Data Structures
Red Black Tree Insert and Delete

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Intro to RBTree Insert and Delete

the insert and delete operations modify the tree such that we must fix the following in order for it to remain a RBTree.

1. colour changes
2. restructuring the links of the tree via. rotation

Rotations

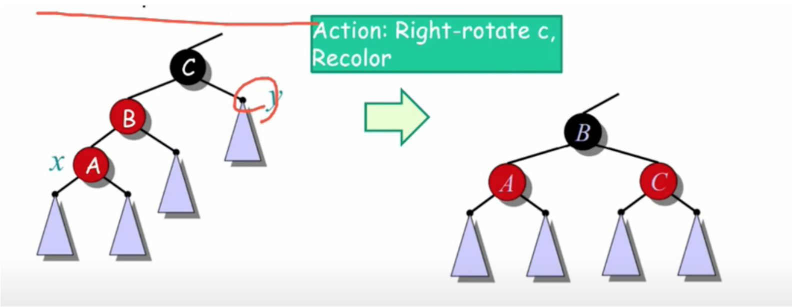
rotations are used to maintain the ordering of the keys. A rotation can be performed in $O(n)$

Insertion

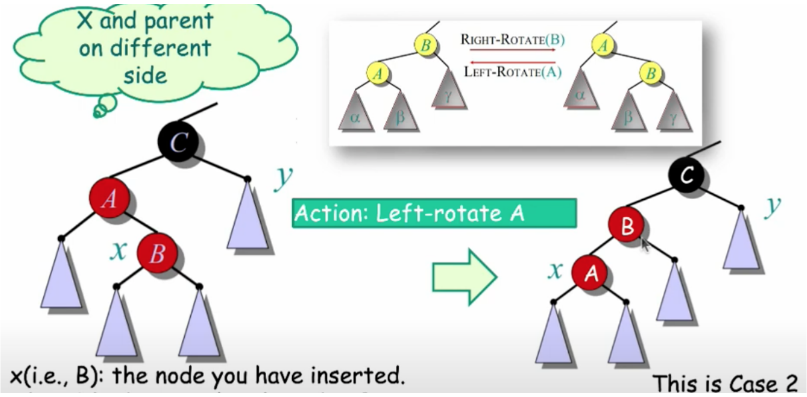
When doing an insertion, the only property that is violated is property 4 (see lesson 5-3-5). This is because nodes are only added as a leaf of the tree (as is the normal). There are 3 cases for this violation given x, the target node:

1. if the Aunt or Uncle node y of x is red then:
   recolour y, the parent of y, and the parent of x

2. if the Aunt or Uncle node is black, and x and it's parent are on the same side, then:
   right-rotate y and recolour the parent of x and y



3. if the Aunt or Uncle node is black, and x and it's parent are on different sides, then:
   left rotate x



X and parent on different side

RIGHT-ROTATE(B)

LEFT-ROTATE(A)

Action: Left-rotate A

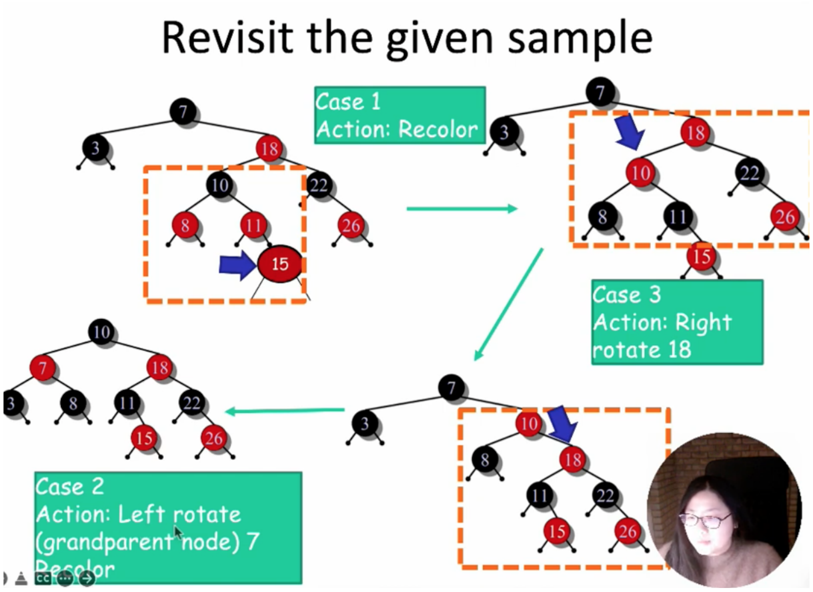
x(i.e., B): the node you have inserted.

This is Case 2

The time complexity of all 3 of these cases are:

$$O(\log n)$$

to see each of these cases in the context of eachother in a real example, observe the following:



Revisit the given sample

Case 1
Action: Recolor

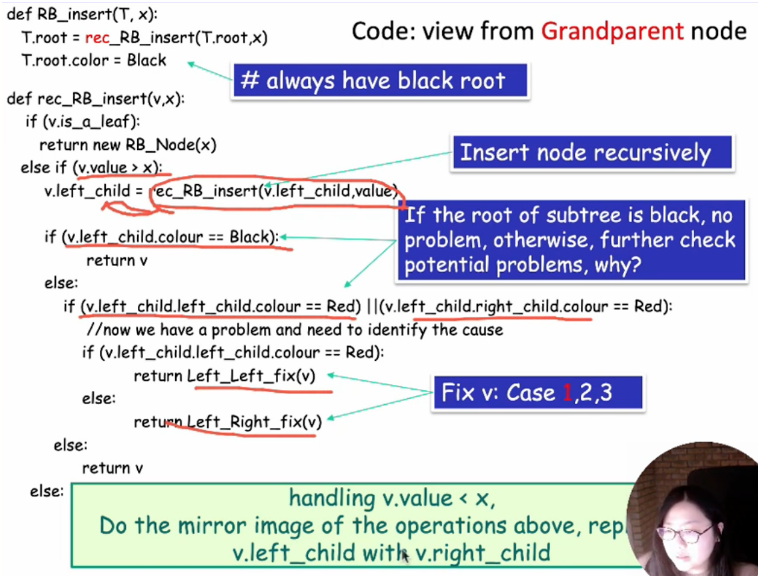
Case 3
Action: Right rotate 18

Case 2
Action: Left rotate (grandparent node) 7
Recolor

Algorith for Insertion from all this information, we can generalize the process for insertion as the following algorithm:

```
if current and parent are both red:
        if grandparent's other child (aunt or uncle) is red:
                colour grandpa red
        else if the current and parent are both on the same side:
                do a single rotation and recolour
        else:
                do a double rotation and recolour
```

An implementation can also be seen in the following figure



Code: view from Grandparent node

```
def RB_insert(T, x):
  T.root = rec_RB_insert(T.root,x)
  T.root.color = Black

def rec_RB_insert(v,x):
   if (v.is_a_leaf):
     return new RB_Node(x)
  else if (v.value > x):
     v.left_child = rec_RB_insert(v.left_child,value)

     if (v.left_child.colour == Black):
           return v
     else:
       if (v.left_child.left_child.colour == Red) ||(v.left_child.right_child.colour == Red):
         //now we have a problem and need to identify the cause
         if (v.left_child.left_child.colour == Red):
               return Left_Left_fix(v)
         else:
               return Left_Right_fix(v)
    else:
         return v
 else:
```

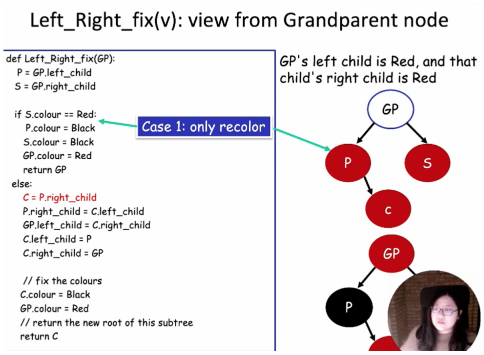
always have black root

Insert node recursively

If the root of subtree is black, no problem, otherwise, further check potential problems, why?

Fix v: Case 1,2,3

handling v.value < x,
Do the mirror image of the operations above, rep
v.left_child with v.right_child

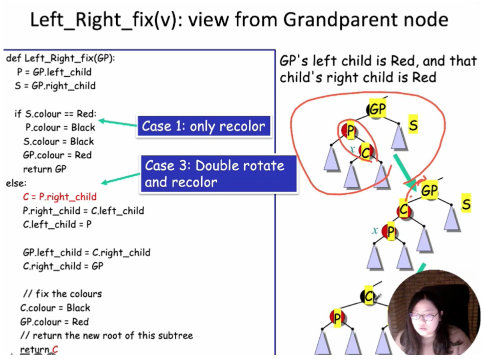


Left_Right_fix(v): view from Grandparent node

```
def Left_Right_fix(GP):
   P = GP.left_child
   S = GP.right_child

   if S.colour == Red:
      P.colour = Black
      S.colour = Black
      GP.colour = Red
      return GP
   else:
      C = P.right_child
      P.right_child = C.left_child
      GP.left_child = C.right_child
      C.left_child = P
      C.right_child = GP

      // fix the colours
      C.colour = Black
      GP.colour = Red
      // return the new root of this subtree
      return C
```

Case 1: only recolor

GP's left child is Red, and that child's right child is Red



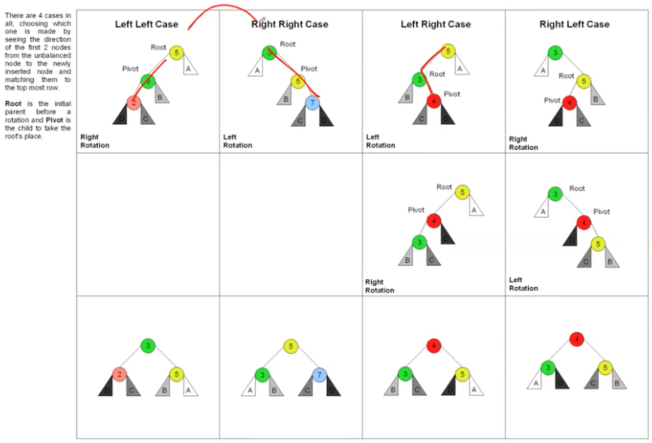
Left_Right_fix(v): view from Grandparent node

```
def Left_Right_fix(GP):
   P = GP.left_child
   S = GP.right_child

   if S.colour == Red:
      P.colour = Black
      S.colour = Black
      GP.colour = Red
      return GP
else:
      C = P.right_child
      P.right_child = C.left_child
      C.left_child = P

      GP.left_child = C.right_child
      C.right_child = GP

      // fix the colours
      C.colour = Black
      GP.colour = Red
      // return the new root of this subtree
      return C
```

Case 1: only recolor

Case 3: Double rotate and recolor

GP's left child is Red, and that child's right child is Red



There are 4 cases in all, choosing which one is made by seeing the direction of the first 2 nodes from the unbalanced node to the newly inserted node and matching them to the top most row.

Root is the initial parent before a rotation and **Pivot** is the child to take the root's place.