

Software Specifications
The Pumping Property of Context Free
Grammars

Cain Susko

Queen's University
School of Computing

February 7, 2022

Limitations of Context Free Grammars

Grammars are very useful and versatile however they have some limitations. Particularly, a limitation of CF Grammars is their pumping property which states that:

if a grammar has a sufficiently long terminal string then the parse tree of said terminal string must repeat a non-terminal string within the said parse tree.

Pumping Lemma for CF Languages

For every context free language L there exists a constant p such that any string $s \in L$ of length at least p can be written in five parts:

$$s = uvwxy.$$

where:

1. $v \neq \epsilon$ or $x \neq \epsilon$
2. $|vwx| \leq p$
3. $uv^iwx^iy \in L$ for all $i \geq 0$

the Context Free Pumping Lemma is used to show that a language is not Context Free. This is done by using proof by contradiction.

Example

Given: $L = \{a^ib^ic^i\}$

Using the CF Pumping Lemma we show that L is not context free. For the sake of contradiction assume that L is Context Free and let p be the constant given by the pumping lemma. We will use the following string in this proof:

$$s = a^pb^pc^p \in L.$$

By the Pumping Lemma, s can be written in five parts: $s = uvwxy$ where the parts satisfy 1, 2, 3 (see pumping lemma above). There are 2 cases to consider in this situation:

- if one of v or x contains more than one type of symbol then
 $uv^2wx^2y \notin a^*b^*v^*$ and $uv^2wx^2y \notin L$
- if v and x both have one type of symbol then, uv^2wx^2y cannot have the same number of all symbols a, b and c Thus, $uv^2wx^2y \notin L$

All cases lead to a contradiction! Thus this means that L is not context free.