Software Specifications
Defining the Pumping Lemma

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Pumping Lemma

the Lemma is as follows:
For every regular language L there is a constant n such that any string $x \in L$ is of length at least n. The lemma can be written in three parts.

$$x = p \cdot q \cdot r$$

1. $q \neq \epsilon$
2. $|pq| \leq n$
3. $p \cdot q^k \cdot r \in L \quad \forall_k \{k \geq 0\}$

The Pumping Lemma is used to show that a language is *not* regular. Simply put, if a language does not contain a string with length n but does contain one with length $n + 1$, the language is not regular. This is normally done using proof by contradiction:

1. assume the language in question is regular
2. show that the Pumping lemma cannot hold for the said language

Example

given the set of balanced strings

$$L_1 = \{a^i b^i | i \geq 0\}$$

we shall show that L_1 is not regular. For the sake of contradiction we will assume that L_1 is regular. Let n be the constant given by the Pumping Lemma.

$$x = a^n b^n \in L_1$$

By the Pumping Lemma x can be written in three parts

$$x = p \cdot q^k \cdot r$$

Where the parts satisfy the conditions of the Pumping Lemma.
Since $|p \cdot q| \leq n$ and $q \neq \epsilon$, we know that $q = a^t \ \forall_t \{t \geq 1\}$
Thus:

$$p \cdot q^i \cdot r = a^{n+t} b^n$$

Which is a contradiction as in order for the Pumping Lemma to hold, we must have an unbalanced number of a or b. Thus, L_1 is not a Regular Language