generalizando:

$$f(x) = x^N$$

$$\frac{df}{dx} \approx \frac{1}{\Delta x}\left[(x+\Delta x)^N - x^N\right]$$

$$\approx \frac{1}{\Delta x}\left[x^N + N x^{N-1}\Delta x + \frac{N(N-1)}{2}x^{N-2}\Delta x^2 + \dots - x^N\right]$$

$$\approx \frac{1}{\Delta x}\left[N x^{N-1}\Delta x + O(\Delta x^2)\right]$$

$$\approx N x^{N-1} + O(\cancel{\Delta x})^{\to 0}$$

$$\boxed{\frac{df}{dx} = N x^{N-1}}$$

Funções trigonométricas

$$f(x) = \text{sen}(x)$$

Dois resultados de limite que serão usados:

$$\boxed{\lim_{x\to 0}\frac{\text{sen}(x)}{x} = 1} \qquad \boxed{\lim_{x\to 0}\frac{\cos(x)-1}{x} = 0}$$

$$\frac{df}{dx} \approx \frac{\text{sen}(x+\Delta x) - \text{sen}(x)}{\Delta x}$$

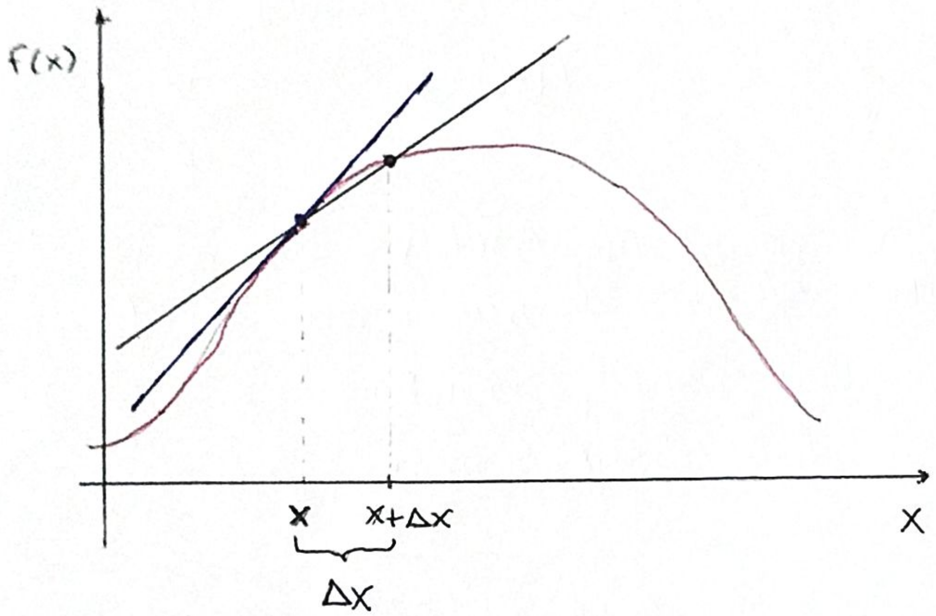
$$\approx \frac{\text{sen}(x)\cos(\Delta x) + \text{sen}(\Delta x)\cos(x) - \text{sen}(x)}{\Delta x}$$

$$\approx \text{sen}(x)\,\cancel{\frac{[\cos(\Delta x)-1]}{\Delta x}}^{\,0} + \cos(x)\,\cancel{\frac{\text{sen}(\Delta x)}{\Delta x}}^{\,1}$$

$$\boxed{\frac{df}{dx} = \cos(x)}$$

Derivada

Taxa de variação de uma função com relação a uma variável independente



$$\boxed{\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}}$$

Lei das Potências

$f(x) = x^2$

$$\frac{df}{dx} \approx \frac{(x+\Delta x)^2 - x^2}{\Delta x}$$

$$\approx \frac{(x^2 + 2x\Delta x + \Delta x^2) - x^2}{\Delta x}$$

$$\approx \frac{2x\Delta x + \Delta x^2}{\Delta x} = 2x + \cancel{\Delta x}^{0}$$

$$\boxed{\frac{df}{dx} = 2x}$$

$F(x) = \cos(x) \rightarrow \boxed{\dfrac{dF}{dx} = -\operatorname{sen}(x)}$

Exercício da lista 1

Exponencial

$$F(x) = b^x$$

$$\frac{dF}{dx} \approx \frac{b^{(x+\Delta x)} - b^x}{\Delta x}$$

$$\approx \frac{b^x \cdot b^{\Delta x} - b^x}{\Delta x} = b^x \left[\frac{b^{\Delta x} - 1}{\Delta x} \right]$$

$$\frac{dF}{dx} = b^x \lim_{\Delta x \to 0} \left[\frac{b^{\Delta x} - 1}{\Delta x} \right]$$

→ A derivada de uma função exponencial é sempre proporcional à própria função

Δx	$\dfrac{2^{\Delta x} - 1}{\Delta x}$	$\dfrac{3^{\Delta x} - 1}{\Delta x}$
0.1	0.71773	1.16123
0.01	0.69556	1.10467
0.001	0.69339	1.09922
0.0001	0.69317	1.09867
0.00001	0.69315	1.09862

$$\lim_{\Delta x \to 0} \frac{2^{\Delta x} - 1}{\Delta x} \approx 0.69315 \longrightarrow \frac{d}{dx}(2^x) = 2^x \cdot (0.69315)$$

$$\lim_{\Delta x \to 0} \frac{2^{\Delta x} - 1}{\Delta x} \approx 1.09862 \longrightarrow \frac{d}{dx}(3^x) = 3^x \cdot (1.09862)$$

Existe alguma base b onde $\displaystyle\lim_{\Delta x \to 0} \frac{b^{\Delta x} - 1}{\Delta x} = 1$?

$e \approx 2.71828$ $\boxed{\displaystyle\lim_{\Delta x \to 0} \frac{e^{\Delta x} - 1}{\Delta x} = 1}$