

# Byte Pair Encoding for Efficient Time Series Forecasting

**Leon Götz<sup>1,2</sup> Marcel Kollovien<sup>2,3</sup> Stephan Günemann<sup>2,3</sup> Leo Schwinn<sup>2,3</sup>**

<sup>1</sup>Volkswagen AG <sup>2</sup>Technical University of Munich <sup>3</sup>Munich Data Science Institute

Correspondence to: [leon.goetz@volkswagen.de](mailto:leon.goetz@volkswagen.de)

## Abstract

Existing time series tokenization methods predominantly encode a constant number of samples into individual tokens. This inflexible approach can generate excessive tokens for even simple patterns like extended constant values, resulting in substantial computational overhead. Inspired by the success of byte pair encoding, we propose the first pattern-centric tokenization scheme for time series analysis. Based on a discrete vocabulary of frequent motifs, our method merges samples with underlying patterns into tokens, compressing time series adaptively. Exploiting our finite set of motifs and the continuous properties of time series, we further introduce conditional decoding as a lightweight yet powerful post-hoc optimization method, which requires no gradient computation and adds no computational overhead. On recent time series foundation models, our motif-based tokenization improves forecasting performance by 36 % and boosts efficiency by 1990 % on average. Conditional decoding further reduces MSE by up to 44 %. In an extensive analysis, we demonstrate the adaptiveness of our tokenization to diverse temporal patterns, its generalization to unseen data, and its meaningful token representations capturing distinct time series properties, including statistical moments and trends.

## 1 Introduction

Transformer architectures have gained increasing relevance in time series processing, demonstrating impressive performance. Here, a key prerequisite for strong performance is effective tokenization — dividing the input into smaller units and embedding them in a high-dimensional space.

Yet, current tokenization schemes in time series processing exhibit considerable limitations: Early works embed each individual time step as a token, creating a fundamentally inefficient representation, where every token captures little temporal information. This results in very long token sequences, imposing a substantial computational burden in the transformer architecture [1]. Splitting the time series into fixed-length subsequences, called patches, mitigates both issues [2]. However, rigid patches can not adapt to diverse temporal patterns in different lengths and complexities [3, 4].

Inspired by adaptive pattern-based tokenization schemes in natural language processing (NLP) [5] and the bio-medical domain [6], we go beyond previous work and propose the first pattern-centric tokenization for time series as in figure 1. Our contribution is threefold:

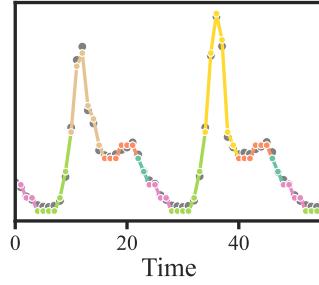


Figure 1: Motif-based tokenization transforms time series data (gray) through a two-step process: 1) quantizing samples into discrete bins, 2) merging recurring patterns of variable length into representative motifs.

**Adaptive tokenization for time series** We provide a novel tokenization strategy based on a discrete vocabulary of frequent time series motifs. Our method merges samples with underlying patterns into tokens, enabling adaptive compression while maintaining a small upper-bounded discretization error. On the recently proposed Chronos foundation model, our tokenization improves forecasting performance by 36 % and boosts efficiency by 1990 % on average.

**Conditional decoding** We introduce conditional decoding as a post-hoc optimization method to further improve forecasting performance by exploiting the continuous properties of time series together with our discrete set of motifs. Conditional decoding is lightweight, requires no gradient computation, introduces no additional overhead during inference, and can be combined with any pretrained time series model with a discrete output vocabulary. We demonstrate its effectiveness in large foundation models, increasing forecasting performance up to 44 %.

**Empirical analysis** In an extensive empirical study, we explore the zero-shot generalization capability of our tokenizer and its ability to automatically adapt to diverse temporal patterns and datasets. We link distinct time series characteristics, including statistical moments and trends to our token representations and show that complex motifs benefit forecasting quality.

## 2 Related work

In recent years, transformer models have shown impressive performance in time series forecasting. While initial work focuses on efficient attention mechanisms and domain-specific architectures [7, 8], universal foundation models have been proposed lately [4, 9, 10, 11, 12, 13, 14, 15]. These models are usually trained on billions of tokens and exhibit high zero-shot performance. However, all these transformer architectures rely on two basic tokenization techniques: using every sample as a token or extracting fixed-length patches from a time series.

**Sample-based tokens** Most early works on transformer models for time series processing extract tokens for every time step, usually as a slice of a multivariate time series [7, 8, 16, 17, 18, 19]. These tokens are linearly transformed into a continuous embedding space. Inspired by the success of discrete token embeddings in NLP, the recently proposed Chronos foundation model [11] quantizes a univariate time series into bins and embeds them using learned vectors. This way, the authors transform forecasting from a regression task to classifying the next time step from a discrete vocabulary [20]. Generating tokens for every time step has two major limitations: First, the large number of tokens imposes a substantial computational burden in transformers, especially for long sequence processing [11, 21]. Second, every token captures only little information about temporal patterns.

**Patch-based tokens** Inspired by the success of patching in computer vision [22], Nie et al. [2] adapt this approach to time series, where multiple samples of an univariate time series are combined into individual tokens. Most subsequent works embed the patches into a continuous space using learned transformations [2, 4, 10, 13, 14, 15, 23, 24, 25]. More advanced approaches learn a discrete codebook of patches [26, 27] using vector quantized variational autoencoder approaches [28]. Masserano et al. [29] apply wavelet transformations to every patch and quantize the coefficients as embedding into discrete space. Patches generally compress the time series and capture local temporal information. However, due to their fixed length and stride, rigid patches can not adapt to varying temporal patterns in a sequence. This is of special importance for foundation models as they try to generalize to previously unseen data in zero-shot settings. To mitigate this, Woo et al. [4] utilize different patch lengths for datasets sampled in different granularities, e.g., minutely or hourly. Their approach requires training of a new embedding transformation for every granularity and fails to capture inter and intra series variations in temporal patterns (see section 5.4).

**Motif-based tokens** Motif-based tokenization utilizes a discrete vocabulary of recurring patterns. In NLP, byte pair encoding hierarchically extracts pairs of character-bytes to tokenize a sentence [5, 30]. Elsner et al. [31] extend this concept from 1d-sequences to tokenizing images. For drug discovery in the bio-medical domain, Hetzel et al. [6] use frequently occurring subgraphs to encode molecules based on discrete shapes. Tokenization based on discrete motifs has proven to be a good inductive bias for high-dimensional distribution learning as it reduces the combinatorial complexity [32]. Yet, such data-dependent tokenization techniques remain unexplored for the time series domain.

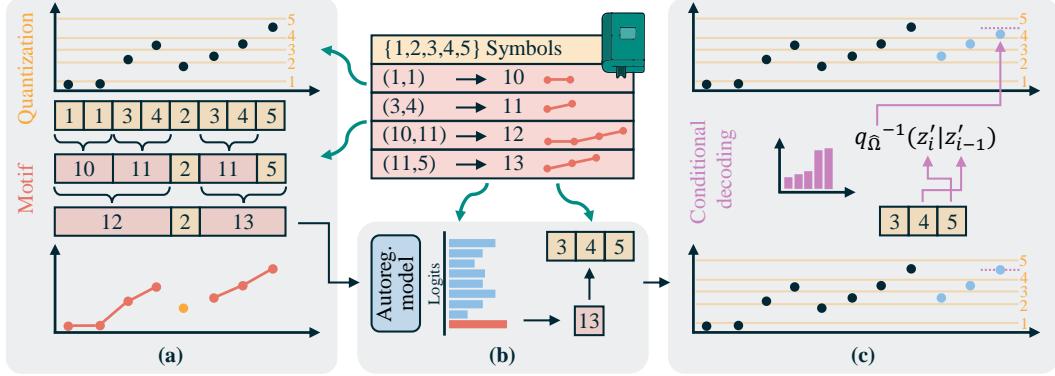


Figure 2: (a) Our motif-based tokenization first quantizes a time series into symbols and finds recurring motifs as tokens, building a discrete vocabulary. (b) Based on the compressed motif sequence, a neural network forecasts the time series through a categorical distribution over our vocabulary. (c) Finally, we propose conditional decoding to reduce the discretization error when transforming tokens back to their continuous representation.

### 3 An adaptive tokenization approach for time series

Despite recent advances in time series processing, current tokenization methods lack efficiency or fail to capture distinct temporal patterns within sequences [3, 4]. We propose an efficient tokenization method using a vocabulary of frequent motifs as depicted in figure 2. Our algorithm combines samples with underlying patterns of varying complexity into single tokens. Its adaptive compression of time series enables efficient long sequence processing. We list pseudocode in appendix A.

Let  $\mathcal{D} = \{z^i\}_{i=1}^N$  be a family index by  $i = 1, \dots, N$  of  $N$  univariate real-valued time series  $z = (z_1, \dots, z_n) \in \mathbb{R}^n$  of length  $n$ . We normalize each series to have zero mean and unit standard deviation. A neural network  $f_\theta : \mathbb{N}^{t_{\text{in}}} \rightarrow \mathbb{N}^{t_{\text{out}}}$  with parameters  $\theta$  predicts  $t_{\text{out}}$  token IDs from  $t_{\text{in}}$  token IDs. Thereby, the tokens are generated by our tokenizer  $g : \mathbb{R}^n \rightarrow \mathbb{N}^t$  from a time series  $z$ . Our tokenization consists of two steps:

$$g(z) = \mathbf{m}_\Psi \circ \mathbf{q}_\Omega(z) \quad (1)$$

where:

- $\mathbf{q}_\Omega : \mathbb{R}^n \rightarrow \mathbb{N}^n$  quantizes the time series into a sequence of discrete symbols,
- $\mathbf{m}_\Psi : \mathbb{N}^n \rightarrow \mathbb{N}^t$  compresses the sequence based on a discrete vocabulary of temporal motifs,
- $\Omega, \Psi$  vocabulary of quantized symbols and motifs, respectively.

#### 3.1 Discretization of real-valued time series

Generalizing the approach from Ansari et al. [11], we sample  $M$  equiprobable discretization intervals  $\Omega = \{C^{-1}(\frac{j}{M})\}_{j=1}^M$ , where  $C^{-1}$  is the inverse cumulative distribution of the probability distribution  $P$ . In practice, we experiment with truncated uniform distributions in  $[\omega_{\text{lb}}, \omega_{\text{ub}}]$ , Gaussian distributions, and the precise data distribution  $P(\mathcal{D})$  for binning. Utilizing the boundaries, we encode the time series  $z$  into a sequence of discrete symbols.

$$\mathbf{q}_\Omega(z) = \{q_\Omega(z_i) \mid z_i \in z\}, \quad \text{where} \quad q_\Omega(z_i) = \begin{cases} 1 & \text{if } z_i \leq \omega_1 \\ j & \text{if } \omega_{j-1} < z_i \leq \omega_j, \omega_j \in \Omega \end{cases} \quad (2)$$

For decoding symbol IDs back to time series samples, we use  $\hat{\Omega} = \{C^{-1}(\frac{j-0.5}{M})\}_{j=1}^M$ , where  $\hat{\omega}_j \in \hat{\Omega}$  is the probabilistic center of  $[\omega_{j-1}, \omega_j]$ . The quantization error can be upper bounded as  $\delta_{\max} = \max_{1 < j \leq M} \max(\hat{\omega}_j - \omega_{j-1}, \omega_j - \hat{\omega}_j)$  within the tokenization range. For uniform binning the probabilistic center is equal to the geometric center and the maximum error simplifies to  $\delta_{\max} = (\omega_{\text{ub}} - \omega_{\text{lb}})(2M)^{-1}$ . Besides the  $M$  token IDs representing quantized time series samples, we introduce two additional tokens: A masking token MASK to account for missing samples in time series and an EOS token we insert at the end of time series.

### 3.2 Vocabulary of temporal motifs

Byte pair encoding, which is commonly used in natural language processing [5] to compress character sequences into subwords, is well suited for pattern recognition in the compressed representation [30]. Originally proposed for compressing sequences of raw bytes [33], we generalize the byte pair compression algorithm to extract temporal patterns from our discretized time series. To this end, we iteratively build a vocabulary  $\Psi$  of frequent time series motifs: Given a dataset  $\mathcal{D}' = \{\mathbf{q}_\Omega(z^i) \mid z^i \in \mathcal{D}\}$  of quantized time series, we extract the most frequent adjacent token IDs  $(z'_i, z'_{i+1})$ , assigning a new token ID  $z'_{\text{new}}$ , which we add to our set of patterns  $\Psi$ :

$$\Psi^{(l+1)} \leftarrow \Psi^{(l)} \cup \{(z'_i, z'_{i+1}) \rightarrow z'_{\text{new}}\}. \quad (3)$$

This process hierarchically finds distinct temporal motifs as discrete tokens and is locally optimal in every step. We build our vocabulary until the new tokens occur less frequent than  $p_{\min}$  in  $\mathcal{D}'$ . This ensures that a minimum number of occurrences are available for a neural network to learn the motifs. Leveraging our vocabulary, we compress a quantized time series into a sequence of motifs:

$$\mathbf{m}_\Psi(z') = \{\psi(z') \mid \psi \in \Psi\}, \quad \mathbf{m}_\Psi : \mathbb{N}^n \rightarrow \mathbb{N}^t. \quad (4)$$

The compression is highly flexible as motifs of different lengths and complexities are mapped to single tokens. To this end, we define the average compression on the sequence level as  $\bar{c} = n/t$ .

### 3.3 Conditional decoding

We propose our novel conditional decoding to universally improve the forecasting quality of models with discrete output vocabularies. To decode a token sequence, such as the predictions of a model, we invert the tokenization  $\mathbf{g}$ . In this process when inverting  $\mathbf{q}_\Omega$ , we previously leveraged the bin centers  $\hat{\omega}_j \in \hat{\Omega}$  to transform a quantized sequence  $z'$  back to a time series  $\hat{z}$ . We introduce conditional decoding to reduce the overall quantization error. Specifically, we decode quantized time series samples  $z'_i$  conditioned on the previous sample  $\hat{z}_i = q_{\hat{\Omega}}^{-1}(z'_i \mid z'_{i-1})$ . To this end, we set parameters  $\hat{\Omega} = \{\hat{\omega}_{j,k} \mid j, k \in \{1, \dots, M\}\}$ , where  $q_{\hat{\Omega}}^{-1}(z'_i = j \mid z'_{i-1} = k) = \hat{\omega}_{j,k}$  to minimize  $\|z_i - \hat{z}_i\|_2^2$ :

$$\min_{\hat{\Omega}} \sum_{(z, z') \in \mathbf{D}} \sum_{i=2}^n \|z_i - q_{\hat{\Omega}}^{-1}(z'_i \mid z'_{i-1})\|_2^2, \quad \text{where } \mathbf{D} = \{(\mathcal{D}_i, \mathcal{D}'_i) \mid i \in \{1, \dots, N\}\} \quad (5)$$

consists of corresponding real-valued and quantized time series. Thereby, a single parameter  $\hat{\omega}_{j,k}$  is given by the mean of the underlying time series samples  $\tilde{z}$  minimizing the squared error:

$$\hat{\omega}_{j,k} = \frac{1}{|\mathcal{D}_{j,k}|} \sum_{\tilde{z} \in \mathcal{D}_{j,k}} \tilde{z}, \quad \text{where } \mathcal{D}_{j,k} = \{z_i \mid (z, z') \in \mathbf{D}, z'_i = j, z'_{i-1} = k\}. \quad (6)$$

Intuitively, we adopt a unigram model to exploit the unique properties of our tokenization: the finite set of discrete symbols and the underlying continuous time series samples. Conditional decoding is lightweight and requires no gradient computation as we solve analytically for the global optimum. Further, it adds no additional inference cost and is very small in practice with only  $M^2$  parameters  $\hat{\omega}_{j,k} \in \hat{\Omega}$ . Conditional decoding can be combined with any pretrained time series model with a discrete output vocabulary and considerably improves forecasting performance in our experiments.

### 3.4 Model architecture

As we represent continuous time series as a sequence of discrete motifs, we can rely on recent advances in transformer architectures in natural language processing. These architectures transform token IDs from our discrete vocabulary  $\mathcal{V} = \Omega \cup \Psi \cup \{\text{MASK}, \text{EOS}\}$  into  $d$ -dimensional space using learned embedding tables  $E \in \mathbb{R}^{|\mathcal{V}| \times d}$ . We optimize the parameters  $\theta \in \Theta$  of our model  $\mathbf{f}_\theta$  on autoregressive next token prediction of our tokenized sequence  $z'' = \mathbf{g}(z)$ . Our model thereby predicts a categorical distribution  $p(z''_{t_{\text{in}}+1} \mid z''_{1:t_{\text{in}}})$  over our finite vocabulary of time series motifs  $\mathcal{V}$ . We impose a cross-entropy loss for distribution learning. To this end, we transform the regression task to a classification [20]. The discrete set of possible motifs reduces the combinatorial complexity and has proven to be a good inductive bias for distribution learning in the bio-medical domain [32]. In

contrast to prior work [11], our tokenizer enhances the efficiency as both model input and generated tokens are compressed time series representations. Models can utilize longer contexts while requiring fewer autoregressive iterations for a given prediction horizon. This is especially important for large foundation models and long sequence processing, imposing substantial computational requirements.

## 4 Experiments

We systematically train different tokenizers and foundation models and evaluate them on 5 time series datasets in zero-shot setting, demonstrating advantages of our motif-based representation over tokenizing every sample or utilizing patches. In appendix B, we provide further experimental details.

**Datasets** For training our models and tokenizers, we utilize the recently proposed Chronos dataset [11]. It contains 11 M time series with over 11 B samples. Due to its diverse nature and size, this dataset is well suited for training foundation models. We base our zero-shot evaluation on 5 commonly used time series datasets: ETTh1, ETTm1, Weather, Electricity, and Traffic.

**Tokenizers** We leverage 3 tokenizers with different numbers of quantization bins  $M$ . Further, we utilize a truncated uniform distribution from  $\omega_{lb} = -5$  to  $\omega_{ub} = 5$  for binning, spanning a range of 5 standard deviations. As a result, our tokenizers in table 1 feature different compression ratios, vocabulary sizes, and discretization errors. We build their vocabulary  $\Psi$  on the same 100 000 randomly selected time series from the Chronos dataset with a total of 100 M samples. To allow the model to learn all tokens, we constrain the motifs to occur at least  $p_{min} = 1000$  times in the compressed data. In sections 5.5 and 5.6 and appendix C.1, we systematically ablate these choices.

Table 1: Tokenizers on the Chronos dataset with different quantization bins, vocabulary size, discretization error, and compression.

Compression	$M$	$ \mathcal{V} $	$\delta_{max}$	$\bar{c}$
low	126	2445	0.040	2.08
medium	37	1675	0.135	3.18
high	22	1373	0.227	4.06

**Models** In our experiments, we explore our tokenization approach in foundation models operating in a zero-shot setting. We compare our motif-based tokenization with single-sample tokenization in Chronos models [11] and patches in MOMENT [13] and Morai [4] foundation models. Following Chronos and MOMENT, we utilize the T5 architecture [34] as backbone, ensuring that tokenization is the primary difference between architectures. We propose models with our tokenizer in 5 sizes ranging from tiny (8 M parameters) to large (710 M parameters). For a fair comparison, we train our models and retrain Chronos models with the same number of tokens and gradient steps. We evaluate on forecasting 64 time series samples, following [11]. As context, we utilize 128 tokens for our and Chronos models and an equivalent input length of 384 time series samples for patch-based models. We restrict our evaluation to models with available data and code to enable us to reproduce the results. Chronos is the only foundation model in literature with sample-wise tokenization.

## 5 Results

We first demonstrate improvements in forecasting performance and efficiency of our motif-based tokenization over existing methods. Next, we explore the adaptiveness of our tokenizer to diverse temporal patterns of different lengths and complexities and its generalization to unseen data. Finally, we link distinct time series properties, including statistical moments and trends, to our token space.

### 5.1 Efficiency improvements of adaptive tokenization

Chronos foundation models tokenize every sample of a time series, resulting in many tokens with little temporal information. Especially for large foundation models, this induces substantial computational requirements. We compare our motif-based tokenization with Chronos models in 5 sizes from tiny to large using 3 tokenizers (see experimental settings in section 4). Chronos and our models are based on the same architecture, training strategy, and dataset. They only differ in tokenization.

In our zero-shot evaluation, our motif-based tokenization finds Pareto optimal points on all 5 datasets. We show in figure 3 that our tokenizer outperforms Chronos models in forecasting quality and efficiency at the same time. We report our results in table 2, choosing

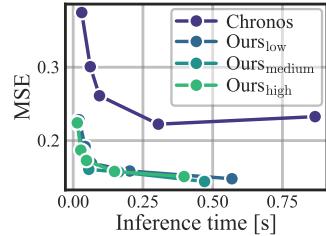


Figure 3: Zero-shot evaluation of our motif-based tokenization and Chronos models tokenizing every sample on Electricity.

the best Chronos model as reference. Among our 3 tokenizers and 5 model sizes, we illustrate two cases: 1) Selecting the best MSE, 2) Selecting the fastest model that is still better than the Chronos reference. Motif-based tokenization without conditional decoding improves MSE by 36.1 % and accelerates models  $19.90\times$  on average. With conditional decoding, the improvements are even more substantial, with forecasting quality increasing by 43.2 % and model acceleration reaching  $26.73\times$ . On the Traffic dataset, Chronos models diverge during zero-shot testing, while our tokenizer still performs well, highlighting the generalization capability of motifs. We show additional results in appendix C.2, where we compare our zero-shot motif-based models with state-of-the-art models that are directly trained on the respective datasets. Remarkably, our approach generates the best forecasts in 19 out of 25 cases without fine-tuning.

Table 2: Motif-based tokenization with conditional decoding (cd) and without improves forecasting quality and accelerates models during zero-shot forecasting. We aim for two extremes: best MSE and fastest acceleration. Among Chronos models, we choose the best as reference. As our tokenization improves MSE while speeding up the model, we are able to choose small models while surpassing forecasting quality of larger ones. **Best** in bold.

Dataset	Chronos		Ours		Ours <sup>cd</sup>
	MSE	MSE <sup>best</sup>	Accel. <sup>fastest</sup>	MSE <sup>best</sup>	Accel. <sup>fastest</sup>
ETTh1	0.717	0.517	24.88 $\times$	<b>0.459</b>	<b>55.74</b> $\times$
ETTm1	1.004	0.637	<b>6.49</b> $\times$	<b>0.449</b>	<b>6.49</b> $\times$
Weather	0.265	0.251	0.26 $\times$	<b>0.236</b>	<b>3.58</b> $\times$
Electricity	0.222	0.150	<b>11.20</b> $\times$	<b>0.144</b>	<b>11.20</b> $\times$
Traffic	2.717	0.591	<b>56.66</b> $\times$	<b>0.574</b>	<b>56.66</b> $\times$

## 5.2 Comparison with patch-based methods

Patching, which involves extracting fixed-length subsequences as tokens, compresses the time series and captures local temporal information [2]. However, tokens are rigid and non-adaptive to diverse time series patterns. Here, we compare our adaptive motif-based tokenization with the patch-based foundation models MOMENT and Morai in zero-shot settings. MOMENT models are most similar to ours as they also utilize T5 models as their backbone. However, both architectures utilize different training strategies and datasets and can not be directly compared with our models.

Our tokenization method outperforms all MOMENT models in table 3. Motif-based tokenization increases forecasting quality by 33.0 % on average. Utilizing conditional decoding, MSE improvements of 41.9 % are substantially enhanced. This indicates that our adaptive tokenization is important. Further, our models generate better forecasts on 2 out of 5 datasets compared to Morai. However, these models utilize different transformer backbone architectures, training strategies, and datasets, making a fair comparison of tokenization methods difficult. In section 5.4 we explore the compression of our motif-based tokenization in more detail.

Table 3: Comparison of our motif-based tokenization with conditional decoding (cd) and without with patch-based models MOMENT and Morai based on zero-shot forecasting quality (MSE). In line with table 2 we report the best among our tokenizers. We highlight values that are worse than our method.

Dataset	Ours	Ours <sup>cd</sup>	MOMENT <sub>small</sub>	MOMENT <sub>base</sub>	MOMENT <sub>large</sub>	Morai <sub>small</sub>	Morai <sub>base</sub>	Morai <sub>large</sub>
ETTh1	0.517	0.459	0.765	0.732	0.693	0.465	0.396	0.397
ETTm1	0.637	0.449	0.700	0.710	0.665	0.710	0.600	0.548
Weather	0.251	0.236	0.275	0.249	0.240	0.193	0.161	0.245
Electricity	0.150	0.144	0.887	0.888	0.852	0.212	0.163	0.146
Traffic	0.591	0.574	1.458	1.534	1.386	0.645	0.406	0.427

## 5.3 Conditional decoding

Recently emerging foundation models show impressive performance but are expensive to train [11]. We propose conditional decoding as a lightweight yet powerful post-hoc optimization method to enhance a model’s forecasting quality. Conditional decoding adds no computational overhead during inference and does not require gradient computation for training. Instead, we analytically compute the global optimum for its few parameters according to equation (6). For our experiments, we utilize 3 tokenizers with different compression (see table 1), 5 datasets, and models in size small. In the

following, we train conditional decoding to dequantize the models’ forecasts on the respective train set and evaluate on the test set.

Conditional decoding consistently improves forecasting quality in figure 4 in all of our experiments. On the ETTm1 dataset and our tokenizer with high compression, conditional decoding reduces MSE by 44.3 % with only 484 trainable parameters. In appendix C.3, we provide additional results and further investigate conditional decoding in a model-independent setting. There, conditional decoding mitigates on average 31.9 % and up to 96.9 % of our tokenizer’s quantization error, enabling us to build tokenizers with even higher compression.

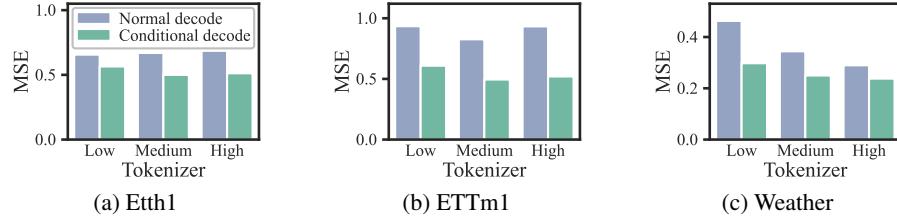


Figure 4: Conditional decoding improves forecasting quality for 3 tokenizers in small models on 3 datasets.

#### 5.4 Adaptive compression of diverse time series

Temporal patterns differ in length and complexity among datasets and within time series [3, 4]. Due to their fixed length, rigid patches are unable to capture these inter and intra series variations. Here, we show that our motif-based tokenization natively adapts to these diverse patterns, compressing them adaptively. To this end, we analyze our medium compression tokenizer (see table 1).

The Weather dataset contains patterns of various complexities, which we illustrate in figures 5b to 5d. Here, our tokenizer compresses motifs of different lengths into single tokens, achieving compressions from 8.13 up to 22.26. Less complex patterns result in higher compression, while more complex patterns are tokenized more fine-grained. Compared to the patch-based MOMENT model with a patch length of 8, we achieve substantially higher compressions. In figure 11, we demonstrate this adaptive intra series compression on 4 other datasets. Among datasets, our tokenizer reaches average compressions of 3.30 on Traffic and 23.15 on Weather in table 4. Further, ETTh1 and ETTm1 are sampled with different frequencies but from the same process. The higher compression on ETTm1 indicates that our tokenizer is agnostic to the sampling frequency. All these results highlight the flexibility of our motif-based tokenization. In appendix C.4, we further investigate relations between compression of input data and generated tokens and find linear dependencies. We also showcase even higher compressions up to 128.

Table 4: Average compression of our medium tokenizer on 5 datasets.

Dataset	$\bar{c}$
ETTh1	3.48
ETTm1	4.59
Weather	23.15
Electricity	3.95
Traffic	3.30

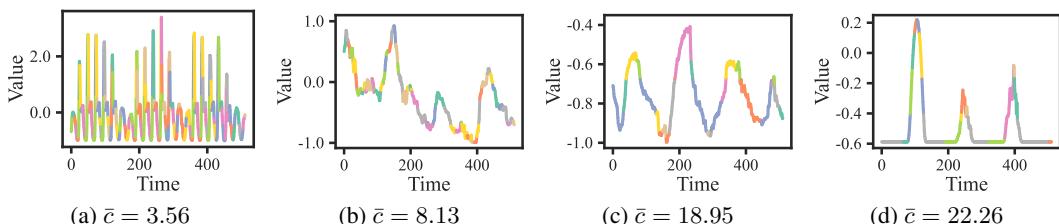


Figure 5: Our adaptive tokenizer (a) exploits periodically recurring motifs on the Traffic dataset and (b-d) compresses time series adaptively depending on pattern complexity on the Weather dataset.

#### 5.5 Vocabulary complexity and generalization

Longer motifs benefit the compression and efficiency of our tokenizer. Here, we systematically explore factors influencing the vocabulary complexity and generalization ability. We show that longer motifs are more expressive and enhance forecasting quality. We list further insights in appendix C.5.

**Quantization granularity** A lower number of quantization bins  $M$  reduces the complexity of the time series, resulting in longer motifs and a smaller vocabulary (see table 1). However, fewer quantization bins also increase the quantization error, potentially failing to capture important nuances and compromising forecasting quality. In figure 9, we utilize 3 tokenizers with different quantization granularities without conditional decoding, on 5 model sizes, and 5 datasets to analyze this tradeoff. In 15 out of 25 settings, our tokenizer with high compression and the largest quantization error leads to best MSE. This experiment indicates that longer, more expressive motifs benefit forecasting, despite higher quantization error. Moreover, as shown in section 5.3, the quantization error can be largely removed with conditional decoding.

**Token occurrence** There is an inherent tradeoff in tokenization: longer, more complex motifs (created by a high number of recursive merges) naturally occur less frequently in the training data. In the limit, the whole dataset can be represented by a single motif. While setting a lower minimum occurrence threshold  $p_{\min}$  allows the vocabulary to capture more complex patterns, these rarer motifs may provide insufficient learning examples for the model to reliably recognize them. Here, we vary  $p_{\min}$  from 1000 to 128 000 training 8 different tokenizers. These tokenizers feature different vocabulary complexity and compression, as in table 5, but have the same quantization error. We base our variations on our medium tokenizer and utilize small models. Our results on Electricity and Traffic in figure 6 indicate that there is an optimal tradeoff. A minimum motif occurrence of  $p_{\min} = 4000$  times among 100 M time series samples represents a good balance. Generally, more complex motifs with higher compression result in best MSE.

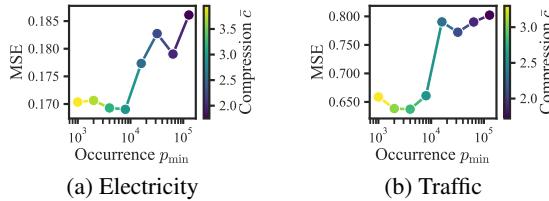


Figure 6: Varying token occurrence  $p_{\min}$  influences forecasting quality for small models. More complex motifs generally improve MSE.

Table 5: Tokenizers on the Chronos dataset with different token occurrence, vocabulary size, and compression.

$p_{\min}$	$ \mathcal{V} $	$\bar{c}$
1000	1675	3.18
2000	993	2.95
4000	604	2.73
8000	373	2.50
16 000	237	2.29
32 000	158	2.08
64 000	108	1.86
128 000	78	1.66

Table 6: Correlation  $\rho$  of token-level compression and MSE.

Dataset	$\rho$
ETTh1	-0.26
ETTm1	-0.24
Weather	-0.26
Electricity	-0.07
Traffic	-0.27

**Token level analysis** Here, we demonstrate on token level that complex motifs are a better representation for time series generation than their simpler counterparts. To this end, we correlate motif length with token-wise MSE of time series forecasts. We utilize our medium compression tokenizer in a small model. On all 5 datasets, we observe negative correlation coefficients  $\rho$  in table 6. Therefore, the generation of longer, more expressive motifs enhances forecasting quality. These results are in line with our previous investigations.

## 5.6 Training dataset size

Here, we explore how much data is required to train an efficient motif-based tokenizer. In general, larger datasets better approximate the true distribution of patterns, resulting in more complete vocabularies of motifs  $\Psi$ . To this end, we train our 3 tokenizers on Chronos dataset subsets ranging from 1000 to 1 M time series and scale  $p_{\min}$  accordingly. Increasing the dataset size improves forecasting quality, as in table 7. As expected, motifs extracted from a larger sample size are less noisy and generalize better. This is also evident in the decreasing vocabulary size and compression, indicating a smaller, more universal set of motifs. With 1 M time series, our tokenizer is still very sample efficient, requiring less than 10 % of Chronos data for vocabulary generation. In appendix C.6, we show full results and similar findings for conditional decoding.

Table 7: Influence of training dataset size  $N$  on tokenizer vocabulary size, compression, and forecasting quality. Results are averaged among 3 tokenizers on 5 evaluation datasets.

$N$	$ \mathcal{V} $	$\bar{c}$	MSE
1 k	2127	3.16	0.569
10 k	1853	3.10	0.560
100 k	1831	3.11	0.555
1 M	1827	3.11	0.533

## 5.7 Learned token representations

Time series have distinct properties such as periodicity, offsets, and trends. A meaningful token representation should model these characteristics.

Our motif-based tokenization captures periodicity by design. Similar patterns at different positions in a time series are mapped to the same token. Qualitatively, this is evident in figures 1 and 5a.

We analyze the token embedding space  $E$  by doing a principal component analysis in figures 7 and 16. The learned embeddings successfully capture the values of quantized symbols in  $\Omega$  (a), which are separated from MASK and EOS tokens. The embedding space further models the mean (b) and standard deviation (c) of motifs in  $\Psi$  in orthogonal dimensions, indicating a good separation of these properties. For motifs with high standard deviation, the model distinguishes between linear and quadratic trends. Finally, motif length is implicitly learned and modeled in the same dimension as the standard deviation, as constant patterns with low standard deviation are likely longer.

Our method finds time series motifs hierarchically. Therefore, a child token is made out of two parents. Intuitively, a child and its first parent should be embedded closely together as the neural network can either predict the child token directly or both parents as a sequence. Here, we analyze this relation in more detail. The average cosine similarity among all embedded token vectors is 0.072, indicating a good utilization of the embedding space. For the first parent tokens and their children, this is significantly higher with 0.475. Qualitatively, we showcase parent-child relations by connecting them in figure 17. Here, parents and children are shifted along the motif length axis. All these results demonstrate that our vocabulary of motifs captures time series characteristics as meaningful representations.

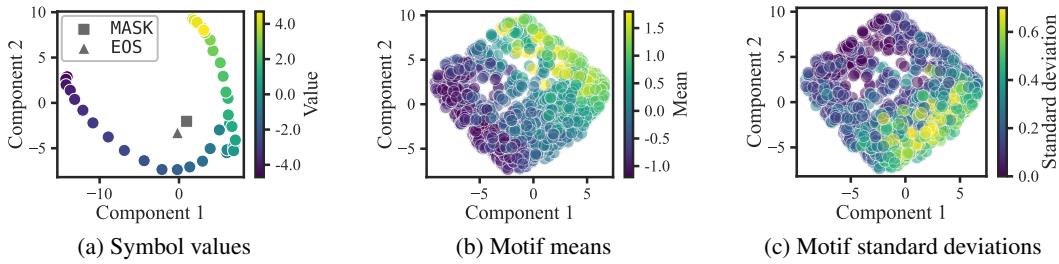


Figure 7: Principal component analysis of token embeddings of our medium tokenizer in a small model.

## 6 Conclusion

In this work, we propose the first pattern-centric tokenization for the time series domain. Our method leverages recurring discrete motifs as tokens and improves forecasting quality and efficiency over existing methods. We further introduce conditional decoding as a lightweight, domain-specific post-hoc optimization method and show its performance gains in large foundation models. We demonstrate our tokenizer’s adaptability to patterns of different complexities and show that the learned token embeddings capture meaningful representations of time series properties, including statistical moments and trends. Finally, our thorough investigation reveals key tradeoffs balancing tokenizer complexity and generalization: discretization granularity presents a dual effect on compression - fewer bins increase discretization error but also make patterns more frequent, potentially improving both learnability and compression; training data size influences how well the discovered motifs generalize, with smaller datasets being insufficient to learn robust representations of rare motifs. However, with sufficient data, longer and more complex motifs can significantly reduce prediction error, ultimately enhancing compression efficiency. We hope our motif-based tokenization will have a positive effect on reducing the resource consumption and environmental impact of time series models.

**Outlook** In future work, we aim to explore efficient tokenization schemes for other architectures, such as LSTMs [35, 36] or state-space models [37], and new generation paradigms including diffusion [38, 39] or flow-matching [40, 41]. Lastly, we plan to evaluate the effect of our tokenization scheme on out-of-distribution robustness [42] and robustness to perturbations [43, 44, 45].

**Limitations** In our work, we do not conduct hyperparameter search for T5 models due to the high computational cost of training large foundation models. We expect even better results with optimized settings. Moreover, future work can utilize more recent transformer architectures.

## References

- [1] Leon Götz, Marcel Kollovieh, Stephan Günemann, and Leo Schwinn. Efficient time series processing for transformers and state-space models through token merging. In *International Conference on Machine Learning*, 2025.
- [2] Yuqi Nie, Nam H Nguyen, Phanwadee Sinthong, and Jayant Kalagnanam. A time series is worth 64 words: Long-term forecasting with transformers. In *International Conference on Learning Representations*, 2023.
- [3] Vijay Ekambaram, Arindam Jati, Pankaj Dayama, Sumanta Mukherjee, Nam H. Nguyen, Wesley M. Gifford, Chandra Reddy, and Jayant Kalagnanam. Tiny time mixers (ttms): Fast pre-trained models for enhanced zero/few-shot forecasting of multivariate time series. In *Advances in Neural Information Processing Systems*, 2024.
- [4] Gerald Woo, Chenghao Liu, Akshat Kumar, Caiming Xiong, Silvio Savarese, and Doyen Sahoo. Unified training of universal time series forecasting transformers. *arXiv:2402.02592*, 2024.
- [5] Rico Sennrich, Barry Haddow, and Alexandra Birch. Neural machine translation of rare words with subword units. In *Annual Meeting of the Association for Computational Linguistics*, 2016.
- [6] Leon Hetzel, Johanna Sommer, Bastian Rieck, Fabian J Theis, and Stephan Günemann. MAGNet: Motif-agnostic generation of molecules from scaffolds. In *International Conference on Learning Representations*, 2025.
- [7] Haixu Wu, Jiehui Xu, Jianmin Wang, and Mingsheng Long. Autoformer: Decomposition transformers with auto-correlation for long-term series forecasting. In *Advances in Neural Information Processing Systems*, 2021.
- [8] Tian Zhou, Ziqing Ma, Qingsong Wen, Xue Wang, Liang Sun, and Rong Jin. Fedformer: Frequency enhanced decomposed transformer for long-term series forecasting. In *International Conference on Machine Learning*, 2022.
- [9] Azul Garza and Max Mergenthaler-Canseco. Timegpt-1. *arXiv:2310.03589*, 2023.
- [10] Abhimanyu Das, Weihao Kong, Rajat Sen, and Yichen Zhou. A decoder-only foundation model for time-series forecasting. *arXiv:2310.10688*, 2023.
- [11] Abdul Fatir Ansari, Lorenzo Stella, Caner Turkmen, Xiyuan Zhang, Pedro Mercado, Huibin Shen, Oleksandr Shchur, Syama Sundar Rangapuram, Sebastian Pineda Arango, Shubham Kapoor, Jasper Zschiegner, Danielle C. Maddix, Hao Wang, Michael W. Mahoney, Kari Torkkola, Andrew Gordon Wilson, Michael Bohlke-Schneider, and Yuyang Wang. Chronos: Learning the language of time series. In *Transactions on Machine Learning Research*, 2024.
- [12] Kashif Rasul, Arjun Ashok, Andrew Robert Williams, Hena Ghonia, Rishika Bhagwatkar, Arian Khorasani, Mohammad Javad Darvishi Bayazi, George Adamopoulos, Roland Riachi, Nadhir Hassen, Marin Biloš, Sahil Garg, Anderson Schneider, Nicolas Chapados, Alexandre Drouin, Valentina Zantedeschi, Yuriy Nevmyvaka, and Irina Rish. Lag-llama: Towards foundation models for probabilistic time series forecasting. *arXiv:2310.08278*, 2023.
- [13] Mononito Goswami, Konrad Szafer, Arjun Choudhry, Yifu Cai, Shuo Li, and Artur Dubrawski. MOMENT: A family of open time-series foundation models. In *International Conference on Machine Learning*, 2024.
- [14] Yong Liu, Haoran Zhang, Chenyu Li, Xiangdong Huang, Jianmin Wang, and Mingsheng Long. Timer: Generative pre-trained transformers are large time series models. In *International Conference on Machine Learning*, 2024.
- [15] Shanghua Gao, Teddy Koker, Owen Queen, Thomas Hartvigsen, Theodoros Tsiligkaridis, and Marinka Zitnik. Units: A unified multi-task time series model. In *Advances in Neural Information Processing Systems*, 2024.
- [16] Haoyi Zhou, Shanghang Zhang, Jieqi Peng, Shuai Zhang, Jianxin Li, Hui Xiong, and Wancai Zhang. Informer: Beyond efficient transformer for long sequence time-series forecasting. In *AAAI Conference on Artificial Intelligence*, 2021.
- [17] Yong Liu, Haixu Wu, Jianmin Wang, and Mingsheng Long. Non-stationary transformers: Exploring the stationarity in time series forecasting. In *Advances in Neural Information Processing Systems*, 2022.
- [18] Shizhan Liu, Hang Yu, Cong Liao, Jianguo Li, Weiyao Lin, Alex X. Liu, and Schahram Dustdar. Pyraformer: Low-complexity pyramidal attention for long-range time series modeling and forecasting. In *International Conference on Learning Representations*, 2022.
- [19] Razvan-Gabriel Cirstea, Chenjuan Guo, Bin Yang, Tung Kieu, Xuanyi Dong, and Shirui Pan. Triformer: Triangular, variable-specific attentions for long sequence multivariate time series forecasting—full version. *arXiv:2204.13767*, 2022.
- [20] Luís Torgo and João Gama. Regression using classification algorithms. In *Intelligent Data Analysis*, 1997.

- [21] Rakshitha Wathsadini Godahewa, Christoph Bergmeir, Geoffrey I. Webb, Rob Hyndman, and Pablo Montero-Manso. Monash time series forecasting archive. In *Neural Information Processing Systems Datasets and Benchmarks Track*, 2021.
- [22] Alexey Dosovitskiy, Lucas Beyer, Alexander Kolesnikov, Dirk Weissenborn, Xiaohua Zhai, Thomas Unterthiner, Mostafa Dehghani, Matthias Minderer, Georg Heigold, Sylvain Gelly, Jakob Uszkoreit, and Neil Houlsby. An image is worth 16x16 words: Transformers for image recognition at scale. In *International Conference on Learning Representations*, 2021.
- [23] Yunhao Zhang and Junchi Yan. Crossformer: Transformer utilizing cross-dimension dependency for multivariate time series forecasting. In *International Conference on Learning Representations*, 2023.
- [24] Yuxuan Wang, Haixu Wu, Jiaxiang Dong, Guo Qin, Haoran Zhang, Yong Liu, Yunzhong Qiu, Jianmin Wang, and Mingsheng Long. Timexer: Empowering transformers for time series forecasting with exogenous variables. In *Advances in Neural Information Processing Systems*, 2024.
- [25] Yuxuan Wang, Haixu Wu, Jiaxiang Dong, Guo Qin, Haoran Zhang, Yong Liu, Yunzhong Qiu, Jianmin Wang, and Mingsheng Long. Timexer: Empowering transformers for time series forecasting with exogenous variables. In *Advances in Neural Information Processing Systems*, 2024.
- [26] Sabera J Talukder, Yisong Yue, and Georgia Gkioxari. TOTEM: TOkenized time series EMbeddings for general time series analysis. In *Transactions on Machine Learning Research*, 2024.
- [27] Zhicheng Chen, Shibo Feng, Zhong Zhang, Xi Xiao, Xingyu Gao, and Peilin Zhao. Sdformer: Similarity-driven discrete transformer for time series generation. In *Advances in Neural Information Processing Systems*, 2024.
- [28] Aaron van den Oord, Oriol Vinyals, and koray kavukcuoglu. Neural discrete representation learning. In *Advances in Neural Information Processing Systems*, 2017.
- [29] Luca Masserano, Abdul Fatir Ansari, Boran Han, Xiyuan Zhang, Christos Faloutsos, Michael W. Mahoney, Andrew Gordon Wilson, Youngsuk Park, Syama Rangapuram, Danielle C. Maddix, and Yuyang Wang. Enhancing foundation models for time series forecasting via wavelet-based tokenization. *arXiv:2412.05244*, 2024.
- [30] Yusuke Shibata, Takuya Kida, Shuichi Fukamachi, Masayuki Takeda, Ayumi Shinohara, and Takeshi Shinohara. Byte pair encoding: A text compression scheme that accelerates pattern matching. 1999.
- [31] Tim Elsner, Paula Usinger, Julius Nehring-Wirxel, Gregor Kobsik, Victor Czech, Yanjiang He, Isaak Lim, and Leif Kobbelt. Multidimensional byte pair encoding: Shortened sequences for improved visual data generation. *arXiv:2411.10281*, 2024.
- [32] Johanna Sommer, Leon Hetzel, David Lüdke, Fabian J Theis, and Stephan Günnemann. The power of motifs as inductive bias for learning molecular distributions. In *ICLR 2023 - Machine Learning for Drug Discovery workshop*, 2023.
- [33] Philip Gage. A new algorithm for data compression. In *C Users J.*, 1994.
- [34] Colin Raffel, Noam Shazeer, Adam Roberts, Katherine Lee, Sharan Narang, Michael Matena, Yanqi Zhou, Wei Li, and Peter J. Liu. Exploring the limits of transfer learning with a unified text-to-text transformer. In *Journal of Machine Learning Research*, 2020.
- [35] Sepp Hochreiter and Jürgen Schmidhuber. Long short-term memory. In *Neural computation*, 1997.
- [36] An Nguyen, Srijeet Chatterjee, Sven Weinzierl, Leo Schwinn, Martin Matzner, and Bjoern M. Eskofier. Time matters: Time-aware lstms for predictive business process monitoring. In *International Workshop on Leveraging Machine Learning in Process Mining*, 2020.
- [37] Albert Gu and Tri Dao. Mamba: Linear-time sequence modeling with selective state spaces. *arXiv:2312.00752*, 2023.
- [38] Marcel Kollovieh, Abdul Fatir Ansari, Michael Bohlke-Schneider, Jasper Zschiegner, Hao Wang, and Yuyang Bernie Wang. Predict, refine, synthesize: Self-guiding diffusion models for probabilistic time series forecasting. In *Advances in Neural Information Processing Systems*, 2023.
- [39] Caspar Meijer and Lydia Y Chen. The rise of diffusion models in time-series forecasting. *arXiv:2401.03006*, 2024.
- [40] Yaron Lipman, Ricky TQ Chen, Heli Ben-Hamu, Maximilian Nickel, and Matt Le. Flow matching for generative modeling. *arXiv:2210.02747*, 2022.
- [41] Marcel Kollovieh, Marten Lienen, David Lüdke, Leo Schwinn, and Stephan Günnemann. Flow matching with gaussian process priors for probabilistic time series forecasting. In *International Conference on Learning Representations*, 2025.
- [42] Leo Schwinn, Leon Bungert, An Nguyen, René Raab, Falk Pulsmeyer, Doina Precup, Björn Eskofier, and Dario Zanca. Improving robustness against real-world and worst-case distribution shifts through decision region quantification. In *International Conference on Machine Learning*, 2022.

- [43] Ian J. Goodfellow, Jonathon Shlens, and Christian Szegedy. Explaining and harnessing adversarial examples. In *International Conference on Learning Representations*, 2015.
- [44] Leo Schwinn, René Raab, An Nguyen, Dario Zanca, and Bjoern M. Eskofier. Exploring misclassifications of robust neural networks to enhance adversarial attacks. In *Applied Intelligence*, 2021.
- [45] Leo Schwinn, An Nguyen, René Raab, Leon Bungert, Daniel Tenbrinck, Dario Zanca, Martin Burger, and Bjoern Eskofier. Identifying untrustworthy predictions in neural networks by geometric gradient analysis. In *Uncertainty in Artificial Intelligence*, 2021.
- [46] Ashish Vaswani, Noam Shazeer, Niki Parmar, Jakob Uszkoreit, Llion Jones, Aidan N Gomez, Łukasz Kaiser, and Illia Polosukhin. Attention is all you need. In *Advances in Neural Information Processing Systems*, 2017.

## A An adaptive tokenization approach for time series

We provide pseudocode for generating a vocabulary of motifs and utilizing the motifs to tokenize a time series.

---

**Algorithm 1** Motif vocabulary generation according to equation (3).

---

**Input:** Dataset of discretized time series  $\mathcal{D}'$ , minimum motif occurrence  $p_{\min}$   
**Output:** Motif vocabulary  $\Psi$

```

 $\Psi \leftarrow \{\}$                                 ▷ Initialize empty vocabulary
 $z'_{\text{new}} \leftarrow M + 2$                   ▷ Account for quantized symbols and {MASK, EOS}
while true do                            ▷ Iteratively find motifs
    pair, cnt  $\leftarrow$  count  $(z'_i, z'_{i+1})$  in  $\mathcal{D}'$       ▷ Most frequent adjacent token pair and its count
    if cnt  $\geq p_{\min}$  then
         $z'_{\text{new}} \leftarrow z'_{\text{new}} + 1$           ▷ Allocate new token ID
         $\Psi[\text{pair}] \leftarrow z'_{\text{new}}$            ▷ Add new token to vocabulary  $(z'_i, z'_{i+1}) \rightarrow z'_{\text{new}}$ 
         $\mathcal{D}' \leftarrow \mathcal{D}' \setminus \{\text{pair}\} \cup \{z'_{\text{new}}\}$   ▷ Replace new token in dataset  $\mathcal{D}'$ 
    else
        return  $\Psi$                                 ▷ Token occurs to infrequent
    end if
end while

```

---

**Algorithm 2** Tokenization of a discretized time series according to equation (4).

---

**Input:** Discretized time series  $z'$ , motif vocabulary  $\Psi$   
**Output:** Tokenized time series  $z''$

```

for  $\psi$  in  $\Psi$  do                          ▷ Iterate over motifs
     $\psi_{\text{key}}, \psi_{\text{value}} \leftarrow \psi$       ▷  $\psi$  made of key value mappings  $(z'_i, z'_{i+1}) \rightarrow z'_{\text{new}}$ 
    for  $(z'_i, z'_{i+1})$  in  $z'$  do          ▷ Iterate over adjacent tokens in  $z'$ 
        if  $(z'_i, z'_{i+1})$  matches  $\psi_{\text{key}}$  then  ▷ Adjacent tokens match motif
            replace  $(z'_i, z'_{i+1})$  with  $\psi_{\text{value}}$  in  $z'$   ▷ Replace tokens with motif: shortens  $z'$  by 1
        end if
    end for
end for
 $z'' \leftarrow z'$ 
return  $z''$ 

```

---

## B Experiments

In this section, we list additional information about our experimental settings and resources.

**Datasets** We train our models and tokenizers on the recently proposed Chronos dataset [11]. It contains 11 M time series with over 11 B samples. Time series are curated from 28 real-world datasets or are generated synthetically. Due to its diverse nature and size, this dataset is well suited for training foundation models. We base our zero-shot evaluation on 5 commonly used time series datasets covering different forecasting applications: *ETTh1* and *ETTm1* measure the power load and temperature of electric transformers in hourly and quarter-hourly granularity [16]. *Weather* consists of meteorological quantities such as air temperature and is recorded every 10 minutes in 2020.<sup>1</sup> *Electricity* measures the energy demand of households in hourly granularity [21]. *Traffic* consists of hourly road occupancies in the San Francisco Bay Area [21].

**Hyperparameters** For training our T5 models, we utilize the hyperparameters of Chronos [11]. We expect even better results of our tokenizer when performing hyperparameter tuning. However, this is very expensive for large foundation models.

**Reproducibility of measurements** In our zero-shot evaluations, we use the same data splits as Wu et al. [7]. We evaluate once and report results on the test set. Regarding efficiency measures, we preferably report the compression of our tokenizer. This is a hardware-independent measure and the metric most related to our work. Needing to process fewer tokens or requiring fewer autoregressions directly translates to improvements in inference time of models, which, however, is a hardware-dependent measure. In cases where we compare models of different sizes, we report the inference time.

**Computational effort** Building the vocabulary of our tokenizer is an iterative process. Computationally, this is rather cheap, and we execute it on a single core of an Intel Xeon w5-3435X CPU. For training the T5 models, we utilize Nvidia H100 GPUs. In total, we train 31 foundation models of different sizes and with different tokenizers. We estimate the computational effort to reproduce our experiments in table 8. Please note that we reuse previously trained tokenizers and models in most of our experiments.

Table 8: Computational effort to reproduce our experiments.

Experiment	Device	Hours
<b>Tokenizer</b>		
low	CPU	9.1
medium	CPU	3.8
high	CPU	2.5
<b>T5 models</b>		
Main experiments	GPU	4800
Vocabulary complexity and generalization	GPU	2240
Training dataset size	GPU	2880

<sup>1</sup><https://www.bgc-jena.mpg.de/wetter/>

## C Results

Here, we show additional experiments and results.

### C.1 Preprocessing strategies

We conduct new experiments exploring different preprocessing strategies before discretizing a time series. Each of these methods features different tradeoffs between signal preservation and noise rejection. The first derivative of a time series  $z$  removes its offset, potentially yielding more similar motifs. However, derivatives generally introduce noise. To counter this, we utilize Gaussian kernels to smooth the time series. We further employ window-based norms. Besides uniform distributions for discretization, we experiment with Gaussian distributions and the precise data distribution  $P(\mathcal{D})$ .

We conduct an extensive search among combinations of preprocessing strategies on 500 tokenizers in figure 8. Uniform discretization with different number of bins  $M$  is Pareto optimal in balancing the average tokenization error  $\delta_{\text{avg}}$  and compression  $\bar{c}$ . We utilize this method throughout our paper.

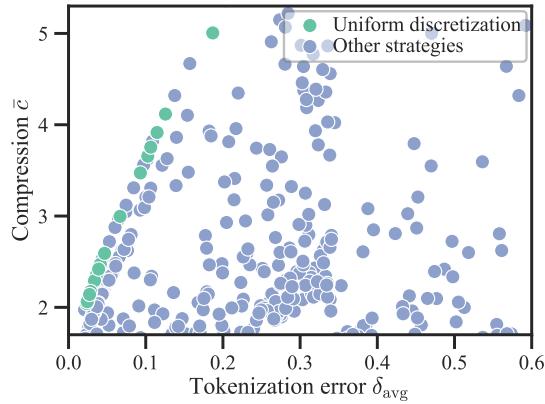


Figure 8: Comparison of uniform discretization with other tokenization preprocessing strategies including derivatives, Gaussian kernels, window-based norms, and Gaussian and data distribution based discretization.

## C.2 Efficiency improvements of adaptive tokenization

In figure 9, we report full results comparing our motif-based tokenization with Chronos foundation models that tokenize every sample. Further, we conduct additional experiments to compare our motif-based tokenization also with non-foundation models that tokenize every sample.

**Non-foundation models** Here, we compare our motif-based tokenization in T5 foundation models with non-foundation models that are specifically trained on the ETTh1, ETTm1, Weather, Electricity, and Traffic datasets. We utilize common time series architectures including Autoformer [7], FEDformer [8], Informer [16], Non-stationary [17], and vanilla transformers [46]. These models extract tokens as a multivariate slice for every time step. For comparison with our motif-based tokenization, we utilize the results of Götz et al. [1] for 2 layer models in table 9. The authors forecast 96 time series samples from 192 context tokens.

In 19 out of 25 cases, our foundation model in zero-shot setting outperforms the specifically trained models in forecasting quality.

Table 9: Comparison of our motif-based tokenization with conditional decoding (cd) and without in zero-shot foundation models with non-foundation models that tokenize every sample, based on forecasting quality (MSE). We highlight values that are worse than our method.

Dataset	Ours	Ours <sup>cd</sup>	Autoformer	FEDformer	Informer	Non-stationary	Transformer
ETTh1	0.52	0.46	0.42	0.38	0.87	0.55	0.75
ETTm1	0.64	0.45	0.44	0.36	0.65	0.42	0.52
Weather	0.25	0.24	0.28	0.27	0.35	0.19	0.25
Electricity	0.15	0.14	0.18	0.20	0.30	0.17	0.25
Traffic	0.59	0.57	0.63	0.59	0.68	0.60	0.66

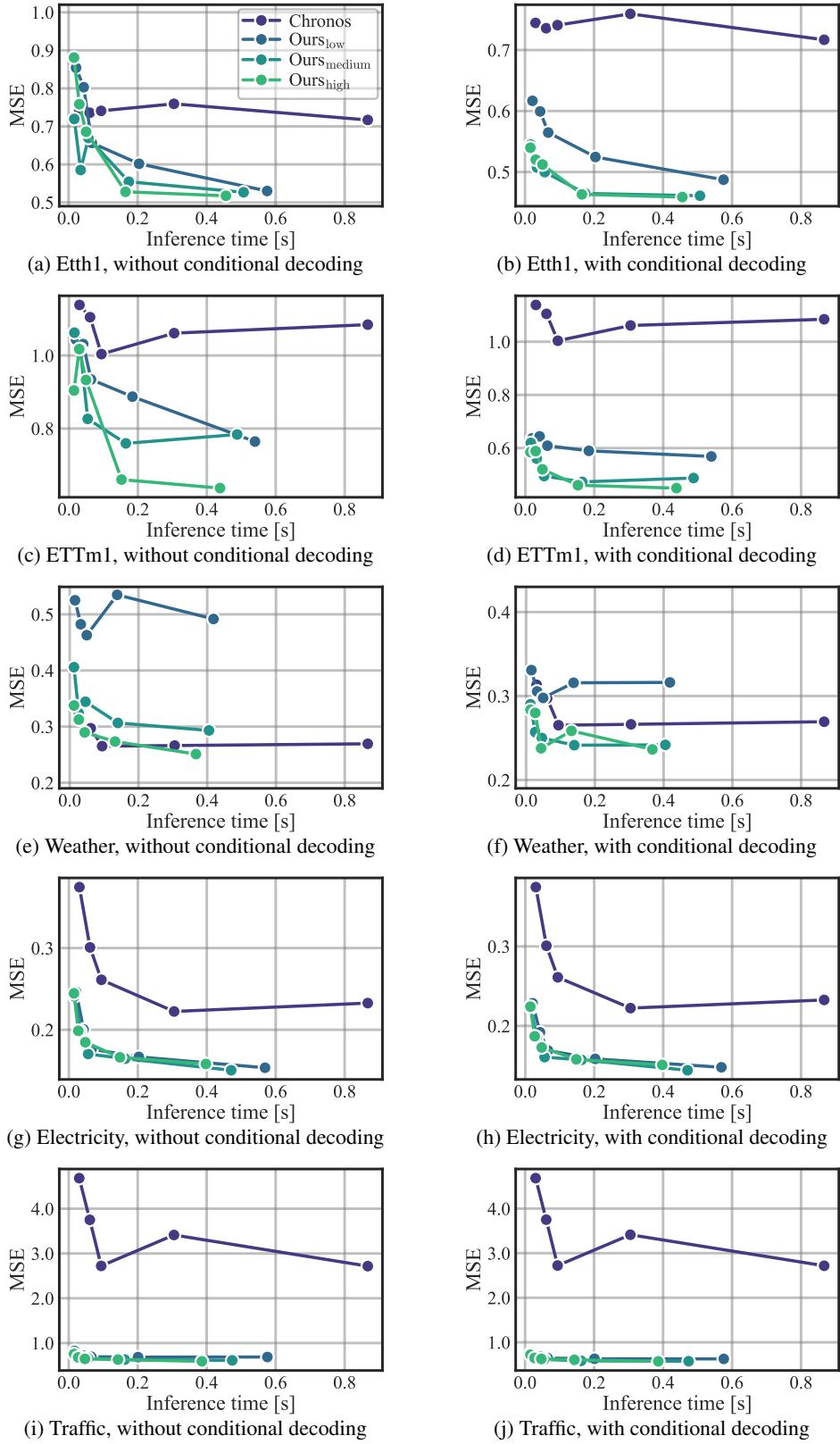


Figure 9: Comparison of our motif-based tokenization with and without conditional decoding with Chronos models tokenizing every sample during zero-shot evaluation on 5 datasets.

### C.3 Conditional decoding

In this section, we provide full results on model-dependent conditional decoding trained on the models' predictions in figure 10. We further explore conditional decoding in a model-independent setting.

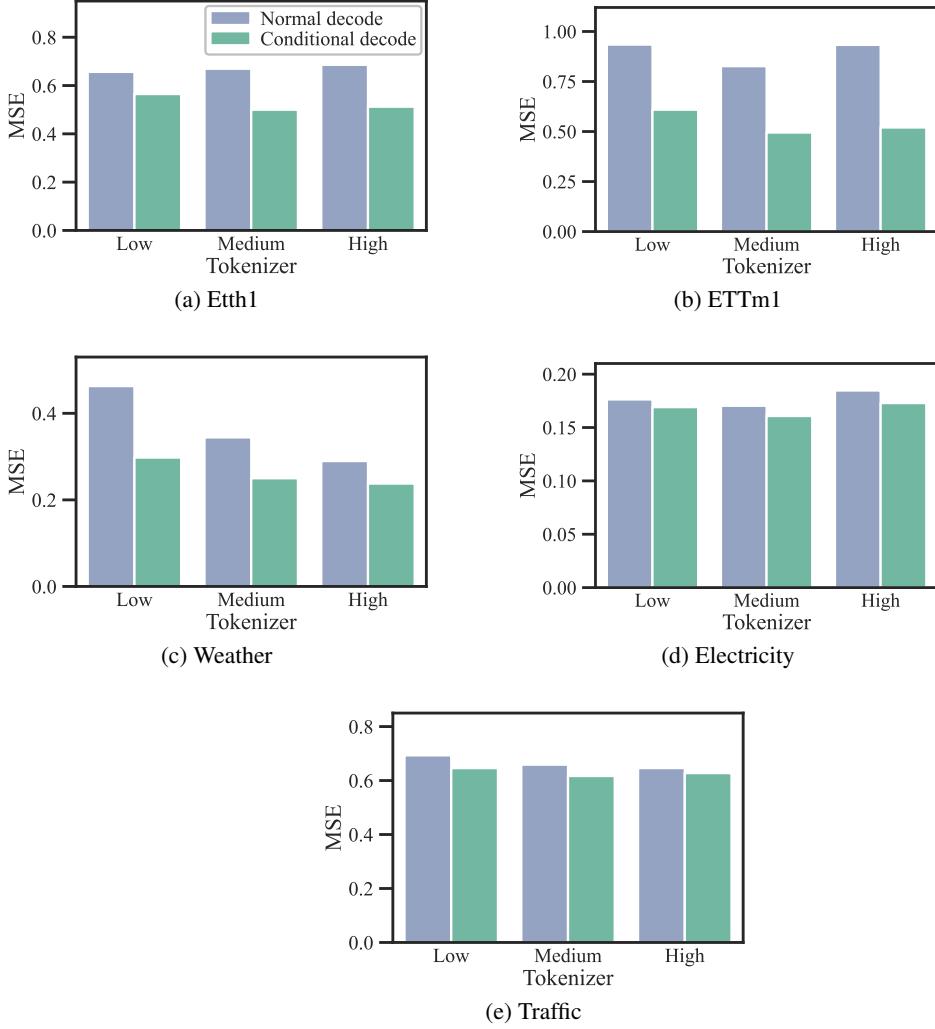


Figure 10: Conditional decoding improves forecasting quality for 3 tokenizers in small models on 5 datasets.

**Model-independent conditional decoding** Here, we investigate conditional decoding in a model-independent setting to universally improve the forecasting quality of foundation models. To this end, we train conditional decoding to dequantize quantized time series  $z' = \mathbf{q}_\Omega(z)$ . Here, conditional decoding is model-independent and can only mitigate the quantization error as this is the only error introduced. This is why we report MSE improvements relative to the average quantization error  $\delta_{\text{avg}}$  on the respective evaluation datasets. We utilize the Chronos dataset for training and 5 datasets for zero-shot evaluation. We demonstrate conditional decoding on our 3 tokenizers and small models.

In 14 out of 15 settings in table 10, conditional decoding improves forecasting quality. On ETTm1, it mitigates up to 96.9 % of the quantization error of our tokenizer with high compression. This enables us to build tokenizers with even higher compression and quantization error, as it can be effectively recovered.

Table 10: Conditional decoding in model-independent setting recovers the quantization errors of our 3 tokenizers by varying degrees on 5 datasets.

Dataset	Compression		
	low	medium	high
ETTh1	22.0 %	21.3 %	49.4 %
ETTm1	61.1 %	16.1 %	96.9 %
Weather	87.6 %	38.9 %	25.0 %
Electricity	13.5 %	23.0 %	21.4 %
Traffic	0.0 %	0.7 %	1.7 %

#### C.4 Adaptive compression of diverse time series

Here, we show the full results of our investigations on adaptive compression. In figure 11, we explore variable compression within the same dataset and show tokenized time series for visual inspection in figure 13. We further investigate relations between input and generation compression.

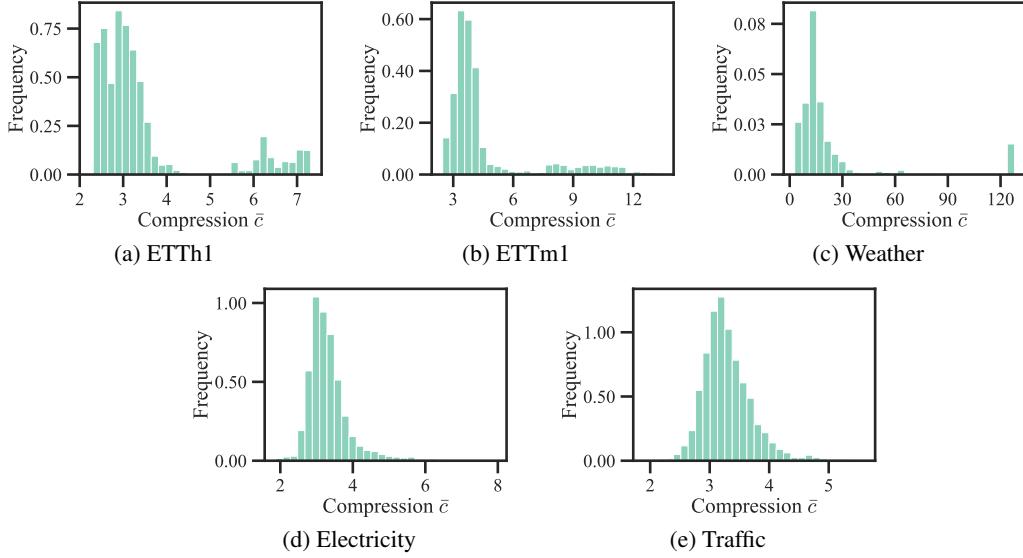


Figure 11: Histograms showing variable compressions of our medium tokenizer within 5 datasets.

**Input and generation compression** We conduct additional experiments to explore relations between input compression and the model’s generations. The model can either predict long motifs with high compression directly or sequences of their shorter components during autoregressive generation as we describe in section 5.7. Therefore, we expect a greater input compression  $\bar{c}_{\text{in}}$  compared to generation compression  $\bar{c}_{\text{out}}$ . We utilize different tokenizers in small models for this experiment. In line with our hypothesis, we find correlations between input and generation compression on all 5 datasets in figure 12. More complex input tokens generally benefit the prediction of longer motifs.

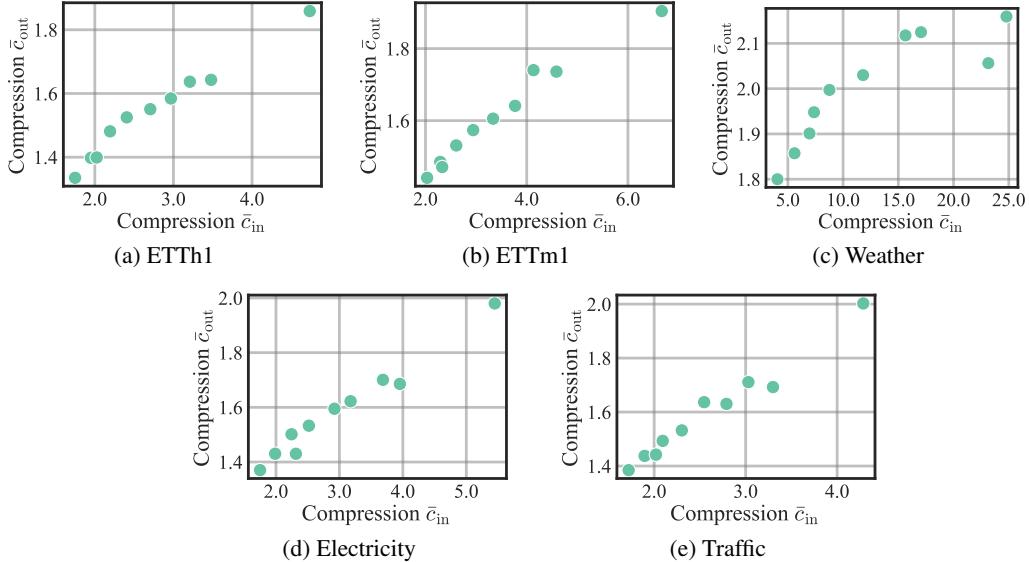


Figure 12: Comparison of input and generation compression of small models and multiple tokenizers on 5 datasets. Please note that efficiency gains in table 2 and figures 3 and 9 relate to the more conservative generation compression.

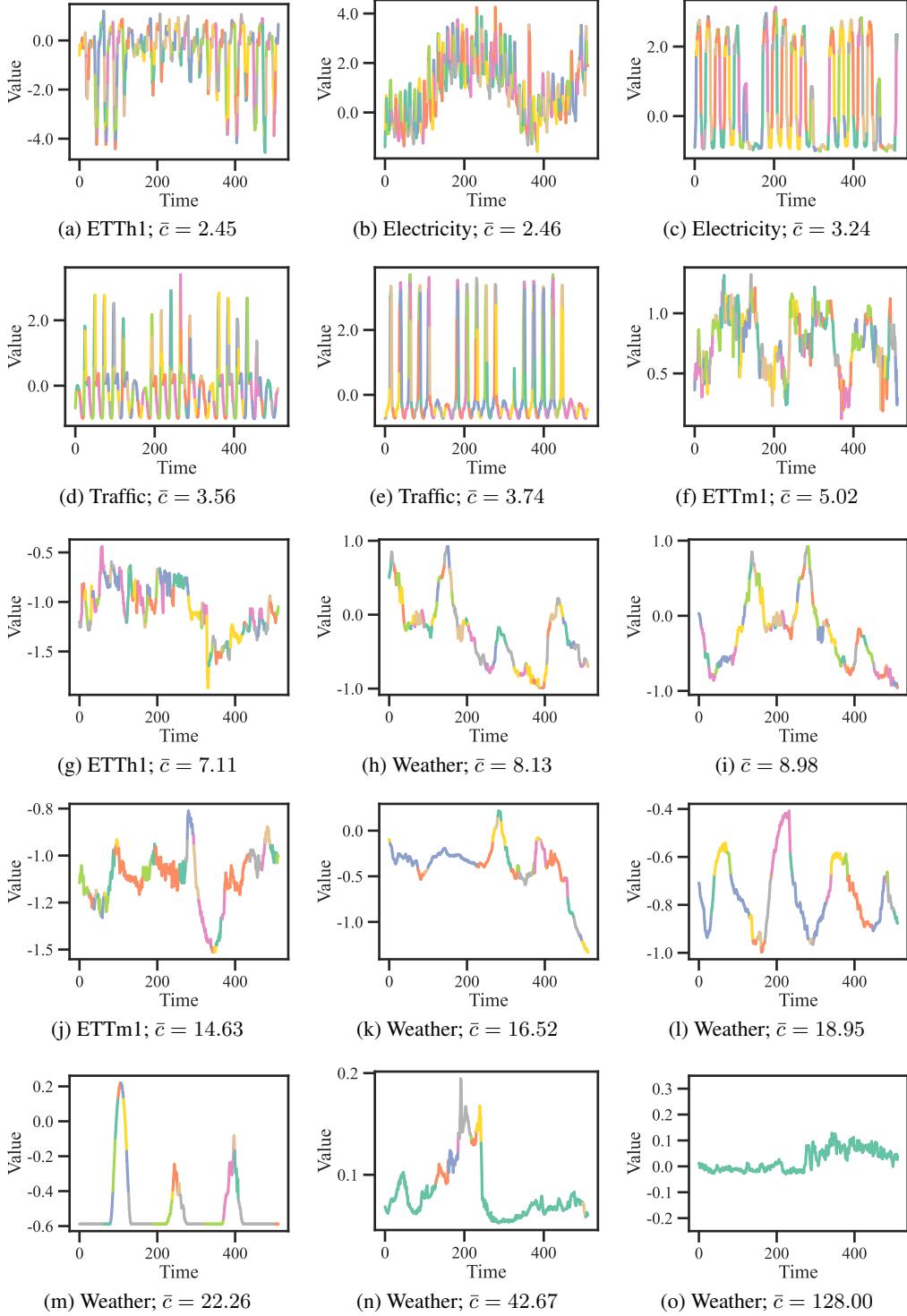


Figure 13: Our medium tokenizer exploits periodically recurring motifs and compresses time series adaptively depending on pattern complexity on 5 datasets. Specifically, (o) highlights the noise rejection ability of discretization.

### C.5 Vocabulary complexity and generalization

In figure 14, we provide full results of our investigations on token occurrence. We further give more insights into the vocabulary generation process.

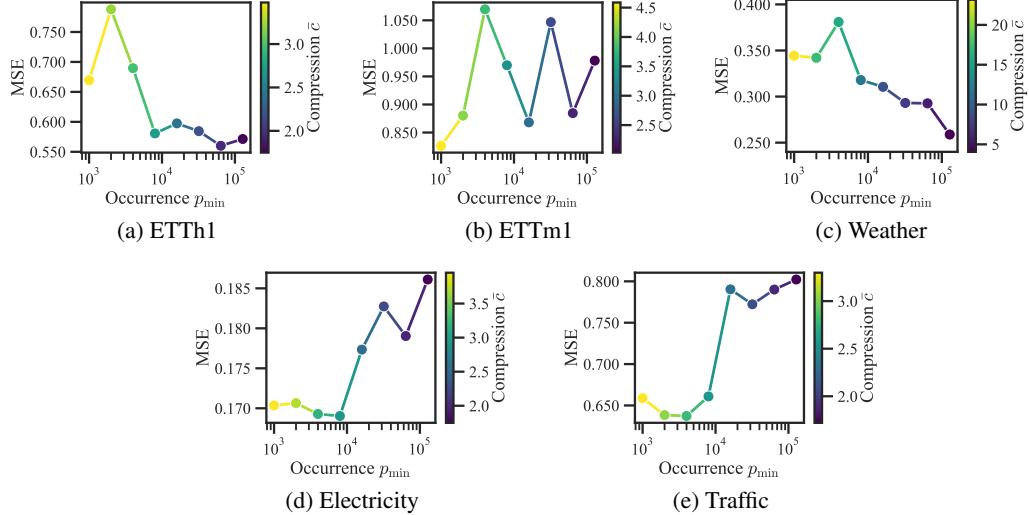


Figure 14: Varying token occurrence  $p_{\min}$  influences forecasting quality for small models on 5 datasets.

**Vocabulary generation process** Here, we further highlight the influence of quantization granularity and token occurrence on compression and vocabulary complexity. In figure 15, we show the iterative process of finding longer motifs with higher compression  $\bar{c}$  during vocabulary generation of  $\Psi$ . These more complex motifs, however, are more specialized and occur less often ( $p_{\min}$ ). A lower number of quantization bins  $M$  results in smaller, less data-specific vocabularies with higher compression.

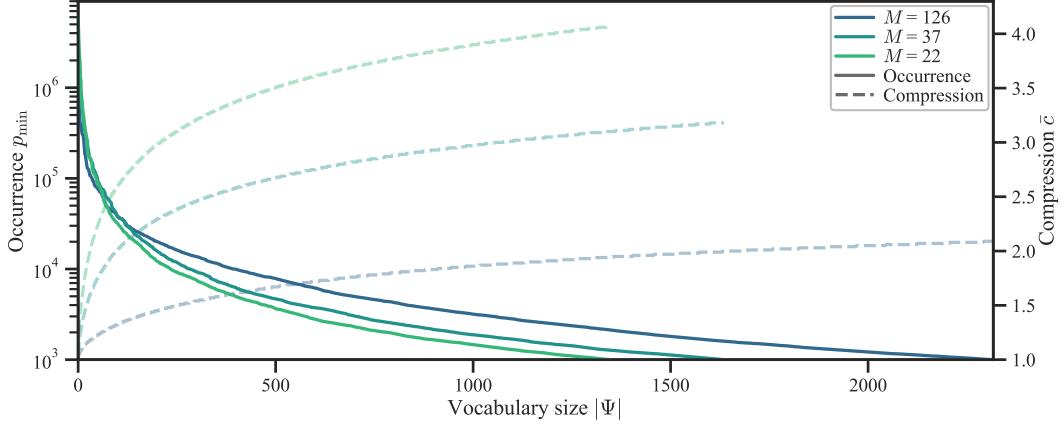


Figure 15: Influence of quantization bins  $M$  and token occurrence  $p_{\min}$  on vocabulary size  $|\Psi|$  and compression  $\bar{c}$  for tokenizers on the Chronos dataset.

### C.6 Training dataset size

We vary the dataset size for building our vocabulary of motifs  $\Psi$  and provide full results here. To this end, we utilize our three tokenizers with low, medium, and high compression and report vocabulary statistics in table 11 and forecasting quality in table 12. For conditional decoding, we observe similar behavior when estimating conditional distributions  $\hat{\Omega}$  in table 13. Here, a larger subset also leads to best representations.

Table 11: Vocabulary statistics for 3 tokenizers trained on Chronos subsets of varying sizes  $N$ .

Compression	$N = 1\text{ k}$		$N = 10\text{ k}$		$N = 100\text{ k}$		$N = 1\text{ M}$	
	$ \mathcal{V} $	$\bar{c}$	$ \mathcal{V} $	$\bar{c}$	$ \mathcal{V} $	$\bar{c}$	$ \mathcal{V} $	$\bar{c}$
low	2789	2.11	2461	2.08	2445	2.08	2441	2.09
medium	1974	3.24	1707	3.16	1675	3.18	1681	3.18
high	1618	4.14	1392	4.05	1373	4.06	1360	4.06

Table 12: Forecasting quality (MSE) on 5 evaluation datasets for 3 tokenizers trained on Chronos subsets of varying sizes  $N$ .

Dataset	Compression	$N = 1\text{ k}$	$N = 10\text{ k}$	$N = 100\text{ k}$	$N = 1\text{ M}$
ETTh1	low	0.712	0.712	0.656	0.659
	medium	0.562	0.698	0.669	0.615
	high	0.751	0.712	0.686	0.593
ETTm1	low	0.944	0.919	0.934	0.913
	medium	0.877	0.898	0.826	0.857
	high	0.985	0.819	0.933	0.821
Weather	low	0.473	0.538	0.463	0.474
	medium	0.342	0.310	0.344	0.333
	high	0.355	0.333	0.290	0.307
Electricity	low	0.178	0.183	0.176	0.175
	medium	0.173	0.167	0.170	0.164
	high	0.191	0.176	0.185	0.178
Traffic	low	0.724	0.680	0.693	0.671
	medium	0.643	0.639	0.659	0.622
	high	0.625	0.621	0.646	0.620

Table 13: Influence of dataset size  $N$  on estimating conditional decoding distributions  $\hat{\Omega}$  for small models on the Electricity dataset.

$N$	MSE
Normal decoding	0.170
1 k	0.168
10 k	0.162
100 k	0.161

### C.7 Learned token representations

Here, we show our full explainability analysis of learned motif representations.

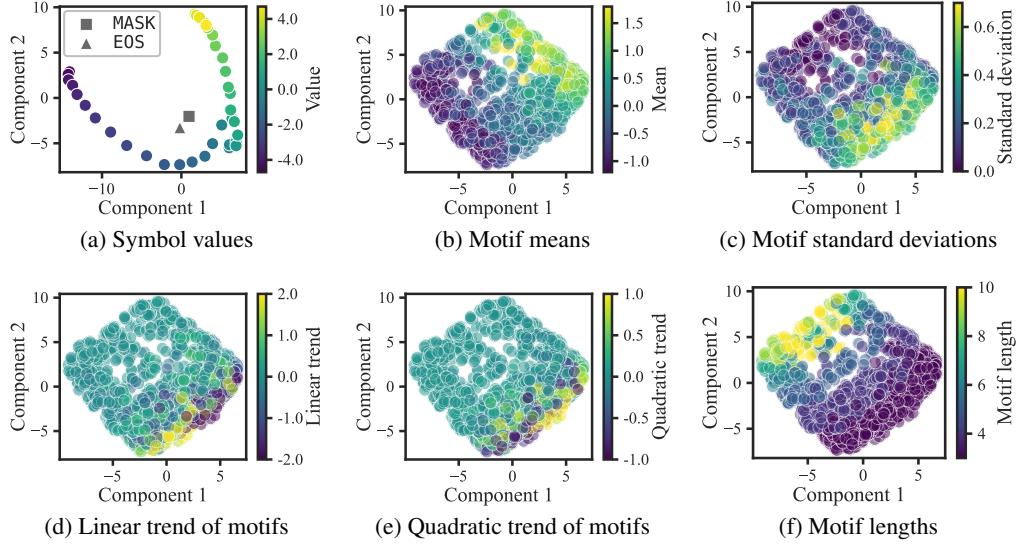


Figure 16: Principal component analysis of token embeddings of our medium tokenizer in a small model.

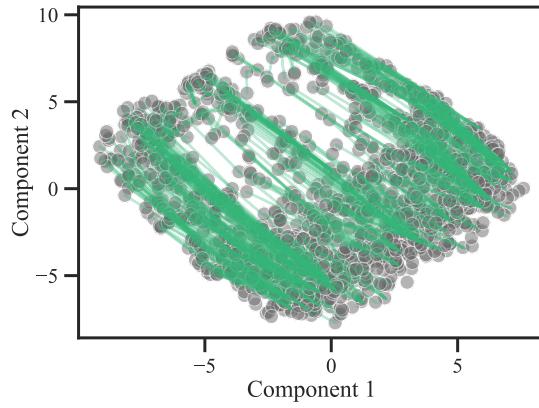


Figure 17: Parent-child token relations analyzed through principal component analysis of token embeddings from our medium tokenizer in a small model. Children and their first parents are connected.