Software Specifications
Closure Properties of Regular Languages

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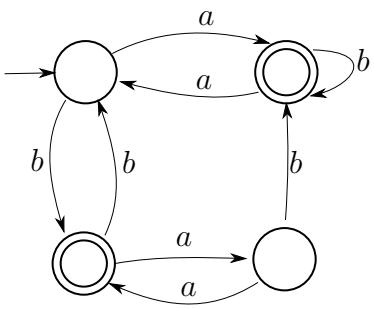
Closure Properties

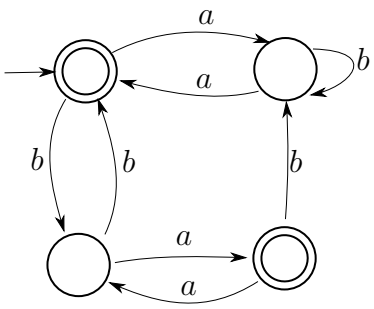
Regular languages are closed under Boolean operations thus, if R and S are regular, then so are:

- $R \cup S$ $(=R+S)$
- $R \cap S$
- $\overline{R} = \Sigma^* - R$ (complement of R)

To illustrate this property we will show the state diagram fir language L:



And then the closure of diagram L,



Such that every time L is in a final state, its closure is not. Note that this can only be done with Deterministic State Diagrams.

Example

Given:

$$A = \{w \in \Sigma \mid w \text{ has equally many many occurrences of symbols a and b}\}$$

Show that A is non-regular
Recall:

$$B = \{a^i b^i \mid i \geq 0\}$$

Note:

$$A \cap a^* \, b^* = B$$

If A were regular, then is B (because the union of 2 regular languages is also regular). We know B is not regular thus: A is not reguar