of each concept

Modeling geography's influence with a GAM

```
# logistic regression: family="binomial"
> geo = gam(NotStd ~ s(Lon,Lat), data=tusc, family="binomial")
> vis.gam(geo,view=c("Lon","Lat"),plot.type="contour",color="terrain",...)
```


Contour plot
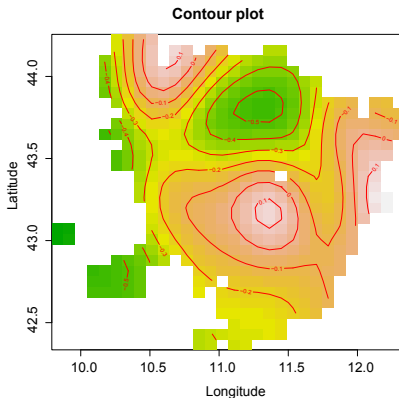
Interpreting logit coefficients

```
> summary(geo)
...
Parametric coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -0.248951   0.003258  -76.42   <2e-16 ***
...

> plogis( coef(geo)["(Intercept)"] ) # = plogis(-0.248951)
[1] 0.4380817 # on average 43.8 percent chance to see non-standard form
```

inv.logit
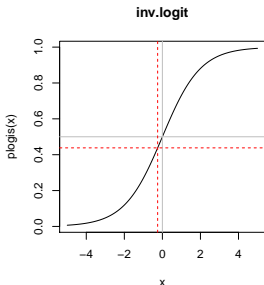
Adding a random intercept to a GAM

```
> model = bam(NotStd ~ s(Lon,Lat) + s(Concept,bs="re"),
              data=tusc, family="binomial")
> summary(model)

Family: binomial
Link function: logit

Parametric coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept)  -0.3614     0.1152  -3.137  0.00171 **

Approximate significance of smooth terms:
               edf Ref.df Chi.sq p-value
s(Lon,Lat)   28.34  28.97   2297  <2e-16 ***
s(Concept)  168.63 169.00  66786  <2e-16 ***

R-sq.(adj) =  0.253   Deviance explained = 20.9%
fREML = 5.452e+05  Scale est. = 1          n = 384454
```

Adding a random slope to a GAM

```
> model2 = bam(NotStd ~ s(Lon,Lat) + CommSize.log.z + s(Concept,bs="re") +
                  s(Concept,CommSize.log.z,bs="re"),
              data=tusc, family="binomial")
> summary(model2)

Family: binomial
Link function: logit

Parametric coefficients:
              Estimate Std. Error z value Pr(>|z|)
(Intercept)   -0.36115    0.11605  -3.112 0.001859 **
CommSize.log.z -0.05519    0.01535  -3.596 0.000323 ***

Approximate significance of smooth terms:
                            edf Ref.df Chi.sq p-value
s(Lon,Lat)                28.31  28.97   2051  <2e-16 ***
s(Concept)               168.63 169.00  81507  <2e-16 ***
s(Concept,CommSize.log.z) 152.50 169.00  32411  <2e-16 ***

R-sq.(adj) =  0.257   Deviance explained = 21.3%
fREML = 5.4484e+05  Scale est. = 1          n = 384454
```

Varying geography's influence based on concept freq.

▶ Wieling, Nerbonne and Baayen (2011, *PLOS ONE*) showed that the effect of word frequency varied depending on geography

▶ Here we explicitly include this in the GAM with `te()`

```
> m = bam(NotStd ~ te(Lon, Lat, Freq, d=c(2,1)) + ...,
          data=tusc, family="binomial")
```

▶ As this pattern may be presumed to differ depending on speaker age, we can integrate this in the model as well

```
> m = bam(NotStd ~ te(Lon, Lat, Freq, Age, d=c(2,1,1)) + ...,
          data=tusc, family="binomial")
```

▶ The results will be discussed next... (Wieling et al., 2014, *Language*)

Results: fixed effects and smooths

	Estimate	Std. Error	z-value	p-value
Intercept	-0.4188	0.1266	-3.31	< 0.001
Community size (log)	-0.0584	0.0224	-2.60	0.009
Male gender	0.0379	0.0128	2.96	0.003
Farmer profession	0.0460	0.0169	2.72	0.006
Education level (log)	-0.0686	0.0126	-5.44	< 0.001

	Est. d.o.f.	Chi. sq.	p-value
Geo × frequency × speaker age	225.9	3295	< 0.001

Interpreting logit coefficients II

```
# chance for a male farmer in a very small village (z-scored population
# size = -2) for which the location is unknown with a very low education
# level (z-score = -2) to use a non-standard lexical form
> plogis(
          coef(m)["(Intercept)"] + coef(m)["SpeakerIsMale"] +
          coef(m)["SpeakerJob_Farmer"] +
          -2 * coef(m)["PopulationSize.log"] + -2 * coef(m)["EduLevel"]
       )
# = plogis(-0.4188 + 0.0379 + 0.0460 + -2 * -0.0584 + -2 * -0.0686)
# = plogis(-0.0809)
[1] 0.479786 # was: 0.438 (43.8%)
```


inv.logit
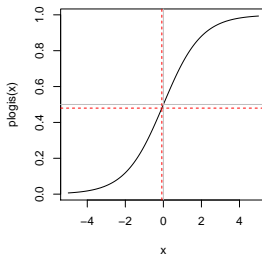
A complex geographical pattern
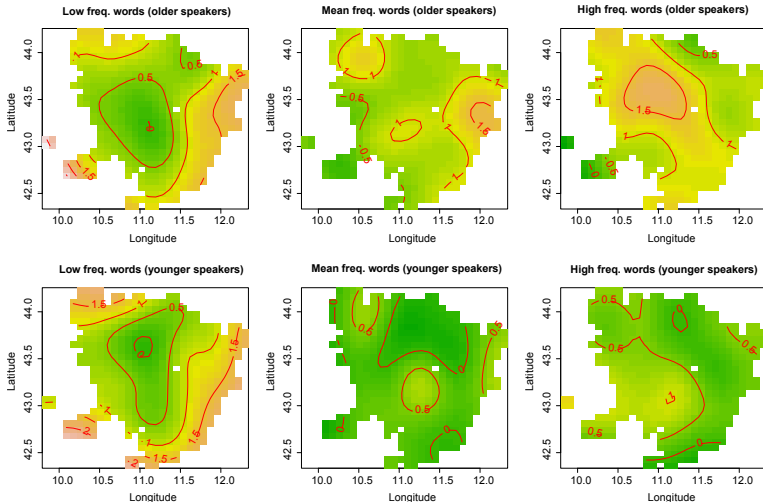
# Animation: increasing frequency for older speakers

Animation: increasing frequency for younger speakers

Results: random effects

| Factors	Random effects	Std. dev.	p-value
Speaker	Intercept	0.0100	0.006
Location	Intercept	0.1874	< 0.001
Concept	Intercept	1.6205	< 0.001
	Year of recording	0.2828	< 0.001
	Community size (log)	0.1769	< 0.001
	Average community income (log)	0.2657	< 0.001
	Average community age (log)	0.2400	< 0.001
	Farmer profession	0.1033	< 0.001
	Executive or auxiliary worker prof.	0.0650	0.002
	Education level (log)	0.1255	< 0.001
	Male gender	0.0797	< 0.001

▶ Complex structure, logistic regression and large dataset: 11 hours of CPU time on 4 processors

By-concept random slopes for community size
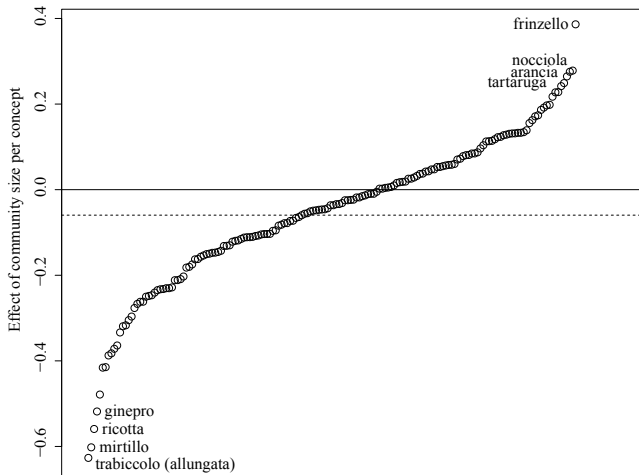
Concepts sorted by the effect of community size

By-concept random slopes for speaker education level

Concepts sorted by the effect of education level
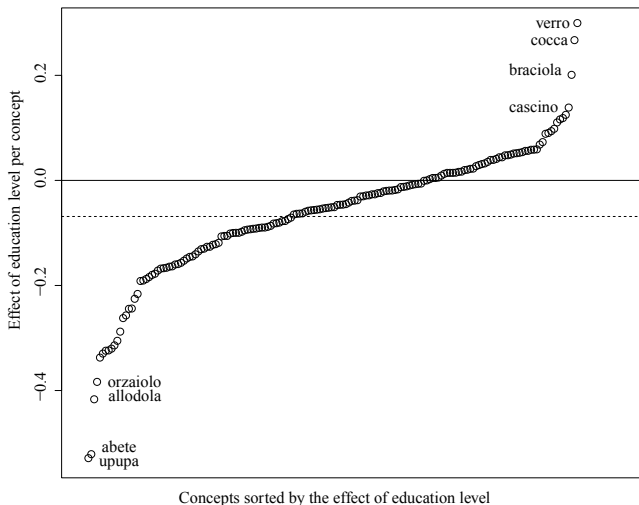
Discussion

- Using a generalized additive mixed-effects regression model (GAMM) to investigate lexical differences between standard Italian and Tuscan dialects revealed interesting dialectal patterns
  - GAMMs are very suitable to model the non-linear influence of geography
  - The regression approach allowed for the simultaneous identification of important social, geographical and lexical predictors
  - By including many concepts, results are less subjective than traditional analyses focusing on only a few pre-selected concepts
  - The mixed-effects regression approach still allows a focus on individual concepts

- There are some drawbacks to GAMMs, however...
  - `bam` and (especially) `gam` are computationally somewhat more expensive than linear mixed-effects modeling using `lmer` (`lme4` package)
  - No correlation parameters in the random-effects structure possible

Conclusion

- ▶ Generalized additive modeling is useful to study non-linear effects
- ▶ Use `bam` if your dataset is large
- ▶ Use `s()` for single / multiple predictors which are on the same scale
- ▶ Use `te()` when predictors are on a different scale
- ▶ (there is also a third option, `ti()`, which will be covered later)

- ▶ We will experiment with these issues in the lab session after the break!
    - ▶ We use a subset of *Dutch* dialect data (faster: no logistic regression)
    - ▶ Similar underlying idea: investigate the effect of geography, word frequency, and location characteristics on pronunciation distances from standard Dutch

- ▶ More interested in Tuscan data and analysis? *Paper package* with all data and analyses available via `http://www.martijnwieling.nl`

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Thank you for your attention!

Any questions?