CH 21: Coulomb's Law

I. Introduction

*. Four forces in Nature

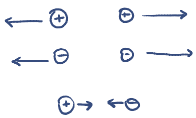
1. Gravity 2. Electromagnetism 3. Weak Nuclear Force 4. Strong Nuclear Force

$$n \rightarrow p^+ + e^- + \bar{\nu}_e$$

II Electric Charge

two types of charges
+ positive
− negative

like charges repell each other
Opposite charges attract each other



* Excess Charge: Most objects are neutral, they have equal amount of + & − charges

III Conductors and Insulators

a. Conductors: material with freely moving charges. eg: metals, salty water

b. Insulators: material with "frozen" charges eg: plastic, glass

c. Semiconductors: material between conductors & insulators eg. Si

* Charge Transfer: material become charged by either gain or loss electrons

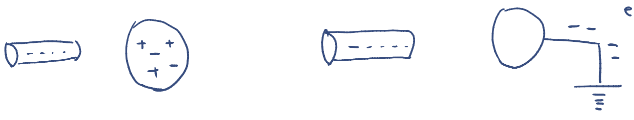
I. Conduction, two materials are rubbed together against another

II Polarization. an already charged. object to have a positive / negative region

III. Induction. Connect a polarized object to a conductor

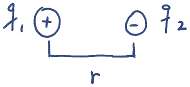
the object lose / gain electron

redistribution.



IV Coulomb's Law

Force between two charges



$k = 8.99 \times 10^{9}\ N\,m^2/c^2$

$$F = k\,\frac{q_1 q_2}{r^2}$$

Recall: Law of Gravitation $F = G\,\dfrac{m_1 m_2}{r^2}$

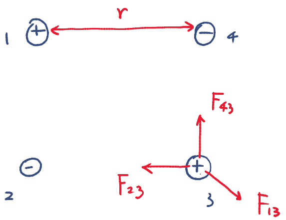
$$K = \frac{1}{4\pi\varepsilon_0} \qquad \varepsilon_0 = 8.85\times 10^{-12}\ \frac{c^2}{N\cdot m^2}$$

$[q] = c$

$[r] = m$

$[F] = N$

* Superposition Principle.



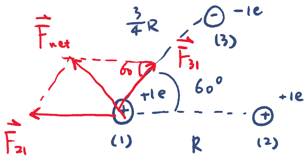
$$\vec{F}_{43} = \vec{F}_{23} = k\frac{q^2}{r^2}$$

$$\vec{F}_{43} + \vec{F}_{23} = \sqrt{2}\,k\,\frac{q^2}{r^2}$$

$$\vec{F}_{13} = k\frac{q^2}{(\sqrt{2}r)^2} = \frac{1}{2}\,k\frac{q^2}{r^2}$$

$$\vec{F}_{total} = \vec{F}_{43} + \vec{F}_{23} - \vec{F}_{13} = \left(\sqrt{2} - \frac{1}{2}\right) k\frac{q^2}{r^2}$$

Example:



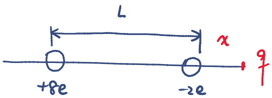
$$\vec{F}_{21} = k\frac{e^2}{R^2} \qquad \vec{F}_{31} = k\frac{e^2}{\left(\frac{3}{4}R\right)^2} = \frac{16}{9}\,k\frac{e^2}{R^2}$$

$$\vec{F}_{net} = \sqrt{\vec{F}_{21}^{\,2} + \vec{F}_{31}^{\,2} - 2|\vec{F}_{21}||\vec{F}_{31}|\cos 60^\circ} = \sqrt{1 + \frac{256}{81} - 2\times\frac{16}{9}\times\frac{1}{2}}$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) \qquad F_{1,net} = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{1 + \frac{256}{81} - \frac{16}{9}} = \sqrt{\frac{256+81-144}{81}}\,k\frac{e^2}{R^2} \approx \frac{\sqrt{193}}{9}\,k\frac{e^2}{R^2}$$

* Example : Finding Equilibrium



Has to be on the right

$$\frac{kq\cdot 2e}{x^2} = \frac{kq\cdot 8e}{(x+L)^2} \Rightarrow \frac{1}{x^2} = \frac{4}{(x+L)^2} \Rightarrow \frac{1}{x} = \frac{2}{x+L} \Rightarrow x + L = 2x$$

$$\Rightarrow x = L.$$

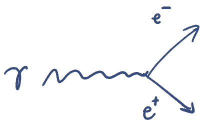
* Electric Current

$$i = \frac{q}{t} = \frac{\text{charge}}{\text{time}} \qquad Q_{total} = n_e \cdot e \Rightarrow n_e = \frac{Q_{total}}{e}$$

$\boxed{\text{Charge is quantized}}$ $\boxed{\text{Charge is conserved}}$

* Noether's law Conservative laws $\longleftrightarrow$ Symmetric of nature

Pair Production:

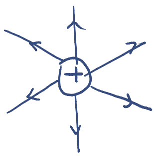


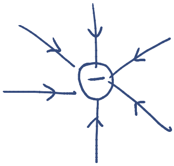
$$\gamma \rightarrow e^- + e^+$$

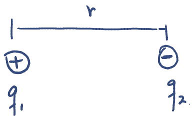
CH 22. Electrical Field

22.1 The Electric Field

* How do charges Communicate a force between one another







$$F = k\frac{q_1 q_2}{r^2}$$

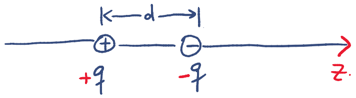
Remove $q_2 \Rightarrow E = \frac{F}{q_2} = k\frac{q_1}{r^2} \Rightarrow E = k\frac{Q}{r^2}$ // $\frac{N}{C}$

for a point charge

* A charged partical in electrical field will feel force

$$\vec{F} = q\vec{E}$$

The electric field of dipole:



$$E_z = E_+ - E_- = k\frac{q}{(z+d/2)^2} - k\frac{q}{(z-d/2)^2}$$

$$= \frac{kq}{z^2}\left[\frac{1}{(1+\frac{d}{2z})^2} - \frac{1}{(1-\frac{d}{2z})^2}\right]$$

$$= \frac{kq}{z^2}\left[\frac{(1-\frac{d}{2z})^2-(1+\frac{d}{2z})^2}{(1+\frac{d}{2z})^2(1-\frac{d}{2z})^2}\right] = \frac{kq}{z^2}\left[\frac{2d/z}{\left[(1+\frac{d}{2z})(1-\frac{d}{2z})\right]^2}\right]$$

$$= \frac{kq}{z^2}\frac{2d/z}{\left[1-(\frac{d}{2z})^2\right]^2} = \frac{2kdq}{z^3}\cdot\frac{1}{\left(1-(\frac{d}{2z})^2\right)^2}$$

when $z \gg d$, $\frac{d}{2z} \to 0$ $\Rightarrow$ $\boxed{E_z = \frac{2kdq}{z^3}}$ E field of a dipole far away

Define the dipole moment $\vec{p} = q\vec{d}$ $\quad\vec{d}$ = negative to positive

$$\therefore \vec{E}_z = \frac{2k\vec{p}}{z^3}$$

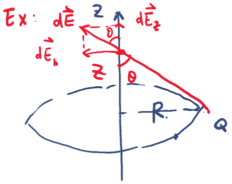
Calculate $\vec{E}$ field of a continuous distribution

1D: $dq = \lambda ds$ $\quad ds$. line element $\quad E = \int dE = \int k\frac{\lambda dl}{r^2}$

2D $dq = \sigma dA$ $\quad dA$ Area element $\quad E = \iint k\frac{\sigma dA}{r^2}$

3D $dq = \rho dV$ $\quad dV$ Volume element $\quad E = \iiint k\frac{\rho dV}{r^2}$

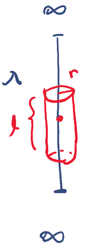
Ex:



Assume the ring has charged positive Q

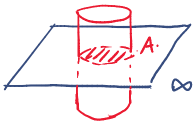
$$d\vec{E} = k\frac{dq}{r^2}\cos\theta = k\cdot\frac{\lambda\, ds}{(z^2+R^2)}\,\frac{z}{\sqrt{z^2+R^2}} = k\cdot\frac{\frac{Q}{2\pi R}\,z\, ds}{(z^2+R^2)^{3/2}}$$

$$E = \int dE = \int \frac{kQz}{2\pi R(z^2+R^2)^{3/2}}\,ds = \frac{kQz}{2\pi R\,(z^2+R^2)^{3/2}}\underbrace{\int ds}_{2\pi R} = \boxed{\frac{kQz}{(z^2+R^2)^{3/2}}}$$



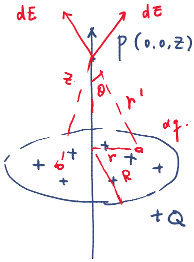
$$\Phi = \oint \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_0} = \frac{\lambda\,\cancel{l}}{\varepsilon_0} = E\,2\pi r\cdot\cancel{l} \Rightarrow \boxed{E = \frac{\lambda}{2\pi\varepsilon_0}\cdot\frac{1}{r}}$$

r is the distance between point and line



$$\Phi = \oint \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_0} = \frac{A\cdot\sigma}{\varepsilon_0} = \underbrace{2E\cdot A}_{\text{(Double face)}} \Rightarrow \boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

Electric field of a disk $\quad \sigma = \frac{Q}{A} = \frac{Q}{\pi R^2}$



$$dq = \sigma\, dA = \sigma\, r\, dr\, d\phi \quad \text{(Jacobian)}$$

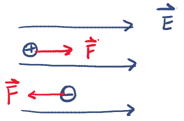
$$dE = \frac{k\,dq}{r'^2} = \frac{k\,dq}{(r^2+z^2)} = \frac{k\sigma r\,dr\,d\phi}{(r^2+z^2)}$$

$$dE_z = dE\cos\theta = \frac{k\sigma r\,dr\,d\phi}{r^2+z^2}\,\frac{z}{(r^2+z^2)^{1/2}} = \frac{k\sigma r z\,dr\,d\phi}{(r^2+z^2)^{3/2}}$$

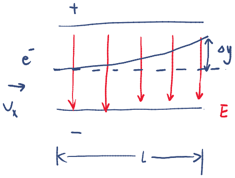
$$E = \iint dE = k\sigma z\int_0^{2\pi}\int_0^{R}\frac{r}{(r^2+z^2)^{3/2}}\,dr\,d\phi = \boxed{\frac{\sigma}{2\varepsilon_0}\left[1-\frac{z}{\sqrt{z^2+R^2}}\right]}$$

① $R\to\infty$, infinite disk $E = \frac{\sigma}{2\varepsilon_0}$ ② $z\to\infty$ $E = 0$

22.6 Point charges in Electric Fields



* How do charges move in a constant $\vec{E}$?



$$\begin{cases} \Delta y = \frac{1}{2} a t^2 & ① \\ L = v_x t & ② \\ a = \frac{F}{m} = \frac{eE}{m} & ③ \end{cases}$$

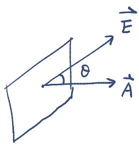
From ② $\Rightarrow$ $t = \frac{L}{v_x}$

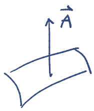
$$\Delta y = \frac{1}{2} \frac{eE}{m} \cdot \frac{L^2}{v_x^2} = \frac{eEL^2}{2mv_x^2}$$

$$\therefore \boxed{\Delta y = \frac{eEL^2}{2mv_x^2}}$$

CH 23. Gauss Law

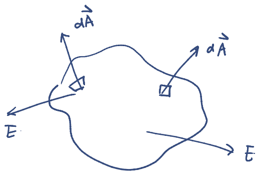
23.1 Electric flux :





$$\Phi_E = \vec{E} \cdot \vec{A} = |\vec{E}|\,|\vec{A}| \cdot \cos\theta$$

* what if E is not constant, and the surface is not flat



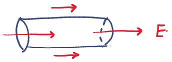
$$\Phi_E = \oint\!\!\oint \vec{E} \cdot d\vec{A}$$ Unit: $\frac{Nm^2}{C}$

Gauss law Another way proof:

$$\oint\!\!\oint \vec{E}\, d\vec{A} = \iiint \operatorname{div} \vec{E}\, dV \text{ (Divergence Theorem)} \quad \frac{Q}{V} \Rightarrow \boxed{\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}}$$

For $\oint\!\!\oint \vec{E}\, d\vec{A} = \frac{1}{\varepsilon_0} \sum q_{inside} = \frac{1}{\varepsilon_0} \iiint \rho\, dV$

Example: Flux through a cylinder

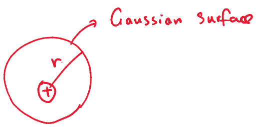


obvisouly we can tell the $\Phi = 0$ because of symmetric

- Gauss proved that the total flux through a closed surface is only non-zero if there is a net charge enclosed by that surface $\boxed{\Phi_E = \oiint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}}$

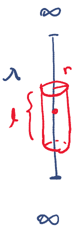
Gaussian Surface (Closed surface)

Proof the electric field of a point charge



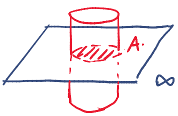
According to Gaussian's Theorem :

$$E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0} \Rightarrow \boxed{E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} = k\frac{q}{r^2}}$$



$$\Phi = \oint \vec{E}\, d\vec{A} = \frac{Q}{\varepsilon_0} = \frac{\lambda \cancel{l}}{\varepsilon_0} = E\ 2\pi r \cdot \cancel{l} \Rightarrow \boxed{E = \frac{\lambda}{2\pi\varepsilon_0} \cdot \frac{1}{r}}$$

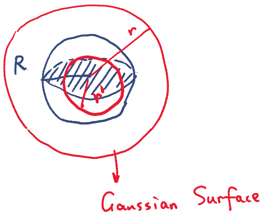
r is the distance between point and line



$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0} = \frac{A \cdot \sigma}{\varepsilon_0} = 2E \cdot A \Rightarrow \boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

(Double face)

Uniform sphere charge



$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4\pi}{3}R^3} = \frac{3Q}{4\pi R^3}$$

$$\oiint \vec{E}\, d\vec{A} = E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0} = \frac{4\pi R^3 \rho}{3\varepsilon_0}$$

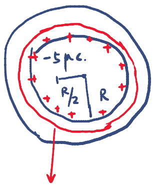
$$\therefore E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \quad (r > R)$$

$$Q' = \rho V' = \rho \cdot \frac{4}{3}\pi r'^3 = \frac{3Q}{4\pi R^3} \cdot \frac{4\pi r'^3}{3} = Q\left(\frac{r'}{R}\right)^3$$

$$\oiint \vec{E}\, d\vec{A} = E \cdot 4\pi r'^2 = \frac{Q'}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q'}{(r')^2} = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{r'}{R^3} = \frac{Q}{4\pi\varepsilon_0 R^3} \cdot r' \quad (r' < R)$$

$$\therefore \boxed{E = \begin{cases} \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} & (r > R) \\ \frac{Q}{4\pi\varepsilon_0 R^3} r & (r < R) \end{cases}}$$

Ex:



Gaussian Surface

Spherical Metal Shell with enclosed charge

Q: How many charge is on the inner & outer Surfaces

A: $\Phi_E = \oiint \vec{E}\, d\vec{A} = \dfrac{q_{enclosed}}{\varepsilon_0} = 0 \Rightarrow q_{enclosed} = 0$

$q_{enclosed} = q_{inner} + q_{-} = 0 \Rightarrow q_{inner} = -q_{-} = 5\mu C$

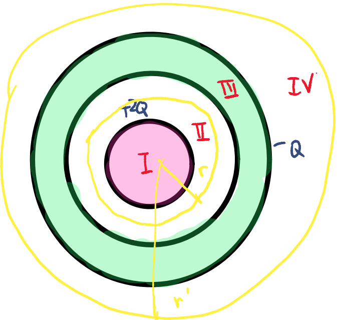
For conductor has to be netural, so $q_{outer} + q_{inner} = 0 \Rightarrow q_{outer} = -5\mu c$

(Condutor can be charged and can Carry the +/- but without excess charge should be netural)

Fact: The $\vec{E}$ field inside a Conductor is 0! And E perpendicular to surface of conductor

According Gauss Theorem no charge can live on the surface of a cavity inside.

Ex: Find electrical field in the of two Conducting object. One sphere, one shell



I: $E=0$ For it's a conductor !

II: Apply Gaussian Law:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{r^2}$$

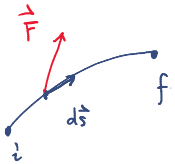
III: $E=0$ For it's a Conductor !

IV: Apply Gaussian Law:

$$E = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q-Q}{r^2}\right) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

CH 24. Electrical Potential

$$\boxed{\Delta U = U_f - U_i = -W_{\text{by force}}}$$



$$dw_e = \vec{F}_e\, d\vec{s} \Rightarrow \int dw_e = W_e = \int_i^f \vec{F}_e \cdot d\vec{s}$$

For $\vec{F} = q\vec{E}$ $\therefore \int_i^f \vec{F}_e\, d\vec{s} = q\int_i^f \vec{E}\, d\vec{s}$

$$\boxed{\Delta U = -q\int_i^f \vec{E}\, d\vec{s}}$$

Potential Energy definition: $W_{p\infty} = -\Delta U = -(U_\infty - U_p) = U_p - U_\infty = U_p$

$$\therefore U_p = +q\int_p^\infty \vec{E}\, d\vec{s} = \boxed{-q\int_\infty^p \vec{E}\, d\vec{s}}$$

For the voltage:

$$V_p = \frac{U_p}{q} = -\int_{\infty}^{P} \vec{E}\, d\vec{s} \qquad \text{Unit (volts)}$$

Let's rearrange: $U = qV \Rightarrow \Delta U = q\Delta V$

$$\therefore \boxed{\Delta V = V_f - V_i = -\int_i^f \vec{E}\, d\vec{s}} \rightarrow \text{obviously a Scalar!}$$

* Conservation of Energy:

$$\boxed{K_i + U_i = K_f + U_f} \Rightarrow \boxed{\Delta K = -\Delta U = -q\Delta V}$$

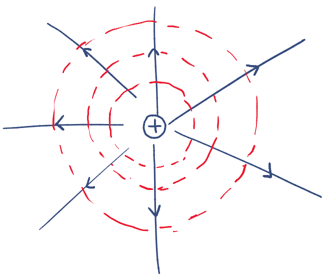
* Involve an extra force

$$K_i + U_i + W = U_f + K_f \Rightarrow \boxed{\Delta K = W - q\Delta V}$$

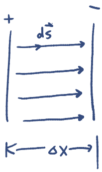
Suppose starts at rest end rest

$$\therefore \boxed{W = q\Delta V}$$

24.2 Equipotential Surface / line



Special case:



$$\Delta V = -\int_i^f \vec{E}\, d\vec{s} = -\vec{E}\int_i^f d\vec{s}$$

$$\therefore \boxed{\Delta V = -E \cdot \Delta x}$$

24.3. Electrical of a potential charge

Using the definition of potential

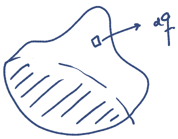
$$\Delta V = V_f - V_i = -\int_i^f \vec{E}\,d\vec{r} = V_\infty - V_r = -\int_r^\infty \vec{E}\,d\vec{r} = -\int_r^\infty \frac{kq}{r^2}\,dr$$

$$= -kq\int_r^\infty \frac{1}{r^2}\,dr = -kq\cdot\left(-\frac{1}{r}\right)\Big|_r^\infty = -kq\left(0-\left(-\frac{1}{r}\right)\right) = \frac{-kq}{r}$$

$$\Rightarrow\ 0 - V_r = -\frac{kq}{r} \ \Rightarrow\ \boxed{V_r = \frac{kq}{r}}$$ potential of a point charge

Superposition Principle: $\boxed{V_p = \sum_{i=1}^{n} V_i}$ directly add value for Scalar!

24.4 Potential of a Continuous charge



$$dV = k\frac{dq}{r} \Rightarrow V = \int k\frac{dq}{r} = k\int\frac{dq}{r}$$

1D: $dq = \lambda\,ds$ $\qquad V = k\lambda\int\frac{ds}{r}$

2D: $dq = \sigma\,dA$ $\qquad V = k\sigma\iint\frac{dA}{r}$

3D: $dq = \rho\,dV$ $\qquad V = k\sigma\iiint\frac{dV}{r}$

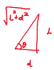
$$k\lambda\int_0^L \frac{ds}{\sqrt{d^2+s^2}} \ \underset{ds = d\sec^2\theta\,d\theta}{\overset{s = d\tan\theta}{=\!=}}\ k\lambda\int_0^{\tan^{-1}(\frac{L}{d})} \frac{1}{d\sec\theta}\,d\sec^2\theta\,d\theta = k\lambda\int_0^{\tan^{-1}(\frac{L}{d})} \sec\theta\,d\theta$$

$0 \le d\tan\theta \le L$

$0 \le \tan\theta \le \frac{L}{d}$

$0 \le \theta \le \tan^{-1}(\frac{L}{d})$

$$= k\lambda \ln\left|\sec\theta + \tan\theta\right|_0^{\tan^{-1}(\frac{L}{d})} \longrightarrow\ = k\lambda\ln\left[\frac{L+(L^2+d^2)^{1/2}}{d}\right]$$



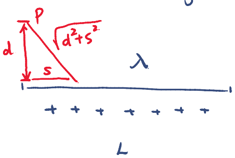
$\sec(\tan^{-1}(\frac{L}{d})) = \frac{1}{\cos\theta} = \frac{\sqrt{L^2+d^2}}{d}$

$\tan(\tan^{-1}(\frac{L}{d})) = \frac{L}{d}$

$\sec 0 = 1$

$\tan 0 = 0$

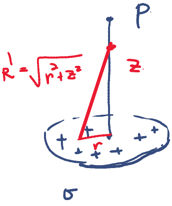
① line of charge



$$V = k\lambda\int\frac{ds}{r} = k\lambda\int_0^L\frac{ds}{\sqrt{d^2+s^2}} = \boxed{k\lambda\ln\left[\frac{L+(L^2+d^2)^{1/2}}{d}\right]}$$

when $d \to \infty$ $\quad V = 0$

② Find the potential of a disk of charge



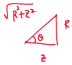
$$V = \int k \frac{dq}{R'} = k \int \frac{\sigma\, dA}{\sqrt{r^2+z^2}} = k \iint \frac{\sigma\, r\, dr\, d\theta}{\sqrt{r^2+z^2}}$$

$$= K \int_0^{2\pi} \int_0^{R} \frac{\sigma r}{\sqrt{r^2+z^2}}\, dr\, d\theta = \boxed{\frac{\sigma}{2\varepsilon_0}\left[\sqrt{z^2+R^2} - z\right]}$$

$$\int_0^R \frac{\sigma r}{\sqrt{r^2+z^2}}\, dr \;\underset{dr = z\sec^2\theta\, d\theta}{\overset{r = z\tan\theta}{=\!=}}\; \int_0^{\tan^{-1}(\frac{R}{z})} \frac{\sigma\, z\tan\theta}{z\sec\theta}\, z\sec^2\theta\, d\theta = \int_0^{\tan^{-1}(\frac{R}{z})} z\sigma\tan\theta\sec\theta\, d\theta = z\sigma \int_0^{\tan^{-1}(\frac{R}{z})} \frac{-d\cos\theta}{\cos^2\theta} = z\sigma\, \frac{1}{p}\Big|_1^{\frac{z}{\sqrt{z^2+R^2}}} = z\sigma\left[\frac{\sqrt{z^2+R^2}}{z} - 1\right]$$

$0 \le z\tan\theta \le R$

$0 \le \theta \le \tan^{-1}(\frac{R}{z})$



$p = \cos\theta$ $\quad \theta = 0 \quad p = 1$

$\theta = \tan^{-1}(\frac{R}{z}) \quad p = \frac{z}{\sqrt{R^2+z^2}}$

$$= \sigma\left[\sqrt{z^2+R^2} - z\right]$$

$$\frac{1}{4\pi\varepsilon_0} \cdot 2\pi \cdot \sigma \left[\sqrt{z^2+R^2} - z\right]$$

24.6 $\vec{E}$ field from the potential

$$dV = -\vec{E}\, d\vec{s} \Rightarrow -\frac{dV}{d\vec{s}} = \vec{E} \Rightarrow \boxed{\vec{E} = -\frac{dV}{d\vec{s}}}$$

For $\boxed{F = qE = q\left(-\frac{dV}{ds}\right) = -\frac{dU}{ds}}$

The component of E in the direction of s is the negative rate of change of potential in that direction " E = negative slope of V"

Cartesian Coordinate $\vec{E} = \langle E_x, E_y, E_z \rangle = \langle -\frac{\partial V}{\partial x}, -\frac{\partial V}{\partial y}, -\frac{\partial V}{\partial z} \rangle$

Recall: For an disk

$$\vec{E}_z = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right) \qquad V_z = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2} - z\right)$$

$$E_z = -\frac{dV_z}{dz} = -\frac{\sigma}{2\varepsilon_0}\left(\frac{2z}{2\sqrt{z^2+R^2}} - 1\right) = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$$

24.7 Energy stored in multiple charges

* How much energy is stored in a pair of charge?

$$\Delta U = q\Delta V$$

$$\Delta U = U_r - \cancel{U_\infty}^{\,0} = q_2\Delta V = q_2(V_r - \cancel{V_\infty}^{\,0}) = q_2 V_2 = \boxed{\frac{kq_1q_2}{r}}$$

* What about the potential energy for multiple charges?

$$U_{1,2,3} = U_{12} + U_{13} + U_{23} = \frac{kq_1q_2}{r_{12}} + \frac{kq_1q_3}{r_{13}} + \frac{kq_2q_3}{r_{23}}$$

For $$U_{1,2,\dots n} = \underbrace{(U_{12}+U_{13}+\dots+U_{1n})}_{n} + \underbrace{(U_{23}+U_{24}+\dots+U_{2n})}_{n-1} + \dots + \underbrace{U_{n-1\,n}}_{1}$$

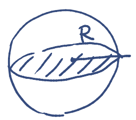
$$= \sum_{i=1}^{n} U_{1i} + \sum_{i=2}^{n} U_{2i} + \dots + \sum_{i=n}^{n} U_{n-1\,i}$$

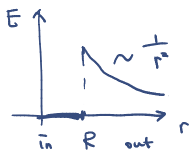
$$= \sum_{j=1}^{n-1}\sum_{i=j}^{n} U_{ji}$$

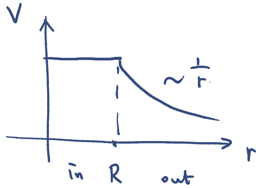
24.8 Potential inside a charged isolated conductor:

* Fact: Excess charge on a Conductor will redistribute itself on the surface so that all points inside the conductor are at equal potential

Spherical Conductor







Potential must be Continuous!

From Previous Study we know

$$\vec{E} = -\frac{dV}{d\vec{s}} \Rightarrow \vec{F} = q\vec{E} = -\frac{d(qV)}{d\vec{s}} = -\frac{du}{d\vec{s}}$$

Therefore : $\boxed{\vec{F} = -\frac{du}{d\vec{s}}}$

Because $\vec{E} = \langle E_x, E_y, E_z \rangle = \langle -\frac{\partial V}{\partial x}, -\frac{\partial V}{\partial y}, -\frac{\partial V}{\partial z} \rangle = -\nabla(V)$

Therefore: $\vec{E} = -\nabla(V)$ also according Gaussian law: $\nabla \cdot \vec{E} = \frac{\rho_{en}}{\varepsilon_0}$

$$\therefore \nabla \cdot [-\nabla(V)] = \frac{\rho}{\varepsilon_0} \Rightarrow \boxed{\nabla^2 V = -\frac{\rho}{\varepsilon_0}}$$ possion equation.

$$\boxed{\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho}{\varepsilon_0}}$$ Another possion equation

gradient : given a scalar function → vector : $f \rightarrow \vec{F} = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle$

Curl : given a vector filed → new vector field

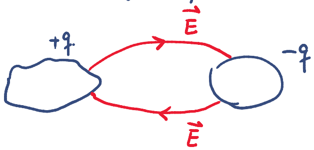
$$\vec{F} = \langle f_x, f_y, f_z \rangle \rightarrow \vec{G} = \langle g_x, g_y, g_z \rangle$$

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_x & f_y & f_z \end{vmatrix} \Rightarrow \begin{aligned} g_x &= \frac{\partial}{\partial y} f_z - \frac{\partial}{\partial z} f_y \\ g_y &= \frac{\partial}{\partial z} f_x - \frac{\partial}{\partial x} f_z \\ g_z &= \frac{\partial}{\partial x} f_y - \frac{\partial}{\partial y} f_x \end{aligned}$$

divergence : Input a Vector → scalar : $\nabla \cdot \vec{F} = \text{div}(\vec{F}) = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y} + \frac{\partial F}{\partial z}$

CH 25 Capacitance

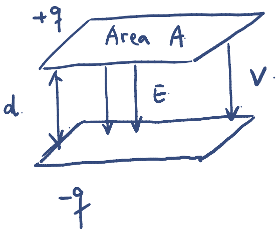
Definition: A capacitor is any pair of conducting object that store equal and opposite charge



- Given a Capacitor, we can define its capacitance as the amount of charge stored for a given voltage. $C = \frac{q}{V}$ (q is charge, V is voltage diff)

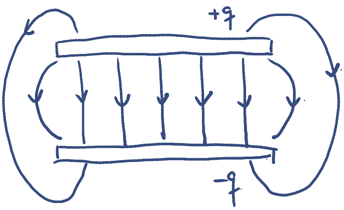
- key point: Capacity is purely a geometric quantity. it only depends on the shape of capacity

- Parralle-plate capacitor



- Fringing E field will be disregarded in this class

- Focus on the uniform $\vec{E}$ field at the center of the parallel-plates.



25.2 Calculating capacitance

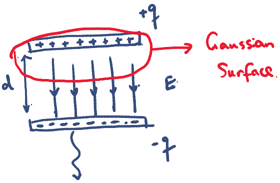
Steps for finding C :

1) Assume each plate has charge $\pm q$
2) Calculate the $\vec{E}$ field use gauss law
3) Calculate the V between the plate

$$V = -\int_{-}^{+} \vec{E}\, d\vec{s}$$

4) Calculate C from $C = \frac{q}{V}$

* Parallel-Plate Capcitance :

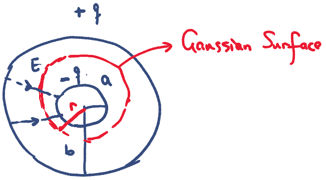


$$\oint \vec{E}\, d\vec{A} = -E \cdot S = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{+q}{A\varepsilon_0}$$

$$V = -\int_{-}^{+} \vec{E}\, d\vec{s} = -\int_{-}^{+} (-E\, ds) = \frac{q}{A\varepsilon_0} \cdot \int_{-}^{+} ds = \frac{qd}{A\varepsilon_0} \Rightarrow \boxed{C = \frac{\varepsilon_0 A}{d}}$$

$$\therefore C = \frac{q}{V} = q \Big/ qd \cdot A\varepsilon_0 = \frac{\varepsilon_0 A}{d} = \frac{A}{4\pi k d}$$

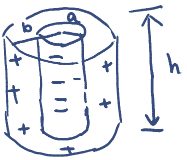
* Cylindrical Capacitance



$$\oint \vec{E}\, d\vec{A} = E \cdot 2\pi r h = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{q}{2\pi r h \varepsilon_0}$$

$$V = -\int_{-}^{+} \vec{E}\, d\vec{s} = \int_{-}^{+} E\, ds = \int_{-}^{+} \frac{q}{2\pi r h \varepsilon_0}\, dh$$

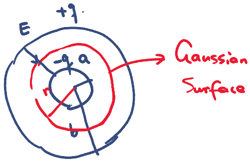
$$= \frac{q}{2\pi h \varepsilon_0} \ln r \Big|_a^b = \frac{q}{2\pi \varepsilon_0 h} \ln\left(\frac{b}{a}\right)$$



$$\therefore C = \frac{q}{V} = \frac{2\pi \varepsilon_0 h}{\ln(b/a)}$$

$$\therefore \boxed{C = \frac{2\pi\varepsilon_0 h}{\ln(b/a)}}$$

* Spherical Capacitance



$$\oiint \vec{E}\, d\vec{A} = E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{q}{4\pi\varepsilon_0 r^2}$$

$$V = -\int_{-}^{+} \vec{E}\, d\vec{s} = \int_{-}^{+} E\, dr = \int_a^b \frac{q}{4\pi\varepsilon_0 r^2}\, dr$$

$$= \frac{q}{4\pi\varepsilon_0}\int_a^b \frac{1}{r^2}\, dr = \frac{q}{4\pi\varepsilon_0}\left(-\frac{1}{r}\right)\Big|_a^b = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$\therefore\ C = \frac{q}{V} = \frac{q}{\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)} = \frac{4\pi\varepsilon_0}{\frac{b-a}{ab}} = \frac{4\pi\varepsilon_0 ab}{b-a}$$

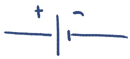
$$\therefore\ \boxed{C = \frac{4\pi\varepsilon_0 ab}{b-a}}$$

* Self-Capacitor: $b \to \infty$ $\quad C = \lim_{b\to\infty} 4\pi\varepsilon_0 \frac{ab}{b-a} = 4\pi\varepsilon_0 a \Rightarrow \boxed{C = 4\pi\varepsilon_0 a}$

25.3 Capacitor in Series and parallel

* Circuit symbol

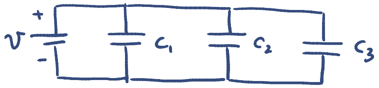
 wire

 battery

 switch

 Capacitor

* Capacity in parallel "Parallel" = "Same voltage drop"



* Can we find one capacitor to replace all of the three capacitors?

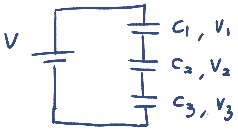
* Principle: The total charge of capacitor is the sum of each charge.

$$q_{total} = q_1 + q_2 + q_3 \Rightarrow C_{total} = \frac{q_{total}}{V} = \frac{q_1}{V} + \frac{q_2}{V} + \frac{q_3}{V} = C_1 + C_2 + C_3$$

Therefore: $\boxed{C_{eq} = C_1 + C_2 + \cdots + C_n}$

* Capacity in Series

"Series" = "Same current / Charge flow"



* Principle: In series, the charge on each capacitor is same

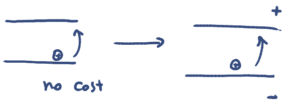
$$q_1 = q_2 = q_3 = \cdots = q_n$$

For $V_1 + V_2 + \cdots + V_n = V_{total}$

$$\frac{q_1}{C_1} + \frac{q_2}{C_2} + \cdots + \frac{q_n}{C_n} = \frac{q_{eq}}{C_{eq}}$$

$$\therefore \boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n}}$$

25.4 Energy Density



feel resistance ....

* To put charge on the capacitor plates, we must do work to move charges against Coulumb repulsion

$$dw = dq\,V = dq\,\frac{q}{C} = \frac{q}{C}\,dq$$

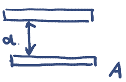
$$w = \int dw = \int_0^Q \frac{q}{C}\,dq = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}V^2C = \frac{1}{2}QV$$

$$\therefore \boxed{w = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}V^2C = \frac{1}{2}QV}$$ energy store in capacitor

* Energy density

- Sometimes it's good to know how much energy is present per volumn of space

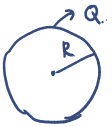
$$u = \frac{U}{V_{olumn}} = \frac{\frac{1}{2}CV^2}{Ad} = \frac{\frac{1}{2}\frac{\varepsilon_0 A}{d}V^2}{Ad} = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2 = \frac{1}{2}\varepsilon_0 E^2$$



$$\therefore \boxed{u = \frac{1}{2}\varepsilon_0 E^2}$$

Therefore $\boxed{u = \frac{1}{2}\varepsilon_0 E^2 \propto E^2}$ u is energy density $[u] = J/m^3$

Eg: potential energy and density of conducting sphere (known Q, R)



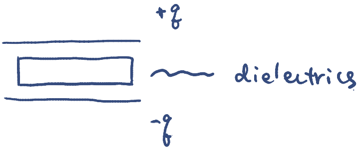
a) $$W = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0 R}$$ (For sphere $C = 4\pi\varepsilon_0 R$)

b) $$u = \frac{W}{V} = \frac{\frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0 R}}{\frac{4}{3}\pi R^3} = \frac{3Q^2}{32\pi^2\varepsilon_0 R^4}$$

method 2:

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left[\frac{Q}{4\pi\varepsilon_0 R^2}\right]^2 = \frac{\varepsilon_0 Q^2}{32\pi^2\varepsilon_0 R^4}$$

25.5 Dielectrics and Capacitor



* The capacitance can be increased if we put a dielectric material betw plates

$$C_{new} = K\, C_{air}$$

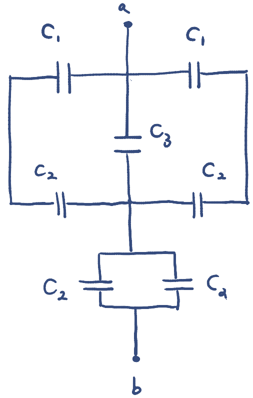
K = Kappa = the strength of dielectric ! = Dielectric constant

* If the region contains a dielectric with constant K, then change $\varepsilon_0 \to K\varepsilon_0$ everywhere !

eg: $$E = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} \rightarrow E = \frac{1}{4\pi \underset{\sim}{K}\varepsilon_0}\frac{q^2}{r^2} = E_{new}$$

$$E = \frac{\sigma}{2\varepsilon_0} \rightarrow E = \frac{\sigma}{2\underset{\sim}{K}\varepsilon_0} = E_{new}$$

Eg. A couple of C_{eq} Review Problem



$C_{12} = \frac{10}{3}$

Know $C_1 = 5\,\mu F$ $C_2 = 10\,\mu F$ $C_3 = 2\,\mu F$

<Sol>: (1) $C_1 + C_2 \rightarrow C_{12} = \frac{C_1 C_2}{C_1 + C_2}$

C_{12}, C_{12} & C_3 are paralle

$$C_{12,12,3} = C_{12} + C_{12} + C_3 = \frac{2C_1C_2}{C_1+C_2} + C_3$$

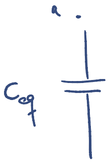
For C_2 & C_2 are paralle $\Rightarrow C_{22} = C_2 + C_2 = 2C_2$

For $C_{12,12,3}$ & C_{22} are series

$$\therefore C_{12,12,3,22} = \frac{1}{\frac{1}{2C_2} + \frac{1}{\frac{2C_1C_2}{C_1+C_2}+C_3}} = \frac{(2C_1C_2 + C_1C_3 + C_2C_3 + 2C_2)\,2C_2}{2C_1C_2 + C_1C_3 + C_2C_3 + C_1 + C_2}$$

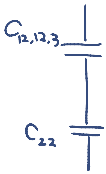
$\Rightarrow C_{eq} = 6.04\,\mu F$

(2) Find charge store in C_3 if ΔV between a and b is $60.0\,V$



$$C_{eq} = \frac{Q_{all}}{V_{ab}} \Rightarrow Q_{all} = C_{eq} \cdot V_{ab} = 362.9\,\mu C$$

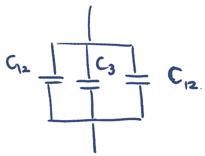
unraval →



For in series $Q_{12,12,3} = Q_{22} = Q_{all} = 362.9\,\mu C$

$$V_{12,12,3} + V_{22} = V_{ab} \Rightarrow V_{12,12,3} = V_{ab} - \frac{Q_{all}}{C_{22}} \quad ①$$

unraval →



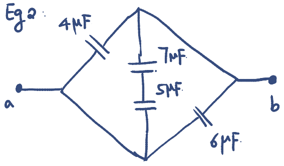
For in paralle $Q_{12} + Q_{12} + Q_3 = Q_{12,12,3} = 362.9\,\mu C$

For $Q_{12} = C_{12}V$ $Q_3 = C_3 V$ $(V = V_{12,12,3})$

$$\Rightarrow Q_3 = Q_{12,12,3} - 2C_{12}V_{12,12,3} \quad ③$$

$$\Rightarrow Q_3 = Q_{12,12,3} - 2C_{12}\left(V_{ab} - \frac{Q_{all}}{C_{22}}\right) = 84.17\,\mu C$$

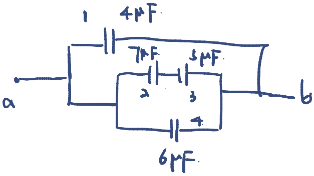
Eg2:



Find C_{eq} for this circuit also find ΔV for the 7μF capacitor if $V_{ab} = 12V$.

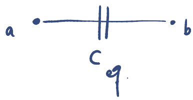
<sol>: $C_{23} = \dfrac{C_2 C_3}{C_2 + C_3} = 2.91\ \mu F$

<Solution>:



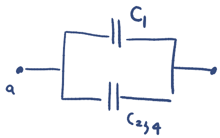
$C_{234} = C_{23} + C_4 = 8.91\ \mu F$

$C_{eq} = C_1 + C_{234} = 12.91\ \mu F$



$\Rightarrow Q_{all} = C_{eq} V_{ab} = 774.6\ \mu C$

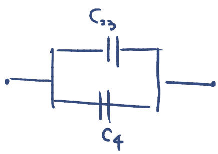
→ unravel



For paralle $V_1 = V_{234} = V_{ab} = 60V$

$Q_{234} = C_{234} V_{234} = 534.6\ \mu C$

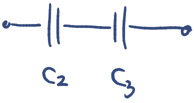
→ unraval



For paralle $V_{23} = V_4 = V_{234} = 60V$

$Q_{23} = C_{23} V_{23} = 174.6\ \mu C$

→ unraval



For series $Q_2 = Q_3 = Q_{23}$

$\therefore Q_2 = 174.6\ \mu C$

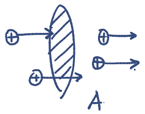
$V_2 = \dfrac{Q_2}{C_2} = 24.9V$

CH 26 Resistance & Resistor

26.1 Electric Current

* Electric current is the steady flow of charges.

* Mathematically $i = \frac{\Delta q}{\Delta t}$



unit

$[i] = \frac{C}{s} = \text{Ampere}$

$= A$

* Instantaneous current $\boxed{i = \frac{dq}{dt}}$

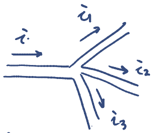
Q : How to get a steady current ?

A : Batteries. A battery generate an $\vec{E}$ field that pushes charge through Conductor.

* Total charge : $i = \frac{dq}{dt} \Rightarrow i\,dt = dq$

$$q_{total} = \int dq = \int_0^{t_f} i\,dt \Rightarrow \boxed{q_{total} = \int_0^{t_f} i\,dt}$$

* Charge Conservation :

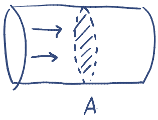


$$i = i_1 + i_2 + i_3 + \dots = \sum_{k=1}^{n} i_k$$

(Kirchoff's current Law) (KCL)

26.2. Current Density

* To understand current distribution over a Cross Section of wire. we need to know the Current density flowing through the wire.



$$J = \frac{i}{A} \quad (\text{uniform current})$$

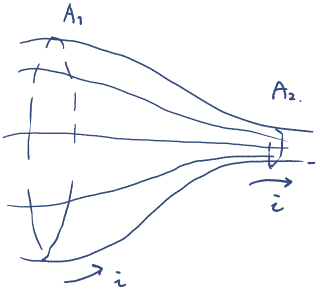
For non-uniform current $\quad di = \vec{J} \cdot d\vec{A}$

$$\therefore \boxed{i = \iint \vec{J} \cdot d\vec{A}} \qquad [\vec{J}] = \frac{\text{Ampere}}{m^2}$$

For divergence theorem: $\iint \vec{J} \cdot d\vec{A} = \iiint (\nabla \cdot \vec{J})\, dV = i$

$$\Rightarrow \boxed{\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}}$$

For $i = -\frac{dQ}{dt} = -\frac{\partial}{\partial t} \iiint \rho\, dV = \iiint \left(-\frac{\partial \rho}{\partial t}\right) dV$



high Current density

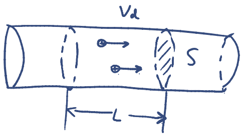
low current density

$$\boxed{\vec{J} = \frac{i}{S} = ne\vec{V}_d}$$

$$\Uparrow$$

$$\Rightarrow \boxed{i = neSV_d}$$

* Draft velocity



$$\Delta t = \frac{L}{V_d} \quad \Rightarrow i = \frac{Q}{\Delta t} = \frac{nLSe}{L/V_d} = neSV_d$$

$Q = Ne = nLSe$

(Suppose N charges in region $n = \frac{N}{\text{Volumn}}$. number density)

☆ Chemistry related: $n = \dfrac{N}{\text{Volumn}} = \dfrac{m N_A}{\text{molar mass} \cdot \text{Volumn}} = \dfrac{\rho N_A}{\text{molar-mass}}$

Eg: Find drift velosity in Copper with wire radius $r = 900\ \mu m$ and $i = 17 mA$

Assume each copper atom contributes one e^- and J is uniform

molar mass $= 63.54 \times 10^{-3}$ kg/mol $\quad \rho = 8.96 \times 10^{3}$ kg/m^3

<Solution> : $n = \dfrac{\rho N_A}{\text{molar mass}} = 8.5 \times 10^{28}$ electrons/m^3

For $\dfrac{\vec{J}}{A} = n e V_d \Rightarrow V_d = \dfrac{\vec{J}}{Ane} = \boxed{4.9 \times 10^{-7}\ m/s}$

26.3 Resistance & Resistivity

* Ohms observes that different materials has different currents through them for a given voltage $\boxed{V = iR}$

$[R] = 1\ \text{ohm} = \Omega = \dfrac{\text{Voltage}}{\text{Amps}}$

Symbol:

* An equivalent definition involves $\vec{E}$ field current density

$$\boxed{\vec{E} = \rho \vec{J}}$$

E = electrical field

J = current density

ρ = resistivity

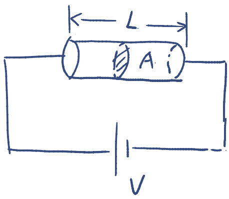
$$[\rho] = \frac{V/m}{A/m^2} = \Omega \cdot m$$

* Only material itself controls the ρ

* We also define inverse of resistivity (conductivity) $\boxed{\sigma = \dfrac{1}{\rho}}$ $\quad [\sigma] = (\text{ohm} \cdot m)^{-1}$ $= \text{mho}$

$$\Rightarrow \boxed{\vec{J} = \sigma \vec{E}}$$

* What is the relationship between R and ρ ?



Assume there is a uniform $\vec{E}$ field.

$$\begin{cases} \vec{E} = \dfrac{V}{L} \\ \vec{J} = i/A \\ \vec{E} = \rho \vec{J} \end{cases} \Rightarrow \frac{V}{L} = \frac{\rho i}{A} \Rightarrow \frac{i}{V} = \frac{\rho L}{A}$$

$$\therefore \boxed{R = \frac{\rho L}{A}}$$

$R = \rho \dfrac{L}{S}$ L = length of wire, A = Cross-section area, R = resistance, ρ = resistivity.

* Resistance and Temperature

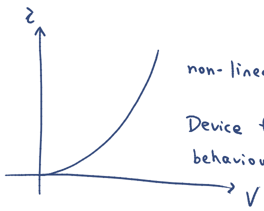
- The resistivity of a material depends on how hot its!

$$\boxed{\rho - \rho_0 = \rho_0 \alpha (T - T_0)}$$

ρ_0 - resistivity at room Temp

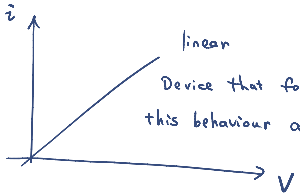
T_0 — room temperature

*



non-linear!

Device follow the behaviour "non-linear"



linear

Device that follow this behaviour are "linear"

☆ Ohm's law is <u>not</u> $V = iR$. it's about linearity between voltage and current.

<u>26.5</u> Power in circuits

* power is that rate of which energy is used / dissipated

* $dU = V\,dq = i\,dt\,V$

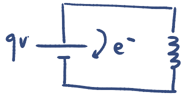
$$\Rightarrow \frac{dU}{dt} = iV \Rightarrow \boxed{P = iV = \frac{dU}{dt}}$$

$[P]$ = power

* Appendix $\boxed{P = iV = i^2R = \frac{V^2}{R}}$ (apply $i = \frac{V}{R}$ into $P = iV$)

CH 27. Circuits

27.1 Single loop circuits



Q: How do batteries maintain same voltage across it's terminal

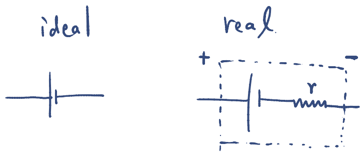
A: Batteries have chemical reaction that pump the charges from + to −

Direct current: whenever a source has constant voltage, we call it DC power

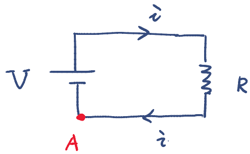
Emf: a device that pumps charges to maintain a fixed voltage!

Emf: Electromotive force

* Ideal emf: no inner resistance
* Real emf: has internal resistance



* Current: in a single loop



☆ Kirchoff's Voltage Law (KVL)

The sum of voltages in any loop must be zero

Proof: $\Delta V = -\int_i^f \vec{E} \cdot d\vec{s}$, ΔV is path independent!

choose $f = i \Rightarrow \Delta V = -\int_i^i \vec{E} \cdot d\vec{s} = V_f - V_i = V_i - V_i = 0$ QED

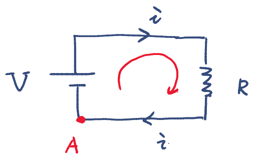
* Resistance Rule : ① move through a resistor in the direction of the current, this gives a change in the voltage of $-iR$

② move in the opposite direction of current. this gives a change in voltage iR

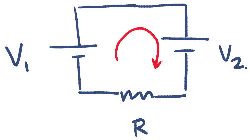
* Emf rule : ① move from $-$ to $+$, the change of voltage is $+V$

② move from $+$ to $-$, the change of voltage is $-V$

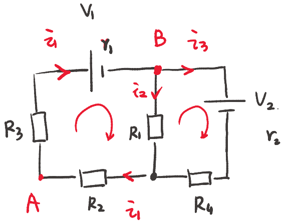
* Apply KVL : choose clockwise path from A to A



$$+V - iR = 0 \Rightarrow i = \frac{V}{R}$$



$$+V_1 - V_2 - iR = 0 \Rightarrow i = \frac{V_1 - V_2}{R}$$



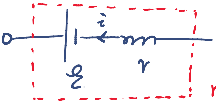
$$\begin{cases} i_1 = i_2 + i_3 & ① \quad (\text{KCL}) \\ -i_1 r_1 - V_1 - i_2 R_1 - i_1 R_2 - i_1 R_3 = 0 & ② \quad (\text{KVL}) \\ -V_2 - i_3 r_2 - i_3 R_4 + i_2 R_1 = 0 & ③ \quad (\text{KVL}) \end{cases}$$

* ΔV in two point : $\boxed{\Delta V = V_B - V_A = -i_1 R_3 - V_1 - i_1 r_1}$

* Resistance in series : $R_{eq} = R_1 + R_2 + R_3 + \cdots + R_n$

* Resistance in paralle : $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$

* Voltage across a ideal emf:



real emf

– The idea voltage is often called ε which is different than the output voltage of an emf V

$$\boxed{V = -ir + \varepsilon}$$

* Power in a real emf:

$$P = iV = i(\varepsilon - ir) = i\varepsilon - i^2 r \Rightarrow \boxed{P = i\varepsilon - i^2 r}$$

Power that is left for the circuit

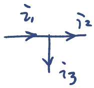
Internal dissipation

Power delivered by idea emf

27.2 Multi-Loop Circuit

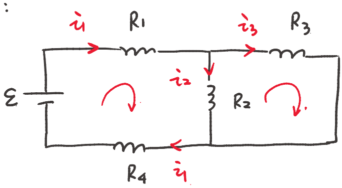
Junction Rule: the current flow into a junction must equal the current flow out

"KCL"



$$i_1 = i_2 + i_3$$

Eg:



$\varepsilon = 12V$

$R_1 = R_2 = 20\Omega$

$R_3 = 30\Omega \quad R_4 = 8\Omega$

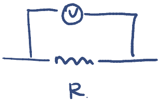
$$\begin{cases} i_1 = i_2 + i_3 & ① \\ \varepsilon - i_1 R_1 - i_2 R_2 - i_1 R_4 = 0 & ② \\ -i_3 R_3 + i_2 R_2 = 0 & ③ \end{cases} \Rightarrow \begin{cases} i_3 = \frac{3}{25} A = 0.12 A \\ i_2 = \frac{9}{50} A = 0.18 A \\ i_1 = \frac{3}{10} A = 0.3 A \end{cases}$$

27.3 Ammeter and Voltmeter

Ammeter ↓ measure current

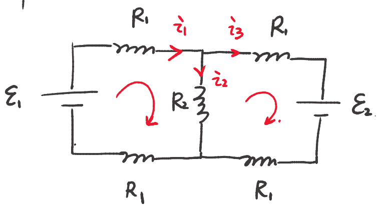
Voltmeter ↓ measure voltage

Voltmeter ($R \to \infty$)



Ammeter ($R \to 0$)

Example:



$\varepsilon_1 = 3V \quad \varepsilon_2 = 6V$

$R_1 = 2\Omega \quad R_2 = 4\Omega$

$$\begin{cases} i_1 = i_2 + i_3 & (1) \\ \varepsilon_1 - i_1 R_1 - i_2 R_2 - i_1 R_1 = 0 & (2) \\ -i_3 R_1 - \varepsilon_2 - i_3 R_1 + i_2 R_2 = 0 & (3) \end{cases}$$

From (2) $\Rightarrow$

$$3 - 2i_1 - 4i_2 - 2i_1 = 0$$

$$\therefore \; i_1 + i_2 = \frac{3}{4} \quad (4)$$

From (3) $\Rightarrow$

$$-2i_3 - 6 - 2i_3 + 4i_2 = 0$$

$$4i_2 - 4i_3 = 6 \Rightarrow i_2 - i_3 = \frac{3}{2} \quad (5)$$

From (4) $\quad 2i_2 + i_3 = \frac{3}{4} \quad (6)$

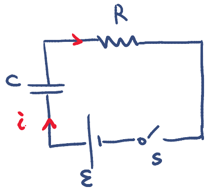
From (5)(6) $\quad i_2 - \left(\frac{3}{4} - 2i_2\right) = \frac{3}{2} \Rightarrow 3i_2 = \frac{3}{4} + \frac{6}{4} = \frac{9}{4} \Rightarrow i_2 = \frac{3}{4}$

$$\therefore \; i_3 = \frac{3}{4} - 2i_2 = \frac{3}{4} - \frac{6}{4} = -\frac{3}{4}\,A$$

$$i_1 = i_2 + i_3 = 0\,A$$

27.4 RC Circuit

Charge mode:



$$\varepsilon - V_C - V_R = 0 \quad ①$$

$$i = \frac{dQ}{dt} \Rightarrow V_R = iR = R\frac{dQ}{dt} \quad ②$$

$$C = \frac{Q}{V_C} \Rightarrow V_C = \frac{Q}{C} \quad ③$$

From ①②③ $\Rightarrow \varepsilon - \frac{Q}{C} - R\frac{dQ}{dt} = 0 \Rightarrow \frac{dQ}{dt} + \frac{Q}{CR} - \frac{\varepsilon}{R} = 0$ ④

$\Rightarrow$ differentiate equation $\frac{dQ}{dt} + \frac{Q}{CR} - \frac{\varepsilon}{R} = 0$

tips: first order $\frac{dy}{dx} + P(x)y = Q(x)$.

then $y = \frac{1}{e^{\int P(x)dx}}\left(\int e^{\int P(x)dx} Q(x)\, dx + C\right)$

Proof: In oure equation $\frac{dQ}{dt} + \frac{1}{CR}\cdot Q = \frac{\varepsilon}{R}$ $\left(P(t) = \frac{1}{CR},\ Q(t) = \frac{\varepsilon}{R}\right)$

$$\therefore Q(t) = \frac{1}{e^{\int \frac{1}{CR}dt}}\left(\int e^{\int \frac{1}{CR}dt} \frac{\varepsilon}{R}\, dt + c'\right)$$

$$\Rightarrow Q(t) = e^{-\frac{t}{CR}}\left(\int e^{\frac{t}{CR}} \frac{\varepsilon}{R} dt + c'\right) = e^{-\frac{t}{CR}}\left(\frac{\varepsilon}{R}\cdot CR\cdot e^{\frac{t}{CR}} + c'\right)$$

$$= e^{-\frac{t}{CR}}\left(C\varepsilon e^{\frac{t}{CR}} + c'\right) \qquad Q(0) = 0$$

$$\Rightarrow 0 = 1\times(C\varepsilon + c') \Rightarrow c' = -\varepsilon C \Rightarrow Q(t) = e^{-\frac{t}{CR}}\left(C\varepsilon e^{\frac{t}{CR}} - C\varepsilon\right)$$

$$\Rightarrow \boxed{Q(t) = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right)}$$

☆

Further: Since $q(t) = \varepsilon C\left(1 - e^{\frac{-t}{RC}}\right)$

$$i(t) = \frac{dq(t)}{dt} = -\varepsilon C e^{\frac{-t}{RC}}\left(-\frac{1}{RC}\right) = \frac{\varepsilon}{R} e^{\frac{-t}{RC}}$$

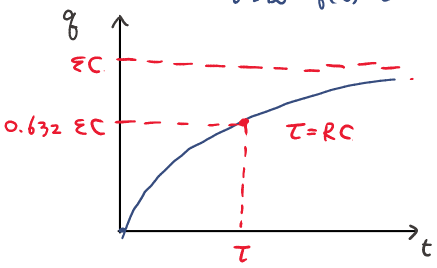
therefore $\boxed{i(t) = \frac{\varepsilon}{R} e^{\frac{-t}{RC}}}$ ☆

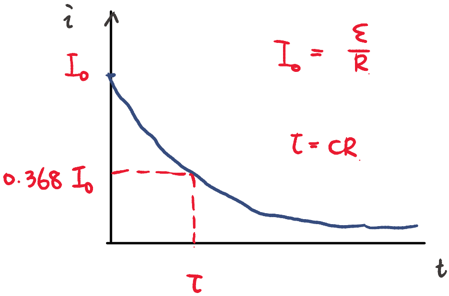
Two important graph:

$q(t) = \varepsilon C\left(1 - e^{\frac{-t}{RC}}\right)$

$i(t) = \frac{\varepsilon}{R} e^{\frac{-t}{RC}}$ $\begin{cases} t \to \infty & i(\infty) = 0 \\ t \to 0 & i(0) = \frac{\varepsilon}{R} \end{cases}$

$t \to \infty \quad q(\infty) = \varepsilon C$

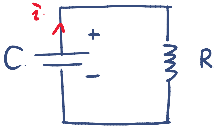




$$\tau = RC \Rightarrow [\tau] = [\Omega]\cdot[F] = \frac{[V]}{[A]}\cdot\frac{[Q]}{[V]} = \frac{[Q]}{\frac{[Q]}{[S]}} = [S]$$

τ is time constant, at this point, the potential difference across resistor is the same as capacitor

* Discharge a capacitor



$-V_c - iR = 0$ ①

$i = \frac{dQ}{dt}$ ② $\Rightarrow$ $\frac{Q}{C} + R\frac{dQ}{dt} = 0$ ④

$C = \frac{Q}{V_c}$ ③

From ④ we can have $\frac{dQ}{dt} + \frac{1}{CR}Q = 0$

tips: first order $\frac{dy}{dx} + P(x)\,y = Q(x)$.

then $y = \frac{1}{e^{\int P(x)dx}}\left(\int e^{\int P(x)dx}\, Q(x)\, dx + c\right)$

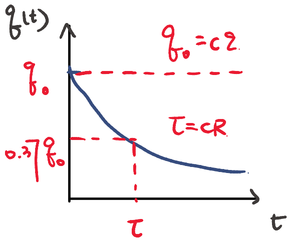
Proof: In this situation, we know $P(t) = \frac{1}{CR}$ $Q(t) = 0$

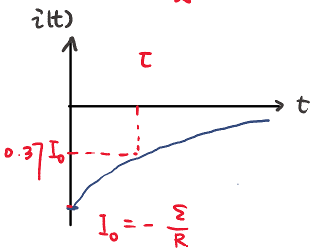
$$\therefore\ Q(t) = \frac{1}{e^{\int \frac{1}{CR}dt}}\left(\int e^{\int \frac{1}{CR}dt}\cdot 0 + c'\right) = e^{\frac{-t}{CR}}\cdot c'$$

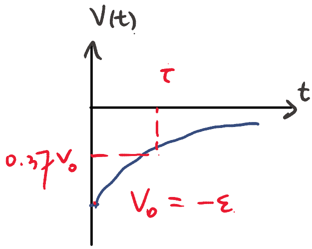
For $Q(0) = Q_0$ $\Rightarrow$ $Q_0 = 1\times c'$ $\Rightarrow$ $c' = Q_0$ ($Q_0 = C\varepsilon$)

$$\boxed{Q(t) = Q_0\, e^{\frac{-t}{CR}} = C\varepsilon e^{-\frac{t}{CR}}}$$

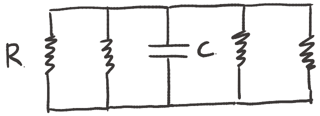
$$\boxed{i(t) = -\frac{\varepsilon}{R}\, e^{-\frac{t}{RC}}}$$







Example:



Consider $R = 100\,G\Omega$ $C = 500\,\mu F$

Q: How long does it take $V_0 = 30\,kV$ the capacitor energy drop to $U = 50\,mJ$

Answer: $U = \frac{1}{2}\,q^2/C$ For discharging: $q(t) = q_0 e^{\frac{-t}{CR_{eq}}}$

$$\therefore\ U(t) = \frac{1}{2C} q_0^2 e^{\frac{-2t}{R_{eq}C}} \Rightarrow \text{Solve for } t$$

$$\frac{2CU}{q_0^2} = e^{-2t/R_{eq}C}$$

$$\Rightarrow \quad \ln \frac{2CU}{q_0^2} = \frac{-2t}{R_{eq}C}$$

$$\Rightarrow \ t = -\frac{R_{eq}C}{2} \ln\left(\frac{2CU}{q_0^2}\right)$$

$$R_{eq} = \left[\frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \frac{1}{R}\right]^{-1} = 25\,G\Omega$$

$$q_0 = CV_0 = 15\,\mu C$$

$$\Rightarrow \boxed{t = 9.4\,s}$$

CH 28 Maganetic Field

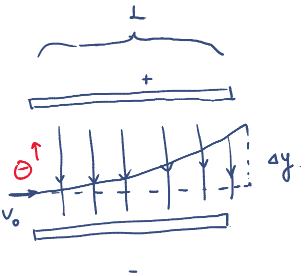
28.1 F_B for a charge moving in a Maganetic Field

$$\vec{F}_B = q\vec{v} \times \vec{B} \qquad |\vec{F}_B| = qvB\sin\theta$$

$$B = \frac{F_b}{qv\sin\theta} = \frac{N}{C\,\frac{m}{s}} = \frac{N}{A\cdot m} = T \qquad 1T = 10{,}000\ G \text{ (Gauss)}$$

28.2 Force on a Current-carrying wire

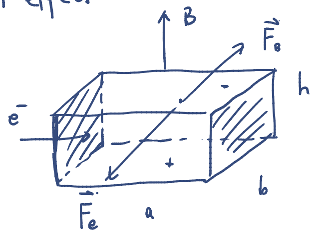
$$\vec{F}_B = i\vec{L} \times \vec{B} \qquad |F_B| = iLB\sin\theta$$



$$a = \frac{F_E}{m} = \frac{eE}{m} \qquad t = \frac{L}{v_0}$$

$$\Delta y = \frac{1}{2}at^2 = \frac{1}{2}\frac{eE}{m}\left(\frac{L}{v_0}\right)^2 = \frac{eEL^2}{2mv_0^2}$$

*Hall effect



$$i = nebh\,v_d$$

$$\Rightarrow v_d = \frac{i}{nebh} \quad ①$$

$$qv_dB = q\frac{V}{b}$$

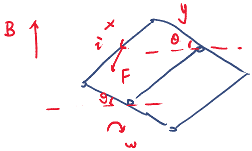
$$\Rightarrow v_d = \frac{V}{Bb} \quad ②$$

From ①②

$$\frac{i}{nebh} = \frac{V}{Bb}$$

$$\Rightarrow \boxed{V = \frac{iB}{neh}}$$

* Torque on a Current loop



$$\vec{F} = i \times B \sin\theta$$

$$\vec{\tau}_1 = \vec{F} \times \vec{r} = i \times B \sin\theta \, y = \frac{1}{2} S \cdot iB\sin\theta$$

$$\vec{\tau} = 2\vec{\tau}_1 = \boxed{iSB\sin\theta}$$

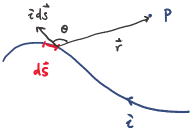
CH 29 : Maganetic Field due to Current

Q_1 : Where do maganetic field come from ?

Answers : 1) Bar / Permanent maganets

2) Electric Current

29.1 Maganetic field due to a Current

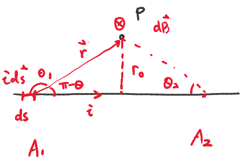


$$\boxed{\text{Biot-Savart law:} \quad d\vec{B} = \frac{\mu_0}{4\pi} \frac{i \, d\vec{s} \times \vec{e}_r}{r^2}}$$ ☆

μ_0 = permeability of free space $= 4\pi \times 10^{-7} \ \frac{T\cdot m}{A}$

i = current. r = the wire element distance to point P

* Long Straight wire



$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{s} \times \vec{e}_r}{r^2} = \frac{\mu_0}{4\pi} \frac{|ids| \sin\theta}{r^2} = \frac{\mu_0}{4\pi} \frac{|ids| \cdot \sin\theta}{r^2}$$

$$B = \int_{A_1}^{A_2} d\vec{B} = \frac{\mu_0}{4\pi} \int_{A_1}^{A_2} \frac{\sin\theta \ ids}{r^2} = \frac{\mu_0}{4\pi} \int_{\theta_1}^{\theta_2} \frac{\sin\theta \ i}{r^2} \cdot \frac{r_0}{\sin^2\theta} d\theta$$

$$\tan(\pi-\theta) = \frac{r_0}{s} = -\tan\theta$$

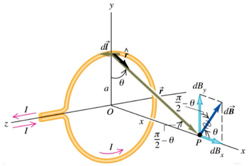
$$\Rightarrow s = -r_0 \operatorname{ctg}\theta \Rightarrow d\vec{s} = \frac{r_0 \, d\theta}{\sin^2\theta}$$

$$= \frac{\mu_0 i}{4\pi} \int_{\theta_1}^{\theta_2} \frac{r_0 \cdot \sin\theta}{r^2 \sin^2\theta} d\theta = \frac{\mu_0 i}{4\pi} \int_{\theta_1}^{\theta_2} \frac{r_0 \sin\theta}{r_0^2} d\theta$$

$$= \frac{\mu_0 i}{4\pi} \int_{\theta_1}^{\theta_2} \frac{1}{r_0} \sin\theta \, d\theta = \frac{\mu_0 i}{4\pi r_0} (\cos\theta_1 - \cos\theta_2)$$

when $\theta_1 \to \pi \quad \theta_2 \to 0 \Rightarrow \boxed{B = \frac{\mu_0 i}{2\pi r_0}}$

* Circle current



$\vec{B}_x = \oint d\vec{B}\cos\theta \qquad x = r\sin\theta$

$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{l}}{r^2}\sin\phi,\quad \phi = \frac{\pi}{2} \Rightarrow d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{l}}{r^2}$

$B_x = \frac{\mu_0}{4\pi}\oint\frac{i\,d\vec{l}}{x^2}\sin^2\theta\cos\theta = \frac{\mu_0}{4\pi}\frac{i\sin^2\theta\cos\theta}{x^2}\oint d\vec{l}$

$\cos\theta = \frac{a}{\sqrt{a^2+x^2}} \qquad \sin\theta = \frac{x}{\sqrt{a^2+x^2}} \qquad \oint d\vec{l} = 2\pi a$

$\therefore \boxed{B_x = \frac{\mu_0}{2}\frac{a^2 i}{(a^2+x^2)^{3/2}}}$

when $x=0$, $B_x = \frac{\mu_0}{2}\frac{i}{a}$

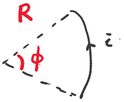
when $x \gg a$, $B_x = \frac{\mu_0 i a^2}{2x^3}$

磁矩 $\vec{m} = i\pi a^2 = i\vec{S}$

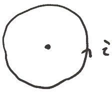
$\vec{B}_x = \frac{\mu_0\vec{m}}{2\pi x^3}$

Compare $\vec{E}_z = \frac{2k\vec{p}}{z^3} = \frac{1}{2\pi\varepsilon_0}\frac{\vec{p}}{z^3}$

* Circular arc curve

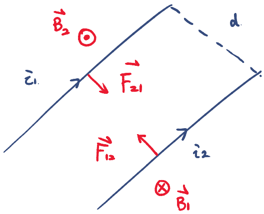


$\boxed{B = \frac{\mu_0 i}{4\pi R}\cdot\phi}$



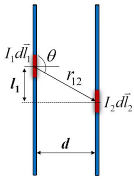
$\boxed{B = \frac{\mu_0 i}{4\pi R}\cdot 2\pi = \frac{\mu_0 i}{2R}}$

29.2 Force between parallel currents



$$\boxed{d\vec{F}_{12} = k\,\frac{(i_2 d\vec{s}_2)\times(i_1 d\vec{s}_1\times\vec{e}_r)}{r_{12}^2}}$$ ☆

$$d\vec{F}_{12} = k\frac{(I_2 d\vec{l}_2)\times(I_1 d\vec{l}_1\times\vec{e}_r)}{r_{12}^2}$$



所有dl_1对dl_2作用力的方向相同

$$dF_2 = kIdl_2\int_{-\infty}^{+\infty}\frac{Idl_1\sin\theta}{r_{12}^2} = kI^2dl_2\int_{-\infty}^{+\infty}\frac{\sin\theta}{r_{12}^2}dl_1$$

$$\because l_1 = -dctg\theta,\quad dl_1 = d\cdot d\theta/\sin^2\theta,\quad r_{12} = d/\sin\theta$$

$$\therefore dF_2 = kI^2 l_2\int_{\theta_1}^{\theta_2}\sin\theta d\theta = kI^2 dl_2(\cos\theta_1 - \cos\theta_2)$$

当 $\theta_1 = 0,\ \theta_2 = \pi$时 $dF_2 = k\frac{2I^2}{d}dl_2$

$\Rightarrow F_1 = \int_L d\vec{F}_2 = k\frac{2I^2}{d}\int_L dl_2 = \boxed{k\frac{2I^2 l}{d}}$ ☆

44

$k = \frac{\mu_0}{4\pi}$

easy method:

$\vec{F} = i_1 l\,\vec{B}_2 = i_1 l\left(\frac{\mu_0 i_2}{2\pi d}\right)$

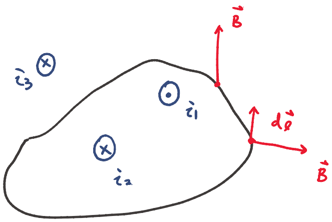
$= \frac{\mu_0 l\, i_1 i_2}{2\pi d} = k\,\frac{2 i_1 i_2 l}{d}$

* Amphere law

- Recall in electro statics, we exploited symmetry using Gaussian's Law to find the electric field ?

$$\boxed{\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \, i_{enclosed}}$$

Closed Loop

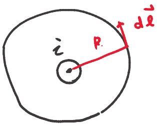


From stokes theorem: $\oint \vec{B} \cdot d\vec{\ell} = \iint (\nabla \times \vec{B}) \, ds$

For $\mu_0 \, i_{enc} = \mu_0 \iint \vec{j} \cdot d\vec{s} = \iint (\mu_0 \cdot \vec{j}) \, d\vec{s}$

$$\Rightarrow \quad \boxed{\nabla \times \vec{B} = \mu_0 \vec{j}}$$

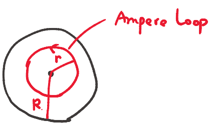
* Ampere's Law on long straight wire :



$$\oint \vec{B} \, d\vec{\ell} = \mu_0 i \Rightarrow B \cdot 2\pi R = \mu_0 i$$

$$\Rightarrow \quad B = \frac{\mu_0 i}{2\pi R}$$

* Ampere's law inside a wire

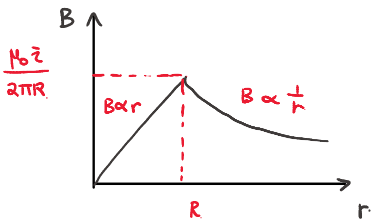


$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 i_{enc} \quad (i_{enc} = \vec{j} \cdot A_{enc})$$

$$B \cdot 2\pi r = \mu_0 \cdot \vec{j} \cdot A_{enc} = \mu_0 \cdot \frac{i}{\pi R^2} \cdot \pi r^2 = \mu_0 \, i \, \frac{r^2}{R^2}$$

$$\Rightarrow \quad B = \frac{\mu_0 i}{2\pi R^2} \cdot r$$

$$B = \begin{cases} \dfrac{\mu_0 i}{2\pi R^2} r & (r < R) \\[2ex] \dfrac{\mu_0 i}{2\pi r} & (r > R) \end{cases}$$

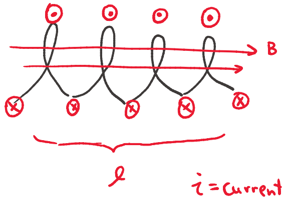


* Appendix Gauss maganetic law

$$\oiint \vec{B} \cdot d\vec{s} = 0 \Rightarrow \oiint \vec{B}\, d\vec{s} = \iiint (\nabla \cdot \vec{B})\, dV = 0 \Rightarrow \boxed{\nabla \cdot \vec{B} = 0}$$

According so far: we have
$$\begin{cases} \nabla \cdot \vec{E} = \dfrac{\rho}{\varepsilon_0} \\ \nabla \cdot \vec{B} = 0 \end{cases} \qquad \nabla \times \vec{B} = \mu_0 \vec{J}$$

* Ampere's law on Solenoids (螺线管)



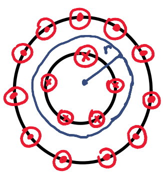
$$\oint \vec{B}\, d\vec{\ell} = B\ell = \mu_0 i = \mu_0 N i$$

$$\Rightarrow B = \mu_0 \frac{N}{\ell} i = \mu_0 n i$$

i = current, N = number of turns, n = turns per unit length

$$\Rightarrow \boxed{B = \mu_0 n i}$$

* Ampere's law in Toroids



$$\oint \vec{B} \cdot d\vec{\ell} = B \cdot 2\pi r = \mu_0 i_{enc} = \mu_0 N i$$

$$\Rightarrow \boxed{B = \frac{\mu_0 i N}{2\pi r}}$$

CH 30 Induction and Inductor (感应与电感器)

30.1 Faraday's law and Lenz's law

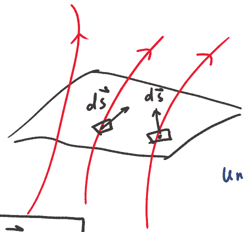
Changing maganetic field Produce electric current

The created Current = induced Current
The created emf = induced emf
The whole process = Maganetic induction

Faraday's law: A changing maganetic flux through a loop creates an emf in the loop

Quantiative treatment:

$$\boxed{\Phi_B = \iint \vec{B} \cdot d\vec{S} = 0}$$



unit wb = Weber

$$\iint \vec{B}\, d\vec{S} = \iiint (\nabla \cdot \vec{B})\, dV = 0 \Rightarrow \boxed{\nabla \cdot \vec{B} = 0}$$

(Divergence Theorem)

Mathamatically: Faraday's law

$$\boxed{\varepsilon = -N \frac{d\Phi_B}{dt}}$$

lenz's law

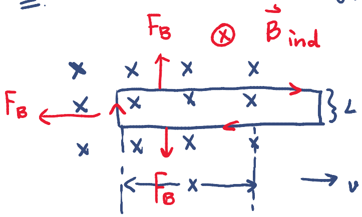
Supposed we have N turns of a Coil

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\iint \vec{B}\, d\vec{s} = -\iint \frac{\partial \vec{B}}{\partial t}\, d\vec{s}$$

* Lenz's Law.

- Faraday's law comes with a minus sign.

30.2 Induction and Energy



$$\Phi_B = BLx$$

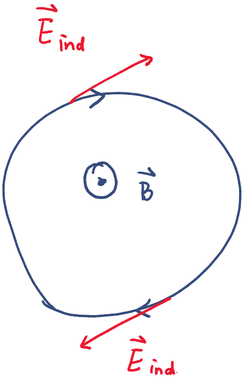
$$\mathcal{E} = \frac{d\Phi_B}{dt} = BL\frac{dx}{dt} = BLv$$

$$i = \frac{\mathcal{E}}{R} = \frac{BLv}{R} \qquad F_B = iBL = \frac{B^2L^2v}{R} \qquad P = Fv = \frac{B^2L^2v^2}{R}$$

* When magnetic field are present, it is no longer 0!

$$\boxed{\mathcal{E} = \oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}}$$

Faraday's Law



For $\oint \vec{E}\cdot d\vec{l} = \iint (\nabla\times\vec{E})\, d\vec{s}$

$$-\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BS) = \iint \frac{-\partial\vec{B}}{\partial t}\, d\vec{s}$$

$$\Rightarrow \boxed{\nabla\times\vec{E}_{ind} = -\frac{\partial\vec{B}}{\partial t}}$$

30.4 Inductors and Inductance

* We've seen how capacitor are devices that store electric field energy. Can we do the same with magnetic field? Yes Inductors!

* Inductors are solenoids of wire : 

* The inductance of an inductor is defined as : $\boxed{L = \frac{N\Phi_B}{i}}$ ☆

"How much flux produced for a given number"

[Unit] $[L] = \frac{[\Phi]}{[i]} = \frac{T \cdot m^2}{A} = \text{Henry} = 1\,H$

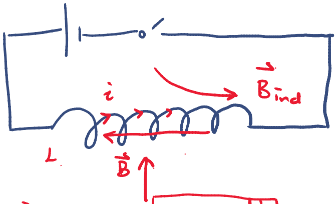
* Inductance of a Solenoid

$$L = \frac{N \cdot \Phi_B}{i} = \frac{N\,(BA)}{i} \xrightarrow[n = N/\ell]{B = \mu_0 i n} \frac{N(\mu_0 i n)A}{i} = N n \mu_0 A$$

$$= \mu_0 \ell n^2 A \Rightarrow \boxed{\frac{L}{\ell} = \mu_0 n^2 A}$$ ☆

Inductance per unit length of a solenoid!

30.5 Self-inductance



voltage of a Inductor : $\boxed{V_L = -L\frac{di}{dt}}$

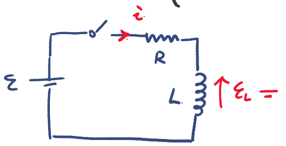
Suppose Switch from 0 to 1.

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} \qquad \text{For } N\Phi_B = Li$$

$$\Rightarrow \mathcal{E} = -\frac{d}{dt}(Li) = -L\frac{di}{dt}$$

$$\Rightarrow \boxed{\mathcal{E} = -L\frac{di}{dt}}$$ ☆

30.6 RL circuit (close switch) $-V_R - V_L + \varepsilon = 0$



$V_R = iR$

$V_L = -L\dfrac{di}{dt}$

$\Rightarrow \varepsilon = iR + L\dfrac{di}{dt} \Rightarrow \dfrac{di}{dt} + \dfrac{R}{L}i - \dfrac{\varepsilon}{L} = 0$

$\Rightarrow$ differentia equation $\dfrac{di}{dt} + \underset{P(t)}{\dfrac{R}{L}} i = \underset{Q(t)}{\dfrac{\varepsilon}{L}}$

tips: first order $\dfrac{dy}{dx} + P(x)\,y = Q(x)$.

then $y = \dfrac{1}{e^{\int P(x)dx}}\left(\int e^{\int P(x)dx} Q(x)\,dx + C\right)$

$$i(t) = \frac{1}{e^{\int \frac{R}{L}dt}}\left(\int e^{\int \frac{R}{L}dt}\,\frac{\varepsilon}{L}\,dt + C\right)$$

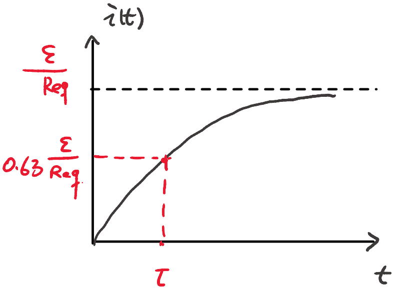
$$= \frac{1}{e^{\frac{R}{L}t}}\left(\int e^{\frac{R}{L}t}\,\frac{\varepsilon}{L}\,dt + C\right)$$

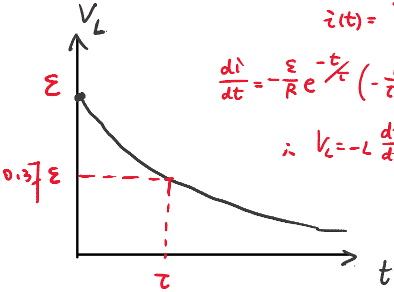
$$= \frac{1}{e^{\frac{R}{L}t}}\left(\frac{\varepsilon}{L}\cdot\frac{L}{R}\,e^{\frac{R}{L}t} + C\right) = \frac{\varepsilon}{R} + \frac{C}{e^{\frac{R}{L}t}}$$

$i(0) = \varepsilon R + C = 0 \Rightarrow C = -\dfrac{\varepsilon}{R} \Rightarrow i(t) = \dfrac{\varepsilon}{R}\left(1 - e^{-\frac{R}{L}t}\right)$

$\Rightarrow i(t) = \dfrac{\varepsilon}{R}\left(1 - e^{\frac{-t}{L/R}}\right)$ we set $\tau = L/R$

$$\Rightarrow \boxed{i(t) = \frac{\varepsilon}{R}\left(1 - e^{\frac{-t}{\tau}}\right)}$$

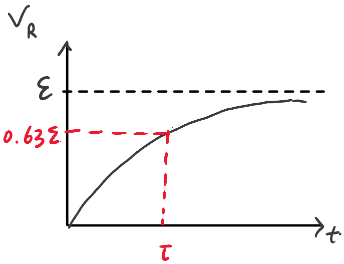




$$i(t) = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right)$$

$$\frac{di}{dt} = -\frac{\varepsilon}{R} e^{-t/\tau}\left(-\frac{1}{\tau}\right) = \frac{\varepsilon}{L} e^{-t/\tau}$$

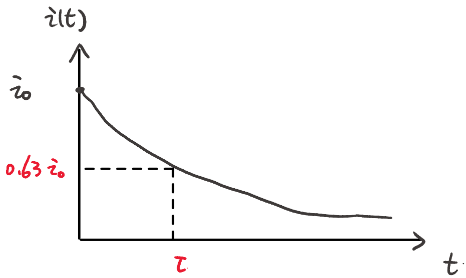
$$\therefore\ V_L = -L\frac{di}{dt} = -\varepsilon e^{-t/\tau}$$



$$V_R = i(t)\,R = \varepsilon\left(1 - e^{-t/\tau}\right)$$

* opening switch

$$i(t) = \frac{\varepsilon}{R} e^{-t/\tau} = i_0\, e^{-t/\tau} \qquad \tau = L/R$$



Maxwell Equation Summary :

From Gauss law: $\oiint \vec{E}\, d\vec{s} = \frac{Q_{enc}}{\varepsilon_0}$ For $\oiint \vec{E}\, d\vec{s} = \iiint \nabla\cdot\vec{E}\, dV$

also $\frac{Q_{enc}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\iiint \rho\, dV = \iiint \frac{\rho}{\varepsilon_0}\, dV \Rightarrow \boxed{\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}}$ ①

From $\oiint \vec{B}\cdot d\vec{s} = 0 \Rightarrow \oiint \vec{B}\, d\vec{s} = \iiint (\nabla\cdot\vec{B})\, dV = 0 \Rightarrow \boxed{\nabla\cdot\vec{B} = 0}$ ②

From $\oint \vec{B}\cdot d\vec{l} = \mu_0 i_{enc}$. For $\oint \vec{B}\cdot d\vec{l} = \iint (\nabla\times\vec{B})\, d\vec{s}$

also $\mu_0 i_{enc} = \mu_0 \iint \vec{j}\, d\vec{s} = \iint \mu_0\vec{j}\, d\vec{s} \Rightarrow \boxed{\nabla\times\vec{B} = \mu_0\vec{j}}$ ③

From $\oint \vec{E}_{ind}\, d\vec{l} = -\frac{d\Phi_B}{dt}$. $\oint \vec{E}_{still}\, d\vec{l} = 0$

$\Rightarrow \oint \vec{E}\, d\vec{l} = \oint (\vec{E}_{ind} + \vec{E}_{still})\, d\vec{l} = -\frac{d\Phi_B}{dt} \Rightarrow \oint \vec{E}\, d\vec{l} = \iint (\nabla\times\vec{E})\, d\vec{s}$

also $-\frac{d}{dt}\Phi_B = -\frac{d}{dt}\iint \vec{B}\cdot d\vec{s} = \iint \frac{-\partial\vec{B}}{\partial t}\, d\vec{s} \Rightarrow \boxed{\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$ ④

30.7 Energy stored in magnetic field

$$P = i V_L = i\left(L \frac{di}{dt}\right) = iL \frac{di}{dt}$$

also $P = \frac{dU}{dt} = iL \frac{di}{dt} \Rightarrow \quad dU = iL\, di$

$$\Rightarrow \int dU = \int iL\, di \Rightarrow \boxed{U = \frac{1}{2} L i^2}$$

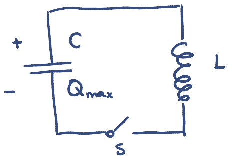
30.8 Energy density in magnetic field

Recall $\quad u_E = \frac{U_E}{V} = \frac{1}{2}\varepsilon_0 E^2$ (electric field energy)

$$u_B = \frac{U_B}{V} = \frac{1}{2\mu_0} B^2$$

CH 31 : Electromagnetic Oscillations and Alternating Current

LC (Inductor - Capacitor) Circuit



Start : capacitor full charged $U_E = \frac{Q_{max}^2}{2C}$ $U_B = 0$

switch close $U_E \downarrow$ $U_B \uparrow$ i reach max

$$U_B = \frac{1}{2} L i_{max}^2$$

$$\boxed{U = U_E + U_B = \frac{q^2}{2C} + \frac{1}{2} L i^2}$$ ☆

$\frac{dU}{dt} = 0$ (U is constant)

According to KVL : $V_L + V_C = 0$ $V_L = L \frac{di}{dt}$ $V_C = \frac{q}{C}$

$$\Rightarrow L \frac{di}{dt} + \frac{q}{C} = 0 \Rightarrow L \frac{d}{dt}\left(\frac{dq}{dt}\right) + \frac{q}{C} = 0$$

$$\Rightarrow L \frac{d^2q}{dt^2} + \frac{q}{C} = 0 \Rightarrow \frac{d^2q}{dt^2} + \frac{1}{LC} q = 0$$

Assume Solution $\frac{dq}{dt} = A\alpha e^{\alpha t}$ $\frac{d^2q}{dt^2} = A\alpha^2 e^{\alpha t}$

$$\Rightarrow A\alpha^2 e^{\alpha t} + \frac{1}{CL} A e^{\alpha t} = 0 \Rightarrow \left(\alpha^2 + \frac{1}{LC}\right) A e^{\alpha t} = 0$$

$$\therefore \alpha^2 + \frac{1}{LC} = 0 \Rightarrow \alpha = \pm j\sqrt{\frac{1}{LC}} \quad (\text{where } j^2 = -1)$$

$$\therefore q(t) = A_1 e^{j\omega_0 t} + A_2 e^{-j\omega_0 t} \quad \left(\omega_0 = \sqrt{\frac{1}{LC}}\right)$$

From $e^{j\theta} = \cos\theta + j\sin\theta$

$$\Rightarrow q(t) = A_1 e^{j\omega_0 t} + A_2 e^{-j\omega_0 t} = A_1(\cos\omega_0 t + j\sin\omega_0 t) + A_2(\cos(-\omega_0 t) + j\sin(-\omega_0 t))$$

$$= B_1 \cos(\omega_0 t) + B_2 \sin(\omega_0 t)$$

$$\therefore i(t) = \frac{dq(t)}{dt} = -B_1\omega_0 \sin(\omega_0 t) + B_2\omega_0 \cos(\omega_0 t)$$

$\therefore$ Solution

$$\boxed{\begin{aligned} q(t) &= B_1\cos(\omega_0 t) + B_2\sin(\omega_0 t) \\ i(t) &= -B_1\omega_0\sin(\omega_0 t) + B_2\omega_0\cos(\omega_0 t) \end{aligned}}$$

$\omega_0 = \frac{1}{\sqrt{CL}}$

B_1, B_2 determined by initial condition of circuit

Also simplifier more

$$\begin{cases} q(t) = Q_{max}\cos(\omega t + \varphi) \\ i(t) = -i_{max}\sin(\omega t + \varphi) \end{cases}$$

$Q_{max} = \varepsilon C$

$i_{max} = \omega Q_{max}$

$$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C}\cos^2(\omega t + \varphi)$$

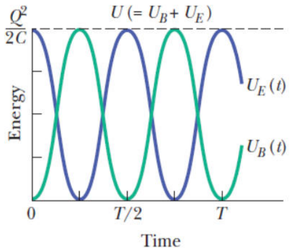
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC}$$

$$U_B = \frac{Q^2}{2C}\sin^2(\omega t + \varphi)$$

Energy In an LC Circuit

The electrical and magnetic energies vary but the total is constant.

$$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C}\cos^2(\omega t + \phi)$$

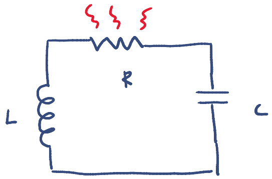


$$U_B = \frac{Q^2}{2C}\sin^2(\omega t + \phi).$$

Note that:

- The maximum values of U_E and U_B are both $\frac{Q^2}{2C}$
- At any instant the sum of U_E and U_B is equal to $\frac{Q^2}{2C}$, a constant.
- When U_E is maximum, U_B is zero, and conversely.

* Damped RLC Circuit



$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{q^2}{2C} + \frac{Li^2}{2}\right) = -i^2R$$

$$\Rightarrow \frac{dU}{dt} = -i^2R \quad (\text{energy consume on } R)$$

According KVL $\Rightarrow$ $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0$

<Solution> : assume $q(t) = e^{\alpha t}$ $\quad \frac{dq}{dt} = \alpha e^{\alpha t}$ $\quad \frac{d^2q}{dt^2} = \alpha^2 e^{\alpha t}$

$$\Rightarrow L\cdot\alpha^2 e^{\alpha t} + R\alpha e^{\alpha t} + \frac{1}{C}e^{\alpha t} = 0 \Rightarrow e^{\alpha t}\left(L\alpha^2 + R\alpha + \frac{1}{C}\right) = 0$$

Solve for α: $\quad L\alpha^2 + R\alpha + \frac{1}{C} = 0$, $\quad \Delta = R^2 - \frac{4L}{C}$

$$\therefore \alpha = \frac{-R \pm \sqrt{R^2 - \frac{4L}{C}}}{2L}$$

① Over damped Case ($\Delta > 0$)

$R^2 > \frac{4L}{C}$ $\quad \alpha_1 = \frac{-R + \sqrt{R^2 - \frac{4L}{C}}}{2L}$, $\quad \alpha_2 = \frac{-R - \sqrt{R^2 - \frac{4L}{C}}}{2L}$

$$\boxed{q(t) = A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t}}$$

② Critically Damped Case ($\Delta = 0$)

$\alpha = \frac{-R}{2L}$ $\quad \boxed{q(t) = (A_1 + A_2 t)\, e^{\alpha t}}$

③ Under damped Case ($\Delta < 0$)

$\alpha = \frac{-R}{2L} \pm j\omega_d$ $\quad \omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$

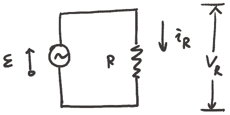
$$\boxed{q(t) = e^{-\frac{R}{2L}t}\left(A_1 \cos(\omega_d t) + A_2 \sin(\omega_d t)\right)}$$

For usual we take underdamped.

$$q(t) = Q_{max}\, e^{-\frac{R}{2L}t} \cos(\omega' t + \varphi) \qquad \omega' = \sqrt{\omega^2 - \left(\frac{R}{2L}\right)^2} \qquad \omega = \frac{1}{\sqrt{LC}}$$

$$U_E = \frac{q^2}{2C} = \frac{\left[Q e^{-\frac{Rt}{2L}} \cos(\omega' t + \phi)\right]^2}{2C} = \frac{Q^2}{2C} e^{-Rt/L} \cos^2(\omega' t + \phi)$$

* Circuit 1 : Resistive Load.



$$\mathcal{E} - V_R = 0$$

$$V_R = \mathcal{E}_m \sin(\omega_d t) = V_R \sin(\omega_d t)$$

$$i_R = \frac{V_R}{R} = \frac{V_R}{R} \sin(\omega_d t) = I_R \sin(\omega_d t)$$

For a purely resistive load the phase constant $\phi = 0°$

Circuit #1: Resistive Load, cont.

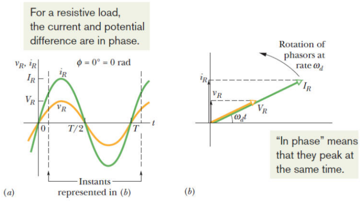
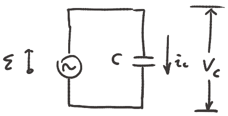


Fig. 31-9 (a) The current i_R and the potential difference v_R across the resistor are plotted on the same graph, both versus time t. They are in phase and complete one cycle in one period T. (b) A phasor diagram shows the same thing as (a).

* Circuit 2 : Capacitive Load.



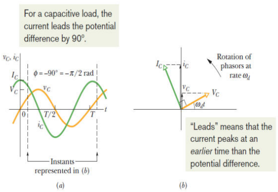
$$V_c = V_c \sin(\omega_d t)$$

$$q_c = C V_c = C V_c \sin(\omega_d t)$$

$$i_c = \frac{dq_c}{dt} = \omega_d C V_c \cos(\omega_d t)$$

$$X_c = \frac{1}{\omega_d C} \quad \text{(capacitive reactance)}$$

Circuit #2: Capacitive Load, cont.



$$\cos \omega_d t = \sin(\omega_d t + 90°).$$

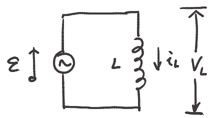
$$i_C = \left(\frac{V_C}{X_C}\right) \sin(\omega_d t + 90°).$$

$$i_C = I_C \sin(\omega_d t - \phi),$$

$$V_C = I_C X_C \quad \text{(capacitor).}$$

Fig. 31-11 (a) The current in the capacitor leads the voltage by 90° (= π/2 rad). (b) A phasor diagram shows the same thing.

* Circuit 3 : Inductive Load



$$V_L = V_L \sin(\omega_d t) \qquad V_L = L\,\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{V_L}{L} = \frac{V_L}{L}\sin(\omega_d t) \Rightarrow \int di_L = \int \frac{V_L}{L}\sin(\omega_d t)\,dt$$

$$\Rightarrow i_L = \frac{V_L}{L}\int \sin(\omega_d t)\,dt = -\frac{V_L}{\omega_d L}\cos(\omega_d t)$$

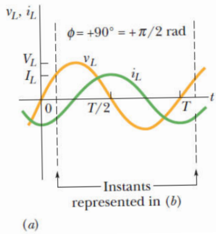
$$i_L = \left(\frac{V_L}{X_L}\right)\sin(\omega_d t - 90^\circ)$$

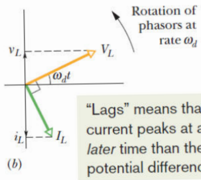
$X_L = \omega_d L$ (inductive reactance)

$$i_L = I_L \sin(\omega_d t - \phi)$$

Circuit #3: Inductive Load, cont.

For an inductive load, the current lags the potential difference by 90°.





"Lags" means that the current peaks at a *later* time than the potential difference.

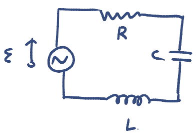
Fig. 31-13 (*a*) The current in the inductor lags the voltage by 90° (= $\pi/2$ rad). (*b*) A phasor diagram shows the same thing.

The summary table for the three simple circuits

Table 31-2

Phase and Amplitude Relations for Alternating Currents and Voltages

Circuit Element	Symbol	Resistance or Reactance	Phase of the Current	Phase Constant (or Angle) ϕ	Amplitude Relation
Resistor	R	R	In phase with v_R	0° (= 0 rad)	$V_R = I_R R$
Capacitor	C	$X_C = 1/\omega_d C$	Leads v_C by 90° (= $\pi/2$ rad)	−90° (= −$\pi/2$ rad)	$V_C = I_C X_C$
Inductor	L	$X_L = \omega_d L$	Lags v_L by 90° (= $\pi/2$ rad)	+90° (= +$\pi/2$ rad)	$V_L = I_L X_L$

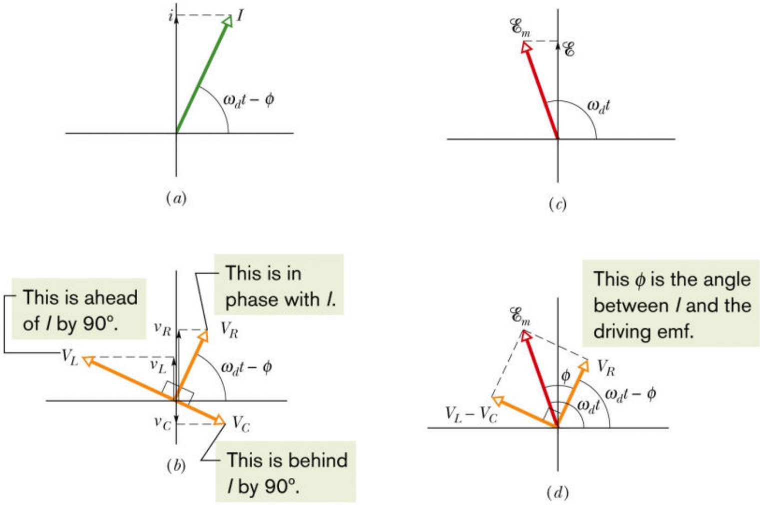


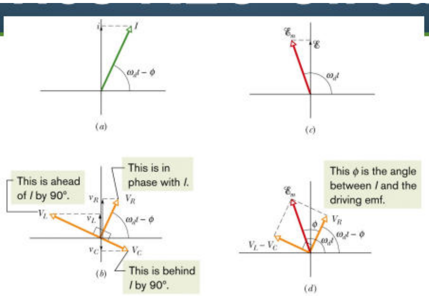
$$\mathcal{E} = \mathcal{E}_m \sin(\omega_d t)$$

$$i = I \sin(\omega_d t)$$

$$\Rightarrow I = \frac{\mathcal{E}_m}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$= \frac{\mathcal{E}_m}{\sqrt{R^2 + \left(\omega_d L - \frac{1}{\omega_d C}\right)^2}}$$





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From the right-hand phasor triangle in Fig.(d) we can write

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR}, \quad \Rightarrow \quad \boxed{\tan\phi = \frac{X_L - X_C}{R}}$$

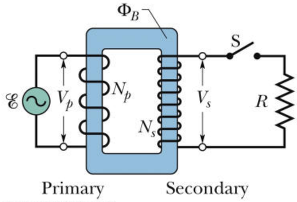
Phase Constant

The current amplitude I is maximum when the driving angular frequency ω_d equals the natural angular frequency ω of the circuit, a condition known as **resonance**. Then $X_C = X_L$, $\phi = 0$, and the current is in phase with the emf.

$$\omega_d = \omega = \frac{1}{\sqrt{LC}} \quad \text{(resonance).}$$

Transformer

Transformers



An ideal transformer (two coils wound on an iron core) in a basic trans- former circuit. An ac generator produces current in the coil at the left (the primary). The coil at the right (the secondary) is connected to the resistive load R when switch S is closed.

A transformer (assumed to be ideal) is an iron core on which are wound a primary coil of N_p turns and a secondary coil of N_s turns. If the primary coil is connected across an alternating-current generator, the primary and secondary voltages are related by

$$V_s = V_p \frac{N_s}{N_p}$$

Energy Transfers. The rate at which the generator transfers energy to the primary is equal to I_pV_p. The rate at which the primary then transfers energy to the secondary (via the alternating magnetic field linking the two coils) is I_sV_s. Because we assume that no energy is lost along the way, conservation of energy requires that

$$I_pV_p = I_sV_s. \longrightarrow \quad I_s = I_p \frac{N_p}{N_s}$$

The equivalent resistance of the (primary) circuit, as “felt” by the generator, is

$$R_{eq} = \left(\frac{N_p}{N_s}\right)^2 R.$$

Note: In this equation, R_{eq} is R_p and R is R_s, assuming no power loss from primary to secondary side.

CH 32. Maxwell Equation

Maxwell Equation Summary :

From Gauss law: $\oiint \vec{E}\, d\vec{s} = \frac{Q_{enc}}{\varepsilon_0}$ For $\oiint \vec{E}\, d\vec{s} = \iiint \nabla\cdot\vec{E}\, dV$

also $\frac{Q_{enc}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\iiint \rho\, dV = \iiint \frac{\rho}{\varepsilon_0}\, dV \Rightarrow \boxed{\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}}$ ①

From $\oiint \vec{B}\cdot d\vec{s} = 0 \Rightarrow \oiint \vec{B}\, d\vec{s} = \iiint (\nabla\cdot\vec{B})\, dV = 0 \Rightarrow \boxed{\nabla\cdot\vec{B} = 0}$ ②

From $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 i_{enc}$. For $\oint \vec{B}\cdot d\vec{\ell} = \iint (\nabla\times\vec{B})\, d\vec{s}$

also $\mu_0 i_{enc} = \mu_0 \iint \vec{j}\, d\vec{s} = \iint \mu_0 \vec{j}\, d\vec{s} \Rightarrow \nabla\times\vec{B} = \mu_0\vec{j}$ ③

also from maxwell's law of Induction $\oint \vec{B}\cdot d\vec{\ell} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}$

$\Rightarrow \oint \vec{B}\cdot d\vec{\ell} = \mu_0 i_{enc} + \varepsilon_0\mu_0 \frac{d\Phi_E}{dt} \Rightarrow \iint (\nabla\times\vec{B})\, d\vec{s} = \mu_0\iint \vec{j}\, d\vec{s} + \varepsilon_0\mu_0 \frac{\partial}{\partial t}\iint \vec{E}\, d\vec{s}$

$\Rightarrow \boxed{\nabla\times\vec{B} = \mu_0\vec{j} + \varepsilon_0\mu_0 \frac{\partial\vec{E}}{\partial t}}$ ③

From $\oint \vec{E}_{ind}\, d\vec{\ell} = -\frac{d\Phi_B}{dt}$. $\oint \vec{E}_{still}\, d\vec{\ell} = 0$

$\Rightarrow \oint \vec{E}\, d\vec{\ell} = \oint (\vec{E}_{ind} + \vec{E}_{still})\, d\vec{\ell} = -\frac{d\Phi_B}{dt} \Rightarrow \oint \vec{E}\, d\vec{\ell} = \iint (\nabla\times\vec{E})\, d\vec{s}$

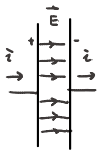
also $-\frac{d}{dt}\Phi_B = -\frac{d}{dt}\iint \vec{B}\cdot d\vec{s} = \iint \frac{-\partial\vec{B}}{\partial t}\, d\vec{s} \Rightarrow \boxed{\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$ ④

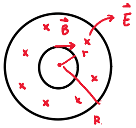
Maxwell's law - Example problem

A parallel plate capacitor with circular plates of radius R is being charged

a) Derive the expression for the induced magnetic field at radius $r \leq R$

b) Derive an expression for the induced magnetic field for case $r \geq R$





a) $i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \pi r^2 \frac{d\vec{E}}{dt}$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \Rightarrow B \cdot 2\pi r = \mu_0 \varepsilon_0 \pi r^2 \frac{d\vec{E}}{dt}$$

$$\Rightarrow B = \frac{\mu_0 \varepsilon_0 r}{2} \frac{d\vec{E}}{dt}$$

b) $\oint \vec{B} \cdot d\vec{l} = \mu_0 i_d = \mu_0 \varepsilon_0 \pi R^2 \frac{d\vec{E}}{dt} = B \cdot 2\pi r \Rightarrow B = \frac{\mu_0 \varepsilon_0 R^2}{2r} \frac{d\vec{E}}{dt}$

$$\therefore \boxed{B = \begin{cases} \frac{\mu_0 \varepsilon_0 r}{2} \frac{d\vec{E}}{dt} & (r \leq R) \\ \frac{\mu_0 \varepsilon_0 R^2}{2r} \frac{d\vec{E}}{dt} & (r \geq R) \end{cases}} \quad ☆$$

Ampere law simplify:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc} \quad \text{(Ampere-Maxwell Equation)}$$

displacement current $\boxed{i_d = \varepsilon_0 \frac{d\Phi_E}{dt}}$ ☆ displacement current density $\boxed{\vec{J}_d = \frac{i_d}{A} = \varepsilon_0 \frac{d\vec{E}}{dt}}$ ☆

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{d,enc} + \mu_0 i_{enc}$$

32.5 Magnetism and Electrons

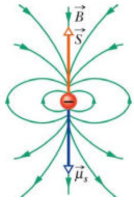
1. Spin Magnetic Dipole Moment. $\vec{S}$: An electron intrinsic spin angular momentum

(自旋磁偶极矩) $\vec{\mu}_s$

$$\boxed{\vec{\mu}_s = -\frac{e}{m}\vec{S}}$$

$S_z = m_s \frac{h}{2\pi}$ $m_s = \pm 1/2$

For an electron, the spin is opposite the magnetic dipole moment.



Therefore $\mu_{s,z} = \pm \frac{eh}{4\pi m} = \pm \mu_B \Rightarrow \mu_B = \frac{eh}{4\pi m} = 9.27\times10^{-24}\ J/T$

Bohr magneton

when electron placed in external magnetic field $\vec{B}_{ext}$

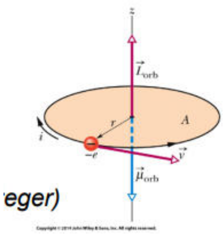
orientation energy $U = -\vec{\mu}_s \cdot \vec{B}_{ext} = -\mu_{s,z} \cdot B_{ext}$

2. Orbital magnetic Dipole Moment. $\vec{L}_{orb}$: electron has additional angular momentum

(旋转电子磁偶极矩 $\vec{\mu}_{orb}$)

$$\boxed{\vec{\mu}_{orb} = -\frac{e}{2m}\vec{L}_{orb}}$$

$L_{orb,z} = m_\ell \frac{h}{2\pi} \left(m_\ell = 0, \pm1, \pm2, \dots\right)$

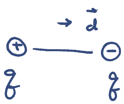


$\therefore \vec{\mu}_{orb} = -m_\ell \frac{eh}{4\pi m} = -m_\ell \mu_B$

place in $\vec{B}_{ext}$ $U = -\vec{\mu}_{orb} \cdot \vec{B}_{ext} = -\mu_{orb,z} B_{ext}$

Recall some knowledge

1. Electric Dipole moment $\vec{p} = q \cdot \vec{d}$



2. Magnetic Dipole moment $\vec{m} = i \cdot \vec{A}$

Special $\vec{\mu}_B$ is $\vec{m}$ for electron

$\vec{\mu}_N$ is $\vec{m}$ for proton / netron

32 Summary

Diamagnetism

- Diamagnetic materials exhibit magnetism only when placed in an external magnetic field; there they form magnetic dipoles directed opposite the external field. In a nonuniform field, they are repelled from the region of greater magnetic field.

Paramagnetism

- Paramagnetic materials have atoms with a permanent magnetic dipole moment but the moments are randomly oriented unless the material is in an external magnetic field. The extent of alignment within a volume V is measured as the magnetization M, given by

$$M = \frac{\text{measured magnetic moment}}{V}, \quad \textbf{Eq. 32-28}$$

- Complete alignment (saturation) of all N dipoles in the volume gives a maximum value $M_{max} = N\mu/V$. At low values of the ratio $\boldsymbol{B}_{ext}/T$,

$$M = C\frac{B_{ext}}{T} \quad \textbf{Eq. 32-39}$$

Ferromagnetism

- The magnetic dipole moments in a ferromagnetic material can be aligned by an external magnetic field and then, after the external field is removed, remain partially aligned in regions (domains). Alignment is eliminated at temperatures above a material's Curie temperature. In a nonuniform external field, a ferromagnetic material is attracted to the region of greater magnetic field.

$\underset{\sim}{M}$ magnetization (磁化强度)　　$\vec{M} = \dfrac{\vec{m}_{net}}{V}$ (净磁偶极矩)

$\underset{\sim}{P}$ polarization (极化强度)　　$\vec{P} = \dfrac{\vec{P}_{net}}{V}$ (净电偶极矩)

$\vec{P} = \varepsilon_0 \chi_e \vec{E}$ (χ_e 是电极化率)　　$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

$\vec{M} = \chi_m \vec{B}$ (χ_m 是磁化率)　　$\vec{B} = \mu_0(\vec{H} + \vec{M})$

$\vec{D}$: electric displacement field / electric induction

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \vec{E} + \varepsilon_0 \chi_e \vec{E} = (1+\chi_e)\varepsilon_0 \vec{E} \xlongequal[\varepsilon_0 \varepsilon_r = \varepsilon]{\varepsilon_r = 1+\chi_e} \varepsilon \vec{E}$$

ε_r: relative permitivity　　ε: permitivity

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) = \mu_0 \vec{H} + \mu_0 \vec{M} = \mu_0 \vec{H} + \mu_0 \chi_m \vec{H} = \mu_0 \vec{H}(1+\chi_m) \xlongequal[\mu_0 \mu_r = \mu]{1+\chi_m = \mu_r} \mu \vec{H}$$

$$\therefore \begin{cases} \vec{D} = \varepsilon \vec{E} \\ \vec{B} = \mu \vec{H} \end{cases}$$

在真空里的麦克斯韦方程组 [编辑]

微观表述[4]:15

名称	微分形式	积分形式
高斯定律	$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$	$\oiint_{\mathbb{S}} \mathbf{E} \cdot \mathrm{d}\mathbf{s} = \frac{Q}{\varepsilon_0}$
高斯磁定律	$\nabla \cdot \mathbf{B} = 0$	$\oiint_{\mathbb{S}} \mathbf{B} \cdot \mathrm{d}\mathbf{s} = 0$
法拉第电磁感应定律	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint_{\mathbb{L}} \mathbf{E} \cdot \mathrm{d}\boldsymbol{\ell} = -\frac{\mathrm{d}\Phi_{\mathbf{B}}}{\mathrm{d}t}$
麦克斯韦-安培定律	$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$	$\oint_{\mathbb{L}} \mathbf{B} \cdot \mathrm{d}\boldsymbol{\ell} = \mu_0 I + \mu_0 \varepsilon_0 \frac{\mathrm{d}\Phi_{\mathbf{E}}}{\mathrm{d}t}$

这种形式的麦克斯韦方程组又称为“微观麦克斯韦方程组”，可以用来推导出宏观麦克斯韦方程组，也可以用来找出原子性质与宏观性质两者之间的关联。[11]:1, 4[5]:2, 248

有电介质时的麦克斯韦方程组 [编辑]

宏观表述[4]:193

名称	微分形式	积分形式
高斯定律	$\nabla \cdot \mathbf{D} = \rho_f$	$\oiint_{\mathbb{S}} \mathbf{D} \cdot \mathrm{d}\mathbf{s} = Q_f$
高斯磁定律	$\nabla \cdot \mathbf{B} = 0$	$\oiint_{\mathbb{S}} \mathbf{B} \cdot \mathrm{d}\mathbf{s} = 0$
法拉第电磁感应定律	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint_{\mathbb{L}} \mathbf{E} \cdot \mathrm{d}\boldsymbol{\ell} = -\frac{\mathrm{d}\Phi_{\mathbf{B}}}{\mathrm{d}t}$
麦克斯韦-安培定律	$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$	$\oint_{\mathbb{L}} \mathbf{H} \cdot \mathrm{d}\boldsymbol{\ell} = I_f + \frac{\mathrm{d}\Phi_{\mathbf{D}}}{\mathrm{d}t}$

Power in AC Circuit Example Problem

$$I_{rms} = \frac{I}{\sqrt{2}} \qquad P_{avg} = I_{rms}^2 R$$

$$V_{rms} = \frac{V}{\sqrt{2}} \quad \& \quad \mathcal{E}_{rms} = \frac{\mathcal{E}_m}{\sqrt{2}}$$

$$P_{avg} = \mathcal{E}_{rms} \cdot i_{rms} \cdot \cos\phi \qquad \cos\phi = \frac{R}{Z} \text{ (power factor)}$$

CH 33: Electromagnetic Waves

$$E(x,t) = E_m \sin(kx - \omega t)$$

$$B(x,t) = B_m \sin(kx - \omega t)$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

displacement, Amplitude, Oscillating term

$$y(x,t) = y_m \sin(kx - \omega t)$$

Time, phase, position

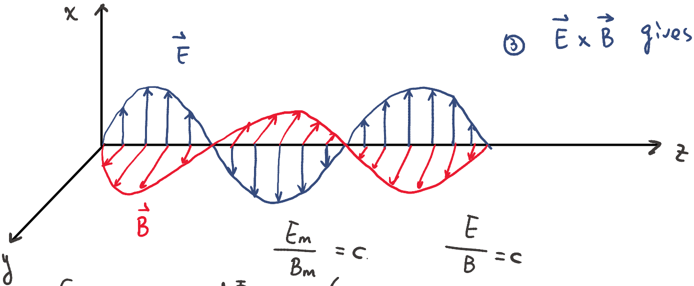
$k = \frac{2\pi}{\lambda}$ is the angular wave number

x is the x-position of the wave element

$\omega = \frac{2\pi}{T}$ is the angular frequency (also: $\omega = 2\pi f$)

t is Time

1. $\vec{E}$ & $\vec{B}$ always $\perp$
2. $\vec{E}$ & $\vec{B}$ $\perp$ to direction of travel
3. $\vec{E} \times \vec{B}$ gives the direction of travel

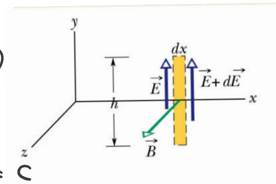


$$\frac{E_m}{B_m} = c \qquad \frac{E}{B} = c$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} = (E + dE)h - Eh = h\,dE$$

$$\frac{d\Phi_B}{dt} = \underbrace{h\,dx}_{S} \frac{dB}{dt} \Rightarrow h\,dE = -h\,dx\,\frac{dB}{dt} \Rightarrow \frac{dE}{dx} = -\frac{dB}{dt}$$

$$\frac{\partial E}{\partial x} = -kE_m \cos(kx - \omega t) \qquad \frac{\partial B}{\partial t} = -\omega B_m \cos(kx - \omega t)$$

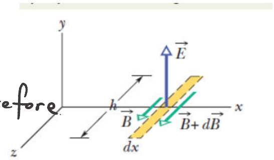


$$\Rightarrow kE_m = \omega B_m \Rightarrow \frac{E_m}{B_m} = \frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = f\lambda = c$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \Rightarrow \oint \vec{B}\cdot d\vec{l} = -(B+dB)h + Bh = -h\,dB$$

$$\frac{d\Phi_E}{dt} = \underbrace{h\,dx}_{S} \frac{dE}{dt} \Rightarrow -h\,dB = \mu_0\varepsilon_0 \left(h\,dx \frac{dE}{dt}\right)$$

$$\therefore \quad -\frac{dB}{dx} = \mu_0\varepsilon_0 \frac{dE}{dt}$$



$$\frac{\partial B}{\partial x} = kB_m \cos(kx-\omega t), \quad \frac{\partial E}{\partial t} = \omega E_m \cos(kx-\omega t) \text{ therefore}$$

$$-kB_m \cos(kx-\omega t) = -\mu_0\varepsilon_0 \omega E_m \cos(kx-\omega t)$$

$$\Rightarrow \frac{E_m}{B_m} = \frac{k}{\omega} \cdot \frac{1}{\mu_0\varepsilon_0} = \frac{1}{\mu_0\varepsilon_0(\omega/k)} = \frac{1}{\mu_0\varepsilon_0\left(E_m/B_m\right)} \Rightarrow \frac{E_m}{B_m} = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

$$\therefore \quad c = \frac{E_m}{B_m} = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

* Energy Transport and the Poynting Vector, cont

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \quad (\text{Poynting Vector}) \qquad \text{unit} \left(\frac{\text{Power}}{\text{area}}\right)$$

$$S = \frac{1}{\mu_0} EB \xrightarrow{c = \frac{E}{B}} \frac{1}{c\mu_0} E^2 \quad (\text{Instantaneous energy flow rate})$$

$$I = S_{avg} = \frac{1}{c\mu_0}(E_{avg})^2 = \frac{1}{c\mu_0}\left[E_m^2 \sin^2(kx-\omega t)\right]_{avg} = \frac{1}{c\mu_0} E_{rms}^2$$

$$\left(E_{rms} = \frac{E_m}{\sqrt{2}}\right)$$

intensity

time-averaged value of energy transport rate

$$u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 (cB)^2 = \frac{1}{2}\varepsilon_0 \frac{1}{\mu_0 \varepsilon_0} B^2 = \frac{B^2}{2\mu_0}$$ (energy density)

$I = c\,u_E$

The simple intensity equation: $I = \dfrac{\text{Power}}{\text{area}} = \dfrac{P_s}{4\pi r^2}$

* Radiation Pressure

Electromagnetic waves have linear momentum and thus can exert a pressure on an object when shining on it.

During Δt an object gain ΔU energy from radiation

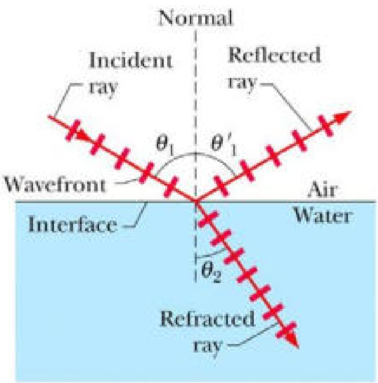
change of momentum $\boxed{\Delta p = \dfrac{\Delta U}{c}}$ (total absorption) ☆

If total reflection, $\Delta p = \dfrac{2\Delta U}{c}$

For $F = \dfrac{\Delta p}{\Delta t}$, and intensity is $I = \dfrac{\text{Power}}{\text{area}} = \dfrac{\text{energy/time}}{\text{area}}$

$\therefore F = \dfrac{IA}{c}$ (absorption) $\quad F = \dfrac{2IA}{c}$ (reflection)

$P_r = \dfrac{I}{c}$ (absorption) $\quad P_r = \dfrac{2I}{c}$ (reflection)



(b)

$$\theta_1 = \theta_1'$$

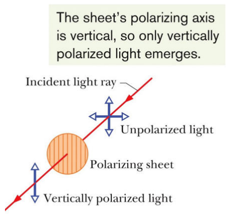
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

n_1, n_2 index of refraction

$n = \frac{c}{v}$ (v is speed of light in this material)

$$n = \frac{\lambda_0}{\lambda} = \frac{\lambda_0 f}{\lambda f} = \frac{c}{v}$$

Polarization, cont.

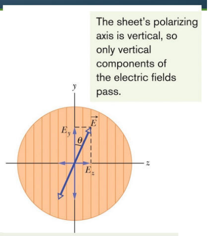


If the original light is initially unpolarized, the transmitted intensity I is half the original intensity I_0:

$$I = \tfrac{1}{2} I_0 \quad \text{(one-half rule).}$$

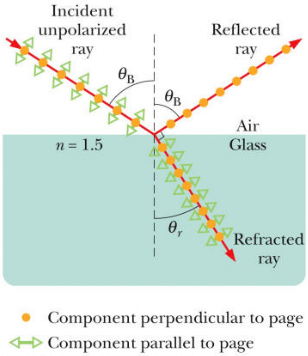
Polarization, cont.

If the original light is initially polarized, the transmitted intensity depends on the angle θ between the polarization direction of the original light and the polarizing direction of the sheet:



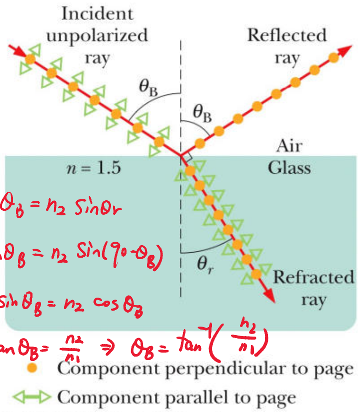
$$I = I_0 \cos^2\theta \quad \text{(cosine-squared rule).}$$

Polarization by Reflection



A ray of unpolarized light in air is incident on a glass surface at the **Brewster angle** θ_B. (Note that this angle is to the NORMAL as well!!). The electric fields along that ray have been resolved into components perpendicular to the page (the plane of incidence, reflection, and refraction) and components parallel to the page. The reflected light consists only of components perpendicular to the page and is thus polarized in that direction. The refracted light consists of the original components parallel to the page and weaker components perpendicular to the page; this light is partially polarized.

Polarization by Reflection



$n_1 \sin\theta_B = n_2 \sin\theta_r$

$n_1 \sin\theta_B = n_2 \sin(90-\theta_B)$

$\Rightarrow n_1 \sin\theta_B = n_2 \cos\theta_B$

$\Rightarrow \tan\theta_B = \frac{n_2}{n_1} \Rightarrow \theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$

As shown in the figure, a **<u>reflected</u>** wave will be fully polarized, with its ***E*** vectors perpendicular to the plane of incidence, if it strikes a boundary at the Brewster angle θ_B, where:

$$\theta_B = \tan^{-1}\frac{n_2}{n_1} \quad \text{(Brewster angle).}$$

n_1 is the incident layer and n_2 is the reflecting layer

$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

CH 34 : Images

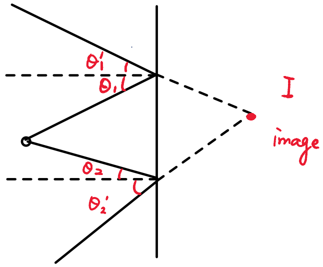
- There are two types of images

① Real images : images that can be formed on a surface. eg: Light is physically present at location

② Virtual images : images that only exists in perception eg: image formed in a mirror

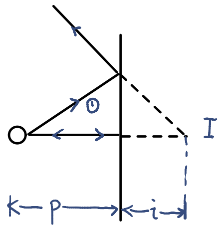
* Plane Mirror

- Mirror obey the law of reflection



* Ray tracing

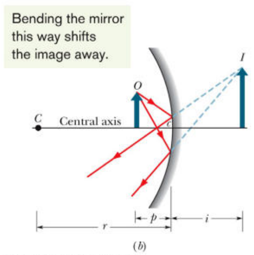
- To find any images, we always need two rays



$$\boxed{i = -p}$$

34.2. Spherical Mirrors

Concave Mirror:
The mirror is curved (caved in) toward the object



(b)

Object distance (p) always + because it's in front of the mirror

Image distance (i) could be + or – because the image could be in front (real image) or behind (virtual image) the mirror

Focal length (f) positive because it's in front of the mirror (inside the curve)

Radius of curvature (r) = 2f also positive

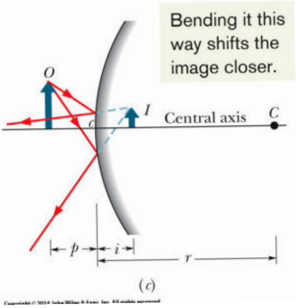
Object distance (p) always + because it's in front of the mirror

Image distance (i) is always negative because the image always forms behind the mirror (virtual image)

Focal length (f) is negative because it's still inside the curve but the curve is toward the back side of the mirror

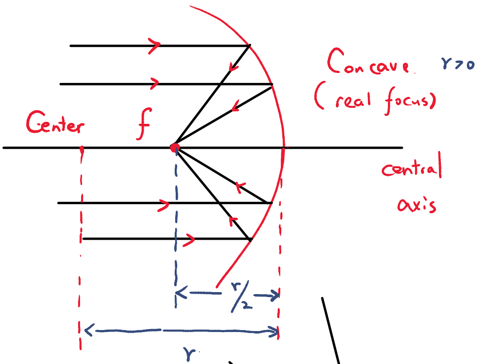
Radius of curvature (r) = 2f also negative

Convex Mirror:
The mirror is curved away from the object



* Focal point of spherical mirror

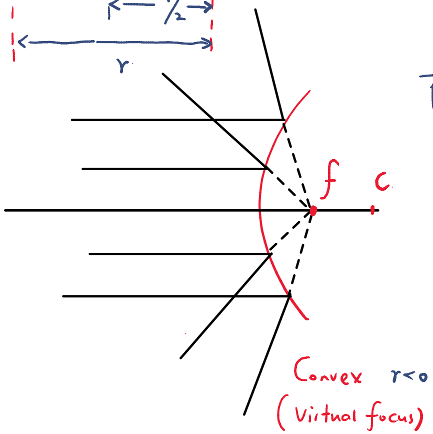
- A focal point f is a point where parallel rays incident on a spherical mirror.



$$\boxed{f = \frac{r}{2}}$$

f = focal point

r = radius



$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$$

$$|m| = \frac{h'}{h}$$

$$m = \frac{-i}{p}$$

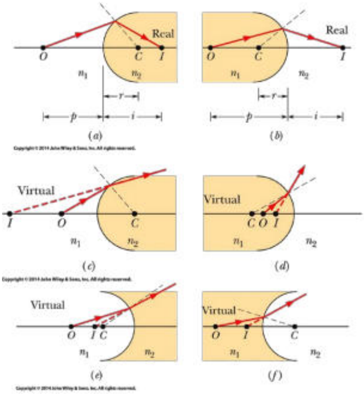
① $0 < m < 1$ upright min

② $-1 < m < 0$ inverted min

③ $m > 1$ upright magnify

④ $m < -1$ inverted magnify

34-3 Spherical Refracting Surface



Real images are formed in (a) and (b); virtual images are formed in the other four situations.

- A single spherical surface that refracts light can form an image.
- The object distance p, the image distance i, and the radius of curvature r of the surface are related by

$$\frac{n_1}{p} + \frac{n_2}{i} = \frac{n_2 - n_1}{r}.$$

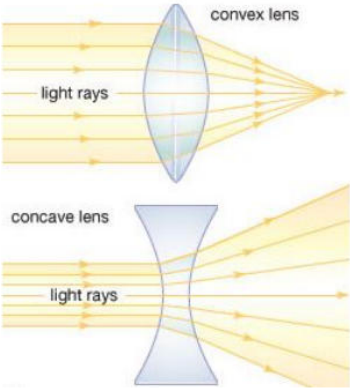
where n_1 is the index of refraction of the material where the object is located and n_2 is the index of refraction on the other side of the surface.

- If the surface faced by the object is convex, *r* is positive, and if it is concave, *r* is negative.

Real images form on the side of a refracting surface that is opposite the object, and virtual images form on the same side as the object.

More on Refracting Surfaces (Lenses)

Just like mirrors, lenses can be concave or convex.



A convex lens is curved away from the object, and it will cause light beams to converge toward one point (the focal point).

A concave lens curves toward the object, and it will cause light beams to converge away from the focal point.

The Basic Thin Lens Equation

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{i} \qquad \text{(thin lens)}$$

Thin Lens Surrounded by Air Equation

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad \text{(thin lens in air)},$$

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Thin Lens in Air Equation

For an object in front of a lens, object distance *p* and image distance *i* are related to the lens's focal length *f*, index of refraction *n*, and radii of curvature r_1 and r_2 by

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad \text{(thin lens in air)},$$

which is often called the lens maker's equation. Here r_1 is the radius of curvature of the lens surface nearer the object and r_2 is that of the other surface. If the lens is surrounded by some medium other than air (say, corn oil) with index of refraction n_{medium}, we replace n in above Eq. with n/n_{medium}.

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} \qquad (1)$$

$$\frac{n_3}{v'} - \frac{n_2}{u'} = \frac{n_3 - n_2}{R_2} \qquad (2)$$

Fact: $u' = v$ because the image of first surface is object of second and hence (2) becomes:

$$\frac{n_3}{v'} - \frac{n_2}{v} = \frac{n_3 - n_2}{R_2} \qquad (3)$$

Adding (3) and (1):

$$\frac{n_3}{v'} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2}$$

If, $n_3 = n_1 = 1$ that is the ambient mediums on both side are air then the previous equation becomes:

$$\frac{1}{v'} - \frac{1}{u} = (n_2 - n_1)\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

first second

$$\boxed{M = m_1 m_2}$$

Multiple lenses

To find focal length, just send $u \to \infty$ which makes:

$$\frac{1}{f} = (n_2 - n_1)\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

Hence,

$$\frac{1}{v'} - \frac{1}{u} = \frac{1}{f}$$

34 Summary

Real and Virtual Images

- If the image can form on a surface, it is a real image and can exist even if no observer is present. If the image requires the visual system of an observer, it is a virtual image.

Image Formation

- Spherical mirrors, spherical refracting surfaces, and thin lenses can form images of a source of light—the object — by redirecting rays emerging from the source.
- **Spherical Mirror:**

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f}=\frac{2}{r},$$ **Eq. 34-3 & 4**

- **Spherical Refracting Surface:**

$$\frac{n_1}{p}+\frac{n_2}{i}=\frac{n_2-n_1}{r}$$ **Eq. 34-8**

- **Thin Lens:**

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right),$$ **Eq. 34-9 & 10**

Optical Instruments

- Three optical instruments that extend human vision are:

1. The simple magnifying lens, which produces an angular magnification m_θ given by

$$m_\theta=\frac{25\text{ cm}}{f}$$ **Eq. 34-12**

2. The compound microscope, which produces an overall magnification M given by

$$M=mm_\theta=-\frac{s}{f_{ob}}\frac{25\text{ cm}}{f_{ey}}$$ **Eq. 34-14**

3. The refracting telescope, which produces an angular magnification mu given by $m_\theta=-\frac{f_{ob}}{f_{ey}}.$ **Eq. 34-15**

Simple Magnifying Lens

The angular magnification of a simple magnifying lens is

$$m_\theta\approx\frac{25\text{ cm}}{f}\quad\text{(simple magnifier)}.$$

where f is the focal length of the lens and 25 cm is a reference value for the near point value.

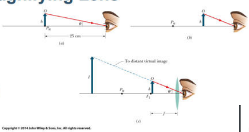


Figure (a) shows an object O placed at the near point P_n of an eye. The size of the image of the object produced on the retina depends on the angle θ that the object occupies in the field of view from that eye. By moving the object closer to the eye, as in Fig.(b), you can increase the angle and, hence, the possibility of distinguishing details of the object. However, because the object is then closer than the near point, it is no longer in focus; that is, the image is no longer clear. You can restore the clarity by looking at O through a converging lens, placed so that O is just inside the focal point F_1 of the lens, which is at focal length f (Fig. c). What you then see is the virtual image of O produced by the lens. That image is farther away than the near point; thus, the eye can see it clearly.

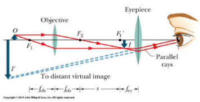
Compound Microscope

Figure shows a thin-lens version of a compound microscope. The instrument consists of an objective (the front lens) of focal length f_{ob} and an eyepiece (the lens near the eye) of focal length f_{ey}. It is used for viewing small objects that are very close to the objective. The object O to be viewed is placed just outside the first focal point F_1 of the objective, close enough to F_1 that we can approximate its distance p from the lens as being f_{ob}. The separation between the lenses is then adjusted so that the enlarged, inverted, real image I produced by the objective is located just inside the first focal point F_1 of the eyepiece. The tube length s shown in the figure is actually large relative to f_{ob}, and therefore we can approximate the distance i between the objective and the image I as being length s.

The overall magnification of a compound microscope is

$$M=mm_\theta=-\frac{s}{f_{ob}}\frac{25\text{ cm}}{f_{ey}},$$

where where m is the lateral magnification of the objective, m_θ is the angular magnification of the eyepiece.

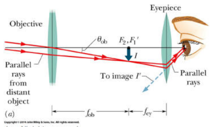


Refracting Telescope

Refracting telescope consists of an objective and an eyepiece; both are represented in the figure with simple lenses, although in practice, as is also true for most microscopes, each lens is actually a compound lens system. The lens arrangements for telescopes and for microscopes are similar, but telescopes are designed to view large objects, such as galaxies, stars, and planets, at large distances, whereas microscopes are designed for just the opposite purpose. This difference requires that in the telescope of the figure the second focal point of the objective F_2 coincide with the first focal point of the eyepiece F'_1, whereas in the microscope these points are separated by the tube length s.

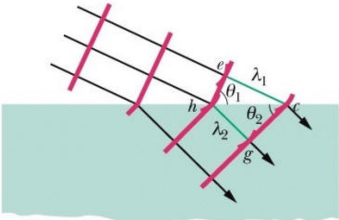
The angular magnification of a refracting telescope is

$$m_\theta=-\frac{f_{ob}}{f_{ey}}.$$



CH 35 Interference

Huygens' Principle, cont.



$v = \frac{\lambda}{T} = \lambda f$ For light f same

$\therefore \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$

$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$, $\sin\theta_1 = \frac{\lambda_1}{hc}$ (for triangle hce) $\sin\theta_2 = \frac{\lambda_2}{hc}$ (for triangle hcg).

$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$. $n_1 = \frac{c}{v_1}$ and $n_2 = \frac{c}{v_2}$. $\frac{\sin\theta_1}{\sin\theta_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}$

Which all leads to: $n_1 \sin\theta_1 = n_2 \sin\theta_2$ (law of refraction),

Snell's Law

$f = f_n \Rightarrow \frac{c}{\lambda} = \frac{v}{\lambda_n} \Rightarrow \lambda_n = \frac{v}{c}\lambda = \frac{1}{n}\lambda \Rightarrow \boxed{\lambda_n = \frac{\lambda}{n}}$

Number of wavelength (N_1 and N_2) there are in the length L of medium 1 & 2

$$\begin{cases} N_1 = \frac{L}{\lambda_{n_1}} = \frac{Ln}{\lambda} \\ N_2 = \frac{L}{\lambda_{n_2}} = \frac{Ln}{\lambda} \end{cases} \Rightarrow N_2 - N_1 = \frac{Ln_2}{\lambda} - \frac{Ln_1}{\lambda} = \frac{L}{\lambda}(n_2 - n_1)$$

phase diff

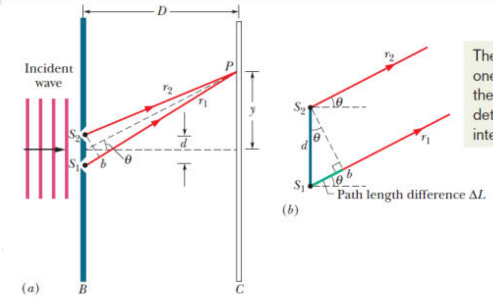
$m = \frac{L}{\lambda}(n_2 - n_1)$

$\frac{\Delta L}{\lambda} = 0, 1, 2 \ldots$ (full constructive interference) ← ① phase diff $= m$ $m = 0, \pm1, \pm2 \ldots$

$\frac{\Delta L}{\lambda} = 0.5, 1.5, 2.5, \ldots$ (full destructive interferce) ← ② phase diff $= m + \frac{1}{2}$ $m = 0, \pm1, \pm2 \ldots$

Young's Interference Experiment, cont.

The phase difference between two waves can change if the waves travel paths of different lengths.



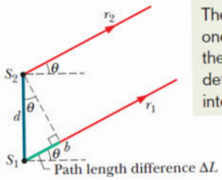
The ΔL shifts one wave from the other, which determines the interference.

What appears at each point on the viewing screen in a Young's double-slit interference experiment is determined by the path length difference ΔL of the rays reaching that point.

Young's Interference Experiment, cont.

Location of bright and dark fringes:

$\Delta L = d \sin \theta$ (path length difference).



The ΔL shifts one wave from the other, which determines the interference.

For a bright fringe, ΔL must be either zero or an integer number of wavelengths. Therefore,

$$\Delta L = d \sin \theta = (\text{integer})(\lambda),$$

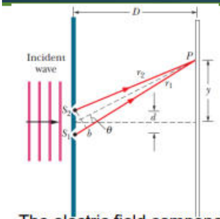
For a dark fringe, ΔL must be an odd multiple of half a wavelength. Therefore.

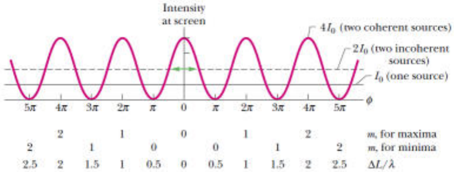
$$\Delta L = d \sin \theta = (\text{odd number})(\tfrac{1}{2}\lambda),$$

$$d \sin \theta = m\lambda, \quad \text{for } m = 0, 1, 2, \ldots \quad \text{(maxima—bright fringes)}.$$

$$d \sin \theta = (m + \tfrac{1}{2})\lambda, \quad \text{for } m = 0, 1, 2, \ldots \quad \text{(minima—dark fringes)}.$$

Intensity in Double Slit Interference





The electric field components of the each of the light waves at point P on the screen can be written as:

$$E_1 = E_0 \sin \omega t$$

$$E_2 = E_0 \sin(\omega t + \phi),$$

The intensity of the pattern at P can be expressed as:

$$I = 4I_0 \cos^2 \tfrac{1}{2}\phi,$$

where I_o is the intensity of the light that arrives at point P when one of the slits is temporarily covered.

And the phase difference can be expressed as:

$$\phi = \frac{2\pi d}{\lambda} \sin \theta.$$

Intensity in Double Slit Interference

$$\phi = \frac{2\pi d}{\lambda} \sin \theta.$$

For a maximum: $\frac{1}{2}\phi = m\pi, \quad \text{for } m = 0, 1, 2, \ldots .$

so: $2m\pi = \frac{2\pi d}{\lambda} \sin \theta,$ **or** $d \sin \theta = m\lambda,$

For a minimum: $\frac{1}{2}\phi = (m + \frac{1}{2})\pi, \quad \text{for } m = 0, 1, 2, \ldots .$

so: $d \sin \theta = (m + \frac{1}{2})\lambda,$

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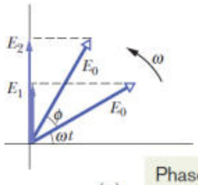
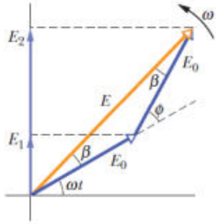
Double Slit Intensity and Phasors



(a)

Phasors that represent waves can be added to find the net wave.



(b)

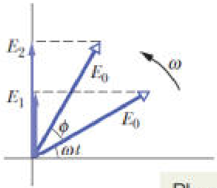
Fig. 35-13 (*a*) Phasors representing, at time *t*, the electric field components given by Eqs. 35-20 and 35-21. Both phasors have magnitude E_0 and rotate with angular speed ω. Their phase difference is ϕ. (*b*) Vector addition of the two phasors gives the phasor representing the resultant wave, with amplitude E and phase constant β.

The values of E_1 and E_2 at any given time are the projections of the two phasors on the vertical axis…

$$E_1 = E_0 \sin \omega t$$
$$E_2 = E_0 \sin(\omega t + \varphi)$$

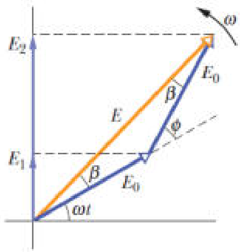
$E_1 + E_2$ = the vertical component of E

Double Slit Intensity and Phasors, cont.



(a)

Phasors that represent waves can be added to find the net wave.



(b)

$$\beta = \tfrac{1}{2}\phi$$

+

$$E = 2(E_0 \cos \beta)$$ HOW??

$$= 2E_0 \cos \tfrac{1}{2}\phi.$$

About angle β: The exterior angle ϕ is equal to the sum of the two opposite interior angles, β.

Square both sides: $E^2 = 4E_0^2 \cos^2 \tfrac{1}{2}\phi.$

And also: $\dfrac{I}{I_0} = \dfrac{E^2}{E_0^2}.$

So: $I = 4I_0 \cos^2 \tfrac{1}{2}\phi,$