Software Specifications
Deterministic State Diagram Minimization Algorithm

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Minimization Algorithm

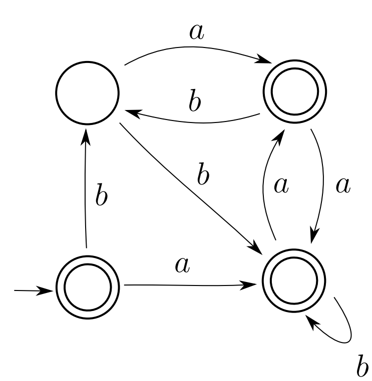
The algorithm for minimizing a state diagram is the following (note: must do preprocessing see lesson 22)

1. mark a pair of states (A, B) where A is final and B is non-final or vice versa. (distinguished by the empty-string ϵ)
   (a) consider pairs (A, B) that remain unmarked. If on a single alphabet symbol $b \in \Sigma$ makes the states (A, B) reach a pair (C, D) and (C, D) was already marked, then mark pair (A, B).
   (b) repeat from step 1 until there are no more pairs (A, B) that are a final and end state and no more pairs (C, D) (see part a).
2. all unmarked state-pairs are indistinguishable and each pair is merged.

This algorithm has a complexity of $O(n^3)$ where n is the number of states. Note: If a pair (p, q) is distinguishable, so is (q, p) thus, the algorithm need only consider half of the n^2 pairs.

Example

consider the given state diagram:



Where the nodes are:
top-left = w, top-right = x, bottom-left = y, bottom-right = z We can then make a matrix representing the pairs, for this problem. Note that because we only need to consider half the pairs, we only need to fill in half the matrix

Stage 0

mark pairs where one state is final and the other is not with 0 (note that the diagonal is filled with X as a state is indistinguishable from its self)

	w	x	y	z
w	X			
x	0	X		
y	0		X	
z	0			X

Stage 1

- $(x, z) \rightarrow^b (w, z)$, (w, z) has been marked, thus mark (x, z)

- $(x, y) \rightarrow^a (z, z) \ \wedge \rightarrow^b (w, w)$, $(z, z) \wedge (w, w)$ has not been marked, thus (x, y) is not marked,

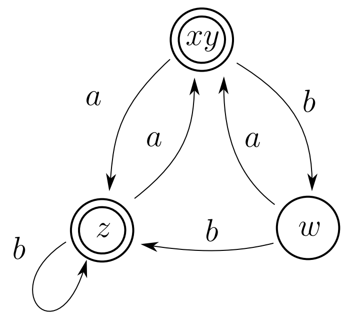
- $(y, z) \rightarrow^b (w, z)$, (w, z) is marked, thus mark (y, z)

thus, we have marked all states that can be marked as per algorithm step 1.b.

	w	x	y	z
w	X			
x	0	X		
y	0		X	
z	0	1	1	X

stage 2

Now we have (x, y) as our only resulting indistinguishable pair. Thus, we will merge x, y to get our final unique minimized Deterministic State Diagram.



Note: there is only one minimal state for each language