

Aula 03

Gradiente Descendente e Otimização de Funções

Agenda

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Função de Custo

02

Minimização da
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Gradiente
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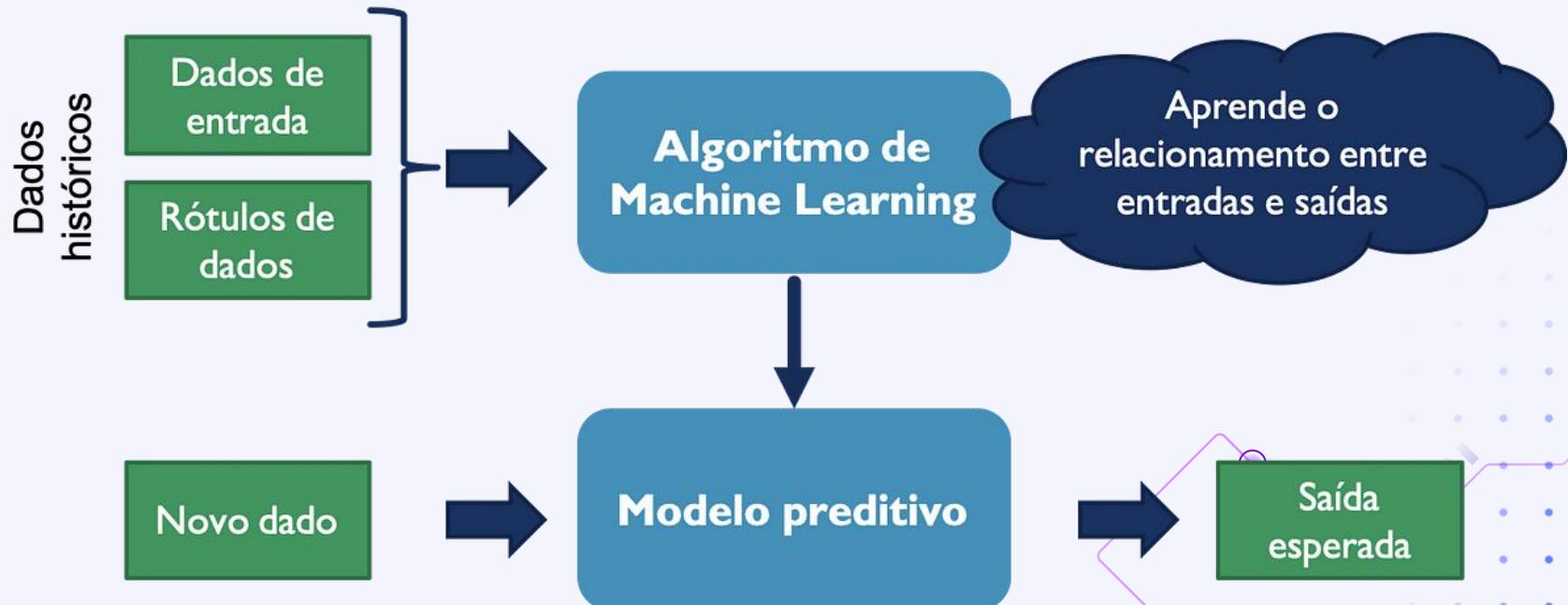
Implementando
os algoritmos

01

Recapitulação: Função de Custo



Modelando um fenômeno



Quão bom é o nosso modelo?

Como esse ajuste é calculado?

Podemos metrificar o quão bom o nosso modelo é?

Qual é o **erro** dele?

O erro do modelo é uma espécie de diferença entre o chute do modelo e a resposta correta.

Quão bom é o nosso modelo?

Como esse ajuste é calculado?

Vamos pensar no nosso erro como **um custo** ou **uma perda**.

Queremos minimizar o custo!

Objetivo: minimizar uma função que representa o custo do nosso modelo.

O que é uma Regressão Linear?

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_p x_p$$

Uma função F é linear se:

$$F(x + y) = F(x) + F(y) \quad \text{e} \quad c * F(x) = F(c * x)$$

Pode ser representada por uma reta, um plano, ...

Qual a Função de Custo?

A nossa métrica de erro será a *Mean Squared Error*:

$$\text{MSE} = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

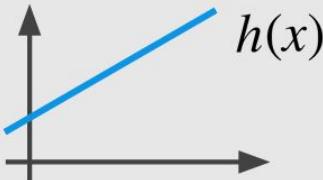
Função de Custo

Hypothesis:

$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

Parameters:

$$\theta_0, \theta_1$$



Cost Function:

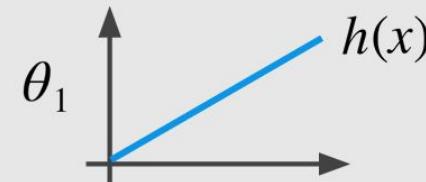
$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

Goal:

$$\underset{\theta_0, \theta_1}{\text{minimize}} J(\theta_0, \theta_1)$$

Simplified

$$h_{\theta}(x) = \theta_1 x$$



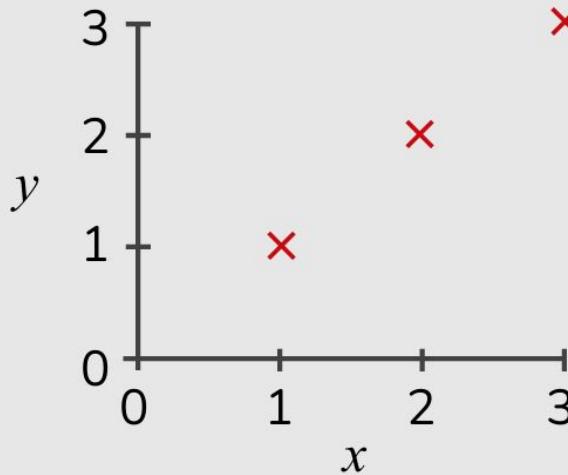
$$J(\theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

$$\underset{\theta_1}{\text{minimize}} J(\theta_1)$$

Função de Custo

$$h_{\theta}(x)$$

(for fixed θ_1 , this is a function of x)



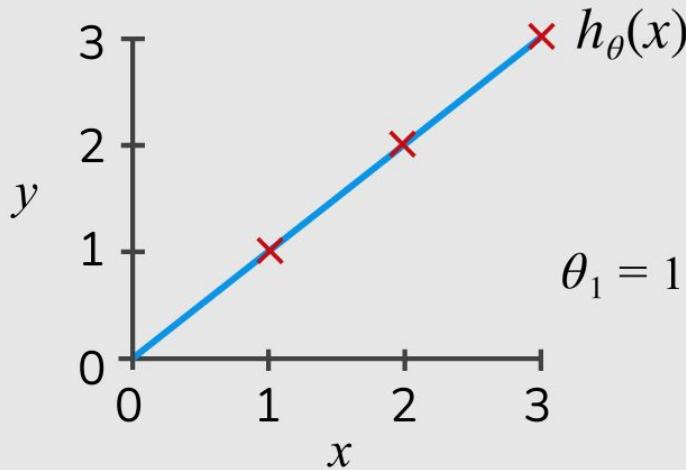
$$J(\theta_1)$$

(function of the parameters θ_1)

Função de Custo

$$h_{\theta}(x)$$

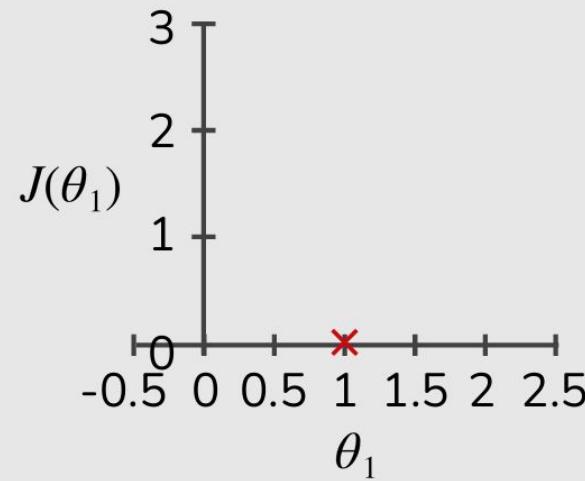
(for fixed θ_1 , this is a function of x)



$$J(\theta_1) = J(1) = 0$$

$$J(\theta_1)$$

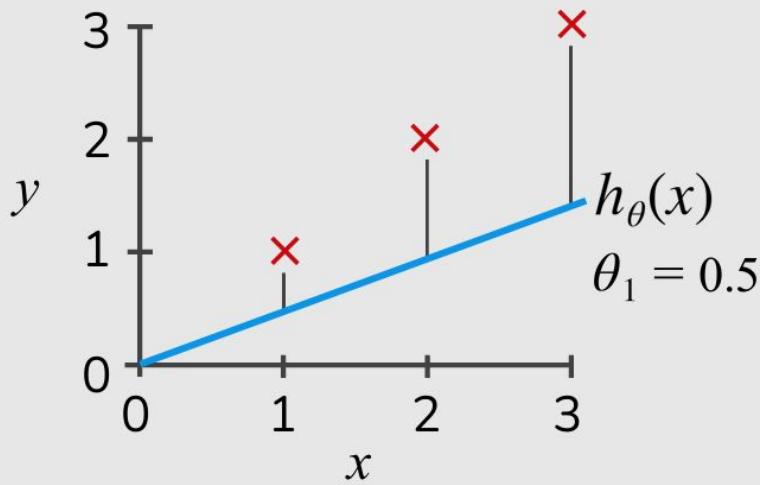
(function of the parameters θ_1)



Função de Custo

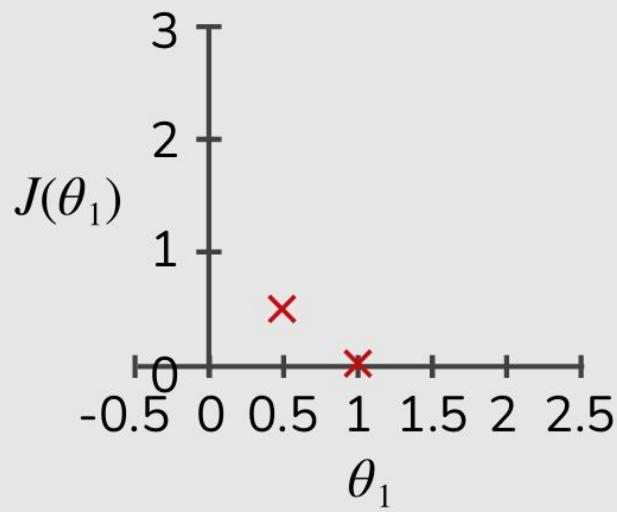
$$h_{\theta}(x)$$

(for fixed θ_1 , this is a function of x)



$$J(\theta_1)$$

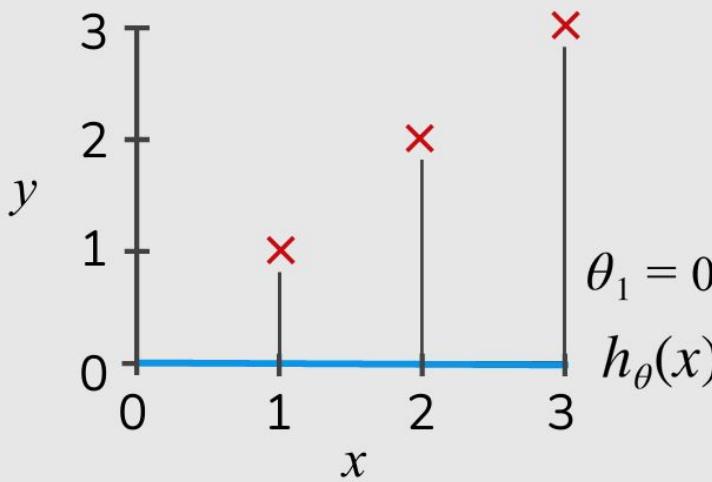
(function of the parameters θ_1)



Função de Custo

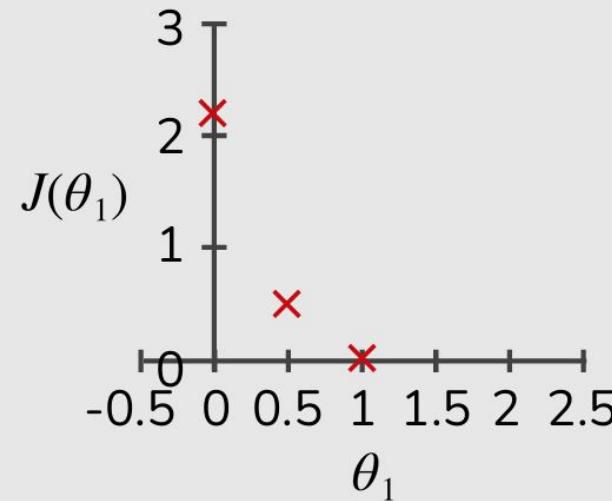
$$h_{\theta}(x)$$

(for fixed θ_1 , this is a function of x)



$$J(\theta_1)$$

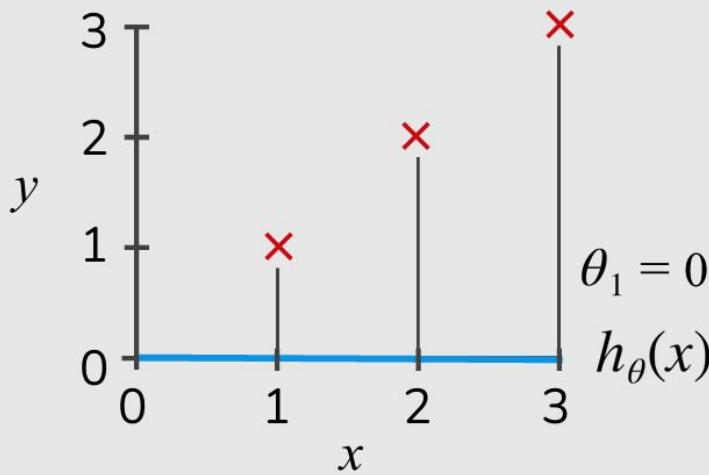
(function of the parameters θ_1)



Função de Custo

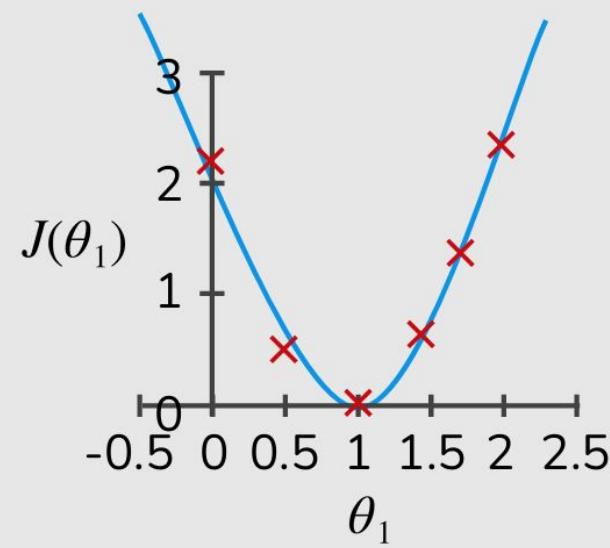
$$h_\theta(x)$$

(for fixed θ_1 , this is a function of x)



$$J(\theta_1)$$

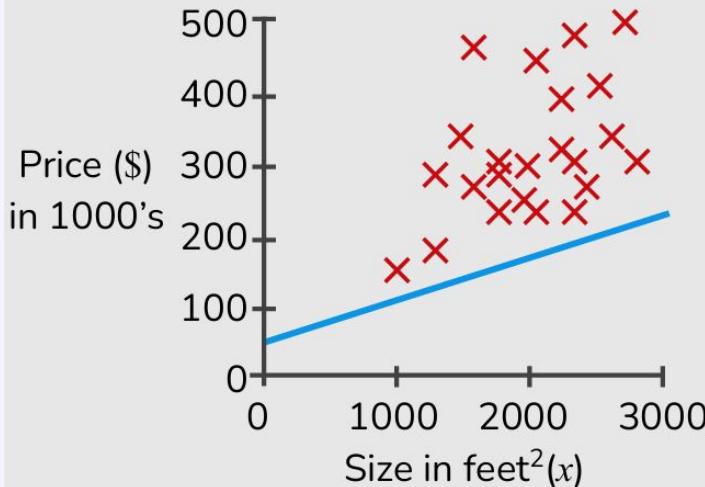
(function of the parameters θ_1)



Função de Custo

$$h_{\theta}(x)$$

(for fixed θ_0, θ_1 , this is a function of x)

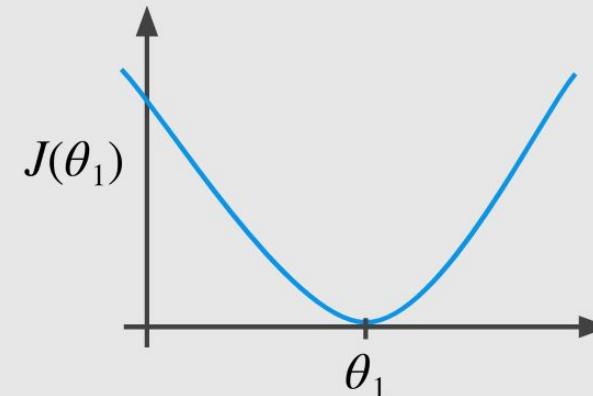


$$h_{\theta}(x) = 50 + 0.06x$$

$$\begin{aligned}\theta_0 &= 50 \\ \theta_1 &= 0.06\end{aligned}$$

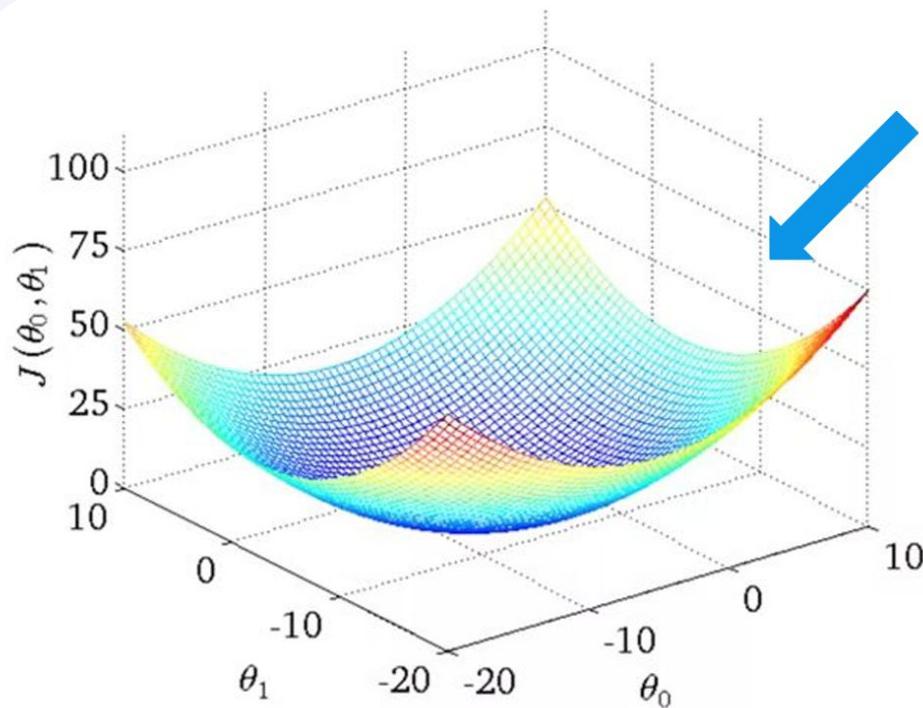
$$J(\theta_0, \theta_1)$$

(function of the parameters θ_0, θ_1)



θ_0 and θ_1 ?

Função de Custo

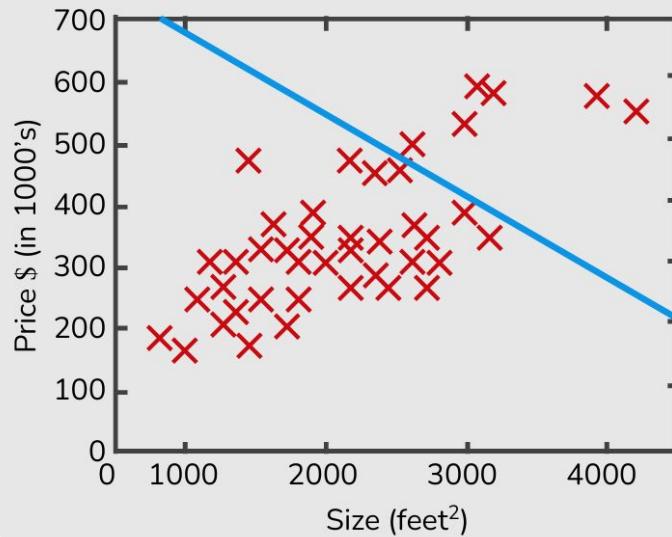


Convex
Function

Função de Custo

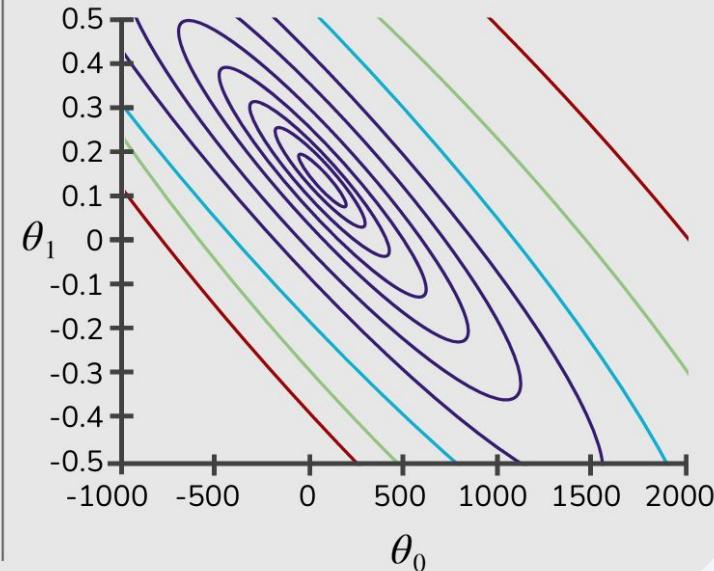
$$h_{\theta}(x)$$

(for fixed θ_0, θ_1 , this is a function of x)



$$J(\theta_0, \theta_1)$$

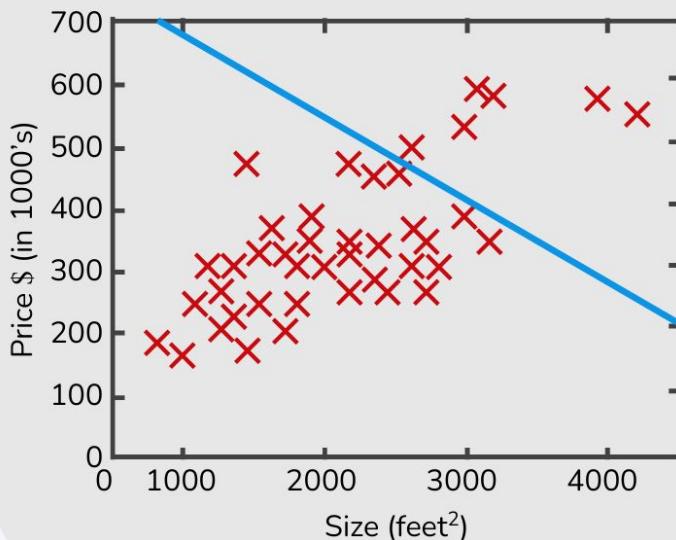
(function of the parameters θ_0, θ_1)



Função de Custo

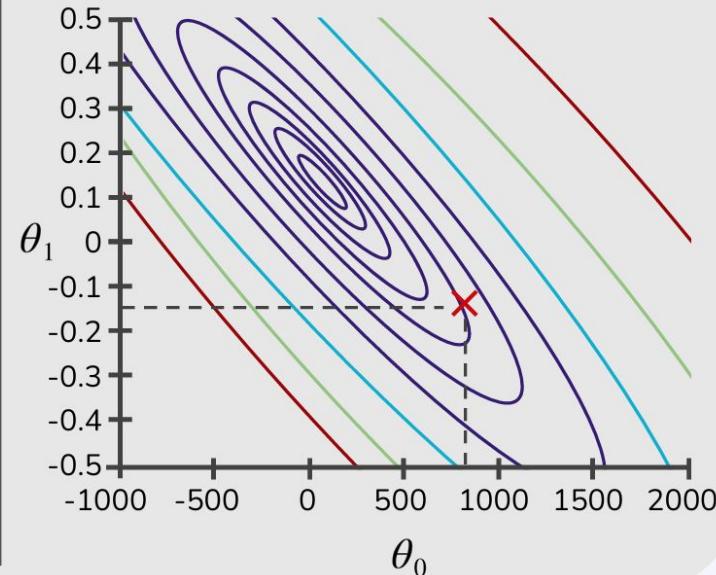
$$h_{\theta}(x)$$

(for fixed θ_0, θ_1 , this is a function of x)



$$J(\theta_0, \theta_1)$$

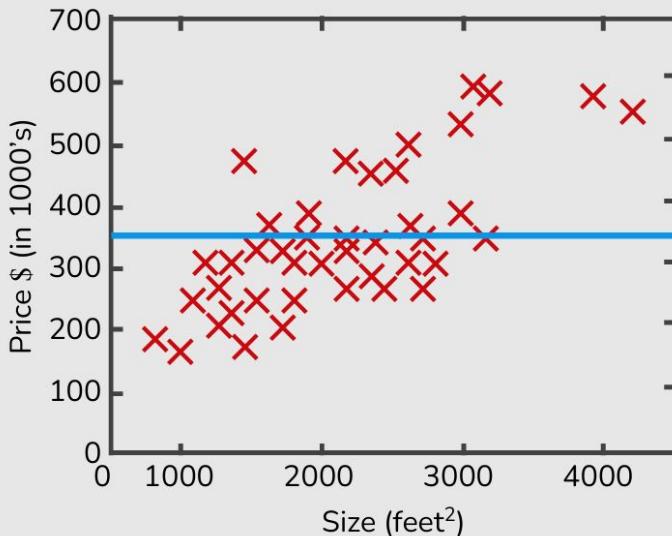
(function of the parameters θ_0, θ_1)



Função de Custo

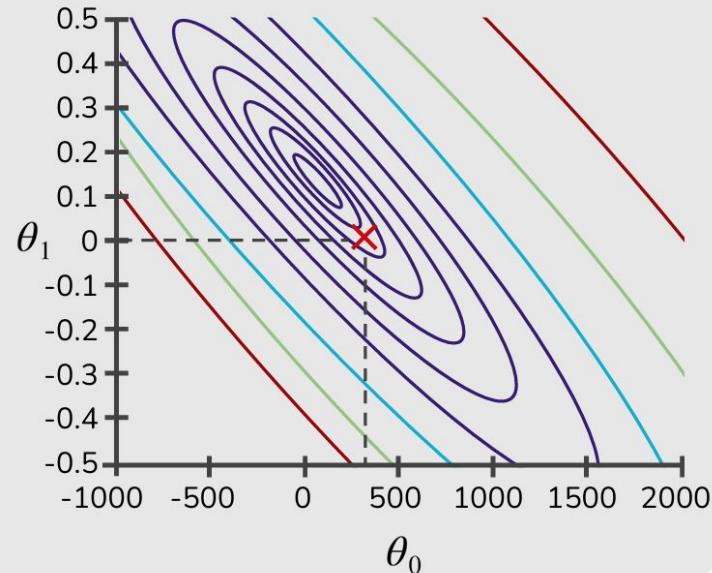
$$h_{\theta}(x)$$

(for fixed θ_0, θ_1 , this is a function of x)



$$J(\theta_0, \theta_1)$$

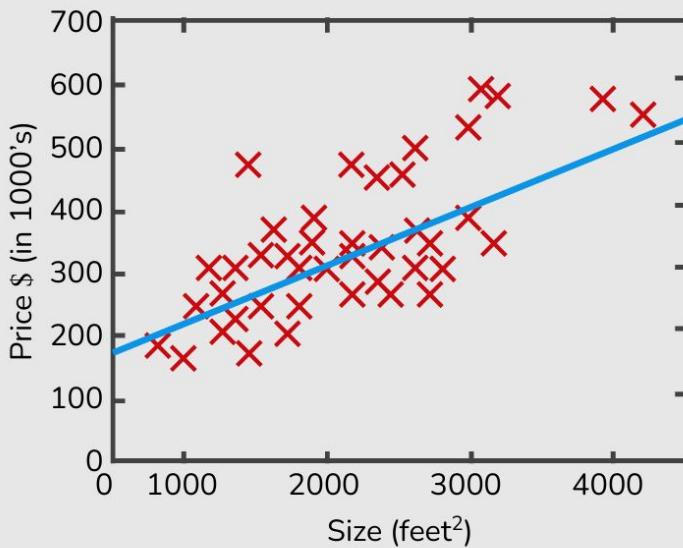
(function of the parameters θ_0, θ_1)



Função de Custo

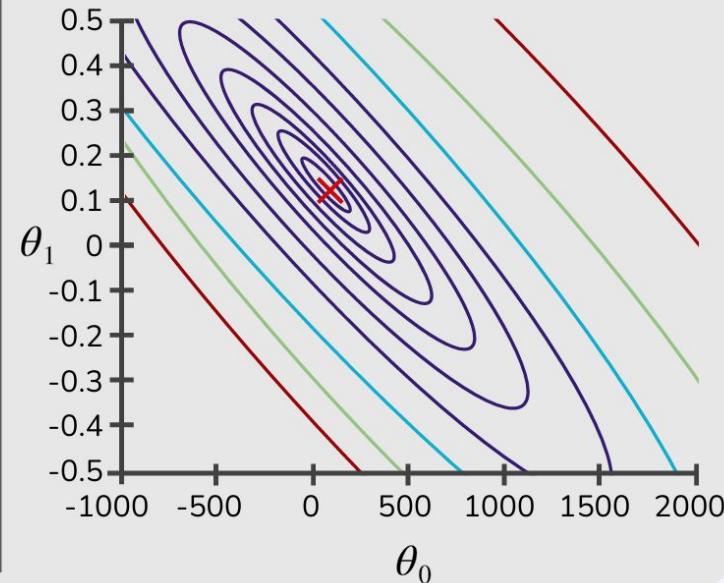
$$h_{\theta}(x)$$

(for fixed θ_0, θ_1 , this is a function of x)



$$J(\theta_0, \theta_1)$$

(function of the parameters θ_0, θ_1)



02

Minimização da Função de Custo



O que é minimizar uma função?

Minimizar uma função é encontrar os valores de entrada que retornam **o menor valor possível** para aquela função.

Este valor também é conhecido como o **mínimo global**.

Achar o mínimo global **nem sempre é possível**.

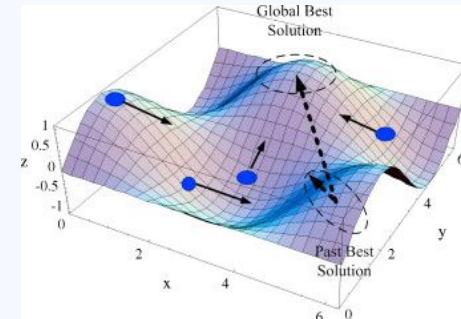
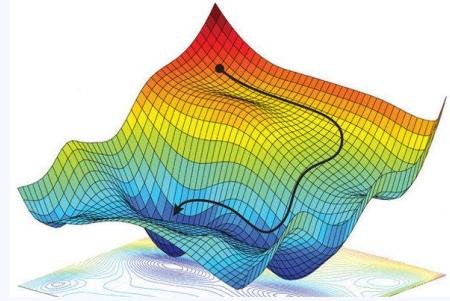
Às vezes, é possível encontrar apenas **mínimos locais**.

Encontrar **o mínimo** ou **o máximo** de funções são **problemas de otimização**.

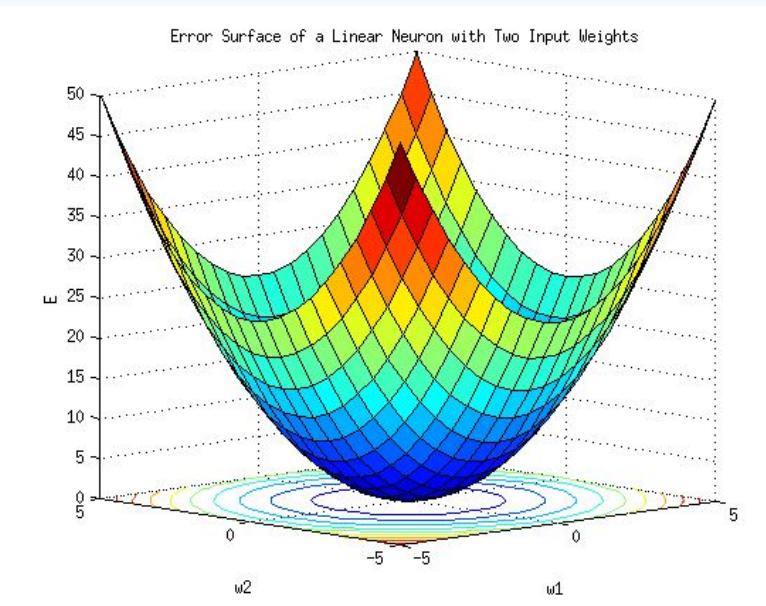
Alguns algoritmos de otimização

- MMQ → Equação Normal
(OLS) (*Normal Equation*)
- **Gradiente Descendente**
(Gradient Descent)
- Otimização por Enxame de Partículas
(*Particle Swarm Optimization*)
- Algoritmo Genético
(*Genetic Algorithm*)

$$\hat{\beta} = (X^T X)^{-1} X^T Y$$

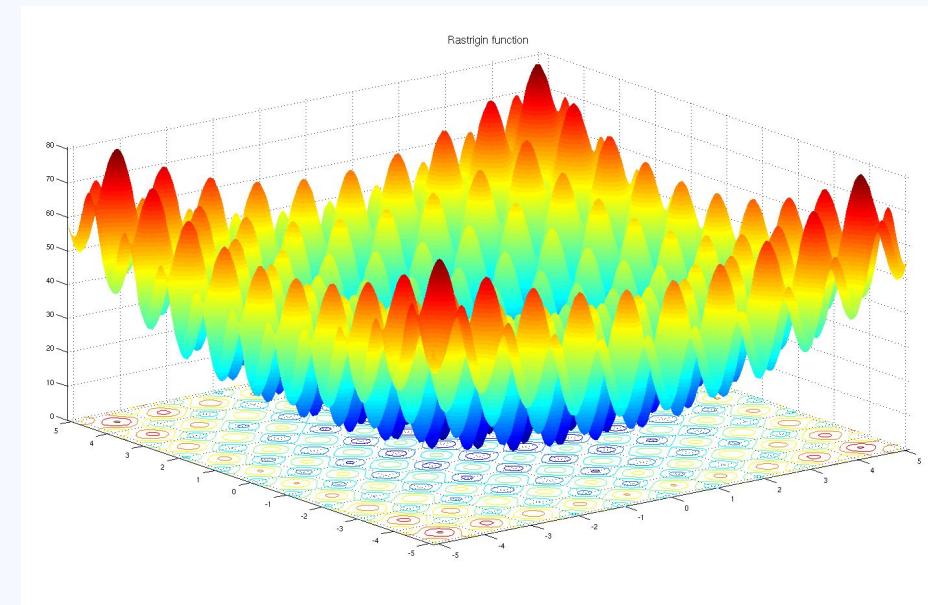


Alguns algoritmos de otimização



Função de Custo em forma de parabolóide
→ **MSE** usado em Regressão Linear

Função de benchmark (Rastrigin) de proc. otimização



Alguns algoritmos de otimização

Exemplo: a classe LinearRegression do *sklearn*.

The screenshot shows the scikit-learn API Reference for the `LinearRegression` class. The page title is "LinearRegression". The code definition is:

```
class sklearn.linear_model.LinearRegression(*, fit_intercept=True,
copy_X=True, n_jobs=None, positive=False)
```

The docstring states: "Ordinary least squares Linear Regression. LinearRegression fits a linear model with coefficients w = (w₁, ..., w_p) to minimize the residual sum of squares between the observed targets in the dataset, and the targets predicted by the linear approximation."

Parameters:

- fit_intercept : bool, default=True**: Whether to calculate the intercept for this model. If set to False, no intercept will be used in calculations (i.e. data is expected to be centered).
- copy_X : bool, default=True**

Notes

From the implementation point of view, this is just plain Ordinary Least Squares (`scipy.linalg.lstsq`) or Non Negative Least Squares (`scipy.optimize.nnls`) wrapped as a predictor object.

Examples

```
>>> import numpy as np
>>> from sklearn.linear_model import LinearRegression
>>> X = np.array([[1, 1], [1, 2], [2, 2], [2, 3]])
>>> # y = 1 * x_0 + 2 * x_1 + 3
>>> y = np.dot(X, np.array([1, 2])) + 3
>>> reg = LinearRegression().fit(X, y)
>>> reg.score(X, y)
1.0
>>> reg.coef_
array([1., 2.])
>>> reg.intercept_
3.0...
>>> reg.predict(np.array([[3, 5]]))
array([16.])
```

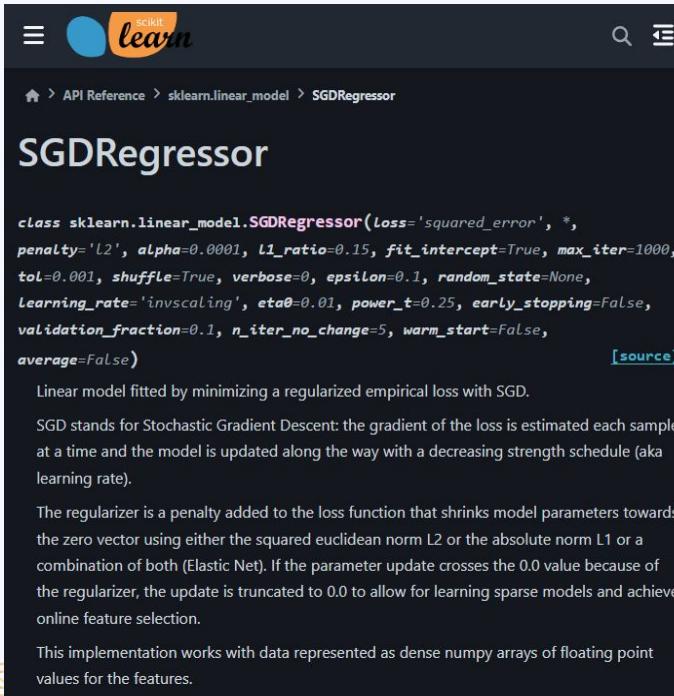
fit(x, y, sample_weight=None)

Fit linear model.

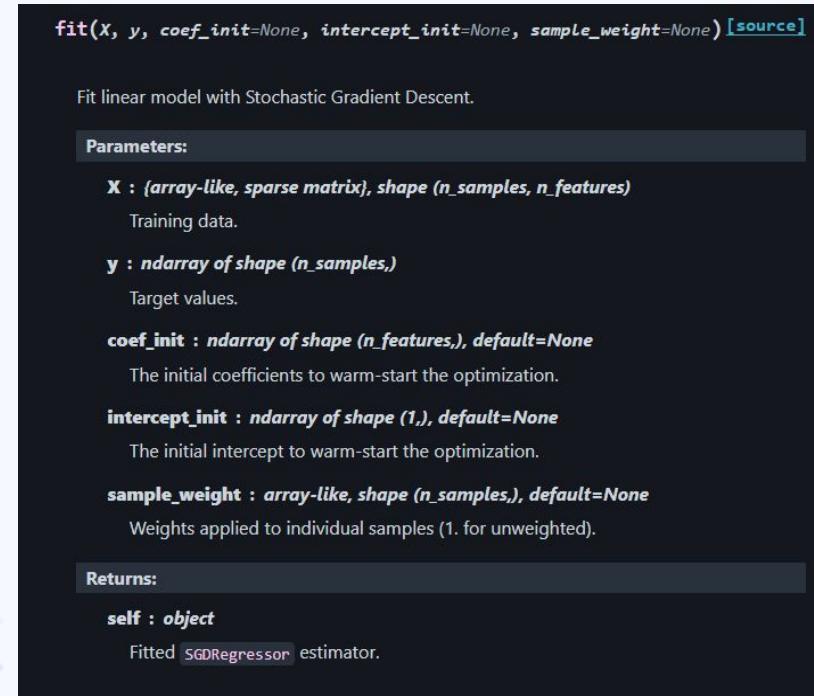
Parameters:

Alguns algoritmos de otimização

Exemplo: a classe SGDRegressor do *sklearn*.



The screenshot shows the scikit-learn API Reference page for the SGDRegressor class. The URL is `https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.SGDRegressor.html`. The page title is "SGDRegressor". The code block defines the SGDRegressor class with various parameters like loss, penalty, alpha, etc. A note below the code says: "Linear model fitted by minimizing a regularized empirical loss with SGD." Another note explains: "SGD stands for Stochastic Gradient Descent: the gradient of the loss is estimated each sample at a time and the model is updated along the way with a decreasing strength schedule (aka learning rate). The regularizer is a penalty added to the loss function that shrinks model parameters towards the zero vector using either the squared euclidean norm L2 or the absolute norm L1 or a combination of both (Elastic Net). If the parameter update crosses the 0.0 value because of the regularizer, the update is truncated to 0.0 to allow for learning sparse models and achieve online feature selection." At the bottom, it states: "This implementation works with data represented as dense numpy arrays of floating point values for the features."



The screenshot shows the SGDRegressor class definition from the scikit-learn source code. It includes the `fit` method and its parameters:

```
fit(X, y, coef_init=None, intercept_init=None, sample_weight=None) [source]
```

Fit linear model with Stochastic Gradient Descent.

Parameters:

- X** : {array-like, sparse matrix}, shape (n_samples, n_features)
Training data.
- y** : ndarray of shape (n_samples,)
Target values.
- coef_init** : ndarray of shape (n_features,), default=None
The initial coefficients to warm-start the optimization.
- intercept_init** : ndarray of shape (1,), default=None
The initial intercept to warm-start the optimization.
- sample_weight** : array-like, shape (n_samples,), default=None
Weights applied to individual samples (1. for unweighted).

Returns:

- self** : object
Fitted SGDRegressor estimator.

Alguns algoritmos de otimização

Exemplo: a classe LogisticRegression do `sklearn`.

The screenshot shows the `sklearn` API Reference page for the `LogisticRegression` class. The URL is `https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.LogisticRegression.html`. The page title is `LogisticRegression`. The code block shows the class definition:

```
class sklearn.linear_model.LogisticRegression(penalty='l2', *, dual=False, tol=0.0001,
C=1.0, fit_intercept=True, intercept_scaling=1, class_weight=None, random_state=None,
solver='lbfgs', max_iter=100, multi_class='deprecated', verbose=0, warm_start=False,
n_jobs=None, l1_ratio=None)
```

The `[source]` button is visible. Below the code, there is a note about the classifier being a logit or MaxEnt classifier. It then discusses the one-vs-rest (OvR) scheme for multiclass cases and the cross-entropy loss for multinomial cases. It also mentions supported solvers: 'lbfgs', 'sag', 'saga', and 'newton-cg'. The text then describes the implementation of regularized logistic regression using the 'liblinear' library and 'lbfgs' solvers. It notes that regularization is applied by default. The input can be C-ordered arrays or CSR matrices. The 'newton-cg', 'sag', and 'lbfgs' solvers support L2 regularization, while 'liblinear' supports both L1 and L2 regularization. Elastic-Net regularization is supported by the 'saga' solver. A link to the User Guide is provided.

Parameters:

- penalty: 'l2', 'l1', 'elasticnet', 'none' (default: 'l2')
- dual: bool (default: False)
- tol: float (default: 0.0001)
- C: float (default: 1.0)
- fit_intercept: bool (default: True)
- intercept_scaling: float (default: 1)
- class_weight: dict or 'balanced' (default: None)
- random_state: int, RandomState instance, or None (default: None)
- solver: 'lbfgs', 'liblinear', 'newton-cg', 'newton-cholesky', 'sag', 'saga' (default: 'lbfgs')
- max_iter: int (default: 100)
- multi_class: 'ovr', 'multinomial', 'deprecated' (default: 'deprecated')
- verbose: int (default: 0)
- warm_start: bool (default: False)
- n_jobs: int or None (default: None)
- l1_ratio: float between 0 and 1 (default: None)

The screenshot shows the detailed documentation for the `LogisticRegression` class. The `solver` parameter is highlighted with a yellow background. The text explains that the choice of solver depends on the penalty type and the number of classes. It lists solvers for different scenarios:

- For small datasets, 'liblinear' is a good choice, whereas 'sag' and 'saga' are faster for large ones;
- For multiclass problems, only 'newton-cg', 'sag', 'saga' and 'lbfgs' handle multinomial loss;
- 'liblinear' and 'newton-cholesky' can only handle binary classification by default. To apply a one-versus-rest scheme for the multiclass setting one can wrap it with the `OneVsRestClassifier`.
- 'newton-cholesky' is a good choice for `n_samples >> n_features`, especially with one-hot encoded categorical features with rare categories. Be aware that the memory usage of this solver has a quadratic dependency on `n_features` because it explicitly computes the Hessian matrix.

Warning

The choice of the algorithm depends on the penalty chosen and on (multinomial) multiclass support:

| solver | penalty | multinomial multiclass |
|-------------------|--------------------------------|------------------------|
| 'lbfgs' | 'l2', None | yes |
| 'liblinear' | 'l1', 'l2' | no |
| 'newton-cg' | 'l2', None | yes |
| 'newton-cholesky' | 'l2', None | no |
| 'sag' | 'l2', None | yes |
| 'saga' | 'elasticnet', 'l1', 'l2', None | yes |

Note

'sag' and 'saga' fast convergence is only guaranteed on features with approximately the same scale. You can preprocess the data with a scaler from `skewer.preprocessing`.

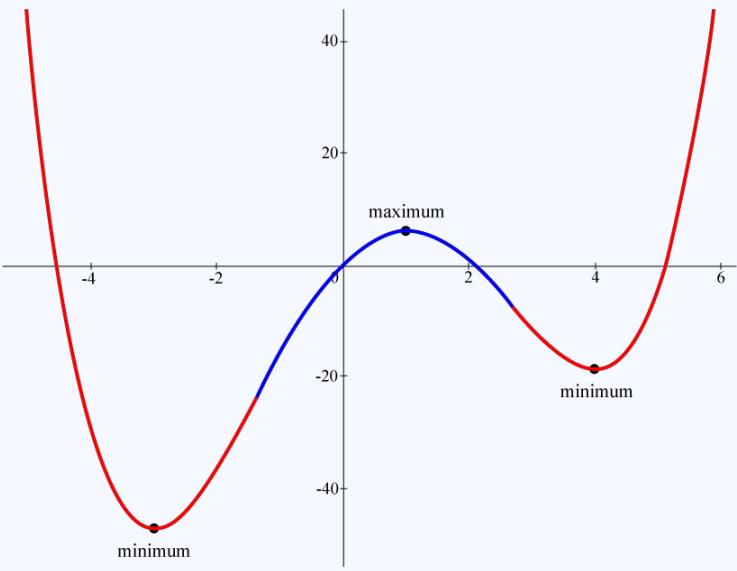
03

Gradiente Descendente

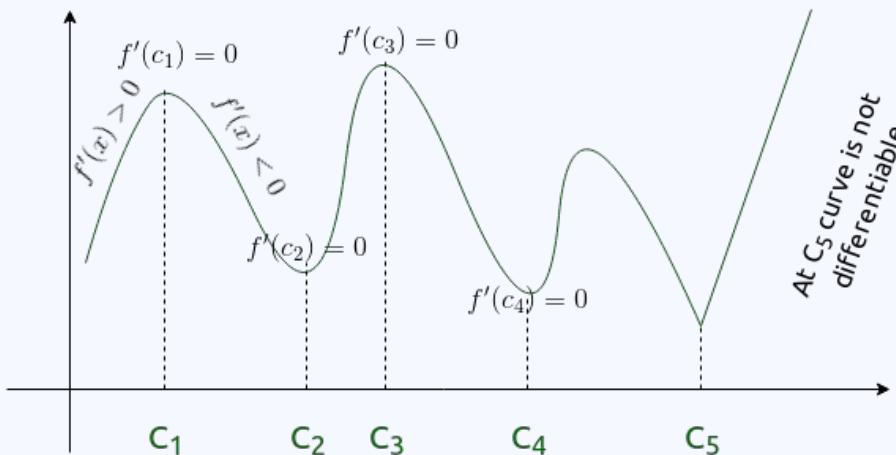


Relembrando derivadas

Como minimizamos funções em Cálculo Diferencial?



Relembrando derivadas



Derivada:

“Taxa” de variação de uma função em certo ponto.

Mínimo local:

Derivada de primeira ordem é igual a 0.

Testes de derivada de primeira e segunda ordem.

Relembrando derivadas

Vetor Gradiente:

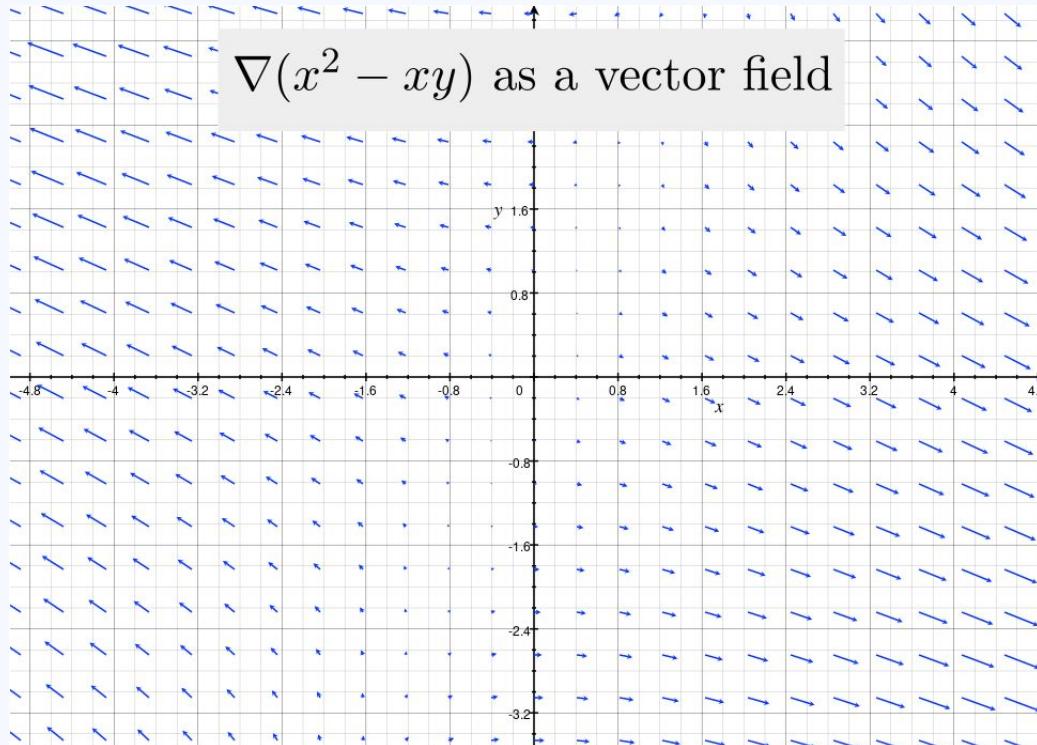
$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

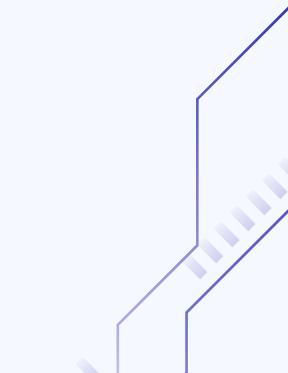
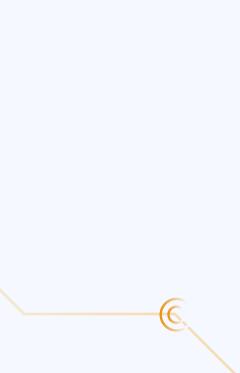
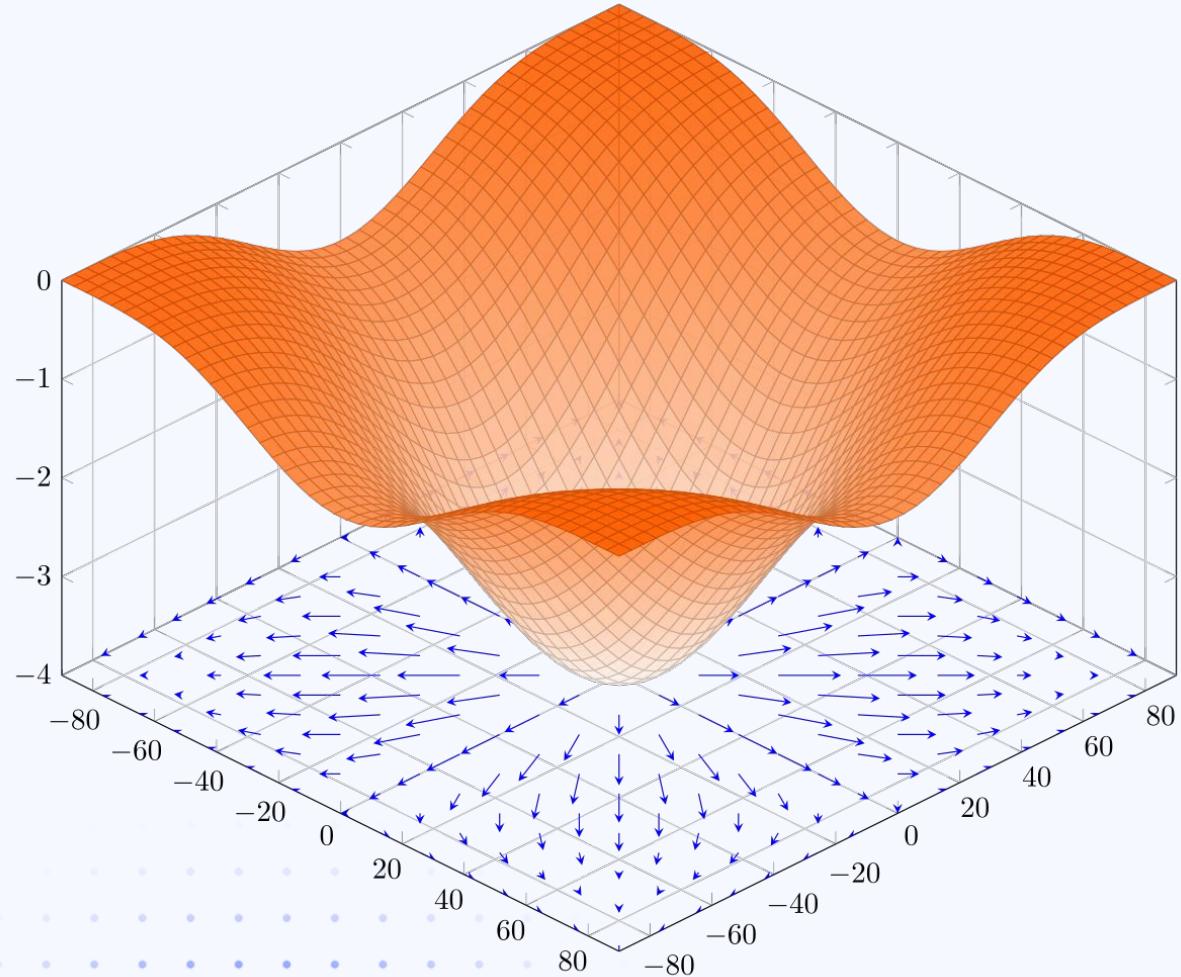
$$F(x, y, z) = x + y^2 + z^3$$

$$\nabla F(x, y, z) = \left(\frac{dF}{dx}, \frac{dF}{dy}, \frac{dF}{dz} \right) = (1, 2y, 3z^2)$$

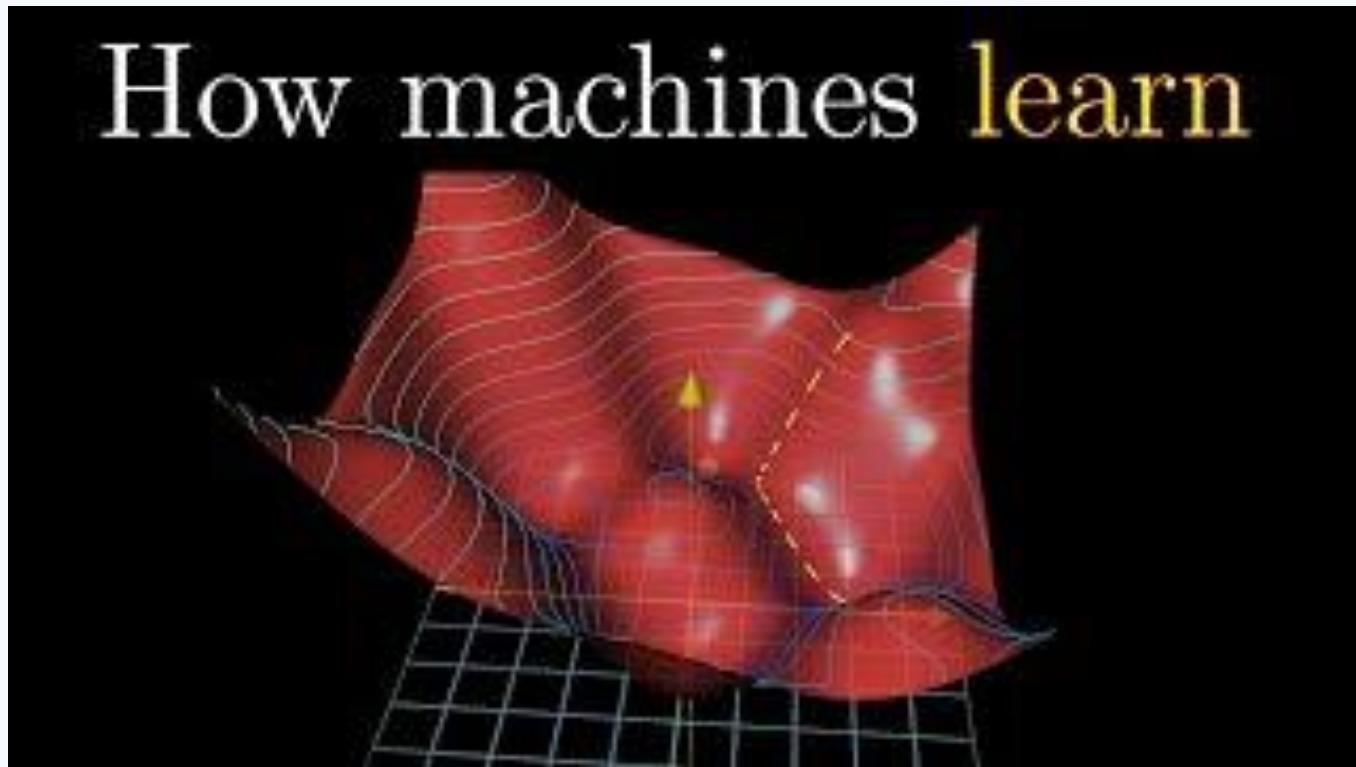
| Name | Symbol | Example |
|--------------------|-------------------------------|---|
| Derivative | $\frac{d}{dx}$ | $\frac{d}{dx}(x^2) = 2x$ |
| Partial derivative | $\frac{\partial}{\partial x}$ | $\frac{\partial}{\partial x}(x^2 - xy) = 2x - y$ |
| Gradient | ∇ | $\nabla(x^2 - xy) = \begin{bmatrix} 2x - y \\ -x \end{bmatrix}$ |

Relembrando derivadas

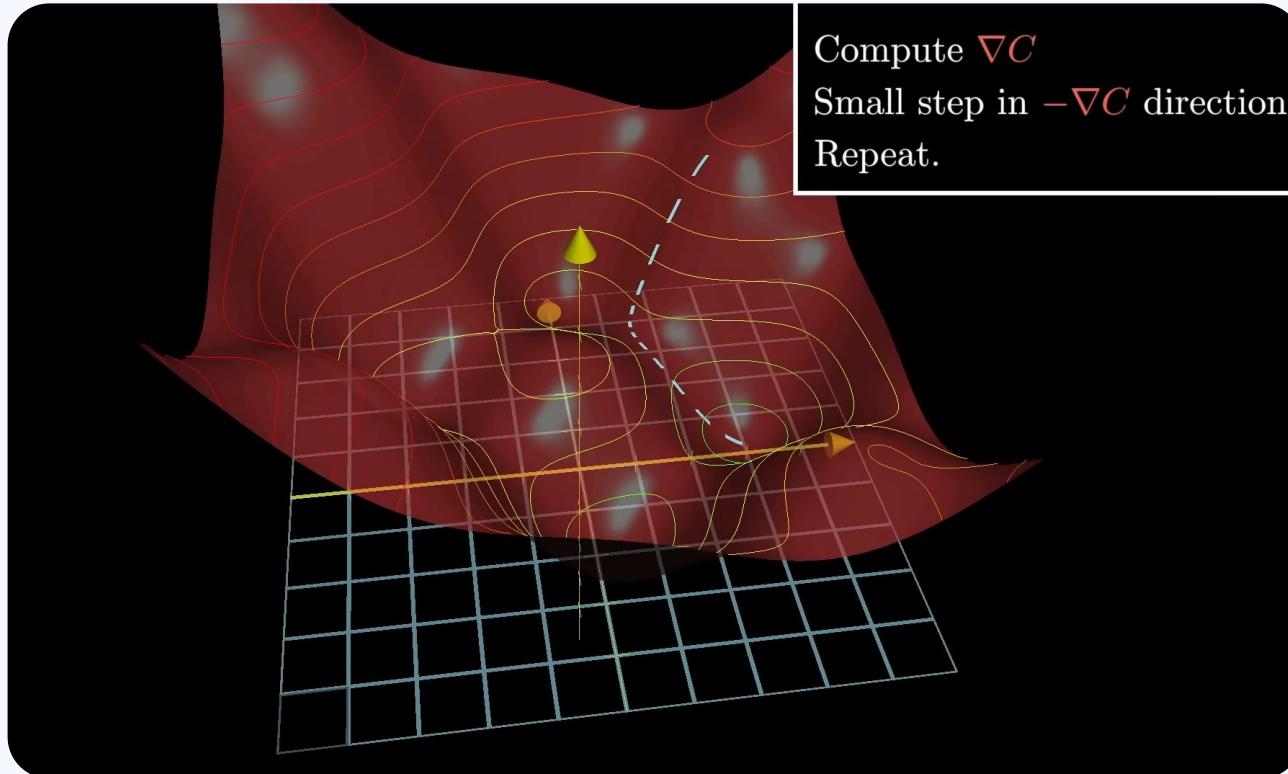




A descida do gradiente



A descida do gradiente



A descida do gradiente

$$\vec{W} = \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ \vdots \\ w_{13,000} \\ w_{13,001} \\ w_{13,002} \end{bmatrix}$$

$$-\nabla C(\vec{W}) = \begin{bmatrix} 0.31 \\ 0.03 \\ -1.25 \\ \vdots \\ 0.78 \\ -0.37 \\ 0.16 \end{bmatrix}$$

w_0 should increase somewhat
 w_1 should increase a little
 w_2 should decrease a lot
 $w_{13,000}$ should increase a lot
 $w_{13,001}$ should decrease somewhat
 $w_{13,002}$ should increase a little

Treinando uma Regressão Linear

Gradient Descent algorithm

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$$

(for $j = 0$ and $j = 1$)

}

Linear Regression Model

$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

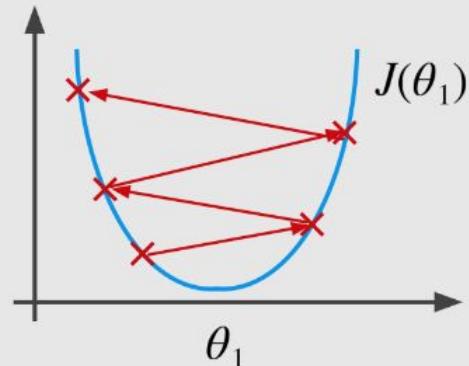
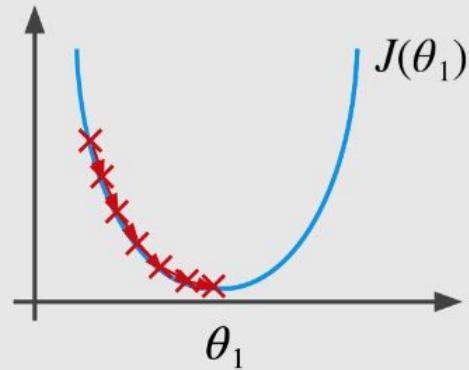
$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

Treinando uma Regressão Linear

$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

If α is too small, gradient descent can be slow.

If α is too large, gradient descent can be overshoot the minimum. It may fail to converge, or even diverge.



Treinando uma Regressão Linear

$$\begin{aligned}\frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) &= \frac{\partial}{\partial \theta_j} \cdot \frac{1}{2m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)})^2 \\ &= \frac{\partial}{\partial \theta_j} \cdot \frac{1}{2m} \sum_{i=1}^m (\theta_0 + \theta_1 x^{(i)} - y^{(i)})^2\end{aligned}$$

$$j = 0: \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)})$$

$$j = 1: \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x^{(i)}$$

Treinando uma Regressão Linear

repeat until convergence {

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)})$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x^{(i)}$$

} update θ_0 and θ_1
simultaneously

}

Batch Gradient Descent

Cada **passo**, leva em conta **todas as amostras** de treinamento:

repeat until convergence {

$$\left. \begin{aligned} \theta_0 &:= \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) \\ \theta_1 &:= \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x^{(i)} \end{aligned} \right\} \text{update } \theta_0 \text{ and } \theta_1 \text{ simultaneously}$$

}

Stochastic Gradient Descent

Cada **passo**, leva em conta **uma amostra** de treinamento:

```
repeat until convergence {
```

```
    for i = 1, ..., m {
```

$$\theta_0 := \theta_0 - \alpha(h_{\theta}(x^{(i)}) - y^{(i)})$$

$$\theta_1 := \theta_1 - \alpha(h_{\theta}(x^{(i)}) - y^{(i)})x^{(i)}$$

```
}
```

```
}
```

Mini-batch Gradient Descent

Cada **passo**, leva em conta **b amostras** de treinamento:

Say $b = 10$, $m = 1000$.

repeat until convergence {

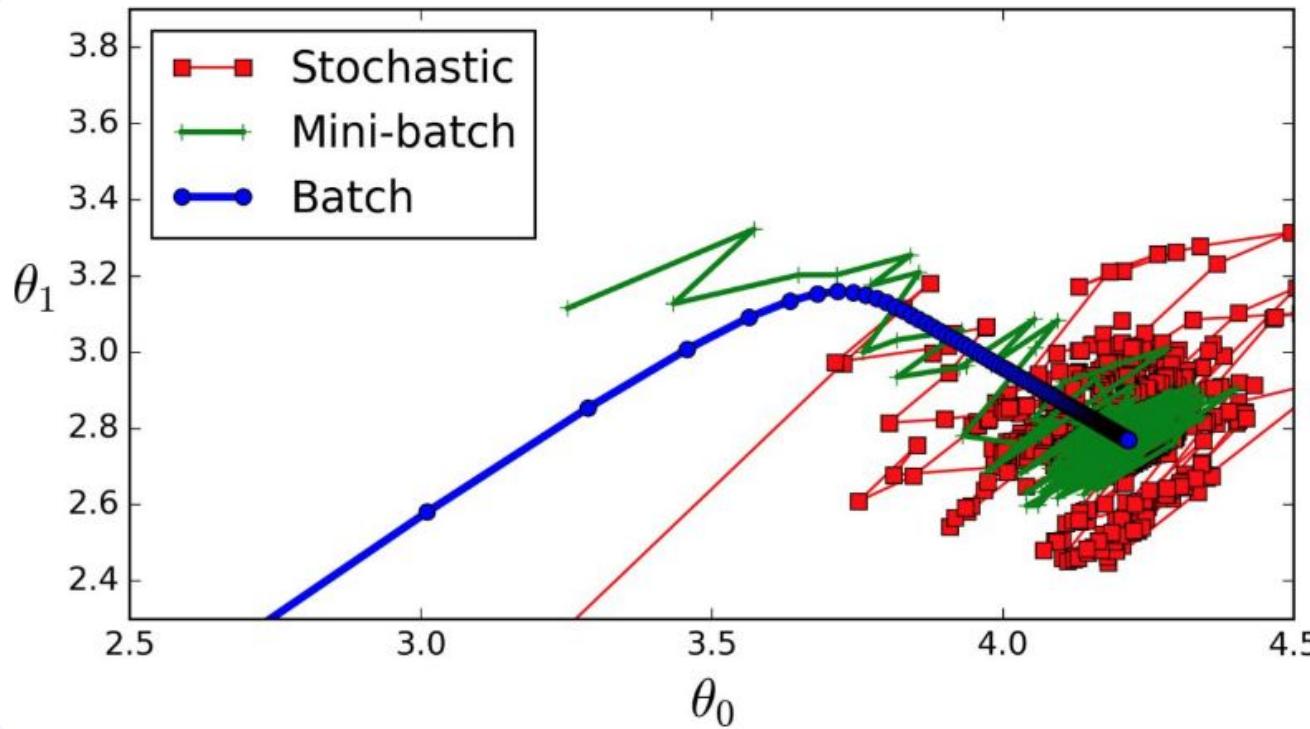
 for $i = 1, 11, 21, \dots, 991$ {

$$\theta_0 := \theta_0 - \alpha \frac{1}{10} \sum_{\substack{i=k \\ i+9}}^{i+9} (h_\theta(x^{(k)}) - y^{(k)})$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{10} \sum_{i=k}^{i+9} (h_\theta(x^{(k)}) - y^{(k)}) x^{(k)}$$

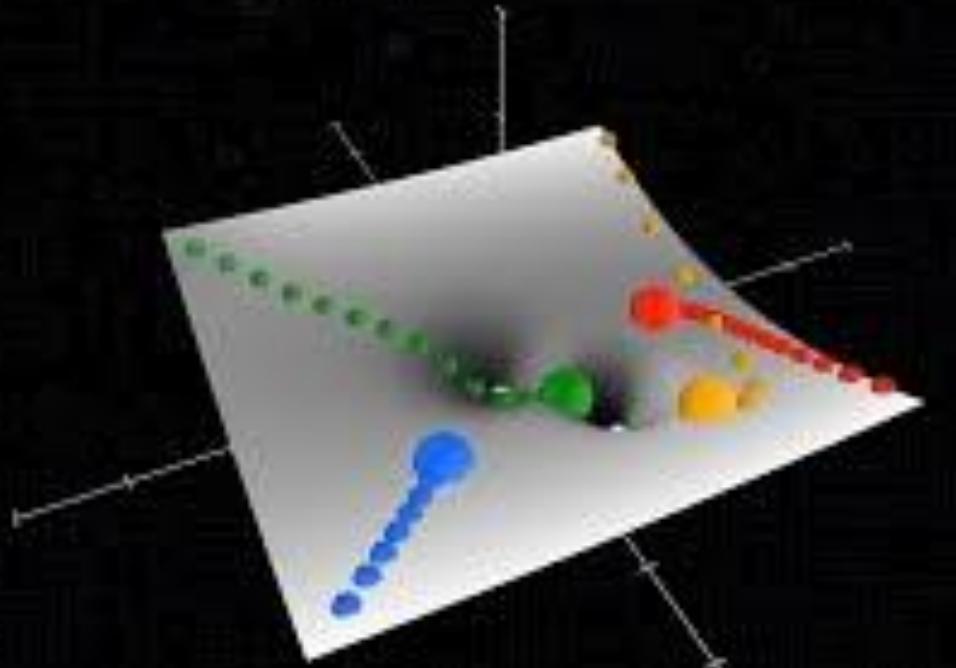
} }

Batch vs Mini-batch vs Stochastic



Optimizers in Deep Learning

- ⚙️ SGD
- ⚙️ Momentum
- ⚙️ RMSProp
- ⚙️ Adam



Instantes: 01:17-04:24; 04:38-04:54; 05:37-07:08; 12:30-12:44; 15:40-16:27; 16:28-16:50.

optimization

An overview of gradient descent optimization algorithms

Gradient descent is the preferred way to optimize neural networks and many other machine learning algorithms but is often used as a black box. This post explores how many of the most popular gradient-based optimization algorithms such as Momentum, Adagrad, and Adam actually work.

**Sebastian Ruder**

Jan 19, 2016 • 28 min read



Link para o artigo: <https://www.ruder.io/optimizing-gradient-descent/>

04

Equação Normal



O que é a Equação Normal?

Basicamente se trata de:

Implementar o Método dos Mínimos Quadrados (MMQ)

Em inglês, *Ordinary Least Squares* (*OLS*)

A **Equação Normal** representa a fórmula para resolver uma Regressão Linear **de maneira analítica**:

$$\frac{d}{d\theta} J(\theta) = \dots = 0 \quad \text{Solve for } \theta$$

Qual a Equação Normal?

Considerando **m exemplos** e **n features**, temos:

| x_0 | x_1 | x_2 | x_3 | x_4 | y |
|-------|-------|-------|-------|-------|-----|
| 1 | 2104 | 5 | 1 | 45 | 460 |
| 1 | 1416 | 3 | 2 | 40 | 232 |
| 1 | 1534 | 3 | 2 | 30 | 315 |
| 1 | 852 | 2 | 1 | 36 | 178 |

$$X = \begin{bmatrix} 1 & 2104 & 5 & 1 & 45 \\ 1 & 1416 & 3 & 2 & 40 \\ 1 & 1534 & 3 & 2 & 30 \\ 1 & 852 & 2 & 1 & 36 \end{bmatrix}$$

Qual a Equação Normal?

Considerando **m exemplos** e **n features**, temos:

| x_0 | Size (feet²) | Number of bedrooms | Number of floors | Age of home (years) | Price (\$) in 1000's |
|-------|------------------------------------|-------------------------------|-----------------------------|--------------------------------|-------------------------------------|
| | x_1 | x_2 | x_3 | x_4 | y |
| 1 | 2104 | 5 | 1 | 45 | 460 |
| 1 | 1416 | 3 | 2 | 40 | 232 |
| 1 | 1534 | 3 | 2 | 30 | 315 |
| 1 | 852 | 2 | 1 | 36 | 178 |

$$X = \begin{bmatrix} 1 & 2104 & 5 & 1 & 45 \\ 1 & 1416 & 3 & 2 & 40 \\ 1 & 1534 & 3 & 2 & 30 \\ 1 & 852 & 2 & 1 & 36 \end{bmatrix} \quad y = \begin{bmatrix} 460 \\ 232 \\ 315 \\ 178 \end{bmatrix}$$

Qual a Equação Normal?

Considerando **m exemplos** e **n features**, temos:

| x_0 | Size (feet²) | Number of bedrooms | Number of floors | Age of home (years) | Price (\$) in 1000's |
|-------|------------------------------------|-------------------------------|-----------------------------|--------------------------------|-------------------------------------|
| | x_1 | x_2 | x_3 | x_4 | y |
| 1 | 2104 | 5 | 1 | 45 | 460 |
| 1 | 1416 | 3 | 2 | 40 | 232 |
| 1 | 1534 | 3 | 2 | 30 | 315 |
| 1 | 852 | 2 | 1 | 36 | 178 |

$$X = \begin{bmatrix} 1 & 2104 & 5 & 1 & 45 \\ 1 & 1416 & 3 & 2 & 40 \\ 1 & 1534 & 3 & 2 & 30 \\ 1 & 852 & 2 & 1 & 36 \end{bmatrix} \quad y = \begin{bmatrix} 460 \\ 232 \\ 315 \\ 178 \end{bmatrix}$$

Qual a Equação Normal?

Considerando **m exemplos** e **n features**, temos:

$$X = \begin{bmatrix} 1 & 2104 & 5 & 1 & 45 \\ 1 & 1416 & 3 & 2 & 40 \\ 1 & 1534 & 3 & 2 & 30 \\ 1 & 852 & 2 & 1 & 36 \end{bmatrix}_{m \times (n+1)} \quad y = \begin{bmatrix} 460 \\ 232 \\ 315 \\ 178 \end{bmatrix}_m$$

Qual a Equação Normal?

Considerando **m exemplos** e **n features**, temos:

$$x^{(i)} = \begin{bmatrix} x_0^{(i)} \\ x_1^{(i)} \\ x_2^{(i)} \\ \vdots \\ x_n^{(i)} \end{bmatrix} \in \mathbb{R}^{n+1}$$

$$X = \begin{bmatrix} \quad (x^{(1)})^T \quad \\ \quad (x^{(2)})^T \quad \\ \vdots \\ \quad (x^{(m)})^T \quad \end{bmatrix}$$

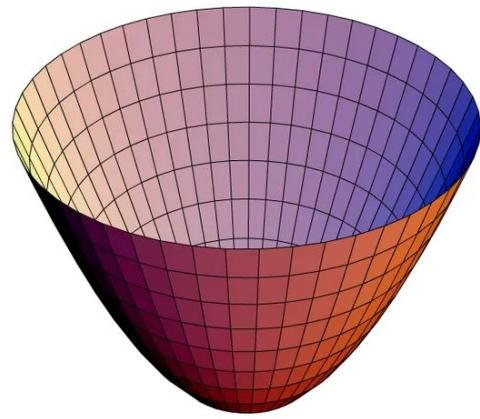
Qual a Equação Normal?

É possível chegar à seguinte expressão:

$$\theta = (X^T X)^{-1} X^T y$$

E se a matriz $X^T X$ não for invertível?

Provavelmente está utilizando duas *features* altamente correlacionadas, ou seja, **linearmente dependentes**.



https://qph.ec.quoracdn.net/main-qimg-99f7ea7d0929ed41dbecf67ec51b80b3?convert_to_webp=true

Rohan #3: Deriving the Normal Equation using matrix calculus

Understanding the analytical solution to linear regression.



Rohan Kapur · Follow

Published in A Year of Artificial Intelligence · 11 min read · Feb 16, 2016



1.2K



23



Link para o artigo:

<https://ayearofai.com/rohan-3-deriving-the-normal-equation-using-matrix-calculus-1a1b16f65dda>

05

Implementando os algoritmos





Obrigado pessoal!

Até próxima aula :)



Iris Data Science UNICAMP



@irisdatascienceunicamp

Referências

Parte do material foi adaptado dos slides da Prof^a. Sandra Avila apresentados na disciplina MC886.

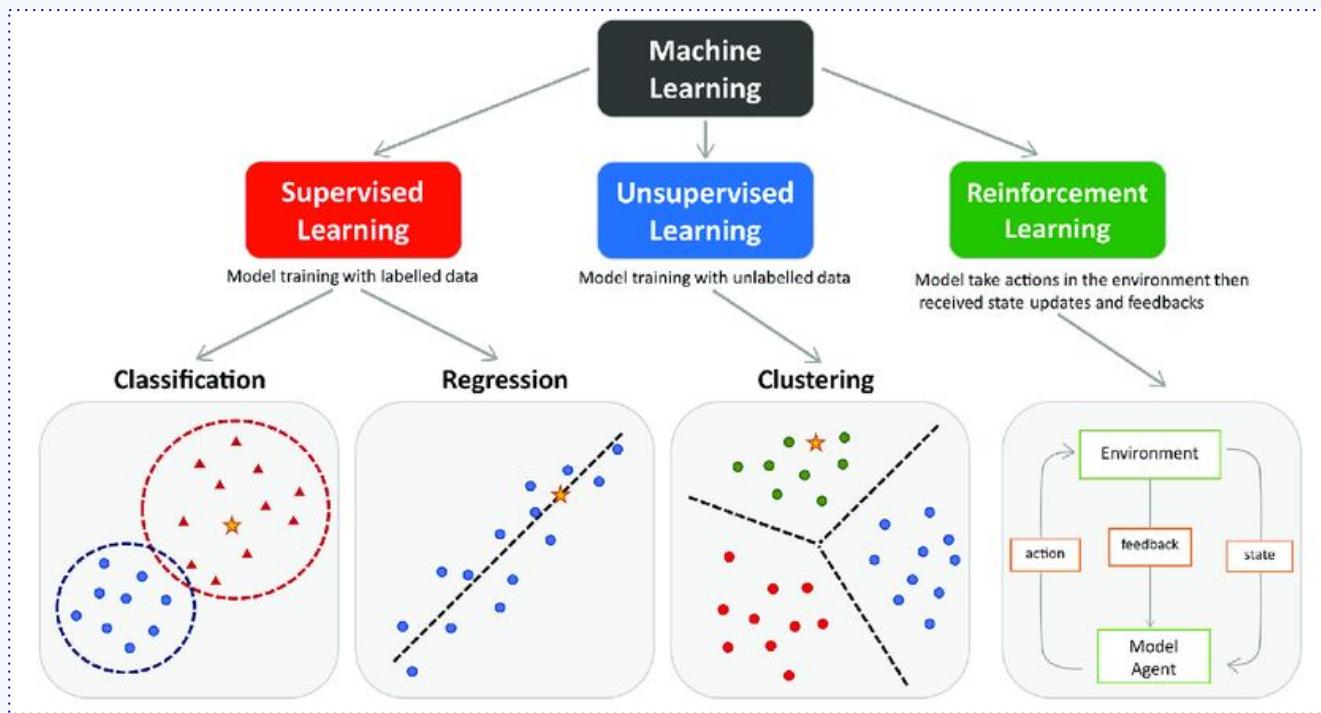
Material online:

- Gradient descent, how neural networks learn | Chapter 2, Deep learning (YouTube)

Livros utilizados:

- An Introduction to Statistical Learning (2023)
- Hands-On Machine Learning with Scikit-Learn, Keras, and Tensor Flow (2017)

Paradigmas de aprendizado



Retirado de:

https://www.researchgate.net/figure/The-main-types-of-machine-learning-Main-approaches-include-classification-and_fig1_354960266

Paradigmas de aprendizado

Aprendizado supervisionado:



O objetivo é “**predizer**” uma saída esperada/alvo.



Saída esperada /alvo é **conhecida**.

Aprendizado Não-supervisionado:



O objetivo é **encontrar padrões** na estrutura dos dados.



Não há uma saída esperada /alvo **conhecida**.

Aprendizado Auto-supervisionado:



O objetivo é extrair e **capturar os padrões** nos dados.



A saída esperada /alvo **faz parte dos dados de entrada**.

Como é o aprendizado supervisionado?

Dados sem
a resposta

Processamento
do modelo

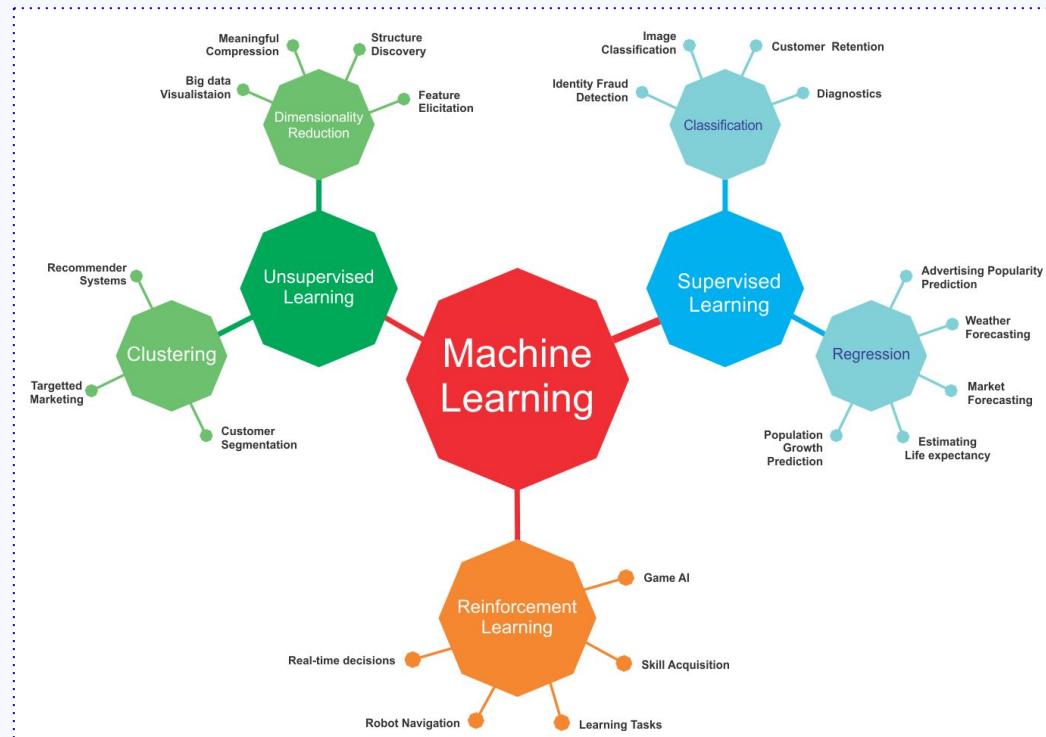
Chute de
resposta

Ajuste das
regras

O chute
está certo
ou errado?



Tarefas (ou “problemas”) em ML



Retirado de: <https://subscription.packtpub.com/book/data/9781789345070/1/ch01lvl1sec04/ml-tasks>

Tarefas supervisionadas

Regressão

O objetivo é **quantificar e inferir a relação** de uma **variável dependente** (*variável de resposta*) com **variáveis independentes** (*variáveis explicativas*).

Classificação

Identificar **a que categoria pertence uma nova observação, com base em um conjunto de dados** contendo observações cujas suas categorias são conhecidas.

Tipos de regressão

- Regressão Linear
- Regressão Não-linear
- Regressão Polinomial

Regressão Logística

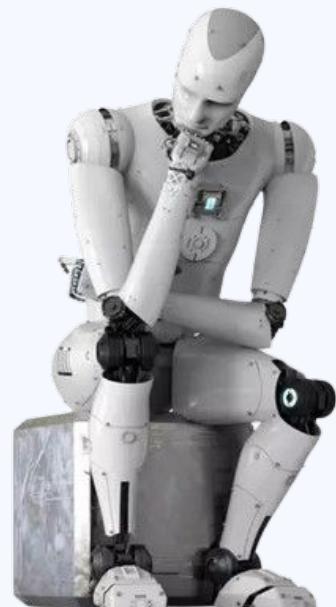


O aprendizado de máquina

Programação tradicional:



Aprendizado de máquina:



Revisão e atividade prática

Ambiente de desenvolvimento: Google Colab.

Atividade de revisão:

- Python;
- Manipulação de dados;
- Análise de dados;
- Operação com matrizes.



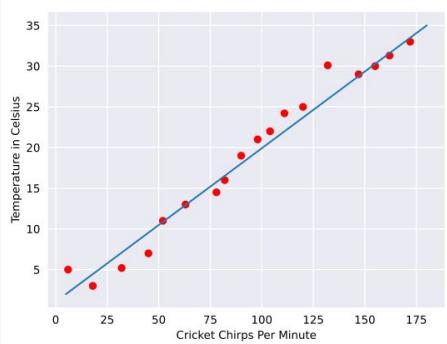
O que é Machine Learning?

“Um programa de computador **aprende a partir de uma experiência E** com respeito a uma **classe de tarefas T** e uma **métrica de performance P**, se a **sua performance em tarefas de T, quando medida por P, melhora com a experiência E.**”

– Tom Mitchell (1997)



O que é Machine Learning?



**Modelos que se
ajustam aos dados:**

- Regressão linear
- Regressão logística
- Algoritmos de Clusterização
- ...

Exemplos:

- Classificadores simples
- Sistemas preditivos
- Sistemas de recomendação
- ...

O que é Deep Learning?

“A aprendizagem profunda é uma forma de aprendizagem de máquina que permite aos computadores aprender com a experiência e **compreender o mundo em termos de uma hierarquia de conceitos**. Como o computador adquire conhecimento a partir da experiência, **não é necessário que um operador de computador humano especifique formalmente todo o conhecimento que o computador precisa**. A hierarquia de conceitos permite que o computador **aprenda conceitos complicados construindo-os a partir de conceitos mais simples.**”

– Goodfellow et al. (2016)



O que é Deep Learning?

**Sistemas que descobrem,
aprendem e combinam
padrões aprendidos:**

- CNNs
- RNNs e LSTMs
- Transformers
- ...

Exemplos:

- Sistemas de visão computacional
- Chatbots modernos
- IA generativa
- ...

Retirado de:

<https://exame.com/inteligencia-artificial/o-que-e-chatgpt-como-usar-a-ia-em-portugues-no-seu-dia-a-dia/>

<https://medium.com/analytics-vidhya/yolo-explained-5b6f4564f31>,

<https://deepmind.google/discover/blog/alphafold-a-solution-to-a-50-year-old-grand-challenge-in-biology/>

Emergência e homogeneização

Emergência

Surgimento de novos comportamentos.
“Soma das partes não é igual ao todo”.

Homogeneização

Uso de uma mesma técnica em várias áreas.

Emergência e homogeneização

Machine Learning

Emergência

Sistemas passam a ter a capacidade de descobrirem como resolver uma tarefa a partir de dados.

Homogeneização

Diversas aplicações passaram a poder se basear em um algoritmo genérico de aprendizado com base em dados.

Deep Learning

Emergência

Aprendizagem de conceitos abstratos sem a especificação explícita. A escala enorme de modelos trouxe características como *in-context learning*.

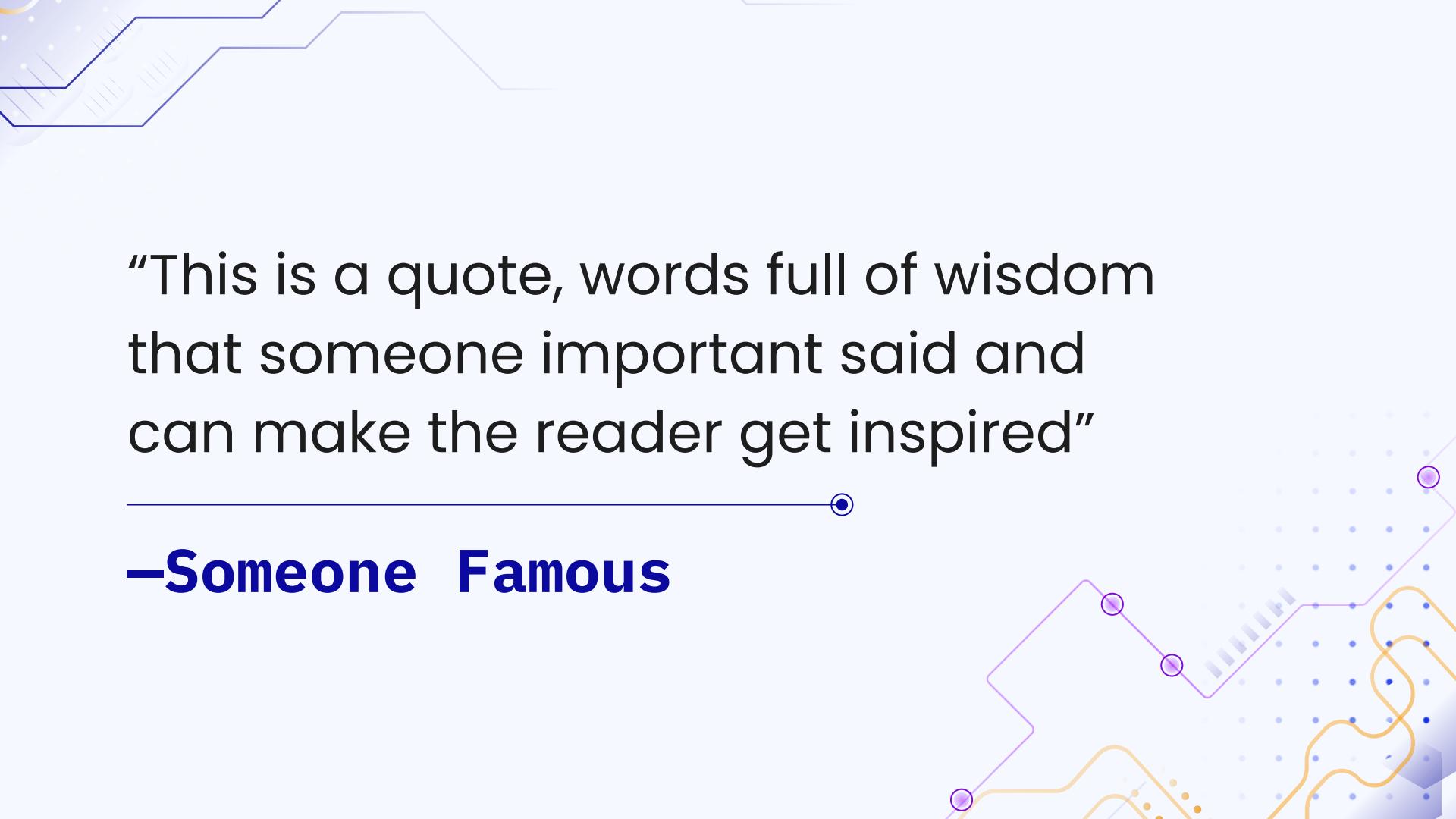
Homogeneização

As mesmas arquiteturas de modelos passaram a ser aplicáveis para diversos tipos de aplicações.

Introduction

Mercury is the closest planet to the Sun and the smallest one in the entire Solar System. **This planet's name has nothing to do with the liquid metal**, since Mercury was named after the Roman messenger god. Mercury's surface is filled with craters

Mercury takes a little more than 58 days to complete its rotation, so try to imagine how long days must be there! **Since the temperatures are so extreme, albeit not as extreme** as on Venus, Mercury has been deemed to be non-habitable for humans



“This is a quote, words full of wisdom
that someone important said and
can make the reader get inspired”

—Someone Famous

Concepts



Mercury

Mercury is the closest planet to the Sun and **the smallest one** in the Solar System—it's only a bit larger than the Moon



Venus

Venus has a beautiful name and is the **second planet from the Sun**. It's hot and has a poisonous atmosphere

What is this topic about?



Mercury

It's the closest planet to the Sun and the **smallest** in the Solar System



Venus

Venus has a beautiful name and is the second planet from the Sun



Mars

Despite being red, Mars is actually a **cold place**. It's full of iron oxide dust

Features of the topic

Mars

Despite **being red**,
Mars is very cold

Neptune

It's the farthest
planet from the Sun

Jupiter

Jupiter is the biggest
planet of them all

Saturn

Saturn is a **gas giant**
and has several rings

Examples



Mercury

It's the closest planet to the Sun and the **smallest** in the Solar System



Venus

Venus has a beautiful name and is the second planet from the Sun



Mars

Despite being red, Mars is actually a **cold place**. It's full of iron oxide dust

Recommendations



Mars

Despite being red,
Mars is very cold



Mercury

Mercury is the closest
planet to the Sun



Venus

Venus is the second
planet from the Sun



Saturn

Saturn is a gas giant
and has several rings



Neptune

Neptune is the farthest
planet from the Sun



Jupiter

Jupiter is the biggest
planet of them all

Image always reinforce the concept

You can give a brief description of the topic you want to talk about here. For example, if you want to talk about Mercury, you can say that it's the smallest planet in the entire Solar System



4,498,300,000

Big numbers catch your audience's attention

9h 55m 23s

Jupiter's rotation period

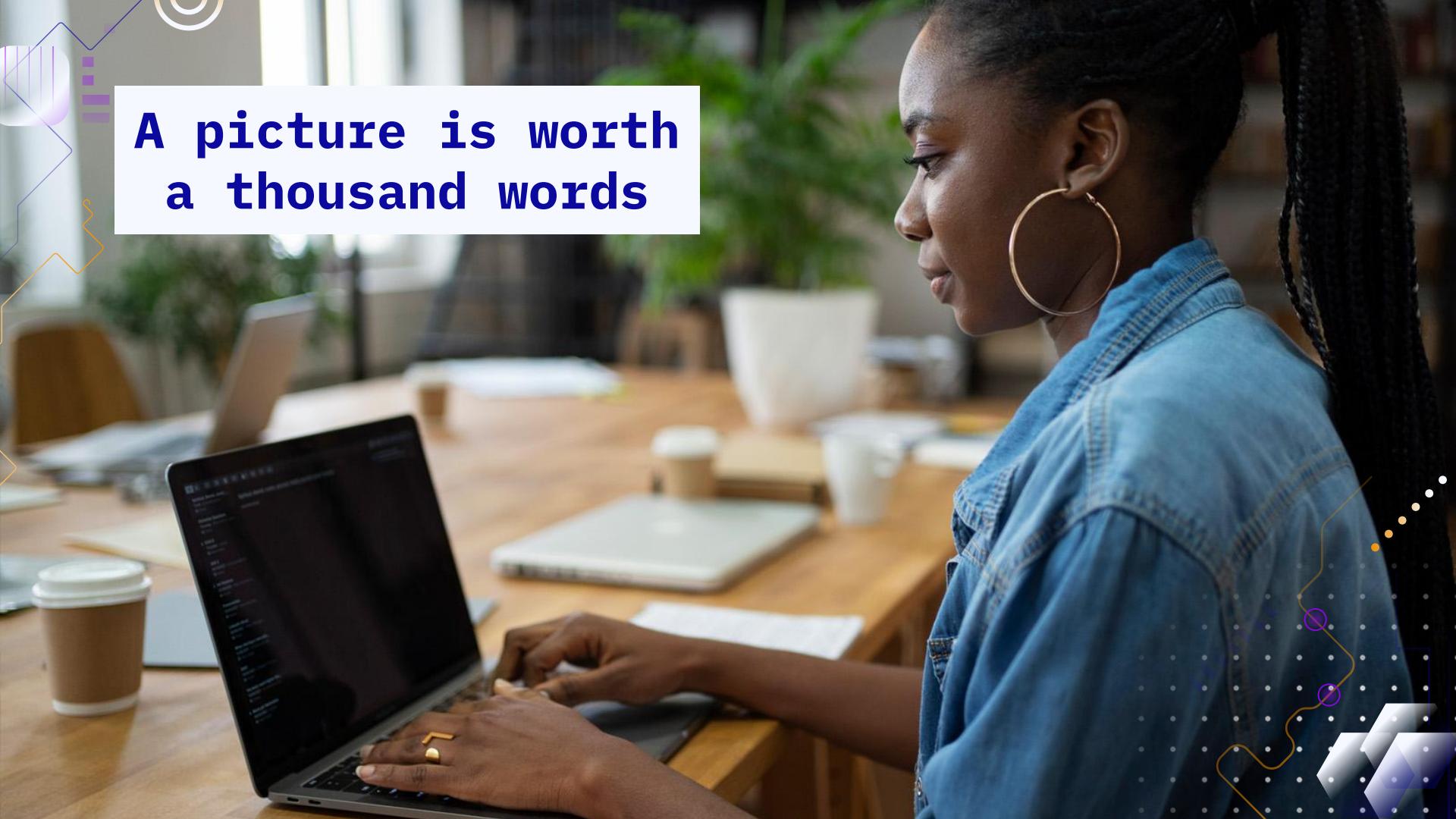
333,000

The Sun's mass compared to Earth's

386,000 km

Distance between the Earth and the Moon

Awesome words



A picture is worth
a thousand words

Practical exercise - calculator

Objective:

Introduce participants to basic coding concepts by building a **simple calculator**

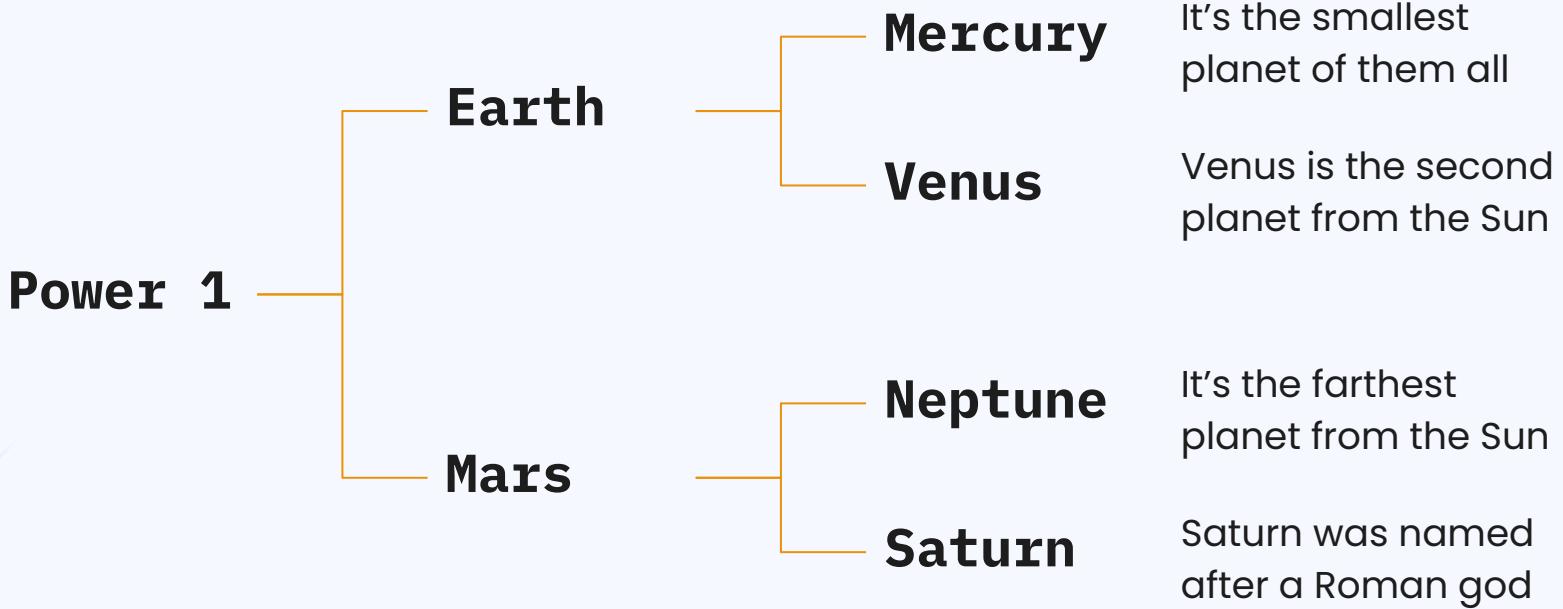
Instructions:

1. Open a Python development environment and write the following code:

```
# Simple Calculator  
num1 = int(input("Enter the first number: "))  
num2 = int(input("Enter the second number: "))  
print("Sum:", num1 + num2)  
print("Difference:", num1 - num2)  
print("Product:", num1 * num2)  
print("Quotient:", num1 / num2)
```

2. Run the program and experiment with different numbers
3. Observe the output

Brainstorm and idea generation



Main topic and details

Mars

Despite being red,
Mars is **very cold**

Jupiter

Jupiter is the biggest
planet of them all

Neptune

It's the farthest
planet from the Sun

Saturn

It's a gas giant and
has **several rings**



Popular programming languages

01

Neptune

Mercury is the closest planet to the Sun and the **smallest** of them all

02

Venus

Venus has a beautiful name and is the **second planet from the Sun**

03

Earth

Earth is the third planet from the Sun and the only one that harbors life in the Solar System

04

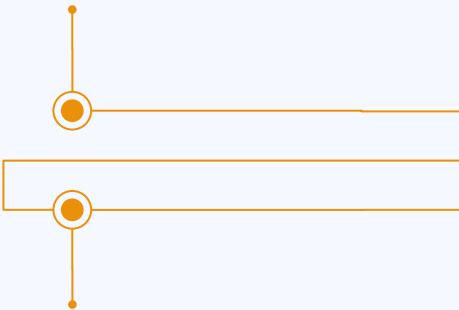
Saturn

Saturn is a gas giant and has several rings. It's composed mostly of hydrogen and helium

Sequences

Saturn is composed of
hydrogen and helium

First

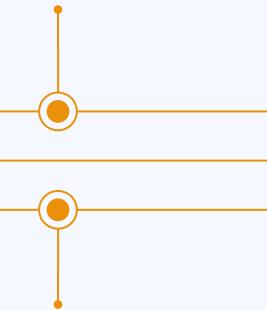


Next

Despite being red,
Mars is **very cold**

Mercury is the **closest**
planet to the Sun

Next



Next

Earth is the third
planet from the Sun



Jupiter was named
after a Roman god

Next



Last

Venus has extremely
high temperatures

Classification

| Mars | Venus | Mercury | Jupiter |
|--|--|--|--|
| <ul style="list-style-type: none">• Small• Red• Cold• Rocky | <ul style="list-style-type: none">• Small• Hot• Dry• Volcanic | <ul style="list-style-type: none">• Small• Hot• Rocky• Cratered | <ul style="list-style-type: none">• Large• Cold• Gassy• Striped |
| Mars is full of iron oxide dust | Venus has high temperatures | Mercury is quite a small planet | Jupiter is a huge gas giant |

Cause and effect

Problem

Mars

Despite being red,
Mars is very cold

Venus

Venus is the second
planet from the Sun

Solution

Mercury

Mercury is the closest
planet to the Sun

Saturn

Saturn is a gas giant
and has several rings

Question and answer

Question

Is Mercury the closest planet to the Sun and the smallest one in the Solar System? **Note that it's a bit larger than the Moon**

Answer

Venus has a beautiful name and is **the second planet from the Sun**. It's hot and has a poisonous atmosphere

Step-by-step coding

01



Earth

It's the only planet known to **harbor life**

02



Mercury

Mercury is the closest planet to the Sun

03



Jupiter

Jupiter is the **biggest** planet of them all

04



Saturn

Saturn was named after a Roman god

Parts and whole

The whole objective

Mercury is the closest planet to the Sun and the smallest one in the entire Solar System

Parts of the object

- Mercury
- Jupiter
- Venus
- Mars
- Earth
- Saturn
- Mercury

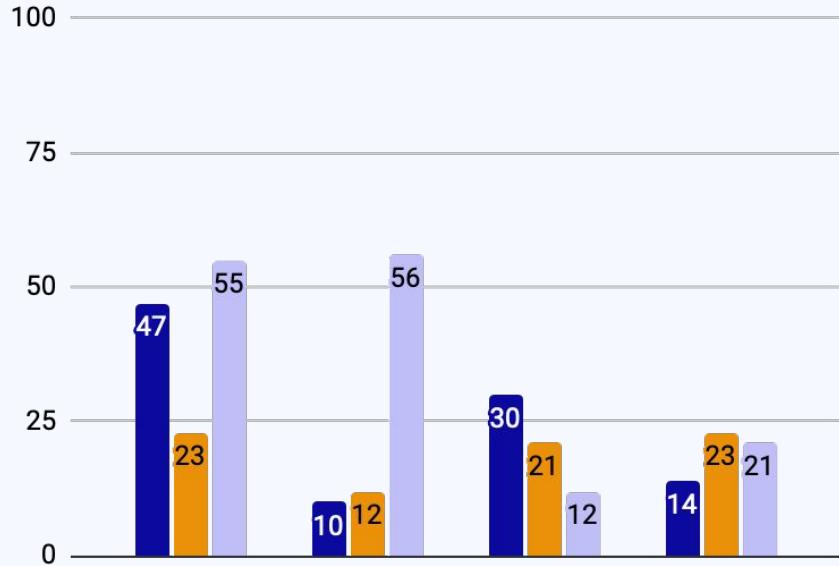
What happens if the parts are missing?

Earth is the third planet from the Sun and the **only one that harbors life in the Solar System**

What's the function of the parts?

Jupiter is a gas giant and the biggest planet in the Solar System

You can use this graph



Follow the link in the graph to modify its data and then paste the new one here. [For more info, click here](#)



Mercury

Mercury is the closest planet to the Sun



Jupiter

Jupiter is the biggest planet of them all



Saturn

Saturn was named after a Roman god

This is a map

USA

Despite being red, Mars is **very cold**

India

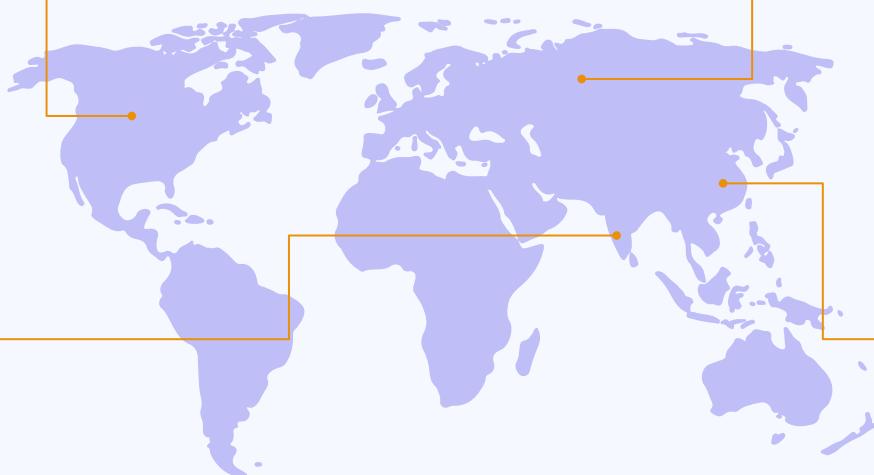
Jupiter is the biggest planet of them all

Russia

Neptune is the farthest planet from the Sun

China

Saturn is **a gas giant** and has several rings



Mockups

You can replace the images on the screen with your own work. Just right-click on them and select “Replace image”



Obrigado!

Do you have any questions?

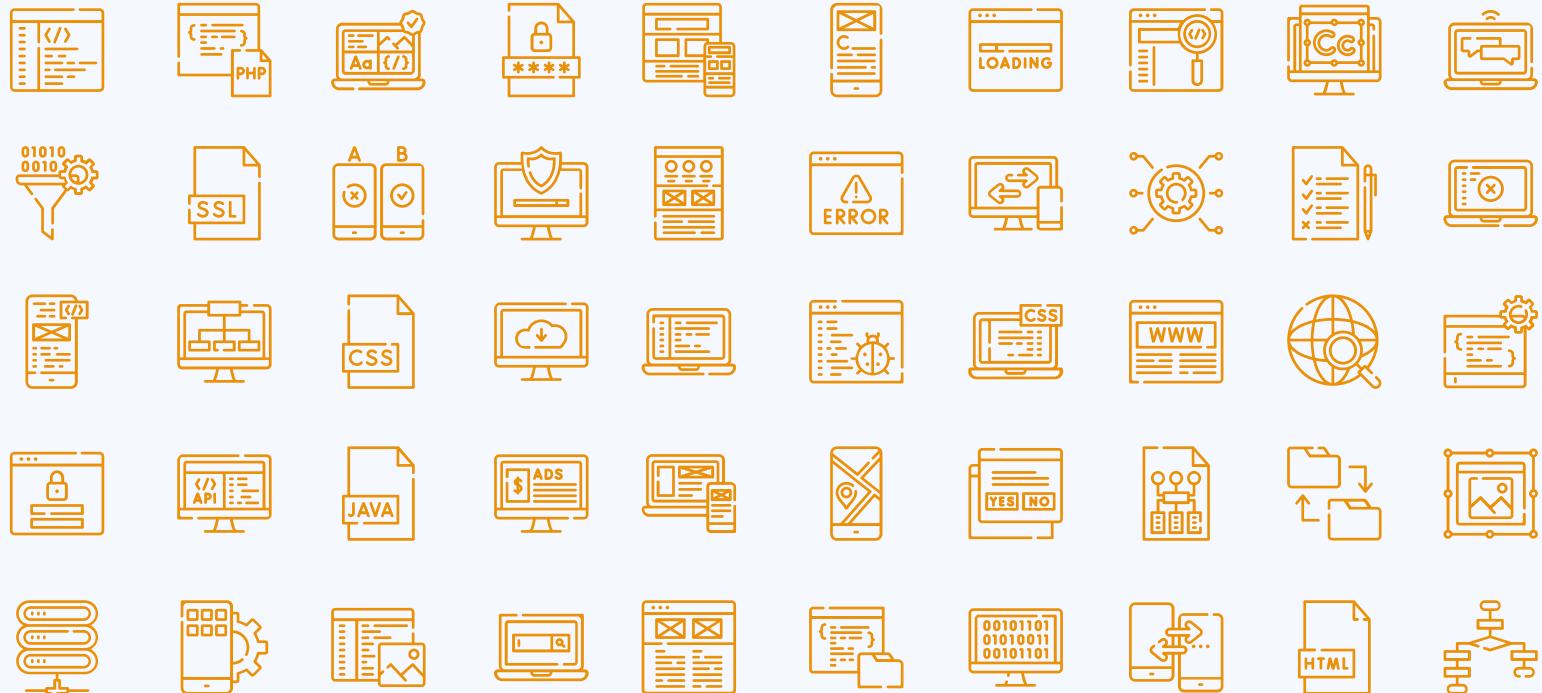
@irisdatascienceunicamp



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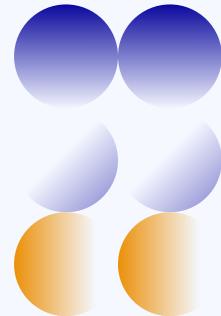
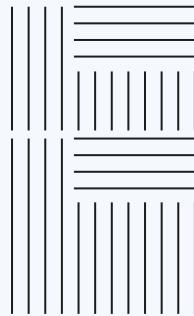


Icon pack



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- [Group of friends planning a trip in a cafe](#)
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- [Side view of men working on laptops at the office](#)
- [Lifestyle of woman in the office](#)
- [Secretary working on laptop](#)

Vectors:

- [Abstract gradient circuit board background](#)

Icon pack:

- [Icon Pack: Coding | Lineal](#)

Agenda

01

Apresentação do curso

You can describe the topic
of the section here

03

Tips

You can describe the topic
of the section here

02

O que é IA, ML e DL?

You can describe the topic
of the section here

04

Problemáticas atuais

You can describe the topic
of the section here

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| <u>Used and alternative resources</u> | An assortment of graphic resources that are suitable for use in this presentation |
| <u>Thanks slide</u> | You must keep it so that proper credits for our design are given |
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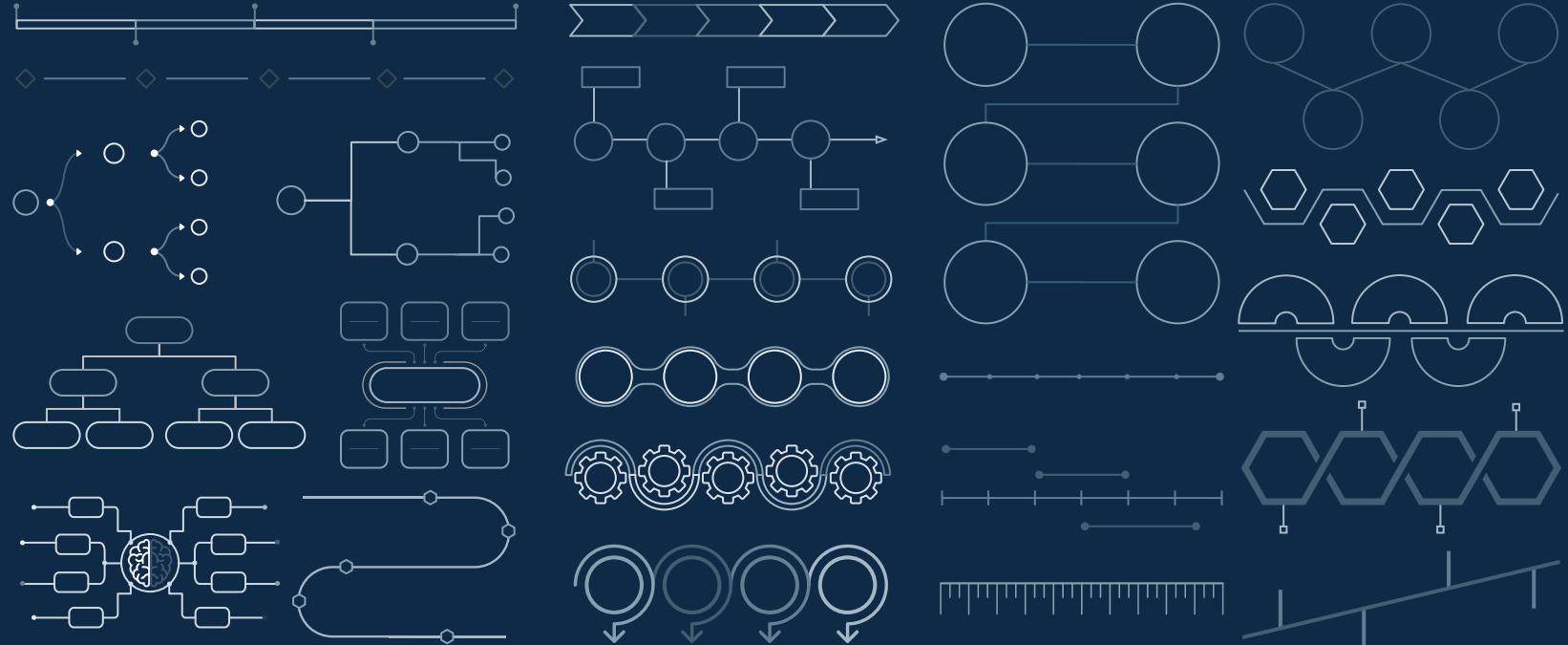
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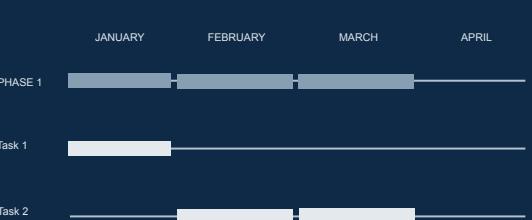
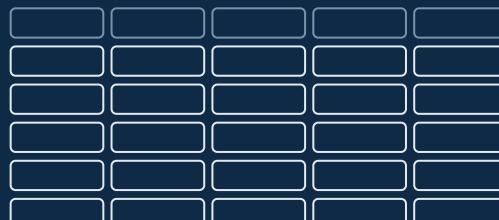
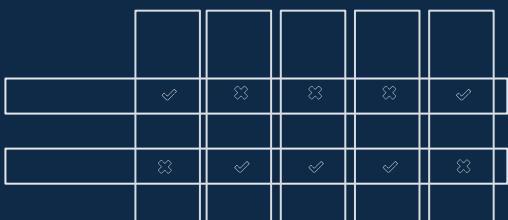
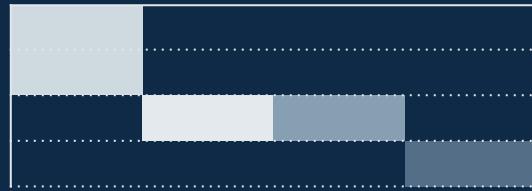
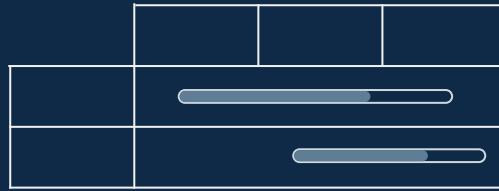
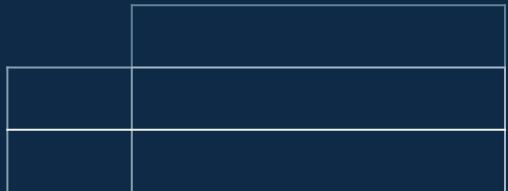
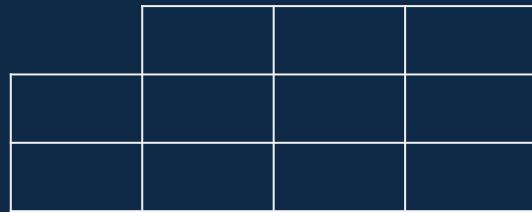
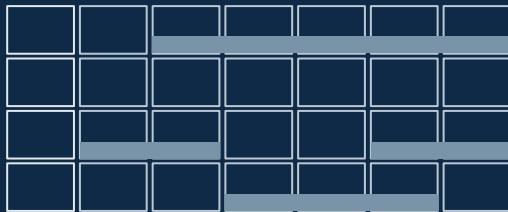
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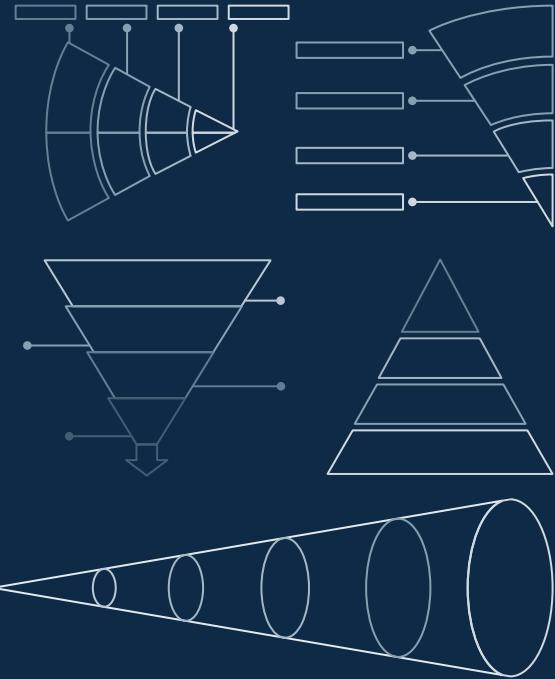
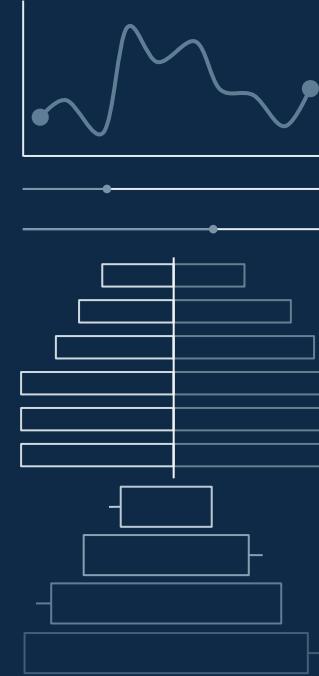
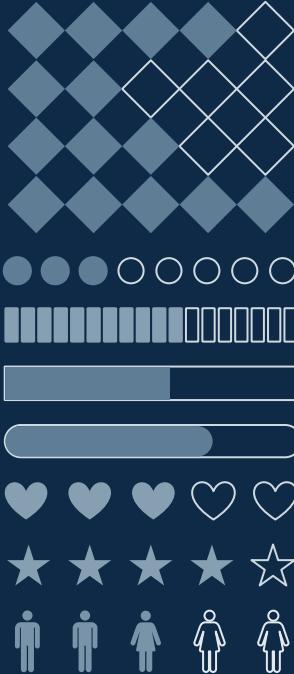
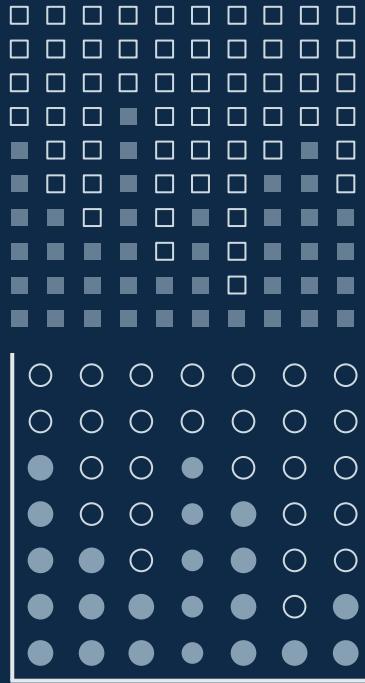












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