CISC/CMPE 223 - Assignment 3 (Winter 2022)
Due: Thursday March 10, 2:00 PM

Regulations on assignments

- The assignments are graded according to the correctness, preciseness and legibility of the solutions. All handwritten parts, including figures, should be clear and legible. This assignment is marked out of 20 possible marks.
- Please submit your solution in onQ before the due time. The submission must be in one of formats: .PDF, .JPG, .PNG, .DOCX.
- **The assignment must be based on individual work.** Copying solutions from other students is a violation of academic integrity. See the course onQ site for more information.

1. (5 marks) Design a deterministic pushdown automaton that recognizes the language

$$L = \{a^k b^i c^{2i} d^{2k} \mid i \geq 1, k \geq 1\}$$

Draw a table that traces the behavior of your pushdown automaton on the input *abbccccdd*. The table should list the current state, currently remaining input, and current stack contents at each step of the computation (see examples on pages 217–218).

Note: Since the pushdown automaton has to be deterministic, we cannot use the general transformation that converts a grammar to a pushdown automaton (as discussed on pp. 218–219 in the text and in video 31).

2. Are the following languages context-free or non-context-free?

   - If a language is context-free, give a context-free grammar that generates it.
   - If a language is not context-free, prove this using the pumping lemma.

   (a) (2.5 marks) $A = \{\ a^k b^i c^{2k} d^{2\ell} \mid k \geq i \geq 0,\ \ \ell \geq 0\ \}$

   (b) (2.5 marks) $B = \{a^{2i+1} b^{k+1} c^{3\ell} d^{2k+1} a^i \mid i \geq 1,\ k \geq 1,\ \ell \geq 1\ \}$

3. Consider the grammar with the following rules (capital letters are nonterminals and S is the start nonterminal):

$S \longrightarrow UV \mid a$

$U \longrightarrow XY$

$V \longrightarrow bUV \mid \varepsilon$

$X \longrightarrow dSd \mid c$

$Y \longrightarrow cXY \mid \varepsilon$

(a) (4 marks) Determine each of the sets

i. FIRST(S)
ii. FIRST(UV)
iii. FIRST(XY)
iv. FOLLOW(S)
v. FOLLOW(U)
vi. FOLLOW(V)
vii. FOLLOW(X)
viii. FOLLOW(Y)

For each element z belonging to a set FOLLOW(W) ($W \in \{S, U, V, X, Y\}$) give a derivation starting from S where z occurs directly after W. If z is `EOS`, give a derivation starting from S where in the resulting string W occurs as the last symbol. (The corresponding derivations are not required for the FIRST-sets.)

(b) (2 marks) Does the grammar allow the use of recursive-descent parsing? Justify your answer using your answers from part (a) – please be specific and give a detailed answer.

4. (4 marks) Use left-factoring and/or eliminate left-recursion to transform each of the below eight grammars into a form where the immediate problems preventing the use of recursive-descent parsing have been removed. As usual capital letters denote variables and lower case letters are terminals.

(a) $S \longrightarrow Sa \mid ba \mid Sdc \mid \varepsilon$

(b) $S \longrightarrow abSc \mid abdSd \mid cbad \mid bdd \mid \varepsilon$

(c) $S \longrightarrow Scb \mid Sac \mid ab \mid Sdb \mid cba$

(d) $S \longrightarrow SA \mid \varepsilon$
$A \longrightarrow Ac \mid b$

(e) $S \longrightarrow bcbdSa \mid bcbcSb \mid cbSca \mid dbba \mid \varepsilon$

(f) $S \longrightarrow aacSb \mid cSb \mid aacbd \mid dcb \mid \varepsilon$

(g) $S \longrightarrow acbSd \mid bdadS \mid bdabS \mid acc \mid dba \mid \varepsilon$

(h) $S \longrightarrow Sba \mid Sbc \mid a \mid c \mid da$
Hint for (h): Here after elimination of left recursion you need to use also left-factoring (or vice versa).