Introduction to statistics

TODO subtitle

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Summary:

- Probability concepts
- Discrete distributions.
- Continuous distributions.
- Calculations on the Normal distribution.
- Convergence
- Inference

Motivation

Figure 1: Dados apontam ... (data shows ...)

Basic concepts of probability:

Sample space Ω

It's the set of all the possible outcomes of a experiment, denoted by S or Ω

Event

It's a subset of the sample space.

Properties needed:

Given a sample sample Ω the class of events denoted by $\mathcal{A}$ need to satisfy the following properties:

1. $\emptyset \in \mathcal{A}$;
2. $a_1, a_2, \ldots, \in \mathcal{A} \Longrightarrow \bigcup_{i=1}^{\infty} a_i \in \mathcal{A}$
3. if $a \in \mathcal{A} \Longrightarrow a^C \in \mathcal{A}$

Basic concepts of probability:

Probability (Definition):

Given a experiment with a sample space Ω and a class of events $\mathcal{A}$, the probability denoted by $\mathbb{P}$ is a function which has $\mathcal{A}$ as domain and associate a numerical value between $[0, 1]$ as image.

Probability properties:

1. $\mathbb{P}(\Omega) = 1$ and $\mathbb{P}(\emptyset) = 0$
2. $0 \leq \mathbb{P}(A) \leq 1$, for every event A
3. For any sequence of mutually exclusive events $A_1, A_2, \ldots$ that's events that $A_i \bigcap A_j$ when $i \neq j$ we have that:

$$\mathbb{P}\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \mathbb{P}(A_i)$$

Basic concepts of probability:

Event independence:

Two events are independent when the occurrence of the first does not influence the second.

Two events A and B are independent if:

$$\mathbb{P}(A \bigcap B) = \mathbb{P}(A)\mathbb{P}(B)$$

Conditional Events:

The probability of a event A to occur given that the event B occurred is:

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(A \bigcap B)}{\mathbb{P}(B)}$$

Basic concepts of probability:

Bayes theorem:

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B)}$$

General case:

$$\mathbb{P}(A_i|B) = \frac{\mathbb{P}(B|A_i)\mathbb{P}(A_i)}{\sum_{j=1}^{n} \mathbb{P}(B|A_j)\mathbb{P}(A_j)}$$

Bayes example (from Veritasium):

You are felling sick, so you go to the doctor, there you run a battery of tests. After getting the results you tested positive for a rare disease (affects 0.1% of the population), the test will correctly identify that you have it 99% of the times.
What's the chances that you actually have the disease? 99%?

Bayes example Solution

Let's denote the event of you have the disease H (stands for hypothesis, the prior) and the test been positive denoted by E (stands for evidence), so we have: $\mathbb{P}(H) = 0.001$ and $\mathbb{P}(E|H) = 0.99$

$$\mathbb{P}(H|E) = \frac{\mathbb{P}(E|H)\mathbb{P}(H)}{\mathbb{P}(E)} = \frac{\mathbb{P}(E|H)\mathbb{P}(H)}{\mathbb{P}(H)\mathbb{P}(E|H) + \mathbb{P}(H^C)\mathbb{P}(E|H^C)} =$$

$$= \frac{0.99 \cdot 0.001}{0.001 \cdot 0.99 + 0.999 \cdot 0.01} = 0.09 = 9\%$$

What if you test again and it's also positive? You can just take the posterior probability we just calculated and use as a prior:

$$= \frac{0.99 \cdot 0.09}{0.09 \cdot 0.99 + 0.91 \cdot 0.01} = 0.907 \approx 91\%$$

- Awesome video: A visual guide to Bayesian thinking

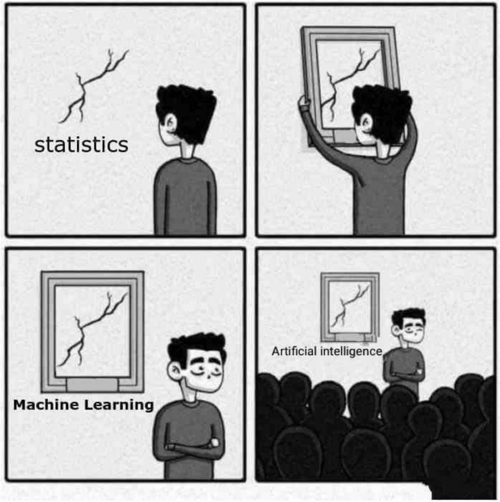

Figure 2: Credits: sandserifcomics

Random Variable (RV)

Consider a experiment with a sample sample Ω associated with it. A function that maps each element $\omega \in \Omega$ to a Real number it's called random variable (RV) ($X : \Omega \to \mathbb{R}$)

- Example: Imagine a experiment that consist of 3 consecutive fair coin tosses, so the sample space of this experiment is: S = {(H,H,H), (H,H,T) , . . . (T,T,T)} . Now we want to create a random variable X that counts the number of heads in each outcome, so X((H,H,H)) = 3 and X((H,H,T)) = 2.

Random Variable:

Probability Mass Function (PMF):

$$f_X(x) = \mathbb{P}[X = x] = \mathbb{P}[\{\omega \in \Omega : X(\omega) = x\}]$$

Probability Density Function (PDF)

$$\mathbb{P}[a \leq X \leq b] = \int_a^b f(x)dx$$

Cumulative Distribution Function (CDF)

$$F_X(x) = \mathbb{P}[X \leq x]$$

Expectation:

- Discrete : $\mathbb{E}[X] = \sum x\mathbb{P}(X = x)$
- Continuous: $\mathbb{E}[X] = \int_{-\infty}^{\infty} xf(x)dx$

Variance:

$$\mathbb{V}[X] = \sigma_X^2 = \mathbb{E}[X^2] - \mathbb{E}^2[X]$$

Sample mean:

$$\overline{X_n} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

Sample variance and standard deviation:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$$

Standard deviation $= s$

Discrete distributions

Bernoulli:

Consider a experiment with has two possible outcomes: success (X=1, with probability p) or failure (X=0), this random variable is called Bernoulli, the PMF is:

$$\mathbb{P}(X = k) = p^k(1-p)^{1-k}$$

Binomial:

Now consider a Bernoulli experiment conducted n times, let X be the random variable that represents the number of successes, X is called Binomial, the PMF is:

$$\mathbb{P}(X = k) = \binom{n}{k} p^k(1-p)^{n-k}$$

Discrete distributions

Geometric:

Again consider a Bernoulli experiment conducted n times, but the first n-1 are failures and the last nth is a success. Let X be number of tries , which is called Geometric, the PMF is:

$$\mathbb{P}(X = k) = (1 - p)^k p$$

- A important property is that Geometric distribution is **memoryless** (TODO definir)

Poisson:

A random variable which value can assume 0,1,2 ... is called Poisson with $\lambda > 0$ parameter if your PMF is:

$$\mathbb{P}(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}$$

Continuous distributions

- Normal
- Exponential memoryless, sunk costs (https://youarenotsosmart.com/2011/03/25/the-sunk-cost-fallacy/)
- Pareto

meme know distributions

Calculations on the Normal distribution

tabela e calcular python, excel, normalizar

order statitics

defs, min, max, median, q1,q3, IQR, pq? estat robusta, boxplot

Convergence

defs, lei dos grandes numeros, teorema do limite central

Inference

metodo da maxima verosimilhanca, e grafico qxq (scipy.stats.probplot)

Further reading:

- Portal action (pt)
- stat cookbook
- havard youtube (https://www.youtube.com/playlist?list=PL2SOU6wwxB0uwwH80KT
- ross, barry james, meyer
- khan academy
- http://www.randomservices.org/random/