



MAC420/5744: Introdução à Computação Gráfica

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Aula #18: Superfícies de subdivisão

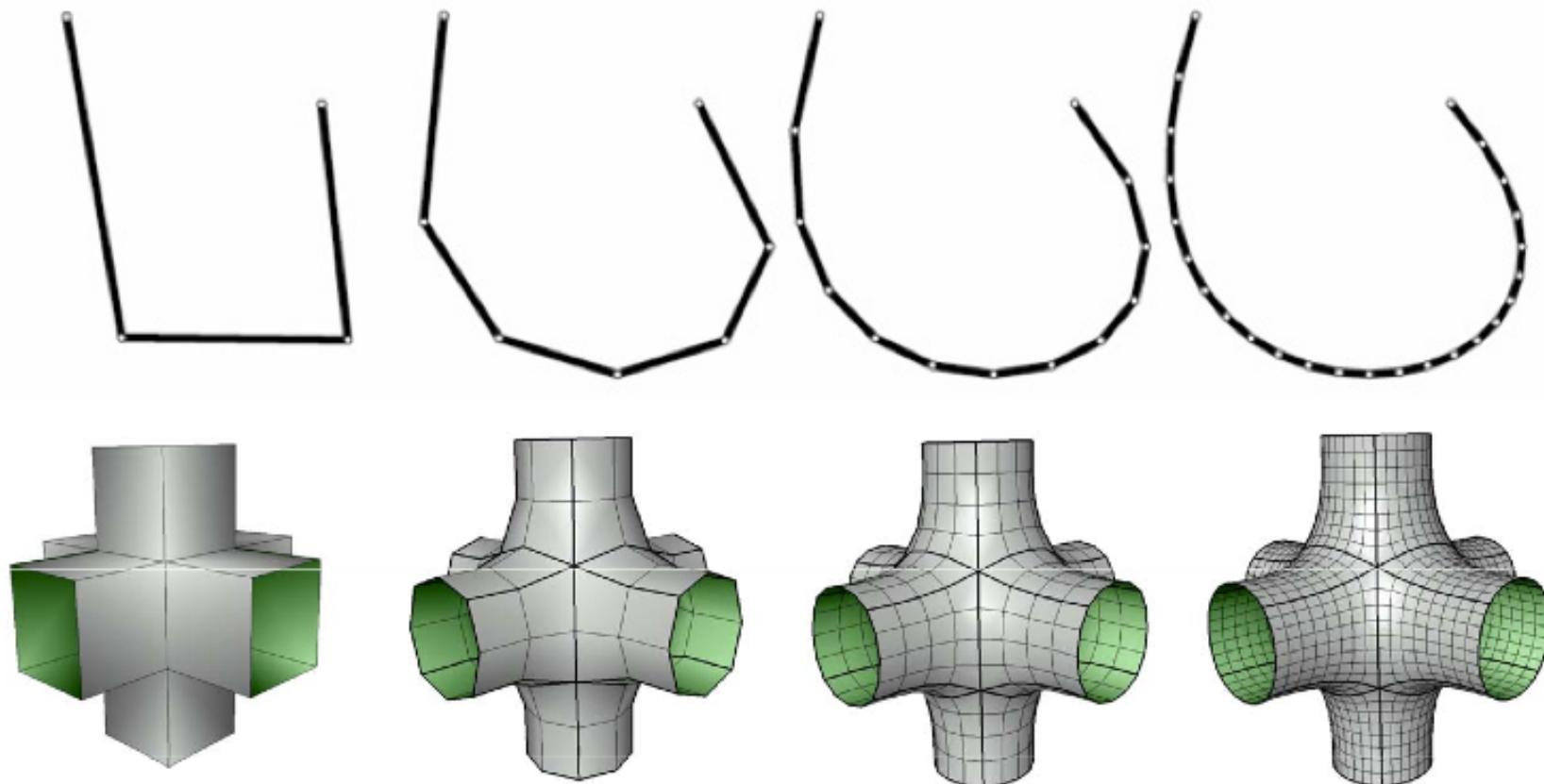
Curvas de subdivisão

- How do you make a smooth curve?



We want to “smooth out” severe angles

Superfícies de subdivisão



Superfícies de subdivisão

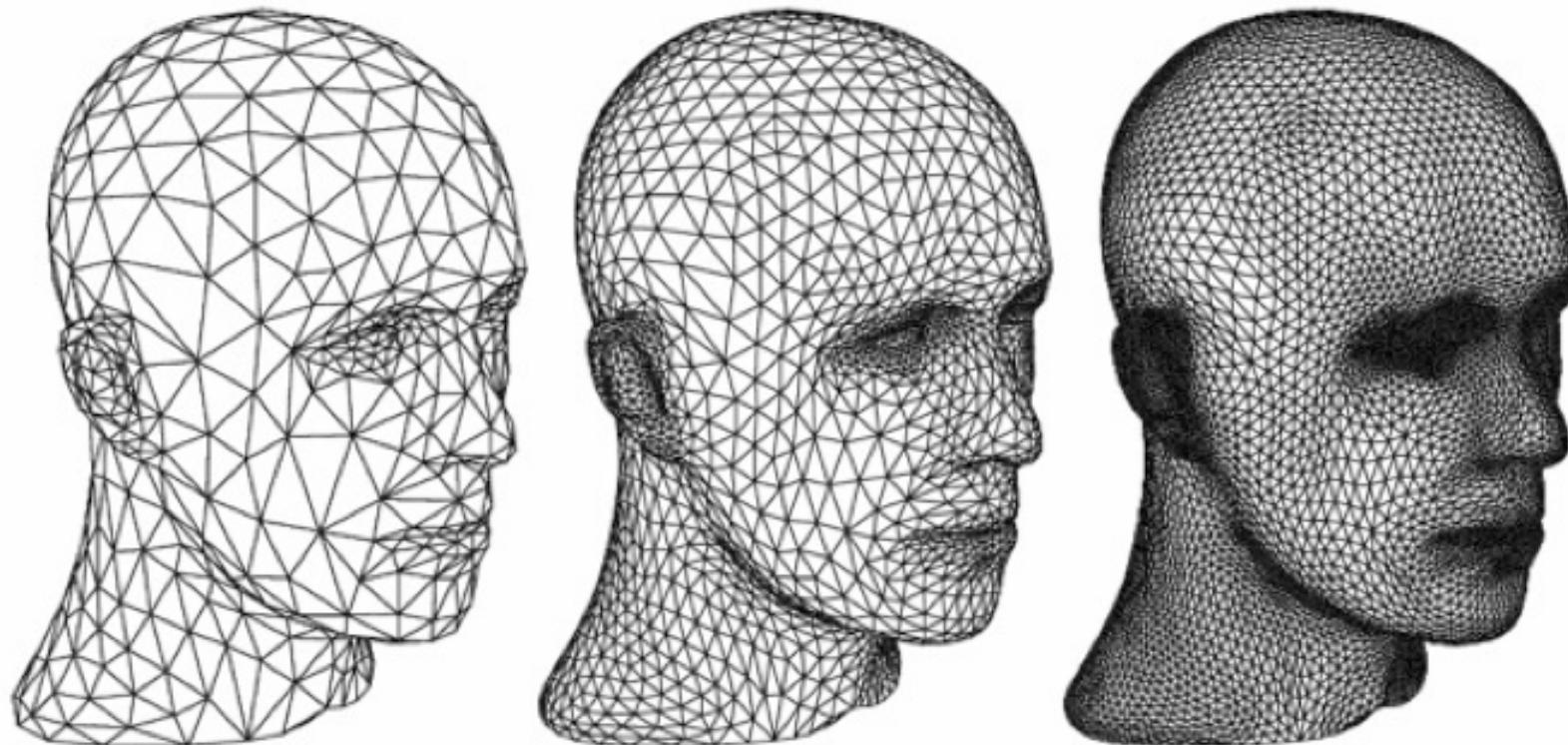
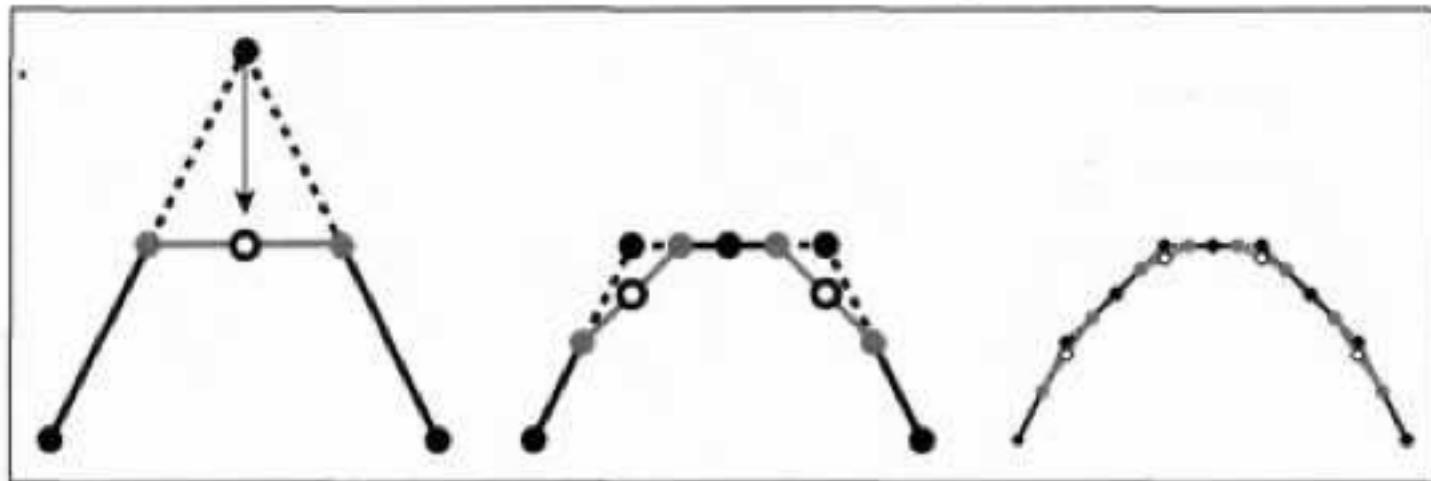


Figure 2.2: Example of subdivision for a surface, showing 3 successive levels of refinement. On the left an initial triangular mesh approximating the surface. Each triangle is split into 4 according to a particular subdivision rule (middle). On the right the mesh is subdivided in this fashion once again.

Exemplo

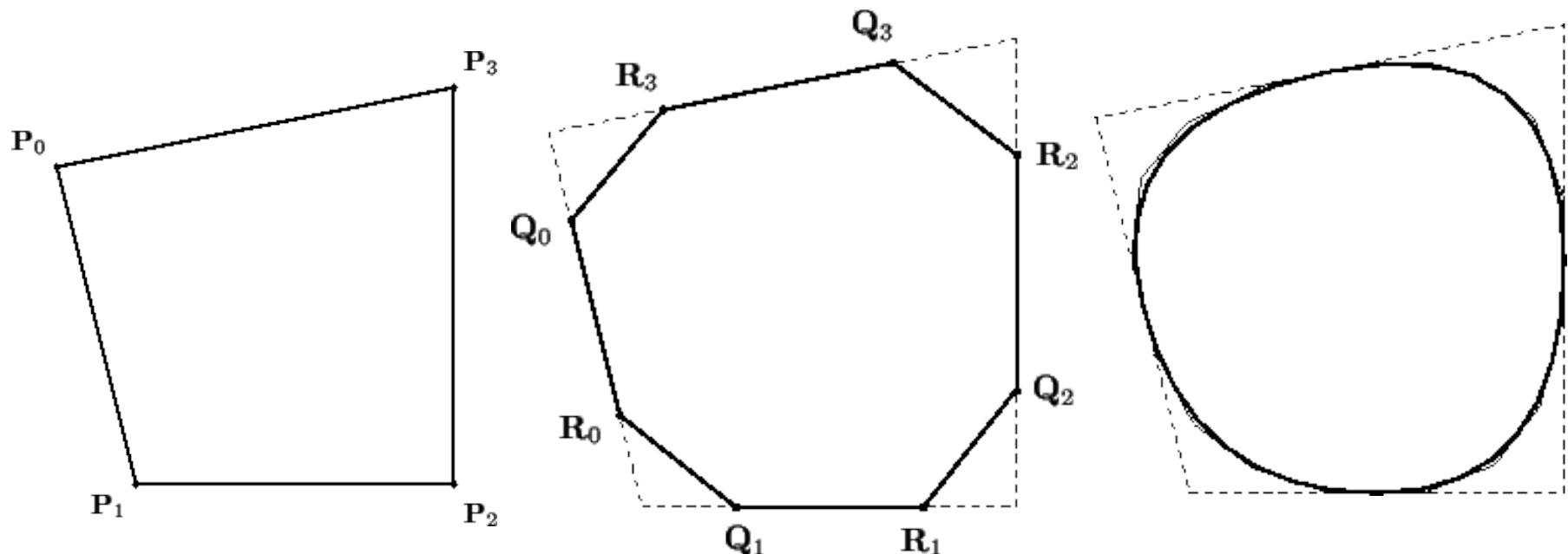


$$\mathbf{p}'_2 = \frac{1}{2}(\frac{1}{2}\mathbf{p}_0 + \frac{1}{2}\mathbf{p}_1) + \frac{1}{2}(\frac{1}{2}\mathbf{p}_1 + \frac{1}{2}\mathbf{p}_2).$$

$$\mathbf{B}_2(u) = (1-u)^2\mathbf{p}_0 + 2u(1-u)\mathbf{p}_1 + u^2\mathbf{p}_2.$$

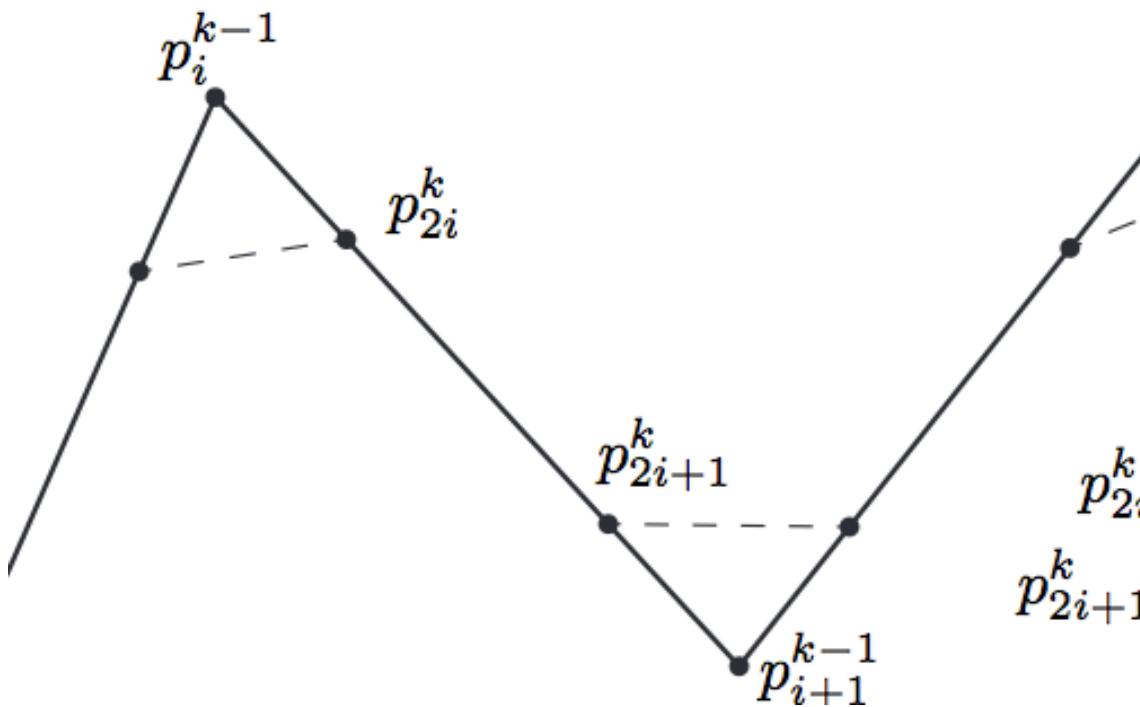
Chaiken

- Publicado em 1974
 - “An algorithm for high speed curve generation”



Recorte de arestas

- New points are linear combinations of old ones
- Different treatment for odd-numbered and even-numbered points.



$$p_{2i}^k = (3p_i^{k-1} + p_{i+1}^{k-1})/4$$

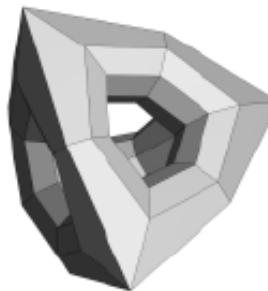
$$p_{2i+1}^k = (p_i^{k-1} + 3p_{i+1}^{k-1})/4$$

Superfícies de subdivisão

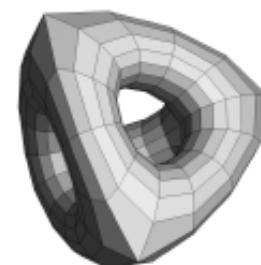
- Coarse mesh & subdivision rule
 - Define smooth surface as limit of sequence of refinements



(a)



(b)



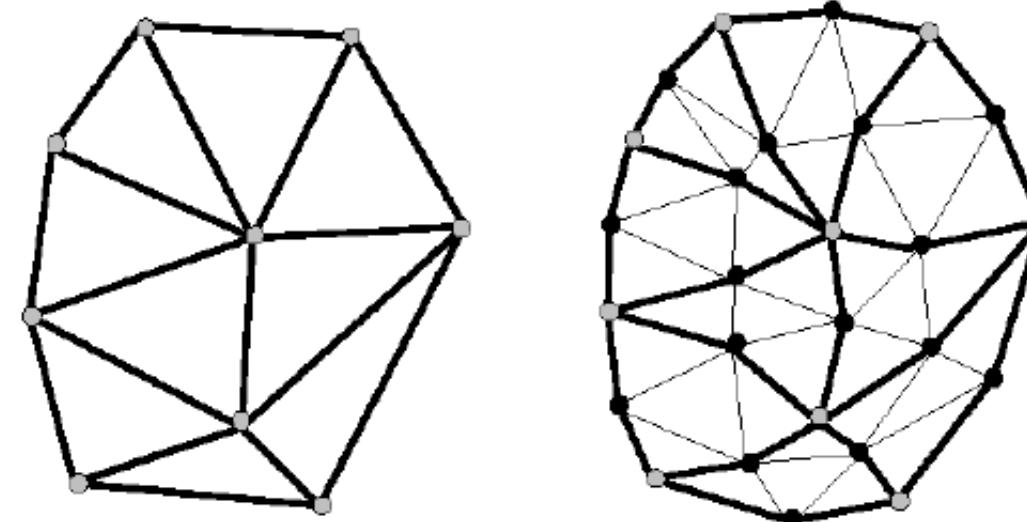
(c)



(d)

Critério de subdivisão

- How to subdivide the mesh?
 - Aim for properties like smoothness
- How to store the mesh?
 - Aim for efficiency of implementing subdivision rules



Vantagens

- Facilidade na criação de geometrias complexas
- Renderização eficiente
- Facilidade no desenvolvimento de ferramentas de animação



O filme *A Bug's Life (Vida de Inseto)* foi a primeira animação a utilizar técnicas de subdivisão

Desvantagens

- Dificuldade na especificação da precisão desejada para o refinamento
- Análise da suavidade se torna complicada na elaboração de novas metodologias de subdivisão
- Ausência de parametrização
 - Dificuldade em avaliar pontos específicos das curvas ou superfícies resultantes.

Esquema geral de subdivisão

- How to subdivide the mesh?

Two parts:

» Refinement:

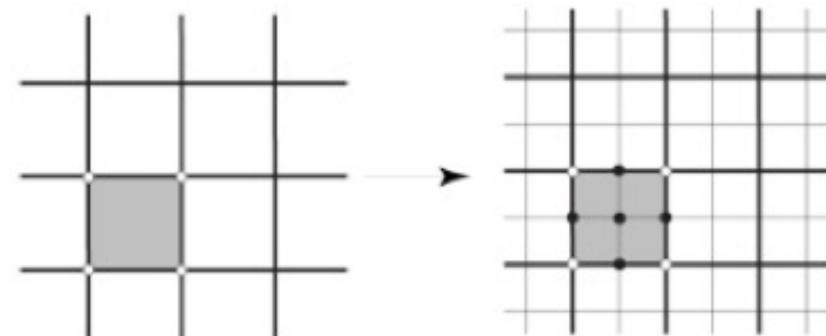
- Add new vertices and connect (topological)

» Smoothing:

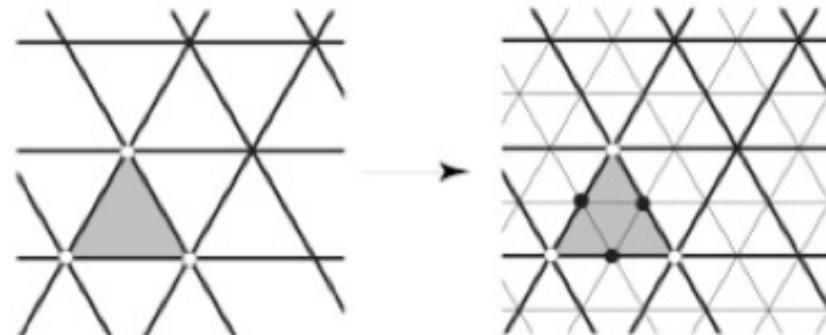
- Move vertex positions (geometric)

Esquemas de subdivisão

- **Quadrilaterals**
 - Catmull-Clark 1978
- **Triangles**
 - Loop 1987



Face split for quads



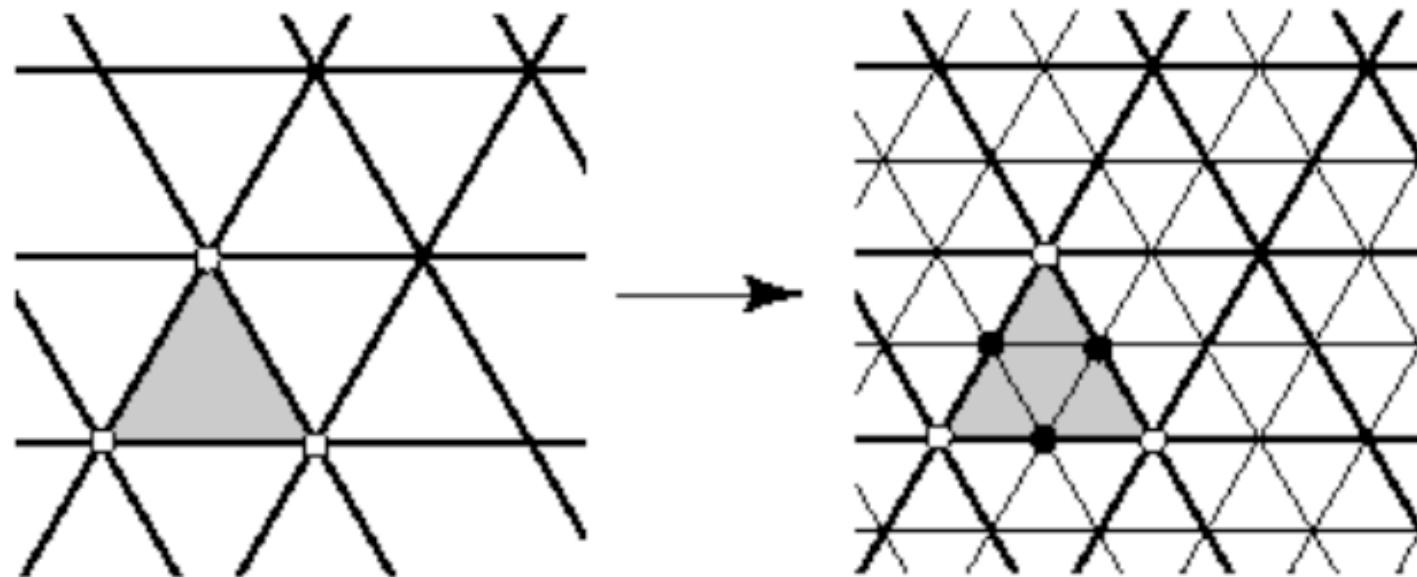
Face split for triangles

Esquemas de subdivisão

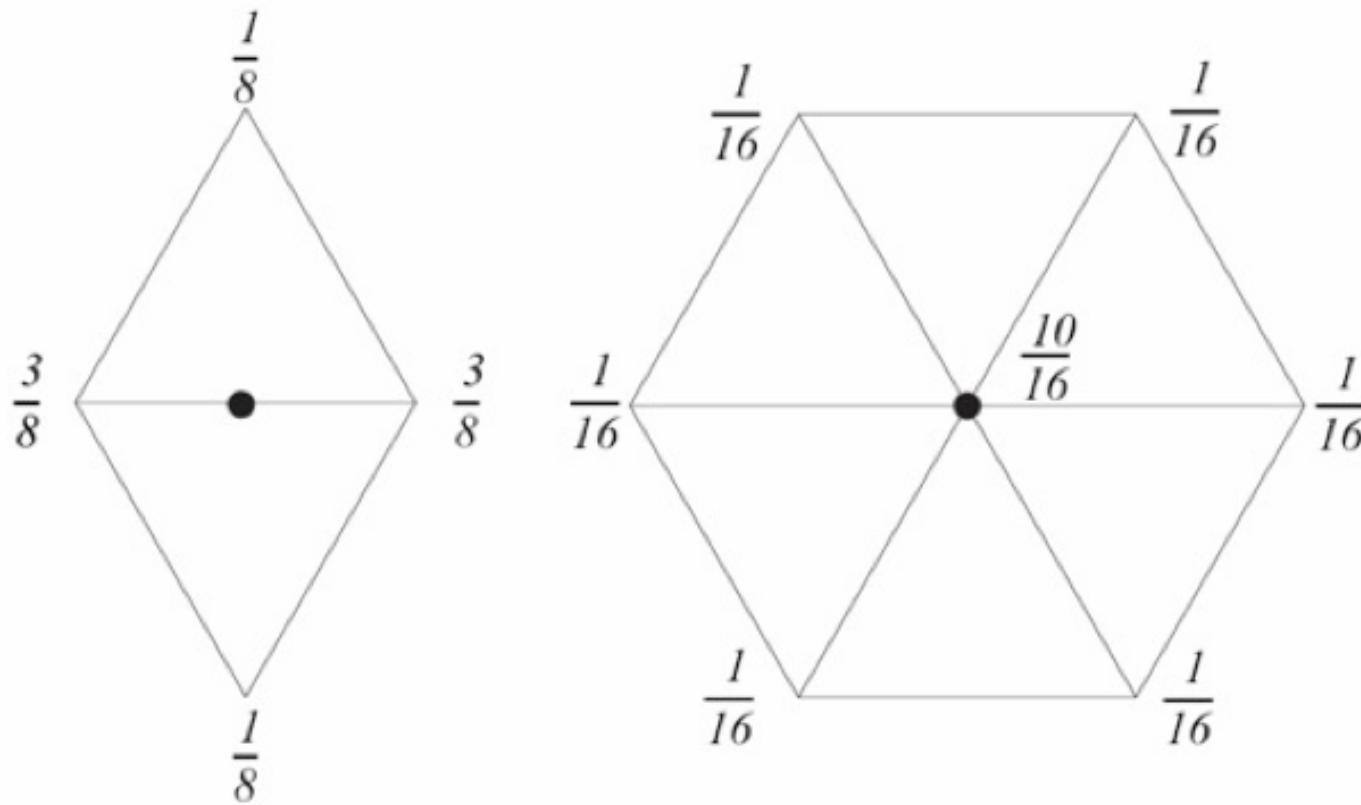
- How to subdivide the mesh?

Refinement:

- » Subdivide each triangle into 4 triangles by splitting each edge and connecting new vertices

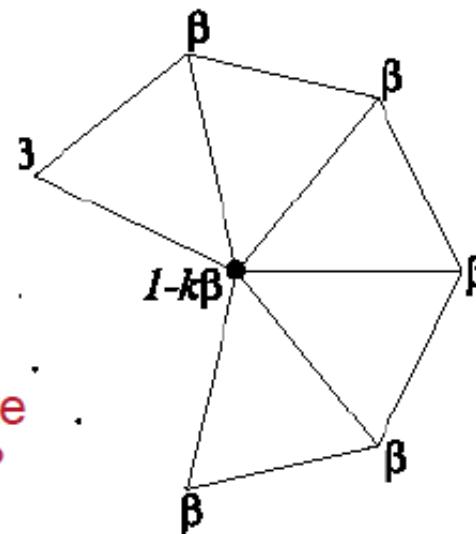


Esquema de subdivisão de Loop



Esquema de subdivisão de Loop

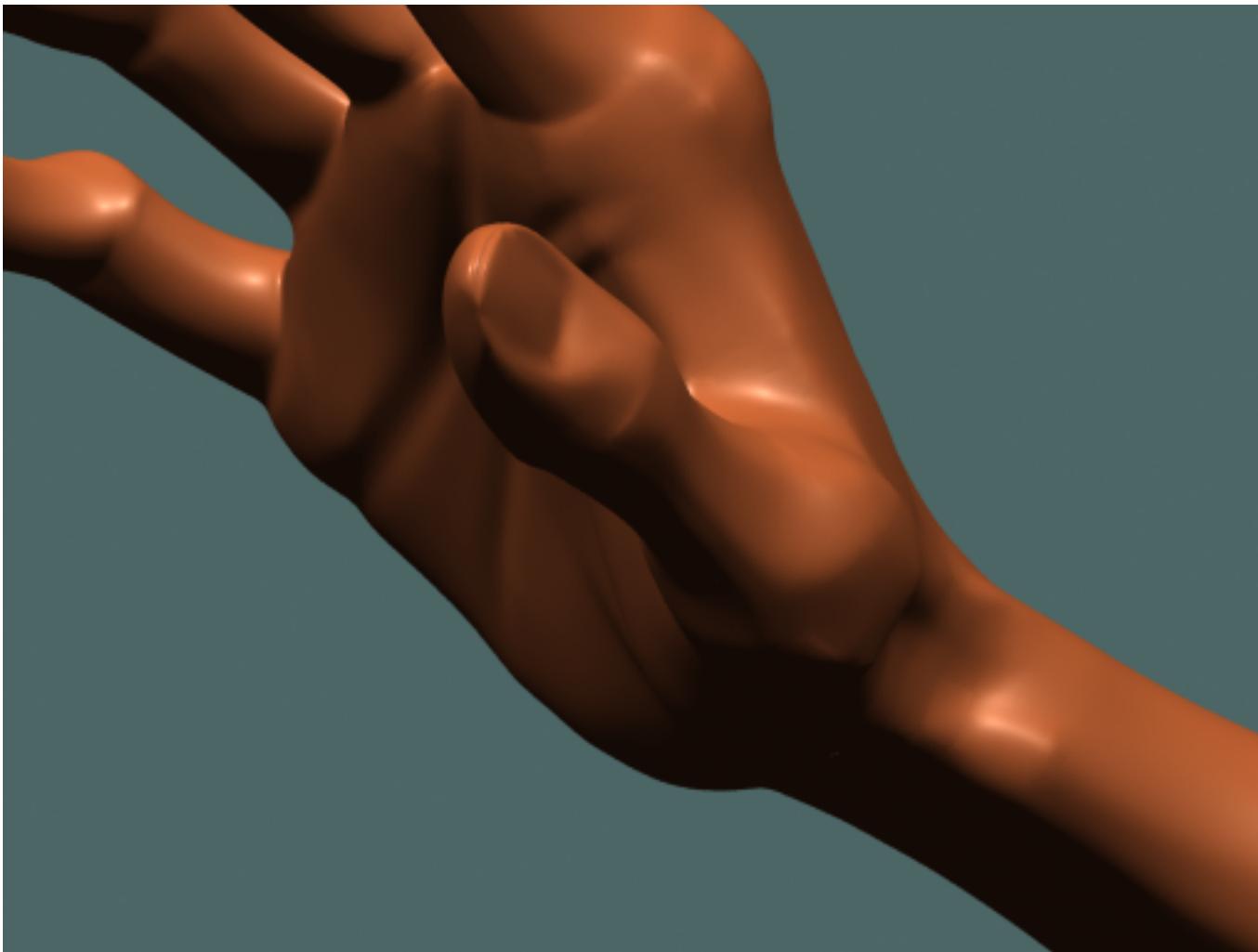
- General rule for moving existing *interior vertices*:



What about vertices that have more
Or less than 6 neighboring faces?

New_position = $(1 - k\beta)$ original_position + sum($\beta * \text{each_original_vertex}$)

Exemplo



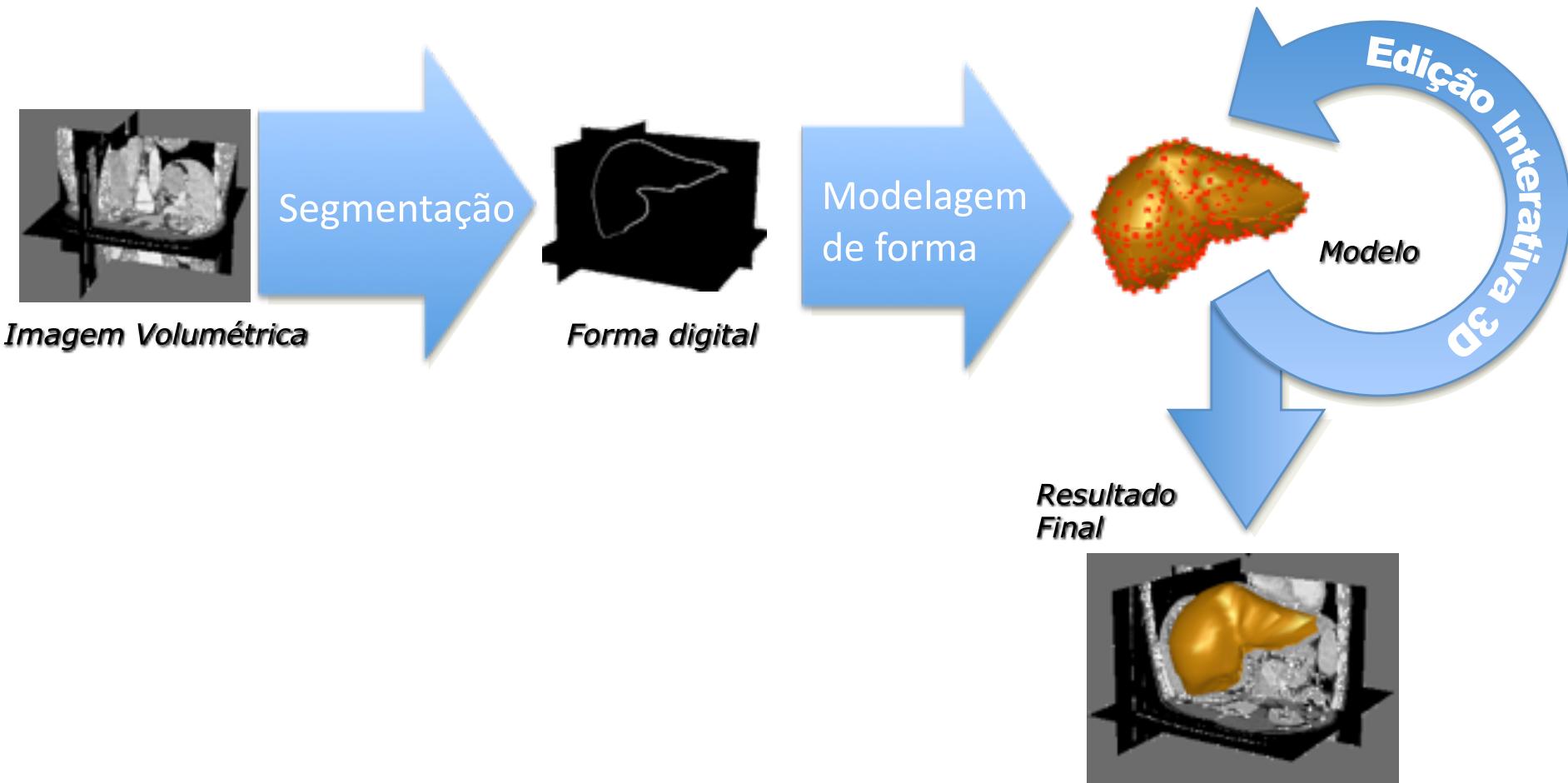
Geri's game

- Pixar short film to test subdivision in production
 - Catmull-Clark (quad mesh) surfaces
 - complex geometry
 - extensive use of creases
 - subdivision surfaces to support cloth dynamics

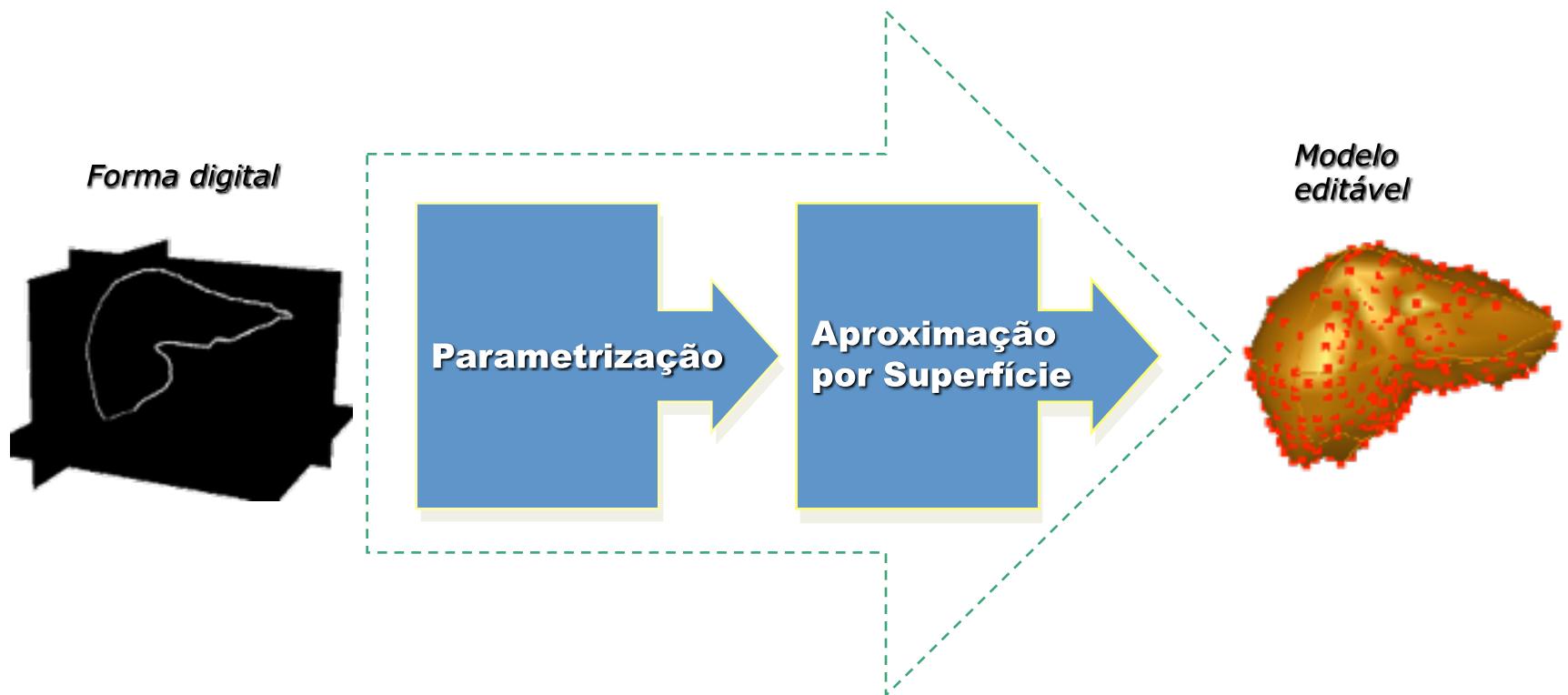


[DeRose et al. SIGGRAPH 1998]

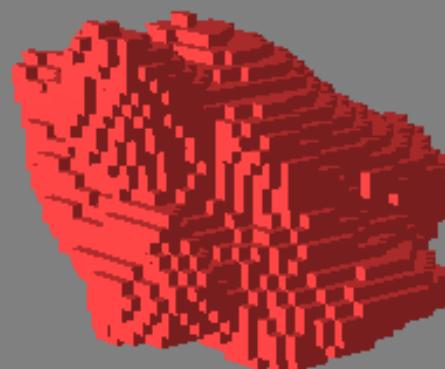
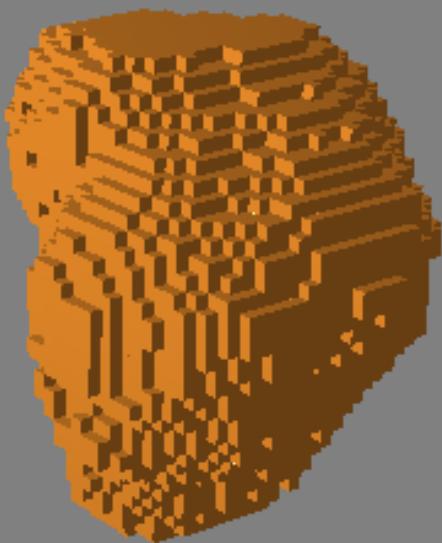
Exemplo de aplicação



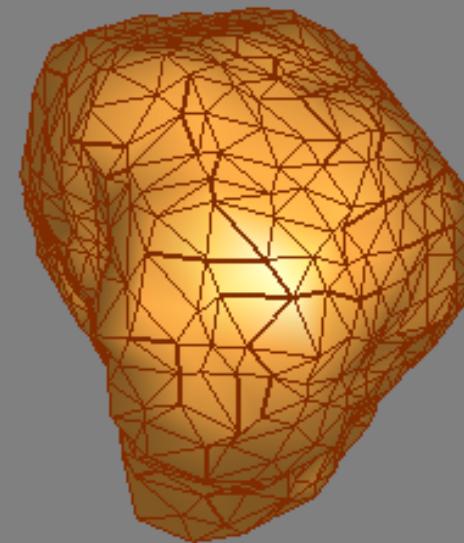
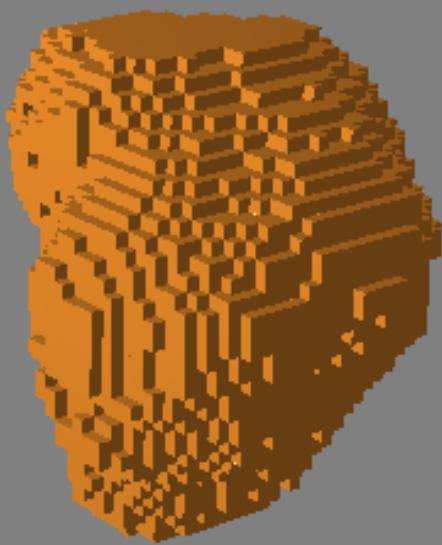
Modelagem de forma



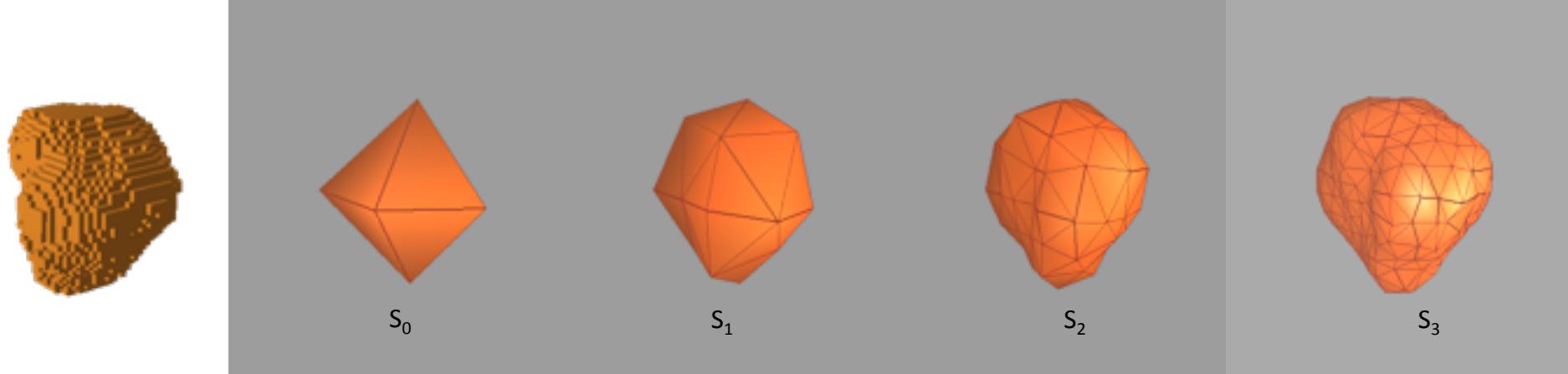
Forma digital



Malha triangular



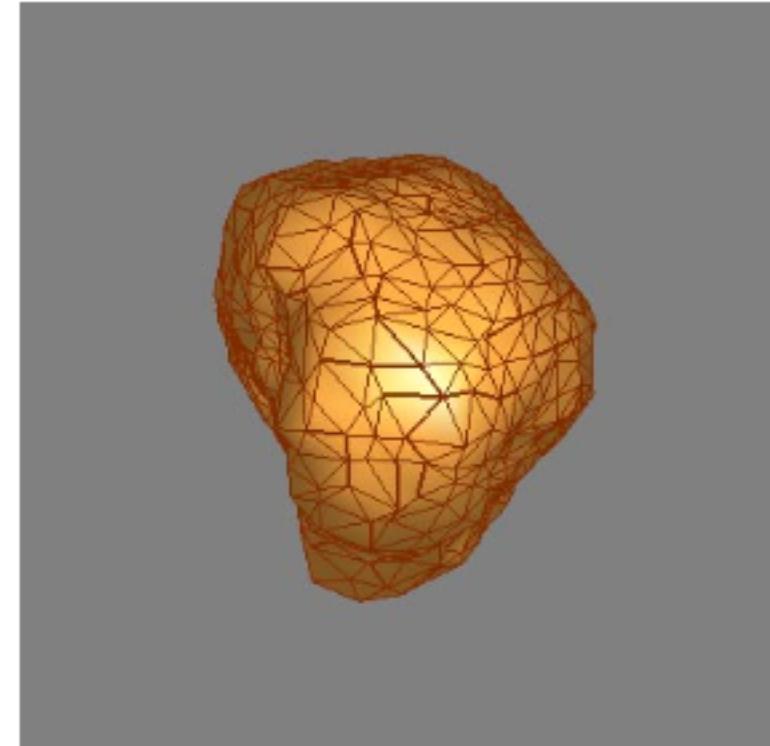
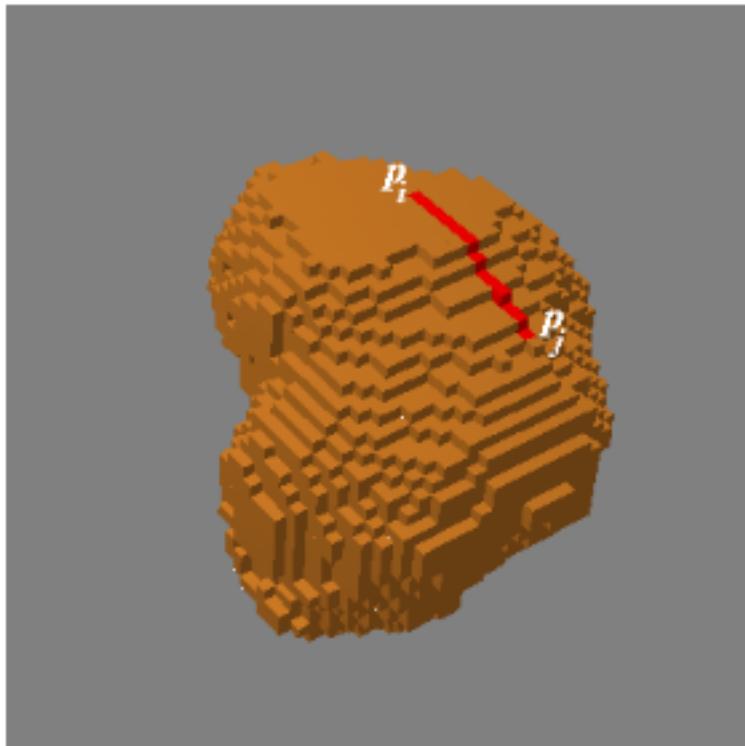
Método de subdivisão



Sucessivas subdivisões de uma forma digital

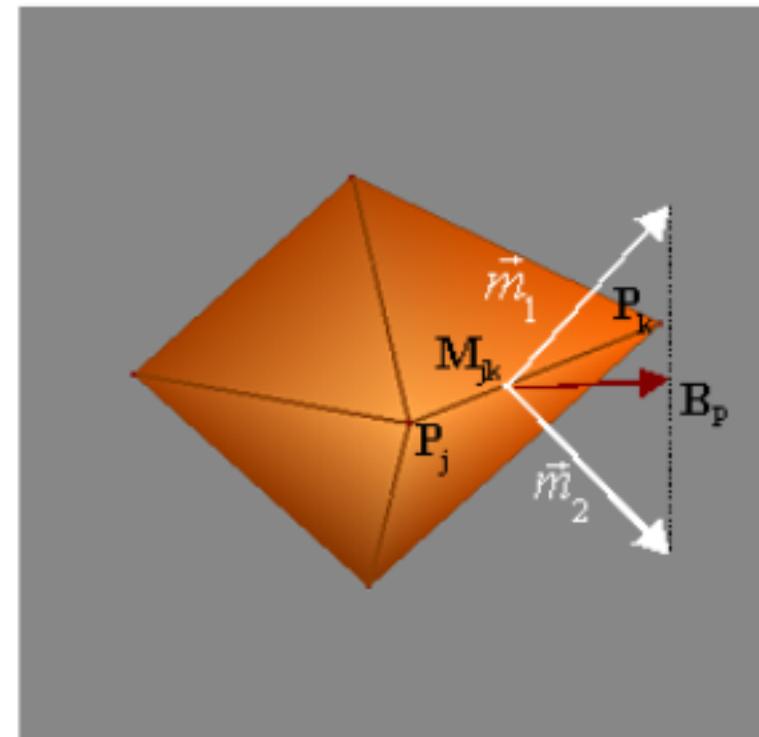
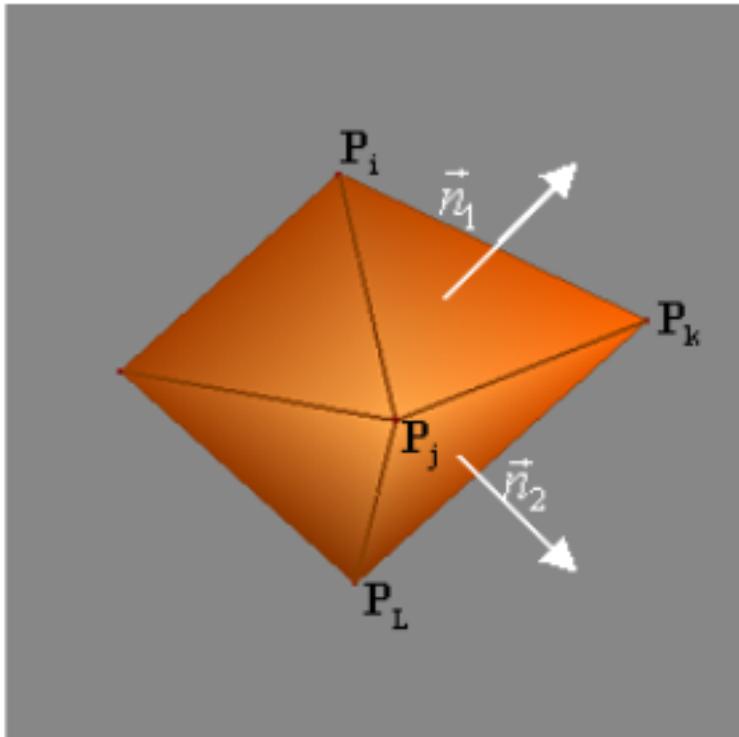
Caminho digital

- Um caminho digital é um conjunto conexo de pontos com início em p_i e fim em p_j , onde nenhum ponto é repetido e cada ponto possui somente dois vizinhos.

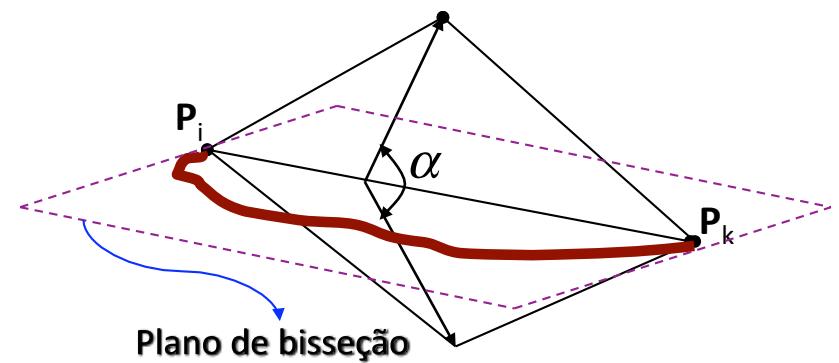
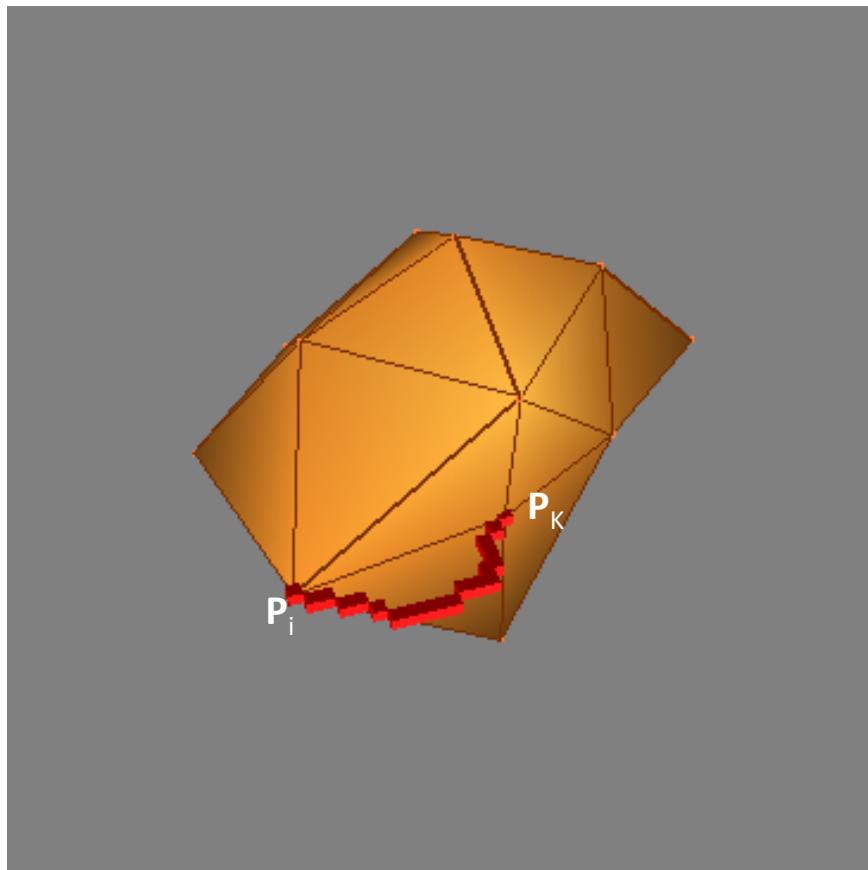


Plano bisetor

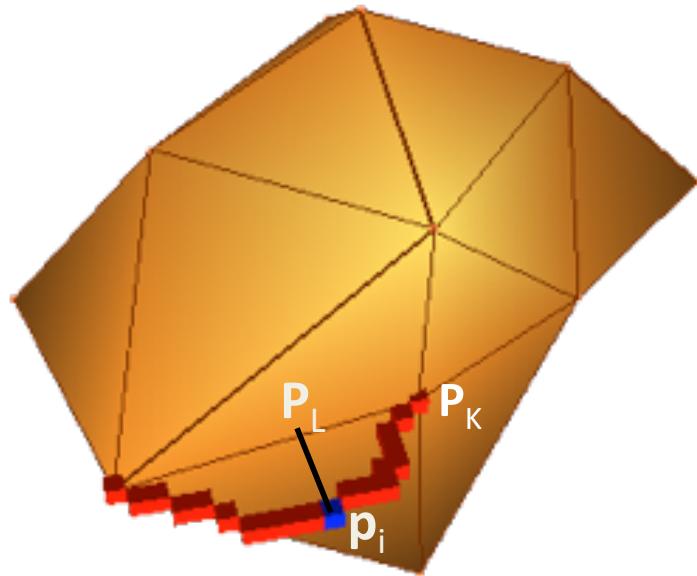
- O ponto de bisseção \mathbf{B}_p juntamente com os vértices \mathbf{P}_j e \mathbf{P}_k determinam o plano de bisseção.



Contorno de aresta



Distância da aresta

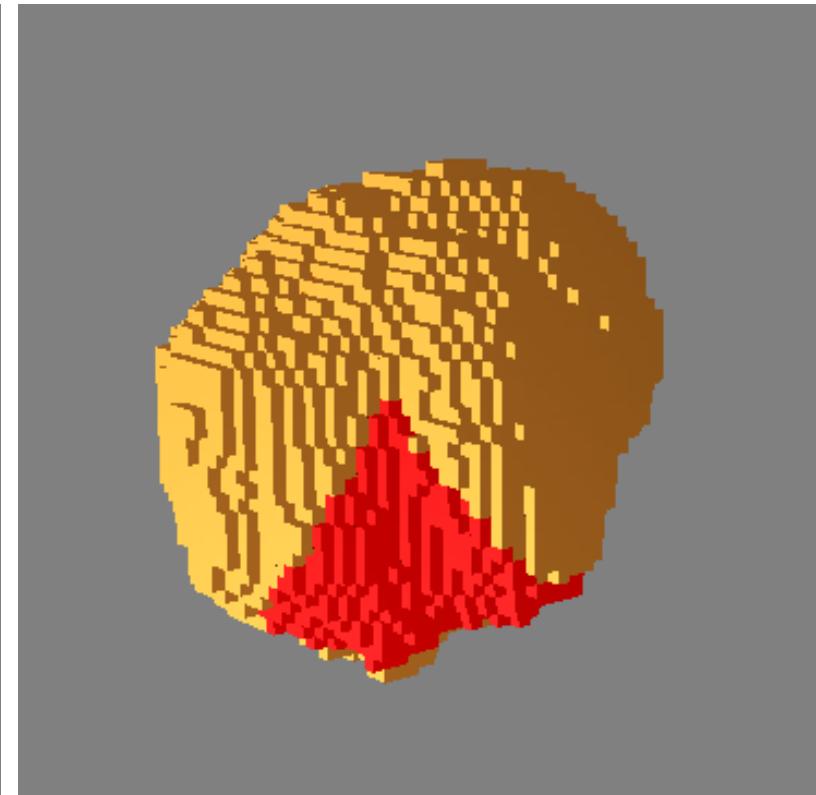
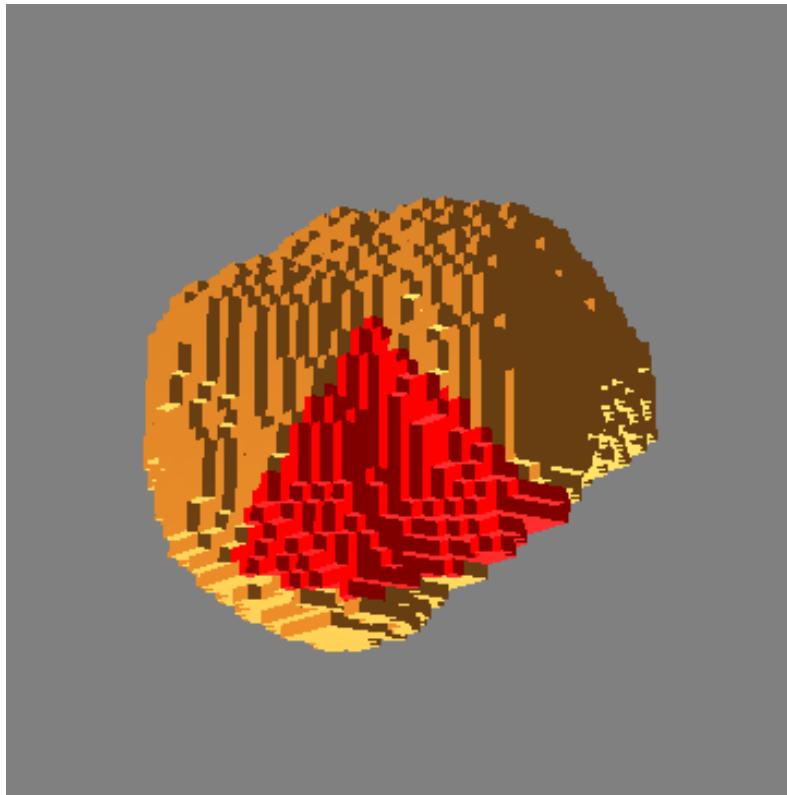


$$d_i = \|p_i - P_L\|$$

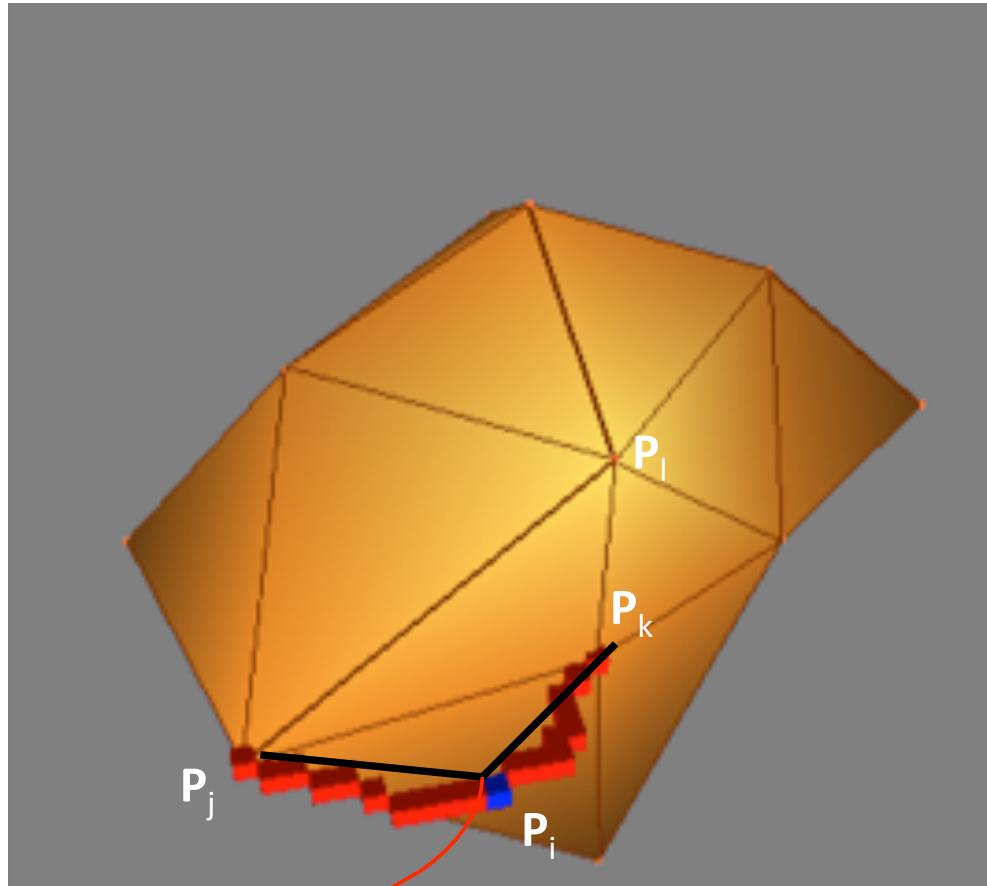
$$D_e = \max_i \{d_i\}$$

“Patch” triangular

- Um retalho triangular é um conjunto conexo de pontos delimitados por três contornos digitais.

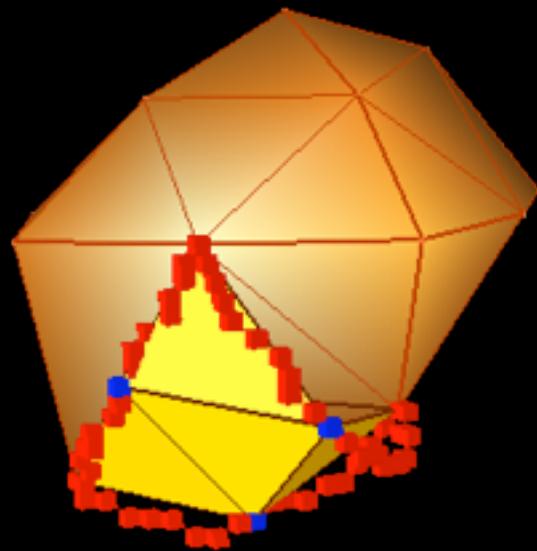


Subdivisão de arestas

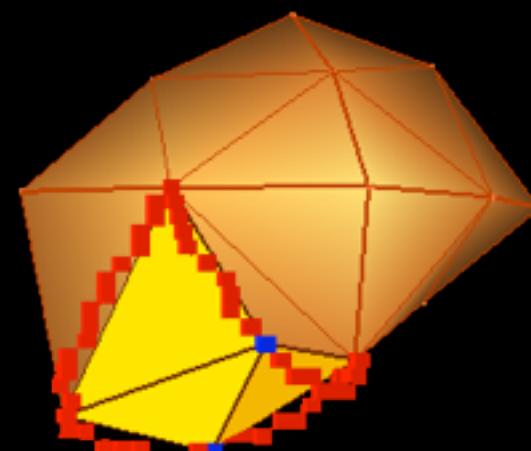


Contorno de aresta

Subdivisão triangular

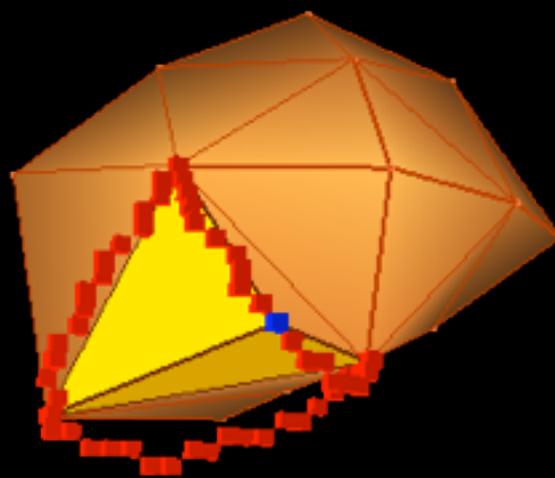


Distâncias entre as três arestas são maiores que o valor de tolerância.

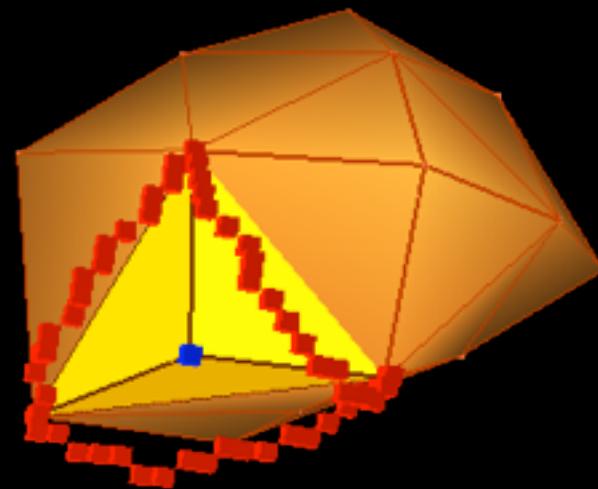


Distâncias entre as duas arestas são maiores que o valor de tolerância.

Subdivisão triangular

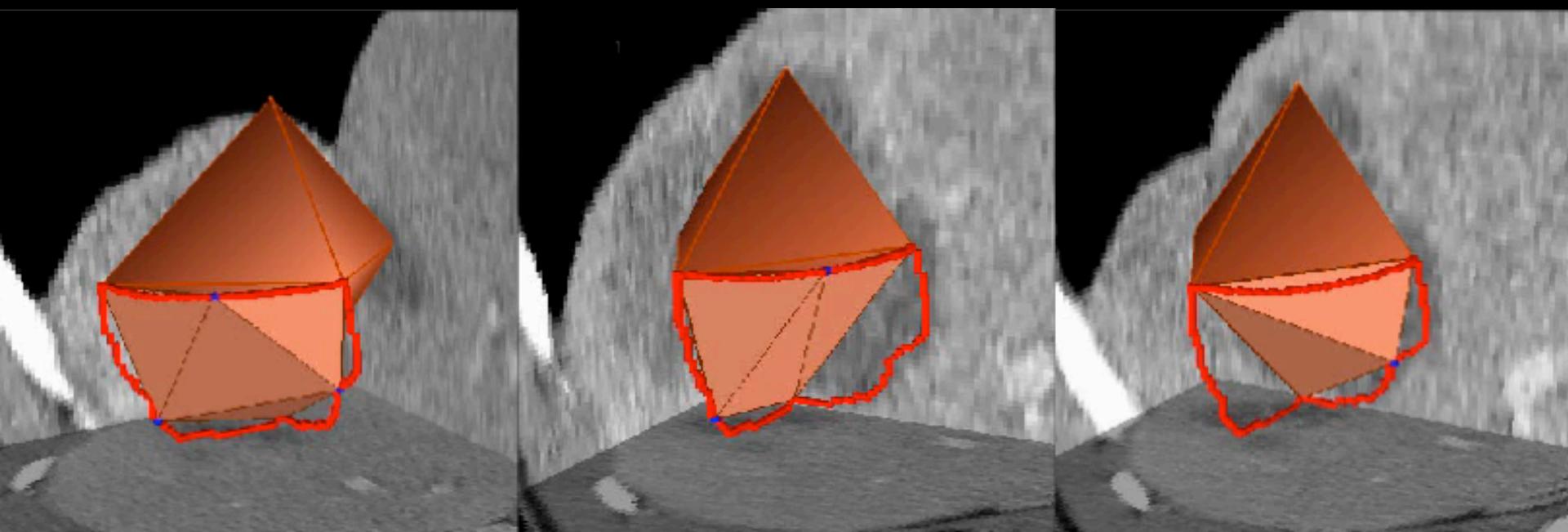


Distâncias entre uma das arestas
s o maiores que o valor de
toler ncia.



Distâncias entre todas as tr s
arestas s o menores que a
toler ncia, por m a dist ncia entre
o retalho e o tri ngulo   maiors.

Exemplo de subdivisão



4 novas faces

3 novas faces

2 novas faces

Algoritmo

- **Inicialização:**

- Determinar os eixos principais da forma e inicializar o modelo inicial através de um octaedro.

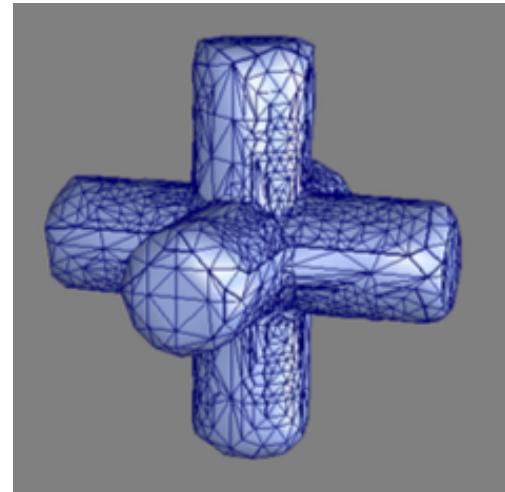
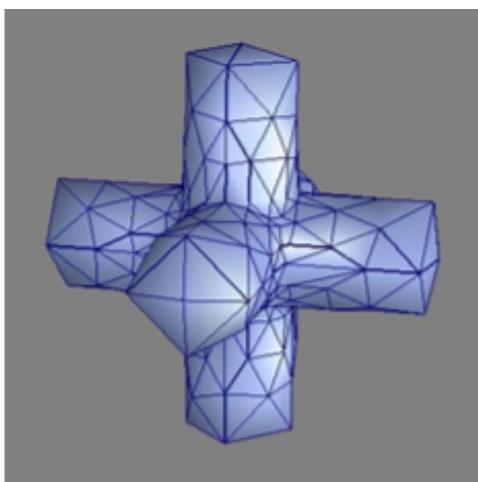
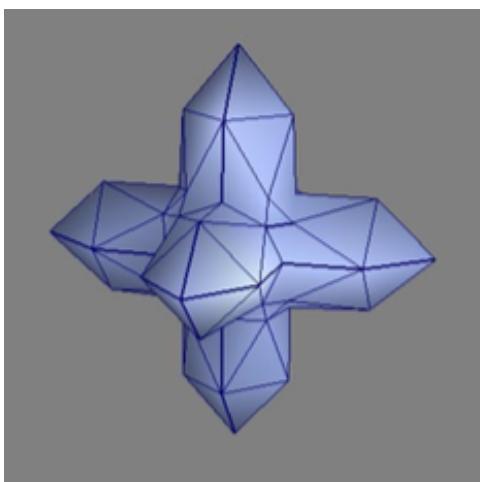
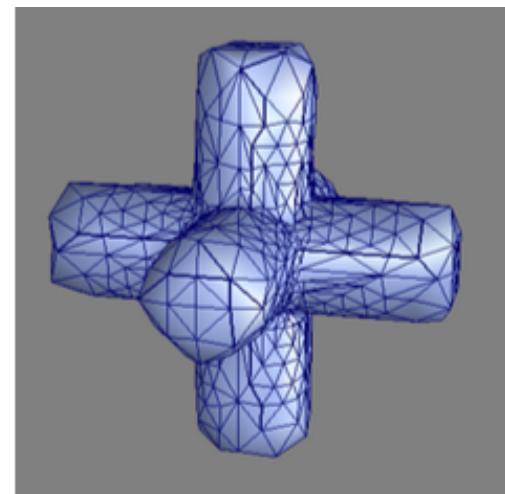
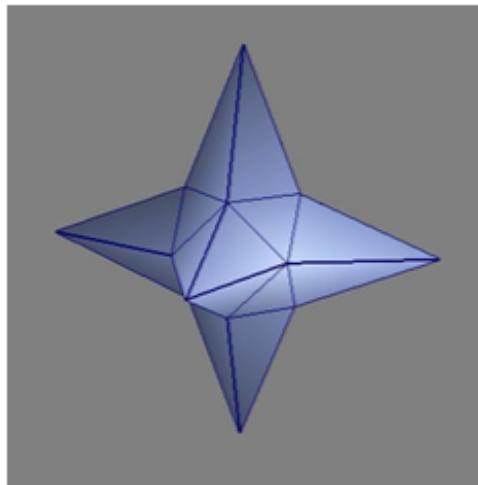
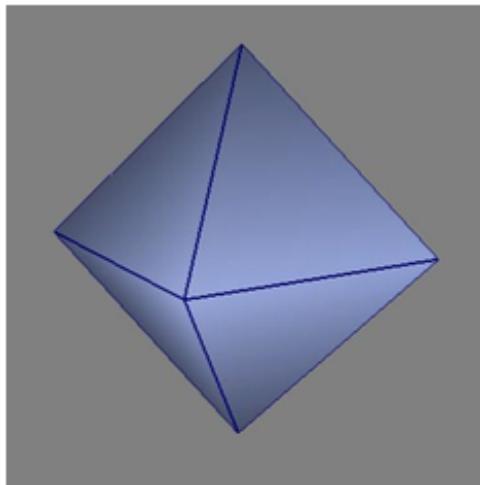
- **Passo principal:**

- Remova um triângulo da lista. Se a distância entre um triângulo e o retalho (“patch”) correspondente é maior que a tolerância, subdivida e insira novos triângulos na lista.

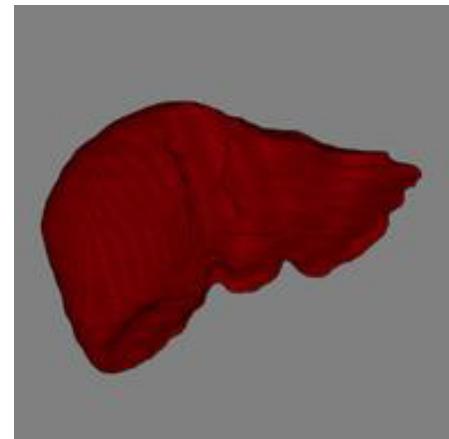
- **Critério de parada:**

- Se a lista estiver vazia, termine a subdivisão; caso contrário retorne ao passo anterior.

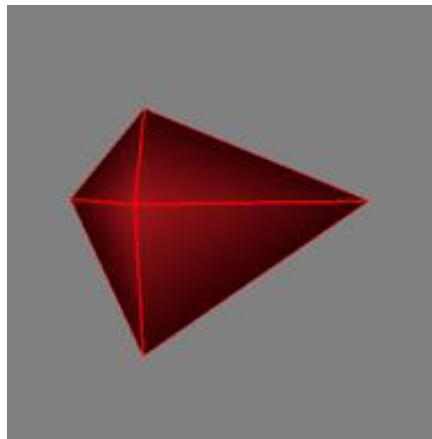
Exemplo



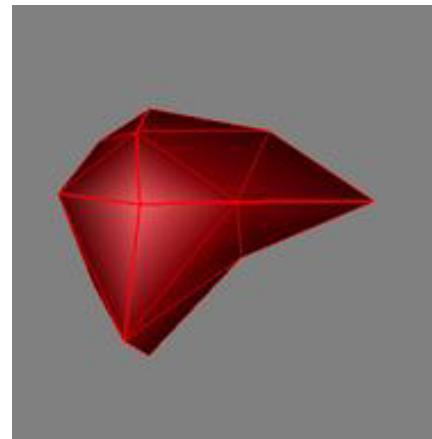
Exemplos



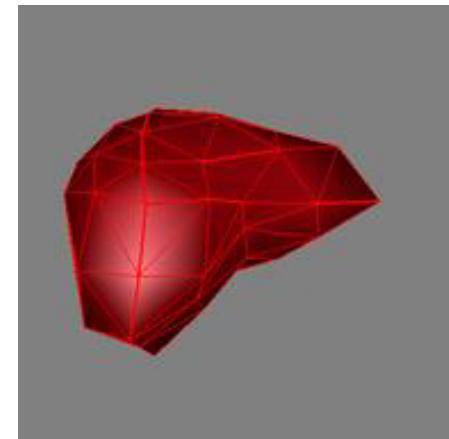
Fígado humano em forma digital (53.268 voxels).



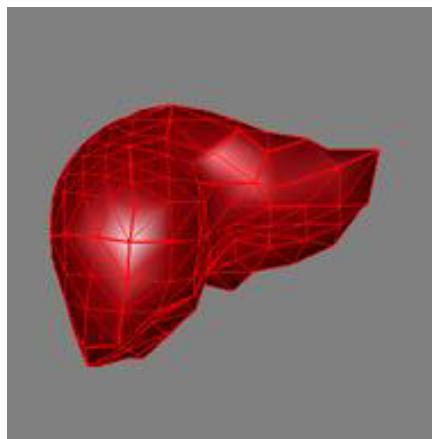
(a) $E=17.55$



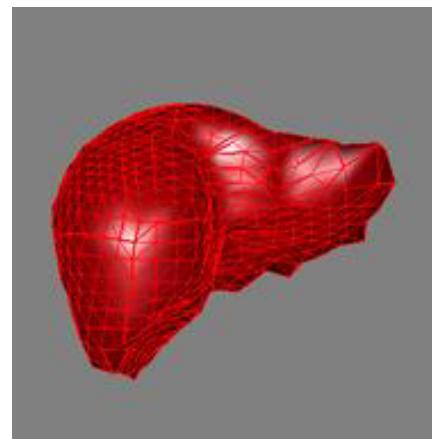
(b) $E=8.16$



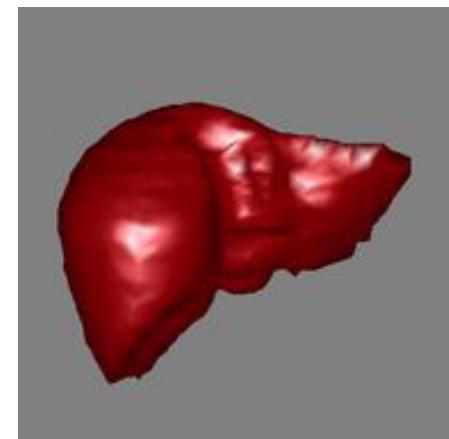
(c) $E=3.79$



(d) $E=1.95$



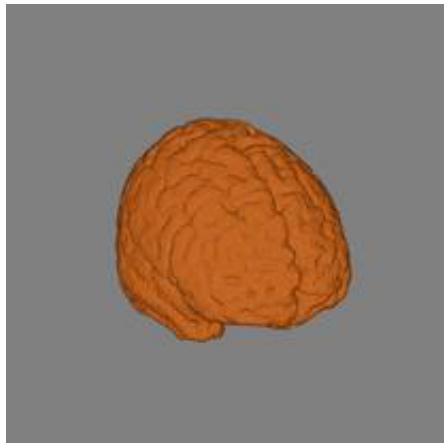
(e) $E=1.20$



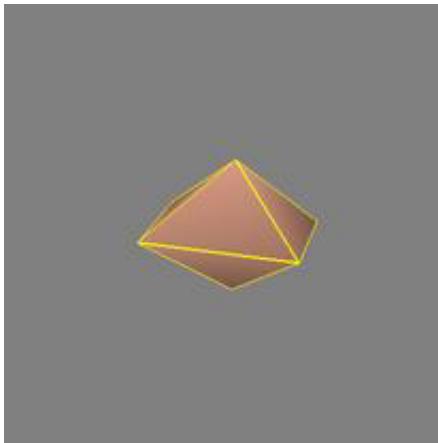
(f) $E=0.83$

Aproximação final contém 4.098 vértices e 8.192 faces triangulares. (Courtesy University of Iowa, Dr. Sonka).

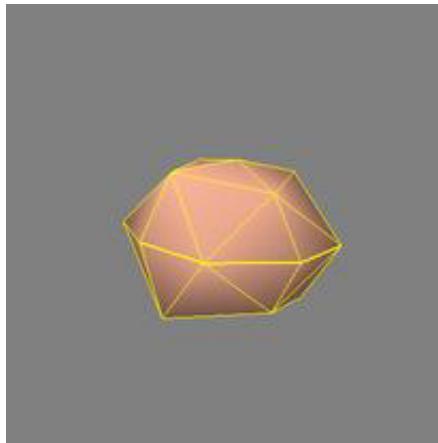
Exemplos



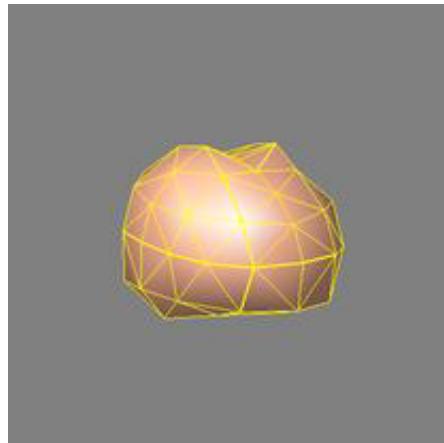
Cérebro humano em forma digital (83.815 voxels).



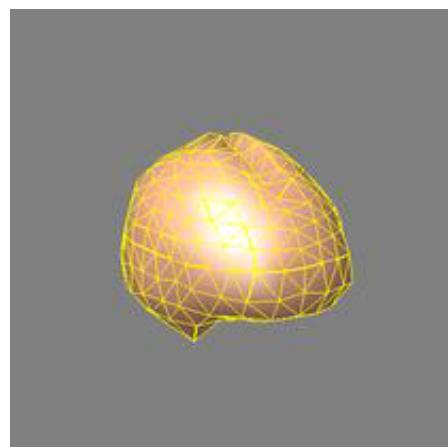
(a) $E=24.35$



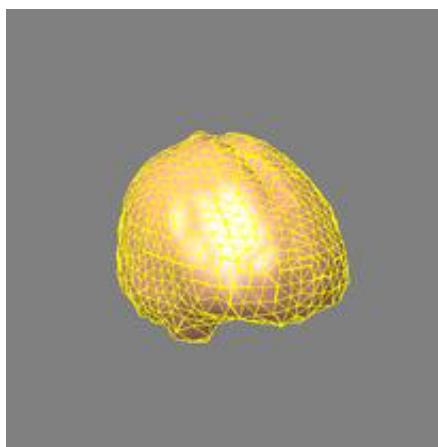
(b) $E=9.75$



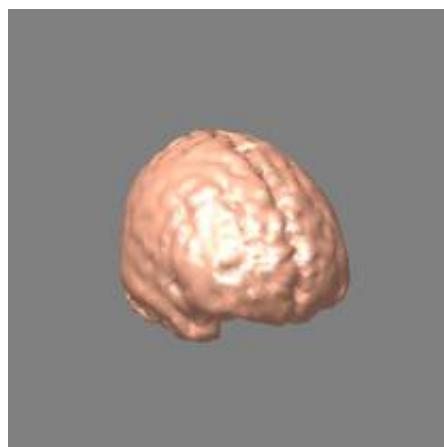
(c) $E=4.28$



(d) $E=1.88$



(e) $E=1.06$



Aproximação final contém 5.828 vértices e 11.652 faces triangulares. (Courtesia da Kettering Medical Center, Dr. Martin Satter)

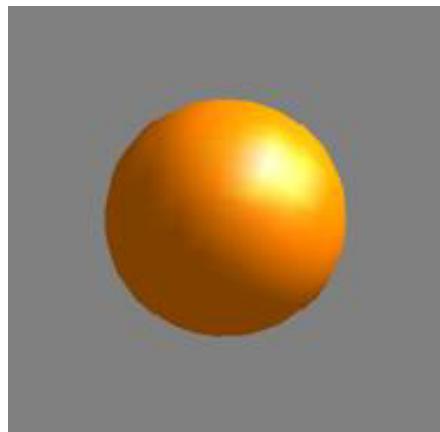
Propriedades

- Uma única subdivisão é obtida independente da orientação ou posição da forma.
- Taxa de compressão depende da complexidade da forma em questão.
- O processo automaticamente ajusta os tamanhos dos triângulos para reproduzir detalhes da forma.

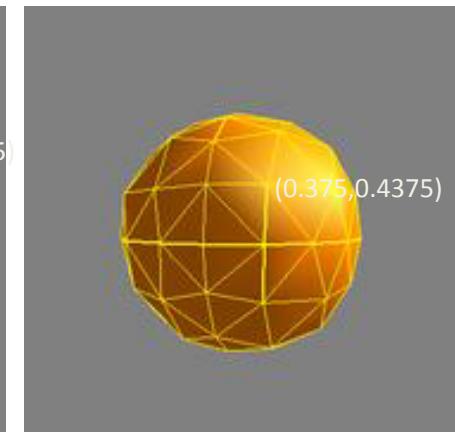
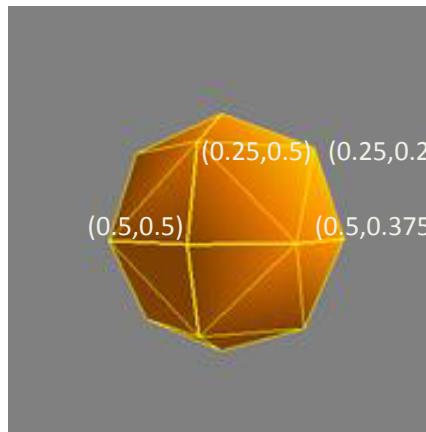
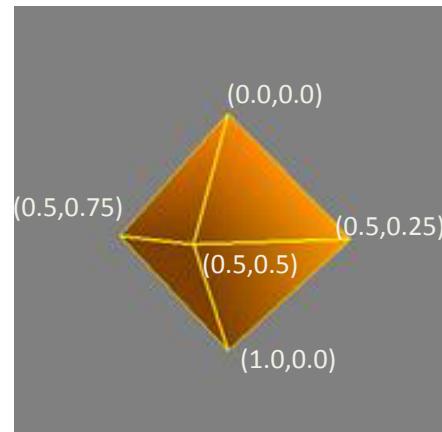
E a parametrização ?

- Para parametrizar vértices da malha, parâmetros do octaedro aproximando a forma são inicialmente determinados.
- A medida que a malha que aproxima a forma é subdividida, cada triângulo correspondente na esfera também é, e parâmetros nos novos pontos serão conhecidos.
- Assim, o processo de subdivisão facilita o cálculo dos parâmetros dos vértices da malha triangular que aproxima a forma digital.

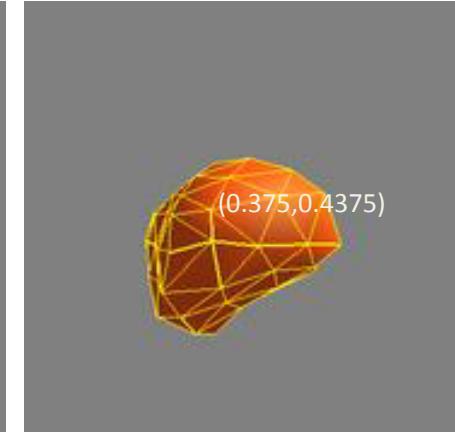
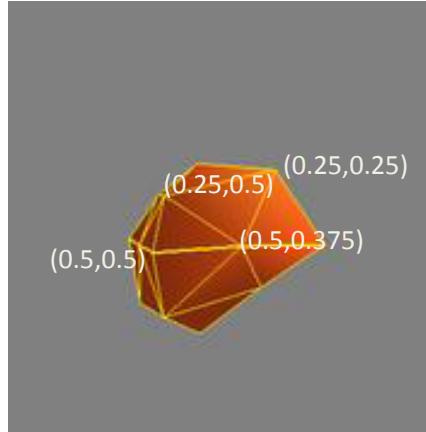
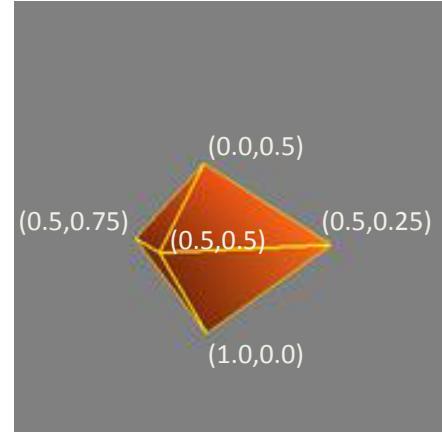
Exemplo de parametrização



esfera



forma digital



Superfícies gaussianas (RaGs)

Assumindo que os vértices obtidos pela subdivisão são representados por P_i

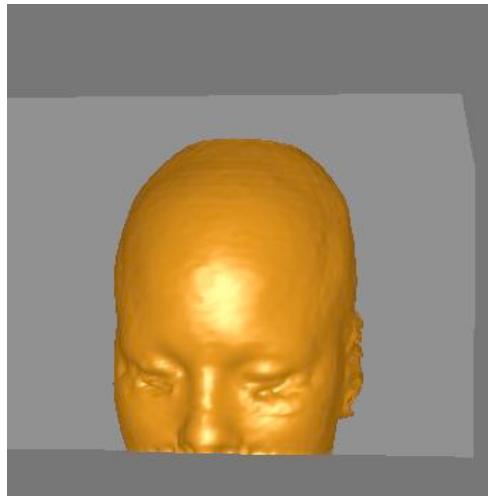
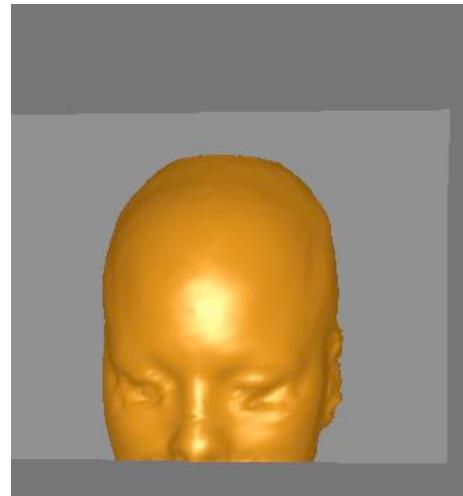
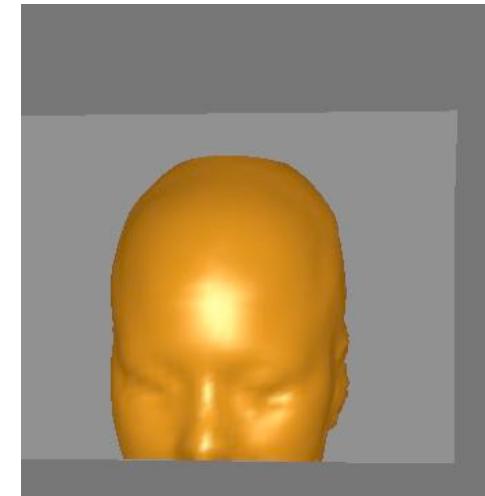
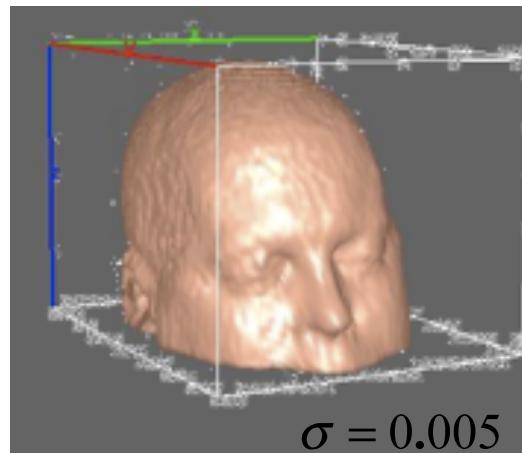
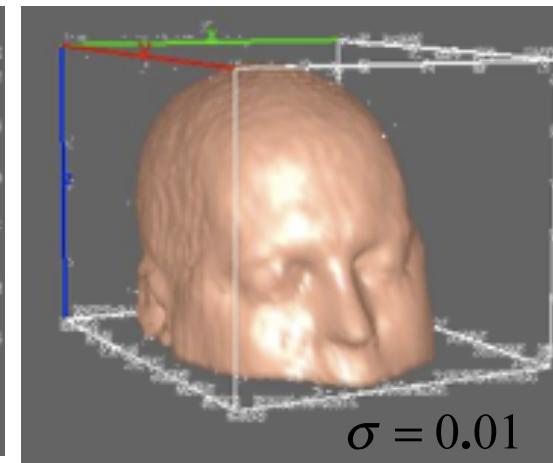
- Uma superfície RaG é definida por

$$P(u, v) = \sum_{i=1}^N P_i g_i(u, v)$$

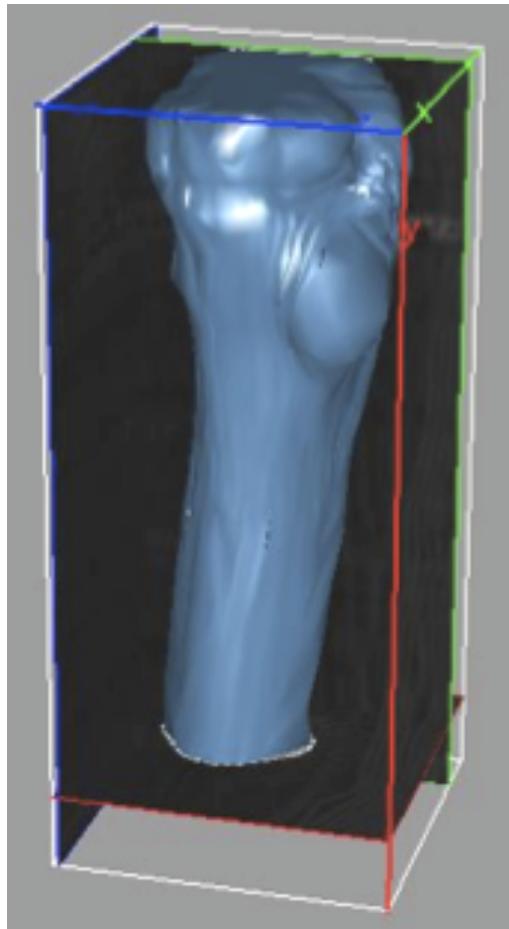
$$g_i(u, v) = \frac{G_i(u - u_i, v - v_i)}{\sum_{j=1}^N G_j(u - u_j, v - v_j)}$$

- Onde G é uma Gaussiana (com altura unitária) em 2D centralizada em cada ponto de controle.

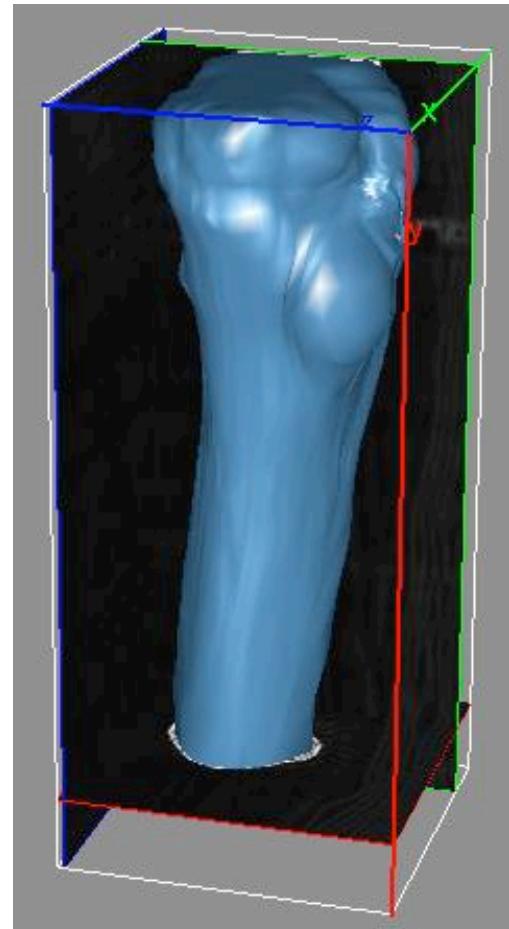
Exemplo

 $\sigma = 0.01$  $\sigma = 0.02$  $\sigma = 0.04$ **forma digital** $\sigma = 0.005$  $\sigma = 0.01$

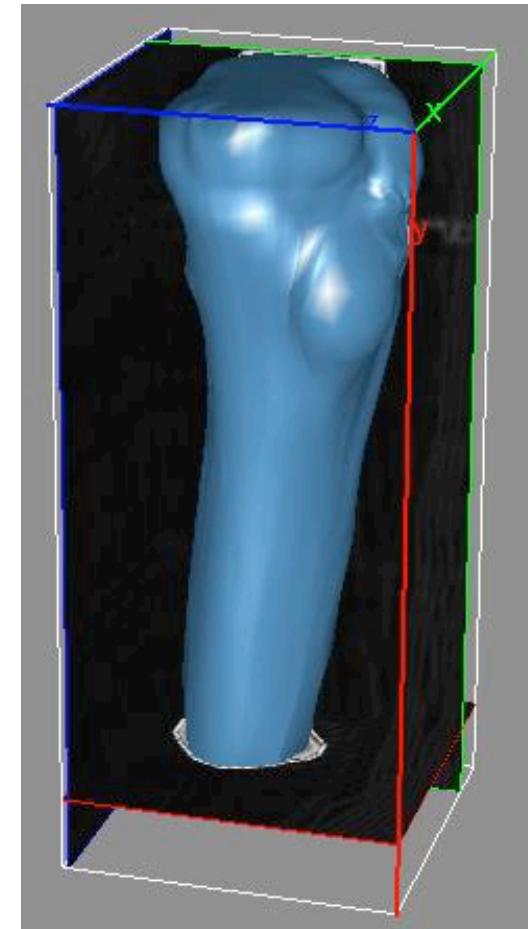
Exemplo



$$\sigma = 0.001$$



$$\sigma = 0.005$$



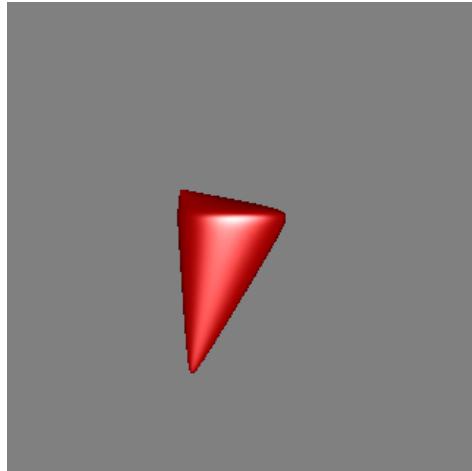
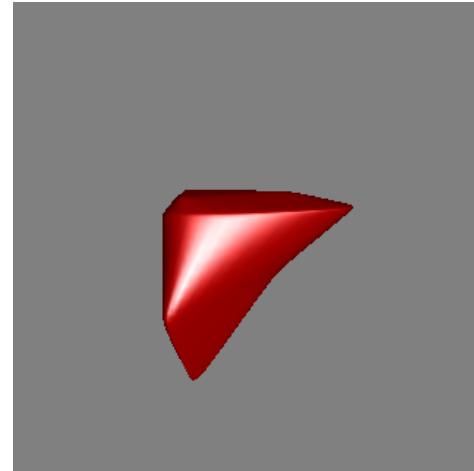
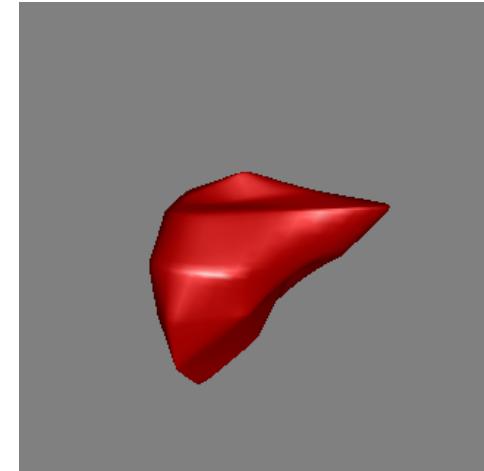
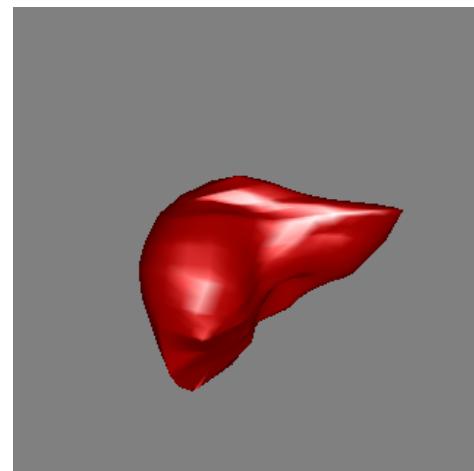
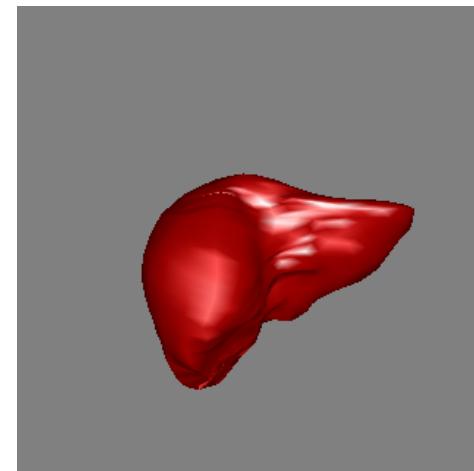
$$\sigma = 0.01$$

$V=2601, F=3187, E=1.024$

Aproximação via RaGs

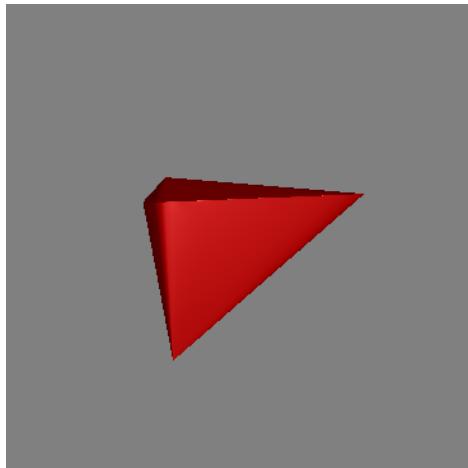
- Já que sabemos os parâmetros de todos os pontos, para cada ponto p_i podemos determinar o ponto correspondente na superfície $P(u_i, v_i)$
- Substituímos a tolerância $D_e = \max_i \{d_i\}$
por $D_t = \max_i \|P(u_i, v_i) - p_i\|$
- Desta forma obteremos um novo algoritmo que aproxima uma forma digital através de uma superfície RaG, com uma tolerância desejada.

Resultados via RaG

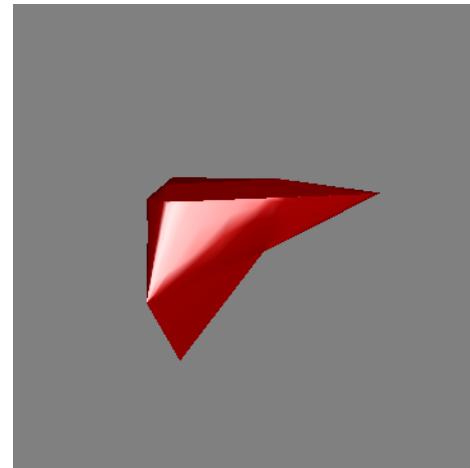
(a) $E=26.5$ (b) $E=13.0$ (c) $E=6.25$ (d) $E=2.95$ (e) $E=1.37$

$s_0=0.2$

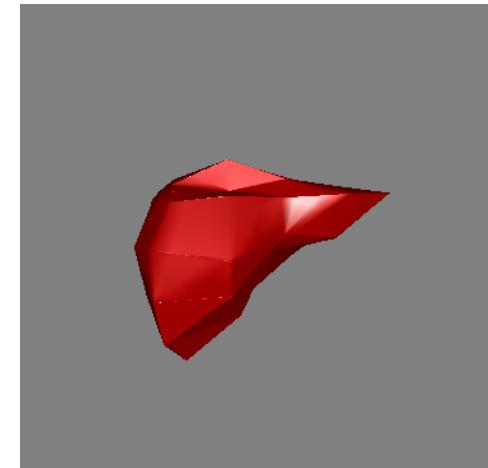
Exemplo



(a) E=10.79

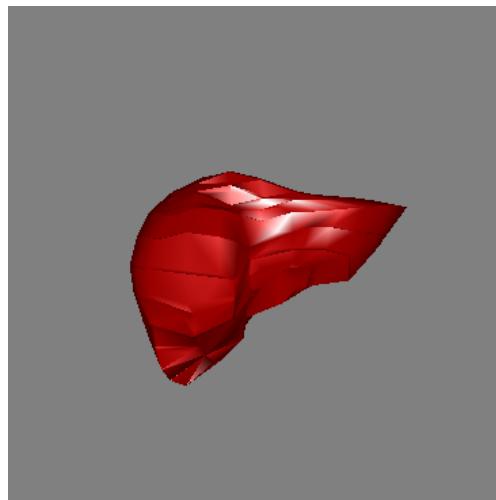


(b) E=8.97

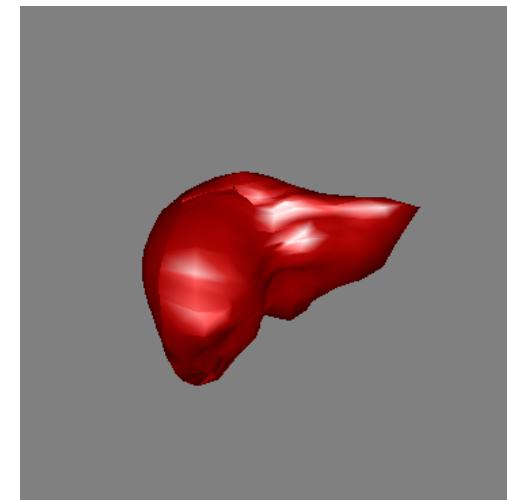


(c) E=4.78

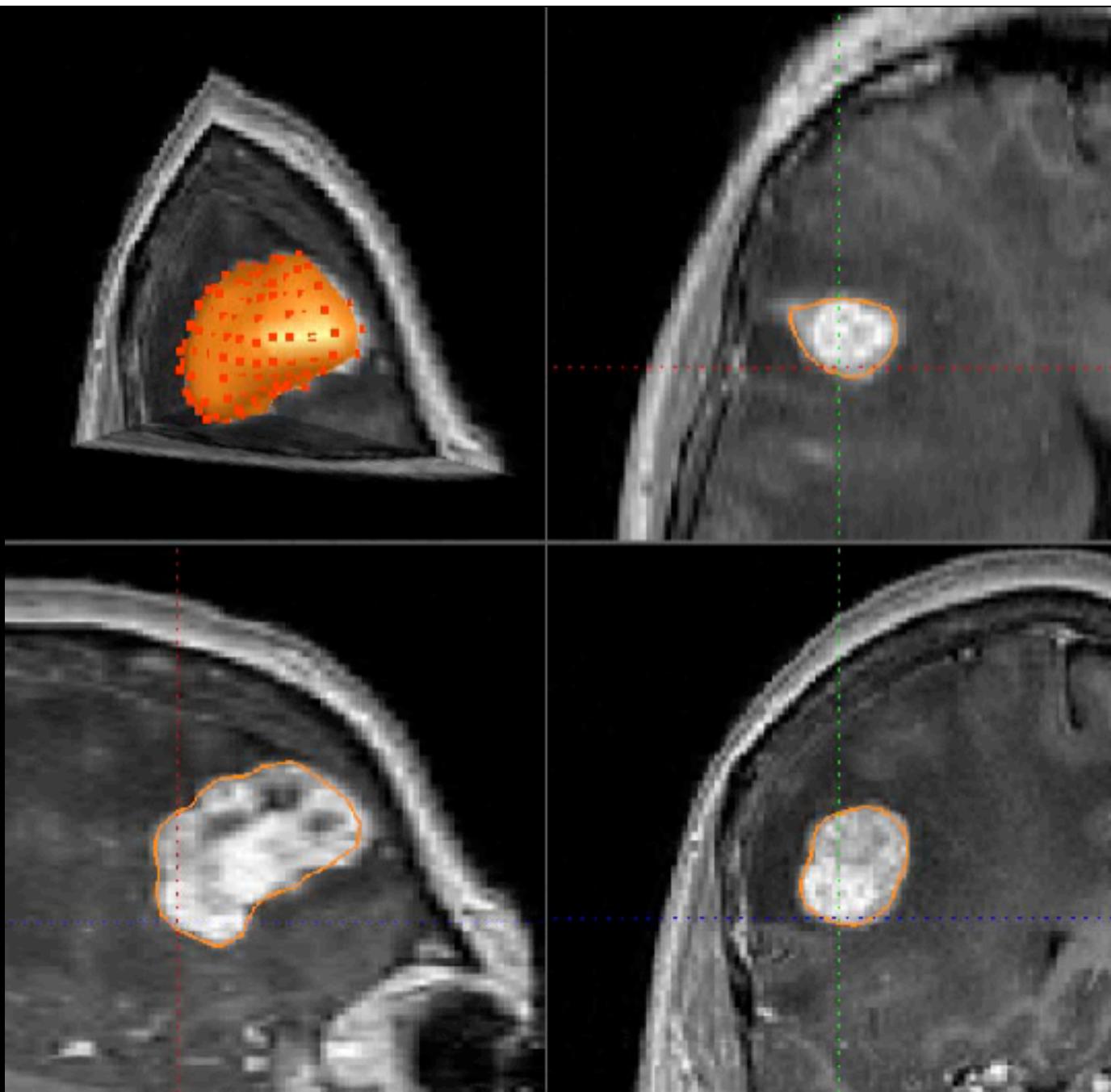
$s_0=0.1$

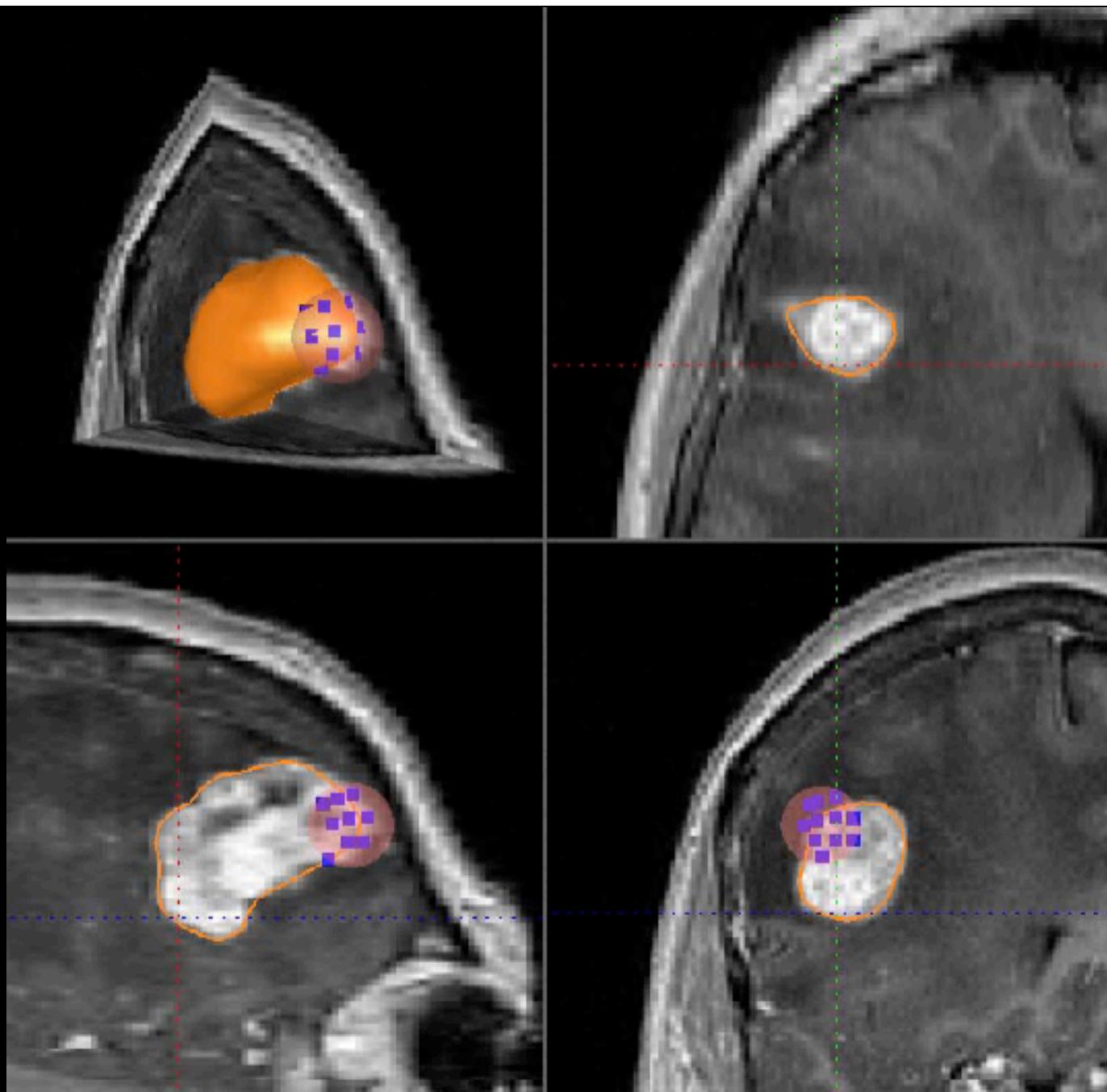


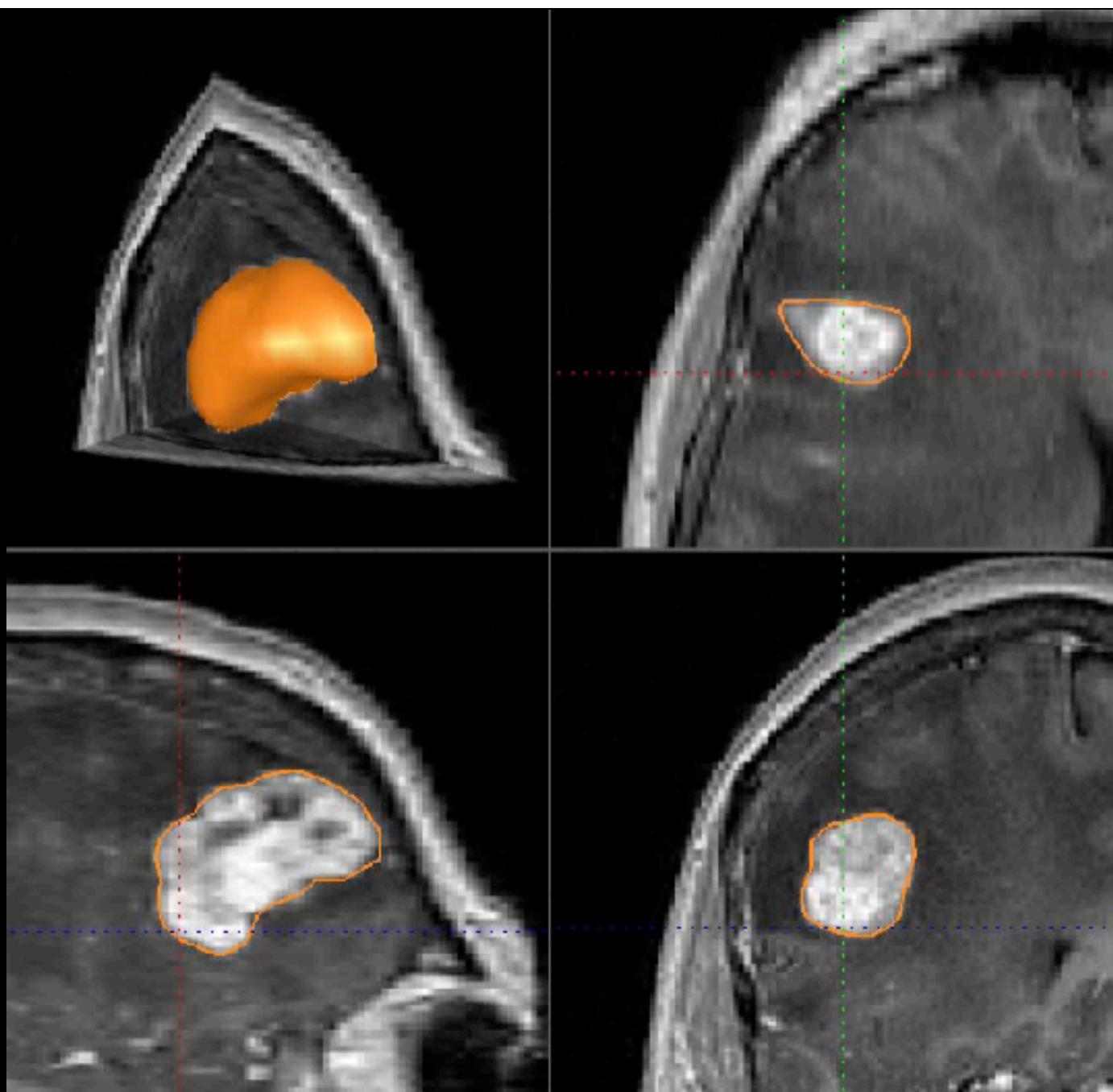
(d) E=2.09



(e) E=1.10







Tarefa de casa

- Leitura artigo
 - A. Goshtasby. *Design and recovery of 2-D and 3-D shapes using rational Gaussian curves and surfaces*, International Journal of Computer, 1993.
 - <http://link.springer.com/article/10.1007%2FBF01539537>