Software Specifications
Context Free Grammar Pumping Lemma Examples

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February 17, 2022

Example

Consider the Language of Squares:

$$L_2 = \{ww \mid w \in \{a, b\}*\}.$$

The claim is that L_2 is not context free.
Note: showing that L_2 is not CF, roughly speaking, shows that variable declarations cannot be specified with CF Grammars.

attempt 1 The Question is what string s should we use to derive a contradiction with the Pumping Lemma? The first (bad) idea for s is the following:

$$s = a^p b a^p b \in L_2.$$

$$s = a^{p-1} a b a^{p-1} b.$$

Where:

$$\begin{aligned} u &= a^{p-1} \\ v &= a \\ w &= b \\ x &= a \\ y &= a^{p-1} b \\ &. \end{aligned}$$

Let p be the constant yielded from the pumping lemma. Additionally as given by the pumping lemma, $s = uvwxy$ for all context free languages.
However, there is **no contradiction** with this s as v and x can be repeated in parallel.

attempt 2 We have to show that **any** way of writing a string in 5 parts, as per the pumping lemma, does *not* satisfy the pumping lemma in order to show that a language is context free. We need to also make sure that the middle part of the string (vwx) with length at most p.
Thus, we will try a better idea for s:

$$s = a^p b^p a^p b^p.$$

Thus, we will start our proof:
For the sake of contradiction assume that L_2 is context free and let p be the constant given by the pumping lemma
MUST COMPLETE NOTE

$$s = a^p b^p a^p b^p.$$