Causal Data Science with Directed Acyclic Graphs (DAGs)

Section 5: Recovering from Selection Bias

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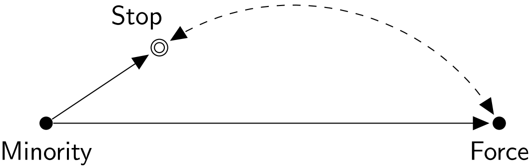
Online Course at Udemy.com

Course Outline

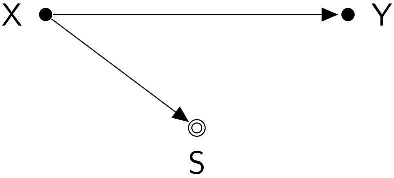
Selection Bias

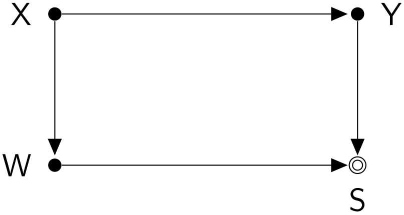
- Non-random, selection-biased data is a frequent problem in economics
- For example, Knox et al. (2019) criticize papers that try to measure the degree of racial-bias in policing with the help of administrative records
    - Problem: An individual only appears in the data, if it was stopped by the police
    - If there is a racial bais in policing, stopping can be the result of minority status
    - There are unobserved confounders, such as officers' suspicion, between the selection variable and outcome



Selection Bias (II)

- We can model the selection mechanism by incorporating a variable S in the DAG, which denotes whether we observe an observation ($S = 1$) or not ($S = 0$)
  - S receives an incoming arrow from those variables in the model that cause the sample selection
  - We denote a DAG that has been augmented with a selection node by G_S





Selection Bias (III)

- How can we recover the causal effect from a selected sample without invoking functional form assumptions such as linearity and normality (as in the famous Heckman selection model Heckman, 1979)?
- In order to recover $P(y|do(x))$, we need to translate it into a do-free expression that also conditions on $S = 1$, because that's all we can observe
- Bareinboim and Pearl (2012), Bareinboim et al. (2014) and Bareinboim and Tian (2015) give this problem a full formal treatment and derive graphical criteria and algorithms for recovering causal effects from selected samples

Recovering from Selection Bias

Theorem 1 Bareinboim and Tian (2015)

The conditional distribution $P(y|t)$ is recoverable from G_S (as $P(y|t, S = 1)$) if and only if $(Y \perp\!\!\!\perp S|T)$.

Corollary 1 Bareinboim and Tian (2015)

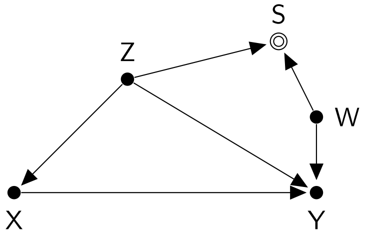
The causal effect $Q = P(y|do(x))$ is recoverable from selection-biased data if using the rules of the do-calculus, Q is reducible to an expression in which no do-operator appears, and recoverability is determined by the previous Theorem.

Recovering from Selection Bias (II)

- In the previous example with $X \to Y$ and $X \to S$, the causal effect is recoverable because $P(y|do(x)) = P(y|x)$ and S is d-separated from Y
  - Thus, $P(y|do(x)) = P(y|x, S = 1)$, which can be estimated from selection-biased data
- An immediate consequence of Theorem 1 is that if S is directly connected to Y, as in the racial policing case, the causal effect will not be recoverable (without strengthening assumptions)
- Note that corollary 1 is a sufficient but not necessary condition for recoverability
  - We can use do-calculus to reduce $P(y|do(x))$ to a do-free expression that conditions on $S = 1$ in cases when corollary 1 is not applicable Example
  - Bareinboim and Tian (2015) develop an algorithm which automatizes this step

Combining Biased and Unbiased Data

- Sometimes we can get at unbiased measurements of covariates, e.g., from census data
- Take the graph on the right. Conditioning on the set $\{Z, W\}$ closes all backdoor paths and d-separates Y from S. Thus



$$
\begin{aligned}
P(y|do(x)) &= \sum_{z,w} P(y|x,z,w)P(z,w) \\
&= \sum_{z,w} P(y|x,z,w,S=1)P(z,w)
\end{aligned}
$$

- $P(Z = z, W = w)$ is not recoverable according to Theorem 1. But if we can get unbiased measurements of Z and Y, this expression is estimable
- Bareinboim et al. (2014) provide more general criteria for this idea

References I

Bareinboim, E. and Pearl, J. (2012). Controlling selection bias in causal inference. In *Proceedings of the Fifteenth International Conference on Artificial Intelligence and Statistics*, pages 100–108.

Bareinboim, E. and Tian, J. (2015). Recovering causal effects from selection bias. In *Proceedings of the Twenty-Ninth AAAI Conference on Artificial Intelligence*.

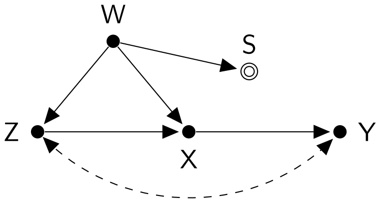
Bareinboim, E., Tian, J., and Pearl, J. (2014). Recovering from selection bias in causal and statistical inference. In *Proceedings of the Twenty-Eighth AAAI Conference on Artificial Intelligence*.

Heckman, J. J. (1979). Sample selection bias as a specification error. *Econometrica*, 47(1):153–161.

Knox, D., Lowe, W., and Mummolo, J. (2019). The bias is built in: How administrative records mask racially biased policing.

Selection Bias Example Derivation

Take the following DAG augmented with selection node S:



By the first rule of do-calculus, since $(S, W \perp\!\!\!\perp Y)$ in $G_{\overline{X}}$ (the resulting graph when all incoming arrows in X are deleted),

$$P(y|do(x)) = P(y|do(x), w, S = 1),$$

$$= \sum_{z} P(y|do(x), z, w, S = 1) P(z|do(x), w, S = 1),$$

where the second line follows from conditioning on Z.

Selection Bias Example Derivation

Applying rule 2, since $(Y \perp\!\!\!\perp X|W, Z)$ in $G_{\underline{X}}$ (the resulting graph when all arrows emitted by X are deleted), we can eliminate the do-operator in the first term

$$= \sum_{z} P(y|x, z, w, S = 1)P(z|do(x), w, S = 1).$$

Finally, because $(Z \perp\!\!\!\perp X|W)$ in $G_{\overline{X}}$, it follows from rule 3 that

$$= \sum_{z} P(y|x, z, w, S = 1)P(z|w, S = 1).$$

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