ECE 202

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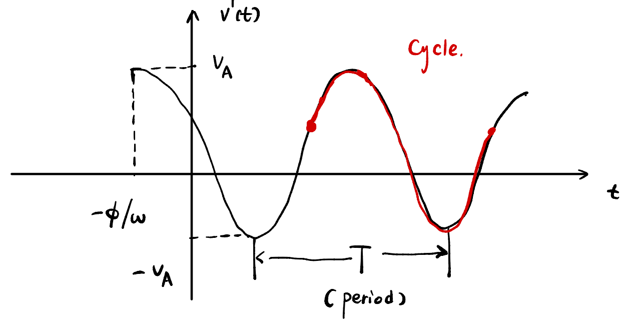
Chapter 8 : Sinusoids Steady-state response.

8.1 sinusoids and phasors

$$v(t) = V_A \cdot \cos(\omega t + \phi) \quad \text{or} \quad i(t) = I_A \cdot \cos(\omega t + \phi)$$



$$f = \frac{1}{T} \quad (Hz)$$

$$\omega = 2\pi f = \frac{2\pi}{T} \quad (rad/s)$$

Conversion between radian & degrees : $x^\circ = \left(\frac{180}{\pi}\right) \times y$ ⟶ in radians

V_A & I_A ⇒ "Amplitude" $2V_A$ & $2I_A$ ⇒ "peak-to-peak amplitude"

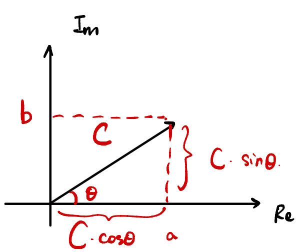
☆ $-\phi$ ($\phi > 0$) the graph shift right ; $+\phi$ ($\phi > 0$) the graph shift left

Complex algebra

① Euler's Formula. $e^{j\theta} = \cos\theta + j\sin\theta$. $j \triangleq \sqrt{-1}$

② Complex number. a. rectangular form. $n = a + jb$

b. polar form. $n = C \cdot e^{j\theta} = C \cdot \angle\theta$



$C = \sqrt{a^2 + b^2}$ = length of phasors

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$ = angle of phasors

$\cos\theta = Re\{e^{j\theta}\}$

$\sin\theta = Im\{e^{j\theta}\}$

phasor transform:

$$v(t) = V_A \cos(\omega t+\phi) = V_A \cdot Re\{e^{j(\omega t+\phi)}\} = V_A \cdot Re\{e^{j\omega t}\cdot e^{j\phi}\}$$

$$= Re\{\underbrace{(V_A\cdot e^{j\phi})}_{\text{phasors}}\, e^{j\omega t}\} \xRightarrow{\text{Phasor: } \vec{V} = V_A\cdot e^{j\phi}} \boxed{v(t) = Re\{\vec{V}\cdot e^{j\omega t}\}}$$

Back transform:

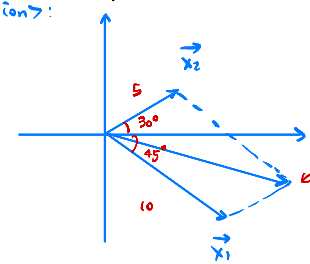
$$v(t) = Re\{\vec{V}\cdot e^{j\omega t}\} = Re\{V_A\cdot e^{j\phi}\cdot e^{j\omega t}\} = Re\{V_A\cdot e^{j(\omega t+\phi)}\} = Re\{V_A(\cos(\omega t+\phi) + j\cdot\sin(\omega t+\phi))\}$$

$$= \cos(\omega t+\phi) \Rightarrow v(t) = \cos(\omega t+\phi)$$

(A). Addition of two complex number

$$\vec{X_1} + \vec{X_2} \qquad \vec{X_1} = 10\angle -45^\circ \qquad \vec{X_2} = 5\angle 30^\circ$$

<Solution>:



$\sim \vec{X_1} + \vec{x_2} = 12.3 \angle -21.8^\circ$

$$\vec{x_1} = 10\cos(-45^\circ) + j\cdot 10\sin(-45^\circ)$$
$$= 7.07 - j\cdot 7.07$$
$$\vec{x_2} = 5\cos(30^\circ) + j\cdot 5\cdot\sin(30^\circ)$$
$$= 4.33 + j\cdot 2.5$$
$$\vec{x_1} + \vec{x_2} = (7.07 - j\,7.07) + (4.33 + j\,2.5)$$
$$= (7.07 + 4.33) + j(-7.07 + 2.5)$$
$$= \underbrace{11.4}_{C\cdot\cos\theta} - j\,\underbrace{4.57}_{C\cdot\sin\theta}$$

$$C = \sqrt{(C\cos\theta)^2 + (C\sin\theta)^2} = \sqrt{(11.4)^2 + (-4.57)^2} = 12.28$$

$$\theta = \tan^{-1}\left(\frac{-4.57}{11.4}\right) = -21.84^\circ$$

(B) Multiplication of two Complex number. (Division just reverse)

$$\vec{X_1}\cdot\vec{X_2} \qquad \vec{X_1} = 7\angle 60^\circ \qquad \vec{X_2} = 11\angle -85^\circ$$

$$\vec{X_1}\cdot\vec{X_2} = (7\cdot e^{j(60^\circ)})\cdot(11\, e^{j(-85^\circ)})$$
$$= 77\, e^{j(60-85)^\circ}$$
$$= 77\, e^{-j(25^\circ)}$$
$$= 77\angle -25^\circ$$

product magnitude $\quad 7\times 11 = 77$

addition angle $\quad 60 - 85 = -25$

8.2 Phasor circuit analysis

(A) Connection Constrains in phasor form.

(1) KVL (Rise = Drop) $\vec{V_1} + \vec{V_2} + \cdots + \vec{V_j} = \vec{V_k} + \vec{V_L} + \cdots + \vec{V_N}$

Proof: If $V_1 \cos(\omega t + \phi_1) + V_2 \cos(\omega t + \phi_2) + \cdots + V_j \cos(\omega t + \phi_j)$

$$= V_K \cos(\omega t + \phi_K) + V_L \cos(\omega t + \phi_L) + \cdots + V_N (\cos \omega t + \phi_N)$$

$$\Rightarrow Re\{ V_1 e^{j\phi_1} e^{j\omega t} \} + Re\{ V_2 e^{j\phi_2} e^{j\omega t} \} + \cdots + Re\{ V_j e^{j\phi_j + j\omega t} \}$$

$$= Re\{ V_K e^{j\phi_K} e^{j\omega t} \} + Re\{ V_L e^{j\phi_L} \cdot e^{j\omega t} \} + \cdots + Re\{ V_N e^{j\phi_N} e^{j\omega t} \}$$

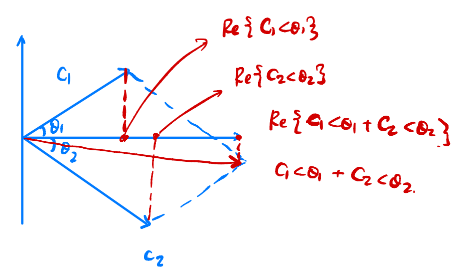
$$\Rightarrow Re\{ (V_1 e^{j\phi_1} + V_2 e^{j\phi_2} + \cdots + V_j e^{j\phi_j}) e^{j\omega t} \} = Re\{ (V_K e^{j\phi_K} + V_L e^{j\phi_L} + \cdots + V_N e^{j\phi_N}) e^{j\omega t} \}$$

$$\therefore \vec{V_1} + \vec{V_2} + \cdots + \vec{V_j} = \vec{V_K} + \vec{V_L} + \cdots + \vec{V_N}$$

(B) KCL (Input = Output) $\vec{I_1} + \vec{I_2} + \cdots + \vec{I_j} = \vec{I_k} + \vec{I_L} + \cdots + \vec{I_N}$ (Same)

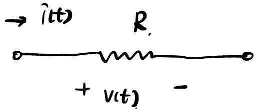
$$Re\{ C_1 \angle \theta_1 \} + Re\{ C_2 \angle \theta_2 \}$$

$$= Re\{ C_1 \angle \theta_1 + C_2 \angle \theta_2 \}$$



(B) Device constraints in phasors form.

(1) Resistance



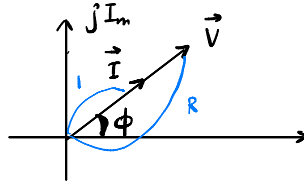
Time Domain elements

$$v(t) = R\left[I_A \cos(\omega t + \phi)\right] = RI_A \cdot \cos(\omega t+\phi) = Re\left\{ RI_A \cdot e^{j(\omega t+\phi)} \right\}$$

$$= Re\left\{ \underbrace{RI_A \cdot e^{j\phi}}_{\vec{V}} \cdot e^{j\omega t} \right\}$$

$\vec{V} = RI_A \cdot e^{j\phi} = RI_A \angle\phi \qquad \vec{I} = I_A \cdot \angle\phi$

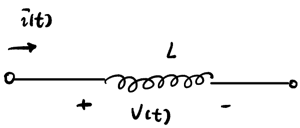
$\therefore \boxed{\vec{V} = R\vec{I}}$



"In-phase"

Phasor diagram of a resistor

(2) Inductance



$$i(t) = I_A \cos(\omega t + \phi) \qquad v(t) = L\frac{di(t)}{dt} = L(-\omega)I_A \sin(\omega t+\phi) = -L\omega I_A \sin(\omega t + \phi)$$

$$\therefore v(t) = -\omega L I_A \cos(\omega t + \phi - 90^\circ) = Re\left\{ -\omega L I_A \cdot e^{j(\omega t+\phi-90^\circ)} \right\} = Re\left\{ -\omega L I_A \cdot e^{j\phi} e^{j\,-90} \cdot e^{j\omega t} \right\}$$

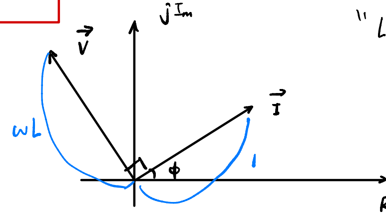
$$e^{j\,-90} = \cos(-90) + j\sin(-90) = -j \Rightarrow v(t) = Re\left\{ \underbrace{+j\omega L I_A \cdot e^{j\phi}}_{\vec{V}}\ e^{j\omega t} \right\}$$

$\therefore \vec{V} = j\omega L \cdot I_A \cdot e^{j\phi}$

$= j\omega L \cdot I_A \angle\phi \Rightarrow \boxed{\vec{V} = j\omega L \cdot \vec{I}}$

As $e^{j90} = \cos 90 + j\sin 90 = j = 1\angle 90^\circ$

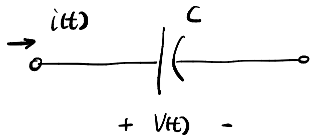
$\hookrightarrow \vec{V} = (\omega L \cdot \angle 90^\circ) \cdot I_A \cdot \angle\phi = (\omega L I_A) \angle(\phi + 90^\circ) \Rightarrow$



"LAGS"

Phasor diagram of Inductance

(3) Capacitance

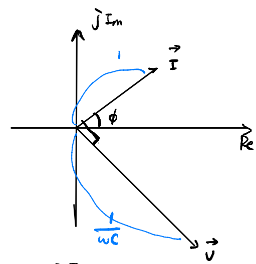


$$v(t) = V_A \cos(\omega t+\phi) \qquad i(t) = C\,\frac{dv(t)}{dt} = C\,V_A(-\omega t)\cdot \sin(\omega t+\phi) = -\omega C\,V_A \sin(\omega t+\phi)$$

$$\Rightarrow i(t) = -\omega C\,V_A \cdot \cos(\omega t+\phi-90^\circ) = Re\left\{-\omega C V_A\, e^{j(\omega t+\phi-90^\circ)}\right\} = Re\left\{\underbrace{j\omega C\,V_A e^{j\phi}}_{\text{phasor}}\cdot e^{j\omega t}\right\}$$

$$\Rightarrow i(t) = Re\left\{\underbrace{j\omega C\,V_A e^{j\phi}}_{\vec{I}}\cdot e^{j\omega t}\right\} \Rightarrow \vec{I} = j\omega C\,V_A\cdot e^{j\phi} = j\omega C\cdot\vec{V}$$

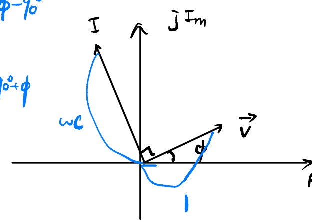
$$\Rightarrow \boxed{\vec{V} = \frac{1}{j\omega C}\vec{I}} \Rightarrow \vec{V} = \frac{-j}{\omega C}\vec{I}$$



As $e^{j(-90^\circ)} = \cos(-90^\circ)+j\sin(-90^\circ) = -j \Rightarrow \angle -90^\circ$

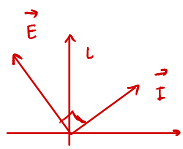
$$\Rightarrow \vec{V} = \left(\frac{1}{\omega C}\cdot\angle -90^\circ\right)(I_A\angle\phi) = \frac{1}{\omega C} I_A\angle\phi-90^\circ$$

Another: $\vec{I} = (j\omega C)\cdot\vec{V} = (\omega C\angle 90^\circ)(V_A\cdot\angle\phi) = \omega C V_A\angle 90^\circ+\phi$



phasor diagram of Capacitor

Tricks



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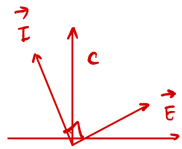


ICE

(c)

Summarize: The Impetance – Admittence Concept.

(1) $\vec{V} = Z\cdot\vec{I}$ (impetance)

For resistor. $\vec{V} = R\vec{I}$. $\quad Z = R$

For inductance $\vec{V} = j\omega L\,\vec{I}$ $\quad Z = j\omega L$

For capacitance $\vec{V} = \frac{1}{j\omega C}\vec{I} = \frac{-j}{\omega C}\vec{I}$ $\quad Z = -j\frac{1}{\omega C}$

(2) $\vec{I} = Y\vec{V}$ (admittance)

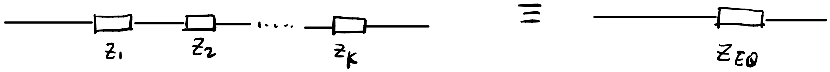
$Y = \frac{1}{R} = G$

$Y = \frac{1}{j\omega L} = -j\frac{1}{\omega L}$

$Y = j\omega C$

8.3 Basic circuit analysis with phasors

(A) Series equivalence of impedance



$$Z_{EQ} = \sum_{i=0}^{k} Z_i$$

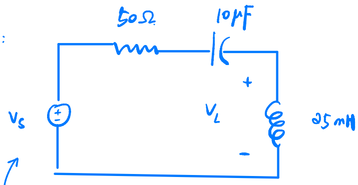
Proof: $\vec{V_s} = \vec{V_1} + \vec{V_2} + \dots + \vec{V_k} = \vec{I_s} Z_1 + \vec{I_s} Z_2 + \dots + \vec{I_s} Z_k = \vec{I_s}(Z_1 + Z_2 + \dots + Z_k) \stackrel{\Delta}{=} \vec{I_s} Z_{EQ}$

$Z_{EQ} = R + jX$ R is resistance.

positive $X \rightarrow$ inductive reactance

negative $X \rightarrow$ capactive reactance

eg:



$V_s = 35\cos(1000t + 0^\circ)$

$\vec{V_s} = 35 \angle 0^\circ$ $\omega = 1000$

$R = 50\Omega$ $Z_L = j(1k)(25m) = j25$

$Z_c = -j\dfrac{1}{(1k)(10\mu)} = -j \cdot 100$

$\Rightarrow Z_{EQ} = 50 - j75$

$= 90.14 \angle -56.31^\circ$

$\vec{I_s} = \dfrac{\vec{V_s}}{Z_{EQ}} = \dfrac{35 \angle 0^\circ}{90.14 \angle -56.31^\circ} = 0.388 \angle 56.31^\circ$ $\Rightarrow$ $i_s(t) = 388m\cos(1kt + 56.31^\circ)\ A$

$\vec{V_L} = (j25) \cdot \vec{I_s} = (25 \angle 90^\circ)(0.388 \angle 56.31^\circ) = 9.707 \angle 146.31^\circ$

$V_L(t) = 9.707\cos(1kt + 146.31^\circ)\ V.$

Voltage divide rule: $\vec{V_j} = \vec{V_s} \cdot \dfrac{Z_j}{Z_1 + Z_2 + \dots + Z_j}$

(B) parallel equivalence and circuit division

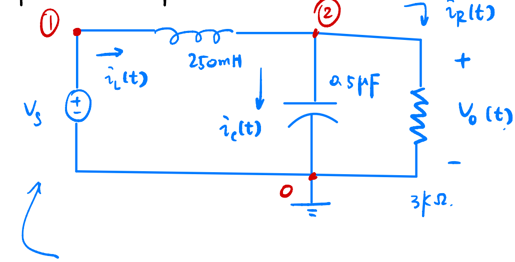
(1) parallel equivalence. $Z_{EQ} = \dfrac{1}{Y_{EQ}} = \dfrac{1}{Y_1 + Y_2 + \dots + Y_j} = \dfrac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \dots + \frac{1}{Z_k}}$

(2) admittance $Y_{EQ} = G + jB$ G is conductance and B is suspectance

(3) Current divide rule: (paralle circuit)

$$\vec{I_j} = \vec{I_s} \cdot \frac{Y_j}{Y_1 + Y_2 + \dots + Y_k}$$

SPICE Example:



$V_s = 100\cos(2000t)$ V 250 mH

$\vec{V}_s = 100\angle 0°$

$Z_L = jωL = j(2000)(250m) = j\,500\,Ω$

$Z_c = -j\dfrac{1}{ωC} = -j\dfrac{1}{(2000)(0.5μ)} = -j\,1000\,Ω.$

$$Z_{EQ} = Z_L + \frac{Z_c \cdot R}{Z_c + R} = j\cdot 500 + \frac{(-j\,1000)(3k)}{3k - j\,1000}$$

$$= 500\angle -53.13°$$

$$\vec{I}_L = \frac{\vec{V}_s}{Z_{EQ}} = \frac{100\angle 0°}{500\angle -53.13°} = 200m\angle 53.13°$$

$$\vec{I}_c = \frac{3k}{3k - j1000}\cdot 200m\angle 53.13° = 189.71m\angle 71.565°$$

$$\Rightarrow \vec{I}_R = \vec{I}_L - \vec{I}_c = 200m\angle 53.13° - 189.71m\angle 71.565° = 63.25m\angle -18.435°$$

By ohm law: $\vec{V}_o = \vec{I}_R \cdot R = (63.25m\angle -18.435°)\cdot(3k) = 189.75\angle -18.435°$

$i_L(t) = 200m\cos(2kt + 53.13°)$ A $i_c(t) = 189.71m\cos(2kt + 71.565°)$ A

$i_R(t) = 63.25m\cos(2kt - 18.435°)$ A $V_o(t) = 189.75m\cos(2kt - 18.435°)$ V

$f = \dfrac{2000}{2\pi}$ DELAY DAMPING FACTOR

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EXAMPLE 8.10
VS 1 0 SIN (0 100 318.31 0 0 90)
L 1 2 250M
C 2 0 0.5U
R 2 0 3K
.PROBE
.TRAN 94.25U 18.845M 0 94.25U
.END
```

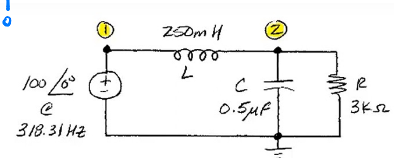
AVE V_A PHASE SHIFT CEILING STEP

$\dfrac{18.845m}{200}$ ← MINIMUM PLOTTED POINTS

18.845M → 6 CYCLES $= 6\cdot\dfrac{1}{318.31}$

Voltage : N+ → N−

Current : tail → head

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EXAMPLE 8.10
VS 1 0 AC 100 0
L 1 2 250M
C 2 0 0.5U
R 2 0 3K
.PROBE
.AC DEC 200 318.31 3.1831K
.END
```

MAGNITUDE

PHASE (DEFAULT IS 0°)

START FREQ.

STOP FREQ.

POINTS/DECADE

SWEEP IN DECADES

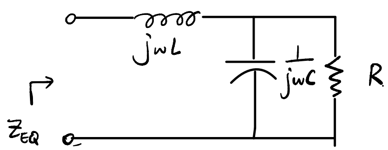
LIN, OCT

X-AXIS points are :

$318.31\cdot 10^{0/200},\ 318.31\cdot 10^{1/200}$

$\ldots\ldots\ 318.31\cdot 10^{200/200}$

Resonance :



$$Z_{RC} = \frac{R\cdot\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}} = \frac{R}{1+(\omega RC)^2} - j\,\frac{\omega R^2 C}{1+(\omega RC)^2}$$

$$Z_{EQ} = j\omega L + Z_{RC} = \underbrace{\frac{R}{1+(\omega RC)^2}}_{R_{EQ}} + j\underbrace{\left[\omega L - \frac{\omega R^2 C}{1+(\omega RC)^2}\right]}_{X_{EQ}}$$

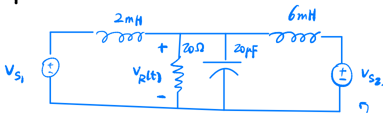
when $X_{EQ}=0 \Rightarrow$ "resonant frequency"

$$X_{EQ}(\omega=\omega_0) = \omega_0 L - \frac{\omega_0 R^2 C}{1+(\omega_0 RC)^2} = 0 \Rightarrow \boxed{\omega_0 = \sqrt{\frac{1}{LC} - \frac{1}{(RC)^2}}}$$

8.4. Circuit theorems with phasors

(A) Superposition

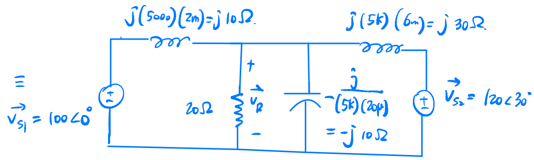
eg :



$v_{s_1} = 100\cos(5000t)$ V

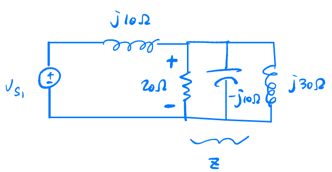
$v_{s_2} = 120\cos(5000t+30^\circ)$ V

≡



Find $V_R(t)$

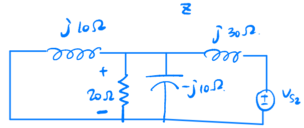
(a).



$$Z = \frac{1}{\frac{1}{20}+\frac{1}{-j20}+\frac{1}{j30}} = 7.2 - j\cdot 9.6$$

$$\vec{V}_R' = 100\angle 0^\circ \cdot \frac{7.2-j\cdot 9.6}{(7.2-j\cdot 9.6)+j10} = 166.4\angle -56.31^\circ = 92.3 - j\,138.5$$

(b)



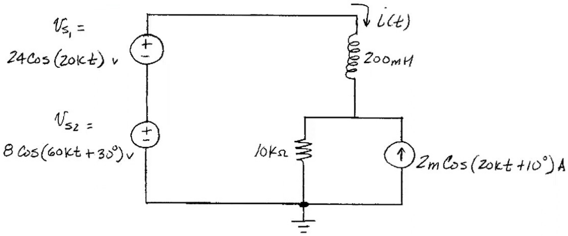
$$Z = \frac{1}{\frac{1}{j10}+\frac{1}{-j10}+\frac{1}{20}} = 20\Omega$$

$$\vec{V}_R'' = 120\angle 30^\circ \cdot \frac{20}{20+j30} = 66.48\angle -26.3^\circ = 59.6 - j\cdot 29.46$$

$$\vec{V}_R = \vec{V}_R' + \vec{V}_R'' = 151.9 - j\cdot 167.96$$
$$= 226.46\angle -47.87^\circ$$

$$v_R(t) = 226.46\cos(5kt - 47.87^\circ)\ V$$

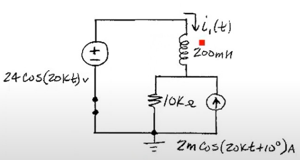
2) EXAMPLE : SOURCES OF DIFFERENT FREQUENCIES

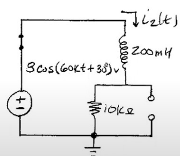


FIND THE STEADY-STATE VALUE OF $i(t)$

SOLUTION :

- APPLY SUPERPOSITION IN THE TIME DOMAIN FOR SOURCES OF THE SAME FREQUENCY, THAT IS,

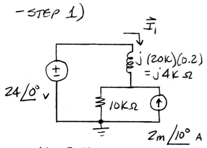




BY SUPERPOSITION, $i(t) = i_1'(t) + i_1''(t) + i_2(t)$

CIRCUIT #1

- STEP 1)



$\omega = 20K$

- STEP 2)

$$\vec{I}_1' = \frac{24\angle 0°}{10K + j4K} = \frac{24\angle 0°}{10.77K\angle 21.8°}$$

$$= 2.228m\ \angle -21.8°$$

$$= 2.069m - j0.8274m$$

Another sub Superposition here

$$\vec{I}_1'' = -2m\angle 10° \ \frac{10K}{10K + j4K} \qquad \frac{\frac{1}{j4K}}{\frac{1}{10K} + \frac{1}{j4K}}$$

$$= 2m\angle 10° \ \frac{10K\angle 180°}{10.77K\angle 21.8°}$$

$$= 1.857m\ \angle 168.2°$$

$$= -1.8178m + j0.3797m$$

$$\therefore \vec{I}_1 = \vec{I}_1' + \vec{I}_1''$$

$$= 0.251m - j0.4477m$$

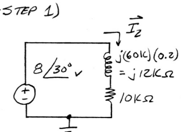
$$= 513.3\mu\ \angle -60.7°$$

- STEP 3)

$$i_1(t) = 513.3\mu\cos(20Kt - 60.7°)$$

CIRCUIT #2

- STEP 1)



$\omega = 60K$

- STEP 2)

$$\vec{I}_2 = \frac{8\angle 30°}{10K + j12K}$$

$$= \frac{8\angle 30°}{15.62K\angle 50.19°}$$

$$= 512\mu\ \angle -20.19°$$

- STEP 3)

$$i_2(t) = 512\mu\cos(60Kt - 20.19°)A$$

$\therefore$

$$\boxed{i(t) = 513.3\mu\cos(20Kt - 60.7°) + 512\mu\cos(60Kt - 20.19°)\ A}$$

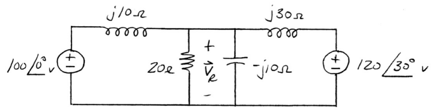
(B) Source Transformation

1) EXAMPLE 8.17

DO EX. 8.14 USING SOURCE TRANSFORMATIONS

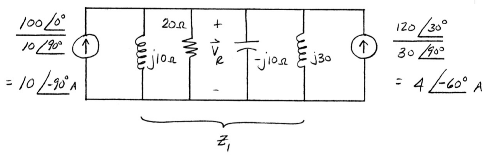
SOLUTION :

- STEP 1)



$\omega = 5000$

- STEP 2)



SINCE $(j10) \parallel (-j10) = \infty$ THEN

$$Z_1 = \frac{20(j30)}{20+j30} = \frac{600\angle 90^\circ}{36.06\angle 56.3^\circ} = 16.639\angle 33.69^\circ$$

$$\vec{V}_R = (10\angle -90^\circ + 4\angle -60^\circ)(16.639\angle 33.69^\circ)$$

$$= (-j10 + 2 - j3.464)(16.639\angle 33.69^\circ)$$

$$\vec{V}_R = (2 - j13.464)(16.639\angle 33.69^\circ)$$

$$= (13.612\angle -81.55^\circ)(16.639\angle 33.69^\circ)$$

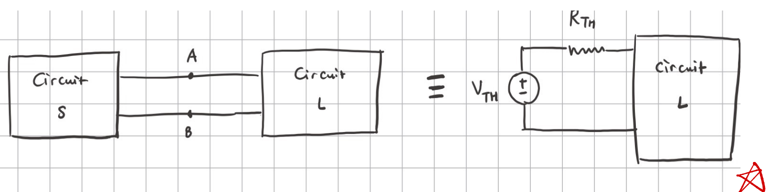
$$= 226.49\angle -47.86^\circ$$

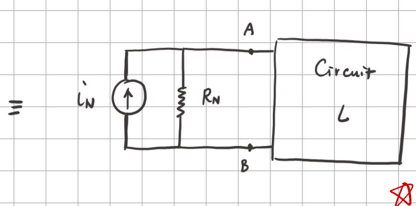
- STEP 3)

$$\boxed{V_R(t) = 226.49\cos(5kt - 47.86^\circ)\ V}$$ ← SAME AS P41

(c) Theven & Norton equivalent circuits

Review :





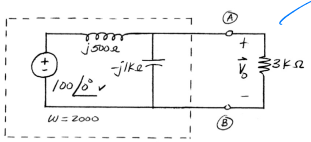
C) Thevenin and Norton Equivalent Circuits

1) Example

Thevenize the circuit of Ex. 8.10 with respect to the $3K\Omega$ load. Find $V_o(t)$ in steady-state.

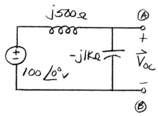
Solution:

- Step 1)



- Step 2)

a) $\vec{V}_{TH}$

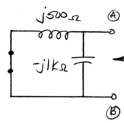


$$\vec{V}_{OC} = \vec{V}_{TH} = 100\angle 0^\circ \frac{-j1K}{-j1K + j500}$$

$$= 100\angle 0^\circ \frac{-j1K}{-j500}$$

$$= 200\angle 0^\circ$$

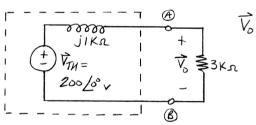
b) Z_{TH}



$$Z_{TH} = \frac{(j500)(-j1K)}{j500 - j1K} = \frac{500K}{-j500}$$

$$= j1000$$

c)



$$\vec{V}_o = 200\angle 0^\circ \frac{3K}{3K + j1K}$$

$$= 200\angle 0^\circ \frac{3K\angle 0^\circ}{3.162K\angle 18.44^\circ}$$

$$= 189.75\angle -18.44^\circ$$

- Step 3)

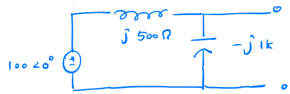
$$\boxed{V_o(t) = 189.75\cos(2Kt - 18.44^\circ)\ \text{V}}$$

← Same as P26

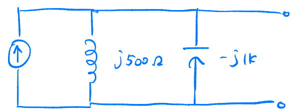
$$1\cdot\angle 90^\circ = 1\cdot e^{j90} = \cos 90 + j\sin 90 = j$$

$$j = \angle 90^\circ \qquad -j = \angle -90^\circ$$

Another understanding

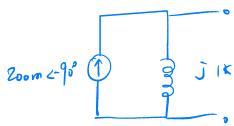


≡

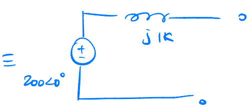


$$\vec{I} = \frac{100\angle 0^\circ}{500\angle 90^\circ} = 200m\angle -90^\circ$$

$$Z_{EQ} = \frac{(j\,0.5K)\cdot(-j\,1K)}{j\,0.5K + j(-1K)} = \frac{\cancel{j}\ \cancel{0.5}\times\cancel{10^6}\ 10^3}{\cancel{-j}\ \cancel{0.5}\times\cancel{10^3}\ j} = j\,1K$$



≡

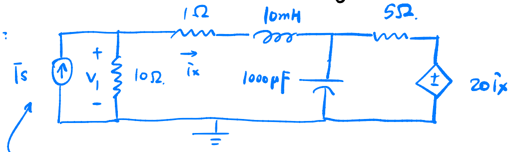


$$\vec{V} = (200m\angle -90^\circ)\cdot(1K\angle 90^\circ)$$

$$= 200\angle 0^\circ$$

8.5 General Circuit Analysis with Phasors (A) Node Voltage

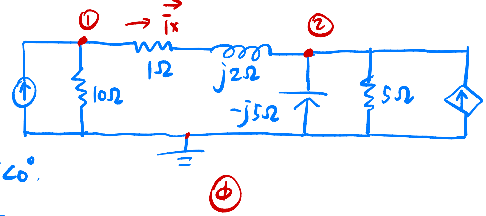
Ex:



$i_s = 10.6\cos(200t)\,A$

Find the steady-state value of $v_1(t)$

<sol>



$\frac{20\vec{I}_x}{5} = 4\vec{I}_x$

$\vec{I}_s = 10.6\angle 0°$

$\omega = 200$

$$\begin{pmatrix} 10.6 \\ 4\vec{I}_x \end{pmatrix} = \begin{pmatrix} \left(\frac{1}{10}+\frac{1}{1+j2}\right) & -\left(\frac{1}{1+j2}\right) \\ -\left(\frac{1}{1+j2}\right) & \left(\frac{1}{1+j2}+\frac{1}{-j5}+\frac{1}{5}\right) \end{pmatrix} \begin{pmatrix} \vec{V}_1 \\ \vec{V}_2 \end{pmatrix}$$

$$\vec{I}_x = \frac{\vec{V}_1-\vec{V}_2}{1+j2} \Rightarrow -4\vec{I}_x = -4\,\frac{\vec{V}_1-\vec{V}_2}{1+j2} = \frac{-4}{1+j2}\vec{V}_1 + \frac{4}{1+j2}\vec{V}_2$$

$$\Rightarrow \begin{pmatrix} 10.6 \\ 0 \end{pmatrix} = \begin{pmatrix} \left(\frac{1}{10}+\frac{1}{1+j2}\right) & -\left(\frac{1}{1+j2}\right) \\ \left(\frac{-5}{1+j2}\right) & \left(\frac{5}{1+j2}+\frac{j}{5}+\frac{1}{5}\right) \end{pmatrix} \begin{pmatrix} \vec{V}_1 \\ \vec{V}_2 \end{pmatrix}$$

$$\begin{bmatrix} 10.6 \\ 0 \end{bmatrix} = \begin{bmatrix} \left(\frac{11+j2}{10+j20}\right) & -\left(\frac{1}{1+j2}\right) \\ -\left(\frac{5}{1+j2}\right) & \left(\frac{24+j3}{5+j10}\right) \end{bmatrix} \begin{bmatrix} \vec{V}_1 \\ \vec{V}_2 \end{bmatrix}$$

$$\vec{V}_1 = \frac{\begin{vmatrix} 10.6 & -\left(\frac{1}{1+j2}\right) \\ 0 & \left(\frac{24+j3}{5+j10}\right) \end{vmatrix}}{\begin{vmatrix} \left(\frac{11+j2}{10+j20}\right) & -\left(\frac{1}{1+j2}\right) \\ -\left(\frac{5}{1+j2}\right) & \left(\frac{24+j3}{5+j10}\right) \end{vmatrix}} = \frac{(10.6)\left(\frac{24+j3}{5+j10}\right)}{\left(\frac{11+j2}{10+j20}\right)\left(\frac{24+j3}{5+j10}\right)-\left(\frac{5}{1+j2}\right)\left(\frac{1}{1+j2}\right)}$$

$$= \frac{10.6\angle 0° \;\frac{24.19\angle 7.13°}{11.18\angle 63.43°}}{\frac{\cancel{11.18}\angle 10.3°}{22.36\angle 63.4°}\,\frac{24.19\angle 7.13°}{\cancel{11.18}\angle 63.43°} - \frac{5\angle 0°}{2.236\angle 63.43°}\,\frac{1\angle 0°}{2.236\angle 63.43°}}$$

$$= \frac{22.94\angle -56.3°}{1.082\angle -109.37° - 1.00\angle -126.86°} = \frac{22.94\angle -56.3°}{(-0.36-j1.02)-(-0.6-j0.8)}$$

$$= \frac{22.94\angle -56.3°}{0.24-j0.22} = \frac{22.94\angle -56.3°}{0.3256\angle -42.5°} = 70.45\angle -13.8°$$

MATLAB:

COEFFICIENT FIRST

SURPRESS PRINTING ON SCREEN

```
Y(1,1) = 1/10 + 1/(1+2j);
Y(1,2) = -1/(1+2j);
Y(2,1) = -5/(1+2j);
Y(2,2) = 5/(1+2j) + 1/5 + j/5;
```

} 2x2 Y MATRIX

```
I = [10.6;0];
V= Y\I
VM=abs(V)
VP=angle(V)*180/pi
```

INVERSE OF Y

MAGNITUDE OF A COMPLEX NUMBER (CASE SENSITIVE – USE LOWER CASE)

ANGLE IN DEGREES

ANGLE IN RADIANS

APPEAR:

```
EDU»
V =

   68.4000 -16.8000i
   68.0000 -26.0000i
```
$= \vec{V}_1$, $= \vec{V}_2$

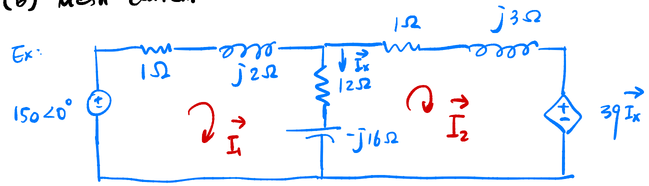
```
VM =

   70.4329
   72.8011
```
$= |\vec{V}_1|$, $= |\vec{V}_2|$

```
VP =

  -13.7995
  -20.9245
```
$= \angle V_1$, $= \angle V_2$

$\therefore \vec{V}_1 = 70.4329 \angle -13.7995^\circ$ ⇐ SAME RESULT AS p50

(B) Mesh Current

Ex:



$$\begin{pmatrix} +150 \\ -39\vec{I}_x \end{pmatrix} = \begin{pmatrix} (1+j2+12-j16) & -(12-j16) \\ -(12-j16) & 12-j16+1+j3 \end{pmatrix} \begin{pmatrix} \vec{I}_1 \\ \vec{I}_2 \end{pmatrix}$$

$$\vec{I}_x = \vec{I}_1 - \vec{I}_2$$

$$\Rightarrow +39\vec{I}_x = +39\vec{I}_1 - 39\vec{I}_2$$

$$\Rightarrow \begin{pmatrix} 150 \\ 0 \end{pmatrix} = \begin{pmatrix} 13-j14 & 12-j16 \\ -12+j16+39 & 13-j13-39 \end{pmatrix} \begin{pmatrix} \vec{I}_1 \\ \vec{I}_2 \end{pmatrix}$$

SCRIPT FILE "mesh.m"

```
Z(1,1) = 13-14j;
Z(1,2) = -12+16j;
Z(2,1) = 27+16j;
Z(2,2) = -26-13j;

V = [150;0];

I= Z\V

IM=abs(I)
IP=angle(I)*180/pi
```

SCREEN OUTPUT

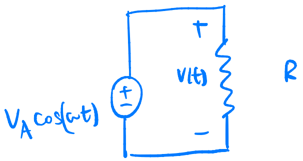
```
EDU»
I =

   -26.0000 -52.0000i
   -24.0000 -58.0000i
```
$= \vec{I}_1$, $= \vec{I}_2$

```
IM =

    58.1378
    62.7694
```
$= |\vec{I}_1|$, $= |\vec{I}_2|$

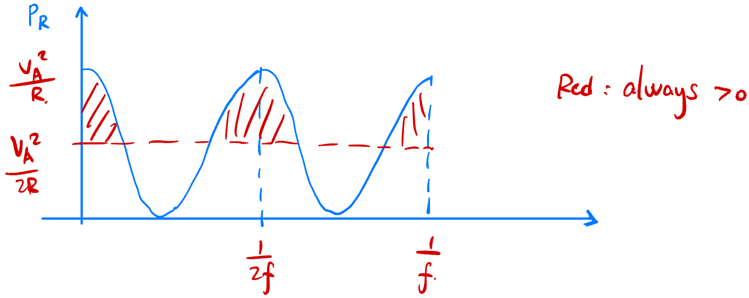
```
IP =

   -116.5651
   -112.4794
```
$= \angle I_1$, $= \angle I_2$

8.6 Energy And Power

(A) Resistance.



$$P_R(t) = \frac{V^2(t)}{R} = \frac{V_A^2 \cos^2(\omega t)}{R} = \frac{V_A^2}{2R}\left(1+\cos(2\omega t)\right)$$



$$P_R(\text{AVG}) \triangleq P_R = \frac{1}{T}\int_0^T \frac{V_A^2}{2R}\left(1+\cos(2\omega t)\right) dt$$

$$= \frac{V_A^2}{2RT}\int_0^T \left(1+\cos(2\omega t)\right) dt$$

$$= \frac{V_A^2}{2RT}\left(t\Big|_0^T + \frac{1}{2\omega}\sin(2\omega t)\Big|_0^T \right)$$

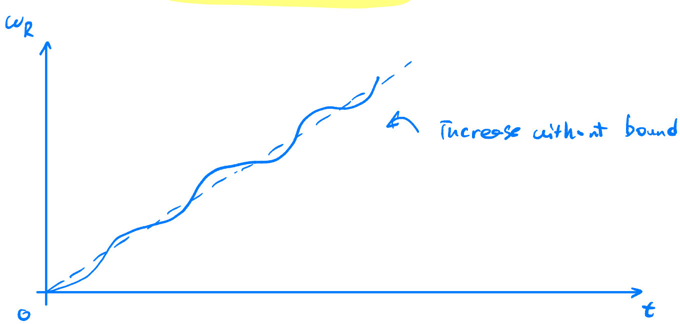
$$= \frac{V_A^2}{2RT}\left[(T-0) + \frac{1}{2\omega}\sin\left(2\cdot 2\pi f\cdot \frac{1}{f}\right)\right]$$

$$= \frac{V_A^2}{2R}$$

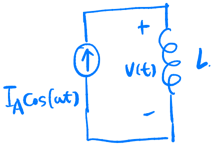
$$\omega_R(t) = \int_0^t P_R(x)\,dx = \int_0^t \frac{V_A^2}{2R}\left(1+\cos 2\omega x\right) dx = \frac{V_A^2}{2R}\int_0^t \left(1+\cos(2\omega x)\right) dx$$

$$= \frac{V_A^2}{2R}\left[x\Big|_0^t + \frac{1}{2\omega}\sin(2\omega x)\Big|_0^t \right] = \frac{V_A^2}{2R}\left[t + \frac{1}{2\omega}\sin(2\omega t)\right]$$

$$= \frac{V_A^2}{2R}t + \frac{V_A^2}{4R\omega}\sin(2\omega t)$$



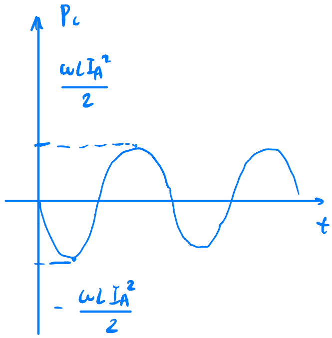
(B) Inductance

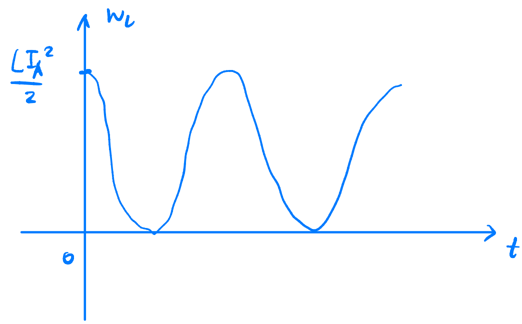


$$W_L(t) = \frac{1}{2} L \left[i(t) \right]^2 = \frac{1}{2} L \cdot I_A^2 \cos^2 \omega t$$

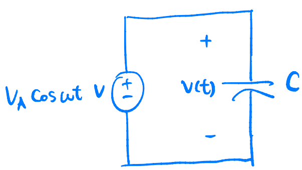
$$= \frac{1}{4} L I_A^2 \left(1 + \cos(2\omega t) \right)$$

$$P_L(t) = \frac{d\,W_L(t)}{dt} = \frac{1}{4} L I_A^2 \left[0 + (-2\omega) \sin(2\omega t) \right] = -\frac{\omega L I_A^2}{2} \sin(2\omega t)$$





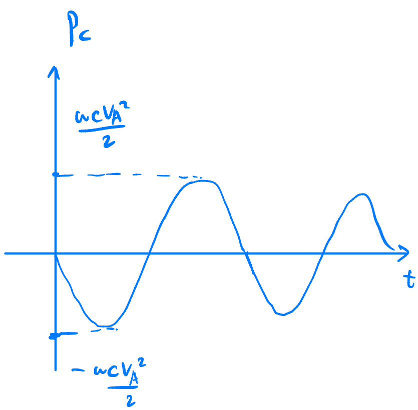
(c) Capacitance

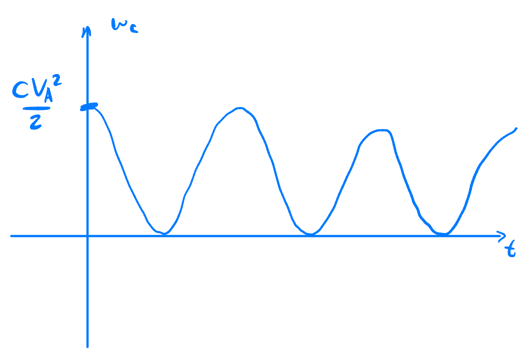


$$W_c(t) = \frac{1}{2} C V(t)^2 = \frac{1}{2} C V_A^2 \cos^2(\omega t) = \frac{1}{4} C V_A^2 \left(1 + \cos(2\omega t) \right)$$

$$P_c(t) = \frac{d\,W_c(t)}{dt} = \frac{1}{4} C V_A^2 \cdot \left[0 + (-2\omega) \sin(2\omega t) \right]$$

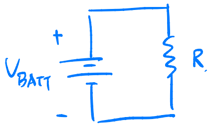
$$= -\frac{\omega C V_A^2}{2} \sin(2\omega t)$$





Equating AC and DC Power

Since we know $P_R = \frac{V_A^2}{2R}$ (Avg)



$$P_R(t) = \frac{V_{BATT}^2}{R} = \frac{V_A^2}{2R} \Rightarrow \boxed{V_{BATT} = \frac{V_A}{\sqrt{2}}}$$

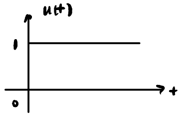
$$\frac{V_{BATT}^2}{R} = \frac{1}{T}\int_0^T \frac{[V_A \cos\omega t]^2}{R}\,dt \Rightarrow V_{BATT} \triangleq V_{EFFECTIVE} \triangleq V_{Rms} = \sqrt{\frac{1}{T}\int_0^T [V_A \cos\omega t]^2\,dt}$$

In general $F_{Rms} = \sqrt{\frac{1}{T}\int_0^T f^2(t)\,dt}$

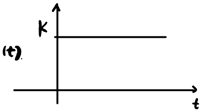
Chapter 9: Laplace Transformation

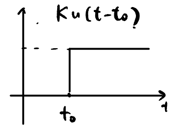
(A) Def: $\mathcal{L}[f(t)] \triangleq F(s) = \int_{0^-}^{\infty} f(t) e^{-st} dt$ where $s = \sigma + j\omega$, and $f(t) = 0$ for $t < 0$

(B) Step wave form. $u(t) \triangleq \begin{cases} 0 & \text{For } t < 0 \\ 1 & \text{For } t > 0 \end{cases}$



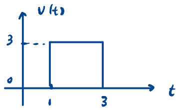
Also $K u(t)$



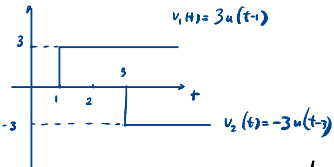


(C) Example 5.1

Ex:



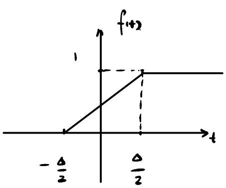
$\Rightarrow$



$v(t) = v_1(t) + v_2(t) = 3u(t-1) - 3u(t-3)$ ✓

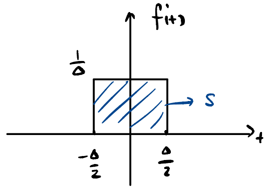
(D) $f(t) = u(t) \Rightarrow F(s) = \frac{1}{s}$ $\quad f(t) = e^{-\alpha t} u(t) \Rightarrow F(s) = \frac{1}{s+\alpha}$

(F) The impulse function



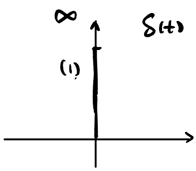
$f(t) \to u(t)$

As $\Delta \to 0$



$S = \frac{1}{\Delta} \cdot \left[\frac{\Delta}{2} - \left(-\frac{\Delta}{2}\right) \right] = 1$

– The unit impulse function $\delta(t)$ is defined as $f'(t)$ as $\Delta \to 0$ and $\delta(t) = 0$ For $t \neq 0$



$$\int_{-\infty}^{t} \delta(x)\, dx = u(t)$$

$f(t) = \delta(t)$

$$F(s) = \int_{0^-}^{\infty} \delta(t) e^{-st} dt = \underbrace{\int_{0^-}^{0^+} \delta(t)}_{\text{area of } 1} \underbrace{e^{-st}}_{\to 1} dt + \cancel{\int_{0^+}^{\infty} \delta(t) e^{-st} dt}^{\,0}$$

$$= \boxed{1}$$

(G) Inverse Transformation:

$$\mathcal{L}^{-1}\{F(s)\} \triangleq f(t) = \frac{1}{2\pi j} \int_{\alpha - j\infty}^{\alpha + j\infty} F(s) \cdot e^{st} ds$$

$Re\{s\} = \alpha$

(I) Uniqueness property $\mathcal{L}^{-1}\{\mathcal{L}[f(t)]\} = \mathcal{L}^{-1}\{F(s)\} = f(t)$

Some Laplace transformation:

$L\{1\} = \frac{1}{s}$

$L\{e^{at}\} = \frac{1}{s-a}$

$L\{\sin(bt)\} = \frac{b}{s^2 + b^2}$

$L\{\cos(bt)\} = \frac{s}{s^2 + b^2}$

$L\{t^n\} = \frac{n!}{s^{n+1}}$

Properties:

$$L\{af(t)+bg(t)\} = aF(s)+bG(s) \iff \mathcal{L}^{-1}\{aF(s)+bG(s)\} = af(t)+bg(t)$$

$$L\{f'(t)\} = sF(s)-f(0) \quad \text{For higher derivative} \quad L\{f^{(n)}(t)\} = s^n F(s) - s^{n-1}f(0) - \dots - f^{(n-1)}(0)$$

$$L\{e^{at}f(t)\} = F(s-a)$$

$$L\{(f*g)(t)\} = F(s)\cdot G(s) \quad \text{where } (f*g)(t) = \int_0^t f(\tau)\, g(t-\tau)\, d\tau$$

Some prep:

① Show that $L\{A(1-e^{-\alpha t})u(t)\} = \dfrac{A\alpha}{S(S+\alpha)}$

$$L\{A(1-e^{-\alpha t})u(t)\} = A\,L\{u(t)\} - A\,L\{e^{-\alpha t}u(t)\} = \frac{A}{S} - \frac{A}{S+\alpha} = \frac{A\alpha}{S(S+\alpha)}$$

② Show that $L\{A[\sin(\beta t)]u(t)\} = \dfrac{A\beta}{S^2+\beta^2}$

For $e^{j\beta t} = \cos\beta t + j\sin\beta t$, $e^{-j\beta t} = \cos\beta t - j\sin\beta t \Rightarrow \sin\beta t = \dfrac{e^{j\beta t}-e^{-j\beta t}}{2j}$

$$f(t) = A\sin\beta t = \frac{A}{2j}e^{j\beta t} - \frac{A}{2j}e^{-j\beta t} \Rightarrow L\{f(t)\} = \frac{A}{2j}\mathcal{L}\{e^{j\beta t}u(t)\} - \frac{A}{2j}L\{e^{-j\beta t}u(t)\}$$

$$\Rightarrow L\{f(t)\} = \frac{A}{2j}\cdot\frac{1}{S-j\beta} - \frac{A}{2j}\cdot\frac{1}{S+j\beta} = \frac{A\beta}{S^2+\beta^2}$$

Intergration Property: $L\left[\int_0^t f(x)\,dx\right] = \dfrac{F(s)}{s}$

Proof: $L\left\{\int_0^t f(x)\,dx\right\} = \int_{0^-}^{\infty}\left[\int_0^t f(x)\,dx\right]e^{-st}\,dt$

Since $\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du \Rightarrow$ let $u = \int_0^t f(x)\,dx \qquad dv = e^{-st}\,dt$

therefore $du = f(t)\,dt \qquad v = -\frac{1}{s}e^{-st} \Rightarrow \text{RHS} = \left[\cancel{\int_0^t f(x)\,dx}\right]\left[\cancel{-\frac{1}{s}e^{-st}}\right]\Big|_{0^-}^{\infty} - \int_{0^-}^{\infty}\left(-\frac{1}{s}\right)e^{-st}f(t)\,dt$

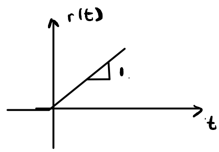
0, For $t=0$ 0, For $t\to\infty$

$$\Rightarrow \text{RHS} = \frac{1}{s}\int_{0^-}^{\infty} f(t)e^{-st}\,dt = \frac{1}{s}\cdot F(s)$$

$\boxed{\text{Q.E.D}}$

The ramp function

$$r(t) \triangleq \int_{-\infty}^{t} u(x)\,dx = t\,u(t)$$



$$L\{r(t)\} = L\left\{\int_{-\infty}^{t} u(x)\,dx\right\} = L\left\{\int_0^t u(x)\,dx\right\}$$

$$= \frac{1}{s}F(s) = \boxed{\frac{1}{s^2}}$$

Differential Property.

$$L\left\{\frac{df(t)}{dt}\right\} = sF(s) - f(0^-)$$

Proof: $L\left\{\frac{df(t)}{dt}\right\} = \int_{0^-}^{\infty} \frac{df(t)}{dt} e^{-st} dt$ let $u = e^{-st}$ $dv = \frac{df(t)}{dt}dt.$ $\Rightarrow du = -se^{-st}$ $v = f(t)$

$$\Rightarrow \int_{0^-}^{\infty} u\,dv = e^{-st}f(t)\Big|_{0^-}^{\infty} - \int_{0^-}^{\infty} f(t)(-s)\cdot e^{-st}dt = -f(0^-) + s\int_{0^-}^{\infty} f(t)e^{-st}dt = \boxed{sF(s) - f(0^-)}$$

n^{th} Derivative: $L\left\{\frac{d^n f(t)}{dt^n}\right\} = s^n F(s) - s^{n-1}f(0^-) - s^{n-2}f'(0^-) - \cdots - f^{(n-1)}(0^-)$

$L\{g(t)\} = L\left\{\frac{df(t)}{dt}\right\}$

Proof: let $g(t) = \frac{df(t)}{dt}$ then $\frac{dg(t)}{dt} = \frac{d^2f(t)}{dt^2}$. $G(s) = sF(s) - f(0^-)$

$$L\left\{\frac{dg(t)}{dt}\right\} = sG(s) - g(0^-) = s\left[sF(s) - f(0^-)\right] - f'(0^-) = s^2F(s) - sf(0^-) - f'(0^-)$$

let $h(t) = \frac{dg(t)}{dt}$ then $L\left\{\frac{dh(t)}{dt}\right\} = sH(s) - h(0^-) = s\left[s^2F(s) - sf(0^-) - f'(0^-)\right] - g'(0^-)$

$\Rightarrow L\left\{\frac{dh(t)}{dt}\right\} = s^3F(s) - s^2f(0^-) - sf'(0^-) - f''(0^-)$ Keep repeating for the answer. $\boxed{\text{Q.E.D}}$

S-Domain translation property

If $L\{f(t)\} = F(s)$ then $L\{e^{-at}f(t)\} = F(s+a)$

Proof: $L\left[e^{-at}f(t)\right] = \int_{0^-}^{\infty} e^{-at}f(t)e^{-st}dt = \int_{0^-}^{\infty} e^{-(s+a)t}f(t)\,dt = F(s+a)$

Time-domain translation property

If $L[f(t)] = F(s)$ then for $a > 0$ $L\left[f(t-a)\,u(t-a)\right] = e^{-as}F(s)$

Proof: $L\left[f(t-a)\,u(t-a)\right] = \int_{0^-}^{\infty} f(t-a)\,u(t-a)e^{-st}dt = \int_{a}^{\infty} f(t-a)\,e^{-st}dt$ let $\uparrow = t-a$.

then $\frac{d\uparrow}{dt} = 1 \Rightarrow d\uparrow = dt$. For $t=a$ $\uparrow = a-a=0$. For $t=\infty$ $\uparrow = \infty - a = \infty$

$$\Rightarrow \int_{a}^{\infty} f(t-a)e^{-st}dt = \int_{0}^{\infty} f(\uparrow)e^{-s(\uparrow+a)}d\uparrow = e^{-sa}\int_{0}^{\infty} f(\uparrow)e^{-s\uparrow}d\uparrow = \boxed{e^{-as}F(s)}$$

Basic laplace transformation

– THE FOLLOWING IS A COLLECTION OF COMMON TRANSFORM PAIRS

SIGNAL	$f(t)$	$F(s)$
1) IMPULSE	$\delta(t)$	1
2) STEP	$u(t)$	$\frac{1}{s}$
3) RAMP	$t\,u(t)$	$\frac{1}{s^2}$
4) EXPONENTIAL	$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$
5) DAMPED RAMP	$t e^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^2}$
6) SINE	$\sin(\beta t)\,u(t)$	$\frac{\beta}{s^2+\beta^2}$
7) COSINE	$\cos(\beta t)\,u(t)$	$\frac{s}{s^2+\beta^2}$
8) DAMPED SINE	$e^{-\alpha t}\sin(\beta t)\,u(t)$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$
9) DAMPED COSINE	$e^{-\alpha t}\cos(\beta t)\,u(t)$	$\frac{(s+\alpha)}{(s+\alpha)^2+\beta^2}$

$\Rightarrow L\{c\} = \frac{c}{s}$

$\Rightarrow L\{t^n\} = \frac{n!}{s^{n+1}}$

9.3 Pole- zero diagrams

In general
$$F(s) = \frac{b_m s^m + b_{m-1} \cdot s^{m-1} + \dots + b_1 s + b_0}{a_n s^n + a_{n-1} \cdot s^{n-1} + \dots + a_1 s + a_0}$$

we factor the number $\Rightarrow$
$$F(s) = K \cdot \frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$
$K = \frac{b_m}{a_n}$ (Scale factor)

$$F(s) = \frac{N(s)}{D(s)}$$

X Poles: value of s makes $D(s)=0$ $F(s) \to \infty$;

O Zeros: values of s that makes $N(s)=0$ $F(s) = 0$

B) MATLAB

– MATLAB CAN FIND THE LAPLACE TRANSFORM OF ANY $f(t)$

```
EDU» syms t s              ← DEFINES t & s AS SYMBOLIC VARIABLES
EDU» f = exp(-2*t) + 4*t - 1     ← EX. ON p14
f =
exp(-2*t)+4*t-1            ← MATLAB RESPONSE
EDU» F = laplace (f, t, s)   ← TRANSFORM f(t) = F(s)
                             ← NOTE ORDER
F =
1/(s+2)+4/s^2-1/s          ← SAME AS p14
EDU» simplify (F)
ans =
2*(s+4)/(s+2)/s^2          ← FINDS COMMON DENOMINATOR
EDU» pretty (ans)
      s + 4
  2 ----------             ← MORE READABLE
            2
    (s + 2) s
```

9.4 Inverse laplace transformation

$$F(s) = K\frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

- if $n>m$ ⇒ $F(s)$ is called a "proper rational function"
- if denominators has no repeated roots ($p_i \neq p_j$ for $i \neq j$) ⇒ $F(s)$ has "simple poles"
- if a proper rational function has only simple poles, then it can be decomposed into a "partial fraction expansion" of the form

$$F(s) = \frac{k_1}{s-p_1} + \frac{k_2}{s-p_2} + \cdots + \frac{k_n}{s-p_n}$$ k is called "residues"

$$\Rightarrow f(t) = \left[k_1 e^{p_1 t} + k_2 e^{p_2 t} + \cdots + k_n e^{p_n t}\right] u(t)$$

ex: $$F(s) = K\frac{(s-z_1)}{(s-p_1)(s-p_2)(s-p_3)} = \frac{k_1}{s-p_1} + \frac{k_2}{s-p_2} + \frac{k_3}{s-p_3}$$

times $(s-p_1)$ both side ⇒ $$(s-p_1)F(s) = \frac{K\cdot(s-z_1)}{(s-p_2)(s-p_3)} = k_1 + \frac{k_2(s-p_1)}{s-p_2} + \frac{k_3(s-p_1)}{s-p_3}$$

So we set $s=p_1$ ⇒ $$K\cdot\frac{p_1-z_1}{(p_1-p_2)(p_1-p_3)} = k_1 + 0 + 0 = k_1$$ Similar find k_2 & k_3

$$\Rightarrow \boxed{k_i = (s-p_i)F(s)\Big|_{s=p_i}}$$

A) EXAMPLE

FIND THE WAVEFORM CORRESPONDING TO THE TRANSFORM

$$F(s) = \frac{2(s+3)}{s(s+1)(s+2)}$$

SOLUTION :

SINCE $n=3 > m=1$, THEN

$$F(s) = \frac{k_1}{s} + \frac{k_2}{s+1} + \frac{k_3}{s+2}$$

$$k_1 = sF(s)\Big|_{s=0} = \cancel{s}\frac{2(s+3)}{\cancel{s}(s+1)(s+2)}\Big|_{s=0} = \frac{2(0+3)}{(0+1)(0+2)} = \frac{6}{2} = 3$$

$$k_2 = (s+1)F(s)\Big|_{s=-1} = \cancel{(s+1)}\frac{2(s+3)}{s\cancel{(s+1)}(s+2)}\Big|_{s=-1} = \frac{2(-1+3)}{(-1)(-1+2)} = \frac{4}{-1} = -4$$

$$k_3 = (s+2)F(s)\Big|_{s=-2} = \cancel{(s+2)}\frac{2(s+3)}{s(s+1)\cancel{(s+2)}}\Big|_{s=-2} = \frac{2(-2+3)}{(-2)(-2+1)} = \frac{2}{2} = 1$$

$$F(s) = \frac{3}{s} - \frac{4}{s+1} + \frac{1}{s+2}$$

$$\boxed{f(t) = 3u(t) - 4e^{-t}u(t) + e^{-2t}u(t)}$$ or

$$= 3 - 4e^{-t} + e^{-2t} \quad t>0$$

Complex poles

$$F(s) = \cdots + \frac{k}{s+\alpha-j\beta} + \frac{k^*}{s+\alpha+j\beta} + \cdots$$

Since $k = (s+\alpha-j\beta)\, F(s)\Big|_{s=-\alpha+j\beta}$ and $k^* = (s+\alpha+j\beta)\cdot F(s)\Big|_{s=-\alpha-j\beta}$.

FIND THE INVERSE TRANSFORM OF

$$F(s) = \frac{20(s+3)}{(s+1)(s^2+2s+5)}$$

ax^2+bx+c

SOLUTION:

s^2+2s+5 HAS ROOTS AT $s = \frac{-2 \pm \sqrt{4-4(5)}}{2} = -1 \pm j2$

$$F(s) = \frac{k_1}{s+1} + \frac{k_2}{s+1-j2} + \frac{k_2^*}{s+1+j2}$$

$$k_1 = (s+1)F(s)\Big|_{s=-1} = \cancel{(s+1)}\frac{20(s+3)}{\cancel{(s+1)}(s^2+2s+5)}\Big|_{s=-1} = \frac{20(-1+3)}{(1-2+5)} = \frac{40}{4} = 10$$

$$f(t) = \left[10e^{-t} + 14.4e^{-t}\cos(2t-135^\circ)\right]u(t)$$

$$k_2 = (s+1-j2)F(s)\Big|_{s=-1+j2}$$

$$= \cancel{(s+1-j2)}\frac{20(s+3)}{(s+1)\cancel{(s+1-j2)}(s+1+j2)}\Big|_{s=-1+j2}$$

$$= \frac{20(-1+j2+3)}{(\cancel{-1}+j2+\cancel{1})(\cancel{-1}+j2+\cancel{1}+j2)} = \frac{\overset{5}{\cancel{20}}(2+j2)}{(j2)(j\cancel{4})} = -5-j5 = 7.07\angle -135^\circ$$

NOTE:

$$k_3 = (s+1+j2)F(s)\Big|_{s=-1-j2} = \cancel{(s+1+j2)}\frac{20(s+3)}{(s+1)(s+1-j2)\cancel{(s+1+j2)}}\Big|_{s=-1-j2}$$

$$= \frac{20(-1-j2+3)}{(\cancel{-1}-j2+\cancel{1})(\cancel{-1}-j2+\cancel{1}-j2)} = \frac{\overset{5}{\cancel{20}}(2-j2)}{(-j2)(-j\cancel{4})} = -5+j5 = k_2^*$$

D) **SUM OF RESIDUES**

$$k_1 + k_2 + \cdots + k_n = \begin{cases} 0 & \text{IF } n > m+1 \\ K & \text{IF } n = m+1 \end{cases}$$

PROOF:

$$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{Ks^{m+1}}{s^n} = \lim_{s\to\infty}\left[\frac{k_1 s}{s-p_1} + \frac{k_2 s}{s-p_2} + \cdots + \frac{k_n s}{s-p_n}\right]$$

$$K \lim_{s\to\infty} \frac{s^{m+1}}{s^n} = k_1 + k_2 + \cdots + k_n$$

IF $n = m+1$, $K \lim_{s\to\infty} \frac{s^{m+1}}{s^n} = K = k_1 + k_2 + \cdots + k_n$

IF $n > m+1$, $K \lim_{s\to\infty} \frac{s^{m+1}}{s^n} = 0 = k_1 + k_2 + \cdots + k_n$

QED

9.6 Laplace transformation solve differential equation.

eg: $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 3x = e^{-2t}$. $\quad x(0)=1 \quad \frac{dx(0)}{dt}=2$.

<Sol>: $\left[s^2x(s) - sx(0) - s^0x'(0)\right] + 4\left[sX(s) - s^0x(0)\right] + 3X(s) = \frac{1}{s+2}$.

$$\left(s^2x(s) - s - 2\right) + 4\left(sx(s) - 1\right) + 3x(s) = \frac{1}{s+2}$$

$$X(s)\left[s^2 + 4s + 3\right] = \frac{1}{s+2} + s + 6 = \frac{(s+6)(s+2)+1}{s+2}$$

$$X(s) = \frac{s^2 + 8s + 13}{(s+1)(s+3)(s+2)} = \frac{k_1}{s+1} + \frac{k_2}{s+2} + \frac{k_3}{s+3}$$

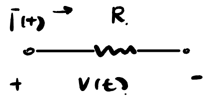
$$\Rightarrow k_1 = (s+1)\cdot X(s)\Big|_{s=-1} = \frac{1-8+13}{2\times 1} = 3.$$ So as $k_2 = -1$. $k_3 = -1$

$$X(s) = \frac{3}{s+1} - \frac{1}{s+2} - \frac{1}{s+3} \Rightarrow x(t) = 3e^{-t} - e^{-2t} - e^{-3t} \quad (t>0)$$

Chapter 10 : S- Domain Circuit.

10.1 Transformed Circuits

(A). Resistance.

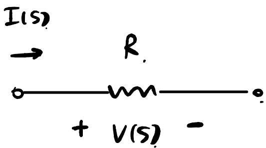


time domain element

$$v(t) = R\, i(t) \qquad \mathcal{L}\{v(t)\} = \mathcal{L}\{R\, i(t)\} = R\, \mathcal{L}\{i(t)\}$$

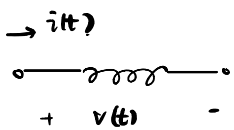
$$\boxed{V(s) = R\, I(s)}$$

S-Domain Ohm's law



S-Domain element.

(B) Inductance

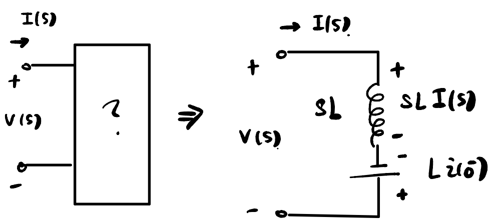


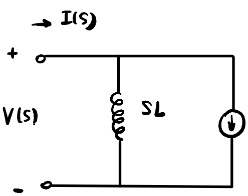
time domain

$$v(t) = L\,\frac{di(t)}{dt}.$$

$$\mathcal{L}\{v(t)\} = \mathcal{L}\left\{L\,\frac{di(t)}{dt}\right\} = L\,\mathcal{L}\left\{\frac{di(t)}{dt}\right\}$$

$$V(s) = L\left[sI(s) - i(0^-)\right] = sL\,I(s) - L\,i(0^-)$$ S Domain.



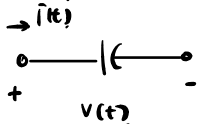


$$\frac{L\,i(0^-)}{sL} = \frac{i(0^-)}{s}$$

Initial Condition is a step function

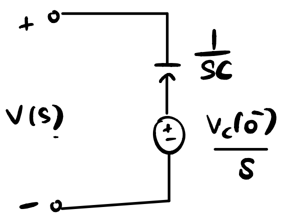
S-Domain model

(3) Capacitance



$$v(t) = \frac{1}{C}\int_0^t i(x)\,dx + V_c(0^-)$$

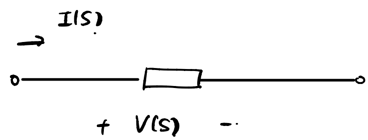
$$\mathcal{L}\{v(t)\} = \mathcal{L}\left\{\frac{1}{C}\int_0^t i(x)\,dx + V_c(0^-)\right\} \Rightarrow V(s) = \frac{1}{C}\cdot\frac{I(s)}{s} + \frac{V_c(0^-)}{s}$$



$$V(s) = \frac{1}{sC}I(s) + \frac{V_c(0^-)}{s}$$

S-Domain model

(B) Impedance and admittance



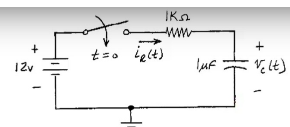
$$V(s) = R\,I(s)$$

$$V(s) = sL\,I(s)$$

$$V(s) = \frac{1}{sC}I(s)$$

$$Z = \begin{cases} R \\ sL \\ \frac{1}{sC} \end{cases}$$

Eg: An example of S-Domain



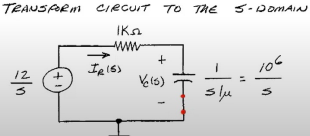
TRANSFORM THIS CIRCUIT INTO THE S-DOMAIN AND USE LAPLACE TRANSFORMS TO SOLVE FOR $V_c(t)$ AND $i_R(t)$.

SOLUTION:

- STEP 1) FIND THE INITIAL CONDITIONS

WITH THE SWITCH OPEN FOR A LONG TIME, $V_c(0^-) = 0$

- STEP 2) TRANSFORM CIRCUIT TO THE S-DOMAIN



- STEP 3) PERFORM S-DOMAIN CIRCUIT ANALYSIS

$$I_R(s) = \frac{\frac{12}{s}}{1K + \frac{10^6}{s}} = \frac{12}{(1K)s + 10^6} = \frac{12m}{s + 1K}$$

$$V_c(s) = \frac{12m}{s+1K}\cdot\frac{10^6}{s} = \frac{12K}{s(s+1K)}$$

- STEP 4) PERFORM PFE* (WHERE NECESSARY)

$$V_c(s) = \frac{k_1}{s} + \frac{k_2}{s+1K}$$

$$k_1 = s\cdot\frac{12K}{s(s+1K)}\bigg|_{s=0} = \frac{12K}{1K} = 12$$

- STEP 5) FIND THE INVERSE LAPLACE TRANSFORM

$$I_R(s) = \frac{12m}{s+1K}$$

$$i_R(t) = 12m\,e^{-1K\cdot t}u(t)$$

$$= \boxed{12m\,e^{-t/1m}u(t)\ A}$$

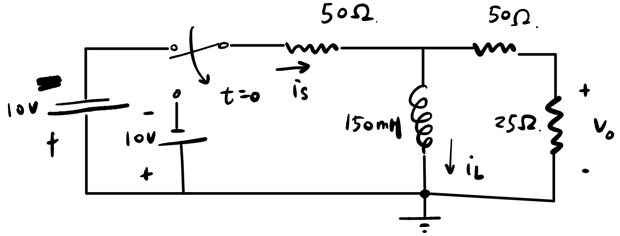
$$V_c(s) = \frac{12}{s} - \frac{12}{s+1K}$$

$$V_c(t) = 12u(t) - 12e^{-1K\cdot t}u(t)$$

$$= \boxed{12(1-e^{-t/1m})u(t)\ V}$$

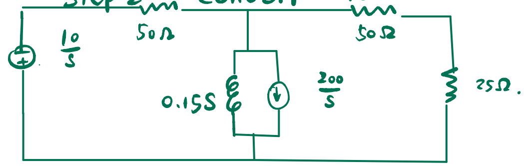
- NOTE: ALWAYS EXPRESS EXPONENTIALS AS $e^{-t/\tau}$ WHERE τ IS THE TIME CONSTANT

Practice Example: *.



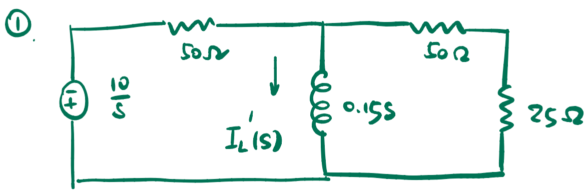
<Solution>: Step 1: Find $i_L(0^-) = \dfrac{10\,V}{50\,\Omega} = 200\,mA$

Step 2: Convert to S- Domain

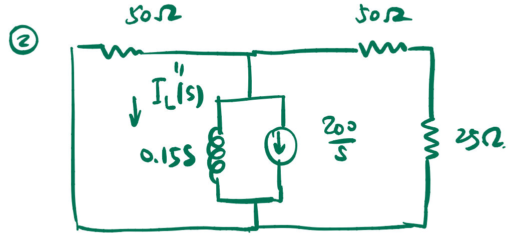


Applied Supper position method:

①.

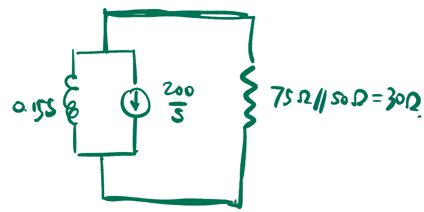


②



$$I_L'(s) = -\left(\frac{\frac{10}{S}}{50\Omega + 0.15S \,//\, (50\Omega + 25\Omega)} \right) \times \frac{\frac{1}{0.15S}}{\frac{1}{0.15S} + \frac{1}{50+25}}$$

$\rightarrow$



$$= \frac{-40}{S(S+200)}$$

$$I_L''(s) = \frac{200m}{S} \times \left(\frac{1}{\frac{1}{0.15S} + \frac{1}{30}} \right) \div 0.15S$$

$$= \frac{200m}{S} \cdot \left(\frac{1}{\frac{1}{0.15S} + \frac{1}{30}} \right) \times \frac{1}{0.15S}$$

$$= \frac{0.2}{S+200}$$

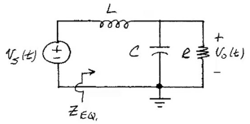
$$I_L(s) = I_L'(s) + I_L''(s) = \frac{0.2(S-200)}{S(S+200)}$$

Step 4: $I_L(s) = \dfrac{-0.2}{S} + \dfrac{0.4}{S+200}$

$$\rightarrow \boxed{i_L(t) = -0.2\,u(t) + 0.4e^{-200t}\,u(t)}$$

10.2 Basic Circuit Analysis In S- Domain

A) EXAMPLE 8.10 - REVISITED



THE INDUCTOR CURRENT AND CAPACITOR VOLTAGE ARE ZERO AT $t=0$.

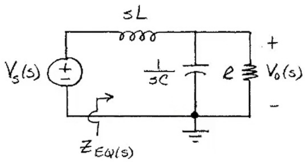
1) TRANSFORM THE CIRCUIT INTO THE S-DOMAIN AND FIND $Z_{EQ}(s)$ AND $V_o(s)$.

2) EVALUATE 1) FOR $L = 250mH$, $C = 0.5\mu F$, $R = 3K$ AND $s = j\omega = j2000$.

3) IF $V_s(t) = 100\cos(2000t)$, FIND $V_o(t)$ USING MATLAB. COMPARE WITH PSPICE.

SOLUTION:

1)



$$Z_{EQ} = sL + \frac{1}{sC + 1/R} = sL + \frac{R}{sCR+1}$$

$$= \boxed{\frac{s^2(LCR) + sL + R}{sCR + 1}}$$

$$V_o(s) = V_s(s)\frac{\frac{R}{sCR+1}}{\frac{R}{sCR+1} + sL}$$

$$= V_s(s)\frac{R}{R + s^2LCR + sL}$$

$$= \boxed{\frac{R}{s^2(LCR) + s(L) + R}V_s(s)}$$

2)

$$Z_{EQ} = \frac{s^2(0.25)(0.5\mu)(3K) + s(0.25) + 3K}{s(0.5\mu)(3K) + 1}$$

$$= \frac{s^2(375\mu) + s(0.25) + 3K}{s(1.5m) + 1}$$

IF $s = j\omega = j2000$ & $s^2 = -\omega^2 = -(2000)^2 = -4(10^6)$ THEN

$$Z_{EQ} = \frac{-4(10^6)(375\mu) + j(2K)(0.25) + 3K}{j(2K)(1.5m) + 1}$$

$$= \frac{-1.5K + j500 + 3K}{j3 + 1}$$

$$= \frac{1.5K + j500}{1 + j3} = \frac{1.5811K\angle 18.43^\circ}{3.162\angle 71.57^\circ}$$

$$= \boxed{500\angle -53.14^\circ\ \Omega}$$ ← SAME AS CH8 p25

$$V_o = \frac{3K}{s^2(0.25)(0.5\mu)(3K) + s(0.25) + 3K}V_s(s)$$

$$= \frac{3K}{s^2(375\mu) + s(0.25) + 3K}V_s(s)$$

IF $s = j2000$ & $s^2 = -4(10^6)$ THEN

$$V_o = \frac{3K}{-4(10^6)375\mu + j(2K)(0.25) + 3K}V_s(j\omega)$$

$$= \frac{3K}{-1.5K + j500 + 3K}V_s(j\omega)$$

$$= \frac{3K}{1.5K + j500}V_s(j\omega)$$

$$= \frac{3K\angle 0^\circ}{1.5811K\angle 18.43^\circ}V_s(j\omega)$$

$$= \boxed{(1.89\angle -18.43^\circ)V_s(j\omega)}$$

IF $V_s(j\omega) = 100\angle 0^\circ$ THEN $V_o = 189\angle -18.43^\circ$ V

↳ SAME AS CH8 p26

3) IF $V_s(t) = 100\cos(2000t)$ THEN

$$V_s(s) = 100\frac{s}{s^2 + (2K)^2}$$

$$= \frac{100s}{s^2 + 4(10^6)}$$

AND

$$V_o(s) = \frac{3K}{s^2(375\mu) + s(0.25) + 3K}V_s(s)$$

$$= \frac{8(10^6)}{s^2 + s(666.6) + 8(10^6)}\ \frac{100s}{s^2 + 4(10^6)}$$

10.3. Circuit Theorems in the S-Domain

A) PROPORTIONALITY

- FOR LINEAR RESISTIVE CIRCUITS THE PROPORTIONALITY THEOREM OF CH 3 STATES THAT ANY OUTPUT y IS PROPORTIONAL TO THE INPUT x, THAT IS,

$$y = K x$$

Network Function $= \dfrac{\text{Output}}{\text{Input}}$

- THE SAME IS TRUE FOR S-DOMAIN CIRCUITS

$$Y(s) = K(s)\, X(s)$$

WHERE $K(s)$ IS AN S-DOMAIN RATIONAL FUNCTION. IT IS SOMETIMES CALLED A "NETWORK FUNCTION".

C) SUPERPOSITION

- FOR LINEAR RESISTIVE CIRCUITS, THE SUPERPOSITION THEOREM OF CH. 3 STATES THAT ANY OUTPUT y OF A LINEAR CIRCUIT CAN BE WRITTEN AS

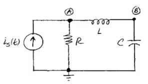
$$y = K_1 x_1 + K_2 x_2 + \cdots + K_n x_n$$

WHERE $x_1, x_2, \cdots, x_n$ ARE CIRCUIT INPUTS AND $K_1, K_2, \cdots, K_n$ ARE SCALE FACTORS THAT DEPEND ON THE CIRCUIT.

- THE SAME IS TRUE FOR S-DOMAIN CIRCUITS. WHERE $K_1(s), K_2(s), \cdots, K_n(s)$ ARE NETWORK FUNCTIONS.
- SUPERPOSITION ALSO APPLIES TO GROUPS OF SOURCES. SINCE INITIAL CONDITIONS WERE SHOWN TO BE SOURCES IN THE S-DOMAIN, WE COULD GROUP

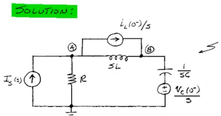
10.4 Node-Voltage Analysis in the S-Domain

A) EXAMPLE 10.10



FORMULATE THE S-DOMAIN EQUATIONS BY INSPECTION FOR THIS CIRCUIT INCLUDING INITIAL CONDITIONS. FIND $V_A(s)$

SOLUTION:



SOURCE TRANSFORM TO CREATE ALL CURRENT SOURCES

$$\begin{bmatrix} I_S - \frac{i_L(0^-)}{s} \\ \frac{i_L(0^-)}{s} + V_C(0^-)C \end{bmatrix} = \begin{bmatrix} G + \frac{1}{sL} & -\frac{1}{sL} \\ -\frac{1}{sL} & sC + \frac{1}{sL} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix}$$

$$V_A = \frac{\Delta_A}{\Delta} = \frac{\begin{vmatrix} I_S - \frac{i_L(0^-)}{s} & -\frac{1}{sL} \\ \frac{i_L(0^-)}{s} + V_C(0^-)C & sC + \frac{1}{sL} \end{vmatrix}}{\begin{vmatrix} G + \frac{1}{sL} & -\frac{1}{sL} \\ -\frac{1}{sL} & sC + \frac{1}{sL} \end{vmatrix}}$$

"CIRCUIT DETERMINANT" DOES NOT DEPEND ON INPUTS OR IC'S.

$$= \frac{\left(I_S - \frac{i_L(0^-)}{s}\right)\left(sC + \frac{1}{sL}\right) - \left(-\frac{1}{sL}\right)\left(\frac{i_L(0^-)}{s} + V_C(0^-)C\right)}{\left(G + \frac{1}{sL}\right)\left(sC + \frac{1}{sL}\right) - \left(-\frac{1}{sL}\right)\left(-\frac{1}{sL}\right)}$$

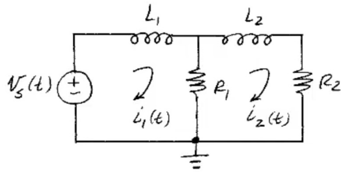
$$= \frac{I_S\left(sC + \frac{1}{sL}\right) - i_L(0^-)\left(C + \cancel{\frac{1}{s^2L}} - \cancel{\frac{1}{s^2L}}\right) + V_C(0^-)\frac{C}{sL}}{sCG + \frac{G}{sL} + \frac{C}{L} + \cancel{\frac{1}{s^2L^2}} - \cancel{\frac{1}{s^2L^2}}} \cdot \frac{sL}{sL}$$

$$= \frac{I_S(s^2LC + 1) - i_L(0^-)(sLC) + V_C(0^-)C}{s^2LCG + sC + G}$$

$$= \underbrace{\frac{I_S(s^2LC + 1)}{s^2LCG + sC + G}}_{\text{ZERO-STATE RESPONSE}} + \underbrace{\frac{-i_L(0^-)(sLC) + V_C(0^-)C}{s^2LCG + sC + G}}_{\text{ZERO-INPUT RESPONSE}}$$

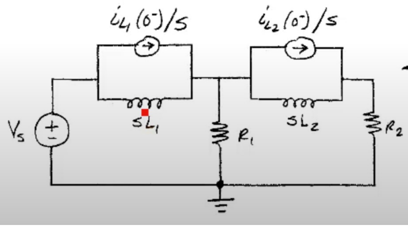
10.5 Mesh Current Analysis In the S-Domain

A) EXAMPLE

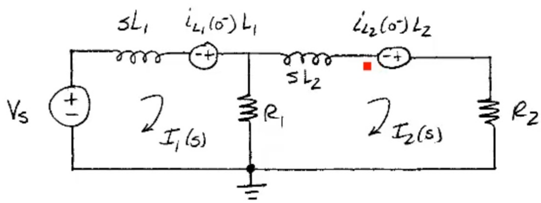


FORMULATE THE S-DOMAIN EQUATIONS BY INSPECTION INCLUDING INITIAL CONDITIONS

SOLUTION:



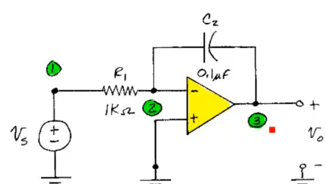
DOING SOURCE TRANSFORMATIONS TO CREATE ALL VOLTAGE SOURCES



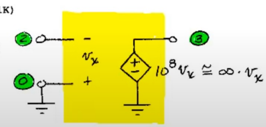
$$\begin{bmatrix} V_s + i_{L_1}(0^-)L_1 \\ i_{L_2}(0^-)L_2 \end{bmatrix} = \begin{bmatrix} sL_1 + R_1 & -R_1 \\ -R_1 & sL_2 + R_1 + R_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

One supplement Pr

S10.6) FOR THE CIRCUIT SHOWN BELOW




```
INTEGRATOR
VS 1 0 AC 2 -90 SIN (0 2 1K)
R1 1 2 1K
C2 2 3 .1U IC=0
EOA 3 0 0 2 100MEG
.PROBE
.AC DEC 200 1K 10K
.TRAN 12.5U 2.5M 0 12.5U
.END
```

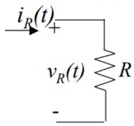


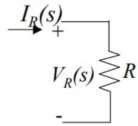
RUN THE ABOVE PSPICE FILE AND COMPLETE:

S-Domain Conversion Order

Element Transformations contd

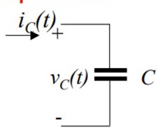
Resistor

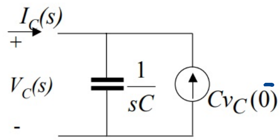


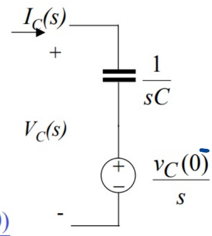


$$V_R(s) = RI_R(s)$$

Capacitor





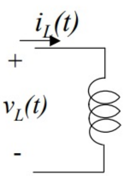


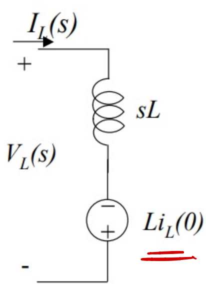
$$I_C(s) = sCV_C(s) - Cv_C(0-) \qquad V_C(s) = \frac{1}{sC}I_C(s) + \frac{v_C(0)}{s}$$

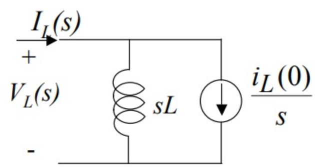
Note the source transformation rules apply!

Element Transformations contd

Inductors $\quad V_L(s) = sLI_L(s) - Li_L(0) \qquad I_L(s) = \frac{1}{sL}V_L(s) + \frac{i_L(0)}{s}$







Complex power: $\vec{I} = I_m \angle \theta \qquad \vec{I}^* = I_m \angle -\theta$

$$\vec{P} = \vec{V}\cdot\vec{I}^* = V_m I_m \angle \varphi_u - \varphi_i = V_m I_m \angle \varphi = V_m I_m \cos\varphi + j V_m I_m \sin\varphi = P + jQ$$

$$\vec{P} = \vec{U}\cdot\vec{I}^* = Z\cdot\vec{I}\cdot\vec{I}^* = Z\cdot\vec{I}^2 = (R+jX)\vec{I}^2 = R\vec{I}^2 + jX\vec{I}^2$$

or $\vec{P} = \vec{V}\cdot\vec{I}^* = \vec{V}\cdot(\vec{V}Y)^* = \vec{V}\,\vec{V}^*\cdot Y^* = \vec{V}^2\, Y^*$

Chapter 11: Network Functions

11.1 Definition of network function

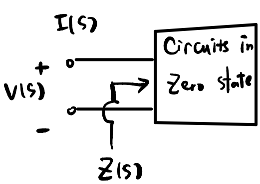
$$\text{Network function} = T(s) = \frac{\text{zero-state response function}}{\text{input signal transform}} = \frac{Y(s)}{X(s)}$$

$$Y(s) = T(s)\cdot X(s) = \underbrace{\sum_{j=1}^{N} \frac{k_j}{s-p_j}}_{\text{natural poles}} + \underbrace{\sum_{\ell=1}^{M} \frac{k_\ell}{s-p_\ell}}_{\text{forced poles}}$$

$$\Rightarrow y(t) = \sum_{j=1}^{N} k_j e^{p_j t} + \sum_{\ell=1}^{M} k_\ell e^{p_\ell t}$$

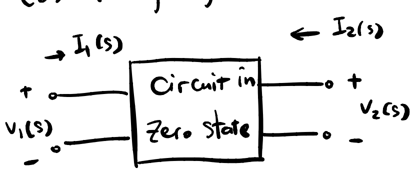
11.2 Network Functions of One & two Port Circuits

(A) Driving point impedance.



$$Z(s) = \frac{V(s)}{I(s)}$$

(B) Transfer function



$$T_V(s) = \frac{V_2(s)}{V_1(s)} \quad \text{voltage transfer function}$$

$$T_I(s) = \frac{I_2(s)}{I_1(s)} \quad \text{Current transfer function}$$

$$T_Y(s) = \frac{I_2(s)}{V_1(s)} \quad \text{transfer admittance.}$$

$$T_Z(s) = \frac{V_2(s)}{I_1(s)} \quad \text{transfer impedance}$$

11.3 Network functions and Impulse response

"Impulse response" when input function $x(t) = \delta(t)$, $y(t) = h(t)$

$Y(s) = H(s)$

$$Y(s) = T(s) \cdot X(s) = T(s) \cdot 1 \triangleq H(s)$$

$$\mathcal{L}\{h(t)\} = T(s) \qquad y(t) = \mathcal{L}^{-1}[H(s) \cdot X(s)]$$

在**信号与系统**或**控制理论**中，不同的符号各自有传统用途，但在不同的教材或论文里可能会出现一些**符号混用或命名差异**。以下是最常见的用法及其含义：

1. $x(t)$、$y(t)$
   - 一般用于表示**时域**下的输入（input）信号和输出（output）信号。
   - 相应地，它们的拉普拉斯变换通常记为**$X(s)$、$Y(s)$**。
2. $h(t)$
   - 一般表示系统的**脉冲响应**（**Impulse Response**），即当输入为单位冲激 $\delta(t)$ 时，系统的输出信号。
   - 它的拉普拉斯变换通常记为**$H(s)$**。
   - 有时在控制系统或某些场合，也可能把系统的脉冲响应记为 $g(t)$ 或其他符号，但 $h(t)$ 是最常见的约定。
3. $H(s)$ **或** $T(s)$ **或** $G(s)$
   - 在**拉普拉斯域**分析中，用于表示**系统的传递函数**（**Transfer Function**）。
   - 在"信号与系统"课程中，常用 $H(s)$ 来代表系统函数；在"控制理论"中，可能更常见到 $G(s)$ 或 $T(s)$ 来表示系统的传递函数。
   - 本质上，这些符号在功能上是等价的：都表示"输出对输入的拉普拉斯变换之比"，即
$$H(s) = \frac{Y(s)}{X(s)}.$$

$X(s) =$

$\mathcal{L}[\delta(t)] = 1$

$$T(s) = H(s) = \frac{Y(s)}{X(s)} = Y(s)$$

拉普拉斯变化 $\qquad s \to$ differentiation

$$\mathcal{L}[h'(t)] = sH(s) - h(0)$$

$\frac{1}{s} \to$ integrator

$$\mathcal{L}\left[\int_0^t h(\tau)\, d\tau\right] = \frac{H(s)}{s}$$

11.4 Network function and Step response.

Step response: when input function $x(t) = u(t)$, $y(t) = g(t)$

$$Y(s) = T(s) \cdot X(s) = T(s) \cdot \frac{1}{s} = G(s) \qquad Y(s) = G(s)$$

$$G(s) = \underbrace{\frac{k_0}{s}}_{\text{forced pole}} + \underbrace{\frac{k_1}{s-p_1} + \frac{k_2}{s-p_2} + \cdots + \frac{k_N}{s-p_N}}_{\text{natural pole}}$$

$$g(t) = \underbrace{k_0 u(t)}_{\text{forced response}} + \Big[\underbrace{k_1 e^{p_1 t} + k_2 e^{p_2 t} + \cdots + k_N e^{p_N t}}_{\text{natural response}} \Big] u(t)$$

11.6 Impulse response and Convolution

"Convolution"

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau$$

个人理解: 对于输入 $x(t) = \delta(t)$ 输出 $y(t) = h(t)$ 来说. 转换函数 $T(s) = Y(s)$

把现在这个函数分成无穷多个小块 每一块 输出 = 输入 × $h(t)$

对于 $x(t) = u(t)$ $y(t) = g(t)$

$$Y(s) = H(s) \cdot \frac{1}{s} \Rightarrow g(t) = \mathcal{L}^{-1}\left\{ \frac{H(s)}{s} \right\} = \int_0^t h(\tau)\, d\tau$$

$$\Rightarrow g(t) = \int_0^t h(\tau)\, d\tau \iff h(t) = \frac{d}{dt} g(t)$$

equivalence of S-Domain and t-domain Convolution

$$Y(s) = H(s)\cdot X(s) = H(s)\left[\int_0^\infty \underbrace{x(\tau)e^{-s\tau}}_{\mathcal{L}[x(\tau)]} d\tau\right]$$

$$= \int_0^\infty \left[H(s)\cdot e^{-s\tau}\right]\cdot x(\tau)\,d\tau$$

$$H(s)\cdot e^{-s\tau} = \mathcal{L}\left[h(t-\tau)\cdot u(t-\tau)\right]$$

$$= \int_0^\infty h(t-\tau)\,u(t-\tau)\cdot e^{-st}\,dt$$

$$\therefore Y(s) = \int_0^\infty \left[\int_0^\infty h(t-\tau)\,u(t-\tau)\,e^{-st}\,dt\right] x(\tau)\,d\tau$$

$$= \int_0^\infty \left[\int_0^\infty h(t-\tau)\,u(t-\tau)\;x(\tau)\,d\tau\right] e^{-st}\,dt$$

$$= \int_0^\infty \left[\int_0^t h(t-\tau)\,x(\tau)\,d\tau\right] e^{-st}\,dt$$

since $u(t-\tau)=0$
$t<\tau$
$\Rightarrow \tau>t$

$$\Rightarrow Y(s) = \mathcal{L}\left[\int_0^t h(t-\tau)\,x(\tau)\,d\tau\right]$$

$\triangle$
$x(t) * h(t)$

$$y(t) = \mathcal{L}^{-1}\left[Y(s)\right] = \mathcal{L}^{-1}\left[H(s)\cdot X(s)\right] = h(t) * x(t)$$

$$F_1(s)\cdot F_2(s) = \mathcal{L}\left[f_1(t) * f_2(t)\right]$$

or $$f_1(t) * f_2(t) = \mathcal{L}^{-1}\left[F_1(s)\cdot F_2(s)\right]$$

12-a

Frequency response

Review. $s \leftrightarrow jw$. $T_v(s) = \frac{V_2(s)}{V_1(s)}$ or $\frac{V_{out}(s)}{V_{in}(s)}$ $H(s) = T_v(s)\Big|_{V_1(s)=1}$ $G(s) = T_v(s)\Big|_{V_1(s)=\frac{1}{s}}$

$$\Rightarrow G(s) = \frac{H(s)}{s}$$

Review of dB:

Decibels (dB): $\text{Bel} \equiv \log_{10}\left(\frac{P_{out}}{P_{in}}\right)$

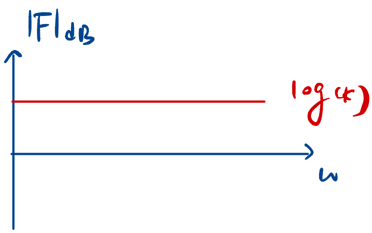
$$\text{Decibel (dB)} = 10 \log_{10}\left(\frac{P_{out}}{P_{in}}\right)$$

Since $P \propto V^2 \Rightarrow$ $\text{Decibel (dB)} = 10 \log_{10}\left(\frac{V_{out}}{V_{in}}\right)^2 = 20 \log_{10}\left(\frac{V_{out}}{V_{in}}\right)$

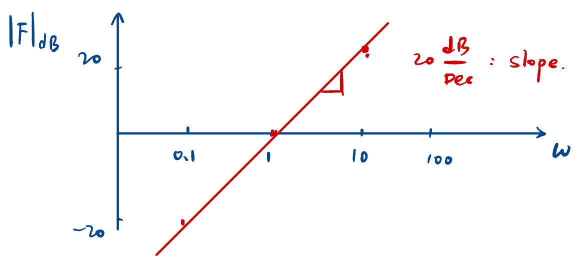
Power ratio: $10 \log_{10}\left(\frac{P_{out}}{P_{in}}\right)$. voltage ratio: $20 \log_{10}\left(\frac{V_{out}}{V_{in}}\right)$ Current ratio: $20 \log_{10}\left(\frac{I_{out}}{I_{in}}\right)$

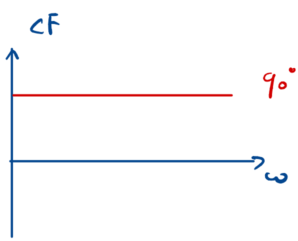
log Function

(a) $F(jw) = K. \Rightarrow |F|_{dB} = 20\log(K)$



(b) $F(jw) = jw.$ $|F|_{dB} = 20\log\sqrt{0^2+w^2} = 20\log(w)$

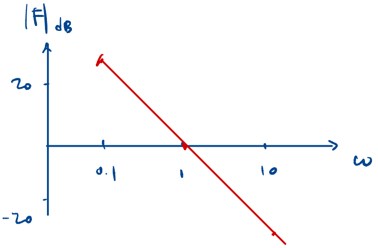


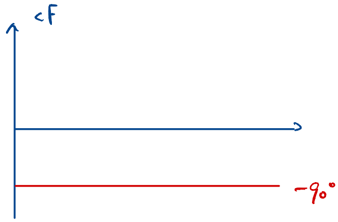


$$\angle F = \tan^{-1}\left(\frac{w}{0}\right) = 90^\circ$$

(c) $F(s)=\frac{1}{s} \Rightarrow F(j\omega)=\frac{1}{j\omega}$

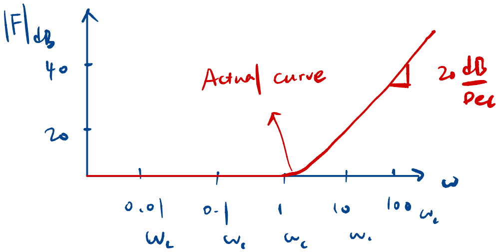
$$|F|_{dB} = 20\log|F| = 20(\log 1-\log\omega) = -20\log\omega$$

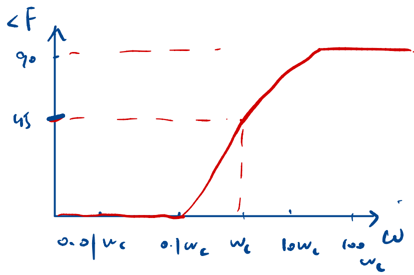




(d) $F(j\omega)=1+j\frac{\omega}{\omega_c}$ $\qquad \omega_c$ = constant.

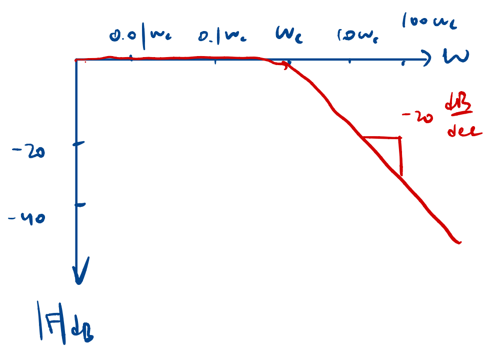
$$|F(j\omega)|_{dB} = 20\log\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2} \qquad \angle F(j\omega) = \tan^{-1}\left(\frac{\omega}{\omega_c}\right)$$

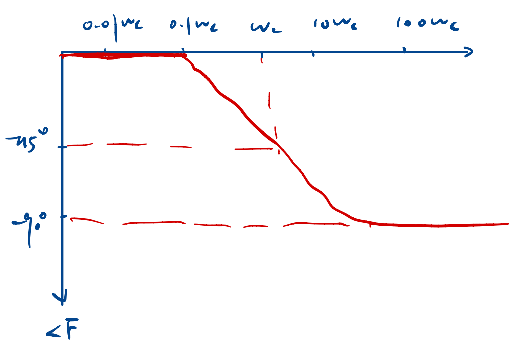




(e) $F(j\omega)=\dfrac{1}{1+j\frac{\omega}{\omega_c}}$

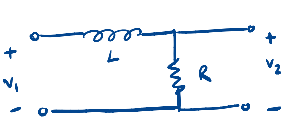
$$|F(j\omega)|_{dB} = 20\log(1) - 20\log\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2} = -20\log\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2} \qquad \angle F = \tan^{-1}(0)-\tan^{-1}\left(\frac{\omega}{\omega_c}\right) = \tan^{-1}\left(\frac{\omega}{\omega_c}\right)$$



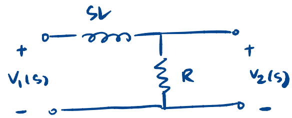


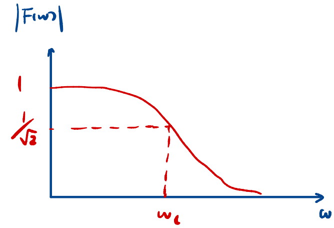
12-b

low-pass filter



S-Domain $\Rightarrow$





$$F(s)=\frac{V_2}{V_1}=\frac{R/L}{s+R/L} \qquad F(j\omega)=\frac{R/L}{j\omega+R/L}$$

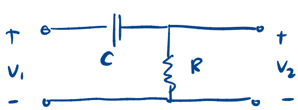
$$|F(j\omega)|=\frac{R/L}{\sqrt{\omega^2+(R/L)^2}} \qquad \theta(j\omega)=\tan^{-1}(0)-\tan^{-1}\left(\frac{\omega}{R/L}\right)=\tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$|F(j\omega_c)|=\frac{R/L}{\sqrt{\omega_c^2+(R/L)^2}}=\frac{1}{\sqrt{2}}|F(j\omega)|_{max}=\frac{1}{\sqrt{2}}\frac{R/L}{\sqrt{\omega^2+(R/L)^2}} \Rightarrow \boxed{\omega_c=\frac{R}{L}}$$

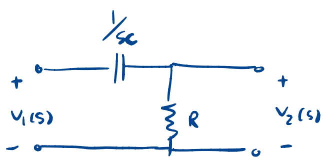
0

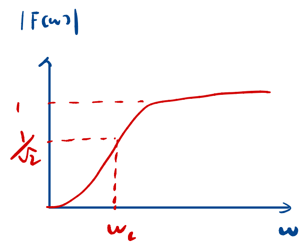
$$F(s)=\frac{\omega_c}{s+\omega_c}$$

high-pass filter



S-Domain $\Longrightarrow$



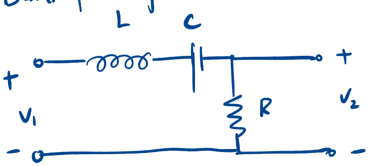


$$F(s)=\frac{V_2}{V_1}=\frac{R}{\frac{1}{sC}+R}=\frac{s}{s+1/RC} \qquad F(j\omega)=\frac{j\omega}{j\omega+1/RC}$$

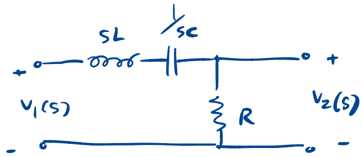
$$|F(j\omega)|=\frac{\omega}{\sqrt{\omega^2+\left(\frac{1}{RC}\right)^2}} \qquad \theta(j\omega)=\tan^{-1}\left(\frac{\omega}{0}\right)-\tan^{-1}(\omega RC)=90^\circ-\tan^{-1}(\omega RC)$$

$$|F(j\omega_c)|=\frac{\omega_c}{\sqrt{\omega_c^2+\left(\frac{1}{RC}\right)^2}}=\frac{1}{\sqrt{2}} \Rightarrow \boxed{\omega_c=\frac{1}{RC}} \qquad F(s)=\frac{s}{s+\omega_c}$$

Band-pass filter.



S-Domain $\Longrightarrow$

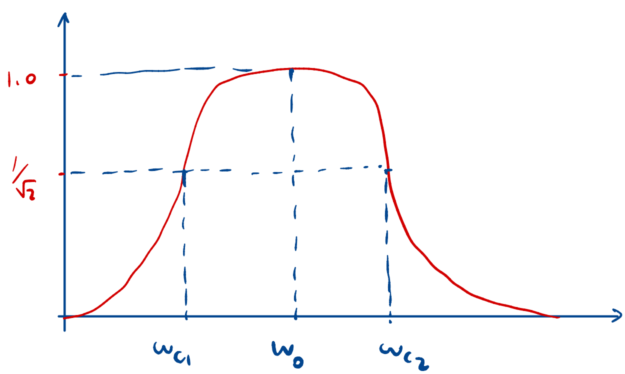


$$F(s) = \frac{V_2}{V_1} = \frac{R}{R + sL + 1/sC} = \frac{\left(\frac{R}{L}\right) \cdot s}{s^2 + \left(\frac{R}{L}\right) \cdot s + \frac{1}{LC}} \Rightarrow F(j\omega) = \frac{j\left(\frac{R}{L}\right)\omega}{-\omega^2 + j\left(\frac{R}{L}\right)\omega + \frac{1}{LC}}$$

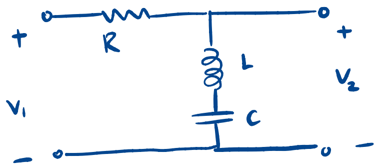
$$|F(j\omega)| = \frac{\omega\left(\frac{R}{L}\right)}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left[\omega \frac{R}{L}\right]^2}} \qquad \theta(j\omega) = 90^\circ - \tan^{-1}\left[\frac{\omega\left(\frac{R}{L}\right)}{\left(\frac{1}{LC}\right) - \omega^2}\right]$$

$$\frac{1}{LC} - \omega^2 = 0 \Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}}} \qquad \omega_0 = \sqrt{\omega_{c_1}\omega_{c_2}}$$

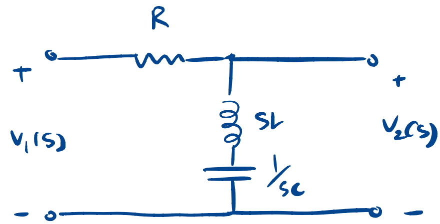
$$|F(j\omega_c)| = \frac{1}{\sqrt{2}}\left|F(j\omega)_{max}\right| = \frac{1}{\sqrt{2}} \Rightarrow \boxed{\begin{aligned} \omega_{c_1} &= -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} \\ \omega_{c_2} &= \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} \end{aligned}}$$



Band-stop filter (or band rejected filter)



S-Domain $\Longrightarrow$

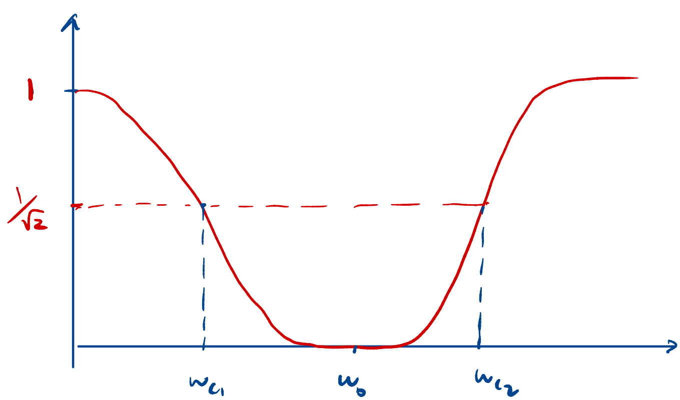


$$F(s) = \frac{V_2}{V_1} = \frac{sL + 1/sC}{sL + 1/sC + R} = \frac{s^2 + \frac{1}{LC}}{s^2 + \frac{R}{L}s + 1/LC} \Rightarrow F(j\omega) = \frac{-\omega^2 + 1/LC}{-\omega^2 + j\omega\frac{R}{L} + 1/LC}$$

$$|F(j\omega)| = \frac{\left|\frac{1}{LC} - \omega^2\right|}{\sqrt{\left[\frac{1}{LC} - \omega^2\right]^2 + \left[\frac{\omega R}{L}\right]^2}} \qquad \theta(j\omega) = -\tan^{-1}\left[\frac{\frac{\omega R}{L}}{\frac{1}{LC} - \omega^2}\right]$$

$$\omega^2 - \frac{1}{LC} = 0 \Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}}} \qquad \omega_0 = \sqrt{\omega_{c1}\omega_{c2}}$$

$$|F(j\omega_c)| = \frac{1}{\sqrt{2}}\,|F(j\omega)|_{max} = \frac{1}{\sqrt{2}} \Rightarrow \boxed{\begin{aligned} \omega_{c1} &= -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \\ \omega_{c2} &= \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \end{aligned}}$$



Band width $= \omega_{c2} - \omega_{c1}$

Quality factor $Q = \dfrac{\sqrt{L/C}}{R}$

$$= \frac{\omega_0}{BW}$$

$$= \frac{\omega_{c1} \cdot \omega_{c2}}{\omega_0}$$

13-a.

Frontier Series periodic function

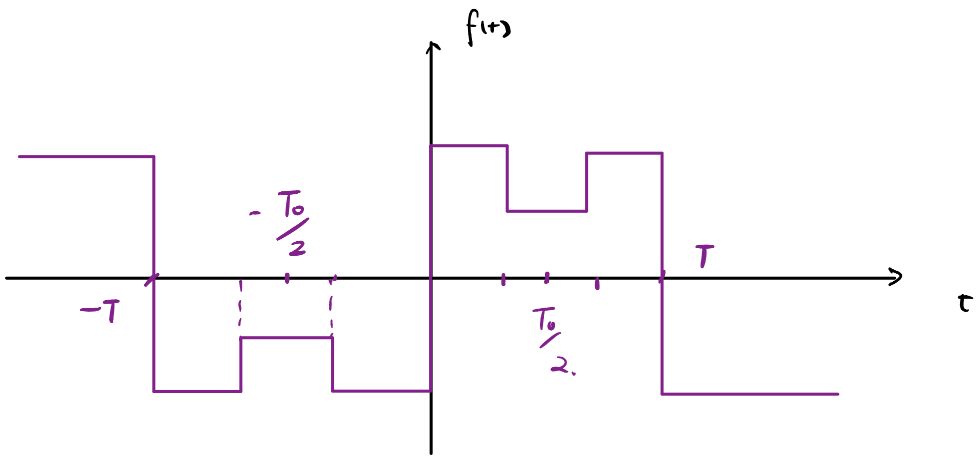
Suppose $f(t)$ with a periodic T_0. $T_0 = \frac{1}{f_0}$, then we can deduct that:

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos(2\pi n f_0 t) + b_n \sin(2\pi n f_0 t) \right]$$

$$a_0 = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} f(t)\, dt \qquad a_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} f(t) \cos(2\pi n t/T_0)\, dt \qquad b_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} f(t) \sin(2\pi n t/T_0)\, dt$$

Quarter wave Periodic Symmetry

every $\frac{1}{4}$ flip or symmetry



Half Wave Symmetry $\quad x(t + T/2) = -x(t)$

For even function $\quad a_0 \neq 0 \quad a_n \neq 0 \quad b_n = 0$

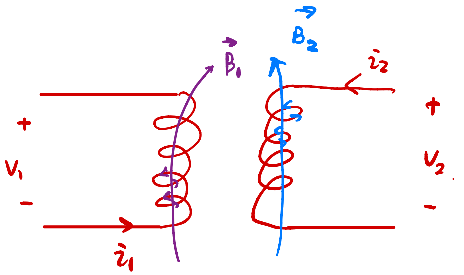
For odd function $\quad a_0 = 0 \quad a_n = 0 \quad b_n \neq 0$

15 a

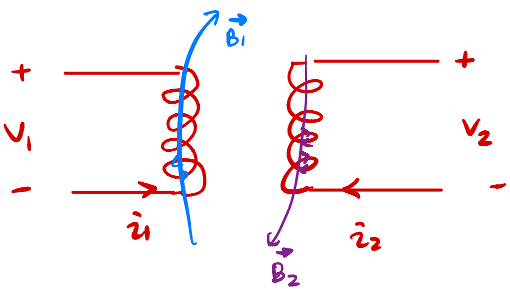
Flux $\lambda = N \Phi$

Voltage $v = -\frac{d\lambda}{dt} = -N\frac{d\Phi}{dt} = -N\frac{d(Li)}{dt} = -NL\frac{di}{dt}$

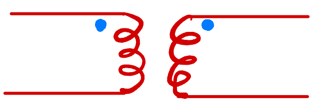
For mutual inductance $v = -M\frac{di}{dt}$

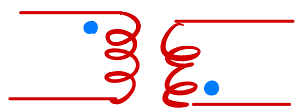


$$\begin{cases} V_1 = L_1\frac{di_1}{dt} - M\frac{di_2}{dt} \\ V_2 = L_2\frac{di_2}{dt} - M\frac{di_1}{dt} \end{cases}$$



$$\begin{cases} V_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \\ V_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}. \end{cases}$$

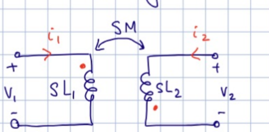


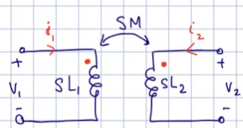


$V_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$.

$V_2 = L_1\frac{di_2}{dt} - M\frac{di_2}{dt}$

In the frequency domain (Sinusoidal Steady State)





$V_1 = SL_1 I_1(s) - SM I_2(s)$

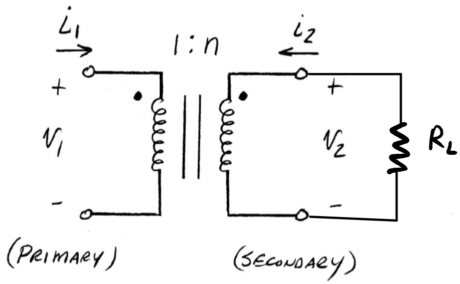
$V_2 = SL_2 I_2(s) - SM I_1(s)$

$V_1 = SL_1 I_1(s) + SM I_2(s)$

$V_2 = SL_2 I_2(s) + SM I_1(s)$

C) IDEAL TRANSFORMER

Direction



$V_2(t) = +n\, V_1(t)$

$i_2(t) = -\frac{1}{n}\, i_1(t)$

$V_2(t) = R_L \cdot (-i_2(t))$

$\Rightarrow \quad \frac{V_2(t)}{i_2(t)} = -R_L$

FOR THIS CASE :

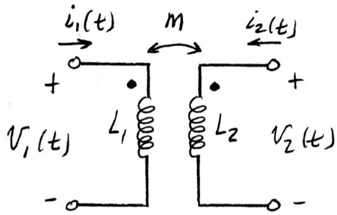
$$\boxed{\frac{V_2}{V_1} = +n \quad \& \quad \frac{i_2}{i_1} = -\frac{1}{n}}$$

$$R_{EQ} = \frac{V_1(t)}{i_1(t)} = \frac{\frac{1}{n} V_2(t)}{-n\, i_2(t)} = -\frac{1}{n^2}\left(\frac{V_2(t)}{i_2(t)}\right)$$

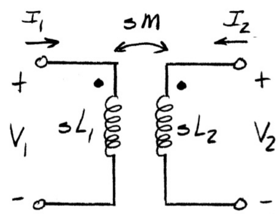
$$= -\frac{1}{n^2}(-R_L) = \frac{R_L}{n^2}$$

$$\boxed{R_{EQ} = \frac{R_L}{n^2}}$$

WHERE $\frac{N_2}{N_1} = \frac{n}{1}$

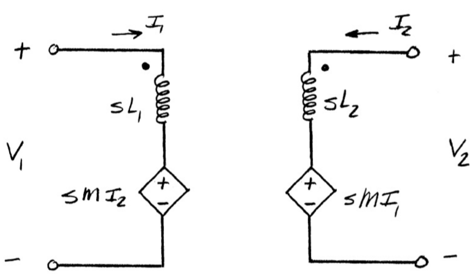


TIME-DOMAIN CIRCUIT

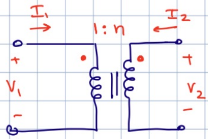


S-DOMAIN CIRCUIT

- THE S-DOMAIN EQUATIONS CAN BE MODELED AS THE FOLLOWING:

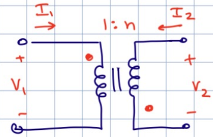


The four possible configurations for an ideal transformer are:



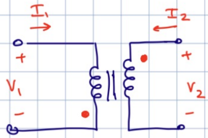
$$I_2 = -\frac{I_1}{n}$$

$$V_2 = n\,V_1$$



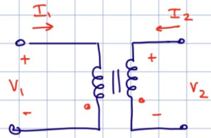
$$I_2 = \frac{I_1}{n}$$

$$V_2 = -nV_1$$



$$I_2 = \frac{I_1}{n}$$

$$V_2 = -n\,V_1$$



$$I_2 = -\frac{I_1}{n}$$

$$V_2 = nV_1$$

Chapter 16.

$$v(t) = V_A \cos(\omega t + \theta_v) \qquad \theta_v = \text{voltage phasor angle.}$$

$$i(t) = I_A \cos(\omega t + \theta_i) \qquad \theta_i = \text{Current phasor angle}$$

$\Rightarrow$

$$i(t) = I_A \cdot \cos(\omega t)$$

$$v(t) = V_A \cdot \cos(\omega t + \theta_v - \theta_i)$$

$$p = vi = V_A I_A \left[\cos(\omega t + \theta_v - \theta_i)\right]\left[\cos(\omega t)\right]$$

tips: $\cos\alpha \cos\beta = \frac{1}{2}\cos(\alpha-\beta) + \frac{1}{2}\cos(\alpha+\beta)$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$$p = \frac{V_A I_A}{2}\left[\cos\left((\omega t + \theta_v - \theta_i) - (\omega t)\right) + \cos\left((\omega t + \theta_v - \theta_i) + \omega t\right)\right]$$

$$= \frac{V_A I_A}{2}\left[\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v - \theta_i)\right]$$

$$= \underbrace{\frac{V_A I_A}{2}\cos(\theta_v - \theta_i)}_{\text{DC Component}} + \underbrace{\frac{V_A I_A}{2}\cos(\theta_v - \theta_i)\cos(2\omega t) - \frac{V_A I_A}{2}\sin(\theta_v - \theta_i)\sin(2\omega t)}_{\text{AC Component}}$$

$$\Rightarrow p = P_{DC} + P\cos(2\omega t) - Q\sin(2\omega t)$$

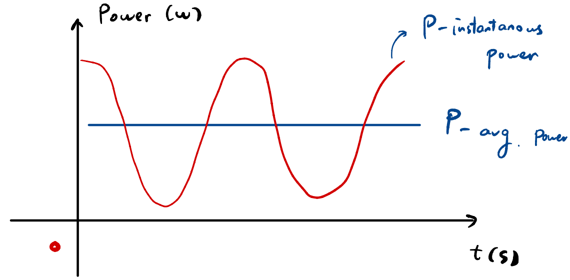
$$P = \frac{V_A I_A}{2}\cos(\theta_v - \theta_i) \qquad Q = \frac{V_A I_A}{2}\sin(\theta_v - \theta_i)$$

P: average power

Q: reactive power

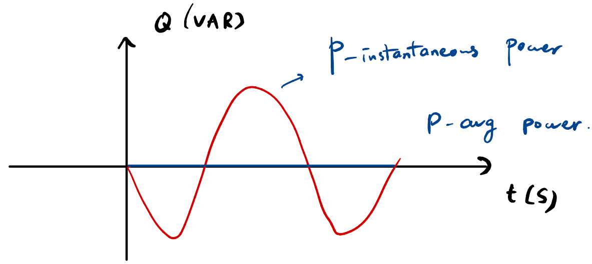
- For Purely resistive Circuits

$$P = P_{DC} + P \cdot \cos(2\omega t)$$



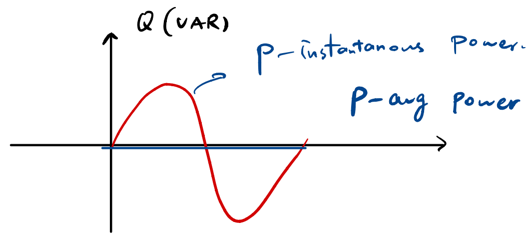
- For Purely Inductive circuits

$$P = -Q \sin(2\omega t)$$



- For purely Capacitive power.

$$P = Q \cdot \sin(2\omega t)$$



power factor. $\quad Pf = \cos(\theta_v - \theta_i)$

$$I_{rms} = \frac{I_A}{\sqrt{2}} \qquad V_{rms} = \frac{V_A}{\sqrt{2}}$$

$$P = V_{rms} \cdot I_{rms} \cos(\theta_v - \theta_i)$$

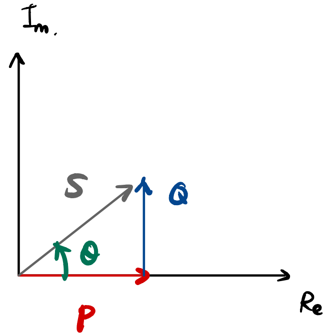
$$Q = V_{rms} \cdot I_{rms} \cdot \sin(\theta_v - \theta_i)$$

Complex power S.

$$S = V \cdot I^{*} = V_{rms} \cdot I_{rms} \cdot e^{j(\theta_v - \theta_i)}$$

$$S = P + jQ = |S| \cdot \angle\theta \quad (\omega)$$

$$\theta = \tan^{-1}\left(\frac{Q}{P}\right)$$



$\theta = \theta_v - \theta_i > 0 \Rightarrow$ lagging power factor. (inductive)

$\theta = \theta_v - \theta_i < 0 \Rightarrow$ leading power factor (capacitive)

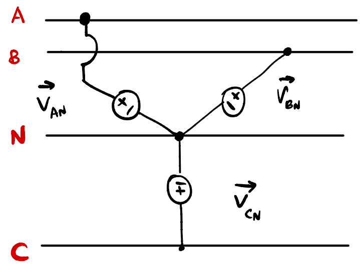
$$|S| = V_{rms} \cdot I_{rms}$$

$$P = V_{rms} \cdot I_{rms} \cos\theta$$

$$Q = V_{rms} \cdot I_{rms} \cdot \sin\theta$$

$$Z = \frac{P + jQ}{I_{rms}^{2}}$$

Three Phase Circuit.



$$\vec{V}_{AB} = \vec{V}_{AN} - \vec{V}_{BN}$$

$$\vec{V}_{BC} = \vec{V}_{BN} - \vec{V}_{CN}$$

$$\vec{V}_{CA} = \vec{V}_{CN} - \vec{V}_{AN}$$

A " Balanced three phase Source" produces phase voltage that satisfy :

$$|\vec{V}_{AN}| = |\vec{V}_{BN}| = |\vec{V}_{CN}| = V_p \qquad \boxed{V_L = \sqrt{3}\, V_p}$$

AND.

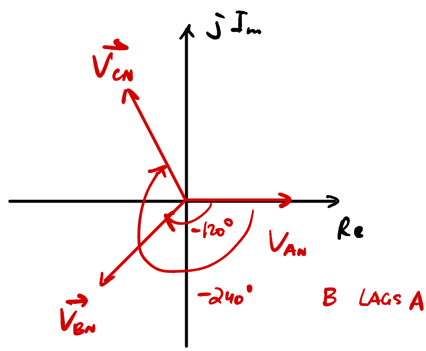
$$\vec{V}_{AN} + \vec{V}_{BN} + \vec{V}_{CN} = 0 + j \cdot 0$$

① positive phase (ABC)

$$\vec{V}_{AN} = V_p \angle 0^\circ$$

$$\vec{V}_{BN} = V_p \angle -120^\circ$$

$$\vec{V}_{CN} = V_p \angle -240^\circ$$

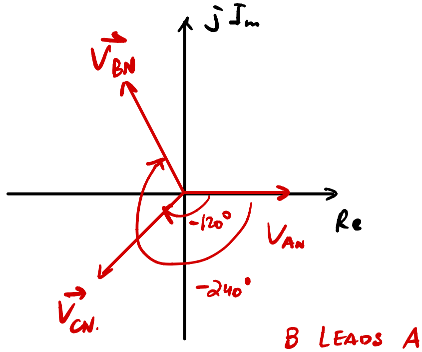


② Negative phase

$$\vec{V}_{AN} = V_p \angle 0^\circ$$

$$\vec{V}_{CN} = V_p \angle -120^\circ$$

$$\vec{V}_{BN} = V_p \angle -240^\circ$$

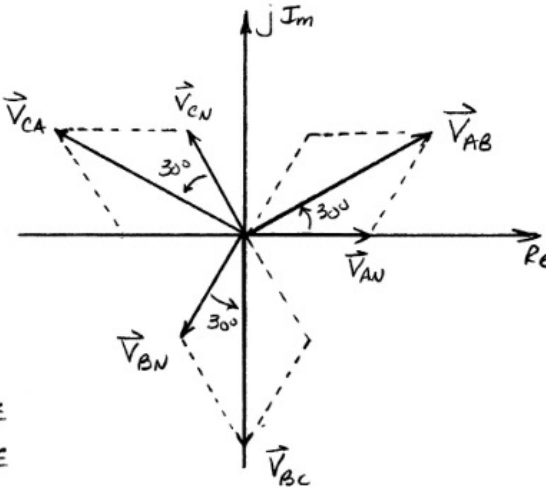


RELATIONSHIPS*

$$\vec{V}_{AB} = \vec{V}_{AN} - \vec{V}_{BN}$$
$$= V_p \angle 0^\circ - V_p \angle -120^\circ$$
$$= V_p (1 + j0) - V_p \left(-\tfrac{1}{2} - j\tfrac{\sqrt{3}}{2}\right)$$
$$= V_p \left(\tfrac{3}{2} + j\tfrac{\sqrt{3}}{2}\right) = \sqrt{3}\, V_p \left(\tfrac{\sqrt{3}}{2} + j\tfrac{1}{2}\right)$$
$$= \underline{\sqrt{3}\ V_p} \angle 30^\circ .$$

$$\vec{V}_{BC} = \vec{V}_{BN} - \vec{V}_{CN}$$
$$= V_p \angle -120^\circ - V_p \angle -240^\circ$$
$$= V_p \left(-\tfrac{1}{2} - j\tfrac{\sqrt{3}}{2}\right) - V_p \left(-\tfrac{1}{2} + j\tfrac{\sqrt{3}}{2}\right)$$
$$= V_p (0 - j\sqrt{3})$$
$$= \sqrt{3}\, V_p \angle -90^\circ .$$

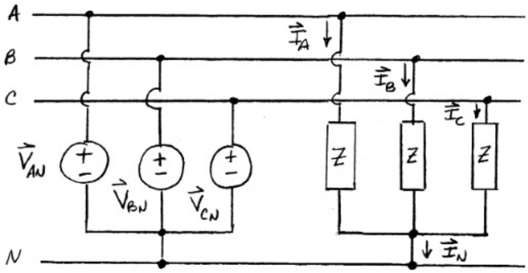
$$\vec{V}_{CA} = \vec{V}_{CN} - \vec{V}_{AN}$$
$$= V_p \angle -240^\circ - V_p \angle 0^\circ$$
$$= V_p \left(-\tfrac{1}{2} + j\tfrac{\sqrt{3}}{2}\right) - V_p (1 + j0)$$
$$= V_p \left(-\tfrac{3}{2} + j\tfrac{\sqrt{3}}{2}\right) = \sqrt{3}\, V_p \left(-\tfrac{\sqrt{3}}{2} + j\tfrac{1}{2}\right)$$
$$= \sqrt{3}\, V_p \angle -210^\circ$$



Y-Connected Source & Y. load

Y 电压√3倍 电流不变

△ 电压不变 电流√3倍



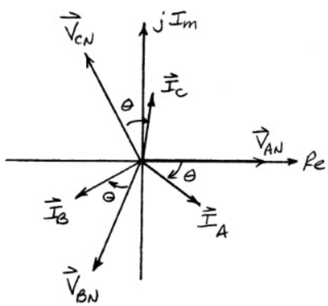
$$\vec{I}_A = \frac{\vec{V}_{AN}}{Z} = \frac{V_P\angle 0^\circ}{|Z|\angle\theta} = \underbrace{\frac{V_P}{|Z|}}_{I_L}\angle -\theta$$

$$\vec{I}_B = \frac{\vec{V}_{BN}}{Z} = \frac{V_P\angle -120^\circ}{|Z|\angle\theta} = \frac{V_P}{|Z|}\angle -120^\circ-\theta$$

$$\vec{I}_C = \frac{\vec{V}_{CN}}{Z} = \frac{V_P\angle -240^\circ}{|Z|\angle\theta} = \frac{V_P}{|Z|}\angle -240^\circ-\theta$$

$$I_L = \frac{V_P}{|Z|}$$

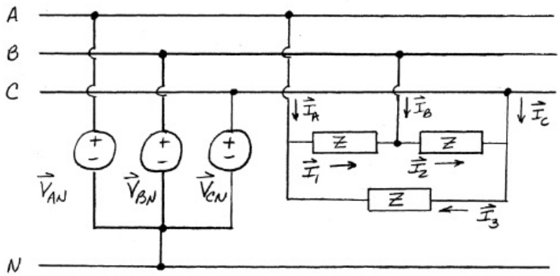
FOR A Y-CONNECTED LOAD



$$\vec{I}_N = \vec{I}_A + \vec{I}_B + \vec{I}_C = 0$$

$$\begin{aligned}
S_L &= \vec{V}_{AN}\cdot\vec{I}_A^{\,*} + \vec{V}_{BN}\cdot\vec{I}_B^{\,*} + \vec{V}_{CN}\,\vec{I}_C^{\,*} \\
&= (V_P\angle 0)(I_L\angle\theta) + (V_P\angle -120^\circ)(I_L\angle 120^\circ+\theta) \\
&\quad + (V_P\angle -240^\circ)(I_L\angle 240^\circ+\theta) \\
&= V_P I_L\angle\theta + V_P I_L\angle\theta + V_P I_L\angle\theta \\
&= \boxed{3V_P I_L\angle\theta} \\
&= 3\frac{V_L}{\sqrt{3}} I_L\angle\theta = \boxed{\sqrt{3}\,V_L I_L\angle\theta}
\end{aligned}$$

Y Connected Source & Δ Circuit Source



- TO FIND THE TOTAL COMPLEX POWER WE NEED TO SOLVE FOR THE PHASE CURRENTS $\vec{I}_1$, $\vec{I}_2$ & $\vec{I}_3$

$$\vec{I}_1 = \frac{\vec{V}_{AB}}{Z} = \frac{V_L \angle 30^\circ}{|Z| \angle \theta} = \overbrace{\frac{V_L}{|Z|}}^{I_p} \angle 30^\circ - \theta$$

$$\vec{I}_2 = \frac{\vec{V}_{BC}}{Z} = \frac{V_L \angle -90^\circ}{|Z| \angle \theta} = \frac{V_L}{|Z|} \angle -90^\circ - \theta$$

$$\vec{I}_3 = \frac{\vec{V}_{CA}}{Z} = \frac{V_L \angle -210^\circ}{|Z| \angle \theta} = \frac{V_L}{|Z|} \angle -210^\circ - \theta$$

$$I_p = \frac{V_L}{|Z|}$$

FOR A Δ-CONNECTED LOAD

$$S_L = \vec{V}_{AB} \cdot \vec{I}_1^{\,*} + \vec{V}_{BC} \cdot \vec{I}_2^{\,*} + \vec{V}_{CA} \cdot \vec{I}_3^{\,*}$$

$$= (V_L \angle 30^\circ)(I_p \angle -30^\circ + \theta) + (V_L \angle -90^\circ)(I_p \angle 90^\circ + \theta) + (V_L \angle -210^\circ)(I_p \angle 210^\circ + \theta)$$

$$= V_L I_p \angle \theta + V_L I_p \angle \theta + V_L I_p \angle \theta$$

$$= \boxed{3\, V_L I_p \angle \theta}$$

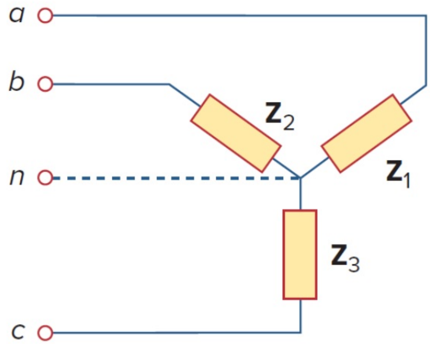
BALANCED THREE-PHASE SYSTEM

Possible Three-Phase Load Configurations:

For a balanced wye-connected load,

$$\boxed{Z_1 = Z_2 = Z_3 = Z_Y}$$

where Z_Y is the load impedance per phase.



Y-connected load

BALANCED THREE-PHASE SYSTEM

Possible Three-Phase Load Configurations:

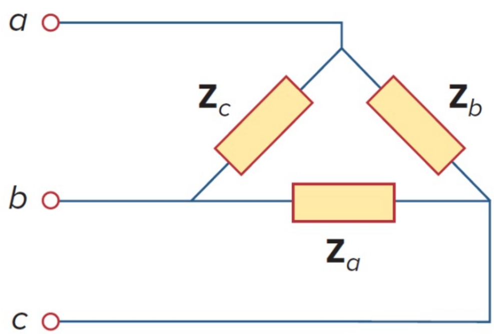
For a balanced delta-connected load,

$$\boxed{Z_a = Z_b = Z_c = Z_\Delta}$$

where Z_Δ is the load impedance per phase.

Impedance relation between Y and Δ connected load:

$$\boxed{Z_\Delta = 3 \times Z_Y \text{ or } Z_Y = \frac{1}{3} \times Z_\Delta}$$



Δ-connected load