# Software Specifications
# Context Free Grammar Pumping Lemma
# Examples

Cain Susko

Queen's University
School of Computing

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# Example

where
(P1) $v \neq \varepsilon$ or $x \neq \varepsilon$
(P2) $|vwx| \leq p$
(P3) for each $i \geq 0$ : $uv^iwx^iy \in L$

Consider the Language of Squares:

$$L_2 = \{ww \mid w \in \{a,b\}*\}.$$

The claim is that $L_2$ is not context free.
Note: showing that $L_2$ is not CF, roughly speaking, shows that variable declarations cannot be specified with CF Grammars.

**attempt 1** The Question is what string $s$ should we use to derive a contradiction with the Pumping Lemma? The first (bad) idea for $s$ is the following:

$$s = a^p b a^p b \in L_2.$$

$$s = a^{p-1} a b a^{p-1} b.$$

Where:

$$\begin{aligned} u &= a^{p-1} \\ v &= a \\ w &= b \\ x &= a \\ y &= a^{p-1} b \end{aligned}$$

.

Let p be the constant yielded from the pumping lemma. Additionally as given by the pumping lemma, $s = uvwxy$ for all context free languages.
However, there is **no contradiction** with $s$ as $v$ and $x$ can be repeated in parallel.

**attempt 2** We have to show that **any** way of writing a string in 5 parts, as per the pumping lemma, does *not* satisfy the pumping lemma in order to show that a language is context free. We need to also make sure that the middle part of the string ($vwx$) with length at most $p$.
Thus, we will try a better idea for $s$:

$$s = a^p b^p a^p b^p.$$

Thus, we will start our proof:
For the sake of contradiction assume that $L_2$ is context free and let $p$ be the constant given by the pumping lemma.

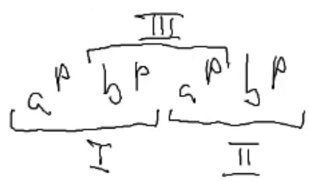
$$s = a^p b^p a^p b^p.$$

by the pumping lemma, $s$ can be written in 5 parts:

$$s = uvwxy$$

where $u, v, w, x, y$ satisfy the 3 properties of the pumping lemma.
we shall divide $s$ into 3 parts such that:

$$I = a^p b^p$$
$$II = a^p b^p$$
$$III = b^p a^p$$




Since $|vwx| \leq p$, the substring $vwx$ must be within one of the parts $I, II, III$.

part I $vwx$ is inside the prefix $a^p b^p$ in the string $uv^2wx^2y$. The first symbol of the second half of the prefix is $b$ and the first symbol of the first half is an $a$. Therefore, $uv^2wx^2y$ is not in $L_2$

part II Similarly, if $vwx$ is inside the suffix $a^p b^p$ in the string $uv^2wx^2y$ then the last symbol of the first half is $a$ and the last symbol of the second half is $b$. Again, $uv^2wx^2y \notin L_2$

part III The last case is that $vxy$ is in the middle part. Now, $uv^0wx^0y = uwy$ is of the form $a^p b^i a^j b^p$ where $i \neq p$ or $j \neq p$. Again, $uv^0wx^0y \notin L_2$, which produces **a contradiction**, and thus, $L_2$ is *not* context free.