

Notebook - Competitive Programming

Anões do TLE

Contents

1 Data structures 2

1.1 Matrix	2
1.2 Merge Sort Tree	3
1.3 Minimal Excluded With Updates (MEX-U)	4
1.4 Minimal Excluded (MEX)	4
1.5 Range Min Query (RMQ)	4
1.6 Segment Tree (Parameterized OP)	5
1.7 Segment Tree 2D	5
1.8 Segment Tree Lazy	5
1.9 Union Find Disjoint Set (UFDS)	6
1.10 Wavelet Tree	6

2 Dynamic programming 7

2.1 Kadane	7
2.2 Longest Increasing Subsequence (LIS)	7

3 Extras 7

3.1 cin/cout __int128_t	7
-----------------------------------	---

4 Geometry 7

4.1 Convex Hull	7
4.2 Point To Segment	8

5 Graphs 8

5.1 Articulation Points	8
5.2 Bellman Ford	8
5.3 BFS 0/1	9
5.4 Bridges	9
5.5 Negative Cycle Bellman Ford	9
5.6 Negative Cycle Floyd Warshall	9
5.7 Dijkstra	10
5.8 Floyd Warshall	10
5.9 Graph	10

5.10 TopSort - Kahn	10
5.11 Kruskal	11
5.12 Minimax	11
5.13 MSF	11
5.14 Minimum Spanning Graph (MSG)	11
5.15 Prim	12
5.16 Retrieve Path 2d	12
5.17 Retrieve Path	12
5.18 Second Best MST	12
5.19 TopSort - Tarjan	13

6 Math 13

6.1 Binomial	13
6.2 Count Divisors Range	13
6.3 Count Divisors	13
6.4 Factorization With Sieve	13
6.5 Factorization	13
6.6 Fast Doubling - Fibonacci	14
6.7 Fast Exp Iterative	14
6.8 Fast Exp	14
6.9 Fast Fourier Transform (FFT)	14
6.10 GCD	15
6.11 Integer Mod	15
6.12 Is prime	15
6.13 LCM	15
6.14 Euler phi $\varphi(n)$	15
6.15 Sieve	16
6.16 Sum Divisors	16
6.17 Sum of difference	16

7 Problems 16

7.1 Kth Digit String (CSES)	16
7.2 Longest Common Substring (LONGCS - SPOJ)	16
7.3 Substring Order II (CSES)	17

8 Strings 17

8.1 Aho-Corasick	17
8.2 Edit Distance	18
8.3 LCP with Suffix Array	18
8.4 Manacher	19
8.5 Rabin Karp	19
8.6 Suffix Array Optimized - O(n)	19
8.7 Suffix Array	20
8.8 Suffix Automaton	20
8.9 Suffix Tree (CP Algo - freopen)	21
8.10 Z Function	21

9 Trees 22

9.1 LCA Binary Lifting (CP Algo)	22
9.2 LCA SegTree (CP Algo)	22
9.3 LCA Sparse Table	23
9.4 Tree Flatten	23
9.5 Tree Isomorph	23

10 Settings and macros 24

10.1 short-macro.cpp	24
10.2 macro.cpp	24

11 Theoretical guide 25

11.1 Number of Different Substrings	25
11.2 String Matching with FFT	25
11.2.1 Wildcards	25
11.3 Modular Multiplicative Inverse	25
11.4 Notable Series	25
11.5 Exponent With Module	25

1 Data structures

1.1 Matrix

```
template <typename T>
struct Matrix {
    vector<vector<T>> d;

    Matrix() : Matrix(0) {}
    Matrix(int n) : Matrix(n, n) {}
    Matrix(int n, int m) : Matrix(vector<vector<T>>(n, vector<T>(m))) {}
    Matrix(const vector<vector<T>> &v) : d(v) {}

    constexpr int n() const { return (int)d.size(); }
    constexpr int m() const { return n() ? (int)d[0].size() : 0; }

    void rotate() { *this = rotated(); }

    Matrix<T> rotated() const {
        Matrix<T> res(m(), n());
        for (int i = 0; i < m(); i++) {
            for (int j = 0; j < n(); j++) {
                res[i][j] = d[n() - j - 1][i];
            }
        }
        return res;
    }

    Matrix<T> pow(int power) const {
        assert(n() == m());

        auto res = Matrix<T>::identity(n());
        auto b = *this;
        while (power) {
            if (power & 1) res *= b;
            b *= b;
            power >>= 1;
        }
        return res;
    }

    Matrix<T> submatrix(int start_i, int start_j, int rows = INT_MAX,
                       int cols = INT_MAX) const {
        rows = min(rows, n() - start_i);
        cols = min(cols, m() - start_j);
        if (rows <= 0 or cols <= 0) return {};

        Matrix<T> res(rows, cols);
        for (int i = 0; i < rows; i++)
            for (int j = 0; j < cols; j++) res[i][j] = d[i + start_i][j + start_j];
        return res;
    }

    Matrix<T> translated(int x, int y) const {
        Matrix<T> res(n(), m());
        for (int i = 0; i < n(); i++) {
            for (int j = 0; j < m(); j++) {
                if (i + x < 0 or i + x >= n() or j + y < 0 or j + y >= m()) continue;

```

```
                res[i + x][j + y] = d[i][j];
            }
        }
        return res;
    }

    static Matrix<T> identity(int n) {
        Matrix<T> res(n);
        for (int i = 0; i < n; i++) res[i][i] = 1;
        return res;
    }

    vector<T> &operator[](int i) { return d[i]; }
    const vector<T> &operator[](int i) const { return d[i]; }
    Matrix<T> &operator+=(T value) {
        for (auto &row : d) {
            for (auto &x : row) x += value;
        }
        return *this;
    }
    Matrix<T> operator+(T value) const {
        auto res = *this;
        for (auto &row : res) {
            for (auto &x : row) x = x + value;
        }
        return res;
    }
    Matrix<T> &operator-=(T value) {
        for (auto &row : d) {
            for (auto &x : row) x -= value;
        }
        return *this;
    }
    Matrix<T> operator-(T value) const {
        auto res = *this;
        for (auto &row : res) {
            for (auto &x : row) x = x - value;
        }
        return res;
    }
    Matrix<T> &operator*=(T value) {
        for (auto &row : d) {
            for (auto &x : row) x *= value;
        }
        return *this;
    }
    Matrix<T> operator*(T value) const {
        auto res = *this;
        for (auto &row : res) {
            for (auto &x : row) x = x * value;
        }
        return res;
    }
    Matrix<T> &operator/=(T value) {
        for (auto &row : d) {
            for (auto &x : row) x /= value;
        }
        return *this;
    }
}
```

```

}
Matrix<T> operator/(T value) const {
    auto res = *this;
    for (auto &row : res) {
        for (auto &x : row) x = x / value;
    }
    return res;
}
Matrix<T> &operator+=(const Matrix<T> &o) {
    assert(n() == o.n() and m() == o.m());
    for (int i = 0; i < n(); i++) {
        for (int j = 0; j < m(); j++) {
            d[i][j] += o[i][j];
        }
    }
    return *this;
}
Matrix<T> operator+(const Matrix<T> &o) const {
    assert(n() == o.n() and m() == o.m());
    auto res = *this;
    for (int i = 0; i < n(); i++) {
        for (int j = 0; j < m(); j++) {
            res[i][j] = res[i][j] + o[i][j];
        }
    }
    return res;
}
Matrix<T> &operator-=(const Matrix<T> &o) {
    assert(n() == o.n() and m() == o.m());
    for (int i = 0; i < n(); i++) {
        for (int j = 0; j < m(); j++) {
            d[i][j] -= o[i][j];
        }
    }
    return *this;
}
Matrix<T> operator-(const Matrix<T> &o) const {
    assert(n() == o.n() and m() == o.m());
    auto res = *this;
    for (int i = 0; i < n(); i++) {
        for (int j = 0; j < m(); j++) {
            res[i][j] = res[i][j] - o[i][j];
        }
    }
    return res;
}
Matrix<T> &operator*=(const Matrix<T> &o) {
    *this = *this * o;
    return *this;
}
Matrix<T> operator*(const Matrix<T> &o) const {
    assert(m() == o.n());
    Matrix<T> res(n(), o.m());
    for (int i = 0; i < res.n(); i++) {
        for (int j = 0; j < res.m(); j++) {
            auto &x = res[i][j];
            for (int k = 0; k < m(); k++) {
                x += (d[i][k] * o[k][j]);
            }
        }
    }
}

```

```

    }
}
return res;
}

friend istream &operator>>(istream &is, Matrix<T> &mat) {
    for (auto &row : mat)
        for (auto &x : row) is >> x;
    return is;
}

friend ostream &operator<<(ostream &os, const Matrix<T> &mat) {
    bool frow = 1;
    for (auto &row : mat) {
        if (not frow) os << '\n';
        bool first = 1;
        for (auto &x : row) {
            if (not first) os << ' ';
            os << x;
            first = 0;
        }

        frow = 0;
    }
    return os;
}

auto begin() { return d.begin(); }
auto end() { return d.end(); }
auto rbegin() { return d.rbegin(); }
auto rend() { return d.rend(); }

auto begin() const { return d.begin(); }
auto end() const { return d.end(); }
auto rbegin() const { return d.rbegin(); }
auto rend() const { return d.rend(); }
};

```

1.2 Merge Sort Tree

Like a segment tree but each node st_i stores a sorted subarray

- $inrange(l, r, a, b)$: counts the number of elements $x \in [l, r]$ such that $a \leq x \leq b$.

Memory: $O(N \log N)$

Build: $O(N \log N)$

inrange: $O(\log^2 N)$

```

template <class T>
struct MergeSortTree {
    int n;
    vector<vector<T>> st;
    MergeSortTree(vector<T>& xs) : n(len(xs)), st(n << 1) {
        for (int i = 0; i < n; i++) st[i + n] = vector<T>({xs[i]});

        for (int i = n - 1; i > 0; i--) {
            st[i].resize(len(st[i << 1]) + len(st[i << 1 | 1]));
            merge(all(st[i << 1]), all(st[i << 1 | 1]), st[i].begin());
        }
    }
}

```

```

int count(int i, T a, T b) {
    return upper_bound(all(st[i]), b) - lower_bound(all(st[i]), a);
}

int intrange(int l, int r, T a, T b) {
    int ans = 0;

    for (l += n, r += n + 1; l < r; l >>= 1, r >>= 1) {
        if (l & 1) ans += count(l++, a, b);
        if (r & 1) ans += count(--r, a, b);
    }

    return ans;
}
};

```

1.3 Minimal Excluded With Updates (MEX-U)

In the problem you need to change individual numbers in the array, and compute the new MEX of the array after each such update.

Pre-compute: $O(N \log N)$

Update: $O(\log N)$

Query: $O(1)$

```

class Mex {
private:
    map<ll, ll> frequency;
    set<ll> missing_numbers;
    vl A;

public:
    Mex(vl const& A) : A(A) {
        for (ll i = 0; i <= A.size(); i++) missing_numbers.insert(i);

        for (ll x : A) {
            ++frequency[x];
            missing_numbers.erase(x);
        }

        ll mex() { return *missing_numbers.begin(); }

        void update(ll idx, ll new_value) {
            if (--frequency[A[idx]] == 0) missing_numbers.insert(A[idx]);
            A[idx] = new_value;
            ++frequency[new_value];
            missing_numbers.erase(new_value);
        }
    };
};

```

1.4 Minimal Excluded (MEX)

Given an array A of size N . You have to find the minimal non-negative element that is not present in the array. That number is commonly called the MEX (minimal excluded).

Time: $O(N)$

```

ll mex(vl const& A) {
    static bool used[MAX + 111] = {0};

    for (ll x : A) {
        if (x <= MAX) used[x] = true;
    }

    ll result = 0;
    while (used[result]) ++result;

    for (ll x : A) {
        if (x <= MAX) used[x] = false;
    }

    return result;
}

```

1.5 Range Min Query (RMQ)

Build: $O(N)$

Query: $O(1)$

```

// @brunomaletta
template <typename T>
struct rmq {
    vector<T> v;
    int n;
    static const int b = 30;
    vector<int> mask, t;

    int op(int x, int y) { return v[x] <= v[y] ? x : y; }
    int msb(int x) { return __builtin_clz(1) - __builtin_clz(x); }
    int small(int r, int sz = b) { return r - msb(mask[r] & ((1 << sz) - 1)); }
    rmq() {}
    rmq(const vector<T>& v_) : v(v_), n(v.size()), mask(n), t(n) {
        for (int i = 0, at = 0; i < n; mask[i++] = at |= 1) {
            at = (at << 1) & ((1 << b) - 1);
            while (at and op(i - msb(at & -at), i) == i) at ^= at & -at;
        }
        for (int i = 0; i < n / b; i++) t[i] = small(b * i + b - 1);
        for (int j = 1; (1 << j) <= n / b; j++)
            for (int i = 0; i + (1 << j) <= n / b; i++)
                t[n / b * j + i] =
                    op(t[n / b * (j - 1) + i], t[n / b * (j - 1) + i + (1 << (j - 1))]);
    }

    int index_query(int l, int r) {
        if (r - l + 1 <= b) return small(r, r - l + 1);
        int x = l / b + 1, y = r / b - 1;
        if (x > y) return op(small(l + b - 1), small(r));
        int j = msb(y - x + 1);
        int ans = op(small(l + b - 1),
                    op(t[n / b * j + x], t[n / b * j + y - (1 << j) + 1]));
        return op(ans, small(r));
    }

    T query(int l, int r) { return v[index_query(l, r)]; }
};

```

1.6 Segment Tree (Parameterized OP)

Query: $O(\log N)$
Update: $O(\log N)$

```
template <typename T, auto op>
class SegTree {
private:
    T e;
    ll N;
    vector<T> seg;

public:
    SegTree(ll N, T e) : e(e), N(N), seg(N + N, e) {}

    void assign(ll i, T v) {
        i += N;
        seg[i] = v;
        for (i >>= 1; i; i >>= 1) seg[i] = op(seg[2 * i], seg[2 * i + 1]);
    }

    T query(ll l, ll r) {
        T la = e, ra = e;
        l += N;
        r += N;

        while (l <= r) {
            if (l & 1) la = op(la, seg[l++]);
            if (~r & 1) ra = op(seg[r--], ra);
            l >>= 1;
            r >>= 1;
        }

        return op(la, ra);
    }
};
```

1.7 Segment Tree 2D

Query: $O(\log N \cdot \log M)$
Update: $O(\log N \cdot \log M)$

```
template <typename T, auto op>
class SegTree {
private:
    T e;
    ll n, m;
    vector<vector<T>> seg;

public:
    SegTree(ll n, ll m, T e)
        : e(e), n(n), m(m), seg(2 * n, vector<T>(2 * m, e)) {}

    void assign(ll x, ll y, T v) {
        ll ny = y += m;
        for (x += n; x; x >>= 1, y = ny) {
            if (x >= n)
                seg[x][y] = v;
            else
```

```
                seg[x][y] = op(seg[2 * x][y], seg[2 * x + 1][y]);

            while (y >>= 1) seg[x][y] = op(seg[x][2 * y], seg[x][2 * y + 1]);
        }

    T query(ll lx, ll rx, ll ly, ll ry) {
        ll ans = e, nx = rx + n, my = ry + m;

        for (lx += n, ly += m; lx <= ly; ++lx >>= 1, --ly >>= 1)
            for (rx = nx, ry = my; rx <= ry; ++rx >>= 1, --ry >>= 1) {
                if (lx & 1 and rx & 1) ans = op(ans, seg[lx][rx]);
                if (lx & 1 and !(ry & 1)) ans = op(ans, seg[lx][ry]);
                if (!(ly & 1) and rx & 1) ans = op(ans, seg[ly][rx]);
                if (!(ly & 1) and !(ry & 1)) ans = op(ans, seg[ly][ry]);
            }

        return ans;
    }
};
```

1.8 Segment Tree Lazy

Query (Range Sum): $O(\log N)$
Update (Sum Value): $O(\log N)$

```
template <typename T>
class SegTreeLazy {
private:
    int N;
    vector<T> seg, lzy;

    void down(int k, int l, int r) {
        seg[k] += (r - l + 1) * lzy[k];
        if (l < r) {
            lzy[k << 1] += lzy[k];
            lzy[k << 1 | 1] += lzy[k];
        }
        lzy[k] = 0;
    }

    void update(int i, int j, int k, int l, int r, T v) {
        if (lzy[k]) down(k, l, r);
        if (i > r or j < l) return;
        if (i <= l and j >= r) {
            seg[k] += (r - l + 1) * v;
            if (l < r) {
                lzy[k << 1] += v;
                lzy[k << 1 | 1] += v;
            }
            return;
        }

        update(i, j, k << 1, l, (l + r) / 2, v);
        update(i, j, k << 1 | 1, (l + r) / 2 + 1, r, v);
        seg[k] = seg[k << 1] + seg[k << 1 | 1];
    }
};
```

```

T query(int i, int j, int k, int l, int r) {
    if (lzy[k]) down(k, l, r);
    if (i > r or j < l) return 0;
    if (i <= l and j >= r) return seg[k];

    T lft = query(i, j, k << 1, l, (l + r) / 2);
    T rgt = query(i, j, k << 1 | 1, (l + r) / 2 + 1, r);
    return lft + rgt;
}

public:
    SegTreeLazy(int N) : N(N), seg(N << 2, 0), lzy(N << 2, 0) {}

    void update(int i, int j, T v) { update(i, j, 1, 0, N - 1, v); }

    T query(int i, int j) { return query(i, j, 1, 0, N - 1); }
};

```

1.9 Union Find Disjoint Set (UFDS)

Uncomment the lines to recover which element belong to each set.

Time: $\approx O(1)$ for everything.

```

class UFDS {
public:
    vi ps, size;
    // vector<unordered_set<int>> sts;

    UFDS(int N) : size(N + 1, 1), ps(N + 1), sts(N) {
        iota(ps.begin(), ps.end(), 0);
        // for (int i = 0; i < N; i++) sts[i].insert(i);
    }

    int find_set(int x) { return x == ps[x] ? x : (ps[x] = find_set(ps[x])); }

    bool same_set(int x, int y) { return find_set(x) == find_set(y); }

    void union_set(int x, int y) {
        if (same_set(x, y)) return;

        int px = find_set(x);
        int py = find_set(y);

        if (size[px] < size[py]) swap(px, py);

        ps[py] = px;
        size[px] += size[py];
        // sts[px].merge(sts[py]);
    }
};

```

1.10 Wavelet Tree

Build: $O(N \cdot \log \sigma)$.

Queries: $O(\log \sigma)$.

σ = alphabet length

```

typedef vector<int>::iterator iter;

class WaveletTree {
public:
    int L, H;
    WaveletTree *l, *r;
    vector<int> frq;

    WaveletTree(iter fr, iter to, int x, int y) {
        L = x, H = y;
        if (fr >= to) return;

        int M = L + ((H - L) >> 1);
        auto F = [M](int x) { return x <= M; };

        frq.reserve(to - fr + 1);
        frq.push_back(0);
        for (auto it = fr; it != to; it++) frq.push_back(frq.back() + F(*it));

        if (H == L) return;
        auto pv = stable_partition(fr, to, F);
        l = new WaveletTree(fr, pv, L, M);
        r = new WaveletTree(pv, to, M + 1, H);
    }

    // Find the k-th smallest element in positions [i,j]
    int quantile(int l, int r, int k) {
        if (l > r) return 0;
        if (L == H) return L;
        int inLeft = frq[r] - frq[l - 1];
        int lb = frq[l - 1], rb = frq[r];
        if (k <= inLeft) return this->l->quantile(lb + 1, rb, k);
        return this->r->quantile(l - lb, r - rb, k - inLeft);
    }

    // Count occurrences of number c until position i -> [0, i].
    int rank(int c, int i) { return until(c, min(i + 1, (int)frq.size() - 1)); }

    int until(int c, int i) {
        if (c > H or c < L) return 0;
        if (L == H) return i;

        int M = L + ((H - L) >> 1);
        int r = frq[i];
        if (c <= M)
            return this->l->until(c, r);
        else
            return this->r->until(c, i - r);
    }

    // Count number of occurrences of numbers in the range [a, b]
    int range(int i, int j, int a, int b) const {
        if (b < a or j < i) return 0;
        return range(i, j + 1, L, H, a, b);
    }

    int range(int i, int j, int a, int b, int L, int U) const {
        if (b < L or U < a) return 0;

```

```

    if (L <= a and b <= U) return j - i;
    int M = a + ((b - a) >> 1);
    int ri = frq[i], rj = frq[j];
    return this->l->range(ri, rj, a, M, L, U) +
           this->r->range(i - ri, j - rj, M + 1, b, L, U);
}

// Number of elements greater than or equal to k in [l, r];
// Can count distinct in a range with aux vector of next pos
int greater(int l, int r, int k) { return _greater(l + 1, r + 1, k); }

int _greater(int l, int r, int k) {
    if (l > r or k > H) return 0;
    if (L >= k) return r - l + 1;

    int ri = frq[l - 1], rj = frq[r];
    return this->l->_greater(ri + 1, rj, k) +
           this->r->_greater(l - ri, r - rj, k);
}
};

```

2 Dynamic programming

2.1 Kadane

```

int kadane(const vi& xs) {
    vi s(xs.size());
    s[0] = xs[0];

    for (size_t i = 1; i < xs.size(); ++i) s[i] = max(xs[i], s[i - 1] + xs[i]);

    return *max_element(all(s));
}

```

2.2 Longest Increasing Subsequence (LIS)

Time: $O(N \cdot \log N)$.

```

int lis(vi const& a) {
    int n = a.size();
    const int INF = 1e9;
    vi d(n + 1, INF);
    d[0] = -INF;

    for (int i = 0; i < n; i++) {
        int l = upper_bound(d.begin(), d.end(), a[i]) - d.begin();
        if (d[l - 1] < a[i] && a[i] < d[l]) d[l] = a[i];
    }

    int ans = 0;
    for (int l = 0; l <= n; l++) {
        if (d[l] < INF) ans = l;
    }

    return ans;
}

```

3 Extras

3.1 cin/cout __int128_t

Allows standard reading and writing with cin/cout for 128-bit integers using `__int128_t` type.

```

ostream& operator<<(ostream& dest, __int128_t value) {
    ostream::sentry s(dest);
    if (s) {
        __uint128_t tmp = value < 0 ? -value : value;
        char buffer[128];
        char* d = end(buffer);
        do {
            --d;
            *d = "0123456789"[tmp % 10];
            tmp /= 10;
        } while (tmp != 0);
        if (value < 0) {
            --d;
            *d = '-';
        }
        int len = end(buffer) - d;
        if (dest.rdbuf()->sputn(d, len) != len) dest.setstate(ios_base::badbit);
    }
    return dest;
}

istream& operator>>(istream& is, __int128_t& value) {
    string s;
    is >> s;

    __int128_t res = 0;
    size_t i = 0;

    bool neg = false;
    if (s[i] == '-') neg = 1, i++;
    for (; i < s.size(); ++i) (res *= 10) += (s[i] - '0');

    value = neg ? -res : res;
    return is;
}

```

4 Geometry

4.1 Convex Hull

Given a set of points find the smallest convex polygon that contains all the given points.

Time: $O(N \cdot \log N)$

By default it removes the collinear points, set the boolean to true if you don't want that

```

struct pt {
    double x, y;
};

int orientation(pt a, pt b, pt c) {
    double v = a.x * (b.y - c.y) + b.x * (c.y - a.y) + c.x * (a.y - b.y);
    if (v < 0) return -1; // clockwise
}

```

```

    if (v > 0) return +1; // counter-clockwise
    return 0;
}

bool cw(pt a, pt b, pt c, bool include_collinear) {
    int o = orientation(a, b, c);
    return o < 0 || (include_collinear && o == 0);
}

bool collinear(pt a, pt b, pt c) { return orientation(a, b, c) == 0; }

void convex_hull(vector<pt>& a, bool include_collinear = false) {
    pt p0 = *min_element(a.begin(), a.end(), [](pt a, pt b) {
        return make_pair(a.y, a.x) < make_pair(b.y, b.x);
    });
    sort(a.begin(), a.end(), [&p0](const pt& a, const pt& b) {
        int o = orientation(p0, a, b);
        if (o == 0)
            return (p0.x - a.x) * (p0.x - a.x) + (p0.y - a.y) * (p0.y - a.y) <
                (p0.x - b.x) * (p0.x - b.x) + (p0.y - b.y) * (p0.y - b.y);
        return o < 0;
    });
    if (include_collinear) {
        int i = (int)a.size() - 1;
        while (i >= 0 && collinear(p0, a[i], a.back())) i--;
        reverse(a.begin() + i + 1, a.end());
    }

    vector<pt> st;
    for (int i = 0; i < (int)a.size(); i++) {
        while (st.size() > 1 &&
            !cw(st[st.size() - 2], st.back(), a[i], include_collinear))
            st.pop_back();
        st.push_back(a[i]);
    }

    a = st;
}

```

4.2 Point To Segment

```
typedef pair<double, double> pdb;
```

```

double pt2segment(pdb A, pdb B, pdb E) {
    pdb AB = {B.fst - A.fst, B.snd - A.snd};
    pdb BE = {E.fst - B.fst, E.snd - B.snd};
    pdb AE = {E.fst - A.fst, E.snd - A.snd};

    double AB_BE = AB.fst * BE.fst + AB.snd * BE.snd;
    double AB_AE = AB.fst * AE.fst + AB.snd * AE.snd;

    double ans;
    if (AB_BE > 0) {
        double y = E.snd - B.snd;
        double x = E.fst - B.fst;
        ans = hypot(x, y);
    } else if (AB_AE < 0) {
        double y = E.snd - A.snd;

```

```

        double x = E.fst - A.fst;
        ans = hypot(x, y);
    } else {
        auto [x1, y1] = AB;
        auto [x2, y2] = AE;
        double mod = hypot(x1, y1);
        ans = abs(x1 * y2 - y1 * x2) / mod;
    }

    return ans;
}

```

5 Graphs

5.1 Articulation Points

```

int dfs_num[MAX], dfs_low[MAX];
vi adj[MAX];

int dfs_articulation_points(int u, int p, int& next, set<int>& points) {
    int children = 0;
    dfs_low[u] = dfs_num[u] = next++;

    for (auto v : adj[u])
        if (not dfs_num[v]) {
            ++children;

            dfs_articulation_points(v, u, next, points);

            if (dfs_low[v] >= dfs_num[u]) points.insert(u);

            dfs_low[u] = min(dfs_low[u], dfs_low[v]);
        } else if (v != p)
            dfs_low[u] = min(dfs_low[u], dfs_num[v]);

    return children;
}

set<int> articulation_points(int N) {
    memset(dfs_num, 0, (N + 1) * sizeof(int));
    memset(dfs_low, 0, (N + 1) * sizeof(int));

    set<int> points;

    for (int u = 1, next = 1; u <= N; ++u)
        if (not dfs_num[u]) {
            auto children = dfs_articulation_points(u, u, next, points);

            if (children == 1) points.erase(u);
        }

    return points;
}

```

5.2 Bellman Ford

Time: $O(V \cdot E)$. Returns the shortest path from s to all other nodes.


```
using edge = tuple<int, int, int>;

pair<vi, vi> bellman_ford(int s, int N, const vector<edge>& edges) {
    vi dist(N + 1, oo), pred(N + 1, oo);

    dist[s] = 0;
    pred[s] = s;

    for (int i = 1; i <= N - 1; i++)
        for (auto [u, v, w] : edges)
            if (dist[u] < oo and dist[v] > dist[u] + w) {
                dist[v] = dist[u] + w;
                pred[v] = u;
            }

    return {dist, pred};
}
```

5.3 BFS 0/1

Time: $O(V + E)$.

```
vii adj[MAX];

vi bfs_01(int s, int N) {
    vi dist(N + 1, oo);
    dist[s] = 0;

    deque<int> q;
    q.emplace_back(s);

    while (not q.empty()) {
        auto u = q.front();
        q.pop_front();

        for (auto [v, w] : adj[u])
            if (dist[v] > dist[u] + w) {
                dist[v] = dist[u] + w;
                w == 0 ? q.emplace_front(v) : q.emplace_back(v);
            }
    }

    return dist;
}
```

5.4 Bridges

```
int dfs_num[MAX], dfs_low[MAX];
vi adj[MAX];

void dfs_bridge(int u, int p, int& next, vii& bridges) {
    dfs_low[u] = dfs_num[u] = next++;

    for (auto v : adj[u])
        if (not dfs_num[v]) {
            dfs_bridge(v, u, next, bridges);
```

```
            if (dfs_low[v] > dfs_num[u]) bridges.emplace_back(u, v);

            dfs_low[u] = min(dfs_low[u], dfs_low[v]);
        } else if (v != p)
            dfs_low[u] = min(dfs_low[u], dfs_num[v]);
    }

vii bridges(int N) {
    memset(dfs_num, 0, (N + 1) * sizeof(int));
    memset(dfs_low, 0, (N + 1) * sizeof(int));

    vii bridges;

    for (int u = 1, next = 1; u <= N; ++u)
        if (not dfs_num[u]) dfs_bridge(u, u, next, bridges);

    return bridges;
}
```

5.5 Negative Cycle Bellman Ford

Time: $O(V \cdot E)$. Detects whether there is a negative cycle in the graph using Bellman Ford.

```
using edge = tuple<int, int, int>;

bool has_negative_cycle(int s, int N, const vector<edge>& edges) {
    vi dist(N + 1, oo);
    dist[s] = 0;

    for (int i = 1; i <= N - 1; i++)
        for (auto [u, v, w] : edges)
            if (dist[u] < oo and dist[v] > dist[u] + w) dist[v] = dist[u] + w;

    for (auto [u, v, w] : edges)
        if (dist[u] < oo and dist[v] > dist[u] + w) return true;

    return false;
}
```

5.6 Negative Cycle Floyd Warshall

Time: $O(n^3)$. Detects whether there is a negative cycle in the graph using Floyd Warshall.

```
int dist[MAX][MAX];
vii adj[MAX];

bool has_negative_cycle(int N) {
    for (int u = 1; u <= N; ++u)
        for (int v = 1; v <= N; ++v) dist[u][v] = u == v ? 0 : oo;

    for (int u = 1; u <= N; ++u)
        for (auto [v, w] : adj[u]) dist[u][v] = w;

    for (int k = 1; k <= N; ++k)
        for (int u = 1; u <= N; ++u)
            for (int v = 1; v <= N; ++v)
                if (dist[u][k] < oo and dist[k][v] < oo)
```

```

        dist[u][v] = min(dist[u][v], dist[u][k] + dist[k][v]);

    for (int i = 1; i <= N; ++i)
        if (dist[i][i] < 0) return true;

    return false;
}

```

5.7 Dijkstra

```

pair<v1, v1> Graph::dijkstra(ll src) {
    v1 pd(this->N, LLONG_MAX), ds(this->N, LLONG_MAX);
    pd[src] = src;
    ds[src] = 0;

    set<pll> st;
    st.emplace(0, src);

    while (!st.empty()) {
        ll u = st.begin()->snd;
        ll wu = st.begin()->fst;
        st.erase(st.begin());

        if (wu != ds[u]) continue;
        for (auto& [v, w] : adj[u]) {
            if (ds[v] > ds[u] + w) {
                ds[v] = ds[u] + w;
                pd[v] = u;
                st.emplace(ds[v], v);
            }
        }

        return {ds, pd};
    }
}

```

5.8 Floyd Warshall

```

vii adj[MAX];

pair<vector<vi>, vector<vi>> floyd_warshall(int N) {
    vector<vi> dist(N + 1, vi(N + 1, oo));
    vector<vi> pred(N + 1, vi(N + 1, oo));

    for (int u = 1; u <= N; ++u) {
        dist[u][u] = 0;
        pred[u][u] = u;
    }

    for (int u = 1; u <= N; ++u)
        for (auto [v, w] : adj[u]) {
            dist[u][v] = w;
            pred[u][v] = u;
        }

    for (int k = 1; k <= N; ++k) {

```

```

        for (int u = 1; u <= N; ++u) {
            for (int v = 1; v <= N; ++v) {
                if (dist[u][k] < oo and dist[k][v] < oo and
                    dist[u][v] > dist[u][k] + dist[k][v]) {
                    dist[u][v] = dist[u][k] + dist[k][v];
                    pred[u][v] = pred[k][v];
                }
            }
        }

    return {dist, pred};
}

```

5.9 Graph

```

class Graph {
private:
    ll N;
    bool undirected;
    vector<vll> adj;

public:
    Graph(ll N, bool is_undirected = true) {
        this->N = N;
        adj.resize(N);
        undirected = is_undirected;
    }

    void add(ll u, ll v, ll w) {
        adj[u].emplace_back(v, w);
        if (undirected) adj[v].emplace_back(u, w);
    }
};

```

5.10 TopSort - Kahn

Works only on Directed Acyclic Graphs (DAGs). For each edge (u,v), u comes before v in the ordering. If the task A is a prerequisite for task B, then A comes before B in the ordering. Time: $O(E \cdot \log(v))$

```

unordered_set<int> in[MAX], out[MAX];

```

```

vi topological_sort(int N) {
    vi o;
    queue<int> q;

    for (int u = 1; u <= N; ++u)
        if (in[u].empty()) q.push(u);

    while (not q.empty()) {
        auto u = q.front();
        q.pop();

        o.emplace_back(u);

        for (auto v : out[u]) {
            in[v].erase(u);

            if (in[v].empty()) q.push(v);
        }
    }
}

```

```

    }
}

return (int)o.size() == N ? o : vi{};
}

```

5.11 Kruskal

Time: $O(e \cdot \log(v))$

```

using edge = tuple<int, int, int>;

int kruskal(int N, vector<edge>& es) {
    sort(es.begin(), es.end());

    int cost = 0;
    UnionFind udfs(N);

    for (auto [w, u, v] : es) {
        if (not udfs.same_set(u, v)) {
            cost += w;
            udfs.union_set(u, v);
        }
    }

    return cost;
}

```

5.12 Minimax

A MST minimizes the maximum weight between the edges in any spanning tree. Time: $O(e \cdot \log(v))$

```

vii adj[MAX];

int minimax(int u, int N) {
    set<int> C;
    C.insert(u);

    priority_queue<ii, vii, greater<ii>> pq;

    for (auto [v, w] : adj[u]) pq.push(ii(w, v));

    int minmax = -oo;

    while ((int)C.size() < N) {
        int v, w;

        do {
            w = pq.top().first, v = pq.top().second;
            pq.pop();
        } while (C.count(v));

        minmax = max(minmax, w);
        C.insert(v);

        for (auto [s, p] : adj[v]) pq.push(ii(p, s));
    }
}

```

```

    return minmax;
}

```

5.13 MSF

Minimum Spanning Forest - a forest of trees of length k that connects all vertices in a graph with minimum total weight. Time: $O(e \cdot \log(v))$

```

using edge = tuple<int, int, int>;

int msf(int k, int N, vector<edge>& es) {
    sort(es.begin(), es.end());

    int cost = 0, cc = N;
    UnionFind udfs(N);

    for (auto [w, u, v] : es) {
        if (not udfs.same_set(u, v)) {
            cost += w;
            udfs.union_set(u, v);

            if (--cc == k) return cost;
        }
    }

    return cost;
}

```

5.14 Minimum Spanning Graph (MSG)

Given some obligatory edges es , find a minimum spanning graph that contains them. Time: $O(e \cdot \log(v))$

```

using edge = tuple<int, int, int>;

const int MAX{100010};

vector<ii> adj[MAX];

int msg(int N, const vector<edge>& es) {
    set<int> C;
    priority_queue<ii, vii, greater<ii>> pq;
    int cost = 0;

    for (auto [u, v, w] : es) {
        cost += w;

        C.insert(u);
        C.insert(v);

        for (auto [r, s] : adj[u]) pq.push(ii(s, r));

        for (auto [r, s] : adj[v]) pq.push(ii(s, r));
    }

    while ((int)C.size() < N) {
        int v, w;

```

```

do {
    w = pq.top().first, v = pq.top().second;
    pq.pop();
} while (C.count(v));

cost += w;
C.insert(v);

for (auto [s, p] : adj[v]) pq.push(ii(p, s));
}

return cost;
}

```

5.15 Prim

A node u is chosen to start a connected component. Time: $O(e \cdot \log(v))$

```

const int MAX{100010};

vector<ii> adj[MAX];

int prim(int u, int N) {
    set<int> C;
    C.insert(u);

    priority_queue<ii, vector<ii>, greater<ii>> pq;

    for (auto [v, w] : adj[u]) pq.push(ii(w, v));

    int mst = 0;

    while ((int)C.size() < N) {
        int v, w;

        do {
            w = pq.top().first, v = pq.top().second;
            pq.pop();
        } while (C.count(v));

        mst += w;
        C.insert(v);

        for (auto [s, p] : adj[v]) pq.push(ii(p, s));
    }

    return mst;
}

```

5.16 Retrieve Path 2d

```

vll Graph::retrieve_path_2d(ll src, ll trg, const vector<vl>& pred) {
    vll p;

    do {

```

```

        p.emplace_back(pred[src][trg], trg);
        trg = pred[src][trg];
    } while (trg != src);

    reverse(all(p));

    return p;
}

```

5.17 Retrieve Path

```

vll Graph::retrieve_path(ll src, ll trg, const vl& pred) {
    vll p;

    do {
        p.emplace_back(pred[trg], trg);
        trg = pred[trg];
    } while (trg != src);

    reverse(all(p));

    return p;
}

```

5.18 Second Best MST

Time: $O(v \cdot e)$

```

using edge = tuple<int, int, int>;

pair<int, vi> kruskal(int N, vector<edge>& es, int blocked = -1) {
    vi mst;
    UnionFind udfs(N);
    int cost = 0;

    for (int i = 0; i < (int)es.size(); ++i) {
        auto [w, u, v] = es[i];

        if (i != blocked and not udfs.same_set(u, v)) {
            cost += w;
            udfs.union_set(u, v);
            mst.emplace_back(i);
        }
    }

    return {(int)mst.size() == N - 1 ? cost : oo, mst};
}

int second_best(int N, vector<edge>& es) {
    sort(es.begin(), es.end());

    auto [_, mst] = kruskal(N, es);
    int best = oo;

    for (auto blocked : mst) {
        auto [cost, __] = kruskal(N, es, blocked);
        best = min(best, cost);
    }
}

```

```

    return best;
}

```

5.19 TopSort - Tarjan

Works only on Directed Acyclic Graphs (DAGs). For each edge (u,v), u comes before v in the ordering. If the task A is a prerequisite for task B, then A comes before B in the ordering. Time: $O(V + E)$

```

enum State { NOT_FOUND, FOUND, PROCESSED };

vi adj[MAX];

bool dfs(int u, vi& o, vi& state) {
    if (state[u] == PROCESSED) return true;

    if (state[u] == FOUND) return false;

    state[u] = FOUND;

    for (auto v : adj[u])
        if (not dfs(v, o, state)) return false;

    state[u] = PROCESSED;
    o.emplace_back(u);

    return true;
}

vi topological_sort(int N) {
    vi o, state(N + 1, NOT_FOUND);

    for (int u = 1; u <= N; ++u)
        if (state[u] == NOT_FOUND and not dfs(u, o, state)) return {};

    reverse(o.begin(), o.end());

    return o;
}

```

6 Math

6.1 Binomial

```

ll binom(ll n, ll k) {
    if (k > n) return 0;
    vll dp(k + 1, 0);
    dp[0] = 1;
    for (ll i = 1; i <= n; i++)
        for (ll j = k; j > 0; j--) dp[j] = dp[j] + dp[j - 1];
    return dp[k];
}

```

6.2 Count Divisors Range

```

vl divisors(MAX, 0);
void count_divisors_range() {
    for (ll i = 1; i <= MAX; i++) {
        for (ll j = 1; j * i <= MAX; j++) divisors[i * j]++;
    }
}

```

6.3 Count Divisors

```

ll count_divisors(ll num) {
    ll count = 1;
    for (int i = 2; (ll)i * i <= num; i++) {
        if (num % i == 0) {
            int e = 0;
            do {
                e++;
                num /= i;
            } while (num % i == 0);
            count *= e + 1;
        }
    }
    if (num > 1) {
        count *= 2;
    }
    return count;
}

```

6.4 Factorization With Sieve

```

map<ll, ll> factorization_with_sieve(ll n, const vl& primes) {
    map<ll, ll> fact;

    for (ll d : primes) {
        if (d * d > n) break;

        ll k = 0;
        while (n % d == 0) {
            k++;
            n /= d;
        }

        if (k) fact[d] = k;
    }

    if (n > 1) fact[n] = 1;
    return fact;
}

```

6.5 Factorization

```

map<ll, ll> factorization(ll n) {
    map<ll, ll> ans;
    for (ll i = 2; i * i <= n; i++) {
        ll count = 0;
        for (; n % i == 0; count++, n /= i)
            ;
        if (count) ans[i] = count;
    }
}

```

```

}
if (n > 1) ans[n]++;
return ans;
}

```

6.6 Fast Doubling - Fibonacci

The Doubling Method can be seen as an improvement to the matrix exponentiation method to find the N -th Fibonacci number.

Time: $O(\log N)$.

```

template <typename T>
class FastDoubling {
public:
    vector<T> sts;
    T a, b, c, d;
    int mod;

    FastDoubling(int mod = 1e9 + 7) : sts(2), mod(mod) {}

    T fib(T x) {
        fill(all(sts), 0);
        a = 0, b = 0, c = 0, d = 0;
        fast_doubling(x, sts);
        return sts[0];
    }

    void fast_doubling(T n, vector<T>& res) {
        if (n == 0) {
            res[0] = 0;
            res[1] = 1;
            return;
        }
        fast_doubling(n >> 1, res);

        a = res[0];
        b = res[1];
        c = (b << 1) - a;

        if (c < 0) c += mod;

        c = (a * c) % mod;
        d = (a * a + b * b) % mod;
        if (n & 1) {
            res[0] = d;
            res[1] = c + d;
        } else {
            res[0] = c;
            res[1] = d;
        }
    }
};

```

6.7 Fast Exp Iterative

```

ll fast_exp_it(ll a, ll n, ll mod = LLONG_MAX) {
    a %= mod;

```

```

    ll res = 1;

    while (n) {
        if (n & 1) (res *= a) %= mod;

        (a *= a) %= mod;
        n >>= 1;
    }

    return res;
}

```

6.8 Fast Exp

```

ll fast_exp(ll a, ll n, ll mod = LLONG_MAX) {
    if (n == 0) return 1;
    if (n == 1) return a;

    ll x = fast_exp(a, n / 2, mod) % mod;

    return ((x * x) % mod * (n & 1 ? a : 1)) % mod;
}

```

6.9 Fast Fourier Transform (FFT)

Time: $O(N \cdot \log N)$

```

using cd = complex<double>;
const double PI = acos(-1);

void fft(vector<cd>& a, bool invert) {
    int n = a.size();

    for (int i = 1, j = 0; i < n; i++) {
        int bit = n >> 1;
        for (; j & bit; bit >>= 1) j ^= bit;
        j ^= bit;

        if (i < j) swap(a[i], a[j]);
    }

    for (int len = 2; len <= n; len <= 1) {
        double ang = 2 * PI / len * (invert ? -1 : 1);
        cd wlen(cos(ang), sin(ang));
        for (int i = 0; i < n; i += len) {
            cd w(1);
            for (int j = 0; j < len / 2; j++) {
                cd u = a[i + j], v = a[i + j + len / 2] * w;
                a[i + j] = u + v;
                a[i + j + len / 2] = u - v;
                w *= wlen;
            }
        }
    }

    if (invert) {
        for (cd& x : a) x /= n;
    }
}

```

```
void fft_2d(vector<vector<cd>>& V, bool invert) {
    for (int i = 0; i < V.size(); i++) fft(V[i], invert);
    for (int i = 0; i < V.front().size(); i++) {
        vector<cd> col(V.size());
        for (int k = 0; k < V.size(); k++) col[k] = V[k][i];
        fft(col, invert);
        for (int k = 0; k < V.size(); k++) V[k][i] = col[k];
    }
}
```

6.10 GCD

The Euclidean algorithm allows to find the greatest common divisor of two numbers a and b in $O(\log \cdot \min(a, b))$.

```
ll gcd(ll a, ll b) { return b ? gcd(b, a % b) : a; }
```

6.11 Integer Mod

```
const ll INF = 1e18;
const ll mod = 998244353;
template <ll MOD = mod>
```

```
struct Modular {
    ll value;
    static const ll MOD_value = MOD;
```

```
    Modular(ll v = 0) {
        value = v % MOD;
        if (value < 0) value += MOD;
    }
```

```
    Modular(ll a, ll b) : value(0) {
        *this += a;
        *this /= b;
    }
```

```
    Modular& operator+=(Modular const& b) {
        value += b.value;
        if (value >= MOD) value -= MOD;
        return *this;
    }
```

```
    Modular& operator--(Modular const& b) {
        value -= b.value;
        if (value < 0) value += MOD;
        return *this;
    }
```

```
    Modular& operator*=(Modular const& b) {
        value = (ll)value * b.value % MOD;
        return *this;
    }
```

```
    friend Modular mexp(Modular a, ll e) {
        Modular res = 1;
        while (e) {
            if (e & 1) res *= a;
```

```
            a *= a;
            e >>= 1;
        }
        return res;
    }
    friend Modular inverse(Modular a) { return mexp(a, MOD - 2); }

    Modular& operator/=(Modular const& b) { return *this *= inverse(b); }
    friend Modular operator+(Modular a, Modular const b) { return a += b; }
    Modular operator++(int) { return this->value = (this->value + 1) % MOD; }
    Modular operator++() { return this->value = (this->value + 1) % MOD; }
    friend Modular operator-(Modular a, Modular const b) { return a -= b; }
    friend Modular operator--(Modular const a) { return 0 - a; }
    Modular operator--(int) {
        return this->value = (this->value - 1 + MOD) % MOD;
    }

    Modular operator--() { return this->value = (this->value - 1 + MOD) % MOD; }
    friend Modular operator*(Modular a, Modular const b) { return a *= b; }
    friend Modular operator/(Modular a, Modular const b) { return a /= b; }
    friend std::ostream& operator<<(std::ostream& os, Modular const& a) {
        return os << a.value;
    }
    friend bool operator==(Modular const& a, Modular const& b) {
        return a.value == b.value;
    }
    friend bool operator!=(Modular const& a, Modular const& b) {
        return a.value != b.value;
    }
};
```

6.12 Is prime

$O(\sqrt{N})$

```
bool isprime(ll n) {
    if (n < 2) return false;
    if (n == 2) return true;
    if (n % 2 == 0) return false;
    for (ll i = 3; i * i <= n; i += 2)
        if (n % i == 0) return false;
    return true;
}
```

6.13 LCM

Calculating the least common multiple (commonly denoted LCM) can be reduced to calculating the GCD with the following simple formula: $\text{lcm}(a, b) = (a \cdot b) / \text{gcd}(a, b)$

Thus, LCM can be calculated using the Euclidean algorithm with the same time complexity:

```
ll lcm(ll a, ll b) { return a / gcd(a, b) * b; }
```

6.14 Euler phi $\varphi(n)$

Computes the number of positive integers less than n that are co-primes with n , in $O(\sqrt{N})$.

```
ll phi(ll n) {
    if (n == 1) return 1;
```

```

auto fs = factorization(n);
auto res = n;

for (auto [p, k] : fs) {
    res /= p;
    res *= (p - 1);
}

return res;
}

```

6.15 Sieve

```

v1 sieve(ll N) {
    bitset<MAX + 1> sieve;
    v1 ps{2, 3};
    sieve.set();

    for (ll i = 5, step = 2; i <= N; i += step, step = 6 - step) {
        if (sieve[i]) {
            ps.push_back(i);

            for (ll j = i * i; j <= N; j += 2 * i) sieve[j] = false;
        }
    }
    return ps;
}

```

6.16 Sum Divisors

```

ll sum_divisors(ll num) {
    ll result = 1;

    for (int i = 2; (ll)i * i <= num; i++) {
        if (num % i == 0) {
            int e = 0;
            do {
                e++;
                num /= i;
            } while (num % i == 0);

            ll sum = 0, pow = 1;
            do {
                sum += pow;
                pow *= i;
            } while (e-- > 0);
            result *= sum;
        }
    }
    if (num > 1) {
        result *= (1 + num);
    }
    return result;
}

```

6.17 Sum of difference

Function to calculate sum of absolute difference of all pairs in array: $\frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N |A_i - A_j|$

```

ll sum_of_difference(v1& arr, ll n) {
    sort(all(arr));

    ll sum = 0;
    for (ll i = 0; i < n; i++) {
        sum += i * arr[i] - (n - 1 - i) * arr[i];
    }

    return sum;
}

```

7 Problems

7.1 Kth Digit String (CSES)

Time: $O(\log_{10} K)$.

Space: $O(1)$.

```

ll kth_digit_string(ll k) {
    if (k < 10) return k;

    ll c = 180, i = 2, u = 10, r = 0, ans = -1, m;
    for (k -= 9; k > c; i++, u *= 10) {
        k -= c;
        c /= i;
        c *= 10 * (i + 1);
    }

    if ((m = k % i))
        r++;
    else
        m = i;

    ll tmp = (k / i) + r + u - 1;
    for (m = i + 1 - m; m--; tmp /= 10) ans = tmp % 10;

    return ans;
}

```

7.2 Longest Common Substring (LONGCS - SPOJ)

Time: $N = \sum_{i=1}^k |S_i|$; $O(N \cdot \log N)$

```

int lcs_ks_strings(vector<string>& sts, int k) {
    vector<int> fml;
    string t;
    for (int i = 0; i < k; i++) {
        t += sts[i];
        for (int j = 0; j < sts[i].size(); j++) fml.push_back(i);
    }

    suffix_array sf(t);
    sf.lcp.insert(sf.lcp.begin(), 0);
}

```



```

int l = 0, r = 0, cnt = 0, lcs = 0, n = sf.sa.size();
vector<int> fr(k + 1);
multiset<int> mst;
while (l < n) {
    while (r < n and cnt < k) {
        mst.insert(sf.lcp[r]);
        if (!fr[fml[sf.sa[r]]]++) cnt++;
        r++;
    }
    mst.erase(mst.find(sf.lcp[l]));
    if (mst.size() and cnt == k) lcs = max(lcs, *mst.begin());
    fr[fml[sf.sa[l]]]--;
    if (!fr[fml[sf.sa[l]]]) cnt--;
    l++;
}

return lcs;
}

```

7.3 Substring Order II (CSES)

Time: $O(M)$
 $M = 2 \cdot N - 1$
 $N = |S|$

```

// ALLOWS REPETITIONS
string kth_smallest_substring(const string& s, ll k) {
    /* uses /strings/suffix-automaton.cpp
    add 'cnt' and 'nmb' to state struct with (0, -1);
    => for new states 'not cloned': cnt = 1

    create 'order' vector to iterate by length in decreasing
    vector<pair<int, int>>: {len, id}
    => for each new state add to 'order' vector

    to do not allow repetitions:
    => remove: kth+=s.size, sort(order) for(l, p : order)
    => add: st[clone].cnt = 1 (sa_extend)
    */
    string ans;
    k += s.size();
    SuffixAutomaton sa(s);

    sort(all(order), greater<pair<int, int>>());
    // count and mark how many times a substring of a state occurs
    for (auto& [l, p] : order) sa.st[sa.st[p].link].cnt += sa.st[p].cnt;

    auto dfs = [&](auto&& self, int u) {
        if (sa.st[u].nmb != -1) return;

        sa.st[u].nmb = sa.st[u].cnt;
        for (int i = 0; i < 26; ++i) {
            if (sa.st[u].next[i]) {
                self(self, sa.st[u].next[i]);
                sa.st[u].cnt += sa.st[sa.st[u].next[i]].cnt;
            }
        }
    }
}

```

```

};
dfs(dfs, 0);

int u = 0;
while (sa.st[u].nmb < k) {
    k -= sa.st[u].nmb;
    for (int i = 0; i < 26; i++) {
        if (sa.st[u].next[i]) {
            int v = sa.st[u].next[i];
            if (sa.st[v].cnt < k)
                k -= sa.st[v].cnt;
            else {
                ans.push_back(i + 'a');
                u = v;
                break;
            }
        }
    }
}

return ans;
}

```

8 Strings

8.1 Aho-Corasick

The Aho-Corasick algorithm allows us to quickly search for multiple patterns in a text. The set of pattern strings is also called a *dictionary*. We will denote the total length of its constituent strings by m and the size of the alphabet by k .

build: $O(m \cdot k)$
occurrences: $O(|s| + ans)$

```

const int K = 26;
struct Vertex {
    char pch;
    int next[K];
    bool check = false;
    int p = -1, lnk = -1, out = -1, ps = -1, d = 0;

    Vertex(int p = -1, char ch = '$') : p(p), pch(ch) {
        fill(begin(next), end(next), -1);
    }
};

class AhoCorasick {
public:
    int sz = 0; // number of strings added
    vector<Vertex> t;

    AhoCorasick() : t(1) {}

    void add_string(string const& s) {
        int v = 0, ds = 0;
        for (char ch : s) {
            int c = ch - 'a';
            if (t[v].next[c] == -1) {

```

```

        t[v].next[c] = t.size();
        t.emplace_back(v, ch);
    }
    v = t[v].next[c];
    t[v].d = ++ds;
}
t[v].check = true;
t[v].ps = sz++;
}

void build() {
    queue<int> qs;
    qs.push(0);
    while (qs.size()) {
        auto u = qs.front();
        qs.pop();

        if (!t[u].p or t[u].p == -1)
            t[u].lnk = 0;
        else {
            int k = t[t[u].p].lnk;
            int c = t[u].pch - 'a';
            while (t[k].next[c] == -1 and k) k = t[k].lnk;
            int ts = t[k].next[c];
            if (ts == -1)
                t[u].lnk = 0;
            else
                t[u].lnk = ts;
        }

        if (t[t[u].lnk].check)
            t[u].out = t[u].lnk;
        else
            t[u].out = t[t[u].lnk].out;

        for (auto v : t[u].next)
            if (v != -1) qs.push(v);
    }
}

void occurrences(string const& s, vector<vector<int>>& res) {
    // to just "count" replace 'res' vector with an int
    res.resize(sz);
    for (int i = 0, v = 0; i < s.size(); i++) {
        int c = s[i] - 'a';
        while (t[v].next[c] == -1 and v) v = t[v].lnk;
        int ts = t[v].next[c];
        if (ts == -1)
            continue;
        else
            v = t[v].next[c];

        int k = v;
        while (t[k].out != -1) {
            k = t[k].out;
            res[t[k].ps].emplace_back(i - t[k].d + 1);
        }
        if (t[v].check) res[t[v].ps].emplace_back(i - t[v].d + 1);
    }
}

```

```

    }
}
};

```

8.2 Edit Distance

Returns the minimum number of operations (insert, delete, replace) to transform string a into string b .
Time: $O(M * N)$

```

int min_value(int x, int y, int z) { return min(min(x, y), z); }

int edit_distance(string str1, string str2) {
    int n = (int)str1.size(), m = (int)str2.size();
    int dp[m + 1][n + 1];

    for (int i = 0; i <= m; i++)
        for (int j = 0; j <= n; j++)
            if (i == 0)
                dp[i][j] = j;
            else if (j == 0)
                dp[i][j] = i;
            else if (str1[i - 1] == str2[j - 1])
                dp[i][j] = dp[i - 1][j - 1];
            else
                dp[i][j] = 1 + min_value(dp[i][j - 1], dp[i - 1][j], dp[i - 1][j - 1]);

    return dp[m][n];
}

```

8.3 LCP with Suffix Array

For a given string s we want to compute the longest common prefix (LCP) of two arbitrary suffixes with position i and j . In fact, let the request be to compute the LCP of the suffixes $p[i]$ and $p[j]$. Then the answer to this query will be $\min(lcp[i], lcp[i + 1], \dots, lcp[j - 1])$. Thus the problem is reduced to the RMQ.
Time: $O(N)$.

```

vector<int> lcp_suffix_array(string const& s, vector<int> const& p) {
    int n = s.size();
    vector<int> rank(n, 0);
    for (int i = 0; i < n; i++) rank[p[i]] = i;

    int k = 0;
    vector<int> lcp(n - 1, 0);
    for (int i = 0; i < n; i++) {
        if (rank[i] == n - 1) {
            k = 0;
            continue;
        }
        int j = p[rank[i] + 1];
        while (i + k < n && j + k < n && s[i + k] == s[j + k]) k++;
        lcp[rank[i]] = k;
        if (k) k--;
    }
    return lcp;
}

```

8.4 Manacher

Given string s with length n . Find all the pairs (i, j) such that substring $s[i \dots j]$ is a palindrome. String t is a palindrome when $t = t_{rev}$ (t_{rev} is a reversed string for t).

Time: $O(N)$

```
vi manacher(string s) {
    string t;
    for (auto c : s) t += string("#") + c;
    t = t + '#';

    int n = t.size();
    t = "$" + t + "~";

    vi p(n + 2);
    int l = 1, r = 1;
    for (int i = 1; i <= n; i++) {
        p[i] = max(0, min(r - i, p[l + (r - i)]));
        while (t[i - p[i]] == t[i + p[i]]) p[i]++;
        if (i + p[i] > r) {
            l = i - p[i], r = i + p[i];
        }
        p[i]--;
    }

    return vi(begin(p) + 1, end(p) - 1);
}
```

8.5 Rabin Karp

```
vector<int> rabin_karp(string const& s, string const& t) {
    const int p = 31;
    const int m = 1e9 + 9;
    int S = s.size(), T = t.size();

    vector<long long> p_pow(max(S, T));
    p_pow[0] = 1;
    for (int i = 1; i < (int)p_pow.size(); i++) p_pow[i] = (p_pow[i - 1] * p) % m;

    vector<long long> h(T + 1, 0);
    for (int i = 0; i < T; i++) {
        h[i + 1] = (h[i] + (t[i] - 'a' + 1) * p_pow[i]) % m;
        long long h_s = 0;
        for (int i = 0; i < S; i++) h_s = (h_s + (s[i] - 'a' + 1) * p_pow[i]) % m;

        vector<int> occurrences;
        for (int i = 0; i + S - 1 < T; i++) {
            long long cur_h = (h[i + S] + m - h[i]) % m;
            if (cur_h == h_s * p_pow[i] % m) occurrences.push_back(i);
        }

        return occurrences;
    }
}
```

8.6 Suffix Array Optimized - $O(n)$

Suffix Array: sa

Rank for LCP: rnk

LCP: lcp

Time: $O(N)$.

```
// @brunomaletta
struct suffix_array {
    string s;
    int n;
    vector<int> sa, cnt, rnk, lcp;
    rmq<int> RMQ; // /data-structures/rmq.cpp

    bool cmp(int a1, int b1, int a2, int b2, int a3 = 0, int b3 = 0) {
        return a1 != b1 ? a1 < b1 : (a2 != b2 ? a2 < b2 : a3 < b3);
    }

    template <typename T>
    void radix(int* fr, int* to, T* r, int N, int k) {
        cnt = vector<int>(k + 1, 0);
        for (int i = 0; i < N; i++) cnt[r[fr[i]]]++;
        for (int i = 1; i <= k; i++) cnt[i] += cnt[i - 1];
        for (int i = N - 1; i + 1; i--) to[--cnt[r[fr[i]]]] = fr[i];
    }

    void rec(vector<int>& v, int k) {
        auto &tmp = rnk, &m0 = lcp;
        int N = v.size() - 3, sz = (N + 2) / 3, sz2 = sz + N / 3;
        vector<int> R(sz2 + 3);
        for (int i = 1, j = 0; j < sz2; i += i % 3) R[j++] = i;

        radix(&R[0], &tmp[0], &v[0] + 2, sz2, k);
        radix(&tmp[0], &R[0], &v[0] + 1, sz2, k);
        radix(&R[0], &tmp[0], &v[0] + 0, sz2, k);

        int dif = 0;
        int l0 = -1, l1 = -1, l2 = -1;
        for (int i = 0; i < sz2; i++) {
            if (v[tmp[i]] != l0 or v[tmp[i] + 1] != l1 or v[tmp[i] + 2] != l2)
                l0 = v[tmp[i]], l1 = v[tmp[i] + 1], l2 = v[tmp[i] + 2], dif++;
            if (tmp[i] % 3 == 1)
                R[tmp[i] / 3] = dif;
            else
                R[tmp[i] / 3 + sz] = dif;
        }

        if (dif < sz2) {
            rec(R, dif);
            for (int i = 0; i < sz2; i++) R[sa[i]] = i + 1;
        } else
            for (int i = 0; i < sz2; i++) sa[R[i] - 1] = i;

        for (int i = 0, j = 0; j < sz2; i++)
            if (sa[i] < sz) tmp[j++] = 3 * sa[i];
        radix(&tmp[0], &m0[0], &v[0], sz, k);
        for (int i = 0; i < sz2; i++)
            sa[i] = sa[i] < sz ? 3 * sa[i] + 1 : 3 * (sa[i] - sz) + 2;

        int at = sz2 + sz - 1, p = sz - 1, p2 = sz2 - 1;
        while (p >= 0 and p2 >= 0) {
```

```

        if ((sa[p2] % 3 == 1 and
            cmp(v[m0[p]], v[sa[p2]], R[m0[p] / 3], R[sa[p2] / 3 + sz])) or
            (sa[p2] % 3 == 2 and
            cmp(v[m0[p]], v[sa[p2]], v[m0[p] + 1], v[sa[p2] + 1],
                R[m0[p] / 3 + sz], R[sa[p2] / 3 + 1])))
            sa[at--] = sa[p2--];
        else
            sa[at--] = m0[p--];
    }
    while (p >= 0) sa[at--] = m0[p--];
    if (N % 3 == 1)
        for (int i = 0; i < N; i++) sa[i] = sa[i + 1];
}

suffix_array(const string& s_)
: s(s_), n(s.size()), sa(n + 3), cnt(n + 1), rnk(n), lcp(n - 1) {
    vector<int> v(n + 3);
    for (int i = 0; i < n; i++) v[i] = i;
    radix(&v[0], &rnk[0], &s[0], n, 256);
    int dif = 1;
    for (int i = 0; i < n; i++)
        v[rnk[i]] = dif += (i and s[rnk[i]] != s[rnk[i - 1]]);
    if (n >= 2) rec(v, dif);
    sa.resize(n);

    for (int i = 0; i < n; i++) rnk[sa[i]] = i;
    for (int i = 0, k = 0; i < n; i++, k -= !!k) {
        if (rnk[i] == n - 1) {
            k = 0;
            continue;
        }
        int j = sa[rnk[i] + 1];
        while (i + k < n and j + k < n and s[i + k] == s[j + k]) k++;
        lcp[rnk[i]] = k;
    }
    RMQ = rmq<int>(lcp);
}

int query(int i, int j) {
    if (i == j) return n - i;
    i = rnk[i], j = rnk[j];
    return RMQ.query(min(i, j), max(i, j) - 1);
}
};

```

8.7 Suffix Array

Let s be a string of length n . The i -th suffix of s is the substring $s[i \dots n - 1]$. A suffix array will contain integers that represent the starting indexes of the all the suffixes of a given string, after the aforementioned suffixes are sorted.
Time: $O(N \log N)$.

```

vector<int> sort_cyclic_shifts(string const& s) {
    int n = s.size();
    const int alphabet = 128;

    vector<int> p(n), c(n), cnt(max(alphabet, n), 0);
    for (int i = 0; i < n; i++) cnt[s[i]]++;

```

```

    for (int i = 1; i < alphabet; i++) cnt[i] += cnt[i - 1];
    for (int i = 0; i < n; i++) p[--cnt[s[i]]] = i;
    c[p[0]] = 0;
    int classes = 1;
    for (int i = 1; i < n; i++) {
        if (s[p[i]] != s[p[i - 1]]) classes++;
        c[p[i]] = classes - 1;
    }

    vector<int> pn(n), cn(n);
    for (int h = 0; (1 << h) < n; ++h) {
        for (int i = 0; i < n; i++) {
            pn[i] = p[i] - (1 << h);
            if (pn[i] < 0) pn[i] += n;
        }
        fill(cnt.begin(), cnt.begin() + classes, 0);
        for (int i = 0; i < n; i++) cnt[c[pn[i]]]++;
        for (int i = 1; i < classes; i++) cnt[i] += cnt[i - 1];
        for (int i = n - 1; i >= 0; i--) p[--cnt[c[pn[i]]]] = pn[i];
        cn[p[0]] = 0;
        classes = 1;
        for (int i = 1; i < n; i++) {
            pair<int, int> cur = {c[p[i]], c[(p[i] + (1 << h)) % n]};
            pair<int, int> prev = {c[p[i - 1]], c[(p[i - 1] + (1 << h)) % n]};
            if (cur != prev) ++classes;
            cn[p[i]] = classes - 1;
        }
        c.swap(cn);
    }

    return p;
}

vector<int> suffix_array(string s) {
    s += "$";
    vector<int> p = sort_cyclic_shifts(s);
    p.erase(p.begin());
    return p;
}

```

8.8 Suffix Automaton

```

class SuffixAutomaton {
public:
    struct state {
        int len, link;
        array<int, 26> next;
    };

    vector<state> st;
    int sz = 0, last;

    SuffixAutomaton(const string& s) : st(s.size() << 1) {
        sa_init();
        for (auto v : s) sa_extend((int)(v - 'a'));
    }

```

```

void sa_init() {
    st[0].len = 0;
    st[0].link = -1;
    sz++;
    last = 0;
}

void sa_extend(int c) {
    int cur = sz++;
    st[cur].len = st[last].len + 1;
    int p = last;
    while (p != -1 && !st[p].next[c]) {
        st[p].next[c] = cur;
        p = st[p].link;
    }
    if (p == -1)
        st[cur].link = 0;
    else {
        int q = st[p].next[c];
        if (st[p].len + 1 == st[q].len)
            st[cur].link = q;
        else {
            int clone = sz++;
            st[clone].len = st[p].len + 1;
            st[clone].link = st[q].link;
            st[clone].next = st[q].next;
            while (p != -1 && st[p].next[c] == q) {
                st[p].next[c] = clone;
                p = st[p].link;
            }
            st[q].link = st[cur].link = clone;
        }
    }
    last = cur;
}

// longest common substring O(N)
int lcs(const string& T) {
    int v = 0, l = 0, best = 0;
    for (int i = 0; i < T.size(); i++) {
        while (v && !st[v].next[T[i] - 'a']) {
            v = st[v].link;
            l = st[v].len;
        }
        if (st[v].next[T[i] - 'a']) {
            v = st[v].next[T[i] - 'a'];
            l++;
        }
        best = max(best, l);
    }
    return best;
}
};

```

8.9 Suffix Tree (CP Algo - freopen)

Build: $O(N)$

Memory: $O(N \cdot k)$

k = alphabet length

```

const int aph = 27; // add $ to final of string
const int N = 2e5 + 31;
class SuffixTree {
public:
    string a;
    vector<array<int, aph>> t;
    vector<int> l, r, p, s, dst;
    int tv, tp, ts, la, b;

    SuffixTree(const string& str, char bs = 'a') : a(str), t(N), l(N),
        r(N, str.size() - 1), p(N), s(N), dst(N), b(bs) {
        build();
    }

    void ukkadd(int c) {
        suff++;
        if (r[tv] < tp) {
            if (t[tv][c] == -1) {
                t[tv][c] = ts; l[ts] = la;
                p[ts++] = tv; tv = s[ts]; tp = r[ts] + 1; goto suff;
            }
            tv = t[tv][c]; tp = l[ts];
        }
        if (tp == -1 || c == a[tp] - b) tp++; else {
            l[ts + 1] = la; p[ts + 1] = ts;
            l[ts] = l[ts]; r[ts] = tp - 1; p[ts] = p[ts];
            t[ts][c] = ts + 1; t[ts][a[tp] - b] = tv; l[ts] = tp;
            p[ts] = ts; t[p[ts]][a[l[ts]] - b] = ts; ts += 2;
            tv = s[p[ts - 2]]; tp = l[ts - 2];
            while (tp <= r[ts - 2]) {
                tv = t[ts][a[tp] - b];
                tp += r[ts] - l[ts] + 1;
            }
            if (tp == r[ts - 2] + 1) s[ts - 2] = tv; else s[ts - 2] = ts;
            tp = r[ts] - (tp - r[ts - 2]) + 2; goto suff;
        }
    }

    void build() {
        ts = 2; tv = 0; tp = 0;
        s[0] = 1; l[0] = -1; r[0] = -1; l[1] = -1; r[1] = -1;
        for (auto& arr : t) { arr.fill(-1); } t[1].fill(0);
        for (la = 0; la < (int)a.size(); ++la) ukkadd(a[la] - b);
    }
};

```

8.10 Z Function

Suppose we are given a string s of length n . The Z-function for this string is an array of length n where the i -th element is equal to the greatest number of characters starting from the position i that coincide with the first characters of s .

Time: $O(N)$

```

vector<int> z_function(string s) {
    int n = s.size();
    vector<int> z(n);

```

```

int l = 0, r = 0;
for (int i = 1; i < n; i++) {
    if (i < r) {
        z[i] = min(r - i, z[i - 1]);
    }
    while (i + z[i] < n && s[z[i]] == s[i + z[i]]) {
        z[i]++;
    }
    if (i + z[i] > r) {
        l = i;
        r = i + z[i];
    }
}
return z;
}

```

9 Trees

9.1 LCA Binary Lifting (CP Algo)

The algorithm described will need $O(N \cdot \log N)$ for preprocessing the tree, and then $O(\log N)$ for each LCA query.

```

ll n, l;
vector<ll> adj[MAX];

ll timer;
vector<ll> tin, tout;
vector<vector<ll>> up;

void dfs(ll v, ll p) {
    tin[v] = ++timer;
    up[v][0] = p;
    for (ll i = 1; i <= l; ++i) up[v][i] = up[up[v][i - 1]][i - 1];

    for (ll u : adj[v]) {
        if (u != p) dfs(u, v);
    }

    tout[v] = ++timer;
}

bool is_ancestor(ll u, ll v) { return tin[u] <= tin[v] && tout[u] >= tout[v];
}

ll lca(ll u, ll v) {
    if (is_ancestor(u, v)) return u;
    if (is_ancestor(v, u)) return v;
    for (ll i = l; i >= 0; --i) {
        if (!is_ancestor(up[u][i], v)) u = up[u][i];
    }
    return up[u][0];
}

void preprocess(ll root) {
    tin.resize(n);
    tout.resize(n);

```

```

timer = 0;
l = ceil(log2(n));
up.assign(n, vector<ll>(l + 1));
dfs(root, root);
}

```

9.2 LCA SegTree (CP Algo)

The algorithm can answer each query in $O(\log N)$ with preprocessing in $O(N)$ time.

```

struct LCA {
    vector<ll> height, euler, first, segtree;
    vector<bool> visited;
    ll n;

    LCA(vector<vector<ll>>& adj, ll root = 0) {
        n = adj.size();
        height.resize(n);
        first.resize(n);
        euler.reserve(n * 2);
        visited.assign(n, false);
        dfs(adj, root);
        ll m = euler.size();
        segtree.resize(m * 4);
        build(1, 0, m - 1);
    }

    void dfs(vector<vector<ll>>& adj, ll node, ll h = 0) {
        visited[node] = true;
        height[node] = h;
        first[node] = euler.size();
        euler.push_back(node);
        for (auto to : adj[node]) {
            if (!visited[to]) {
                dfs(adj, to, h + 1);
                euler.push_back(node);
            }
        }
    }

    void build(ll node, ll b, ll e) {
        if (b == e) {
            segtree[node] = euler[b];
        } else {
            ll mid = (b + e) / 2;
            build(node << 1, b, mid);
            build(node << 1 | 1, mid + 1, e);
            ll l = segtree[node << 1], r = segtree[node << 1 | 1];
            segtree[node] = (height[l] < height[r]) ? l : r;
        }
    }

    ll query(ll node, ll b, ll e, ll L, ll R) {
        if (b > R || e < L) return -1;
        if (b >= L && e <= R) return segtree[node];
        ll mid = (b + e) >> 1;

        ll left = query(node << 1, b, mid, L, R);

```

```

    ll right = query(node << 1 | 1, mid + 1, e, L, R);
    if (left == -1) return right;
    if (right == -1) return left;
    return height[left] < height[right] ? left : right;
}

ll lca(ll u, ll v) {
    ll left = first[u], right = first[v];
    if (left > right) swap(left, right);
    return query(1, 0, euler.size() - 1, left, right);
}
};

```

9.3 LCA Sparse Table

The algorithm described will need $O(N)$ for preprocessing, and then $O(1)$ for each LCA query.
0 indexed!

```

typedef vector<vl> vl2d;
#define all(a) a.begin(), a.end()
#define len(x) (int)x.size()

template <typename T>
struct SparseTable {
    vector<T> v;
    ll n;
    static const ll b = 30;
    vl mask, t;

    ll op(ll x, ll y) { return v[x] < v[y] ? x : y; }
    ll msb(ll x) { return __builtin_clz(1) - __builtin_clz(x); }
    SparseTable() {}
    SparseTable(const vector<T>& v_) : v(v_), n(v.size()), mask(n), t(n) {
        for (ll i = 0, at = 0; i < n; mask[i++] = at |= 1) {
            at = (at << 1) & ((1 << b) - 1);
            while (at and op(i, i - msb(at & -at)) == i) at ^= at & -at;
        }
        for (ll i = 0; i < n / b; i++)
            t[i] = b * i + b - 1 - msb(mask[b * i + b - 1]);
        for (ll j = 1; (1 << j) <= n / b; j++)
            for (ll i = 0; i + (1 << j) <= n / b; i++)
                t[n / b * j + i] =
                    op(t[n / b * (j - 1) + i], t[n / b * (j - 1) + i + (1 << (j - 1))]);
    }
    ll small(ll r, ll sz = b) { return r - msb(mask[r] & ((1 << sz) - 1)); }
    T query(ll l, ll r) {
        if (r - l + 1 <= b) return small(r, r - l + 1);
        ll ans = op(small(l + b - 1), small(r));
        ll x = l / b + 1, y = r / b - 1;
        if (x <= y) {
            ll j = msb(y - x + 1);
            ans = op(ans, op(t[n / b * j + x], t[n / b * j + y - (1 << j) + 1]));
        }
        return ans;
    }
};

struct LCA {

```

```

    SparseTable<ll> st;
    ll n;
    vl v, pos, dep;

    LCA(const vl2d& g, ll root) : n(len(g)), pos(n) {
        dfs(root, 0, -1, g);
        st = SparseTable<ll>(vector<ll>(all(dep)));
    }

    void dfs(ll i, ll d, ll p, const vl2d& g) {
        v.emplace_back(len(dep)) = i, pos[i] = len(dep), dep.emplace_back(d);
        for (auto j : g[i])
            if (j != p) {
                dfs(j, d + 1, i, g);
                v.emplace_back(len(dep)) = i, dep.emplace_back(d);
            }
    }

    ll lca(ll a, ll b) {
        ll l = min(pos[a], pos[b]);
        ll r = max(pos[a], pos[b]);
        return v[st.query(l, r)];
    }
    ll dist(ll a, ll b) {
        return dep[pos[a]] + dep[pos[b]] - 2 * dep[pos[lca(a, b)]];
    }
};

```

9.4 Tree Flatten

```

vll tree_flatten(ll root) {
    vl pre;
    pre.reserve(N);

    vll flat(N);
    ll timer = -1;
    auto dfs = [&](auto&& self, ll u, ll p) -> void {
        timer++;
        pre.push_back(u);
        for (auto [v, w] : adj[u])
            if (v != p) {
                self(self, v, u);
            }
        flat[u].second = timer;
    };
    dfs(dfs, root, -1);
    for (ll i = 0; i < (ll)N; i++) flat[pre[i]].first = i;
    return flat;
}

```

9.5 Tree Isomorph

Checks whether two trees are isomorphic. The function `thash()` returns the hash of the tree (using centroids as special vertices). Two trees are isomorphic if their hash are the same.

```
map<vector<int>, int> mhash;
```

```

struct tree {
    int n;
    vector<vector<int>> g;
    vector<int> sz, cs;

    tree(int n_) : n(n_), g(n_), sz(n_) {}

    void dfs_centroid(int v, int p) {
        sz[v] = 1;
        bool cent = true;
        for (int u : g[v])
            if (u != p) {
                dfs_centroid(u, v), sz[v] += sz[u];
                if (sz[u] > n / 2) cent = false;
            }
        if (cent and n - sz[v] <= n / 2) cs.push_back(v);
    }

    int fhash(int v, int p) {
        vector<int> h;
        for (int u : g[v])
            if (u != p) h.push_back(fhash(u, v));
        sort(h.begin(), h.end());
        if (!mhash.count(h)) mhash[h] = mhash.size();
        return mhash[h];
    }

    ll thash() {
        cs.clear();
        dfs_centroid(0, -1);
        if (cs.size() == 1) return fhash(cs[0], -1);
        ll h1 = fhash(cs[0], cs[1]), h2 = fhash(cs[1], cs[0]);
        return (min(h1, h2) << 30) + max(h1, h2);
    }

    void add(int a, int b) {
        g[a].emplace_back(b);
        g[b].emplace_back(a);
    }
};

```

10 Settings and macros

10.1 short-macro.cpp

```

#include <bits/stdc++.h>

using namespace std;

#ifdef DEBUG
#include "../settings-and-macros/debug.cpp"
#else
#define dbg(...)
#endif

typedef long long ll;
typedef pair<int, int> ii;

#define all(x) x.begin(), x.end()

```

```

#define vin(vt) for (auto &e : vt) cin >> e

auto solve() { }

int main() {
    ios_base::sync_with_stdio(0);
    cin.tie(0);

    ll t = 1;
    //cin >> t;

    while (t--) solve();

    return 0;
}

```

10.2 macro.cpp

```

#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>

using namespace __gnu_pbds;
#define ordered_set tree<int, null_type, less<int>, rb_tree_tag,
    tree_order_statistics_node_update>

using namespace std;

#ifdef DEBUG
#include "../settings-and-macros/debug.cpp"
#else
#define dbg(...)
#endif

typedef long long ll;
typedef pair<int, int> pii;
typedef pair<ll, ll> pll;
typedef vector<int> vi;
typedef vector<ll> vl;
typedef vector<pii> vii;
typedef vector<pll> vll;

#define fst first
#define snd second
#define all(x) x.begin(), x.end()
#define len(vt) (int)vt.size()
#define vin(vt) for (auto &e : vt) cin >> e
#define LSONe(S) ((S) & ~(S))
#define MSONe(S) (1ull << (63 - __builtin_clzll(S)))
#define fastio ios_base::sync_with_stdio(0); \
    cin.tie(0); \
    cout.tie(0)

const vii dir4 {{1,0},{-1,0},{0,1},{0,-1}};

auto solve() { }

int main() {

```



```

fastio;

ll t = 1;
//cin >> t;

while (t--) solve();

return 0;
}

```

11 Theoretical guide

11.1 Number of Different Substrings

$$\sum_{i=0}^{n-1} (n - p[i]) - \sum_{i=0}^{n-2} \text{lcp}[i] = \frac{n^2 + n}{2} - \sum_{i=0}^{n-2} \text{lcp}[i]$$

11.2 String Matching with FFT

We are given two strings, a text T and a pattern P , consisting of lowercase letters. We have to compute all the occurrences of the pattern in the text.

We create a polynomial for each string ($T[i]$ and $P[i]$ are numbers between 0 and 25 corresponding to the 26 letters of the alphabet):

$$A(x) = a_0x^0 + a_1x^1 + \dots + a_{n-1}x^{n-1}, \quad n = |T|$$

with

$$a_i = \cos(\alpha_i) + i \sin(\alpha_i), \quad \alpha_i = \frac{2\pi T[i]}{26}$$

And

$$B(x) = b_0x^0 + b_1x^1 + \dots + b_{m-1}x^{m-1}, \quad m = |P|$$

with

$$b_i = \cos(\beta_i) - i \sin(\beta_i), \quad \beta_i = \frac{2\pi P[m - i - 1]}{26}$$

Notice that with the expression $P[m - i - 1]$ explicitly reverses the pattern.

The $(m - 1 + i)$ th coefficients of the product of the two polynomials $C(x) = A(x) \cdot B(x)$ will tell us, if the pattern appears in the text at position i .

If there isn't a match, then at least a character is different, which leads that one of the products $a_{i+1} \cdot b_{m-1-j}$ is not equal to 1, which leads to the coefficient $c_{m-1+i} \neq m$.

11.2.1 Wildcards

This is an extension of the previous problem. This time we allow that the pattern contains the wildcard character $*$, which can match every possible letter.

We create the exact same polynomials, except that we set $b_i = 0$ if $P[m - i - 1] = *$. If x is the number of wildcards in P , then we will have a match of P in T at index i if $c_{m-1+i} = m - x$.

11.3 Modular Multiplicative Inverse

A modular multiplicative inverse of an integer a is an integer x such that $a \cdot x$ is congruent to 1 modular some modulus m . To write it in a formal way:

$$a \cdot x \equiv 1 \pmod{m}.$$

Euler's theorem, which states that the following congruence is true if a and m are co-primes:

$$a^{\phi(m)} \equiv 1 \pmod{m}$$

Multiply both sides of the above equations by a^{-1} , and we get:

- For an arbitrary (but coprime) modulus m : $a^{\phi(m)-1} \equiv a^{-1} \pmod{m}$
- For a prime modulus m : $a^{m-2} \equiv a^{-1} \pmod{m}$

From these results, we can easily find the modular inverse using the binary exponentiation algorithm, which works in $O(\log m)$ time.

11.4 Notable Series

1. Sum of the first n naturals:

$$S_n = \sum_{i=1}^n i = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

2. Sum of the squares of the first n naturals:

$$S_n = \sum_{i=1}^n i^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

3. Sum of the cubes of the first natural n :

$$S_n = \sum_{i=1}^n i^3 = 1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2$$

4. Sum of the first n odd numbers:

$$S_n = \sum_{i=1}^n 2i - 1 = 1 + 3 + 5 + \dots + (2n - 1) = n^2$$

11.5 Exponent With Module

If a and m are coprime, then

$$a^n \equiv a^{n \pmod{\phi(m)}} \pmod{m}$$

Generally, if $n \geq \log_2 m$, then

$$a^n \equiv a^{\phi(m) + [n \pmod{\phi(m)}]} \pmod{m}$$