Portfolio efficiency tests with conditioning information using empirical likelihood estimation

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Motivation

- We may say that any asset pricing model may be defined following the basic pricing equation:

$$p_t = E_t(m_{t+1}x_{t+1})$$

- As the GMM estimators does not impose any restrictions on the data distribution, only being based on assumptions about the moments, this method is widely used in finance.
- Cochrane (2009) even says that GMM structure fits naturally for the stochastic discount factors formulation of asset pricing theories, due to the easiness on the use of sample moments in the place of population moments.

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- However, GMM estimators can be **suboptimal in finite samples**.
- In special, in two-stage and iterated GMM estimators are affected by the presence of a finite sample bias component proportional to the number of moment conditions.
- Besides, the use of higher order moment conditions makes these estimators sensitive to the presence of outliers in distributions with heavy tails.
- Using simulations, Ferson and Foerster (1994) showed evidences that GMM may not be a reliable method for asset pricing models in small samples.
- Chaussé (2010) also discuss the severe effects that weak instruments may cause on GMM under small samples.

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Introduction

Goals

- This study analyses the use of Generalized Empirical Likelihood (GEL) to circumvent the deficiencies existing on the use of usual estimators in testing portfolio efficiency in the presence of conditional information.
- Smith (1997) and Owen (2001) recently introduced this family of estimators that just as GMM, it is only based on moment conditions.
- This class of estimators has some special characteristics that confer it to have better statistical properties, such as **robustness to outliers** and **heavy tails** distributions, and **better finite and asymptotic properties** compared to the usual methods (as GMM).

Main Results

- We found some evidences that estimators from GEL class really performs differently in small samples.
- In general, efficiency tests using GEL generate lower estimates compared to tests using the standard approach with GMM.
- With simulation experiments we assess the robustness of the tests to data contaminations, as outliers and the presence of heavy tails in the innovation structure.
  - Experiments show that GEL has better performance when heavy tails are present.
  - There is no conclusive results regarding the presence of outliers.

- Take the fundamental pricing equation:

$$p_t = E_t(m_{t+1}x_{t+1})$$

- Notice that the models need to portray the prices taking into account conditional moments.
- Defining Z_t as the set of available information at t, the equation may be written as $p_t = E(m_{t+1}x_{t+1}|Z_t)$ or simply $E(m_{t+1}R_{t+1}|Z_t) = 1$.

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- Assuming that exist a subset of variables $\tilde{Z}_t$ such that $\tilde{Z}_t \subset Z_t$, multiplying both sides by the elements of $\tilde{Z}_t$, and taking the unconditional expectation it is possible to get:

$$E(m_{t+1}R_{t+1} \otimes \tilde{Z}_t) = E(1 \otimes \tilde{Z}_t)$$

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Moments Conditions Derivation

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\begin{aligned}
& E(m_{t+1}R_{t+1}|Z_t) = 1 \\
\Rightarrow \quad & E(m_{t+1}R_{t+1}) = E(1) \\
\Rightarrow \quad & E(m_{t+1}R_{t+1} - 1) = 0
\end{aligned}
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The asset pricing under unconditional moments must be a specific case of pricing under conditional moments.

Unconditional moments conditions for *managed portfolios*:

$$E[(m_{t+1}R_{t+1} - 1) \otimes \tilde{Z}_t] = 0 \Rightarrow E[m_{t+1}(R_{t+1} \otimes \tilde{Z}_t) - (1 \otimes \tilde{Z}_t)] = 0$$

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- As the returns of the assets may not be an i.i.d. process (or other violations, such as fat tails, or even non-linearity of returns), this fact creates the need to work under serial correlation or even with dependence on returns.
- **Long-run covariance matrix:** We may use the widely known class of non-parametric estimators for the optimal long-run covariance matrix consistent to heteroskedasticity and autocorrelation (HAC):

$$\mathbf{\Omega}_{HAC}(\boldsymbol{\theta}_0) = \sum_{j=-(T-1)}^{T-1} k(j/b)\mathbf{\Gamma}_j$$

- Therefore, denoting by θ as the vector of parameters to be estimated, the GMM estimator $\hat{\theta}_T$ may be defined as:

$$\hat{\boldsymbol{\theta}}(\hat{\mathbf{\Omega}}_{HAC}(\boldsymbol{\theta}_0)) \equiv arg \min_{\hat{\theta}} g_T(\hat{\theta})'\hat{\mathbf{\Omega}}_{HAC}^{-1}(\boldsymbol{\theta}_0) g_T(\hat{\theta})$$

We may use instead the GEL class, that is only based on moment conditions.

In short, suppose a system of restrictions on unconditional moments, such as:

$$E[g(w, \theta_0)] = 0$$

We may write the *empirical likelihood* problem as:

$$\begin{aligned} \max_{p,\theta} \quad & \frac{1}{n}\sum_{i=1}^{n} log(p_i) \\ \text{subject to} \quad & \sum_{i=1}^{n} p_i g(w_i, \theta) = 0 \\ & \sum_{i=1}^{n} p_i = 1 \end{aligned}$$

From this constraint maximization we obtain the *saddlepoint problem*, which its solution provide the $\hat{\theta}$ empirical likelihood estimator.

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Take the following pricing equation in *Beta* format:

$$R_{i,t} = \alpha + \beta_i \mathbf{f}_t + \varepsilon_t, \quad t = 1, \ldots, T \quad ; \text{e} \quad i = 1, \ldots, N$$

where, for practicality $R_{i,t}$ and $\mathbf{f}_t$ already represent, respectively, excess returns for the N securities and for the K factors.

For a system with N assets we have:

$$\boldsymbol{R}_t = \boldsymbol{\alpha} + \boldsymbol{\beta} \boldsymbol{f}_t + \boldsymbol{\varepsilon}_t$$

Under the correct pricing assumption, the theoretical framework for these asset pricing models imply that the vector $\boldsymbol{\alpha} = \mathbf{0}$. Thus:

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The null and alternative hypotheses from a test of efficiency to assess whether all pricing errors u_t are jointly equal to zero are given by:

$$H_0 : \boldsymbol{\alpha} = \mathbf{0}$$
$$H_A : \boldsymbol{\alpha} \neq \mathbf{0}$$

$$J_{Wald} = \hat{\alpha}'[Cov(\hat{\alpha})]^{-1}\hat{\alpha} \sim \chi^2_N$$

- Underlies in the large samples distribution theory;
- Relying on the CLT, the test is based primarily on the fact that $\hat{\alpha}$ has normal distribution.

$$J_{GRS} = \frac{\frac{T-N-K}{N}}{\left(1 + E_T(\boldsymbol{f})'\hat{\boldsymbol{\Omega}}^{-1}E_T(\boldsymbol{f})\right)^{-1} \hat{\alpha}'\hat{\boldsymbol{\Sigma}}^{-1}\hat{\alpha}}$$

- $J_{GRS} \sim F_{(N,T-N-K)}$
- From Gibbons, Ross and Shanken (1989), the test requires that the errors are normally distributed, homoskedastic and uncorrelated.

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2. We compare both methods when (i) no conditional information is used, and when (ii) the *managed portfolios* structure is used.
3. The assessment is made by comparing the test results for different sample sizes and portfolios types, as well as for two asset pricing models (CAPM and the Fama-French three-factors).
4. Firstly, the performance is evaluated when the standard long-run covariance HAC matrix is used during the parameters estimations.
5. In the second part we assess the efficiency tests when fixed-b estimators are used (Vogelsang-Kiefer method).
6. Finally, we perform Monte Carlo experiments to evaluate the effects that data contaminations, as outliers and the presence of heavy tails in the innovation structure, may cause on the results of efficiency tests when GEL and GMM estimators are used in a finite sample context.

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Data - Instruments

- Following previous studies, we selected a limited number of instruments from those commonly used (standard set) to measure the state of the economy:
  1. lagged value of a 3-month Treasury-bill yield
  2. spread between corporate bond yields with different ratings (Moody's Baa - Aaa)
  3. spread between the 10-year and 1-year Treasury-bill yield with constant maturity
  4. percentage change in the U.S. inflation (*Consumer Price Index* - CPI)
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- Market portfolio $\rightarrow$ consists of the weighted return of the value of all companies listed on the NYSE, AMEX and NASDAQ;

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Data - Portfolios

- Selected four portfolios with increasing number of assets;
- Three of them have their composition based on size and book-to-market;
- While one is composed with categories derived from industries classification according to the business segment.

1. *6 Portfolios Formed on Size and Book-to-Market (2 x 3);*
2. *25 Portfolios Formed on Size and Book-to-Market (5 x 5);*
3. *49 Industry Portfolios;*
4. *100 Portfolios Formed on Size and Book-to-Market (10 x 10).*

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Results - Conventional HAC Estimation

Tests of portfolio efficiency using 6 portfolios formed on size and book-to-market for selected periods of time

- **No Conditional Information**

	Months	Wald Test		GRS Test		Wald Test		GRS Test	
		Statistic	p-value	Statistic	p-value	Statistic	p-value	Statistic	p-value
		CAPM				**FF**			
GMM	120	15.0	0.020	2.4	0.035	14.6	0.023	2.3	0.043
	240	28.4	0.000	4.6	0.000	64.4	0.000	10.3	0.000
	360	60.6	0.000	9.9	0.000	137.7	0.000	22.4	0.000
	480	73.0	0.000	12.0	0.000	219.3	0.000	35.9	0.000
	600	74.2	0.000	12.2	0.000	274.8	0.000	45.1	0.000
	720	78.5	0.000	13.0	0.000	285.8	0.000	47.0	0.000
	840	81.6	0.000	13.5	0.000	277.6	0.000	45.8	0.000
	960	83.2	0.000	13.8	0.000	268.5	0.000	44.3	0.000
	1020	70.2	0.000	11.6	0.000	242.7	0.000	40.1	0.000
GEL	120	7.2	0.304	1.1	0.351	12.5	0.052	1.9	0.083
	240	26.0	0.000	4.2	0.000	51.6	0.000	8.3	0.000
	360	54.0	0.000	8.8	0.000	109.6	0.000	17.8	0.000
	480	61.0	0.000	10.0	0.000	185.6	0.000	30.4	0.000
	600	51.5	0.000	8.5	0.000	221.6	0.000	36.4	0.000
	720	56.8	0.000	9.4	0.000	236.6	0.000	38.9	0.000
	840	56.6	0.000	9.4	0.000	227.4	0.000	37.5	0.000
	960	72.7	0.000	12.0	0.000	176.6	0.000	29.2	0.000
	1020	47.6	0.000	7.9	0.000	171.9	0.000	28.4	0.000

Results - Conventional HAC Estimation

Tests of portfolio efficiency using 6 portfolios formed on size and book-to-market under *scaled payoffs* for selected periods of time

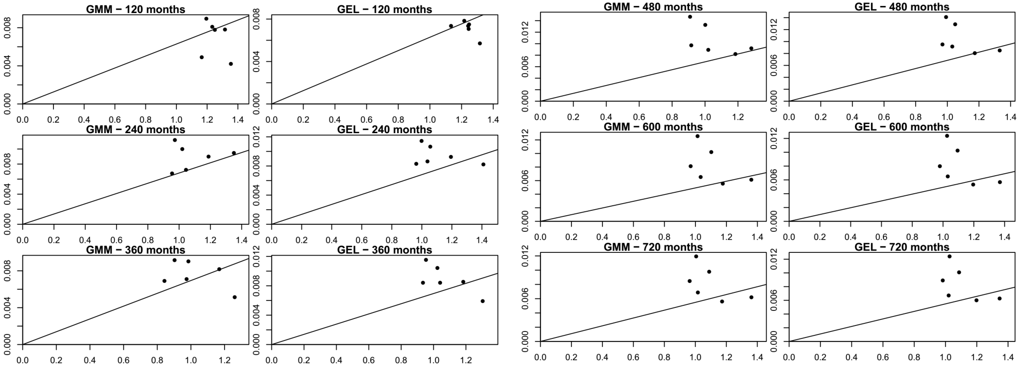
- ***Managed Portfolios***

	Months	Wald Test		GRS Test		Wald Test		GRS Test	
		Statistic	p-value	Statistic	p-value	Statistic	p-value	Statistic	p-value
		CAPM				**FF**			
GMM	120	128.0	0.000	20.1	0.000	15.7	0.015	2.4	0.031
	240	32.1	0.000	5.2	0.000	97.5	0.000	15.6	0.000
	360	79.0	0.000	12.9	0.000	209.7	0.000	34.1	0.000
	480	111.5	0.000	18.3	0.000	310.4	0.000	50.8	0.000
	600	111.1	0.000	18.3	0.000	398.1	0.000	65.3	0.000
	720	102.4	0.000	16.9	0.000	380.0	0.000	62.5	0.000
GEL	120	5.6	0.466	0.9	0.509	16.3	0.012	2.5	0.025
	240	31.6	0.000	5.1	0.000	82.3	0.000	13.2	0.000
	360	63.3	0.000	10.3	0.000	174.9	0.000	28.4	0.000
	480	78.9	0.000	13.0	0.000	275.2	0.000	45.0	0.000
	600	72.2	0.000	11.9	0.000	241.8	0.000	39.7	0.000
	720	76.0	0.000	12.5	0.000	332.5	0.000	54.7	0.000

Results - Conventional HAC Estimation

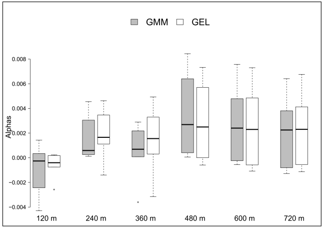
CAPM Model - Comparison of GMM and GEL estimated betas under *scaled returns* against the sample mean of monthly excess returns for the portfolio with 6 assets

- ***Managed Portfolios***

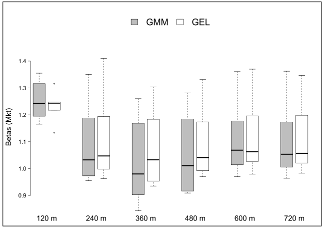


Results - Conventional HAC Estimation

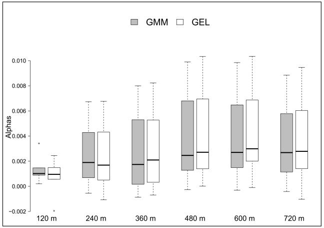
Boxplots for comparison of estimations by GMM and GEL using 6 portfolios formed on size and book-to-market under *scaled returns* for selected periods of time



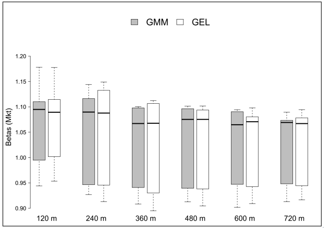
(a) CAPM Model: Alphas estimated by GMM and GEL



(b) CAPM Model: $Betas_{Mkt}$ estimated by GMM and GEL



(c) Fama-French Model: Alphas estimated by GMM and GEL



(d) Fama-French Model: $Betas_{Mkt}$ estimated by GMM and GEL

HAC & Fixed-b estimators

- The econometric literature has developed a number of methods to circumvent the problems caused by serial correlation, heteroskedasticity and observation or measurement errors.
- Despite the important outcome of consistency in HAC estimators, this methodology has some weaknesses.
- Müller (2007) shows that this asymptotic estimator class does not performs well in case of finite samples.
- Kiefer and Vogelsang (2005) discuss that HAC robust tests have a tendency to over-reject in finite samples under the null hypothesis.
- To circumvent this deficiency Kiefer and Vogelsang (2002) propose a new HAC class of robust tests that have bandwidth equal to sample size.
- Under this approach, the tests have pivotal asymptotic distributions

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- The econometric literature has developed a number of methods to circumvent the problems caused by serial correlation, heteroskedasticity and observation or measurement errors.
- Despite the important outcome of consistency in HAC estimators, this methodology has some weaknesses.
- Müller (2007) shows that this asymptotic estimator class does not performs well in case of finite samples.
- Kiefer and Vogelsang (2005) discuss that HAC robust tests have a tendency to over-reject in finite samples under the null hypothesis.
- To circumvent this deficiency Kiefer and Vogelsang (2002) propose a new HAC class of robust tests that have bandwidth equal to sample size.
- Under this approach, the tests have pivotal asymptotic distributions

Results - Fixed-b Estimators

Tests of portfolio efficiency (VK method) using 6 portfolios formed on size and book-to-market for selected periods of time

- **No Conditional Information**

	Months	Wald Test		GRS Test		Wald Test		GRS Test	
		Statistic	p-value	Statistic	p-value	Statistic	p-value	Statistic	p-value
		CAPM				**FF**			
GMM	120	139.4	0.000	21.9	0.000	125.7	0.000	19.4	0.000
	240	262.2	0.000	42.4	0.000	256.5	0.000	41.1	0.000
	360	332.8	0.000	54.4	0.000	470.2	0.000	76.4	0.000
	480	417.6	0.000	68.6	0.000	532.5	0.000	87.1	0.000
	600	528.6	0.000	87.1	0.000	994.4	0.000	163.2	0.000
	720	233.7	0.000	38.6	0.000	390.5	0.000	64.3	0.000
	840	276.1	0.000	45.6	0.000	665.4	0.000	109.7	0.000
	960	302.6	0.000	50.1	0.000	699.0	0.000	115.4	0.000
	1020	294.6	0.000	48.8	0.000	497.3	0.000	82.2	0.000

- ***Managed Portfolios***

	Months	Wald Test		GRS Test		Wald Test		GRS Test	
		Statistic	p-value	Statistic	p-value	Statistic	p-value	Statistic	p-value
		CAPM				**FF**			
GMM	120	610.9	0.000	95.9	0.000	783.7	0.000	120.8	0.000
	240	917.8	0.000	148.5	0.000	4,184.1	0.000	671.2	0.000
	360	4,902.3	0.000	801.2	0.000	6,532.9	0.000	1,061.6	0.000
	480	3,258.6	0.000	535.2	0.000	10,250.2	0.000	1,676.3	0.000
	600	4,021.4	0.000	662.4	0.000	16,048.9	0.000	2,634.7	0.000
	720	5,396.1	0.000	890.6	0.000	10,124.1	0.000	1,666.3	0.000

Evaluating Robustness with Monte Carlo Simulations

Goal: to analyze the robustness of the tests on each of the two different estimation methodologies.
As previous results had shown some evidence, we assess the tests under:

1. heavy-tailed distributions
2. outliers

Structure: it is analyzed the tests simulating

1. 4 different innovations scenarios
2. used the portfolio with 6 assets, and 120 months period ($N = 6$, $T = 120$)
3. Fama-French model with Conditional Information (*managed portfolios* approach)
4. 500 artificial returns datasets simulations for each of the 4 scenarios (due GEL has high computational cost)

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Evaluating Robustness with Monte Carlo Simulations

Each artificial dataset assume the following DGP:

$$R_{i,t}^{\text{Sim}*} = \hat{\beta}_{i,1}^{OLS} Mkt_t + +\hat{\beta}_{i,2}^{OLS} SMB_t + \hat{\beta}_{i,3}^{OLS} HML_t + \hat{\varepsilon}_{i,t}^{\text{Sim}*} \quad , \quad t = 1, \ldots, 120$$
$$, \quad i = 1, \ldots, 6$$

Sampling Distributions of the Test Statistics

To analyze the results of the Monte Carlo experiments, we used the graphical method proposed by Davidson and MacKinnon (1998):

1. *P-value plot*: plots $\hat{F}(x_i)$ against x_i
2. *P-value discrepancy plot*: plots $\hat{F}(x_i) - x_i$ against x_i

Evaluating Robustness with Monte Carlo Simulations

Scenario 1 - Gaussian Shocks:

$$\hat{\varepsilon}_{i,t}^{\text{Sim*}} = \hat{\xi}_{i,t}^{\text{Sim1}}, \quad t = 1, \ldots, 120 \quad ; \quad i = 1, \ldots, 6$$
$$\hat{\xi}_{i,t}^{\text{Sim1}} \sim N(0, \hat{\sigma}_i^{2\ \text{OLS}})$$

Scenario 2 - Shocks from a t distribution:

$$\hat{\varepsilon}_{i,t}^{\text{Sim*}} = \hat{\nu}_{i,t}^{\text{Sim2}}, \quad t = 1, \ldots, 120 \quad ; \quad i = 1, \ldots, 6$$
$$\hat{\nu}_{i,t}^{\text{Sim2}} \sim t(4)$$

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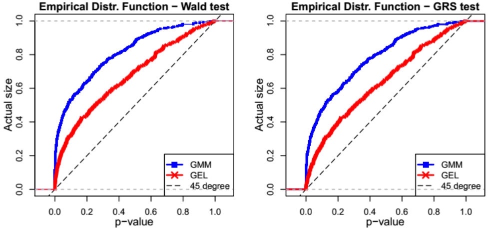
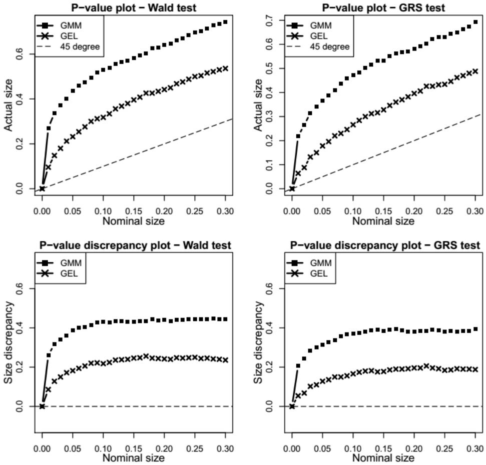
$$\hat{\nu}_{i,t}^{\text{Sim2}} \sim t(4)$$

Simulated scenario 2 with shocks from a t distribution ($\hat{\varepsilon}_{i,t}^{\text{Sim}*} = \hat{\nu}_{i,t}^{\text{Sim2}}$) in Wald and GRS tests (Model=Fama-French, N=6, T=120, 500 simulations)





Evaluating Robustness with Monte Carlo Simulations

Scenario 3 - Outlier on a fixed date:

$$\hat{\varepsilon}^{\text{Sim}*}_{i,t} = 1_{t=T/2}(\hat{\kappa}^{\text{Sim3}}_{i,t}), \qquad t = 1, \ldots, 120 \quad ; \quad i = 1, \ldots, 6$$

$$1_{t=T/2}(\kappa^{\text{Sim3}}_{i,t}) = \begin{cases} -\hat{\kappa}^{\text{Sim3}}_{i,t} & \text{, if } t = T/2 \\ 0 & \text{, if } t \neq T/2 \end{cases}$$

$$\hat{\kappa}^{\text{Sim3}}_{i,t} \sim N(0, 5\hat{\sigma}^{2\ \text{OLS}}_{i})$$

Evaluating Robustness with Monte Carlo Simulations

Scenario 3 - Outlier on a fixed date:

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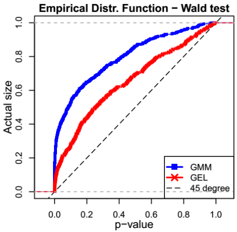
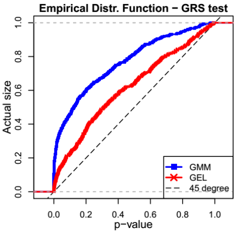
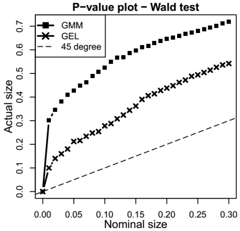
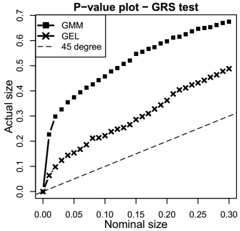
$$\hat{\kappa}_{i,t}^{\text{Sim3}} \sim N(0, 5\hat{\sigma}_i^{2\ \text{OLS}})$$

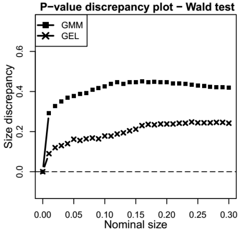
Simulated scenario 3 with shocks at t=T/2 defined by $\hat{\varepsilon}_{i,t}^{\text{Sim}*} = 1_{t=T/2}(\hat{\kappa}_{i,t}^{\text{Sim3}})$ in Wald and GRS tests (Model=Fama-French, N=6, T=120, 500 simulations)

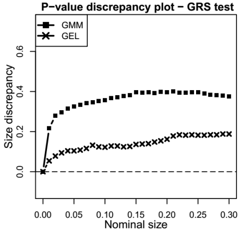












Evaluating Robustness with Monte Carlo Simulations

Scenario 4 - Outlier with 5% probability:

$$\hat{\varepsilon}_{i,t}^{\text{Sim*}} = \hat{\xi}_{i,t}^{\text{Sim4}} - 1_{\hat{p}_{i,t}<0.05}(\hat{\kappa}_{i,t}^{\text{Sim4}}), \qquad t = 1, \ldots, 120 \quad ; \quad i = 1, \ldots, 6$$

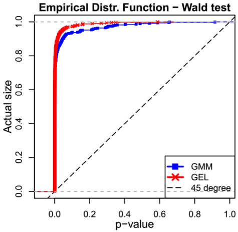
$$1_{\hat{p}_{i,t}<0.05}(\kappa_{i,t}^{\text{Sim4}}) = \begin{cases} \hat{\kappa}_{i,t}^{\text{Sim4}} & \text{, if } \hat{p}_{i,t}^{\text{Sim4}} < 0.05 \\ 0 & \text{, if } \hat{p}_{i,t}^{\text{Sim4}} \geq 0.05 \end{cases}$$

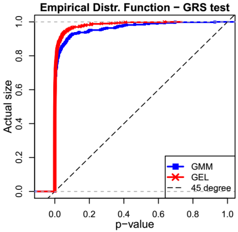
$$\hat{p}_{i,t}^{\text{Sim4}} \sim unif(0,1)$$

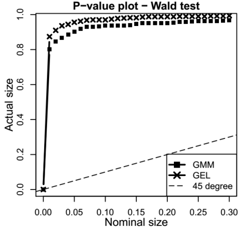
$$\hat{\xi}_{i,t}^{\text{Sim4}} \sim N(0, \hat{\sigma}_i^{2\ \text{OLS}})$$

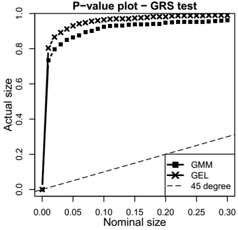
$$\hat{\kappa}_{i,t}^{\text{Sim4}} \sim N(0, 5\hat{\sigma}_i^{2\ \text{OLS}})$$

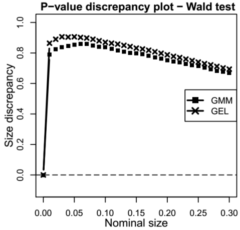
Simulated scenario 4 with shocks defined by $\hat{\varepsilon}_{i,t}^{\text{Sim}*} = \hat{\xi}_{i,t}^{\text{Sim4}} - 1_{\hat{p}_{i,t}<0.05}(\hat{\kappa}_{i,t}^{\text{Sim4}})$ in Wald and GRS tests (Model=Fama-French, N=6, T=120, 500 simulations)

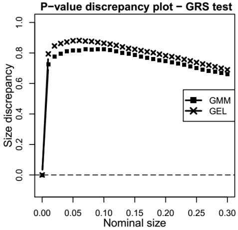












Conclusions

Comparing the results for (a) different portfolios sizes, (b) different composition methods, as well as (c) increasing periods of time:

1. in general, efficiency tests using GEL generate lower estimates compared to tests using the standard approach with GMM;
2. when the sample may be characterized as finite, with low n and T , we note that the results are conflicting among the methodologies.

- These results may be an evidence that estimators from GEL class really performs differently in small samples.
- Results apply **(i)** for both models (CAPM and the Fama-French), as well as **(ii)** both approaches evaluated in this work (unconditional efficiency structure and the multiplicative approach).

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