Software Specifications
The Pumping Lemma-Examples

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Example 1

Given

$$L = \{a^{2^m} | m \geq 1\}$$

Show that L is not regular. Assume L is regular and let n be the constant specified by the Pumping Lemma.
We will first consider k such that

$$2^k > n \ \land \ x = a^{2^k} \in L$$

By the Pumping Lemma we can write $x = p \cdot q \cdot r$ where each part satisfies the Pumping Lemma.
Note that $1 \leq |q| \leq n < 2^k$. Which expands to:

$$2^k < |p \cdot q^2 \cdot r| < 2^k + 2^k = 2^{k+1}$$

Thus we have reached a contradiction as in order for a string to be in L it must have a length equal to a power of 2. From the equation above, it states that for a language of this form to be regular, it must have a length between 2^k and 2^{k+1}. Therefore, L is not regular.

Example 2

Given

$$L = \{a^i b^k | 0 \leq k \leq i\}$$

We shall prove that this language L is not regular via the Pumping Lemma. Let n be the constant given by the pumping lemma. We shall use the following string $x \in L$ in this proof

$$x = a^n b^n \in L$$

By the Pumping Lemma we can write $x = p \cdot q \cdot r$ where x is any string in L and the parts satisfy the conditions of the Pumping Lemma.
We know that $|p \cdot q| \leq n \ \land \ q \neq \epsilon$. This implies that $q = a^t \ \ \{t \geq 1\}$.
Thus we can then derive the following Regex.

$$pq^2 r = a^{n+t} b^n \in L$$

$$pq^3 r = a^{n+2t} b^n \in L$$

Thus we arrive at no contradiction.
However, consider the Regex below instead:

$$pq^0r = p \cdot r = a^{n-t}b^n$$

And Thus, we have reached a contradiction as the number of 'a' cannot be less than the number of 'b' in a string

Notes

- We use the Pumping Lemma in order to show that a language is *not* regular
- Showing that a language satisfies the Pumping Lemma does *not* imply that the language is regular as there exists non-regular languages that satisfy the Pumping Lemma
- if one wants to show that a language is regular, one should produce a Regex or State Diagram for said language