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Using fuzzy inference system for architectural space analysis

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ABSTRACT

Though architectural space is the main source and the only indispensable component of any architectural construction, in many cases its boundaries are uncertain, leading intuitive spatial design. Creating a mathematical model of architectural space with concrete results will offer many possibilities for design process in analysing spatial organization, independently from in architect's experience and intuitions. This paper presents a fuzzy inference system based spatial analysis model for spatial analysis for architectural design which brings many advantages to design process. The aim of this article is to investigate the potential of a fuzzy system with a Mamdami inference engine, considering different numbers of membership functions. Two venues have been selected and the fuzzy inference system based spatial analysis model is applied. For better judgement, outcomes of the model have been compared to depthmap analysis model. The results of the model indicate that fuzzy inference system based spatial analysis model performs very well, even with the limited and imprecise data. Such prototype can evolve into a tool for identifying spatial formations for improvements during the architectural design process.

1. Introduction

The concept of architectural space, which the architecture profession is based on, is the main source and the only indispensable component of any architectural structure. However, this most indispensable component is not limited with certain boundaries most of the time. On the contrary, it is usually impossible to say where an architectural space exactly starts or ends. Architectural spaces are usually connected with each other by soft outlines. A part of an architectural space may be a part of another architectural space at the same time [1,2]. Especially in architectural constructions where many of such spaces are to be designed, an architect behaves intuitively depending on his professional knowledge and experience. In most cases it is possible to see whether the space is as desired or not, only after the construction is finished. While an architect with enough knowledge and experience will be able to design the space with desired properties, an inexperienced architect with less professional knowledge will not succeed in designing the space with desired properties and the results will mostly be irrecoverable.

The reason for creating a mathematical model of the architectural space is of particular significance in that such a model will offer many possibilities in analysing whether the design has enough components to form the desired spatial organization or not. Thus it will be possible to analyse and model an architectural space with uncertain boundaries independently from an architect's experience and intuitions. Through this model, any designed space can be cross-checked by its spatial properties and modified to provide the desired spatial organization prior to its construction.

Thanks to the architectural Computer Aided Design (CAD) technologies which have become a standart in architectural design process within the last decades, it is possible to visualize any architectural design before it has been constructed. However the results of these systems are not three-dimensional and in their original scales or most importantly in some cases they can rather be misleading than being supportive in the evaluation process [3,5]. In addition to the misleading results like deformed perspectives, most of the time the evaluation process can only be based on visual sense. Space syntax [6–9] and its main tools, axial map [6,7] and depthmap [10–13], are the most common tools used for architectural space analysis today [14]. Yet as it will be explained in further sections, spatial perception is a multi-sensored process [15], results of those systems are not totaly satisfactory enough for spatial analysis.

One of the mathematical models to compute uncertainties is fuzzy logic [16]. Fuzzy logic and sets were first introduced by Zadeh in 1965 to represent and manipulate imprecise data. Fuzzy logic and fuzzy sets were first introduced by Zadeh as a generalization of conventional sets theory [18,19]. This approach has been an alternative to classical Boolean logic for problems where uncertainties in means of imprecision, vagueness, imperfect knowledge exist. In Zadeh's theory, objects of the sets are represented with their membership degrees to that set [17,20–23]. The membership degree of an object determines the state of its belonging to a set. In fuzzy logic, elements of the sets are expressed by membership degrees.

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When x is a defined element of the fuzzy variable space X and $\mu_A(x)$, membership degree of this element to a fuzzy set A which is defined in this fuzzy variable space X, fuzzy set A will be expressed as

$$A : \{(x, \mu_A(x));\ \ x \in X\}. \qquad (1)$$

The membership degree $\mu_A(x)$ takes a positive value between 0 and 1 for every element of the fuzzy set A [27,28]. Fuzzy set A is defined by the μ_x membership function which shows the membership degree of the set's elements expressed by x. Membership functions show one element's interest in one fuzzy set. The most important advantage of the fuzzy sets is the possibility of defining blurry limits between different sets, which means that an object may belong to different sets with different degrees of membership. Because of architectural spaces' uncertain and fuzzy characteristics making a mathematical model and analysis of it will be possible by fuzzy sets and fuzzy logic [24–26]. Although today the usage of fuzzy logic and fuzzy sets are very limited and relatively new in architecture, it will provide great advantages on spatial modeling and analysis in design process.

The aim of this article is to investigate the potential of a fuzzy system with a Mamdami inference engine, considering different numbers of membership functions. The paper is organized as follows. First, well known spatial analysis models in literature are explained shortly. Definitions of architectural space, spatial perception and characteristics of the spatial elements are briefly explained in Section 3. In Section 4, general information about the applied model is explained. The fuzzy inference system based spatial analysis model is applied on two selected venues in Section 5. For better judgement, outcomes of the model have been compared to the results of the depthmap tool which is commonly used in space syntax to analyse physical properties of architectural venues. Finally, advantages of the proposed spatial analysis model will be discussed.

2. The subject of spatial analysis and well known models

In this section, the subject of spatial analysis and the well known models are explained.

2.1. Spatial analysis, space syntax and axial map

Originally the term of space syntax was conceived by Bill Hillier, Julien Hanson and colleagues at University College London in late 1970s as a tool to help architects simulate the likely social effects of their design [7]. The main idea of space syntax was to divide the spaces into components and analyse to represent with graphs and maps, called axial map, that describe the relative connectivity and integration of those spaces. By the help of these maps and graphs a designer will have the opportunity to analyse any space by the means of social relations and properties. After this tool other concepts are also developed to analyse a space by its different properties.

2.2. The concepts of isovist and visibility graph analysis

In addition to the social properties of a space visual properties also effect the spatial perception strongly. For analyzing a space by its visual properties the concept of isovist was developed and popularized by Michael Benedikt at University of Texas [11].

An isovist is basicly the polygon of the field of view from any particular point (Fig. 1). Isovists are naturally three-dimensional, but they may also be studied in two dimensions: either in plan or in vertical sections through the three-dimensional isovist. Every point in physical space has an isovist associated with it [11].

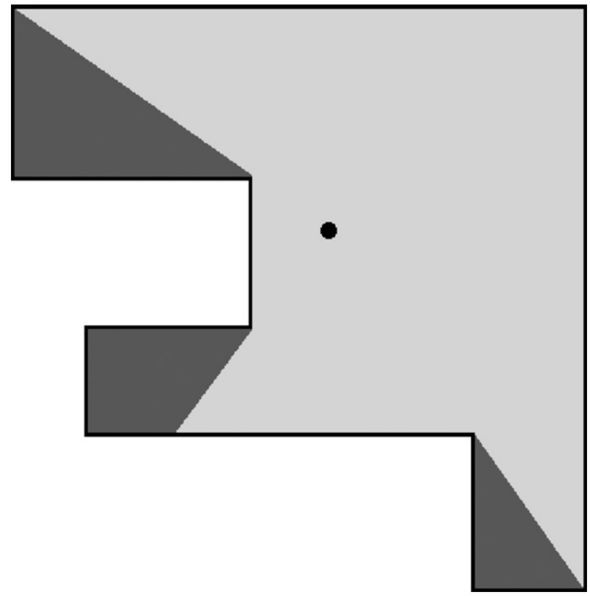

Fig. 1. An isovist from a point where lighter grey areas define the visible parts and the darker areas define invisible areas.

The boundary-shape of an isovist may or may not vary with location in, say, a room. If the room is convex, like a rectangle or circle, then the boundary-shape of every isovist in that room is the same. But if the room were non-convex, like an L-shaped room, or a rectangular room with partitions, then there would be many isovists whose area would be less than the whole room's. One can also think of the isovist as the volume of space illuminated by a point source of light.

The visibility graph analysis applications were first introduced by Braaksma and Cook [65,12]. Braaksma and Cook calculate the co-visibility of various units to produce an adjacency matrix to represent these relationships, placing a 1 in the matrix where two locations are mutually visible, and a 0 where they are not. From this matrix they propose a metric to compare the number of existing visibility relationships with the number which could possibly exist, in order to quantify how usefully a plan satisfies a

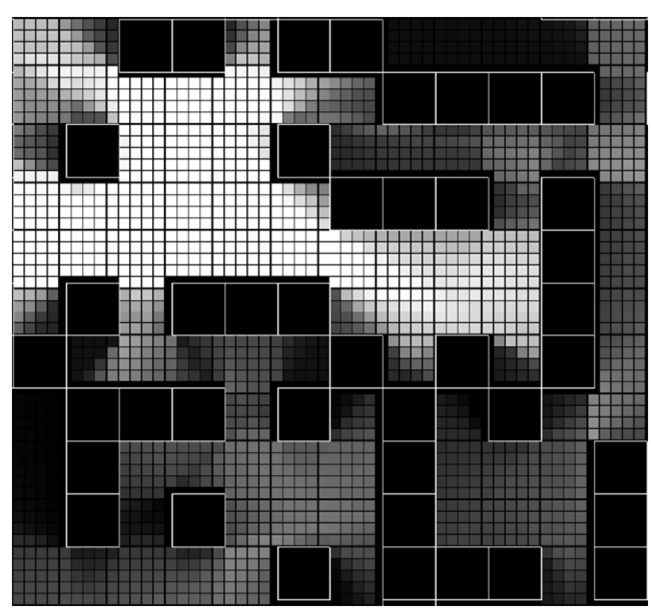

Fig. 2. A sample layout calculation with the depthmap tool [12].

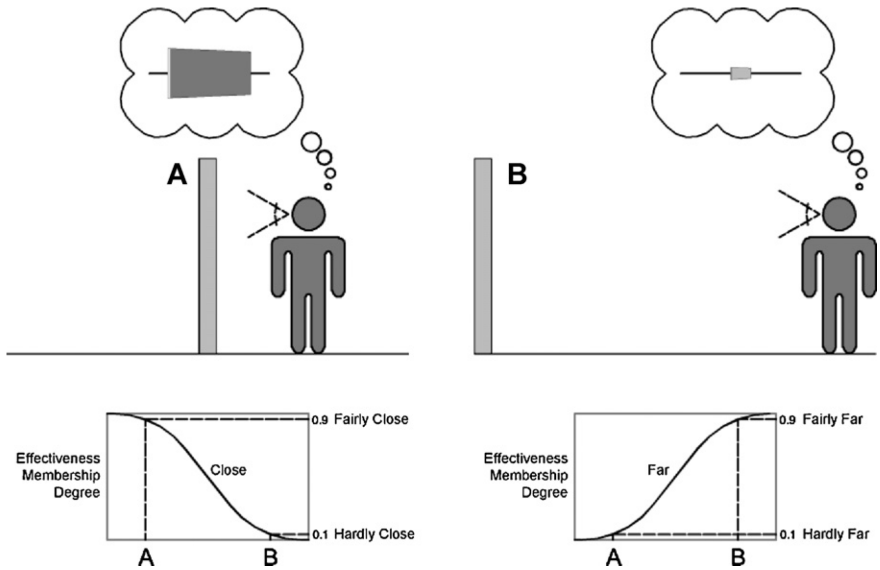

Fig. 3. Effectiveness of an element is dependent to its distance from the observer.

goal of total mutual visibility of locations [12]. Turner and Penn rediscovered the subject after almost 20 years in 1999 and Turner et al. developed a software for visibility graph analysis in 2001 [8,11].

2.3. The depthmap tool

Depthmap tool is the first and most popular software that is used for visual spatial analysis today which is developed by Alasdair Turner and his colleagues at University College London [12]. They recast the adjacency matrix as a visibility graph of locations, where a graph edge connects vertices representing mutually visible locations. Turner et al. then use several graph metrics as a few representative measures which could be performed on such graphs [12]. They presented a tool which enables a user to perform visibility graph analysis on both building and urban environments, allowing the user to perform the kind of analysis proposed by Turner et al. [11].

The program attempts to find all the visible locations from each grid location in the layout one by one, and uses a simple point visibility test radiating from the current location to achieve this. As each location is considered, a vertex is added to the graph for this point, and the set of visible vertices is stored. The simplified form of the algorithm in pseudo code as follows. In the algorithm: $V(G)$ is the set of all locations or vertices that exist, and v_i an individual location or vertex in the graph. Each vertex v_i will have a set of vertices connected to it, which will labelled the set $V(G_i)$, otherwise known as the vertex's neighbourhood [12].

```
FOR v_i in V(G)
  BEGIN
    FOR v_j in V(G)
    BEGIN
      IF v_i 'can see' v_j THEN add v_j to V(G_i)
    END
  END
```

In graph theory, the neighbourhood size for a vertex is commonly written k_i, and may be expressed as in the following equation:

$$k_i = \left|V(G_i)\right| = \left|v_j : \{v_i, v_j\} \in E(G)\right| \tag{2}$$

where $E(G)$ is the set of all edges in the graph [12]. Fig. 2 shows a simple layout after the visibility graph has been made using depthmap.

Depthmap colours k_i values using a spectral range from indigo for low values through blue, cyan, green, yellow, orange, red to magenta for high values. The user may change the bounds for this range, or choose to use a greyscale instead. Since this paper is reproduced in greyscale, all the figures shown have used the greyscale option, where black represents low values and white represents high values.

2.4. Drawbacks of the present models

Axial map is used to map the social relations of the convex spaces [14] and depthmap is used to map visual depth and isovist [10–13] properties. As it is not always possible to map convex

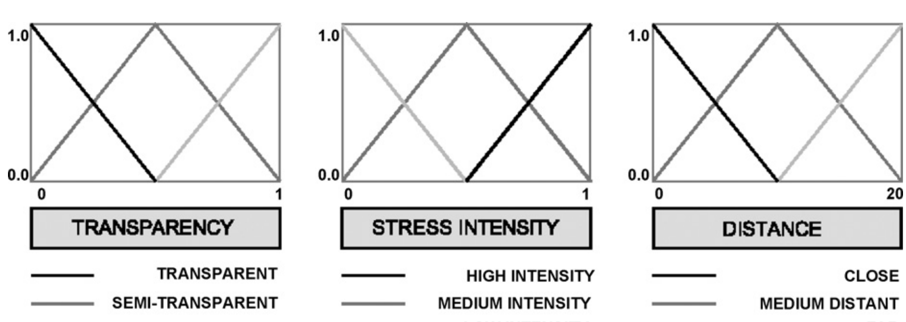

Fig. 4. The membership functions of an element's boundary, stress and effectiveness characteristics.

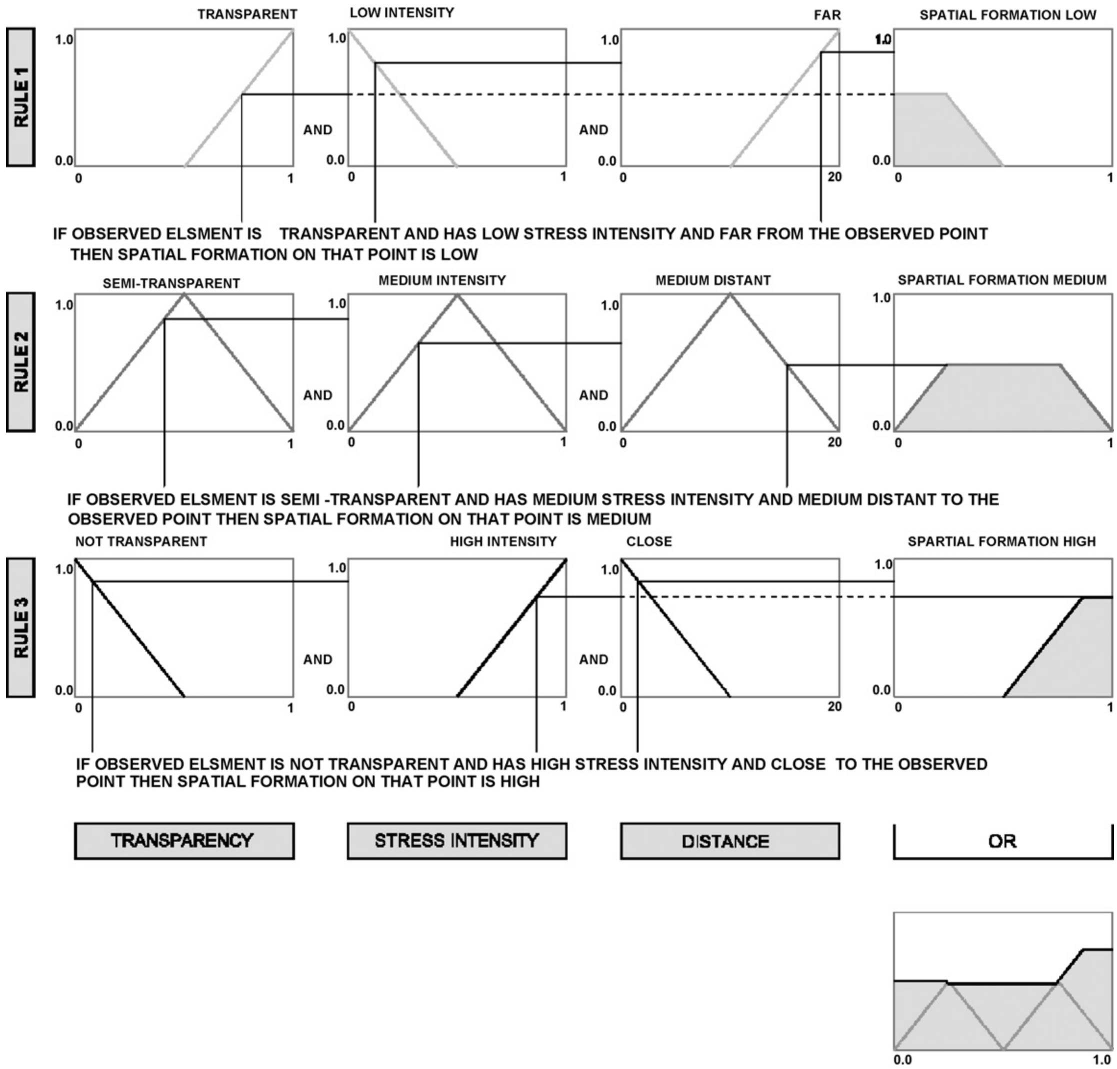

Fig. 5. Application of fuzzy operators.

spaces for any architectural layout and it is occasionally possible to define an architectural space without a function, space syntax tool which needs clear and exact convex spaces to analyse social relation between spaces, may not function for every architectural space. Similar to the CAD systems the depthmap tool used to analyse isovists and visual depth of an architectural space, is based only on the visual sense. In addition it analyses boundary elements, like walls or windows in real life, without their transparentness property, which is a matter of priority in architectural design as they will affect spatial formation in different ways depending on their transparentness.

3. A brief review of architectural space and elements affecting its perception

In this section, definitions of architectural space, spatial perception and characteristics of the spatial elements are briefly explained.

3.1. Architectural space and spatial perception

Architectural design, like in other spatial design fields, is a knowledge-rich activity. During the design process, an architect has to be concerned with issues all of which require specializations. Despite all inseparable branches of knowledge, architecture has not changed much since the date Vitrivius described these principles as a special construction activity based on spatial factors [29,30]. All the principles are aimed to form, define, and structurally carry the architectural space in one way. Architectural space is the main subject of the architecture profession, the only indispensable component of architecture and the element that calls architecture into being. Without architectural space, it is impossible to imagine architecture [31].

An architect intervenes in the spatial formation geometrically and the space forms the defined zone. In other words the architectural space is closed. Though architectural spaces are generally undertaken as the emptiness between constructional components they are actually formed with their own formal characteristics between them. The variabilities of the interior and the exterior compose the architecture's heart. Being inside is always preferred by an observer because of the privacy and protection factors [32].

Spatial need has emerged mainly from the sheltering and protection needs of human beings. According to Kuban [33], constructiveness of livings as a result of protection instinct is basically a process of isolating them from the environment. As a first step of architectural activity human beings created a limited space where they could feel protected. Boundary elements of an architectural

Fig. 6. Fallingwater, designed by Frank Lloyd Wright [61].

space, like walls, columns, transparent or semi-transparent separators of buildings, psychologically relax human beings [34]. They limit the universal environment to create a concave, more private space, which they were in difficulty to conceive in one or more directions [33]. Hasol [35] indicates that architectural space is a limited part of nature perceived by the human being. As an architectural approach, architectural space is a limited part of the universe which an observer can perceive [36,37]. The existence of boundaries is inevitable for the observer to perceive and define this space. However these boundaries that human brain can perceive easily may not be clear and certain. Even though these boundaries do not close a volumetrical space they may be sufficient to perceive a closed architectural space.

It is not always necessary to limit an architectural space with closed boundaries in all directions [37]. Although limitations of architectural space can be physical, it can also be provided by other senses. The only important factor is the presence of certain or uncertain perceptual boundaries. Meis [38] emphasizes sight and visual perception on spatial perception before everything else when defining architectural elements. Rapoport [2]

Fig. 7. Vitra Firestation, designed by Zaha Hadid [62].

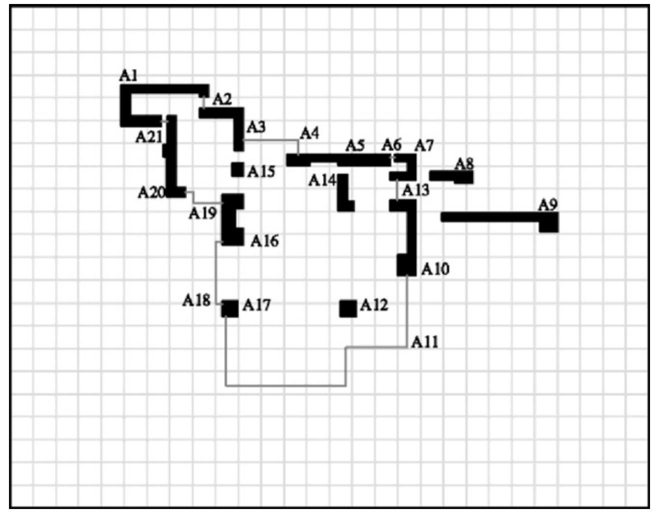

Fig. 8. Living room floor plan of the Fallingwater.

defines sensuous recognition of the knowledge that is incoming from the environment for perception. Although visual perception is emphasized and other sensorial factors are ignored, in many cases perception is a multi-sensored process [15]. Perception has to be taken into consideration as a result of a combination of many senses and perceptual factors.

The fact that an architectural space may be clear with unclear boundaries causes many uncertainties and issues for architects to solve. During the design process architects form many main and sub-spaces. This is an intuitive process and within these uncertainties, architects' success in designing desired spaces depends mostly on their professional knowledge and experience [39,40].

3.2. Boundary characteristic of elements forming architectural space

The definition of elements forming architectural space depends on their position and function of the elements in an environmental system. Elements may take several roles according to its position in the composition. These roles provide indicators for the observer to conceive the spatial organization [32]. Boundary elements play the most important role in a spatial formation. If the interior of a building is considered as an example, boundary elements are usually elements such as walls and ceilings [41]. They give the space meanings ranging from privacy to publicity. Dividing and limiting boundaries are graded due to their effects on the privacy of the space they surround [42].

In some cases at urban scale, paths such as highways and railroads can also be perceived as boundaries because of their differences related to texture, density and movement speeds [43]. In a scale of human being a volumetrically closed room that has four walls and a ceiling is such an architectural space without dispute [44]. Boundary elements are emphasized with their intransigent properties. A boundary element is as effective as its intransigent characteristics. As the transparency of the boundary element increases, it loses its intransigent characteristic in an inverse proportional way. This transparency and intransigent property exists in many ways. Observers perceive the boundary elements by their senses. Every limit, affecting visual, auditory, thermal or smelling senses seem to be a boundary element for an observer. Although these factors do not affect the senses equally, perception process will occur in a multi-sensored way.

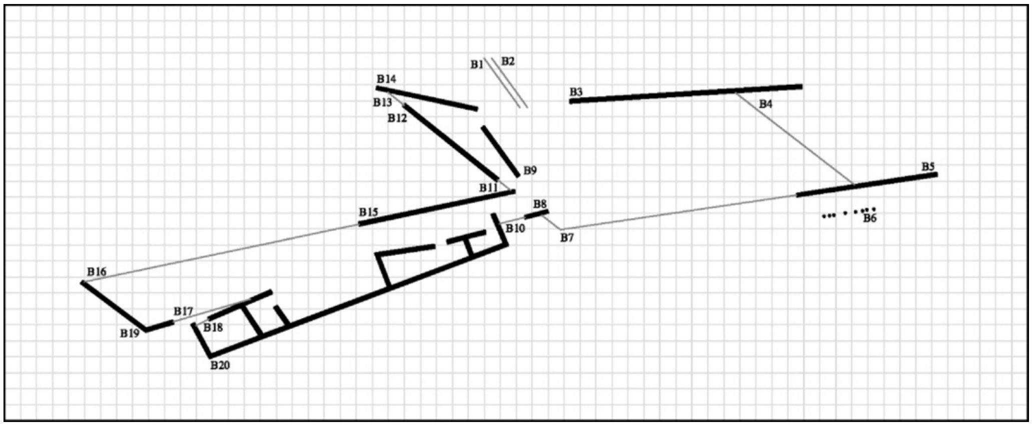

Fig. 9. Ground floor plan of the Vitra Firestation.

3.3. *Stress characteristic of elements forming architectural space*

Stress elements have a different effect on spatial formation compared to boundary elements. Stress elements do not usually limit the observer's movement or his visual, auditory, thermal or smelling senses. Furthermore, they are stronger elements in a composition functionally, symbolically and formally; drawing the attraction toward an architectural space and strengthen the spatial feel. These elements being one, two or three-dimensional are easily separable during the perception process and therefore clarify the observer's spatial perception by adding a spatial definition to it [45].

Monumental sculptures and statues in public squares are the most common examples for such processes. It is their scales, sizes and the meanings that strengthen the spatial feel around them. They raise attention by giving a focal effect to the point which they are placed. Also, a television in a living room can be another example for the visual and audio stress element. Likewise a fireplace can be a thermal and visual stress element. None of these examples restrict the observer's senses or his mobility. Nor, they raise any feeling of protection. They only clarify and strengthen the observer's spatial perception. The most important difference between stress elements and boundary elements is that the stress elements are not indispensable as boundary elements are in a spatial formation.

Stress intensity is another important conception concerning the stress element's effect on spatial formation. It is calculated through the stress element's level of dominance. A stress element is at

Table 1
Transparency and stress intensity values based on the material properties of spatial elements for both venues.

Element ID number (Ref.: Figs. 8 and 9)	Overall transparency degree (0.0–1.0)	Overall stress intensity degree (0.0–1.0)
A1	0.0	0.3
A2	0.7	0.0
A3	0.0	0.3
A4	0.9	0.0
A5	0.0	0.5
A6	0.7	0.0
A7	0.0	0.3
A8	0.0	0.6
A9	0.0	0.6
A10	0.0	0.5
A11	1.0	0.0
A12	0.0	0.7
A13	0.7	0.0
A14	0.0	0.7
A15	0.0	0.7
A16	0.0	1.0
A17	0.0	0.7
A18	1.0	0.0
A19	1.0	0.0
A20	0.0	0.3
A21	0.7	0.0
B1	0.5	0.5
B2	0.5	0.5
B3	0.0	0.7
B4	1.0	0.0
B5	0.0	0.7
B6	0.0	0.9
B7	1.0	0.0
B8	0.0	0.6
B9	0.0	0.7
B10	1.0	0.0
B11	1.0	0.0
B12	0.0	0.7
B13	1.0	0.0
B14	0.0	0.7
B15	0.0	0.7
B16	1.0	0.0
B17	1.0	0.0
B18	1.0	0.0
B19	0.0	0.7
B20	0.0	0.9

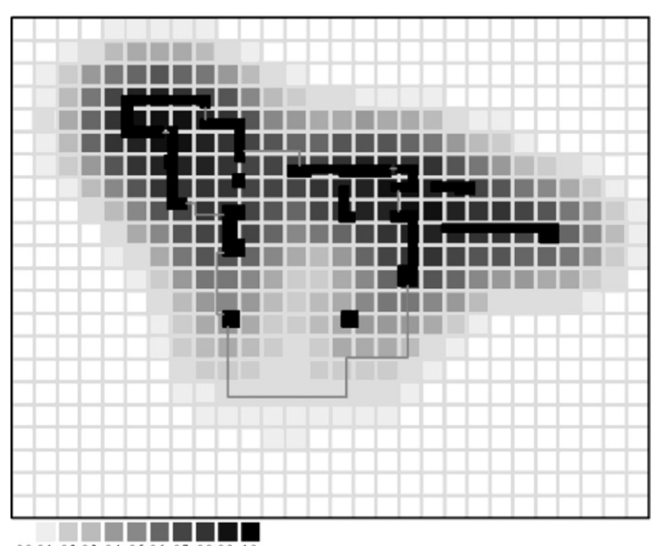

Fig. 10. Results for the Fallingwater.

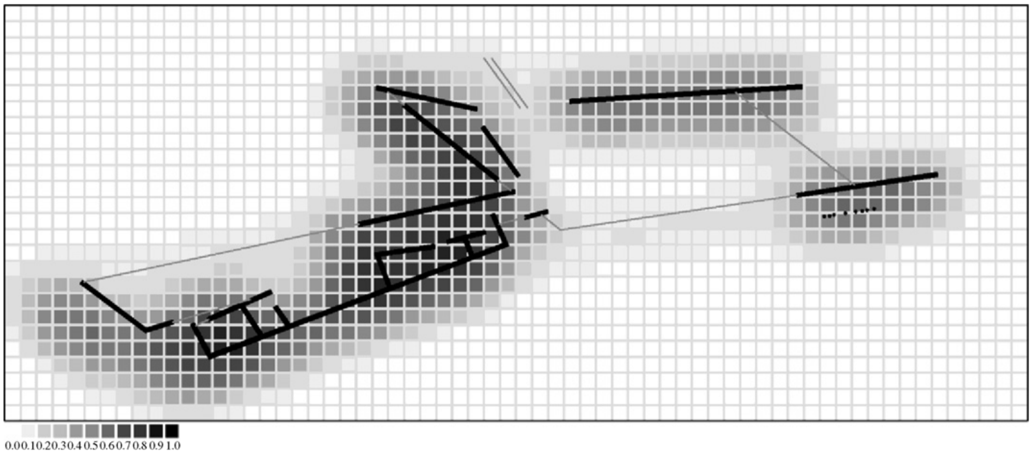

Fig. 11. Results for the Vitra Firestation.

the same level of dominance and intensity with its contrast in an architectural space. This contrast can be maintained by the stress element's volume, area, color, illumination or by any audio or thermal properties.

3.4. Effectiveness and distance between the observer and the element

Besides an element's boundary and stress characteristic, an element can contribute to spatial formation as long as it is perceived by an observer. The dominance of this contribution by any element is related to the distance between the observer and that element. When one observer is closer to an element, the effectiveness of the element will be sensed more whereas when he goes far from that element, its effectiveness will be sensed less. In other words membership degrees of the points in a general effectiveness set are diminished as the observer recedes from the element (Fig. 3). This change in effectiveness is related with the ratio that the observer allocates in his perception. The further the observer is, the less the ratio he will allocate for an element and the element will be less effective in a spatial formation. On the contrary when the observer comes closer, the effectiveness of the element will increase.

4. Fuzzy system development

A fuzzy inference system has three main sections, fuzzifier, fuzzy database and defuzzifier. First section converts input information into fuzzy data, which is done by membership functions. The precise amount value becomes as fuzzy values by membership functions in this section. The second section, fuzzy database, is consisted of two sub-sections. First sub-section is fuzzy rule base where rules related to fuzzy propositions are described and the second sub-section is inference engine where analysis operation is applied. The output values are entered into the defuzzifier section where fuzzy values are converted into crisp output values. In this section, general information about the applied model is explained.

4.1. The algorithm of the proposed model

The simplified form of the proposed model's algorithm in pseudo code is as follows:

FOR v_i in $V(G)$

  BEGIN

    BEGIN

      SET $r = 0$
      IF $r < 3$ THEN
      REPEAT

        FOR rule r

        BEGIN

        GET transparency value of e_i
        DETERMINE t_{MFi} = transparency membership function (MF) of e_i
        GET stress intensity value of e_i
        DETERMINE c_{MFi} = stress intensity membership function (MF) of e_i
        GET distance value of e_i *to* v_i
        DETERMINE d_{MFi} = distance membership function (MF) of e_i *to* v_i
        CALCULATE fuzzy operation $s_{MFr} = t_{MFi}$ AND c_{MFi} AND d_{MFi}

        END

      CALCULATE $r = r + 1$
      UNTIL $r = 3$

    ELSE END

  CALCULATE fuzzy operation $s_{MF} = s_{MF1} + s_{MF2} + s_{MF3}$
  CALCULATE s_i = centroid average (CA) of s_{MF}

END

In the algorithm above the $V(G)$ is the set of all locations and vertices that exist, v_i defines every individual location or vertex in the graph, r is the rule, s_i is the spatial intensity membership function for v_i and e_i is the individual boundary or stress element. Details of the steps of this algorithm are explained in the following sections.

4.2. Fuzzification of input variables

The process of transforming crisp input values into linguistic values is called fuzzification and it involves two processes. First, the input values are translated into linguistic concepts represented by fuzzy sets. Then membership functions are applied to the measurements and the degree of truth in each premise is determined. The input values were examined to determine the degree to which they belong to each of the appropriate fuzzy sets via membership functions. The major task in fuzzy logic approach lies in defining the membership functions for each input variable, as the efficiency and accuracy of fuzzy logic systems depend on accuracy of the membership functions. There are several methods for generating

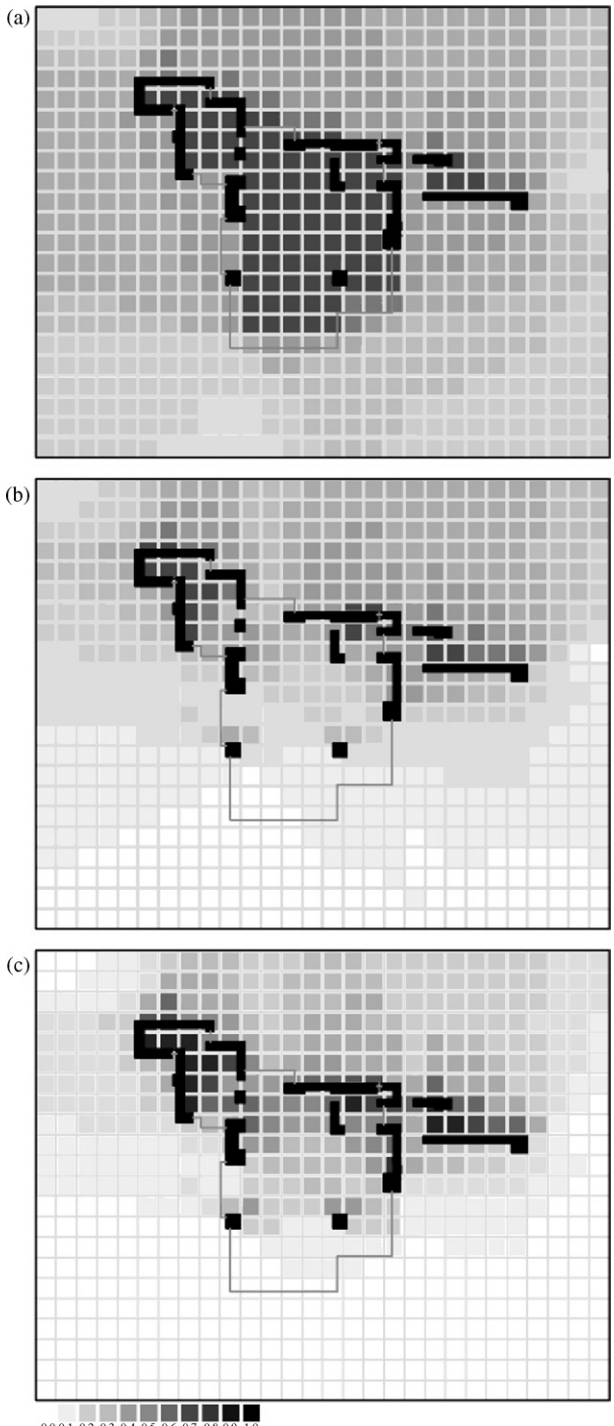

Fig. 12. Depthmap analyses for the Fallingwater: (a) first analysis results, (b) second analysis results, (c) average results.

of membership functions, which includes but are not limited to expert knowledge, subjective evaluation, statistical and learning and adaptation.

Each of the input variables was fuzzified into three fuzzy sets: low, medium, and high that are described in detail below. There are several types of membership functions such as triangular, trapezoidal, and Gaussian, to name a few. The shapes of the membership functions were a tradeoff between accuracy, simplicity, convenience, and efficiency. Literature search shows that trapezoidal and triangular membership functions are quite simple and have been used extensively in many applications with successful results [46–48]. Therefore, membership functions are generated from the above methodology and were fitted with triangular and trapezoidal shapes as much as possible.

The transparency of a boundary element observed from the point that is being analysed is the basic property that determines the effectivity of that element. The less transparent the boundary element is, the more it will have blocking characteristics and the more it will solidify the spatial formation for that point.

The contribution of stress elements in the spatial formation is directly proportional with their stress intensities. The intensity will increase as the contrast of the stress element being observed form the point that is being analysed increases. The stress intensity set will include points with membership degrees proportional with this contrast. If stress intensity of a point is too high, then it will solidify the spatial formation for that point. On the contrary if stress intensity of a point is exiguous then it will become ineffective.

The effectiveness of an element also depends on the distance between the element and the point that is being analysed. As the point being analysed, the observer in real life, comes closer to an element, the element's contribution to spatial formation for that point will ascend. In the direct contradiction, if the point being analysed goes farther to an element, the element's contribution to spatial formation for that point will descend.

While determining the membership functions, the material properties, which are being widely used in literature [49,50] are taken into account for transparency and stress intensity values. The determination of the membership function for effectiveness is based on the studies focusing on perspective and perception [51–59]. Fig. 4 shows the membership functions of elements' boundary, stress and effectiveness characteristics.

4.3. Fuzzy data base

4.3.1. Application of fuzzy operators

This step determines the degree to which each part of the antecedent has been satisfied for each rule. If the antecedent of a given rule has more than one part, a fuzzy operator is applied to obtain one number that represents the result of the antecedent for that rule as shown in Fig. 5. The input to the fuzzy operator is the three membership values from fuzzified input variables and the output is a single truth value which is applied to the output function. The fuzzy operator AND was applied to combine the antecedent of each rule to obtain a single degree of truth value which represents the weight of that element's rule as shown in the following equation:

$$W_i = \mu_{Bi} \Lambda \mu_{Si} \Lambda \mu_{Ei} \tag{3}$$

where, W_i is the weight of an element's rule i, μ_{Bi} is the degree of membership of boundary characteristic, μ_{Si} is the degree of membership of stress characteristic, and μ_{Ei} is the degree of membership of effectiveness. The fuzzy AND operator simply selects the minimum of the three values. Fig. 5 shows an example for application of these fuzzy rules. The weight of the element's rule i, W_i is the one used in implementation process described below to truncate the output membership function.

4.3.2. Implication of rule

The implication process uses the degree of truth for the entire rule, W_i, to shape the output fuzzy set as described in Fig. 5. A multi-rule fuzzy inference system was designed based on understanding of the system since spatial formation occurs at different combinations of the input variables. Consequently, several rules were formulated to cover all of those combinations and were evaluated in parallel using fuzzy reasoning "if-then rules". The input for the implication process is the weight of an element's rule and the output is a fuzzy set represented by an output membership function

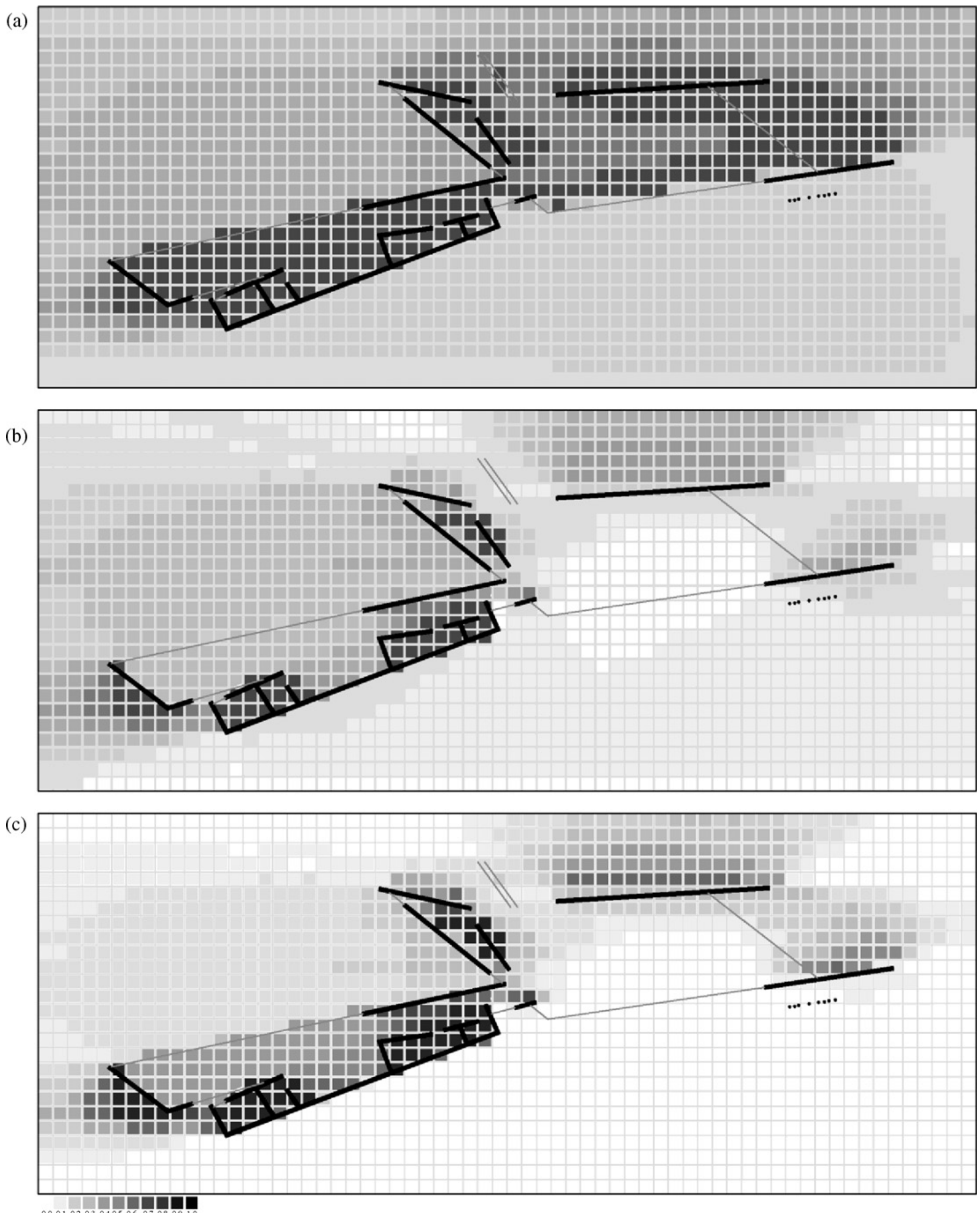

Fig. 13. Depthmap analyses for the Vitra Firestation: (a) first analysis results, (b) second analysis results, and (c) average results.

that has been truncated using the weight of that element (Fig. 5). If the antecedent is only partially true, the degree of truth value or weight is assigned a value less than *1*, which is then used to truncated the output membership function. Mathematically, it is expressed as

$$\Delta i = \mu_{Ti}(X) = W_i \mu_{Oi}(X) \tag{4}$$

where, i is the implication of element's rule i, W_i is the weight of element's rule i that is used to truncate the output membership function, $\mu_{Oi}(X)$ is the output membership function for element's rule i, $\mu_{Ti}(X)$ is the truncated membership function for element's rule i.

4.3.3. Aggregation of outputs

The decisions made from fuzzy logic models are based on the testing of all of the rules in a fuzzy inference system. Because there are many rules for are many rules for many elements involved, they must be combined in some manner in order to make a decision. Aggregation is the process through which the fuzzy sets that represent the outputs of each rule obtained at the implication step are combined into a single fuzzy set as shown in Fig. 5. Aggregation occurs once for each output variable, prior to defuzzification. The fuzzy OR operator selects the maximum of the input values. The input of the aggregation process is the list of truncated output membership functions returned by the implication process for each rule and the output is one fuzzy set for each output variable. In

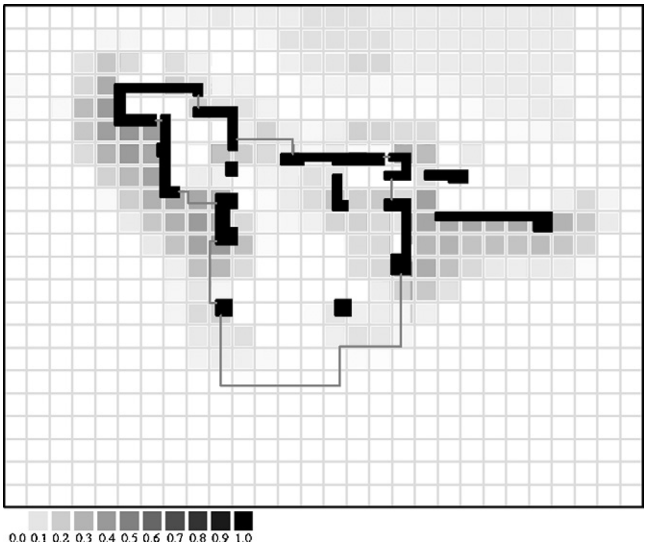

Fig. 14. Differences of the results of both models for the Fallingwater.

Fig. 5, all rules have been placed together to show how the output of each rule is aggregated, into a single fuzzy set whose membership function assigns a weighting for every output value. Aggregation operation on n fuzzy sets can be defined by the function:

$$h : [0, 1]^n \rightarrow [0, 1] (2 \leq n) \tag{5}$$

4.4. Defuzzification

The aggregate fuzzy set encompasses a range of output values, so it must be defuzzified into a single output (crisp) value for decision making. The input for the defuzzification process is the aggregate fuzzy set and the output is a single (crisp) number. There are several defuzzification methods which include but are not limited to: centroid average (CA), bisector (B), mean of maximum (MOM), smallest of maximum (SOM), and largest of maximum (LOM). Centroid defuzzification returns the center of area under the curve of the aggregate membership function and along the x-axis about which this shape would balance. The popular defuzzification method, the centroid calculation, which returns the center of area under the membership curve, was used [60]. The center of the area defined by the truncated membership function is simple to calculate when triangular and trapezoidal functions are used. Mathematically, the defuzzification process is expressed as the following equation:

$$W_P = \frac{\int \mu_C(X) \cdot XdX}{\int \mu_C(X) \cdot dX} \tag{6}$$

5. Applications of the proposed model

In this section the fuzzy inference system based spatial analysis model is applied on two selected venues. Two venues are selected with almost opposite architectural styles, one with rational and second with irrational planimetric approach, to reflect the model's flexibility in different layouts.

5.1. Study area and data set

First one is the 'Fallingwater' (Fig. 6), a country retreat located in Pennsylvania, United States of America, which is accepted by the architecture community as one of the masterpieces of famous architect Frank Lloyd Wright [61]. This venue is one of the most well known examples of modern architecture and has rational layout design.

The second venue is the 'Vitra Firestation' (Fig. 7), located in Weil Am Rhein, Germany and designed by architect Zaha Hadid [62]. This venue is one of the well known examples of irrational layout design.

Both venues offer different elements with different transparency and stress intensity properties. Elements being considered in analysis are marked and numbered on plans in Figs. 8 and 9 for venues one and two respectively. Transparency and stress intensity values based on the material properties of those elements are listed in Table 1. Although spatial formation analysis for every point is possible with the fuzzy inference system based spatial analysis model, the depthmap tool can only make analysis on a grid system. Therefore calculations on the plans of both venues are made on a virtual grid similar to the depthmap tool uses. This virtual grid is not perceived by the observer and provides the same platform with the depthmap tool to make comparisons of both model's results possible. Calculations on both models are done for every square's center point on this grid and resulting values are represented with greyscale spectrum on the plans. The virtual grid also allows the results to be read on the superposed plans easily. This virtual grid can also be seen in Figs. 8 and 9 for the venues one and two, respectively.

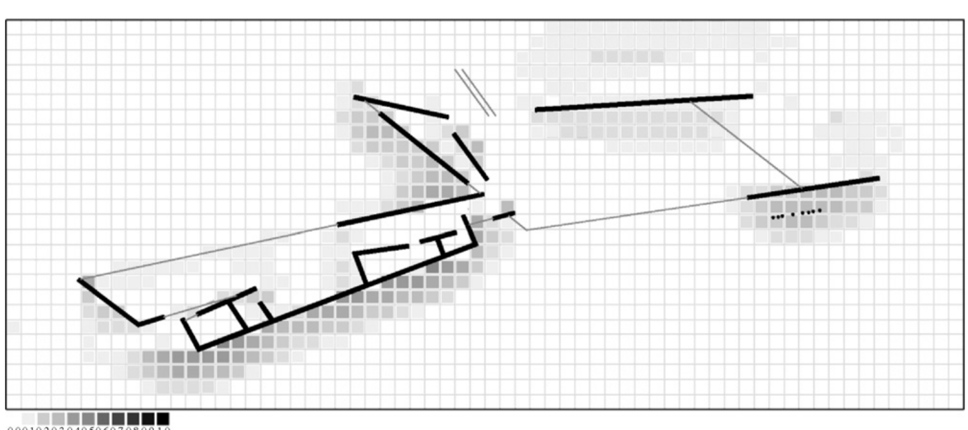

Fig. 15. Differences of the results of both models for the Vitra Firestation.

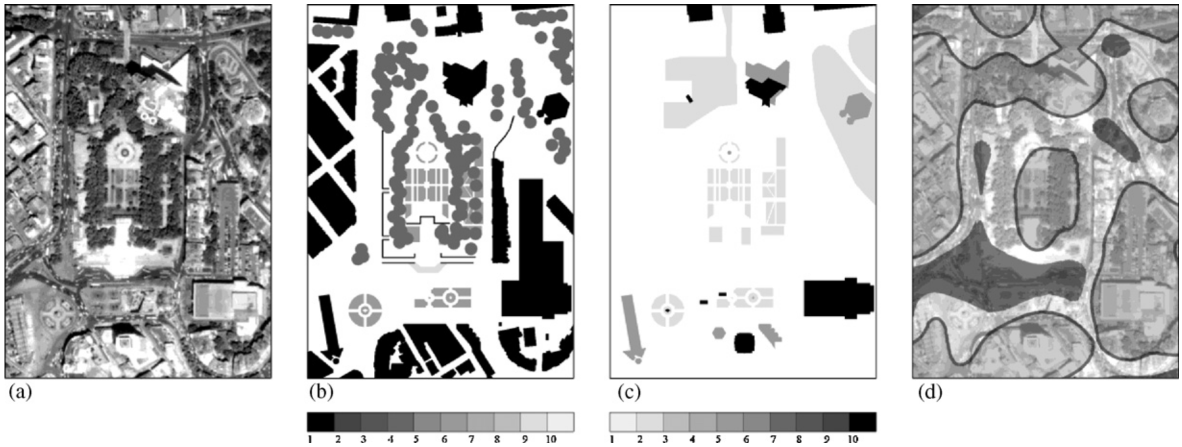

Fig. 16. Urban scale application of the proposed model: (a) study area map, (b) transparency values of the elements, (c) stress intensity values of the elements, and (d) results superposed on the map [4].

5.2. Results and discussion

The results of the fuzzy inference system based spatial analysis model for both venues are presented in this section. Darker squares represent the areas where spatial formation is sensed more intense, while lighter squares meaning the opposite. The membership value equivalents of the grayscale spectrum, in a range of 0 to 1, are also presented below Figs. 10 and 11.

5.2.1. Fallingwater

Obtained results for the Fallingwater are presented in Fig. 10 on the virtual grid. The results of the model, showing the membership degrees of the points in the architectural space, express how boundary and stress elements contribute to the spatial formation with their different properties. Through aggregating and defuzzifying all elements' outputs, it is easy to see where an architectural space is formed and how intense this spatial effect is on a point. The points on the virtual grid with membership degrees between 0 and 1 indicate that there is a spatial formation including them with an intensity related to their membership degrees. Points with membership degrees of 0 indicates that there are no perceivable spatial formation on them with this layout of these elements with given material properties.

5.2.2. Vitra Firestation

Obtained results for the Vitra Firestation are presented in Fig. 11 on the virtual grid. Equally to the results of the Fallingwater, the points on the virtual grid with membership degrees between 0 and 1 indicate that there is a spatial formation including them with an intensity related to their membership degrees.

5.2.3. Comparison with fuzzy inference system based spatial analysis model results and depthmap results

For better judgement, outcomes of the model have been compared to depthmap tool's results which are widely used in space syntax to analyse physical properties of architectural venues. Same virtual grid is also used for the depthmap analysis for both venues. For it is not possible to analyse elements with transparency or stress intensity in depthmap tool, two analyses are done and their results averages are taken to simulate the effect of semi-transparent elements. First analysis includes all elements as they are not transparent, and the second excluding the transparent elements like windows or other openings which do not fully block the visibility. Results of the first and second analyses and the average results for the Fallingwater and the Vitra Firestation obtained from the depthmap tool are presented in Figs. 12 and 13, respectively. Darker squares represent the center points where isovist depth, the total visible area limited with the boundary elements from that point, is lower and therefore the spatial formation is sensed more, while the lighter squares meaning the opposite.

The differences of both models' results are analysed and presented in Figs. 14 and 15 for the Fallingwater and the Vitra Firestation respectively. Absolute values of the differences of both models for every square on the grid for both venues are calculated and presented with grayscale spectrum on the virtual grid. The absolute difference value equivalents of the grayscale spectrum, in a range of 0 to 1, are also presented below Figs. 14 and 15. The differences of the two models mostly appear near the elements which have higher transparency or stress intensity values, mostly on the exterior parts of both of the buildings where stress intensity characters of the elements affect more then their boundary characters. It is obvious that the applied fuzzy inference system based spatial analysis model provides more realistic and reliable results than the depthmap tool, depending on the facts that the spatial perception is not only based on the visual sense but is a multi-sensored process and also transparency and stress intensity can not only be graded with crisp values but it should be graded with several degrees. While it is not possible to visualize the different transparency and stress properties in the depthmap tool, the results may become even misleading in venues including elements with such different properties. Therefore the differences should not be considered as an error but a result of difference of only-visual and multi-sensored perceptions. The differences of the two models lessen near the elements which have transparency or stress intensity properties. This shows that in the rare cases where only visual sense is taken into account, fuzzy inference system based spatial analysis model may still give more reliable results then the depthmap, because of its capability of considering spatial elements with their realistic degreed transparency and stress intensity properties.

6. Conclusion

This work presents a new and original mathematical model for spatial analysis, based on a fuzzy inference system. The results of this model on the applications visualize that the spatial formations occured as expected and the architectural space itself can be modeled with this fuzzy inference system based spatial analysis model. Because of the flexible characteristics of the model it can also easily be reshaped for other purposes, like privacy-publicity analysis or inclusivity–exclusivity analysis of architectural spaces. The fuzzy inference system based spatial analysis model is also capable of analyzing spaces in different scales from interior design to urban planning [4,5].

Fig. 16 shows another application of the proposed model on an urban scale which also represents the model's flexibility on different architectural scales. A city square is analysed with the model to see the spatial formation. Unless the results are not compared with depthmap tool's results in this example the virtual grid is no more necessary. This example also shows model's capability of working on non-gridal layout.

The mathematical analysis approach of spatial formation with fuzzy logic and sets can bring such concrete results independent from the designer's professional experience and knowledge. The results of the model can show how architectural space is formed, where it is intense and where it is weak. With such results of the model, a designer may add or remove elements, change properties of the elements to balance spatial formation in order to conclude the design with desired spatial properties. These results show that mathematical analysis of spatial formation with fuzzy logic is a very beneficial alternative in an architectural design process and in future this model can be used in many related branches as an automatic spatial analysis tool like improved architectural CAD softwares [63]. In addition to the use of the model in design process, it will be used to analyse existing spaces' current situations or evolutions in time so it can serve as an analysis tool for the historians of architecture or urban planning. While the 'digital design' technologies [64] support the complexity of designs, a model with such concrete results will definitely help the architects to form the desired spatial properties in complex designs.

References

[1] A. Rapoport, Human Aspects of Urban Form, Pergamon Press, Oxford, 1977.
[2] A. Rapoport, Culture Architecture Design, YEM Publications, Istanbul, 2004.
[3] B.C. Arabacioglu, Evaluation of today's computer aided 3D design areas in barches of architecture and interior design, 2nd Congress of information technologies, vol. 1, Denizli, 2003, pp. 232–234.
[4] B.C. Arabacioglu, A fuzzy inference system based model for architectural spatial analysis, Arkitekt 74 (2007), 512/513, 32–45.
[5] B.C. Arabacioglu, Fuzzy inference system based spatial analysis in the process of interior architectural design, in: National Symposium of Interior Architecture vol. 1, Istanbul, 2008, pp. 306–323.
[6] B. Hillier, In defense of space, RIBA Journal (Royal Institute of British Architects Journal) (1973) 539–544.
[7] B. Hillier, et al., Space syntax, Environment and Planning B: Planning and Design 3 (1976) 147–185.
[8] A. Penn, A. Turner, Space syntax based agent simulation, in: Proceedings of the 1st International Conference on Pedestrian and Evacuation Dynamics, University of Duisburg, Springer, Berlin, 2002, pp. 99–114.
[9] D.R. Conroy, Space syntax and spatial cognition, in: D. Wu (Ed.), World Architecture: Space Syntax Monograph, 2005.
[10] D. O'Sullivan, A. Turner, Visibility graphs and landscape visibility analysis, International Journal of Geographical Information Science 15 (2001) 221–237.
[11] A. Turner, et al., From isovists to visibility graphs: a methodology for the analysis of architectural space, Environment and Planning B: Planning and Design 28 (2001) 103–121.
[12] A. Turner, Depthmap, in: Proceedings of 3rd International Space Syntax Symposium, Atlanta, 2001, pp. 31.1–31.9.
[13] B. Hillier, The architectures of seeing and going: or, are cities shaped by bodies or minds? And is there a syntax of spatial cognition?, in: 4th International Space Syntax Symposium, Londra, 2003.
[14] S. Bafna, Space syntax: a brief introduction to its logic and analytical techniques, Environment and Behavior 35 (2003) 17–29.
[15] A. Gallace, C. Spence, The cognitive and neural correlates of "tactile consciousness": a multisensory perspective, Consciousness and Cognition 17 (1) (2008) 370–407.
[16] H. Hillenbrand, On the relation between probabilistic inference and fuzzy sets in visual scene analysis, Pattern Recognition Letters 25 (2004) 1691–1699.
[17] A. Petrosino, G. Salvi, Rough fuzzy set based scale space transforms and their use in image analysis, International Journal of Approximate Reasoning 41 (2006) 212–228.
[18] L.A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.
[19] L.A. Zadeh, Fuzzy algorithms, Information and Control 5 (1968) 94–102.
[20] O. Akyilmaz, Flexible Calculation Methods on Geodesical Applications, Ph.D. thesis, Institute of Natural Sciences, Istanbul Technical University, 2005.
[21] L.A. Zadeh, Outline of a new approach to the analysis of complex systems and decision processes, IEEE Transactions on Systems Man and Cybernetics 3 (1973) 28–44.
[22] L.A. Zadeh, Fuzzy sets as a basis for a theory of possibility, Fuzzy Sets and Systems 1 (1978) 3–28.
[23] L.A. Zadeh, A Fuzzy Logic: Advanced Concepts and Structures, IEEE Picataway, New York, 1992.
[24] G. Klir, et al., Fuzzy Set Theory: Foundations and Applications, Prentice Hall PTR, Indiana, 1997.
[25] G.J. Klir, et al., Fuzzy Sets and Fuzzy Logic: Theory and Applications, Prentice Hall PTR, Indiana, 1995.
[26] G. Klir, Fuzzy Sets Uncertainty and Information, Pearson Education Ltd, Cambridge, 1993.
[27] H.J. Zimmermann, Fuzzy Technologien, VDI Verlag, Dusseldorf, 1993.
[28] H.J. Zimmermann, Fuzzy Set Theory and Its Applications, Springer, New York, 2001.
[29] A. Heylighen, H. Neuckermans, Walking on a thin line — between passive knowledge and active knowing of components and concepts in architectural design, Design Studies 20 (2) (1999) 211–235.
[30] B.R. Lawson, Design in Mind, Butterworth Architecture Press, London, 1994.
[31] L.M. Roth, The Story of Architecture, Kabalci Publications, Istanbul, 2000.
[32] S.O. Gur, Spatial Organization, Gur Publications, Trabzon, 1996.
[33] D. Kuban, Architectural Concepts — An Introduction to Theoretical Dictionary of Architecture in a Historical Perspective, YEM Publications, Istanbul, 1992.
[34] I.M. Ozdemir, Spatial Organization Concept in Evaluation of Architectural Space: Living Spaces in Houses, BTU Publications, Trabzon, 1994.
[35] D. Hasol, Encyclopedic Dictionary of Architecture, YEM Publications, Istanbul, 1975.
[36] Space, in: J. Simpson, E. Weiner (Eds.), The Oxford English Reference Dictionary, vol. 16, Clarendon Press, Oxford, UK, 1989, pp. 87–92.
[37] M. Sozen, U. Tanyeli, Dictionary of Art Terms and Concepts, Remzi Publications, Istanbul, 1994.
[38] P.V. Meis, Elements of Architecture: From Form to Place, Spon Press, New York, 2002.
[39] C.J. Atman, A comparison of freshman and senior engineering design processes, Design Studies 20 (1999) 131–152.
[40] C.J. Atman, et al., Comparing freshman and senior engineering design processes: an in-depth follow-up study, Design Studies 26 (2004) 325–357.
[41] Y. Ashihara, Exterior Design in Architecture, Van Nostrand Reinhold Press, New York, 1981.
[42] J. Lang, U. Ark, Nature of spatial imagery, informing styles and aesthetic perforations, BTU Architectural Bulletin 7 (1982) 67–94.
[43] K Lynch, The Image of City, MIT Press, Massachusetts, 1960.
[44] J. Gympel, The Story of Architecture, Könemann Press, Hong Kong, 1996.
[45] S.O. Gur, Dimensions And Principles That Effect Living Quality in Houses: A Case Study on After Usage Evaluation in Eastern Blacksea Region, BTU Press, Trabzon, 1993.
[46] S. Cafiso, et al., A Fuzzy Model for Safety Evaluation of New and Old Roads in Transportation Research Board, 2004 Annual Meeting CD-ROM, Washington, DC, 2004.
[47] D.I. Brubaker, Cedric Sheerer, Fuzzy-Logic System Solves Control Problem, EDN, Waltham, Massachusetts, 1992, pp. 121–127.
[48] N. Gulley, J.S. Jang, The Mathworks in Fuzzy Logic Toolbox For Use with MATLAB User Guide, Version 2, pp. 2–20.
[49] M. Eric, Construction Physics and Material, Literatur Press, Istanbul, 1994.
[50] N. Toydemir, E. Gurdal, L. Tanacan, Material in Construction Element Design, Literatur Press, Istanbul, 2000.
[51] S.S. Bosbach, W. Prinz, Perceptual resonance: action-induced modulation of perception, Trends in Cognitive Sciences 11 (2007) 349–355.
[52] R.G. Cook, S. Roberts, The role of video coherence on object-based motion discriminations by pigeons, Journal of Experimental Psychology: Animal Behavior Processes 33 (2007) 287–298.
[53] H. Bruggeman, et al., The processing of linear perspective and binocular information for action and perception, Neuropsychologia 45 (2007) 1420–1426.
[54] H. Zhong, et al., The visual perception of plane tilt from motion in small field and large field: psychophysics and theory, Vision Research 46 (2006) 3494–3513.
[55] H. Sakata, et al., Toward an understanding of the neural processing for 3D shape perception, Neuropsychologia 43 (2005) 151–161.
[56] I. Juricevic, J.M. Kennedy, Looking at perspective pictures from too far, too close, and just right, Journal of Experimental Psychology: General 135 (2006) 448–461.
[57] J. Bräuer, et al., Visual perspective taking in dogs (*Canis familiaris*) in the presence of barriers, Applied Animal Behaviour Science 88 (2004) 299–317.
[58] A. Jacobs, et al., Experience, context, and the visual perception of human movement, Journal of Experimental Psychology: Human Perception and Performance 30 (2004) 822–835.
[59] M. Kubovy, D.V. Valkenburg, Auditory and visual objects, Cognition 80 (2001) 97–126.
[60] F.M. Mcneill, Wrap around universe of discourse in a fuzzy system with a defuzzification method in Fuzzy Logic '93, Fuzzy Systems Engineering, Wilsey Way, Poway, 1993.
[61] W. Curtis Jr., Modern Architecture Since 1900, Phaidon Press, Singapore, 1982.
[62] M. Ekincioglu, Zaha Hadid, Boyut Publishing Group, Istanbul, 2000.
[63] B.C. Arabacioglu, Changing values in computer aided spatial design, Mimar Sinan University of Fine Arts, Interior architecture seminars, Istanbul, 2003.
[64] R. Oxman, Theory and design in the first digital age, Design Studies 27 (3) (2006) 229–265.
[65] J.P. Braaksma, W.J. Cook, Human orientation in transportation terminals, Transportation Engineering Journal 106 (1980) 189–203.