$(x-2)(x-2)$
$= x^2 - 4x + 4$

Lista 2 - C3:

1) $F(x,y) = \left\langle \underbrace{\frac{y}{x^2+y^2} + \frac{y}{(x-2)^2+y^2}}_{P} , \underbrace{\frac{-x}{x^2+y^2} + \frac{2-x}{(x-2)^2+y^2}}_{Q} \right\rangle$

$$\frac{\partial Q}{\partial x} = \frac{\partial}{\partial x}\left(\frac{-x}{x^2+y^2} + \frac{2-x}{(x-2)^2+y^2}\right) = \frac{-1.(x^2+y^2) - (-x)(2x)}{(x^2+y^2)^2} + \frac{(-1)\overbrace{[(x-2)^2+y^2]}^{x^2-4x+4+y^2} - \overbrace{(2-x)(2x-4)}^{8x-8-2x^2}}{[(x-2)^2+y^2]^2}$$

$$= \frac{-x^2-y^2+2x^2}{(x^2+y^2)^2} + \frac{-x^2+4x-4-y^2-8x+8+2x^2}{[(x-2)^2+y^2]^2}$$

$$= \frac{x^2-y^2}{(x^2+y^2)^2} + \frac{x^2-4x+4-y^2}{[(x-2)^2+y^2]^2} = \boxed{\frac{x^2-y^2}{(x^2+y^2)^2} + \frac{(x-2)^2-y^2}{[(x-2)^2+y^2]}}$$

$$\frac{\partial P}{\partial y} = \frac{\partial}{\partial y}\left(\frac{y}{x^2+y^2} + \frac{y}{(x-2)^2+y^2}\right) = \frac{1(x^2+y^2) - y(2y)}{(x^2+y^2)^2} + \frac{1[(x-2)^2+y^2] - y(2y)}{[(x-2)^2+y^2]^2}$$

$$= \frac{x^2+y^2-2y^2}{(x^2+y^2)^2} + \frac{(x-2)^2+y^2-2y^2}{[(x-2)^2+y^2]^2} = \boxed{\frac{x^2-y^2}{(x^2+y^2)^2} + \frac{(x-2)^2-y^2}{[(x-2)^2+y^2]^2}}$$

TEOREMA DE GREEN PARA REGIÕES COM FUROS:

$$\int_C P.dx + Q.dy + \int_{-C'} P.dx + Q.dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

$$= \iint_D \left[\frac{x^2-y^2}{(x^2+y^2)^2} + \frac{(x-2)^2-y^2}{[(x-2)^2+y^2]} - \left(\frac{x^2-y^2}{(x^2+y^2)^2} + \frac{(x-2)^2-y^2}{[(x-2)^2+y^2]^2}\right)\right] dA = 0$$

Logo, $\int_C P \cdot dx + Q \cdot dy = \int_{C'} P \cdot dx + Q \cdot dy$

isto é, $\int_C F \cdot dr = \int_{C'} F \cdot dr$

Dividindo o campo em 2: $\int_C F \cdot dr = \int_{C_1} F_1 \cdot dr_1 + \int_{C_2} F_2 \, dr_2$

parametrizando: $x = a\cos t$ $\quad r_1(t) = a\cos t\, i + a\,\text{sen}\,t\, j$

$y = a\,\text{sen}\,t$ $\quad r_2(t) = b\cos t\, i + b\,\text{sen}\,t\, j$

Temos que: $F_1 = \left\langle \dfrac{\cancel{a}\,\text{sen}\,t}{a^{\cancel{2}}\underbrace{(\cos^2 t + \text{sen}^2 t)}_{1}}, \dfrac{-\cancel{a}\cos t}{a^{\cancel{2}}(\cos^2 t + \text{sen}^2 t)} \right\rangle$

$$F_1 = \left\langle \frac{\text{sen}\,t}{a}, \frac{-\cos t}{a} \right\rangle$$

$$r_1'(t) = \langle -a\,\text{sen}\,t, a\cos t \rangle$$

$$\int_{C_1} \left\langle \frac{\text{sen}\,t}{a}, \frac{-\cos t}{a} \right\rangle \langle -a\,\text{sen}\,t, a\cos t \rangle \, dt = \int_{2\pi}^{0} \left(\frac{-\cancel{a}\,\text{sen}^2 t}{\cancel{a}} - \frac{\cancel{a}\cos^2 t}{\cancel{a}} \right) dt$$

$$= \int_{2\pi}^{0} -(\text{sen}^2 t + \cos^2 t)\, dt = -\int_{2\pi}^{0} dt = -(0 - 2\pi) = \boxed{2\pi}$$

Fazendo o mesmo para F_2:

$$F_2 = \left\langle \frac{b\,\text{sen}\,t}{(b\cos t + \cancel{2} - \cancel{2})^2 + (b\,\text{sen}\,t)^2}, \frac{\cancel{2} - b\cos t - \cancel{2}}{(b\cos t + \cancel{2} - \cancel{2})^2 + (b\,\text{sen}\,t)^2} \right\rangle$$

$$F_2 = \left\langle \frac{\text{sen}\,t}{b}, \frac{-\cos t}{b} \right\rangle \qquad r_2'(t) = \langle -b\,\text{sen}\,t, b\cos t \rangle$$

$$\int_{C_2} \left\langle \frac{sen\,t}{b}, \frac{-\cos t}{b} \right\rangle \langle -b\,sen\,t, b\cos t \rangle \, dt$$

$$\int_{2\pi}^{0} \left(\frac{-\cancel{b}\, sen^2 t}{\cancel{b}} - \frac{\cancel{b}\cos^2 t}{\cancel{b}} \right) dt = \int_{2\pi}^{0} -(sen^2 t + \cos^2 t)\, dt$$

$$= 2\pi$$

Portanto:

$$\int_C F \cdot dr = \int_{C_1} F_1 \, dr_1 + \int_{C_2} F_2 \cdot dr_2$$

$$= 2\pi + 2\pi = \boxed{4\pi}$$