Exchange Rates in South America's Emerging Markets

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Abstract

Since Meese and Rogoff (1983) results showed that no model could outperform a driftless random walk in predicting exchange rates, there have been many papers which have tried to find some forecasting methodology that could beat the random walk, at least for certain forecasting periods. The present paper compares the Purchasing Power Parity, the Uncovered Interest Rate, the Sticky Price, the Bayesian Model Averaging, and the Bayesian Vector Autoregression models to the random walk benchmark in forecasting exchange rates between the Paraguayan Guarani and the US Dollar, the Brazilian Real and the Argentinian Peso. Forecasts are evaluated under the criteria of Root Mean Square Error, Direction of Change, and the Diebold-Mariano statistic. The results indicate that in shorter horizon forecasting BMA and BVAR can perform better but other models outperform the random walk at longer horizons.

Keywords: Forecasting, Exchange rates, Bayesian Model Averaging, Bayesian Vector Autoregression, Purchasing Power Parity, Uncovered Interest Rate, Sticky Price.

1 Introduction

Exchange rate forecasting is a complicated matter. It has been the subject of many studies which have yielded promising results only to be subsequently refuted by others. Attempts at formal exchange forecasting have existed for over a century. For instance, Purchasing Power Parity (PPP) was developed into a theory of exchange rate behavior already in the early twentieth century Cassel (1918). By the 1960s, as discussed in the next section, it was the subject of "appraisals". In the same vein, the Uncovered Interest Rate Parity (UIP) has been part of the discussion since the twenties with Keynes (1923). Both of these models are still working horses in international finance courses all over the world. The 1970s saw a burst of activity in exchange rate forecasting theory, with several models being developed one after the other - an activity which spilled into the early 1980s. And yet, a definitive model or framework remains elusive. The idea of efficient markets and the impossibility to predict asset prices consistently dates back to Malkiel and Fama (1970), but it took a few years before it made its way into international finance. In particular, since Meese and Rogoff (1983) argued that no model outperforms a driftless random-walk in forecasting exchange rates, researchers have been forced to go back to the drawing board to come up with more solid alternatives. For the following three decades, economists would go back and forth in arguing for and against the possibility of forecasting exchange rates. For instance, Lothian and Wu (2011) shows that Uncovered Interest Parity (UIP) has remarkable forecasting power in longer time horizons. But even recently, Cheung et al. (2005) have reinforced the idea that no model can consistently beat a random walk.

The objective of the present work is to evaluate the predictability of these standards models, in addition to two Bayesian approaches to the Paraguayan case. We make a modest contribution to the literature by expanding on the results obtained by Wright (2008) and extending the framework of Lam et al. (2008) to a regional application. Both studies used Bayesian Model Averaging (BMA) to forecast the exchange rates of the U.S. Dollar (USD) with respect to several other currencies and then compare them to the performance of a benchmark model, namely, the driftless random walk. In particular, Lam et al. (2008) also added three other structural models and compared them to the random walk as well. These models are the aforementioned PPP, and UIP and Sticky Price (SP). They are well-known models in the literature and have been extensively discussed, both in the past and in recent years. It is this latter approach that we have followed for this paper, in conjunction with a Bayesian Vector Auto-regression (BVAR) model with a Minnesota prior. As Carriero et al. (2009) shows, BVAR models perform well in the short run. To assess the performance of each model, we evaluate Root Mean Square Error (RMSE) ratio generated by each model, the Direction of Change (DoC) ratio and we also perform the Diebold-Mariano (DM) test.

More specifically, we have used the above mentioned models to forecast the exchange rates between the Paraguayan Guarani (PYG) and the USD, the Brazilian Real (BRL) and the Argentinian Peso (ARS) using monthly data. Unlike Wright (2008), we do not separate the variables into a financial and a macroeconomic data set in order to estimate monthly and quarterly exchange rates, respectively – all variables are monthly. Lam et al. (2008) only produced forecasts based on quarterly data. The forecasting periods are 3, 6, 9, and 12 months ahead. The results are encouraging and in line with Wright's and Lothian and Wu's works, as well as Carriero et al. (2009): under the RMSE criterion, in the cases of the USD and BRL, BMA and BVAR outperform all other models in the 3-month and 6-month horizons; UIP outperforms all other models in the subsequent horizons; and, results are similar under the DoC criterion. In the case of Argentina, PPP and SP appear to fare far better under both criteria, owing perhaps to Argentina's recent and complex history of inflation, and price and exchange rate volatility. Under the DM criterion, forecasts are statistically significant improvements in the 3-month horizon in the cases of BMA and BVAR for the PYG/USD, and only in the case of BVAR for PYG/BRL. UIP produces improved forecasts that are statistically significant only beyond 36 periods ahead for both exchange rates. As for the Argentinian peso, BVAR and SP forecasts are the statistically significant improvements over the random walk.

The rest of the paper proceeds as follows: in section 2, we discuss the previous literature related to exchange rate forecasting; in section 3, we describe the models and the motivation for their choice; in section 4, we describe the data and their sources; in section 5, we present the results, briefly discussing them. Finally, in section 6 we conclude and suggest possible further research.

2 Literature Review

Exchange rate forecasting models have been around for over one hundred years now and the literature on forecasting theory and applications is extensive, to say the least. Models such as Purchasing Power Parity (PPP) and Uncovered Interest-Rate Parity (UIP) have been thoroughly analyzed time and again (see, for instance, Balassa (1964) for PPP and the aforementioned Lothian and Wu (2011) paper for UIP). Dornbusch (1976) proposed a Sticky Price (SP) model based on monetary fundamentals and Frankel (1979) further developed this framework by emphasizing the role of expectations. However, Meese and Rogoff (1983) wrote a seminal paper in which they argued that no exchange rate model can outperform a driftless random walk in out-of-sample forecasting. Since then, Mark (1995) proposed that at longer horizons a monetary fundamentals model could provide with better out-of-sample forecasts. This model has been subject to criticism by Kilian and Taylor (2003) and Faust et al. (2003) where they argue that improvements occur only with a two-year window and disappear afterwards. Interestingly, Kilian and Taylor (2003) finds that ESTAR models are helpful in explaining real exchange rate behavior. Also of interest to the present study is Barnett et al. (2005) work which shows that the use of Divisia monetary aggregates and the user cost price dramatically improve the forecasting power of structural models. Many papers empirically tested these standard models for the main currency currency pairs exchange rate. For instance, Hsing and Sergi (2009) analyzed the behavior of the USD/EUR using the PPP, the UIP, and two extended Mundell-Fleming model with several focuses, finding several policy implications.

Several authors have had some success in forecasting using large datasets as in Stock and Watson (2002) for the Index of Industrial Production, and Bernanke and Boivin (2003) for inflation. Moreover, Stock and Watson (2004) have used the combination of forecast methods to approximate output growth with encouraging results[1]. It is worth noting that forecast combination methods can be dated back to Bates and Granger (1969).

Bayesian Model Averaging (BMA) was first introduced by Leamer (1978) and was further developed by Raftery et al. (1997) and Hoeting et al. (1999). BMA was first used for exchange rate forecasting by Wright (2008) and subsequently by Lam et al. (2008). Both papers find that BMA produces improvements in out-of-sample forecasts when compared with a driftless random walk.

BVAR was used in forecasting as far back as Litterman (1986). Sarantis (2006) showed that a BVAR model outperforms a random walk in forecasting daily exchange rates. Banbura et al. (2007) used BVAR for forecasting employment, the Consumer Price Index (CPI) and the Fed Funds Rate with positive results for first-quarter predictions. Recently, Schüssler et al. (2018) have used VAR-based models with Bayesian estimation methods for exchange rate forecasting with some success.

3 Methodology

In this section we discuss the four different models that we have used to estimate the PYG/USD, PYG/BRL and PYG/ARS exchange rate forecasts and their respective specifications. Regarding the choice of the models, we followed Lam et al. (2008) and, partly, Cheung et al. (2019). The final topic 3.6, we discuss how the assessment of the performance of each model is done.

[1] Bernanke and Boivin (2003) also utilizes forecast combination for inflation measures.

3.1 Purchasing Power Parity

As mentioned in the previous section, PPP is a well-known and widely discussed theoretical model which gives a clear and intuitive explanation for exchange rate determination. The PPP model is expressed in the following manner:

$$\ln e_t = \ln p_t - \ln p_t^* \tag{1}$$

where e_t is the nominal exchange rate, p_t is the domestic price and p_t^* is the foreign price. These are, of course, price indexes and not price levels.

The PPP specification used in this work follows Lam et al. (2008). This involves an error-correction restriction and no short-run dynamics. What this means is that the variation from the exchange rate is a correction of the deviation from a long-run equilibrium in the previous period. Then, the form of the equation is

$$\ln e_{t+h} - \ln e_t = \alpha_0 + \alpha_1(\ln e_t - \beta_0 - \beta_1 \ln \tilde{p}_t) + \epsilon_t \tag{2}$$

where $\tilde{p}_t$ is the relative price level of the domestic economy relative to the foreign one, h is the forecast horizon and ϵ_t is the error term.

3.2 Uncovered Interest-rate Parity

UIP is another model that has been studied repeatedly as an approximation to forecasting exchange rates. This model entails the no-arbitrage condition that the expected return of the exchange rate h periods ahead is equal to the interest rate differential, which can be expressed as:

$$E_t(\ln e_{t+h} - \ln e_t) = i_t - i_t^* \tag{3}$$

where $E_t(\cdot)$ is the expectation at time t, and i_t and i_t^* are the domestic and foreign interest rates, respectively.

In a similar specification as the one above for the PPP model, also including an error-correction restriction, we write the equation as

$$\ln e_{t+h} - \ln e_t = \alpha_0 + \alpha_1(\ln e_t - \beta_0 - \beta_1 \ln \tilde{i}_t) + \epsilon_t \tag{4}$$

where $\tilde{i}_t$ is the relative interest rate (domestic to foreign).

3.3 Sticky Price

As in Frankel (1979) and we can expand the PPP framework so that exchange rates are also determined by money supply, output and interest rates. This is given by the following equation

$$\ln e_t = \ln m_t - \ln m_t^* - \phi(\ln y_t - \ln y_t^*) + \lambda(\ln i_t - \ln i_t^*) + \beta(\ln \pi_t - ln\pi_t^*) \tag{5}$$

where m_t and m_t^*, y_t and y_t^*, i_t and i_t^*, and π_t and π_t^* are, respectively, domestic and foreign money supply, domestic and foreign output, domestic and foreign interest rates and domestic and foreign current long-run expected rates of inflation.

As in the above cases, we use a restrictive error correction form of the model:

$$\ln e_{t+h} - \ln e_t = \alpha_0 + \alpha_1(\ln e_t - \beta_0 - \beta_1 \ln \tilde{m}_t - \beta_2 \ln \tilde{y}_t - \beta_3 \ln \tilde{i}_t - \beta_4 \ln \tilde{p}_t) + \epsilon_t \tag{6}$$

where $\tilde{m}_t$, $\tilde{y}_t$, and $\tilde{i}_t$ are domestic to foreign relative money demand, output and short-term interest rates, respectively. Notice that we have replaced long-run expected rates of inflation with the only proxy available - relative prices. The reason for this choice will be explained in the next section.

3.4 Bayesian Model Averaging (BMA)

BMA is a forecasting method that utilizes large datasets and many different models. Say there are M_i models, i=*1,...,n* each of which has a parameter θ_i. One does not know which model is the true model but assumes that one of them is. Assumes that the ith model is the true model based on some prior belief $P(M_i)$, the posterior probabilities are computed starting from a prior about which model is the true one. Thus, if D is the available data, the posterior probability that the ith model is the true model is given by

$$P(M_i|D) = \frac{P(D|M_i)P(M_i)}{\sum_{j=1}^{n} P(D|M_j)P(M_j)} \tag{7}$$

where,

$$P(D|M_i) = \int P(D|\theta_i, M_i)P(\theta_i|M_i)d\theta_i \tag{8}$$

being $P(D|M_i)$ the marginal likelihood for the model M_i, $P(\theta_i|M_i)$ is the prior density of the parameters and the likelihood is given by $P(D|\theta_i, M_i)$[2]. The forecasts from each of the different models are then weighted by their respective posteriors. The model is assumed to be linear. And so, one has

$$y = X_i\beta_i + \epsilon \tag{9}$$

where y is the vector of exchange rates (in this case), X_i are the predictors, β_i are parameters and ϵ is the mean zero, i.i.d. error with variance σ^2 and $\theta_i = (\beta_i', \sigma^2)$. We also assume that the regressors are strict exogenous. As Wright (2008) argues, even dealing with time series is possible to make this assumption and still obtain good forecasting power.

As for the coefficients, we assume a prior mean of zero. The structure of their variance is given by Zellner's g so that

$$\beta_i|g \sim N\left(0, \sigma^2\left(\frac{1}{gX_i'X_i}\right)^{-1}\right) \tag{10}$$

where the hyperparameter g is set to the default "unit information prior" $g{=}n$ (the number of models).

The forecasting model is then given by

$$\ln e_{t+h} - \ln e_t = \beta_i' X_{i,t} + \epsilon_t \tag{11}$$

where $X_{i,t}$ is the vector of regressors at time t for model i. For each model we have a forecast $\tilde{\beta}_i' X_{i,t}$ where $\tilde{\beta}_i'$ is the posterior mean of β_i. Each model is weighted by their posterior probabilities so that the forecast is given by $\sum_{i=1}^{n} P(M_i|D)\tilde{\beta}_i' X_{i,t}$ where $P(M_i|D)$ is the posterior probability of the ith model and D is the dataset.

3.5 Bayesian Vector Autoregression (BVAR)

The BVAR model with a Minnesota prior was introduced by Litterman (1986). As previously described, it has been widely used in forecasting. If the model is as follows

$$y = (I_m \otimes X)\alpha + \epsilon, \qquad \epsilon \sim (0, \Sigma_\epsilon \otimes I_T) \tag{12}$$

then y and ϵ are $mT \times 1$ vectors of dependent variables and errors, respectively, and where m is the number of variables and T, the time periods. I_m is the identity matrix, X is the matrix of independent variables and α is a $ml \times 1$ vector where l is the number of lags. More specifically, $\alpha = \bar{\alpha} + \xi_\alpha$ with $\xi_\alpha \sim N(0, \Sigma_\alpha)$, where in the Minnesota prior $\bar{\alpha} = 0$ except

[2] Wright (2008) discuss about the distinction between model uncertainty and parameter uncertainty.

$\bar{\alpha}_{1i} = 1, i = 1, ..., m$, Σ_α is diagonal and each element $\sigma_{ij,l}$ (equation i, variable j, and lag l) is as follows

$$\sigma_{ij,l} = \phi_0/h(l), \qquad i = j \tag{13}$$

If j is endogenous, then

$$\sigma_{ij,l} = \phi_0 \times \phi_1/h(l) \times (\sigma_j/\sigma_i)^2, \qquad i \neq j \tag{14}$$

And if j is exogenous, then

$$\sigma_{ij,l} = \phi_0 \times \phi_2 \tag{15}$$

In this case $\phi_0, \phi_1, \phi_2, (\sigma_j/\sigma_i)^2$ and $h(l)$ are, respectively, hyperparameters, a scaling factor, and a function of lags l. Note that ϕ_0 measures the tightness of the first lag's variance, ϕ_1 is the relative tightness of any other variables, and ϕ_2 is the relative tightness of exogenous variables. Finally, $h(l)$ is a measure of the relative tightness of the variance of the lags.

The error correction model follows a similar process to the one laid out for the SP model, using the same variables. The number of lags is 1.

3.6 Out-of-Sample Performance Evaluation: Root Mean Square Error Ratio and Direction of Change

In order to evaluate our models out-of-sample (OOS), we use the recursive window approach. Given our division of in-sample (IS) and out-of-sample (OOS) periods, we use our IS data to start the recursive estimation. In this approach, at each $t \in \tau_{OOS}$, where τ_{OOS} represents the OOS subset of the entire sample, we use all the available information from the set of predictors ranging from the first observation in-sample up to t to form the matrix $X_{i,t}$ to forecast h periods ahead from t, the variation of the log of the exchange rate. With the fitted model at t, we can obtain the residuals for each framework. This procedure is repeated in next period $t+1$, updating the estimated coefficient and obtaining the residuals, until the end of the OOS observations.

In order to evaluate the accuracy of each model in the OOS period, we compare each one to a benchmark model which in this case is the driftless random-walk given by

$$\ln e_{t+h} - \ln e_t = \epsilon_t \tag{16}$$

Following Meese and Rogoff (1983) methodology, we take the expectation of the random walk so that it becomes a martingale process, so that the predictor of the exchange rate h periods ahead is whatever the exchange rate is at time t.

First, we start with the root mean square error (RMSE) of each of the models and dividing them by the RMSE from the random-walk. Doing so, we can interpret this ratio as how the OOS RMSE generated by each model compares to the OOS RMSE of a simulated random walk. A ratio of less than one indicates that the model is performing better than the random-walk and vice versa. We assess the OOS performance of each model considering four horizons $h = 3, 6, 9$ and 12 months ahead.

We also evaluate the Direction of Change (DoC) ratio. The DoC measures the proportion of times each model correctly predicts whether the actual exchange rate increases or decreases (direction). Assuming that the expected value of random walk predicting the right DoC is 0.5, values above 0.5 indicate that a model is outperforming the random walk. The higher the proportion is, the better the model is performing.

The third method is the statistic produced by Diebold & Mariano (1995), which allows for the comparison of forecasts in terms of whether the difference between two forecasts for the same forecasting period is statistically significant and whether the improvement is statistically significant (and thus, one forecast is "better" than another). If $g(e_{it})$ is the loss function of a forecast error, the loss differential function is defined as $d_t = g(e_{1t}) - g(e_{2t})$. If d_t is zero, then the forecasts under examination are equally accurate. Under the null, the expected value of d_t is zero. The DM statistic itself takes the form

$$DM = \frac{\bar{d}}{\sqrt{2\pi \hat{f}_{d(0)}/T}} \tag{17}$$

where $\bar{d}$ is the sample mean of the loss differential function and $\hat{f}_{d(0)}$ is a consistent estimate of the spectral density. Under the null, $DM \rightarrow N(0, 1)$. The null is rejected if $|DM| > z_{\alpha/2}$.

4 Data

The data for this study are monthly series of the bilateral exchange rate between Paraguay × USD (PYG/US), Paraguay × Brazil (PYG/BRL), and Paraguay × Argentina (PYG/ARS). These series start in January, 1994 extending up to July, 2017 for the PYG/USD exchange rate; January, 1994 to December, 2016 for Brazil; and, January, 1997 to December, 2016 for Argentina. The reason we chose to start the series at these particular dates is that before January 1994 there were scarce to none monthly Paraguayan data available and in the case of Argentina, because there was no monthly exchange rate data available prior to

1997. The choice of in-sample and out-of-sample data was done based on the conclusion of the Paraguayan Stand-by agreement with the International Monetary Fund (IMF) in 2004 regarding the payment of its foreign debt. Therefore, the out-of-sample prediction period starts in January 2005.

Following Wright (2008) and Lam et al. (2008), we consider the following monthly variables as potential predictors: (i) short-term interest rates and relative short-term interest rates, (ii) log of output and log of relative output (domestic to foreign), (iii) log of money supply and log of relative money supply (domestic to foreign), (iv) log of price levels and log of relative price levels (domestic to foreign), (v) oil price, and for the particular case of Paraguay, (vi) soy price. This gives a total of 2^6 possible models.

The choice of short-term interest rates for the SP model was done not only following Frankel's methodology but also out of necessity: it is the only interest rate that has been consistently reported since 1994. We should also point out that these are interest rates on the bonds that the Central Bank of Paraguay (BCP) trades with Paraguayan private banks as a monetary policy tool. For the same reason, they are the interest rates used in the UIP model. On a similar note, the choice of Consumer Price Index (CPI) as a proxy for expected inflation was also done following Frankel and out of necessity: of all the possible proxies used by Frankel, it was the only one available. The proxy for monthly Paraguayan, Argentinian and Brazilian GDP were, respectively: the Monthly Economic Activity Index (IMAEP) produced by the BCP which tracks the performance of the most relevant Paraguayan industries; the Monthly Estimator of Economic Activity (EMAE) produced by the National Institute of Statistics and Census (INDEC) in Argentina; and, the Index of Monthly Monetary Activity (IBC-Br) produced by the Central Bank of Brazil (BCB). Finally, soy prices were included in the BMA model because they are the main Paraguayan export and they represent a significant portion of Argentinean and Brazilian exports as well. Also, the soy market is highly dollarized.

All the data pertaining to Paraguay, as well as oil and soy prices, were obtained from the Statistical Annex of the yearly Economic Report published by the BCP. M1 monetary aggregates, CPI and three-month Treasury bill interest rates for the US were retrieved from the Federal Reserve Bank of Saint Louis' Federal Reserve Economic Data (FRED). As for US monthly GDP data, they were obtained from the Macroeconomic Advisers data bank. The Brazilian data were taken from the BCB statistical bulletin, except the CPI which was retrieved from the Getulio Vargas Foundation website.

For the Argentinian data, an observation should be made. Their M1 aggregates were taken from the FRED, and interest rates, from the Central Bank of the Argentine Republic's (BCRA) site. But the CPI had to be constructed from four different sources: the original

CPI series from INDEC (base year 2008), a second CPI series from INDEC (base year 2014), a third CPI series from INDEC (base year 2016) and a parallel series put together by the Argentine Congress, as the INDEC stopped producing its CPI series from November, 2015 to November, 2016. This series was retrieved from *Ambito.com*, the internet website which compiled it.

5 Results

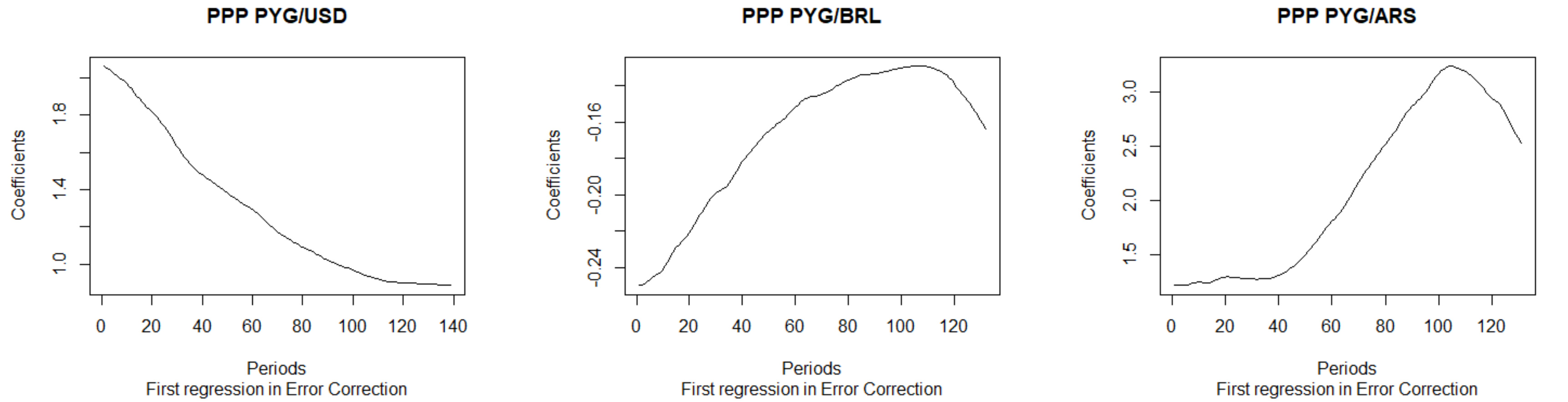
5.1 Coefficients in the PPP and UIP models

Before discussing the results, figure 1 depicts the behavior of the coefficients for the PPP and UIP models in the first regression of the error correction. As might be expected, they almost always differ from 1. The PPP coefficients for the PYG/USD exchange rate follow a neat downward trajectory ranging from 2.06 in the first period to 0.885 at its lowest in the last period. The coefficients for the PYG/BRL and PYG/ARS range from a low of -0.25 (in the first period) to a high of -0.13 (November, 2013) and from 1.22 (in the first period) to a high of 3.24 (September, 2015). The graphs also show that the latter two exchange rate coefficients behave somewhat similarly but differ greatly from the PYG/USD coefficients.

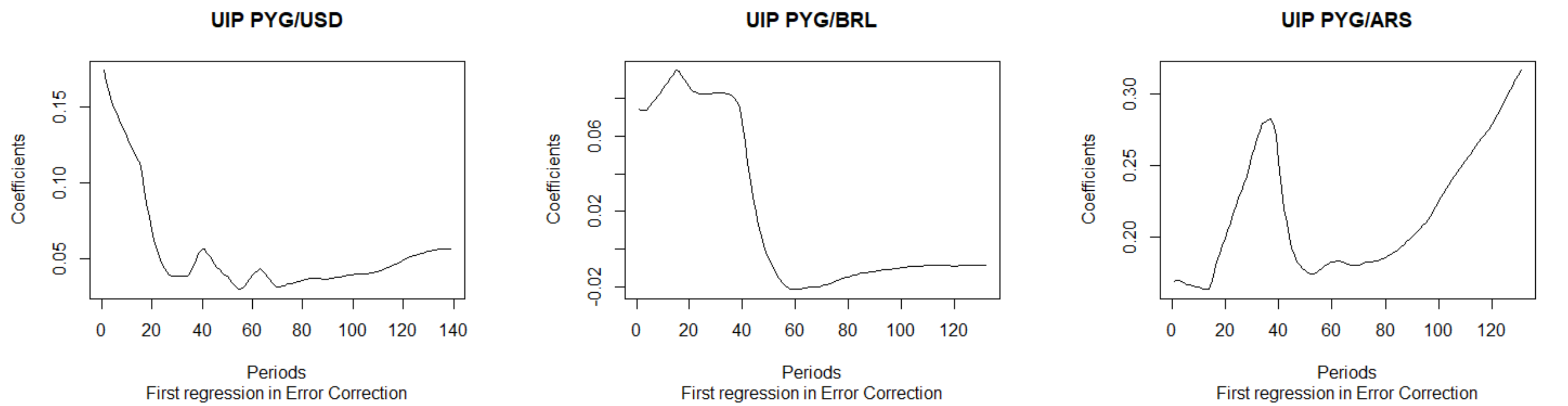
UIP coefficients for the PYG/USD exchange rate vary from a low of 0.039 (September, 2007) to a high of 0.17 in the first period. The PYG/BRL coefficients range from -0.022 (November, 2009) to 0.095 (March, 2007). As for the PYG/ARS coefficients, they hit a low of 0.163 (December, 2005) and high of 0.316 in the last period.

What all of this does is to underscore the need to adjust the models by using an error-correcting approach, as these relations are clearly not stable across time and can vary greatly from model to model and from case to case.

Figure 1: PPP and UIP Estimated Coefficients for the PYG/USD, PYG/BRL and PYG/ARS



Panel A: This panel reports the OOS estimated coefficients using PPP model for the three bilateral exchange rates for Paraguay. The x-axis represents the OOS t in the recursive window approach. The left graph, plots the PYG/USD, the center one plots PYG/BRL, while the right one plots the PYG/ARS.)



Panel B: This panel reports the OOS estimated coefficients using UIP model for the three bilateral exchange rates for Paraguay. The x-axis represents the OOS t in the recursive window approach. The left graph, plots the PYG/USD, the center one plots PYG/BRL, while the right one plots the PYG/ARS.)

5.2 Comparing Models using RMSE

As discussed in section 3, we are using the ratio of the RMSE of each model to the RMSE of the random-walk in order to assess their performance. Recall that a ratio of less than 1 is an improvement over the random walk, and vice-versa. Table 1 shows the RMSE ratios of the PYG/USD, PYG/ARS and the PYG/BRL. In panels A and C (US and Brazil) we can see that BMA and BVAR outperform all other models in the 3-month and 6-month forecasting horizons and UIP outperforms all other models in the following forecasting horizons. PPP appears to also do well in longer forecasting horizons.

In the case of PYG/ARS, results are quite different. PPP and SP outperform UIP and BMA, but BVAR in particular does exceedingly well. UIP, on the other hand, does not do better than the random walk in any of the forecasting horizons, in contrast with what is observed in panels A and C. As we will see in the following subsections, this pattern will keep repeating itself.

5.3 Comparing Models using DoC

Table 2 shows the ratios of DoC for the PYG/USD, PYG/ARS and PYG/BRL. There are two ways in which we can analyze results under this criterion. Assuming, as previously mentioned, that a random walk will correctly predict the direction of change half the time (an expected value of 0.5), then all models outperform it in the first and last forecasting horizons for PYG/USD in panel A (except BVAR in the last period), with BMA having the largest ratio in the 3-month period and SP, in the 12-month period. In panel B (PYG/ARS), all models outperform the random walk in all forecasting periods, especially in the 12-month horizon where UIP and PPP give the highest proportions. In panel C (PYG/BRL), all models beat the random walk in the first period, but only BMA and BVAR outperform it in the second period, only BVAR in the third period, and in the fourth forecasting period, SP produces the highest proportion.

Notice though that if the actual random walk forecasts are taken as reference, what the results say changes. Using this metric, in panel A (PYG/USD) BMA and BVAR outperform all models in the first two forecasting horizons, BVAR and PPP in the third and all except BVAR do better in the 12-month horizon. In panel B (PYG/ARS) only PPP outperforms the random walk in the first three forecasting horizons, and BVAR, in the first and third horizons, and none in the last horizon. In panel C (PYG/BRL), in the first forecasting horizon BMA and PPP outperform the random walk; in the second, all except SP; in the third, SP and BVAR; and in the fourth, UIP, SP, and BMA.

It is clear that if enough iterations of the experiment were made, the random walk would

Table 1: Ratio of Model's RMSE over Random Walk RMSE

Panel A: PYG/USD				
	3 months	6 months	9 months	12 months
PPP	0.9939	1.0295	1.0820	1.1585
UIP	1.0011	0.9878	0.9729	0.9580
SP	1.0464	1.1019	1.1611	1.2419
BMA	0.8805	0.9646	1.0275	1.0381
BVAR	0.8426	0.9772	1.0652	1.1288
Panel B: PYG/ARS				
	3 months	6 months	9 months	12 months
PPP	0.9549	0.9651	0.9501	0.8795
UIP	1.0094	1.1505	1.2001	1.1629
SP	0.9946	0.9158	0.8580	0.8116
BMA	1.0634	0.9685	0.9465	0.9301
BVAR	0.7370	0.8314	0.8119	0.7830
Panel C: PYG/BRL				
	3 months	6 months	9 months	12 months
PPP	1.0108	1.0042	0.9979	0.9930
UIP	1.0086	1.0002	0.9926	0.9871
SP	1.0998	1.1724	1.2583	1.3933
BMA	0.9339	0.9947	1.0362	1.0170
BVAR	0.7949	0.9569	1.0498	1.1119

Table 1 summarizes the out-of-sample (OOS) ratio of each model's RMSE over Random Walk RMSE. The models assessed are: PPP, UIP, SP, BMA and BVAR. Panel A reports the ratio for the PYG/USD (OOS ranging from 1994:01 to 2017:07). Panel B reports the ratio for the PYG/ARS (OOS ranging from 1997:01 to 2016:12). Panel C reports the ratio for the PYG/BRL (OOS ranging from 1994:01 to 2016:12). The table summarizes the result for four different horizons: $h = 3, 6, 9$ and 12 months. A number less than 1 means that the model in the row predicts better than the random walk benchmark.

tend to its expected value of 0.5 but it is nonetheless interesting to make both comparisons, as they give slightly different interpretations. If we do not constantly repeat the very same experiment, the realized values of the random walk do not exactly behave as a random walk.

Table 2: Ratio of Direction of Change (DoC)

Panel A: PYG/USD	3 months	6 months	9 months	12 months
RW	0.5147	0.4286	0.4769	0.5276
PPP	0.5147	0.4361	0.4923	0.5354
UIP	0.5147	0.4286	0.4615	0.5354
SP	0.5221	0.4286	0.4692	0.5591
BMA	0.5662	0.4812	0.4462	0.5039
BVAR	0.5515	0.4812	0.5462	0.4646
Panel B: PYG/ARS				
	3 months	6 months	9 months	12 months
RW	0.5469	0.5440	0.5246	0.6135
PPP	0.5547	0.5600	0.5492	0.6135
UIP	0.5625	0.5680	0.5164	0.6050
SP	0.5313	0.5760	0.5000	0.6050
BMA	0.5547	0.5360	0.5082	0.5462
BVAR	0.5781	0.5280	0.5492	0.5546
Panel C: PYG/BRL				
	3 months	6 months	9 months	12 months
RW	0.5349	0.4286	0.4472	0.4833
PPP	0.5426	0.4444	0.4390	0.4750
UIP	0.5349	0.4365	0.4390	0.4917
SP	0.5194	0.4286	0.4634	0.5500
BMA	0.5659	0.5079	0.4228	0.5083
BVAR	0.5271	0.5318	0.5122	0.4333

Table 2 summarizes the out-of-sample (OOS) Direction of Change (DoC) ratios. The models assessed are: PPP, UIP, SP, BMA and BVAR. Panel A reports the ratio for the PYG/USD (OOS ranging from 1994:01 to 2017:07). Panel B reports the ratio for the PYG/ARS (OOS ranging from 1997:01 to 2016:12). Panel C reports the ratio for the PYG/BRL (OOS ranging from 1994:01 to 2016:12). The table summarizes the result for four different horizons: $h = 3, 6, 9$ and 12 months. The DoC measures the proportion of times each model correctly predicts whether the actual exchange rate increases or decreases (direction). Values above 0.5 indicate that a model is outperforming the random walk.

5.4 Comparing models using the DM statistic

In order to see if the forecasts produced by some of the models are statistically significantly different and better than those produced by the random walk, these forecasts are compared using the Diebold-Mariano (DM) statistic. PPP and SP statistics for the

PYG/USD and PYG/BRL are not presented as they do not improve on random walk forecasts. In the case of the PYG/ARS exchange rate, all models are below as the behavior differs greatly from that of the first two exchange rates. Also, the forecasting periods for UIP now reach up to 48 months so as to show that as forecasting horizons become larger, forecasts produced by UIP are statistically significant improvements on the random walk.

In the long run, table 3 show that by the 48th forecasting horizon, UIP forecasts are statistically significant improvements over the random walk forecasts at the 1% level for both the PYG/USD and PYG/BRL forecasts. In the three-month horizon, BMA and BVAR produce forecasts that also improve on the random walk and are statistically significant at the 5% level for the PYG/USD exchange rate. In this same horizon, BMA does not produce a statistically significant improvement for the PYG/BRL exchange rate but BVAR does at the 1% level.

Table 4 show the DM statistics and p-values for the PYG/ARS exchange rate. They are the opposite of the results in the previous tables. Although PPP again does not do any better than random walk forecasts, UIP produces no improvement either. Instead, it is the SP model that produces the statistically significant improvements in the longer horizons. BMA does not work well in the short run but BVAR does, both in the short run and the long run.

5.5 Discussion

The above results are encouraging but also puzzling. The performance of BMA is consistent with Wright's findings in that BMA's forecasting power diminishes in longer forecasting horizons but can outperform the random-walk in the 3 and 6-month periods. This is particularly true in this study of the PYG/USD and PYG/BRL exchange rates, whose forecasts seem to behave similarly. This is also true of the forecasts produced by BVAR which in the short-run perform better than those produced by BMA, except at the 6-month horizon in the case of the PYG/USD exchange rate.

If we consider Lothian and Wu's findings that UIP performs well in longer time horizons, it is consistent that UIP outperforms all models in 9-month and/or 12 month-ahead periods (under RMSE) and beyond (where the improvement actually becomes statistically significant) in the US and Brazil case. These results are mostly congruent using the three evaluation criteria if under DoC the reference is the actual forecast produced by the random walk and not its assumed expected value. In the latter case, results are much less informative.

The case of Argentina is different and somewhat puzzling. Under the RMSE criterion, BMA and BVAR forecasting power gets better, not worse, in longer time horizons, and UIP

Table 3: Diebold-Mariano Test for PYG/USD and PYG/BRL

Panel A: UIP

	PYG/USD		PYG/BRL	
	DM Statistic	p-value	DM Statistic	p-value
3 months	0.39654	0.65410	0.80299	0.78900
6 months	0.02622	0.51050	0.40647	0.65780
9 months	-0.24012	0.40510	0.20580	0.58150
12 months	-0.41955	0.33740	0.09330	0.53720
24 months	-0.90075	0.18390	-0.80289	0.21100
36 months	-1.03720	0.14980	-0.92069	0.17860
48 months	-4.40530	0.00001	-3.93600	0.00004

Panel B: BMA

	PYG/USD		PYG/BRL	
	DM Statistic	p-value	DM Statistic	p-value
3 months	-1.69310	0.04522	-0.79202	0.21420
6 months	-1.15690	0.12370	-0.26083	0.39710
9 months	0.24600	0.59720	0.18570	0.57370
12 months	0.38388	0.64950	-0.03754	0.48500

Panel C: BVAR

	PYG/USD		PYG/BRL	
	DM Statistic	p-value	DM Statistic	p-value
3 months	-1.73600	0.04128	-2.33640	0.00973
6 months	0.03673	0.51460	-0.42412	0.33570
9 months	0.97573	0.83540	0.85064	0.80250
12 months	1.16230	0.87740	1.06440	0.85640

Table 3 summarizes the out-of-sample (OOS) Diebold-Mariano (DM) tests for the PYG/USD and PYG/BRL. The OOS for the PYG/USD ranges from 1994:01 to 2017:07, while the OOS for the PYG/BRL ranges from 1997:01 to 2016:12. Panel A reports the tests for the UIP for seven different horizons: $h = 3, 6, 9, 12, 24, 36$ and 48 months. Panel B reports the DM tests for the BMA and Panel C for the BVAR for four different horizons: $h = 3, 6, 9$ and 12 months.

Table 4: Diebold-Mariano Test for PYG/ARS

	UIP		PPP	
	DM Statistic	p-value	DM Statistic	p-value
3 months	1.85080	0.96790	-0.11886	0.45270
6 months	0.96790	0.97270	1.06470	0.85650
9 months	1.68280	0.95380	0.89264	0.81400
12 months	1.43750	0.92470	0.24573	0.59710
	SP		BMA	
	DM Statistic	p-value	DM Statistic	p-value
3 months	-0.10973	0.45630	2.05080	0.97990
6 months	-1.64470	0.05002	0.95237	0.82950
9 months	-4.19740	0.00001	0.74075	0.77060
12 months	-7.22450	0	0.46682	0.67970
	BVAR			
	DM Statistic	p-value		
3 months	-2.66800	0.00382		
6 months	-2.91120	0.00180		
9 months	-4.22160	0.00001		
12 months	-5.43310	0		

Table 4 summarizes the out-of-sample (OOS) Diebold-Mariano (DM) tests for the PYG/ARS. The OOS period ranges from 1997:01 to 2016:12. The table reports the DM tests for five different models (UIP, PPP, SP, BMA, and BVAR) for four different horizons: $h = 3, 6, 9$ and 12 months.

never outperforms the random walk. However, under DoC and comparing actual forecasts BMA and BVAR forecasts improve on the random walk in the 3-month horizon, as expected. Here, again, UIP does not outperform random walk in the longer run. This is in direct contrast with the above-discussed results. One explanation may be that Argentina had a contentious history with inflation during the period under study and its monetary and fiscal policies were rather volatile. So much so that publications such as *The Economist* refused to publish governmental data on inflation because it was considered highly inaccurate[3]. Eventually, as mentioned earlier, even the government stopped publishing its own CPI for a year.

It is perhaps because of this, that the model that focuses on prices and the monetary aggregates do better in forecasting exchange rates: it is what economic agents payed most attention to when dealing with the Argentinian Peso. Finally, the Diebold-Mariano criterion evaluation is largely consistent with the RMSE criterion. UIP forecasts' improvement becomes statistically significant in the long-run and most of the short-run improvements produced by BMA and BVAR are also statistically significant. Something that calls the attention is that BVAR p-values become more and more statistically significant as the forecasting horizon is extended. This is the opposite of what we see in the case of the other exchange rate forecasts. Perhaps this owes something to faulty data produced by Argentina in the period under discussion.

6 Conclusion

The academic discussion on exchange rate forecasting has existed for over a century. In all this time, the arguments for and against the possibility of actually being able to produce significant forecasts has gone back and forth, and the strength of either argument has ebbed and flowed, depending on the context. The conventional wisdom in the profession seems to be anchored in the idea that markets are indeed efficient and that a consistent method of forecasting does not exist. In that sense, the present study "goes against the grain" as it were, in arguing that we can contribute to finding the elusive forecasting methodology that will fit different contexts for Paraguayan exchange rates case.

Even if improvements are only contextual, these methods can still be relevant for policy. For the Paraguayan economy, exchange rates are a very important matter. Its main exports are traded in a highly dollarized market. As a small and open economy, Paraguay also imports a number of goods traded in dollars, among them, of course, oil. Paraguay's largest

[3]See for instance The Economist. *Don't lie to me, Argentina* at `https://www.economist.com/node/21548242`

neighbors and trading partners are Argentina and Brazil, and these exchange rates are indeed relevant. Therefore, the ability to forecast the PYG/USD, PYG/ARS and PYG/BRL exchange rates would be a powerful tool for both monetary and fiscal policy. In that spirit, it has been the objective of this paper to compare and assess the performance of five different models of exchange rate determination: PPP, UIP, SP, BMA and BVAR. In order to do so, we evaluated the three standards models and the two Bayesian generated ones in light of the RMSE and DoC ratios and the Diebold-Mariano test.

The obtained results seem to indicate that, in the case of PYG/USD and PYG/BRL, UIP, BMA and BVAR forecasts improve over all other models, although neither produces the best forecast for every period. We see that BMA and BVAR have more accuracy in the shorter forecasting horizons and UIP in the longer ones. This is in line with previous studies, in particular Wright (2008) and Lothian and Wu (2011). Argentina's case is different, but this may have to do with its recent economic history or faulty data, or both.

Further research could shed more light on the forecastability of exchange rates in the different parts of the world. The present study could be expanded in different directions. First, the same models could be used to forecast the behavior of the PYG exchange rate with respect to other relevant currencies.

Another avenue of research could be the inclusion of other forecasting models and compare their performances to the benchmark model. Some possibilities include GARCH models (see for instance Pilbeam and Langeland (2015)) or copula models (see Cerrato et al. (2015)).

A more laborious possibility could involve the calculation of Divisia Monetary Aggregates for Paraguay and then include them in the pertinent models. Barnett et al. (2005) have already used these aggregates in the several structural models to forecast the exchange rate between the US Dollar and the British Pound. A similar experiment would be to include Divisia monetary aggregates in the Bayesian models to verify if they have a greater weight in forecasting exchange rates or if they can help in their improvement.

A Appendix: MSE Tables

Table 5: Mean Square Error (MSE)

Panel A: PYG/USD	3 months	6 months	9 months	12 months
PPP	0.0035	0.0091	0.0144	0.0197
UIP	0.0036	0.0083	0.0116	0.0135
SP	0.0039	0.0104	0.0165	0.0226
BMA	0.0028	0.0079	0.0129	0.0158
BVAR	0.0025	0.0082	0.0139	0.0187
Random Walk	0.0036	0.0085	0.0123	0.0147
Panel B: PYG/ARS	3 months	6 months	9 months	12 months
PPP	0.0069	0.0172	0.0285	0.0347
UIP	0.0077	0.0244	0.0454	0.0606
SP	0.0075	0.0155	0.0232	0.0295
BMA	0.0086	0.0173	0.0283	0.0388
BVAR	0.0041	0.0127	0.0208	0.0275
Random Walk	0.0076	0.0184	0.0315	0.0448
Panel C: PYG/BRL	3 months	6 months	9 months	12 months
PPP	0.0037	0.0087	0.0124	0.0148
UIP	0.0037	0.0086	0.0123	0.0147
SP	0.0044	0.0119	0.0197	0.0292
BMA	0.0031	0.0085	0.0134	0.0156
BVAR	0.0023	0.0079	0.0137	0.0186
Random Walk	0.0036	0.0086	0.0124	0.0150

Table 5 summarizes the out-of-sample (OOS) the Mean Square Error (MSE). The models assessed are: PPP, UIP, SP, BMA, BVAR and the Random Walk (benchmark). Panel A reports the MSE for the PYG/USD (OOS ranging from 1994:01 to 2017:07). Panel B reports the MSE for the PYG/ARS (OOS ranging from 1997:01 to 2016:12). Panel C reports the MSE for the PYG/BRL (OOS ranging from 1994:01 to 2016:12). The table summarizes the result for four different horizons: $h = 3, 6, 9$ and 12 months.

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