

# A Machine Learning Factor-Based Interpretation for the Bond Risk Premia in the U.S.

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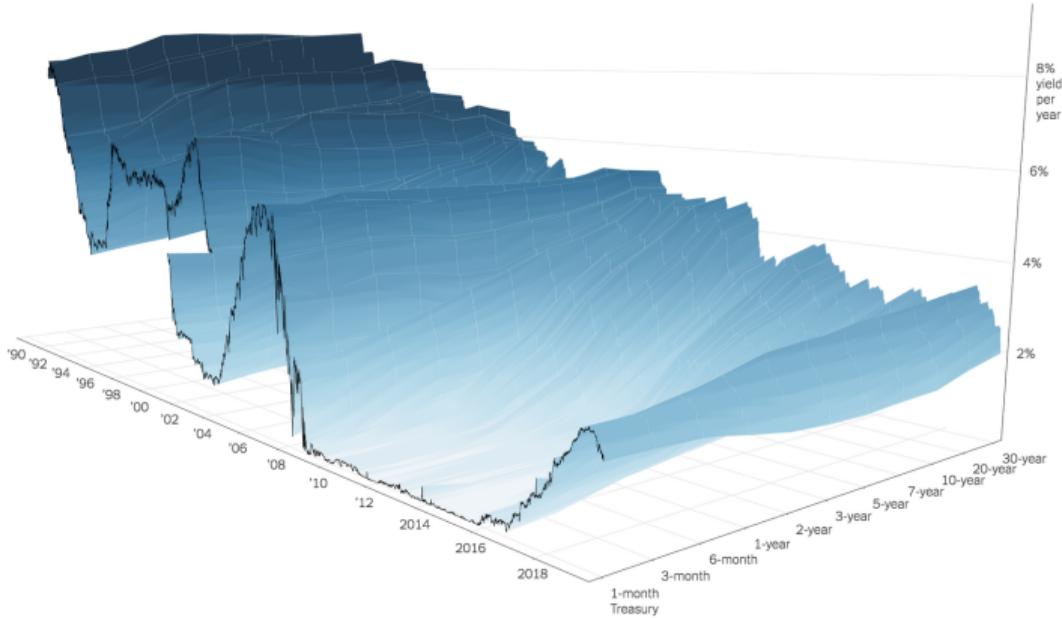
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April 22, 2021

# Motivation

- I study the time variation of the **risk premia in U.S. Treasuries bonds**.
- Treasury bonds play an important role in financial markets  $\Rightarrow$  its risk and return dynamics is of **central economic importance**.
  - ↳ major importance for monetary police
  - ↳ strategic in investors' portfolios
  - ↳ understanding of financial events: e.g., zero rates in 2008, 2020
- Understanding pricing of U.S. Treasuries is a central question in the study of bond markets.
  - ↳ The U.S. Treasury market is the largest government debt market in the world with an estimated value of **\$14 trillion (2019)**.
  - ↳  $\approx 30\%$  of the entire U.S. bond market (corporate debt + mortgage and municipal bonds + money market instruments + asset-backed securities)

# Treasuries Yields



**risk premia:**  
*difference between the current long rate and the expected average of future short rates.*

# Motivation

- **Bond Premia**

- ↳ Long literature. Back from Fama and Bliss (1987)
- ↳ Nonetheless, never fully answered/understood
- ↳ Many **factors** were proposed in the literature:
  - ↳ Fama and Bliss (1987) → *forward spreads*
  - ↳ Cochrane and Piazzesi (2005) → *linear combination of forward rates*
  - ↳ Ludvigson and Ng (2009) → *linear combination of macro PC loadings*
  - ↳ Cieslak and Povala (2015) and Lee (2018) → *trend inflation*

## Bauer and Hamilton (2018)

- ↳ evidence against the use factors not derived from the yield curve (non spanning) →  
**“spanning puzzle” literature**
- ↳ raised methodological issues: econometric problems when overlapping returns is used.

# Research Question

- An important question that could assist to elucidate the whole bond premia problem is related with the factor structure of expected returns. **Is there a factor representation? If so, what is its structure?**
- Recently, Cochrane (2015) argued that it is possible that there is a dominant single factor structure for bond returns, in such a way that risk premiums rise and fall together.
  - ↳ *A parsimonious number is key here.*

## Central Question

- *What is the linear combination of forecasting variables that captures common movement in expected returns across assets?*

## A Different Route

It is possible that this search for deriving, building and estimating factors that represent state variables in macro-finance models may be limited.

- The process done by financial economists of manually discovering and hand picking this list of factors may be leaving unseen relationships between the state variables out in their derivation.
- To do so, I make use of one of the most powerful approaches in machine learning: **deep neural network** to uncover relationships in the full set of information from the yield curve.

# Contribution

## Methodological/Theory

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- ↳ I propose a novel approach for deriving a **parsimonious number** of state factor consistent with a **dynamic term-structure with unspanned risks** theoretically motivated model.
- ↳ I use **deep neural networks** to uncover relationships in the full set of information from the yield curve, I derive a single state variable factor that provide a better approximation to the spanned space of all the information from the term-structure.
- ↳ I also introduce a way to obtain **unspanned risks** from the yield curve that is used to complete the state space.

## Empirical Findings

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- ↳ I show that this parsimonious number of state variables have predictive power for excess returns of bonds over 1-month holding period (in/out-of-sample).
- ↳ I provide an intuitive interpretation of derived factors, and show what information from macroeconomic variables and sentiment-based measures they can capture.

# Contribution - Discussion

- **First**, through DNNs, we can introduce **nonlinearities** when modeling the bond risk premia in our first step of the recursive process.
  - ↳ while still making use of a linear combination of the latent factors in the second step,
  - ↳ and generating a parsimonious number of factors (state variables).
    - ↳ With neural networks we can introduce **flexible and complex nonlinear relationships** from the inputs while approximating arbitrarily well a rich set of smooth functions.
    - ↳ Consistent with recent findings (e.g., Gu et al. (2018); Bianchi et al. (2019)) → importance of allowing for **nonlinearities**.
    - ↳ The approach is at the intersection of bond premia and **sequential learning** as in Gargano et al. (2019) and Dubiel-Teleszynski et al. (2019).
- **Second**, the approach avoids hand-picking the variables from the yield curve
  - ↳ as through a DNN we are able to **recursively learn the best-approximating function** that condenses the yield curve into a single latent factor.

# Contribution - Discussion

- **Third,** I overcome some of the issues raised by Bauer and Hamilton (2018)
  - ↳ use of non-overlapping returns, as done by the most recent literature (Gargano et al., 2019)
  - ↳ I make use of the term structure at the higher frequency of 1-month holding period with maturities ranging up to 60 months ahead.
- **Fourth,** we start our process with only information from the term structure.
- **Fifth,** we take a broader interpretation of the unspanning factor.
  - ↳ we can link with other sources of risks (macroeconomics and sentiment-based variables)

## Log yields

$$y_t^{(n)} \equiv -\frac{1}{n} p_t^{(n)}$$

where,  $y_t^{(n)}$  denote the log yield of a  $n$ -maturity bond at time  $t$   
 $p_t^{(n)}$  denote the natural logarithm price of this bond

- **holding period returns**

$$r_{t+\Delta}^{(n)} \equiv p_{t+\Delta}^{(n-\Delta)} - p_t^{(n)}$$

- **Excess Returns**

$$rx_{t+h/12}^{(n)} \equiv \text{holding period return } r_{t+h/12}^{(n)} - \text{1-period yield}$$

# Spanning Hypothesis

- SH is a central issue in macro-finance models (Gürkaynak et al., 2007; Duffee, 2013; Bauer and Hamilton, 2018)

EH

## Spanning Hypothesis

- All relevant information to forecast yields and excess returns can be found on the term-structure.
- The yields curve fully spans all necessary information, and thus, no other variable already present in the term-structure should be necessary.
- It does not rule out the importance of macro variables (current or future).
- Yield curve completely reflects and spans this information.
- Influential works/factors:
  - ↳ Spanning: Fama and Bliss (1987) [FB Details](#) and Cochrane and Piazzesi (2005) [CP Details](#)
  - ↳ Not spanning: Ludvigson and Ng (2009) [LN Details](#)

# A Deep-Learning Structure for Bond Premia

**Proposition 1.** *The state vector  $Z_t$  that encompasses all risks in the economy can be partitioned as  $Z_t = \{Z_t^y, Z_t^{y^c}\}$ , in such a way that  $Z_t^y$  contains information solely from the yield curve, and  $Z_t^{y^c}$  any other information not found in the term structure.*

$Z_t^y$  contains only yield curve variables [yields, forward rates]

$Z_t^{y^c}$  contains any other variable (complement) [e.g., macro and sentiment-based variables]

We can summarize previous approaches with the following predictive regression:

$$rx_{t+h/12}^{(n)} = \beta^\top Z_t + \epsilon_{t+h/12} \quad (1)$$

- **Spanning hypothesis**  $\Rightarrow Z_t = \{Z_t^y\}$  (only yield curve information).
- Evidence against the **spanning hypothesis**  $\Rightarrow Z_t^{y^c} \neq \emptyset$ .

# Artificial Neural Network: A Primer

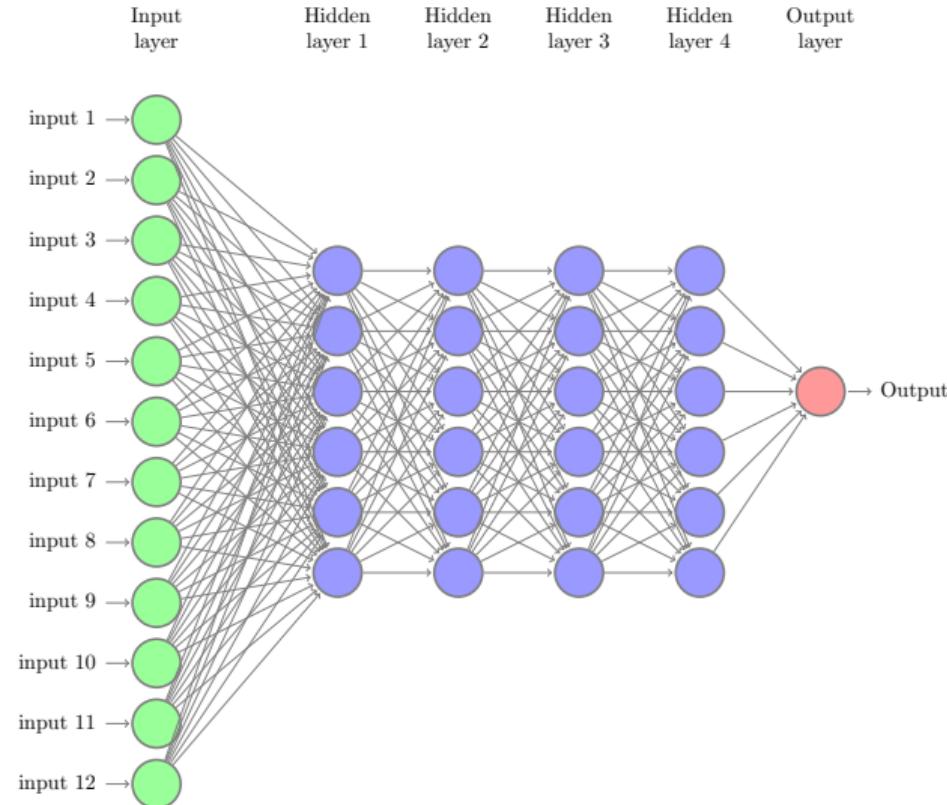
**Idea:** Attempt to replicate the brain architecture

↳ Many levels of processing information

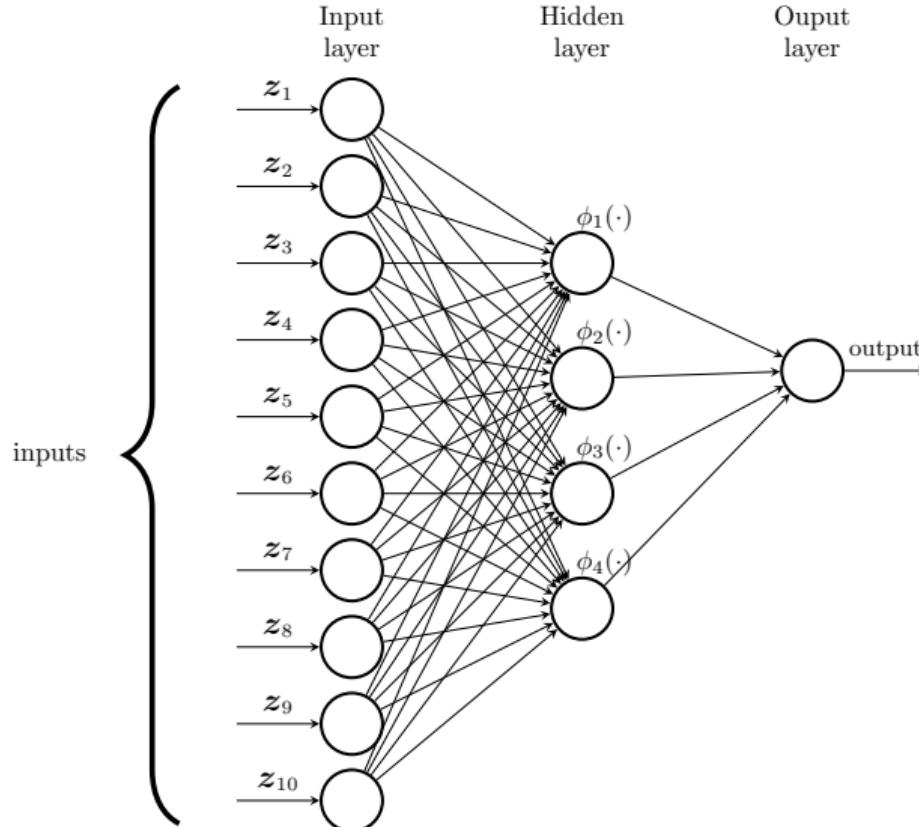
**Goal:** Extract complex non-linear combinations of the input

↳ Supervised Learning

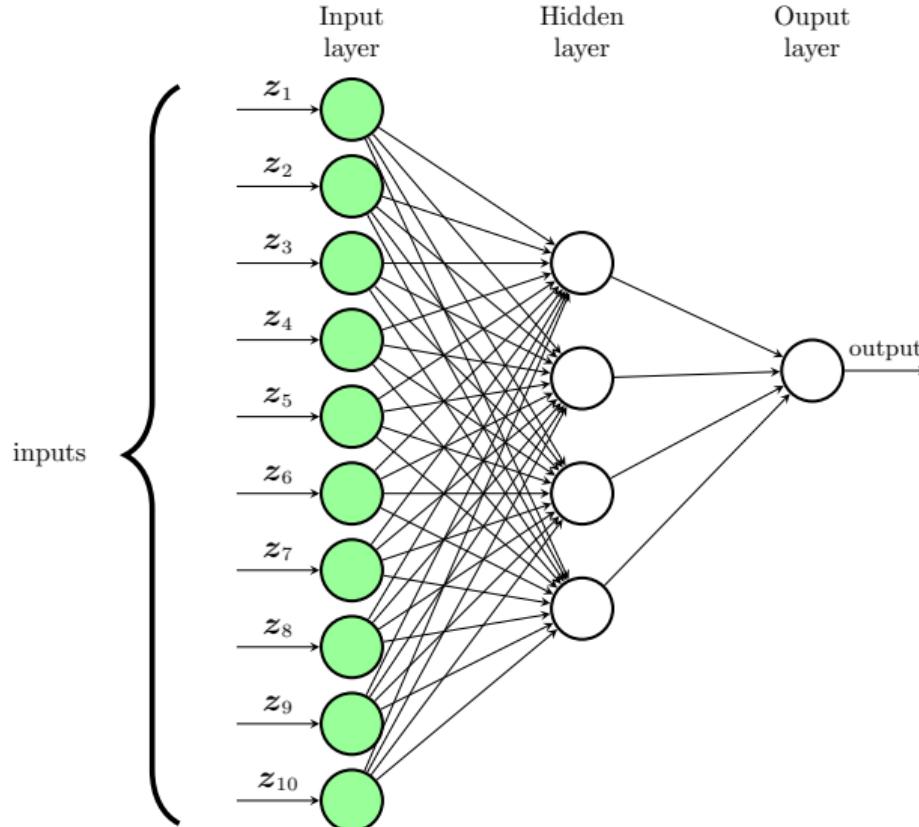
↳ Conditioning on target (here,  $rx_{t+h/12}^{(n)}$ ) and the inputs (here,  $Z_t$ )



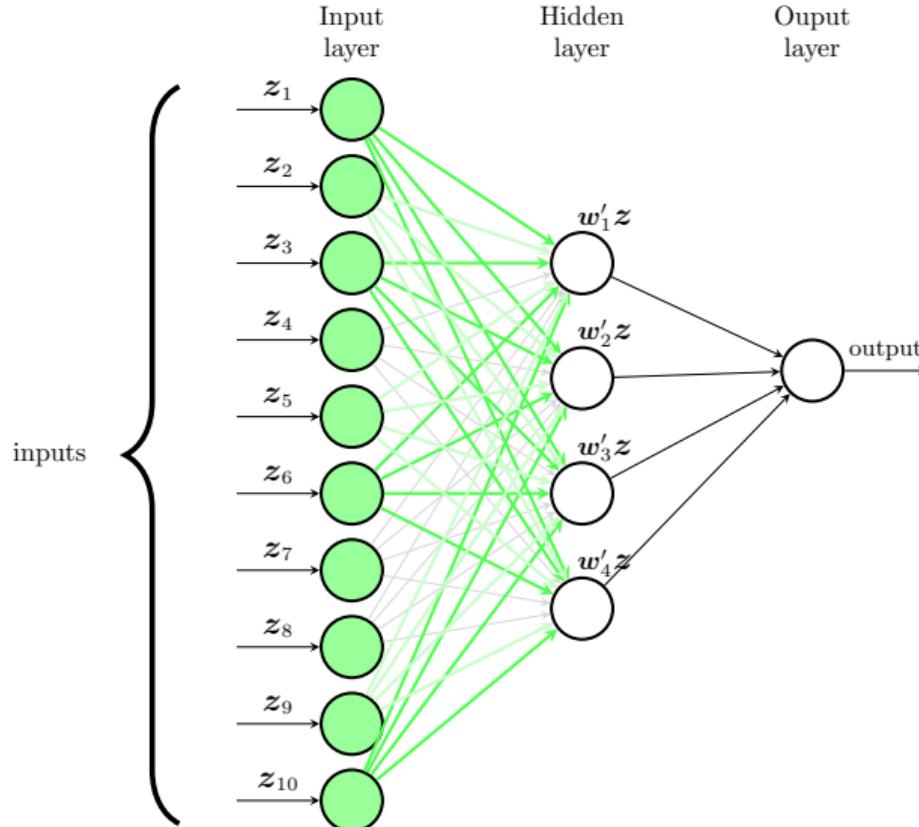
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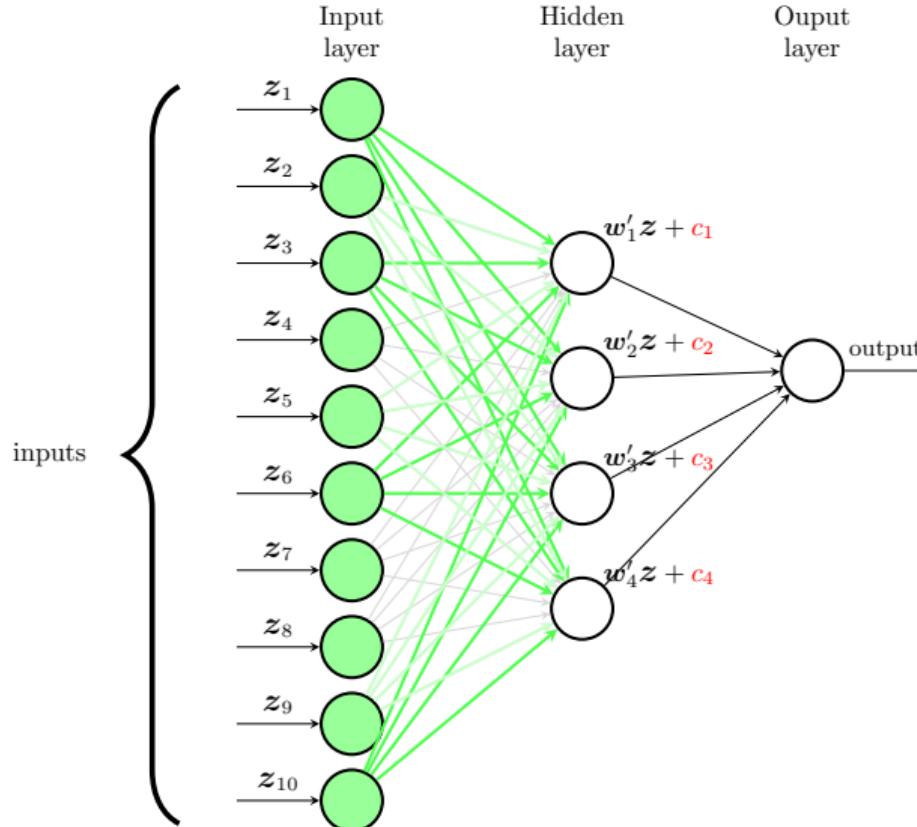
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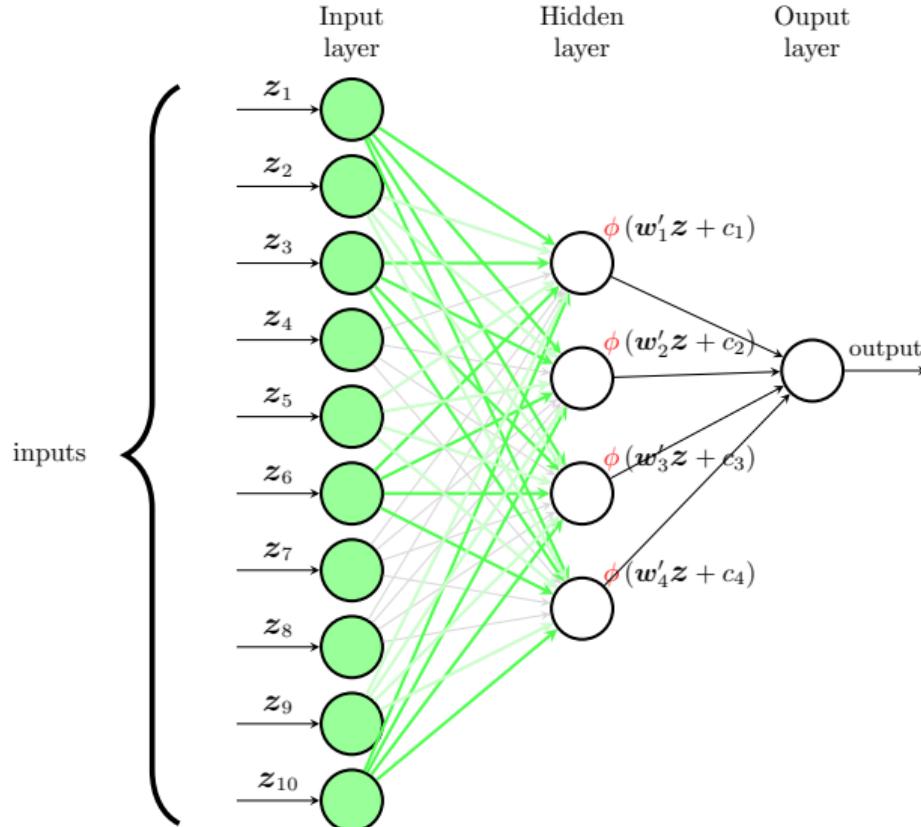
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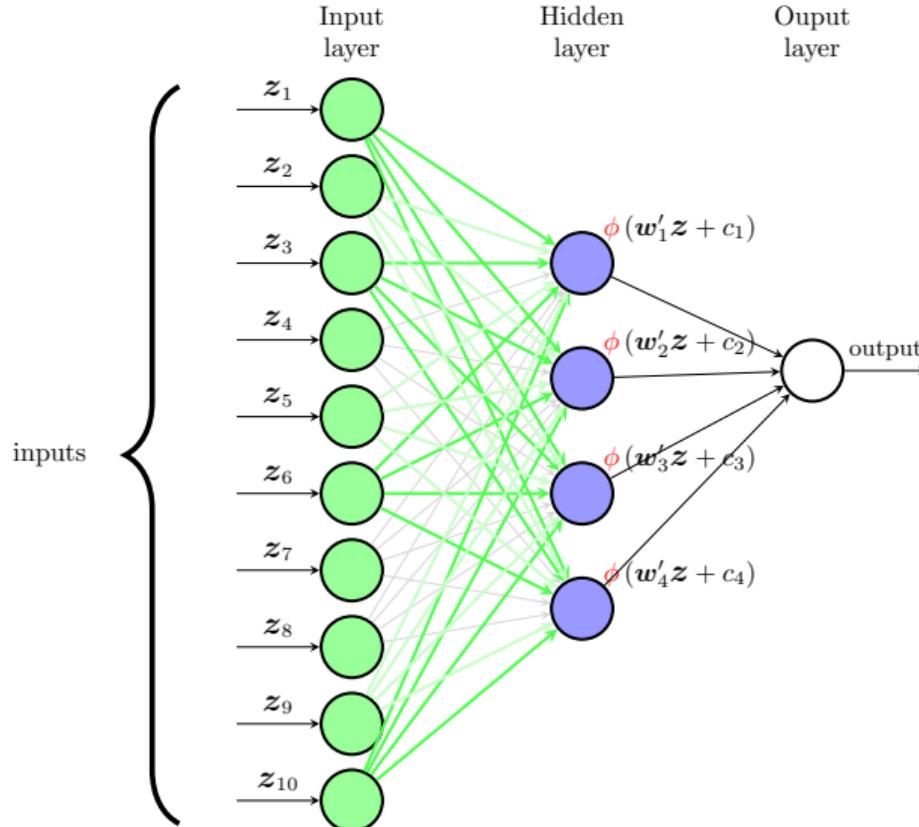
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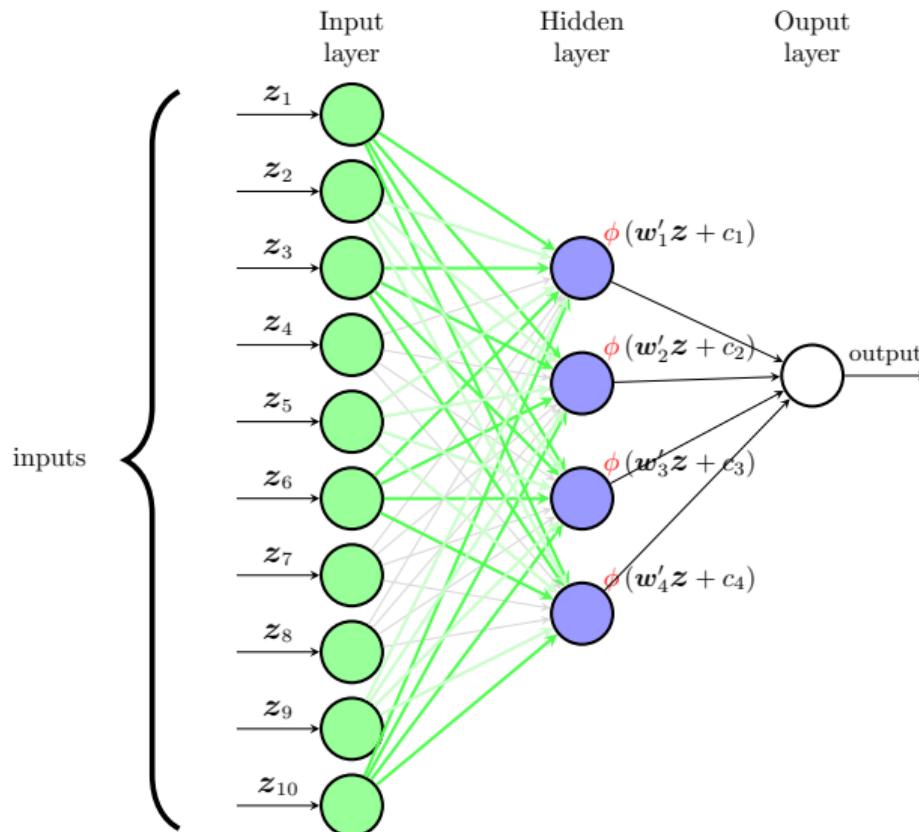
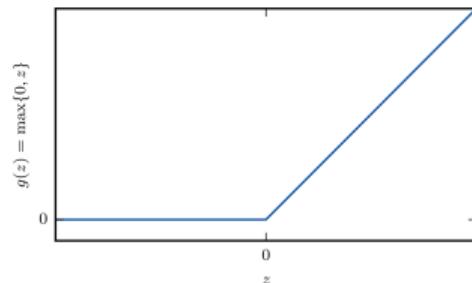
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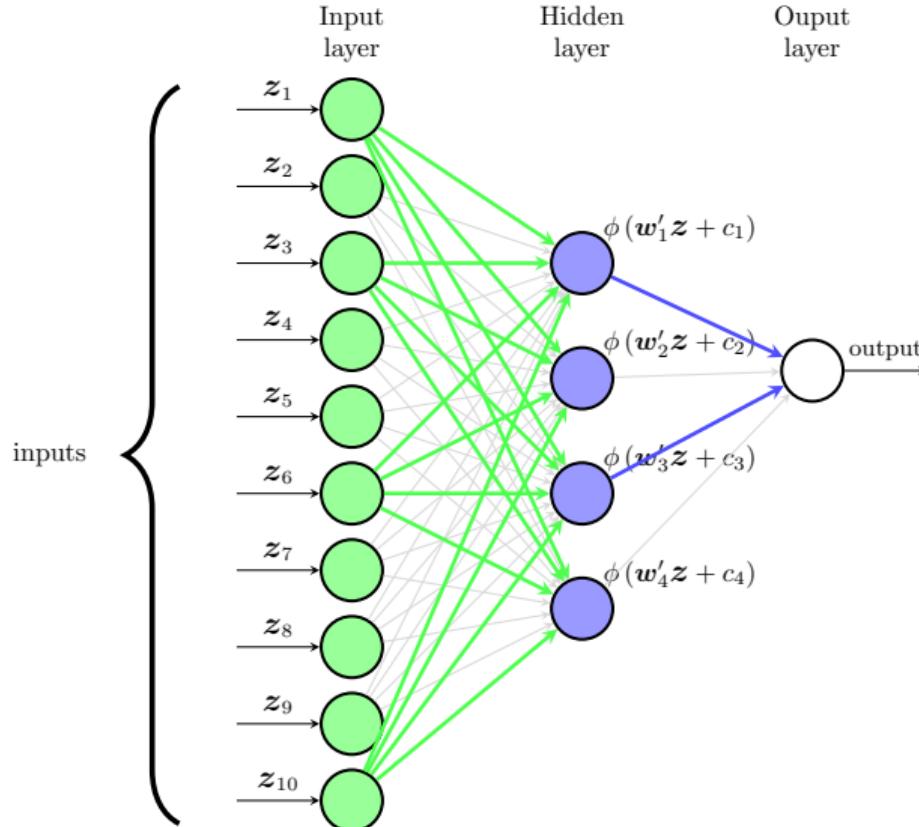
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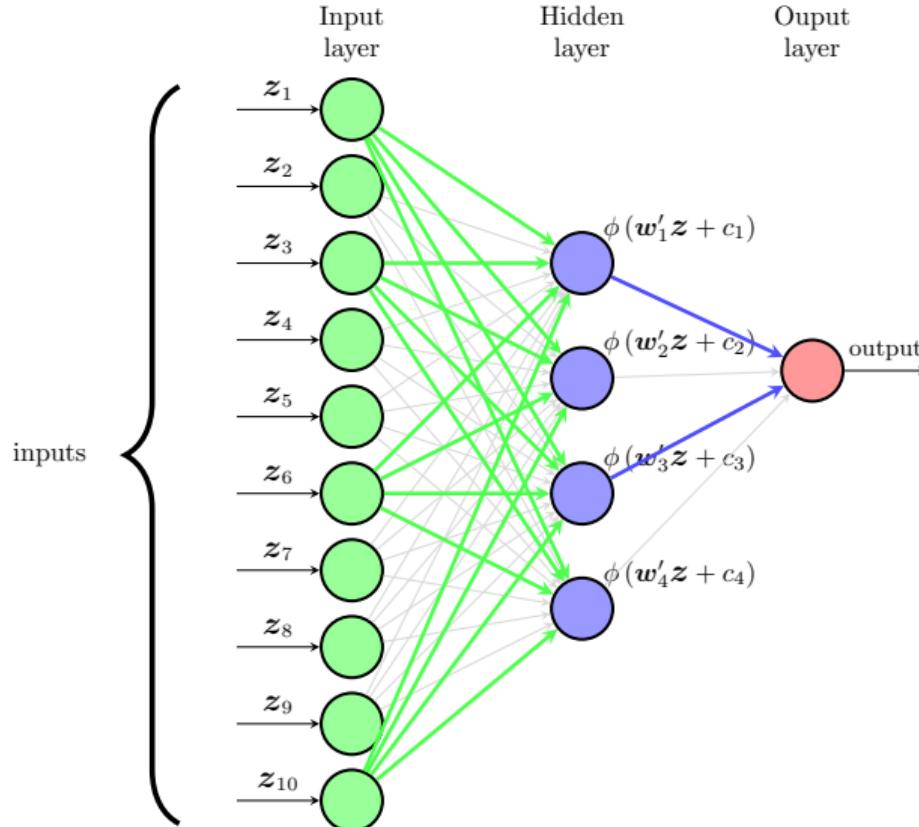
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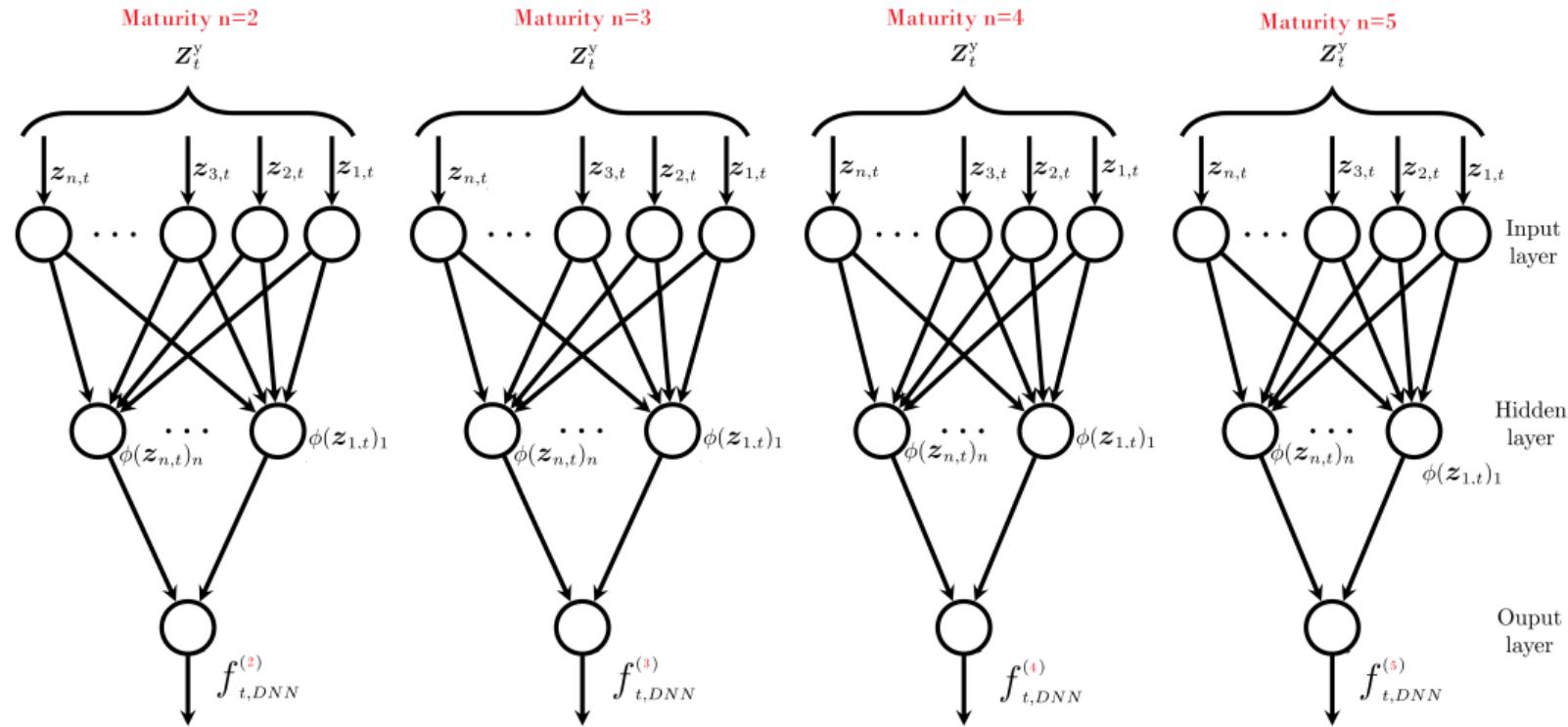
# A Deep-Learning Structure for Bond Premia

- DNN defines a mapping such as  $rx_{t+h/12}^{(n)} = g(\mathbf{Z}_t, \theta_t)$  to learn the parameter  $\theta_t$  that provides the best function approximation.
- Represented in a direct acyclic graph with a chain of functions  
$$g(\mathbf{Z}_t) = g^{(L)} \left( \dots \left( g^{(2)} \left( g^{(1)}(\mathbf{Z}_t) \right) \right) \right).$$

## Universal Approximation Theorem (Hornik et al., 1989; Cybenko, 1989)

- Feedforward network with a linear output layer and **at least one hidden layer** with any activation function can approximate **any function**<sup>1</sup> from one finite-dimensional space to another with any desired nonzero amount of error.
  - ↳ **Implication:** there exists a network large enough to achieve any degree of accuracy.
  - ↳ In short, a simple neural network can represent a wide variety of functions.

# A Deep-Learning Structure for Bond Premia



## DNN Factors

$$\begin{aligned} \frac{1}{4} \sum_{n=2}^5 r x_{t+h/12}^{(n)} &= \tau_0 + \tau_1 f_{t,DNN}^{(2),h} + \tau_2 f_{t,DNN}^{(3),h} + \tau_3 f_{t,DNN}^{(4),h} + \tau_4 f_{t,DNN}^{(5),h} + \bar{\epsilon}_{t+h/12} \\ &= \boldsymbol{\tau}^\top \widehat{\mathfrak{F}}_t^h + \bar{\epsilon}_{t+h/12} \end{aligned} \quad (2)$$

where  $\widehat{\mathfrak{F}}_t$  and  $\boldsymbol{\tau}$  are  $5 \times 1$  vectors given by  $\widehat{\mathfrak{F}}_t \equiv [1 \quad f_{t,DNN}^{(2),h} \quad f_{t,DNN}^{(3),h} \quad f_{t,DNN}^{(4),h} \quad f_{t,DNN}^{(5),h}]^\top$ , and  $\boldsymbol{\tau} \equiv [\tau_0 \quad \tau_1 \quad \tau_2 \quad \tau_3 \quad \tau_4]^\top$ .

- We **recursively** orthogonalize the excess returns generated by the deep neural network factor  $f_{t,DNN}^{(n)}$ , and denote it by  $\xi_t^{(n),h}$ .
- The factor  $\xi_{t+h/12}^{(n),h}$  that lies in an orthogonal vector to the space spanned by  $f_{t,DNN}^{(n)}$ , can be seen as all the information not spanned by the term-structure captured by  $f_{t,DNN}^{(n)}$ .

# A Deep-Learning Structure for Bond Premia

## Linear Rotation of the State Space

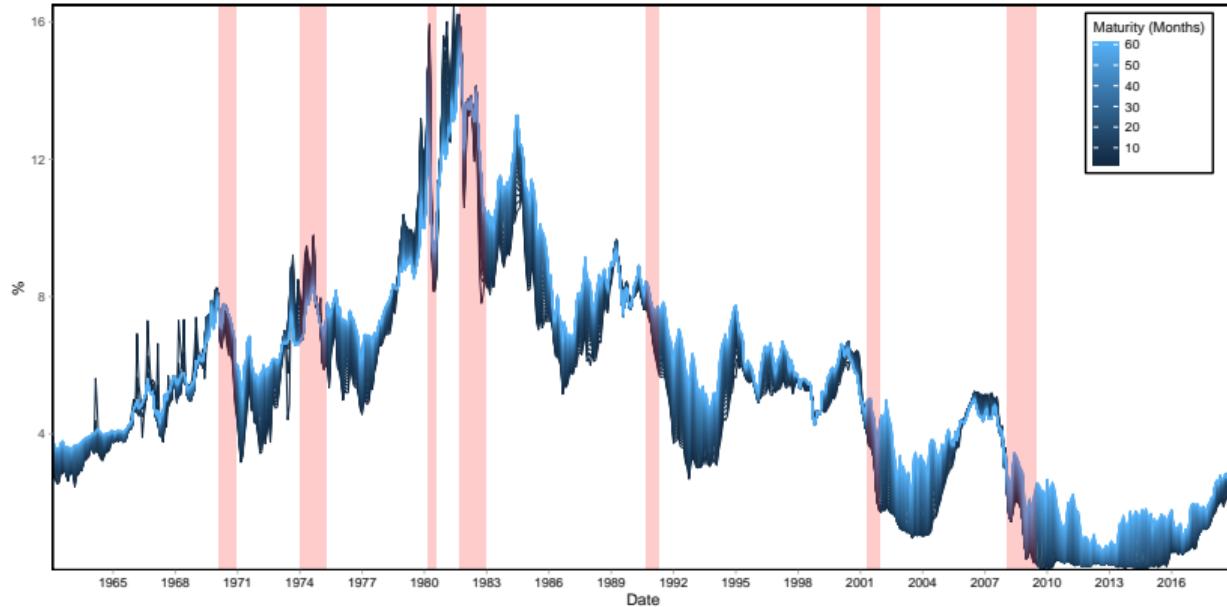
**Proposition 2.** As in the dynamic term structure model of Joslin et al. (2014),  $f(\xi_{t+h/12}^h)$  complete and fill the unspanned factor in the state space, in a such a way that  $\left[ \left( \tau^\top \widehat{\mathfrak{F}}_t \right)_t^h, f(\xi_{t+h/12}^h) \right]$  and  $Z_t$  represent linear rotations of the same economy-wide risks underlying all tradable assets available to agents in the economy.

- Analogous to Joslin et al. (2014), we argue
  - ↳ that the unspanned information in  $\widehat{\xi}_{t+h/12}^h$  could be capturing macroeconomic information or sentiment measures not spanned by the term-structure.

Details

## Data & Strategy

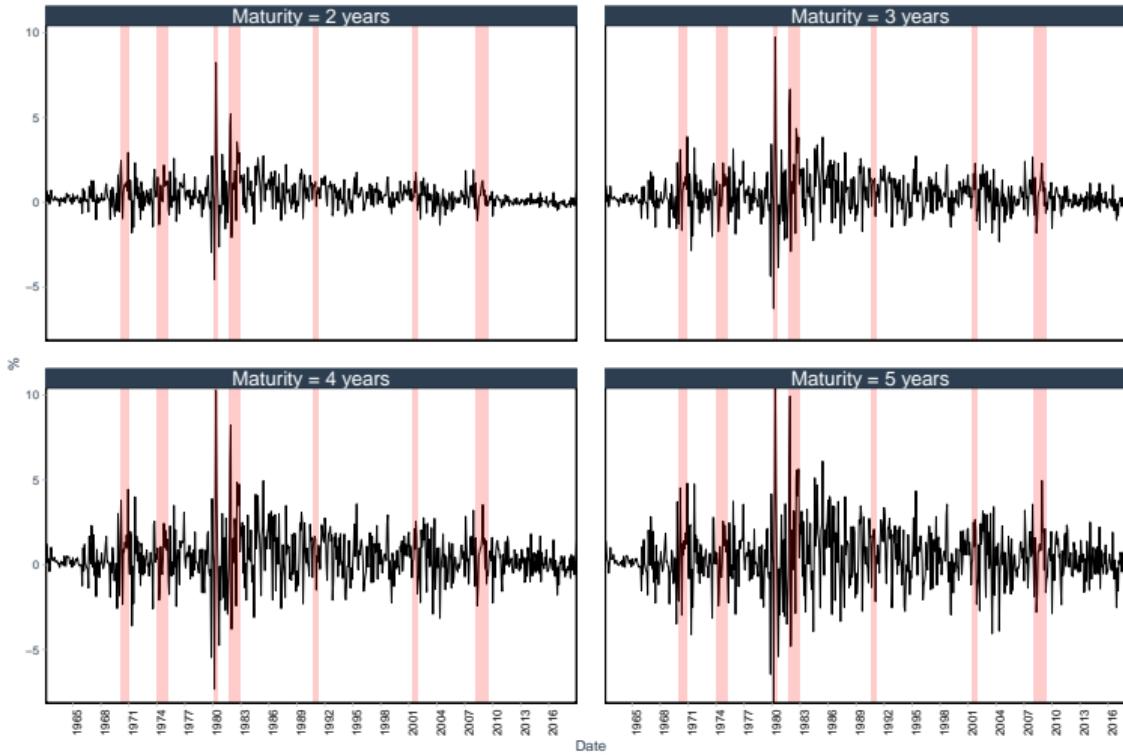
Derived zero-coupon bonds log yields for maturities ( $n$ ) up to 60 months



- Full period:  
1962:01 to 2017:12
- Period of evaluation:  
1993:01 to 2017:12

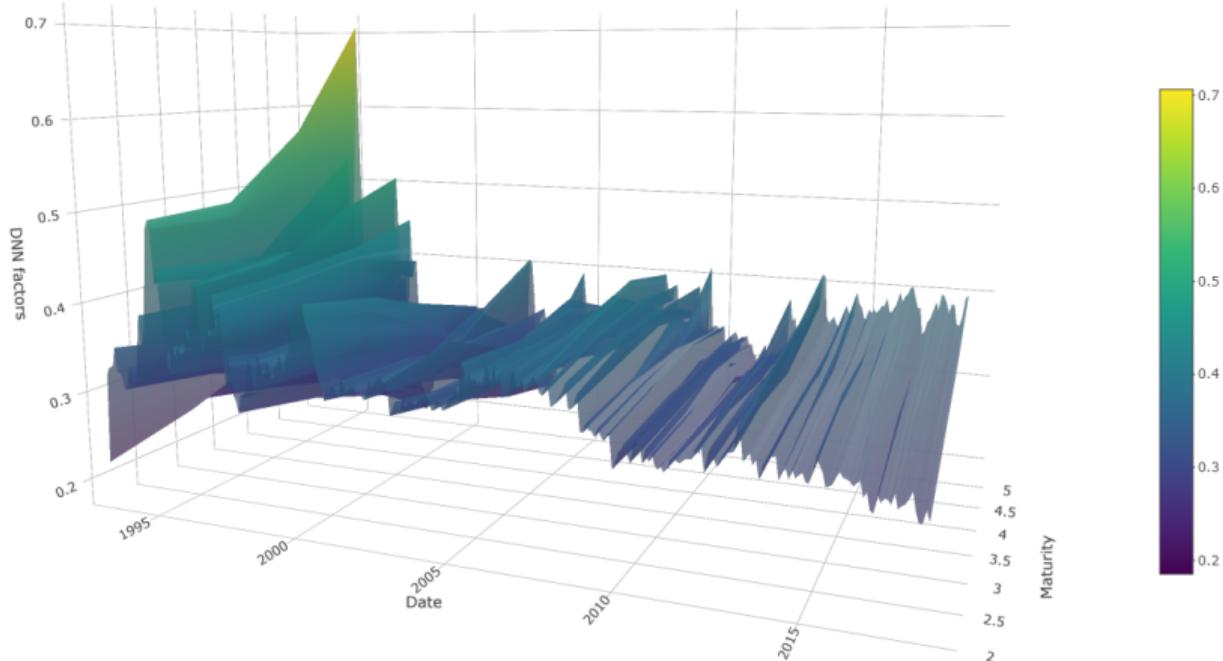
# Treasuries Excess Returns

## 1-Month Bonds Excess Returns (1962-2017)



## Empirical Results

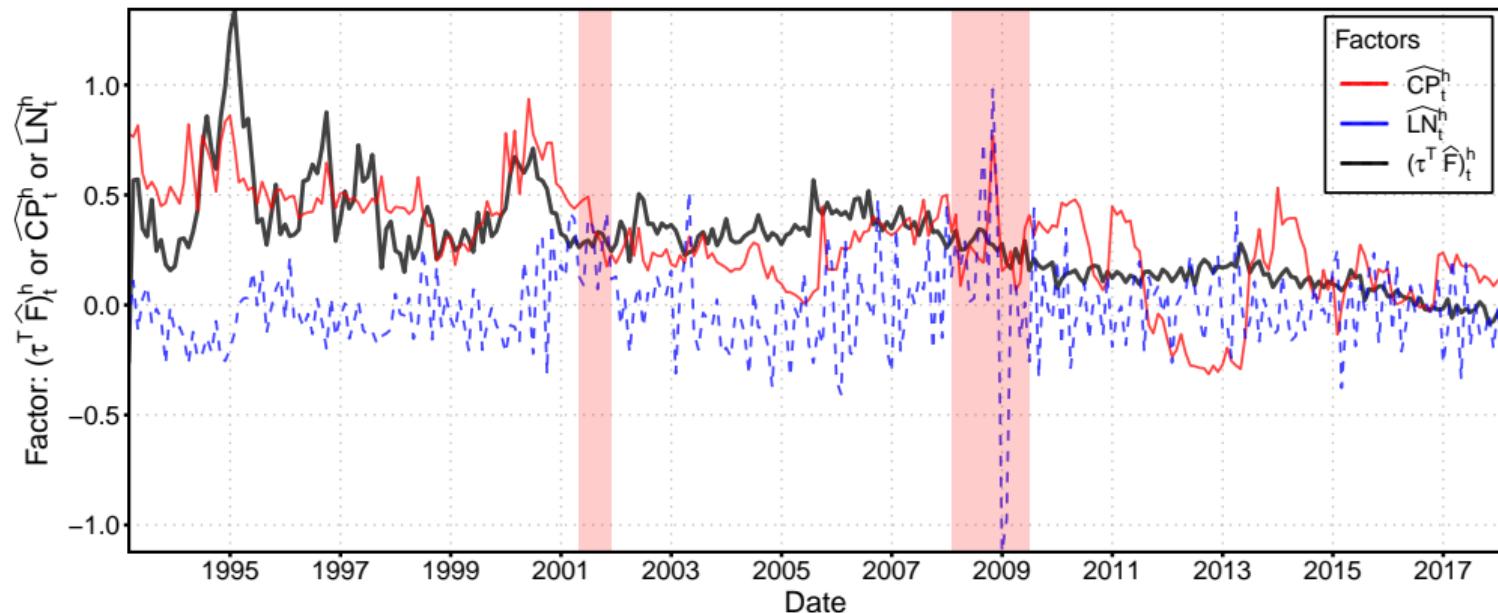
Derived Factors  $f_{t,DNN}^{(n),h}$  for **DNN 2** Generated Using the Set of Yields



# Empirical Results

## Comparison with Other Factors from the Literature

Figure 1: Time Series of our Derived Factor  $(\tau^\top \widehat{F}_t)^h$ , along with  $\widehat{CP}_t^h$  and  $\widehat{LN}_t^h$

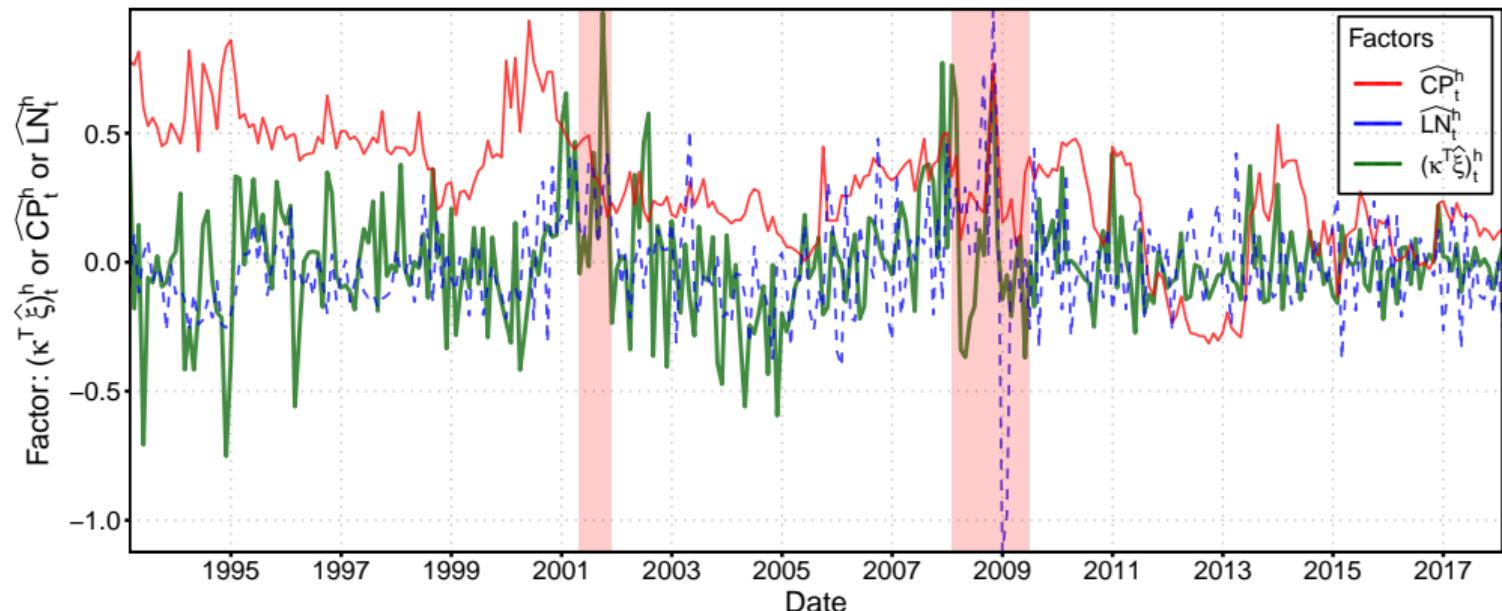


# Empirical Results

## Comparison with Other Factors from the Literature

Correlation

Figure 2: Time Series of our Derived Factor  $(\kappa^\top \hat{\xi})_t^h$ , along with  $\widehat{CP}_t^h$  and  $\widehat{LN}_t^h$



# Empirical Results - Predictive Regressions Using $(\tau^\top \widehat{\mathfrak{F}}_t)_t^h$ and $(\kappa^\top \widehat{\xi})_t^{(-n),h}$ as State Variables

Details

	$rx_{t+h/12}^{(2)}$	$rx_{t+h/12}^{(3)}$	$rx_{t+h/12}^{(4)}$	$rx_{t+h/12}^{(5)}$				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$(\tau^\top \widehat{\mathfrak{F}})_t^h$	0.811*** (0.131)	0.811*** (0.119)	0.943*** (0.199)	0.943*** (0.188)	1.065*** (0.264)	1.065*** (0.253)	1.181*** (0.325)	1.181*** (0.312)
$M_{\tau^\top \widehat{\mathfrak{F}}} (\kappa^\top \widehat{\xi})_t^{(-n),h}$		0.779*** (0.180)		0.789*** (0.219)		0.807*** (0.288)		0.848*** (0.318)
Constant	-0.010 (0.039)	-0.010 (0.035)	-0.003 (0.063)	-0.003 (0.060)	0.004 (0.088)	0.004 (0.086)	0.010 (0.114)	0.010 (0.111)
Observations	300	300	300	300	300	300	300	300
Adjusted R <sup>2</sup>	0.119	0.178	0.063	0.100	0.042	0.069	0.032	0.060

Note:

\* p<0.1; \*\* p<0.05; \*\*\* p<0.01

# Empirical Results - Predictive Regressions with $(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}_t)_t^h$ and $(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}})_t^{(-n),h}$ , along with the Cochrane-Piazzesi and Ludvingson-Ng factors, and Fama-Bliss Regressions with Forward Spreads

Details

**Panel A:**

$rX_{t+h/12}^{(2)}$

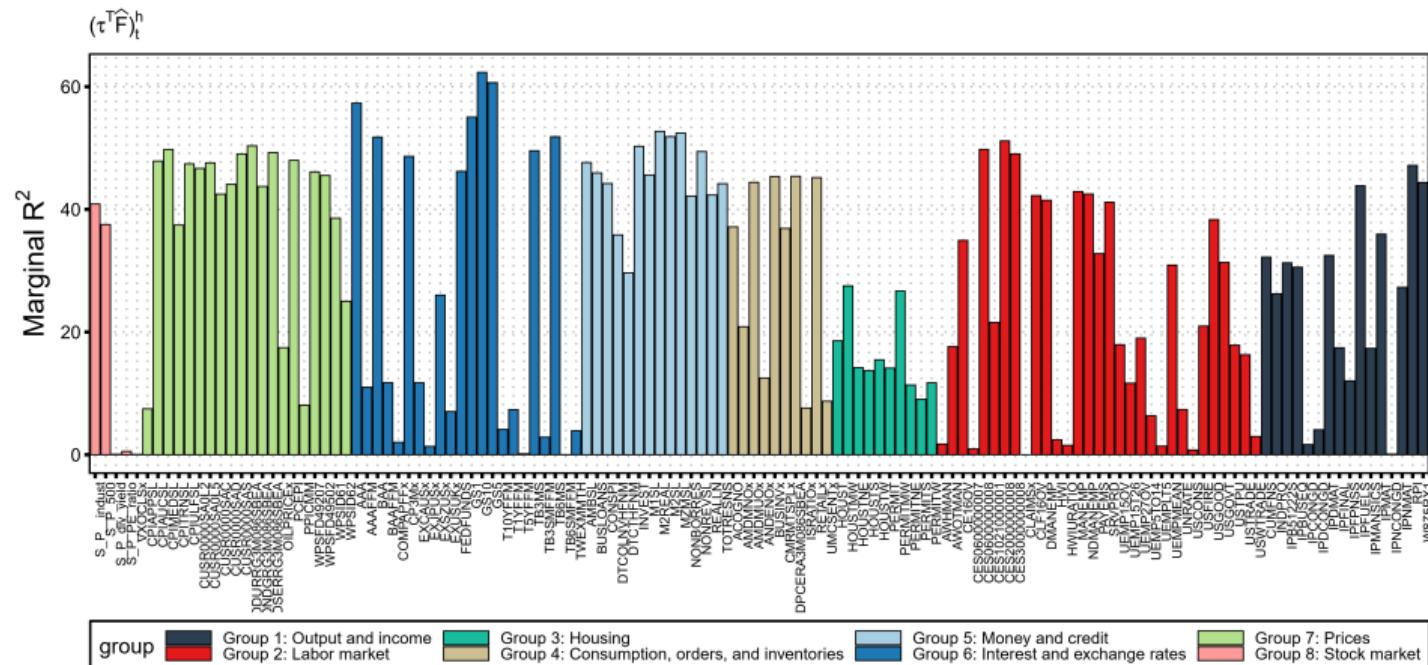
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}})_t^h$	0.847*** (0.124)	0.842*** (0.115)	0.853*** (0.128)	0.824*** (0.117)	0.525*** (0.154)	0.582*** (0.140)	0.582*** (0.145)	0.614*** (0.135)
$M_{\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}} (\boldsymbol{\kappa}^\top \boldsymbol{\xi})_t^{(-2),h}$		0.658*** (0.172)		0.745*** (0.182)		0.704*** (0.182)		0.558*** (0.185)
$\bar{LN}_t^h$	0.617*** (0.127)	0.529*** (0.120)					0.559*** (0.110)	0.518*** (0.110)
$f_{st}^{(n,h)}$			-0.746 (0.476)	-0.225 (0.438)			-0.570 (0.437)	-0.172 (0.429)
$\bar{CP}_t^h$					0.454*** (0.126)	0.364*** (0.112)	0.465*** (0.112)	0.375*** (0.109)
Constant	-0.013 (0.037)	-0.012 (0.034)	0.031 (0.051)	0.002 (0.047)	-0.060 (0.039)	-0.050 (0.036)	-0.031 (0.045)	-0.044 (0.043)
Observations	300	300	300	300	300	300	300	300
Adjusted R <sup>2</sup>	0.183	0.223	0.128	0.177	0.150	0.197	0.215	0.240

Note:

\* p<0.1; \*\* p<0.05; \*\*\* p<0.01

## Empirical Results - Economic Interpretation

Marginal  $R^2$  of the factor  $(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}_t)^h_t$

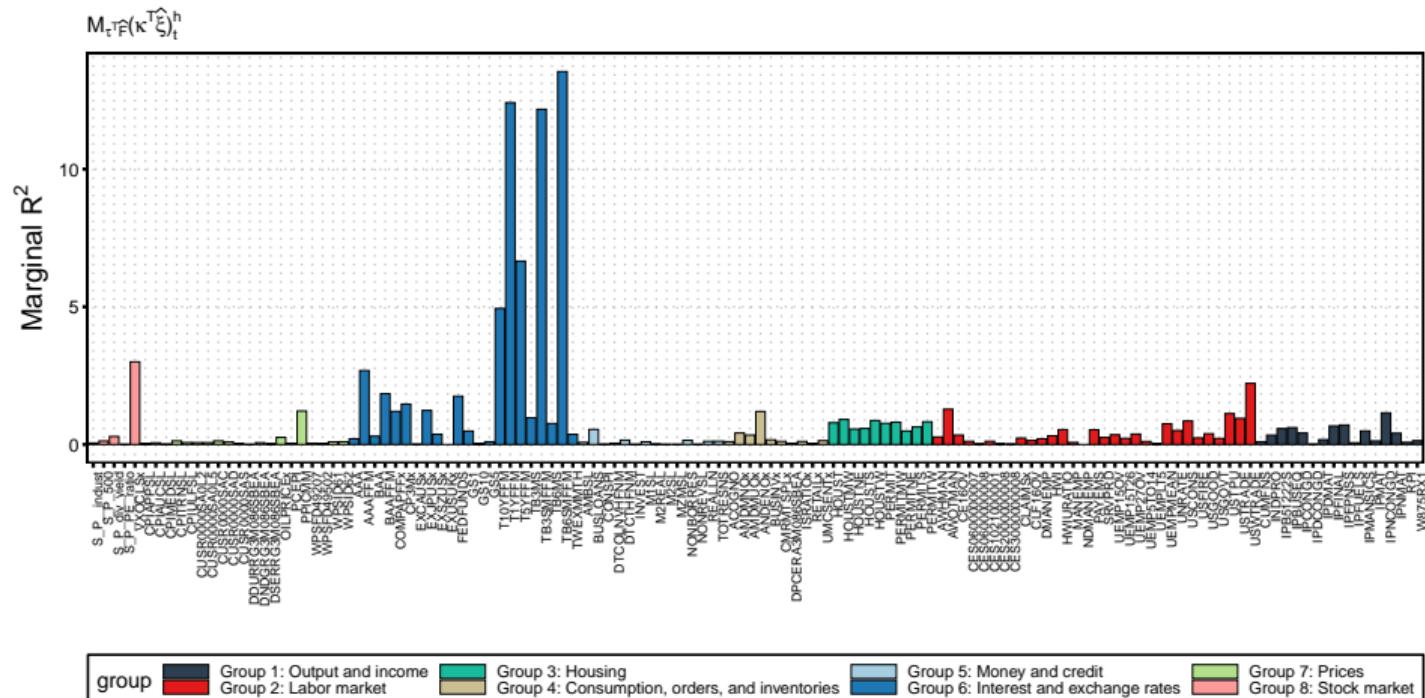


# Empirical Results - Economic Interpretation

Sentiment-based Results

Marginal  $R^2$  of the factor  $\mathbf{M}_{\tau} \hat{\mathbf{F}}(\kappa^\top \hat{\boldsymbol{\xi}})_{t+h/12}^h$

Additional Results



Overview

Introduction

Framework

Data & Empirical Strategy

Empirical Results

References

Appendix

# Empirical Results

## Out-of-Sample Forecasting Performance

- Set the out-of-sample period to range from 1997 : 01 to 2017 : 12, where the data from 1993 : 01 to 1996 : 12 is used to initiate the analysis.
- At each  $\tau \in \tau_{OoS}$ , we use all the previous information up to  $\tau - 1$  to obtain the point forecast of  $rx^{(n)}$  for the month  $\tau$ .

Out-of-Sample  $R^2$

(Campbell and Thompson, 2007; Gargano et al., 2019)

The out-of-sample  $R^2$  is computed as

$$R_{OoS,i}^{2(n)} = 1 - \frac{\sum_{\tau \in \tau_{OoS}} \left( rx_{t+h/12|t}^{(n)} - \hat{rx}_{t+h/12|t}^{(n)} \right)^2}{\sum_{\tau \in \tau_{OoS}} \left( rx_{t+h/12|t}^{(n)} - \bar{rx}_{t+h/12|t}^{(n)} \right)^2} \quad (3)$$

# Empirical Results

## Out-of-Sample Forecasting Performance ( $R^2$ )

Additional Results

Regression	Maturity $n = 2$	Maturity $n = 3$	Maturity $n = 4$	Maturity $n = 5$
$rx_{t+h/12}^{(n)} = \beta_0 + \beta_1(\tau^\top \hat{\delta}_t)^h + \epsilon_{t+h/12}$	0.17	0.03	-0.02	-0.04
$rx_{t+h/12}^{(n)} = \beta_0 + \beta_1 M_{\tau^\top \hat{\delta}} (\kappa^\top \hat{\xi})_t^h + \epsilon_{t+h/12}$	0.22	0.05	-0.01	-0.03
$rx_{t+h/12}^{(n)} = \beta_0 + \beta_1 \widehat{LN}_t^h + \epsilon_{t+h/12}$	0.12	-0.02	-0.06	-0.07
$rx_{t+h/12}^{(n)} = \beta_0 + \beta_1 fs_t^{(n,h)} + \epsilon_{t+h/12}$	0.18	0.05	0.00	-0.01
$rx_{t+h/12}^{(n)} = \beta_0 + \beta_1 \widehat{CP}_t^h + \epsilon_{t+h/12}$	0.15	-0.02	-0.08	-0.10

# Empirical Results

## Out-of-Sample Forecasting Performance ( $R^2$ )

[Additional Results](#)

Regression	Maturity $n = 2$	Maturity $n = 3$	Maturity $n = 4$	Maturity $n = 5$
$r\alpha_{t+h/12}^{(n)} = \beta_0 + \beta_1(\tau^\top \hat{\mathbf{f}})_t^h + \beta_2 \hat{LN}_t^h + \epsilon_{t+h/12}$	0.21	0.04	-0.03	-0.05
$r\alpha_{t+h/12}^{(n)} = \beta_0 + \beta_1(\tau^\top \hat{\mathbf{f}})_t^h + \beta_2 \mathbf{M}_{\tau^\top \hat{\mathbf{f}}}(\kappa^\top \hat{\mathbf{f}})_t^{(-n),h} + \beta_3 \hat{LN}_t^h + \epsilon_{t+h/12}$	0.23	0.04	-0.02	-0.05
$r\alpha_{t+h/12}^{(n)} = \beta_0 + \beta_1(\tau^\top \hat{\mathbf{f}})_t^h + \beta_2 f\mathbf{s}_t^{(n,h)} + \epsilon_{t+h/12}$	0.26	0.08	0.02	-0.00
$r\alpha_{t+h/12}^{(n)} = \beta_0 + \beta_1(\tau^\top \hat{\mathbf{f}})_t^h + \beta_2 \mathbf{M}_{\tau^\top \hat{\mathbf{f}}}(\kappa^\top \hat{\mathbf{f}})_t^{(-n),h} + \beta_3 f\mathbf{s}_t^{(n,h)} + \epsilon_{t+h/12}$	0.27	0.08	0.02	-0.00
$r\alpha_{t+h/12}^{(n)} = \beta_0 + \beta_1(\tau^\top \hat{\mathbf{f}})_t^h + \beta_2 \hat{CP}_t^h + \epsilon_{t+h/12}$	0.20	0.01	-0.06	-0.09
$r\alpha_{t+h/12}^{(n)} = \beta_0 + \beta_1(\tau^\top \hat{\mathbf{f}})_t^h + \beta_2 \mathbf{M}_{\tau^\top \hat{\mathbf{f}}}(\kappa^\top \hat{\mathbf{f}})_t^{(-n),h} + \beta_3 \hat{CP}_t^h + \epsilon_{t+h/12}$	0.22	0.01	-0.06	-0.08
$r\alpha_{t+h/12}^{(n)} = \beta_0 + \beta_1(\tau^\top \hat{\mathbf{f}})_t^h + \beta_2 \hat{LN}_t^h + \beta_3 f\mathbf{s}_t^{(n,h)} + \beta_4 \hat{CP}_t^h + \epsilon_{t+h/12}$	0.19	-0.03	-0.10	-0.13
$r\alpha_{t+h/12}^{(n)} = \beta_0 + \beta_1(\tau^\top \hat{\mathbf{f}})_t^h + \beta_2 \mathbf{M}_{\tau^\top \hat{\mathbf{f}}}(\kappa^\top \hat{\mathbf{f}})_t^{(-n),h} + \beta_3 \hat{LN}_t^h + \beta_4 f\mathbf{s}_t^{(n,h)} + \beta_5 \hat{CP}_t^h + \epsilon_{t+h/12}$	0.19	-0.04	-0.11	-0.13

## Conclusion

- I proposed a novel approach for deriving a **single state factor** consistent with a dynamic term-structure with unspanned risks.
- Making use of **deep neural networks** to uncover relationships in the term-structure, I build a **single state factor** that provides a good approximation to the space that spans all the information from the term-structure.
- I also introduced a way to obtain **unspanned risks from the yield curve** that is used to complete the state space.
- I show that this parsimonious number of state variables have predictive power for excess returns of bonds over 1-month holding period.
- Additionally, I provide an **intuitive interpretation of derived factors**, and show what information from macroeconomic variables and sentiment-based measures they can capture.

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# Notation

- **holding period returns**

$$\begin{aligned} r_{t+\Delta}^{(n)} &\equiv p_{t+\Delta}^{(n-\Delta)} - p_t^{(n)} \\ r_{t+h/12}^{(n)} &\equiv p_{t+h/12}^{(n-h/12)} - p_t^{(n)} = ny_t^{(n)} - (n - h/12)y_{t+h/12}^{(n-h/12)} \end{aligned} \quad (4)$$

- **Excess Returns**

$$\begin{aligned} rx_{t+h/12}^{(n)} &\equiv \text{holding period return } r_{t+h/12}^{(n)} - \text{1-period yield} \\ &= ny_t^{(n)} - (n - h/12)y_{t+h/12}^{(n-h/12)} - (h/12)y_t^{(h/12)} \end{aligned} \quad (5)$$

- **Forward rates** at time  $t$  for loans between time  $t + n - h/12$  and  $t + n$  as

$$\begin{aligned} f_t^{(n)} &\equiv p_t^{(n-h/12)} - p_t^{(n)} \\ &= ny_t^{(n)} - (n - h/12)y_t^{(n-h/12)} \end{aligned} \quad (6)$$

# Expectation Hypothesis

**Risk Premium:** difference between a long rate and the expected average of future short rates.

$$y_t^{(n)} \equiv \underbrace{\frac{1}{n} \mathbb{E}_t \left( y_t^{(1/12)} + y_{t+1/12}^{(1/12)} + \dots + y_{t+n-1/12}^{(1/12)} \right)}_{\text{expectations component}} + \underbrace{\frac{1}{n} \mathbb{E}_t \left( rx_{t+1/12}^{(n)} + rx_{t+2/12}^{(n-1/12)} + rx_{t+3/12}^{(n-2/12)} + \dots + rx_{t+n-1/12}^{(2/12)} \right)}_{\text{yield risk premium}}$$
(7)

# Expectation Hypothesis

Assuming that the agents' information set at time  $t$  can be summarized by a state vector  $\mathbf{Z}_t$

$$y_t^{(n)} = \frac{1}{n} \left( \sum_{j=0}^{12 \cdot n/h - 1} \mathbb{E} \left[ y_{t+j \cdot h/12}^{(h/12)} | \mathbf{Z}_t \right] \right) + \frac{1}{n} \left( \sum_{j=0}^{12 \cdot n/h - 1} \left[ r_{t+h/12(j+1)}^{(n-j \cdot h/12)} | \mathbf{Z}_t \right] \right). \quad (8)$$

$\mathbf{Z}_t$  should contain all the information used by investors to forecast at time  $t$  the excess-returns for all future periods.

# Fama and Bliss (1987)

- Fama and Bliss (1987) builds forward rates spreads and use these variables as covariates.
- Forward rate spread between of a  $n$ -year maturity bond:  $fs_t^{(n,h)} \equiv f_t^{(n)} - y_t^{(h/12)}(h/12)$ .

## Predictive Regression

$$rx_{t+h/12}^{(n)} = \beta_0 + \beta_1 fs_t^{(n,h)} + \epsilon_{t+h/12} \quad . \quad (9)$$

# Cochrane and Piazzesi (2005)

- Cochrane and Piazzesi (2005) derive a single factor to use as predictor ( $CP_t^h$ ).
- First, they estimate ( $CP_t^h$ ) as

$$\begin{aligned} \frac{1}{4} \sum_{n=2}^5 rx_{t+h/12}^{(n)} &= \gamma_0 + \gamma_1 f_t^{(1)} + \gamma_2 f_t^{(2)} + \gamma_3 f_t^{(3)} + \gamma_4 f_t^{(4)} + \gamma_5 f_t^{(5)} + \bar{\epsilon}_{t+h/12} \\ \overline{rx}_{t+h/12} &= \underbrace{\gamma^\top \mathbf{f}_t}_{CP_t^h} + \bar{\epsilon}_{t+h/12} \end{aligned} \quad (10)$$

where  $\mathbf{f}$  and  $\gamma$  are  $6 \times 1$  vectors given by  $\mathbf{f} \equiv [1 \quad f_t^{(1)} \quad f_t^{(2)} \quad f_t^{(3)} \quad f_t^{(4)} \quad f_t^{(5)}]^\top$ , and  $\gamma \equiv [\gamma_0 \quad \gamma_1 \quad \gamma_2 \quad \gamma_3 \quad \gamma_4 \quad \gamma_5]^\top$ .

## Predictive Regression

$$rx_{t+h/12}^{(n)} = \beta_0 + \beta_1 \widehat{CP}_t^h + \epsilon_{t+h/12} \quad . \quad (11)$$

# Ludvigson and Ng (2009)

- Ludvigson and Ng (2009) use a large panel of macro variables, and build a single linear combination ( $LN_t^h$ ) out of the first  $i$  estimated principal components ( $\hat{g}_{i,t}$ ).
- First, they estimate ( $LN_t^h$ ) as

$$\begin{aligned} \frac{1}{4} \sum_{n=2}^5 rx_{t+h/12}^{(n)} &= \lambda_0 + \lambda_1 \hat{g}_{1,t} + \lambda_2 \hat{g}_{1,t}^3 + \lambda_3 \hat{g}_{3,t} + \lambda_4 \hat{g}_{4,t} + \lambda_5 \hat{g}_{8,t} + \bar{\epsilon}_{t+h/12} \\ \bar{rx}_{t+h/12} &= \underbrace{\lambda^\top \hat{\mathbf{G}}_t}_{LN_t^h} + \bar{\epsilon}_{t+h/12} \end{aligned} \quad (12)$$

where  $\hat{\mathbf{G}}_t$  and  $\lambda$  are  $5 \times 1$  vectors given by  $\hat{\mathbf{G}}_t \equiv [\hat{g}_{1,t} \quad \hat{g}_{1,t}^3 \quad \hat{g}_{3,t} \quad \hat{g}_{5,t} \quad \hat{g}_{8,t}]^\top$ , and  $\lambda \equiv [\lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4 \quad \lambda_5]^\top$ .

## Predictive Regression

$$rx_{t+h/12}^{(n)} = \beta_0 + \beta_1 \widehat{LN}_t^h + \epsilon_{t+h/12} \quad . \quad (13)$$

## Algorithm 1: Recursively generated factors with updated parameters

[Return](#)

**Initialization:** Start with a set of information from the term structure collected in  $Z^y$ . Partitionate your sample  $\{t_0, \dots, t_{split}, \tau, \tau + 1, \dots, T\}$  between the data to be used to initialize the process  $\{t_0, \dots, t_{split}\}$ , and to obtain the recursively generated factors  $\{\tau, \tau + 1, \dots, T\}$ ;

**for**  $n \in \{2, 3, 4, 5\}$  **do**

**for**  $t \in \{\tau, \tau + 1, \dots, T\}$  **do**

        Feed  $DNN_i$  with lagged data  $Z_{t-1}^y = \{z_{t_0}^y, z_{t_0+1}^y, \dots, z_{t-1}^y\}$  to learn/approximate with output  $rx_t^{(n)}$ ,  
        and use the last 10% of the data for validation;

        Obtain the learned parameters;

$$\hat{f}_{t,DNN}^{(n),h} \leftarrow g(Z_{t-1}^y, \theta_{t-1})$$

        Obtain the  $t$ -th element that lies in the orthogonal vector from the space generated by the  
 $f_{t-1,DNN}^{(n),h}$  through:

$$\hat{\xi}_t^{(n),h} \leftarrow rx_t^{(n)} - \hat{\beta}_0 - \hat{\beta}_1 \hat{f}_{t-1,DNN}^{(n),h}$$

**end**

**end**

---

**Algorithm 2:** Recursively generated factors with updated parameters
 

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**Result:**

$$\hat{\mathfrak{F}}_{t,DNN_i} \equiv \begin{bmatrix} \hat{\mathfrak{f}}_{t,DNN_i}^{(2),h} \\ \hat{\mathfrak{f}}_{t,DNN_i}^{(3),h} \\ \hat{\mathfrak{f}}_{t,DNN_i}^{(4),h} \\ \hat{\mathfrak{f}}_{t,DNN_i}^{(5),h} \\ \hat{\mathfrak{f}}_{t,DNN_i} \end{bmatrix} = \begin{bmatrix} \hat{\mathfrak{f}}_{\tau,DNN_i}^{(2),h} & \hat{\mathfrak{f}}_{\tau,DNN_i}^{(3),h} & \hat{\mathfrak{f}}_{\tau,DNN_i}^{(4),h} & \hat{\mathfrak{f}}_{\tau,DNN_i}^{(5),h} \\ \hat{\mathfrak{f}}_{\tau+1,DNN_i}^{(2),h} & \hat{\mathfrak{f}}_{\tau+1,DNN_i}^{(3),h} & \hat{\mathfrak{f}}_{\tau+1,DNN_i}^{(4),h} & \hat{\mathfrak{f}}_{\tau+1,DNN_i}^{(5),h} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{\mathfrak{f}}_{T,DNN_i}^{(2),h} & \hat{\mathfrak{f}}_{T,DNN_i}^{(3),h} & \hat{\mathfrak{f}}_{T,DNN_i}^{(4),h} & \hat{\mathfrak{f}}_{T,DNN_i}^{(5),h} \end{bmatrix}$$

And,

$$\hat{\xi}_t^h \equiv \begin{bmatrix} \hat{\xi}_{\tau,DNN_i}^{(2),h} & \hat{\xi}_{\tau,DNN_i}^{(3),h} & \hat{\xi}_{\tau,DNN_i}^{(4),h} & \hat{\xi}_{\tau,DNN_i}^{(5),h} \\ \hat{\xi}_{\tau+1,DNN_i}^{(2),h} & \hat{\xi}_{\tau+1,DNN_i}^{(3),h} & \hat{\xi}_{\tau+1,DNN_i}^{(4),h} & \hat{\xi}_{\tau+1,DNN_i}^{(5),h} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{\xi}_{T,DNN_i}^{(2),h} & \hat{\xi}_{T,DNN_i}^{(3),h} & \hat{\xi}_{T,DNN_i}^{(4),h} & \hat{\xi}_{T,DNN_i}^{(5),h} \end{bmatrix}$$


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# An Illustrative Term-Structure Model

The no-arbitrage assumption rely on the fundamental asset pricing equation:

$$P_t^{(n)} = \mathbb{E}_t \left( \mathcal{M}_{t+1} P_{t+1}^{(n-1)} \right) \quad (14)$$

where

- $P_t^{(n)}$  is the price of a bond,
- $\mathcal{M}_{t+h/12}$  is the stochastic discount factor (SDF).

**SDF:**

$$\mathcal{M}_{t+h/12} = \exp^{-r_t \frac{1}{2} \Lambda_t^\top \Lambda_t - \Lambda_t^\top \epsilon_{t+h/12}} \quad (15)$$

where  $\Lambda_t$  is the market prices of the risks, i.e., the amount of compensation required by investors to face the unit normal shock  $\epsilon_{t+h/12}$ .

$$r_t = \rho_0 + \rho_1 Z_t \quad . \quad (16)$$

# An Illustrative Term-Structure Model

- Define  $\mathbf{Z}_t = \{\mathbf{Z}_t^y, \mathbf{Z}_t^{y^c}\}$
- Dynamics of  $\mathbf{Z}_t$  that capture all the risks of the economy following a Gaussian VAR process given by:

$$\begin{bmatrix} \mathbf{Z}_t^y \\ \mathbf{Z}_t^{y^c} \end{bmatrix} = \boldsymbol{\mu} + \boldsymbol{\Phi} \begin{bmatrix} \mathbf{Z}_{t-1}^y \\ \mathbf{Z}_{t-1}^{y^c} \end{bmatrix} + \boldsymbol{\Sigma} \boldsymbol{\epsilon}_t \quad (17)$$

$$\mathbf{Z}_t = \boldsymbol{\mu} + \boldsymbol{\Phi} \mathbf{Z}_{t-1} + \boldsymbol{\Sigma} \boldsymbol{\epsilon}_t \quad \boldsymbol{\epsilon}_t \sim N(0, \mathbf{I})$$

where  $\boldsymbol{\mu}$  is a  $k \times 1$  vector, and  $\boldsymbol{\Phi}$  and  $\boldsymbol{\Sigma}$  are  $k \times k$  matrices, being  $k$  the number of state variables.

# An Illustrative Term-Structure Model

Details

- In a similar fashion to Joslin et al. (2014), we can write:

$$\mathbf{Z}_t^{y^C} = \gamma_0 + \gamma_1 \mathbf{Z}_t^y + \mathbf{M}_{\mathbf{Z}_t^y} \mathbf{Z}_t^{y^C} \quad (18)$$

where  $\mathbf{M}_{\mathbf{Z}_t^y} \mathbf{Z}_t^{y^C}$  is the annihilator matrix of the space spanned by  $\mathbf{Z}_t^y$ , i.e.,

$$\mathbf{M}_{\mathbf{Z}_t^y} \mathbf{Z}_t^{y^C} \equiv \mathbf{Z}_t^{y^C} - \text{Proj} \left[ \mathbf{Z}_t^{y^C} | \mathbf{Z}_t^y \right] \quad (19)$$

In our methodology,

- $\mathbf{Z}_t^y$  is given by the derived factor  $\left( \boldsymbol{\tau}^\top \widehat{\mathfrak{F}}_t \right)_t^h$
- $\mathbf{Z}_t^{y^C}$  by a function of  $\xi_{t+h/12}^h$  as  $f(\xi_{t+h/12}^h)$

# Empirical Results

## Correlation Matrix

[Return](#)

	$(\tau^\top \widehat{\mathfrak{F}})_t^h$	$M_{\tau^\top \widehat{\mathfrak{F}}}(\kappa^\top \widehat{\xi})_t^h$	$M_{\tau^\top \widehat{\mathfrak{F}}}(\kappa^\top \widehat{\xi})_t^{(-2),h}$	$M_{\tau^\top \widehat{\mathfrak{F}}}(\kappa^\top \widehat{\xi})_t^{(-3),h}$	$M_{\tau^\top \widehat{\mathfrak{F}}}(\kappa^\top \widehat{\xi})_t^{(-4),h}$	$M_{\tau^\top \widehat{\mathfrak{F}}}(\kappa^\top \widehat{\xi})_t^{(-5),h}$	$\widehat{CP}_t^h$	$\widehat{LN}_t^h$
$(\tau^\top \widehat{\mathfrak{F}})_t^h$	1	0	0	0	0	0	0.556	-0.059
$M_{\tau^\top \widehat{\mathfrak{F}}}(\kappa^\top \widehat{\xi})_t^h$	0	1	0.995	0.912	0.904	0.919	0.129	0.171
$M_{\tau^\top \widehat{\mathfrak{F}}}(\kappa^\top \widehat{\xi})_t^{(-2),h}$	0	0.995	1	0.938	0.900	0.888	0.135	0.174
$M_{\tau^\top \widehat{\mathfrak{F}}}(\kappa^\top \widehat{\xi})_t^{(-3),h}$	0	0.912	0.938	1	0.947	0.849	0.170	0.203
$M_{\tau^\top \widehat{\mathfrak{F}}}(\kappa^\top \widehat{\xi})_t^{(-4),h}$	0	0.904	0.900	0.947	1	0.959	0.173	0.204
$M_{\tau^\top \widehat{\mathfrak{F}}}(\kappa^\top \widehat{\xi})_t^{(-5),h}$	0	0.919	0.888	0.849	0.959	1	0.146	0.178
$\widehat{CP}_t^h$	0.556	0.129	0.135	0.170	0.173	0.146	1	-0.007
$\widehat{LN}_t^h$	-0.059	0.171	0.174	0.203	0.204	0.178	-0.007	1

# Empirical Results - Predictive Regressions Using $(\boldsymbol{\tau}^\top \widehat{\boldsymbol{\delta}}_t)^h$ , $(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}})^h$ and $(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}})^{(-n),h}$ as State Variables

[Return](#)

Panel A:

$rX_{t+h/12}^{(2)}$

	DNN 1		DNN 2		DNN 3				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$(\boldsymbol{\tau}^\top \widehat{\boldsymbol{\delta}})^h_t$	0.810*** (0.160)	0.810*** (0.149)	0.810*** (0.147)	0.811*** (0.131)	0.811*** (0.119)	0.811*** (0.119)	1.419*** (0.414)	1.419*** (0.377)	1.419*** (0.356)
$M_{\boldsymbol{\tau}^\top \widehat{\boldsymbol{\delta}}} (\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}})^{(-2),h}_t$		0.760*** (0.204)			0.779*** (0.180)			0.875*** (0.211)	
$M_{\boldsymbol{\tau}^\top \widehat{\boldsymbol{\delta}}} (\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}})^h_t$			0.591*** (0.139)			0.525*** (0.126)			0.679*** (0.138)
Constant	-0.010 (0.054)	-0.010 (0.050)	-0.010 (0.049)	-0.010 (0.039)	-0.010 (0.035)	-0.010 (0.035)	-0.189* (0.110)	-0.189* (0.101)	-0.189** (0.094)
Observations	300	300	300	300	300	300	300	300	300
Adjusted R <sup>2</sup>	0.100	0.148	0.159	0.119	0.178	0.175	0.046	0.105	0.124

Note:

\* p<0.1; \*\* p<0.05; \*\*\* p<0.01

# Empirical Results - Predictive Regressions Using $(\tau^\top \widehat{\mathfrak{F}}_t)^h$ , $(\kappa^\top \widehat{\xi})^h_t$ and $(\kappa^\top \widehat{\xi})^{(-n),h}_t$ as State Variables

[Return](#)

Panel B:

	$rX_{t+h/12}^{(3)}$									
$(\tau^\top \widehat{\mathfrak{F}}_t)^h$	0.959*** (0.248)	0.959*** (0.234)	0.959*** (0.233)	0.943*** (0.199)	0.943*** (0.188)	0.943*** (0.184)	1.175* (0.630)	1.175** (0.566)	1.175** (0.559)	
$M_{\tau^\top \widehat{\mathfrak{F}}}(\kappa^\top \widehat{\xi})_{t+h/12}^{(-3),h}$		0.799*** (0.234)			0.789*** (0.219)			0.984*** (0.236)		
$M_{\tau^\top \widehat{\mathfrak{F}}}(\kappa^\top \widehat{\xi})_{t+h/12}^h$			0.765*** (0.225)			0.757*** (0.205)			0.929*** (0.224)	
Constant	-0.008 (0.087)	-0.008 (0.082)	-0.008 (0.082)	-0.003 (0.063)	-0.003 (0.060)	-0.003 (0.059)	-0.072 (0.169)	-0.072 (0.153)	-0.072 (0.150)	
Observations	300	300	300	300	300	300	300	300	300	
Adjusted R <sup>2</sup>	0.055	0.092	0.093	0.063	0.100	0.109	0.010	0.067	0.067	

Note:

\*  $p < 0.1$ ; \*\*  $p < 0.05$ ; \*\*\*  $p < 0.01$

# Empirical Results - Predictive Regressions Using $(\tau^\top \widehat{\mathfrak{F}}_t)^h$ , $(\kappa^\top \widehat{\xi})^h_t$ and $(\kappa^\top \widehat{\xi})^{(-n),h}_t$ as State Variables

[Return](#)

Panel C:

	$rX_{t+h/12}^{(4)}$								
$(\tau^\top \widehat{\mathfrak{F}})_t^h$	1.073*** (0.334)	1.073*** (0.320)	1.073*** (0.317)	1.065*** (0.264)	1.065*** (0.253)	1.065*** (0.248)	0.864 (0.835)	0.864 (0.759)	0.864 (0.755)
$M_{\tau^\top \widehat{\mathfrak{F}}}(\kappa^\top \widehat{\xi})_t^{(-4),h}$		0.795*** (0.291)			0.807*** (0.288)			1.038*** (0.289)	
$M_{\tau^\top \widehat{\mathfrak{F}}}(\kappa^\top \widehat{\xi})_t^h$			0.902*** (0.312)			0.945*** (0.284)			1.144*** (0.313)
Constant	0.002 (0.120)	0.002 (0.116)	0.002 (0.115)	0.004 (0.088)	0.004 (0.086)	0.004 (0.085)	0.063 (0.228)	0.063 (0.209)	0.063 (0.207)
Observations	300	300	300	300	300	300	300	300	300
Adjusted R <sup>2</sup>	0.036	0.060	0.063	0.042	0.069	0.080	0.001	0.046	0.046

Note:

\* p<0.1; \*\* p<0.05; \*\*\* p<0.01

# Empirical Results - Predictive Regressions Using $(\tau^\top \widehat{\mathfrak{F}}_t)^h$ , $(\kappa^\top \widehat{\xi})^h_t$ and $(\kappa^\top \widehat{\xi})^{(-n),h}_t$ as State Variables

[Return](#)

Panel D:

	$rx_{t+h/12}^{(5)}$								
$(\tau^\top \widehat{\mathfrak{F}})_t^h$	1.158*** (0.415)	1.158*** (0.395)	1.158*** (0.398)	1.181*** (0.325)	1.181*** (0.312)	1.181*** (0.309)	0.542 (1.025)	0.542 (0.949)	0.542 (0.939)
$M_{\tau^\top \widehat{\mathfrak{F}}}(\kappa^\top \widehat{\xi})_t^{(-5),h}$		0.854** (0.336)			0.848*** (0.318)			1.069*** (0.339)	
$M_{\tau^\top \widehat{\mathfrak{F}}}(\kappa^\top \widehat{\xi})_t^h$			1.000** (0.398)			1.081*** (0.363)			1.322*** (0.404)
Constant	0.017 (0.152)	0.017 (0.146)	0.017 (0.147)	0.010 (0.114)	0.010 (0.111)	0.010 (0.111)	0.198 (0.284)	0.198 (0.267)	0.198 (0.263)
Observations	300	300	300	300	300	300	300	300	300
Adjusted R <sup>2</sup>	0.025	0.049	0.046	0.032	0.060	0.062	-0.002	0.033	0.036

Note:

\* p<0.1; \*\* p<0.05; \*\*\* p<0.01

# Empirical Results - Predictive Regressions with $(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}_t)_t^h$ and $(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}})_t^{(-n),h}$ , along with the Cochrane-Piazzesi and Ludvingson-Ng factors, and Fama-Bliss Regressions with Forward Spreads

[Return](#)

**Panel B:**

$rX_{t+h/12}^{(3)}$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}})_t^h$	0.996*** (0.190)	0.989*** (0.184)	0.940*** (0.199)	0.947*** (0.188)	0.559** (0.245)	0.648*** (0.234)	0.626*** (0.238)	0.719*** (0.237)
$M_{\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}} (\boldsymbol{\kappa}^\top \boldsymbol{\xi})_t^{(-3),h}$		0.620*** (0.209)		0.852*** (0.228)		0.692*** (0.226)		0.585** (0.237)
$\bar{LN}_t^h$	0.921*** (0.209)	0.800*** (0.201)					0.900*** (0.194)	0.823*** (0.191)
$f_{\mathfrak{s}t}^{(n,h)}$			-0.215 (0.554)	0.410 (0.532)			-0.053 (0.525)	0.394 (0.542)
$\bar{CP}_t^h$					0.608*** (0.205)	0.467** (0.195)	0.583*** (0.188)	0.437** (0.198)
Constant	-0.007 (0.060)	-0.006 (0.059)	0.021 (0.091)	-0.049 (0.087)	-0.070 (0.063)	-0.054 (0.061)	-0.064 (0.082)	-0.098 (0.082)
Observations	300	300	300	300	300	300	300	300
Adjusted R <sup>2</sup>	0.120	0.141	0.060	0.099	0.084	0.111	0.136	0.151

**Note:**

\* p<0.1; \*\* p<0.05; \*\*\* p<0.01

# Empirical Results - Predictive Regressions with $(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}_t)_t^h$ and $(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}})_t^{(-n),h}$ , along with the Cochrane-Piazzesi and Ludvingson-Ng factors, and Fama-Bliss Regressions with Forward Spreads

[Return](#)

Panel C:

$rx_{t+h/12}^{(4)}$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}})_t^h$	1.135*** (0.254)	1.127*** (0.247)	1.082*** (0.270)	1.108*** (0.257)	0.547 (0.335)	0.651** (0.323)	0.685** (0.329)	0.790** (0.329)
$M_{\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}} (\boldsymbol{\kappa}^\top \boldsymbol{\xi})_t^{(-4),h}$		0.609** (0.262)		0.872*** (0.289)		0.688** (0.291)		0.555** (0.274)
$\bar{LN}_t^h$	1.218*** (0.307)	1.079*** (0.287)					1.222*** (0.285)	1.118*** (0.273)
$f_{\mathfrak{s}_t}^{(n,h)}$			0.260 (0.622)	0.665 (0.595)			0.386 (0.593)	0.655 (0.587)
$\bar{CP}_t^h$					0.822*** (0.290)	0.657** (0.276)	0.755*** (0.265)	0.606** (0.272)
Constant	-0.0003 (0.085)	0.0002 (0.084)	-0.038 (0.130)	-0.103 (0.124)	-0.085 (0.089)	-0.068 (0.087)	-0.144 (0.121)	-0.171 (0.118)
Observations	300	300	300	300	300	300	300	300
Adjusted R <sup>2</sup>	0.095	0.108	0.039	0.070	0.063	0.081	0.112	0.122

Note:

\* p<0.1; \*\* p<0.05; \*\*\* p<0.01

# Empirical Results - Predictive Regressions with $(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}_t)_t^h$ and $(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}})_t^{(-n),h}$ , along with the Cochrane-Piazzesi and Ludvingson-Ng factors, and Fama-Bliss Regressions with Forward Spreads

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Panel D:

$rX_{t+h/12}^{(5)}$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}})_t^h$	1.268*** (0.315)	1.258*** (0.305)	1.247*** (0.334)	1.263*** (0.318)	0.511 (0.422)	0.626 (0.401)	0.736* (0.409)	0.834** (0.400)
$M_{\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}} (\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}})_t^{(-5),h}$		0.673** (0.281)		0.872*** (0.312)		0.738** (0.315)		0.590** (0.279)
$\bar{LN}_t^h$	1.501*** (0.421)	1.337*** (0.381)					1.518*** (0.387)	1.386*** (0.360)
$f_{\mathfrak{s}_t}^{(n,h)}$			0.633 (0.698)	0.789 (0.656)			0.739 (0.658)	0.848 (0.632)
$\bar{CP}_t^h$					1.064*** (0.380)	0.882** (0.352)	0.967*** (0.343)	0.818** (0.337)
Constant	0.005 (0.111)	0.005 (0.109)	-0.116 (0.166)	-0.147 (0.158)	-0.106 (0.117)	-0.086 (0.115)	-0.248 (0.158)	-0.253* (0.152)
Observations	300	300	300	300	300	300	300	300
Adjusted R <sup>2</sup>	0.082	0.098	0.031	0.062	0.054	0.074	0.103	0.114

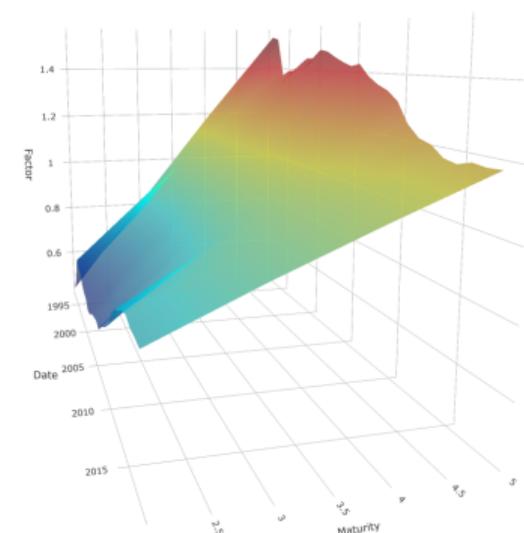
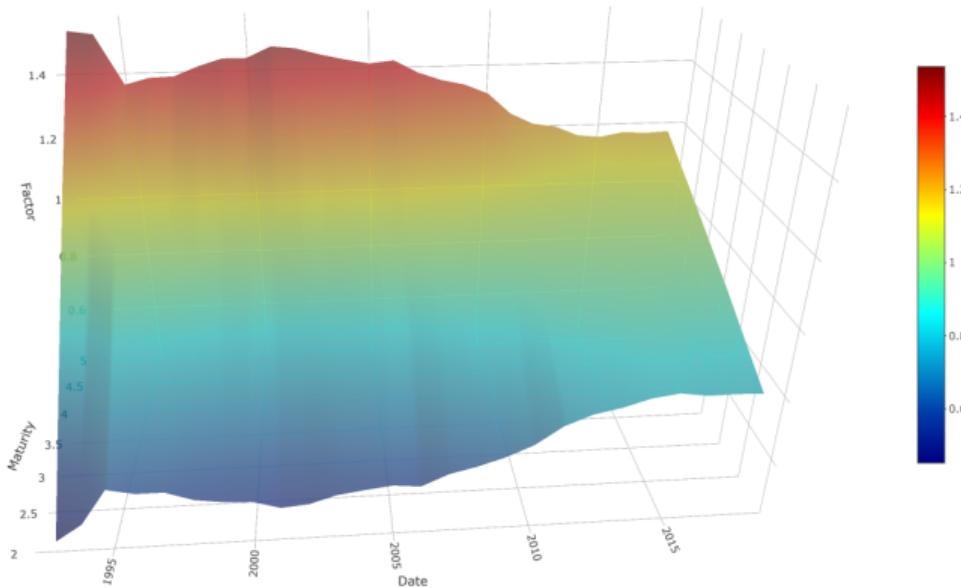
Note:

\* p<0.1; \*\* p<0.05; \*\*\* p<0.01

# Empirical Results

Regression Coefficients of  $\left(\boldsymbol{\tau}^\top \widehat{\boldsymbol{\delta}}_t\right)_t^h$  Over Time as a Function of Maturity ( $n$ )

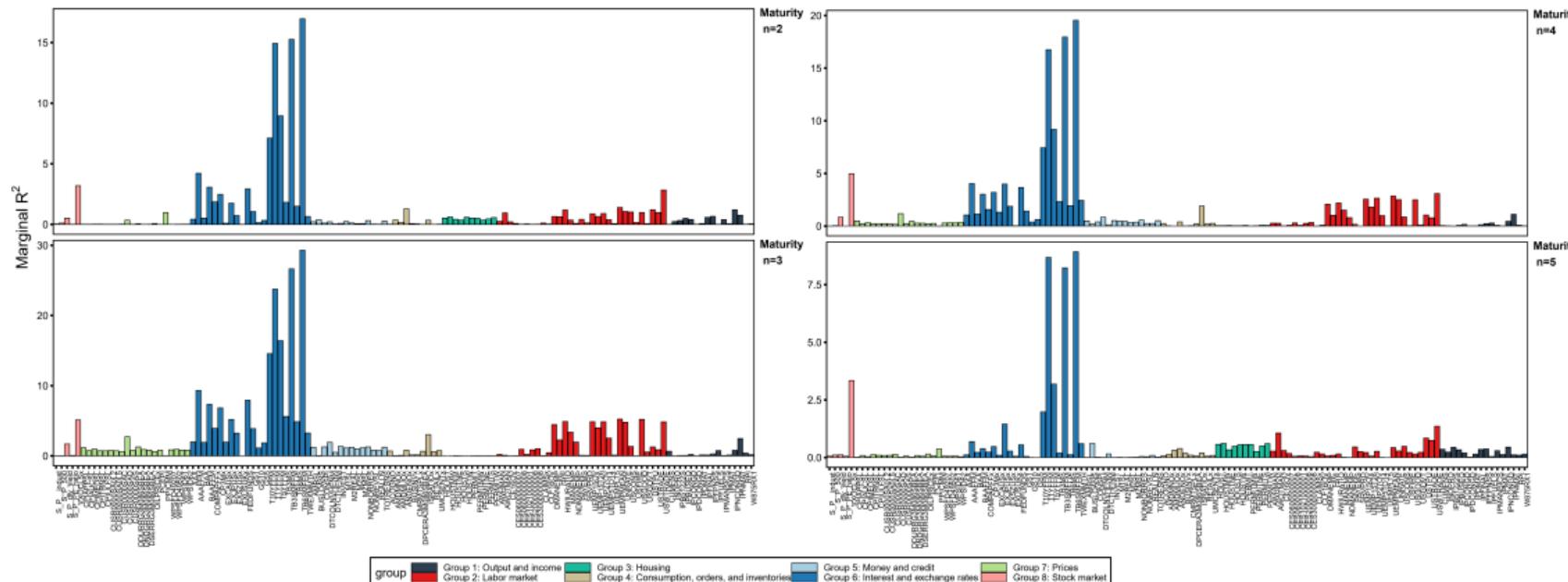
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# Empirical Results - Economic Interpretation

Marginal  $R^2$  of the factors  $\mathbf{M}_{\tau^\top \widehat{\mathfrak{F}}} (\kappa^\top \widehat{\boldsymbol{\xi}})^{(-n), h}$

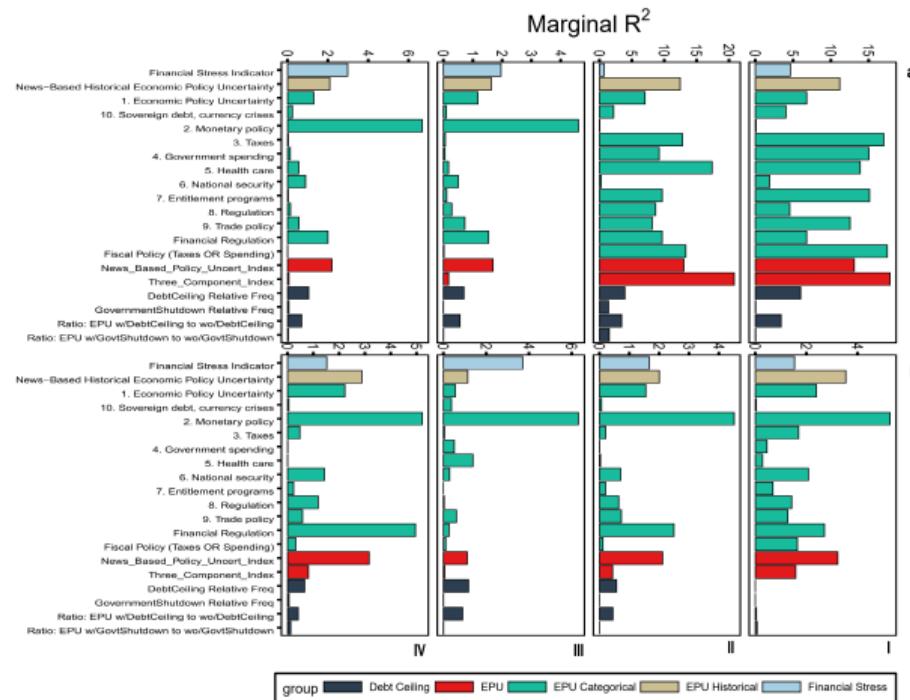
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# Empirical Results - Economic Interpretation

## Marginal $R^2$ Using Sentiment-Based Measures

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# Empirical Results

Regression Coefficients of  $M_{\tau^+} \hat{\delta} (\kappa^\top \hat{\xi})_{t+h/12}^{(-n), h}$  Over Time as a Function of Maturity ( $n$ )

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