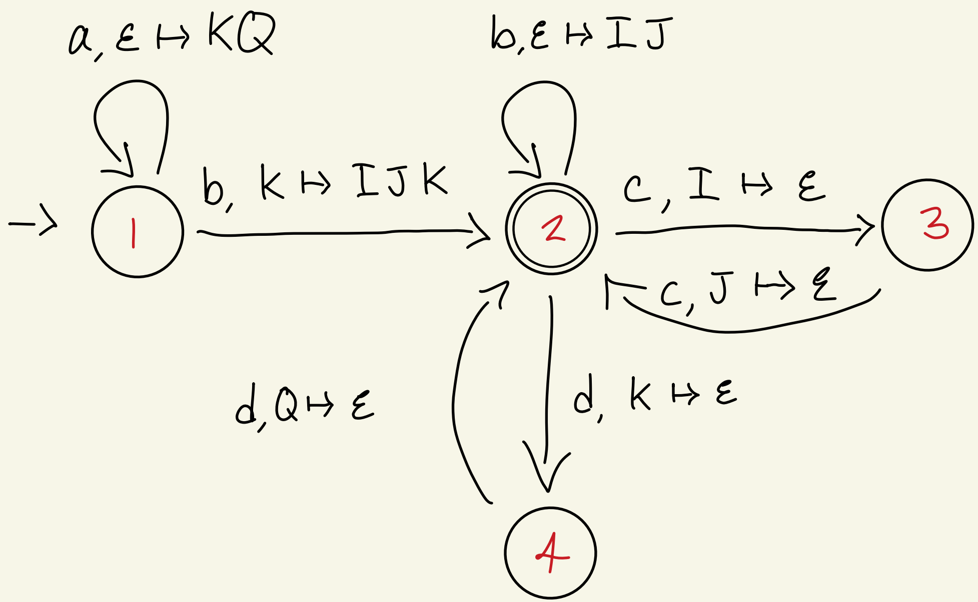
$L = \{ a^k b^i c^{2i} d^{2k} \}$



State	Stack	Input
1	ε	abbccccdd
1	KQ	bbccccdd
2	IJKQ	bccccdd
2	IJIJKQ	ccccdd
3	JIJKQ	cccdd
2	IJKQ	ccdd
3	JKQ	cdd
2	KQ	dd
4	Q	d
2	ε	EOS

and thus, the string is accepted by the Deterministic Pushdown Automata as the stack is empty once reaching the final State

$A = \{ a^k b^i c^{2k} d^{2l} \mid k \geq i \geq 0,\ l \geq 0 \}$

A is not Context free

For the sake of argument, assume A is Context Free. Let p be the constant given by the pumping lemma. We will choose $S = a^{\frac{p}{2}} b^i c^p d^{2l}$ as our string for the proof. By the pumping lemma, S can be written in 5 parts such that: $S = uv^i wx^i y \in A$ for each $i \geq 0$.

If $a^{\frac{p}{2}} b^i$ is in u & c^p is in $v^j w$, with x^j being ε & y being d^{2l}, thus $S = uv^2 wx^2 y = a^{\frac{p}{2}} b^i (c^p c^p) d^{2l}$. $(c^p c^p)$ is within vwx & $p + p > p$, thus we have reached a contradiction & therefore: A is not Context Free

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$$B = \{ a^{2i+1} b^{k+1} c^{3l} d^{2k+1} a^{i} \mid i, k, l \geq 1 \}$$

B is context free

Grammar:

$S \to aaSa \mid aQ$

$Q \to bQdd \mid bPd$

$P \to cccP \mid \varepsilon$

$S \to aaSa \to aaaaSaa$

$\to aaaaaQaa$

$aaaaabQddaa$

$aaaaabbPdddaa$

$aaaaabbcccPdddaa$

$\underbrace{aaaaa}_{4+1}\underbrace{bb}_{1+1}\underbrace{ccc}_{3}\underbrace{ddd}_{2+1}\underbrace{aa}_{2}$

$\downarrow$

$2i+1 \quad k+1 \quad 3l \quad 2k+1 \quad i$

$i = 2$
$k = 1$
$l = 1$

thus, $L(B) = B$ ✱
B is Context free

Grammar: $\Sigma = \{a, b, c, d\}$

$S \rightarrow UV \mid a$

$U \rightarrow XY$

$V \rightarrow bUV \mid \varepsilon$

$X \rightarrow dSd \mid c$

$Y \rightarrow cXY \mid \varepsilon$

i) $\text{First}(S) = \{d, a, \varepsilon\}$

ii) $\text{First}(UV) = \{d, b, \varepsilon\}$

iii) $\text{First}(XY) = \{d, c, \varepsilon\}$

iv) $\text{Follow}(S) = \{d\} \longrightarrow S \rightarrow UV \rightarrow XY \rightarrow dSd$

v) $\text{Follow}(U) = \{b, d, \text{EOS}\} \longrightarrow$

$S \rightarrow UV \rightarrow UbUV$

$S \rightarrow UV \rightarrow XY \rightarrow dSd \rightarrow dUVd \rightarrow dUd$

$S \rightarrow UV \rightarrow U$

vi) $\text{Follow}(V) = \{\text{EOS}\} \longrightarrow S \rightarrow UV \rightarrow XYV \rightarrow dSdV \rightarrow dadV$

vii) $\text{Follow}(X) = \{c, d, \text{EOS}\} \longrightarrow$

$S \rightarrow UV \rightarrow XY \rightarrow XcXY$

$S \rightarrow UV \rightarrow XY \rightarrow dSd \rightarrow dUVd \rightarrow dXYd \rightarrow dXd$

$S \rightarrow UV \rightarrow XY \rightarrow X$

viii) $\text{Follow}(Y) = \{\text{EOS}\} \longrightarrow S \rightarrow UV \rightarrow XY$

Grammar: $\Sigma = \{a, b, c, d\}$

$S \to UV \mid a$

$U \to XY$

$V \to bUV \mid \varepsilon$

$X \to dSd \mid c$

$Y \to cXY \mid \varepsilon$

we know from class that in order for a grammar to allow recursive descent parsing it must satisfy 2 conditions for any 2 productions having the same variable on the left:

$N \to \alpha \mid \beta$

RD1 $\text{First}(\alpha) \cap \text{First}(\beta) = \emptyset$

RD2 if $\beta \to^* \varepsilon$ then: $\text{First}(\alpha) \cap \text{Follow}(N) = \emptyset$

thus, if we take the intersection of sets for each non-terminal symbol we get:

$S' = \{d, b, \varepsilon\} \cap \{a\} = \emptyset$

$V' = \{d, b, \varepsilon\} \cap \{EOS\} = \emptyset$

$X' = \{d, b, \varepsilon\} \cap \{c\} = \emptyset$

$Y' = \{d, c, \varepsilon\} \cap \{EOS\} = \emptyset$

thus, for each pair of productions w the same non-terminal on the left satisfy RD1 & RD2 therefore, the given grammar allows recursive descent

Eliminate Left recursion in the following Grammars:

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a) $S \to Sa \mid ba \mid Sdc \mid \varepsilon$

$S \to baS' \mid \varepsilon$

$S' \to aS' \mid dcS' \mid \varepsilon$

b) $S \to abSc \mid abdSd \mid cbad \mid bdd \mid \varepsilon$

$S \to abS' \mid cbadS \mid bddS \mid \varepsilon$

$S' \to bSc \mid dSd$

c) $S \to Scb \mid Sac \mid ab \mid Sdb \mid cba$

$S \to abS' \mid cbaS'$

$S' \to cbS' \mid acS' \mid dbS'$

d) $S \to SA \mid \varepsilon \qquad A \to Ac \mid b$

$S \to SA \mid \varepsilon$

$A \to bA'$

$A' \to cA$

e) $S \to bcbdSa \mid bcbcSb \mid cbSca \mid dbba \mid \varepsilon$

$S \to bcbS' \mid cbSca \mid dbbaS \mid \varepsilon$

$S' \to dSa \mid cSb$

f) $S \to aacSb \mid cSb \mid aacbd \mid dcb \mid \varepsilon$

$S \to aaS' \mid dcbS' \mid aacbdS' \mid \varepsilon$

$S' \to cSb$

cont.

g) $S \to acbSd \mid bdadS \mid dbabS \mid acc \mid dba \mid \varepsilon$

$S \to acS' \mid dbS'' \mid bdadS$

$S' \to bSd \mid c$

$S'' \to abS \mid a$

h) $S \to Sba \mid Sbc \mid a \mid c \mid da$

$S \to bS' \mid S' \mid daS$

$S' \to aS \mid cS$