Causal Data Science with Directed Acyclic Graphs (DAGs)

Section 6: Transportability of Causal Knowledge Across Domains

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Online Course at Udemy.com

Course Outline

Transportability

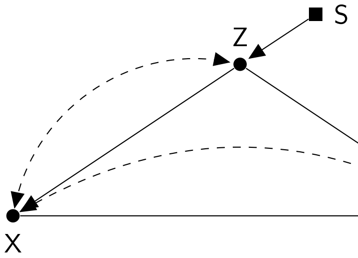
- Causal knowledge is usually acquired in different contexts than it is supposed to be used (e.g., in a laboratory experiment)
- If domains differ structurally in important ways, how can we be sure that causal knowledge remains valid across contexts?
- This problem is known under the rubric of *"transportability"* in the causal artificial intelligence field
  - How can an artificial learner transport causal knowledge obtained in one domain to another?
- Social scientists use the term *"external validity"* more frequently
  - How can a human learner (researcher) transport causal knowledge obtained in one domain to another?

Motivating Example: Banerjee et al. (2007)

- Banerjee et al. (2007) study the effect of a randomized remedial education program for third and fourth graders in two Indian cities: Mumbai and Vadodara
  - They find similar effects on math skills, but effect positive impact on language proficiency is much smaller in Mumbai compared to Vadodara
- Banerjee et al. (2007) explain this result by baseline reading skills that were higher in Mumbai, because families are wealthier there and schools are better equipped
- What do we do if we do not have a second experiment to validate our results?
  - Naïve extrapolation clearly would have gotten them into trouble

Selection Diagram

- We can incorporate knowledge about structural differences across domains by a selection node (■) in a causal diagram
- Captures the notion that domains differ either in the distribution of background factors $P(U_i)$ or causal mechanisms f_i in the underlying structural causal model
  - Differences across domains can be in arbitrary ways (akin to the nonparametric nature of DAGs)



Selection Diagram (II)

Definition: Selection Diagram (Pearl and Bareinboim, 2011)

Let $\langle M, M^* \rangle$ be a pair of structural causal models relative to domains $\langle \Pi, \Pi^* \rangle$, sharing a causal diagram G. $\langle M, M^* \rangle$ is said to induce a selection diagram D if D is constructed as follows:

(i) Every edge in G is also an edge in D.

(ii) D contains an extra edge $S_i \to V_i$ whenever there might exist a discrepancy $f_i \neq f_i^*$ or $P(U_i) \neq P^*(U_i)$ between M and M^*.

- Switching across domains Π and Π^* is denoted by conditioning on different values of S
- Compared to selection bias case, now S points into other variables

Transportability Task

- The transportability problem: We have experimental results from a source domain Π, how can we transport them to a target Π^* where we only have passive observations?
  - I.e., we know $P(y|do(x))$ but would like to know $P^*(y|do(x))$
- Note that Π and Π^* share the same causal diagram G. Thus, if the causal effect in Π would be identified from observational data alone (i.e., no experimental data needed) then it would also be identified in Π^* and there would be no need for transportation ("trivial transportability", Pearl and Bareinboim, 2011)
- So transportability theory is (mainly) concerned with transporting experimental results across domains
  - Although observational / statistical transportability can be useful to economize on data collection efforts (Pearl and Bareinboim, 2011)

S-Admissibility

Theorem 2 in Pearl and Bareinboim (2011)

Let D be the selection diagram characterizing two populations, Π and Π^*, and S the set of selection variables in D. The strata-specific causal effect $P^*(y|do(x), z)$ is transportable from Π to Π^* if Z d-separates Y from S in the X-manipulated version of D, that is, Z satisfies $(Y \perp\!\!\!\perp S|Z)_{D_{\overline{X}}}$.

- The set of variables Z is then called *s-admissible*
- Note that $D_{\overline{X}}$ is the result of experimentally manipulating X in the source domain
  - Since S has to be d-separated from Y in $D_{\overline{X}}$, it follows that every S pointing into X does not threaten transportability and can be ignored

S-Admissibility (II)

Corollary 1 in Pearl and Bareinboim (2011)

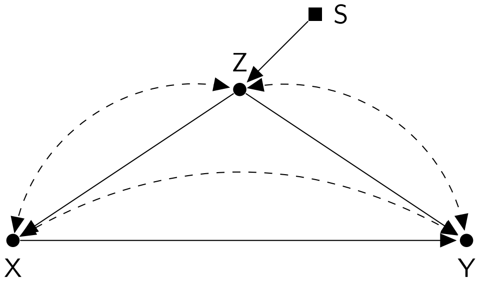
The causal effect $P^*(y|do(x))$ is transportable from Π to Π^* if there exists a set Z of observed pretreatment covariates that is s-admissible. Moreover, the transport formula is given by the weighting

$$P^*(y|do(x)) = \sum_z P(y|do(x), z)P^*(z).$$

- This *transport formula* says that we reweight the z-specific causal effect in the source domain by the distribution of z in the target domains
  - E.g., find experimental results for several income levels in Mumbai and weight by income distribution in Vadodara

S-Admissibility (III)

- Consider this selection diagram, which is the same as before expect for the added edge $Z \leftarrow\text{-}\text{-}\text{-}\text{-}\rightarrow Y$
- Is the causal effect transportable in this case?



Transportability – The General Case

- Transportability formula is well-known in economics (Hotz et al., 2005; Andrews and Oster, 2018), but these papers focus on s-admissible *pretreatment* variables. Solutions based on do-calculus are more general

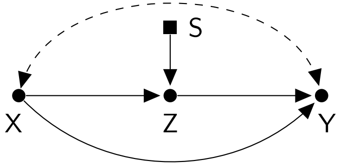
Theorem 1 in Pearl and Bareinboim (2011)

Let D be the selection diagram characterizing two populations, Π and Π^*, and S as set of selection variables in D. The relation $R = P^*(y|do(x))$ is transportable from Π to Π^* if the expression $P(y|do(x), s)$ is reducible, using the rules of do-calculus, to an expression in which S appears only as a conditioning variable in do-free terms.

- Bareinboim and Pearl (2013b) develop a complete algorithm for automatization

Example: Selection Affecting Post-Treatment Variables

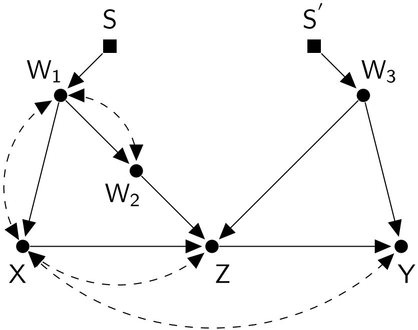
- Duflo et al. (2008) discuss the problem that the effectiveness of development programs, X, is mediated by the quality of program officials, Z



  - Pilot studies are often conducted with a particular level of care and high-quality officials, at a level that is impossible to replicate when the program is supposed to be scaled up
  - How can we transport experimental results from the pilot study to populations where Z is distributed differently?
    - With the help of the transportability algorithm developed by Bareinboim and Pearl (2013b) we find the transport formula

$$P^*(y|do(x)) = \sum_z P(y|do(x), z)P^*(z|x)$$

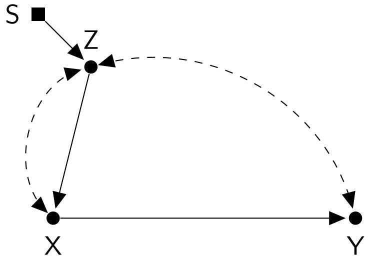
Example: A Complex Selection Diagram



- Here the relevant transport formula is found to be

$$P^*(y|do(x)) = \sum_{z,w_2,w_3} P(y|do(x), z, w_2, w_3) P(z|do(x), w_2, w_3) P^*(w_2, w_3)$$

z-Transportability



- What if we do not have the possibility to run experiments on the treatment X, but we can conduct a surrogate experiment on Z instead (as in the encouragement design from development economics discussed previously)
- This gives rise to the idea of z-transportability

z-Transportability (II)

Theorem 1 in Bareinboim and Pearl (2013a)

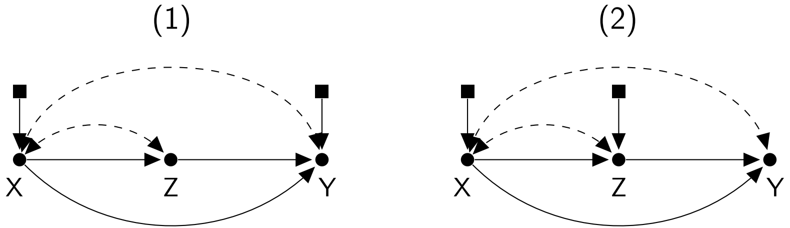
Let D be the selection diagram characterizing two populations, Π and Π^*, and S as set of selection variables in D. The relation $R = P^*(y|do(x))$ is z-transportable from Π to Π^* in D if the expression $P(y|do(x), s)$ is reducible, using the rules of do-calculus, to an expression in which all do-operators apply to subsets of Z, and the S-variables are separated from these do-operators.

- Again, this theorem is only procedural. Bareinboim and Pearl (2013a) develop a complete algorithm for the z-transportability case

Meta-Transportability

- Transportability techniques are particularly valuable we can combine results from several source domains
- This strategy is generally known under the rubric of "meta-analysis"
  - Meta-analyses become increasingly popular in economics (Card et al., 2010; Dehejia et al., 2015)
- The problem with standard meta-analytic tools is that they do not take domian heterogeneity into account but instead aim to "average out" differences across populations
- Bareinboim and Pearl (2013c) extend the transportability idea, which captures domain-specific heterogeneity by the ■-nodes in the causal diagram, to the case with multiple source domains

Meta-Transportability (II)



- Both selection diagrams D_1 and D_2 depict how domains π_1 and π_2 differ from the target domain π^*
- Here, the causal effect would not be individually transportable from a single source domain. But it is transportable using combined information from both domains as

$$P^*(y|do(x)) = \sum_z P^{(2)}(y|do(x), do(z)) P^{(1)}(z|do(x)).$$

Meta-Transportability (III)

- Bareinboim and Pearl (2013c) develop a complete algorithm for deciding about meta-transportability
- Bareinboim and Pearl (2014) combine the idea of meta-transportability with z-transportability to what they call *mz*-transportability
  - Bareinboim and Pearl (2014) develop a complete algorithm for *mz*-transportability
- These results will hopefully allow to combine more flexibly and make more effective use of results from a whole body of empirical literature

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