MATH 111 A7

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1 Problem 3

Given:

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \\ z_{n+1} \end{bmatrix} = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix} \text{ where: } \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

find the solution to this dynamical equation by finding the Eigen Values.

The Characteristic polynomial is:

$$det(A - \lambda I) = det \begin{bmatrix} -1-\lambda & 4 & -2 \\ -3 & 4-\lambda & 0 \\ -3 & 1 & 3-\lambda \end{bmatrix}$$

Thus:

$$= -\lambda^3 + 6\lambda^2 - 11\lambda + 6 = -(\lambda - 1)(\lambda - 2)(\lambda - 3)$$

the zeroes the characterisitc equation are $1, 2, 3$. therefore, these are the Eigen Values for A
now we must find the Eigen Vectors of A using the Eigen Values:

$\lambda = 1$:

$$A - I = \begin{bmatrix} -2 & 4 & -2 \\ -3 & 3 & 0 \\ -3 & 1 & 2 \end{bmatrix} : x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\lambda = 2$:

$$A - 2I = \begin{bmatrix} -3 & 4 & -2 \\ -3 & 2 & 0 \\ -3 & 1 & 1 \end{bmatrix} : x = \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}$$

$\lambda = 3$:

$$A - 3I = \begin{bmatrix} -4 & 4 & -2 \\ -3 & 1 & 0 \\ -3 & 1 & 0 \end{bmatrix} : x = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$$

now we must write the the starting vector as a linear combination of the just solved Eigen Vectors:

$$\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = a \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + b \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix} + c \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$$

which solves to $a = 1, b = 1, c = -1$
apply A^k:

$$\begin{bmatrix} x_k \\ y_k \\ z_k \end{bmatrix} = A^k \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = (1^k) \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + (2^k) \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix} - 1(3^k) \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$$

2 Problem 4

Given:

$$S(n) = u_1u_2 + u_2u_3 + u_3u_4 + ... + u_{n-1}u_n$$

find a simple form of $S(n)$.

$S(2) = 1 * 1 = 1$
$S(3) = S(2) + 2 * 1 = 3$
$S(4) = S(3) + 3 * 2 = 9$
$S(5) = S(4) + 5 * 3 = 24$
$S(6) = S(5) + 8 * 5 = 64$
$S(7) = S(6) + 13 * 8 = 168$

Thus we can see that, when cross referenced with the table of u_n numbers, the equation for the n^{th} sum given n is:

if n is even:

$$S(n) = u_{n-1}u_{n+1} - 1$$

if n is odd:

$$S(n) = u_{n-1}u_{n+1}$$

Assuming $S(n)$ is true, we must establish $S(n+2)$ for both the even and odd equation

EVEN:
$P(n+2) : u_1u_2 + u_3u_4 + ... + u_{2n-1}u_{2n} + u_{2n+1}u_{2n+2} = u_{2n+1}u_{2n+3} - 1$
$LHS : (u_1u_2 + u_3u_4 + ... + u_{2n-1}u_{2n}) + u_{2n+1}u_{2n+2}$
$LHS : (u_{2n-1}u_{2n+1} - 1) + u_{2n+1}u_{2n+2}$ - by induction hypothesis

$$RHS = u_{n-1}u_{n+1} - 1$$

by law of the Fibonacci sequence therefore the above equation is true.

ODD:
$P(n+2) : u_2u_3 + u_4u_5 + ... + u_{n-1}u_n + u_{n+1}u_{n+3} = u_{n+1}u_{n+3}$
$LHS : (u_2u_3 + u_4u_5 + ... + u_{n-1}u_n) + u_{n+1}u_{n+3}$
$LHS : (u_{n-1}u_{n+1}) + u_{n+1}u_{n+3}$ - by induction hypothesis

$$RHS = u_{n-1}u_{n+1}$$

by the law of the Fibonacci sequence therefore the above equation is true.