Bad Reputation with Simple Rating Systems

Caio Lorecchio
Universitat de Barcelona
School of Economics

Daniel Monte
Fundação Getúlio Vargas
São Paulo School of Economics

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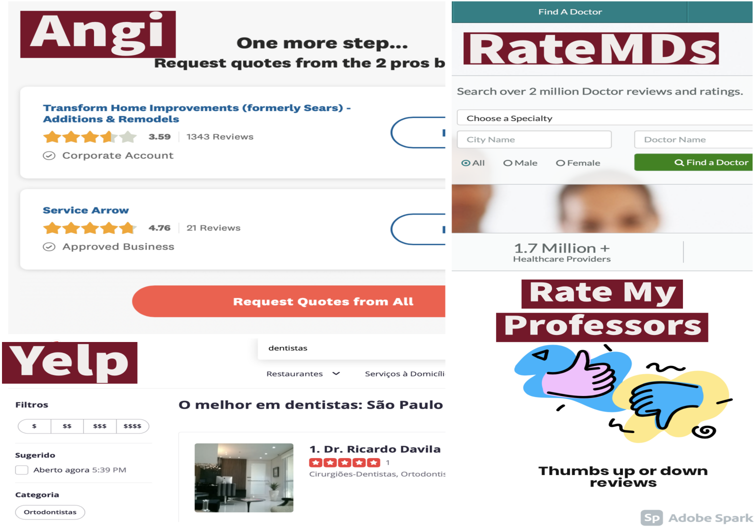
Motivation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Transform Home Improvements (formerly Sears) - Additions & Remodels
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Environment

A long-run expert interacts with a sequence of short-run customers.

In each period, a customer has the option of hiring the expert's services.

If hired, the expert observes a problem and proposes a treatment.

- A severe problem requires a high-cost treatment,
- a mild one is solvable through a low-cost treatment.

Environment

The customer, however, cannot determine how severe the problem is.

Moreover, each customer has a common prior belief that the expert is a "**bad**" commitment type that always provides the expensive treatment.

Without info censoring, reputational effects are so strong that the market collapses (Ely and Välimäki, 2003).

We add an uninformed intermediary committed to a simple rating system.

Simple rating systems

A **rating system** consists of

1. A finite message space (ratings);
2. an initial distribution over messages;
3. a transition rule from current messages and observable outcomes to a distribution over future messages.

Past info about outcomes cannot be used directly in the transition rule.

The current rating is the only source of info for each customer.

Motivations: storage/legal constraints, plausibility, complexity ...

Results

How can simple rating systems benefit customers and experts?

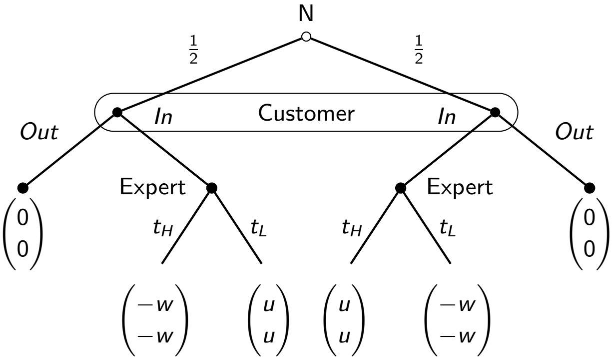
1. To maximize customers' equilibrium payoff, more ratings are better.
   - Rating inflation: the system almost never issues intermediary ratings.
   - Rating persistence: expert's intertemporal incentives matter for the construction (higher patience $\rightarrow$ more persistent ratings).

2. To maximize expert's equilibrium payoff, it suffices two ratings.
   - It improves upon the no-censoring and full-censoring environments.
   - No bad reputation effect: expert gets hired even in the long run.

This talk

1. Model
2. Optimal systems for customers
   a. An upper bound on the equilibrium payoff
   b. Construction of an optimal system
3. Conclusion

Model - stage game



Model - stage game

$\rho \in (0, 1)$: prior belief about expert being bad (always plays t_H).

In a one-shot game, the expert tells the truth and a customer hires if

$$\rho \left(\frac{u - w}{2} \right) + (1 - \rho) u \geq 0 \Rightarrow \rho \leq \rho^* := \frac{2u}{u + w}.$$

Model - bad reputation

No customer hires the expert if $\rho > \rho^*$.

What if $\rho \leq \rho^*$?

Theorem (Ely and Välimäki, 2003)

Let $\bar{V}(\rho, \delta)$ represent the supremum of all discounted payoffs in a Nash equilibrium of the repeated game. It must be that

$$\lim_{\delta \to 1} \bar{V}(\rho, \delta) = 0.$$

Model - rating systems

There is a trivial way to avoid bad reputation: full censoring.

It does not tempt the expert to lie...

... but it does not bring any additional benefit to customers.

Is there a rating system that improves upon full memory and no memory?

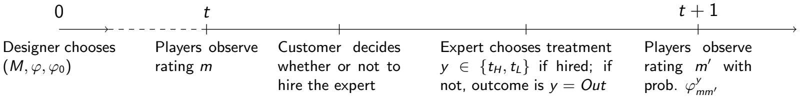
What is the optimal rating system?

Model - rating systems

A rating system is a triple (M, φ, φ_0) where

- M is a finite rating set;
- $\varphi : M \times \{t_H, t_L, Out\} \to \Delta(M)$ is a Markov transition rule;
- $\varphi_0 \in \Delta(M)$ is an initial distribution over ratings.

Customers only observe current ratings.



Model - strategies

The hiring probability α depends exclusively on ratings.

The expert offers the correct treatment to each problem according to:

$$\beta^H : M \to [0,1],$$
$$\beta^L : M \to [0,1].$$

Model - transitions

Probability of observing t_H in $m \in M$ if the expert is strategic:

$$\gamma_m := \frac{1}{2}\beta_m^H + \frac{1}{2}(1 - \beta_m^L).$$

Probability of moving from m to m' for every type of expert:

$$\begin{aligned}
\tau_{mm'}^S &= \alpha_m[\gamma_m \varphi_{mm'}^H + (1-\gamma_m)\varphi_{mm'}^L] + (1-\alpha_m)\varphi_{mm'}^{Out}, \\
\tau_{mm'}^B &= \alpha_m \varphi_{mm'}^H + (1-\alpha_m)\varphi_{mm'}^{Out}.
\end{aligned}$$

Model - posteriors

A rating system and strategies for the players induce Markov transition matrices defined over the rating set.

For this talk, we assume that all ratings will be visited in equilibrium.

- Two stationary distributions: $(f_m^S)_{m \in M}$ e $(f_m^B)_{m \in M}$.

Probability of expert being bad after observing *rating m*:

$$\rho_m = \frac{f_m^B \rho}{f_m^B \rho + f_m^S (1 - \rho)}.$$

Model - equilibrium

Each customer's expected payoff in equilibrium, in rating m:

$$\nu(\rho_m) := \rho_m \left(\frac{u-w}{2}\right) + (1-\rho_m)[\beta_m u - (1-\beta_m)w].$$

$\beta_m := \frac{1}{2}\beta_m^H + \frac{1}{2}\beta_m^L$ is probability of getting the right treatment at m.

Each customer hires after observing m if and only if $\nu(\rho_m) \geq 0$.

Model - equilibrium

Expert's continuation value for each problem, after observing rating m:

$$V_m^H(\rho,\delta) := (1-\delta)[\beta_m^H u - (1-\beta_m^H)w] + \\ \delta \sum_{m' \in M} [\beta_m^H \varphi_{mm'}^H + (1-\beta_m^H)\varphi_{mm'}^L] V_{m'}(\rho,\delta),$$

$$V_m^L(\rho,\delta) := (1-\delta)[\beta_m^L u - (1-\beta_m^L)w] + \\ + \delta \sum_{m' \in M} [\beta_m^L \varphi_{mm'}^L + (1-\beta_m^L)\varphi_{mm'}^H] V_{m'}(\rho,\delta).$$

Expert's strategy must maximize each continuation value, at every rating.

Model - optimal systems

Let $f_m := f_m^B \rho + f_m^S (1-\rho)$.

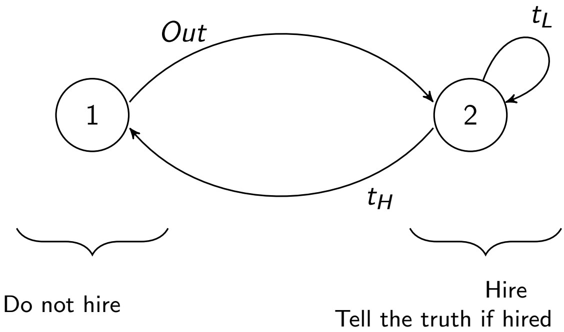
Customers' stationary value from the interaction (equilibrium payoff):

$$v(\rho, \delta) = \sum_{m \in M} f_m \alpha_m \nu(\rho_m).$$

An **informative rating system** has $|M| \geq 2$ and induces an eq. such that customers hire in at least one rating and do not hire in at least another.

<u>Our goal</u>: an informative system that maximizes customers' eq. payoff.

Example of a rating system



Example of a rating system

Transition matrices are

$$T^B = \begin{array}{c} \\ 1 \\ 2 \end{array}\!\!\begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \end{array}, \qquad T^S = \begin{array}{c} \\ 1 \\ 2 \end{array}\!\!\begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \end{array}.$$

Stationary distributions are $f^B = (\frac{1}{2}, \frac{1}{2})$ and $f^S = (\frac{1}{3}, \frac{2}{3})$.

The equilibrium beliefs are $\rho_1 = \frac{3\rho}{3\rho+2(1-\rho)}$ and $\rho_2 = \frac{3\rho}{3\rho+4(1-\rho)}$.

For $\frac{2\rho^*}{3-\rho^*} < \rho \leq \frac{4\rho^*}{3+\rho^*}$, we get $\rho_2 \leq \rho^*$ and $\rho_1 > \rho^*$.

Example of a rating system

Continuation values are

$$V_1(\rho, \delta) = \frac{2u\delta}{2+\delta}, \qquad\qquad V_2(\rho, \delta) = \frac{2u}{2+\delta}.$$

It is optimal for the expert to tell the truth whenever hired.

For a range of prior beliefs, the system eliminates the bad reputation effect and improves upon no-memory and full-memory settings.

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Optimal systems - belief upper bound

The following lemma is a well-known result in Markov processes.

Lemma

If the rating space M is partitioned into two sets C and C', then in the steady state the probability of transitioning from C to C' must equal the probability of transitioning from C' to C, for $\omega \in \{S, B\}$:

$$\sum_{m \in C} \sum_{n \in C'} f_m^\omega \tau_{mn}^\omega = \sum_{n \in C'} \sum_{m \in C} f_n^\omega \tau_{nm}^\omega.$$

Optimal rating systems - belief upper bound

Proposition

In any informative rating system, all non-hiring ratings i obey

$$\frac{f_i^B}{f_i^S} \leq 2\left[\frac{\rho^*(1-\rho)}{\rho(1-\rho^*)}\right].$$

It implies that all induced beliefs must be at most $\bar{\rho}$, defined by

$$\bar{\rho} := \frac{2\rho^*}{1+\rho^*}.$$

Optimal systems - belief upper bound

Proof sketch (binary rating systems with truth-telling):

- If customer hires in rating 2, but not in rating 1, it must be that

$$\frac{f_1^B}{f_1^S} = \frac{f_1^B \varphi_{12}^{Out}}{f_1^S \varphi_{12}^{Out}} = \frac{f_2^B \tau_{21}^B}{f_2^S \tau_{21}^S}.$$

- If there is a positive probability of the bad expert transitioning from 2 to 1, there is a positive probability for the strategic expert as well:

$$\tau_{21}^S = \frac{1}{2}\varphi_{21}^H + \frac{1}{2}\varphi_{21}^L \geq \frac{1}{2}\tau_{21}^B.$$

Optimal systems - belief upper bound

- In rating 2, it must that $\rho_2 \leq \rho^*$ or $\frac{f_2^B}{f_2^S} \leq \frac{\rho^*(1-\rho)}{\rho(1-\rho^*)}$.

- Collecting results,

$$\frac{f_1^B}{f_1^S} = \frac{f_1^B \varphi_{12}^{Out}}{f_1^S \varphi_{12}^{Out}} = \frac{f_2^B \tau_{21}^B}{f_2^S \tau_{21}^S} \leq 2 \left[\frac{\rho^*(1-\rho)}{\rho(1-\rho^*)}\right].$$

- We prove that in a binary system, it is optimal for the expert to be honest whenever hired.

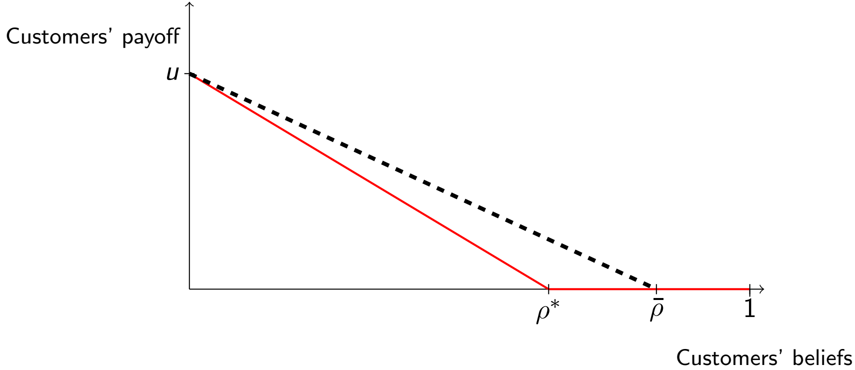
Optimal systems - payoff upper bound

Proposition

There is an upper bound on the equilibrium payoff that customers can achieve in equilibrium, given every $\rho \in (0, \bar{\rho})$*:*

$$v(\rho, \delta) \leq u \left[\frac{\rho - \bar{\rho}}{\bar{\rho}} \right].$$

Optimal systems - payoff upper bound



Optimal systems - construction

Theorem

For any prior belief $\rho \in (0, \bar{\rho})$ and any discount factor $\delta \in (0, 1)$,

a. *there is a sequence of informative rating systems in which the customers' equilibrium payoff approaches the maximum bound.*
b. *This sequence of rating systems has customers hiring in every rating except 1 and the expert telling the truth whenever hired.*

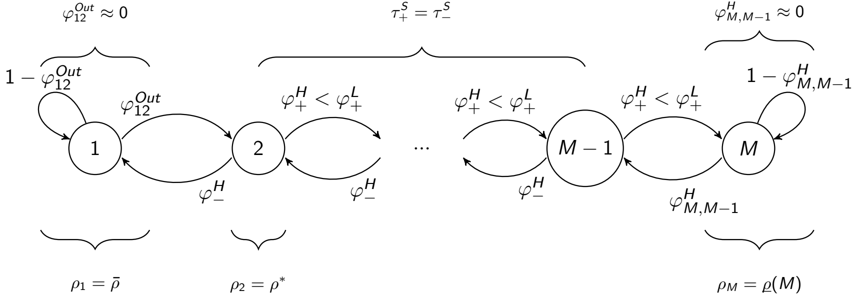
Optimal systems - construction

We need the auxiliary result.

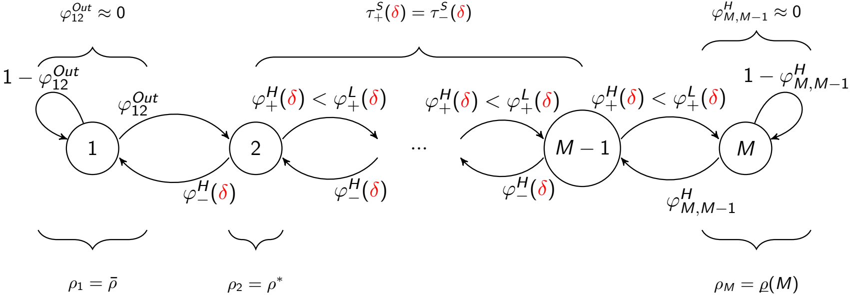
Lemma

In any rating system, in equilibrium, at least one rating must have a belief (weakly) lower than the prior, and at least one rating must have a belief (weakly) higher than the prior.

Optimal systems - construction



Optimal systems - construction



Optimal system - construction

Features:

- Higher the number of ratings, lower the beliefs in hiring ratings.
- Rating inflation: the system almost never issues intermediary ratings.
- Rating persistence: expert's intertemporal incentives matter for the construction (higher patience $\rightarrow$ more persistent ratings).

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Summary

People rely on online reputation systems to hire experts, but such systems can generate wrong incentives for experts and mistrust from customers.

We characterize what can be achieved with simple systems.

They overcome bad reputation and improve upon extreme info settings.

- From the customers' perspective, more ratings are better.
- Equilibrium characteristics: inflation and persistence.

Related literature

Learning with limited memory: Hellman and Cover (1970); Wilson (2014); Monte (2013).

Rating systems: Ekmekci (2011); Spagnolo and Kovbasyuk (2021); Vong (2021); Monte and Lorecchio (2022).

Informational restrictions to overcome bad reputation: Ely and Välimäki (2003); Lillethun (2017); Sperisen (2018).

Plausibility of simple rules: Compte and Postlewaite (2015).