Software Specifications
Context-Free Grammar Definition

Cain Susko

Queen's University
School of Computing

February 4, 2022

Definition

A context free grammar is a tuple G such that

$$G = (V, \Sigma, S, P)\,.$$

Where:

V is a finite set of variables or non-terminals

Σ is a finite set of terminals, $V \cap \Sigma \neq \emptyset$

$S \in V$ is the starting variable

P is a finite set of productions of the form $N \to w$ where $N \in V$ and $w \in (\Sigma \cup V)*$

Derivation Step

Consider two strings such that:

$$w_1, w_2 \in (\Sigma \cup V)$$

String w_1 derives w_2 in one step,

$$w_1 \implies w_2$$

if we are able to write:

$$w_1 = uNv \wedge w_2 = uwv$$

such that $N \to w \in P$; $n, v, w \in (\Sigma \cup V*)$; $N \in V$

Furthermore, the Operator for reflexive trasitive closure on $\Rightarrow$ is denoted by $\Rightarrow^*$ which can be used if $w_1 = w_2$ or if $w_1 \implies n_1 \implies \ldots \implies n_k \implies w_2$ Additionally, a language generated by Grammar G is:

$$L(G) = \{w \in \Sigma \mid S \Rightarrow^* w\}.$$

Example

Given:

$$A = \{a^{3i}b^k c^{2i+3} \mid i \geq i, l \geq 1\}.$$

The grammar for A:

$$\begin{aligned} S &\to a^3Sc^2 \\ S &\to a^3Xc^5 \\ X &\to bX \\ X &\to b. \end{aligned}$$

We can then derive the terminal string for A

$$S \implies a^3Sc^2 \implies a^6Xc^7 \implies a^6bXc^7 \implies a^6b^2c^7.$$

Sort Hand Notation

an easier way of writing the grammar for A in shorthand notation is as follows:

$$\begin{aligned} S &\to a^3Sc^2 \mid a^3Xc^5 \\ X &\to bX \mid b. \end{aligned}$$