Computer Architecture
Numbers

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Question 1

Encode the following decimal numbers with 8-bit two's complement binary, or indicate that number would overflow the range:

1. 49_{10}
2. -31_{10}
3. 120_{10}
4. -128_{10}
5. 128_{10}

Answer

A 2's complement number is a number encoded in w bits with a range of:

$$(-2^{w-1}, 2^{w-1}]$$

Where the most significant digit is the sign and the remaining $w - 1$ bits represent the actual number.

1. 00110001
2. 11100001
3. 01111000
4. 10000000
5. Overflow

Question 2

Around 250 B.C., the Greek mathematician Archimedes proved that:

$$\frac{223}{71} < \pi < \frac{22}{7}$$

Had he had access to a computer and the standard library `<math.h>` he would have been able to determine that the single precision floating-point approximation of π has the hexadecimal representation:

`0x40490FDB`

Of course, all of these are just approximations since π is not rational.

1. What is the Fractional binary number denoted by this floating point value?
2. What is the Fractional binary number representation of $\frac{22}{7}$?
3. At what bit positions relative to the binary point do these two approximations of π diverge?

Answer

This question both requires the knowledge of converting hexadecimal to binary as well as converting binary to integer. Furthermore one must also know how to read encoded floating point binary. If we have a Fractional binary number with w bits, a digit at position i relative to the ones position represents the value 2^i. Given a w-bit binary string which represents the value x, the Fractional decimal representation N of this binary string is

$$N = \frac{x}{2^w}$$

1. 01000000010010010000111111011011
2. 01000000010010010010010010010010
3. they differ at the 10^{th} position from the binary decimal (if counting up from 1)