

Examples of Using *fmincon* in MATLAB

fmincon finds a minimum of a constrained nonlinear multivariable function, and by default is based on the SQP (Sequential Quadratic Programming) algorithm.

fmincon solves problems of the form:

$$\begin{array}{ll}
 \min_{\mathbf{x}} f(\mathbf{x}) \text{ subject to:} & \\
 \left. \begin{array}{l} \mathbf{A}^* \mathbf{x} \leq \mathbf{B} \\ \mathbf{Aeq}^* \mathbf{x} = \mathbf{Beq} \end{array} \right\} & \text{linear constraints} \\
 \left. \begin{array}{l} \mathbf{C}(\mathbf{x}) \leq 0 \\ \mathbf{Ceq}(\mathbf{x}) = 0 \end{array} \right\} & \text{(nonlinear constraints)} \\
 \mathbf{LB} \leq \mathbf{x} \leq \mathbf{UB} \rightarrow & \text{bounding of variables}
 \end{array}$$

Common Syntax:

$$\mathbf{x} = \mathbf{fmincon}(\mathbf{fun}, \mathbf{x0}, \mathbf{A}, \mathbf{b})$$

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$$[\mathbf{x}, \mathbf{fval}, \mathbf{exitflag}, \mathbf{output}, \mathbf{lambda}] = \mathbf{fmincon}(\mathbf{fun}, \mathbf{x0}, \mathbf{A}, \mathbf{b}, \mathbf{Aeq}, \mathbf{Beq}, \mathbf{lb}, \mathbf{ub}, \mathbf{nonlcon})$$

where $\mathbf{x0}$ is the initial guess for the minimum, **nonlcon** is an M-file function call for the nonlinear constraints and defines $\mathbf{C}(\mathbf{x})$ and $\mathbf{Ceq}(\mathbf{x})$. Note that if any argument is not present, it should be set to a null set []. If **lb** and **ub** are not known, they should be set to null sets as well. **Lambda** is the Lagrange multipliers.

Example 1: Minimize $f(\mathbf{x}) = -x_1 x_2 x_3$
s.t. $0 \leq x_1 + 2x_2 + 2x_3 \leq 72$

$$\mathbf{x0} = [10 \ 10 \ 10]^T$$

Rewrite the linear inequality constraints as:

$$-x_1 - 2x_2 - 2x_3 \leq 0 \quad \text{and} \quad x_1 + 2x_2 + 2x_3 \leq 72$$

Write an M-file for $\mathbf{f}(\mathbf{x})$:

```
function f = myfun(x)
f = -x(1)*x(2)*x(3);
```

The MATLAB input is:

```
>> A = [-1 -2 -2; 1 2 2];
>> b = [0; 72];
>> x0 = [10; 10; 10];
>> [x, fval] = fmincon(@myfun, x0, A, b)
```

x =

```
24.0000 12.0000 12.0000
```

fval =

```
-3.456e+03
```

Example 2: Minimize $\mathbf{f}(\mathbf{x}) = e^{x_1}[4x_1^2 + 2x_2^2 + 4x_1x_2 + 2x_2 + 1]$
s.t. $1.5 + x_1x_2 - x_1 - x_2 \leq 0$
 $-x_1x_2 \leq 10$

$$\mathbf{x}_0 = [-1 \ -1]^T$$

Write an M-file for the objective function:

```
function f = objfun(x)
f = exp(x(1))*(4*x(1)^2+2*x(2)^2+4*x(1)*x(2)+2*x(2)+1);
```

Write an M-file for the nonlinear constraints:

```
function [c, ceq] = confun(x)
c = [1.5+x(1)*x(2) -x(1) -x(2); -x(1)*x(2) -10];
ceq = [];
```

The MATLAB input is:

```
>> x0 = [-1; -1];
>> [x, fval] = fmincon(@objfun, x0, [], [], [], [], [], @confun)
```

x =

-9.5474 1.0474

fval =

0.0236