

# Software Specifications Context-Free Grammar Example

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February 4, 2022

## Example

Given the following, find the grammar for  $B$ :

$$\begin{aligned}\Sigma &= \{a, b, c, d\} \\ B &= \{a^i b^{2k} c^k d^{3i} \mid i, k \geq 1\} \cup \{a^r b^{2r} c^s d^{3s} \mid r, s \geq 1\}.\end{aligned}$$

We can see that in both sets,  $a, b$  and  $c, d$  are related in the first set and  $a, b$  and  $c, d$  are related in the second set because of the exponents. Note that union is like saying *or*. Thus the first rule is

$$S \rightarrow X \mid Y$$

now we can continue with defining  $X$  and  $Y$ .

We can then define  $X$  as an option between its two related parts:

$$S \rightarrow aXd^3 \mid aZd^3$$

Note that  $X$  is in the first option because it can continue to make  $a, d$  until it switches to the  $Z$  option where it will make the inner part of the string.

$$Z \rightarrow b^2 Zc \mid b^2.$$

Where the second option is the terminal string for  $X$

Now we can move on to  $Y$

$$Y \rightarrow UW.$$

Note that capital letters like  $W, U$  are variables. The above rule represents the concatenation of the two strings  $U, W$

Now, to define  $U$ :

$$U \rightarrow aUb^2 \mid ab^2.$$

which represents  $U$ 's recursive and terminal strings.

Finally,  $W$  is written as

$$W \rightarrow cWd^3 \mid cd^3.$$

Which has a similar form to  $U$

Thus we have found the recursive grammar rules for the union of sets  $B$