

Data Structures

Quiz 3 Review

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Priority Queue

we should know the following things to do with Priority Queue:

- priority queue vs. BST
- Properties - ie. for a complete binary tree, what is the priority level
- Operations and time complexity
- Implementations
 - max heap, min heap (array based, linked list, tree)
 - check, put (add, insert), pop (delete)
 - heapify
 - heapsort
- Applications
 - use priority queue for task scheduling
 - heapsort

Operations

a priority queue has three operations:

- i insert a new item into a queue
- ii get the value of the highest priority item
- iii remove the highest priority item from the queue

Additionally, when we implement a priority queue, we must define what is the priority of each possible item in the queue (normally based on its value)

Heaps

There are 2 types of heaps that we can use (which implement the priority queue)

- maxheap
- minheap

maxheap has the following advantages:

- Quickly insert items into the heap
- quickly extract the largest item from the heap

minheap has the following advantages

- quickly insert a new item into the heap
- quickly extract the smallest value from the heap

Implementation

When implementing a heap, we find the best way is by using an array. We do this like so:

1. the root of the heap goes in `array[0]`
2. if the data for the node appears in `array[i]`, it's children, if they exist, are in the locations:
 - left child: `array[2i+1]`
 - right child: `array[2i+2]`
3. if the data for a non-root node is `array[i]`, then its parent is always at `array[(i-1)/2]` (using integer division)

Extracting from the Heap

Adding Node to Heap

Complexity the complexity of the Heap, when both inserting and extracting is

$$O(\log_2(n))$$

But, if you cannot find a given element in a heap, it takes

$$O(n)$$

time. In order to avoid this case we can implement efficient heapsort.

1. If the `len(heap) == 0` (it's an empty tree), return error.
2. Otherwise, `heap[0]` holds the biggest value. Remember it for later.
3. If the `len(heap) == 1` (that was the only node) then **delete the only value** and return the saved value.
4. Copy the value from the right-most, bottom-most node to the root node:
`heap[0] = heap[-1]`
5. Delete the right-most node in the bottom-most row: `del heap[-1]`
6. Repeatedly swap the just-moved value with the larger of its two children:
Starting with `i=0`, compare and swap:
`heap[i]` with `heap[2*i+1]` and `heap[2*i+2]`
7. Return the saved value to the user.

	heap
0	12
1	7
2	10
3	3
4	2
5	8
6	4
7	2
8	1
9	
10	
11	
12	
13	
...	

Heapsort

Heapsort is done in 2 steps:

1. convert input array into maxheap
2. reheapify the array:

Complexity the time complexity for transferring a list into a maxheap is:

$$O(n)$$

and the time complexity for extracting a element is :

$$O(\log n) * n$$

1. Insert a new node in the bottom-most, left-most open slot:
`heap.append(value)`
2. Compare the new value `heap[-1]` with its parent's value: `heap[(len(heap)-1 -1)/2]`
3. If the new value is greater than its parent's value, then swap them. (*we don't need to care other nodes, why?*)
4. Repeat steps 2-3 until the new value rises to its proper place or we reach the top of the array.

0	10
1	7
2	8
3	3
4	

...

Hash Tables

Let's update our shuffling algorithm slightly!

$\text{startNode} = N/2 - 1$

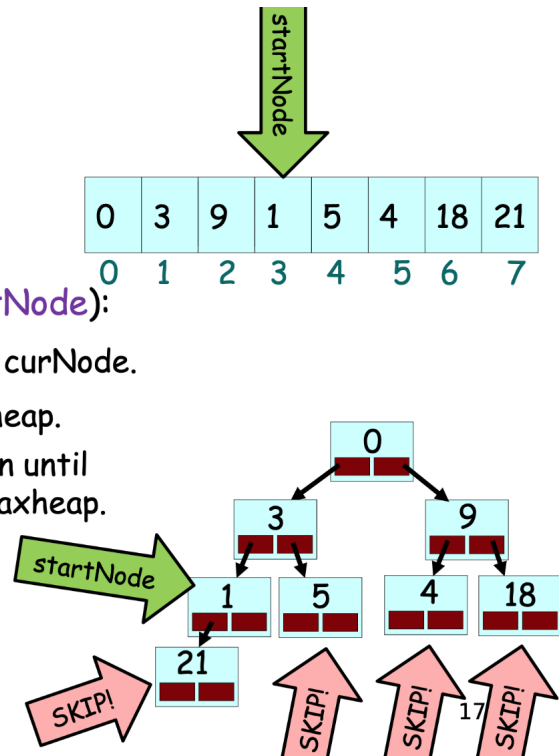
for ($\text{curNode} = \text{startNode}$ till rootNode):

Focus on the subtree rooted at curNode .

Think of this subtree as a maxheap.

Keep shifting the top value down until your subtree becomes a valid maxheap.

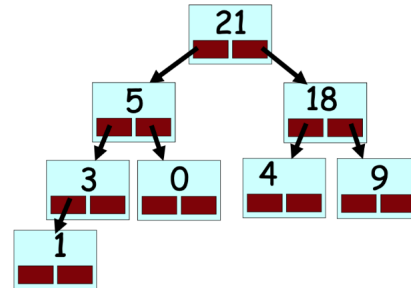
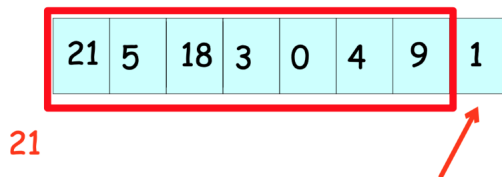
This is the complete version of the efficient shuffling algorithm!



Types

There are 2 types of hash tables:

So we've completed Step #1 and our input array now holds a valid maxheap. On to Step #2!



Reheapification Algorithm (same as before)

1. → Copy the value from the right-most node in the bottom-most row to the root node.
2. → Delete the right-most node in the bottom-most row.
3. → Repeatedly swap the just-moved value with the larger of its two children until the value is greater than or equal to both of its children.

maxheap and re-heapify
(go)
y
(in it)
freed-up slot of the array.

Chaining

Chaining is a way of searching for an item:

- 1) **Concepts:**
 - closed hash table, open hash table, open addressing, linear probing, quadratic probing, double hashing, load factor, rehash, hash function.
- 2) **Operations** (status after operations) and time complexity
 - Hash table (search, insert, delete)
 - Solve collisions (separate chaining, open address)
 - Average steps for different hash tables (open addressing vs. chaining).
- 3) **Implementation** (writing code)
 - Closed hash table and open hash table (insertion/search/deletion).
- 4) Understand **how to design** a good hash function. Identify whether a given hash function is good or bad.

There are many schemes for dealing with collisions, and today we'll learn **two** of the most popular...

Closed Hash Table

Open Hash Table

Closed Hash Table: all elements are stored in the hash table itself



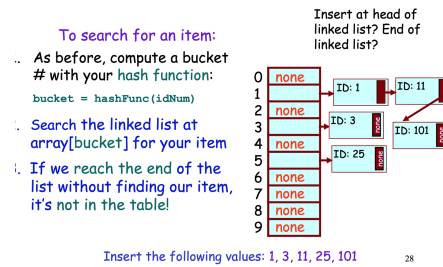
Closed Hashing
= Open Addressing

Open Hashing
= Separate chaining

Open addressing: Put everything into the table, but not necessarily into cell $h(k)$.

Implementation

The following is a possible implementation of a hash table in python.



```

Chained-Hash-Insert( T, x )
    insert x at the head of list T[ h( key[x] ) ]

def insert(new_object):
    hash_val = h(new_object.key)
    new_object.next = T[hash_val]
    T[hash_val] = new_object

Chained-Hash-Search( T, k )
    search for an element with key k in list T[ h(k) ]

Chained-Hash-Delete( T, x )
    delete x from the list T[ h( key[x] ) ]

```

Collisions

collisions are when the hash table probes into a bucket that is already full, there are 2 ways we can avoid that:

- quadratic probing - using a quadratic function to dictate the sequence in which buckets are checked
- double hasing - using 2 hash functions in order to add more randomness to the initial probing and avoid collisions.