Contents

1 Data structures

1.1 Ufds

```
class UFDS {
 private:
  vector<int> size, ps;

 public:
  UFDS(int N) : size(N + 1, 1), ps(N + 1) { iota(ps.begin(), ps.end(), 0); }

  int find_set(int x) { return x == ps[x] ? x : (ps[x] = find_set(ps[x])); }

  bool same_set(int x, int y) { return find_set(x) == find_set(y); }

  void union_set(int x, int y) {
    if (same_set(x, y)) return;

    int p = find_set(x);
    int q = find_set(y);

    if (size[p] < size[q]) swap(p, q);

    ps[q] = p;
    size[p] += size[q];
  }
};
```

2 Dynamic programming

2.1 Kadane

```
int kadane(const vi& xs) {
  vi s(xs.size());
  s[0] = xs[0];

  for (size_t i = 1; i < xs.size(); ++i) s[i] = max(xs[i], s[i - 1] + xs[i]);

  return *max_element(all(s));
}
```

2.2 Longest Increasing Subsequence (LIS)

Time: $O(N \cdot \log N)$.

```
int lis(vi const& a) {
  int n = a.size();
  const int INF = 1e9;
  vi d(n + 1, INF);
  d[0] = -INF;

  for (int i = 0; i < n; i++) {
    int l = upper_bound(d.begin(), d.end(), a[i]) - d.begin();
    if (d[l - 1] < a[i] && a[i] < d[l]) d[l] = a[i];
  }

  int ans = 0;
  for (int l = 0; l <= n; l++) {
    if (d[l] < INF) ans = l;
  }

  return ans;
}
```

3 Graphs

3.1 Dijkstra

```
vector<pll> adj[MAX];
class Graph {
 public:
  void add(ll u, ll v, ll w) {
    adj[u].emplace_back(v, w);
    // Undirected Graph
    // adj[u].emplace_back(v, w);
  }

  vl dijkstra(ll src, ll n) {
    vl ds(n, LLONG_MAX);
    ds[src] = 0;
    set<pll> pq;
    pq.emplace(0, src);

    while (!pq.empty()) {
      ll u = pq.begin()->second;
      ll wu = pq.begin()->first;
      pq.erase(pq.begin());

      if (wu != ds[u]) continue;
      for (auto [v, w] : adj[u]) {
        if (ds[v] > ds[u] + w) {
          ds[v] = ds[u] + w;
          pq.emplace(ds[v], v);
        }
      }
    }

    return ds;
  }
};
```

4 Math

4.1 Convex Hull

Given a set of points find the smallest convex polygon that contains all the given points.
Time: $O(N \cdot \log N)$
By default it removes the collinear points, set the boolean to true if you don't want that

```
struct pt {
  double x, y;
};
```

```cpp
int orientation(pt a, pt b, pt c) {
  double v = a.x * (b.y - c.y) + b.x * (c.y - a.y) + c.x * (a.y - b.y);
  if (v < 0) return -1;  // clockwise
  if (v > 0) return +1;  // counter-clockwise
  return 0;
}

bool cw(pt a, pt b, pt c, bool include_collinear) {
  int o = orientation(a, b, c);
  return o < 0 || (include_collinear && o == 0);
}
bool collinear(pt a, pt b, pt c) { return orientation(a, b, c) == 0; }

void convex_hull(vector<pt>& a, bool include_collinear = false) {
  pt p0 = *min_element(a.begin(), a.end(), [](pt a, pt b) {
    return make_pair(a.y, a.x) < make_pair(b.y, b.x);
  });
  sort(a.begin(), a.end(), [&p0](const pt& a, const pt& b) {
    int o = orientation(p0, a, b);
    if (o == 0)
      return (p0.x - a.x) * (p0.x - a.x) + (p0.y - a.y) * (p0.y - a.y) <
             (p0.x - b.x) * (p0.x - b.x) + (p0.y - b.y) * (p0.y - b.y);
    return o < 0;
  });
  if (include_collinear) {
    int i = (int)a.size() - 1;
    while (i >= 0 && collinear(p0, a[i], a.back())) i--;
    reverse(a.begin() + i + 1, a.end());
  }

  vector<pt> st;
  for (int i = 0; i < (int)a.size(); i++) {
    while (st.size() > 1 &&
           !cw(st[st.size() - 2], st.back(), a[i], include_collinear))
      st.pop_back();
    st.push_back(a[i]);
  }

  a = st;
}
```

4.2 Factorization With Sieve

```cpp
map<ll, ll> factorization_with_sieve(ll n, const vl& primes) {
  map<ll, ll> fact;

  for (ll d : primes) {
    if (d * d > n) break;

    ll k = 0;
    while (n % d == 0) {
      k++;
      n /= d;
    }

    if (k) fact[d] = k;
  }
  if (n > 1) fact[n] = 1;
  return fact;
}
```

4.3 Factorization

```cpp
map<ll, ll> factorization(ll n) {
  map<ll, ll> ans;
  for (ll i = 2; i * i <= n; i++) {
    ll count = 0;
    for (; n % i == 0; count++, n /= i)
      ;
    if (count) ans[i] = count;
  }
  if (n > 1) ans[n]++;
  return ans;
}
```

4.4 Euler phi $\varphi(n)$

Computes the number of positive integers less than n that are coprimes with n, in $O(\sqrt{N})$.

```cpp
ll phi(ll n) {
  if (n == 1) return 1;

  auto fs = factorization(n);
  auto res = n;

  for (auto [p, k] : fs) {
    res /= p;
    res *= (p - 1);
  }

  return res;
}
```

4.5 Point To Segment

```cpp
typedef pair<double, double> pdb;

#define fst first
#define snd second

double pt2segment(pdb A, pdb B, pdb E) {
  pdb AB = {B.fst - A.fst, B.snd - A.snd};
  pdb BE = {E.fst - B.fst, E.snd - B.snd};
  pdb AE = {E.fst - A.fst, E.snd - A.snd};

  double AB_BE = AB.fst * BE.fst + AB.snd * BE.snd;
  double AB_AE = AB.fst * AE.fst + AB.snd * AE.snd;

  double ans;
  if (AB_BE > 0) {
    double y = E.snd - B.snd;
    double x = E.fst - B.fst;
    ans = sqrt(x * x + y * y);
```

```
  } else if (AB_AE < 0) {
    double y = E.snd - A.snd;
    double x = E.fst - A.fst;
    ans = sqrt(x * x + y * y);
  } else {
    auto [x1, y1] = AB;
    auto [x2, y2] = AE;
    double mod = sqrt(x1 * x1 + y1 * y1);
    ans = abs(x1 * y2 - y1 * x2) / mod;
  }

  return ans;
}
```

4.6 Sieve

```
vl sieve(ll N) {
  bitset<MAX + 1> sieve;
  vl ps{2, 3};
  sieve.set();

  for (ll i = 5, step = 2; i <= N; i += step, step = 6 - step) {
    if (sieve[i]) {
      ps.push_back(i);

      for (ll j = i * i; j <= N; j += 2 * i) sieve[j] = false;
    }
  }
  return ps;
}
```

5 Problems

5.1 Kth Digit String (CSES)

Time: $O(\log_{10} K)$.
Space: $O(1)$.

```
ll kth_digit_string(ll k) {
  if (k < 10) return k;

  ll c = 180, i = 2, u = 10, r = 0, ans = -1, m;
  for (k -= 9; k > c; i++, u *= 10) {
    k -= c;
    c /= i;
    c *= 10 * (i + 1);
  }

  if ((m = k % i))
    r++;
  else
    m = i;

  ll tmp = (k / i) + r + u - 1;
  for (m = i + 1 - m; m--; tmp /= 10) ans = tmp % 10;

  return ans;
}
```

6 Strings

6.1 Manacher

Given string s with length n. Find all the pairs (i, j) such that substring $s[i \dots j]$ is a palindrome. String t is a palindrome when $t = t_{rev}$ (t_{rev} is a reversed string for t).
Time: $O(N)$

```
vi manacher(string s) {
  string t;
  for (auto c : s) t += string("#") + c;
  t = t + '#';

  int n = t.size();
  t = "$" + t + "^";

  vi p(n + 2);
  int l = 1, r = 1;
  for (int i = 1; i <= n; i++) {
    p[i] = max(0, min(r - i, p[l + (r - i)]));
    while (t[i - p[i]] == t[i + p[i]]) p[i]++;
    if (i + p[i] > r) {
      l = i - p[i], r = i + p[i];
    }
    p[i]--;
  }

  return vi(begin(p) + 1, end(p) - 1);
}
```

7 Trees

7.1 LCA Binary Lifting (CP Algo)

The algorithm described will need $O(N \cdot \log N)$ for preprocessing the tree, and then $O(\log N)$ for each LCA query.

```
ll n, l;
vector<ll> adj[MAX];

ll timer;
vector<ll> tin, tout;
vector<vector<ll>> up;

void dfs(ll v, ll p) {
  tin[v] = ++timer;
  up[v][0] = p;
  for (ll i = 1; i <= l; ++i) up[v][i] = up[up[v][i - 1]][i - 1];

  for (ll u : adj[v]) {
    if (u != p) dfs(u, v);
  }

  tout[v] = ++timer;
}
```

```
bool is_ancestor(ll u, ll v) { return tin[u] <= tin[v] && tout[u] >= tout[v];
    }

ll lca(ll u, ll v) {
  if (is_ancestor(u, v)) return u;
  if (is_ancestor(v, u)) return v;
  for (ll i = l; i >= 0; --i) {
    if (!is_ancestor(up[u][i], v)) u = up[u][i];
  }
  return up[u][0];
}

void preprocess(ll root) {
  tin.resize(n);
  tout.resize(n);
  timer = 0;
  l = ceil(log2(n));
  up.assign(n, vector<ll>(l + 1));
  dfs(root, root);
}
```

7.2 LCA SegTree (CP Algo)

The algorithm can answer each query in $O(\log N)$ with preprocessing in $O(N)$ time.

```
struct LCA {
  vector<ll> height, euler, first, segtree;
  vector<bool> visited;
  ll n;

  LCA(vector<vector<ll>>& adj, ll root = 0) {
    n = adj.size();
    height.resize(n);
    first.resize(n);
    euler.reserve(n * 2);
    visited.assign(n, false);
    dfs(adj, root);
    ll m = euler.size();
    segtree.resize(m * 4);
    build(1, 0, m - 1);
  }

  void dfs(vector<vector<ll>>& adj, ll node, ll h = 0) {
    visited[node] = true;
    height[node] = h;
    first[node] = euler.size();
    euler.push_back(node);
    for (auto to : adj[node]) {
      if (!visited[to]) {
        dfs(adj, to, h + 1);
        euler.push_back(node);
      }
    }
  }

  void build(ll node, ll b, ll e) {
    if (b == e) {
      segtree[node] = euler[b];
    } else {
      ll mid = (b + e) / 2;
      build(node << 1, b, mid);
      build(node << 1 | 1, mid + 1, e);
      ll l = segtree[node << 1], r = segtree[node << 1 | 1];
      segtree[node] = (height[l] < height[r]) ? l : r;
    }
  }

  ll query(ll node, ll b, ll e, ll L, ll R) {
    if (b > R || e < L) return -1;
    if (b >= L && e <= R) return segtree[node];
    ll mid = (b + e) >> 1;

    ll left = query(node << 1, b, mid, L, R);
    ll right = query(node << 1 | 1, mid + 1, e, L, R);
    if (left == -1) return right;
    if (right == -1) return left;
    return height[left] < height[right] ? left : right;
  }

  ll lca(ll u, ll v) {
    ll left = first[u], right = first[v];
    if (left > right) swap(left, right);
    return query(1, 0, euler.size() - 1, left, right);
  }
};
```

7.3 LCA Sparse Table

The algorithm described will need $O(N)$ for preprocessing, and then $O(1)$ for each LCA query.
0 indexed !

```
#define len(__x) (int)__x.size()

using ll = long long;
using pll = pair<ll, ll>;
using vi = vector<int>;
using vi2d = vector<vi>;
#define all(a) a.begin(), a.end()
#define pb(___x) push_back(___x)
#define mp(___a, ___b) make_pair(___a, ___b)
#define eb(___x) emplace_back(___x)

template <typename T>
struct SparseTable {
  vector<T> v;
  ll n;
  static const ll b = 30;
  vi mask, t;

  ll op(ll x, ll y) { return v[x] < v[y] ? x : y; }
  ll msb(ll x) { return __builtin_clz(1) - __builtin_clz(x); }
  SparseTable() {}
  SparseTable(const vector<T>& v_) : v(v_), n(v.size()), mask(n), t(n) {
    for (ll i = 0, at = 0; i < n; mask[i++] = at |= 1) {
      at = (at << 1) & ((1 << b) - 1);
      while (at and op(i, i - msb(at & -at)) == i) at ^= at & -at;
```

```
    }
    for (ll i = 0; i < n / b; i++)
      t[i] = b * i + b - 1 - msb(mask[b * i + b - 1]);
    for (ll j = 1; (1 << j) <= n / b; j++)
      for (ll i = 0; i + (1 << j) <= n / b; i++)
        t[n / b * j + i] =
          op(t[n / b * (j - 1) + i], t[n / b * (j - 1) + i + (1 << (j - 1))]);
  }
  ll small(ll r, ll sz = b) { return r - msb(mask[r] & ((1 << sz) - 1)); }
  T query(ll l, ll r) {
    if (r - l + 1 <= b) return small(r, r - l + 1);
    ll ans = op(small(l + b - 1), small(r));
    ll x = l / b + 1, y = r / b - 1;
    if (x <= y) {
      ll j = msb(y - x + 1);
      ans = op(ans, op(t[n / b * j + x], t[n / b * j + y - (1 << j) + 1]));
    }
    return ans;
  }
};

struct LCA {
  SparseTable<ll> st;
  ll n;
  vi v, pos, dep;

  LCA(const vi2d& g, ll root) : n(len(g)), pos(n) {
    dfs(root, 0, -1, g);
    st = SparseTable<ll>(vector<ll>(all(dep)));
  }

  void dfs(ll i, ll d, ll p, const vi2d& g) {
    v.eb(len(dep)) = i, pos[i] = len(dep), dep.eb(d);
    for (auto j : g[i])
      if (j != p) {
        dfs(j, d + 1, i, g);
        v.eb(len(dep)) = i, dep.eb(d);
      }
  }

  ll lca(ll a, ll b) {
    ll l = min(pos[a], pos[b]);
    ll r = max(pos[a], pos[b]);
    return v[st.query(l, r)];
  }
  ll dist(ll a, ll b) {
    return dep[pos[a]] + dep[pos[b]] - 2 * dep[pos[lca(a, b)]];
  }
};
```

8 Settings and macros

8.1 macro.cpp

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>

using namespace __gnu_pbds;
#define ordered_set tree<int, null_type, less<int>, rb_tree_tag,
    tree_order_statistics_node_update>

using namespace std;

typedef long long ll;
typedef pair<int, int> pii;
typedef pair<ll, ll> pll;
typedef vector<int> vi;
typedef vector<ll> vl;
typedef vector<pii> vii;

#define all(x) x.begin(), x.end()
#define vin(vt) for (auto &e : vt) cin >> e
#define LSOne(S) ((S) & -(S))
#define MSOne(S) (1ull << (63 - __builtin_clzll(S)))

const vii dir4{ {1,0},{-1,0},{0,1},{0,-1} };

auto solve() {  }

int main() {
    ios_base::sync_with_stdio(0);
    cin.tie(0);

    ll t = 1;
    //cin >> t;

    while (t--) solve();

    return 0;
}
```

8.2 short-macro.cpp

```
#include <bits/stdc++.h>

using namespace std;

typedef long long ll;
typedef pair<int, int> ii;

#define all(x) x.begin(), x.end()
#define vin(vt) for (auto &e : vt) cin >> e

auto solve() {  }

int main() {
    ios_base::sync_with_stdio(0);
    cin.tie(0);

    ll t = 1;
    //cin >> t;

    while (t--) solve();
```

```
return 0;
}
```