## Chapter 14

Caio Geraldes

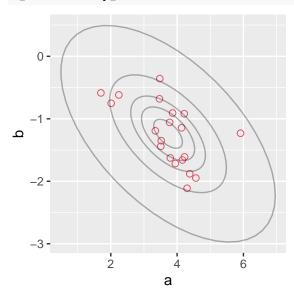
## Generative model

```
a <- 3.5
b <- (-1)
sigma_a <- 1
sigma_b <- 0.5
rho <- (-0.7)

Mu <- c(a, b)
sigmas <- c(sigma_a, sigma_b)
Rho <- matrix(c(1, rho, rho, 1), nrow=2) # correlation matrix
Sigma <- diag(sigmas) %*% Rho %*% diag(sigmas)

N_cafes <- 20
set.seed(5)
vary_effects <- mvrnorm(N_cafes, Mu, Sigma)

a_cafe <- vary_effects[,1]
b_cafe <- vary_effects[,2]</pre>
```



## Simulating observations

```
set.seed(22)
N_visits <- 10</pre>
```

```
afternoon <- rep(0:1, N_visits*N_cafes/2)</pre>
cafe_id <- rep(1:N_cafes, each = N_visits)</pre>
mu <- a_cafe[cafe_id] + b_cafe[cafe_id]*afternoon</pre>
sigma <- 0.5
wait <- rnorm(N_visits*N_cafes, mu, sigma)</pre>
d <- tibble(cafe = cafe_id, afternoon = afternoon, wait = wait)</pre>
d %>% sample_n(10)
## # A tibble: 10 x 3
##
       cafe afternoon wait
##
      <int>
              <int> <dbl>
                    1 1.22
##
    1
          2
                    1 2.98
##
    2
         14
## 3
         10
                    0 3.67
##
        20
                    1 3.63
## 5
         4
                    0 3.61
## 6
       13
                    1 1.21
##
  7
                    0 1.59
        5
##
  8
        10
                    0 3.69
                    1 2.76
## 9
         1
## 10
                    0 4.08
```

## Varying slopes model

$$W_{i} \sim \text{Normal}(\mu_{i}, \sigma)$$

$$\mu_{i} = \alpha_{C_{i}} + \beta_{C_{i}} A_{i}$$

$$\begin{bmatrix} \alpha_{C} \\ \beta_{C} \end{bmatrix} \sim \text{MVNormal} \begin{pmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}, \mathbf{S} \end{pmatrix}$$

$$\mathbf{S} = \begin{pmatrix} \sigma_{\alpha} & 0 \\ 0 & \sigma_{\beta} \end{pmatrix} \mathbf{R} \begin{pmatrix} \sigma_{\alpha} & 0 \\ 0 & \sigma_{\beta} \end{pmatrix}$$

$$\alpha \sim \text{Normal}(5, 2)$$

$$\beta \sim \text{Normal}(-1, 0.5)$$

$$\sigma \sim \text{Exponential}(1)$$

$$\sigma_{\alpha} \sim \text{Exponential}(1)$$

$$\sigma_{\beta} \sim \text{Exponential}(1)$$

$$\mathbf{R} \sim \text{LKJcorr}(2)$$

```
set.seed(867530)

f.14.1 <- alist(
    wait ~ normal(mu, sigma),
    mu <- a_cafe[cafe] + b_cafe[cafe]*afternoon,

# adaptative priors
    c(a_cafe, b_cafe)[cafe] ~ multi_normal(c(a, b), Rho, sigma_cafe),

# fixed priors
    a ~ normal(5,2),
    b ~ normal(-1,0.5),
    sigma_cafe ~ exponential(1),</pre>
```

```
Rho ~ lkj_corr(2)
m14.1 <- ulam(
 flist = f.14.1,
  data = d,
  chains = 4,
  cores =4
## Warning in '/tmp/RtmpoMvR2s/model-511a4d321072.stan', line 3, column 4: Declaration
       of arrays by placing brackets after a variable name is deprecated and
       will be removed in Stan 2.32.0. Instead use the array keyword before the
##
       type. This can be changed automatically using the auto-format flag to
##
##
       stanc
## Warning in '/tmp/RtmpoMvR2s/model-511a4d321072.stan', line 4, column 4: Declaration
##
       of arrays by placing brackets after a variable name is deprecated and
##
       will be removed in Stan 2.32.0. Instead use the array keyword before the
##
       type. This can be changed automatically using the auto-format flag to
##
       stanc
## Warning in '/tmp/RtmpoMvR2s/model-511a4d321072.stan', line 23, column 4: Declaration
##
       of arrays by placing brackets after a variable name is deprecated and
##
       will be removed in Stan 2.32.0. Instead use the array keyword before the
##
       type. This can be changed automatically using the auto-format flag to
##
       stanc
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: lkj_corr_lpdf: Correlation matrix is not positive definite. (in '/tmp/RtmpoMvR2s/
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
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```

sigma ~ exponential(1),

```
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## Chain 1
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: lkj_corr_lpdf: Correlation matrix is not positive definite. (in '/tmp/RtmpoMvR2s/
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## Chain 3
## Chain 4 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 4 Exception: lkj_corr_lpdf: Correlation matrix is not positive definite. (in '/tmp/RtmpoMvR2s/s
## Chain 4 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 4 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 4
```