Análise e Modelagem de Sistemas Dinâmicos

Prova 1 – 07/03/2023

Nome: **Ra:**

1 – Utilize a definição para calcular a derivada das seguintes funções:

a) $\frac{1}{x}$
b) $\frac{1-x}{2x}$

2 – Utilizando a lei das potências, calcule a derivada das seguintes funções:

a) $\frac{1}{x^2}$
b) $\sqrt{x^{2+\pi}}$

3 – A função $cot(x)$ é definida como $cot(x) = \frac{cos(x)}{\text{sen}(x)}$. E a função $\text{cossec}(x)$ é definida como $\sec(x) = \frac{1}{\text{sen}(x)}$. Calcule a derivada da função $cot(x)$.

4 – Calcule a derivada das seguintes funções:

a) $2x^3\,\text{sen}(x) - e^{2x}$
b) $\frac{xcos(x)}{sen(x)} - e^{\text{sen}(x)}$

①

$$\frac{dF(x)}{dx} = \lim_{\Delta x \to 0} \frac{F(x+\Delta x) - F(x)}{\Delta x}$$

a) $F(x) = \frac{1}{x}$

$$\frac{df}{dx} \approx \frac{1}{\Delta x} \cdot \left[\frac{1}{(x+\Delta x)} - \frac{1}{x} \right]$$

$$\approx \frac{1}{\Delta x} \cdot \frac{x - (x+\Delta x)}{x(x+\Delta x)}$$

$$\approx - \frac{1}{x(x+\Delta x)}$$

$$\Delta x \to 0 \Rightarrow \boxed{\frac{df}{dx} = -\frac{1}{x^2}}$$

b) $F(x) = \frac{1-x}{2x}$

$$\frac{df}{dx} \approx \frac{1}{\Delta x} \left[\frac{1-(x+\Delta x)}{2(x+\Delta x)} - \left(\frac{1-x}{2x} \right) \right]$$

$$\approx \frac{1}{2\Delta x} \left[\frac{x - x(x+\Delta x) - (1-x)(x+\Delta x)}{x(x+\Delta x)} \right]$$

$$\approx \frac{1}{2\Delta x} \left[\frac{x - x^2 - x\Delta x - x - \Delta x + x^2 + x\Delta x}{x(x+\Delta x)} \right]$$

$$\approx \frac{1}{2\Delta x} \left[\frac{-\Delta x}{x(x+\Delta x)} \right] = \frac{-1}{2x(x+\Delta x)}$$

$$\Delta x \to 0 \Rightarrow \boxed{\frac{df}{dx} = -\frac{1}{2x^2}}$$

(2)

a) $F(x) = \frac{1}{x^2} = x^{-2}$

$$\frac{df}{dx} = -2x^{-3} \Rightarrow \boxed{\frac{df}{dx} = -\frac{2}{x^3}}$$

b) $F(x) = \sqrt{x^{2+\pi}} = x^{\frac{2+\pi}{2}} = x^{1+\pi/2}$

$$\frac{df}{dx} = (1+\pi/2)\, x^{(1+\pi/2)-1} = \left(1+\frac{\pi}{2}\right) x^{\pi/2}$$

$$\boxed{\frac{df}{dx} = \left(1+\frac{\pi}{2}\right)\sqrt{x^{\pi}}}$$

(3) $\cot(x) = \frac{\cos(x)}{\operatorname{sen}(x)} = \cos(x)\, \operatorname{sen}(x)^{-1}$

$$\frac{df}{dx} = -\operatorname{sen}(x)\, \operatorname{sen}(x)^{-1} + \cos(x)\,(-1)\,\operatorname{sen}(x)^{-2} . \cos(x)$$

$$= -1 - \frac{\cos^2(x)}{\operatorname{sen}^2(x)} = -\frac{\operatorname{sen}^2(x) - \cos^2(x)}{\operatorname{sen}^2(x)}$$

$$= -\frac{1}{\operatorname{sen}^2(x)}$$

$$\boxed{\frac{df}{dx} = -\operatorname{cossec}^2(x)}$$

(4)

a) $\left(2x^3 \operatorname{sen}(x) - e^{2x}\right)' = 2\left[3x^2 \operatorname{sen}(x) + x^3 \cos(x)\right] - 2xe^{2x}$

$= \boxed{6x^2 \operatorname{sen}(x) + 2x^3 \cos(x) - 2xe^{2x}}$

b) $\left(\frac{x\cos(x)}{\operatorname{sen}(x)} - e^{\operatorname{sen}(x)}\right)' = \left(x\cot(x) - e^{\operatorname{sen}(x)}\right)'$

$= \boxed{\cot(x) - x\operatorname{cossec}^2(x) - \cos(x).\, e^{\operatorname{sen}(x)}}$