ECE 201: Circuit & System I

Chapter 1:

SI system.

1) Length m
2) Mass kg
3) Time s
4) Ampere A
5) Temperature K.
6) Luminous intensity Cd.
7) Voltage V.

A) Current $\boxed{i = \frac{dq}{dt}}$

B) voltage $\boxed{V = \frac{dw}{dq}}$

C) energy $p = \frac{dw}{dt} = \left(\frac{dw}{dq}\right) \cdot \left(\frac{dq}{dt}\right) = \boxed{V \cdot i}$

$$\int_{t_0}^{t_1} p\,dt = \int_{t_0}^{t_1} \frac{dw}{dt}\,dt = \int_{t_0}^{t_1} dw = w(t_1) - w(t_0)$$

$$\therefore \Delta w = w(t_1) - w(t_0) = \int_{t_0}^{t_1} p\,dt$$

$$\boxed{w(t_1) = \int_{t_0}^{t_1} p\,dt + w(t_0)}$$

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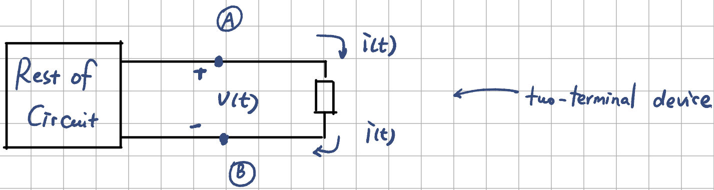
* Standard Decimal Prefixes

10^{-18}	atto	a	10^{-3}	milli	m
10^{-15}	femto	f	10^{3}	kilo	K
10^{-12}	pico	p	10^{6}	mega	M
10^{-9}	nano	n	10^{9}	giga	G
10^{-6}	micro	μ	10^{12}	tera	T

tips: when convert, the value should be 1~1000

(D) Passive Sign Convention



(1) If $u(t)$ & $i(t)$ is positive

(a) A voltage drop exists from terminal A to terminal B

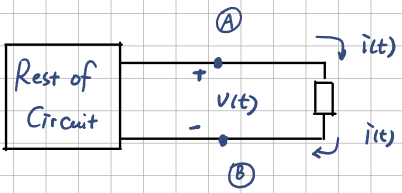
(b) A voltage rise exists from terminal B to terminal A

(c) A net positive charge is flowing from terminal A to terminal B in circuit

(d) A net negative charge is flowing from terminal B to terminal A in circuit

(2) If current move through a drop in voltage. it charges lose energe

If current move through a rise in voltage. it charges gain energe



Charges gain energy. Power is generated ; Charges lose energy, power is absorbed

(3) Power Absorbed

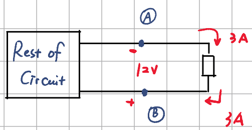
- Whenever the reference direction of current into a two-terminal device is in the direction of the reference voltage drop across the device then the power absorbed or dissipated is positive

$$p(t) = v(t) \cdot i(t) > 0 \qquad (\text{Absorbed})$$

$$p(t) = v(t) \cdot i(t) < 0 \qquad (\text{Generate})$$

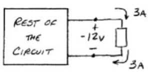
✳ Negative power absorbed means power is generated

Q : Find Power absorbed/generate



Solution :

- REDRAWING THE CIRCUIT TO LOOK LIKE passive sign convention

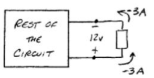


POWER ABSORBED = p

$= [-12V][3A] = \boxed{-36\,W}$

Negative Power Absorbed means Power is Generated.

- (OR) WE COULD DO THE FOLLOWING



POWER ABSORBED = p

$= [12V][-3A] = \boxed{-36\,W}$

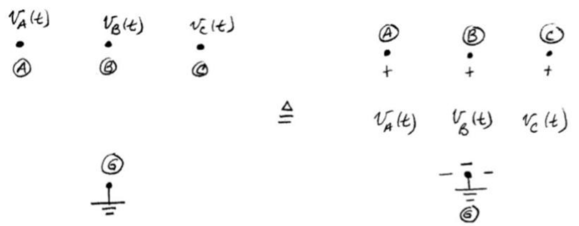
CURRENT IS ENTERING THE + TERMINAL WHICH IS THE DEFINITION OF POWER ABSORBED

Ground

- Since voltage exists between two points it is often useful to define a common voltage reference point called ground.

- Common Symbols: 

1) EXAMPLE



Ground (cont.)

- THE GROUND OR COMMON POINT IS ASSUMED TO BE THE − TERMINAL.

- A TWO TERMINAL DEVICE MAY BE BETWEEN POINTS A AND G OR MAY <u>NOT</u> BE.

- ALTHOUGH WE ARE ASSIGNING A POLARITY TO v_A, v_B, v_C, THEIR ACTUAL VALUE MAY BE NEGATIVE.

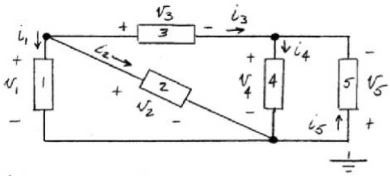
Conservation of Power

p, v, and i are related:

$$p = v * i$$

Also note passive sign convention

1) EXAMPLE : GIVEN



← NOTE: PASSIVE SIGN CONVENTION

WHERE THE FOLLOWING QUANTITIES WERE MEASURED:

	DEVICE 1	DEVICE 2	DEVICE 3	DEVICE 4	DEVICE 5
v	+100V	?	+25V	+75V	−75V
i	?	+5mA	+5mA	?	+5mA
P	−1W	+500mW	?	+750mW	?

a) FIND THE MISSING QUANTITY

b) SUM THE ABSORBED POWER

SOLUTION:

$$i_1 = \frac{P_1}{V_1} = \frac{-1}{100} = \boxed{-10mA}$$

✩ Change i_1 to positive, also need change direction

$$V_2 = \frac{P_2}{i_2} = \frac{500m}{5m} = \boxed{100V}$$

$$P_3 = V_3 i_3 = (25)(5m) = \boxed{125mW}$$

$$i_4 = \frac{P_4}{V_4} = \frac{750m}{75} = \boxed{10mA}$$

$$P_5 = V_5 i_5 = (-75)(5m) = \boxed{-375mW}$$

We could say that P1 and P5 generated 1.375 W and that P2, P3 and P4 absorbed 1.375 W.

b)

$P_1 = -1W$
$P_5 = -0.375W$

$-1.375W$

$P_2 = 0.5W$
$P_3 = 0.125W$
$P_4 = 0.750W$

$1.375W$

May be, P1 and P5 are voltage sources and P2, P3 and P4 are resistances.

$$\Sigma \text{ POWER ABSORBED } = \boxed{0W}$$

Postulates

- Postulates are things that are assumed to be true. Given a set of postulates we can then derive theorems or properties

(a) Conservation of energy postulate

Energy cannot be created or destroyed. Although it can be changed from one to another, that is

$$\sum \omega_{absorb} = \sum \omega_{generate}$$

$$\sum_{i=1}^{j} \int P_{absorb} \cdot dt = \sum_{i=j+1}^{n} \int P_{generate} \cdot dt$$

$$\int \sum_{i=1}^{j} P_{absorb} \cdot dt = \int \sum_{i=j+1}^{n} P_{generate} \cdot dt$$

$$\therefore \sum_{i=1}^{j} P_{absorb} \cdot dt = \sum_{i=j+1}^{n} P_{generate} \cdot dt$$

QED *

$\Rightarrow$ Conservation of power

In a circuit. $\sum_{i=1}^{n} P_{AB_i} = 0$

one device

Supplemental Problems

S1.1) THE CURRENT THROUGH A CIRCUIT ELEMENT IS 20mA. IF THE CHARGE OF 1 ELECTRON IS 1.6×10^{-19} C THEN THERE ARE $1/1.6\times10^{-19} = 6.25 \times 10^{18}$ ELECTRONS IN 1 COULOMB OF CHARGE.

FIND THE TOTAL CHARGE AND THE NUMBER OF ELECTRONS TRANSFERRED DURING AN INTERVAL OF $1\mu s$.

SOLUTION:

$$i = \frac{dq}{dt} \Rightarrow \int_{t_0}^{t_1} i\,dt = \int_{t_0}^{t_1} \frac{dq}{dt}\,dt = \int_{t_0}^{t_1} dq = q(t_1) - q(t_0)$$

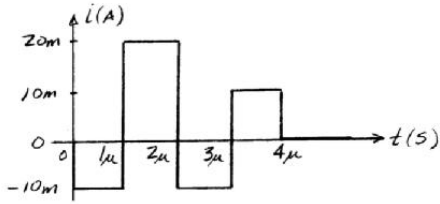
– THIS IS THE TOTAL CHARGE TRANSFERRED* BETWEEN t_0 & t_1

a) $\therefore \Delta q = \int_{t_0}^{t_1} i\,dt = \int_0^{1\mu} (20m)\,dt = 20m(1\mu - 0) = \boxed{20nC}$

b) NUMBER OF ELECTRONS $\triangleq n_E = 6.25(10^{18})\,\frac{ELECT}{C} \cdot 20nC$

$= \boxed{125(10^9)\ \text{ELECTRONS}}$

S1.2) GIVEN A PLOT OF CURRENT FLOWING PAST A REFERENCE POINT VERSUS TIME, PLOT THE CHARGE TRANSFERRED PAST THE REFERENCE POINT VERSUS TIME IF $q(0) = 0C$.



SOLUTION: $q(t) = \int_{t_0}^{t} i(x)\,dx + q(t_0)$

a) $0 \le t \le 1\mu$

$$q(t) = \int_0^t (-10m)\,dx + 0 = (-10m)\,x\Big|_0^t = -10m\,t$$

& $q(1\mu) = (-10m)(1\mu) = -10nC$ ⟵ INITIAL CONDITION FOR THE NEXT TIME INTERVAL

b) $1\mu \le t \le 2\mu$

$$q(t) = \int_{1\mu}^{t} (20m)\,dx + (-10n) = 20m(t-1\mu) - 10n$$

$$= 20m\,t - 20n - 10n = (20m)t - 30n \quad \& \; q(2\mu) = 10nC$$

c) $2\mu \le t \le 3\mu$

$$q(t) = \int_{2\mu}^{t} (-10m)\,dx + 10n = (-10m)(t-2\mu) + 10n$$

$$= (-10m)t + 20n + 10n = (-10m)t + 30n \quad \& \; q(3\mu) = 0C$$

d) $3\mu \le t \le 4\mu$

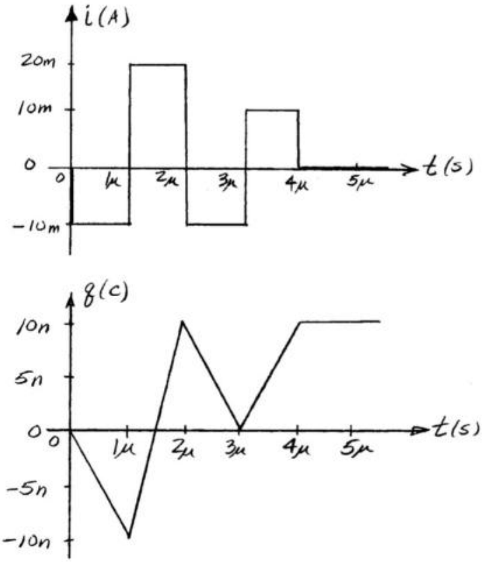
$$q(t) = \int_{3\mu}^{t} (10m)\,dx + 0 = 10m(t-3\mu)$$

$$= 10m\,t - 30n \quad \& \; q(4\mu) = 10nC$$

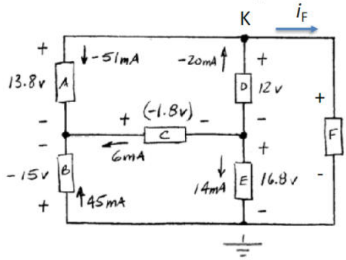
e) $t \geq 4\mu$

$$q(t) = \int_{4\mu}^{t} 0\,dt + 10n = 10nC$$

— GRAPHING THE RESULTS



51.4) USE THE CONSERVATION OF POWER TO FIND THE POWER ABSORBED BY ELEMENT F.



Check:
v_F = 16.8+12 = 28.8 V
i_F = 892.8m / 28.8
= 31 mA
Alternative:
KCL: i_F = -20m – (-51m)
@K = 31 mA

Power = 28.8x31m W
= 892.8 mW

SOLUTION: $\Sigma P_{AB} = 0$

a)

ELEMENT	POWER ABSORBED
A	$(13.8)(-51m) = -703.8mW$
B	$(-15)(45m) = -675mW$
C	$(-1.8)(-6m) = 10.8mW$
D	$(12)(20m) = 240mW$
E	$(16.8)(14m) = 235.2mW$

b) $\Sigma P_{AB} = (10.8m + 240m + 235.2m) - (703.8m + 675m) + P_F$

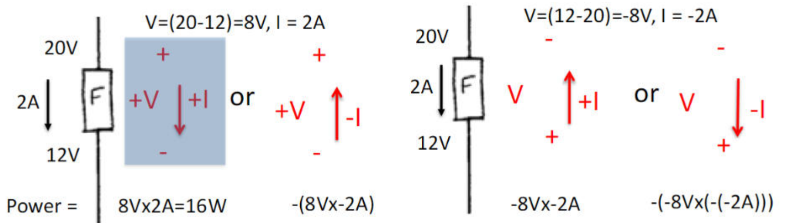
$= 486m - 1378.8m + P_F$

$= -892.8\,mW + P_F$

SINCE $\Sigma P_{AB} = 0$, $\boxed{P_F = 892.8mW}$

It is an absorbing device,
Like a resistor.

Voltage and current possibilities on a Device F:



All are correct. It is better to choose +ve values of Voltage and Current and adjust the polarity of voltages and direction of current.

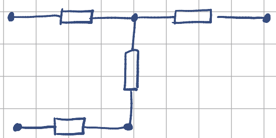
Chapter 2

Element Constraints

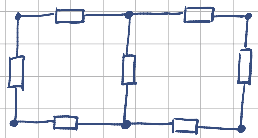
(1) The interconnection of two or more electrical device is called an electrical network

(2) If an electrical network contains at least one closed path, then it is called a circuit

closed path example :



Not a circuit

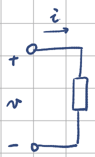


a circuit

(3) Device :

- An electrical device is a component that is treated as a seperate entity

- Two-terminal devices are represented in general, with a rectangle box

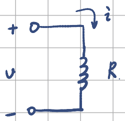


(4) circuit element

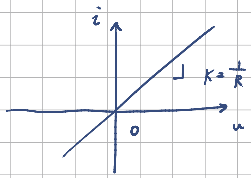
- To distinguish between the real device and its idealized model, the model will be called the circuit elements

Resistance Element

(1)



$$\boxed{\begin{array}{l} v = iR \\ i = \dfrac{v}{R} \end{array}}$$



R is called resistance $\Omega = \frac{V}{A}$

(2) Sometimes it is convenient to use a called conductance * which is the reciprocal of resistance.

$$\boxed{G = \frac{1}{R}}$$

[G is conductance and has units of $\frac{A}{V}$ or siemens* $S = \Omega^{-1} = A/V$

(3) Power

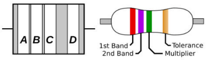
$$P = v \cdot i = (iR) \cdot i = \boxed{i^2 R}$$

$$= v \cdot \frac{v}{R} = \boxed{\frac{v^2}{R}}$$

(always positives)

4) RESISTANCE DEVICES (OR RESISTOR)

- THE RESISTOR IS THE PHYSICAL DEVICE WHILE RESISTANCE IS THE IDEAL CONCEPT. *
- RESISTORS CAN BE APPROXIMATED OR MODELED UNDER MANY CONDITIONS AS AN IDEAL RESISTANCE
- MANY TYPES OF MATERIALS ARE USED TO MAKE RESISTORS. CARBON IS THE MOST COMMON.
- A TYPICAL CARBON COMPOSITION OR CARBON FILM RESISTOR IS CYLINDRICAL WITH MULTICOLORED BANDS (OR STAMPED NUMBERS) INDICATING THE NOMINAL VALUE



a) COLOR CODE

$$R = (\underline{a}\ \underline{b} \times 10^{\underline{c}}) \pm \underline{d}$$

i)

COLOR[†]	VALUE	COLOR	VALUE
SILVER*	-2	YELLOW	4
GOLD*	-1	GREEN	5
BLACK	0	BLUE	6
BROWN	1	VIOLET	7
RED	2	GRAY	8
ORANGE	3	WHITE	9

† FOR BANDS a, b, c

* ONLY OCCUR IN BAND c

ii) FOR BAND d

NO COLOR	=	20%
SILVER	=	10%
GOLD	=	5%

iii) EXAMPLE : YELLOW - VIOLET - RED - GOLD

$$R = (\underline{4}\ \underline{7} \times 10^{2}) \pm \underline{5\ \%} = 4.7K\Omega \pm 5\%$$

$$\therefore \quad 4.7K(1-0.05) \le R \le 4.7K(1+0.05)$$

$$\boxed{4.465K\Omega \le R \le 4.935K\Omega}$$

b) STANDARD VALUES

Any other resistor will cost more!

i) ±5, ±10, ±20 % RESISTORS

VALUE	TOL.	VALUE	TOL.	VALUE	TOL.
10	5, 10, 20	22	5, 10, 20	47	5, 10, 20
11	5	24	5	51	5
12	5, 10	27	5, 10	56	5, 10
13	5	30	5	62	5
15	5, 10, 20	33	5, 10, 20	68	5, 10, 20
16	5	36	5	75	5
18	5, 10	39	5, 10	82	5, 10
20	5	43	5	91	5

WHERE "VALUE" ARE THE POSSIBLE COMBINATIONS OF BANDS a, b.

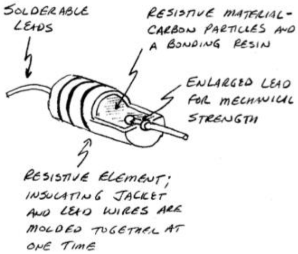
ii) ± 1% RESISTORS

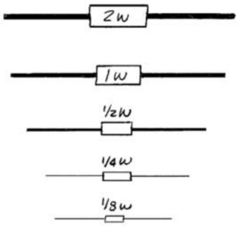
VALUE	VALUE	VALUE	VALUE	VALUE	VALUE
10.0	14.7	21.5	31.6	46.4	68.1
10.5	15.4	22.6	33.2	48.7	71.5
11.0	16.2	23.7	34.8	51.1	75.0
11.5	16.9	24.9	36.5	53.6	78.7
12.1	17.8	26.1	38.3	56.2	82.5
12.7	18.7	27.4	40.2	59.0	86.6
13.3	19.6	28.7	42.2	61.9	90.9
14.0	20.5	30.1	44.2	64.9	95.3

More expensive

c) SOME TYPES

i) CARBON COMPOSITION, $1\Omega - 20M\Omega$, ±5%, ±10%, ±20% TOL.

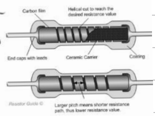




APPROXIMATE PHYSICAL SIZE

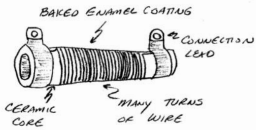
ii) CARBON-FILM, $1\Omega - 20M\Omega$, ±1%, ±2%, ±5% TOL.

- MADE BY PLACING CERAMIC RODS IN A METHANE-FILLED FLASK AND HEATING IT UNTIL A CARBON FILM IS DEPOSITED ON THE CORES.



- AVAILABLE IN ½, 1, AND 2 WATT SIZES LIKE ABOVE

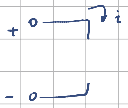
iii) WIRE-WOUND, $100m\Omega - 200k\Omega$, ±0.1% - ±2% TOL.



- AVAILABLE IN WATTAGES FROM 1W TO 250W

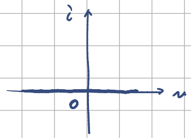
Large power absorbing resistors

(1) Open circuit



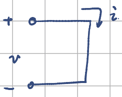
$$i = \frac{v}{R}\Big|_{R\to\infty} = 0$$

u = unspecified



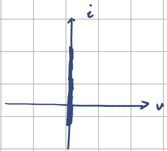
v-i Characteristic

(2) Short circuit



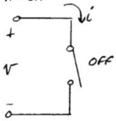
$$v = iR\Big|_{R\to 0} = 0$$

i = unspecified



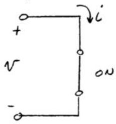
3) IDEAL SWITCH

Ideally, $R_{OFF} = \infty$



OFF = OPEN CIRCUIT

Ideally, $R_{ON} = 0$



ON = SHORT CIRCUIT

b) $R_{OFF} \neq \infty$ BUT A LARGE VALUE

R_{OFF} (WALL-SWITCH) = R_{OFF} (RADIO-SWITCH)

= RESISTANCE OF AIR

$\approx 10^{15}\,\Omega$

4) NON-IDEAL OR REAL SWITCH

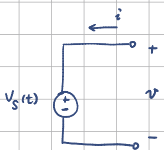
a) $R_{ON} \neq 0$ BUT A SMALL VALUE

R_{ON} (WALL-SWITCH) $\approx 50\,m\Omega$

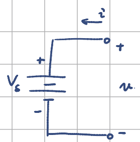
R_{ON} (RADIO-SWITCH) $\approx 30\,m\Omega$

Ideal Sources

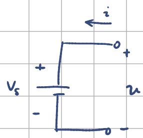
(1) ideal voltage Source



General Symbol

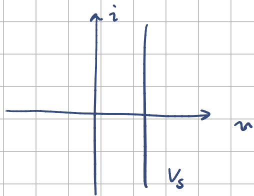


Special case: A battery



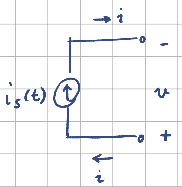
Special case: A DC Sourse

V_s / t

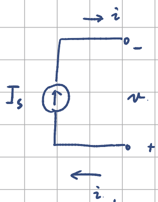


v-i Characteristic

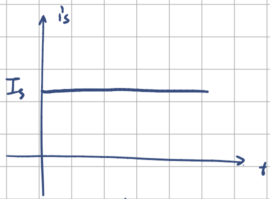
(2) Ideal Current Source



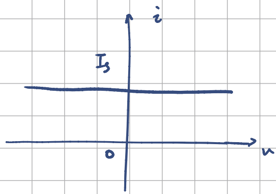
General Symbol

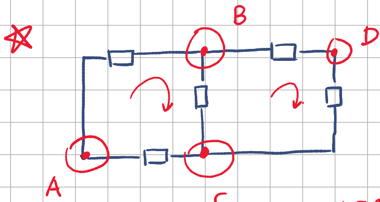


Special case: A DC Source



i_s / t



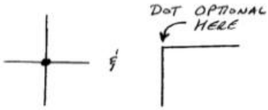


n: # of nodes

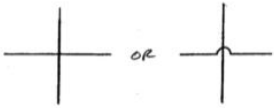
m: # of meshes (a closed loops that has no other loop)

b: # of branches (path that connects two nodes)

n = 4 m = 2 b = 5 ⇒ $\boxed{b = m + n - 1}$ △

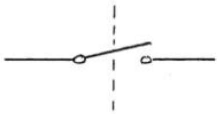


ELECTRICAL CONNECTION



CROSSOVER WITH NO CONNECTION

JACK CONNECTION



CONTROL LINE USED TO OPEN OR CLOSE A SWITCH

KCL (Inside node $\sum i$ = Outside node $\sum i$)

KVL (Rise of voltage = Drop of voltage)

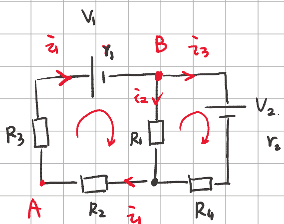
* Resistance Rule : ① move through a resistor in the direction of the current,

this gives a change in the voltage of $-iR$

② move in the opposite direction of current. this gives a change in voltage iR

* Emf rule : ① move from $-$ to $+$, the change of voltage is $+V$

② move from $+$ to $-$, the change of voltage is $-V$



$$\begin{cases} i_1 = i_2 + i_3 & ① \quad (\text{KCL}) \\ -i_1 r_1 - V_1 - i_2 R_1 - i_1 R_2 - i_1 R_3 = 0 & ② \quad (\text{KVL}) \\ -V_2 - i_3 r_2 - i_3 R_4 + i_2 R_1 = 0 & ③ \quad (\text{KVL}) \end{cases}$$

* ΔV in two point : $\boxed{\Delta V = V_B - V_A = -i_1 R_3 - V_1 - i_1 r_1}$

Equivalent Circuit

(1) Series Resistance $R_{EQ} = \sum_{j=1}^{k} R_j$

(2) Parallel Resistance $R_{EQ} = \dfrac{1}{\sum_{j=1}^{k} \left(\frac{1}{R_j}\right)}$

k equal resistance $R_{eq} = \dfrac{1}{\underbrace{\frac{1}{R} + \cdots + \frac{1}{R}}_{k}} = \dfrac{R}{k}$

Proof : $R_{EQ} = \dfrac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_k}}$ Suppose R_3 = Smallest R

$R_{EQ} = \dfrac{1}{\frac{1}{R_3}\left[\frac{R_3}{R_1} + \frac{R_3}{R_2} + 1 + \cdots + \frac{R_3}{R_k}\right]} \overset{?}{<} R_3$

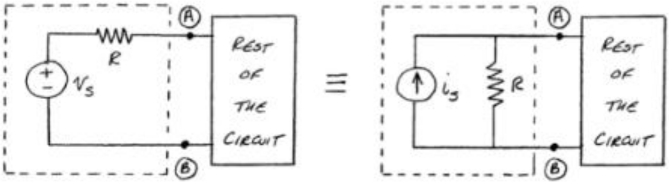
$= \dfrac{R_3}{\left[\frac{R_3}{R_1} + \frac{R_3}{R_2} + \cdots + \frac{R_3}{R_k}\right]} \overset{?}{<} R_3$

$= \dfrac{1}{\underbrace{\left[\frac{R_3}{R_1} + \frac{R_3}{R_2} + 1 + \cdots + \frac{R_3}{R_k}\right]}_{>1}} \overset{?}{<} 1$ ✓. QED.

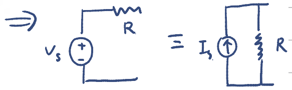
3.4 Equivalent Circuit

3) Source Transformations

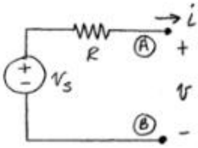
Back to Next Topic...



WHERE $V_s = i_s R$ AND $i_s = V_s / R$ $\Rightarrow$



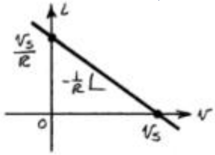
Proof :



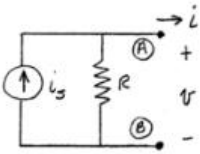
$V_s = iR + v$

$\therefore i = \frac{V_s - v}{R}$

$\boxed{i = \left(-\frac{1}{R}\right)v + \frac{V_s}{R}}$



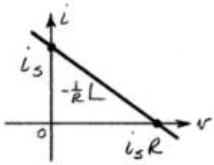
CIRCUIT EQNS V-i CHARACTERISTICS



$i_s = \frac{v}{R} + i$

$\therefore i = -\frac{v}{R} + i_s$

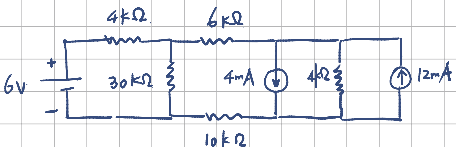
$\boxed{i = \left(-\frac{1}{R}\right)v + i_s}$

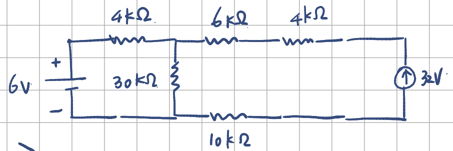


For these circuit to be equivalent we need to have same $v-i$ characteristics

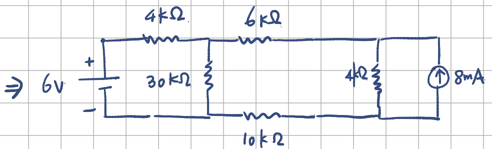
at terminal A—B we should let $\frac{V_s}{R} = i_s$ And $V_s = i_s R$

Example : Find the power absorbed by 6v Power Source

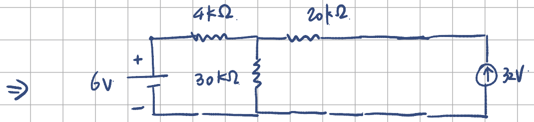


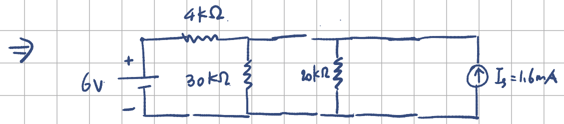


$\Rightarrow$

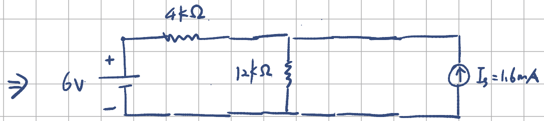


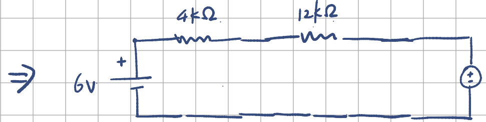
$\Rightarrow$





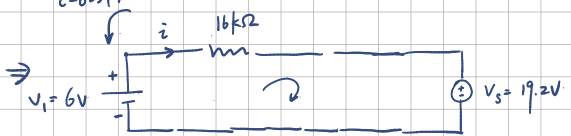
$$R_{eq} = \frac{30 \times 20}{30+20} k\Omega = \frac{600}{50} k\Omega = 12 k\Omega$$





$V_s = 12k\Omega \cdot 1.6mA = 19.2V$

$i = 825\mu A$

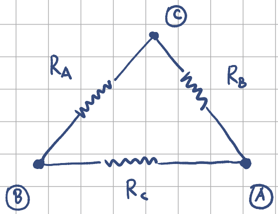


$$+V_1 - iR - V_s = 0$$

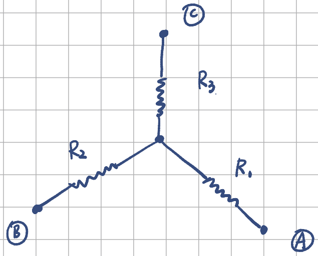
$$6V - i \times 16k\Omega - 19.2V = 0 \Rightarrow i = \frac{6-19.2}{16} mA = -0.825mA = -825\mu A$$

$$P = V_1 i = 6V \times (825\mu A) = 4.95 mW > 0 \quad \text{(Absorbed)}$$

$Y \rightleftharpoons \Delta$ Transformation



$\equiv$



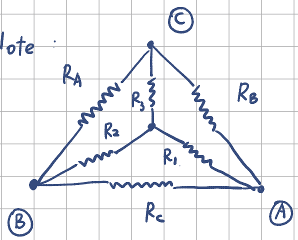
where

$$R_A = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_1} \qquad R_1 = \frac{R_BR_C}{R_A + R_B + R_C}$$

$$R_B = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_2} \qquad R_2 = \frac{R_AR_C}{R_A + R_B + R_C}$$

$$R_C = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_3} \qquad R_3 = \frac{R_AR_B}{R_A + R_B + R_C}$$

Note :

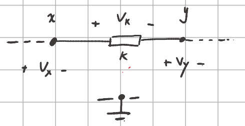


$$\Delta\text{-Resistance} = \frac{\text{Sum of product of } y\text{-resistance in pairs}}{\text{The opposite } y\text{-resistance}}$$

$$Y\text{-Resistance} = \frac{\text{Product of adjacent } \Delta\text{-resistance}}{\text{Sum of } \Delta\text{-resistance}}$$

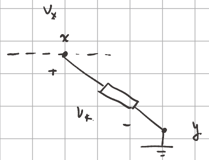
Chapter 3: Circuit Analysis Techniques

3.1 Node Voltage analysis



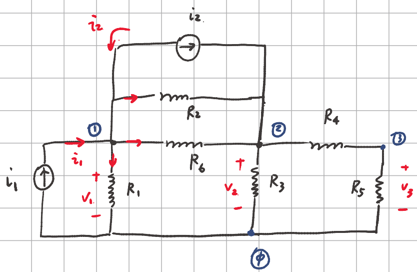
$\Rightarrow \quad V_k = V_x - V_y$

Special case:



$V_k = V_x - \underset{\overset{\|}{0}}{V_y} = V_x$

Writing node equation by inspection



Node 1: $i_1 - i_2 = v_1\left(\frac{1}{R_1}\right) + (v_1 - v_2)\left(\frac{1}{R_2}\right) + (v_1 - v_2)\frac{1}{R_6}$

$= G_1 v_1 + (v_1 - v_2)G_2 + (v_1 - v_2)G_6$

$= (G_1 + G_2 + G_6)V_1 - (G_2 + G_6)V_2 + 0 \cdot V_3$

Node 2: $i_2 = (V_2 - V_1)(G_2 + G_6) + (V_2 - V_3)G_4 + V_2 G_3$

$= -(G_2 + G_6)V_1 + (G_2 + G_3 + G_4 + G_6)V_2 - G_4 V_3$

Node 3: $0 = (V_3 - V_2)G_4 + V_3 G_5$

$= 0 \cdot V_1 - G_4 V_2 + (G_4 + G_5)V_3$

Summarize

$$i_1 - i_2 = (G_1+G_2+G_6)V_1 - (G_2+G_6)V_2 + 0\cdot V_3$$

$$i_2 = -(G_2+G_6)V_1 + (G_2+G_3+G_4+G_6)V_2 - G_4 V_3$$

$$0 = 0\cdot V_1 - G_4\cdot V_2 + (G_4+G_5)V_3$$

⇒ Matrix form

$$\begin{pmatrix} i_1 - i_2 \\ i_2 \\ 0 \end{pmatrix} = \begin{pmatrix} G_1+G_2+G_6 & -(G_2+G_6) & 0 \\ -(G_2+G_6) & (G_2+G_3+G_4+G_6) & -G_4 \\ 0 & -G_4 & G_4+G_5 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

⇒ In the general form

$$\begin{pmatrix} j_1 \\ j_2 \\ j_3 \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

二、 结点方程

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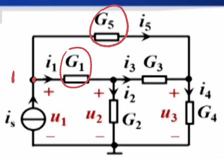

$$\begin{cases} G_1(u_1-u_2)+G_5(u_1-u_3)=i_S \\ -G_1(u_1-u_2)+G_2u_2+G_3(u_2-u_3)=0 \\ -G_5(u_1-u_3)+G_4u_3-G_3(u_2-u_3)=0 \end{cases}$$

$$\begin{cases} (G_1+G_5)u_1 - G_1u_2 - G_5u_3 = i_S \\ -G_1u_1+(G_1+G_2+G_3)u_2-G_3u_3=0 \\ -G_5u_1-G_3u_2+(G_3+G_4+G_5)u_3=0 \end{cases}$$

自电导

互电导

流入结点 电压升 取正值

电子科技大学

电路分析基础

三、 举例说明

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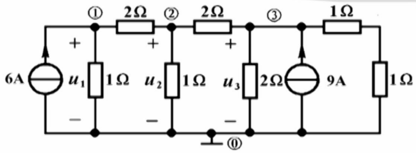


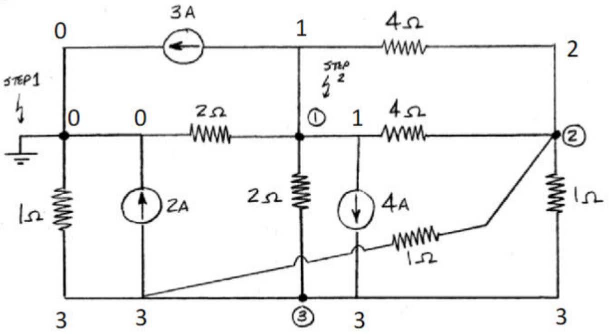
解：

$$\begin{cases} (1+0.5)u_1 - 0.5u_2 = 6 \\ -0.5u_1 + (0.5+1+0.5)u_2 - 0.5u_3 = 0 \\ -0.5u_2 + (0.5+0.5+0.5)u_3 = 9 \end{cases}$$

电子科技大学

电路分析基础

EXAMPLE



$$
\begin{array}{c} \\ ① \\ ② \\ ③ \end{array}
\begin{array}{c} \\ \begin{bmatrix} -3-4 \\ 0 \\ -2+4 \end{bmatrix} \end{array}
=
\begin{array}{c} \begin{array}{ccc} ① & ② & ③ \end{array} \\ \begin{bmatrix} (\frac{1}{2}+\frac{1}{4}+\frac{1}{4}+\frac{1}{2}) & -(\frac{1}{4}+\frac{1}{4}) & -(\frac{1}{2}) \\ -(\frac{1}{4}+\frac{1}{4}) & (\frac{1}{4}+\frac{1}{4}+1+1) & -(1+1) \\ -(\frac{1}{2}) & -(1+1) & (\frac{1}{2}+1+1+1) \end{bmatrix} \end{array}
\begin{array}{c} \\ \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \end{array}
$$

$$
\begin{bmatrix} -7 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 1.5 & -0.5 & -0.5 \\ -0.5 & 2.5 & -2 \\ -0.5 & -2 & 3.5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}
$$

1) CRAMER'S RULE

GIVEN A SET OF n-LINEAR EQUATIONS IN n-UNKNOWNS

$$
\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}
$$

$$
[B] = [A][x]
$$

IF THE DETERMINANT (Δ) OF A IS NOT ZERO, THEN THE SYSTEM HAS A UNIQUE SOLUTION

$$
x_j = \frac{\Delta A_j}{\Delta A} \qquad j = 1, 2, \cdots, n
$$

⊘ Tips from linear algebra

1.5 cramer 法则

(条件: ① # of function = # unknown ② $D \neq 0$)

Eg: $\begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 - x_2 + 5x_3 = 6 \\ -x_1 + x_2 + 6x_3 = 9 \end{cases}$ $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 5 \\ -1 & 1 & 6 \end{vmatrix}$ 系数行列式 $x_j = \dfrac{D_j}{D}$

$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 5 \\ -1 & 1 & 6 \end{vmatrix}$ $D_1 = \begin{vmatrix} 1 & 1 & 1 \\ 6 & -1 & 5 \\ 9 & 1 & 6 \end{vmatrix}$ $= \begin{vmatrix} 1 & 1 & 1 \\ 1 & 6 & 5 \\ -1 & 9 & 6 \end{vmatrix}$ $D_3 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 6 \\ -1 & 1 & 9 \end{vmatrix}$

$x_1 = \dfrac{D_1}{D}$ $x_2 = \dfrac{D_2}{D}$ $x_3 = \dfrac{D_3}{D}$

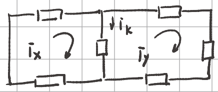
⊘ Tips: minor m_{ij} Cofactor C_{ij} $C_{ij} = (-1)^{i+j} m_{ij}$

Value of $\Delta A = a_{i1} C_{i1} + a_{i2} C_{i2} + \cdots + a_{in} C_{in}$ (按行展开)

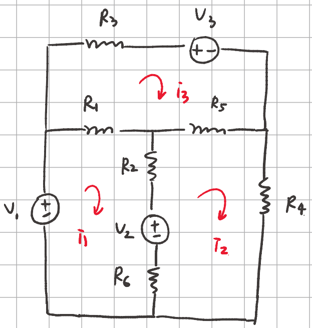
Example: $\Delta A = \begin{vmatrix} 1 & 1 & 2 \\ -1 & 3 & 1 \\ 3 & -1 & -3 \end{vmatrix}$

$$= 1 \times (-1)^{1+1} \begin{vmatrix} 3 & 1 \\ -1 & -3 \end{vmatrix} + 1 \times (-1)^{1+2} \begin{vmatrix} -1 & 1 \\ 3 & -3 \end{vmatrix} + 2 \times (-1)^{3+1} \begin{vmatrix} -1 & 3 \\ 3 & -1 \end{vmatrix}$$

3.2 Mesh Current Analysis



$$i_x - i_y = i_k$$



mesh 1:

$$v_1 - (i_1 - i_3) R_1 - (i_1 - i_2)(R_2 + R_6) + V_2 = 0$$

$$\therefore \; V_1 - V_2 = (R_1 + R_2 + R_6)\, i_1 - (R_2 + R_6)\, i_2 - R_1 i_3$$

mesh 2:

$$V_2 = -(R_2 + R_6)\, i_1 + (R_2 + R_6 + R_4)\, i_2 - R_5 i_3$$

mesh 3:

$$-V_3 = -R_1 i_1 - R_5 i_2 + (R_1 + R_3 + R_5)\, i_3$$

In matrix form

$$\begin{pmatrix} V_1 - V_2 \\ V_2 \\ -V_3 \end{pmatrix} = \begin{pmatrix} R_1 + R_2 + R_6 & -(R_2 + R_6) & -R_1 \\ -(R_2 + R_6) & R_2 + R_4 + R_5 + R_6 & -R_5 \\ -R_1 & -R_5 & R_1 + R_3 + R_5 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix}$$

In general

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix}$$

3.3 Superposition Principle

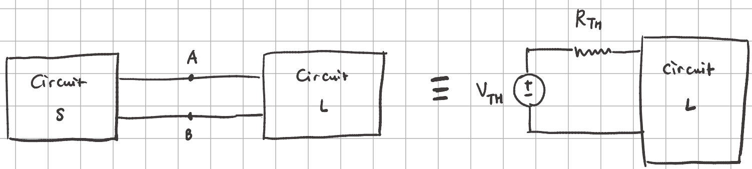
The response (voltage or current) from a number of independent sources acting simultaneously is simply the sum of the responses which would be produced by each of the independent sources acting alone with all the other independent sonces set equal to zero

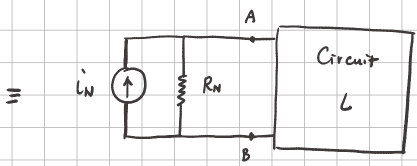
Zero Source

(a) Setting Voltage source to be zero. like a short circuit $V=0$ $i=\infty$

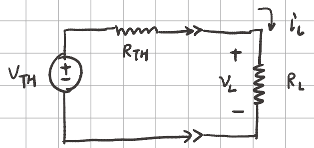
(b) Setting Voltage source to be zero. like a open Circuit. $i=0$ $V=\infty$

3.4 Thevenin and Norton Equivalent Circuit.





3.5 Maximum Signal Transfer



tips: Second derivative test

$f''(x)$ continuous $\quad f'(x_0)=0$

(a) $f''(x_0) < 0 \quad f(x)$ has relative max

(b) $f''(x_0) > 0 \quad f(x)$ has relative min

(c) $f''(x_0) = 0 \quad$ No Conclusion

$$P_{R_L} = i_L^2 \cdot R_L = \left(\frac{V_{TH}}{R_{TH}+R_L}\right)^2 R_L$$

$$\frac{dP_{R_L}}{dR_L} = 0 \Rightarrow \left\{\frac{d}{dR_L}\left[R_{TH}+R_L\right]^{-2}\right\}(V_{TH}^2 R_L) + (R_{TH}+R_L)^{-2}\frac{d}{dR_L}\left[V_{TH}^2 \cdot R_L\right] = 0$$

$$\Rightarrow R_{TH} = R_L \qquad f''(x_0) = \left.\frac{d^2 P_{R_L}}{dR_L^2}\right|_{R_L=R_{TH}} < 0 \Rightarrow \text{Relative max}$$

(1) $$P_{R_L(max)} = \left.\frac{V_{TH}^2}{(R_{TH}+R_L)^2}\cdot R_L\right|_{R_L=R_{TH}} = \frac{V_{TH}^2}{4R_{TH}}$$

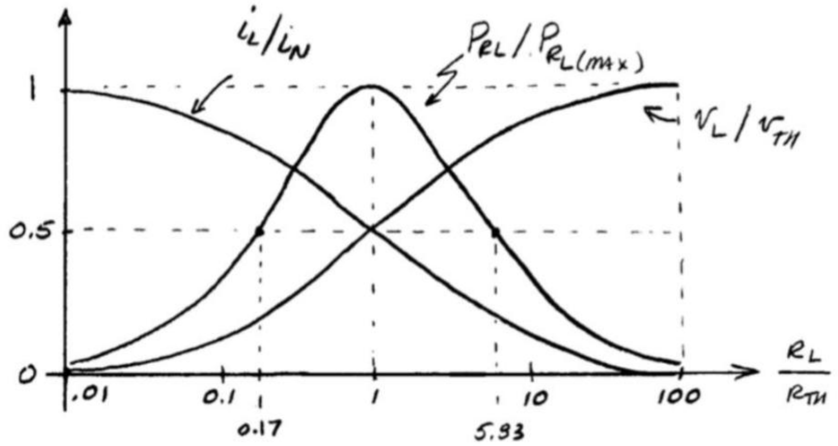
(2) Since $i_N = \dfrac{V_{TH}}{R_{TH}}$ $\quad P_{R_L(max)} = \dfrac{i_N^2 R_{TH}^2}{4R_{TH}} = \dfrac{i_N^2 R_{TH}}{4}$

(3) Also $P_{R_L(max)} = \dfrac{V_{TH}^2}{4R_{TH}} = \dfrac{V_{TH}\cdot V_{TH}}{4R_{TH}} = \dfrac{V_{TH}\cdot(i_N \cdot R_{TH})}{4R_{TH}} = \dfrac{V_{TH}}{2}\cdot\dfrac{i_N}{2}$

(4) $$\frac{i_L}{i_N} = \frac{\frac{V_{TH}}{R_{TH}+R_L}}{i_N} = \frac{R_{TH}}{R_L+R_{TH}} = \frac{1}{\left(\frac{R_L}{R_{TH}}\right)+1}$$

$$\frac{V_L}{V_{TH}} = \frac{\left(\frac{V_{TH}}{R_{TH}+R_L}\right)^2\cdot R_L}{\frac{V_{TH}^2}{4R_{TH}}} = \frac{\left(\frac{R_L}{R_{TH}}\right)}{\left(\frac{R_L}{R_{TH}}\right)+1}$$

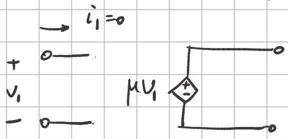
(5) $$\frac{P_{RL}}{P_{RL(max)}} = \frac{\left(\frac{V_{TH}}{R_L+R_{TH}}\right)^2 R_L}{\frac{V_{TH}^2}{4R_{TH}}} = \frac{4\left(\frac{R_L}{R_{TH}}\right)}{\left[\left(\frac{R_L}{R_{TH}}\right)+1\right]^2}$$



Chapter 4.

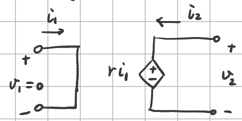
A) Dependent Voltage Sources

VCVS (Voltage Control voltage Source)



μ: 电压放大倍数.

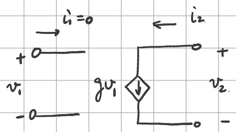
CCVS (Current Control Voltage Source)



r: 转移电阻

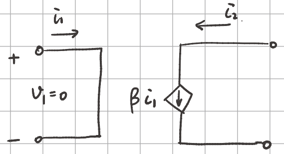
B) Dependent Current Sources

VCCS (Voltage Control Current Source)



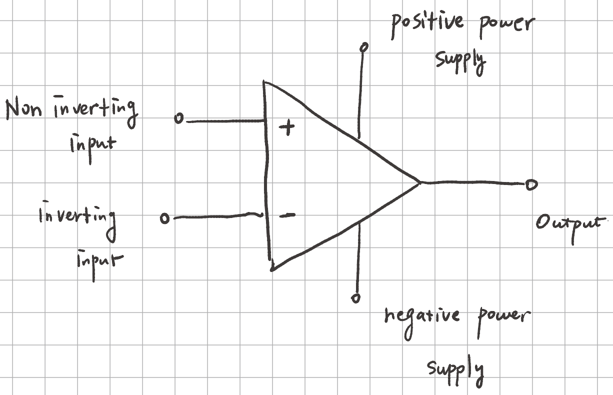
g: 转移电导

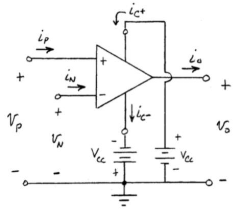
CCCS (current control Current source)



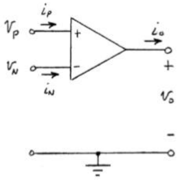
β: 电流放大倍数.

Operational Amplifier (OP-AMP)

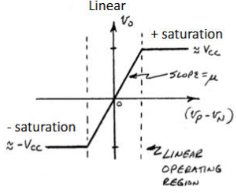




Connections needed with reference directions



Short-hand notation



Transfer characteristic

Desired property of an OP-AMP, its operational range is when v_O is within $[-V_{cc}, V_{cc}]$.

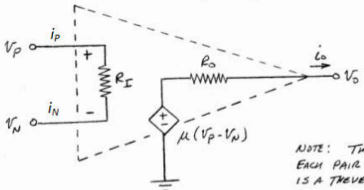
$$-V_{CC} \le v_O \le V_{CC}$$

$$-V_{CC} \le A(v_P - v_N) \le V_{CC}$$

$$-\frac{V_{CC}}{A} \le (v_P - v_N) \le \frac{V_{CC}}{A}$$

V_{CC} is typically 15 V, and A is a very large number, so $(v_P - v_N)$ is small.

Dependent Source Model of an OP-AMP Operating at Linear Mode



NOTE: THE MODEL FOR EACH PAIR OF TERMINALS IS A THEVENIN EQ. CIRCUIT

Linear region model

B) Commercial Op-Amps

$$1\,M\Omega < R_I < 1\,T\Omega$$

$$10\,\Omega < R_o < 100\,\Omega$$

$$100K < \mu < 100M$$

C) Ideal Op-Amps

- If we could make the perfect op-amp

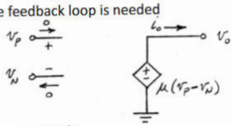
$$R_I \to \infty\,\Omega$$

$$R_o \to 0\,\Omega$$

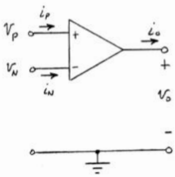
$$\mu \to \infty$$

- We know how to model infinite and zero resistance, but what about infinite gain?

-- Some feedback loop is needed



$\mu \to \infty$



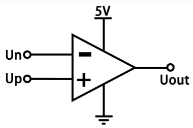
Open circuit

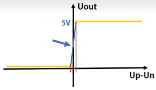
Also open circuit

$v_P = v_N$

$i_P = i_N = 0$

运算放大器原理



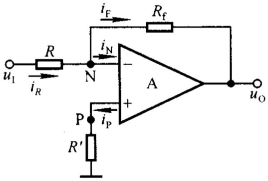


Up>Un Uout=5V

Up<Un Uout=0V

Up≈Un Uout=A(Up-Un)+2.5

反相比例运算电路



根据虚短虚断:

$U_n = U_p = 0$ $ip = i_N = 0$

计算节点N的电流:

$$i_R = i_f = \frac{U_i}{R}$$

计算输出电压:

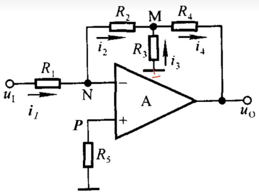
$$Uo = -i_f \cdot R_f = -\frac{U_i}{R} \times R_f$$

$$U_0 = -\frac{R_f}{R} U_i$$

计算输入平衡电阻:

$$R' = R \parallel R_f$$

T型网络反比例



根据虚短虚断有: $i_1 = i_2$ $U_N = U_P$

第一步: $i_1 = i_2 = \frac{U_i}{R_1}$ $i_4 = i_2 + i_3$

第二步: $U_m = -i_2R_2 = -i_3R_3$ $i_3 = i_2\frac{R_2}{R_3} = \frac{U_iR_2}{R_1R_3}$

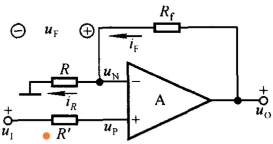
第三步: $U_0 = -i_2R_2 - i_4R_4$ $= -i_2R_2 - (i_2 + i_3)R_4$

$= -i_2R_2 - i_2R_4 - i_3R_4$ $= -i_2(R_2 + R_4) - i_3R_4$

$= -U_i\frac{R_2 + R_4}{R_1} - \frac{U_iR_2}{R_1R_3}R_4$ $= -U_i\left(\frac{R_2 + R_4}{R_1} + \frac{R_2R_4}{R_1R_3}\right)$

第四步: R5=R1//[R2+(R3//R4)]

同相比例运算电路



根据虚短虚断:

$U_N = U_p = U_i$ $ip = i_N = 0$

$$\therefore I_R = I_F = \frac{U_i}{R}$$

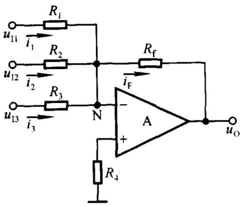
$$\therefore U_o = U_N + I_FR_f = U_i + \frac{U_i}{R}R_f$$

$$\therefore U_o = U_i\left(1 + \frac{R_f}{R}\right)$$

计算输入平衡电阻:

$$R' = R \parallel R_f$$

反相求和运算电路



$$i_1 + i_2 + i_3 = i_F$$

$$U_o = -R_fI_f = -R_f(i_1 + i_2 + i_3)$$

$$u_O = -R_f\left(\frac{u_{I1}}{R_1} + \frac{u_{I2}}{R_2} + \frac{u_{I3}}{R_3}\right)$$

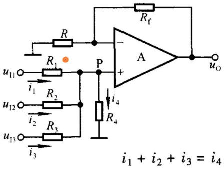
$$= -\frac{R_f}{R_1}u_{I1} - \frac{R_f}{R_2}u_{I2} - \frac{R_f}{R_3}u_{I3}$$

当$R_1=R_2=R_3$时

$$U_o = -\frac{R_f}{R_1}(U_{i1} + U_{i2} + U_{i3})$$

$$R_4 = R_1 // R_2 // R_3 // R_f$$

同相求和运算电路



$$i_1 + i_2 + i_3 = i_4$$

$$\frac{u_{I1}-u_P}{R_1}+\frac{u_{I2}-u_P}{R_2}+\frac{u_{I3}-u_P}{R_3}=\frac{u_P}{R_4}$$

$$\therefore \frac{U_{i1}}{R_1}-\frac{U_p}{R_1}+\frac{U_{i2}}{R_2}-\frac{U_p}{R_2}+\frac{U_{i3}}{R_3}-\frac{U_p}{R_3}=\frac{U_p}{R_4}$$

$$\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right)u_P=\frac{u_{I1}}{R_1}+\frac{u_{I2}}{R_2}+\frac{u_{I3}}{R_3}$$

令：$R_1 \parallel R_2 \parallel R_3 \parallel R_4 = R_p$

$$\therefore \frac{1}{R_1}+\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{R_4}=\frac{1}{R_p}$$

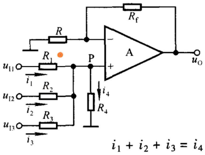
$$u_P = R_P\left(\frac{u_{I1}}{R_1}+\frac{u_{I2}}{R_2}+\frac{u_{I3}}{R_3}\right)$$

$$\because U_N = U_p$$

$$\therefore U_o = U_N\left(1+\frac{R_f}{R}\right)$$

$$=\left(\frac{R_p}{R_1}U_{i1}+\frac{R_p}{R_2}U_{i2}+\frac{R_p}{R_3}U_{i3}\right)\left(1+\frac{R_f}{R}\right)$$

加减运算电路



$$i_1 + i_2 + i_3 = i_4$$

$$\frac{u_{I1}-u_P}{R_1}+\frac{u_{I2}-u_P}{R_2}+\frac{u_{I3}-u_P}{R_3}=\frac{u_P}{R_4}$$

$$\therefore \frac{U_{i1}}{R_1}-\frac{U_p}{R_1}+\frac{U_{i2}}{R_2}-\frac{U_p}{R_2}+\frac{U_{i3}}{R_3}-\frac{U_p}{R_3}=\frac{U_p}{R_4}$$

$$\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right)u_P=\frac{u_{I1}}{R_1}+\frac{u_{I2}}{R_2}+\frac{u_{I3}}{R_3}$$

令：$R_1 \parallel R_2 \parallel R_3 \parallel R_4 = R_p$

$$\therefore \frac{1}{R_1}+\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{R_4}=\frac{1}{R_p}$$

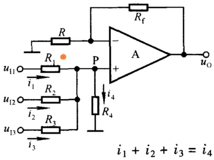
$$u_P = R_P\left(\frac{u_{I1}}{R_1}+\frac{u_{I2}}{R_2}+\frac{u_{I3}}{R_3}\right)$$

$$\because U_N = U_p$$

$$\therefore U_o = U_N\left(1+\frac{R_f}{R}\right)$$

$$=\left(\frac{R_p}{R_1}U_{i1}+\frac{R_p}{R_2}U_{i2}+\frac{R_p}{R_3}U_{i3}\right)\left(1+\frac{R_f}{R}\right)$$

差分比例运算电路



$$i_1 + i_2 + i_3 = i_4$$

$$\frac{u_{I1}-u_P}{R_1}+\frac{u_{I2}-u_P}{R_2}+\frac{u_{I3}-u_P}{R_3}=\frac{u_P}{R_4}$$

$$\therefore \frac{U_{i1}}{R_1}-\frac{U_p}{R_1}+\frac{U_{i2}}{R_2}-\frac{U_p}{R_2}+\frac{U_{i3}}{R_3}-\frac{U_p}{R_3}=\frac{U_p}{R_4}$$

$$\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right)u_P=\frac{u_{I1}}{R_1}+\frac{u_{I2}}{R_2}+\frac{u_{I3}}{R_3}$$

令：$R_1 \parallel R_2 \parallel R_3 \parallel R_4 = R_p$

$$\therefore \frac{1}{R_1}+\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{R_4}=\frac{1}{R_p}$$

$$u_P = R_P\left(\frac{u_{I1}}{R_1}+\frac{u_{I2}}{R_2}+\frac{u_{I3}}{R_3}\right)$$

$$\because U_N = U_p$$

$$\therefore U_o = U_N\left(1+\frac{R_f}{R}\right)$$

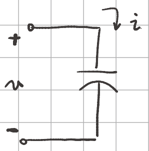
$$=\left(\frac{R_p}{R_1}U_{i1}+\frac{R_p}{R_2}U_{i2}+\frac{R_p}{R_3}U_{i3}\right)\left(1+\frac{R_f}{R}\right)$$

积分运算

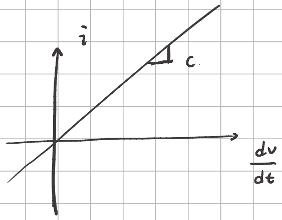
Chapter 6.

$$i(t) = \frac{dq}{dt} = \frac{d(Cv)}{dt} = C\,\frac{dv(t)}{dt}$$

6.1 The capacity



$$i(t) = C\,\frac{dv(t)}{dt}$$



$$\int i(t)\,dt = \int C\,\frac{dv(t)}{dt}\,dt$$

$$\int_{t_0}^{t_1} i(t)\,dt = C\int_{t_0}^{t_1} dv(t) = C\left[v(t_1) - v(t_0)\right]$$

$$\Rightarrow \boxed{V(t_1) = \frac{1}{C}\int_{t_0}^{t_1} i(t)\,dt + v(t_0)}$$

$$p(t) = \frac{dw(t)}{dt} \Rightarrow \int p(t)\,dt = \int \frac{dw(t)}{dt}\cdot dt$$

$$\Rightarrow \int_{t_0}^{t_1} p(t)\,dt = \int_{t_0}^{t_1} dw(t) = w(t_1) - w(t_0)$$

$$\therefore \boxed{w(t_1) = \int_{t_0}^{t_1} p(t)\,dt + w(t_0) = \int_{t_0}^{t_1} i(t)\cdot v(t)\,dt + w(t_0)}$$

$$w(t) = \int_{t_1}^{t_0} i(t)\,v(t)\,dt + w(t_0) = \int_{t_0}^{t}\left[C\,\frac{dv(t)}{dt}\right] v(t)\,dt + w(t_0)$$

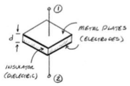
$$= C\int_{t_0}^{t} v(t)\,d(v(t)) + w(t_0) = \frac{1}{2}C\left[v(t)\right]^2\Big|_{t_0}^{t} + w(t_0)$$

$$= \frac{1}{2}C\,v^2(t) - \frac{1}{2}C\,v^2(t_0) + w(t_0) = \boxed{\frac{1}{2}C\,v^2(t)} = \frac{1}{2}\,\frac{q^2(t)}{C}$$

$$p(t) = \frac{dw(t)}{dt} = \boxed{\frac{1}{2}C\,\frac{dv^2(t)}{dt}}$$

E) PHYSICAL DEVICE - CAPACITOR (SEE APPENDIX A)

1)



$C = \epsilon \frac{A}{d}$

A CAPACITOR CONSISTS OF TWO METAL PLATES SEPARATED BY AN INSULATOR.

CAPACITANCE INCREASES WHEN THE AREA OF THE PLATES, A, INCREASES AND/OR THE DISTANCE, d, BETWEEN THE PLATES DECREASES.

2) SOME TYPES OF CAPACITORS

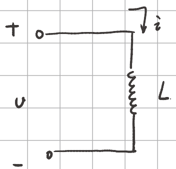
3) COLOR CODE - 3 NUMBERS AND 1 LETTER STAMPED

$\underline{a}\ \underline{b}\ \underline{c}\ \underline{d} \Rightarrow C = (\underline{a}\,\underline{b} \times 10^{\underline{c}})\ pF \pm \underline{d}$

WHERE

d: $M = \pm 20\%$ $K = \pm 10\%$ $J = \pm 5\%$
$G = \pm 2\%$ $F = \pm 1\%$

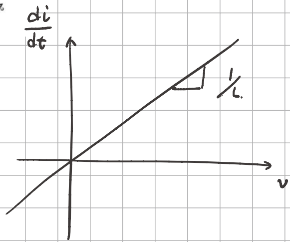
6.2. The Inductor



$$(\Phi(t) = L\, i(t))$$

$$\lambda(t) = L\, i(t)$$

$$v(t) = L \frac{di(t)}{dt}$$



$$\text{Faraday law: } v(t) = \frac{d\Phi_B(t)}{dt} = \frac{d(L\, i(t))}{dt} = L \cdot \frac{di(t)}{dt}$$

$\lambda(t)$ represents magnetic flux leakage

unit of L is H $\quad 1\text{H} = 1\ \frac{\text{wb}}{\text{A}} \quad$ $\lambda(t)$ unit is Wb

$$v(t) = L \cdot \frac{di(t)}{dt} \Rightarrow \int v(t)\,dt = \int L\, di(t)$$

$$\Rightarrow \int_{t_0}^{t_1} v(t)\,dt = \int_{t_0}^{t_1} L\, di(t) = L\left[i(t_1) - i(t_0)\right]$$

$$\Rightarrow \boxed{i(t_1) = \frac{1}{L} \cdot \int_{t_0}^{t_1} v(t)\,dt + i(t_0)}$$

$$p(t) = \frac{d\,w(t)}{dt} \Rightarrow \int p(t)\,dt = \int d\,w(t)$$

$$\int_{t_0}^{t_1} p(t)\,dt = \int_{t_0}^{t_1} d\,w(t) = w(t_1) - w(t_0)$$

$$\Rightarrow w(t_1) = \int_{t_0}^{t_1} p(t)\,dt + w(t_0) = \int_{t_0}^{t_1} v(t)\,i(t)\,dt + w(t_0)$$

$$= \int_{t_0}^{t_1} \left[L \frac{d\,i(t)}{dt}\right] \cdot i(t)\,dt + w(t_0)$$

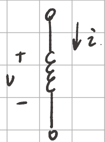
$$= L \cdot \int_{t_0}^{t_1} i(t)\,d\,i(t) + w(t_0)$$

$$= L \cdot \frac{1}{2} i^2(t) \Big|_{t_0}^{t_1} + w(t_0)$$

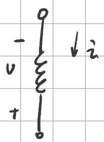
$$= L \left[\frac{1}{2} i^2(t_1) - \frac{1}{2} i^2(t_0)\right] + w(t_0)$$

$$= \boxed{\frac{1}{2} L\, i^2(t)}$$

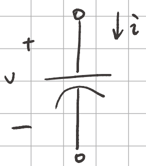
$$p(t) = \frac{d\,w(t)}{dt} = \boxed{\frac{1}{2} L \cdot \frac{d\, i^2(t)}{dt}}$$



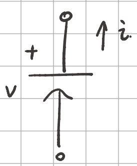
Absorbing power



generating power



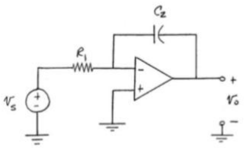
Absorbing power



generating power

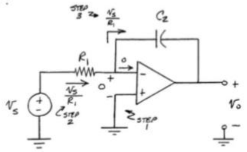
CH. 6 : SUPPLEMENTAL PROBLEMS AND SOLUTIONS

S6.1) INTEGRATOR



FIND $v_o(t)$

SOLUTION :



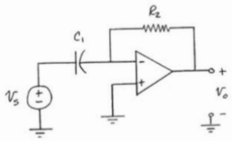
$$v_o(t_1) = -v_{C_2} + 0 = -\left[\frac{1}{C_2}\int_{t_0}^{t_1}\frac{v_s}{R_1}\,dt + \underbrace{v_{C_2}(t_0)}_{= -v_o(t_0)}\right] + 0$$

$$= -\frac{1}{R_1C_2}\int_{t_0}^{t_1} v_s(t)\,dt + v_o(t_0)$$

OR DOING A CHANGE OF VARIABLE

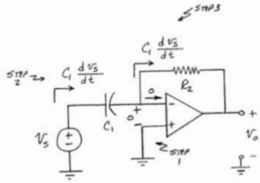
$$\boxed{v_o(t) = -\frac{1}{R_1C_2}\int_{t_0}^{t} v_s(x)\,dx + v_o(t_0)}$$

S6.2) DIFFERENTIATOR



FIND $v_o(t)$

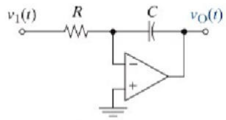
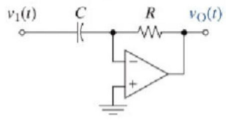
SOLUTION:



$$v_o = -C_1\frac{dv_s(t)}{dt}R_2 + 0$$

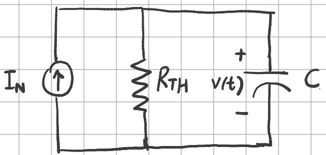
$$\therefore \boxed{v_o = -R_2C_1\frac{dv_s(t)}{dt}}$$

OP-AMP for Differentiator and Integrator

 Integrator	$v_1(t) \rightarrow \boxed{K} \rightarrow \boxed{\int} \rightarrow v_O(t)$	$K = -\frac{1}{RC}$
 Differentiator	$v_1(t) \rightarrow \boxed{K} \rightarrow \boxed{\frac{d}{dt}} \rightarrow v_O(t)$	$K = -RC$

Chapter 7: First and Second-Order Circuits

RC Circuits



$$I_N = \frac{V(t)}{R_{TH}} + C\frac{dv(t)}{dt} = C\left[\frac{V(t)}{R_{TH}\cdot C} + \frac{dv(t)}{dt}\right]$$

$$\Rightarrow \frac{dv(t)}{dt} = -\frac{V(t)}{R_{TH}C} + \frac{I_N}{C} = -\frac{1}{R_{TH}C}\left[V(t) - I_N R_{TH}\right]$$

$$\Rightarrow \frac{1}{V(t) - I_N R_{TH}}\, dv(t) = -\frac{1}{R_{TH}C}\, dt$$

$$\Rightarrow \int_{t_0}^{t_1}\left(\frac{1}{V(t) - I_N R_{TH}}\right) dv(t) = \int_{t_0}^{t_1}\left(\frac{-1}{R_{TH}\cdot C}\right) dt$$

$$\Rightarrow \ln\left(v(t) - I_N R_{TH}\right)\Big|_{t_0}^{t_1} = -\frac{1}{R_{TH}\cdot C}(t_1 - t_0)$$

$$\Rightarrow \ln\left[\frac{V(t_1) - I_N R_{TH}}{V(t_0) - I_N R_{TH}}\right] = -\frac{1}{R_{TH}C}(t_1 - t_0)$$

$$\Rightarrow \frac{v(t_1) - I_N R_{TH}}{V(t_0) - I_N R_{TH}} = e^{-\frac{1}{R_{TH}C}(t_1 - t_0)}$$

$$\Rightarrow \boxed{V(t_1) = I_N R_{TH} + \left[V(t_0) - I_N R_{TH}\right] e^{-(t_1 - t_0)/R_{TH}C}}$$

Assume $A_1 = I_N R_{TH}$ $\quad B_1 = V(t_0) - I_N R_{TH}$ $\quad \uparrow = R_{TH} C$ $\quad A_1 + B_1 = I_N R_{TH}$

(a) $t = t_0$ $\quad v(t) = A_1 + B_1 e^{-(t_1 - t_0)/\uparrow} = V(t_0)$

(b) $t = \infty$ $\quad v(t) = A_1 + B_1 e^{-\infty} = A_1 = I_N R_{TH}$

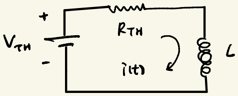
More general $\quad i_x(t) \quad V_x(t) = A_2 + B_2 e^{-(t_1 - t_0)/\uparrow}$

$$A_2 + B_2 = i_x(t_0) \quad V_x(t_0)$$

$$A_2 = i_x(t \to \infty) \quad V_x(t \to \infty)$$

$$\uparrow = R_{TH} C$$

RL Circuit



$$V_{TH} = R_{TH} \cdot i(t) + L \cdot \frac{di(t)}{dt}$$

For $V_L = L \cdot \frac{di(t)}{dt}$ (Formula)

$$\Rightarrow \quad V_{TH} = L\left[\frac{R_{TH}}{L} i(t) + \frac{di(t)}{dt}\right] \Rightarrow \frac{di(t)}{dt} = -\frac{R_{TH}}{L} i(t) + \frac{V_{TH}}{L}$$

$$\Rightarrow \quad \frac{di(t)}{dt} = -\frac{R_{TH}}{L}\left[i(t) - \frac{V_{TH}}{R_{TH}}\right]$$

$$\Rightarrow \int_{t_0}^{t_1} \frac{1}{i(t) - \frac{V_{TH}}{R_{TH}}} \frac{di(t)}{dt} = \int_{t_0}^{t_1} -\frac{R_{TH}}{L}$$

$$\Rightarrow \quad \ln\left[i(t_1) - \frac{V_{TH}}{R_{TH}}\right] - \ln\left[i(t_0) - \frac{V_{TH}}{R_{TH}}\right] = -\frac{R_{TH}}{L}(t_1 - t_0)$$

$$\Rightarrow \quad \frac{i(t_1) - \frac{V_{TH}}{R_{TH}}}{i(t_0) - \frac{V_{TH}}{R_{TH}}} = e^{-\frac{R_{TH}}{L}(t_1 - t_0)}$$

$$\Rightarrow \quad i(t_1) = \frac{V_{TH}}{R_{TH}} + \left[i(t_0) - \frac{V_{TH}}{R_{TH}}\right] \cdot e^{-\frac{R_{TH}}{L}(t_1 - t_0)}$$

$$f(t) = A_1 + B_1 \cdot e^{-(t_1 - t_0)/\tau} \qquad A_1 = V_{TH}/R_{TH} \qquad B_1 = i(t_0) - V_{TH}/R_{TH}$$

$$\tau = L/R_{TH}$$

Formula $\quad f(t) = A + B \cdot e^{-(t - t_0)/\tau}$

(a) $t = t_0 \qquad f(t_0) = A + B$

(b) $t = \infty \qquad f(\infty) = A$

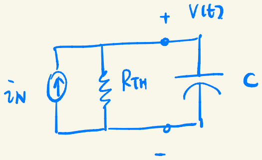
$$\tau = L/R_{TH}$$

For L. the current i (can't jump) but u (can jump)

For C the current i (can jump) but u (can't jump)

RC Circuit. [Period sin input]

$i_N = I_A \cos\omega t$



Based on formula. $\frac{dv(t)}{dt} + \frac{1}{R_{TH}C} v(t) = \frac{I_A}{C}\cos\omega t$

$v(t) = Ae^{-t/\tau} + B\cos(\omega t + \theta)$ For $t \geq 0$

$\frac{dv(t)}{dt} = -\frac{A}{\tau}e^{-t/\tau} - B\omega\sin(\omega t + \theta)$

Therefore: $-\frac{A}{\tau}e^{-t/\tau} - B\omega\sin(\omega t + \theta) + \frac{1}{R_{TH}C}\cdot\left[Ae^{-t/\tau} + B\cos(\omega t + \theta)\right] = \frac{I_A}{C}\cos\omega t$

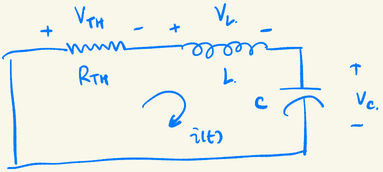
Also $V(0) = A + B\cos\theta$ Form from trig & math calculation.

$\theta = \tan^{-1}(-\omega\tau)$ $B = \frac{I_A}{C}\cdot\frac{\tau\cdot\frac{1}{\cos\theta}}{\omega^2\tau^2+1}$

$A = V(0) - B\cos\theta = V_0 - \frac{I_A}{C}\cdot\frac{\tau}{\omega^2\tau^2+1}$

$$\Rightarrow v(t) = \underbrace{\left[V(0) - \frac{I_A}{C}\frac{\tau}{1+\omega^2\tau^2}\right]e^{-t/\tau}}_{\text{Natural response}} + \underbrace{\frac{I_A}{C}\frac{\tau}{\sqrt{1+\omega^2\tau^2}}\cos\left[\omega t - \tan^{-1}(\omega\tau)\right]}_{\text{forced response}}$$

RLC Circuit



$i_L(0^+) = i_L(0^-) = i(0^-) = i(0^+) = i_R(0^+)$

$= i_R(0^-) = i_c(0^+) = i_c(0^-)$

$V_c(0^+) = V_c(0^-)$ $\quad V_R(0^+) = i(0^-)\, R_{TH}$

$V_L(0^+) = -\left[i(0^-)\cdot R_{TH} + V_c(0^-) \right]$

$i \cdot R_{TH} + L \cdot \frac{di}{dt} + V_c = 0$ $\quad$ For $i_c = \frac{dq}{dt} = C\,\frac{dV}{dt} \Rightarrow i_c\, dt = C\, dV \Rightarrow V(t) = \frac{1}{C}\int_{t_0}^{t_1} i(x)\,dx + V(t_0)$

$\Rightarrow i \cdot R_{TH} + L\cdot \frac{di}{dt} + \frac{1}{C}\int_0^t i(x)\,dx + V_c(0) = 0$ $\quad$ Differential from both side.

$\Rightarrow R_{TH}\cdot \frac{di}{dt} + L\cdot \frac{d^2 i}{dt^2} + \frac{1}{C}\cdot i = 0 \Rightarrow \boxed{\frac{d^2 i}{dt^2} + \frac{R_{TH}}{C}\cdot\frac{di}{dt} + \frac{1}{LC}\cdot i = 0}$

$s^2 + \frac{R}{L}\cdot s + \frac{1}{LC} = 0$ $\quad \Delta = \frac{R^2}{L^2} - 4\cdot\frac{1}{LC} = \frac{R^2C^2 - 4LC}{L^2C^2}$

① $\Delta > 0$ $\quad R^2C^2 - 4LC > 0 \Rightarrow R^2 > 4\frac{L}{C}$ $\quad$ Overdamped. $\quad S_1, S_2$

② $\Delta = 0$ $\quad R^2C^2 - 4LC = 0 \Rightarrow R^2 = 4\frac{L}{C}$ $\quad$ Critically damped $\quad S_1$

③ $\Delta < 0$ $\quad R^2C^2 - 4LC = 0 \Rightarrow R^2 < 4\frac{L}{C}$ $\quad$ Underdamped

$\alpha_0 = \frac{R_{TH}}{2L}$ $\quad \omega_0 = \frac{1}{\sqrt{LC}}$ $\quad S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$ $\quad S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$.

For Current $i(t)$

Overdamped. $\quad i(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}$

Critically damped $\quad i(t) = (A_1 + A_2 t)\, e^{S_1 t}$

Underdamped. $\quad i(t) = e^{-\alpha t}\left[A_1 \cos \omega_d t + A_2 \sin \omega_d t \right]$

where $\alpha = \frac{R}{2L}$ $\quad \omega_d = \sqrt{\frac{1}{LC} - \alpha^2} = \sqrt{\omega_0^2 - \alpha^2}$

RCL Circuit Key points

Equation: $\frac{d^2 i}{dt^2} + \frac{R_{TH}}{L}\frac{di}{dt} + \frac{1}{LC} i = 0$

① $\alpha^2 > \omega_0^2$. overdamped $\quad S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \quad S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

$\alpha = \frac{R_{TH}}{2L} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$

$i(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t} \qquad i(0) = A_1 + A_2$ ①

$\frac{di(t)}{dt} = A_1 S_1 e^{S_1 t} + A_2 S_2 e^{S_2 t} \qquad \frac{di(0)}{dt} = A_1 S_1 + A_2 S_2$ ②

Also $\frac{di(0^+)}{dt} = \frac{V_L(0^+)}{L} = \frac{1}{L}\left[-V_{R_{TH}}(0^+) - V_c(0^-)\right]$

Based on those two equation find A_1 & A_2

For $V_c(t)$. ① method 1. $V_c(t) = \frac{1}{C}\int_0^t i_c(t)\,dt + V_c(0^-)$

② method 2 $\quad V_R(t) + V_L(t) + V_c(t) = 0 \Rightarrow V_c(t) = -i(t)R - L\cdot\frac{di(t)}{dt}$

② $\alpha^2 = \omega_0^2$. Critically damped. $\quad S_1 = S_2 = -\alpha = \frac{R_{TH}}{2L}$

$i(t) = (A_1 t + A_2) e^{-\alpha t} \qquad i(0^-) = A_2$

$\frac{di(t)}{dt} = A_1 e^{-\alpha t} + (A_1 t + A_2)(-\alpha) e^{-\alpha t} \qquad \frac{di(0^+)}{dt} = (A_1 - \alpha A_2) = \frac{V_L(0^-)}{L} = \frac{1}{L}\left[-i(0^-) R_{TH} - V_c(0^-)\right]$

③ $\alpha^2 < \omega_0^2 \qquad S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -\alpha + \sqrt{(-1)(\omega_0^2 - \alpha^2)} = -\alpha + j\sqrt{\omega_0^2 - \alpha^2} = -\alpha + j\cdot\omega_d$.

Same. $S_2 = -\alpha - j\omega_d$. where $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

$i(t) = B_1 e^{-\alpha t}\cos(\omega_d t) + B_2 e^{-\alpha t}\sin(\omega_d t) = C_1 e^{-\alpha t}\cos(\omega_d t + \theta)$

$\frac{di(t)}{dt} = \left[C_1(-\alpha) e^{-\alpha t}\cos(\omega_d t + \theta) + C_1 e^{-\alpha t}\omega_d \cos(\omega_d t + \theta)\right]$

$i(0^-) = C_1 \cdot \cos\theta$ ①

$\frac{di(0)}{dt} = -\alpha C_1 \cos\theta - C_1 \omega_d \sin\theta = \frac{V_L(0^-)}{L} = \frac{1}{L}\left[-i(0^-) R_{TH} - V_c(0^-)\right]$ ②