Processos Estocásticos
Componentes: - Bruno Oliveira
- Caio Cristiano
- Lara Dourado

1) a) $P(A_1 \cap A_2 \cap A_3 \cap A_4)$

$$= \frac{4}{52} \cdot \frac{4}{52} \cdot \frac{4}{52} \cdot \frac{4}{52} = \frac{256}{7311616} = \frac{1}{28561} = 3,501 \cdot 10^{-5}$$

b) $$\frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} \cdot \frac{1}{49} = \frac{24}{6497400} = \frac{1}{270725} = 3,69 \cdot 10^{-6}$$

2) $A_1 = \{3\}$ $\quad A_2 = \{2, 4, 6\}$
$A_3 = \{1, 3, 5\}$

a) • $P(A_1 \cap A_3) = \frac{1}{6}$

• $P(A_2 \cup A_3) = P(A_2) + P(A_3) - P(A_2 \cap A_3)$

$$= \frac{3}{6} + \frac{3}{6} - \emptyset = 1$$

↳ Mutuamente Excludentes

• $P(A_2 \cap A_3) = \{\ \}$ → Conjunto vazio

• $$P(A_1 | A_3) = \frac{P(A_1 \cap A_3)}{P(A_3)} = \frac{\cancel{1/6}}{\frac{3}{\cancel{6}}} = \frac{1}{3}$$

b) A_2 e A_3 → não são independentes segundo a definição

$$P(A_2 A_3) = P(A_2 \cap A_3) \neq P(A_2) \cdot P(A_3)$$

$$\emptyset \neq \frac{3}{6} \cdot \frac{3}{6}$$

$$\emptyset \neq \frac{1}{4}$$

3) $C: \{R_1, R_2, R_3, R_4, R_5\}$

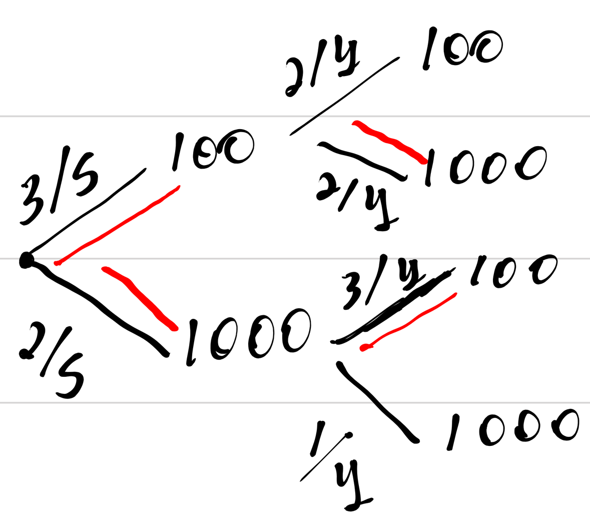
$R_1 = R_2 = R_3 = 100$

$R_4 = R_5 = 1000$

Eventos dependentes

a) $P(100 \text{ e } 100) = \frac{3}{5} \cdot \frac{2}{4} = \frac{6}{20} = 0,3$ ou 30%

b) $P(100 \text{ e } 1000) = \frac{3}{5} \cdot \frac{2}{4} + \frac{2}{5} \cdot \frac{3}{4} = \frac{6}{20} + \frac{6}{20} = \frac{12}{20} = \frac{3}{5}$



c) $\frac{3}{5} \cdot \frac{2}{4} = \frac{6}{20} = \frac{3}{10} = 0,3$ ou 30%

4) $P(A|B) = \frac{P(B \cap A)}{P(B)}$ (I)

$P(B|A) = \frac{P(A \cap B)}{P(A)} \longrightarrow P(A \cap B) = P(B|A) \cdot P(A)$ (II)

$P(A \cap B) = P(B \cap A)$, Substituindo (II) em (I)

$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$ (III)

Se $P(B|A) = P(A)$, Substituindo em (III):

$P(A) = \frac{P(B|A) \cdot P(A)}{P(B)} \therefore P(B|A) = P(B)$, c.q.d.

5) $P(ABC) = P(CAB) = P(C|AB) \cdot P(AB) =$

$P(C|AB) \cdot P(BA) =$

$P(BA) \cdot P(C|AB) =$

$P(B|A) \cdot P(A) \cdot P(C|AB) =$

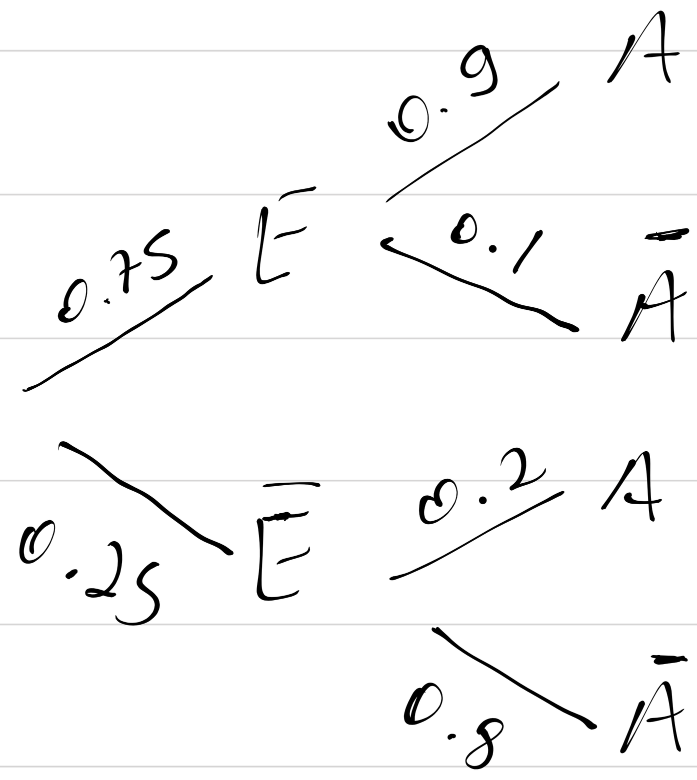
$P(A) \cdot P(B|A) \cdot P(C|AB)$ c.q.d.

Outra forma de provar que $P(ABC) = P(A)P(B|A)P(C|AB)$

$$P(ABC) = \cancel{P(A)} \cdot \frac{\cancel{P(AB)}}{\cancel{P(A)}} \; \frac{P(CAB)}{\cancel{P(AB)}}$$

$$P(ABC) = P(CAB)$$

6) E - estudado
A - Aprovado



$P(A|E) = 0.9$
$P(A|\bar{E}) = 0.2$
$P(E) = 0.75$

$$P(E|A) = P(E) \cdot \frac{P(A|E)}{P(A)}$$

$$P(A) = P(A|E)P(E) + P(A|\bar{E})P(\bar{E})$$

$$P(A) = 0{,}9 \cdot 0{,}75 + 0{,}2 \cdot 0{,}25$$

$$P(A) = 0.725$$

$$P(E|A) = \frac{0{,}75 \cdot 0{,}9}{0.725} = 0{,}931$$

93,1%