Séries de taylor

$\cos(x) \longrightarrow \quad C_0 + C_1 x + C_2 x^2$

Primeiro Termo:

$\cos(0) = 1 \qquad C_0 + C_1.(0) + C_2.(0)^2 = 1 \Rightarrow \boxed{C_0 = 1}$

Segundo Termo:

$\frac{d}{dx}\cos(x) = -\operatorname{sen}(x) \qquad C_1 + 2.C_2\, x$

$-\operatorname{sen}(0) = 0 \qquad C_1 + 2.C_2.(0) = 0 \Rightarrow \boxed{C_1 = 0}$

Terceiro Termo:

$\frac{d^2}{dx^2}\cos(x) = -\cos(x) \qquad 2\,C_2$

$-\cos(0) = -1 \qquad 2\,C_2 = -1 \Rightarrow \boxed{C_2 = -1/2}$

Assim:

$$\cos(x) \approx 1 + 0x - \frac{1}{2}x^2$$

Incluindo mais termos

$$\cos(x) \approx 1 - \frac{1}{2}x^2 + \underbrace{C_3 x^3}_{\text{Quarto termo}} + C_4 x^4 + C_5 x^5 + \dots$$

$\frac{d^3}{dx^3}\cos(x) = \operatorname{sen}(x) \qquad (3.2.1).C_3 + \cancel{O(x)}^{0} = 0$

$\operatorname{sen}(0) = \varphi \qquad C_3 = \frac{1}{3.2.1}.(0) \Rightarrow \boxed{C_3 = 0}$

$$\frac{d^4}{dx^4}\cos(x) = \cos(x) \qquad (4.3.2.1).C_4 + \cancel{0(x)}^{0}$$

$$\cos(0) = 1 \qquad \underbrace{(4.3.2.1)}_{4!}.C_4 = 1 \Rightarrow \boxed{C_4 = \frac{1}{4!}}$$

Definição Geral

$$\boxed{f(x) = f(a) + \frac{df(a)}{dx}(x-a) + \frac{1}{2!}\frac{d^2}{dx^2}f(a)(x-a)^2 + \frac{1}{3!}\frac{d^2}{dx^2}f(a)(x-a)^3 + \dots}$$

↳ Serie de Taylor

Exemplo:

$$F(x) = e^x$$

expandindo em torno de $x=0$:

$$e^x = e^0 + \frac{de^{(0)}}{dx}x + \frac{1}{2!}\frac{d^2e^{(0)}}{dx^2}x^2 + \frac{1}{3!}\frac{d^3}{dx^3}e^{(0)}x^3 + \dots$$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots = \sum_{N=0}^{\infty}\frac{x^N}{N!}$$

$$\boxed{e^x = \sum_{N=0}^{\infty}\frac{x^N}{N!}}$$

Para valores pequenos de x:

$$\boxed{e^x \approx 1 + x} \qquad e^{(0.01)} = 1.010050167\dots$$

$\longrightarrow 1+0.01 = 1.01$

$$F(x) = e^{ix}$$

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$$e^{ix} = e^{i0} + i e^{i0} x + \frac{1}{2} i^2 e^{i0} x^2 + \frac{1}{3!} i^3 e^{i0} x^3 + \frac{1}{4!} i^4 e^{i0} x^4 + \frac{1}{5!} i^5 e^{i0} x^5 + \dots$$

$$= 1 + ix - \frac{x^2}{2} - \frac{ix^3}{3!} + \frac{x^4}{4!} + \frac{ix^5}{5!} - \frac{x^6}{6!} - \frac{ix^7}{7!} + \frac{x^8}{8!} + \dots$$

$$= \underbrace{\left[1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} + \dots\right]}_{\cos(x)} + i \underbrace{\left[x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots\right]}_{\operatorname{sen}(x)}$$

$$\boxed{e^{ix} = \cos(x) + i \operatorname{sen}(x)} \rightarrow$$ Formula de Euler

$$\boxed{e^{-ix} = \cos(x) - i \operatorname{sen}(x)}$$

$$e^{ix} + e^{-ix} = 2\cos(x) \Rightarrow \boxed{\cos(x) = \frac{e^{ix} + e^{-ix}}{2}}$$

$$e^{ix} - e^{-ix} = 2i \operatorname{sen}(x) \Rightarrow \boxed{\operatorname{sen}(x) = \frac{e^{ix} - e^{-ix}}{2i}}$$