Data Structures
Red And Black Trees

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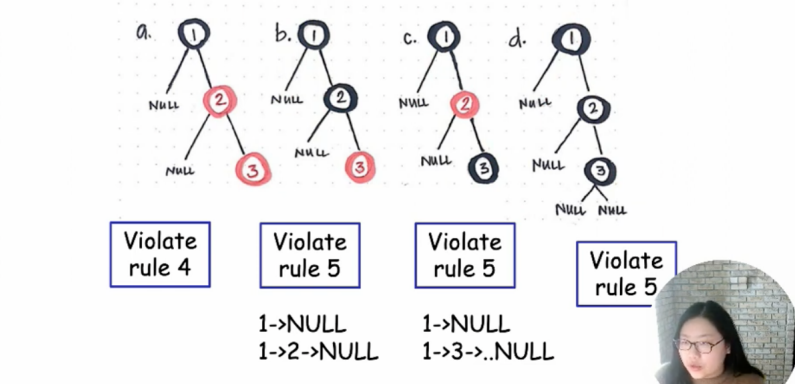
February 28, 2022

Definition of RBT

An Red Black Tree is another type of binary search tree that is self balancing. Each node in the tree is either coloured *red* or *black*. In order for a tree to be a RBT it must satisfy the following conditions

1. Every node is red or black
2. the root is black
3. every leaf (NIL) is black
4. if a node is red, then both of it's children are black. thus no 2 consecitive nodes are red.
5. for each node x, all simple paths from the node to it's descendant leaves contain the same ammount of black nodes.



An RBTree also has the following properties:

- The path from any node to the farthest leaf is no more than *twice* as long as the path from the node to the nearest leaf
- the sortest path for node n down to a leaf is equal to $k' \geq b \geq \frac{k}{2}$ where k' is the length of the shortest path and b is the number of black nodes in all paths (as this number must be the same for all paths).
- a red black tree with nodes has a height of:

$$h \geq 2\log_2(n+1)$$

- the number of leaves in an RBTree is $n+1$